Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.3-Tangent/220-4.3.7

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**501**]. This is test number [220].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.80 (500)	0.20 (1)
Mathematica	99.60 (499)	0.40 (2)
Fricas	83.03 (416)	16.97 (85)
Maple	82.24 (412)	17.76 (89)
Giac	60.08 (301)	39.92 (200)
Mupad	56.49 (283)	43.51 (218)
Maxima	53.69 (269)	46.31 (232)
Reduce	46.51 (233)	53.49 (268)
Sympy	19.16 (96)	80.84 (405)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

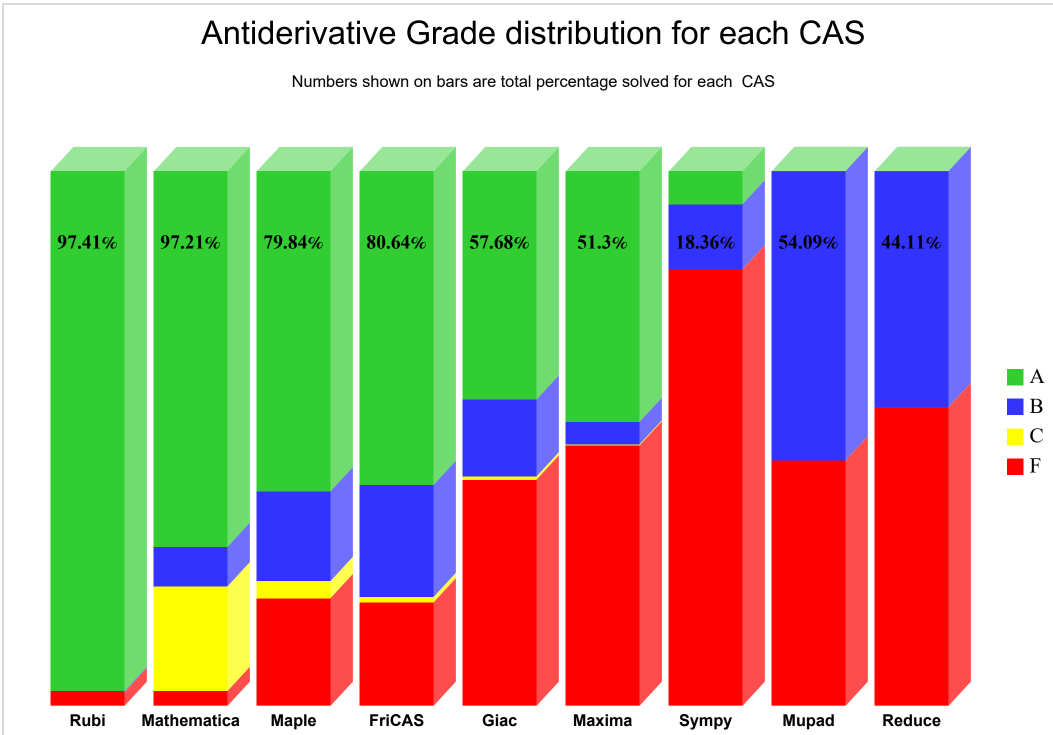
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

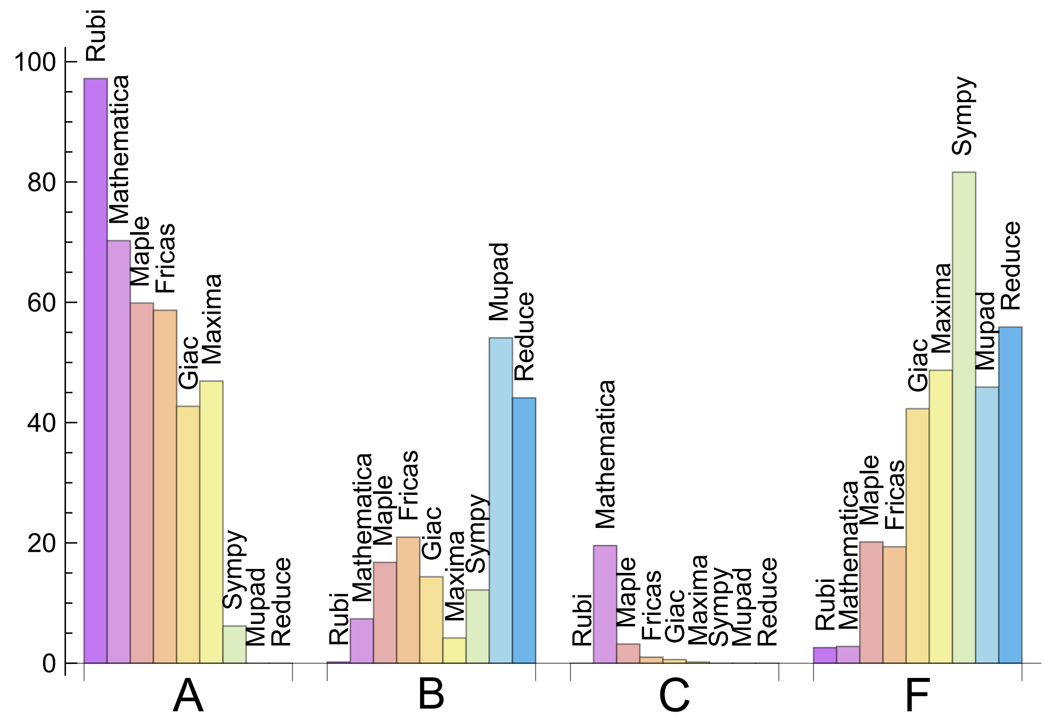
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.206	0.200	0.000	2.595
Mathematica	70.259	7.385	19.561	2.794
Maple	59.880	16.766	3.194	20.160
Fricas	58.683	20.958	0.998	19.361
Maxima	46.906	4.192	0.200	48.703
Giac	42.715	14.371	0.599	42.315
Sympy	6.188	12.176	0.000	81.637
Mupad	0.000	54.092	0.000	45.908
Reduce	0.000	44.112	0.000	55.888

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	85	88.24	4.71	7.06
Maple	89	100.00	0.00	0.00
Giac	200	62.50	16.00	21.50
Mupad	218	0.00	100.00	0.00
Maxima	232	69.83	9.48	20.69
Reduce	268	100.00	0.00	0.00
Sympy	405	76.54	23.46	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Reduce	0.28
Maxima	0.42
Rubi	0.48
Fricas	1.25
Mathematica	1.78
Giac	2.04
Mupad	8.73
Maple	10.55
Sympy	12.41

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	110.67	1.02	92.00	1.00
Maxima	128.25	1.51	83.00	1.11
Mathematica	197.27	1.83	97.00	1.00
Reduce	357.24	3.12	132.00	1.98
Fricas	417.79	3.21	241.50	2.61
Giac	549.54	6.32	107.00	1.32
Sympy	1061.28	9.35	135.50	1.95
Maple	1237.49	10.71	111.00	1.10
Mupad	1598.88	13.55	108.00	1.31

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

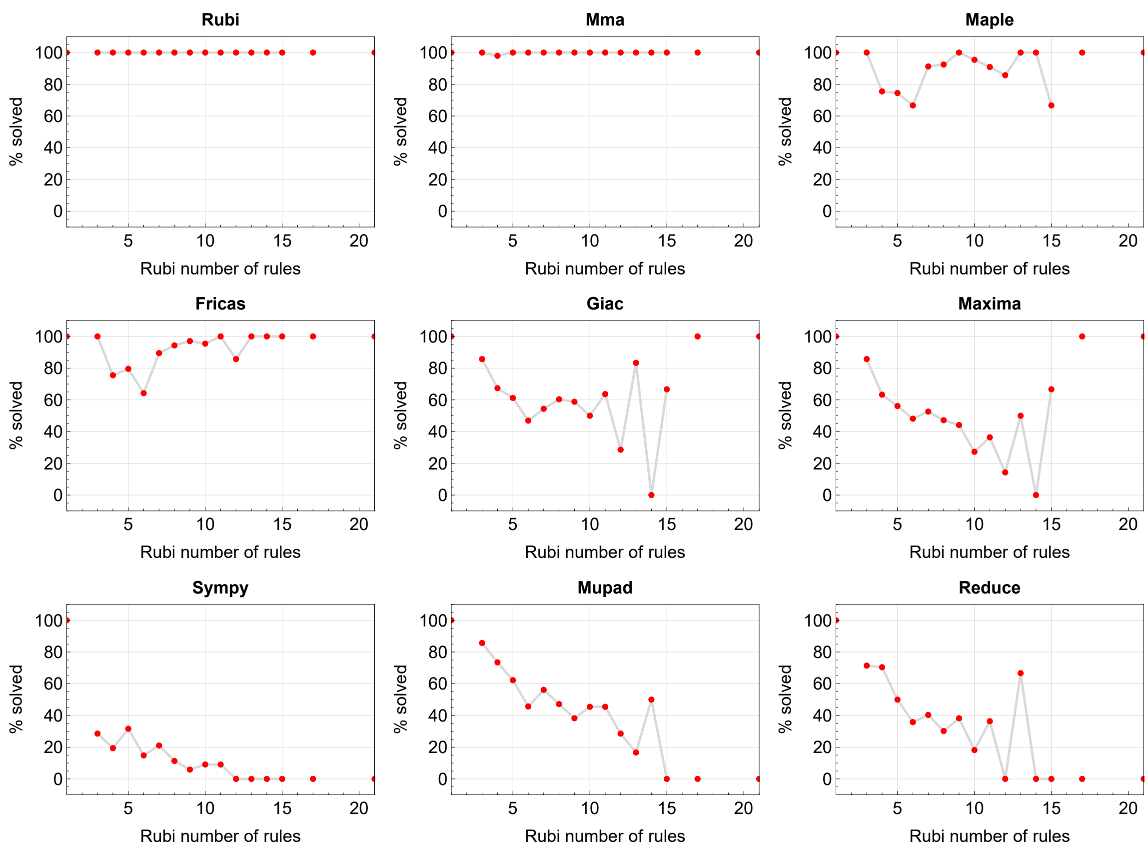


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

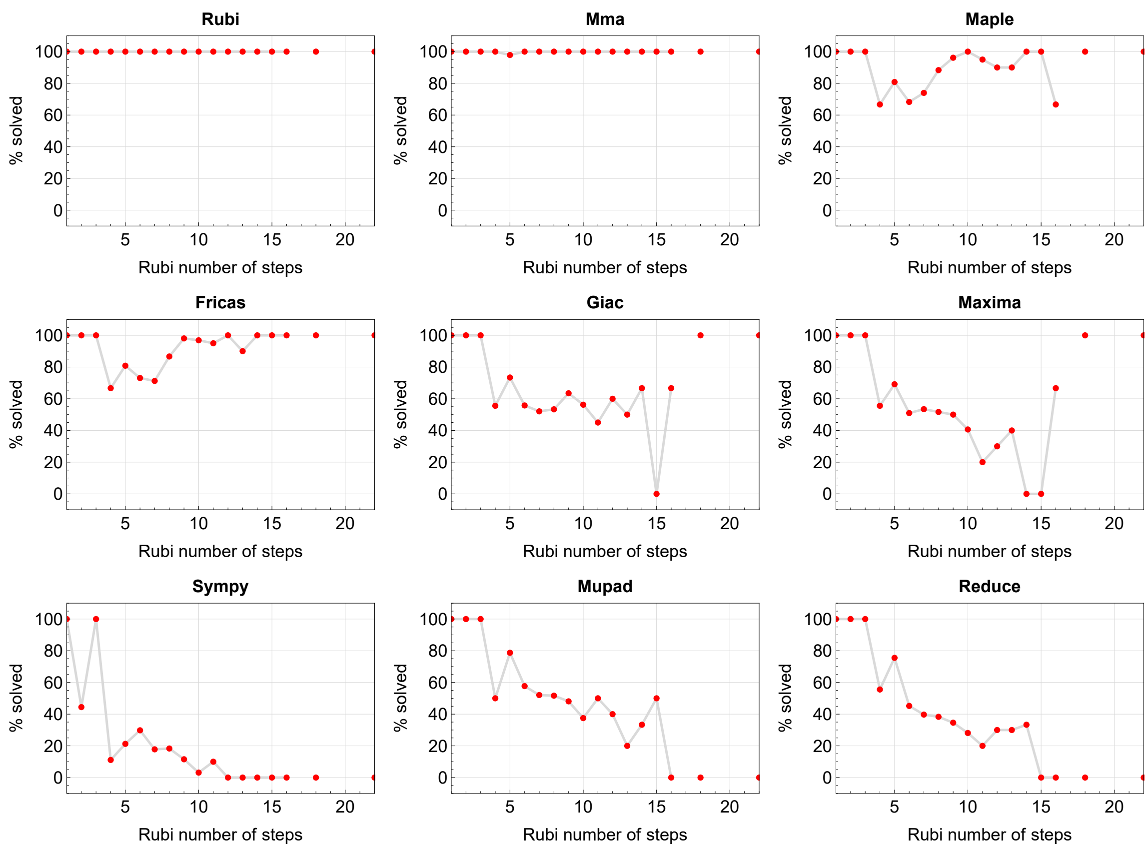


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

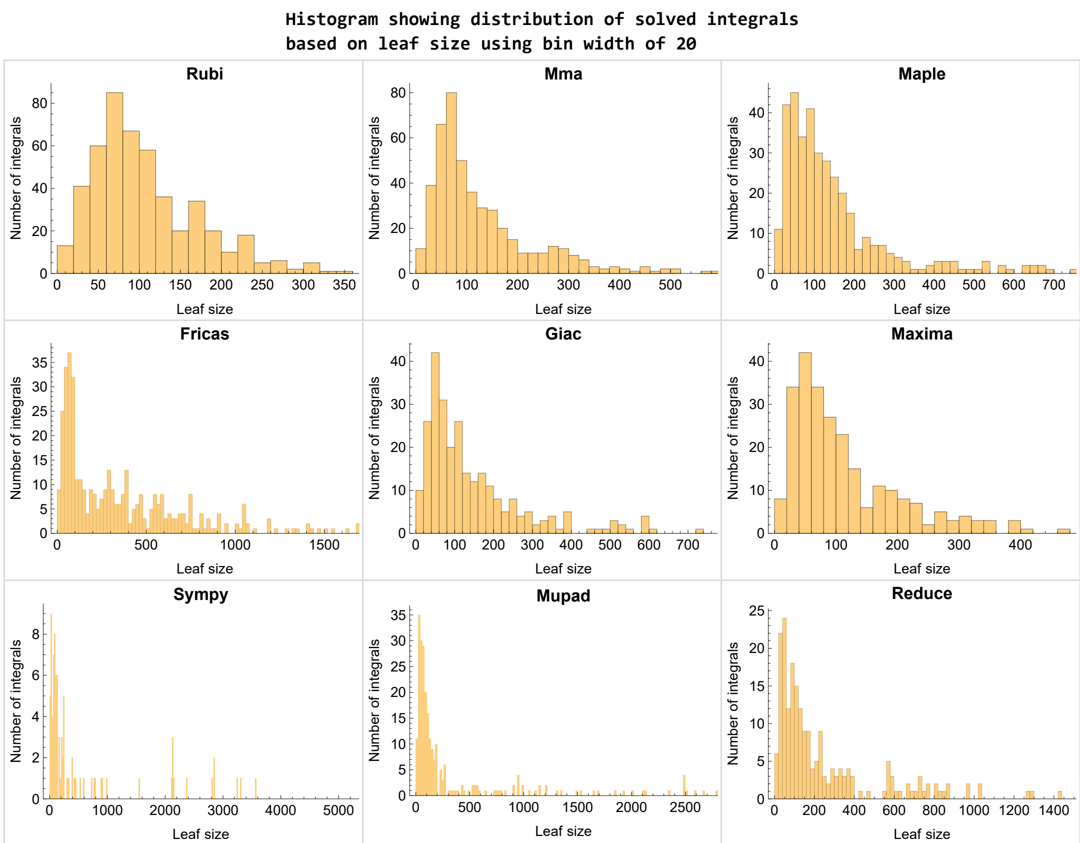


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

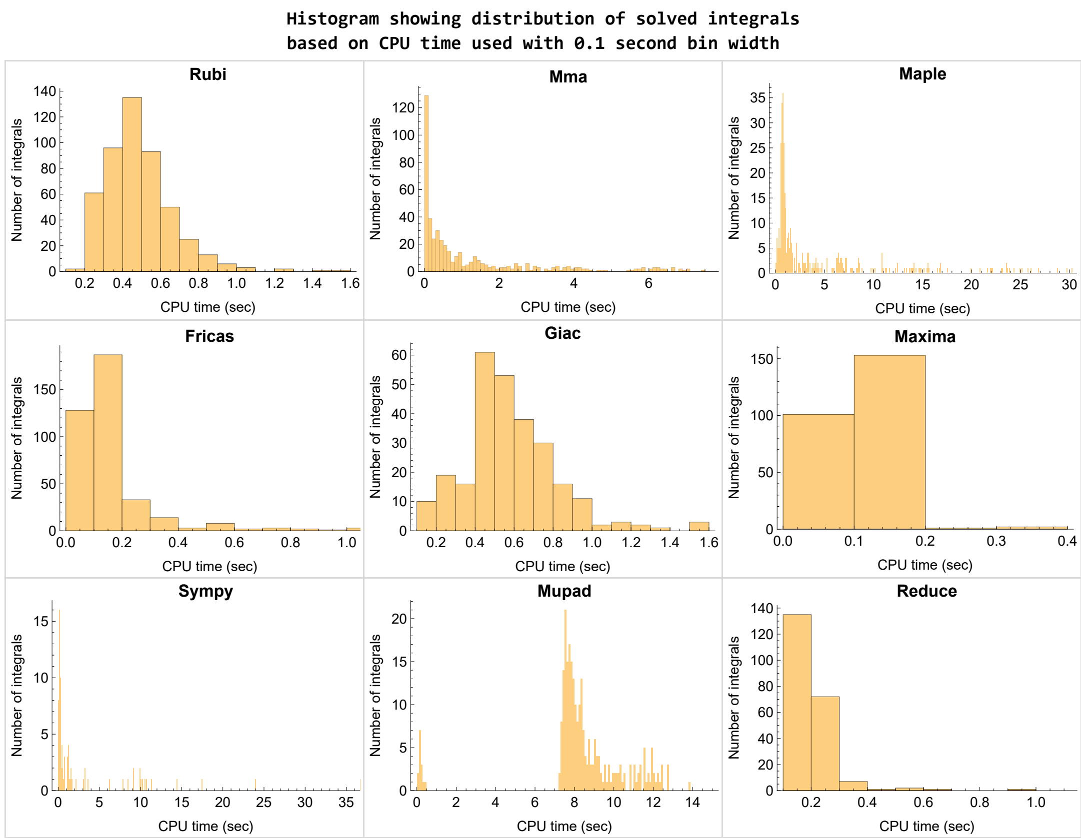


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

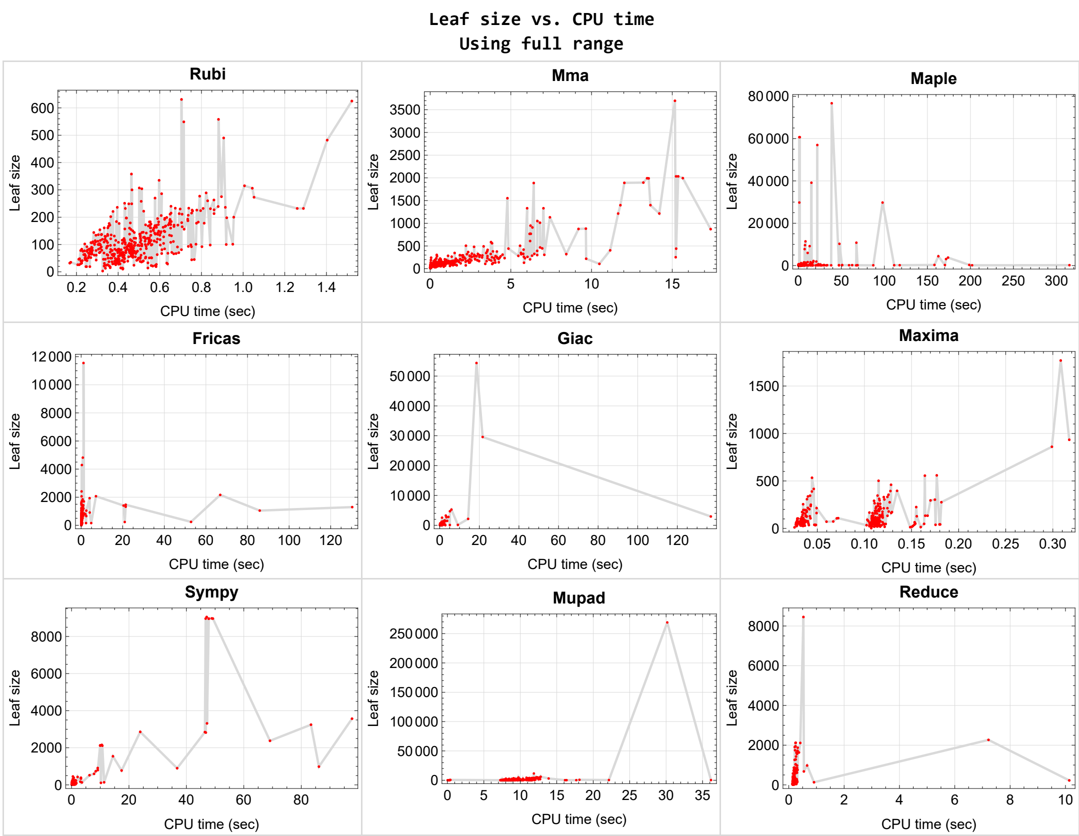


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{174, 178, 422, 426, 488, 489, 490, 491, 492, 496, 497, 501}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {202, 203, 215, 216, 228, 229, 241, 242, 297, 298, 310, 311, 324, 325, 337, 350, 351, 365, 366, 396, 397, 408, 409, 410, 411}

Mathematica {12, 92, 93, 96, 97, 104, 105, 106, 108, 109, 136, 144, 145, 148, 152, 153, 154, 155, 157, 158, 159, 160, 164, 170, 171, 173, 176, 299, 304, 305, 312, 313, 318, 326, 330, 331, 332, 342, 343, 344, 345, 354, 355, 356, 357, 368, 369, 370, 371, 372, 392, 401, 424, 439, 440, 477, 486, 487, 498, 499, 500}

Maple {29, 95, 96, 97, 107, 108, 109, 119, 120, 131, 132, 133, 143, 144, 145, 167, 168, 169, 179, 180, 181, 296, 297, 298, 309, 310, 311, 323, 324, 325, 331, 336, 337, 338, 343, 344, 349, 350, 351, 374, 375, 376, 479, 480}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

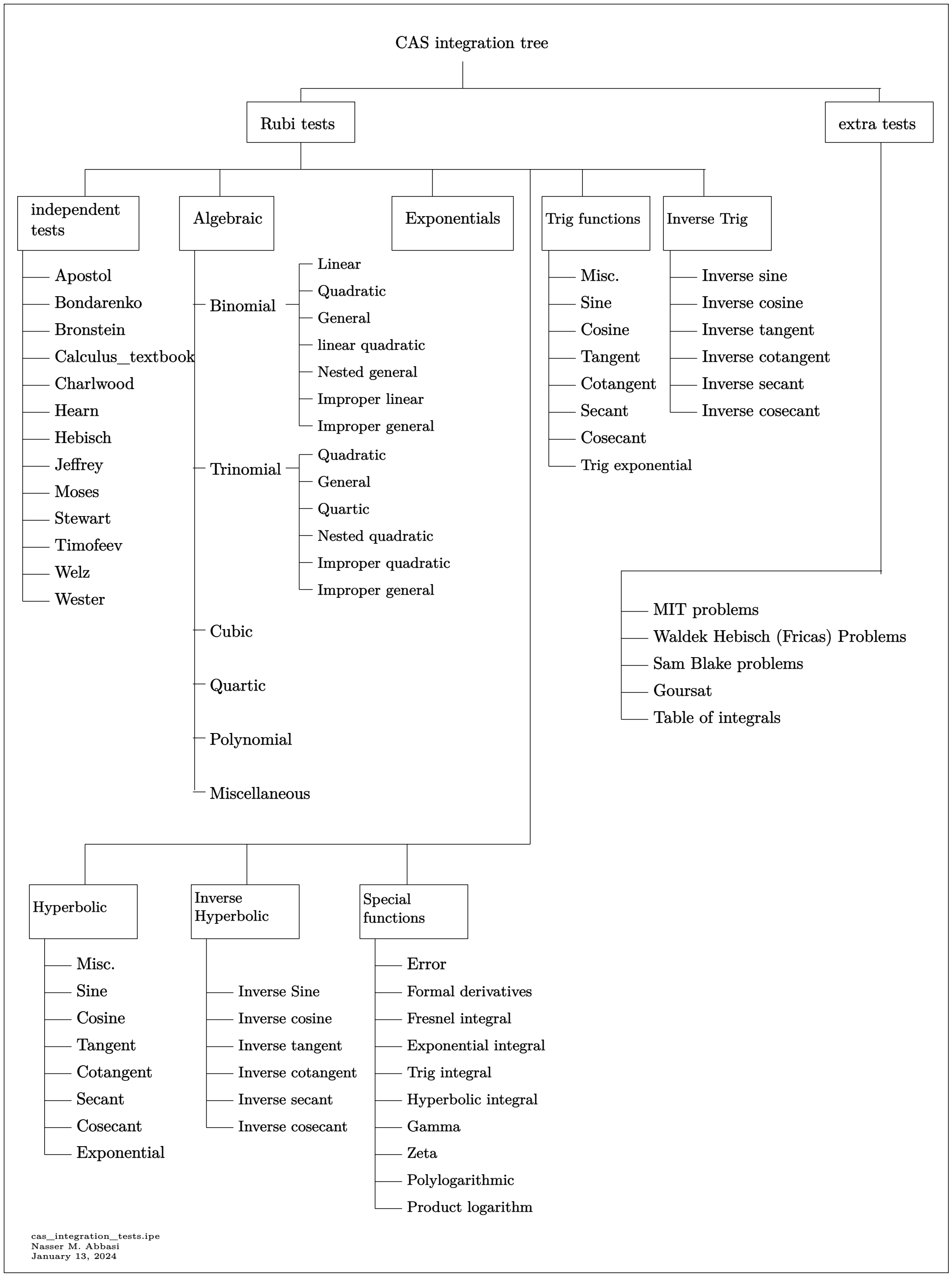
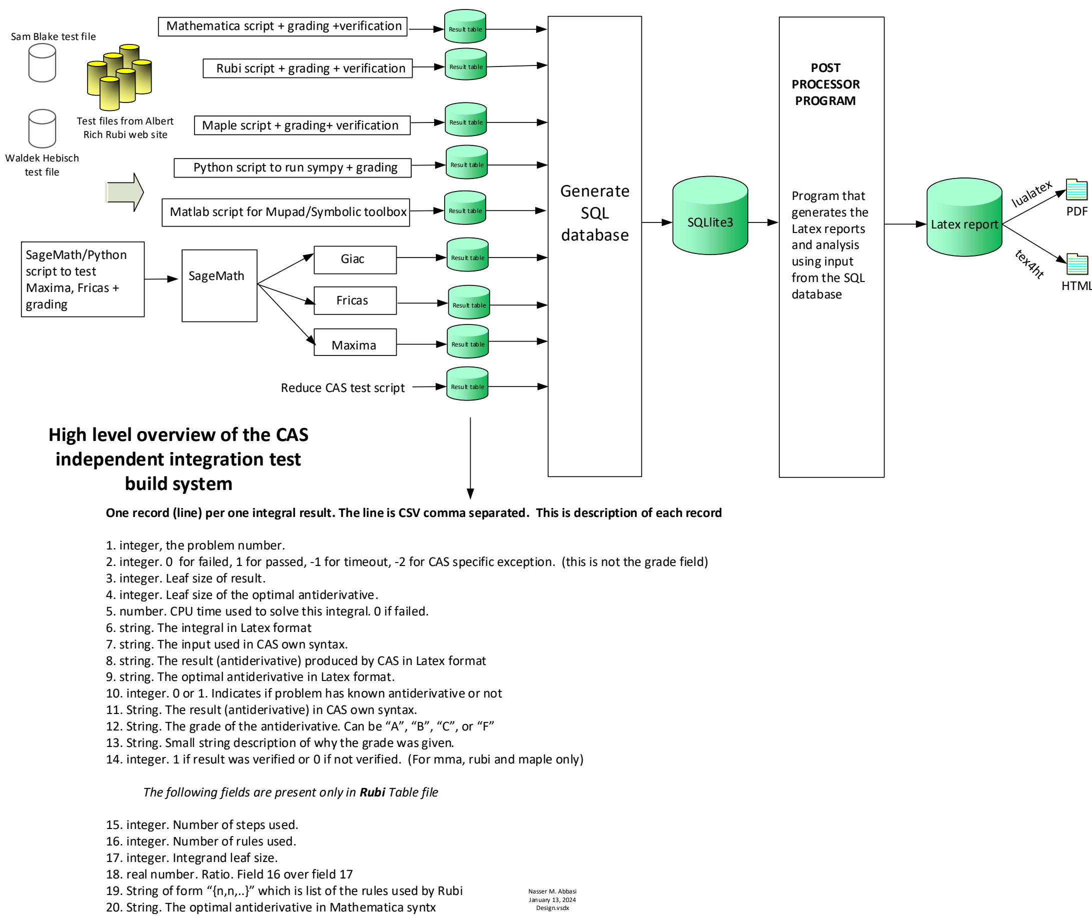


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 493, 494, 495, 498, 499, 500 }

B grade { 386 }

C grade { }

F normal fail { 338 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 104, 105, 106, 109, 112, 116, 117, 118, 121, 124, 125, 126, 127, 128, 129, 130, 133, 137, 138, 139, 140, 141, 142, 149, 150, 151, 154, 155, 156, 161, 162, 163, 165, 166, 167, 168, 169, 172, 175, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 206, 207, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 302, 306, 307, 308, 309, 310, 311, 315, 319, 320, 321, 322, 323, 324, 325, 329, 334, 341, 353, 359, 360, 361, 362, 363, 364, 365, 366, 377, 381, 382, 383, 384, 385, 389, 390, 391, 393, 394, 395, 398, 399, 400, 402, 403, 405, 406, 408, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 493, 494, 495 }

B grade { 34, 47, 48, 57, 58, 59, 60, 95, 96, 97, 107, 108, 119, 120, 131, 132, 143, 144, 145, 153, 157, 158, 160, 176, 204, 205, 249, 288, 368, 369, 370, 371, 372, 424, 452, 477, 499 }

C grade { 16, 17, 18, 98, 99, 101, 102, 103, 110, 111, 113, 114, 115, 122, 123, 134, 135, 136, 146, 147, 148, 152, 159, 164, 170, 171, 173, 195, 196, 197, 208, 210, 269, 270, 299, 300, 301, 303, 304, 305, 312, 313, 314, 316, 317, 318, 326, 327, 328, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 374, 375, 376, 378, 379, 380, 386, 387, 388, 392, 396, 397, 401, 404, 407, 409, 439, 440, 451, 486, 487, 498, 500 }

F normal fail { 367, 373 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 101, 113, 116, 117, 118, 122, 124, 125, 126, 127, 128, 129, 130, 136, 137, 138, 139, 140, 141, 142, 148, 149, 150, 151, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 339, 340, 341, 342, 346, 347, 348, 354, 355, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 386, 389, 390, 398, 399, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 481 }

B grade { 44, 45, 50, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 114, 115, 119, 120, 121, 123, 131, 132, 133, 134, 135, 143, 144, 145, 146, 147, 279, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 323, 324, 325, 330, 331, 336, 337, 338, 343, 349, 350, 351, 352, 353, 393, 394, 402, 403, 405, 406, 449 }

C grade { 29, 110, 167, 168, 169, 332, 344, 345, 378, 385, 387, 388, 392, 401, 479, 480 }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 391, 395, 396, 397, 400, 404, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 476, 477, 478, 482, 483, 484, 485, 486, 487, 493, 494, 495, 498, 499, 500 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 68, 69, 70, 74, 75, 76, 92, 93, 94, 95, 97, 100, 104, 105, 106, 107, 108, 109, 112, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 128, 129, 130, 136, 137, 138, 139, 140, 141, 142, 149, 150, 167, 168, 169, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 230, 231, 233, 234, 235, 236, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 277, 278, 282, 283, 284, 285, 286, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 338, 339, 340, 341, 342, 343, 344, 345, 353, 357, 358, 374, 375, 376, 377, 380, 381, 382, 383, 384, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 462, 463, 464, 465, 466, 467, 474, 475, 479, 480, 481 }

B grade { 33, 34, 35, 47, 48, 60, 65, 66, 67, 71, 72, 73, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 98, 99, 101, 102, 103, 110, 111, 113, 122, 123, 131, 132, 133, 134, 135, 143, 144, 145, 147, 148, 224, 228, 229, 232, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 256, 274, 276, 279, 280, 281, 287, 288, 321, 322, 333, 334, 335, 336, 337, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 385, 386, 402, 403, 404, 405, 406, 407, 460, 461, 468, 469, 470, 471, 472, 473 }

C grade { 290, 291, 292, 378, 379 }

F normal fail { 25, 26, 27, 28, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 388, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 476, 477, 478, 482, 483, 484, 485, 486, 487, 493, 494, 495, 498, 499, 500 }

F(-1) timedout fail { 146, 151, 387, 392 }

F(-2) exception fail { 19, 20, 21, 22, 23, 24 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 101, 102, 103, 104, 105, 106, 113, 114, 115, 116, 117, 118, 125, 126, 127, 129, 130, 137, 138, 139, 141, 142, 149, 150, 151, 167, 168, 169, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 262, 264, 273, 274, 276, 278, 279, 281, 283, 284, 285, 286, 287, 288, 289, 291, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 458, 459, 460, 461, 462, 463, 470, 471, 472, 473, 474, 475, 479, 480, 481 }

B grade { 128, 140, 179, 180, 238, 239, 242, 257, 258, 259, 261, 263, 265, 266, 267, 269, 270, 271, 272, 277, 282 }

C grade { 290 }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 95, 96, 97, 98, 99, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 131, 134, 135, 143, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 172, 173, 175, 176, 177, 260, 268, 275, 280, 292, 293, 294, 295, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 327, 328, 330, 331, 335, 336, 340, 348, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 408, 409, 410, 412, 413, 414, 415, 416, 418, 419, 420, 421, 423, 424, 425, 476, 477, 478, 482, 483, 484, 485, 486, 487, 493, 494, 495, 498, 499, 500 }

F(-1) timedout fail { 132, 133, 144, 145, 325, 326, 332, 333, 337, 338, 339, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 417 }

F(-2) exception fail { 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 100, 124, 136, 148, 296, 302, 329, 334, 341, 342, 346, 347, 353, 354, 355, 404, 407, 411, 452, 453, 454, 455, 456, 457, 464, 465, 466, 467, 468, 469 }

Giac

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 31, 32, 36, 37, 38, 39, 40, 41, 42, 44, 45, 49, 50, 51, 52, 53, 54, 57, 61, 62, 63, 64, 65, 66, 67, 70, 74, 75, 76, 77, 78, 79, 82, 86, 87, 88, 89, 90, 91, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 289, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 389, 390, 393, 394, 396, 397, 399, 402, 403, 427, 428, 429, 430, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 470, 471, 472, 473, 474, 475, 479, 480, 481 }

B grade { 4, 30, 33, 34, 35, 46, 47, 48, 55, 56, 58, 59, 60, 68, 69, 71, 72, 73, 80, 81, 83, 84, 85, 92, 93, 94, 104, 105, 106, 108, 116, 117, 118, 120, 121, 128, 129, 130, 132, 133, 140, 141, 142, 144, 145, 189, 190, 237, 240, 242, 262, 264, 265, 266, 270, 271, 272, 283, 284, 285, 286, 288, 385, 386, 405, 406, 431, 432, 443, 456, 457, 469 }

C grade { 290, 291, 292 }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 98, 99, 100, 101, 102, 103, 110, 111, 113, 114, 115, 122, 123, 124, 125, 126, 127, 134, 135, 137, 138, 139, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 302, 303, 304, 305, 328, 329, 330, 331, 332, 337, 347, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 392, 401, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 476, 477, 478, 482, 483, 484, 485, 486, 487, 493, 494, 495, 498, 499, 500 }

F(-1) timedout fail { 43, 136, 148, 320, 321, 322, 326, 327, 333, 334, 335, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 444, 445 }

F(-2) exception fail { 95, 96, 97, 107, 109, 112, 119, 131, 143, 293, 294, 295, 296, 297, 298, 299, 300, 301, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 323, 324, 325, 336, 340, 391, 395, 398, 400, 404, 407 }

Mupad

A grade { }

B grade { 4, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 124, 125, 126, 127, 137, 138, 149, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 306, 307, 308, 309, 310, 311, 320, 321, 322, 323, 324, 325, 329, 333, 334, 335, 336, 337, 338, 346, 347, 348, 349, 350, 351, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 481, 495 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 257, 259, 267, 270, 271, 272, 274, 282, 299, 300, 301, 302, 303, 304, 305, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 330, 331, 332, 339, 340, 341, 342, 343, 344, 345, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 476, 477, 478, 479, 480, 482, 483, 484, 485, 486, 487, 493, 494, 498, 499, 500 }

F(-2) exception fail { }

Sympy

A grade { 39, 51, 179, 180, 181, 185, 191, 192, 193, 194, 207, 251, 252, 253, 260, 268, 275, 278, 280, 289, 348, 374, 375, 376, 377, 380, 381, 382, 383, 384, 435 }

B grade { 64, 76, 88, 182, 183, 184, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 237, 238, 239, 243, 244, 245, 246, 250, 254, 255, 256, 335 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 57, 58, 59, 60, 65, 66, 70, 71, 72, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 106, 107, 111, 112, 113, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 160, 164, 165, 166, 167, 171, 172, 173, 175, 177, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 279, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 368, 369, 370, 378, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 425, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 446, 447, 448, 449, 450, 452, 453, 454, 455, 458, 459, 460, 461, 464, 465, 466, 467, 468, 471, 472, 473, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 495, 498, 500 }

F(-1) timedout fail { 54, 55, 56, 61, 62, 63, 67, 68, 69, 73, 74, 75, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 104, 105, 108, 109, 110, 114, 115, 116, 117, 128, 129, 140, 141, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 168, 169, 170, 174, 176, 178, 229, 235, 236, 240, 241, 242, 247, 248, 249, 318, 365, 366, 367, 371, 372, 373, 379, 424, 426, 444, 445, 451, 456, 457, 462, 463, 469, 470, 474, 475, 477, 487, 489, 491, 492, 493, 494, 497, 499, 501 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 258, 260, 266, 268, 273, 275, 278, 279, 280, 283, 284, 285, 286, 289, 292, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475 }

C grade { }

F normal fail { 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 216, 222, 223, 229, 235, 236, 242, 248, 249, 257, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 274, 276, 277, 281, 282, 287, 288, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 493, 494, 495, 498, 499, 500 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	67	58	58	47	74	0	55	42	0
N.S.	1	0.68	0.59	0.59	0.48	0.76	0.00	0.56	0.43	0.00
time (sec)	N/A	0.609	0.120	1.583	0.123	0.097	0.000	0.473	0.136	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	49	47	48	34	52	0	41	30	0
N.S.	1	0.80	0.77	0.79	0.56	0.85	0.00	0.67	0.49	0.00
time (sec)	N/A	0.506	0.049	1.308	0.117	0.137	0.000	0.653	0.144	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	19	38	0	26	18	0
N.S.	1	1.00	1.00	1.16	0.59	1.19	0.00	0.81	0.56	0.00
time (sec)	N/A	0.390	0.023	1.471	0.117	0.099	0.000	0.574	0.143	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	31	45	33	50	0	59	33	34
N.S.	1	1.06	1.00	1.45	1.06	1.61	0.00	1.90	1.06	1.10
time (sec)	N/A	0.395	0.039	1.380	0.118	0.118	0.000	0.648	0.151	7.991

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	54	47	64	46	69	0	93	57	0
N.S.	1	0.82	0.71	0.97	0.70	1.05	0.00	1.41	0.86	0.00
time (sec)	N/A	0.533	0.099	1.296	0.115	0.099	0.000	0.558	0.143	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	70	58	74	66	82	0	106	68	0
N.S.	1	0.72	0.60	0.76	0.68	0.85	0.00	1.09	0.70	0.00
time (sec)	N/A	0.691	0.103	1.316	0.115	0.097	0.000	0.719	0.150	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	232	204	266	178	310	0	289	92	0
N.S.	1	0.78	0.68	0.89	0.60	1.04	0.00	0.97	0.31	0.00
time (sec)	N/A	1.260	0.663	3.060	0.126	0.099	0.000	0.414	0.149	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	198	115	236	140	248	0	253	22	0
N.S.	1	0.89	0.52	1.06	0.63	1.11	0.00	1.13	0.10	0.00
time (sec)	N/A	0.921	0.441	2.770	0.116	0.089	0.000	0.483	0.160	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	178	162	208	133	264	0	195	37	0
N.S.	1	0.92	0.84	1.08	0.69	1.37	0.00	1.01	0.19	0.00
time (sec)	N/A	0.776	0.217	2.951	0.124	0.092	0.000	0.489	0.143	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	178	87	210	126	257	0	251	24	0
N.S.	1	0.93	0.45	1.09	0.66	1.34	0.00	1.31	0.12	0.00
time (sec)	N/A	0.813	0.246	2.904	0.118	0.117	0.000	0.567	0.156	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	200	98	233	163	270	0	279	24	0
N.S.	1	0.86	0.42	1.00	0.70	1.16	0.00	1.20	0.10	0.00
time (sec)	N/A	0.955	0.213	2.714	0.128	0.117	0.000	0.689	0.146	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	232	139	272	172	290	0	305	24	0
N.S.	1	0.78	0.46	0.91	0.58	0.97	0.00	1.02	0.08	0.00
time (sec)	N/A	1.289	0.394	2.792	0.129	0.096	0.000	0.779	0.155	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	100	86	84	79	96	0	85	63	0
N.S.	1	0.55	0.47	0.46	0.43	0.53	0.00	0.47	0.35	0.00
time (sec)	N/A	0.919	0.515	1.544	0.104	0.108	0.000	0.452	0.139	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	68	66	64	53	62	0	59	41	0
N.S.	1	0.62	0.60	0.58	0.48	0.56	0.00	0.54	0.37	0.00
time (sec)	N/A	0.639	0.520	1.357	0.112	0.106	0.000	0.444	0.155	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	37	41	42	26	37	0	26	17	0
N.S.	1	0.74	0.82	0.84	0.52	0.74	0.00	0.52	0.34	0.00
time (sec)	N/A	0.403	0.071	1.430	0.113	0.103	0.000	0.390	0.138	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	38	43	40	27	39	0	33	29	0
N.S.	1	0.75	0.84	0.78	0.53	0.76	0.00	0.65	0.57	0.00
time (sec)	N/A	0.399	0.057	1.442	0.112	0.133	0.000	0.394	0.155	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	71	45	63	50	62	0	56	52	0
N.S.	1	0.60	0.38	0.53	0.42	0.52	0.00	0.47	0.44	0.00
time (sec)	N/A	0.657	0.037	1.355	0.108	0.090	0.000	0.446	0.141	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	101	45	83	70	82	0	76	72	0
N.S.	1	0.55	0.25	0.45	0.38	0.45	0.00	0.42	0.39	0.00
time (sec)	N/A	0.950	0.027	1.326	0.113	0.087	0.000	0.452	0.151	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	0	0	0	18	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.467	0.109	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	16	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.469	0.096	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	70	56	0	0	0	0	0	15	0
N.S.	1	1.25	1.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.464	0.077	0.000	0.000	0.000	0.000	0.000	0.140	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0	20	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.463	0.087	0.000	0.000	0.000	0.000	0.000	0.141	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	0	0	0	0	0	20	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.476	0.098	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	0	0	0	20	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.470	0.101	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.468	0.087	0.000	0.000	0.000	0.000	0.000	0.140	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	0	0	0	0	16	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.470	0.083	0.000	0.000	0.000	0.000	0.000	0.138	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.463	0.080	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	0	0	0	0	16	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.466	0.074	0.000	0.000	0.000	0.000	0.000	0.140	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	5979	0	23	0	0	21	0
N.S.	1	1.00	1.00	186.84	0.00	0.72	0.00	0.00	0.66	0.00
time (sec)	N/A	0.399	0.019	10.801	0.000	0.082	0.000	0.000	0.148	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	62	104	92	62	64	0	54384	103	92
N.S.	1	0.89	1.49	1.31	0.89	0.91	0.00	776.91	1.47	1.31
time (sec)	N/A	0.410	0.241	7.125	0.033	0.095	0.000	18.569	0.145	7.599

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	43	72	72	44	46	0	71	80	68
N.S.	1	0.90	1.50	1.50	0.92	0.96	0.00	1.48	1.67	1.42
time (sec)	N/A	0.393	0.162	3.876	0.028	0.093	0.000	0.484	0.142	7.451

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	46	52	31	31	0	38	57	39
N.S.	1	0.93	1.64	1.86	1.11	1.11	0.00	1.36	2.04	1.39
time (sec)	N/A	0.350	0.119	2.146	0.032	0.109	0.000	0.481	0.144	7.334

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	23	25	34	40	56	0	58	41	37
N.S.	1	0.92	1.00	1.36	1.60	2.24	0.00	2.32	1.64	1.48
time (sec)	N/A	0.335	0.015	0.785	0.033	0.090	0.000	0.424	0.139	7.306

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	58	123	68	76	124	0	171	135	95
N.S.	1	1.14	2.41	1.33	1.49	2.43	0.00	3.35	2.65	1.86
time (sec)	N/A	0.378	0.183	2.173	0.036	0.096	0.000	0.443	0.152	7.336

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	94	131	102	101	178	0	239	157	138
N.S.	1	1.19	1.66	1.29	1.28	2.25	0.00	3.03	1.99	1.75
time (sec)	N/A	0.446	5.815	3.395	0.032	0.540	0.000	0.552	0.140	7.421

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	128	89	122	111	90	0	107	140	105
N.S.	1	1.25	0.87	1.20	1.09	0.88	0.00	1.05	1.37	1.03
time (sec)	N/A	0.618	0.444	10.948	0.114	0.104	0.000	0.476	0.158	8.165

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	94	58	102	82	72	0	85	117	79
N.S.	1	1.27	0.78	1.38	1.11	0.97	0.00	1.15	1.58	1.07
time (sec)	N/A	0.447	0.389	4.348	0.110	0.096	0.000	0.451	0.142	7.361

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	59	43	81	51	54	0	62	93	41
N.S.	1	1.28	0.93	1.76	1.11	1.17	0.00	1.35	2.02	0.89
time (sec)	N/A	0.379	0.308	2.184	0.106	0.093	0.000	0.426	0.151	7.329

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	20	23	21	20	28	22	21
N.S.	1	1.00	1.47	1.05	1.21	1.11	1.05	1.47	1.16	1.11
time (sec)	N/A	0.264	0.006	0.369	0.105	0.086	0.079	0.394	0.140	7.436

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	22	24	25	24	37	0	24	44	26
N.S.	1	0.92	1.00	1.04	1.00	1.54	0.00	1.00	1.83	1.08
time (sec)	N/A	0.357	0.054	0.977	0.028	0.087	0.000	0.446	0.152	7.595

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	37	60	54	40	66	0	49	69	41
N.S.	1	0.88	1.43	1.29	0.95	1.57	0.00	1.17	1.64	0.98
time (sec)	N/A	0.370	0.232	4.019	0.032	0.085	0.000	0.413	0.141	7.615

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	56	106	83	59	91	0	73	91	59
N.S.	1	0.88	1.66	1.30	0.92	1.42	0.00	1.14	1.42	0.92
time (sec)	N/A	0.402	0.187	6.352	0.028	0.134	0.000	0.418	0.142	7.903

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	96	97	185	104	105	0	0	280	183
N.S.	1	0.90	0.91	1.73	0.97	0.98	0.00	0.00	2.62	1.71
time (sec)	N/A	0.502	0.964	9.945	0.030	0.103	0.000	0.000	0.155	8.927

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	72	72	155	80	80	0	135	187	128
N.S.	1	0.90	0.90	1.94	1.00	1.00	0.00	1.69	2.34	1.60
time (sec)	N/A	0.456	0.655	6.747	0.034	0.099	0.000	0.837	0.140	11.623

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	48	125	71	59	0	88	260	126
N.S.	1	0.91	0.89	2.31	1.31	1.09	0.00	1.63	4.81	2.33
time (sec)	N/A	0.400	0.382	4.060	0.031	0.093	0.000	0.683	0.157	10.120

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	47	66	97	68	87	0	139	168	86
N.S.	1	0.90	1.27	1.87	1.31	1.67	0.00	2.67	3.23	1.65
time (sec)	N/A	0.404	0.474	2.168	0.033	0.097	0.000	0.659	0.142	8.883

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	231	85	111	168	0	236	328	188
N.S.	1	1.05	3.12	1.15	1.50	2.27	0.00	3.19	4.43	2.54
time (sec)	N/A	0.459	5.875	2.842	0.035	0.109	0.000	0.631	0.154	8.959

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	134	447	145	163	284	0	391	424	243
N.S.	1	1.17	3.89	1.26	1.42	2.47	0.00	3.40	3.69	2.11
time (sec)	N/A	0.600	6.946	5.769	0.035	0.101	0.000	0.590	0.147	8.106

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	137	96	199	130	120	0	162	366	128
N.S.	1	1.18	0.83	1.72	1.12	1.03	0.00	1.40	3.16	1.10
time (sec)	N/A	0.590	1.428	7.226	0.108	0.114	0.000	0.531	0.151	7.939

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	84	71	168	87	94	0	120	328	114
N.S.	1	1.09	0.92	2.18	1.13	1.22	0.00	1.56	4.26	1.48
time (sec)	N/A	0.443	0.881	4.148	0.108	0.088	0.000	0.474	0.162	7.868

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	73	49	58	51	68	69	59	76
N.S.	1	1.09	1.59	1.07	1.26	1.11	1.48	1.50	1.28	1.65
time (sec)	N/A	0.364	0.479	0.568	0.108	0.077	0.108	0.443	0.144	7.486

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	44	48	41	71	0	41	102	67
N.S.	1	0.89	0.96	1.04	0.89	1.54	0.00	0.89	2.22	1.46
time (sec)	N/A	0.396	0.843	3.822	0.027	0.090	0.000	0.466	0.158	7.564

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	62	59	81	66	92	0	78	125	69
N.S.	1	0.89	0.84	1.16	0.94	1.31	0.00	1.11	1.79	0.99
time (sec)	N/A	0.429	1.229	6.816	0.029	0.099	0.000	0.474	0.152	7.469

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	82	88	136	88	137	0	119	178	90
N.S.	1	0.88	0.95	1.46	0.95	1.47	0.00	1.28	1.91	0.97
time (sec)	N/A	0.469	1.371	10.802	0.035	0.092	0.000	0.492	0.163	7.859

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	109	177	144	0	294	0	356	335	643
N.S.	1	0.93	1.51	1.23	0.00	2.51	0.00	3.04	2.86	5.50
time (sec)	N/A	0.585	2.341	21.999	0.000	0.169	0.000	0.502	0.181	10.515

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	87	149	87	0	206	0	173	202	382
N.S.	1	1.04	1.77	1.04	0.00	2.45	0.00	2.06	2.40	4.55
time (sec)	N/A	0.461	0.733	5.263	0.000	0.111	0.000	0.540	0.178	9.056

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	121	61	0	158	0	78	120	112
N.S.	1	0.97	2.02	1.02	0.00	2.63	0.00	1.30	2.00	1.87
time (sec)	N/A	0.379	0.345	1.627	0.000	0.114	0.000	0.509	0.155	7.794

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	144	70	0	184	0	108	109	91
N.S.	1	0.97	2.40	1.17	0.00	3.07	0.00	1.80	1.82	1.52
time (sec)	N/A	0.402	0.361	1.231	0.000	0.119	0.000	0.466	0.172	7.798

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	195	117	0	327	0	201	154	591
N.S.	1	1.09	2.19	1.31	0.00	3.67	0.00	2.26	1.73	6.64
time (sec)	N/A	0.480	0.645	1.699	0.000	0.195	0.000	0.451	0.157	8.447

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	149	326	185	0	630	0	354	311	740
N.S.	1	1.15	2.51	1.42	0.00	4.85	0.00	2.72	2.39	5.69
time (sec)	N/A	0.620	6.454	2.750	0.000	0.166	0.000	0.537	0.163	10.069

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	223	140	185	305	521	0	277	386	4910
N.S.	1	1.25	0.79	1.04	1.71	2.93	0.00	1.56	2.17	27.58
time (sec)	N/A	0.758	0.671	47.227	0.121	0.161	0.000	0.477	0.201	12.382

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	161	99	131	183	383	0	176	236	3588
N.S.	1	1.25	0.77	1.02	1.42	2.97	0.00	1.36	1.83	27.81
time (sec)	N/A	0.572	0.458	10.803	0.113	0.128	0.000	0.426	0.171	11.549

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	101	69	83	93	274	0	100	125	190
N.S.	1	1.23	0.84	1.01	1.13	3.34	0.00	1.22	1.52	2.32
time (sec)	N/A	0.464	0.371	2.862	0.110	0.112	0.000	0.443	0.177	9.306

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	50	48	182	240	49	42	948
N.S.	1	1.00	0.98	1.00	0.96	3.64	4.80	0.98	0.84	18.96
time (sec)	N/A	0.506	0.058	0.643	0.110	0.093	1.383	0.379	0.148	8.747

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	46	48	44	42	257	0	45	99	40
N.S.	1	0.96	1.00	0.92	0.88	5.35	0.00	0.94	2.06	0.83
time (sec)	N/A	0.390	0.340	1.388	0.121	0.106	0.000	0.400	0.168	8.145

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	71	73	67	68	373	0	77	227	67
N.S.	1	0.93	0.96	0.88	0.89	4.91	0.00	1.01	2.99	0.88
time (sec)	N/A	0.424	0.423	1.987	0.112	0.142	0.000	0.428	0.158	8.221

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	97	103	99	104	543	0	128	379	115
N.S.	1	0.92	0.98	0.94	0.99	5.17	0.00	1.22	3.61	1.10
time (sec)	N/A	0.494	0.804	3.270	0.116	0.116	0.000	0.451	0.174	8.223

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	213	215	197	0	593	0	530	1032	1049
N.S.	1	1.31	1.33	1.22	0.00	3.66	0.00	3.27	6.37	6.48
time (sec)	N/A	0.860	2.777	62.513	0.000	0.145	0.000	0.668	0.247	11.926

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	134	182	152	0	456	0	354	818	737
N.S.	1	1.01	1.37	1.14	0.00	3.43	0.00	2.66	6.15	5.54
time (sec)	N/A	0.647	2.553	15.545	0.000	0.126	0.000	0.588	0.217	11.307

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	146	103	0	307	0	147	544	436
N.S.	1	1.03	1.45	1.02	0.00	3.04	0.00	1.46	5.39	4.32
time (sec)	N/A	0.420	0.861	3.765	0.000	0.114	0.000	0.604	0.174	9.852

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	122	184	128	0	470	0	255	725	1140
N.S.	1	1.11	1.67	1.16	0.00	4.27	0.00	2.32	6.59	10.36
time (sec)	N/A	0.508	0.955	1.581	0.000	0.164	0.000	0.633	0.177	10.050

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	154	218	156	0	672	0	390	866	917
N.S.	1	1.05	1.48	1.06	0.00	4.57	0.00	2.65	5.89	6.24
time (sec)	N/A	0.568	4.240	2.187	0.000	0.155	0.000	0.628	0.183	8.350

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	226	392	232	0	1052	0	516	874	1113
N.S.	1	1.08	1.87	1.10	0.00	5.01	0.00	2.46	4.16	5.30
time (sec)	N/A	0.755	6.632	3.344	0.000	0.175	0.000	0.713	0.211	8.673

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	230	136	159	312	705	0	258	805	4616
N.S.	1	1.17	0.69	0.81	1.59	3.60	0.00	1.32	4.11	23.55
time (sec)	N/A	0.698	1.419	30.216	0.116	0.153	0.000	0.517	0.216	12.211

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	161	111	120	185	568	0	180	701	3301
N.S.	1	1.17	0.80	0.87	1.34	4.12	0.00	1.30	5.08	23.92
time (sec)	N/A	0.556	1.411	7.651	0.113	0.143	0.000	0.705	0.192	11.045

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	116	88	93	114	390	2125	113	227	2489
N.S.	1	1.20	0.91	0.96	1.18	4.02	21.91	1.16	2.34	25.66
time (sec)	N/A	0.431	0.792	0.730	0.108	0.107	9.963	0.599	0.142	9.571

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	81	83	69	73	373	0	73	331	70
N.S.	1	0.99	1.01	0.84	0.89	4.55	0.00	0.89	4.04	0.85
time (sec)	N/A	0.415	0.747	1.706	0.112	0.111	0.000	0.588	0.184	7.778

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	112	100	115	587	0	121	574	108
N.S.	1	1.02	0.97	0.86	0.99	5.06	0.00	1.04	4.95	0.93
time (sec)	N/A	0.601	0.959	2.445	0.116	0.122	0.000	0.622	0.189	7.826

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	187	151	144	161	855	0	187	839	178
N.S.	1	1.24	1.00	0.95	1.07	5.66	0.00	1.24	5.56	1.18
time (sec)	N/A	0.735	1.453	3.950	0.111	0.151	0.000	0.432	0.212	9.118

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	273	278	282	0	1018	0	834	2112	1536
N.S.	1	1.22	1.24	1.26	0.00	4.54	0.00	3.72	9.43	6.86
time (sec)	N/A	1.052	4.054	158.198	0.000	0.211	0.000	0.816	0.416	12.495

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	182	230	196	0	775	0	541	1627	1154
N.S.	1	1.01	1.28	1.09	0.00	4.31	0.00	3.01	9.04	6.41
time (sec)	N/A	0.785	4.171	51.003	0.000	0.167	0.000	0.811	0.275	11.377

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	150	170	132	0	556	0	215	8454	780
N.S.	1	1.09	1.23	0.96	0.00	4.03	0.00	1.56	61.26	5.65
time (sec)	N/A	0.470	1.391	6.407	0.000	0.158	0.000	0.768	0.529	10.590

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	192	247	183	0	1050	0	503	1906	1844
N.S.	1	1.16	1.49	1.10	0.00	6.33	0.00	3.03	11.48	11.11
time (sec)	N/A	0.649	3.216	1.204	0.000	0.287	0.000	0.804	0.215	11.991

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	233	286	206	0	1419	0	583	2114	1652
N.S.	1	1.14	1.40	1.00	0.00	6.92	0.00	2.84	10.31	8.06
time (sec)	N/A	0.739	6.313	2.316	0.000	0.292	0.000	0.739	0.243	9.422

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	275	468	265	0	1693	0	861	2123	1357
N.S.	1	1.06	1.81	1.02	0.00	6.54	0.00	3.32	8.20	5.24
time (sec)	N/A	0.896	6.812	3.363	0.000	0.320	0.000	0.765	0.251	8.798

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	289	194	209	460	1191	0	380	1874	5965
N.S.	1	1.16	0.78	0.84	1.84	4.76	0.00	1.52	7.50	23.86
time (sec)	N/A	0.822	1.006	50.740	0.128	0.227	0.000	0.518	0.323	12.798

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	233	164	172	346	1076	0	273	1744	4997
N.S.	1	1.21	0.85	0.89	1.79	5.58	0.00	1.41	9.04	25.89
time (sec)	N/A	0.674	2.225	12.493	0.114	0.208	0.000	0.505	0.227	12.191

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	183	138	142	227	742	8964	198	577	3901
N.S.	1	1.22	0.92	0.95	1.51	4.95	59.76	1.32	3.85	26.01
time (sec)	N/A	0.554	1.322	0.526	0.113	0.143	47.796	0.443	0.151	11.229

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	144	83	105	555	0	91	695	102
N.S.	1	1.04	1.29	0.74	0.94	4.96	0.00	0.81	6.21	0.91
time (sec)	N/A	0.460	1.069	1.596	0.112	0.158	0.000	0.476	0.213	7.732

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	156	146	123	158	857	0	152	1032	147
N.S.	1	1.01	0.95	0.80	1.03	5.56	0.00	0.99	6.70	0.95
time (sec)	N/A	0.717	1.527	2.542	0.111	0.155	0.000	0.462	0.245	8.773

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	236	346	175	212	1199	0	235	1429	199
N.S.	1	1.17	1.71	0.87	1.05	5.94	0.00	1.16	7.07	0.99
time (sec)	N/A	0.915	1.516	4.293	0.112	0.185	0.000	0.551	0.262	9.279

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	173	208	645	203	395	0	2554	24	0
N.S.	1	1.07	1.29	4.01	1.26	2.45	0.00	15.86	0.15	0.00
time (sec)	N/A	0.593	2.449	8.506	0.115	0.226	0.000	1.164	0.150	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	108	170	443	132	292	0	1182	24	0
N.S.	1	0.96	1.50	3.92	1.17	2.58	0.00	10.46	0.21	0.00
time (sec)	N/A	0.478	0.967	6.440	0.119	0.219	0.000	0.923	0.151	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	140	404	97	219	0	389	22	0
N.S.	1	0.97	1.94	5.61	1.35	3.04	0.00	5.40	0.31	0.00
time (sec)	N/A	0.398	0.590	6.421	0.110	0.224	0.000	0.867	0.158	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	82	170	406	0	579	0	0	22	0
N.S.	1	0.98	2.02	4.83	0.00	6.89	0.00	0.00	0.26	0.00
time (sec)	N/A	0.470	1.643	6.280	0.000	0.169	0.000	0.000	0.151	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	127	134	586	761	0	915	0	0	24	0
N.S.	1	1.06	4.61	5.99	0.00	7.20	0.00	0.00	0.19	0.00
time (sec)	N/A	0.557	3.771	6.486	0.000	0.580	0.000	0.000	0.149	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	187	207	1049	1150	0	1340	0	0	24	0
N.S.	1	1.11	5.61	6.15	0.00	7.17	0.00	0.00	0.13	0.00
time (sec)	N/A	0.736	6.643	6.302	0.000	0.860	0.000	0.000	0.161	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	213	330	562	0	2068	0	0	24	0
N.S.	1	1.13	1.75	2.97	0.00	10.94	0.00	0.00	0.13	0.00
time (sec)	N/A	0.693	2.586	23.128	0.000	7.130	0.000	0.000	0.153	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	132	273	325	0	1847	0	0	24	0
N.S.	1	1.03	2.13	2.54	0.00	14.43	0.00	0.00	0.19	0.00
time (sec)	N/A	0.506	2.709	14.306	0.000	0.728	0.000	0.000	0.166	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	108	169	0	394	0	0	15	0
N.S.	1	0.98	1.27	1.99	0.00	4.64	0.00	0.00	0.18	0.00
time (sec)	N/A	0.401	0.319	0.542	0.000	0.147	0.000	0.000	0.148	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	156	93	47	331	0	0	24	0
N.S.	1	0.97	2.36	1.41	0.71	5.02	0.00	0.00	0.36	0.00
time (sec)	N/A	0.423	1.605	0.506	0.028	0.186	0.000	0.000	0.150	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	95	204	241	76	435	0	0	24	0
N.S.	1	0.95	2.04	2.41	0.76	4.35	0.00	0.00	0.24	0.00
time (sec)	N/A	0.452	2.793	12.429	0.039	0.235	0.000	0.000	0.156	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	145	287	316	131	587	0	0	24	0
N.S.	1	1.03	2.04	2.24	0.93	4.16	0.00	0.00	0.17	0.00
time (sec)	N/A	0.521	2.447	13.322	0.038	0.668	0.000	0.000	0.148	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	206	233	1637	330	443	0	4702	61	0
N.S.	1	0.91	1.03	7.21	1.45	1.95	0.00	20.71	0.27	0.00
time (sec)	N/A	0.607	3.962	10.865	0.121	0.357	0.000	5.139	0.202	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	168	188	888	296	322	0	2688	61	0
N.S.	1	0.90	1.01	4.77	1.59	1.73	0.00	14.45	0.33	0.00
time (sec)	N/A	0.545	1.667	8.803	0.116	0.300	0.000	3.391	0.182	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	111	170	684	176	285	0	1366	57	0
N.S.	1	0.98	1.50	6.05	1.56	2.52	0.00	12.09	0.50	0.00
time (sec)	N/A	0.428	1.146	8.497	0.119	0.289	0.000	1.979	0.192	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	127	124	492	666	0	809	0	0	57	0
N.S.	1	0.98	3.87	5.24	0.00	6.37	0.00	0.00	0.45	0.00
time (sec)	N/A	0.550	3.313	8.361	0.000	0.651	0.000	0.000	0.183	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	168	1012	1032	0	1059	0	1467	61	0
N.S.	1	1.01	6.06	6.18	0.00	6.34	0.00	8.78	0.37	0.00
time (sec)	N/A	0.628	6.805	8.477	0.000	0.767	0.000	2.592	0.197	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	223	231	415	1386	0	1431	0	0	61	0
N.S.	1	1.04	1.86	6.22	0.00	6.42	0.00	0.00	0.27	0.00
time (sec)	N/A	0.843	3.560	8.515	0.000	1.039	0.000	0.000	0.189	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	226	278	2156	0	2163	0	0	61	0
N.S.	1	1.02	1.25	9.71	0.00	9.74	0.00	0.00	0.27	0.00
time (sec)	N/A	0.778	3.551	17.561	0.000	66.886	0.000	0.000	0.182	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	163	324	623	0	1931	0	0	61	0
N.S.	1	0.99	1.96	3.78	0.00	11.70	0.00	0.00	0.37	0.00
time (sec)	N/A	0.616	3.755	15.526	0.000	4.076	0.000	0.000	0.189	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	123	142	297	0	521	0	0	44	0
N.S.	1	0.98	1.14	2.38	0.00	4.17	0.00	0.00	0.35	0.00
time (sec)	N/A	0.484	0.547	0.336	0.000	0.361	0.000	0.000	0.162	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	98	220	121	73	387	0	0	61	0
N.S.	1	0.98	2.20	1.21	0.73	3.87	0.00	0.00	0.61	0.00
time (sec)	N/A	0.449	2.174	0.423	0.040	0.319	0.000	0.000	0.206	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	144	177	361	175	497	0	0	61	0
N.S.	1	0.89	1.09	2.23	1.08	3.07	0.00	0.00	0.38	0.00
time (sec)	N/A	0.515	1.499	13.984	0.039	0.711	0.000	0.000	0.189	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	175	213	459	202	655	0	0	61	0
N.S.	1	0.89	1.09	2.34	1.03	3.34	0.00	0.00	0.31	0.00
time (sec)	N/A	0.558	1.857	15.484	0.042	2.482	0.000	0.000	0.201	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	151	112	131	214	124	0	1303	38	0
N.S.	1	1.05	0.78	0.91	1.49	0.86	0.00	9.05	0.26	0.00
time (sec)	N/A	0.569	0.998	4.756	0.049	0.132	0.000	1.594	0.149	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	86	74	78	106	75	0	586	38	0
N.S.	1	0.98	0.84	0.89	1.20	0.85	0.00	6.66	0.43	0.00
time (sec)	N/A	0.466	1.416	4.460	0.043	0.098	0.000	1.529	0.156	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	52	51	35	45	0	231	36	0
N.S.	1	1.00	1.41	1.38	0.95	1.22	0.00	6.24	0.97	0.00
time (sec)	N/A	0.356	0.635	4.704	0.041	0.126	0.000	1.222	0.150	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	221	302	0	151	0	0	36	0
N.S.	1	1.00	5.26	7.19	0.00	3.60	0.00	0.00	0.86	0.00
time (sec)	N/A	0.397	1.592	3.137	0.000	0.200	0.000	0.000	0.149	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	91	97	367	647	0	301	0	464	38	0
N.S.	1	1.07	4.03	7.11	0.00	3.31	0.00	5.10	0.42	0.00
time (sec)	N/A	0.478	2.356	3.333	0.000	0.187	0.000	1.133	0.166	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	160	273	1167	0	454	0	861	38	0
N.S.	1	1.12	1.91	8.16	0.00	3.17	0.00	6.02	0.27	0.00
time (sec)	N/A	0.572	2.871	3.201	0.000	0.211	0.000	1.341	0.146	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	163	314	228	0	788	0	0	38	0
N.S.	1	1.12	2.15	1.56	0.00	5.40	0.00	0.00	0.26	0.00
time (sec)	N/A	0.584	2.976	21.662	0.000	2.060	0.000	0.000	0.163	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	270	177	0	696	0	0	38	0
N.S.	1	1.04	2.90	1.90	0.00	7.48	0.00	0.00	0.41	0.00
time (sec)	N/A	0.462	2.548	12.204	0.000	0.293	0.000	0.000	0.149	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	122	0	0	30	40
N.S.	1	1.00	1.00	1.46	0.00	2.65	0.00	0.00	0.65	0.87
time (sec)	N/A	0.338	0.071	0.605	0.000	0.118	0.000	0.000	0.146	9.556

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	49	31	30	49	0	0	38	36
N.S.	1	1.00	1.63	1.03	1.00	1.63	0.00	0.00	1.27	1.20
time (sec)	N/A	0.379	0.453	0.509	0.033	0.111	0.000	0.000	0.161	9.344

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	68	84	87	90	0	0	38	145
N.S.	1	0.97	0.92	1.14	1.18	1.22	0.00	0.00	0.51	1.96
time (sec)	N/A	0.437	0.483	5.892	0.035	0.183	0.000	0.000	0.151	16.135

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	126	90	135	173	141	0	0	38	761
N.S.	1	1.02	0.73	1.10	1.41	1.15	0.00	0.00	0.31	6.19
time (sec)	N/A	0.522	1.507	7.158	0.034	0.484	0.000	0.000	0.156	18.118

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	196	186	299	389	233	0	3260	54	0
N.S.	1	1.06	1.01	1.62	2.10	1.26	0.00	17.62	0.29	0.00
time (sec)	N/A	0.624	1.941	198.661	0.044	0.286	0.000	3.070	0.154	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	129	106	177	216	158	0	1414	54	0
N.S.	1	1.02	0.83	1.39	1.70	1.24	0.00	11.13	0.43	0.00
time (sec)	N/A	0.507	3.415	6.769	0.040	0.188	0.000	2.855	0.150	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	72	116	83	104	0	445	52	0
N.S.	1	0.97	0.95	1.53	1.09	1.37	0.00	5.86	0.68	0.00
time (sec)	N/A	0.403	2.207	6.720	0.037	0.156	0.000	2.278	0.163	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	82	505	752	0	370	0	0	52	0
N.S.	1	0.98	6.01	8.95	0.00	4.40	0.00	0.00	0.62	0.00
time (sec)	N/A	0.447	5.570	5.943	0.000	0.208	0.000	0.000	0.155	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	127	133	304	934	0	472	0	731	54	0
N.S.	1	1.05	2.39	7.35	0.00	3.72	0.00	5.76	0.43	0.00
time (sec)	N/A	0.556	2.738	6.206	0.000	0.263	0.000	1.707	0.152	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	187	211	345	1414	0	722	0	1250	54	0
N.S.	1	1.13	1.84	7.56	0.00	3.86	0.00	6.68	0.29	0.00
time (sec)	N/A	0.707	3.472	6.312	0.000	0.296	0.000	1.871	0.166	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	211	325	10276	0	1046	0	0	54	0
N.S.	1	1.13	1.74	54.95	0.00	5.59	0.00	0.00	0.29	0.00
time (sec)	N/A	0.649	2.954	47.732	0.000	85.882	0.000	0.000	0.151	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	142	282	1597	0	908	0	0	54	0
N.S.	1	1.06	2.10	11.92	0.00	6.78	0.00	0.00	0.40	0.00
time (sec)	N/A	0.535	2.777	14.125	0.000	4.137	0.000	0.000	0.165	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	232	102	0	307	0	0	46	0
N.S.	1	0.98	2.73	1.20	0.00	3.61	0.00	0.00	0.54	0.00
time (sec)	N/A	0.384	4.615	0.642	0.000	0.158	0.000	0.000	0.147	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	74	59	58	90	0	0	54	2978
N.S.	1	0.97	1.19	0.95	0.94	1.45	0.00	0.00	0.87	48.03
time (sec)	N/A	0.429	1.647	0.767	0.033	0.372	0.000	0.000	0.151	13.887

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	106	119	132	141	155	0	0	54	269040
N.S.	1	0.96	1.08	1.20	1.28	1.41	0.00	0.00	0.49	2445.82
time (sec)	N/A	0.483	0.928	11.615	0.034	2.349	0.000	0.000	0.161	30.158

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	135	192	255	232	0	0	54	0
N.S.	1	1.00	0.84	1.19	1.58	1.44	0.00	0.00	0.34	0.00
time (sec)	N/A	0.554	1.293	14.141	0.035	20.972	0.000	0.000	0.151	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	240	294	334	534	370	0	5261	70	0
N.S.	1	1.02	1.25	1.42	2.27	1.57	0.00	22.39	0.30	0.00
time (sec)	N/A	0.654	1.843	4.360	0.044	0.479	0.000	5.908	0.168	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	172	205	251	308	270	0	2800	70	0
N.S.	1	0.98	1.16	1.43	1.75	1.53	0.00	15.91	0.40	0.00
time (sec)	N/A	0.532	6.191	170.361	0.039	0.285	0.000	4.382	0.149	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	122	124	161	135	202	0	1471	68	0
N.S.	1	1.03	1.05	1.36	1.14	1.71	0.00	12.47	0.58	0.00
time (sec)	N/A	0.428	3.903	68.839	0.038	0.214	0.000	3.739	0.212	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	136	148	300	3062	0	711	0	0	68	0
N.S.	1	1.09	2.21	22.51	0.00	5.23	0.00	0.00	0.50	0.00
time (sec)	N/A	0.557	6.633	171.668	0.000	0.299	0.000	0.000	0.152	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	177	188	380	3762	0	906	0	1111	70	0
N.S.	1	1.06	2.15	21.25	0.00	5.12	0.00	6.28	0.40	0.00
time (sec)	N/A	0.649	3.530	173.997	0.000	0.300	0.000	2.158	0.153	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	237	260	1132	4402	0	1054	0	1648	70	0
N.S.	1	1.10	4.78	18.57	0.00	4.45	0.00	6.95	0.30	0.00
time (sec)	N/A	0.830	7.421	162.724	0.000	0.357	0.000	2.492	0.166	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	277	378	10810	0	0	0	0	70	0
N.S.	1	1.13	1.54	43.94	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.791	4.135	67.438	0.000	0.000	0.000	0.000	0.153	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	199	309	1895	0	1294	0	0	70	0
N.S.	1	1.10	1.71	10.47	0.00	7.15	0.00	0.00	0.39	0.00
time (sec)	N/A	0.633	4.487	20.869	0.000	130.347	0.000	0.000	0.167	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	144	1331	163	0	558	0	0	62	0
N.S.	1	1.07	9.93	1.22	0.00	4.16	0.00	0.00	0.46	0.00
time (sec)	N/A	0.478	7.021	0.677	0.000	0.168	0.000	0.000	0.167	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	133	90	85	156	0	0	70	324
N.S.	1	1.00	1.37	0.93	0.88	1.61	0.00	0.00	0.72	3.34
time (sec)	N/A	0.464	2.409	0.778	0.036	4.847	0.000	0.000	0.155	22.153

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	138	140	162	195	240	0	0	70	0
N.S.	1	0.91	0.92	1.07	1.28	1.58	0.00	0.00	0.46	0.00
time (sec)	N/A	0.506	1.317	15.694	0.035	52.845	0.000	0.000	0.164	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	195	174	237	337	0	0	0	70	0
N.S.	1	0.94	0.84	1.14	1.62	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.589	1.742	14.725	0.040	0.000	0.000	0.000	0.153	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	292	0	0	0	0	0	28	0
N.S.	1	1.00	3.17	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.802	1.975	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	147	275	0	0	0	0	0	29	0
N.S.	1	1.20	2.25	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.606	2.458	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	215	283	0	0	0	0	0	25	0
N.S.	1	1.01	1.33	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.654	5.465	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	138	184	0	0	0	0	0	25	0
N.S.	1	0.96	1.28	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.504	2.939	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	79	80	0	0	0	0	0	23	0
N.S.	1	0.95	0.96	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.391	0.789	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	88	1215	0	0	0	0	0	23	0
N.S.	1	0.96	13.21	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.431	14.201	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	92	252	0	0	0	0	0	25	0
N.S.	1	0.96	2.62	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.466	15.216	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	3698	0	0	0	0	0	25	0
N.S.	1	0.99	44.02	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.444	15.169	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	192	0	0	0	0	0	16	0
N.S.	1	0.99	2.43	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.372	0.329	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	68	0	0	0	0	0	25	0
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.411	0.491	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	118	111	0	0	0	0	0	25	0
N.S.	1	0.98	0.92	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.459	0.829	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	183	141	0	0	0	0	0	25	0
N.S.	1	1.01	0.78	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.594	0.588	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	295	0	0	0	0	0	33	0
N.S.	1	1.00	3.01	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.842	2.034	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	30	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.588	0.442	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.510	0.075	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	9186	37	51	0	0	30	0
N.S.	1	1.00	0.94	278.36	1.12	1.55	0.00	0.00	0.91	0.00
time (sec)	N/A	0.543	0.349	13.118	0.047	0.091	0.000	0.000	0.174	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	84	59	29777	73	104	0	0	30	0
N.S.	1	1.22	0.86	431.55	1.06	1.51	0.00	0.00	0.43	0.00
time (sec)	N/A	0.623	0.385	97.884	0.067	0.124	0.000	0.000	0.159	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	89	60670	108	180	0	0	30	0
N.S.	1	1.07	0.86	583.37	1.04	1.73	0.00	0.00	0.29	0.00
time (sec)	N/A	0.665	0.479	1.693	0.072	0.095	0.000	0.000	0.172	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	506	0	0	0	0	0	30	0
N.S.	1	1.00	5.44	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.737	2.217	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	284	0	0	0	0	0	28	0
N.S.	1	1.00	3.12	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.685	1.109	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	28	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.704	0.317	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	1399	0	0	0	0	0	30	0
N.S.	1	1.00	15.21	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.752	13.650	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	29	27
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.16	1.08
time (sec)	N/A	0.354	2.885	1.421	6.877	0.106	0.000	6.421	0.183	8.828

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	0	0	0	0	0	28	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.749	0.561	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	108	2033	0	0	0	0	0	29	0
N.S.	1	0.99	18.65	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.652	15.369	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	0	0	0	33	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.774	0.648	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	32	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.19	1.07
time (sec)	N/A	0.606	2.566	1.489	6.611	0.145	0.000	45.178	0.170	8.139

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	51	46	43	157	56	68	56	46	53
N.S.	1	0.78	0.71	0.66	2.42	0.86	1.05	0.86	0.71	0.82
time (sec)	N/A	0.430	0.138	0.560	0.113	0.072	0.198	0.619	0.161	7.557

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	41	38	35	102	43	54	43	36	36
N.S.	1	0.82	0.76	0.70	2.04	0.86	1.08	0.86	0.72	0.72
time (sec)	N/A	0.418	0.074	0.520	0.115	0.131	0.159	0.575	0.144	7.462

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	29	26	25	59	29	37	29	24	24
N.S.	1	0.91	0.81	0.78	1.84	0.91	1.16	0.91	0.75	0.75
time (sec)	N/A	0.409	0.032	0.519	0.105	0.070	0.093	0.407	0.144	7.636

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	26	25	36	40	87	40	41	26
N.S.	1	0.94	0.84	0.81	1.16	1.29	2.81	1.29	1.32	0.84
time (sec)	N/A	0.374	0.019	0.606	0.106	0.082	0.287	0.527	0.157	7.544

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	42	67	84	248	53	77	35
N.S.	1	1.00	0.65	0.76	1.22	1.53	4.51	0.96	1.40	0.64
time (sec)	N/A	0.483	0.032	0.671	0.107	0.088	0.383	0.597	0.144	7.621

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	81	46	59	90	120	454	63	109	51
N.S.	1	1.03	0.58	0.75	1.14	1.52	5.75	0.80	1.38	0.65
time (sec)	N/A	0.610	0.030	0.711	0.115	0.093	0.480	0.817	0.146	7.730

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	66	65	73	99	67	116	98	89	68
N.S.	1	0.89	0.88	0.99	1.34	0.91	1.57	1.32	1.20	0.92
time (sec)	N/A	0.667	0.157	0.932	0.035	0.104	0.170	0.565	0.158	7.633

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	50	47	55	70	51	88	63	66	57
N.S.	1	0.94	0.89	1.04	1.32	0.96	1.66	1.19	1.25	1.08
time (sec)	N/A	0.531	0.153	0.668	0.035	0.080	0.111	0.731	0.146	7.925

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	35	37	36	60	36	43	37
N.S.	1	1.00	1.18	1.03	1.09	1.06	1.76	1.06	1.26	1.09
time (sec)	N/A	0.393	0.042	0.635	0.035	0.092	0.082	0.739	0.147	7.900

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	25	31	46	58	37	80	36
N.S.	1	1.08	1.00	0.96	1.19	1.77	2.23	1.42	3.08	1.38
time (sec)	N/A	0.402	0.024	1.081	0.028	0.089	0.201	0.790	0.163	7.890

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	42	37	31	61	97	79	119	54
N.S.	1	1.06	1.24	1.09	0.91	1.79	2.85	2.32	3.50	1.59
time (sec)	N/A	0.429	0.027	0.852	0.029	0.080	0.584	0.695	0.146	8.049

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	71	58	52	85	124	100	153	74
N.S.	1	0.98	1.34	1.09	0.98	1.60	2.34	1.89	2.89	1.40
time (sec)	N/A	0.559	0.031	0.904	0.033	0.132	1.233	1.210	0.147	8.012

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	67	129	74	72	69	109	116	89	70
N.S.	1	0.84	1.61	0.92	0.90	0.86	1.36	1.45	1.11	0.88
time (sec)	N/A	0.678	0.036	0.705	0.110	0.083	0.181	1.080	0.164	7.682

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	51	97	57	57	54	82	87	67	53
N.S.	1	0.85	1.62	0.95	0.95	0.90	1.37	1.45	1.12	0.88
time (sec)	N/A	0.550	0.025	0.672	0.109	0.087	0.130	1.126	0.145	7.576

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	37	65	38	41	38	54	59	44	37
N.S.	1	0.92	1.62	0.95	1.02	0.95	1.35	1.48	1.10	0.92
time (sec)	N/A	0.431	0.019	0.688	0.107	0.078	0.105	0.905	0.143	7.716

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	20	23	21	20	28	22	21
N.S.	1	1.00	1.47	1.05	1.21	1.11	1.05	1.47	1.16	1.11
time (sec)	N/A	0.245	0.001	0.552	0.107	0.075	0.068	0.933	0.145	7.527

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	34	24	27	29	42	30	43	21
N.S.	1	1.00	1.62	1.14	1.29	1.38	2.00	1.43	2.05	1.00
time (sec)	N/A	0.333	0.012	0.770	0.114	0.073	0.291	0.904	0.143	7.930

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	65	41	46	49	66	54	83	40
N.S.	1	1.00	1.67	1.05	1.18	1.26	1.69	1.38	2.13	1.03
time (sec)	N/A	0.446	0.027	0.813	0.108	0.085	0.753	0.635	0.161	7.666

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	51	69	55	61	64	94	77	117	57
N.S.	1	0.84	1.13	0.90	1.00	1.05	1.54	1.26	1.92	0.93
time (sec)	N/A	0.561	0.035	0.842	0.116	0.074	1.679	0.710	0.149	8.228

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	98	99	114	162	107	206	175	167	113
N.S.	1	0.93	0.94	1.09	1.54	1.02	1.96	1.67	1.59	1.08
time (sec)	N/A	0.496	0.172	0.760	0.049	0.123	0.234	0.861	0.159	7.600

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	79	78	90	127	86	160	128	129	97
N.S.	1	0.96	0.95	1.10	1.55	1.05	1.95	1.56	1.57	1.18
time (sec)	N/A	0.470	0.109	0.710	0.039	0.085	0.196	0.669	0.146	7.888

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	60	52	66	82	64	112	74	90	68
N.S.	1	0.97	0.84	1.06	1.32	1.03	1.81	1.19	1.45	1.10
time (sec)	N/A	0.413	0.169	0.646	0.036	0.082	0.118	0.571	0.161	7.460

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	53	57	49	59	69	97	61	369	62
N.S.	1	1.04	1.12	0.96	1.16	1.35	1.90	1.20	7.24	1.22
time (sec)	N/A	0.424	0.068	0.947	0.035	0.090	0.466	0.632	0.149	7.522

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	63	53	51	93	129	93	206	68
N.S.	1	0.98	1.12	0.95	0.91	1.66	2.30	1.66	3.68	1.21
time (sec)	N/A	0.451	0.043	0.976	0.030	0.087	1.171	0.744	0.150	7.746

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	73	91	69	61	99	172	133	217	91
N.S.	1	0.96	1.20	0.91	0.80	1.30	2.26	1.75	2.86	1.20
time (sec)	N/A	0.489	0.030	0.986	0.034	0.105	3.059	0.928	0.164	7.729

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	108	243	125	118	115	212	210	172	155
N.S.	1	0.96	2.15	1.11	1.04	1.02	1.88	1.86	1.52	1.37
time (sec)	N/A	0.506	0.059	0.770	0.112	0.080	0.290	0.623	0.147	7.511

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	190	98	97	94	165	162	134	127
N.S.	1	0.98	2.09	1.08	1.07	1.03	1.81	1.78	1.47	1.40
time (sec)	N/A	0.459	0.044	0.735	0.112	0.078	0.211	0.920	0.161	7.467

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	70	137	77	76	73	117	116	96	100
N.S.	1	1.01	1.99	1.12	1.10	1.06	1.70	1.68	1.39	1.45
time (sec)	N/A	0.437	0.035	0.671	0.117	0.081	0.160	0.914	0.148	7.790

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	73	49	58	51	68	69	59	76
N.S.	1	1.09	1.59	1.07	1.26	1.11	1.48	1.50	1.28	1.65
time (sec)	N/A	0.364	0.350	0.518	0.108	0.102	0.112	0.785	0.167	7.331

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	66	38	46	50	71	51	106	70
N.S.	1	1.11	1.74	1.00	1.21	1.32	1.87	1.34	2.79	1.84
time (sec)	N/A	0.421	0.071	0.883	0.120	0.083	0.657	0.891	0.148	7.257

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	48	71	45	57	60	88	65	106	58
N.S.	1	1.09	1.61	1.02	1.30	1.36	2.00	1.48	2.41	1.32
time (sec)	N/A	0.431	0.845	0.849	0.106	0.093	1.593	0.991	0.147	7.545

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	104	62	78	81	133	104	162	76
N.S.	1	1.01	1.53	0.91	1.15	1.19	1.96	1.53	2.38	1.12
time (sec)	N/A	0.457	0.080	0.960	0.117	0.080	3.617	1.073	0.183	7.589

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	67	64	67	76	92	338	74	73	74
N.S.	1	0.94	0.90	0.94	1.07	1.30	4.76	1.04	1.03	1.04
time (sec)	N/A	0.484	0.098	1.284	0.040	0.123	3.264	0.728	0.146	8.016

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	53	41	46	53	65	230	54	45	54
N.S.	1	1.06	0.82	0.92	1.06	1.30	4.60	1.08	0.90	1.08
time (sec)	N/A	0.470	0.026	0.669	0.041	0.096	1.543	0.879	0.161	8.001

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	49	37	40	30	38	133	54	39	66
N.S.	1	1.36	1.03	1.11	0.83	1.06	3.69	1.50	1.08	1.83
time (sec)	N/A	0.386	0.019	0.681	0.034	0.094	1.139	0.743	0.146	7.825

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	59	58	49	72	388	76	133	68
N.S.	1	1.03	0.92	0.91	0.77	1.12	6.06	1.19	2.08	1.06
time (sec)	N/A	0.466	0.034	0.954	0.032	0.104	3.272	0.793	0.153	7.502

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	83	63	81	68	128	733	121	226	89
N.S.	1	0.93	0.71	0.91	0.76	1.44	8.24	1.36	2.54	1.00
time (sec)	N/A	0.508	0.172	0.969	0.038	0.125	8.415	0.948	10.137	7.589

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	106	83	101	96	163	898	171	25	118
N.S.	1	0.92	0.72	0.88	0.83	1.42	7.81	1.49	0.22	1.03
time (sec)	N/A	0.551	0.238	1.033	0.036	0.159	36.780	0.884	200.015	7.539

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	100	92	88	83	278	595	107	98	1310
N.S.	1	1.18	1.08	1.04	0.98	3.27	7.00	1.26	1.15	15.41
time (sec)	N/A	0.562	0.520	0.849	0.113	0.104	7.839	0.810	0.161	7.749

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	74	70	65	65	220	427	67	65	1212
N.S.	1	1.17	1.11	1.03	1.03	3.49	6.78	1.06	1.03	19.24
time (sec)	N/A	0.458	0.174	0.769	0.115	0.110	2.110	0.856	0.190	7.882

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	57	49	50	47	181	252	49	42	135
N.S.	1	1.14	0.98	1.00	0.94	3.62	5.04	0.98	0.84	2.70
time (sec)	N/A	0.423	0.019	0.718	0.110	0.138	1.241	0.822	0.158	7.449

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	50	48	182	240	49	42	948
N.S.	1	1.00	0.98	1.00	0.96	3.64	4.80	0.98	0.84	18.96
time (sec)	N/A	0.497	0.018	0.534	0.113	0.102	1.217	0.720	0.164	7.800

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	75	68	68	65	243	522	70	132	438
N.S.	1	1.17	1.06	1.06	1.02	3.80	8.16	1.09	2.06	6.84
time (sec)	N/A	0.454	0.186	0.914	0.119	0.097	6.260	0.734	0.911	7.819

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	101	92	88	86	308	775	98	25	484
N.S.	1	1.20	1.10	1.05	1.02	3.67	9.23	1.17	0.30	5.76
time (sec)	N/A	0.543	0.466	0.976	0.115	0.102	17.434	0.895	200.023	8.689

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	132	121	111	112	352	984	141	25	524
N.S.	1	1.17	1.07	0.98	0.99	3.12	8.71	1.25	0.22	4.64
time (sec)	N/A	0.656	1.301	1.002	0.109	0.111	86.160	0.831	200.017	10.301

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	85	73	83	128	186	1542	146	220	90
N.S.	1	0.94	0.81	0.92	1.42	2.07	17.13	1.62	2.44	1.00
time (sec)	N/A	0.535	0.452	0.715	0.042	0.109	14.422	0.648	0.199	7.997

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	76	61	74	88	98	910	107	157	270
N.S.	1	1.10	0.88	1.07	1.28	1.42	13.19	1.55	2.28	3.91
time (sec)	N/A	0.493	0.363	0.717	0.036	0.095	9.123	0.613	0.210	8.299

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	71	57	78	88	98	796	96	169	195
N.S.	1	1.09	0.88	1.20	1.35	1.51	12.25	1.48	2.60	3.00
time (sec)	N/A	0.455	0.385	0.715	0.036	0.094	9.147	0.613	0.192	7.971

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	100	90	102	124	197	2377	182	775	104
N.S.	1	0.97	0.87	0.99	1.20	1.91	23.08	1.77	7.52	1.01
time (sec)	N/A	0.530	1.257	1.174	0.039	0.189	69.113	0.620	0.198	8.186

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	120	98	122	187	292	3568	240	976	144
N.S.	1	0.91	0.74	0.92	1.42	2.21	27.03	1.82	7.39	1.09
time (sec)	N/A	0.589	0.531	1.465	0.039	0.129	97.685	0.720	0.662	8.415

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	147	121	148	236	347	0	280	25	191
N.S.	1	0.91	0.75	0.92	1.47	2.16	0.00	1.74	0.16	1.19
time (sec)	N/A	0.624	0.710	1.549	0.043	0.135	0.000	0.577	200.021	8.615

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	142	118	104	135	474	2859	135	292	2581
N.S.	1	1.09	0.91	0.80	1.04	3.65	21.99	1.04	2.25	19.85
time (sec)	N/A	0.603	0.852	0.911	0.117	0.155	23.928	0.635	0.284	9.256

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	113	94	90	114	381	2157	111	227	2358
N.S.	1	1.19	0.99	0.95	1.20	4.01	22.71	1.17	2.39	24.82
time (sec)	N/A	0.466	0.513	0.846	0.110	0.109	10.546	0.598	0.233	9.918

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	104	87	83	97	393	2113	100	229	2136
N.S.	1	1.16	0.97	0.92	1.08	4.37	23.48	1.11	2.54	23.73
time (sec)	N/A	0.454	0.356	0.816	0.114	0.122	10.724	0.566	0.251	8.909

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	116	88	93	114	390	2125	113	227	2489
N.S.	1	1.20	0.91	0.96	1.18	4.02	21.91	1.16	2.34	25.66
time (sec)	N/A	0.420	0.654	0.570	0.114	0.126	9.990	0.581	0.243	9.579

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	142	117	106	151	503	3245	155	677	2674
N.S.	1	1.11	0.91	0.83	1.18	3.93	25.35	1.21	5.29	20.89
time (sec)	N/A	0.575	1.890	1.010	0.119	0.177	83.398	0.735	0.545	10.360

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	180	137	126	193	596	0	163	25	2000
N.S.	1	1.07	0.81	0.75	1.14	3.53	0.00	0.96	0.15	11.83
time (sec)	N/A	0.705	2.468	1.099	0.116	0.126	0.000	0.571	200.023	11.073

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	224	165	152	239	672	0	207	25	3030
N.S.	1	1.03	0.76	0.70	1.10	3.08	0.00	0.95	0.11	13.90
time (sec)	N/A	0.796	3.497	1.154	0.115	0.160	0.000	0.589	200.025	11.521

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	112	97	117	189	206	3315	208	346	577
N.S.	1	1.04	0.90	1.08	1.75	1.91	30.69	1.93	3.20	5.34
time (sec)	N/A	0.550	0.690	0.894	0.038	0.146	47.184	0.682	0.165	8.382

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	87	101	194	212	2819	150	346	532
N.S.	1	1.04	0.90	1.04	2.00	2.19	29.06	1.55	3.57	5.48
time (sec)	N/A	0.514	0.497	0.842	0.037	0.105	46.775	0.711	0.172	8.230

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	96	82	108	192	206	2846	148	384	375
N.S.	1	1.03	0.88	1.16	2.06	2.22	30.60	1.59	4.13	4.03
time (sec)	N/A	0.478	0.459	0.778	0.037	0.128	46.454	0.665	0.160	8.059

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	142	126	149	250	422	0	290	1945	181
N.S.	1	0.96	0.85	1.01	1.69	2.85	0.00	1.96	13.14	1.22
time (sec)	N/A	0.590	1.104	1.961	0.043	0.138	0.000	0.716	0.242	7.903

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	166	144	173	345	545	0	340	2266	229
N.S.	1	0.92	0.80	0.96	1.91	3.01	0.00	1.88	12.52	1.27
time (sec)	N/A	0.668	1.340	2.485	0.042	0.151	0.000	0.642	7.218	9.152

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	193	178	199	416	611	0	483	25	269
N.S.	1	0.92	0.85	0.95	1.98	2.91	0.00	2.30	0.12	1.28
time (sec)	N/A	0.734	1.630	2.138	0.046	0.206	0.000	0.648	200.022	9.259

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	184	142	142	229	743	8974	199	577	3838
N.S.	1	1.20	0.93	0.93	1.50	4.86	58.65	1.30	3.77	25.08
time (sec)	N/A	0.655	1.511	1.040	0.113	0.129	48.851	0.668	0.202	11.776

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	169	136	132	212	749	8957	183	580	3667
N.S.	1	1.17	0.94	0.91	1.46	5.17	61.77	1.26	4.00	25.29
time (sec)	N/A	0.577	1.302	1.028	0.120	0.124	47.800	0.747	0.203	11.529

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	171	139	137	213	759	9051	186	582	3817
N.S.	1	1.19	0.97	0.95	1.48	5.27	62.85	1.29	4.04	26.51
time (sec)	N/A	0.553	1.337	0.944	0.124	0.145	47.067	0.689	0.243	11.726

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	183	138	142	227	742	8964	198	577	3901
N.S.	1	1.22	0.92	0.95	1.51	4.95	59.76	1.32	3.85	26.01
time (sec)	N/A	0.554	1.207	0.668	0.117	0.122	46.710	0.577	0.210	11.945

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	215	174	153	275	881	0	219	1631	915
N.S.	1	1.14	0.92	0.81	1.46	4.66	0.00	1.16	8.63	4.84
time (sec)	N/A	0.753	1.445	1.200	0.127	0.139	0.000	0.622	0.368	11.956

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	263	184	173	332	1006	0	244	25	986
N.S.	1	1.10	0.77	0.72	1.38	4.19	0.00	1.02	0.10	4.11
time (sec)	N/A	0.860	2.928	1.333	0.124	0.158	0.000	0.575	200.027	12.082

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	315	949	199	396	1114	0	288	25	2507
N.S.	1	1.06	3.20	0.67	1.33	3.75	0.00	0.97	0.08	8.44
time (sec)	N/A	1.007	6.216	0.859	0.135	0.190	0.000	0.654	200.027	12.464

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	113	137	133	162	134	209	205	178	164
N.S.	1	0.98	1.19	1.16	1.41	1.17	1.82	1.78	1.55	1.43
time (sec)	N/A	0.467	1.240	0.444	0.109	0.080	0.219	0.608	0.199	7.863

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	102	88	104	90	126	130	111	115
N.S.	1	1.01	1.32	1.14	1.35	1.17	1.64	1.69	1.44	1.49
time (sec)	N/A	0.410	0.613	0.399	0.108	0.111	0.151	0.557	0.151	7.631

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	73	49	58	51	68	69	59	76
N.S.	1	1.09	1.59	1.07	1.26	1.11	1.48	1.50	1.28	1.65
time (sec)	N/A	0.369	0.378	0.360	0.107	0.073	0.116	0.519	0.211	7.969

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	20	23	21	20	28	22	21
N.S.	1	1.00	1.47	1.05	1.21	1.11	1.05	1.47	1.16	1.11
time (sec)	N/A	0.248	0.001	0.420	0.108	0.096	0.066	0.495	0.161	7.809

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	50	48	182	240	49	42	948
N.S.	1	1.00	0.98	1.00	0.96	3.64	4.80	0.98	0.84	18.96
time (sec)	N/A	0.501	0.063	0.711	0.121	0.101	1.230	0.482	0.174	7.698

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	116	88	93	114	390	2125	113	227	2489
N.S.	1	1.20	0.91	0.96	1.18	4.02	21.91	1.16	2.34	25.66
time (sec)	N/A	0.426	0.697	0.829	0.119	0.105	10.004	0.558	0.162	8.879

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	183	138	142	227	742	8964	198	577	3901
N.S.	1	1.22	0.92	0.95	1.51	4.95	59.76	1.32	3.85	26.01
time (sec)	N/A	0.550	1.282	0.576	0.116	0.118	49.238	0.647	0.201	9.764

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	44	32	56	860	56	0	48	17	0
N.S.	1	0.81	0.59	1.04	15.93	1.04	0.00	0.89	0.31	0.00
time (sec)	N/A	0.753	0.061	0.510	0.299	0.080	0.000	0.576	0.172	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	38	20	29	276	18	0	29	17	19
N.S.	1	1.27	0.67	0.97	9.20	0.60	0.00	0.97	0.57	0.63
time (sec)	N/A	0.506	0.045	0.394	0.182	0.073	0.000	0.511	0.166	0.259

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	29	24	39	295	47	0	40	17	0
N.S.	1	0.81	0.67	1.08	8.19	1.31	0.00	1.11	0.47	0.00
time (sec)	N/A	0.637	0.034	0.661	0.170	0.083	0.000	0.563	0.166	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	10	10	10	10	10
N.S.	1	1.00	1.00	1.10	0.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.422	0.009	0.589	0.000	0.078	0.233	0.477	0.178	7.527

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	20	38	60	0	24	15	12
N.S.	1	1.00	0.71	0.83	1.58	2.50	0.00	1.00	0.62	0.50
time (sec)	N/A	0.476	0.011	1.678	0.175	0.089	0.000	0.505	0.196	0.180

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	17	16	0	32	17	25
N.S.	1	1.00	1.00	0.93	1.21	1.14	0.00	2.29	1.21	1.79
time (sec)	N/A	0.543	0.012	1.642	0.121	0.089	0.000	0.522	0.172	7.829

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	53	41	29	303	58	0	50	17	37
N.S.	1	1.18	0.91	0.64	6.73	1.29	0.00	1.11	0.38	0.82
time (sec)	N/A	0.507	0.096	1.592	0.175	0.083	0.000	0.586	0.200	7.697

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	27	22	26	29	24	0	59	17	40
N.S.	1	0.79	0.65	0.76	0.85	0.71	0.00	1.74	0.50	1.18
time (sec)	N/A	0.584	0.017	1.564	0.112	0.072	0.000	0.548	0.172	7.677

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	38	31	34	65	85	0	66	16	41
N.S.	1	1.06	0.86	0.94	1.81	2.36	0.00	1.83	0.44	1.14
time (sec)	N/A	0.406	0.023	1.071	0.153	0.087	0.000	0.795	0.203	7.549

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	40	22	29	559	29	0	72	24	19
N.S.	1	1.25	0.69	0.91	17.47	0.91	0.00	2.25	0.75	0.59
time (sec)	N/A	0.521	0.062	0.664	0.177	0.074	0.000	0.512	0.167	7.694

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	45	34	54	934	57	0	49	33	0
N.S.	1	0.76	0.58	0.92	15.83	0.97	0.00	0.83	0.56	0.00
time (sec)	N/A	0.766	0.050	0.546	0.317	0.084	0.000	0.532	0.204	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	12	12	12	18	11
N.S.	1	1.00	1.00	0.93	0.00	0.86	0.86	0.86	1.29	0.79
time (sec)	N/A	0.412	0.011	0.555	0.000	0.070	0.757	0.499	0.170	0.176

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	42	24	44	134	49	0	42	33	33
N.S.	1	1.14	0.65	1.19	3.62	1.32	0.00	1.14	0.89	0.89
time (sec)	N/A	0.489	0.046	0.722	0.165	0.099	0.000	0.556	0.209	7.283

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	23	27	54	134	53	0	62	37	0
N.S.	1	0.70	0.82	1.64	4.06	1.61	0.00	1.88	1.12	0.00
time (sec)	N/A	0.578	0.018	0.729	0.167	0.080	0.000	0.505	0.183	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	43	60	556	72	0	533	40	0
N.S.	1	1.03	0.63	0.88	8.18	1.06	0.00	7.84	0.59	0.00
time (sec)	N/A	0.420	0.049	0.898	0.164	0.091	0.000	0.960	0.187	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	59	90	1769	91	0	1153	66	0
N.S.	1	1.04	0.60	0.92	18.05	0.93	0.00	11.77	0.67	0.00
time (sec)	N/A	0.461	0.158	0.754	0.308	0.082	0.000	1.555	0.179	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	36	23	26	37	17	0	27	27	22
N.S.	1	1.44	0.92	1.04	1.48	0.68	0.00	1.08	1.08	0.88
time (sec)	N/A	0.512	0.057	0.654	0.049	0.075	0.000	0.513	0.167	0.283

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	21	22	38	42	64	0	40	28	0
N.S.	1	0.68	0.71	1.23	1.35	2.06	0.00	1.29	0.90	0.00
time (sec)	N/A	0.566	0.017	0.622	0.180	0.106	0.000	0.536	0.197	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	12	14	12	22	11
N.S.	1	1.00	1.00	1.08	0.00	1.00	1.17	1.00	1.83	0.92
time (sec)	N/A	0.423	0.019	0.638	0.000	0.074	0.229	0.419	0.175	7.599

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	45	36	25	42	66	0	34	26	31
N.S.	1	1.29	1.03	0.71	1.20	1.89	0.00	0.97	0.74	0.89
time (sec)	N/A	0.495	0.051	1.589	0.180	0.087	0.000	0.478	0.202	0.140

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	19	22	19	128	33	0	47	28	40
N.S.	1	0.61	0.71	0.61	4.13	1.06	0.00	1.52	0.90	1.29
time (sec)	N/A	0.582	0.019	1.575	0.156	0.082	0.000	0.473	0.163	7.547

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	38	23	29	38	43	39	26	36	29
N.S.	1	1.27	0.77	0.97	1.27	1.43	1.30	0.87	1.20	0.97
time (sec)	N/A	0.518	0.051	0.556	0.047	0.078	1.442	0.457	0.183	7.576

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	56	14	39	0	16	32	28
N.S.	1	1.00	0.78	2.43	0.61	1.70	0.00	0.70	1.39	1.22
time (sec)	N/A	0.559	0.012	0.554	0.160	0.099	0.000	0.492	0.163	7.546

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	35	15	12	28	23
N.S.	1	1.00	1.00	0.93	0.00	2.50	1.07	0.86	2.00	1.64
time (sec)	N/A	0.421	0.009	0.540	0.000	0.071	1.118	0.362	0.161	0.147

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	68	59	36	48	94	0	53	32	46
N.S.	1	1.28	1.11	0.68	0.91	1.77	0.00	1.00	0.60	0.87
time (sec)	N/A	0.525	0.065	1.724	0.163	0.095	0.000	0.511	0.196	7.446

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	32	31	32	225	52	0	55	34	0
N.S.	1	0.53	0.52	0.53	3.75	0.87	0.00	0.92	0.57	0.00
time (sec)	N/A	0.633	0.037	1.638	0.155	0.081	0.000	0.464	0.167	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	26	24	25	13	38	0	47	38	55
N.S.	1	1.08	1.00	1.04	0.54	1.58	0.00	1.96	1.58	2.29
time (sec)	N/A	0.389	0.052	0.781	0.149	0.076	0.000	0.625	0.177	7.772

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	64	40	57	26	70	0	144	61	35
N.S.	1	1.10	0.69	0.98	0.45	1.21	0.00	2.48	1.05	0.60
time (sec)	N/A	0.415	0.043	0.797	0.152	0.078	0.000	1.734	0.166	7.451

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	101	52	88	39	94	0	248	81	47
N.S.	1	1.15	0.59	1.00	0.44	1.07	0.00	2.82	0.92	0.53
time (sec)	N/A	0.435	0.068	0.816	0.153	0.080	0.000	2.552	0.164	0.199

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	138	62	119	50	118	0	384	101	161
N.S.	1	1.17	0.53	1.01	0.42	1.00	0.00	3.25	0.86	1.36
time (sec)	N/A	0.455	0.123	0.759	0.154	0.076	0.000	2.308	0.191	7.400

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	24	23	19	18	72	0	29	24	18
N.S.	1	1.09	1.05	0.86	0.82	3.27	0.00	1.32	1.09	0.82
time (sec)	N/A	0.343	0.014	0.701	0.110	0.077	0.000	0.560	0.167	0.103

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	14	4	3	60	0	16	9	3
N.S.	1	1.00	4.67	1.33	1.00	20.00	0.00	5.33	3.00	1.00
time (sec)	N/A	0.326	0.004	0.754	0.107	0.077	0.000	0.475	0.177	0.057

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	13	11	12	11	11	12	11	18	9
N.S.	1	1.18	1.00	1.09	1.00	1.00	1.09	1.00	1.64	0.82
time (sec)	N/A	0.339	0.017	0.410	0.026	0.075	0.196	0.505	0.181	0.028

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	39	23	32	20	73	0	29	27	31
N.S.	1	1.11	0.66	0.91	0.57	2.09	0.00	0.83	0.77	0.89
time (sec)	N/A	0.383	0.013	0.664	0.114	0.076	0.000	0.497	0.180	0.105

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	18	16	17	17	19	0	16	11	16
N.S.	1	1.12	1.00	1.06	1.06	1.19	0.00	1.00	0.69	1.00
time (sec)	N/A	0.362	0.004	0.514	0.150	0.093	0.000	0.487	0.162	7.303

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	15	13	14	0	12	0	12	20	13
N.S.	1	1.15	1.00	1.08	0.00	0.92	0.00	0.92	1.54	1.00
time (sec)	N/A	0.362	0.016	0.507	0.000	0.097	0.000	0.496	0.166	7.763

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	109	157	0	346	0	0	209	157
N.S.	1	0.97	0.93	1.34	0.00	2.96	0.00	0.00	1.79	1.34
time (sec)	N/A	0.555	0.920	0.909	0.000	0.127	0.000	0.000	0.225	16.347

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	87	82	114	0	280	0	0	135	76
N.S.	1	0.99	0.93	1.30	0.00	3.18	0.00	0.00	1.53	0.86
time (sec)	N/A	0.474	0.237	0.846	0.000	0.183	0.000	0.000	0.195	10.188

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	64	59	91	0	240	0	0	109	54
N.S.	1	1.03	0.95	1.47	0.00	3.87	0.00	0.00	1.76	0.87
time (sec)	N/A	0.410	0.035	0.805	0.000	0.149	0.000	0.000	0.221	8.360

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	72	569	0	362	0	0	22	83
N.S.	1	1.03	0.97	7.69	0.00	4.89	0.00	0.00	0.30	1.12
time (sec)	N/A	0.468	0.043	6.562	0.000	0.101	0.000	0.000	0.171	0.286

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	115	112	115	934	0	571	0	0	24	238
N.S.	1	0.97	1.00	8.12	0.00	4.97	0.00	0.00	0.21	2.07
time (sec)	N/A	0.529	0.255	6.857	0.000	0.114	0.000	0.000	0.228	7.473

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	163	166	138	1354	0	708	0	0	25	542
N.S.	1	1.02	0.85	8.31	0.00	4.34	0.00	0.00	0.15	3.33
time (sec)	N/A	0.625	0.888	7.267	0.000	0.121	0.000	0.000	200.032	0.416

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	232	420	429	0	810	0	0	24	0
N.S.	1	1.05	1.89	1.93	0.00	3.65	0.00	0.00	0.11	0.00
time (sec)	N/A	0.808	5.789	0.842	0.000	1.003	0.000	0.000	0.267	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	173	767	307	0	655	0	0	24	0
N.S.	1	1.02	4.54	1.82	0.00	3.88	0.00	0.00	0.14	0.00
time (sec)	N/A	0.653	6.163	0.898	0.000	0.511	0.000	0.000	0.245	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	120	251	223	0	523	0	0	24	0
N.S.	1	0.98	2.04	1.81	0.00	4.25	0.00	0.00	0.20	0.00
time (sec)	N/A	0.532	3.831	0.815	0.000	0.235	0.000	0.000	0.221	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	108	169	0	394	0	0	15	0
N.S.	1	0.98	1.27	1.99	0.00	4.64	0.00	0.00	0.18	0.00
time (sec)	N/A	0.404	0.193	0.821	0.000	0.130	0.000	0.000	0.227	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	73	64	263	0	257	0	0	24	0
N.S.	1	0.97	0.85	3.51	0.00	3.43	0.00	0.00	0.32	0.00
time (sec)	N/A	0.466	0.211	22.101	0.000	0.136	0.000	0.000	0.238	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	115	241	248	0	311	0	0	25	0
N.S.	1	0.98	2.06	2.12	0.00	2.66	0.00	0.00	0.21	0.00
time (sec)	N/A	0.558	4.307	24.319	0.000	0.156	0.000	0.000	200.020	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	171	321	324	0	375	0	0	25	0
N.S.	1	1.02	1.92	1.94	0.00	2.25	0.00	0.00	0.15	0.00
time (sec)	N/A	0.669	8.435	24.995	0.000	0.143	0.000	0.000	200.018	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	138	139	256	0	440	0	0	360	233
N.S.	1	0.95	0.96	1.77	0.00	3.03	0.00	0.00	2.48	1.61
time (sec)	N/A	0.578	0.894	0.683	0.000	0.190	0.000	0.000	0.246	36.156

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	112	204	0	360	0	0	255	156
N.S.	1	0.99	0.97	1.76	0.00	3.10	0.00	0.00	2.20	1.34
time (sec)	N/A	0.509	0.598	0.654	0.000	0.137	0.000	0.000	0.264	17.676

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	91	80	181	0	281	0	0	203	91
N.S.	1	1.01	0.89	2.01	0.00	3.12	0.00	0.00	2.26	1.01
time (sec)	N/A	0.441	0.269	0.614	0.000	0.129	0.000	0.000	0.263	11.079

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	817	0	647	0	0	57	546
N.S.	1	1.00	0.95	8.60	0.00	6.81	0.00	0.00	0.60	5.75
time (sec)	N/A	0.514	0.171	13.686	0.000	0.515	0.000	0.000	0.292	8.324

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	113	109	1080	0	563	0	0	25	447
N.S.	1	0.97	0.94	9.31	0.00	4.85	0.00	0.00	0.22	3.85
time (sec)	N/A	0.573	0.227	6.668	0.000	0.125	0.000	0.000	200.027	0.327

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	160	140	1506	0	727	0	0	25	578
N.S.	1	0.99	0.87	9.35	0.00	4.52	0.00	0.00	0.16	3.59
time (sec)	N/A	0.652	0.945	6.602	0.000	0.141	0.000	0.000	200.019	7.610

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	306	908	669	0	1043	0	0	53	0
N.S.	1	1.04	3.09	2.28	0.00	3.55	0.00	0.00	0.18	0.00
time (sec)	N/A	1.044	6.337	0.756	0.000	2.515	0.000	0.000	0.203	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	227	442	510	0	845	0	0	53	0
N.S.	1	1.01	1.97	2.28	0.00	3.77	0.00	0.00	0.24	0.00
time (sec)	N/A	0.861	4.851	0.691	0.000	1.500	0.000	0.000	0.242	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	171	771	386	0	692	0	0	53	0
N.S.	1	0.99	4.48	2.24	0.00	4.02	0.00	0.00	0.31	0.00
time (sec)	N/A	0.687	6.201	0.694	0.000	0.541	0.000	0.000	0.202	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	123	142	297	0	521	0	0	44	0
N.S.	1	0.98	1.14	2.38	0.00	4.17	0.00	0.00	0.35	0.00
time (sec)	N/A	0.476	0.385	0.680	0.000	0.326	0.000	0.000	0.192	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	109	256	439	0	710	0	0	25	0
N.S.	1	0.96	2.25	3.85	0.00	6.23	0.00	0.00	0.22	0.00
time (sec)	N/A	0.548	4.132	23.592	0.000	0.591	0.000	0.000	200.023	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	113	78	436	0	308	0	0	25	0
N.S.	1	0.98	0.68	3.79	0.00	2.68	0.00	0.00	0.22	0.00
time (sec)	N/A	0.573	0.248	23.003	0.000	0.147	0.000	0.000	200.017	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	534	0	385	0	0	25	0
N.S.	1	1.00	0.85	3.24	0.00	2.33	0.00	0.00	0.15	0.00
time (sec)	N/A	0.707	4.253	23.850	0.000	0.157	0.000	0.000	200.012	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	169	461	0	687	0	0	76	0
N.S.	1	1.00	0.99	2.71	0.00	4.04	0.00	0.00	0.45	0.00
time (sec)	N/A	0.583	0.681	1.059	0.000	1.089	0.000	0.000	0.201	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	93	87	103	0	340	0	0	92	97
N.S.	1	0.98	0.92	1.08	0.00	3.58	0.00	0.00	0.97	1.02
time (sec)	N/A	0.534	1.580	0.832	0.000	0.148	0.000	0.000	0.176	9.043

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	62	56	0	274	0	0	38	56
N.S.	1	1.05	0.97	0.88	0.00	4.28	0.00	0.00	0.59	0.88
time (sec)	N/A	0.471	0.194	0.823	0.000	0.130	0.000	0.000	0.170	8.719

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	0	211	0	0	36	35
N.S.	1	1.00	1.00	0.85	0.00	5.15	0.00	0.00	0.88	0.85
time (sec)	N/A	0.403	0.023	1.048	0.000	0.118	0.000	0.000	0.192	8.718

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	72	444	0	426	0	0	36	232
N.S.	1	1.03	0.97	6.00	0.00	5.76	0.00	0.00	0.49	3.14
time (sec)	N/A	0.478	0.062	6.280	0.000	0.101	0.000	0.000	0.177	8.583

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	117	135	875	0	676	0	0	25	830
N.S.	1	1.01	1.16	7.54	0.00	5.83	0.00	0.00	0.22	7.16
time (sec)	N/A	0.548	0.520	6.560	0.000	0.116	0.000	0.000	200.018	8.097

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F(-2)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	166	175	162	1235	0	836	0	0	25	1215
N.S.	1	1.05	0.98	7.44	0.00	5.04	0.00	0.00	0.15	7.32
time (sec)	N/A	0.648	1.268	6.628	0.000	0.141	0.000	0.000	200.023	8.327

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	185	768	248	0	801	0	0	38	0
N.S.	1	1.05	4.34	1.40	0.00	4.53	0.00	0.00	0.21	0.00
time (sec)	N/A	0.649	6.237	0.911	0.000	0.990	0.000	0.000	0.152	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	127	271	157	0	631	0	0	38	0
N.S.	1	1.02	2.17	1.26	0.00	5.05	0.00	0.00	0.30	0.00
time (sec)	N/A	0.526	4.364	0.872	0.000	0.511	0.000	0.000	0.153	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	84	149	100	0	463	0	0	38	0
N.S.	1	0.98	1.73	1.16	0.00	5.38	0.00	0.00	0.44	0.00
time (sec)	N/A	0.463	0.530	0.892	0.000	0.160	0.000	0.000	0.164	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	67	0	122	0	0	30	40
N.S.	1	1.00	1.00	1.46	0.00	2.65	0.00	0.00	0.65	0.87
time (sec)	N/A	0.330	0.027	1.033	0.000	0.097	0.000	0.000	0.153	8.270

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	179	166	0	289	0	0	38	0
N.S.	1	0.97	2.29	2.13	0.00	3.71	0.00	0.00	0.49	0.00
time (sec)	N/A	0.454	3.783	23.519	0.000	0.144	0.000	0.000	12.544	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	123	262	225	0	359	0	0	25	0
N.S.	1	1.02	2.18	1.88	0.00	2.99	0.00	0.00	0.21	0.00
time (sec)	N/A	0.542	5.806	25.162	0.000	0.153	0.000	0.000	200.017	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	179	1214	773	0	437	0	0	25	0
N.S.	1	1.05	7.14	4.55	0.00	2.57	0.00	0.00	0.15	0.00
time (sec)	N/A	0.646	11.650	13.852	0.000	0.153	0.000	0.000	200.016	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	84	130	0	458	0	0	199	112
N.S.	1	1.00	0.86	1.33	0.00	4.67	0.00	0.00	2.03	1.14
time (sec)	N/A	0.559	0.264	0.743	0.000	0.144	0.000	0.000	0.187	9.782

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	77	87	0	384	0	0	160	90
N.S.	1	1.03	1.05	1.19	0.00	5.26	0.00	0.00	2.19	1.23
time (sec)	N/A	0.475	0.329	0.689	0.000	0.136	0.000	0.000	0.193	9.098

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	56	66	0	358	110	0	52	85
N.S.	1	1.03	0.81	0.96	0.00	5.19	1.59	0.00	0.75	1.23
time (sec)	N/A	0.420	0.059	0.664	0.000	0.137	10.237	0.000	0.164	9.168

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	106	123	91	7772	0	900	0	0	52	1922
N.S.	1	1.16	0.86	73.32	0.00	8.49	0.00	0.00	0.49	18.13
time (sec)	N/A	0.530	0.122	7.260	0.000	0.147	0.000	0.000	0.188	8.364

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	172	115	9634	0	1229	0	0	25	2483
N.S.	1	1.10	0.73	61.36	0.00	7.83	0.00	0.00	0.16	15.82
time (sec)	N/A	0.641	0.291	7.729	0.000	0.153	0.000	0.000	200.024	8.475

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F(-1)	A	F	F(-1)	F	B
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	215	0	142	11429	0	1501	0	0	25	2118
N.S.	1	0.00	0.66	53.16	0.00	6.98	0.00	0.00	0.12	9.85
time (sec)	N/A	0.000	0.794	8.267	0.000	0.141	0.000	0.000	200.019	9.204

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	192	327	270	0	1191	0	0	54	0
N.S.	1	1.05	1.80	1.48	0.00	6.54	0.00	0.00	0.30	0.00
time (sec)	N/A	0.623	5.770	0.721	0.000	1.125	0.000	0.000	0.192	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	136	250	182	0	958	0	0	54	0
N.S.	1	1.11	2.03	1.48	0.00	7.79	0.00	0.00	0.44	0.00
time (sec)	N/A	0.319	2.026	0.717	0.000	0.600	0.000	0.000	0.208	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	79	154	126	0	282	0	0	153	0
N.S.	1	0.98	1.90	1.56	0.00	3.48	0.00	0.00	1.89	0.00
time (sec)	N/A	0.277	2.196	0.750	0.000	0.097	0.000	0.000	0.200	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	232	102	0	307	0	0	46	0
N.S.	1	0.98	2.73	1.20	0.00	3.61	0.00	0.00	0.54	0.00
time (sec)	N/A	0.225	3.549	0.654	0.000	0.126	0.000	0.000	0.170	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	128	129	882	499	0	471	0	0	54	0
N.S.	1	1.01	6.89	3.90	0.00	3.68	0.00	0.00	0.42	0.00
time (sec)	N/A	0.340	9.649	25.713	0.000	0.184	0.000	0.000	1.702	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	1398	1080	0	579	0	0	25	0
N.S.	1	1.00	7.60	5.87	0.00	3.15	0.00	0.00	0.14	0.00
time (sec)	N/A	0.392	11.790	14.812	0.000	0.168	0.000	0.000	200.019	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	249	1994	1474	0	687	0	0	25	0
N.S.	1	0.99	7.91	5.85	0.00	2.73	0.00	0.00	0.10	0.00
time (sec)	N/A	0.465	15.658	16.821	0.000	0.187	0.000	0.000	200.016	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	113	91	155	0	634	0	0	336	148
N.S.	1	0.98	0.79	1.35	0.00	5.51	0.00	0.00	2.92	1.29
time (sec)	N/A	0.344	0.284	0.741	0.000	0.155	0.000	0.000	0.161	11.550

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	84	110	0	598	0	0	294	138
N.S.	1	1.09	0.82	1.07	0.00	5.81	0.00	0.00	2.85	1.34
time (sec)	N/A	0.303	0.226	0.770	0.000	0.160	0.000	0.000	0.205	11.359

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	107	58	89	0	570	138	0	68	131
N.S.	1	1.08	0.59	0.90	0.00	5.76	1.39	0.00	0.69	1.32
time (sec)	N/A	0.269	0.093	0.699	0.000	0.164	11.342	0.000	0.170	11.507

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	147	177	94	39148	0	1627	0	0	68	2788
N.S.	1	1.20	0.64	266.31	0.00	11.07	0.00	0.00	0.46	18.97
time (sec)	N/A	0.357	0.267	15.177	0.000	0.141	0.000	0.000	0.175	8.021

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	206	232	138	56970	0	2061	0	0	25	3429
N.S.	1	1.13	0.67	276.55	0.00	10.00	0.00	0.00	0.12	16.65
time (sec)	N/A	0.435	0.426	22.032	0.000	0.216	0.000	0.000	200.025	9.080

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	272	304	165	76697	0	2411	0	0	25	4652
N.S.	1	1.12	0.61	281.97	0.00	8.86	0.00	0.00	0.09	17.10
time (sec)	N/A	0.514	1.341	38.757	0.000	0.213	0.000	0.000	200.028	9.735

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	197	295	382	0	1698	0	0	70	0
N.S.	1	1.15	1.73	2.23	0.00	9.93	0.00	0.00	0.41	0.00
time (sec)	N/A	0.440	3.082	0.563	0.000	1.336	0.000	0.000	0.196	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	137	260	291	0	495	0	0	307	0
N.S.	1	1.05	1.98	2.22	0.00	3.78	0.00	0.00	2.34	0.00
time (sec)	N/A	0.349	6.031	0.204	0.000	0.117	0.000	0.000	0.202	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	133	305	212	0	526	0	0	70	0
N.S.	1	1.04	2.38	1.66	0.00	4.11	0.00	0.00	0.55	0.00
time (sec)	N/A	0.350	7.074	0.197	0.000	0.122	0.000	0.000	0.202	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	144	1331	163	0	558	0	0	62	0
N.S.	1	1.07	9.93	1.22	0.00	4.16	0.00	0.00	0.46	0.00
time (sec)	N/A	0.283	6.009	0.136	0.000	0.128	0.000	0.000	0.205	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-1)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	197	1890	0	0	753	0	0	70	0
N.S.	1	1.06	10.16	0.00	0.00	4.05	0.00	0.00	0.38	0.00
time (sec)	N/A	0.439	12.034	0.000	0.000	0.199	0.000	0.000	0.854	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-1)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	249	258	871	0	0	879	0	0	25	0
N.S.	1	1.04	3.50	0.00	0.00	3.53	0.00	0.00	0.10	0.00
time (sec)	N/A	0.511	17.381	0.000	0.000	0.204	0.000	0.000	200.022	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-1)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	335	441	0	0	1023	0	0	25	0
N.S.	1	1.02	1.35	0.00	0.00	3.13	0.00	0.00	0.08	0.00
time (sec)	N/A	0.597	15.246	0.000	0.000	0.371	0.000	0.000	200.023	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	70	0	0	0	0	0	21	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.362	0.107	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	100	101	0	0	0	0	0	29	0
N.S.	1	0.99	1.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.287	0.248	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	120	106	0	0	0	0	0	507	0
N.S.	1	0.93	0.82	0.00	0.00	0.00	0.00	0.00	3.93	0.00
time (sec)	N/A	0.337	0.533	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	90	73	0	0	0	0	0	245	0
N.S.	1	0.95	0.77	0.00	0.00	0.00	0.00	0.00	2.58	0.00
time (sec)	N/A	0.272	0.095	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	119	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.89	0.00
time (sec)	N/A	0.235	0.061	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	114	98	0	0	0	0	0	23	0
N.S.	1	0.96	0.82	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.290	0.132	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	154	142	0	0	0	0	0	25	0
N.S.	1	0.97	0.89	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.312	0.456	0.000	0.000	0.000	0.000	0.000	200.020	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	221	172	0	0	0	0	0	25	0
N.S.	1	1.01	0.79	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.375	1.764	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	0	0	0	0	0	0	25	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.263	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	1896	0	0	0	0	0	25	0
N.S.	1	0.99	22.57	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.269	13.212	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	1992	0	0	0	0	0	25	0
N.S.	1	0.99	23.71	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.261	13.444	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	192	0	0	0	0	0	16	0
N.S.	1	0.99	2.43	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.220	0.288	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	1989	0	0	0	0	0	25	0
N.S.	1	0.99	24.86	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.272	13.540	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	1887	0	0	0	0	0	25	0
N.S.	1	0.99	22.46	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.263	6.419	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	0	0	0	0	0	0	25	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.265	0.000	0.000	0.000	0.000	0.000	0.000	200.018	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	236	224	227	260	225	301	293	256	310
N.S.	1	0.93	0.88	0.89	1.02	0.88	1.18	1.15	1.00	1.22
time (sec)	N/A	0.396	0.615	1.020	0.117	0.101	0.421	0.369	0.182	7.559

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	168	161	160	146	183	148	194	184	160	174
N.S.	1	0.96	0.95	0.87	1.09	0.88	1.15	1.10	0.95	1.04
time (sec)	N/A	0.333	0.305	0.235	0.116	0.094	0.251	0.331	0.186	7.498

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	88	107	82	83	85	94	104	84	117
N.S.	1	0.99	1.20	0.92	0.93	0.96	1.06	1.17	0.94	1.31
time (sec)	N/A	0.275	0.326	0.135	0.109	0.086	0.139	0.237	0.187	7.358

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	33	36	36	37	36	35	34
N.S.	1	1.00	0.94	1.03	1.12	1.12	1.16	1.12	1.09	1.06
time (sec)	N/A	0.168	0.035	0.056	0.033	0.085	0.089	0.174	0.167	7.722

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	270	278	265	291	4817	0	335	298	342
N.S.	1	1.05	1.09	1.04	1.14	18.82	0.00	1.31	1.16	1.34
time (sec)	N/A	0.577	0.424	0.572	0.110	0.822	0.000	0.224	0.180	8.422

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	560	411	502	11554	0	584	1262	988
N.S.	1	1.00	1.00	0.74	0.90	20.71	0.00	1.05	2.26	1.77
time (sec)	N/A	0.882	3.835	0.778	0.115	1.142	0.000	0.256	0.210	8.251

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	43	57	34	33	48	34	34	33	41
N.S.	1	1.16	1.54	0.92	0.89	1.30	0.92	0.92	0.89	1.11
time (sec)	N/A	0.295	0.019	0.385	0.105	0.085	0.077	0.181	0.172	7.629

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	202	196	231	265	225	386	375	318	271
N.S.	1	0.94	0.91	1.07	1.23	1.04	1.79	1.74	1.47	1.25
time (sec)	N/A	0.364	3.666	0.510	0.111	0.096	0.774	0.374	0.169	8.397

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	136	128	149	167	145	224	227	192	180
N.S.	1	0.94	0.89	1.03	1.16	1.01	1.56	1.58	1.33	1.25
time (sec)	N/A	0.305	0.660	0.284	0.105	0.082	0.440	0.331	0.164	7.652

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	80	75	83	91	81	116	117	98	109
N.S.	1	0.98	0.91	1.01	1.11	0.99	1.41	1.43	1.20	1.33
time (sec)	N/A	0.254	0.367	0.150	0.103	0.103	0.326	0.350	0.195	7.846

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	32	34	32	32	46	35	31
N.S.	1	1.00	1.26	0.91	0.97	0.91	0.91	1.31	1.00	0.89
time (sec)	N/A	0.173	0.013	0.051	0.102	0.082	0.092	0.184	0.185	8.319

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	300	228	157	261	1541	0	335	347	4038
N.S.	1	1.35	1.03	0.71	1.18	6.94	0.00	1.51	1.56	18.19
time (sec)	N/A	0.466	0.378	0.294	0.114	0.132	0.000	0.246	0.194	12.260

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	631	609	356	394	4291	0	502	1586	11516
N.S.	1	2.08	2.00	1.17	1.30	14.12	0.00	1.65	5.22	37.88
time (sec)	N/A	0.704	6.193	0.764	0.128	0.356	0.000	0.284	0.290	11.861

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	549	219	531	0	0	0	0	15	0
N.S.	1	1.06	0.42	1.03	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.715	9.667	1.796	0.000	0.000	0.000	0.000	0.196	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	358	106	123	0	0	0	0	30	0
N.S.	1	1.03	0.30	0.35	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.464	10.487	0.872	0.000	0.000	0.000	0.000	0.218	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	107	145	181	0	555	0	18	111	0
N.S.	1	0.93	1.26	1.57	0.00	4.83	0.00	0.16	0.97	0.00
time (sec)	N/A	0.350	2.422	1.211	0.000	0.274	0.000	0.192	0.226	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	86	86	139	0	475	0	89	14	0
N.S.	1	0.96	0.96	1.54	0.00	5.28	0.00	0.99	0.16	0.00
time (sec)	N/A	0.314	0.030	0.317	0.000	0.199	0.000	0.214	0.278	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	98	98	0	0	1073	0	0	14	0
N.S.	1	0.96	0.96	0.00	0.00	10.52	0.00	0.00	0.14	0.00
time (sec)	N/A	0.383	0.050	0.000	0.000	0.301	0.000	0.000	0.618	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	490	404	537	0	0	0	0	16	0
N.S.	1	1.07	0.88	1.17	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.907	11.155	0.734	0.000	0.000	0.000	0.000	0.277	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	157	189	307	0	758	0	70	237	0
N.S.	1	0.92	1.11	1.80	0.00	4.43	0.00	0.41	1.39	0.00
time (sec)	N/A	0.432	4.002	0.203	0.000	0.368	0.000	0.221	0.405	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	131	166	245	0	593	0	138	174	0
N.S.	1	0.96	1.21	1.79	0.00	4.33	0.00	1.01	1.27	0.00
time (sec)	N/A	0.382	3.033	0.111	0.000	0.299	0.000	0.241	0.394	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	160	190	0	0	1321	0	0	37	0
N.S.	1	1.09	1.29	0.00	0.00	8.99	0.00	0.00	0.25	0.00
time (sec)	N/A	0.455	2.034	0.000	0.000	21.434	0.000	0.000	68.952	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	222	248	0	0	1404	0	173	17	0
N.S.	1	1.59	1.77	0.00	0.00	10.03	0.00	1.24	0.12	0.00
time (sec)	N/A	0.523	2.115	0.000	0.000	20.398	0.000	0.218	200.021	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	286	317	0	0	1464	0	170	17	0
N.S.	1	1.89	2.10	0.00	0.00	9.70	0.00	1.13	0.11	0.00
time (sec)	N/A	0.609	5.642	0.000	0.000	21.452	0.000	0.167	200.016	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	74	91	0	483	0	0	26	0
N.S.	1	0.97	1.00	1.23	0.00	6.53	0.00	0.00	0.35	0.00
time (sec)	N/A	0.313	0.044	0.305	0.000	0.213	0.000	0.000	0.211	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	65	0	150	0	46	24	0
N.S.	1	1.00	1.00	1.59	0.00	3.66	0.00	1.12	0.59	0.00
time (sec)	N/A	0.251	0.013	0.689	0.000	0.177	0.000	0.186	0.205	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	69	70	0	0	475	0	0	24	0
N.S.	1	0.99	1.00	0.00	0.00	6.79	0.00	0.00	0.34	0.00
time (sec)	N/A	0.332	0.037	0.000	0.000	0.204	0.000	0.000	0.250	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	307	122	179	0	0	0	0	26	0
N.S.	1	1.05	0.42	0.62	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.502	1.084	0.954	0.000	0.000	0.000	0.000	0.213	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	67	267	0	292	0	103	116	0
N.S.	1	0.99	0.94	3.76	0.00	4.11	0.00	1.45	1.63	0.00
time (sec)	N/A	0.310	0.214	29.779	0.000	0.239	0.000	0.186	0.276	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	73	73	248	0	319	0	119	36	0
N.S.	1	0.99	0.99	3.35	0.00	4.31	0.00	1.61	0.49	0.00
time (sec)	N/A	0.272	0.185	0.152	0.000	0.303	0.000	0.200	0.263	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	115	108	0	0	954	0	0	36	0
N.S.	1	1.12	1.05	0.00	0.00	9.26	0.00	0.00	0.35	0.00
time (sec)	N/A	0.389	0.395	0.000	0.000	0.326	0.000	0.000	75.849	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	121	104	638	0	556	0	597	214	0
N.S.	1	1.08	0.93	5.70	0.00	4.96	0.00	5.33	1.91	0.00
time (sec)	N/A	0.365	0.537	0.431	0.000	0.291	0.000	0.213	0.422	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	130	113	586	0	599	0	618	48	0
N.S.	1	1.11	0.97	5.01	0.00	5.12	0.00	5.28	0.41	0.00
time (sec)	N/A	0.337	0.513	0.145	0.000	0.282	0.000	0.214	0.234	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	179	149	0	0	1749	0	0	48	0
N.S.	1	1.20	1.00	0.00	0.00	11.74	0.00	0.00	0.32	0.00
time (sec)	N/A	0.458	1.096	0.000	0.000	0.490	0.000	0.000	1.957	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	239	151	0	0	0	0	0	87	0
N.S.	1	1.13	0.71	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.880	0.944	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	149	304	0	0	0	0	0	33	0
N.S.	1	1.23	2.51	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.595	0.408	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	460	482	263	0	0	0	0	0	638	0
N.S.	1	1.05	0.57	0.00	0.00	0.00	0.00	0.00	1.39	0.00
time (sec)	N/A	1.404	3.052	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	617	625	381	0	0	0	0	0	0	0
N.S.	1	1.01	0.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.522	3.891	0.000	0.000	0.000	0.000	0.000	0.703	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.399	0.098	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	30	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.370	0.092	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.285	0.018	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	30	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.380	0.076	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	0	0	0	0	0	30	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.383	0.090	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	0	0	0	0	0	30	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.377	0.102	0.000	0.000	0.000	0.000	0.000	51.652	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	128	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	2.03	0.00
time (sec)	N/A	0.373	0.079	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	55	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.347	0.080	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	28	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.351	0.035	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	0	0	0	0	0	30	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.377	0.076	0.000	0.000	0.000	0.000	0.000	0.414	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	32	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.19	1.07
time (sec)	N/A	0.240	2.967	1.610	6.616	0.089	36.503	1.233	0.280	9.171

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	0	0	28	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.437	0.101	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	107	265	0	0	0	0	0	29	0
N.S.	1	0.99	2.45	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.434	1.333	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	0	0	0	33	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.452	0.105	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	32	29
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.19	1.07
time (sec)	N/A	0.386	7.129	1.478	6.496	0.102	0.000	1.491	0.278	8.835

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	77	93	102	95	95	0	98	308	147
N.S.	1	1.10	1.33	1.46	1.36	1.36	0.00	1.40	4.40	2.10
time (sec)	N/A	0.238	0.028	3.023	0.035	0.115	0.000	0.281	0.206	10.827

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	48	48	67	62	76	0	64	177	79
N.S.	1	1.14	1.14	1.60	1.48	1.81	0.00	1.52	4.21	1.88
time (sec)	N/A	0.215	0.018	0.391	0.036	0.112	0.000	0.279	0.210	8.594

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	47	39	46	44	0	48	51	32
N.S.	1	0.93	1.68	1.39	1.64	1.57	0.00	1.71	1.82	1.14
time (sec)	N/A	0.213	0.012	0.598	0.034	0.150	0.000	0.257	0.211	7.876

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	30	44	36	29	30	0	36	36	47
N.S.	1	0.94	1.38	1.12	0.91	0.94	0.00	1.12	1.12	1.47
time (sec)	N/A	0.221	0.008	4.179	0.037	0.094	0.000	0.280	0.220	7.991

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	52	72	47	47	0	2147	59	71
N.S.	1	0.91	0.96	1.33	0.87	0.87	0.00	39.76	1.09	1.31
time (sec)	N/A	0.246	0.137	14.090	0.036	0.093	0.000	14.347	0.209	8.239

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	68	75	92	64	63	0	29589	81	95
N.S.	1	0.89	0.99	1.21	0.84	0.83	0.00	389.33	1.07	1.25
time (sec)	N/A	0.258	0.193	47.216	0.031	0.088	0.000	21.672	0.246	8.425

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	60	75	55	56	74	0	70	121	56
N.S.	1	0.88	1.10	0.81	0.82	1.09	0.00	1.03	1.78	0.82
time (sec)	N/A	0.242	0.314	11.333	0.028	0.084	0.000	0.264	0.267	7.723

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	53	38	39	56	0	48	89	40
N.S.	1	0.89	1.15	0.83	0.85	1.22	0.00	1.04	1.93	0.87
time (sec)	N/A	0.228	0.192	3.261	0.034	0.100	0.000	0.190	0.274	7.782

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	28	25	25	37	36	25	57	25
N.S.	1	0.93	1.00	0.89	0.89	1.32	1.29	0.89	2.04	0.89
time (sec)	N/A	0.206	0.017	0.668	0.035	0.084	0.592	0.186	0.216	7.941

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	46	32	40	39	30	0	49	43	32
N.S.	1	1.39	0.97	1.21	1.18	0.91	0.00	1.48	1.30	0.97
time (sec)	N/A	0.211	0.064	1.260	0.111	0.089	0.000	0.236	0.275	8.456

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	75	46	55	69	49	0	73	79	67
N.S.	1	1.23	0.75	0.90	1.13	0.80	0.00	1.20	1.30	1.10
time (sec)	N/A	0.231	0.096	7.133	0.109	0.091	0.000	0.228	0.219	7.852

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	102	74	100	97	66	0	96	114	93
N.S.	1	1.17	0.85	1.15	1.11	0.76	0.00	1.10	1.31	1.07
time (sec)	N/A	0.240	0.151	26.058	0.114	0.097	0.000	0.258	0.205	8.245

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	137	875	199	156	137	0	167	678	269
N.S.	1	1.21	7.74	1.76	1.38	1.21	0.00	1.48	6.00	2.38
time (sec)	N/A	0.321	9.193	4.169	0.037	0.110	0.000	0.408	0.214	11.915

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	110	347	146	119	116	0	120	470	177
N.S.	1	1.36	4.28	1.80	1.47	1.43	0.00	1.48	5.80	2.19
time (sec)	N/A	0.279	5.628	0.857	0.033	0.141	0.000	0.392	0.206	10.442

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	65	66	100	105	106	0	104	252	148
N.S.	1	1.05	1.06	1.61	1.69	1.71	0.00	1.68	4.06	2.39
time (sec)	N/A	0.275	0.251	1.758	0.034	0.100	0.000	0.369	0.271	10.304

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	51	70	76	72	79	0	96	100	136
N.S.	1	0.91	1.25	1.36	1.29	1.41	0.00	1.71	1.79	2.43
time (sec)	N/A	0.257	0.339	8.412	0.029	0.100	0.000	0.471	0.277	9.998

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	52	52	89	56	71	0	2946	80	119
N.S.	1	0.91	0.91	1.56	0.98	1.25	0.00	51.68	1.40	2.09
time (sec)	N/A	0.269	0.102	32.619	0.029	0.086	0.000	136.942	0.210	8.078

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	78	77	153	81	95	0	0	118	160
N.S.	1	0.91	0.90	1.78	0.94	1.10	0.00	0.00	1.37	1.86
time (sec)	N/A	0.294	0.238	111.356	0.033	0.087	0.000	0.000	0.241	8.238

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	103	116	183	104	117	0	0	156	188
N.S.	1	0.90	1.02	1.61	0.91	1.03	0.00	0.00	1.37	1.65
time (sec)	N/A	0.311	0.404	315.120	0.029	0.094	0.000	0.000	0.293	8.443

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	85	106	157	85	114	0	118	206	80
N.S.	1	0.89	1.10	1.64	0.89	1.19	0.00	1.23	2.15	0.83
time (sec)	N/A	0.272	0.926	28.787	0.029	0.100	0.000	0.437	0.285	8.148

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	66	83	111	66	94	0	80	158	60
N.S.	1	0.89	1.12	1.50	0.89	1.27	0.00	1.08	2.14	0.81
time (sec)	N/A	0.261	0.831	9.719	0.028	0.086	0.000	0.469	0.309	8.345

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	44	49	57	42	69	0	42	110	40
N.S.	1	0.90	1.00	1.16	0.86	1.41	0.00	0.86	2.24	0.82
time (sec)	N/A	0.242	0.102	3.594	0.033	0.090	0.000	0.427	0.223	8.356

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	65	55	111	66	69	0	85	154	91
N.S.	1	1.18	1.00	2.02	1.20	1.25	0.00	1.55	2.80	1.65
time (sec)	N/A	0.270	0.676	5.267	0.109	0.104	0.000	0.423	0.226	8.366

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	106	65	103	97	75	0	114	132	93
N.S.	1	1.32	0.81	1.29	1.21	0.94	0.00	1.42	1.65	1.16
time (sec)	N/A	0.273	0.925	16.838	0.110	0.098	0.000	0.486	0.298	8.304

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	134	87	166	131	98	0	150	188	126
N.S.	1	1.20	0.78	1.48	1.17	0.88	0.00	1.34	1.68	1.12
time (sec)	N/A	0.293	0.787	58.924	0.108	0.101	0.000	0.509	0.303	8.591

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	99	207	129	0	292	0	131	594	268
N.S.	1	1.10	2.30	1.43	0.00	3.24	0.00	1.46	6.60	2.98
time (sec)	N/A	0.315	0.908	14.689	0.000	0.158	0.000	0.414	0.230	9.093

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	53	75	0	169	0	88	131	67
N.S.	1	0.97	0.90	1.27	0.00	2.86	0.00	1.49	2.22	1.14
time (sec)	N/A	0.260	0.063	3.226	0.000	0.188	0.000	0.512	0.219	8.119

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	36	0	122	0	47	96	32
N.S.	1	1.00	1.00	0.90	0.00	3.05	0.00	1.18	2.40	0.80
time (sec)	N/A	0.221	0.029	0.626	0.000	0.103	0.000	0.394	0.223	7.974

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	60	61	0	182	0	73	134	61
N.S.	1	0.97	1.00	1.02	0.00	3.03	0.00	1.22	2.23	1.02
time (sec)	N/A	0.243	0.071	1.085	0.000	0.131	0.000	0.491	0.301	8.117

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	83	115	98	0	276	0	161	204	251
N.S.	1	0.94	1.31	1.11	0.00	3.14	0.00	1.83	2.32	2.85
time (sec)	N/A	0.306	0.360	5.217	0.000	0.128	0.000	0.416	0.245	11.163

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	118	148	165	0	395	0	319	297	1493
N.S.	1	0.94	1.17	1.31	0.00	3.13	0.00	2.53	2.36	11.85
time (sec)	N/A	0.337	1.167	23.645	0.000	0.141	0.000	0.469	0.241	10.806

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	100	103	123	110	425	0	153	1280	136
N.S.	1	0.93	0.95	1.14	1.02	3.94	0.00	1.42	11.85	1.26
time (sec)	N/A	0.304	3.843	87.083	0.108	0.121	0.000	0.383	0.250	8.312

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	72	74	74	69	339	0	89	614	90
N.S.	1	0.94	0.96	0.96	0.90	4.40	0.00	1.16	7.97	1.17
time (sec)	N/A	0.273	0.461	26.816	0.112	0.149	0.000	0.371	0.299	8.580

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	52	44	45	267	0	46	180	44
N.S.	1	0.96	1.00	0.85	0.87	5.13	0.00	0.88	3.46	0.85
time (sec)	N/A	0.241	0.304	6.928	0.109	0.117	0.000	0.626	0.292	7.834

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	23	205	0	23	68	24
N.S.	1	1.00	1.00	0.75	0.72	6.41	0.00	0.72	2.12	0.75
time (sec)	N/A	0.225	0.270	1.473	0.113	0.104	0.000	0.645	0.227	7.774

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	103	78	85	95	290	0	102	141	254
N.S.	1	1.24	0.94	1.02	1.14	3.49	0.00	1.23	1.70	3.06
time (sec)	N/A	0.286	0.440	2.331	0.113	0.125	0.000	0.572	0.200	9.646

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	161	113	130	185	401	0	176	256	3681
N.S.	1	1.25	0.88	1.01	1.43	3.11	0.00	1.36	1.98	28.53
time (sec)	N/A	0.351	0.593	11.260	0.111	0.136	0.000	0.481	0.291	12.075

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	177	254	175	0	635	0	245	1718	4304
N.S.	1	1.06	1.52	1.05	0.00	3.80	0.00	1.47	10.29	25.77
time (sec)	N/A	0.405	2.571	117.788	0.000	0.204	0.000	0.737	0.317	10.849

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	115	191	124	0	407	0	153	720	946
N.S.	1	1.06	1.75	1.14	0.00	3.73	0.00	1.40	6.61	8.68
time (sec)	N/A	0.315	0.550	38.135	0.000	0.185	0.000	0.736	0.256	10.290

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	75	80	0	266	0	91	382	187
N.S.	1	0.97	0.95	1.01	0.00	3.37	0.00	1.15	4.84	2.37
time (sec)	N/A	0.251	0.172	10.126	0.000	0.134	0.000	0.713	0.218	10.287

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	92	114	102	0	337	0	112	628	239
N.S.	1	0.98	1.21	1.09	0.00	3.59	0.00	1.19	6.68	2.54
time (sec)	N/A	0.250	0.154	1.932	0.000	0.117	0.000	0.553	0.219	8.878

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	109	140	118	0	451	0	152	730	269
N.S.	1	0.96	1.23	1.04	0.00	3.96	0.00	1.33	6.40	2.36
time (sec)	N/A	0.332	0.282	3.510	0.000	0.146	0.000	0.820	0.297	11.267

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	135	147	164	0	600	0	329	855	1690
N.S.	1	0.94	1.03	1.15	0.00	4.20	0.00	2.30	5.98	11.82
time (sec)	N/A	0.366	0.971	16.765	0.000	0.157	0.000	0.770	0.298	12.719

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	119	135	137	137	597	0	176	1562	167
N.S.	1	0.94	1.06	1.08	1.08	4.70	0.00	1.39	12.30	1.31
time (sec)	N/A	0.324	4.041	201.967	0.109	0.153	0.000	0.722	0.297	9.090

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	99	104	97	100	479	0	115	764	119
N.S.	1	0.95	1.00	0.93	0.96	4.61	0.00	1.11	7.35	1.14
time (sec)	N/A	0.308	0.829	67.250	0.112	0.183	0.000	0.628	0.258	8.159

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	83	69	69	367	0	77	376	65
N.S.	1	0.97	1.08	0.90	0.90	4.77	0.00	1.00	4.88	0.84
time (sec)	N/A	0.250	0.530	19.956	0.108	0.137	0.000	0.639	0.219	8.040

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	63	55	53	327	0	56	279	54
N.S.	1	0.97	0.95	0.83	0.80	4.95	0.00	0.85	4.23	0.82
time (sec)	N/A	0.240	0.517	4.648	0.114	0.116	0.000	0.742	0.253	8.030

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	171	116	128	209	614	0	206	754	3843
N.S.	1	1.16	0.78	0.86	1.41	4.15	0.00	1.39	5.09	25.97
time (sec)	N/A	0.360	1.121	7.883	0.117	0.164	0.000	0.627	0.319	12.437

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	249	148	173	355	801	0	270	975	5272
N.S.	1	1.17	0.70	0.82	1.67	3.78	0.00	1.27	4.60	24.87
time (sec)	N/A	0.451	1.783	33.467	0.126	0.160	0.000	0.914	0.300	12.705

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	0	0	0	0	0	28	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.351	0.122	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	110	2033	0	0	0	0	0	29	0
N.S.	1	1.01	18.65	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.292	15.240	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	89	0	0	0	0	0	33	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.363	0.104	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	99	108	122	60672	106	107	0	130	30	0
N.S.	1	1.09	1.23	612.85	1.07	1.08	0.00	1.31	0.30	0.00
time (sec)	N/A	0.401	1.437	1.101	0.071	0.139	0.000	9.158	0.229	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	82	87	29779	71	75	0	85	30	0
N.S.	1	1.26	1.34	458.14	1.09	1.15	0.00	1.31	0.46	0.00
time (sec)	N/A	0.364	1.378	1.110	0.060	0.095	0.000	5.112	0.220	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	36	35	49	0	35	30	31
N.S.	1	1.00	1.00	1.16	1.13	1.58	0.00	1.13	0.97	1.00
time (sec)	N/A	0.334	0.018	16.592	0.048	0.100	0.000	2.834	0.213	9.159

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.295	0.019	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	30	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.351	0.472	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0	30	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.346	0.078	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	0	0	0	0	0	28	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.319	0.051	0.000	0.000	0.000	0.000	0.000	0.295	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	482	0	0	0	0	0	28	0
N.S.	1	1.00	6.10	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.322	2.720	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	87	1552	0	0	0	0	0	30	0
N.S.	1	1.06	18.93	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.344	4.789	0.000	0.000	0.000	0.000	0.000	0.336	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	32	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.19	1.15
time (sec)	N/A	0.236	2.810	1.266	6.702	0.113	103.157	1.308	0.246	9.424

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	28	27
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.12	1.08
time (sec)	N/A	0.233	3.485	1.089	10.420	0.114	0.000	4.514	0.268	9.058

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	26	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.13	1.17
time (sec)	N/A	0.211	1.507	0.944	5.711	0.101	35.261	3.329	0.296	8.056

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	0	25	26	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.13	1.09
time (sec)	N/A	0.222	1.847	0.982	8.045	0.102	0.000	69.277	0.242	8.560

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	28	27
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.12	1.08
time (sec)	N/A	0.232	4.612	3.676	10.241	0.128	0.000	65.780	0.226	10.882

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	251	165	0	0	0	0	0	28	0
N.S.	1	1.02	0.67	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.431	0.927	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	167	122	0	0	0	0	0	28	0
N.S.	1	1.03	0.75	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.354	0.675	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	75	75	0	0	0	0	0	28	76
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	0.37	1.00
time (sec)	N/A	0.266	0.336	0.000	0.000	0.000	0.000	0.000	0.225	8.736

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	19	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.19	1.12
time (sec)	N/A	0.185	0.650	0.888	5.799	0.100	2.337	1.153	0.276	9.178

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	28	27
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.12	1.08
time (sec)	N/A	0.230	4.557	1.427	7.395	0.104	0.000	1.633	0.229	8.973

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	299	0	0	0	0	0	28	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.602	1.843	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	155	292	0	0	0	0	0	29	0
N.S.	1	1.21	2.28	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.462	3.237	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	319	0	0	0	0	0	33	0
N.S.	1	1.00	3.07	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.618	2.279	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	0	29	32	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	1.07	1.19	1.15
time (sec)	N/A	0.372	3.158	1.342	7.776	0.106	0.000	1.362	0.301	9.083

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [7] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	8	0.68	14	0.571
2	A	6	6	0.80	14	0.429
3	A	4	4	1.00	14	0.286
4	A	5	5	1.06	14	0.357
5	A	9	9	0.82	14	0.643
6	A	13	13	0.72	14	0.929
7	A	22	21	0.78	14	1.500
8	A	18	17	0.89	14	1.214
9	A	16	15	0.92	14	1.071
10	A	16	15	0.93	14	1.071
11	A	18	17	0.86	14	1.214
12	A	22	21	0.78	14	1.500
13	A	13	13	0.55	14	0.929
14	A	9	9	0.62	14	0.643
15	A	5	5	0.74	14	0.357
16	A	5	5	0.75	14	0.357
17	A	9	9	0.60	14	0.643
18	A	13	13	0.55	14	0.929
19	A	6	5	1.00	14	0.357
20	A	6	5	1.00	14	0.357
21	A	6	5	1.25	14	0.357

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	6	5	1.00	14	0.357
23	A	6	5	1.00	14	0.357
24	A	6	5	1.00	14	0.357
25	A	6	5	1.00	12	0.417
26	A	6	5	1.00	12	0.417
27	A	6	5	1.00	12	0.417
28	A	6	5	1.00	12	0.417
29	A	4	4	1.00	14	0.286
30	A	5	4	0.89	21	0.190
31	A	6	5	0.90	21	0.238
32	A	5	4	0.93	19	0.211
33	A	6	5	0.92	19	0.263
34	A	6	5	1.14	21	0.238
35	A	9	8	1.19	21	0.381
36	A	10	9	1.25	21	0.429
37	A	7	6	1.27	21	0.286
38	A	7	6	1.28	21	0.286
39	A	1	1	1.00	12	0.083
40	A	5	4	0.92	21	0.190
41	A	5	4	0.88	21	0.190
42	A	5	4	0.88	21	0.190
43	A	5	4	0.90	23	0.174
44	A	6	5	0.90	23	0.217
45	A	5	4	0.91	21	0.190
46	A	6	5	0.90	21	0.238
47	A	7	6	1.05	23	0.261
48	A	9	8	1.17	23	0.348
49	A	7	6	1.18	23	0.261
50	A	7	6	1.09	23	0.261
51	A	5	4	1.09	14	0.286
52	A	5	4	0.89	23	0.174
53	A	5	4	0.89	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	0.88	23	0.174
55	A	5	4	0.93	23	0.174
56	A	7	6	1.04	23	0.261
57	A	5	4	0.97	21	0.190
58	A	7	6	0.97	21	0.286
59	A	7	6	1.09	23	0.261
60	A	10	9	1.15	23	0.391
61	A	10	9	1.25	23	0.391
62	A	8	7	1.25	23	0.304
63	A	7	6	1.23	23	0.261
64	A	6	5	1.00	14	0.357
65	A	5	4	0.96	23	0.174
66	A	6	5	0.93	23	0.217
67	A	5	4	0.92	23	0.174
68	A	9	8	1.31	23	0.348
69	A	8	7	1.01	23	0.304
70	A	6	5	1.03	21	0.238
71	A	9	8	1.11	21	0.381
72	A	9	8	1.05	23	0.348
73	A	12	11	1.08	23	0.478
74	A	11	10	1.17	23	0.435
75	A	9	8	1.17	23	0.348
76	A	7	6	1.20	14	0.429
77	A	6	5	0.99	23	0.217
78	A	7	6	1.02	23	0.261
79	A	7	6	1.24	23	0.261
80	A	10	9	1.22	23	0.391
81	A	9	8	1.01	23	0.348
82	A	7	6	1.09	21	0.286
83	A	11	10	1.16	21	0.476
84	A	11	10	1.14	23	0.435
85	A	14	13	1.06	23	0.565

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	12	11	1.16	23	0.478
87	A	10	9	1.21	23	0.391
88	A	8	7	1.22	14	0.500
89	A	7	6	1.04	23	0.261
90	A	8	7	1.01	23	0.304
91	A	9	8	1.17	23	0.348
92	A	9	8	1.07	25	0.320
93	A	8	7	0.96	25	0.280
94	A	6	5	0.97	23	0.217
95	A	9	8	0.98	23	0.348
96	A	9	8	1.06	25	0.320
97	A	12	11	1.11	25	0.440
98	A	10	9	1.13	25	0.360
99	A	9	8	1.03	25	0.320
100	A	8	7	0.98	16	0.438
101	A	6	5	0.97	25	0.200
102	A	7	6	0.95	25	0.240
103	A	8	7	1.03	25	0.280
104	A	10	9	0.91	25	0.360
105	A	9	8	0.90	25	0.320
106	A	7	6	0.98	23	0.261
107	A	11	10	0.98	23	0.435
108	A	11	10	1.01	25	0.400
109	A	15	14	1.04	25	0.560
110	A	13	12	1.02	25	0.480
111	A	11	10	0.99	25	0.400
112	A	9	8	0.98	16	0.500
113	A	7	6	0.98	25	0.240
114	A	8	7	0.89	25	0.280
115	A	10	9	0.89	25	0.360
116	A	7	6	1.05	25	0.240
117	A	6	5	0.98	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	3	1.00	23	0.130
119	A	6	5	1.00	23	0.217
120	A	7	6	1.07	25	0.240
121	A	9	8	1.12	25	0.320
122	A	8	7	1.12	25	0.280
123	A	7	6	1.04	25	0.240
124	A	5	4	1.00	16	0.250
125	A	4	3	1.00	25	0.120
126	A	5	4	0.97	25	0.160
127	A	6	5	1.02	25	0.200
128	A	8	7	1.06	25	0.280
129	A	7	6	1.02	25	0.240
130	A	5	4	0.97	23	0.174
131	A	7	6	0.98	23	0.261
132	A	9	8	1.05	25	0.320
133	A	12	11	1.13	25	0.440
134	A	10	9	1.13	25	0.360
135	A	8	7	1.06	25	0.280
136	A	6	5	0.98	16	0.312
137	A	5	4	0.97	25	0.160
138	A	6	5	0.96	25	0.200
139	A	7	6	1.00	25	0.240
140	A	9	8	1.02	25	0.320
141	A	8	7	0.98	25	0.280
142	A	6	5	1.03	23	0.217
143	A	10	9	1.09	23	0.391
144	A	11	10	1.06	25	0.400
145	A	14	13	1.10	25	0.520
146	A	11	10	1.13	25	0.400
147	A	10	9	1.10	25	0.360
148	A	8	7	1.07	16	0.438
149	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	7	6	0.91	25	0.240
151	A	8	7	0.94	25	0.280
152	A	6	6	1.00	23	0.261
153	A	6	5	1.20	25	0.200
154	A	8	7	1.01	23	0.304
155	A	7	6	0.96	23	0.261
156	A	5	4	0.95	21	0.190
157	A	6	5	0.96	21	0.238
158	A	5	4	0.96	23	0.174
159	A	5	4	0.99	23	0.174
160	A	5	4	0.99	14	0.286
161	A	5	4	0.99	23	0.174
162	A	6	5	0.98	23	0.217
163	A	7	6	1.01	23	0.261
164	A	6	6	1.00	25	0.240
165	A	6	5	1.00	23	0.217
166	A	6	5	1.00	14	0.357
167	A	6	5	1.00	23	0.217
168	A	7	6	1.22	23	0.261
169	A	7	6	1.07	23	0.261
170	A	6	6	1.00	23	0.261
171	A	6	6	1.00	21	0.286
172	A	6	6	1.00	21	0.286
173	A	6	6	1.00	23	0.261
174	N/A	2	0	1.00	25	0.000
175	A	6	6	1.00	23	0.261
176	A	7	6	0.99	25	0.240
177	A	6	6	1.00	25	0.240
178	N/A	4	0	1.00	27	0.000
179	A	7	6	0.78	14	0.429
180	A	7	6	0.82	14	0.429
181	A	7	6	0.91	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	6	6	0.94	14	0.429
183	A	8	8	1.00	14	0.571
184	A	10	10	1.03	14	0.714
185	A	8	8	0.89	21	0.381
186	A	6	6	0.94	21	0.286
187	A	4	4	1.00	19	0.211
188	A	5	5	1.08	19	0.263
189	A	7	7	1.06	21	0.333
190	A	11	11	0.98	21	0.524
191	A	9	9	0.84	21	0.429
192	A	7	7	0.85	21	0.333
193	A	5	5	0.92	21	0.238
194	A	1	1	1.00	12	0.083
195	A	3	3	1.00	21	0.143
196	A	7	7	1.00	21	0.333
197	A	9	9	0.84	21	0.429
198	A	6	5	0.93	23	0.217
199	A	6	5	0.96	23	0.217
200	A	6	5	0.97	21	0.238
201	A	6	5	1.04	21	0.238
202	A	6	5	0.98	23	0.217
203	A	6	5	0.96	23	0.217
204	A	5	4	0.96	23	0.174
205	A	5	4	0.98	23	0.174
206	A	5	4	1.01	23	0.174
207	A	5	4	1.09	14	0.286
208	A	5	4	1.11	23	0.174
209	A	5	4	1.09	23	0.174
210	A	5	4	1.01	23	0.174
211	A	6	5	0.94	23	0.217
212	A	6	5	1.06	23	0.217
213	A	6	5	1.36	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	6	5	1.03	21	0.238
215	A	6	5	0.93	23	0.217
216	A	6	5	0.92	23	0.217
217	A	9	8	1.18	23	0.348
218	A	7	6	1.17	23	0.261
219	A	6	5	1.14	23	0.217
220	A	6	5	1.00	14	0.357
221	A	8	7	1.17	23	0.304
222	A	9	8	1.20	23	0.348
223	A	11	10	1.17	23	0.435
224	A	6	5	0.94	23	0.217
225	A	6	5	1.10	23	0.217
226	A	6	5	1.09	21	0.238
227	A	6	5	0.97	21	0.238
228	A	6	5	0.91	23	0.217
229	A	6	5	0.91	23	0.217
230	A	8	7	1.09	23	0.304
231	A	7	6	1.19	23	0.261
232	A	7	6	1.16	23	0.261
233	A	7	6	1.20	14	0.429
234	A	8	7	1.11	23	0.304
235	A	10	9	1.07	23	0.391
236	A	12	11	1.03	23	0.478
237	A	6	5	1.04	23	0.217
238	A	6	5	1.04	23	0.217
239	A	6	5	1.03	21	0.238
240	A	6	5	0.96	21	0.238
241	A	6	5	0.92	23	0.217
242	A	6	5	0.92	23	0.217
243	A	9	8	1.20	23	0.348
244	A	9	8	1.17	23	0.348
245	A	8	7	1.19	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	7	1.22	14	0.500
247	A	9	8	1.14	23	0.348
248	A	11	10	1.10	23	0.435
249	A	13	12	1.06	23	0.522
250	A	5	4	0.98	14	0.286
251	A	5	4	1.01	14	0.286
252	A	5	4	1.09	14	0.286
253	A	1	1	1.00	12	0.083
254	A	6	5	1.00	14	0.357
255	A	7	6	1.20	14	0.429
256	A	8	7	1.22	14	0.500
257	A	10	10	0.81	17	0.588
258	A	8	7	1.27	17	0.412
259	A	8	8	0.81	17	0.471
260	A	6	5	1.00	15	0.333
261	A	8	7	1.00	15	0.467
262	A	9	8	1.00	17	0.471
263	A	8	7	1.18	17	0.412
264	A	9	8	0.79	17	0.471
265	A	7	6	1.06	16	0.375
266	A	8	7	1.25	17	0.412
267	A	10	10	0.76	17	0.588
268	A	6	5	1.00	15	0.333
269	A	9	8	1.14	15	0.533
270	A	10	9	0.70	17	0.529
271	A	8	7	1.03	16	0.438
272	A	9	8	1.04	16	0.500
273	A	8	7	1.44	17	0.412
274	A	9	8	0.68	17	0.471
275	A	6	5	1.00	15	0.333
276	A	9	8	1.29	15	0.533
277	A	9	8	0.61	17	0.471

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	8	7	1.27	17	0.412
279	A	8	7	1.00	17	0.412
280	A	6	5	1.00	15	0.333
281	A	10	9	1.28	15	0.600
282	A	9	8	0.53	17	0.471
283	A	6	5	1.08	16	0.312
284	A	7	6	1.10	16	0.375
285	A	8	7	1.15	16	0.438
286	A	9	8	1.17	16	0.500
287	A	7	6	1.09	10	0.600
288	A	6	5	1.00	10	0.500
289	A	6	5	1.18	10	0.500
290	A	8	7	1.11	12	0.583
291	A	7	6	1.12	12	0.500
292	A	6	5	1.15	12	0.417
293	A	6	5	0.97	25	0.200
294	A	8	7	0.99	25	0.280
295	A	7	6	1.03	23	0.261
296	A	7	6	1.03	23	0.261
297	A	9	8	0.97	25	0.320
298	A	11	10	1.02	25	0.400
299	A	12	11	1.05	25	0.440
300	A	11	10	1.02	25	0.400
301	A	9	8	0.98	25	0.320
302	A	8	7	0.98	16	0.438
303	A	8	7	0.97	25	0.280
304	A	9	8	0.98	25	0.320
305	A	10	9	1.02	25	0.360
306	A	6	5	0.95	25	0.200
307	A	9	8	0.99	25	0.320
308	A	8	7	1.01	23	0.304
309	A	8	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	9	8	0.97	25	0.320
311	A	11	10	0.99	25	0.400
312	A	14	13	1.04	25	0.520
313	A	13	12	1.01	25	0.480
314	A	11	10	0.99	25	0.400
315	A	9	8	0.98	16	0.500
316	A	10	9	0.96	25	0.360
317	A	9	8	0.98	25	0.320
318	A	11	10	1.00	25	0.400
319	A	10	9	1.00	16	0.562
320	A	6	5	0.98	25	0.200
321	A	7	6	1.05	25	0.240
322	A	6	5	1.00	23	0.217
323	A	7	6	1.03	23	0.261
324	A	9	8	1.01	25	0.320
325	A	11	10	1.05	25	0.400
326	A	10	9	1.05	25	0.360
327	A	9	8	1.02	25	0.320
328	A	8	7	0.98	25	0.280
329	A	5	4	1.00	16	0.250
330	A	8	7	0.97	25	0.280
331	A	9	8	1.02	25	0.320
332	A	10	9	1.05	25	0.360
333	A	6	5	1.00	25	0.200
334	A	7	6	1.03	25	0.240
335	A	7	6	1.03	23	0.261
336	A	8	7	1.16	23	0.304
337	A	11	10	1.10	25	0.400
338	F	0	0	N/A	0.000	N/A
339	A	10	9	1.05	25	0.360
340	A	9	8	1.11	25	0.320
341	A	6	5	0.98	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	6	5	0.98	16	0.312
343	A	8	7	1.01	25	0.280
344	A	9	8	1.00	25	0.320
345	A	10	9	0.99	25	0.360
346	A	6	5	0.98	25	0.200
347	A	8	7	1.09	25	0.280
348	A	8	7	1.08	23	0.304
349	A	10	9	1.20	23	0.391
350	A	13	12	1.13	25	0.480
351	A	15	14	1.12	25	0.560
352	A	12	11	1.15	25	0.440
353	A	8	7	1.05	25	0.280
354	A	8	7	1.04	25	0.280
355	A	8	7	1.07	16	0.438
356	A	9	8	1.06	25	0.320
357	A	11	10	1.04	25	0.400
358	A	12	11	1.02	25	0.440
359	A	7	6	1.00	23	0.261
360	A	5	4	0.99	25	0.160
361	A	6	5	0.93	23	0.217
362	A	6	5	0.95	23	0.217
363	A	5	4	1.00	21	0.190
364	A	7	6	0.96	21	0.286
365	A	8	7	0.97	23	0.304
366	A	9	8	1.01	23	0.348
367	A	5	4	0.99	23	0.174
368	A	5	4	0.99	23	0.174
369	A	5	4	0.99	23	0.174
370	A	5	4	0.99	14	0.286
371	A	5	4	0.99	23	0.174
372	A	5	4	0.99	23	0.174
373	A	5	4	0.99	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	5	4	0.93	14	0.286
375	A	5	4	0.96	14	0.286
376	A	5	4	0.99	14	0.286
377	A	1	1	1.00	12	0.083
378	A	5	4	1.05	14	0.286
379	A	5	4	1.00	14	0.286
380	A	5	4	1.16	8	0.500
381	A	5	4	0.94	14	0.286
382	A	5	4	0.94	14	0.286
383	A	5	4	0.98	14	0.286
384	A	1	1	1.00	12	0.083
385	A	5	4	1.35	14	0.286
386	B	5	4	2.08	14	0.286
387	A	10	9	1.06	16	0.562
388	A	7	6	1.03	16	0.375
389	A	11	10	0.93	17	0.588
390	A	10	9	0.96	15	0.600
391	A	13	12	0.96	15	0.800
392	A	13	12	1.07	17	0.706
393	A	13	12	0.92	17	0.706
394	A	12	11	0.96	15	0.733
395	A	16	15	1.09	15	1.000
396	A	6	5	1.59	17	0.294
397	A	6	5	1.89	17	0.294
398	A	9	8	0.97	17	0.471
399	A	6	5	1.00	15	0.333
400	A	6	5	0.99	15	0.333
401	A	7	6	1.05	17	0.353
402	A	8	7	0.99	17	0.412
403	A	9	8	0.99	15	0.533
404	A	6	5	1.12	15	0.333
405	A	10	9	1.08	17	0.529

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	10	9	1.11	15	0.600
407	A	6	5	1.20	15	0.333
408	A	7	6	1.13	29	0.207
409	A	7	6	1.23	27	0.222
410	A	7	6	1.05	29	0.207
411	A	7	6	1.01	29	0.207
412	A	7	6	1.00	25	0.240
413	A	7	6	1.00	23	0.261
414	A	6	5	1.00	14	0.357
415	A	7	6	1.00	23	0.261
416	A	7	6	1.00	23	0.261
417	A	7	6	1.00	23	0.261
418	A	7	6	1.00	23	0.261
419	A	7	6	1.00	21	0.286
420	A	7	6	1.00	21	0.286
421	A	7	6	1.00	23	0.261
422	N/A	2	0	1.00	27	0.000
423	A	8	7	1.00	23	0.304
424	A	7	6	0.99	25	0.240
425	A	8	7	1.00	25	0.280
426	N/A	4	0	1.00	27	0.000
427	A	6	5	1.10	21	0.238
428	A	5	4	1.14	19	0.211
429	A	5	4	0.93	19	0.211
430	A	4	3	0.94	21	0.143
431	A	5	4	0.91	21	0.190
432	A	5	4	0.89	21	0.190
433	A	5	4	0.88	21	0.190
434	A	5	4	0.89	21	0.190
435	A	4	3	0.93	21	0.143
436	A	5	4	1.39	21	0.190
437	A	6	5	1.23	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	7	6	1.17	21	0.286
439	A	8	7	1.21	23	0.304
440	A	7	6	1.36	21	0.286
441	A	5	4	1.05	21	0.190
442	A	5	4	0.91	23	0.174
443	A	5	4	0.91	23	0.174
444	A	5	4	0.91	23	0.174
445	A	5	4	0.90	23	0.174
446	A	5	4	0.89	23	0.174
447	A	5	4	0.89	23	0.174
448	A	5	4	0.90	23	0.174
449	A	5	4	1.18	23	0.174
450	A	6	5	1.32	23	0.217
451	A	7	6	1.20	23	0.261
452	A	8	7	1.10	23	0.304
453	A	6	5	0.97	23	0.217
454	A	4	3	1.00	21	0.143
455	A	5	4	0.97	21	0.190
456	A	5	4	0.94	23	0.174
457	A	5	4	0.94	23	0.174
458	A	5	4	0.93	23	0.174
459	A	5	4	0.94	23	0.174
460	A	5	4	0.96	23	0.174
461	A	4	3	1.00	23	0.130
462	A	8	7	1.24	23	0.304
463	A	10	9	1.25	23	0.391
464	A	10	9	1.06	23	0.391
465	A	8	7	1.06	23	0.304
466	A	5	4	0.97	23	0.174
467	A	5	4	0.98	21	0.190
468	A	5	4	0.96	21	0.190
469	A	5	4	0.94	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	5	4	0.94	23	0.174
471	A	5	4	0.95	23	0.174
472	A	5	4	0.97	23	0.174
473	A	5	4	0.97	23	0.174
474	A	10	9	1.16	23	0.391
475	A	12	11	1.17	23	0.478
476	A	4	4	1.00	23	0.174
477	A	5	4	1.01	25	0.160
478	A	4	4	1.00	25	0.160
479	A	7	6	1.09	23	0.261
480	A	7	6	1.26	23	0.261
481	A	6	5	1.00	23	0.217
482	A	6	5	1.00	14	0.357
483	A	6	5	1.00	23	0.217
484	A	4	4	1.00	23	0.174
485	A	4	4	1.00	21	0.190
486	A	4	4	1.00	21	0.190
487	A	4	4	1.06	23	0.174
488	N/A	2	0	1.00	27	0.000
489	N/A	2	0	1.00	25	0.000
490	N/A	2	0	1.00	23	0.000
491	N/A	2	0	1.00	23	0.000
492	N/A	2	0	1.00	25	0.000
493	A	5	4	1.02	25	0.160
494	A	5	4	1.03	25	0.160
495	A	5	4	0.99	25	0.160
496	N/A	2	0	1.00	16	0.000
497	N/A	2	0	1.00	25	0.000
498	A	8	8	1.00	23	0.348
499	A	8	7	1.21	25	0.280
500	A	8	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
501	N/A	4	0	1.00	27	0.000

CHAPTER 3

Listing of integrals

3.1 $\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx = -\frac{b^2\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f} - \frac{b^2\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f} + \frac{b^2\tan^3(e+fx)\sqrt{b\tan^2(e+fx)}}{4f}$$

output
```
-b^2*cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f-1/2*b^2*tan(f*x+e)
*(b*tan(f*x+e)^2)^(1/2)/f+1/4*b^2*tan(f*x+e)^3*(b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx = \frac{b^2\cot(e+fx)\left(-4\log(\cos(e+fx))-4\sec^2(e+fx)+\sec^4(e+fx)\right)\sqrt{b\tan^2(e+fx)}}{4f}$$

input
```
Integrate[(b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(b^2*Cot[e + f*x]*(-4*Log[Cos[e + f*x]] - 4*Sec[e + f*x]^2 + Sec[e + f*x]^
4)*Sqrt[b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(e+fx)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4141}$$

$$b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\int \tan^5(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\int \tan(e+fx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\left(\frac{\tan^4(e+fx)}{4f}-\int \tan^3(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\left(\frac{\tan^4(e+fx)}{4f}-\int \tan(e+fx)^3dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\left(\int \tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\left(\int\tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)$$

$$\Big\downarrow \text{3956}$$

$$b^2\cot(e+fx)\sqrt{b\tan^2(e+fx)}\left(\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}-\frac{\log(\cos(e+fx))}{f}\right)$$

input
```
Int[(b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
b^2*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^2]*(-(Log[Cos[e + f*x]]/f) - Tan[e +
f*x]^2/(2*f) + Tan[e + f*x]^4/(4*f))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\left(b\tan(fx+e)^2\right)^{\frac{5}{2}}\left(\tan(fx+e)^4-2\tan(fx+e)^2+2\ln\left(1+\tan(fx+e)^2\right)\right)}{4f\tan(fx+e)^5}$
default	$\frac{\left(b\tan(fx+e)^2\right)^{\frac{5}{2}}\left(\tan(fx+e)^4-2\tan(fx+e)^2+2\ln\left(1+\tan(fx+e)^2\right)\right)}{4f\tan(fx+e)^5}$
risch	$\frac{b^2\left(e^{2i(fx+e)}+1\right)\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,x}{e^{2i(fx+e)}-1}-\frac{2b^2\left(e^{2i(fx+e)}+1\right)\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,(fx+e)}{\left(e^{2i(fx+e)}-1\right)f}-\frac{4ib^2\sqrt{-\frac{b\left(e^{2i(fx+e)}\right.}{\left(e^{2i(fx+e)}+\right.}}}{\left(e^{2i(fx\right.}}$

input
```
int((b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(b*tan(f*x+e)^2)^(5/2)*(tan(f*x+e)^4-2*tan(f*x+e)^2+2*ln(1+tan(f*x+e
)^2))/tan(f*x+e)^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx=\frac{\left(b^2\tan\left(fx+e\right)^4-2\,b^2\tan\left(fx+e\right)^2-2\,b^2\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-3\,b^2\right)\sqrt{b\tan\left(fx+e\right)^2}}{4\,f\tan\left(fx+e\right)}$$

input
```
integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(b^2*tan(f*x + e)^4 - 2*b^2*tan(f*x + e)^2 - 2*b^2*log(1/(tan(f*x + e)
^2 + 1)) - 3*b^2)*sqrt(b*tan(f*x + e)^2)/(f*tan(f*x + e))
```

Sympy [F]

$$\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx = \int \left(b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

$$\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx = \frac{b^{\frac{5}{2}}\tan\left(fx+e\right)^4 - 2\,b^{\frac{5}{2}}\tan\left(fx+e\right)^2 + 2\,b^{\frac{5}{2}}\log\left(\tan\left(fx+e\right)^2+1\right)}{4\,f}$$

input
```
integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(b^(5/2)*tan(f*x + e)^4 - 2*b^(5/2)*tan(f*x + e)^2 + 2*b^(5/2)*log(tan
(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \left(b\tan^2(e+fx)\right)^{5/2}\,dx = \frac{1}{4}\,b^{\frac{5}{2}}\left(\frac{2\,\log\left(\tan\left(fx+e\right)^2+1\right)}{f} + \frac{f\tan\left(fx+e\right)^4 - 2\,f\tan\left(fx+e\right)^2}{f^2}\right)\mathrm{sgn}(\tan\left(fx+e\right))$$

input
```
integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/4*b^(5/2)*(2*log(tan(f*x + e)^2 + 1)/f + (f*tan(f*x + e)^4 - 2*f*tan(f*x
 + e)^2)/f^2)*sgn(tan(f*x + e))
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(e+fx)\right)^{5/2} dx = \int \left(b\tan(e+f\,x)^2\right)^{5/2} dx$$

input
```
int((b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int((b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \left(b\tan^2(e+fx)\right)^{5/2} dx = \frac{\sqrt{b}\,b^2\left(2\log\left(\tan\left(fx+e\right)^2+1\right)+\tan\left(fx+e\right)^4-2\tan\left(fx+e\right)^2\right)}{4f}$$

input
```
int((b*tan(f*x+e)^2)^(5/2),x)
```

output
```
(sqrt(b)*b**2*(2*log(tan(e + f*x)**2 + 1) + tan(e + f*x)**4 - 2*tan(e + f*
x)**2))/(4*f)
```

3.2 $\int \left(b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \left(b\tan^2(e+fx)\right)^{3/2} dx = \frac{b\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f} + \frac{b\tan(e+fx)\sqrt{b\tan^2(e+fx)}}{2f}$$

output
```
b*cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f+1/2*b*tan(f*x+e)*(b*t
an(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \left(b\tan^2(e+fx)\right)^{3/2} dx = \frac{\cot^3(e+fx)\left(2\log(\cos(e+fx))+\sec^2(e+fx)\right)\left(b\tan^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Integrate[(b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Cot[e + f*x]^3*(2*Log[Cos[e + f*x]] + Sec[e + f*x]^2)*(b*Tan[e + f*x]^2)^
(3/2))/(2*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4141, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b \tan^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b \tan(e+fx)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4141}$$

$$b \cot(e+fx)\sqrt{b \tan^2(e+fx)} \int \tan^3(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b \cot(e+fx)\sqrt{b \tan^2(e+fx)} \int \tan(e+fx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$b \cot(e+fx)\sqrt{b \tan^2(e+fx)} \left(\frac{\tan^2(e+fx)}{2f} - \int \tan(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b \cot(e+fx)\sqrt{b \tan^2(e+fx)} \left(\frac{\tan^2(e+fx)}{2f} - \int \tan(e+fx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$b \cot(e+fx)\sqrt{b \tan^2(e+fx)} \left(\frac{\tan^2(e+fx)}{2f} + \frac{\log(\cos(e+fx))}{f}\right)$$

input
```
Int[(b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
b*Cot[e + f*x]*Sqrt[b*Tan[e + f*x]^2]*(Log[Cos[e + f*x]]/f + Tan[e + f*x]^
2/(2*f))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{\left(b\tan(fx+e)^2\right)^{\frac{3}{2}}\left(-\tan(fx+e)^2+\ln\left(1+\tan(fx+e)^2\right)\right)}{2f\tan(fx+e)^3}$
default	$-\frac{\left(b\tan(fx+e)^2\right)^{\frac{3}{2}}\left(-\tan(fx+e)^2+\ln\left(1+\tan(fx+e)^2\right)\right)}{2f\tan(fx+e)^3}$
risch	$\frac{b\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,(ie^{4i(fx+e)}\ln(e^{2i(fx+e)}+1)+e^{4i(fx+e)}fx+2\,e^{4i(fx+e)}e+2ie^{2i(fx+e)}\ln(e^{2i(fx+e)}+1)+2\,e^{2i(f}}{(e^{2i(fx+e)}-1)\,(e^{2i(fx+e)}+1)\,f}$

input
```
int((b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(b*tan(f*x+e)^2)^(3/2)*(-tan(f*x+e)^2+ln(1+tan(f*x+e)^2))/tan(f*x+e
)^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\left(b\tan\left(fx+e\right)^2+b\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+b\right)\sqrt{b\tan\left(fx+e\right)^2}}{2\,f\tan\left(fx+e\right)}$$

input
```
integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b*tan(f*x + e)^2 + b*log(1/(tan(f*x + e)^2 + 1)) + b)*sqrt(b*tan(f*x
+ e)^2)/(f*tan(f*x + e))
```

Sympy [F]

$$\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{b^{\frac{3}{2}}\tan{(fx+e)}^2-b^{\frac{3}{2}}\log{\left(\tan{(fx+e)}^2+1\right)}}{2\,f}$$

input
```
integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b^(3/2)*tan(f*x + e)^2 - b^(3/2)*log(tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{1}{2}\,b^{\frac{3}{2}}\left(\frac{\tan{(fx+e)}^2}{f}-\frac{\log{\left(\tan{(fx+e)}^2+1\right)}}{f}\right)\operatorname{sgn}(\tan{(fx+e)})$$

input
```
integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*b^(3/2)*(tan(f*x + e)^2/f - log(tan(f*x + e)^2 + 1)/f)*sgn(tan(f*x + e
))
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan(e+f\,x)^2\right)^{3/2}\,dx$$

input
```
int((b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int((b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \left(b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{b}\,b\left(-\log\left(\tan\left(fx+e\right)^2+1\right)+\tan\left(fx+e\right)^2\right)}{2f}$$

input
```
int((b*tan(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(b)*b*( - log(tan(e + f*x)**2 + 1) + tan(e + f*x)**2))/(2*f)
```

3.3 $\int \sqrt{b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \sqrt{b\tan^2(e+fx)}\,dx = -\frac{\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}$$

output
```
-cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sqrt{b\tan^2(e+fx)}\,dx = -\frac{\cot(e+fx)\log(\cos(e+fx))\sqrt{b\tan^2(e+fx)}}{f}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4141, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\cot(e+fx)\sqrt{b\tan^2(e+fx)}\int \tan(e+fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cot(e+fx)\sqrt{b\tan^2(e+fx)}\int \tan(e+fx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)}\log(\cos(e+fx))}{f}$$

input
```
Int[Sqrt[b*Tan[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Log[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]^2])/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\sqrt{b\tan(fx+e)^2}\,\ln\left(1+\tan(fx+e)^2\right)}{2f\tan(fx+e)}$
default	$\frac{\sqrt{b\tan(fx+e)^2}\,\ln\left(1+\tan(fx+e)^2\right)}{2f\tan(fx+e)}$
risch	$\frac{\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,(e^{2i(fx+e)}+1)x}{e^{2i(fx+e)}-1} - \frac{2\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,(e^{2i(fx+e)}+1)(fx+e)}{(e^{2i(fx+e)}-1)f} - \frac{i\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^2}{\left(e^{2i(fx+e)}+1\right)^2}}\,(e^{2}}{(e^{2i(f}}$

input
```
int((b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(b*tan(f*x+e)^2)^(1/2)/tan(f*x+e)*ln(1+tan(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \sqrt{b\tan^2(e+fx)}\,dx = -\frac{\sqrt{b\tan\left(fx+e\right)^2}\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f\tan\left(fx+e\right)}$$

input
```
integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*tan(f*x + e)^2)*log(1/(tan(f*x + e)^2 + 1))/(f*tan(f*x + e))
```

Sympy [F]

$$\int \sqrt{b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan^2\left(e+fx\right)}\,dx$$

input
```
integrate((b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \sqrt{b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

input
```
integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b)*log(tan(f*x + e)^2 + 1)/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \sqrt{b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\log\left(\tan\left(fx+e\right)^2+1\right)\operatorname{sgn}(\tan\left(fx+e\right))}{2\,f}$$

input
```
integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b)*log(tan(f*x + e)^2 + 1)*sgn(tan(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan\left(e+f\,x\right)^2}\,dx$$

input
```
int((b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int((b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.56

$$\int \sqrt{b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\log\left(\tan\left(fx+e\right)^2+1\right)}{2f}$$

input
```
int((b*tan(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(b)*log(tan(e + f*x)**2 + 1))/(2*f)
```

3.4 $\int \frac{1}{\sqrt{b \tan^2(e+fx)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{b \tan^2(e+fx)}} \, dx = \frac{\log(\sin(e+fx)) \tan(e+fx)}{f\sqrt{b \tan^2(e+fx)}}$$

output
```
ln(sin(f*x+e))*tan(f*x+e)/f/(b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b \tan^2(e+fx)}} \, dx = \frac{\log(\sin(e+fx)) \tan(e+fx)}{f\sqrt{b \tan^2(e+fx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[e + f*x]^2],x]
```

output
```
(Log[Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4141, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan(e+fx)\int \cot(e+fx)dx}{\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx}{\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx}{\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan(e+fx)\log(-\sin(e+fx))}{f\sqrt{b\tan^2(e+fx)}}$$

input
```
Int[1/Sqrt[b*Tan[e + f*x]^2],x]
```

output
```
(Log[-Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[b*Tan[e + f*x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{\tan(fx+e)\left(\ln\left(1+\tan(fx+e)^{2}\right)-2\ln(\tan(fx+e))\right)}{2f\sqrt{b\tan(fx+e)^{2}}}$
default	$-\frac{\tan(fx+e)\left(\ln\left(1+\tan(fx+e)^{2}\right)-2\ln(\tan(fx+e))\right)}{2f\sqrt{b\tan(fx+e)^{2}}}$
risch	$\frac{(e^{2i(fx+e)}-1)x}{\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^{2}}{\left(e^{2i(fx+e)}+1\right)^{2}}}\,(e^{2i(fx+e)}+1)}-\frac{2(e^{2i(fx+e)}-1)(fx+e)}{\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^{2}}{\left(e^{2i(fx+e)}+1\right)^{2}}}\,(e^{2i(fx+e)}+1)f}-\frac{i(e^{2i(fx+e)}-1)\ln(e^{2i(fx+e)}-1}{\sqrt{-\frac{b\left(e^{2i(fx+e)}-1\right)^{2}}{\left(e^{2i(fx+e)}+1\right)^{2}}}\,(e^{2i(fx+e)}+}$

input
```
int(1/(b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*tan(f*x+e)*(ln(1+tan(f*x+e)^2)-2*ln(tan(f*x+e)))/(b*tan(f*x+e)^2)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx = \frac{\sqrt{b\tan\left(fx+e\right)^2}\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)}{2\,bf\tan\left(fx+e\right)}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))/(b*f*t
an(f*x + e))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan^2\left(e+fx\right)}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx = -\frac{\frac{\log\left(\tan(fx+e)^2+1\right)}{\sqrt{b}} - \frac{2\log(\tan(fx+e))}{\sqrt{b}}}{2f}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(log(tan(f*x + e)^2 + 1)/sqrt(b) - 2*log(tan(f*x + e))/sqrt(b))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx = -\frac{1}{2}\sqrt{b}\left(\frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{bf\mathrm{sgn}\left(\tan\left(fx+e\right)\right)} - \frac{\log\left(\tan\left(fx+e\right)^2\right)}{bf\mathrm{sgn}\left(\tan\left(fx+e\right)\right)}\right)$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(b)*(log(tan(f*x + e)^2 + 1)/(b*f*sgn(tan(f*x + e))) - log(tan(f*
x + e)^2)/(b*f*sgn(tan(f*x + e))))
```

Mupad [B] (verification not implemented)

Time = 7.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{-b}\tan(e+f\,x)}{\sqrt{b\tan(e+f\,x)^2}}\right)}{\sqrt{-b}\,f}$$

input
```
int(1/(b*tan(e + f*x)^2)^(1/2),x)
```

output
```
atan(((-b)^(1/2)*tan(e + f*x))/(b*tan(e + f*x)^2)^(1/2))/((-b)^(1/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b\tan^2(e+fx)}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(fx+e\right)^2+1\right)+2\log\left(\tan\left(fx+e\right)\right)\right)}{2bf}$$

input
```
int(1/(b*tan(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(b)*( - log(tan(e + f*x)**2 + 1) + 2*log(tan(e + f*x))))/(2*b*f)
```

3.5 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\cot(e+fx)}{2bf\sqrt{b\tan^2(e+fx)}}-\frac{\log(\sin(e+fx))\tan(e+fx)}{bf\sqrt{b\tan^2(e+fx)}}$$

output
```
-1/2*cot(f*x+e)/b/f/(b*tan(f*x+e)^2)^(1/2)-ln(sin(f*x+e))*tan(f*x+e)/b/f/(
b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\left(\csc^2(e+fx)+2\log(\sin(e+fx))\right)\tan^3(e+fx)}{2f\left(b\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[(b*Tan[e + f*x]^2)^(-3/2),x]
```

output
```
-1/2*((Csc[e + f*x]^2 + 2*Log[Sin[e + f*x]])*Tan[e + f*x]^3)/(f*(b*Tan[e +
 f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan(e+fx)\int \cot^3(e+fx)dx}{b\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)^3dx}{b\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx}{b\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}-\int -\cot(e+fx)dx\right)}{b\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(\int \cot(e+fx)dx+\frac{\cot^2(e+fx)}{2f}\right)}{b\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tan(e+fx)\left(\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\cot^2(e+fx)}{2f}\right)}{b\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow 25$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}-\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx\right)}{b\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow 3956$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}+\frac{\log(-\sin(e+fx))}{f}\right)}{b\sqrt{b\tan^2(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^2)^(-3/2),x]
```

output
```
-(((Cot[e + f*x]^2/(2*f) + Log[-Sin[e + f*x]]/f)*Tan[e + f*x])/(b*Sqrt[b*T
an[e + f*x]^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{\tan(fx+e)\left(2\ln(\tan(fx+e))\tan(fx+e)^{2}-\ln\left(1+\tan(fx+e)^{2}\right)\tan(fx+e)^{2}+1\right)}{2f\left(b\tan(fx+e)^{2}\right)^{\frac{3}{2}}}$
default	$-\frac{\tan(fx+e)\left(2\ln(\tan(fx+e))\tan(fx+e)^{2}-\ln\left(1+\tan(fx+e)^{2}\right)\tan(fx+e)^{2}+1\right)}{2f\left(b\tan(fx+e)^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{i\,\mathrm{e}^{4i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)+\mathrm{e}^{4i(fx+e)}fx+2\,\mathrm{e}^{4i(fx+e)}e-2i\,\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)-2\,\mathrm{e}^{2i(fx+e)}fx-2i\,\mathrm{e}^{2i(fx+e)}-4}{b\left(\mathrm{e}^{2i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{-\frac{b\left(\mathrm{e}^{2i(fx+e)}-1\right)^{2}}{\left(\mathrm{e}^{2i(fx+e)}+1\right)^{2}}}\,f}$

input
```
int(1/(b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*tan(f*x+e)*(2*ln(tan(f*x+e))*tan(f*x+e)^2-ln(1+tan(f*x+e)^2)*tan(f*
x+e)^2+1)/(b*tan(f*x+e)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{b\tan\left(fx+e\right)^{2}}\left(\log\left(\frac{\tan(fx+e)^{2}}{\tan(fx+e)^{2}+1}\right)\tan\left(fx+e\right)^{2}+\tan\left(fx+e\right)^{2}+1\right)}{2\,b^{2}f\tan\left(fx+e\right)^{3}}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*tan(f*x + e)^2)*(log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(
f*x + e)^2 + tan(f*x + e)^2 + 1)/(b^2*f*tan(f*x + e)^3)
```

Sympy [F]

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{\log\left(\tan(fx+e)^2+1\right)}{b^{\frac{3}{2}}} - \frac{2\log(\tan(fx+e))}{b^{\frac{3}{2}}} - \frac{1}{b^{\frac{3}{2}}\tan(fx+e)^2}}{2f}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(log(tan(f*x + e)^2 + 1)/b^(3/2) - 2*log(tan(f*x + e))/b^(3/2) - 1/(b^
(3/2)*tan(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{\log\left(\tan(fx+e)^2+1\right)}{bf\mathrm{sgn}(\tan(fx+e))} - \frac{\log\left(\tan(fx+e)^2\right)}{bf\mathrm{sgn}(\tan(fx+e))} + \frac{\tan(fx+e)^2-1}{bf\mathrm{sgn}(\tan(fx+e))\tan(fx+e)^2}}{2\sqrt{b}}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(log(tan(f*x + e)^2 + 1)/(b*f*sgn(tan(f*x + e))) - log(tan(f*x + e)^2)
/(b*f*sgn(tan(f*x + e))) + (tan(f*x + e)^2 - 1)/(b*f*sgn(tan(f*x + e))*tan
(f*x + e)^2))/sqrt(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{b}\left(\log\left(\tan\left(fx+e\right)^2+1\right)\tan\left(fx+e\right)^2 - 2\log\left(\tan\left(fx+e\right)\right)\tan\left(fx+e\right)^2 -}{2\tan\left(fx+e\right)^2 b^2 f}$$

input
```
int(1/(b*tan(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(b)*(log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2 - 2*log(tan(e + f*x))*t
an(e + f*x)**2 - 1))/(2*tan(e + f*x)**2*b**2*f)
```

3.6 $\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx=\frac{\cot(e+fx)}{2b^2f\sqrt{b\tan^2(e+fx)}}-\frac{\cot^3(e+fx)}{4b^2f\sqrt{b\tan^2(e+fx)}}+\frac{\log(\sin(e+fx))\tan(e+fx)}{b^2f\sqrt{b\tan^2(e+fx)}}$$

output
```
1/2*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^3/b^2/f/(b*tan(
f*x+e)^2)^(1/2)+ln(sin(f*x+e))*tan(f*x+e)/b^2/f/(b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx=-\frac{\cot(e+fx)\left(-4\csc^2(e+fx)+\csc^4(e+fx)-4\log(\sin(e+fx))\right)\sqrt{b\tan^2(e+fx)}}{4b^3f}$$

input
```
Integrate[(b*Tan[e + f*x]^2)^(-5/2),x]
```

output
```
-1/4*(Cot[e + f*x]*(-4*Csc[e + f*x]^2 + Csc[e + f*x]^4 - 4*Log[Sin[e + f*x
]])*Sqrt[b*Tan[e + f*x]^2])/(b^3*f)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan(e+fx)\int \cot^5(e+fx)dx}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)^5dx}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5dx}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\int -\cot^3(e+fx)dx\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\left(\int \cot^3(e+fx)dx+\frac{\cot^4(e+fx)}{4f}\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tan(e+fx)\left(\int -\tan\left(e+fx+\frac{\pi}{2}\right)^3dx+\frac{\cot^4(e+fx)}{4f}\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$-\frac{\tan(e+fx)\left(\int -\cot(e+fx)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(-\int \cot(e+fx)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tan(e+fx)\left(-\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}-\frac{\log(-\sin(e+fx))}{f}\right)}{b^2\sqrt{b\tan^2(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^2)^(-5/2),x]
```

output
```
-(((-1/2*Cot[e + f*x]^2/f + Cot[e + f*x]^4/(4*f) - Log[-Sin[e + f*x]]/f)*T
an[e + f*x])/(b^2*Sqrt[b*Tan[e + f*x]^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\tan(fx+e)\left(4\ln(\tan(fx+e))\tan(fx+e)^4-2\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^4+2\tan(fx+e)^2-1\right)}{4f\left(b\tan(fx+e)^2\right)^{\frac{5}{2}}}$
default	$\frac{\tan(fx+e)\left(4\ln(\tan(fx+e))\tan(fx+e)^4-2\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^4+2\tan(fx+e)^2-1\right)}{4f\left(b\tan(fx+e)^2\right)^{\frac{5}{2}}}$
risch	$\frac{(e^{2i(fx+e)}-1)x}{b^2(e^{2i(fx+e)}+1)\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}}-\frac{2(e^{2i(fx+e)}-1)(fx+e)}{b^2(e^{2i(fx+e)}+1)\sqrt{-\frac{b(e^{2i(fx+e)}-1)^2}{(e^{2i(fx+e)}+1)^2}}\,f}+\frac{4i(e^{6i(fx+e)}-e^{4i(fx}}{b^2(e^{2i(fx+e)}-1)^3(e^{2i(fx+e)}+}$

input
```
int(1/(b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*tan(f*x+e)*(4*ln(tan(f*x+e))*tan(f*x+e)^4-2*ln(1+tan(f*x+e)^2)*tan(f
*x+e)^4+2*tan(f*x+e)^2-1)/(b*tan(f*x+e)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{(b\tan^2(e+fx))^{5/2}}\,dx = \frac{\left(2\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan(fx+e)^4+3\tan(fx+e)^4+2\tan(fx+e)^2-1\right)\sqrt{}}{4b^3f\tan(fx+e)^5}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + 3*tan(f*x
 + e)^4 + 2*tan(f*x + e)^2 - 1)*sqrt(b*tan(f*x + e)^2)/(b^3*f*tan(f*x + e)
^5)
```

Sympy [F]

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan^2{(e+fx)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\frac{2\log\left(\tan(fx+e)^2+1\right)}{b^{\frac{5}{2}}} - \frac{4\log(\tan(fx+e))}{b^{\frac{5}{2}}} - \frac{2\sqrt{b}\tan(fx+e)^2-\sqrt{b}}{b^3\tan(fx+e)^4}}{4f}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(2*log(tan(f*x + e)^2 + 1)/b^(5/2) - 4*log(tan(f*x + e))/b^(5/2) - (2
*sqrt(b)*tan(f*x + e)^2 - sqrt(b))/(b^3*tan(f*x + e)^4))/f
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\frac{2\log\left(\tan(fx+e)^2+1\right)}{bf\mathrm{sgn}(\tan(fx+e))} - \frac{2\log\left(\tan(fx+e)^2\right)}{bf\mathrm{sgn}(\tan(fx+e))} + \frac{3\tan(fx+e)^4-2\tan(fx+e)^2+1}{bf\mathrm{sgn}(\tan(fx+e))\tan(fx+e)^4}}{4b^{\frac{3}{2}}}$$

input
```
integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(2*log(tan(f*x + e)^2 + 1)/(b*f*sgn(tan(f*x + e))) - 2*log(tan(f*x +
e)^2)/(b*f*sgn(tan(f*x + e))) + (3*tan(f*x + e)^4 - 2*tan(f*x + e)^2 + 1)/
(b*f*sgn(tan(f*x + e))*tan(f*x + e)^4))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\tan^2(e+fx)\right)^{5/2}}\,dx=\frac{\sqrt{b}\left(-2\log\left(\tan\left(fx+e\right)^2+1\right)\tan\left(fx+e\right)^4+4\log\left(\tan\left(fx+e\right)\right)\tan\left(fx+e\right.}{4\tan\left(fx+e\right)^4b^3f}$$

input
```
int(1/(b*tan(f*x+e)^2)^(5/2),x)
```

output
```
(sqrt(b)*( - 2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4 + 4*log(tan(e + f*
x))*tan(e + f*x)**4 + 2*tan(e + f*x)**2 - 1))/(4*tan(e + f*x)**4*b**3*f)
```

3.7 $\int (b\tan^3(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 298

$$\int (b\tan^3(e+fx))^{5/2}\,dx = -\frac{2b^2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} - \frac{b^2\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{b^2\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{b^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(e+fx)}}{1+\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{2b^2\tan(e+fx)\sqrt{b\tan^3(e+fx)}}{5f} - \frac{2b^2\tan^3(e+fx)\sqrt{b\tan^3(e+fx)}}{9f} + \frac{2b^2\tan^5(e+fx)\sqrt{b\tan^3(e+fx)}}{13f}$$

output
```
-2*b^2*cot(f*x+e)*(b*tan(f*x+e)^3)^(1/2)/f+1/2*b^2*arctan(-1+2^(1/2)*tan(f
*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)*2^(1/2)/f/tan(f*x+e)^(3/2)+1/2*b^2*arc
tan(1+2^(1/2)*tan(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)*2^(1/2)/f/tan(f*x+e
)^(3/2)+1/2*b^2*arctanh(2^(1/2)*tan(f*x+e)^(1/2)/(1+tan(f*x+e)))*(b*tan(f*
x+e)^3)^(1/2)*2^(1/2)/f/tan(f*x+e)^(3/2)+2/5*b^2*tan(f*x+e)*(b*tan(f*x+e)^
3)^(1/2)/f-2/9*b^2*tan(f*x+e)^3*(b*tan(f*x+e)^3)^(1/2)/f+2/13*b^2*tan(f*x+
e)^5*(b*tan(f*x+e)^3)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int (b\tan^3(e+fx))^{5/2}\,dx = \frac{(b\tan^3(e+fx))^{5/2}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+t\cdots\right.}{2\sqrt{2}}\cdots\right.}{\cdots}$$

input
```
Integrate[(b*Tan[e + f*x]^3)^(5/2),x]
```

output
```
((b*Tan[e + f*x]^3)^(5/2)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2
]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt
[Tan[e + f*x]] + Tan[e + f*x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[e +
f*x]] + Tan[e + f*x]]/(2*Sqrt[2]) - 2*Sqrt[Tan[e + f*x]] + (2*Tan[e + f*x]
^(5/2))/5 - (2*Tan[e + f*x]^(9/2))/9 + (2*Tan[e + f*x]^(13/2))/13))/(f*Tan
[e + f*x]^(15/2))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.78, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^3(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(e+fx)^3)^{5/2}\,dx$$

$$\Big\downarrow \; 4141$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\int\tan^{\frac{15}{2}}(e+fx)dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\int\tan(e+fx)^{15/2}dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3954}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\int\tan^{\frac{11}{2}}(e+fx)dx\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\int\tan(e+fx)^{11/2}dx\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3954}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\int\tan^{\frac{7}{2}}(e+fx)dx+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\int\tan(e+fx)^{7/2}dx+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3954}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(-\int\tan^{\frac{3}{2}}(e+fx)dx+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}+\frac{2\tan^{\frac{5}{2}}(e+fx)}{5f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(-\int\tan(e+fx)^{3/2}dx+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}+\frac{2\tan^{\frac{5}{2}}(e+fx)}{5f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \textbf{3954}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\int \frac{1}{\sqrt{\tan(e+fx)}}dx+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}+\frac{2\tan^{\frac{5}{2}}(e+fx)}{5f}-\frac{2\sqrt{\tan(e+fx)}}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\int \frac{1}{\sqrt{\tan(e+fx)}}dx+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}+\frac{2\tan^{\frac{5}{2}}(e+fx)}{5f}-\frac{2\sqrt{\tan(e+fx)}}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{\int \frac{1}{\sqrt{\tan(e+fx)}\left(\tan^2(e+fx)+1\right)}d\tan(e+fx)}{f}+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}+\frac{2\tan^{\frac{5}{2}}(e+fx)}{5f}-\frac{2\sqrt{\tan(e+fx)}}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{266}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\int \frac{1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}}{f}+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}+\frac{2\tan^{\frac{5}{2}}(e+fx)}{5f}-\frac{2\sqrt{\tan(e+fx)}}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{755}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int \frac{\tan(e+fx)+1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}-\frac{2\tan^{\frac{9}{2}}(e+fx)}{9f}+2\tan\ldots\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+f\ldots}}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 217

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)\right)}{f}+\frac{2\tan^{\frac{13}{2}}(e+fx)}{13f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 1479

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 25

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 27

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(e+fx)}+1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 1103

$$\frac{b^2\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(e+fx)-\sqrt{\ldots}\right.}{2\ldots}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

input
```
Int[(b*Tan[e + f*x]^3)^(5/2),x]
```

output
```
(b^2*Sqrt[b*Tan[e + f*x]^3]*((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]
/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2])/2 + (-1/2*Log[
1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]/Sqrt[2] + Log[1 + Sqrt[2]*S
qrt[Tan[e + f*x]] + Tan[e + f*x]]/(2*Sqrt[2]))/2))/f - (2*Sqrt[Tan[e + f*x
]])/f + (2*Tan[e + f*x]^(5/2))/(5*f) - (2*Tan[e + f*x]^(9/2))/(9*f) + (2*T
an[e + f*x]^(13/2))/(13*f)))/Tan[e + f*x]^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\left(b\tan(fx+e)^3\right)^{\frac{5}{2}}\left(360(b\tan(fx+e))^{\frac{13}{2}}-520b^2(b\tan(fx+e))^{\frac{9}{2}}+936b^4(b\tan(fx+e))^{\frac{5}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\ln\left(-\frac{b\tan(f}{(b^2)^{\frac{1}{4}}}\right.\right.}{}$
default	$\frac{\left(b\tan(fx+e)^3\right)^{\frac{5}{2}}\left(360(b\tan(fx+e))^{\frac{13}{2}}-520b^2(b\tan(fx+e))^{\frac{9}{2}}+936b^4(b\tan(fx+e))^{\frac{5}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\ln\left(-\frac{b\tan(f}{(b^2)^{\frac{1}{4}}}\right.\right.}{}$

input
```
int((b*tan(f*x+e)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2340/f*(b*tan(f*x+e)^3)^(5/2)*(360*(b*tan(f*x+e))^(13/2)-520*b^2*(b*tan(
f*x+e))^(9/2)+936*b^4*(b*tan(f*x+e))^(5/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln(
-(b*tan(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2
)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2)))+1170*b^6*(
b^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)
^(1/4))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-
(b^2)^(1/4))/(b^2)^(1/4))-4680*(b*tan(f*x+e))^(1/2)*b^6)/tan(f*x+e)^5/(b*t
an(f*x+e))^(5/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04

$$\int (b\tan^3(e+fx))^{5/2}\,dx = \frac{1170\sqrt{2}b^{\frac{5}{2}}\arctan\left(\frac{b\tan(fx+e)+\sqrt{2}\sqrt{b\tan(fx+e)^3}\sqrt{b}}{b\tan(fx+e)}\right)\tan(fx+e)+1170\sqrt{2}b^{\frac{5}{2}}\arctan\left(-\frac{b\tan(f}{}\right.}{}$$

input
```
integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/2340*(1170*sqrt(2)*b^(5/2)*arctan((b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f
*x + e)^3)*sqrt(b))/(b*tan(f*x + e)))*tan(f*x + e) + 1170*sqrt(2)*b^(5/2)*
arctan(-(b*tan(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e)^3)*sqrt(b))/(b*tan(f
*x + e)))*tan(f*x + e) + 585*sqrt(2)*b^(5/2)*log((b*tan(f*x + e)^2 + b*tan
(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e)^3)*sqrt(b))/tan(f*x + e))*tan(f*x
+ e) - 585*sqrt(2)*b^(5/2)*log((b*tan(f*x + e)^2 + b*tan(f*x + e) - sqrt(2
)*sqrt(b*tan(f*x + e)^3)*sqrt(b))/tan(f*x + e))*tan(f*x + e) + 8*(45*b^2*t
an(f*x + e)^6 - 65*b^2*tan(f*x + e)^4 + 117*b^2*tan(f*x + e)^2 - 585*b^2)*
sqrt(b*tan(f*x + e)^3))/(f*tan(f*x + e))
```

Sympy [F]

$$\int (b\tan^3(e+fx))^{5/2}\,dx = \int \left(b\tan^3(e+fx)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**3)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**3)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.60

$$\int \left(b\tan^3(e+fx)\right)^{5/2}\,dx = \frac{360\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{13}{2}} - 520\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{9}{2}} + 936\,b^{\frac{5}{2}}\tan\left(fx+e\right)^{\frac{5}{2}} + 585\left(2\sqrt{2}\sqrt{b}\arctan\right.}{}$$

input
```
integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

output
```
1/2340*(360*b^(5/2)*tan(f*x + e)^(13/2) - 520*b^(5/2)*tan(f*x + e)^(9/2) +
 936*b^(5/2)*tan(f*x + e)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2
)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan
(f*x + e)) + tan(f*x + e) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x
 + e)) + tan(f*x + e) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.97

$$\int \left(b\tan^3(e+fx)\right)^{5/2}\,dx = \frac{1}{2340}\,b^2\left(\frac{1170\,\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\,\sqrt{b\tan(fx+e)}\right)}{2\,\sqrt{|b|}}\right)}{f} + \frac{1170\,\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.}{f}\right.$$

input
```
integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

output
```
1/2340*b^2*(1170*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(b)) + 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + 1170*sqrt(2)*sqrt(abs(b))
*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(
abs(b)))/f + 585*sqrt(2)*sqrt(abs(b))*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*
tan(f*x + e))*sqrt(abs(b)) + abs(b))/f - 585*sqrt(2)*sqrt(abs(b))*log(b*ta
n(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/f + 8*(45
*sqrt(b*tan(f*x + e))*b^78*f^12*tan(f*x + e)^6 - 65*sqrt(b*tan(f*x + e))*b
^78*f^12*tan(f*x + e)^4 + 117*sqrt(b*tan(f*x + e))*b^78*f^12*tan(f*x + e)^
2 - 585*sqrt(b*tan(f*x + e))*b^78*f^12)/(b^78*f^13))*sgn(tan(f*x + e))
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(e+fx)\right)^{5/2}\,dx=\int \left(b\tan(e+f\,x)^3\right)^{5/2}\,dx$$

input
```
int((b*tan(e + f*x)^3)^(5/2),x)
```

output
```
int((b*tan(e + f*x)^3)^(5/2), x)
```

Reduce [F]

$$\int \left(b\tan^3(e+fx)\right)^{5/2}\,dx=\frac{\sqrt{b}\,b^2\Big(90\sqrt{\tan\left(fx+e\right)}\,\tan\left(fx+e\right)^6-130\sqrt{\tan\left(fx+e\right)}\,\tan\left(fx+e\right)^4+234\sqrt{\tan\left(fx\right.}}{585f}$$

input
```
int((b*tan(f*x+e)^3)^(5/2),x)
```

output
```
(sqrt(b)*b**2*(90*sqrt(tan(e + f*x))*tan(e + f*x)**6 - 130*sqrt(tan(e + f*
x))*tan(e + f*x)**4 + 234*sqrt(tan(e + f*x))*tan(e + f*x)**2 - 1170*sqrt(t
an(e + f*x)) + 585*int(sqrt(tan(e + f*x))/tan(e + f*x),x)*f))/(585*f)
```

3.8 $\int (b\tan^3(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 223

$$\int (b\tan^3(e+fx))^{3/2}\,dx = -\frac{2b\sqrt{b\tan^3(e+fx)}}{3f} - \frac{b\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{b\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(e+fx)}}{1+\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} + \frac{2b\tan^2(e+fx)\sqrt{b\tan^3(e+fx)}}{7f}$$

output
```
-2/3*b*(b*tan(f*x+e)^3)^(1/2)/f+1/2*b*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*
(b*tan(f*x+e)^3)^(1/2)*2^(1/2)/f/tan(f*x+e)^(3/2)+1/2*b*arctan(1+2^(1/2)*t
an(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)*2^(1/2)/f/tan(f*x+e)^(3/2)-1/2*b*a
rctanh(2^(1/2)*tan(f*x+e)^(1/2)/(1+tan(f*x+e)))*(b*tan(f*x+e)^3)^(1/2)*2^(
1/2)/f/tan(f*x+e)^(3/2)+2/7*b*tan(f*x+e)^2*(b*tan(f*x+e)^3)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx = \frac{b\sqrt{b\tan^3(e+fx)}\left(21\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)\sqrt[4]{-\tan(e+fx)} - 21\text{arctanh}\left(\sqrt[4]{-\tan\right.\right.}{21f\tan^{\frac{7}{4}}(e+fx)}$$

input

```
Integrate[(b*Tan[e + f*x]^3)^(3/2),x]
```

output

```
(b*Sqrt[b*Tan[e + f*x]^3]*(21*ArcTan[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*
x])^(1/4) - 21*ArcTanh[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x])^(1/4) + 2*
Tan[e + f*x]^(7/4)*(-7 + 3*Tan[e + f*x]^2)))/(21*f*Tan[e + f*x]^(7/4))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^3(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{b\sqrt{b\tan^3(e+fx)}\int \tan^{\frac{9}{2}}(e+fx)dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan^3(e+fx)}\int \tan(e+fx)^{9/2}dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 3954

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\int \tan^{\frac{5}{2}}(e+fx)dx\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\int \tan(e+fx)^{5/2}dx\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 3954

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\int \sqrt{\tan(e+fx)}dx+\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\frac{2\tan^{\frac{3}{2}}(e+fx)}{3f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\int \sqrt{\tan(e+fx)}dx+\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\frac{2\tan^{\frac{3}{2}}(e+fx)}{3f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 3957

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{\int \frac{\sqrt{\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}+\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\frac{2\tan^{\frac{3}{2}}(e+fx)}{3f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 266

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\int \frac{\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}}{f}+\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\frac{2\tan^{\frac{3}{2}}(e+fx)}{3f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 826

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\int \frac{\tan(e+fx)+1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}-\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}+\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\frac{2\tan^{\frac{3}{2}}(e+fx)}{3f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 1476

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int\frac{1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)-\frac{1}{2}\int\frac{1-\tan(e+}{\tan^2(e+f}\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 1082

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 217

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}+\frac{2\tan^{\frac{7}{2}}(e+fx)}{7f}-\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 1479

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 25

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 27

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(e+fx)}+1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\ldots}\right.}{\ldots}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\Big\downarrow$ 1103

$$\frac{b\sqrt{b\tan^3(e+fx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(e+fx)+\sqrt{2}\ldots\right.}{2\sqrt{\ldots}}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

input
```
Int[(b*Tan[e + f*x]^3)^(3/2),x]
```

output
```
(b*Sqrt[b*Tan[e + f*x]^3]*((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]/S
qrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2])/2 + (Log[1 - Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt
[Tan[e + f*x]] + Tan[e + f*x]]/(2*Sqrt[2]))/2))/f - (2*Tan[e + f*x]^(3/2))
/(3*f) + (2*Tan[e + f*x]^(7/2))/(7*f)))/Tan[e + f*x]^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\left(b\tan(fx+e)^3\right)^{\frac{3}{2}}\left(24(b\tan(fx+e))^{\frac{7}{2}}\left(b^2\right)^{\frac{1}{4}}+21b^4\sqrt{2}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\,\mathrm{arct}}{84f\tan(fx+e)^3(b\tan(fx}$
default	$\frac{\left(b\tan(fx+e)^3\right)^{\frac{3}{2}}\left(24(b\tan(fx+e))^{\frac{7}{2}}\left(b^2\right)^{\frac{1}{4}}+21b^4\sqrt{2}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\,\mathrm{arct}}{84f\tan(fx+e)^3(b\tan(fx}$

input
```
int((b*tan(f*x+e)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84/f*(b*tan(f*x+e)^3)^(3/2)*(24*(b*tan(f*x+e))^(7/2)*(b^2)^(1/4)+21*b^4*
2^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(
1/2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))
+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1
/4))+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2
)^(1/4))-56*b^2*(b*tan(f*x+e))^(3/2)*(b^2)^(1/4))/tan(f*x+e)^3/(b*tan(f*x+
e))^(3/2)/b^2/(b^2)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.11

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx = \frac{42\sqrt{2}b^{\frac{3}{2}}\arctan\left(\frac{b\tan(fx+e)+\sqrt{2}\sqrt{b\tan(fx+e)^3}\sqrt{b}}{b\tan(fx+e)}\right)+42\sqrt{2}b^{\frac{3}{2}}\arctan\left(-\frac{b\tan(fx+e)-\sqrt{2}\sqrt{b\tan(fx+e)}}{b\tan(fx+e)}\right.}{}$$

input
```
integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/84*(42*sqrt(2)*b^(3/2)*arctan((b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x +
 e)^3)*sqrt(b))/(b*tan(f*x + e))) + 42*sqrt(2)*b^(3/2)*arctan(-(b*tan(f*x
+ e) - sqrt(2)*sqrt(b*tan(f*x + e)^3)*sqrt(b))/(b*tan(f*x + e))) - 21*sqrt
(2)*b^(3/2)*log((b*tan(f*x + e)^2 + b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*
x + e)^3)*sqrt(b))/tan(f*x + e)) + 21*sqrt(2)*b^(3/2)*log((b*tan(f*x + e)^
2 + b*tan(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e)^3)*sqrt(b))/tan(f*x + e))
 + 8*sqrt(b*tan(f*x + e)^3)*(3*b*tan(f*x + e)^2 - 7*b))/f
```

Sympy [F]

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx = \int \left(b\tan^3\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**3)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int (b\tan^3(e+fx))^{3/2}\, dx = \frac{24\, b^{\frac{3}{2}}\tan(fx+e)^{\frac{7}{2}} - 56\, b^{\frac{3}{2}}\tan(fx+e)^{\frac{3}{2}} + 21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right.\right.\right.}{}$$

input
```
integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")
```

output
```
1/84*(24*b^(3/2)*tan(f*x + e)^(7/2) - 56*b^(3/2)*tan(f*x + e)^(3/2) + 21*(
2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*log(sqrt(
2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(
f*x + e)) + tan(f*x + e) + 1))*b^(3/2))/f
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.13

$$\int (b\tan^3(e+fx))^{3/2}\, dx = \frac{1}{84}\, b\left(\frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{bf} + \frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan}\right.}{2\sqrt{|b|}}\right.}{bf}\right.$$

input
```
integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

output
```
1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b*f) + 42*sqrt(2)*abs(b)^(3/2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs(
b)))/(b*f) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*t
an(f*x + e))*sqrt(abs(b)) + abs(b))/(b*f) + 21*sqrt(2)*abs(b)^(3/2)*log(b*
tan(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b*f) +
 8*(3*sqrt(b*tan(f*x + e))*b^21*f^6*tan(f*x + e)^3 - 7*sqrt(b*tan(f*x + e)
)*b^21*f^6*tan(f*x + e))/(b^21*f^7))*sgn(tan(f*x + e))
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx=\int \left(b\tan(e+f\,x)^3\right)^{3/2}dx$$

input
```
int((b*tan(e + f*x)^3)^(3/2),x)
```

output
```
int((b*tan(e + f*x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(b\tan^3(e+fx)\right)^{3/2}\,dx=\sqrt{b}\left(\int \sqrt{\tan(fx+e)}\,\tan(fx+e)^4\,dx\right)b$$

input
```
int((b*tan(f*x+e)^3)^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(tan(e + f*x))*tan(e + f*x)**4,x)*b
```

3.9 $\int \sqrt{b\tan^3(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 193

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \frac{2\cot(e+fx)\sqrt{b\tan^3(e+fx)}}{f} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(e+fx)}}{1+\tan(e+fx)}\right)\sqrt{b\tan^3(e+fx)}}{\sqrt{2}f\tan^{\frac{3}{2}}(e+fx)}$$

output

```
2*cot(f*x+e)*(b*tan(f*x+e)^3)^(1/2)/f-1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/
2))*(b*tan(f*x+e)^3)^(1/2)*2^(1/2)/f/tan(f*x+e)^(3/2)-1/2*arctan(1+2^(1/2)
*tan(f*x+e)^(1/2))*(b*tan(f*x+e)^3)^(1/2)*2^(1/2)/f/tan(f*x+e)^(3/2)-1/2*a
rctanh(2^(1/2)*tan(f*x+e)^(1/2)/(1+tan(f*x+e)))*(b*tan(f*x+e)^3)^(1/2)*2^(
1/2)/f/tan(f*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.84

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(e+fx)}+\tan(e+fx)\right)}{2\sqrt{2}}\right.}{f\tan^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]^3],x]
```

output
```
((ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[e + f*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x
]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]]/(2*Sqr
t[2]) + 2*Sqrt[Tan[e + f*x]])*Sqrt[b*Tan[e + f*x]^3])/(f*Tan[e + f*x]^(3/2
))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^3(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\int \tan^{\frac{3}{2}}(e+fx)dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\int \tan(e+fx)^{3/2}dx}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\int \frac{1}{\sqrt{\tan(e+fx)}}dx\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\int \frac{1}{\sqrt{\tan(e+fx)}}dx\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{\int \frac{1}{\sqrt{\tan(e+fx)}\left(\tan^2(e+fx)+1\right)}d\tan(e+fx)}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\int \frac{1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{755}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int \frac{\tan(e+fx)+1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int \frac{}{\tan(e+f}\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(\sqrt{2}\right.}{\sqrt{2}}\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 217

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)\right)}{f}\right)}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 1479

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 25

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 27

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(e+fx)}+1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

$\downarrow$ 1103

$$\frac{\sqrt{b\tan^3(e+fx)}\left(\frac{2\sqrt{\tan(e+fx)}}{f}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}}-\right.\right.}{f}\right.}{\tan^{\frac{3}{2}}(e+fx)}$$

input
```
Int[Sqrt[b*Tan[e + f*x]^3],x]
```

output
```
(((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f
*x]] + Tan[e + f*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e
+ f*x]]/(2*Sqrt[2]))/2))/f + (2*Sqrt[Tan[e + f*x]])/f)*Sqrt[b*Tan[e + f*x]
^3])/Tan[e + f*x]^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{\sqrt{b\tan(fx+e)^3}\left((b^2)^{\frac14}\sqrt{2}\ln\left(-\frac{b\tan(fx+e)+(b^2)^{\frac14}\sqrt{b\tan(fx+e)}\sqrt{2}+\sqrt{b^2}}{(b^2)^{\frac14}\sqrt{b\tan(fx+e)}\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}\right)+2(b^2)^{\frac14}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(fx+e)}}{(b^2)^{\frac14}}\right.\right.}{4f\tan(fx+e)\sqrt{b\tan(fx+e)}}$
default	$-\frac{\sqrt{b\tan(fx+e)^3}\left((b^2)^{\frac14}\sqrt{2}\ln\left(-\frac{b\tan(fx+e)+(b^2)^{\frac14}\sqrt{b\tan(fx+e)}\sqrt{2}+\sqrt{b^2}}{(b^2)^{\frac14}\sqrt{b\tan(fx+e)}\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}\right)+2(b^2)^{\frac14}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(fx+e)}}{(b^2)^{\frac14}}\right.\right.}{4f\tan(fx+e)\sqrt{b\tan(fx+e)}}$

input
```
int((b*tan(f*x+e)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(b*tan(f*x+e)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln(-(b*tan(f*x+e)+(b^2)
^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4)*(b*tan(f*x+e
))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*(b^2)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan
(f*x+e))^(1/2))/tan(f*x+e)/(b*tan(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.37

$$\int \sqrt{b\tan^3(e+fx)}\,dx = -\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{b\tan(fx+e)+\sqrt{2}\sqrt{b\tan(fx+e)^3}\sqrt{b}}{b\tan(fx+e)}\right)\tan(fx+e)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{b\tan(fx+e)-\sqrt{2}\sqrt{b\tan(fx+e)^3}\sqrt{b}}{b\tan(fx+e)}\right.}{}$$

input
```
integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan((b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x +
e)^3)*sqrt(b))/(b*tan(f*x + e)))*tan(f*x + e) + 2*sqrt(2)*sqrt(b)*arctan(-
(b*tan(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e)^3)*sqrt(b))/(b*tan(f*x + e))
)*tan(f*x + e) + sqrt(2)*sqrt(b)*log((b*tan(f*x + e)^2 + b*tan(f*x + e) +
sqrt(2)*sqrt(b*tan(f*x + e)^3)*sqrt(b))/tan(f*x + e))*tan(f*x + e) - sqrt(
2)*sqrt(b)*log((b*tan(f*x + e)^2 + b*tan(f*x + e) - sqrt(2)*sqrt(b*tan(f*x
 + e)^3)*sqrt(b))/tan(f*x + e))*tan(f*x + e) - 8*sqrt(b*tan(f*x + e)^3))/(
f*tan(f*x + e))
```

Sympy [F]

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \int \sqrt{b\tan^3{(e+fx)}}\,dx$$

input
```
integrate((b*tan(f*x+e)**3)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.69

$$\int \sqrt{b\tan^3(e+fx)}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(fx+e)}\right)\right)}{}$$

input
```
integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e))
)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e))
)) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) -
sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - 8*sq
rt(b)*sqrt(tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01

$$\int \sqrt{b\tan^3(e+fx)}\,dx =$$

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{f}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{f}+\cdots\right.$$

$$+e))$$

input
```
integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*
sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/f + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/f +
 sqrt(2)*sqrt(abs(b))*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sq
rt(abs(b)) + abs(b))/f - sqrt(2)*sqrt(abs(b))*log(b*tan(f*x + e) - sqrt(2)
*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/f - 8*sqrt(b*tan(f*x + e))/f)
*sgn(tan(f*x + e))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \int \sqrt{b\tan\left(e+f\,x\right)^3}\,dx$$

input
```
int((b*tan(e + f*x)^3)^(1/2),x)
```

output
```
int((b*tan(e + f*x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\tan^3(e+fx)}\,dx = \frac{\sqrt{b}\left(2\sqrt{\tan\left(fx+e\right)}-\left(\int \frac{\sqrt{\tan(fx+e)}}{\tan(fx+e)}dx\right)f\right)}{f}$$

input
```
int((b*tan(f*x+e)^3)^(1/2),x)
```

output
```
(sqrt(b)*(2*sqrt(tan(e + f*x)) - int(sqrt(tan(e + f*x))/tan(e + f*x),x)*f)
)/f
```

3.10 $\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 192

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = {} & -\frac{2\tan(e+fx)}{f\sqrt{b\tan^3(e+fx)}} \\ & + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}} \\ & - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(e+fx)}}{1+\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}f\sqrt{b\tan^3(e+fx)}}\end{aligned}$$

output
```
-2*tan(f*x+e)/f/(b*tan(f*x+e)^3)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1
/2))*tan(f*x+e)^(3/2)*2^(1/2)/f/(b*tan(f*x+e)^3)^(1/2)-1/2*arctan(1+2^(1/2
)*tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)*2^(1/2)/f/(b*tan(f*x+e)^3)^(1/2)+1/2*
arctanh(2^(1/2)*tan(f*x+e)^(1/2)/(1+tan(f*x+e)))*tan(f*x+e)^(3/2)*2^(1/2)/
f/(b*tan(f*x+e)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx$$
$$= \frac{\tan(e+fx)\left(-2-\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)\sqrt[4]{-\tan^2(e+fx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(e+fx)}\right)\sqrt[4]{-\right.}{f\sqrt{b\tan^3(e+fx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[e + f*x]^3],x]
```

output
```
(Tan[e + f*x]*(-2 - ArcTan[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x]^2)^(1/4
) + ArcTanh[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x]^2)^(1/4)))/(f*Sqrt[b*T
an[e + f*x]^3])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{b\tan(e+fx)^3}}dx$$
$$\Big\downarrow \text{ 4141}$$
$$\frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{3}{2}}(e+fx)}dx}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan(e+fx)^{3/2}}dx}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 3955}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int \sqrt{\tan(e+fx)}dx-\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int \sqrt{\tan(e+fx)}dx-\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{\int \frac{\sqrt{\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}-\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\int \frac{\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}}{f}-\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\int \frac{\tan(e+fx)+1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}-\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}-\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)-\frac{1}{2}\int \frac{1-\tan(e+fx}{\tan^2(e+fx)+}\right.}{f}}{\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(e+fx)-1}d\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(e+fx)-1}d\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}\right.}{\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 217

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}-\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e\right.}{\sqrt{2}}\right.}{f}\right.}{\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+\right.}{\sqrt{2}}\right.}{f}\right.}{\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(e+fx)}+1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{ta\right.}{\sqrt{2}}\right.}{f}\right.}{\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 1103

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}\cdots\right)}{2\sqrt{2}}\cdots\right)}{f}\cdots\right)}{\sqrt{b\tan^3(e+fx)}}$$

input
```
Int[1/Sqrt[b*Tan[e + f*x]^3],x]
```

output
```
(((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]
+ Tan[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]]/(2*Sqrt[2]))/2))/f - 2/(f*Sqrt[Tan[e + f*x]]))*Tan[e + f*x]^(3/2))/
Sqrt[b*Tan[e + f*x]^3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{\tan(fx+e)\left(\sqrt{2}\,\sqrt{b\tan(fx+e)}\,\ln\left(-\frac{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^{2}}}{b\tan(fx+e)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\sqrt{2}\,\sqrt{b\tan(fx+e)}\,\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\cdots}{4f\sqrt{b\tan(fx+e)^{3}}\,\left(b^{2}\right)^{\frac{1}{4}}}$
default	$-\frac{\tan(fx+e)\left(\sqrt{2}\,\sqrt{b\tan(fx+e)}\,\ln\left(-\frac{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^{2}}}{b\tan(fx+e)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\sqrt{2}\,\sqrt{b\tan(fx+e)}\,\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\cdots}{4f\sqrt{b\tan(fx+e)^{3}}\,\left(b^{2}\right)^{\frac{1}{4}}}$

input
```
int(1/(b*tan(f*x+e)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*tan(f*x+e)*(2^(1/2)*(b*tan(f*x+e))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(f*
x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^2)^(1/4)*(b
*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(f*x+e))^(1/2)*ar
ctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))-2*2^(1/2)*(b*
tan(f*x+e))^(1/2)*arctan((-2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)
^(1/4))+8*(b^2)^(1/4))/(b*tan(f*x+e)^3)^(1/2)/(b^2)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx =$$
$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(fx+e)^3}}{\sqrt{b}}+\tan(fx+e)}{\tan(fx+e)}\right)\tan(fx+e)^2+2\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(fx+e)^3}}{\sqrt{b}}-\tan(fx+e)}{\tan(fx+e)}\right)\mathrm{ta}}{}$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b) + t
an(f*x + e))/tan(f*x + e))*tan(f*x + e)^2 + 2*sqrt(2)*sqrt(b)*arctan((sqrt
(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b) - tan(f*x + e))/tan(f*x + e))*tan(f*x +
 e)^2 - sqrt(2)*sqrt(b)*log((tan(f*x + e)^2 + sqrt(2)*sqrt(b*tan(f*x + e)^
3)/sqrt(b) + tan(f*x + e))/tan(f*x + e))*tan(f*x + e)^2 + sqrt(2)*sqrt(b)*
log((tan(f*x + e)^2 - sqrt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b) + tan(f*x + e
))/tan(f*x + e))*tan(f*x + e)^2 + 8*sqrt(b*tan(f*x + e)^3))/(b*f*tan(f*x +
 e)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(e + f*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(fx+e)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(fx+e)}+\tan(fx+e)+1\right)+\sqrt{2}\log}{\sqrt{b}}}{4f}$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt(2)*lo
g(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt(2)*sq
rt(tan(f*x + e)) + tan(f*x + e) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(f*x +
e))))/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = -\frac{1}{4}b^2\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{b^4f\operatorname{sgn}(\tan(fx+e))}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{b^4f\operatorname{sgn}(\tan(fx+e))}-\right.$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
+ 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^4*f*sgn(tan(f*x + e))) + 2*sqrt
(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(
f*x + e)))/sqrt(abs(b)))/(b^4*f*sgn(tan(f*x + e))) - sqrt(2)*abs(b)^(3/2)*
log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(
b^4*f*sgn(tan(f*x + e))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(f*x + e) - sqrt(
2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^4*f*sgn(tan(f*x + e))) +
 8/(sqrt(b*tan(f*x + e))*b^2*f*sgn(tan(f*x + e))))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)^3}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^3)^(1/2),x)
```

output
```
int(1/(b*tan(e + f*x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\tan^3(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}}{\tan(fx+e)^2}dx\right)}{b}$$

input
```
int(1/(b*tan(f*x+e)^3)^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(e + f*x))/tan(e + f*x)**2,x))/b
```

3.11 $\int \frac{1}{(b\tan^3(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 232

$$\int \frac{1}{(b\tan^3(e+fx))^{3/2}}\,dx = \frac{2}{3bf\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^2(e+fx)}{7bf\sqrt{b\tan^3(e+fx)}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(e+fx)}}{1+\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}bf\sqrt{b\tan^3(e+fx)}}$$

output
```
2/3/b/f/(b*tan(f*x+e)^3)^(1/2)-2/7*cot(f*x+e)^2/b/f/(b*tan(f*x+e)^3)^(1/2)
+1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)*2^(1/2)/b/f/(b*t
an(f*x+e)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(f*x+e)^(1/2))*tan(f*x+e)^(3/2)
*2^(1/2)/b/f/(b*tan(f*x+e)^3)^(1/2)+1/2*arctanh(2^(1/2)*tan(f*x+e)^(1/2)/(
1+tan(f*x+e)))*tan(f*x+e)^(3/2)*2^(1/2)/b/f/(b*tan(f*x+e)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{14-6\cot^2(e+fx)-21\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)\left(-\tan^2(e+fx)\right)^{3/4}-2}{21bf\sqrt{b\tan^3(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x]^3)^(-3/2),x]
```

output
```
(14 - 6*Cot[e + f*x]^2 - 21*ArcTan[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x]
^2)^(3/4) - 21*ArcTanh[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x]^2)^(3/4))/(
21*b*f*Sqrt[b*Tan[e + f*x]^3])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.86, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(e+fx)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{9}{2}}(e+fx)}dx}{b\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\int\frac{1}{\tan(e+fx)^{9/2}}dx}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 3955

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int\frac{1}{\tan^{\frac{5}{2}}(e+fx)}dx-\frac{2}{7f\tan^{\frac{7}{2}}(e+fx)}\right)}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int\frac{1}{\tan(e+fx)^{5/2}}dx-\frac{2}{7f\tan^{\frac{7}{2}}(e+fx)}\right)}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 3955

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\int\frac{1}{\sqrt{\tan(e+fx)}}dx+\frac{2}{3f\tan^{\frac{3}{2}}(e+fx)}-\frac{2}{7f\tan^{\frac{7}{2}}(e+fx)}\right)}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\int\frac{1}{\sqrt{\tan(e+fx)}}dx+\frac{2}{3f\tan^{\frac{3}{2}}(e+fx)}-\frac{2}{7f\tan^{\frac{7}{2}}(e+fx)}\right)}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 3957

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{\int\frac{1}{\sqrt{\tan(e+fx)}\left(\tan^2(e+fx)+1\right)}d\tan(e+fx)}{f}+\frac{2}{3f\tan^{\frac{3}{2}}(e+fx)}-\frac{2}{7f\tan^{\frac{7}{2}}(e+fx)}\right)}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 266

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\int\frac{1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}}{f}+\frac{2}{3f\tan^{\frac{3}{2}}(e+fx)}-\frac{2}{7f\tan^{\frac{7}{2}}(e+fx)}\right)}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 755

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int\frac{\tan(e+fx)+1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}+\frac{2}{3f\tan^{\frac{3}{2}}(e+fx)}-\frac{2}{7f\tan^{\frac{7}{2}}(e+fx)}\right)}{b\sqrt{b\tan^3(e+fx)}}$$

$\Big\downarrow$ 1476

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int\frac{1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\right.}{f}}{b\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 1082

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(e+fx)-1}d\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}\right)\right)}{f}+\right.}{b\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 217

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)\right)}{f}+\frac{2}{3f\tan^{\frac{3}{2}}(e+fx)}-\frac{}{7f\right.}}{b\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e\right.}{\sqrt{2}}\right.}{f}\right.}{b\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}\right.}{\sqrt{2}}\right.}{f}\right.}{b\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(e+fx)}+1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+\right.}{\sqrt{2}}\right.}{f}\right.}{b\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 1103

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(e+fx)-\sqrt{2}\sqrt{\tan\right.}{2\sqrt{2}}\right.}{f}\right.}{b\sqrt{b\tan^3(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^3)^(-3/2),x]
```

output
```
(((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*
x]] + Tan[e + f*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
 f*x]]/(2*Sqrt[2]))/2))/f - 2/(7*f*Tan[e + f*x]^(7/2)) + 2/(3*f*Tan[e + f*
x]^(3/2)))*Tan[e + f*x]^(3/2))/(b*Sqrt[b*Tan[e + f*x]^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\tan(fx+e)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(fx+e))^{\frac{7}{2}}\ln\left(\frac{b\tan(fx+e)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}{-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+b\tan(fx+e)+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(fx+e))}{84f\,b^4}$
default	$\frac{\tan(fx+e)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(fx+e))^{\frac{7}{2}}\ln\left(\frac{b\tan(fx+e)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}{-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+b\tan(fx+e)+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(fx+e))}{84f\,b^4}$

input
```
int(1/(b*tan(f*x+e)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84/f*tan(f*x+e)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*ln((b*t
an(f*x+e)+(b^2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2))/(-(b^2)^(1
/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+b*tan(f*x+e)+(b^2)^(1/2)))+42*(b^2)^(1/4)
*2^(1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(
1/4))/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(f*x+e))^(7/2)*arctan((2^(
1/2)*(b*tan(f*x+e))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+56*b^4*tan(f*x+e)^2-24
*b^4)/(b*tan(f*x+e)^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{42\,\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(fx+e)^3}}{\sqrt{b}}+\tan(fx+e)}{\tan(fx+e)}\right)\tan\left(fx+e\right)^5+42\,\sqrt{2}\sqrt{b}\arctan\left(\cdots\right.}{\cdots}$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/84*(42*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b) +
tan(f*x + e))/tan(f*x + e))*tan(f*x + e)^5 + 42*sqrt(2)*sqrt(b)*arctan((sq
rt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b) - tan(f*x + e))/tan(f*x + e))*tan(f*x
 + e)^5 + 21*sqrt(2)*sqrt(b)*log((tan(f*x + e)^2 + sqrt(2)*sqrt(b*tan(f*x
+ e)^3)/sqrt(b) + tan(f*x + e))/tan(f*x + e))*tan(f*x + e)^5 - 21*sqrt(2)*
sqrt(b)*log((tan(f*x + e)^2 - sqrt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b) + tan
(f*x + e))/tan(f*x + e))*tan(f*x + e)^5 + 8*sqrt(b*tan(f*x + e)^3)*(7*tan(
f*x + e)^2 - 3))/(b^2*f*tan(f*x + e)^5)
```

Sympy [F]

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^{3}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**3)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**3)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(fx+e)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\right.}\right.}{b^{\frac{3}{2}}}$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="maxima")
```

output
```
1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) + sqrt(2)
*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) - sqrt(2)*log(-sqrt(2)
*sqrt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(3/2) + 8*(21*sqrt(tan(f*x + e)
) + 7/tan(f*x + e)^(3/2) - 3/tan(f*x + e)^(7/2))/b^(3/2) - 168*sqrt(tan(f*
x + e))/b^(3/2))/f
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{1}{84}\,b^4\left(\frac{42\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{b^6 f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{42\sqrt{2}\sqrt{|b|}\arctan\left(-\right.}{b^6 f\operatorname{sgn}\left(\mathrm{ta}\right.}\right.$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

output
```
1/84*b^4*(42*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
 + 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^6*f*sgn(tan(f*x + e))) + 42*sq
rt(2)*sqrt(abs(b))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*ta
n(f*x + e)))/sqrt(abs(b)))/(b^6*f*sgn(tan(f*x + e))) + 21*sqrt(2)*sqrt(abs
(b))*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(
b))/(b^6*f*sgn(tan(f*x + e))) - 21*sqrt(2)*sqrt(abs(b))*log(b*tan(f*x + e)
 - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^6*f*sgn(tan(f*x
+ e))) + 8*(7*b^2*tan(f*x + e)^2 - 3*b^2)/(sqrt(b*tan(f*x + e))*b^7*f*sgn(
tan(f*x + e))*tan(f*x + e)^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^3)^(3/2),x)
```

output
```
int(1/(b*tan(e + f*x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}}{\tan(fx+e)^5}dx\right)}{b^2}$$

input
```
int(1/(b*tan(f*x+e)^3)^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(e + f*x))/tan(e + f*x)**5,x))/b**2
```

3.12 $\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 299

$$\begin{aligned}\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx &= -\frac{2\cot(e+fx)}{5b^2f\sqrt{b\tan^3(e+fx)}} \\ &+ \frac{2\cot^3(e+fx)}{9b^2f\sqrt{b\tan^3(e+fx)}} - \frac{2\cot^5(e+fx)}{13b^2f\sqrt{b\tan^3(e+fx)}} \\ &+ \frac{2\tan(e+fx)}{b^2f\sqrt{b\tan^3(e+fx)}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}} \\ &+ \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}} \\ &- \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(e+fx)}}{1+\tan(e+fx)}\right)\tan^{\frac{3}{2}}(e+fx)}{\sqrt{2}b^2f\sqrt{b\tan^3(e+fx)}}\end{aligned}$$

output

```
-2/5*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^3)^(1/2)+2/9*cot(f*x+e)^3/b^2/f/(b*tan
(f*x+e)^3)^(1/2)-2/13*cot(f*x+e)^5/b^2/f/(b*tan(f*x+e)^3)^(1/2)+2*tan(f*x+
e)/b^2/f/(b*tan(f*x+e)^3)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(f*x+e)^(1/2))*ta
n(f*x+e)^(3/2)*2^(1/2)/b^2/f/(b*tan(f*x+e)^3)^(1/2)+1/2*arctan(1+2^(1/2)*t
an(f*x+e)^(1/2))*tan(f*x+e)^(3/2)*2^(1/2)/b^2/f/(b*tan(f*x+e)^3)^(1/2)-1/2
*arctanh(2^(1/2)*tan(f*x+e)^(1/2)/(1+tan(f*x+e)))*tan(f*x+e)^(3/2)*2^(1/2)
/b^2/f/(b*tan(f*x+e)^3)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.46

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{-234\cot(e+fx)+130\cot^3(e+fx)-90\cot^5(e+fx)+585\text{arctanh}\left(\sqrt[4]{-\text{ta}\cdots}\right.}{\cdots}$$

input
```
Integrate[(b*Tan[e + f*x]^3)^(-5/2),x]
```

output
```
(-234*Cot[e + f*x] + 130*Cot[e + f*x]^3 - 90*Cot[e + f*x]^5 + 585*ArcTanh[
(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x])^(5/4)*Tan[e + f*x]^(1/4) + 1170*T
an[e + f*x] + 585*ArcTan[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x])^(1/4)*Ta
n[e + f*x]^(5/4))/(585*b^2*f*Sqrt[b*Tan[e + f*x]^3])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.78, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(b\tan(e+fx)^3\right)^{5/2}}dx$$

$$\Big\downarrow \ 4141$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan^{\frac{15}{2}}(e+fx)}dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\int \frac{1}{\tan(e+fx)^{15/2}}dx}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int \frac{1}{\tan^{\frac{11}{2}}(e+fx)}dx-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int \frac{1}{\tan(e+fx)^{11/2}}dx-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\int \frac{1}{\tan^{\frac{7}{2}}(e+fx)}dx+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\int \frac{1}{\tan(e+fx)^{7/2}}dx+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int \frac{1}{\tan^{\frac{3}{2}}(e+fx)}dx-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(-\int \frac{1}{\tan(e+fx)^{3/2}}dx-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\int \sqrt{\tan(e+fx)}dx-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}+\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\int \sqrt{\tan(e+fx)}dx-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}+\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 3957

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{\int \frac{\sqrt{\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}+\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 266

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\int \frac{\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}}{f}-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{13}{2}}(e+fx)}+\frac{2}{f\sqrt{\tan(e+fx)}}\right)}{b^2\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 826

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\int \frac{\tan(e+fx)+1}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}-\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{2}{9f\tan^{\frac{9}{2}}(e+fx)}-\frac{2}{13f\tan^{\frac{1}{2}}}\right.}{b^2\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 1476

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)-\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\right.}{f}\right.}{b^2\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 1082

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(e+fx)-1}d\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(e+fx)-1}d\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}-\right.}{b^2\sqrt{b\tan^3(e+fx)}}$$

$\downarrow$ 217

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(e+fx)}{\tan^2(e+fx)+1}d\sqrt{\tan(e+fx)}\right)}{f}-\frac{2}{5f\tan^{\frac{5}{2}}(e+fx)}+\frac{}{9f}\right.}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\downarrow \text{1479}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+\right.}{\sqrt{2}}\right.}{f}\right.}{b^2\sqrt{b\tan^3(e+fx}}$$

$$\downarrow \text{25}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+f\right.}{\sqrt{2}}\right.}{f}\right.}{b^2\sqrt{b\tan^3(e+fx)}}$$

$$\downarrow \text{27}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(e+fx)}}{\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(e+fx)}+1}{\tan(e+fx)+\sqrt{2}\sqrt{\tan(e+fx)}+1}d\sqrt{\tan(e+fx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}}\right.}{f}\right.}{b^2\sqrt{b\tan^3(e+f}}$$

$$\downarrow \text{1103}$$

$$\frac{\tan^{\frac{3}{2}}(e+fx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(e+fx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(e+fx)-\sqrt{2}\sqrt{\tan(e+fx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(e+fx)+\sqrt{2}\sqrt{\tan\right.}{2\sqrt{2}}\right.}{f}\right.}{b^2\sqrt{b\tan^3(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^3)^(-5/2),x]
```

output
```
(((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[e + f*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] +
 Tan[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e +
f*x]]/(2*Sqrt[2]))/2))/f - 2/(13*f*Tan[e + f*x]^(13/2)) + 2/(9*f*Tan[e + f
*x]^(9/2)) - 2/(5*f*Tan[e + f*x]^(5/2)) + 2/(f*Sqrt[Tan[e + f*x]]))*Tan[e
+ f*x]^(3/2))/(b^2*Sqrt[b*Tan[e + f*x]^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\tan(fx+e)\left(585\sqrt{2}\,(b\tan(fx+e))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(fx+e))^{\frac{13}{2}}\,\mathrm{arct}}{}$
default	$\frac{\tan(fx+e)\left(585\sqrt{2}\,(b\tan(fx+e))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}-b\tan(fx+e)-\sqrt{b^2}}{b\tan(fx+e)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(fx+e)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(fx+e))^{\frac{13}{2}}\,\mathrm{arct}}{}$

input
```
int(1/(b*tan(f*x+e)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2340/f*tan(f*x+e)/b^6*(585*2^(1/2)*(b*tan(f*x+e))^(13/2)*ln(-((b^2)^(1/4
)*(b*tan(f*x+e))^(1/2)*2^(1/2)-b*tan(f*x+e)-(b^2)^(1/2))/(b*tan(f*x+e)+(b^
2)^(1/4)*(b*tan(f*x+e))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(f*
x+e))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4)
)+1170*2^(1/2)*(b*tan(f*x+e))^(13/2)*arctan((2^(1/2)*(b*tan(f*x+e))^(1/2)-
(b^2)^(1/4))/(b^2)^(1/4))+4680*b^6*tan(f*x+e)^6*(b^2)^(1/4)-936*b^6*tan(f*
x+e)^4*(b^2)^(1/4)+520*b^6*tan(f*x+e)^2*(b^2)^(1/4)-360*b^6*(b^2)^(1/4))/(
b*tan(f*x+e)^3)^(5/2)/(b^2)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{1170\,\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(fx+e)^3}}{\sqrt{b}}+\tan(fx+e)}{\tan(fx+e)}\right)\tan\left(fx+e\right)^8 + 1170\,\sqrt{2}\sqrt{b}\arcta}{}$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/2340*(1170*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b
) + tan(f*x + e))/tan(f*x + e))*tan(f*x + e)^8 + 1170*sqrt(2)*sqrt(b)*arct
an((sqrt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(b) - tan(f*x + e))/tan(f*x + e))*t
an(f*x + e)^8 - 585*sqrt(2)*sqrt(b)*log((tan(f*x + e)^2 + sqrt(2)*sqrt(b*t
an(f*x + e)^3)/sqrt(b) + tan(f*x + e))/tan(f*x + e))*tan(f*x + e)^8 + 585*
sqrt(2)*sqrt(b)*log((tan(f*x + e)^2 - sqrt(2)*sqrt(b*tan(f*x + e)^3)/sqrt(
b) + tan(f*x + e))/tan(f*x + e))*tan(f*x + e)^8 + 8*(585*tan(f*x + e)^6 -
117*tan(f*x + e)^4 + 65*tan(f*x + e)^2 - 45)*sqrt(b*tan(f*x + e)^3))/(b^3*
f*tan(f*x + e)^8)
```

Sympy [F]

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan^3\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**3)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**3)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{\frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(fx+e)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(fx+e)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\right.}{b^{\frac{5}{2}}}}{}$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="maxima")
```

output
```
1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(f*x + e)))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(f*x + e)))) - sqrt
(2)*log(sqrt(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1) + sqrt(2)*log(-sqrt
(2)*sqrt(tan(f*x + e)) + tan(f*x + e) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(
tan(f*x + e)) - 117*sqrt(b)/tan(f*x + e)^(5/2) + 65*sqrt(b)/tan(f*x + e)^(
9/2) - 45*sqrt(b)/tan(f*x + e)^(13/2))/b^3)/f
```

Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{1}{2340}\,b^6\left(\frac{1170\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{b^{10}f\operatorname{sgn}\left(\tan\left(fx+e\right)\right)}+\frac{1170\sqrt{2}|b|^{\frac{3}{2}}\arctan}{b^{10}f\operatorname{sg}}\right.$$

input
```
integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

output
```
1/2340*b^6*(1170*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(b)) + 2*sqrt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^10*f*sgn(tan(f*x + e))) +
1170*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sq
rt(b*tan(f*x + e)))/sqrt(abs(b)))/(b^10*f*sgn(tan(f*x + e))) - 585*sqrt(2)
*abs(b)^(3/2)*log(b*tan(f*x + e) + sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b
)) + abs(b))/(b^10*f*sgn(tan(f*x + e))) + 585*sqrt(2)*abs(b)^(3/2)*log(b*t
an(f*x + e) - sqrt(2)*sqrt(b*tan(f*x + e))*sqrt(abs(b)) + abs(b))/(b^10*f*
sgn(tan(f*x + e))) + 8*(585*b^6*tan(f*x + e)^6 - 117*b^6*tan(f*x + e)^4 +
65*b^6*tan(f*x + e)^2 - 45*b^6)/(sqrt(b*tan(f*x + e))*b^14*f*sgn(tan(f*x +
 e))*tan(f*x + e)^6))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)^3\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^3)^(5/2),x)
```

output
```
int(1/(b*tan(e + f*x)^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(b\tan^3(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}}{\tan(fx+e)^8}dx\right)}{b^3}$$

input
```
int(1/(b*tan(f*x+e)^3)^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(e + f*x))/tan(e + f*x)**8,x))/b**3
```

3.13 $\int \left(b \tan^4(e+fx)\right)^{5/2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 182

$$\int \left(b \tan^4(e+fx)\right)^{5/2} \, dx = \frac{b^2 \cot(e+fx)\sqrt{b \tan^4(e+fx)}}{f} - b^2 x \cot^2(e+fx)\sqrt{b \tan^4(e+fx)} - \frac{b^2 \tan(e+fx)\sqrt{b \tan^4(e+fx)}}{3f} + \frac{b^2 \tan^3(e+fx)\sqrt{b \tan^4(e+fx)}}{5f} - \frac{b^2 \tan^5(e+fx)\sqrt{b \tan^4(e+fx)}}{7f} + \frac{b^2 \tan^7(e+fx)\sqrt{b \tan^4(e+fx)}}{9f}$$

output
```
b^2*cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-b^2*x*cot(f*x+e)^2*(b*tan(f*x+e)^4
)^(1/2)-1/3*b^2*tan(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f+1/5*b^2*tan(f*x+e)^3*(
b*tan(f*x+e)^4)^(1/2)/f-1/7*b^2*tan(f*x+e)^5*(b*tan(f*x+e)^4)^(1/2)/f+1/9*
b^2*tan(f*x+e)^7*(b*tan(f*x+e)^4)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.47

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx=\frac{\cot(e+fx)\left(35-45\cot^2(e+fx)+63\cot^4(e+fx)-105\cot^6(e+fx)+315\cot^8(e+fx)\right.}{315f}$$

input
```
Integrate[(b*Tan[e + f*x]^4)^(5/2),x]
```

output
```
(Cot[e + f*x]*(35 - 45*Cot[e + f*x]^2 + 63*Cot[e + f*x]^4 - 105*Cot[e + f*
x]^6 + 315*Cot[e + f*x]^8 - 315*ArcTan[Tan[e + f*x]]*Cot[e + f*x]^9)*(b*Ta
n[e + f*x]^4)^(5/2))/(315*f)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.55, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^4\right)^{5/2}dx$$

$$\Big\downarrow \text{4141}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\int \tan^{10}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\int \tan(e+fx)^{10}dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\frac{\tan^9(e+fx)}{9f}-\int\tan^8(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\frac{\tan^9(e+fx)}{9f}-\int\tan(e+fx)^8dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\int\tan^6(e+fx)dx+\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\int\tan(e+fx)^6dx+\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(-\int\tan^4(e+fx)dx+\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}+\frac{\tan^5(e+fx)}{5f}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(-\int\tan(e+fx)^4dx+\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}+\frac{\tan^5(e+fx)}{5f}\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\int\tan^2(e+fx)dx+\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\int\tan(e+fx)^2dx+\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(-\int 1dx+\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}\right)$$

$$\Big\downarrow \text{24}$$

$$b^2\left(\frac{\tan^9(e+fx)}{9f}-\frac{\tan^7(e+fx)}{7f}+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}-x\right)\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

input
```
Int[(b*Tan[e + f*x]^4)^(5/2),x]
```

output
```
b^2*Cot[e + f*x]^2*Sqrt[b*Tan[e + f*x]^4]*(-x + Tan[e + f*x]/f - Tan[e + f
*x]^3/(3*f) + Tan[e + f*x]^5/(5*f) - Tan[e + f*x]^7/(7*f) + Tan[e + f*x]^9
/(9*f))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

method	result
derivativedivides	$-\frac{\left(b\tan(fx+e)^{4}\right)^{\frac{5}{2}}\left(-35\tan(fx+e)^{9}+45\tan(fx+e)^{7}-63\tan(fx+e)^{5}+105\tan(fx+e)^{3}+315\arctan(\tan(fx+e))-3\right.}{315f\tan(fx+e)^{10}}$
default	$-\frac{\left(b\tan(fx+e)^{4}\right)^{\frac{5}{2}}\left(-35\tan(fx+e)^{9}+45\tan(fx+e)^{7}-63\tan(fx+e)^{5}+105\tan(fx+e)^{3}+315\arctan(\tan(fx+e))-3\right.}{315f\tan(fx+e)^{10}}$
risch	$\frac{b^{2}\left(e^{2i(fx+e)}+1\right)^{2}\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^{4}}{\left(e^{2i(fx+e)}+1\right)^{4}}}\,x}{\left(e^{2i(fx+e)}-1\right)^{2}}-\frac{2ib^{2}\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^{4}}{\left(e^{2i(fx+e)}+1\right)^{4}}}\left(1575e^{16i(fx+e)}+6300e^{14i(fx+e)}+21000e^{12i(fx\right.}{315(e^{2}}$

input
```
int((b*tan(f*x+e)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315/f*(b*tan(f*x+e)^4)^(5/2)*(-35*tan(f*x+e)^9+45*tan(f*x+e)^7-63*tan(f
*x+e)^5+105*tan(f*x+e)^3+315*arctan(tan(f*x+e))-315*tan(f*x+e))/tan(f*x+e)
^10
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.53

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx=\frac{\left(35\,b^2\tan\left(fx+e\right)^9-45\,b^2\tan\left(fx+e\right)^7+63\,b^2\tan\left(fx+e\right)^5-105\,b^2\tan\left(fx+e\right)^3-315\right.}{315\,f\tan\left(fx+e\right)^2}$$

input
```
integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 - 45*b^2*tan(f*x + e)^7 + 63*b^2*tan(f*x + e)
^5 - 105*b^2*tan(f*x + e)^3 - 315*b^2*f*x + 315*b^2*tan(f*x + e))*sqrt(b*t
an(f*x + e)^4)/(f*tan(f*x + e)^2)
```

Sympy [F]

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx=\int \left(b\tan^4{(e+fx)}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**4)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**4)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.43

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx=\frac{35\,b^{\frac{5}{2}}\tan{(fx+e)}^9-45\,b^{\frac{5}{2}}\tan{(fx+e)}^7+63\,b^{\frac{5}{2}}\tan{(fx+e)}^5-105\,b^{\frac{5}{2}}\tan{(fx+e)}^3-31}{315\,f}$$

input
```
integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")
```

output
```
1/315*(35*b^(5/2)*tan(f*x + e)^9 - 45*b^(5/2)*tan(f*x + e)^7 + 63*b^(5/2)*
tan(f*x + e)^5 - 105*b^(5/2)*tan(f*x + e)^3 - 315*(f*x + e)*b^(5/2) + 315*
b^(5/2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.47

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx=-\frac{1}{315}\,b^{\frac{5}{2}}\left(\frac{315\,(fx+e)}{f}-\frac{35\,f^8\tan{(fx+e)}^9-45\,f^8\tan{(fx+e)}^7+63\,f^8\tan{(fx+e)}^5-105\,f^8\tan{(fx}}{f^9}\right.$$

input
```
integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/315*b^(5/2)*(315*(f*x + e)/f - (35*f^8*tan(f*x + e)^9 - 45*f^8*tan(f*x
+ e)^7 + 63*f^8*tan(f*x + e)^5 - 105*f^8*tan(f*x + e)^3 + 315*f^8*tan(f*x
+ e))/f^9)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx=\int \left(b\tan(e+f\,x)^4\right)^{5/2}dx$$

input
```
int((b*tan(e + f*x)^4)^(5/2),x)
```

output
```
int((b*tan(e + f*x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \left(b\tan^4(e+fx)\right)^{5/2}\,dx=\frac{\sqrt{b}\,b^2\left(35\tan\left(fx+e\right)^9-45\tan\left(fx+e\right)^7+63\tan\left(fx+e\right)^5-105\tan\left(fx+e\right)^3+315\tan}{315f}$$

input
```
int((b*tan(f*x+e)^4)^(5/2),x)
```

output
```
(sqrt(b)*b**2*(35*tan(e + f*x)**9 - 45*tan(e + f*x)**7 + 63*tan(e + f*x)**
5 - 105*tan(e + f*x)**3 + 315*tan(e + f*x) - 315*f*x))/(315*f)
```

3.14 $\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx=\frac{b\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-bx\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}-\frac{b\tan(e+fx)\sqrt{b\tan^4(e+fx)}}{3f}+\frac{b\tan^3(e+fx)\sqrt{b\tan^4(e+fx)}}{5f}$$

output
```
b*cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-b*x*cot(f*x+e)^2*(b*tan(f*x+e)^4)^(1
/2)-1/3*b*tan(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f+1/5*b*tan(f*x+e)^3*(b*tan(f*
x+e)^4)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx=\frac{\cot(e+fx)\left(3-5\cot^2(e+fx)+15\cot^4(e+fx)-15\arctan(\tan(e+fx))\cot^5(e+fx)\right)(}{15f}$$

input
```
Integrate[(b*Tan[e + f*x]^4)^(3/2),x]
```

output
```
(Cot[e + f*x]*(3 - 5*Cot[e + f*x]^2 + 15*Cot[e + f*x]^4 - 15*ArcTan[Tan[e
+ f*x]]*Cot[e + f*x]^5)*(b*Tan[e + f*x]^4)^(3/2))/(15*f)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{4141}$$

$$b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\int \tan^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\int \tan(e+fx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\frac{\tan^5(e+fx)}{5f}-\int \tan^4(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\frac{\tan^5(e+fx)}{5f}-\int \tan(e+fx)^4dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\int \tan^2(e+fx)dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\int \tan(e+fx)^2dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3954}$$

$$b\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(-\int 1dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}\right)$$

$$\Big\downarrow \text{24}$$

$$b\left(\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}-x\right)\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

input
```
Int[(b*Tan[e + f*x]^4)^(3/2),x]
```

output
```
b*Cot[e + f*x]^2*Sqrt[b*Tan[e + f*x]^4]*(-x + Tan[e + f*x]/f - Tan[e + f*x
]^3/(3*f) + Tan[e + f*x]^5/(5*f))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{\left(b\tan(fx+e)^4\right)^{\frac{3}{2}}\left(-3\tan(fx+e)^5+5\tan(fx+e)^3+15\arctan(\tan(fx+e))-15\tan(fx+e)\right)}{15f\tan(fx+e)^6}$
default	$-\frac{\left(b\tan(fx+e)^4\right)^{\frac{3}{2}}\left(-3\tan(fx+e)^5+5\tan(fx+e)^3+15\arctan(\tan(fx+e))-15\tan(fx+e)\right)}{15f\tan(fx+e)^6}$
risch	$\frac{b\left(e^{2i(fx+e)}+1\right)^2\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\,x}{\left(e^{2i(fx+e)}-1\right)^2}-\frac{2ib\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\left(45\,e^{8i(fx+e)}+90\,e^{6i(fx+e)}+140\,e^{4i(fx+e)}+70\,e^{2i(f\right.}{15\left(e^{2i(fx+e)}-1\right)^2\left(e^{2i(fx+e)}+1\right)^3f}$

input
```
int((b*tan(f*x+e)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f*(b*tan(f*x+e)^4)^(3/2)*(-3*tan(f*x+e)^5+5*tan(f*x+e)^3+15*arctan(t
an(f*x+e))-15*tan(f*x+e))/tan(f*x+e)^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx=\frac{\left(3\,b\tan\left(fx+e\right)^5-5\,b\tan\left(fx+e\right)^3-15\,bfx+15\,b\tan\left(fx+e\right)\right)\sqrt{b\tan\left(fx+e\right)^4}}{15\,f\tan\left(fx+e\right)^2}$$

input
```
integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*b*tan(f*x + e)^5 - 5*b*tan(f*x + e)^3 - 15*b*f*x + 15*b*tan(f*x +
e))*sqrt(b*tan(f*x + e)^4)/(f*tan(f*x + e)^2)
```

Sympy [F]

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx=\int \left(b\tan^4\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**4)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx=\frac{3\,b^{\frac{3}{2}}\tan\left(fx+e\right)^5-5\,b^{\frac{3}{2}}\tan\left(fx+e\right)^3-15\,(fx+e)b^{\frac{3}{2}}+15\,b^{\frac{3}{2}}\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

output
```
1/15*(3*b^(3/2)*tan(f*x + e)^5 - 5*b^(3/2)*tan(f*x + e)^3 - 15*(f*x + e)*b
^(3/2) + 15*b^(3/2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx = -\frac{1}{15}\,b^{\frac{3}{2}}\left(\frac{15\,(fx+e)}{f}-\frac{3\,f^4\tan\left(fx+e\right)^5-5\,f^4\tan\left(fx+e\right)^3+15\,f^4\tan\left(fx+e\right)}{f^5}\right)$$

input
```
integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/15*b^(3/2)*(15*(f*x + e)/f - (3*f^4*tan(f*x + e)^5 - 5*f^4*tan(f*x + e)
^3 + 15*f^4*tan(f*x + e))/f^5)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx = \int {\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^4\right)}^{3/2} \,d x$$

input
```
int((b*tan(e + f*x)^4)^(3/2),x)
```

output
```
int((b*tan(e + f*x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int \left(b\tan^4(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}\,b\left(3\tan\left(fx+e\right)^5-5\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)-15fx\right)}{15f}$$

input
```
int((b*tan(f*x+e)^4)^(3/2),x)
```

output
```
(sqrt(b)*b*(3*tan(e + f*x)**5 - 5*tan(e + f*x)**3 + 15*tan(e + f*x) - 15*f
*x))/(15*f)
```

3.15 $\int \sqrt{b \tan^4(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \sqrt{b \tan^4(e+fx)}\,dx=\frac{\cot(e+fx)\sqrt{b\tan^4(e+fx)}}{f}-x\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

output
```
cot(f*x+e)*(b*tan(f*x+e)^4)^(1/2)/f-x*cot(f*x+e)^2*(b*tan(f*x+e)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \sqrt{b \tan^4(e+fx)}\,dx=-\frac{\cot(e+fx)(-1+\arctan(\tan(e+fx))\cot(e+fx))\sqrt{b\tan^4(e+fx)}}{f}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]^4],x]
```

output
```
-((Cot[e + f*x]*(-1 + ArcTan[Tan[e + f*x]]*Cot[e + f*x])*Sqrt[b*Tan[e + f*
x]^4])/f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^4(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4141}$$

$$\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\int \tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\int \tan(e+fx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}\left(\frac{\tan(e+fx)}{f}-\int 1dx\right)$$

$$\Big\downarrow \text{24}$$

$$\left(\frac{\tan(e+fx)}{f}-x\right)\cot^2(e+fx)\sqrt{b\tan^4(e+fx)}$$

input
```
Int[Sqrt[b*Tan[e + f*x]^4],x]
```

output
```
Cot[e + f*x]^2*Sqrt[b*Tan[e + f*x]^4]*(-x + Tan[e + f*x]/f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{\sqrt{b\tan(fx+e)^{4}}\,(-\tan(fx+e)+\arctan(\tan(fx+e)))}{f\tan(fx+e)^{2}}$	42
default	$-\frac{\sqrt{b\tan(fx+e)^{4}}\,(-\tan(fx+e)+\arctan(\tan(fx+e)))}{f\tan(fx+e)^{2}}$	42
risch	$\frac{\sqrt{\frac{b\left(\mathrm{e}^{2i(fx+e)}-1\right)^{4}}{\left(\mathrm{e}^{2i(fx+e)}+1\right)^{4}}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)^{2}x}{\left(\mathrm{e}^{2i(fx+e)}-1\right)^{2}}-\frac{2i\sqrt{\frac{b\left(\mathrm{e}^{2i(fx+e)}-1\right)^{4}}{\left(\mathrm{e}^{2i(fx+e)}+1\right)^{4}}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)}{\left(\mathrm{e}^{2i(fx+e)}-1\right)^{2}f}$	120

input
```
int((b*tan(f*x+e)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(b*tan(f*x+e)^4)^(1/2)*(-tan(f*x+e)+arctan(tan(f*x+e)))/tan(f*x+e)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \sqrt{b\tan^4(e+fx)}\,dx = -\frac{\sqrt{b\tan\left(fx+e\right)^4}(fx-\tan\left(fx+e\right))}{f\tan\left(fx+e\right)^2}$$

input
```
integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*tan(f*x + e)^4)*(f*x - tan(f*x + e))/(f*tan(f*x + e)^2)
```

Sympy [F]

$$\int \sqrt{b\tan^4(e+fx)}\,dx = \int \sqrt{b\tan^4\left(e+fx\right)}\,dx$$

input
```
integrate((b*tan(f*x+e)**4)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^4(e+fx)}\,dx = -\frac{(fx+e)\sqrt{b}-\sqrt{b}\tan\left(fx+e\right)}{f}$$

input
```
integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

output
```
-((f*x + e)*sqrt(b) - sqrt(b)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^4(e+fx)}\,dx = -\sqrt{b}\left(\frac{fx+e}{f} - \frac{\tan(fx+e)}{f}\right)$$

input
```
integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*((f*x + e)/f - tan(f*x + e)/f)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^4(e+fx)}\,dx = \int \sqrt{b\tan(e+f\,x)^4}\,dx$$

input
```
int((b*tan(e + f*x)^4)^(1/2),x)
```

output
```
int((b*tan(e + f*x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.34

$$\int \sqrt{b\tan^4(e+fx)}\,dx = \frac{\sqrt{b}\,(\tan(fx+e) - fx)}{f}$$

input
```
int((b*tan(f*x+e)^4)^(1/2),x)
```

output
```
(sqrt(b)*(tan(e + f*x) - f*x))/f
```

3.16 $\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = -\frac{\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}} - \frac{x\tan^2(e+fx)}{\sqrt{b\tan^4(e+fx)}}$$

output
```
-tan(f*x+e)/f/(b*tan(f*x+e)^4)^(1/2)-x*tan(f*x+e)^2/(b*tan(f*x+e)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)\tan(e+fx)}{f\sqrt{b\tan^4(e+fx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[e + f*x]^4],x]
```

output
```
-((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*Sqrt[
b*Tan[e + f*x]^4]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4141, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(e+fx)^4}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^2(e+fx)\int \cot^2(e+fx)dx}{\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx}{\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(-\int 1dx-\frac{\cot(e+fx)}{f}\right)}{\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^2(e+fx)\left(-\frac{\cot(e+fx)}{f}-x\right)}{\sqrt{b\tan^4(e+fx)}}$$

input
```
Int[1/Sqrt[b*Tan[e + f*x]^4],x]
```

output
```
((-x - Cot[e + f*x]/f)*Tan[e + f*x]^2)/Sqrt[b*Tan[e + f*x]^4]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{\tan(fx+e)(\arctan(\tan(fx+e))\tan(fx+e)+1)}{f\sqrt{b\tan(fx+e)^4}}$	40
default	$-\frac{\tan(fx+e)(\arctan(\tan(fx+e))\tan(fx+e)+1)}{f\sqrt{b\tan(fx+e)^4}}$	40
risch	$\frac{\left(e^{2i(fx+e)}-1\right)^2 x}{\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\left(e^{2i(fx+e)}+1\right)^2}+\frac{2i\left(e^{2i(fx+e)}-1\right)}{\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^4}{\left(e^{2i(fx+e)}+1\right)^4}}\left(e^{2i(fx+e)}+1\right)^2 f}$	120

input
```
int(1/(b*tan(f*x+e)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*tan(f*x+e)*(arctan(tan(f*x+e))*tan(f*x+e)+1)/(b*tan(f*x+e)^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = -\frac{\sqrt{b\tan{(fx+e)}^4}(fx\tan{(fx+e)}+1)}{bf\tan{(fx+e)}^3}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*tan(f*x + e)^4)*(f*x*tan(f*x + e) + 1)/(b*f*tan(f*x + e)^3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan^4{(e+fx)}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(e + f*x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = -\frac{\frac{fx+e}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan(fx+e)}}{f}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="maxima")
```

output
```
-((f*x + e)/sqrt(b) + 1/(sqrt(b)*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = -\sqrt{b}\left(\frac{fx+e}{bf} + \frac{1}{bf\tan(fx+e)}\right)$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*((f*x + e)/(b*f) + 1/(b*f*tan(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan(e+f\,x)^4}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^4)^(1/2),x)
```

output
```
int(1/(b*tan(e + f*x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{b\tan^4(e+fx)}}\,dx = -\frac{\sqrt{b}\,(\tan(fx+e)\,fx+1)}{\tan(fx+e)\,bf}$$

input
```
int(1/(b*tan(f*x+e)^4)^(1/2),x)
```

output
```
( - sqrt(b)*(tan(e + f*x)*f*x + 1))/(tan(e + f*x)*b*f)
```

3.17 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 119

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = \frac{\cot(e+fx)}{3bf\sqrt{b\tan^4(e+fx)}} - \frac{\cot^3(e+fx)}{5bf\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{bf\sqrt{b\tan^4(e+fx)}} - \frac{x\tan^2(e+fx)}{b\sqrt{b\tan^4(e+fx)}}$$

output
```
1/3*cot(f*x+e)/b/f/(b*tan(f*x+e)^4)^(1/2)-1/5*cot(f*x+e)^3/b/f/(b*tan(f*x+
e)^4)^(1/2)-tan(f*x+e)/b/f/(b*tan(f*x+e)^4)^(1/2)-x*tan(f*x+e)^2/b/(b*tan(
f*x+e)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)\tan(e+fx)}{5f\left(b\tan^4(e+fx)\right)^{3/2}}$$

input
```
Integrate[(b*Tan[e + f*x]^4)^(-3/2),x]
```

output
```
-1/5*(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(
b*Tan[e + f*x]^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b \tan^4(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b \tan(e+fx)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^2(e+fx)\int \cot^6(e+fx)dx}{b\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\int \tan\left(e+fx+\frac{\pi}{2}\right)^6dx}{b\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(-\int \cot^4(e+fx)dx-\frac{\cot^5(e+fx)}{5f}\right)}{b\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\left(-\int \tan\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{\cot^5(e+fx)}{5f}\right)}{b\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(\int \cot^2(e+fx)dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}\right)}{b\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\left(\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}\right)}{b\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(-\int 1dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}\right)}{b\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^2(e+fx)\left(-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}-x\right)}{b\sqrt{b\tan^4(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^4)^(-3/2),x]
```

output
```
((-x - Cot[e + f*x]/f + Cot[e + f*x]^3/(3*f) - Cot[e + f*x]^5/(5*f))*Tan[e
 + f*x]^2)/(b*Sqrt[b*Tan[e + f*x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$-\frac{\tan(fx+e)\left(15\arctan(\tan(fx+e))\tan(fx+e)^{5}+15\tan(fx+e)^{4}-5\tan(fx+e)^{2}+3\right)}{15f\left(b\tan(fx+e)^{4}\right)^{\frac{3}{2}}}$	63
default	$-\frac{\tan(fx+e)\left(15\arctan(\tan(fx+e))\tan(fx+e)^{5}+15\tan(fx+e)^{4}-5\tan(fx+e)^{2}+3\right)}{15f\left(b\tan(fx+e)^{4}\right)^{\frac{3}{2}}}$	63
risch	$\frac{\left(e^{2i(fx+e)}-1\right)^{2}x}{b\left(e^{2i(fx+e)}+1\right)^{2}\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^{4}}{\left(e^{2i(fx+e)}+1\right)^{4}}}}+\frac{2i\left(45\,e^{8i(fx+e)}-90\,e^{6i(fx+e)}+140\,e^{4i(fx+e)}-70\,e^{2i(fx+e)}+23\right)}{15b\left(e^{2i(fx+e)}-1\right)^{3}\left(e^{2i(fx+e)}+1\right)^{2}\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^{4}}{\left(e^{2i(fx+e)}+1\right)^{4}}}\,f}$	174

input
```
int(1/(b*tan(f*x+e)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f*tan(f*x+e)*(15*arctan(tan(f*x+e))*tan(f*x+e)^5+15*tan(f*x+e)^4-5*t
an(f*x+e)^2+3)/(b*tan(f*x+e)^4)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.52

$$\int \frac{1}{(b\tan^4(e+fx))^{3/2}}\,dx = -\frac{\left(15\,fx\tan\left(fx+e\right)^{5}+15\,\tan\left(fx+e\right)^{4}-5\,\tan\left(fx+e\right)^{2}+3\right)\sqrt{b\tan\left(fx+e\right)^{4}}}{15\,b^{2}f\tan\left(fx+e\right)^{7}}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(15*f*x*tan(f*x + e)^5 + 15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)*s
qrt(b*tan(f*x + e)^4)/(b^2*f*tan(f*x + e)^7)
```

Sympy [F]

$$\int \frac{1}{(b\tan^4(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan^4{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**4)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{1}{(b\tan^4(e+fx))^{3/2}}\,dx = -\frac{\frac{15\,(fx+e)}{b^{\frac{3}{2}}} + \frac{15\,\tan(fx+e)^4 - 5\,\tan(fx+e)^2 + 3}{b^{\frac{3}{2}}\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(f*x + e)/b^(3/2) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(
b^(3/2)*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = -\frac{\frac{15\,(fx+e)}{bf} + \frac{15\,\tan(fx+e)^4 - 5\,\tan(fx+e)^2 + 3}{bf\tan(fx+e)^5}}{15\,\sqrt{b}}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/15*(15*(f*x + e)/(b*f) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/(b*
f*tan(f*x + e)^5))/sqrt(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)^4\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^4)^(3/2),x)
```

output
```
int(1/(b*tan(e + f*x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{b}\left(-15\tan\left(fx+e\right)^5 fx - 15\tan\left(fx+e\right)^4 + 5\tan\left(fx+e\right)^2 - 3\right)}{15\tan\left(fx+e\right)^5 b^2 f}$$

input
```
int(1/(b*tan(f*x+e)^4)^(3/2),x)
```

output
```
(sqrt(b)*( - 15*tan(e + f*x)**5*f*x - 15*tan(e + f*x)**4 + 5*tan(e + f*x)*
*2 - 3))/(15*tan(e + f*x)**5*b**2*f)
```

3.18 $\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 183

$$\begin{aligned}\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx &= \frac{\cot(e+fx)}{3b^2f\sqrt{b\tan^4(e+fx)}}\\ &\quad - \frac{\cot^3(e+fx)}{5b^2f\sqrt{b\tan^4(e+fx)}} + \frac{\cot^5(e+fx)}{7b^2f\sqrt{b\tan^4(e+fx)}}\\ &\quad - \frac{\cot^7(e+fx)}{9b^2f\sqrt{b\tan^4(e+fx)}} - \frac{\tan(e+fx)}{b^2f\sqrt{b\tan^4(e+fx)}} - \frac{x\tan^2(e+fx)}{b^2\sqrt{b\tan^4(e+fx)}}\end{aligned}$$

output
```
1/3*cot(f*x+e)/b^2/f/(b*tan(f*x+e)^4)^(1/2)-1/5*cot(f*x+e)^3/b^2/f/(b*tan(
f*x+e)^4)^(1/2)+1/7*cot(f*x+e)^5/b^2/f/(b*tan(f*x+e)^4)^(1/2)-1/9*cot(f*x+
e)^7/b^2/f/(b*tan(f*x+e)^4)^(1/2)-tan(f*x+e)/b^2/f/(b*tan(f*x+e)^4)^(1/2)-
x*tan(f*x+e)^2/b^2/(b*tan(f*x+e)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.25

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)\tan(e+fx)}{9f\left(b\tan^4(e+fx)\right)^{5/2}}$$

input
```
Integrate[(b*Tan[e + f*x]^4)^(-5/2),x]
```

output
```
-1/9*(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(
b*Tan[e + f*x]^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(e+fx)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^2(e+fx)\int \cot^{10}(e+fx)dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\int \tan\left(e+fx+\frac{\pi}{2}\right)^{10}dx}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(-\int \cot^8(e+fx)dx-\frac{\cot^9(e+fx)}{9f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\left(-\int \tan\left(e+fx+\frac{\pi}{2}\right)^8dx-\frac{\cot^9(e+fx)}{9f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(\int \cot^6(e+fx)dx-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\left(\int \tan\left(e+fx+\frac{\pi}{2}\right)^6dx-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(-\int \cot^4(e+fx)dx-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}-\frac{\cot^5(e+fx)}{5f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\left(-\int \tan\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}-\frac{\cot^5(e+fx)}{5f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(\int \cot^2(e+fx)dx-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(e+fx)\left(\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(e+fx)\left(-\int 1dx-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

$$\Big\downarrow 24$$

$$\frac{\tan^2(e+fx)\left(-\frac{\cot^9(e+fx)}{9f}+\frac{\cot^7(e+fx)}{7f}-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}-x\right)}{b^2\sqrt{b\tan^4(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^4)^(-5/2),x]
```

output
```
((-x - Cot[e + f*x]/f + Cot[e + f*x]^3/(3*f) - Cot[e + f*x]^5/(5*f) + Cot[
e + f*x]^7/(7*f) - Cot[e + f*x]^9/(9*f))*Tan[e + f*x]^2)/(b^2*Sqrt[b*Tan[e
 + f*x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result
derivativedivides	$-\frac{\tan(fx+e)\left(315\arctan(\tan(fx+e))\tan(fx+e)^{9}+315\tan(fx+e)^{8}-105\tan(fx+e)^{6}+63\tan(fx+e)^{4}-45\tan(fx+e\right.}{315f\left(b\tan(fx+e)^{4}\right)^{\frac{5}{2}}}$
default	$-\frac{\tan(fx+e)\left(315\arctan(\tan(fx+e))\tan(fx+e)^{9}+315\tan(fx+e)^{8}-105\tan(fx+e)^{6}+63\tan(fx+e)^{4}-45\tan(fx+e\right.}{315f\left(b\tan(fx+e)^{4}\right)^{\frac{5}{2}}}$
risch	$\frac{\left(e^{2i(fx+e)}-1\right)^{2}x}{b^{2}\left(e^{2i(fx+e)}+1\right)^{2}\sqrt{\frac{b\left(e^{2i(fx+e)}-1\right)^{4}}{\left(e^{2i(fx+e)}+1\right)^{4}}}}+\frac{2i\left(1575\,e^{16i(fx+e)}-6300\,e^{14i(fx+e)}+21000\,e^{12i(fx+e)}-31500\,e^{10i(fx+e)}+\right.}{315b^{2}\left(e^{2i(fx+e)}-1\right)^{7}\left(e^{2i(fx+\right.}}$

input
```
int(1/(b*tan(f*x+e)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315/f*tan(f*x+e)*(315*arctan(tan(f*x+e))*tan(f*x+e)^9+315*tan(f*x+e)^8-
105*tan(f*x+e)^6+63*tan(f*x+e)^4-45*tan(f*x+e)^2+35)/(b*tan(f*x+e)^4)^(5/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx =$$
$$-\frac{\left(315\,fx\tan\left(fx+e\right)^{9}+315\,\tan\left(fx+e\right)^{8}-105\,\tan\left(fx+e\right)^{6}+63\,\tan\left(fx+e\right)^{4}-45\,\tan\left(fx+e\right)^{2}+\right.}{315\,b^{3}f\tan\left(fx+e\right)^{11}}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/315*(315*f*x*tan(f*x + e)^9 + 315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
 63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)*sqrt(b*tan(f*x + e)^4)/(b^3*f
*tan(f*x + e)^11)
```

Sympy [F]

$$\int \frac{1}{(b\tan^4(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\tan^4{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**4)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{1}{(b\tan^4(e+fx))^{5/2}}\,dx = -\frac{\frac{315\,(fx+e)}{b^{\frac{5}{2}}} + \frac{315\,\tan(fx+e)^8 - 105\,\tan(fx+e)^6 + 63\,\tan(fx+e)^4 - 45\,\tan(fx+e)^2 + 35}{b^{\frac{5}{2}}\tan(fx+e)^9}}{315\,f}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="maxima")
```

output
```
-1/315*(315*(f*x + e)/b^(5/2) + (315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 +
 63*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)/(b^(5/2)*tan(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.42

$$\int \frac{1}{(b\tan^4(e+fx))^{5/2}}\,dx = -\frac{\frac{315\,(fx+e)}{bf} + \frac{315\,\tan(fx+e)^8 - 105\,\tan(fx+e)^6 + 63\,\tan(fx+e)^4 - 45\,\tan(fx+e)^2 + 35}{bf\tan(fx+e)^9}}{315\,b^{\frac{3}{2}}}$$

input
```
integrate(1/(b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/315*(315*(f*x + e)/(b*f) + (315*tan(f*x + e)^8 - 105*tan(f*x + e)^6 + 6
3*tan(f*x + e)^4 - 45*tan(f*x + e)^2 + 35)/(b*f*tan(f*x + e)^9))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)^4\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^4)^(5/2),x)
```

output
```
int(1/(b*tan(e + f*x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(b\tan^4(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{b}\left(-315\tan\left(fx+e\right)^9 fx-315\tan\left(fx+e\right)^8+105\tan\left(fx+e\right)^6-63\tan\left(\right.\right.}{315\tan\left(fx+e\right)^9 b^3 f}$$

input
```
int(1/(b*tan(f*x+e)^4)^(5/2),x)
```

output
```
(sqrt(b)*( - 315*tan(e + f*x)**9*f*x - 315*tan(e + f*x)**8 + 105*tan(e + f
*x)**6 - 63*tan(e + f*x)**4 + 45*tan(e + f*x)**2 - 35))/(315*tan(e + f*x)*
*9*b**3*f)
```

3.19 $\int (b\tan^n(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \frac{2b^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5n),\frac{1}{4}(6+5n),-\tan^2(e+fx)\right)\tan^{1+2n}(e+fx)\sqrt{b\tan^n(}}{f(2+5n)}$$

output
```
2*b^2*hypergeom([1, 1/2+5/4*n],[3/2+5/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1+2*
n)*(b*tan(f*x+e)^n)^(1/2)/f/(2+5*n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5n),\frac{1}{4}(6+5n),-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^n(e+fx)\right)}{f\left(1+\frac{5n}{2}\right)}$$

input
```
Integrate[(b*Tan[e + f*x]^n)^(5/2),x]
```

output
```
(Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*Tan[e + f*x]^n)^(5/2))/(f*(1 + (5*n)/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^n(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx)^n)^{5/2}\,dx$$

$$\Big\downarrow \text{4142}$$

$$b^2\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \tan^{\frac{5n}{2}}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \tan(e+fx)^{5n/2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b^2\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \frac{\tan^{\frac{5n}{2}}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{2b^2\tan^{2n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(5n+2),\frac{1}{4}(5n+6),-\tan^2(e+fx)\right)}{f(5n+2)}$$

input
```
Int[(b*Tan[e + f*x]^n)^(5/2),x]
```

output
```
(2*b^2*Hypergeometric2F1[1, (2 + 5*n)/4, (6 + 5*n)/4, -Tan[e + f*x]^2]*Tan
[e + f*x]^(1 + 2*n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 5*n))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b\tan(fx+e)^n)^{\frac{5}{2}}\,dx$$

input
```
int((b*tan(f*x+e)^n)^(5/2),x)
```

output
```
int((b*tan(f*x+e)^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \int (b\tan^n{(e+fx)})^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**n)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**n)**(5/2), x)
```

Maxima [F]

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \int (b\tan{(fx+e)^n})^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^n)^(5/2), x)
```

Giac [F]

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \int (b\tan(fx+e)^n)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x)^n)^{5/2}\,dx$$

input
```
int((b*tan(e + f*x)^n)^(5/2),x)
```

output
```
int((b*tan(e + f*x)^n)^(5/2), x)
```

Reduce [F]

$$\int (b\tan^n(e+fx))^{5/2}\,dx = \sqrt{b}\left(\int \tan(fx+e)^{\frac{5n}{2}}\,dx\right)b^2$$

input
```
int((b*tan(f*x+e)^n)^(5/2),x)
```

output
```
sqrt(b)*int(tan(e + f*x)**((5*n)/2),x)*b**2
```

3.20 $\int (b\tan^n(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int (b\tan^n(e+fx))^{3/2}\,dx = \frac{2b\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3n),\frac{3(2+n)}{4},-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)\sqrt{b\tan^n(e+}}{f(2+3n)}$$

output
```
2*b*hypergeom([1, 1/2+3/4*n],[3/2+3/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1+n)*(
b*tan(f*x+e)^n)^(1/2)/f/(2+3*n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (b\tan^n(e+fx))^{3/2}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3n),\frac{3(2+n)}{4},-\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^{3/2}}{f\left(1+\frac{3n}{2}\right)}$$

input
```
Integrate[(b*Tan[e + f*x]^n)^(3/2),x]
```

output
```
(Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan[e +
 f*x]*(b*Tan[e + f*x]^n)^(3/2))/(f*(1 + (3*n)/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^n(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx)^n)^{3/2}\,dx$$

$$\Big\downarrow \text{4142}$$

$$b\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \tan^{\frac{3n}{2}}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \tan(e+fx)^{3n/2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \frac{\tan^{\frac{3n}{2}}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{2b\tan^{n+1}(e+fx)\sqrt{b\tan^n(e+fx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3n+2),\frac{3(n+2)}{4},-\tan^2(e+fx)\right)}{f(3n+2)}$$

input
```
Int[(b*Tan[e + f*x]^n)^(3/2),x]
```

output
```
(2*b*Hypergeometric2F1[1, (2 + 3*n)/4, (3*(2 + n))/4, -Tan[e + f*x]^2]*Tan
[e + f*x]^(1 + n)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + 3*n))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b\tan (fx + e)^{n})^{\frac{3}{2}}\,dx$$

input
```
int((b*tan(f*x+e)^n)^(3/2),x)
```

output
```
int((b*tan(f*x+e)^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(b\tan^n(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(b\tan^n(e+fx)\right)^{3/2}\,dx = \int \left(b\tan^n\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)**n)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**n)**(3/2), x)
```

Maxima [F]

$$\int \left(b\tan^n(e+fx)\right)^{3/2}\,dx = \int \left(b\tan\left(fx+e\right)^n\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^n)^(3/2), x)
```

Giac [F]

$$\int (b\tan^n(e+fx))^{3/2}\,dx = \int (b\tan(fx+e)^n)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^n(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x)^n)^{3/2}\,dx$$

input
```
int((b*tan(e + f*x)^n)^(3/2),x)
```

output
```
int((b*tan(e + f*x)^n)^(3/2), x)
```

Reduce [F]

$$\int (b\tan^n(e+fx))^{3/2}\,dx = \sqrt{b}\left(\int \tan(fx+e)^{\frac{3n}{2}}\,dx\right)b$$

input
```
int((b*tan(f*x+e)^n)^(3/2),x)
```

output
```
sqrt(b)*int(tan(e + f*x)**((3*n)/2),x)*b
```

3.21 $\int \sqrt{b \tan^n(e+fx)}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \sqrt{b \tan^n(e+fx)}\, dx = \frac{2 \, \text{Hypergeometric2F1}\left(1, \frac{2+n}{4}, \frac{6+n}{4}, -\tan^2(e+fx)\right) \tan(e+fx) \sqrt{b \tan^n(e+fx)}}{f(2+n)}$$

output
```
2*hypergeom([1, 1/2+1/4*n],[3/2+1/4*n],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*
x+e)^n)^(1/2)/f/(2+n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \sqrt{b \tan^n(e+fx)}\, dx = \frac{2 \, \text{Hypergeometric2F1}\left(1, \frac{2+n}{4}, \frac{6+n}{4}, -\tan^2(e+fx)\right) \tan(e+fx) \sqrt{b \tan^n(e+fx)}}{f(2+n)}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]^n],x]
```

output
```
(2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x
]*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b \tan^n(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{b \tan(e+fx)^n}dx$$
$$\Big\downarrow \text{4142}$$
$$\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \tan^{\frac{n}{2}}(e+fx)dx$$
$$\Big\downarrow \text{3042}$$
$$\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \tan(e+fx)^{n/2}dx$$
$$\Big\downarrow \text{3957}$$
$$\frac{\tan^{-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\int \frac{\tan^{\frac{n}{2}}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{278}$$
$$\frac{2\tan^{\frac{n+2}{2}-\frac{n}{2}}(e+fx)\sqrt{b\tan^n(e+fx)}\,\text{Hypergeometric2F1}\left(1,\frac{n+2}{4},\frac{n+6}{4},-\tan^2(e+fx)\right)}{f(n+2)}$$

input
```
Int[Sqrt[b*Tan[e + f*x]^n],x]
```

output
```
(2*Hypergeometric2F1[1, (2 + n)/4, (6 + n)/4, -Tan[e + f*x]^2]*Tan[e + f*x
]^(-1/2*n + (2 + n)/2)*Sqrt[b*Tan[e + f*x]^n])/(f*(2 + n))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sqrt{b \tan \left(f x + e\right)^{n}}dx$$

input
```
int((b*tan(f*x+e)^n)^(1/2),x)
```

output
```
int((b*tan(f*x+e)^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \int \sqrt{b\tan^n{(e+fx)}}\,dx$$

input
```
integrate((b*tan(f*x+e)**n)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x)**n), x)
```

Maxima [F]

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^n}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^n), x)
```

Giac [F]

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \int \sqrt{b\tan\left(fx+e\right)^n}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \int \sqrt{b\tan\left(e+f\,x\right)^n}\,dx$$

input
```
int((b*tan(e + f*x)^n)^(1/2),x)
```

output
```
int((b*tan(e + f*x)^n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\tan^n(e+fx)}\,dx = \sqrt{b}\left(\int \tan\left(fx+e\right)^{\frac{n}{2}}dx\right)$$

input
```
int((b*tan(f*x+e)^n)^(1/2),x)
```

output
```
sqrt(b)*int(tan(e + f*x)**(n/2),x)
```

3.22 $\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2-n}{4},\frac{6-n}{4},-\tan^2(e+fx)\right)\tan(e+fx)}{f(2-n)\sqrt{b\tan^n(e+fx)}}$$

output
```
2*hypergeom([1, 1/2-1/4*n],[3/2-1/4*n],-tan(f*x+e)^2)*tan(f*x+e)/f/(2-n)/(
b*tan(f*x+e)^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2-n}{4},\frac{6-n}{4},-\tan^2(e+fx)\right)\tan(e+fx)}{f(-2+n)\sqrt{b\tan^n(e+fx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[e + f*x]^n],x]
```

output
```
(-2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*
x])/(f*(-2 + n)*Sqrt[b*Tan[e + f*x]^n])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b \tan^n(e+fx)}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{b \tan(e+fx)^n}} dx$$

$$\Big\downarrow \text{ 4142}$$

$$\frac{\tan^{\frac{n}{2}}(e+fx) \int \tan^{-\frac{n}{2}}(e+fx) dx}{\sqrt{b \tan^n(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^{\frac{n}{2}}(e+fx) \int \tan(e+fx)^{-n/2} dx}{\sqrt{b \tan^n(e+fx)}}$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{\tan^{\frac{n}{2}}(e+fx) \int \frac{\tan^{-\frac{n}{2}}(e+fx)}{\tan^2(e+fx)+1} d\tan(e+fx)}{f \sqrt{b \tan^n(e+fx)}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{2 \tan(e+fx) \text{Hypergeometric2F1}\left(1, \frac{2-n}{4}, \frac{6-n}{4}, -\tan^2(e+fx)\right)}{f(2-n) \sqrt{b \tan^n(e+fx)}}$$

input
```
Int[1/Sqrt[b*Tan[e + f*x]^n],x]
```

output
```
(2*Hypergeometric2F1[1, (2 - n)/4, (6 - n)/4, -Tan[e + f*x]^2]*Tan[e + f*x
])/(f*(2 - n)*Sqrt[b*Tan[e + f*x]^n])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\sqrt{b\tan\left(fx+e\right)^{n}}}dx$$

input
```
int(1/(b*tan(f*x+e)^n)^(1/2),x)
```

output
```
int(1/(b*tan(f*x+e)^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**n)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(e + f*x)**n), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan(fx+e)^n}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(fx+e\right)^n}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*tan(f*x + e)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)^n}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^n)^(1/2),x)
```

output
```
int(1/(b*tan(e + f*x)^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\tan^n(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\tan(fx+e)^{\frac{n}{2}}}dx\right)}{b}$$

input
```
int(1/(b*tan(f*x+e)^n)^(1/2),x)
```

output
```
(sqrt(b)*int(1/tan(e + f*x)**(n/2),x))/b
```

3.23 $\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx=\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3n),\frac{3(2-n)}{4},-\tan^2(e+fx)\right)\tan^{1-n}(e+fx)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

output
```
2*hypergeom([1, 1/2-3/4*n],[3/2-3/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1-n)/b/f
/(2-3*n)/(b*tan(f*x+e)^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx=\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3n),-\frac{3}{4}(-2+n),-\tan^2(e+fx)\right)\tan(e+fx)}{f\left(1-\frac{3n}{2}\right)(b\tan^n(e+fx))^{3/2}}$$

input
```
Integrate[(b*Tan[e + f*x]^n)^(-3/2),x]
```

output
```
(Hypergeometric2F1[1, (2 - 3*n)/4, (-3*(-2 + n))/4, -Tan[e + f*x]^2]*Tan[e
 + f*x])/(f*(1 - (3*n)/2)*(b*Tan[e + f*x]^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx)^n)^{3/2}}dx$$

$$\Big\downarrow \text{4142}$$

$$\frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{3n}{2}}(e+fx)dx}{b\sqrt{b\tan^n(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{n}{2}}(e+fx)\int \tan(e+fx)^{-3n/2}dx}{b\sqrt{b\tan^n(e+fx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{\frac{n}{2}}(e+fx)\int \frac{\tan^{-\frac{3n}{2}}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{bf\sqrt{b\tan^n(e+fx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\tan^{1-n}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3n),\frac{3(2-n)}{4},-\tan^2(e+fx)\right)}{bf(2-3n)\sqrt{b\tan^n(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^n)^(-3/2),x]
```

output
```
(2*Hypergeometric2F1[1, (2 - 3*n)/4, (3*(2 - n))/4, -Tan[e + f*x]^2]*Tan[e
 + f*x]^(1 - n))/(b*f*(2 - 3*n)*Sqrt[b*Tan[e + f*x]^n])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{(b\tan(fx+e)^n)^{\frac{3}{2}}}\,dx$$

input
```
int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

output
```
int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan^n(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**n)**(3/2),x)
```

output
```
Integral((b*tan(e + f*x)**n)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan(fx+e)^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan(fx+e)^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^n)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan(e+f\,x)^n)^{3/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^n)^(3/2),x)
```

output
```
int(1/(b*tan(e + f*x)^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\tan(fx+e)^{\frac{3n}{2}}}dx\right)}{b^2}$$

input
```
int(1/(b*tan(f*x+e)^n)^(3/2),x)
```

output
```
(sqrt(b)*int(1/tan(e + f*x)**((3*n)/2),x))/b**2
```

3.24 $\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5n),\frac{1}{4}(6-5n),-\tan^2(e+fx)\right)\tan^{1-2n}(e+f}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

output
```
2*hypergeom([1, 1/2-5/4*n],[3/2-5/4*n],-tan(f*x+e)^2)*tan(f*x+e)^(1-2*n)/b
^2/f/(2-5*n)/(b*tan(f*x+e)^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5n),\frac{1}{4}(6-5n),-\tan^2(e+fx)\right)\tan(e+fx)}{f\left(1-\frac{5n}{2}\right)(b\tan^n(e+fx))^{5/2}}$$

input
```
Integrate[(b*Tan[e + f*x]^n)^(-5/2),x]
```

output
```
(Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e + f
*x])/(f*(1 - (5*n)/2)*(b*Tan[e + f*x]^n)^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(b\tan(e+fx)^n)^{5/2}}dx$$
$$\Big\downarrow \text{ 4142}$$
$$\frac{\tan^{\frac{n}{2}}(e+fx)\int \tan^{-\frac{5n}{2}}(e+fx)dx}{b^2\sqrt{b\tan^n(e+fx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\tan^{\frac{n}{2}}(e+fx)\int \tan(e+fx)^{-5n/2}dx}{b^2\sqrt{b\tan^n(e+fx)}}$$
$$\Big\downarrow \text{ 3957}$$
$$\frac{\tan^{\frac{n}{2}}(e+fx)\int \frac{\tan^{-\frac{5n}{2}}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{b^2f\sqrt{b\tan^n(e+fx)}}$$
$$\Big\downarrow \text{ 278}$$
$$\frac{2\tan^{1-2n}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5n),\frac{1}{4}(6-5n),-\tan^2(e+fx)\right)}{b^2f(2-5n)\sqrt{b\tan^n(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x]^n)^(-5/2),x]
```

output
```
(2*Hypergeometric2F1[1, (2 - 5*n)/4, (6 - 5*n)/4, -Tan[e + f*x]^2]*Tan[e +
 f*x]^(1 - 2*n))/(b^2*f*(2 - 5*n)*Sqrt[b*Tan[e + f*x]^n])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{(b\tan (fx+e)^n)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*tan(f*x+e)^n)^(5/2),x)
```

output
```
int(1/(b*tan(f*x+e)^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\tan^n(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)**n)**(5/2),x)
```

output
```
Integral((b*tan(e + f*x)**n)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\tan(fx+e)^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\tan{(fx+e)}^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(f*x+e)^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^n)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\tan{(e+f\,x)}^n)^{5/2}}\,dx$$

input
```
int(1/(b*tan(e + f*x)^n)^(5/2),x)
```

output
```
int(1/(b*tan(e + f*x)^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\tan^n(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\tan(fx+e)^{\frac{5n}{2}}}dx\right)}{b^3}$$

input
```
int(1/(b*tan(f*x+e)^n)^(5/2),x)
```

output
```
(sqrt(b)*int(1/tan(e + f*x)**((5*n)/2),x))/b**3
```

3.25 $\int (b\tan^n(e+fx))^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^n(e+fx))^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+np), \frac{1}{2}(3+np), -\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^p}{f(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(
f*x+e)^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (b\tan^n(e+fx))^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+np), \frac{1}{2}(3+np), -\tan^2(e+fx)\right)\tan(e+fx)\,(b\tan^n(e+fx))^p}{f(1+np)}$$

input
```
Integrate[(b*Tan[e + f*x]^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*Tan[e + f*x]^n)^p)/(f*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^n(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx)^n)^p dx$$

$$\Big\downarrow \text{4142}$$

$$\tan^{-np}(e+fx)\,(b\tan^n(e+fx))^p\int \tan^{np}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-np}(e+fx)\,(b\tan^n(e+fx))^p\int \tan(e+fx)^{np}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-np}(e+fx)\,(b\tan^n(e+fx))^p\int \frac{\tan^{np}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)\,(b\tan^n(e+fx))^p\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)}{f(np+1)}$$

input
```
Int[(b*Tan[e + f*x]^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*Tan[e + f*x]^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b \tan (fx + e)^n)^p \, dx$$

input
```
int((b*tan(f*x+e)^n)^p,x)
```

output
```
int((b*tan(f*x+e)^n)^p,x)
```

Fricas [F]

$$\int (b\tan^n(e+fx))^p\,dx = \int (b\tan(fx+e)^n)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^n)^p, x)
```

Sympy [F]

$$\int (b\tan^n(e+fx))^p\,dx = \int (b\tan^n(e+fx))^p\,dx$$

input
```
integrate((b*tan(f*x+e)**n)**p,x)
```

output
```
Integral((b*tan(e + f*x)**n)**p, x)
```

Maxima [F]

$$\int (b\tan^n(e+fx))^p\,dx = \int (b\tan(fx+e)^n)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^n)^p, x)
```

Giac [F]

$$\int (b\tan^n(e+fx))^p \, dx = \int (b\tan(fx+e)^n)^p \, dx$$

input
```
integrate((b*tan(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^n)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^n(e+fx))^p \, dx = \int (b\tan(e+f\,x)^n)^p \, dx$$

input
```
int((b*tan(e + f*x)^n)^p,x)
```

output
```
int((b*tan(e + f*x)^n)^p, x)
```

Reduce [F]

$$\int (b\tan^n(e+fx))^p \, dx = b^p \left(\int \tan(fx+e)^{np} \, dx\right)$$

input
```
int((b*tan(f*x+e)^n)^p,x)
```

output
```
b**p*int(tan(e + f*x)**(n*p),x)
```

3.26 $\int \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \left(b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+2p),\frac{1}{2}(3+2p),-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1+2p)}$$

output
```
hypergeom([1, 1/2+p],[3/2+p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/
f/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \left(b\tan^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+p,\frac{3}{2}+p,-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^2(e+fx)\right)^p}{f(1+2p)}$$

input
```
Integrate[(b*Tan[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1, 1/2 + p, 3/2 + p, -Tan[e + f*x]^2]*Tan[e + f*x]*(b*T
an[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b \tan^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b \tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2p}(e+fx) \left(b \tan^2(e+fx)\right)^p \int \tan^{2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx) \left(b \tan^2(e+fx)\right)^p \int \tan(e+fx)^{2p}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-2p}(e+fx) \left(b \tan^2(e+fx)\right)^p \int \frac{\tan^{2p}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx) \left(b \tan^2(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(2p+1),\frac{1}{2}(2p+3),-\tan^2(e+fx)\right)}{f(2p+1)}$$

input
```
Int[(b*Tan[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + 2*p)/2, (3 + 2*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b\tan\left(fx+e\right)^{2}\right)^{p}dx$$

input
```
int((b*tan(f*x+e)^2)^p,x)
```

output
```
int((b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \left(b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2)^p, x)
```

Sympy [F]

$$\int \left(b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan^2\left(e+fx\right)\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((b*tan(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int \left(b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2)^p, x)
```

Giac [F]

$$\int \left(b\tan^2(e+fx)\right)^p\,dx = \int \left(b\tan\left(fx+e\right)^2\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(e+fx)\right)^p\,dx = \int \left(b\tan(e+f\,x)^2\right)^p\,dx$$

input
```
int((b*tan(e + f*x)^2)^p,x)
```

output
```
int((b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(b\tan^2(e+fx)\right)^p\,dx = b^p\left(\int \tan\left(fx+e\right)^{2p}\,dx\right)$$

input
```
int((b*tan(f*x+e)^2)^p,x)
```

output
```
b**p*int(tan(e + f*x)**(2*p),x)
```

3.27 $\int \left(b\tan^3(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \left(b\tan^3(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3p),\frac{3(1+p)}{2},-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^3(e+fx)\right)^p}{f(1+3p)}$$

output
```
hypergeom([1, 1/2+3/2*p],[3/2*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+
e)^3)^p/f/(1+3*p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \left(b\tan^3(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3p),\frac{3(1+p)}{2},-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^3(e+fx)\right)^p}{f(1+3p)}$$

input
```
Integrate[(b*Tan[e + f*x]^3)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
 f*x]*(b*Tan[e + f*x]^3)^p)/(f*(1 + 3*p))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^3(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^3\right)^p dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\int \tan^{3p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\int \tan(e+fx)^{3p}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-3p}(e+fx)\left(b\tan^3(e+fx)\right)^p\int \frac{\tan^{3p}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)\left(b\tan^3(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(3p+1),\frac{3(p+1)}{2},-\tan^2(e+fx)\right)}{f(3p+1)}$$

input
```
Int[(b*Tan[e + f*x]^3)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + 3*p)/2, (3*(1 + p))/2, -Tan[e + f*x]^2]*Tan[e +
 f*x]*(b*Tan[e + f*x]^3)^p)/(f*(1 + 3*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \tan \left(f x +e \right)^{3}\right)^{p}d x$$

input
```
int((b*tan(f*x+e)^3)^p,x)
```

output
```
int((b*tan(f*x+e)^3)^p,x)
```

Fricas [F]

$$\int \left(b\tan^3(e+fx)\right)^p\,dx = \int \left(b\tan\left(fx+e\right)^3\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^3)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^3)^p, x)
```

Sympy [F]

$$\int \left(b\tan^3(e+fx)\right)^p\,dx = \int \left(b\tan^3\left(e+fx\right)\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)**3)**p,x)
```

output
```
Integral((b*tan(e + f*x)**3)**p, x)
```

Maxima [F]

$$\int \left(b\tan^3(e+fx)\right)^p\,dx = \int \left(b\tan\left(fx+e\right)^3\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^3)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^3)^p, x)
```

Giac [F]

$$\int \left(b\tan^3(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^3\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^3)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(e+fx)\right)^p\,dx=\int \left(b\tan(e+f\,x)^3\right)^p\,dx$$

input
```
int((b*tan(e + f*x)^3)^p,x)
```

output
```
int((b*tan(e + f*x)^3)^p, x)
```

Reduce [F]

$$\int \left(b\tan^3(e+fx)\right)^p\,dx=b^p\left(\int \tan\left(fx+e\right)^{3p}dx\right)$$

input
```
int((b*tan(f*x+e)^3)^p,x)
```

output
```
b**p*int(tan(e + f*x)**(3*p),x)
```

3.28 $\int (b\tan^4(e+fx))^p \, dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^4(e+fx))^p \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+4p),\frac{1}{2}(3+4p),-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^4(e+fx)\right)^p}{f(1+4p)}$$

output
```
hypergeom([1, 1/2+2*p],[3/2+2*p],-tan(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^4
)^p/f/(1+4*p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (b\tan^4(e+fx))^p \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+2p,\frac{3}{2}+2p,-\tan^2(e+fx)\right)\tan(e+fx)\left(b\tan^4(e+fx)\right)^p}{f(1+4p)}$$

input
```
Integrate[(b*Tan[e + f*x]^4)^p,x]
```

output
```
(Hypergeometric2F1[1, 1/2 + 2*p, 3/2 + 2*p, -Tan[e + f*x]^2]*Tan[e + f*x]*
(b*Tan[e + f*x]^4)^p)/(f*(1 + 4*p))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^4\right)^p dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p \int \tan^{4p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p \int \tan(e+fx)^{4p}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-4p}(e+fx)\left(b\tan^4(e+fx)\right)^p \int \frac{\tan^{4p}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)\left(b\tan^4(e+fx)\right)^p \text{Hypergeometric2F1}\left(1,\frac{1}{2}(4p+1),\frac{1}{2}(4p+3),-\tan^2(e+fx)\right)}{f(4p+1)}$$

input
```
Int[(b*Tan[e + f*x]^4)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + 4*p)/2, (3 + 4*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*Tan[e + f*x]^4)^p)/(f*(1 + 4*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b \tan \left(f x + e\right)^{4}\right)^{p} dx$$

input
```
int((b*tan(f*x+e)^4)^p,x)
```

output
```
int((b*tan(f*x+e)^4)^p,x)
```

Fricas [F]

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = \int \left(b\tan\left(fx+e\right)^4\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^4)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^4)^p, x)
```

Sympy [F]

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = \int \left(b\tan^4\left(e+fx\right)\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)**4)**p,x)
```

output
```
Integral((b*tan(e + f*x)**4)**p, x)
```

Maxima [F]

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = \int \left(b\tan\left(fx+e\right)^4\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^4)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^4)^p, x)
```

Giac [F]

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = \int \left(b\tan\left(fx+e\right)^4\right)^p\,dx$$

input
```
integrate((b*tan(f*x+e)^4)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^4)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = \int \left(b\tan(e+f\,x)^4\right)^p\,dx$$

input
```
int((b*tan(e + f*x)^4)^p,x)
```

output
```
int((b*tan(e + f*x)^4)^p, x)
```

Reduce [F]

$$\int \left(b\tan^4(e+fx)\right)^p\,dx = b^p\left(\int \tan\left(fx+e\right)^{4p}\,dx\right)$$

input
```
int((b*tan(f*x+e)^4)^p,x)
```

output
```
b**p*int(tan(e + f*x)**(4*p),x)
```

3.29 $\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx = -\frac{\cot(e+fx)\log(\cos(e+fx))\,(b\tan^n(e+fx))^{\frac{1}{n}}}{f}$$

output
```
-cot(f*x+e)*ln(cos(f*x+e))*(b*tan(f*x+e)^n)^(1/n)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx = -\frac{\cot(e+fx)\log(\cos(e+fx))\,(b\tan^n(e+fx))^{\frac{1}{n}}}{f}$$

input
```
Integrate[(b*Tan[e + f*x]^n)^n^(-1),x]
```

output
```
-((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4142, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx)^n)^{\frac{1}{n}}dx$$

$$\Big\downarrow \text{4142}$$

$$\cot(e+fx)\,(b\tan^n(e+fx))^{\frac{1}{n}}\int \tan(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cot(e+fx)\,(b\tan^n(e+fx))^{\frac{1}{n}}\int \tan(e+fx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\cot(e+fx)\log(\cos(e+fx))\,(b\tan^n(e+fx))^{\frac{1}{n}}}{f}$$

input
```
Int[(b*Tan[e + f*x]^n)^n^(-1),x]
```

output
```
-((Cot[e + f*x]*Log[Cos[e + f*x]]*(b*Tan[e + f*x]^n)^n^(-1))/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.80 (sec) , antiderivative size = 5979, normalized size of antiderivative = 186.84

method	result	size
risch	Expression too large to display	5979

input
```
int((b*tan(f*x+e)^n)^(1/n),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx = -\frac{b^{\left(\frac{1}{n}\right)}\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f}$$

input
```
integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="fricas")
```

output
```
-1/2*b^(1/n)*log(1/(tan(f*x + e)^2 + 1))/f
```

Sympy [F]

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx = \int (b\tan^n{(e+fx)})^{\frac{1}{n}}\,dx$$

input
```
integrate((b*tan(f*x+e)**n)**(1/n),x)
```

output
```
Integral((b*tan(e + f*x)**n)**(1/n), x)
```

Maxima [F]

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx = \int (b\tan(fx+e)^n)^{\left(\frac{1}{n}\right)}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^n)^(1/n), x)
```

Giac [F]

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx=\int (b\tan(fx+e)^n)^{\left(\frac{1}{n}\right)}\,dx$$

input
```
integrate((b*tan(f*x+e)^n)^(1/n),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^n)^(1/n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx=\int (b\tan(e+f\,x)^n)^{1/n}\,dx$$

input
```
int((b*tan(e + f*x)^n)^(1/n),x)
```

output
```
int((b*tan(e + f*x)^n)^(1/n), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int (b\tan^n(e+fx))^{\frac{1}{n}}\,dx=\frac{b^{\frac{1}{n}}\log\left(\tan(fx+e)^2+1\right)}{2f}$$

input
```
int((b*tan(f*x+e)^n)^(1/n),x)
```

output
```
(b**(1/n)*log(tan(e + f*x)**2 + 1))/(2*f)
```

3.30 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a-3b)\cos(e+fx)}{f}+\frac{(2a-3b)\cos^3(e+fx)}{3f}-\frac{(a-b)\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a-3*b)*cos(f*x+e)/f+1/3*(2*a-3*b)*cos(f*x+e)^3/f-1/5*(a-b)*cos(f*x+e)^5/
f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{5a\cos(e+fx)}{8f}+\frac{19b\cos(e+fx)}{8f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{3b\cos(3(e+fx))}{16f}-\frac{a\cos(5(e+fx))}{80f}+\frac{b\cos(5(e+fx))}{80f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-5*a*Cos[e + f*x])/(8*f) + (19*b*Cos[e + f*x])/(8*f) + (5*a*Cos[3*(e + f*
x)])/(48*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (a*Cos[5*(e + f*x)])/(80*f)
+ (b*Cos[5*(e + f*x)])/(80*f) + (b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4147, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^5 \left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^6(e+fx) \left(1-\sec^2(e+fx)\right)^2 \left(b\sec^2(e+fx)+a-b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left((a-b)\cos^6(e+fx)+(3b-2a)\cos^4(e+fx)+(a-3b)\cos^2(e+fx)+b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{5}(a-b)\cos^5(e+fx)+\frac{1}{3}(2a-3b)\cos^3(e+fx)-(a-3b)\cos(e+fx)+b\sec(e+fx)}{f}$$

input
```
Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((a - 3*b)*Cos[e + f*x]) + ((2*a - 3*b)*Cos[e + f*x]^3)/3 - ((a - b)*Cos
[e + f*x]^5)/5 + b*Sec[e + f*x])/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 7.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)\right)-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+}{5}}{f}$
default	$\frac{b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)\right)-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+}{5}}{f}$
risch	$-\frac{5\,e^{i(fx+e)}a}{16f}+\frac{19\,e^{i(fx+e)}b}{16f}-\frac{5\,e^{-i(fx+e)}a}{16f}+\frac{19\,e^{-i(fx+e)}b}{16f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\cos(5fx+5e)a}{80f}+\frac{\cos(5f}{8}$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(sin(f*x+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*si
n(f*x+e)^2)*cos(f*x+e))-1/5*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+
e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx = -\frac{3 (a-b) \cos (fx+e)^6 - 5 (2 a-3 b) \cos (fx+e)^4 + 15 (a-3 b) \cos (fx+e)^2 - 15 b}{15 f \cos (fx+e)}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/15*(3*(a - b)*cos(f*x + e)^6 - 5*(2*a - 3*b)*cos(f*x + e)^4 + 15*(a - 3
*b)*cos(f*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx = \int \left(a+b\tan^2{(e+fx)}\right)\sin^5{(e+fx)}\, dx$$

input
```
integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{3\,(a-b)\cos\left(fx+e\right)^5-5\,(2\,a-3\,b)\cos\left(fx+e\right)^3+15\,(a-3\,b)\cos\left(fx+e\right)-\frac{15\,b}{\cos(fx+e)}}{15\,f}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/15*(3*(a - b)*cos(f*x + e)^5 - 5*(2*a - 3*b)*cos(f*x + e)^3 + 15*(a - 3
*b)*cos(f*x + e) - 15*b/cos(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54384 vs. 2(66) = 132.

Time = 18.57 (sec) , antiderivative size = 54384, normalized size of antiderivative = 776.91

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/1920*(315*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/
2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^
2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)
^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^12 + 315*pi*b*sgn(tan(
1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - t
an(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/
2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*tan(1/2*f*x) - 1)*
tan(1/2*f*x)^12*tan(1/2*e)^12 - 315*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 +
 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f
*x) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^12 + 315*pi*b*sgn(tan(1/2*f*x)^2*tan(1
/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 - 2*
tan(1/2*f*x) - 1)*tan(1/2*f*x)^12*tan(1/2*e)^12 - 630*pi*b*sgn(tan(1/2*f*x
)^2*tan(1/2*e)^2 - tan(1/2*f*x)^2 - 4*tan(1/2*f*x)*tan(1/2*e) - tan(1/2*e)
^2 + 1)*tan(1/2*f*x)^12*tan(1/2*e)^12 + 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1
/2*e)^2 + 2*tan(1/2*f*x)^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 + 2*
tan(1/2*e) - 1)*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*tan(1/2*f*x)*tan(1/2*e
)^2 - tan(1/2*f*x)^2 + tan(1/2*e)^2 + 2*tan(1/2*f*x) - 1)*tan(1/2*f*x)^12*
tan(1/2*e)^10 + 1260*pi*b*sgn(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)
^2*tan(1/2*e) + tan(1/2*f*x)^2 - tan(1/2*e)^2 - 2*tan(1/2*e) - 1)*sgn(tan(
1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)^2 - tan(1/2*f*x)^2 ...
```

Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx =$$

$$-\frac{\frac{5\,a}{16}-\frac{35\,b}{16}+\frac{25\,a\,\cos(2\,e+2\,f\,x)}{96}-\frac{11\,a\,\cos(4\,e+4\,f\,x)}{240}+\frac{a\,\cos(6\,e+6\,f\,x)}{160}-\frac{35\,b\,\cos(2\,e+2\,f\,x)}{32}+\frac{7\,b\,\cos(4\,e+4\,f\,x)}{80}-\frac{b\,\cos(6\,e+6\,f\,x)}{160}}{f\,\cos\left(e+f\,x\right)}$$

input
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

output
```
-((5*a)/16 - (35*b)/16 + (25*a*cos(2*e + 2*f*x))/96 - (11*a*cos(4*e + 4*f*
x))/240 + (a*cos(6*e + 6*f*x))/160 - (35*b*cos(2*e + 2*f*x))/32 + (7*b*cos
(4*e + 4*f*x))/80 - (b*cos(6*e + 6*f*x))/160)/(f*cos(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{8\cos(fx+e)\,a-48\cos(fx+e)\,b+3\sin(fx+e)^6\,a-3\sin(fx+e)^6\,b+\sin(fx+e)^4\,a-6\sin(fx+$$
$$15\cos(fx+e)\,f$$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

output
```
(8*cos(e + f*x)*a - 48*cos(e + f*x)*b + 3*sin(e + f*x)**6*a - 3*sin(e + f*
x)**6*b + sin(e + f*x)**4*a - 6*sin(e + f*x)**4*b + 4*sin(e + f*x)**2*a -
24*sin(e + f*x)**2*b - 8*a + 48*b)/(15*cos(e + f*x)*f)
```

3.31 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a-2b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a-2*b)*cos(f*x+e)/f+1/3*(a-b)*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{3a\cos(e+fx)}{4f}+\frac{7b\cos(e+fx)}{4f}+\frac{a\cos(3(e+fx))}{12f}-\frac{b\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-3*a*Cos[e + f*x])/(4*f) + (7*b*Cos[e + f*x])/(4*f) + (a*Cos[3*(e + f*x)]
)/(12*f) - (b*Cos[3*(e + f*x)])/(12*f) + (b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left((a-b)\cos^4(e+fx)+(2b-a)\cos^2(e+fx)-b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}(a-b)\cos^3(e+fx)-(a-2b)\cos(e+fx)+b\sec(e+fx)}{f}$$

input
```
Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((a - 2*b)*Cos[e + f*x]) + ((a - b)*Cos[e + f*x]^3)/3 + b*Sec[e + f*x])/
f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)-\frac{a\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}}{f}$
default	$\frac{b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)-\frac{a\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}}{f}$
risch	$-\frac{3\,\mathrm{e}^{i(fx+e)}a}{8f}+\frac{7\,\mathrm{e}^{i(fx+e)}b}{8f}-\frac{3\,\mathrm{e}^{-i(fx+e)}a}{8f}+\frac{7\,\mathrm{e}^{-i(fx+e)}b}{8f}+\frac{2b\,\mathrm{e}^{i(fx+e)}}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)}+\frac{\cos(3fx+3e)a}{12f}-\frac{\cos(3fx\text{-}}{12f}$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(sin(f*x+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*
x+e))-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{(a-b)\cos\left(fx+e\right)^4-3\left(a-2\,b\right)\cos\left(fx+e\right)^2+3\,b}{3\,f\cos\left(fx+e\right)}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*((a - b)*cos(f*x + e)^4 - 3*(a - 2*b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x
 + e))
```

Sympy [F]

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)\sin^3\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{(a-b)\cos\left(fx+e\right)^3-3\,(a-2\,b)\cos\left(fx+e\right)+\frac{3\,b}{\cos(fx+e)}}{3\,f}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*((a - b)*cos(f*x + e)^3 - 3*(a - 2*b)*cos(f*x + e) + 3*b/cos(f*x + e))
/f
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.48

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b}{f\cos\left(fx+e\right)} + \frac{af^5\cos\left(fx+e\right)^3-bf^5\cos\left(fx+e\right)^3-3\,af^5\cos\left(fx+e\right)+6\,bf^5\cos\left(fx+e\right)}{3\,f^6}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - b*f^5*cos(f*x + e)^3 - 3*
a*f^5*cos(f*x + e) + 6*b*f^5*cos(f*x + e))/f^6
```

Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=-\frac{\frac{3\,a}{8}-\frac{15\,b}{8}+\frac{a\,\cos(2\,e+2\,f\,x)}{3}-\frac{a\,\cos(4\,e+4\,f\,x)}{24}-\frac{5\,b\,\cos(2\,e+2\,f\,x)}{6}+\frac{b\,\cos(4\,e+4\,f\,x)}{24}}{f\,\cos\left(e+f\,x\right)}$$

input
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

output
```
-((3*a)/8 - (15*b)/8 + (a*cos(2*e + 2*f*x))/3 - (a*cos(4*e + 4*f*x))/24 -
(5*b*cos(2*e + 2*f*x))/6 + (b*cos(4*e + 4*f*x))/24)/(f*cos(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{2\cos\left(fx+e\right)a-8\cos\left(fx+e\right)b+\sin\left(fx+e\right)^4a-\sin\left(fx+e\right)^4b+\sin\left(fx+e\right)^2a-4\sin\left(fx+e\right)^2}{3\cos\left(fx+e\right)f}$$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

output
```
(2*cos(e + f*x)*a - 8*cos(e + f*x)*b + sin(e + f*x)**4*a - sin(e + f*x)**4
*b + sin(e + f*x)**2*a - 4*sin(e + f*x)**2*b - 2*a + 8*b)/(3*cos(e + f*x)*
f)
```

3.32 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right) dx = -\frac{(a-b)\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a-b)*cos(f*x+e)/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right) dx = -\frac{a\cos(e)\cos(fx)}{f}+\frac{b\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}+\frac{a\sin(e)\sin(fx)}{f}$$

input
```
Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((a*Cos[e]*Cos[f*x])/f) + (b*Cos[e + f*x])/f + (b*Sec[e + f*x])/f + (a*Si
n[e]*Sin[f*x])/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left((a-b)\cos^2(e+fx)+b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sec(e+fx)-(a-b)\cos(e+fx)}{f}$$

input
```
Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((a - b)*Cos[e + f*x]) + b*Sec[e + f*x])/f
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result	size
derivativedivides	$\frac{-a\cos(fx+e)+b\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e)\right)}{f}$	52
default	$\frac{-a\cos(fx+e)+b\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e)\right)}{f}$	52
risch	$-\frac{e^{i(fx+e)}a}{2f}+\frac{e^{i(fx+e)}b}{2f}-\frac{e^{-i(fx+e)}a}{2f}+\frac{e^{-i(fx+e)}b}{2f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}$	90

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f*x+e)^2)*cos(f*x+e))
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a-b)\cos{(fx+e)}^2-b}{f\cos{(fx+e)}}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-((a - b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\int \left(a+b\tan^2{(e+fx)}\right)\sin{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\left(\frac{1}{\cos{(fx+e)}}+\cos{(fx+e)}\right)-a\cos{(fx+e)}}{f}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b*(1/cos(f*x + e) + cos(f*x + e)) - a*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = b\left(\frac{\cos(fx+e)}{f}+\frac{1}{f\cos(fx+e)}\right)-\frac{a\cos(fx+e)}{f}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
b*(cos(f*x + e)/f + 1/(f*cos(f*x + e))) - a*cos(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{(\cos(e+f\,x)+1)\,(b-a\,\cos(e+f\,x)+b\,\cos(e+f\,x))}{f\,\cos(e+f\,x)}$$

input
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

output
```
((cos(e + f*x) + 1)*(b - a*cos(e + f*x) + b*cos(e + f*x)))/(f*cos(e + f*x)
)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.04

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\cos(fx+e)\,a-2\cos(fx+e)\,b+\sin(fx+e)^2\,a-\sin(fx+e)^2\,b-a+2b}{\cos(fx+e)\,f}$$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

output
```
(cos(e + f*x)*a - 2*cos(e + f*x)*b + sin(e + f*x)**2*a - sin(e + f*x)**2*b
 - a + 2*b)/(cos(e + f*x)*f)
```

3.33 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\text{arctanh}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

output
```
-a*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\text{arctanh}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((a*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4147, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{b\sec^2(e+fx)+a-b}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b\sec^2(e+fx)+a-b}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{b\sec(e+fx)-a\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\sec(e+fx)-a\text{arctanh}(\sec(e+fx))}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-(a*ArcTanh[Sec[e + f*x]]) + b*Sec[e + f*x])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$\frac{\frac{b}{\cos(fx+e)}+a\ln(\csc(fx+e)-\cot(fx+e))}{f}$	34
default	$\frac{\frac{b}{\cos(fx+e)}+a\ln(\csc(fx+e)-\cot(fx+e))}{f}$	34
risch	$\frac{2b\,\mathrm{e}^{i(fx+e)}}{f(\mathrm{e}^{2i(fx+e)}+1)}+\frac{a\ln(\mathrm{e}^{i(fx+e)}-1)}{f}-\frac{a\ln(\mathrm{e}^{i(fx+e)}+1)}{f}$	65

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/cos(f*x+e)+a*ln(csc(f*x+e)-cot(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{a\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-a\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2\,b}{2\,f\cos\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*(a*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - a*cos(f*x + e)*log(-1/2
*cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)\csc\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{a\log\left(\cos\left(fx+e\right)+1\right)-a\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\,b}{\cos(fx+e)}}{2\,f}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*log(cos(f*x + e) + 1) - a*log(cos(f*x + e) - 1) - 2*b/cos(f*x + e)
)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{a\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{4\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2\,f}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 4*b/((cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{2\,b}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b*tan(e + f*x)^2)/sin(e + f*x),x)
```

output
```
(a*log(tan(e/2 + (f*x)/2)))/f - (2*b)/(f*(tan(e/2 + (f*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a-\cos(fx+e)\,b+b}{\cos(fx+e)\,f}$$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

output
```
(cos(e + f*x)*log(tan((e + f*x)/2))*a - cos(e + f*x)*b + b)/(cos(e + f*x)*
f)
```

3.34 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a+2b)\text{arctanh}(\cos(e+fx))}{2f}-\frac{a\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

output
```
-1/2*(a+2*b)*arctanh(cos(f*x+e))/f-1/2*a*cot(f*x+e)*csc(f*x+e)/f+b*sec(f*x
+e)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(51) = 102.

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.41

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\sec(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/8*(a*Csc[(e + f*x)/2]^2)/f - (a*Log[Cos[(e + f*x)/2]])/(2*f) - (b*Log[C
os[(e + f*x)/2]])/f + (a*Log[Sin[(e + f*x)/2]])/(2*f) + (b*Log[Sin[(e + f*
x)/2]])/f + (a*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sec[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4147, 360, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{ 4147}$$

$$\frac{\int \frac{\sec^2(e+fx)(b\sec^2(e+fx)+a-b)}{(1-\sec^2(e+fx))^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 360}$$

$$\frac{\frac{a\sec(e+fx)}{2(1-\sec^2(e+fx))}-\frac{1}{2}\int \frac{2b\sec^2(e+fx)+a}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{1}{2}\left(2b\sec(e+fx)-(a+2b)\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)\right)+\frac{a\sec(e+fx)}{2(1-\sec^2(e+fx))}}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}(2b\sec(e+fx)-(a+2b)\text{arctanh}(\sec(e+fx)))+\frac{a\sec(e+fx)}{2(1-\sec^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
((-((a + 2*b)*ArcTanh[Sec[e + f*x]]) + 2*b*Sec[e + f*x])/2 + (a*Sec[e + f*
x])/(2*(1 - Sec[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
risch	$\frac{e^{i(fx+e)}\left(a\,e^{4i(fx+e)}+2b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-4b\,e^{2i(fx+e)}+a+2b\right)}{f\left(e^{2i(fx+e)}-1\right)^{2}\left(e^{2i(fx+e)}+1\right)}-\frac{a\ln\left(e^{i(fx+e)}+1\right)}{2f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b}{f}+$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e)))+a*(-1/2*csc(f*x+e)*cot(f*x
+e)+1/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(47) = 94$.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \frac{2\,(a+2\,b)\cos\left(fx+e\right)^2 - \left((a+2\,b)\cos\left(fx+e\right)^3 - (a+2\,b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right) + (}{4\left(f\cos\left(fx+e\right)^3 - f\cos\left(fx\right.\right.}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(2*(a + 2*b)*cos(f*x + e)^2 - ((a + 2*b)*cos(f*x + e)^3 - (a + 2*b)*co
s(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 2*b)*cos(f*x + e)^3 - (a +
 2*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3
- f*cos(f*x + e))
```

Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)\csc^3\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(a+2\,b)\log\left(\cos\left(fx+e\right)+1\right)-(a+2\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+2\,b)\cos(fx+e)^2-2\,b\right)}{\cos(fx+e)^3-\cos(fx+e)}}{4\,f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/4*((a + 2*b)*log(cos(f*x + e) + 1) - (a + 2*b)*log(cos(f*x + e) - 1) -
2*((a + 2*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(47) = 94.

Time = 0.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.35

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{2\,(a+2\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a+\frac{14\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{2\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+\frac{(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}}{8\,f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*(2*(a + 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - a*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + (a + 14*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x +
e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + (
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2))/f
```

Mupad [B] (verification not implemented)

Time = 7.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{8f} - \frac{\frac{a}{2}-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a}{2}+8b\right)}{f\left(4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\right)} + \frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{a}{2}+b\right)}{f}$$

input
```
int((a + b*tan(e + f*x)^2)/sin(e + f*x)^3,x)
```

output
```
(a*tan(e/2 + (f*x)/2)^2)/(8*f) - (a/2 - tan(e/2 + (f*x)/2)^2*(a/2 + 8*b))/
(f*(4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^4)) + (log(tan(e/2 + (f*
x)/2))*(a/2 + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.65

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \frac{4\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^2 a + 8\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^2 b - \cos(}{8\cos(fx+e)\sin}$$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

output
```
(4*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**2*a + 8*cos(e + f*x)*l
og(tan((e + f*x)/2))*sin(e + f*x)**2*b - cos(e + f*x)*sin(e + f*x)**2*a -
8*cos(e + f*x)*sin(e + f*x)**2*b + 4*sin(e + f*x)**2*a + 8*sin(e + f*x)**2
*b - 4*a)/(8*cos(e + f*x)*sin(e + f*x)**2*f)
```

3.35 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{3(a+4b)\operatorname{arctanh}(\cos(e+fx))}{8f}-\frac{(5a+4b)\cot(e+fx)\csc(e+fx)}{8f}-\frac{a\cot^3(e+fx)\csc(e+fx)}{4f}+\frac{b\sec(e+fx)}{f}$$

output
```
-3/8*(a+4*b)*arctanh(cos(f*x+e))/f-1/8*(5*a+4*b)*cot(f*x+e)*csc(f*x+e)/f-1
/4*a*cot(f*x+e)^3*csc(f*x+e)/f+b*sec(f*x+e)/f
```

Mathematica [A] (verified)

Time = 5.81 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{-2(3a+4b)\csc^2\left(\frac{1}{2}(e+fx)\right)-a\csc^4\left(\frac{1}{2}(e+fx)\right)-24(a+4b)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+\right.\right.\right.}{64f}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-2*(3*a + 4*b)*Csc[(e + f*x)/2]^2 - a*Csc[(e + f*x)/2]^4 - 24*(a + 4*b)*(
Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]) + (6*a + 8*b)*Sec[(e + f*x)
/2]^2 + a*Sec[(e + f*x)/2]^4 + 128*b*Sec[e + f*x]*Sin[(e + f*x)/2]^2)/(64*
f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4147, 25, 360, 25, 1471, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a-b\right)}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a-b\right)}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{-\frac{1}{4}\int -\frac{4b\sec^4(e+fx)+4a\sec^2(e+fx)+a}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{a\sec(e+fx)}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\int \frac{4b\sec^4(e+fx)+4a\sec^2(e+fx)+a}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{a\sec(e+fx)}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{1}{4}\left(\frac{(5a+4b)\sec(e+fx)}{2(1-\sec^2(e+fx))}-\frac{1}{2}\int\frac{8b\sec^2(e+fx)+3a+4b}{1-\sec^2(e+fx)}d\sec(e+fx)\right)-\frac{a\sec(e+fx)}{4(1-\sec^2(e+fx))^2}}{f}$$

$\Big\downarrow$ 299

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8b\sec(e+fx)-3(a+4b)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)\right)+\frac{(5a+4b)\sec(e+fx)}{2(1-\sec^2(e+fx))}\right)-\frac{a\sec(e+fx)}{4(1-\sec^2(e+fx))^2}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8b\sec(e+fx)-3(a+4b)\text{arctanh}(\sec(e+fx)))+\frac{(5a+4b)\sec(e+fx)}{2(1-\sec^2(e+fx))}\right)-\frac{a\sec(e+fx)}{4(1-\sec^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-1/4*(a*Sec[e + f*x])/(1 - Sec[e + f*x]^2)^2 + ((-3*(a + 4*b)*ArcTanh[Sec
[e + f*x]] + 8*b*Sec[e + f*x])/2 + ((5*a + 4*b)*Sec[e + f*x])/(2*(1 - Sec[
e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln\cdots}{}\right.}{f}$
default	$\frac{b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln\cdots}{}\right.}{f}$
risch	$\frac{e^{i(fx+e)}\left(3a\,e^{8i(fx+e)}+12b\,e^{8i(fx+e)}-8a\,e^{6i(fx+e)}-32b\,e^{6i(fx+e)}-22a\,e^{4i(fx+e)}+40b\,e^{4i(fx+e)}-8a\,e^{2i(fx+e)}-32b\,e\cdots\right.}{4f\left(e^{2i(fx+e)}-1\right)^4\left(e^{2i(fx+e)}+1\right)}$

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(-1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(
f*x+e)))+a*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e
)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(73) = 146$.

Time = 0.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.25

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{6\,(a+4\,b)\cos\left(fx+e\right)^4-10\,(a+4\,b)\cos\left(fx+e\right)^2-3\left((a+4\,b)\cos\left(fx+e\right)^5-2\,(a+4\,b)\cos\left(fx+e\right.\right.}{}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/16*(6*(a + 4*b)*cos(f*x + e)^4 - 10*(a + 4*b)*cos(f*x + e)^2 - 3*((a + 4
*b)*cos(f*x + e)^5 - 2*(a + 4*b)*cos(f*x + e)^3 + (a + 4*b)*cos(f*x + e))*
log(1/2*cos(f*x + e) + 1/2) + 3*((a + 4*b)*cos(f*x + e)^5 - 2*(a + 4*b)*co
s(f*x + e)^3 + (a + 4*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b
)/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

Sympy [F]

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)\csc^5\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{3\,(a+4\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+4\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+4\,b)\cos(fx+e)^4-5\,(a+4\,b)\cos(fx+e)^2+8\,b\right)}{\cos(fx+e)^5-2\,\cos(fx+e)^3+\cos(fx+e)}}{16\,f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/16*(3*(a + 4*b)*log(cos(f*x + e) + 1) - 3*(a + 4*b)*log(cos(f*x + e) -
1) - 2*(3*(a + 4*b)*cos(f*x + e)^4 - 5*(a + 4*b)*cos(f*x + e)^2 + 8*b)/(co
s(f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(73) = 146$.

Time = 0.55 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.03

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{12\,(a+4\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{\left(a-\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{72\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}}{64\,f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/64*(12*(a + 4*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - (a
- 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 18*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 72*b*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e)
- 1)^2 - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 128*b
/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

Mupad [B] (verification not implemented)

Time = 7.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.75

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a}{8}+\frac{b}{8}\right)}{f}-\frac{(-2a-34b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+\left(\frac{7a}{4}+2b\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+\frac{a}{4}}{f\left(16\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-16\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\right)}$$
$$+\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{3a}{8}+\frac{3b}{2}\right)}{f}+\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{64f}$$

input
```
int((a + b*tan(e + f*x)^2)/sin(e + f*x)^5,x)
```

output
```
(tan(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (a/4 + tan(e/2 + (f*x)/2)^2*((7*a)/
4 + 2*b) - tan(e/2 + (f*x)/2)^4*(2*a + 34*b))/(f*(16*tan(e/2 + (f*x)/2)^4
- 16*tan(e/2 + (f*x)/2)^6)) + (log(tan(e/2 + (f*x)/2))*((3*a)/8 + (3*b)/2)
)/f + (a*tan(e/2 + (f*x)/2)^4)/(64*f)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^4a+12\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^4b-\cos}{}$$

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

output
```
(3*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*a + 12*cos(e + f*x)*
log(tan((e + f*x)/2))*sin(e + f*x)**4*b - cos(e + f*x)*sin(e + f*x)**4*a -
 9*cos(e + f*x)*sin(e + f*x)**4*b + 3*sin(e + f*x)**4*a + 12*sin(e + f*x)*
*4*b - sin(e + f*x)**2*a - 4*sin(e + f*x)**2*b - 2*a)/(8*cos(e + f*x)*sin(
e + f*x)**4*f)
```

3.36 $\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$
\begin{aligned}
\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx &= \frac{5}{16}(a-7b)x-\frac{(11a-29b)\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad+\frac{(13a-19b)\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad-\frac{(a-b)\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{b\tan(e+fx)}{f}
\end{aligned}
$$

output
```
5/16*(a-7*b)*x-1/16*(11*a-29*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-19*b)*c
os(f*x+e)^3*sin(f*x+e)/f-1/6*(a-b)*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/
f
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{60ae-420be+60afx-420bfx+(-45a+141b)\sin(2(e+fx))+3(3a-5b)\sin(4(e+fx))-a\sin(6}{192f}$$

input
```
Integrate[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
(60*a*e - 420*b*e + 60*a*f*x - 420*b*f*x + (-45*a + 141*b)*Sin[2*(e + f*x)
] + 3*(3*a - 5*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + b*Sin[6*(e + f*x
)] + 192*b*Tan[e + f*x])/(192*f)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4146, 360, 25, 2345, 27, 1471, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(e+fx)^6\left(a+b\tan(e+fx)^2\right)dx$$
$$\Big\downarrow \text{4146}$$
$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{360}$$
$$\frac{-\frac{1}{6}\int -\frac{6b\tan^6(e+fx)+6(a-b)\tan^4(e+fx)-6(a-b)\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{(a-b)\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{6}\int \frac{6b\tan^6(e+fx)+6(a-b)\tan^4(e+fx)-6(a-b)\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{(a-b)\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 2345$$

$$\frac{\frac{1}{6}\left(\frac{(13a-19b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{3\left(-8b\tan^4(e+fx)-8(a-2b)\tan^2(e+fx)+3a-5b\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)\right)-\frac{(a-b)\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{6}\left(\frac{(13a-19b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{3}{4}\int \frac{-8b\tan^4(e+fx)-8(a-2b)\tan^2(e+fx)+3a-5b}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)\right)-\frac{(a-b)\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 1471$$

$$\frac{\frac{1}{6}\left(\frac{(13a-19b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{3}{4}\left(\frac{(11a-29b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}-\frac{1}{2}\int \frac{16b\tan^2(e+fx)+5a-19b}{\tan^2(e+fx)+1}d\tan(e+fx)\right)\right)-\frac{(a-b)\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{1}{6}\left(\frac{(13a-19b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-5(a-7b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-16b\tan(e+fx)\right)+\frac{(11a-29b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)\right)}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{6}\left(\frac{(13a-19b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-5(a-7b)\arctan(\tan(e+fx))-16b\tan(e+fx)\right)+\frac{(11a-29b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)\right)-\frac{(a-b)}{6(\tan^2(}}{f}$$

input
```
Int[Sin[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-1/6*((a - b)*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^3 + (((13*a - 19*b)*Tan[
e + f*x])/(4*(1 + Tan[e + f*x]^2)^2) - (3*((-5*(a - 7*b)*ArcTan[Tan[e + f*
x]] - 16*b*Tan[e + f*x])/2 + ((11*a - 29*b)*Tan[e + f*x])/(2*(1 + Tan[e +
f*x]^2))))/4)/6)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 10.95 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{b\left(\frac{\sin(fx+e)^9}{\cos(fx+e)}+\left(\sin(fx+e)^7+\frac{7\sin(fx+e)^5}{6}+\frac{35\sin(fx+e)^3}{24}+\frac{35\sin(fx+e)}{16}\right)\cos(fx+e)-\frac{35fx}{16}-\frac{35e}{16}\right)+a\left(-\frac{\left(\sin(fx+e)^5\right.}{}\right.}{f}$
default	$\frac{b\left(\frac{\sin(fx+e)^9}{\cos(fx+e)}+\left(\sin(fx+e)^7+\frac{7\sin(fx+e)^5}{6}+\frac{35\sin(fx+e)^3}{24}+\frac{35\sin(fx+e)}{16}\right)\cos(fx+e)-\frac{35fx}{16}-\frac{35e}{16}\right)+a\left(-\frac{\left(\sin(fx+e)^5\right.}{}\right.}{f}$
risch	$\frac{5ax}{16}-\frac{35bx}{16}+\frac{15ie^{2i(fx+e)}a}{128f}-\frac{47ie^{2i(fx+e)}b}{128f}-\frac{15ie^{-2i(fx+e)}a}{128f}+\frac{47ie^{-2i(fx+e)}b}{128f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}-$ si

input
```
int(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(sin(f*x+e)^9/cos(f*x+e)+(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f
*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)-35/16*f*x-35/16*e)+a*(-1/6*(sin(f*x+e
)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{15\,(a-7\,b)fx\cos\left(fx+e\right)-\left(8\,(a-b)\cos\left(fx+e\right)^6-2\,(13\,a-19\,b)\cos\left(fx+e\right)^4+3\,(11\,a-29\,b)\cos\right.}{48\,f\cos\left(fx+e\right)}$$

input
```
integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/48*(15*(a - 7*b)*f*x*cos(f*x + e) - (8*(a - b)*cos(f*x + e)^6 - 2*(13*a
- 19*b)*cos(f*x + e)^4 + 3*(11*a - 29*b)*cos(f*x + e)^2 - 48*b)*sin(f*x +
e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\int \left(a+b\tan^2\left(e+fx\right)\right)\sin^6\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{15\,(fx+e)(a-7\,b)+48\,b\tan\left(fx+e\right)-\frac{3\,(11\,a-29\,b)\tan(fx+e)^5+8\,(5\,a-17\,b)\tan(fx+e)^3+3\,(5\,a-19\,b)\tan(fx+e)}{\tan(fx+e)^6+3\,\tan(fx+e)^4+3\,\tan(fx+e)^2+1}}{48\,f}$$

input
```
integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/48*(15*(f*x + e)*(a - 7*b) + 48*b*tan(f*x + e) - (3*(11*a - 29*b)*tan(f*
x + e)^5 + 8*(5*a - 17*b)*tan(f*x + e)^3 + 3*(5*a - 19*b)*tan(f*x + e))/(t
an(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{5\,(fx+e)(a-7\,b)}{16\,f} + \frac{b\tan\left(fx+e\right)}{f} - \frac{33\,a\tan\left(fx+e\right)^5 - 87\,b\tan\left(fx+e\right)^5 + 40\,a\tan\left(fx+e\right)^3 - 136\,b\tan\left(fx+e\right)^3 + 15\,a\tan\left(fx+e\right) \cdots}{48\left(\tan\left(fx+e\right)^2+1\right)^3 f}$$

input
```
integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
5/16*(f*x + e)*(a - 7*b)/f + b*tan(f*x + e)/f - 1/48*(33*a*tan(f*x + e)^5
- 87*b*tan(f*x + e)^5 + 40*a*tan(f*x + e)^3 - 136*b*tan(f*x + e)^3 + 15*a*
tan(f*x + e) - 57*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*f)
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = x\left(\frac{5\,a}{16} - \frac{35\,b}{16}\right) - \frac{\left(\frac{11\,a}{16} - \frac{29\,b}{16}\right)\,\tan(e+f\,x)^5 + \left(\frac{5\,a}{6} - \frac{17\,b}{6}\right)\,\tan(e+f\,x)^3 + \left(\frac{5\,a}{16} - \frac{19\,b}{16}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6 + 3\tan\left(e+f\,x\right)^4 + 3\tan\left(e+f\,x\right)^2 + 1\right)} + \frac{b\tan(e+f\,x)}{f}$$

input
```
int(sin(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

output
```
x*((5*a)/16 - (35*b)/16) - (tan(e + f*x)^3*((5*a)/6 - (17*b)/6) + tan(e +
f*x)^5*((11*a)/16 - (29*b)/16) + tan(e + f*x)*((5*a)/16 - (19*b)/16))/(f*(
3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*
x))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.37

$$\int \sin^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \frac{15\cos(fx+e)\,ae+15\cos(fx+e)\,afx-105\cos(fx+e)\,be-105\cos(fx+e)\,bfx+8\sin(fx+e)^7\,a}{}$$

input
```
int(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

output
```
(15*cos(e + f*x)*a*e + 15*cos(e + f*x)*a*f*x - 105*cos(e + f*x)*b*e - 105*
cos(e + f*x)*b*f*x + 8*sin(e + f*x)**7*a - 8*sin(e + f*x)**7*b + 2*sin(e +
 f*x)**5*a - 14*sin(e + f*x)**5*b + 5*sin(e + f*x)**3*a - 35*sin(e + f*x)*
*3*b - 15*sin(e + f*x)*a + 105*sin(e + f*x)*b)/(48*cos(e + f*x)*f)
```

3.37 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{3}{8}(a-5b)x-\frac{(5a-9b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{(a-b)\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

output
```
3/8*(a-5*b)*x-1/8*(5*a-9*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*(a-b)*cos(f*x+e)^3
*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{12(a-5b)(e+fx)-8(a-2b)\sin(2(e+fx))+(a-b)\sin(4(e+fx))+32b\tan(e+fx)}{32f}$$

input
```
Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
(12*(a - 5*b)*(e + f*x) - 8*(a - 2*b)*Sin[2*(e + f*x)] + (a - b)*Sin[4*(e
+ f*x)] + 32*b*Tan[e + f*x])/(32*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4146, 360, 1471, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{(a-b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{-4b\tan^4(e+fx)-4(a-b)\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b\tan^2(e+fx)+3a-7b}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{(5a-9b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{(a-b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a-5b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+8b\tan(e+fx)\right)-\frac{(5a-9b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{(a-b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3(a-5b)\arctan(\tan(e+fx))+8b\tan(e+fx))-\frac{(5a-9b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{(a-b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
(((a - b)*Tan[e + f*x])/(4*(1 + Tan[e + f*x]^2)^2) + ((3*(a - 5*b)*ArcTan[
Tan[e + f*x]] + 8*b*Tan[e + f*x])/2 - ((5*a - 9*b)*Tan[e + f*x])/(2*(1 + T
an[e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)-\frac{15fx}{8}-\frac{15e}{8}\right)+a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)}{4}\right.}{f}$
default	$\frac{b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)-\frac{15fx}{8}-\frac{15e}{8}\right)+a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)}{4}\right.}{f}$
risch	$\frac{3ax}{8}-\frac{15bx}{8}+\frac{i e^{2i(fx+e)}a}{8f}-\frac{i e^{2i(fx+e)}b}{4f}-\frac{i e^{-2i(fx+e)}a}{8f}+\frac{i e^{-2i(fx+e)}b}{4f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\sin(4fx+4e}{32f}$

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*
x+e))*cos(f*x+e)-15/8*f*x-15/8*e)+a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{3\,(a-5\,b)fx\cos\left(fx+e\right)+\left(2\,(a-b)\cos\left(fx+e\right)^4-(5\,a-9\,b)\cos\left(fx+e\right)^2+8\,b\right)\sin\left(fx+e\right)}{8\,f\cos\left(fx+e\right)}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/8*(3*(a - 5*b)*f*x*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^4 - (5*a - 9*b
)*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)\sin^4\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{3\,(fx+e)(a-5\,b)+8\,b\tan\left(fx+e\right)-\frac{(5\,a-9\,b)\tan(fx+e)^3+(3\,a-7\,b)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/8*(3*(f*x + e)*(a - 5*b) + 8*b*tan(f*x + e) - ((5*a - 9*b)*tan(f*x + e)^
3 + (3*a - 7*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{3\,(fx+e)(a-5\,b)}{8\,f}+\frac{b\tan\left(fx+e\right)}{f} - \frac{5\,a\tan\left(fx+e\right)^3-9\,b\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)-7\,b\tan\left(fx+e\right)}{8\,\left(\tan\left(fx+e\right)^2+1\right)^2f}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
3/8*(f*x + e)*(a - 5*b)/f + b*tan(f*x + e)/f - 1/8*(5*a*tan(f*x + e)^3 - 9
*b*tan(f*x + e)^3 + 3*a*tan(f*x + e) - 7*b*tan(f*x + e))/((tan(f*x + e)^2
+ 1)^2*f)
```

Mupad [B] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = x\left(\frac{3\,a}{8}-\frac{15\,b}{8}\right)-\frac{\left(\frac{5\,a}{8}-\frac{9\,b}{8}\right)\,\tan(e+f\,x)^3+\left(\frac{3\,a}{8}-\frac{7\,b}{8}\right)\,\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)} + \frac{b\tan(e+f\,x)}{f}$$

input
```
int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

output
```
x*((3*a)/8 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/8 - (9*b)/8) + tan(e + f*x
)*((3*a)/8 - (7*b)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*ta
n(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{3\cos(fx+e)\,ae+3\cos(fx+e)\,afx-15\cos(fx+e)\,be-15\cos(fx+e)\,bfx+2\sin(fx+e)^5\,a-2}{8\cos(fx+e)\,f}$$

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

output
```
(3*cos(e + f*x)*a*e + 3*cos(e + f*x)*a*f*x - 15*cos(e + f*x)*b*e - 15*cos(
e + f*x)*b*f*x + 2*sin(e + f*x)**5*a - 2*sin(e + f*x)**5*b + sin(e + f*x)*
*3*a - 5*sin(e + f*x)**3*b - 3*sin(e + f*x)*a + 15*sin(e + f*x)*b)/(8*cos(
e + f*x)*f)
```

3.38 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{1}{2}(a-3b)x-\frac{(a-b)\cos(e+fx)\sin(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

output
```
1/2*(a-3*b)*x-1/2*(a-b)*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{2(a-3b)(e+fx)+(-a+b)\sin(2(e+fx))+4b\tan(e+fx)}{4f}$$

input
```
Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
(2*(a - 3*b)*(e + f*x) + (-a + b)*Sin[2*(e + f*x)] + 4*b*Tan[e + f*x])/(4*
f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4146, 360, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2 \left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{-\frac{1}{2}\int -\frac{2b\tan^2(e+fx)+a-b}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{(a-b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{2b\tan^2(e+fx)+a-b}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{(a-b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{2}\left((a-3b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+2b\tan(e+fx)\right)-\frac{(a-b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}((a-3b)\arctan(\tan(e+fx))+2b\tan(e+fx))-\frac{(a-b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

input
```
Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
(((a - 3*b)*ArcTan[Tan[e + f*x]] + 2*b*Tan[e + f*x])/2 - ((a - b)*Tan[e +
f*x])/(2*(1 + Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$	81
default	$\frac{a\left(-\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$	81
risch	$\frac{ax}{2}-\frac{3bx}{2}+\frac{i\,e^{2i(fx+e)}a}{8f}-\frac{i\,e^{2i(fx+e)}b}{8f}-\frac{i\,e^{-2i(fx+e)}a}{8f}+\frac{i\,e^{-2i(fx+e)}b}{8f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	94

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e)+b*(sin(f*x+e)^5/cos(f*x+
e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{(a-3\,b)fx\cos\left(fx+e\right)-\left((a-b)\cos\left(fx+e\right)^2-2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*((a - 3*b)*f*x*cos(f*x + e) - ((a - b)*cos(f*x + e)^2 - 2*b)*sin(f*x +
 e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\int \left(a+b\tan^2{(e+fx)}\right)\sin^2{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{(fx+e)(a-3\,b)+2\,b\tan{(fx+e)}-\frac{(a-b)\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a - 3*b) + 2*b*tan(f*x + e) - (a - b)*tan(f*x + e)/(tan(f*
x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{(fx+e)(a-3\,b)}{2\,f}+\frac{b\tan{(fx+e)}}{f}-\frac{a\tan{(fx+e)}-b\tan{(fx+e)}}{2\left(\tan{(fx+e)}^2+1\right)f}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(f*x + e)*(a - 3*b)/f + b*tan(f*x + e)/f - 1/2*(a*tan(f*x + e) - b*tan
(f*x + e))/((tan(f*x + e)^2 + 1)*f)
```

Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)-\sin\left(2\,e+2\,f\,x\right)\,\left(\frac{a}{4}-\frac{b}{4}\right)+f\,x\,\left(\frac{a}{2}-\frac{3\,b}{2}\right)}{f}$$

input
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

output
```
(b*tan(e + f*x) - sin(2*e + 2*f*x)*(a/4 - b/4) + f*x*(a/2 - (3*b)/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\cos\left(fx+e\right)ae+\cos\left(fx+e\right)afx-3\cos\left(fx+e\right)be-3\cos\left(fx+e\right)bfx+\sin\left(fx+e\right)^3a-\sin\left(fx+\right.}{2\cos\left(fx+e\right)f}$$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

output
```
(cos(e + f*x)*a*e + cos(e + f*x)*a*f*x - 3*cos(e + f*x)*b*e - 3*cos(e + f*
x)*b*f*x + sin(e + f*x)**3*a - sin(e + f*x)**3*b - sin(e + f*x)*a + 3*sin(
e + f*x)*b)/(2*cos(e + f*x)*f)
```

3.39 $\int (a+b\tan^2(e+fx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int (a+b\tan^2(e+fx))\,dx = ax-bx+\frac{b\tan(e+fx)}{f}$$

output
```
a*x-b*x+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int (a+b\tan^2(e+fx))\,dx = ax-\frac{b\arctan(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[a + b*Tan[e + f*x]^2,x]
```

output
```
a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

input
```
Int[a + b*Tan[e + f*x]^2,x]
```

output
```
a*x - b*x + (b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
norman	$(a-b)x+\frac{b\tan(fx+e)}{f}$	20
parallelrisch	$-\frac{b(fx-\tan(fx+e))}{f}+ax$	23
default	$ax+\frac{b(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}$	26
parts	$ax+\frac{b(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}$	26
derivativedivides	$\frac{b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$	27
risch	$ax-bx+\frac{2ib}{f(e^{2i(fx+e)}+1)}$	29

input
```
int(a+b*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(a-b)*x+b*tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=\frac{(a-b)fx+b\tan\left(fx+e\right)}{f}$$

input
```
integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
((a - b)*f*x + b*tan(f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=ax+b\left(\begin{cases}-x+\frac{\tan(e+fx)}{f} & \text{for } f\neq 0\\ x\tan^2(e) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*tan(f*x+e)**2,x)
```

output
```
a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=ax-\frac{(fx+e-\tan(fx+e))b}{f}$$

input
```
integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x - (f*x + e - tan(f*x + e))*b/f
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=ax-b\left(\frac{fx+e}{f}-\frac{\tan(fx+e)}{f}\right)$$

input
```
integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x - b*((f*x + e)/f - tan(f*x + e)/f)
```

Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)+f\,x\,(a-b)}{f}$$

input
```
int(a + b*tan(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + f*x*(a - b))/f
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=\frac{\tan\left(fx+e\right)b+afx-bfx}{f}$$

input
```
int(a+b*tan(f*x+e)^2,x)
```

output
```
(tan(e + f*x)*b + a*f*x - b*f*x)/f
```

3.40 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

output
```
-a*cot(f*x+e)/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((a*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(a\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\tan(e+fx)-a\cot(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-(a*Cot[e + f*x]) + b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{b\tan(fx+e)-\frac{a}{\tan(fx+e)}}{f}$	25
default	$\frac{b\tan(fx+e)-\frac{a}{\tan(fx+e)}}{f}$	25
risch	$-\frac{2i\left(a\,\mathrm{e}^{2i(fx+e)}-b\,\mathrm{e}^{2i(fx+e)}+a+b\right)}{f\left(\mathrm{e}^{2i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)}$	59

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*tan(f*x+e)-a/tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a+b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-((a + b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\int\left(a+b\tan^2\left(e+fx\right)\right)\csc^2\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\tan\left(fx+e\right)-\frac{a}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b*tan(f*x + e) - a/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\tan(fx+e)-\frac{a}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
(b*tan(f*x + e) - a/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)}{f}-\frac{a}{f\tan(e+f\,x)}$$

input
```
int((a + b*tan(e + f*x)^2)/sin(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x))/f - a/(f*tan(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{\sin(fx+e)^2 a+\sin(fx+e)^2 b-a}{\cos(fx+e)\sin(fx+e)f}$$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

output
```
(sin(e + f*x)**2*a + sin(e + f*x)**2*b - a)/(cos(e + f*x)*sin(e + f*x)*f)
```

3.41 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a+b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

output
```
-(a+b)*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{2a\cot(e+fx)}{3f}-\frac{b\cot(e+fx)}{f}-\frac{a\cot(e+fx)\csc^2(e+fx)}{3f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-2*a*Cot[e + f*x])/(3*f) - (b*Cot[e + f*x])/f - (a*Cot[e + f*x]*Csc[e + f
*x]^2)/(3*f) + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\tan(e+fx)^2}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 355$$

$$\frac{\int \left(a\cot^4(e+fx)+(a+b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{-(a+b)\cot(e+fx)-\frac{1}{3}a\cot^3(e+fx)+b\tan(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((a + b)*Cot[e + f*x]) - (a*Cot[e + f*x]^3)/3 + b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

method	result	size
derivativedivides	$\frac{b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)}{f}$	54
default	$\frac{b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)}{f}$	54
risch	$\frac{4i\left(3a\,e^{4i(fx+e)}-3b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}-a-3b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)}$	88

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+a*(-2/3-1/3*csc(f*x+e)^2)*co
t(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= -\frac{2\,(a+3\,b)\cos\left(fx+e\right)^4 - 3\,(a+3\,b)\cos\left(fx+e\right)^2 + 3\,b}{3\,\left(f\cos\left(fx+e\right)^3 - f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a + 3*b)*cos(f*x + e)^4 - 3*(a + 3*b)*cos(f*x + e)^2 + 3*b)/((f*c
os(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)\csc^4\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{3\,b\tan\left(fx+e\right) - \frac{3\,(a+b)\tan(fx+e)^2+a}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*b*tan(f*x + e) - (3*(a + b)*tan(f*x + e)^2 + a)/tan(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan(fx+e)^2+3\,b\tan(fx+e)^2+a}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 3*b*tan(f*x + e)^2 + a)/tan(
f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)}{f}-\frac{(a+b)\,\tan(e+f\,x)^2+\frac{a}{3}}{f\tan\left(e+f\,x\right)^3}$$

input
```
int((a + b*tan(e + f*x)^2)/sin(e + f*x)^4,x)
```

output
```
(b*tan(e + f*x))/f - (a/3 + tan(e + f*x)^2*(a + b))/(f*tan(e + f*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{2\sin\left(fx+e\right)^4a+6\sin\left(fx+e\right)^4b-\sin\left(fx+e\right)^2a-3\sin\left(fx+e\right)^2b-a}{3\cos\left(fx+e\right)\sin\left(fx+e\right)^3f}$$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

output
```
(2*sin(e + f*x)**4*a + 6*sin(e + f*x)**4*b - sin(e + f*x)**2*a - 3*sin(e +
 f*x)**2*b - a)/(3*cos(e + f*x)*sin(e + f*x)**3*f)
```

3.42 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a+2b)\cot(e+fx)}{f}-\frac{(2a+b)\cot^3(e+fx)}{3f}-\frac{a\cot^5(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

output
```
-(a+2*b)*cot(f*x+e)/f-1/3*(2*a+b)*cot(f*x+e)^3/f-1/5*a*cot(f*x+e)^5/f+b*ta
n(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{8a\cot(e+fx)}{15f}-\frac{5b\cot(e+fx)}{3f}-\frac{4a\cot(e+fx)\csc^2(e+fx)}{15f}-\frac{b\cot(e+fx)\csc^2(e+fx)}{3f}-\frac{a\cot(e+fx)\csc^4(e+fx)}{5f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-8*a*Cot[e + f*x])/(15*f) - (5*b*Cot[e + f*x])/(3*f) - (4*a*Cot[e + f*x]*
Csc[e + f*x]^2)/(15*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) - (a*Cot[e
+ f*x]*Csc[e + f*x]^4)/(5*f) + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left(a\cot^6(e+fx)+(2a+b)\cot^4(e+fx)+(a+2b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}(2a+b)\cot^3(e+fx)-(a+2b)\cot(e+fx)-\frac{1}{5}a\cot^5(e+fx)+b\tan(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((a + 2*b)*Cot[e + f*x]) - ((2*a + b)*Cot[e + f*x]^3)/3 - (a*Cot[e + f*x
]^5)/5 + b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

method	result	s
derivativedivides	$\frac{b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+a\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)}{f}$	8
default	$\frac{b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+a\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)}{f}$	8
risch	$-\frac{16i\left(10a\,e^{6i(fx+e)}-10b\,e^{6i(fx+e)}+5a\,e^{4i(fx+e)}+25b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-20b\,e^{2i(fx+e)}+a+5b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5\left(e^{2i(fx+e)}+1\right)}$	1

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x
+e))+a*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{8\,(a+5\,b)\cos\left(fx+e\right)^6-20\,(a+5\,b)\cos\left(fx+e\right)^4+15\,(a+5\,b)\cos\left(fx+e\right)^2-15\,b}{15\,\left(f\cos\left(fx+e\right)^5-2\,f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a + 5*b)*cos(f*x + e)^6 - 20*(a + 5*b)*cos(f*x + e)^4 + 15*(a +
5*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*co
s(f*x + e))*sin(f*x + e))
```

Sympy [F]

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)\csc^6\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)*csc(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{15\,b\tan\left(fx+e\right) - \frac{15\,(a+2\,b)\tan(fx+e)^4+5\,(2\,a+b)\tan(fx+e)^2+3\,a}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*b*tan(f*x + e) - (15*(a + 2*b)*tan(f*x + e)^4 + 5*(2*a + b)*tan(f
*x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{15\,b\tan\left(fx+e\right) - \frac{15\,a\tan(fx+e)^4+30\,b\tan(fx+e)^4+10\,a\tan(fx+e)^2+5\,b\tan(fx+e)^2+3\,a}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 30*b*tan(f*x + e)^4 + 10*
a*tan(f*x + e)^2 + 5*b*tan(f*x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{b\tan(e+f\,x)}{f}-\frac{(a+2\,b)\,\tan(e+f\,x)^4+\left(\frac{2\,a}{3}+\frac{b}{3}\right)\,\tan(e+f\,x)^2+\frac{a}{5}}{f\tan\left(e+f\,x\right)^5}$$

input
```
int((a + b*tan(e + f*x)^2)/sin(e + f*x)^6,x)
```

output
```
(b*tan(e + f*x))/f - (a/5 + tan(e + f*x)^2*((2*a)/3 + b/3) + tan(e + f*x)^
4*(a + 2*b))/(f*tan(e + f*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{8\sin\left(fx+e\right)^6a+40\sin\left(fx+e\right)^6b-4\sin\left(fx+e\right)^4a-20\sin\left(fx+e\right)^4b-\sin\left(fx+e\right)^2a-5\sin\left(fx\right.}{15\cos\left(fx+e\right)\sin\left(fx+e\right)^5f}$$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

output
```
(8*sin(e + f*x)**6*a + 40*sin(e + f*x)**6*b - 4*sin(e + f*x)**4*a - 20*sin
(e + f*x)**4*b - sin(e + f*x)**2*a - 5*sin(e + f*x)**2*b - 3*a)/(15*cos(e
+ f*x)*sin(e + f*x)**5*f)
```

3.43 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{(a^2-6ab+6b^2)\cos(e+fx)}{f} + \frac{2(a-2b)(a-b)\cos^3(e+fx)}{3f} - \frac{(a-b)^2\cos^5(e+fx)}{5f} + \frac{2(a-2b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-(a^2-6*a*b+6*b^2)*cos(f*x+e)/f+2/3*(a-2*b)*(a-b)*cos(f*x+e)^3/f-1/5*(a-b)
^2*cos(f*x+e)^5/f+2*(a-2*b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{-30(5a^2-38ab+41b^2)\cos(e+fx)+5(5a-13b)(a-b)\cos(3(e+fx))-3(a-b)^2\cos(5(e+fx))+}{240f}$$

input
```
Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-30*(5*a^2 - 38*a*b + 41*b^2)*Cos[e + f*x] + 5*(5*a - 13*b)*(a - b)*Cos[3
*(e + f*x)] - 3*(a - b)^2*Cos[5*(e + f*x)] + 480*(a - 2*b)*b*Sec[e + f*x]
+ 80*b^2*Sec[e + f*x]^3)/(240*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4147, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^5\left(a+b\tan(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^2d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left((a-b)^2\cos^6(e+fx)+2(a-2b)(b-a)\cos^4(e+fx)+\left(a^2-6ba+6b^2\right)\cos^2(e+fx)+b^2\sec^2(e+fx)+2(}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-(a^2-6ab+6b^2)\cos(e+fx)-\frac{1}{5}(a-b)^2\cos^5(e+fx)+\frac{2}{3}(a-2b)(a-b)\cos^3(e+fx)+2b(a-2b)\sec(e+f}{f}$$

input
```
Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a^2 - 6*a*b + 6*b^2)*Cos[e + f*x]) + (2*(a - 2*b)*(a - b)*Cos[e + f*x]
^3)/3 - ((a - b)^2*Cos[e + f*x]^5)/5 + 2*(a - 2*b)*b*Sec[e + f*x] + (b^2*S
ec[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 9.94 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(f\right.}{}$
default	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(f\right.}{}$
risch	$-\frac{e^{5i(fx+e)}a^2}{160f}+\frac{e^{5i(fx+e)}ab}{80f}-\frac{e^{5i(fx+e)}b^2}{160f}+\frac{5\,e^{3i(fx+e)}a^2}{96f}-\frac{3\,e^{3i(fx+e)}ab}{16f}+\frac{13\,e^{3i(fx+e)}b^2}{96f}-\frac{5\,e^{i(fx+e)}}{16f}$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*
x+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*co
s(f*x+e))+b^2*(1/3*sin(f*x+e)^10/cos(f*x+e)^3-7/3*sin(f*x+e)^10/cos(f*x+e)
-7/3*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*
x+e)^2)*cos(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^8-10\left(a^2-3\,ab+2\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2-6\,ab+6\,b^2\right)\cos\left(fx+e\right.}{15\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^8 - 10*(a^2 - 3*a*b + 2*b^2)*cos
(f*x + e)^6 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e)^4 - 30*(a*b - 2*b^2)*c
os(f*x + e)^2 - 5*b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\int \left(a+b\tan^2\left(e+fx\right)\right)^2\sin^5\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=$$

$$-\frac{3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-10\left(a^2-3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3+15\left(a^2-6\,ab+6\,b^2\right)\cos\left(fx+e\right.}{15\,f}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(a^2 - 3*a*b + 2*b^2)*cos
(f*x + e)^3 + 15*(a^2 - 6*a*b + 6*b^2)*cos(f*x + e) - 5*(6*(a*b - 2*b^2)*c
os(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Giac [F(-1)]

Timed out.

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.71

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \frac{2\,a^2\cos\left(e+f\,x\right)^3}{3\,f} - \frac{6\,b^2\,\cos\left(e+f\,x\right)}{f} - \frac{a^2\,\cos\left(e+f\,x\right)}{f} - \frac{a^2\cos\left(e+f\,x\right)^5}{5\,f} - \frac{4\,b^2}{f\,\cos\left(e+f\,x\right)} + \frac{b^2}{3\,f\cos\left(e+f\,x\right)^3} + \frac{4\,b^2\cos\left(e+f\,x\right)^3}{3\,f} - \frac{b^2\cos\left(e+f\,x\right)^5}{5\,f} + \frac{6\,a\,b\,\cos\left(e+f\,x\right)}{f} + \frac{2\,a\,b}{f\,\cos\left(e+f\,x\right)} - \frac{2\,a\,b\cos\left(e+f\,x\right)^3}{f} + \frac{2\,a\,b\cos\left(e+f\,x\right)^5}{5\,f}$$

input
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(2*a^2*cos(e + f*x)^3)/(3*f) - (6*b^2*cos(e + f*x))/f - (a^2*cos(e + f*x))
/f - (a^2*cos(e + f*x)^5)/(5*f) - (4*b^2)/(f*cos(e + f*x)) + b^2/(3*f*cos(
e + f*x)^3) + (4*b^2*cos(e + f*x)^3)/(3*f) - (b^2*cos(e + f*x)^5)/(5*f) +
(6*a*b*cos(e + f*x))/f + (2*a*b)/(f*cos(e + f*x)) - (2*a*b*cos(e + f*x)^3)
/f + (2*a*b*cos(e + f*x)^5)/(5*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.62

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$$

$$= \frac{8\cos\left(fx+e\right)\sin\left(fx+e\right)^2a^2 - 96\cos\left(fx+e\right)\sin\left(fx+e\right)^2ab + 128\cos\left(fx+e\right)\sin\left(fx+e\right)^2b^2 - 8\,\mathrm{c}}{}$$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(8*cos(e + f*x)*sin(e + f*x)**2*a**2 - 96*cos(e + f*x)*sin(e + f*x)**2*a*b
 + 128*cos(e + f*x)*sin(e + f*x)**2*b**2 - 8*cos(e + f*x)*a**2 + 96*cos(e
+ f*x)*a*b - 128*cos(e + f*x)*b**2 + 3*sin(e + f*x)**8*a**2 - 6*sin(e + f*
x)**8*a*b + 3*sin(e + f*x)**8*b**2 - 2*sin(e + f*x)**6*a**2 - 6*sin(e + f*
x)**6*a*b + 8*sin(e + f*x)**6*b**2 + 3*sin(e + f*x)**4*a**2 - 36*sin(e + f
*x)**4*a*b + 48*sin(e + f*x)**4*b**2 - 12*sin(e + f*x)**2*a**2 + 144*sin(e
 + f*x)**2*a*b - 192*sin(e + f*x)**2*b**2 + 8*a**2 - 96*a*b + 128*b**2)/(1
5*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.44 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{(a-3b)(a-b)\cos(e+fx)}{f}+\frac{(a-b)^2\cos^3(e+fx)}{3f}+\frac{(2a-3b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-(a-3*b)*(a-b)*cos(f*x+e)/f+1/3*(a-b)^2*cos(f*x+e)^3/f+(2*a-3*b)*b*sec(f*x
+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{(-9a^2+42ab-33b^2)\cos(e+fx)+(a-b)^2\cos(3(e+fx))+4b\sec(e+fx)\left(6a-9b+b\sec^2(e+fx)\right)}{12f}$$

input
```
Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((-9*a^2 + 42*a*b - 33*b^2)*Cos[e + f*x] + (a - b)^2*Cos[3*(e + f*x)] + 4*
b*Sec[e + f*x]*(6*a - 9*b + b*Sec[e + f*x]^2))/(12*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left((a-b)^2\cos^4(e+fx)+(a-3b)(b-a)\cos^2(e+fx)-b^2\sec^2(e+fx)+b(3b-2a)\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}(a-b)^2\cos^3(e+fx)-(a-3b)(a-b)\cos(e+fx)+b(2a-3b)\sec(e+fx)+\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a - 3*b)*(a - b)*Cos[e + f*x]) + ((a - b)^2*Cos[e + f*x]^3)/3 + (2*a -
 3*b)*b*Sec[e + f*x] + (b^2*Sec[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(76) = 152.

Time = 6.75 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(fx+e)^8}{3\cos(fx+e)^3}-\frac{5\sin(fx+e)^8}{3\cos(fx+e)}-\frac{5\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{3}\right)+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin\right.\right.}{f}$
default	$\frac{b^2\left(\frac{\sin(fx+e)^8}{3\cos(fx+e)^3}-\frac{5\sin(fx+e)^8}{3\cos(fx+e)}-\frac{5\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{3}\right)+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin\right.\right.}{f}$
risch	$\frac{e^{3i(fx+e)}a^2}{24f}-\frac{e^{3i(fx+e)}ab}{12f}+\frac{e^{3i(fx+e)}b^2}{24f}-\frac{3\,e^{i(fx+e)}a^2}{8f}+\frac{7\,e^{i(fx+e)}ab}{4f}-\frac{11\,e^{i(fx+e)}b^2}{8f}-\frac{3\,e^{-i(fx+e)}a^2}{8f}$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2*(1/3*sin(f*x+e)^8/cos(f*x+e)^3-5/3*sin(f*x+e)^8/cos(f*x+e)-5/3*(1
6/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e))+2*a*b*(sin
(f*x+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))-1/3*a
^2*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{(a^2-2\,ab+b^2)\cos\left(fx+e\right)^6-3\left(a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)^4+3\left(2\,ab-3\,b^2\right)\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x
+ e)^4 + 3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\int \left(a+b\tan^2\left(e+fx\right)\right)^2\sin^3\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^3-3\left(a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)+\frac{3\left(2\,ab-3\,b^2\right)\cos(fx+e)^2+b^2}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - 3*(a^2 - 4*a*b + 3*b^2)*cos(f*x
+ e) + (3*(2*a*b - 3*b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.69

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{6\,ab\cos\left(fx+e\right)^2-9\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$
$$+\frac{a^2f^{11}\cos\left(fx+e\right)^3-2\,abf^{11}\cos\left(fx+e\right)^3+b^2f^{11}\cos\left(fx+e\right)^3-3\,a^2f^{11}\cos\left(fx+e\right)+12\,abf^{11}\cos}{3\,f^{12}}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(6*a*b*cos(f*x + e)^2 - 9*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
 + 1/3*(a^2*f^11*cos(f*x + e)^3 - 2*a*b*f^11*cos(f*x + e)^3 + b^2*f^11*cos
(f*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 12*a*b*f^11*cos(f*x + e) - 9*b^2*f
^11*cos(f*x + e))/f^12
```

Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{32\,a\,b+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(64\,a\,b-32\,a^2\right)+12\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(24\,a^2-96\,a\,b+96\,b^2\right)-}{f\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-9\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+9\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\right)}$$

input
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
-(32*a*b + tan(e/2 + (f*x)/2)^6*(64*a*b - 32*a^2) + 12*a^2*tan(e/2 + (f*x)
/2)^8 + tan(e/2 + (f*x)/2)^4*(24*a^2 - 96*a*b + 96*b^2) - 4*a^2 - 32*b^2)/
(f*(9*tan(e/2 + (f*x)/2)^4 - 9*tan(e/2 + (f*x)/2)^8 + 3*tan(e/2 + (f*x)/2)
^12 - 3))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.34

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8a^2-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8ab+8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8b^2-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a^2+24\tan\right.}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+\right.}$$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(4*tan((e + f*x)/2)**4*(tan((e + f*x)/2)**8*a**2 - 8*tan((e + f*x)/2)**8*a
*b + 8*tan((e + f*x)/2)**8*b**2 - 6*tan((e + f*x)/2)**4*a**2 + 24*tan((e +
 f*x)/2)**4*a*b - 24*tan((e + f*x)/2)**4*b**2 + 8*tan((e + f*x)/2)**2*a**2
 - 16*tan((e + f*x)/2)**2*a*b - 3*a**2))/(3*f*(tan((e + f*x)/2)**12 - 3*ta
n((e + f*x)/2)**8 + 3*tan((e + f*x)/2)**4 - 1))
```

3.45 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = -\frac{(a-b)^2\cos(e+fx)}{f} + \frac{2(a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-(a-b)^2*cos(f*x+e)/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx = \frac{-3(a-b)^2\cos(e+fx)+b\sec(e+fx)\left(6a-6b+b\sec^2(e+fx)\right)}{3f}$$

input
```
Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-3*(a - b)^2*Cos[e + f*x] + b*Sec[e + f*x]*(6*a - 6*b + b*Sec[e + f*x]^2)
)/(3*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx) \left(a+b\tan(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 4147}$$

$$\frac{\int \cos^2(e+fx) \left(b\sec^2(e+fx)+a-b\right)^2 d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left((a-b)^2\cos^2(e+fx)+b^2\sec^2(e+fx)+2(a-b)b\right) d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-(a-b)^2\cos(e+fx)+2b(a-b)\sec(e+fx)+\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a - b)^2*Cos[e + f*x]) + 2*(a - b)*b*Sec[e + f*x] + (b^2*Sec[e + f*x]^
3)/3)/f
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(52) = 104$.

Time = 4.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.31

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(fx+e)^6}{3\cos(fx+e)^3}-\frac{\sin(fx+e)^6}{\cos(fx+e)}-\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+2ab\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos\cdots}{f}$
default	$\frac{b^2\left(\frac{\sin(fx+e)^6}{3\cos(fx+e)^3}-\frac{\sin(fx+e)^6}{\cos(fx+e)}-\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+2ab\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos\cdots}{f}$
risch	$-\frac{e^{i(fx+e)}a^2}{2f}+\frac{e^{i(fx+e)}ab}{f}-\frac{e^{i(fx+e)}b^2}{2f}-\frac{e^{-i(fx+e)}a^2}{2f}+\frac{e^{-i(fx+e)}ab}{f}-\frac{e^{-i(fx+e)}b^2}{2f}-\frac{4b\,e^{i(fx+e)}(-3a\,e\cdots}{\cdots}$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2*(1/3*sin(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f
*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+2*a*b*(sin(f*x+e)^4/cos(f*x+e)+(2+si
n(f*x+e)^2)*cos(f*x+e))-cos(f*x+e)*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{3\,(a^2-2\,ab+b^2)\cos\left(fx+e\right)^4-6\,(ab-b^2)\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 6*(a*b - b^2)*cos(f*x + e)^2
- b^2)/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^2\sin\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{6\,ab\left(\frac{1}{\cos(fx+e)}+\cos\left(fx+e\right)\right)-b^2\left(\frac{6\,\cos(fx+e)^2-1}{\cos(fx+e)^3}+3\,\cos\left(fx+e\right)\right)-3\,a^2\cos\left(fx+e\right)}{3\,f}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(6*a*b*(1/cos(f*x + e) + cos(f*x + e)) - b^2*((6*cos(f*x + e)^2 - 1)/c
os(f*x + e)^3 + 3*cos(f*x + e)) - 3*a^2*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{a^2f^3\cos\left(fx+e\right)-2\,abf^3\cos\left(fx+e\right)+b^2f^3\cos\left(fx+e\right)}{f^4} + \frac{6\,ab\cos\left(fx+e\right)^2-6\,b^2\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-(a^2*f^3*cos(f*x + e) - 2*a*b*f^3*cos(f*x + e) + b^2*f^3*cos(f*x + e))/f^
4 + 1/3*(6*a*b*cos(f*x + e)^2 - 6*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e
)^3)
```

Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.33

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{8\,a\,b+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(8\,a\,b-6\,a^2\right)+2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(6\,a^2-16\,a\,b+\frac{32\,b^2}{3}\right)-2\,a^2}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

input
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
-(8*a*b + tan(e/2 + (f*x)/2)^4*(8*a*b - 6*a^2) + 2*a^2*tan(e/2 + (f*x)/2)^
6 + tan(e/2 + (f*x)/2)^2*(6*a^2 - 16*a*b + (32*b^2)/3) - 2*a^2 - (16*b^2)/
3)/(f*(2*tan(e/2 + (f*x)/2)^2 - 2*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2
)^8 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 4.81

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{-\cos(fx+e)\tan(fx+e)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4b^2+\cos(fx+e)\tan(fx+e)^2b^2-6\cos(fx+e)\tan\left(\frac{fx}{2}+\right.}{}$$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - cos(e + f*x)*tan(e + f*x)**2*tan((e + f*x)/2)**4*b**2 + cos(e + f*x)*t
an(e + f*x)**2*b**2 - 6*cos(e + f*x)*tan((e + f*x)/2)**4*a**2 + 2*cos(e +
f*x)*tan((e + f*x)/2)**4*b**2 + 6*cos(e + f*x)*a**2 - 2*cos(e + f*x)*b**2
+ 2*sin(e + f*x)*tan(e + f*x)**3*tan((e + f*x)/2)**4*b**2 - 2*sin(e + f*x)
*tan(e + f*x)**3*b**2 + 2*sin(e + f*x)*tan(e + f*x)*tan((e + f*x)/2)**4*b*
*2 - 2*sin(e + f*x)*tan(e + f*x)*b**2 - 48*tan((e + f*x)/2)**4*a*b + 36*ta
n((e + f*x)/2)**4*b**2)/(6*f*(tan((e + f*x)/2)**4 - 1))
```

3.46 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{a^2\operatorname{arctanh}(\cos(e+fx))}{f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-a^2*arctanh(cos(f*x+e))/f+(2*a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{3a^2\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+3(2a-b)b\sec(e+fx)+b^2\sec^3(e+fx)}{3f}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(3*a^2*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]) + 3*(2*a - b)*b*Se
c[e + f*x] + b^2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4147, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\left(b\sec^2(e+fx)+a-b\right)^2}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(b\sec^2(e+fx)+a-b\right)^2}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(\frac{a^2}{1-\sec^2(e+fx)}-b^2\sec^2(e+fx)-(2a-b)b\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2\text{arctanh}(\sec(e+fx))+b(2a-b)\sec(e+fx)+\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(a^2*ArcTanh[Sec[e + f*x]]) + (2*a - b)*b*Sec[e + f*x] + (b^2*Sec[e + f*
x]^3)/3)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.87

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(fx+e)^4}{3\cos(fx+e)^3}-\frac{\sin(fx+e)^4}{3\cos(fx+e)}-\frac{\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}\right)+\frac{2ab}{\cos(fx+e)}+a^2\ln(\csc(fx+e)-\cot(fx+e))}{f}$
default	$\frac{b^2\left(\frac{\sin(fx+e)^4}{3\cos(fx+e)^3}-\frac{\sin(fx+e)^4}{3\cos(fx+e)}-\frac{\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}\right)+\frac{2ab}{\cos(fx+e)}+a^2\ln(\csc(fx+e)-\cot(fx+e))}{f}$
risch	$-\frac{2b\,e^{i(fx+e)}\left(-6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-12a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}-6a+3b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}-\frac{a^2\ln\left(e^{i(fx+e)}+1\right)}{f}+\frac{a^2\ln(e^{i(}}{}$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2*(1/3*sin(f*x+e)^4/cos(f*x+e)^3-1/3*sin(f*x+e)^4/cos(f*x+e)-1/3*(2
+sin(f*x+e)^2)*cos(f*x+e))+2*a*b/cos(f*x+e)+a^2*ln(csc(f*x+e)-cot(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{3\,a^2\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-3\,a^2\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-6\left(2\,ab-\right.}{6\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*a^2*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*a^2*cos(f*x + e
)^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b - b^2)*cos(f*x + e)^2 - 2*b^2)
/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=-\frac{3\,a^2\log\left(\cos\left(fx+e\right)+1\right)-3\,a^2\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(2\,ab-b^2)\cos(fx+e)^2+b^2\right)}{\cos(fx+e)^3}}{6\,f}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*a^2*log(cos(f*x + e) + 1) - 3*a^2*log(cos(f*x + e) - 1) - 2*(3*(2*
a*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(50) = 100.

Time = 0.66 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.67

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,a^2\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{8\left(3\,ab-b^2+\frac{6\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6\,f}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*a^2*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 8*(3*a*b -
b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3*b^2*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.65

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2\,\ln\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{4\,a\,b-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(8\,a\,b-4\,b^2\right)-\frac{4\,b^2}{3}+4\,a\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

input
```
int((a + b*tan(e + f*x)^2)^2/sin(e + f*x),x)
```

output
```
(a^2*log(tan(e/2 + (f*x)/2)))/f - (4*a*b - tan(e/2 + (f*x)/2)^2*(8*a*b - 4
*b^2) - (4*b^2)/3 + 4*a*b*tan(e/2 + (f*x)/2)^4)/(f*(tan(e/2 + (f*x)/2)^2 -
 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.23

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{3\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(fx+e\right)^2a^2-3\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a^2-6\cos\left(fx+e\right)\mathrm{s}}{3}$$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(3*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**2 - 3*cos(e + f*x
)*log(tan((e + f*x)/2))*a**2 - 6*cos(e + f*x)*sin(e + f*x)**2*a*b + 2*cos(
e + f*x)*sin(e + f*x)**2*b**2 + 6*cos(e + f*x)*a*b - 2*cos(e + f*x)*b**2 +
 6*sin(e + f*x)**2*a*b - 3*sin(e + f*x)**2*b**2 - 6*a*b + 2*b**2)/(3*cos(e
 + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.47 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{a(a+4b)\operatorname{arctanh}(\cos(e+fx))}{2f}-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-1/2*a*(a+4*b)*arctanh(cos(f*x+e))/f-1/2*a^2*cot(f*x+e)*csc(f*x+e)/f+2*a*b
*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 231 vs. 2(74) = 148.

Time = 5.88 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.12

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$=\frac{\left(24ab+8b^2+24ab\cos(2(e+fx))-12ab\cos(3(e+fx))-b^2\cos(3(e+fx))-12a^2\cos^2(e+fx)\cot^2(\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((24*a*b + 8*b^2 + 24*a*b*Cos[2*(e + f*x)] - 12*a*b*Cos[3*(e + f*x)] - b^2
*Cos[3*(e + f*x)] - 12*a^2*Cos[e + f*x]^2*Cot[e + f*x]^2 - 3*a^2*Cos[3*(e
+ f*x)]*Log[Cos[(e + f*x)/2]] - 12*a*b*Cos[3*(e + f*x)]*Log[Cos[(e + f*x)/
2]] + 3*a^2*Cos[3*(e + f*x)]*Log[Sin[(e + f*x)/2]] + 12*a*b*Cos[3*(e + f*x
)]*Log[Sin[(e + f*x)/2]] - 3*Cos[e + f*x]*(b*(12*a + b) + 3*a*(a + 4*b)*Lo
g[Cos[(e + f*x)/2]] - 3*a*(a + 4*b)*Log[Sin[(e + f*x)/2]]))*Sec[e + f*x]^3
)/(24*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4147, 366, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\sec^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^2}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{a^2\sec^3(e+fx)}{2\left(1-\sec^2(e+fx)\right)}-\frac{1}{2}\int \frac{\sec^2(e+fx)\left(a^2+4ba-2b^2+2b^2\sec^2(e+fx)\right)}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\sec^3(e+fx)-a(a+4b)\int \frac{\sec^2(e+fx)}{1-\sec^2(e+fx)}d\sec(e+fx)\right)+\frac{a^2\sec^3(e+fx)}{2\left(1-\sec^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\sec^3(e+fx)-a(a+4b)\left(\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)-\sec(e+fx)\right)\right)+\frac{a^2\sec^3(e+fx)}{2(1-\sec^2(e+fx))}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a^2\sec^3(e+fx)}{2(1-\sec^2(e+fx))}+\frac{1}{2}\left(\frac{2}{3}b^2\sec^3(e+fx)-a(a+4b)(\text{arctanh}(\sec(e+fx))-\sec(e+fx))\right)}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((a^2*Sec[e + f*x]^3)/(2*(1 - Sec[e + f*x]^2)) + (-(a*(a + 4*b)*(ArcTanh[S
ec[e + f*x]] - Sec[e + f*x])) + (2*b^2*Sec[e + f*x]^3)/3)/2)/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{b^{2}}{3\cos\left(fx+e\right)^{3}}+2ab\left(\frac{1}{\cos\left(fx+e\right)}+\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)\right)+a^{2}\left(-\frac{\csc\left(fx+e\right)\cot\left(fx+e\right)}{2}+\frac{\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2}\right)}{f}$
default	$\frac{\frac{b^{2}}{3\cos\left(fx+e\right)^{3}}+2ab\left(\frac{1}{\cos\left(fx+e\right)}+\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)\right)+a^{2}\left(-\frac{\csc\left(fx+e\right)\cot\left(fx+e\right)}{2}+\frac{\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2}\right)}{f}$
risch	$\frac{e^{i(fx+e)}\left(3a^{2}e^{8i(fx+e)}+12ab\,e^{8i(fx+e)}+12a^{2}e^{6i(fx+e)}+8b^{2}e^{6i(fx+e)}+18a^{2}e^{4i(fx+e)}-24ab\,e^{4i(fx+e)}-16b^{2}e^{4i(fx+e)}\right.}{3f\left(e^{2i(fx+e)}-1\right)^{2}\left(e^{2i(fx+e)}+1\right)^{3}}$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b^2/cos(f*x+e)^3+2*a*b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e)))+a
^2*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(68) = 136$.

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.27

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{6\left(a^2+4\,ab\right)\cos\left(fx+e\right)^4-4\left(6\,ab-b^2\right)\cos\left(fx+e\right)^2-4\,b^2-3\left(\left(a^2+4\,ab\right)\cos\left(fx+e\right)^5-\left(a^2+4\,ab\right.\right.}{12\left(f\cos\right.}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/12*(6*(a^2 + 4*a*b)*cos(f*x + e)^4 - 4*(6*a*b - b^2)*cos(f*x + e)^2 - 4*
b^2 - 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*cos(f*x + e)^3)*log(
1/2*cos(f*x + e) + 1/2) + 3*((a^2 + 4*a*b)*cos(f*x + e)^5 - (a^2 + 4*a*b)*
cos(f*x + e)^3)*log(-1/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*
x + e)^3)
```

Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^2\csc^3\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\left(a^2+4ab\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+4ab\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^2+4ab\right)\cos(fx+e)^4-2\left(6ab-\right.\right.}{\cos(fx+e)^5-\cos(f}}{12f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(3*(a^2 + 4*a*b)*log(cos(f*x + e) + 1) - 3*(a^2 + 4*a*b)*log(cos(f*x
 + e) - 1) - 2*(3*(a^2 + 4*a*b)*cos(f*x + e)^4 - 2*(6*a*b - b^2)*cos(f*x +
 e)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(68) = 136.

Time = 0.63 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.19

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx =$$

$$-\frac{\frac{3a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-6\left(a^2+4ab\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{3\left(a^2-\frac{2a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8ab(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}-}{24f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/24*(3*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 6*(a^2 + 4*a*b)*log(a
bs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 3*(a^2 - 2*a^2*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(
cos(f*x + e) + 1)/(cos(f*x + e) - 1) - 16*(6*a*b + b^2 + 12*a*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.54

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{a^2}{2}+2\,b\,a\right)}{f}+\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{8\,f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{3\,a^2}{2}+32\,b\,a\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(\frac{a^2}{2}+16\,a\,b+8\,b^2\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{3\,a^2}{2}+16\,a\,b+\frac{8\,b^2}{3}\right.}{f\left(-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+12\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-12\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\right)}$$

input
```
int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

output
```
(log(tan(e/2 + (f*x)/2))*(2*a*b + a^2/2))/f + (a^2*tan(e/2 + (f*x)/2)^2)/(
8*f) - (tan(e/2 + (f*x)/2)^4*(32*a*b + (3*a^2)/2) - tan(e/2 + (f*x)/2)^6*(
16*a*b + a^2/2 + 8*b^2) - tan(e/2 + (f*x)/2)^2*(16*a*b + (3*a^2)/2 + (8*b^
2)/3) + a^2/2)/(f*(4*tan(e/2 + (f*x)/2)^2 - 12*tan(e/2 + (f*x)/2)^4 + 12*t
an(e/2 + (f*x)/2)^6 - 4*tan(e/2 + (f*x)/2)^8))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.43

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{12\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^4a^2+48\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^4ab-}{}$$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(12*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*a**2 + 48*cos(e + f
*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*a*b - 12*cos(e + f*x)*log(tan((e
 + f*x)/2))*sin(e + f*x)**2*a**2 - 48*cos(e + f*x)*log(tan((e + f*x)/2))*s
in(e + f*x)**2*a*b - 9*cos(e + f*x)*sin(e + f*x)**4*a**2 - 48*cos(e + f*x)
*sin(e + f*x)**4*a*b - 8*cos(e + f*x)*sin(e + f*x)**4*b**2 + 9*cos(e + f*x
)*sin(e + f*x)**2*a**2 + 48*cos(e + f*x)*sin(e + f*x)**2*a*b + 8*cos(e + f
*x)*sin(e + f*x)**2*b**2 + 12*sin(e + f*x)**4*a**2 + 48*sin(e + f*x)**4*a*
b - 24*sin(e + f*x)**2*a**2 - 48*sin(e + f*x)**2*a*b - 8*sin(e + f*x)**2*b
**2 + 12*a**2)/(24*cos(e + f*x)*sin(e + f*x)**2*f*(sin(e + f*x)**2 - 1))
```

3.48 $\int \csc^5(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \csc^5(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = -\frac{(3a^2+24ab+8b^2)\operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{a(5a+8b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{a^2\cot^3(e+fx)\csc(e+fx)}{4f} + \frac{b(2a+b)\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-1/8*(3*a^2+24*a*b+8*b^2)*arctanh(cos(f*x+e))/f-1/8*a*(5*a+8*b)*cot(f*x+e)
*csc(f*x+e)/f-1/4*a^2*cot(f*x+e)^3*csc(f*x+e)/f+b*(2*a+b)*sec(f*x+e)/f+1/3
*b^2*sec(f*x+e)^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 447 vs. 2(115) = 230.

Time = 6.95 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.89

$$
\begin{aligned}
&\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx \\
&= \frac{(-3a^2-8ab)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32f} - \frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)}{64f} \\
&\quad + \frac{(-3a^2-24ab-8b^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8f} \\
&\quad + \frac{(3a^2+24ab+8b^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8f} + \frac{(3a^2+8ab)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32f} \\
&\quad + \frac{a^2\sec^4\left(\frac{1}{2}(e+fx)\right)}{64f} + \frac{b^2}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} \\
&\quad + \frac{b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} \\
&\quad - \frac{b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} \\
&\quad + \frac{b^2}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} \\
&\quad + \frac{-12ab\sin\left(\frac{1}{2}(e+fx)\right)-7b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)} \\
&\quad + \frac{12ab\sin\left(\frac{1}{2}(e+fx)\right)+7b^2\sin\left(\frac{1}{2}(e+fx)\right)}{6f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}
\end{aligned}
$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((-3*a^2 - 8*a*b)*Csc[(e + f*x)/2]^2)/(32*f) - (a^2*Csc[(e + f*x)/2]^4)/(6
4*f) + ((-3*a^2 - 24*a*b - 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*f) + ((3*a^2 +
 24*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*f) + ((3*a^2 + 8*a*b)*Sec[(e +
f*x)/2]^2)/(32*f) + (a^2*Sec[(e + f*x)/2]^4)/(64*f) + b^2/(12*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2) + (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^3) - (b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3) + b^2/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^2) + (-12*a*b*Sin[(e + f*x)/2] - 7*b^2*Sin[(e + f*x)/2])/(6*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])) + (12*a*b*Sin[(e + f*x)/2] + 7*b^2*Sin[(e + f*
x)/2])/(6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4147, 25, 366, 360, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a-b\right)^2}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a-b\right)^2}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{1}{4}\int \frac{\sec^4(e+fx)\left(a^2+8ba-4b^2+4b^2\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{a^2\sec^5(e+fx)}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int -\frac{8b^2\sec^4(e+fx)+2a(a+8b)\sec^2(e+fx)+a(a+8b)}{1-\sec^2(e+fx)}d\sec(e+fx)+\frac{a(a+8b)\sec(e+fx)}{2(1-\sec^2(e+fx))}\right)-\frac{a^2\sec^5(e+fx)}{4(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\left(\frac{a(a+8b)\sec(e+fx)}{2(1-\sec^2(e+fx))}-\frac{1}{2}\int \frac{8b^2\sec^4(e+fx)+2a(a+8b)\sec^2(e+fx)+a(a+8b)}{1-\sec^2(e+fx)}d\sec(e+fx)\right)-\frac{a^2\sec^5(e+fx)}{4(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\frac{1}{4}\left(\frac{a(a+8b)\sec(e+fx)}{2(1-\sec^2(e+fx))}-\frac{1}{2}\int \left(-8b^2\sec^2(e+fx)-2\left(a^2+8ba+4b^2\right)+\frac{3a^2+24ba+8b^2}{1-\sec^2(e+fx)}\right)d\sec(e+fx)\right)-\frac{a^2\sec^5(e+fa}{4(1-\sec^2(e+fa}}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(-\left(3a^2+24ab+8b^2\right)\text{arctanh}(\sec(e+fx))+2\left(a^2+8ab+4b^2\right)\sec(e+fx)+\frac{8}{3}b^2\sec^3(e+fx)\right)+\frac{a(a+8b)\,s}{2(1-\sec^2}}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/4*(a^2*Sec[e + f*x]^5)/(1 - Sec[e + f*x]^2)^2 + ((a*(a + 8*b)*Sec[e +
f*x])/(2*(1 - Sec[e + f*x]^2)) + (-((3*a^2 + 24*a*b + 8*b^2)*ArcTanh[Sec[e
 + f*x]]) + 2*(a^2 + 8*a*b + 4*b^2)*Sec[e + f*x] + (8*b^2*Sec[e + f*x]^3)/
3)/2)/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 5.77 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-}{2}\right.}{f}$
default	$\frac{b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-}{2}\right.}{f}$
risch	$\frac{e^{i(fx+e)}(9a^2e^{12i(fx+e)}+72ab\,e^{12i(fx+e)}+24b^2e^{12i(fx+e)}-6a^2e^{10i(fx+e)}-48ab\,e^{10i(fx+e)}-16b^2e^{10i(fx+e)}-105a^2e^{8}}{}$

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2*(1/3/cos(f*x+e)^3+1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e)))+2*a*b*(
-1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e)))
+a^2*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-cot(
f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(107) = 214.

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.47

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{6\left(3a^2+24ab+8b^2\right)\cos(fx+e)^6-10\left(3a^2+24ab+8b^2\right)\cos(fx+e)^4+16\left(6ab+b^2\right)\cos(fx+e)^2}{}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/48*(6*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 24*a*b + 8*b
^2)*cos(f*x + e)^4 + 16*(6*a*b + b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^2
+ 24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)
^5 + (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2)
+ 3*((3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 24*a*b + 8*b^2)*
cos(f*x + e)^5 + (3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x
 + e) + 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\int \left(a+b\tan^2{(e+fx)}\right)^2\csc^5{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=$$
$$-\frac{3\left(3a^2+24ab+8b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(3a^2+24ab+8b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(3a^2+\cdots\right.\right.}{\cdots}}{48f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/48*(3*(3*a^2 + 24*a*b + 8*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 24*a*
b + 8*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x +
 e)^6 - 5*(3*a^2 + 24*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(6*a*b + b^2)*cos(f*
x + e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(107) = 214$.

Time = 0.59 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx =$$

$$-\frac{\frac{24\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3\,a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-12\left(3\,a^2+24\,ab+8\,b^2\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\cdots}{\cdots}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/192*(24*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a*b*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) - 3*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 - 12*(3*a^2 + 24*a*b + 8*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e
) + 1)) + 3*(a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x +
 e) - 1)^2 - 256*(3*a*b + 2*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + 3*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.11

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$$
$$= \frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{64 f} + \frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(\frac{3a^2}{8}+3ab+b^2\right)}{f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{5a^2}{4}+4ba\right) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(2a^2+68ab+64b^2\right) - \tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\frac{21a^2}{4}+76ab+12\right.}{f\left(-16\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+48\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8-48\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+16\right.}$$
$$+ \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a^2}{8}+\frac{ba}{4}\right)}{f}$$

input
```
int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

output
```
(a^2*tan(e/2 + (f*x)/2)^4)/(64*f) + (log(tan(e/2 + (f*x)/2))*(3*a*b + (3*a
^2)/8 + b^2))/f - (tan(e/2 + (f*x)/2)^2*(4*a*b + (5*a^2)/4) - tan(e/2 + (f
*x)/2)^8*(68*a*b + 2*a^2 + 64*b^2) - tan(e/2 + (f*x)/2)^4*(76*a*b + (21*a^
2)/4 + (128*b^2)/3) + tan(e/2 + (f*x)/2)^6*(140*a*b + (23*a^2)/4 + 64*b^2)
 + a^2/4)/(f*(16*tan(e/2 + (f*x)/2)^4 - 48*tan(e/2 + (f*x)/2)^6 + 48*tan(e
/2 + (f*x)/2)^8 - 16*tan(e/2 + (f*x)/2)^10)) + (tan(e/2 + (f*x)/2)^2*((a*b
)/4 + a^2/8))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.69

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$$
$$= \frac{72\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^6a^2+576\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin(fx+e)^6ab\ -}{}$$

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

output

```
(72*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**6*a**2 + 576*cos(e +
f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**6*a*b + 192*cos(e + f*x)*log(tan(
(e + f*x)/2))*sin(e + f*x)**6*b**2 - 72*cos(e + f*x)*log(tan((e + f*x)/2))
*sin(e + f*x)**4*a**2 - 576*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x
)**4*a*b - 192*cos(e + f*x)*log(tan((e + f*x)/2))*sin(e + f*x)**4*b**2 - 6
3*cos(e + f*x)*sin(e + f*x)**6*a**2 - 528*cos(e + f*x)*sin(e + f*x)**6*a*b
 - 256*cos(e + f*x)*sin(e + f*x)**6*b**2 + 63*cos(e + f*x)*sin(e + f*x)**4
*a**2 + 528*cos(e + f*x)*sin(e + f*x)**4*a*b + 256*cos(e + f*x)*sin(e + f*
x)**4*b**2 + 72*sin(e + f*x)**6*a**2 + 576*sin(e + f*x)**6*a*b + 192*sin(e
 + f*x)**6*b**2 - 96*sin(e + f*x)**4*a**2 - 768*sin(e + f*x)**4*a*b - 256*
sin(e + f*x)**4*b**2 - 24*sin(e + f*x)**2*a**2 + 192*sin(e + f*x)**2*a*b +
 48*a**2)/(192*cos(e + f*x)*sin(e + f*x)**4*f*(sin(e + f*x)**2 - 1))
```

3.49 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{1}{8}\left(3a^2-30ab+35b^2\right)x-\frac{(5a-13b)(a-b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{(a-b)^2\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{(2a-3b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
1/8*(3*a^2-30*a*b+35*b^2)*x-1/8*(5*a-13*b)*(a-b)*cos(f*x+e)*sin(f*x+e)/f+1
/4*(a-b)^2*cos(f*x+e)^3*sin(f*x+e)/f+(2*a-3*b)*b*tan(f*x+e)/f+1/3*b^2*tan(
f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{12(3a^2-30ab+35b^2)(e+fx)-24(a^2-4ab+3b^2)\sin(2(e+fx))+3(a-b)^2\sin(4(e+fx))+32b(6a}{96f}$$

input
```
Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(12*(3*a^2 - 30*a*b + 35*b^2)*(e + f*x) - 24*(a^2 - 4*a*b + 3*b^2)*Sin[2*(
e + f*x)] + 3*(a - b)^2*Sin[4*(e + f*x)] + 32*b*(6*a - 10*b + b*Sec[e + f*
x]^2)*Tan[e + f*x])/(96*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4146, 366, 360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)^2dx$$
$$\Big\downarrow \text{ 4146}$$
$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{ 366}$$

$$\frac{\frac{(a-b)^2\tan^5(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{1}{4}\int\frac{\tan^4(e+fx)(a^2-10ba+5b^2-4b^2\tan^2(e+fx))}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int\frac{8b^2\tan^4(e+fx)-2(a-9b)(a-b)\tan^2(e+fx)+(a-9b)(a-b)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{(a-9b)(a-b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{(a-b)^2\tan^5(e+fx)}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int\left(8b^2\tan^2(e+fx)-2(a^2-10ba+13b^2)+\frac{3a^2-30ba+35b^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)-\frac{(a-9b)(a-b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{(a-b)^2}{4(\tan^2(}}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left((3a^2-30ab+35b^2)\arctan(\tan(e+fx))-2(a^2-10ab+13b^2)\tan(e+fx)+\frac{8}{3}b^2\tan^3(e+fx)\right)-\frac{(a-9b)(}{2(\tan}}{f}$$

input
```
Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(((a - b)^2*Tan[e + f*x]^5)/(4*(1 + Tan[e + f*x]^2)^2) + (-1/2*((a - 9*b)*
(a - b)*Tan[e + f*x])/(1 + Tan[e + f*x]^2) + ((3*a^2 - 30*a*b + 35*b^2)*Ar
cTan[Tan[e + f*x]] - 2*(a^2 - 10*a*b + 13*b^2)*Tan[e + f*x] + (8*b^2*Tan[e
 + f*x]^3)/3)/2)/4)/f
```

Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.23 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\right.}{}$
risch	$\frac{3x\,a^2}{8}-\frac{15xab}{4}+\frac{35x\,b^2}{8}-\frac{i\mathrm{e}^{4i(fx+e)}a^2}{64f}+\frac{i\mathrm{e}^{2i(fx+e)}a^2}{8f}+\frac{i\mathrm{e}^{-4i(fx+e)}b^2}{64f}-\frac{4ib\left(-3a\,\mathrm{e}^{4i(fx+e)}+6b\,\mathrm{e}^{4i(fx+e)}\right.}{3f\left(\mathrm{e}^{2i}\right.}$

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a
*b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e)
)*cos(f*x+e)-15/8*f*x-15/8*e)+b^2*(1/3*sin(f*x+e)^9/cos(f*x+e)^3-2*sin(f*x
+e)^9/cos(f*x+e)-2*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16
*sin(f*x+e))*cos(f*x+e)+35/8*f*x+35/8*e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{3\left(3\,a^2-30\,ab+35\,b^2\right)fx\cos\left(fx+e\right)^3+\left(6\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^6-3\left(5\,a^2-18\,ab+13\,b^2\right)\cos\right.}{24\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(3*a^2 - 30*a*b + 35*b^2)*f*x*cos(f*x + e)^3 + (6*(a^2 - 2*a*b + b
^2)*cos(f*x + e)^6 - 3*(5*a^2 - 18*a*b + 13*b^2)*cos(f*x + e)^4 + 16*(3*a*
b - 5*b^2)*cos(f*x + e)^2 + 8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\int \left(a+b\tan^2{(e+fx)}\right)^2\sin^4{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{8\,b^2\tan\left(fx+e\right)^3+3\,(3\,a^2-30\,ab+35\,b^2)(fx+e)+24\,(2\,ab-3\,b^2)\tan\left(fx+e\right)-\frac{3\left((5\,a^2-18\,ab+13\,b^2)\,\mathrm{ta}\right.}{\tan(f}}{24\,f}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 30*a*b + 35*b^2)*(f*x + e) + 24*(2
*a*b - 3*b^2)*tan(f*x + e) - 3*((5*a^2 - 18*a*b + 13*b^2)*tan(f*x + e)^3 +
 (3*a^2 - 14*a*b + 11*b^2)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^
2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.40

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{(3\,a^2-30\,ab+35\,b^2)(fx+e)}{8\,f}$$
$$+\frac{b^2f^2\tan\left(fx+e\right)^3+6\,abf^2\tan\left(fx+e\right)-9\,b^2f^2\tan\left(fx+e\right)}{3\,f^3}$$
$$-\frac{5\,a^2\tan\left(fx+e\right)^3-18\,ab\tan\left(fx+e\right)^3+13\,b^2\tan\left(fx+e\right)^3+3\,a^2\tan\left(fx+e\right)-14\,ab\tan\left(fx+e\right)}{8\left(\tan\left(fx+e\right)^2+1\right)^2f}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/8*(3*a^2 - 30*a*b + 35*b^2)*(f*x + e)/f + 1/3*(b^2*f^2*tan(f*x + e)^3 +
6*a*b*f^2*tan(f*x + e) - 9*b^2*f^2*tan(f*x + e))/f^3 - 1/8*(5*a^2*tan(f*x
+ e)^3 - 18*a*b*tan(f*x + e)^3 + 13*b^2*tan(f*x + e)^3 + 3*a^2*tan(f*x + e
) - 14*a*b*tan(f*x + e) + 11*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*f)
```

Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = x\left(\frac{3\,a^2}{8}-\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)+\frac{\tan(e+f\,x)\,\left(2\,a\,b-3\,b^2\right)}{f}+\frac{b^2\tan(e+f\,x)^3}{3\,f} - \frac{\left(\frac{5\,a^2}{8}-\frac{9\,a\,b}{4}+\frac{13\,b^2}{8}\right)\tan(e+f\,x)^3+\left(\frac{3\,a^2}{8}-\frac{7\,a\,b}{4}+\frac{11\,b^2}{8}\right)\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4+2\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
x*((3*a^2)/8 - (15*a*b)/4 + (35*b^2)/8) + (tan(e + f*x)*(2*a*b - 3*b^2))/f
 + (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*((3*a^2)/8 - (7*a*b)/4 + (11
*b^2)/8) + tan(e + f*x)^3*((5*a^2)/8 - (9*a*b)/4 + (13*b^2)/8))/(f*(2*tan(
e + f*x)^2 + tan(e + f*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.16

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{9\cos\left(fx+e\right)\sin\left(fx+e\right)^2a^2e+9\cos\left(fx+e\right)\sin\left(fx+e\right)^2a^2fx-90\cos\left(fx+e\right)\sin\left(fx+e\right)^2abe-}{}$$

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(9*cos(e + f*x)*sin(e + f*x)**2*a**2*e + 9*cos(e + f*x)*sin(e + f*x)**2*a*
*2*f*x - 90*cos(e + f*x)*sin(e + f*x)**2*a*b*e - 90*cos(e + f*x)*sin(e + f
*x)**2*a*b*f*x + 105*cos(e + f*x)*sin(e + f*x)**2*b**2*e + 105*cos(e + f*x
)*sin(e + f*x)**2*b**2*f*x - 9*cos(e + f*x)*a**2*e - 9*cos(e + f*x)*a**2*f
*x + 90*cos(e + f*x)*a*b*e + 90*cos(e + f*x)*a*b*f*x - 105*cos(e + f*x)*b*
*2*e - 105*cos(e + f*x)*b**2*f*x + 6*sin(e + f*x)**7*a**2 - 12*sin(e + f*x
)**7*a*b + 6*sin(e + f*x)**7*b**2 - 3*sin(e + f*x)**5*a**2 - 18*sin(e + f*
x)**5*a*b + 21*sin(e + f*x)**5*b**2 - 12*sin(e + f*x)**3*a**2 + 120*sin(e
+ f*x)**3*a*b - 140*sin(e + f*x)**3*b**2 + 9*sin(e + f*x)*a**2 - 90*sin(e
+ f*x)*a*b + 105*sin(e + f*x)*b**2)/(24*cos(e + f*x)*f*(sin(e + f*x)**2 -
1))
```

3.50 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{1}{2}(a-5b)(a-b)x-\frac{(a-b)^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{2(a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
1/2*(a-5*b)*(a-b)*x-1/2*(a-b)^2*cos(f*x+e)*sin(f*x+e)/f+2*(a-b)*b*tan(f*x+
e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{6(a^2-6ab+5b^2)(e+fx)-3(a-b)^2\sin(2(e+fx))+4b(6a-7b+b\sec^2(e+fx))\tan(e+fx)}{12f}$$

input
```
Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(6*(a^2 - 6*a*b + 5*b^2)*(e + f*x) - 3*(a - b)^2*Sin[2*(e + f*x)] + 4*b*(6
*a - 7*b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(12*f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4146, 366, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2\left(a+b\tan(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{(a-b)^2\tan^3(e+fx)}{2\left(\tan^2(e+fx)+1\right)}-\frac{1}{2}\int \frac{\tan^2(e+fx)\left(a^2-6ba+3b^2-2b^2\tan^2(e+fx)\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx)-(a-5b)(a-b)\int \frac{\tan^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)\right)+\frac{(a-b)^2\tan^3(e+fx)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx)-(a-5b)(a-b)\left(\tan(e+fx)-\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)\right)\right)+\frac{(a-b)^2\tan^3(e+fx)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx)-(a-5b)(a-b)(\tan(e+fx)-\arctan(\tan(e+fx)))\right)+\frac{(a-b)^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(((a - b)^2*Tan[e + f*x]^3)/(2*(1 + Tan[e + f*x]^2)) + ((2*b^2*Tan[e + f*x
]^3)/3 - (a - 5*b)*(a - b)*(-ArcTan[Tan[e + f*x]] + Tan[e + f*x]))/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(71) = 142$.

Time = 4.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(fx+e)^7}{3\cos(fx+e)^3}-\frac{4\sin(fx+e)^7}{3\cos(fx+e)}-\frac{4\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{3}+\frac{5fx}{2}+\frac{5e}{2}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\right.}{f}$
default	$\frac{b^2\left(\frac{\sin(fx+e)^7}{3\cos(fx+e)^3}-\frac{4\sin(fx+e)^7}{3\cos(fx+e)}-\frac{4\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{3}+\frac{5fx}{2}+\frac{5e}{2}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\right.}{f}$
risch	$\frac{x\,a^2}{2}-3xab+\frac{5x\,b^2}{2}+\frac{i\mathrm{e}^{2i(fx+e)}a^2}{8f}-\frac{i\mathrm{e}^{2i(fx+e)}ab}{4f}+\frac{i\mathrm{e}^{2i(fx+e)}b^2}{8f}-\frac{i\mathrm{e}^{-2i(fx+e)}a^2}{8f}+\frac{i\mathrm{e}^{-2i(fx+e)}ab}{4f}-$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2*(1/3*sin(f*x+e)^7/cos(f*x+e)^3-4/3*sin(f*x+e)^7/cos(f*x+e)-4/3*(s
in(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/2*f*x+5/2*e)+2*
a*b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*
f*x-3/2*e)+a^2*(-1/2*cos(f*x+e)*sin(f*x+e)+1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{3\left(a^2-6ab+5b^2\right)fx\cos\left(fx+e\right)^3-\left(3\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^4-2\left(6ab-7b^2\right)\cos\left(fx+e\right)^2-2\right.}{6f\cos\left(fx+e\right)^3}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*(a^2 - 6*a*b + 5*b^2)*f*x*cos(f*x + e)^3 - (3*(a^2 - 2*a*b + b^2)*c
os(f*x + e)^4 - 2*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f
*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\int \left(a+b\tan^2\left(e+fx\right)\right)^2\sin^2\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{2b^2\tan\left(fx+e\right)^3+3\left(a^2-6ab+5b^2\right)\left(fx+e\right)+12\left(ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2-2ab+b^2\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{6f}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*tan(f*x + e)^3 + 3*(a^2 - 6*a*b + 5*b^2)*(f*x + e) + 12*(a*b -
b^2)*tan(f*x + e) - 3*(a^2 - 2*a*b + b^2)*tan(f*x + e)/(tan(f*x + e)^2 + 1
))/f
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.56

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{(a^2-6\,ab+5\,b^2)(fx+e)}{2\,f} - \frac{a^2\tan(fx+e)-2\,ab\tan(fx+e)+b^2\tan(fx+e)}{2\left(\tan(fx+e)^2+1\right)f} + \frac{b^2f^2\tan(fx+e)^3+6\,abf^2\tan(fx+e)-6\,b^2f^2\tan(fx+e)}{3\,f^3}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2 - 6*a*b + 5*b^2)*(f*x + e)/f - 1/2*(a^2*tan(f*x + e) - 2*a*b*tan(
f*x + e) + b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)*f) + 1/3*(b^2*f^2*tan(f
*x + e)^3 + 6*a*b*f^2*tan(f*x + e) - 6*b^2*f^2*tan(f*x + e))/f^3
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\tan(e+f\,x)\,\left(2\,a\,b-2\,b^2\right)}{f} + \frac{b^2\tan(e+f\,x)^3}{3\,f} - \frac{\sin\left(2\,e+2\,f\,x\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{2\,f} + \frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)\,(a-5\,b)}{2\,\left(\frac{a^2}{2}-3\,a\,b+\frac{5\,b^2}{2}\right)}\right)\,(a-b)\,(a-5\,b)}{2\,f}$$

input
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)*(2*a*b - 2*b^2))/f + (b^2*tan(e + f*x)^3)/(3*f) - (sin(2*e +
 2*f*x)*(a^2/2 - a*b + b^2/2))/(2*f) + (atan((tan(e + f*x)*(a - b)*(a - 5*
b))/(2*(a^2/2 - 3*a*b + (5*b^2)/2)))*(a - b)*(a - 5*b))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.26

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{3\cos(fx+e)\sin(fx+e)^2 a^2e+3\cos(fx+e)\sin(fx+e)^2 a^2fx-18\cos(fx+e)\sin(fx+e)^2 abe-}{}$$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**2*a**2*e + 3*cos(e + f*x)*sin(e + f*x)**2*a*
*2*f*x - 18*cos(e + f*x)*sin(e + f*x)**2*a*b*e - 18*cos(e + f*x)*sin(e + f
*x)**2*a*b*f*x + 15*cos(e + f*x)*sin(e + f*x)**2*b**2*e + 15*cos(e + f*x)*
sin(e + f*x)**2*b**2*f*x - 3*cos(e + f*x)*a**2*e - 3*cos(e + f*x)*a**2*f*x
 + 18*cos(e + f*x)*a*b*e + 18*cos(e + f*x)*a*b*f*x - 15*cos(e + f*x)*b**2*
e - 15*cos(e + f*x)*b**2*f*x + 3*sin(e + f*x)**5*a**2 - 6*sin(e + f*x)**5*
a*b + 3*sin(e + f*x)**5*b**2 - 6*sin(e + f*x)**3*a**2 + 24*sin(e + f*x)**3
*a*b - 20*sin(e + f*x)**3*b**2 + 3*sin(e + f*x)*a**2 - 18*sin(e + f*x)*a*b
 + 15*sin(e + f*x)*b**2)/(6*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.51 $\int \left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=(a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
(a-b)^2*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\tan(e+fx)\left(\frac{3(a-b)^2\operatorname{arctanh}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}+b(6a-b(3-\tan^2(e+fx)))\right)}{3f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e +
f*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{(a-b)^2}{\tan^2(e+fx)+1}+b^2\tan^2(e+fx)+(2a-b)b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a-b)^2\arctan(\tan(e+fx))+b(2a-b)\tan(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((a - b)^2*ArcTan[Tan[e + f*x]] + (2*a - b)*b*Tan[e + f*x] + (b^2*Tan[e +
f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
norman	$(a^2-2ab+b^2)\,x+\frac{(2a-b)b\tan(fx+e)}{f}+\frac{b^2\tan(fx+e)^3}{3f}$	49
derivativedivides	$\frac{\frac{\tan(fx+e)^3b^2}{3}+2\tan(fx+e)ab-\tan(fx+e)b^2+(a^2-2ab+b^2)\arctan(\tan(fx+e))}{f}$	59
default	$\frac{\frac{\tan(fx+e)^3b^2}{3}+2\tan(fx+e)ab-\tan(fx+e)b^2+(a^2-2ab+b^2)\arctan(\tan(fx+e))}{f}$	59
parallelrisch	$\frac{\tan(fx+e)^3b^2+3a^2fx-6abfx+3b^2fx+6\tan(fx+e)ab-3\tan(fx+e)b^2}{3f}$	60
parts	$x\,a^2+\frac{b^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{2ab(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}$	63
risch	$x\,a^2-2xab+x\,b^2-\frac{4ib\left(-3a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+3b\,e^{2i(fx+e)}-3a+2b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	92

input
```
int((a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(a^2-2*a*b+b^2)*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{b^2\tan\left(fx+e\right)^3+3\left(a^2-2\,ab+b^2\right)fx+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(
f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = \begin{cases} a^2x-2abx+\frac{2ab\tan\left(e+fx\right)}{f}+b^2x+\frac{b^2\tan^3\left(e+fx\right)}{3f}-\frac{b^2\tan\left(e+fx\right)}{f} & \text{for } f\neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
 f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2,
True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = a^2x-\frac{2\,(fx+e-\tan(fx+e))ab}{f}+\frac{\left(\tan(fx+e)^3+3\,fx+3\,e-3\,\tan(fx+e)\right)b^2}{3\,f}$$

input
```
integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3
*e - 3*tan(f*x + e))*b^2/f
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{(a^2-2\,ab+b^2)(fx+e)}{f}+\frac{b^2f^2\tan(fx+e)^3+6\,abf^2\tan(fx+e)-3\,b^2f^2\tan(fx+e)}{3\,f^3}$$

input
```
integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
(a^2 - 2*a*b + b^2)*(f*x + e)/f + 1/3*(b^2*f^2*tan(f*x + e)^3 + 6*a*b*f^2*
tan(f*x + e) - 3*b^2*f^2*tan(f*x + e))/f^3
```

Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\tan(e+f\,x)\,\left(2\,a\,b-b^2\right)}{f}+\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f}+\frac{b^2\,\tan(e+f\,x)^3}{3\,f}$$

input
```
int((a + b*tan(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)*(2*a*b - b^2))/f + (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a
*b + b^2))*(a - b)^2)/f + (b^2*tan(e + f*x)^3)/(3*f)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\tan(fx+e)^3b^2+6\tan(fx+e)ab-3\tan(fx+e)b^2+3a^2fx-6abfx+3b^2fx}{3f}$$

input
```
int((a+b*tan(f*x+e)^2)^2,x)
```

output
```
(tan(e + f*x)**3*b**2 + 6*tan(e + f*x)*a*b - 3*tan(e + f*x)*b**2 + 3*a**2*
f*x - 6*a*b*f*x + 3*b**2*f*x)/(3*f)
```

3.52 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{a^2\cot(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-a^2*cot(f*x+e)/f+2*a*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{-3a^2\cot(e+fx)+b(6a-b+b\sec^2(e+fx))\tan(e+fx)}{3f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-3*a^2*Cot[e + f*x] + b*(6*a - b + b*Sec[e + f*x]^2)*Tan[e + f*x])/(3*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(a^2\cot^2(e+fx)+b^2\tan^2(e+fx)+2ab\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2\cot(e+fx)+2ab\tan(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(a^2*Cot[e + f*x]) + 2*a*b*Tan[e + f*x] + (b^2*Tan[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{b^2 \sin(fx+e)^3}{3\cos(fx+e)^3}+2\tan(fx+e)ab-\cot(fx+e)a^2}{f}$
default	$\frac{\frac{b^2 \sin(fx+e)^3}{3\cos(fx+e)^3}+2\tan(fx+e)ab-\cot(fx+e)a^2}{f}$
risch	$-\frac{2i\left(3a^2 e^{6i(fx+e)}-6ab\,e^{6i(fx+e)}+3b^2 e^{6i(fx+e)}+9a^2 e^{4i(fx+e)}-6ab\,e^{4i(fx+e)}-3b^2 e^{4i(fx+e)}+9a^2 e^{2i(fx+e)}+6ab\,e^{2i(\cdots}{3f\left(e^{2i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)^3}$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3+2*tan(f*x+e)*a*b-cot(f*x+e)*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \csc^2(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = -\frac{(3a^2+6ab-b^2)\cos(fx+e)^4 - 2(3ab-b^2)\cos(fx+e)^2 - b^2}{3f\cos(fx+e)^3\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a*b - b^2)*cos(f*x + e)^
2 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \int \left(a+b\tan^2(e+fx)\right)^2 \csc^2(e+fx)\, dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \csc^2(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \frac{b^2\tan(fx+e)^3 + 6ab\tan(fx+e) - \frac{3a^2}{\tan(fx+e)}}{3f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)-\frac{3\,a^2}{\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) - 3*a^2/tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{-3\,a^2\cos\left(e+f\,x\right)^4+6\,a\,b\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2+b^2\sin\left(e+f\,x\right)^4}{3\,f\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)}$$

input
```
int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^2,x)
```

output
```
(b^2*sin(e + f*x)^4 - 3*a^2*cos(e + f*x)^4 + 6*a*b*cos(e + f*x)^2*sin(e +
f*x)^2)/(3*f*cos(e + f*x)^3*sin(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{3\sin\left(fx+e\right)^4a^2+6\sin\left(fx+e\right)^4ab-\sin\left(fx+e\right)^4b^2-6\sin\left(fx+e\right)^2a^2-6\sin\left(fx+e\right)^2ab+3a^2}{3\cos\left(fx+e\right)\sin\left(fx+e\right)f\left(\sin\left(fx+e\right)^2-1\right)}$$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(3*sin(e + f*x)**4*a**2 + 6*sin(e + f*x)**4*a*b - sin(e + f*x)**4*b**2 - 6
*sin(e + f*x)**2*a**2 - 6*sin(e + f*x)**2*a*b + 3*a**2)/(3*cos(e + f*x)*si
n(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.53 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{a(a+2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-a*(a+2*b)*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)^3/f+b*(2*a+b)*tan(f*x+e)/f+1/3*
b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{-a\cot(e+fx)\left(2a+6b+a\csc^2(e+fx)\right)+b\left(6a+2b+b\sec^2(e+fx)\right)\tan(e+fx)}{3f}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(a*Cot[e + f*x]*(2*a + 6*b + a*Csc[e + f*x]^2)) + b*(6*a + 2*b + b*Sec[e
 + f*x]^2)*Tan[e + f*x])/(3*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2 d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left(a^2\cot^4(e+fx)+a(a+2b)\cot^2(e+fx)+b^2\tan^2(e+fx)+b(2a+b)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^2\cot^3(e+fx)+b(2a+b)\tan(e+fx)-a(a+2b)\cot(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(a*(a + 2*b)*Cot[e + f*x]) - (a^2*Cot[e + f*x]^3)/3 + b*(2*a + b)*Tan[e
+ f*x] + (b^2*Tan[e + f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 6.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)}{f}$
default	$\frac{-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)}{f}$
risch	$\frac{4i(3a^2e^{8i(fx+e)}-6ab\,e^{8i(fx+e)}+3b^2e^{8i(fx+e)}+8a^2e^{6i(fx+e)}-8b^2e^{6i(fx+e)}+6a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+6b^2e^{4i(fx}}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)^3}$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e
)-2*cot(f*x+e))+a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{2\,(a^2+6\,ab+b^2)\cos\left(fx+e\right)^6-3\,(a^2+6\,ab+b^2)\cos\left(fx+e\right)^4+6\,ab\cos\left(fx+e\right)^2+b^2}{3\left(f\cos\left(fx+e\right)^5-f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2 + 6*a*b + b^2)*cos(f*x + e)^6 - 3*(a^2 + 6*a*b + b^2)*cos(f*x
 + e)^4 + 6*a*b*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos(f*x + e)^
3)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^2\csc^4\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**2*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{b^2\tan\left(fx+e\right)^3+3\,(2\,ab+b^2)\tan\left(fx+e\right)-\frac{3\,(a^2+2\,ab)\tan(fx+e)^2+a^2}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + b^2)*tan(f*x + e) - (3*(a^2 + 2*a*b)*
tan(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \csc^4(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \frac{b^2 \tan(fx+e)^3 + 6\,ab\tan(fx+e) + 3\,b^2\tan(fx+e) - \frac{3\,a^2\tan(fx+e)^2 + 6\,ab\tan(fx+e)^2 + a^2}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 3*b^2*tan(f*x + e) - (3*a^2
*tan(f*x + e)^2 + 6*a*b*tan(f*x + e)^2 + a^2)/tan(f*x + e)^3)/f
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \csc^4(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \frac{b^2 \tan(e+f\,x)^3}{3\,f} - \frac{\tan(e+f\,x)^2\,(a^2+2\,b\,a) + \frac{a^2}{3}}{f\tan(e+f\,x)^3} + \frac{b\tan(e+f\,x)\,(2\,a+b)}{f}$$

input
```
int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)^2*(2*a*b + a^2) + a^2/3)/(f*tan
(e + f*x)^3) + (b*tan(e + f*x)*(2*a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.79

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{2\sin(fx+e)^6a^2+12\sin(fx+e)^6ab+2\sin(fx+e)^6b^2-3\sin(fx+e)^4a^2-18\sin(fx+e)^4ab-3}{3\cos(fx+e)\sin(fx+e)^3f\left(\sin(fx+e)^2-1\right)}$$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(2*sin(e + f*x)**6*a**2 + 12*sin(e + f*x)**6*a*b + 2*sin(e + f*x)**6*b**2
- 3*sin(e + f*x)**4*a**2 - 18*sin(e + f*x)**4*a*b - 3*sin(e + f*x)**4*b**2
 + 6*sin(e + f*x)**2*a*b + a**2)/(3*cos(e + f*x)*sin(e + f*x)**3*f*(sin(e
+ f*x)**2 - 1))
```

3.54 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{(a^2+4ab+b^2)\cot(e+fx)}{f}-\frac{2a(a+b)\cot^3(e+fx)}{3f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-(a^2+4*a*b+b^2)*cot(f*x+e)/f-2/3*a*(a+b)*cot(f*x+e)^3/f-1/5*a^2*cot(f*x+e
)^5/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{-\cot(e+fx)\left(8a^2+50ab+15b^2+2a(2a+5b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)+5b(6a+5b+b\sec^2(}{15f}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(Cot[e + f*x]*(8*a^2 + 50*a*b + 15*b^2 + 2*a*(2*a + 5*b)*Csc[e + f*x]^2
+ 3*a^2*Csc[e + f*x]^4)) + 5*b*(6*a + 5*b + b*Sec[e + f*x]^2)*Tan[e + f*x]
)/(15*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left(a^2\cot^6(e+fx)+2a(a+b)\cot^4(e+fx)+\left(a^2+4ba+b^2\right)\cot^2(e+fx)+b^2\tan^2(e+fx)+2b(a+b)\right)d\tan}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2+4ab+b^2\right)\cot(e+fx)-\frac{1}{5}a^2\cot^5(e+fx)+2b(a+b)\tan(e+fx)-\frac{2}{3}a(a+b)\cot^3(e+fx)+\frac{1}{3}b^2\tan^3(e}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a^2 + 4*a*b + b^2)*Cot[e + f*x]) - (2*a*(a + b)*Cot[e + f*x]^3)/3 - (a
^2*Cot[e + f*x]^5)/5 + 2*b*(a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3)/3)/
f
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 10.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{b^{2}\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^{3}}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+2ab\left(-\frac{1}{3\sin(fx+e)^{3}\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\right.}{f}$
default	$\frac{b^{2}\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^{3}}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+2ab\left(-\frac{1}{3\sin(fx+e)^{3}\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\right.}{f}$
risch	$-\frac{16i\left(10a^{2}\mathrm{e}^{10i(fx+e)}-20ab\,\mathrm{e}^{10i(fx+e)}+10b^{2}\mathrm{e}^{10i(fx+e)}+25a^{2}\mathrm{e}^{8i(fx+e)}+10ab\,\mathrm{e}^{8i(fx+e)}-35b^{2}\mathrm{e}^{8i(fx+e)}+16a^{2}\mathrm{e}^{6i(fx\right.}{15}$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2*(1/3/sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*
x+e))+2*a*b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*co
t(f*x+e))+a^2*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.47

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{8\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+10\,ab+5\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+10\,ab+5\,b^2\right)\cos\left(f\right.}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin\left(f\right.}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*(8*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^8 - 20*(a^2 + 10*a*b + 5*b^2)
*cos(f*x + e)^6 + 15*(a^2 + 10*a*b + 5*b^2)*cos(f*x + e)^4 - 10*(3*a*b + b
^2)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*co
s(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{5\,b^2\tan\left(fx+e\right)^3+30\left(ab+b^2\right)\tan\left(fx+e\right)-\frac{15\left(a^2+4\,ab+b^2\right)\tan\left(fx+e\right)^4+10\left(a^2+ab\right)\tan\left(fx+e\right)^2+3\,a^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + b^2)*tan(f*x + e) - (15*(a^2 + 4*a*
b + b^2)*tan(f*x + e)^4 + 10*(a^2 + a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x +
 e)^5)/f
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{5\,b^2\tan\left(fx+e\right)^3+30\,ab\tan\left(fx+e\right)+30\,b^2\tan\left(fx+e\right)-\frac{15\,a^2\tan\left(fx+e\right)^4+60\,ab\tan\left(fx+e\right)^4+15\,b^2\tan\left(fx+e\right)^4+}{\tan\left(fx+e\right)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 30*b^2*tan(f*x + e) - (
15*a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10
*a^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

Mupad [B] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{b^2\tan(e+f\,x)^3}{3\,f} - \frac{\tan(e+f\,x)^4\,(a^2+4\,a\,b+b^2)+\frac{a^2}{5}+\tan(e+f\,x)^2\left(\frac{2\,a^2}{3}+\frac{2\,b\,a}{3}\right)}{f\tan\left(e+f\,x\right)^5}$$
$$+ \frac{2\,b\tan(e+f\,x)\,(a+b)}{f}$$

input
```
int((a + b*tan(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)^4*(4*a*b + a^2 + b^2) + a^2/5 +
 tan(e + f*x)^2*((2*a*b)/3 + (2*a^2)/3))/(f*tan(e + f*x)^5) + (2*b*tan(e +
 f*x)*(a + b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.91

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{8\sin\left(fx+e\right)^8a^2+80\sin\left(fx+e\right)^8ab+40\sin\left(fx+e\right)^8b^2-12\sin\left(fx+e\right)^6a^2-120\sin\left(fx+e\right)^6ab}{15\cos\left(fx+\right.}$$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(8*sin(e + f*x)**8*a**2 + 80*sin(e + f*x)**8*a*b + 40*sin(e + f*x)**8*b**2
 - 12*sin(e + f*x)**6*a**2 - 120*sin(e + f*x)**6*a*b - 60*sin(e + f*x)**6*
b**2 + 3*sin(e + f*x)**4*a**2 + 30*sin(e + f*x)**4*a*b + 15*sin(e + f*x)**
4*b**2 - 2*sin(e + f*x)**2*a**2 + 10*sin(e + f*x)**2*a*b + 3*a**2)/(15*cos
(e + f*x)*sin(e + f*x)**5*f*(sin(e + f*x)**2 - 1))
```

3.55 $\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx=-\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{7/2}f}-\frac{a^2\cos(e+fx)}{(a-b)^3f}+\frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2f}-\frac{\cos^5(e+fx)}{5(a-b)f}$$

output
```
-a^2*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(7/2)/f-a^2*cos(
f*x+e)/(a-b)^3/f+1/3*(2*a-b)*cos(f*x+e)^3/(a-b)^2/f-1/5*cos(f*x+e)^5/(a-b)
/f
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx=\frac{120a^2\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+120a^2\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{a-b}\cos(e+fx)\,(}{120(a-b)^{7/2}f}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
(120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]]
+ 120*a^2*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]]
 + Sqrt[a - b]*Cos[e + f*x]*(-89*a^2 - 42*a*b + 11*b^2 + 4*(7*a^2 - 9*a*b
+ 2*b^2)*Cos[2*(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)]))/(120*(a - b)^(7
/2)*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4147, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^5}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\cos^6(e+fx)}{a-b}+\frac{(b-2a)\cos^4(e+fx)}{(a-b)^2}+\frac{a^2\cos^2(e+fx)}{(a-b)^3}-\frac{a^2b}{(a-b)^3\left(b\sec^2(e+fx)+a-b\right)}\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{7/2}}-\frac{a^2\cos(e+fx)}{(a-b)^3}-\frac{\cos^5(e+fx)}{5(a-b)}+\frac{(2a-b)\cos^3(e+fx)}{3(a-b)^2}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a - b)^(7/2))
 - (a^2*Cos[e + f*x])/(a - b)^3 + ((2*a - b)*Cos[e + f*x]^3)/(3*(a - b)^2)
 - Cos[e + f*x]^5/(5*(a - b)))/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 22.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{\frac{a^2\cos(fx+e)^5}{5}-\frac{2ab\cos(fx+e)^5}{5}+\frac{b^2\cos(fx+e)^5}{5}-\frac{2a^2\cos(fx+e)^3}{3}+ab\cos(fx+e)^3-\frac{b^2\cos(fx+e)^3}{3}+\cos(fx+e)a^2}{(a-b)^3}+\frac{a^2b\arctan}{(a}}{f}$
default	$\frac{-\frac{\frac{a^2\cos(fx+e)^5}{5}-\frac{2ab\cos(fx+e)^5}{5}+\frac{b^2\cos(fx+e)^5}{5}-\frac{2a^2\cos(fx+e)^3}{3}+ab\cos(fx+e)^3-\frac{b^2\cos(fx+e)^3}{3}+\cos(fx+e)a^2}{(a-b)^3}+\frac{a^2b\arctan}{(a}}{f}$
risch	$-\frac{5\,e^{i(fx+e)}a^2}{16(a-b)^3f}-\frac{e^{i(fx+e)}ab}{4(a-b)^3f}+\frac{e^{i(fx+e)}b^2}{16(a-b)^3f}-\frac{5\,e^{-i(fx+e)}a^2}{16(a-b)(a^2-2ab+b^2)f}-\frac{e^{-i(fx+e)}ab}{4(a-b)(a^2-2ab+b^2)f}+\frac{e^{-i(fx}}{16(a-b)(a^2}$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)^3*(1/5*a^2*cos(f*x+e)^5-2/5*a*b*cos(f*x+e)^5+1/5*b^2*cos(f*x
+e)^5-2/3*a^2*cos(f*x+e)^3+a*b*cos(f*x+e)^3-1/3*b^2*cos(f*x+e)^3+cos(f*x+e
)*a^2)+a^2*b/(a-b)^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.51

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \left[-\frac{6\,(a^2-2\,ab+b^2)\cos(fx+e)^5-10\,(2\,a^2-3\,ab+b^2)\cos(fx+e)^3+15\,a^2\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos(fx}{}\right.}{30\,(a^3-3\,a^2b+3\,ab^2-b^3)f}\right.$$

$$-\frac{3\,(a^2-2\,ab+b^2)\cos(fx+e)^5-5\,(2\,a^2-3\,ab+b^2)\cos(fx+e)^3+15\,a^2\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\,c}{b}\right.}{15\,(a^3-3\,a^2b+3\,ab^2-b^3)f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/30*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 10*(2*a^2 - 3*a*b + b^2)*co
s(f*x + e)^3 + 15*a^2*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a
 - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 3
0*a^2*cos(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f), -1/15*(3*(a^2 - 2
*a*b + b^2)*cos(f*x + e)^5 - 5*(2*a^2 - 3*a*b + b^2)*cos(f*x + e)^3 + 15*a
^2*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*a^
2*cos(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(105) = 210$.

Time = 0.50 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.04

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{15\,a^2b\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab-b^2}} - \frac{2\left(8\,a^2+9\,ab-2\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{30\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{10\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a^2(\cos(\ldots}{(\cos(\ldots}\right.}{(a^3-3\,a^2b+3\,a\ldots}}{15\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/15*(15*a^2*b*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b -
b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqr
t(a*b - b^2)) - 2*(8*a^2 + 9*a*b - 2*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 30*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 10*b^2*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 10*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*b^2*(cos(f*x + e) - 1)^3/(cos(
f*x + e) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^3
- 3*a^2*b + 3*a*b^2 - b^3)*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))
/f
```

Mupad [B] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 643, normalized size of antiderivative = 5.50

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{2\,(8\,a^2+9\,a\,b-2\,b^2)}{15\,(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4(8\,a^2+b^2)}{3\,(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2(4\,a^2+3\,a\,b-b^2)}{3\,(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{4\,b\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6(3\,a-b)}{(a-b)\,(a^2-2\,a\,b+b^2)}+\frac{2\,a\,b\tan\left(\frac{e}{2}+\ldots\right.}{(a-b)\,(a^2-2\ldots}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\ldots\right.}$$

$$-\frac{a^2\sqrt{b}\,\mathrm{atan}\left(\frac{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{a\sqrt{b}\,(16\,a^{10}\,b-96\,a^9\,b^2+240\,a^8\,b^3-320\,a^7\,b^4+240\,a^6\,b^5-96\,a^5\,b^6+16\,a^4\,b^7)}{2\,(a-b)^{13/2}}+\frac{a^3\sqrt{b}\,(a-2\,b)\,(16\,a^{12}-176\,a^{11}\,b+86\ldots}{\ldots}\right.\right.}{\ldots}\right.}{\ldots}$$

input
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

output
```
- ((2*(9*a*b + 8*a^2 - 2*b^2))/(15*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(e
/2 + (f*x)/2)^4*(8*a^2 + b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(e/
2 + (f*x)/2)^2*(3*a*b + 4*a^2 - b^2))/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (4
*b*tan(e/2 + (f*x)/2)^6*(3*a - b))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*a*b*
tan(e/2 + (f*x)/2)^8)/((a - b)*(a^2 - 2*a*b + b^2)))/(f*(5*tan(e/2 + (f*x)
/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f
*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1)) - (a^2*b^(1/2)*atan(((tan(e/2 + (f*
x)/2)^2*((a*b^(1/2)*(16*a^10*b + 16*a^4*b^7 - 96*a^5*b^6 + 240*a^6*b^5 - 3
20*a^7*b^4 + 240*a^8*b^3 - 96*a^9*b^2))/(2*(a - b)^(13/2)) + (a^3*b^(1/2)*
(a - 2*b)*(16*a^12 - 176*a^11*b + 32*a^2*b^10 - 304*a^3*b^9 + 1296*a^4*b^8
 - 3264*a^5*b^7 + 5376*a^6*b^6 - 6048*a^7*b^5 + 4704*a^8*b^4 - 2496*a^9*b^
3 + 864*a^10*b^2))/(8*(a - b)^(21/2))) + (a^3*b^(1/2)*(a - 2*b)*(144*a^11*
b - 16*a^12 + 16*a^3*b^9 - 144*a^4*b^8 + 576*a^5*b^7 - 1344*a^6*b^6 + 2016
*a^7*b^5 - 2016*a^8*b^4 + 1344*a^9*b^3 - 576*a^10*b^2))/(8*(a - b)^(21/2))
)*(a - b)^7)/(4*a^12*b + 4*a^6*b^7 - 24*a^7*b^6 + 60*a^8*b^5 - 80*a^9*b^4
+ 60*a^10*b^3 - 24*a^11*b^2)))/(f*(a - b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.86

$$\int \frac{\sin^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \frac{15\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a^2+15\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a^2-3\cos\left(fx+e\right.}{}$$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

output
```
(15*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*a**2 + 15*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*
x)/2))/sqrt(b))*a**2 - 3*cos(e + f*x)*sin(e + f*x)**4*a**3 + 9*cos(e + f*x
)*sin(e + f*x)**4*a**2*b - 9*cos(e + f*x)*sin(e + f*x)**4*a*b**2 + 3*cos(e
 + f*x)*sin(e + f*x)**4*b**3 - 4*cos(e + f*x)*sin(e + f*x)**2*a**3 + 7*cos
(e + f*x)*sin(e + f*x)**2*a**2*b - 2*cos(e + f*x)*sin(e + f*x)**2*a*b**2 -
 cos(e + f*x)*sin(e + f*x)**2*b**3 - 8*cos(e + f*x)*a**3 - cos(e + f*x)*a*
*2*b + 11*cos(e + f*x)*a*b**2 - 2*cos(e + f*x)*b**3 + 8*a**3 + a**2*b - 11
*a*b**2 + 2*b**3)/(15*f*(a**4 - 4*a**3*b + 6*a**2*b**2 - 4*a*b**3 + b**4))
```

3.56 $\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx=-\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}-\frac{a\cos(e+fx)}{(a-b)^2f}+\frac{\cos^3(e+fx)}{3(a-b)f}$$

output
```
-a*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(5/2)/f-a*cos(f*x+
e)/(a-b)^2/f+1/3*cos(f*x+e)^3/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{6a\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+6a\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+(a-b)\cos(e}{6(a-b)^3f}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
(6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/S
qrt[b]] + 6*a*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f
*x)/2])/Sqrt[b]] + (a - b)*Cos[e + f*x]*(-5*a - b + (a - b)*Cos[2*(e + f*x
)]))/(6*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4147, 25, 359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^3}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4147}$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\frac{a\int \frac{\cos^2(e+fx)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a-b}+\frac{\cos^3(e+fx)}{3(a-b)}}{f}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\frac{a\left(-\frac{b\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{a-b}\right)}{a-b}+\frac{\cos^3(e+fx)}{3(a-b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{a\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{\cos(e+fx)}{a-b}\right)}{a-b}+\frac{\cos^3(e+fx)}{3(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Cos[e + f*x]^3/(3*(a - b)) + (a*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])
/Sqrt[a - b]])/(a - b)^(3/2)) - Cos[e + f*x]/(a - b)))/(a - b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 5.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\frac{b\cos(fx+e)^3}{3}-a\cos(fx+e)}{(a-b)^2}+\frac{ab\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)^2\sqrt{b(a-b)}}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\frac{b\cos(fx+e)^3}{3}-a\cos(fx+e)}{(a-b)^2}+\frac{ab\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)^2\sqrt{b(a-b)}}}{f}$
risch	$-\frac{3\,e^{i(fx+e)}a}{8(-a+b)^2f}-\frac{e^{i(fx+e)}b}{8(-a+b)^2f}-\frac{3\,e^{-i(fx+e)}a}{8(-a+b)^2f}-\frac{e^{-i(fx+e)}b}{8(-a+b)^2f}+\frac{i\sqrt{b(a-b)}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{b(a-b)}\,e^{i(fx+e)}}{a-b}+\right.}{2(a-b)^3f}$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^2*(1/3*a*cos(f*x+e)^3-1/3*b*cos(f*x+e)^3-a*cos(f*x+e))+a*b/(a
-b)^2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.45

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \left[\frac{2\,(a-b)\cos\left(fx+e\right)^3+3\,a\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos(fx+e)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)-6\,a\cos\left(fx+e\right)\ \ (a}{6\,(a^2-2\,ab+b^2)f},\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/6*(2*(a - b)*cos(f*x + e)^3 + 3*a*sqrt(-b/(a - b))*log(((a - b)*cos(f*x
 + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x +
e)^2 + b)) - 6*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f), 1/3*((a - b)*cos(f
*x + e)^3 - 3*a*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x +
e)/b) - 3*a*cos(f*x + e))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(74) = 148$.

Time = 0.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.06

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{ab\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{(a^2-2ab+b^2)\sqrt{ab-b^2}f} + \frac{a^2f^5\cos(fx+e)^3 - 2abf^5\cos(fx+e)^3 + b^2f^5\cos(fx+e)^3 - 3a^2f^5\cos(fx+e) + 3abf^5\cos(fx-}{3(a^3f^6-3a^2bf^6+3ab^2f^6-b^3f^6)}$$

input

```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output

```
a*b*arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*
b + b^2)*sqrt(a*b - b^2)*f) + 1/3*(a^2*f^5*cos(f*x + e)^3 - 2*a*b*f^5*cos(
f*x + e)^3 + b^2*f^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) + 3*a*b*f^5*c
os(f*x + e))/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.55

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{2(2a+b)}{3(a-b)^2} + \frac{4a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{(a-b)^2} + \frac{2b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{(a-b)^2}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6 + 3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 + 3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 + 1\right)} - \frac{a\sqrt{b}\,\mathrm{atan}\left(\frac{\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{\sqrt{b}\left(8a^7b-32a^6b^2+48a^5b^3-32a^4b^4+8a^3b^5\right)}{(a-b)^{9/2}} - \frac{a\sqrt{b}(a-2b)\left(-16a^9+128a^8b-432a^7b^2+800a^6b^3-880a^5b^4+\right.}{8(a-b)^{15/2}}\right.}{4a^8b-16a^7b^2+24a^6b^3-}\right.}{f(a-b)^{5/2}}$$

input

```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

output
```
- ((2*(2*a + b))/(3*(a - b)^2) + (4*a*tan(e/2 + (f*x)/2)^2)/(a - b)^2 + (2
*b*tan(e/2 + (f*x)/2)^4)/(a - b)^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2
 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (a*b^(1/2)*atan(((tan(e/2 + (
f*x)/2)^2*((b^(1/2)*(8*a^7*b + 8*a^3*b^5 - 32*a^4*b^4 + 48*a^5*b^3 - 32*a^
6*b^2))/(a - b)^(9/2) - (a*b^(1/2)*(a - 2*b)*(128*a^8*b - 16*a^9 + 32*a^2*
b^7 - 208*a^3*b^6 + 576*a^4*b^5 - 880*a^5*b^4 + 800*a^6*b^3 - 432*a^7*b^2)
)/(8*(a - b)^(15/2))) - (a*b^(1/2)*(a - 2*b)*(16*a^9 - 96*a^8*b + 16*a^3*b
^6 - 96*a^4*b^5 + 240*a^5*b^4 - 320*a^6*b^3 + 240*a^7*b^2))/(8*(a - b)^(15
/2)))*(a - b)^5)/(4*a^8*b + 4*a^4*b^5 - 16*a^5*b^4 + 24*a^6*b^3 - 16*a^7*b
^2)))/(f*(a - b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.40

$$\int \frac{\sin^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a+3\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a-\cos\left(fx+e\right)\sin\left(\right.}{}$$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

output
```
(3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*a + 3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2)
)/sqrt(b))*a - cos(e + f*x)*sin(e + f*x)**2*a**2 + 2*cos(e + f*x)*sin(e +
f*x)**2*a*b - cos(e + f*x)*sin(e + f*x)**2*b**2 - 2*cos(e + f*x)*a**2 + co
s(e + f*x)*a*b + cos(e + f*x)*b**2 + 2*a**2 - a*b - b**2)/(3*f*(a**3 - 3*a
**2*b + 3*a*b**2 - b**3))
```

3.57 $\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{\cos(e+fx)}{(a-b)f}$$

output
```
-b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(3/2)/f-cos(f*x+e)/(
a-b)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 121 vs. 2(60) = 120.

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.02

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+(-a+b)\cos(e+ }{(a-b)^2f}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/
Sqrt[b]] + (-a + b)*Cos[e + f*x])/((a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4147, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\cos^2(e+fx)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{-\frac{b\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{a-b}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{\cos(e+fx)}{a-b}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a - b)^(3/2)) - C
os[e + f*x]/(a - b))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)}{a-b}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)\sqrt{b(a-b)}}}{f}$
default	$\frac{-\frac{\cos(fx+e)}{a-b}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{(a-b)\sqrt{b(a-b)}}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2(a-b)f}-\frac{e^{-i(fx+e)}}{2(a-b)f}-\frac{i\sqrt{b(a-b)}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{b(a-b)}\,e^{i(fx+e)}}{a-b}+1\right)}{2(a-b)^2 f}+\frac{i\sqrt{b(a-b)}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{b(c}{}\right)}{2(a-b)^2 f}$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-cos(f*x+e)/(a-b)+b/(a-b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*
(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.63

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = \left[-\frac{\sqrt{-\frac{b}{a-b}}\log\left(-\frac{(a-b)\cos(fx+e)^2-2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)+2\,\cos\left(fx+e\right)}{2\,(a-b)f},\ -\frac{\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+\cos\left(fx+e\right)}{(a-b)f}\right]$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(
a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*cos(f*x + e))/
((a - b)*f), -(sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e
)/b) + cos(f*x + e))/((a - b)*f)]
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = \int \frac{\sin\left(e+fx\right)}{a+b\tan^2\left(e+fx\right)}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{f\cos(fx+e)}{af^2-bf^2} + \frac{b\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}(a-b)f}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-f*cos(f*x + e)/(a*f^2 - b*f^2) + b*arctan((a*cos(f*x + e) - b*cos(f*x + e
))/sqrt(a*b - b^2))/(sqrt(a*b - b^2)*(a - b)*f)
```

Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.87

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{-a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a^2+a\,b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-3\,a\,b+2\,b^2}{2\sqrt{b}\,(a-b)^{3/2}}\right)}{f\,(a-b)^{3/2}} - \frac{2\sqrt{a-b}}{f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(a-b)^{3/2}+(a-b)^{3/2}\right)}$$

input
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

output
```
(b^(1/2)*atan((a^2 - a^2*tan(e/2 + (f*x)/2)^2 - 3*a*b + 2*b^2 + a*b*tan(e/
2 + (f*x)/2)^2)/(2*b^(1/2)*(a - b)^(3/2))))/(f*(a - b)^(3/2)) - (2*(a - b)
^(1/2))/(f*(tan(e/2 + (f*x)/2)^2*(a - b)^(3/2) + (a - b)^(3/2)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.00

$$\int \frac{\sin(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a+\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a-\cos\left(fx+e\right)a^2+\mathrm{co}}{af\left(a^2-2ab+b^2\right)}$$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

output
```
(sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*a + sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sq
rt(b))*a - cos(e + f*x)*a**2 + cos(e + f*x)*a*b + a*b - b**2)/(a*f*(a**2 -
 2*a*b + b**2))
```

3.58 $\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}f}-\frac{\operatorname{arctanh}(\cos(e+fx))}{af}$$

output
```
-b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/a/(a-b)^(1/2)/f-arctanh(co
s(f*x+e))/a/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(60) = 120.

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.40

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-(a-b)\left(\log\left(\cos\left(\right.\right.\right.}{a(a-b)f}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[
b]] + Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/
Sqrt[b]] - (a - b)*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))/(a*(a
- b)*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4147, 25, 303, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 303$$

$$\frac{-\frac{b\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}-\frac{\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}-\frac{\operatorname{arctanh}(\sec(e+fx))}{a}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a*Sqrt[a - b])) -
 ArcTanh[Sec[e + f*x]]/a)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a}+\frac{\ln(\cos(fx+e)-1)}{2a}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a\sqrt{b(a-b)}}}{f}$
default	$\frac{-\frac{\ln(\cos(fx+e)+1)}{2a}+\frac{\ln(\cos(fx+e)-1)}{2a}+\frac{b\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a\sqrt{b(a-b)}}}{f}$
risch	$-\frac{\ln(e^{i(fx+e)}+1)}{af}+\frac{\ln(e^{i(fx+e)}-1)}{af}+\frac{i\sqrt{b(a-b)}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{b(a-b)}\,e^{i(fx+e)}}{a-b}+1\right)}{2(a-b)fa}-\frac{i\sqrt{b(a-b)}\,\ln\left(e^{2i(f}\right.}{}$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/a*ln(cos(f*x+e)+1)+1/2/a*ln(cos(f*x+e)-1)+b/a/(b*(a-b))^(1/2)*ar
ctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.07

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos(fx+e)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)-\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos\left(fx+e\right)\right.}{2\,af}\right.$$
$$\left.-\frac{2\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\,af}\right]$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a
- b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x +
e) + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/(a*f), -1/2*(2*sqrt(b/(a - b))*a
rctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + log(1/2*cos(f*x + e) + 1/
2) - log(-1/2*cos(f*x + e) + 1/2))/(a*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = \int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(52) = 104$.

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.80

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{2\,b\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}a} - \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a}}{2\,f}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(2*b*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b - b^2)*c
os(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a) - log(abs(-cos(f*x + e
) + 1)/abs(cos(f*x + e) + 1))/a)/f
```

Mupad [B] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\ln\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{a\,f} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{b-a\,\cos(e+f\,x)+b\,\cos(e+f\,x)}{2\,\sqrt{b}\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sqrt{a-b}}\right)}{a\,f\,\sqrt{a-b}}$$

input
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)),x)
```

output
```
log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(a*f) - (b^(1/2)*atan((b - a*co
s(e + f*x) + b*cos(e + f*x))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(1/2)
)))/(a*f*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.82

$$\int \frac{\csc(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)+\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)+\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a-}{af\,(a-b)}$$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

output
```
(sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
) + sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt
(b)) + log(tan((e + f*x)/2))*a - log(tan((e + f*x)/2))*b)/(a*f*(a - b))
```

3.59 $\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx=-\frac{\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^2f}-\frac{(a-2b)\text{arctanh}(\cos(e+fx))}{2a^2f}-\frac{\cot(e+fx)\csc(e+fx)}{2af}$$

output
```
-(a-b)^(1/2)*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/a^2/f-1/2*(a-2
*b)*arctanh(cos(f*x+e))/a^2/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(89) = 178.

Time = 0.64 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$=\frac{8\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)+8\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-a\csc^2\left(\frac{1}{2}(e+f\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
(8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2]])/Sqr
t[b]] + 8*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/
2])/Sqrt[b]] - a*Csc[(e + f*x)/2]^2 - 4*a*Log[Cos[(e + f*x)/2]] + 8*b*Log[
Cos[(e + f*x)/2]] + 4*a*Log[Sin[(e + f*x)/2]] - 8*b*Log[Sin[(e + f*x)/2]]
+ a*Sec[(e + f*x)/2]^2)/(8*a^2*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4147, 373, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\int \frac{-b\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\frac{(a-2b)\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{2b(a-b)\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}}{2a}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))}-\frac{\frac{(a-2b)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{2\sqrt{b}\sqrt{a-b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a}}{2a}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))}-\frac{\frac{2\sqrt{b}\sqrt{a-b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a}+\frac{(a-2b)\text{arctanh}(\sec(e+fx))}{a}}{2a}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-1/2*((2*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/
a + ((a - 2*b)*ArcTanh[Sec[e + f*x]])/a)/a + Sec[e + f*x]/(2*a*(1 - Sec[e
+ f*x]^2)))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{1}{4a(\cos(fx+e)+1)}+\frac{(-a+2b)\ln(\cos(fx+e)+1)}{4a^2}+\frac{b(a-b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a^2\sqrt{b(a-b)}}+\frac{1}{4a(\cos(fx+e)-1)}+\frac{(a-2b)\ln(\cos(fx+e)-1)}{4a^2}}{f}$
default	$\frac{\frac{1}{4a(\cos(fx+e)+1)}+\frac{(-a+2b)\ln(\cos(fx+e)+1)}{4a^2}+\frac{b(a-b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{a^2\sqrt{b(a-b)}}+\frac{1}{4a(\cos(fx+e)-1)}+\frac{(a-2b)\ln(\cos(fx+e)-1)}{4a^2}}{f}$
risch	$\frac{e^{3i(fx+e)}+e^{i(fx+e)}}{fa\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{2af}+\frac{\ln\left(e^{i(fx+e)}+1\right)b}{a^2f}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{2af}-\frac{\ln\left(e^{i(fx+e)}-1\right)b}{a^2f}-\frac{i\sqrt{ab-b^2}}{}$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4/a/(cos(f*x+e)+1)+1/4/a^2*(-a+2*b)*ln(cos(f*x+e)+1)+b*(a-b)/a^2/(b
*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))+1/4/a/(cos(f*x+e)-1
)+1/4*(a-2*b)/a^2*ln(cos(f*x+e)-1))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.67

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{2\sqrt{-ab+b^2}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{-ab+b^2}\cos(fx+e)-b}{(a-b)\cos(fx+e)^2+b}\right)+2a\cos\left(fx+e\right)-\left((a-2\right.}{4\left(a^2f\right.}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a*b + b^2)*(cos(f*x + e)^2 - 1)*log(-((a - b)*cos(f*x + e)^2
 + 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*
a*cos(f*x + e) - ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e)
 + 1/2) + ((a - 2*b)*cos(f*x + e)^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2
))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/4*(4*sqrt(a*b - b^2)*(cos(f*x + e)^2
- 1)*arctan(sqrt(a*b - b^2)*cos(f*x + e)/b) + 2*a*cos(f*x + e) - ((a - 2*b
)*cos(f*x + e)^2 - a + 2*b)*log(1/2*cos(f*x + e) + 1/2) + ((a - 2*b)*cos(f
*x + e)^2 - a + 2*b)*log(-1/2*cos(f*x + e) + 1/2))/(a^2*f*cos(f*x + e)^2 -
 a^2*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(77) = 154$.

Time = 0.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.26

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{2\,(a-2\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2} - \frac{8\sqrt{ab-b^2}\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{a^2} + \frac{\left(a-\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+}{a^2(\cos(fx+e)-1)}}{8\,f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*(2*(a - 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^2 - 8
*sqrt(a*b - b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b -
 b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/a^2 + (a - 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x +
e) + 1)/(a^2*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/(a*(cos(f*x + e) + 1
)))/f
```

Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 591, normalized size of antiderivative = 6.64

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)),x)
```

output
```
-(a*(cos(e/2 + (f*x)/2)^2 - 2*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/
cos(e/2 + (f*x)/2)) + 2*cos(e/2 + (f*x)/2)^4*log(sin(e/2 + (f*x)/2)/cos(e/
2 + (f*x)/2)) - 1/2) + 4*atan((6*b^5*cos(e/2 + (f*x)/2)^2 + 3*a*b^4 - a^4*
b - 6*a^2*b^3 + 4*a^3*b^2 + 20*a^2*b^3*cos(e/2 + (f*x)/2)^2 - 10*a^3*b^2*c
os(e/2 + (f*x)/2)^2 - 18*a*b^4*cos(e/2 + (f*x)/2)^2 + 2*a^4*b*cos(e/2 + (f
*x)/2)^2)/(6*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(5/2) - 2*a^2*cos(e/2 + (f*x
)/2)^2*(a*b - b^2)^(3/2)))*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(1/2) - 4*atan
((6*b^5*cos(e/2 + (f*x)/2)^2 + 3*a*b^4 - a^4*b - 6*a^2*b^3 + 4*a^3*b^2 + 2
0*a^2*b^3*cos(e/2 + (f*x)/2)^2 - 10*a^3*b^2*cos(e/2 + (f*x)/2)^2 - 18*a*b^
4*cos(e/2 + (f*x)/2)^2 + 2*a^4*b*cos(e/2 + (f*x)/2)^2)/(6*cos(e/2 + (f*x)/
2)^2*(a*b - b^2)^(5/2) - 2*a^2*cos(e/2 + (f*x)/2)^2*(a*b - b^2)^(3/2)))*co
s(e/2 + (f*x)/2)^4*(a*b - b^2)^(1/2) + 4*b*cos(e/2 + (f*x)/2)^2*log(sin(e/
2 + (f*x)/2)/cos(e/2 + (f*x)/2)) - 4*b*cos(e/2 + (f*x)/2)^4*log(sin(e/2 +
(f*x)/2)/cos(e/2 + (f*x)/2)))/(4*a^2*f*cos(e/2 + (f*x)/2)^2 - 4*a^2*f*cos(
e/2 + (f*x)/2)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.73

$$\int \frac{\csc^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{2\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx+e)^2+2\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx+}{2\sin(fx+e)^2}$$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

output
```
(2*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**2 + 2*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*ta
n((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2 - cos(e + f*x)*a + log(tan((e + f
*x)/2))*sin(e + f*x)**2*a - 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*b)/(2*
sin(e + f*x)**2*a**2*f)
```

3.60 $\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{(a-b)^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a^3f} - \frac{(3a^2-12ab+8b^2)\,\text{arctanh}(\cos(e+fx))}{8a^3f} - \frac{(5a-4b)\cot(e+fx)\csc(e+fx)}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af}$$

output

```
-(a-b)^(3/2)*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/a^3/f-1/8*(3*a
^2-12*a*b+8*b^2)*arctanh(cos(f*x+e))/a^3/f-1/8*(5*a-4*b)*cot(f*x+e)*csc(f*
x+e)/a^2/f-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 326 vs. $2(130) = 260$.

Time = 6.45 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.51

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{(a-b)^{3/2}\sqrt{b}\arctan\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{a^3f} + \frac{(a-b)^{3/2}\sqrt{b}\arctan\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{a^3f} + \frac{(-3a+4b)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^2f} - \frac{\csc^4\left(\frac{1}{2}(e+fx)\right)}{64af} + \frac{(-3a^2+12ab-8b^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8a^3f} + \frac{(3a^2-12ab+8b^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^3f} + \frac{(3a-4b)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32a^2f} + \frac{\sec^4\left(\frac{1}{2}(e+fx)\right)}{64af}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
((a - b)^(3/2)*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)
/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(a^3*f) + ((a - b)^(3/2)*Sqrt[b
]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e
+ f*x)/2]))/Sqrt[b]])/(a^3*f) + ((-3*a + 4*b)*Csc[(e + f*x)/2]^2)/(32*a^2*
f) - Csc[(e + f*x)/2]^4/(64*a*f) + ((-3*a^2 + 12*a*b - 8*b^2)*Log[Cos[(e +
 f*x)/2]])/(8*a^3*f) + ((3*a^2 - 12*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8
*a^3*f) + ((3*a - 4*b)*Sec[(e + f*x)/2]^2)/(32*a^2*f) + Sec[(e + f*x)/2]^4
/(64*a*f)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4147, 25, 372, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\sec^4(e+fx)}{\left(1-\sec^2(e+fx)\right)^3\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(e+fx)}{\left(1-\sec^2(e+fx)\right)^3\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(4a-3b)\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{4a}-\frac{\sec(e+fx)}{4a\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int -\frac{(3a-4b)(a-b)-(5a-4b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a}+\frac{(5a-4b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)}}{4a}-\frac{\sec(e+fx)}{4a\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{(5a-4b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\int \frac{(3a-4b)(a-b)-(5a-4b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a}}{4a}-\frac{\sec(e+fx)}{4a\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{(5a-4b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\frac{\left(3a^2-12ab+8b^2\right)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{8b(a-b)^2\int\frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(5a-4b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\frac{\left(3a^2-12ab+8b^2\right)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{8\sqrt{b}(a-b)^{3/2}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a}}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{(5a-4b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\frac{\left(3a^2-12ab+8b^2\right)\text{arctanh}(\sec(e+fx))}{a}+\frac{8\sqrt{b}(a-b)^{3/2}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a}}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-1/4*Sec[e + f*x]/(a*(1 - Sec[e + f*x]^2)^2) + (-1/2*((8*(a - b)^(3/2)*Sq
rt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/a + ((3*a^2 - 12*a*b + 8
*b^2)*ArcTanh[Sec[e + f*x]])/a)/a + ((5*a - 4*b)*Sec[e + f*x])/(2*a*(1 - S
ec[e + f*x]^2)))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{1}{16a(\cos(fx+e)-1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)-1)}+\frac{(3a^2-12ab+8b^2)\ln(\cos(fx+e)-1)}{16a^3}+\frac{1}{16a(\cos(fx+e)+1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)+1)}+}{f}$
default	$\frac{-\frac{1}{16a(\cos(fx+e)-1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)-1)}+\frac{(3a^2-12ab+8b^2)\ln(\cos(fx+e)-1)}{16a^3}+\frac{1}{16a(\cos(fx+e)+1)^2}-\frac{-3a+4b}{16a^2(\cos(fx+e)+1)}+}{f}$
risch	$\frac{3a\,e^{7i(fx+e)}-4b\,e^{7i(fx+e)}-11a\,e^{5i(fx+e)}+4b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}+4b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}-4b\,e^{i(fx+e)}}{4fa^2(e^{2i(fx+e)}-1)^4}-\frac{3}{}$

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/16/a/(cos(f*x+e)-1)^2-1/16*(-3*a+4*b)/a^2/(cos(f*x+e)-1)+1/16*(3*a
^2-12*a*b+8*b^2)/a^3*ln(cos(f*x+e)-1)+1/16/a/(cos(f*x+e)+1)^2-1/16*(-3*a+4
*b)/a^2/(cos(f*x+e)+1)+1/16/a^3*(-3*a^2+12*a*b-8*b^2)*ln(cos(f*x+e)+1)+b*(
a^2-2*a*b+b^2)/a^3/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(116) = 232.

Time = 0.17 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.85

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a^2 - 4*a*b)*cos(f*x + e)^3 - 8*((a - b)*cos(f*x + e)^4 - 2*(a
 - b)*cos(f*x + e)^2 + a - b)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2
 - 2*sqrt(-a*b + b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 2*
(5*a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(1/
2*cos(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^
2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2)*log(-1/2*cos(
f*x + e) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f),
1/16*(2*(3*a^2 - 4*a*b)*cos(f*x + e)^3 + 16*((a - b)*cos(f*x + e)^4 - 2*(a
 - b)*cos(f*x + e)^2 + a - b)*sqrt(a*b - b^2)*arctan(sqrt(a*b - b^2)*cos(f
*x + e)/b) - 2*(5*a^2 - 4*a*b)*cos(f*x + e) - ((3*a^2 - 12*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b
+ 8*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 12*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 - 12*a*b + 8*b^2
)*log(-1/2*cos(f*x + e) + 1/2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x +
e)^2 + a^3*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx=\int \frac{\csc^5{(e+fx)}}{a+b\tan^2{(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(116) = 232$.

Time = 0.54 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.72

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2}-\frac{4\,(3\,a^2-12\,ab+8\,b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3}+\frac{64\,(a^2b-2\,ab^2+b^3)\arctan\left(-\frac{a\cos}{\sqrt{ab-}}\right.}{\sqrt{ab-b^2}a^3}}{}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/64*((8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 8*b*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 4
*(3*a^2 - 12*a*b + 8*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)
)/a^3 + 64*(a^2*b - 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e
) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*
a^3) + (a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 - 72*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^2*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(a^3*(cos(f*x + e
) - 1)^2))/f
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 740, normalized size of antiderivative = 5.69

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)),x)
```

output
```
(a^2*((3*cos(3*e + 3*f*x))/4 - (11*cos(e + f*x))/4 + (9*log(sin(e/2 + (f*x
)/2)/cos(e/2 + (f*x)/2)))/8 - (3*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/c
os(e/2 + (f*x)/2)))/2 + (3*cos(4*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2
 + (f*x)/2)))/8) + 3*b^2*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) - a*(b
*cos(3*e + 3*f*x) - b*cos(e + f*x) + (9*b*log(sin(e/2 + (f*x)/2)/cos(e/2 +
 (f*x)/2)))/2 - 6*b*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x
)/2)) + (3*b*cos(4*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/
2) - 4*b^2*cos(2*e + 2*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + b
^2*cos(4*e + 4*f*x)*log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)) + 3*b^(1/2)
*atan((a^4*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*a^2
*b^2 + 6*a^2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f*x
))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*(a - b)^(3/2) - 4*b^(1/
2)*atan((a^4*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e + f*x) + b^4 + 3*a
^2*b^2 + 6*a^2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) - 4*a^3*b*cos(e + f
*x))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*cos(2*e + 2*f*x)*(a -
 b)^(3/2) + b^(1/2)*atan((a^4*cos(e + f*x) - a^3*b - 3*a*b^3 + b^4*cos(e +
 f*x) + b^4 + 3*a^2*b^2 + 6*a^2*b^2*cos(e + f*x) - 4*a*b^3*cos(e + f*x) -
4*a^3*b*cos(e + f*x))/(2*b^(1/2)*cos(e/2 + (f*x)/2)^2*(a - b)^(7/2)))*cos(
4*e + 4*f*x)*(a - b)^(3/2))/(3*a^3*f - 4*a^3*f*cos(2*e + 2*f*x) + a^3*f*co
s(4*e + 4*f*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.39

$$\int \frac{\csc^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \frac{8\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx+e)^4\,a-8\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx}{}$$

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

output
```
(8*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**4*a - 8*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*b + 8*sqrt(b)*sqrt(a - b)*atan(
(sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a - 8*sq
rt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*s
in(e + f*x)**4*b - 3*cos(e + f*x)*sin(e + f*x)**2*a**2 + 4*cos(e + f*x)*si
n(e + f*x)**2*a*b - 2*cos(e + f*x)*a**2 + 3*log(tan((e + f*x)/2))*sin(e +
f*x)**4*a**2 - 12*log(tan((e + f*x)/2))*sin(e + f*x)**4*a*b + 8*log(tan((e
 + f*x)/2))*sin(e + f*x)**4*b**2)/(8*sin(e + f*x)**4*a**3*f)
```

3.61 $\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{(5a^3+15a^2b-5ab^2+b^3)\,x}{16(a-b)^4} - \frac{a^{5/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^4 f} - \frac{(11a^2-4ab+b^2)\cos(e+fx)\sin(e+fx)}{16(a-b)^3 f} + \frac{(3a-b)\cos^3(e+fx)\sin(e+fx)}{8(a-b)^2 f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6(a-b)f}$$

output
```
1/16*(5*a^3+15*a^2*b-5*a*b^2+b^3)*x/(a-b)^4-a^(5/2)*b^(1/2)*arctan(b^(1/2)
*tan(f*x+e)/a^(1/2))/(a-b)^4/f-1/16*(11*a^2-4*a*b+b^2)*cos(f*x+e)*sin(f*x+
e)/(a-b)^3/f+1/8*(3*a-b)*cos(f*x+e)^3*sin(f*x+e)/(a-b)^2/f+1/6*cos(f*x+e)^
3*sin(f*x+e)^3/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.79

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{-12(5a^3+15a^2b-5ab^2+b^3)(e+fx)+192a^{5/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+3(a-b)(5a-b)(3a+b)}{192(a-b)^4f}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/192*(-12*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(e + f*x) + 192*a^(5/2)*Sqr
t[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + 3*(a - b)*(5*a - b)*(3*a + b
)*Sin[2*(e + f*x)] - 3*(a - b)^2*(3*a - b)*Sin[4*(e + f*x)] + (a - b)^3*Si
n[6*(e + f*x)])/((a - b)^4*f)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4146, 372, 27, 440, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)^6}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\frac{\tan^3(e+fx)}{6(a-b)(\tan^2(e+fx)+1)^3}-\frac{\int \frac{3\tan^2(e+fx)\left(a-(2a-b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{6(a-b)}}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6(a-b)(\tan^2(e+fx)+1)^3}-\frac{\int \frac{\tan^2(e+fx)\left(a-(2a-b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2(a-b)}}{f}$$

$\big\downarrow$ 440

$$\frac{\frac{\tan^3(e+fx)}{6(a-b)(\tan^2(e+fx)+1)^3}-\frac{\frac{\int \frac{a(3a-b)-\left(8a^2-3ba+b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{4(a-b)}-\frac{(3a-b)\tan(e+fx)}{4(a-b)\left(\tan^2(e+fx)+1\right)^2}}{2(a-b)}}{f}$$

$\big\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6(a-b)(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(11a^2-4ab+b^2\right)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{a(5a-b)(a+b)-b\left(11a^2-4ba+b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2(a-b)}}{4(a-b)}-\frac{(3a-b)\tan(e+fx)}{4(a-b)\left(\tan^2(e+fx)+1\right)^2}}{2(a-b)}}{f}$$

$\big\downarrow$ 397

$$\frac{\frac{\tan^3(e+fx)}{6(a-b)(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(11a^2-4ab+b^2\right)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(5a^3+15a^2b-5ab^2+b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{16a^3b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}}{4(a-b)}-}{2(a-b)}}{f}$$

$\big\downarrow$ 216

$$\frac{\frac{\tan^3(e+fx)}{6(a-b)(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{\left(11a^2-4ab+b^2\right)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(5a^3+15a^2b-5ab^2+b^3\right)\arctan(\tan(e+fx))}{a-b}-\frac{16a^3b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}}{4(a-b)}-\frac{(3a-b}{4(a-b)(\text{ta}}}{2(a-b)}}{f}$$

$\big\downarrow$ 218

$$\frac{\frac{\tan^3(e+fx)}{6(a-b)(\tan^2(e+fx)+1)^3}-\frac{\frac{\frac{(11a^2-4ab+b^2)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\frac{(5a^3+15a^2b-5ab^2+b^3)\arctan(\tan(e+fx))}{a-b}-\frac{16a^{5/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a-b}}{2(a-b)}}{4(a-b)}-\frac{(3a-b)\,\mathrm{ta}\cdots}{4(a-b)\left(\tan^2\cdots\right.}}{2(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]^3/(6*(a - b)*(1 + Tan[e + f*x]^2)^3) - (-1/4*((3*a - b)*Tan[
e + f*x])/((a - b)*(1 + Tan[e + f*x]^2)^2) + (-1/2*(((5*a^3 + 15*a^2*b - 5
*a*b^2 + b^3)*ArcTan[Tan[e + f*x]])/(a - b) - (16*a^(5/2)*Sqrt[b]*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a - b))/(a - b) + ((11*a^2 - 4*a*b + b^2)
*Tan[e + f*x])/(2*(a - b)*(1 + Tan[e + f*x]^2)))/(4*(a - b)))/(2*(a - b)))
/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 47.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-\frac{a^3 b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^4 \sqrt{ab}}+\frac{\frac{\left(-\frac{11}{16}a^3+\frac{15}{16}a^2 b-\frac{5}{16}a b^2+\frac{1}{16}b^3\right)\tan(fx+e)^5+\left(-\frac{5}{6}a^3+\frac{1}{2}a^2 b+\frac{1}{2}a b^2-\frac{1}{6}b^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a\right.}{\left(1+\tan(fx+e)^2\right)^3}}{(a-b)^4}}{f}$
default	$\frac{-\frac{a^3 b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^4 \sqrt{ab}}+\frac{\frac{\left(-\frac{11}{16}a^3+\frac{15}{16}a^2 b-\frac{5}{16}a b^2+\frac{1}{16}b^3\right)\tan(fx+e)^5+\left(-\frac{5}{6}a^3+\frac{1}{2}a^2 b+\frac{1}{2}a b^2-\frac{1}{6}b^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a\right.}{\left(1+\tan(fx+e)^2\right)^3}}{(a-b)^4}}{f}$
risch	$\frac{5x a^3}{16(a-b)^4}+\frac{15x a^2 b}{16(a-b)^4}-\frac{5xa b^2}{16(a-b)^4}+\frac{x b^3}{16(a-b)^4}+\frac{15i e^{2i(fx+e)} a^2}{128(a-b)^3 f}+\frac{i e^{2i(fx+e)} ab}{64(a-b)^3 f}-\frac{i e^{2i(fx+e)} b^2}{128(a-b)^3 f}-\frac{15}{128(a^2}$

input
```
int(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^3*b/(a-b)^4/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))+1/(a-b)^4
*(((-11/16*a^3+15/16*a^2*b-5/16*a*b^2+1/16*b^3)*tan(f*x+e)^5+(-5/6*a^3+1/2
*a^2*b+1/2*a*b^2-1/6*b^3)*tan(f*x+e)^3+(-5/16*a^3+1/16*a^2*b+5/16*a*b^2-1/
16*b^3)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a^3+15*a^2*b-5*a*b^2+b^3)*a
rctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.93

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \left[\frac{12\sqrt{-ab}a^2\log\left(\frac{(a^2+6ab+b^2)\cos(fx+e)^4-2(3ab+b^2)\cos(fx+e)^2+4\left((a+b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab}\sin(fx+e)+b^2}{(a^2-2ab+b^2)\cos(fx+e)^4+2(ab-b^2)\cos(fx+e)^2+b^2}\right)+\right.$$

input
```
integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/48*(12*sqrt(-a*b)*a^2*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*
b + b^2)*cos(f*x + e)^2 + 4*((a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt
(-a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b -
b^2)*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f*x - (
8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^2*b +
27*a*b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 - b^3)*c
os(f*x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f)
, 1/48*(24*sqrt(a*b)*a^2*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)
/(a*b*cos(f*x + e)*sin(f*x + e))) + 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*f
*x - (8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - 2*(13*a^3 - 33*a^
2*b + 27*a*b^2 - 7*b^3)*cos(f*x + e)^3 + 3*(11*a^3 - 15*a^2*b + 5*a*b^2 -
b^3)*cos(f*x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.71

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{48\,a^3b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)\sqrt{ab}} - \frac{3\,(5\,a^3+15\,a^2b-5\,ab^2+b^3)(fx+e)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} + \frac{3\,(11\,a^2-4\,ab+b^2)\tan(fx+e)^5+8\,(5\,a^2+2}{(a^3-3\,a^2b+3\,ab^2-b^3)\tan(fx+e)^6+3\,(a^3-3\,a^2b+3\,ab^2-b^3)\tan}}{48\,f}$$

input
```
integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/48*(48*a^3*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 4*a^3*b + 6*a^2*b
^2 - 4*a*b^3 + b^4)*sqrt(a*b)) - 3*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*(f*x
 + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*(11*a^2 - 4*a*b + b
^2)*tan(f*x + e)^5 + 8*(5*a^2 + 2*a*b - b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 4
*a*b - b^2)*tan(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^6
+ 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^4 + a^3 - 3*a^2*b + 3*a*b
^2 - b^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.56

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{a^3b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4f-4\,a^3bf+6\,a^2b^2f-4\,ab^3f+b^4f)\sqrt{ab}}$$
$$+\frac{(5\,a^3+15\,a^2b-5\,ab^2+b^3)(fx+e)}{16\,(a^4f-4\,a^3bf+6\,a^2b^2f-4\,ab^3f+b^4f)}$$
$$-\frac{33\,a^2\tan(fx+e)^5-12\,ab\tan(fx+e)^5+3\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+16\,ab\tan(fx+}{48\,(a^3f-3\,a^2bf+3\,ab^2f-b^3f)\big(\mathrm{ta}}$$

input
```
integrate(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-a^3*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4*f - 4*a^3*b*f + 6*a^2*b^2*f
- 4*a*b^3*f + b^4*f)*sqrt(a*b)) + 1/16*(5*a^3 + 15*a^2*b - 5*a*b^2 + b^3)*
(f*x + e)/(a^4*f - 4*a^3*b*f + 6*a^2*b^2*f - 4*a*b^3*f + b^4*f) - 1/48*(33
*a^2*tan(f*x + e)^5 - 12*a*b*tan(f*x + e)^5 + 3*b^2*tan(f*x + e)^5 + 40*a^
2*tan(f*x + e)^3 + 16*a*b*tan(f*x + e)^3 - 8*b^2*tan(f*x + e)^3 + 15*a^2*t
an(f*x + e) + 12*a*b*tan(f*x + e) - 3*b^2*tan(f*x + e))/((a^3*f - 3*a^2*b*
f + 3*a*b^2*f - b^3*f)*(tan(f*x + e)^2 + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 4910, normalized size of antiderivative = 27.58

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

output
```
(atan(-((((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4 + 32*a^4*b^9 - (77*
a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5 + (107*a^9*b^4)/4 -
9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 +
 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (tan(
e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(1024*b^11 - 7168*a*b^10
 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 14336*a^5*b^6 - 28672*a
^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^2))/(4096*(a^4 - 4*a^3*
b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4
 - 20*a^3*b^3 + 15*a^4*b^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i))/(3
2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (tan(e + f*x)*(b^9 - 10*a
*b^8 + 55*a^2*b^7 - 140*a^3*b^6 + 175*a^4*b^5 + 150*a^5*b^4 + 281*a^6*b^3)
)/(128*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b
^2)))*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*1i)/(32*(a^4 - 4*a^3*b - 4*
a*b^3 + b^4 + 6*a^2*b^2)) - (((((3*a^2*b^11 - (a*b^12)/4 - (55*a^3*b^10)/4
 + 32*a^4*b^9 - (77*a^5*b^8)/2 + 14*a^6*b^7 + (49*a^7*b^6)/2 - 40*a^8*b^5
+ (107*a^9*b^4)/4 - 9*a^10*b^3 + (5*a^11*b^2)/4)/(9*a*b^8 - 9*a^8*b + a^9
- b^9 - 36*a^2*b^7 + 84*a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 +
 36*a^7*b^2) + (tan(e + f*x)*(a^2*b*15i - a*b^2*5i + a^3*5i + b^3*1i)*(102
4*b^11 - 7168*a*b^10 + 20480*a^2*b^9 - 28672*a^3*b^8 + 14336*a^4*b^7 + 143
36*a^5*b^6 - 28672*a^6*b^5 + 20480*a^7*b^4 - 7168*a^8*b^3 + 1024*a^9*b^...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.17

$$\int \frac{\sin^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{48\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a^2-48\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a^2-8\cos\left(fx+e\right)\sin\left(fx\right.}{}$$

input
```
int(sin(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

output
```
(48*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*a**2 - 48*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/s
qrt(b))*a**2 - 8*cos(e + f*x)*sin(e + f*x)**5*a**3 + 24*cos(e + f*x)*sin(e
 + f*x)**5*a**2*b - 24*cos(e + f*x)*sin(e + f*x)**5*a*b**2 + 8*cos(e + f*x
)*sin(e + f*x)**5*b**3 - 10*cos(e + f*x)*sin(e + f*x)**3*a**3 + 18*cos(e +
 f*x)*sin(e + f*x)**3*a**2*b - 6*cos(e + f*x)*sin(e + f*x)**3*a*b**2 - 2*c
os(e + f*x)*sin(e + f*x)**3*b**3 - 15*cos(e + f*x)*sin(e + f*x)*a**3 + 3*c
os(e + f*x)*sin(e + f*x)*a**2*b + 15*cos(e + f*x)*sin(e + f*x)*a*b**2 - 3*
cos(e + f*x)*sin(e + f*x)*b**3 + 15*a**3*e + 15*a**3*f*x + 45*a**2*b*e + 4
5*a**2*b*f*x - 15*a*b**2*e - 15*a*b**2*f*x + 3*b**3*e + 3*b**3*f*x)/(48*f*
(a**4 - 4*a**3*b + 6*a**2*b**2 - 4*a*b**3 + b**4))
```

3.62 $\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{(3a^2+6ab-b^2)\,x}{8(a-b)^3} - \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^3 f} - \frac{(5a-b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2 f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f}$$

output
```
1/8*(3*a^2+6*a*b-b^2)*x/(a-b)^3-a^(3/2)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/
a^(1/2))/(a-b)^3/f-1/8*(5*a-b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f+1/4*cos(f*x
+e)^3*sin(f*x+e)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{4(3a^2+6ab-b^2)(e+fx) - 32a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - 8a(a-b)\sin(2(e+fx)) + (a-b)^2\sin(4}{32(a-b)^3 f}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(4*(3*a^2 + 6*a*b - b^2)*(e + f*x) - 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a]] - 8*a*(a - b)*Sin[2*(e + f*x)] + (a - b)^2*Sin[4*(e +
 f*x)])/(32*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4146, 372, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^4}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\int \frac{a-(4a-b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{4(a-b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{a(3a+b)-(5a-b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+6ab-b^2\right)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{8a^2b\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}}{4(a-b)}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+6ab-b^2\right)\arctan(\tan(e+fx))}{a-b}-\frac{8a^2b\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}}{4(a-b)}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+6ab-b^2\right)\arctan(\tan(e+fx))}{a-b}-\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a-b}}{2(a-b)}}{4(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]/(4*(a - b)*(1 + Tan[e + f*x]^2)^2) - (-1/2*(((3*a^2 + 6*a*b
- b^2)*ArcTan[Tan[e + f*x]])/(a - b) - (8*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/(a - b))/(a - b) + ((5*a - b)*Tan[e + f*x])/(2*(a
- b)*(1 + Tan[e + f*x]^2)))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 10.80 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{5}{8}a^2+\frac{3}{4}ab-\frac{1}{8}b^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2+\frac{1}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+6ab-b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^3}-\frac{a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{5}{8}a^2+\frac{3}{4}ab-\frac{1}{8}b^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2+\frac{1}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+6ab-b^2\right)\arctan(\tan(fx+e))}{8}}{(a-b)^3}-\frac{a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}}{f}$
risch	$\frac{3xa^2}{8(a-b)^3}+\frac{3xab}{4(a-b)^3}-\frac{xb^2}{8(a-b)^3}+\frac{ia\,e^{2i(fx+e)}}{8(a-b)^2f}-\frac{ia\,e^{-2i(fx+e)}}{8(a^2-2ab+b^2)f}+\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2(a-b)^3f}-$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^3*(((-5/8*a^2+3/4*a*b-1/8*b^2)*tan(f*x+e)^3+(-3/8*a^2+1/4*a*b
+1/8*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)^2+1/8*(3*a^2+6*a*b-b^2)*arctan(tan(
f*x+e)))-a^2*b/(a-b)^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.97

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{(3a^2+6ab-b^2)fx-2\sqrt{-ab}a\log\left(\frac{(a^2+6ab+b^2)\cos(fx+e)^4-2(3ab+b^2)\cos(fx+e)^2-4\left((a+b)\cos(fx+e)^3-b\cos(fx+e)\right.}{(a^2-2ab+b^2)\cos(fx+e)^4+2(ab-b^2)\cos(fx+e)^2+b^2}\right.}{8(a^3-3a^2b+}\right.$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/8*((3*a^2 + 6*a*b - b^2)*f*x - 2*sqrt(-a*b)*a*log(((a^2 + 6*a*b + b^2)*
cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a + b)*cos(f*x + e)^
3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*co
s(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) + (2*(a^2 - 2*a*b + b^
2)*cos(f*x + e)^3 - (5*a^2 - 6*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/((a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*f), 1/8*((3*a^2 + 6*a*b - b^2)*f*x + 4*sqrt(a
*b)*a*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*
sin(f*x + e))) + (2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 6*a*b +
b^2)*cos(f*x + e))*sin(f*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.42

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{8\,a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab}} - \frac{(3\,a^2+6\,ab-b^2)(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{(5\,a-b)\tan(fx+e)^3+(3\,a+b)\tan(fx+e)}{(a^2-2\,ab+b^2)\tan(fx+e)^4+2\,(a^2-2\,ab+b^2)\tan(fx+e)^2+a^2-2\,ab+b^2}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/8*(8*a^2*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*sqrt(a*b)) - (3*a^2 + 6*a*b - b^2)*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) + ((5*a - b)*tan(f*x + e)^3 + (3*a + b)*tan(f*x + e))/((a^2 - 2*a
*b + b^2)*tan(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*tan(f*x + e)^2 + a^2 - 2*
a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.36

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= -\frac{a^2b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3f-3\,a^2bf+3\,ab^2f-b^3f)\sqrt{ab}}+\frac{(3\,a^2+6\,ab-b^2)(fx+e)}{8\,(a^3f-3\,a^2bf+3\,ab^2f-b^3f)}$$
$$-\frac{5\,a\tan{(fx+e)}^3-b\tan{(fx+e)}^3+3\,a\tan{(fx+e)}+b\tan{(fx+e)}}{8\,(a^2f-2\,abf+b^2f)\left(\tan{(fx+e)}^2+1\right)^2}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-a^2*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*f - 3*a^2*b*f + 3*a*b^2*f -
b^3*f)*sqrt(a*b)) + 1/8*(3*a^2 + 6*a*b - b^2)*(f*x + e)/(a^3*f - 3*a^2*b*f
 + 3*a*b^2*f - b^3*f) - 1/8*(5*a*tan(f*x + e)^3 - b*tan(f*x + e)^3 + 3*a*t
an(f*x + e) + b*tan(f*x + e))/((a^2*f - 2*a*b*f + b^2*f)*(tan(f*x + e)^2 +
 1)^2)
```

Mupad [B] (verification not implemented)

Time = 11.55 (sec) , antiderivative size = 3588, normalized size of antiderivative = 27.81

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

output
```
(atan(((((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*
b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((32*a*b^9 - 96*
a^2*b^8 - 96*a^3*b^7 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7
*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^
3*b^3 + 15*a^4*b^2)) - (tan(e + f*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9
- 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3
 - 256*a^7*b^2))/(64*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*
b^3 + b^4 + 6*a^2*b^2)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3
)))*(-a^3*b)^(1/2)*1i)/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (((tan(e + f*
x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a^3*b^4 + 73*a^4*b^3))/(32*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((32*a*b^9 - 96*a^2*b^8 - 96*a^3*b^7
 + 800*a^4*b^6 - 1440*a^5*b^5 + 1248*a^6*b^4 - 544*a^7*b^3 + 96*a^8*b^2)/(
64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2))
 + (tan(e + f*x)*(-a^3*b)^(1/2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 128
0*a^3*b^6 + 1280*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(64
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^
2)))*(-a^3*b)^(1/2))/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(-a^3*b)^(1/2)*1
i)/(2*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/((a^2*b^6 - 11*a^3*b^5 + 27*a^4*b^
4 + 15*a^5*b^3)/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b
^3 + 15*a^4*b^2)) + (((tan(e + f*x)*(b^7 - 12*a*b^6 + 30*a^2*b^5 + 36*a...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.83

$$\int \frac{\sin^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a-8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)a-2\cos\left(fx+e\right)\sin\left(fx+e\right.}{}$$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

output
```
(8*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
a - 8*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*a - 2*cos(e + f*x)*sin(e + f*x)**3*a**2 + 4*cos(e + f*x)*sin(e + f*x)**
3*a*b - 2*cos(e + f*x)*sin(e + f*x)**3*b**2 - 3*cos(e + f*x)*sin(e + f*x)*
a**2 + 2*cos(e + f*x)*sin(e + f*x)*a*b + cos(e + f*x)*sin(e + f*x)*b**2 +
3*a**2*e + 3*a**2*f*x + 6*a*b*e + 6*a*b*f*x - b**2*e - b**2*f*x)/(8*f*(a**
3 - 3*a**2*b + 3*a*b**2 - b**3))
```

3.63 $\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{(a+b)x}{2(a-b)^2} - \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^2 f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f}$$

output
```
1/2*(a+b)*x/(a-b)^2-a^(1/2)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-
b)^2/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{2(a+b)(e+fx) - 4\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) + (-a+b)\sin(2(e+fx))}{4(a-b)^2 f}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
(2*(a + b)*(e + f*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqr
t[a]] + (-a + b)*Sin[2*(e + f*x)])/(4*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4146, 373, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\int \frac{a-b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{2ab\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{(a+b)\arctan(\tan(e+fx))}{a-b}-\frac{2ab\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(a+b)\arctan(\tan(e+fx))}{a-b}-\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a-b}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
((((a + b)*ArcTan[Tan[e + f*x]])/(a - b) - (2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a]])/(a - b))/(2*(a - b)) - Tan[e + f*x]/(2*(a - b)
*(1 + Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan(fx+e)^2}+\frac{(a+b)\arctan(\tan(fx+e))}{2}}{(a-b)^2}-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan(fx+e)^2}+\frac{(a+b)\arctan(\tan(fx+e))}{2}}{(a-b)^2}-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}}{f}$
risch	$\frac{xa}{2(a-b)^2}+\frac{xb}{2(a-b)^2}+\frac{i\mathrm{e}^{2i(fx+e)}}{8(a-b)f}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8(a-b)f}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2(a-b)^2f}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}\right.}{2(a-b}$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^2*((-1/2*a+1/2*b)*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+b)*arcta
n(tan(f*x+e)))-a*b/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.34

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$=\left[\frac{2\,(a+b)fx-2\,(a-b)\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos\left(fx+e\right)^4-2\,\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2+\cdots}{\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^4\cdots}\right.}{4\,\left(a^2-2\,ab+b^2\right)f}\cdots\right.$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)*f*x - 2*(a - b)*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b)*log
(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*
((a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)/(
(a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)))
/((a^2 - 2*a*b + b^2)*f), 1/2*((a + b)*f*x - (a - b)*cos(f*x + e)*sin(f*x
+ e) + sqrt(a*b)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*co
s(f*x + e)*sin(f*x + e))))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{2\,ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}} - \frac{(fx+e)(a+b)}{a^2-2\,ab+b^2} + \frac{\tan(fx+e)}{(a-b)\tan(fx+e)^2+a-b}}{2\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b
)) - (f*x + e)*(a + b)/(a^2 - 2*a*b + b^2) + tan(f*x + e)/((a - b)*tan(f*x
 + e)^2 + a - b))/f
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2f-2\,abf+b^2f)\sqrt{ab}} + \frac{(fx+e)(a+b)}{2\,(a^2f-2\,abf+b^2f)} - \frac{\tan\left(fx+e\right)}{2\,(af-bf)\left(\tan\left(fx+e\right)^2+1\right)}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-a*b*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*f - 2*a*b*f + b^2*f)*sqrt(a*b)
) + 1/2*(f*x + e)*(a + b)/(a^2*f - 2*a*b*f + b^2*f) - 1/2*tan(f*x + e)/((a
*f - b*f)*(tan(f*x + e)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.32

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{b\sin(2e+2fx)-a\sin(2e+2fx)+2a\,\mathrm{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)+2b\,\mathrm{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)-\mathrm{atan}\left(\frac{b^3\sin(e+fx)\sqrt{-}}{\cos(e+}\right.}{4fa^2-8fab+4fb^2}$$

input
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

output
```
(b*sin(2*e + 2*f*x) - a*sin(2*e + 2*f*x) + 2*a*atan(sin(e + f*x)/cos(e + f
*x)) + 2*b*atan(sin(e + f*x)/cos(e + f*x)) - atan((b^3*sin(e + f*x)*(-a*b)
^(1/2)*1i - a*b^2*sin(e + f*x)*(-a*b)^(1/2)*2i + a^2*b*sin(e + f*x)*(-a*b)
^(1/2)*1i)/(a*b^3*cos(e + f*x) - 2*a^2*b^2*cos(e + f*x) + a^3*b*cos(e + f*
x)))*(-a*b)^(1/2)*4i)/(4*a^2*f + 4*b^2*f - 8*a*b*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \frac{\sin^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{2\sqrt{b}\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)-2\sqrt{b}\sqrt{a}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)-\cos(fx+e)\sin(fx+e)a+}{2f(a^2-2ab+b^2)}$$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
- 2*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))
 - cos(e + f*x)*sin(e + f*x)*a + cos(e + f*x)*sin(e + f*x)*b + a*e + a*f*x
 + b*e + b*f*x)/(2*f*(a**2 - 2*a*b + b**2))
```

3.64 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{x}{a-b} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)f}$$

output
```
x/(a-b)-b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(1/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{\arctan(\tan(e+fx)) - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

output
```
(ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/S
qrt[a])/(a*f - b*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4143, 3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tan^2(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\tan(e+fx)^2}dx$$
$$\Big\downarrow \text{4143}$$
$$\frac{x}{a-b}-\frac{b\int \frac{\sec^2(e+fx)}{b\tan^2(e+fx)+a}dx}{a-b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{x}{a-b}-\frac{b\int \frac{\sec(e+fx)^2}{b\tan(e+fx)^2+a}dx}{a-b}$$
$$\Big\downarrow \text{4158}$$
$$\frac{x}{a-b}-\frac{b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{f(a-b)}$$
$$\Big\downarrow \text{218}$$
$$\frac{x}{a-b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}f(a-b)}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

output
```
x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
 b)*f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4143
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Simp[b/(a - b)   Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x
] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
default	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)f}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)f}$	120

input
```
int(1/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)*arctan(tan(f*x+e))-b/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.64

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{4\,fx-\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{4\,(a-b)f}, \frac{2\,fx-\sqrt{\frac{b}{a}}\arctan\left(\frac{(\right.}{2\,(a-}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
 e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.

Time = 1.38 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.80

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\tan^2(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{-x-\frac{1}{f\tan(e+fx)}}{b} & \text{for } a=0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{fx}{2bf\tan^2(e+fx)+2bf}+\frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x}{a+b\tan^2(e)} & \text{for } f=0 \\ \frac{2fx\sqrt{-\frac{a}{b}}}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}-\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a, Eq(b, 0
)), ((-x - 1/(f*tan(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*t
an(e + f*x)**2 + 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*
x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f,
 0)), (2*f*x*sqrt(-a/b)/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) - log(-sqrt(
-a/b) + tan(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) + log(sqrt(-a/
b) + tan(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)}-\frac{fx+e}{a-b}}{f}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a -
b))/f
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = -\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(af-bf)} + \frac{fx+e}{af-bf}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a*f - b*f)) + (f*x + e)/(a
*f - b*f)
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 948, normalized size of antiderivative = 18.96

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(e + f*x)^2),x)
```

output
```
(atan((((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3
+ 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*
a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/
2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan
(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b
- a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(e +
f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(
1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b
- a^2))))/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*
(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5
 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2)
))*(-a*b)^(1/2)*1i)/(a*f*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 +
 (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*
1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4
*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*
a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b))/(((((4*b^4 -
8*a*b^3 + 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b
^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b)
 - ((((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^
2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx=\frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)+afx}{af\,(a-b)}$$

input
```
int(1/(a+b*tan(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a))) + a*f*x)/(a*f
*(a - b))
```

3.65 $\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

output
```
-b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)/f-cot(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(3/2)*f)) - Cot[e +
f*x]/(a*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4146, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 264$$

$$\frac{-\frac{b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(e+fx)}{a}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(3/2)) - Cot[e + f*x
]/a)/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\tan(fx+e)}}{f}$	44
default	$\frac{-\frac{b \arctan\left(\frac{b \tan(fx+e)}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{a\tan(fx+e)}}{f}$	44
risch	$-\frac{2i}{fa\left(\mathrm{e}^{2i(fx+e)}-1\right)}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a^2f}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a^2f}$	119

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))-1/a/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 257, normalized size of antiderivative = 5.35

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(fx+e)^4-2\,(3\,ab+b^2)\cos(fx+e)^2+4\left((a^2+ab)\cos(fx+e)^3-ab\cos(fx+e)\right)\sqrt{-\frac{b}{a}}\sin(fx+e)+b^2}{(a^2-2\,ab+b^2)\cos(fx+e)^4+2\,(ab-b^2)\cos(fx+e)^2+b^2}\right)\sin(}{4\,af\sin(fx+e)}\right.$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)
*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-
b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^
2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/(a*f*sin(f*x + e)
), 1/2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos
(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x + e))/(a*f*sin(f*x + e))
]
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{1}{a\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}af} - \frac{1}{af\tan(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a*f) - 1/(a*f*tan(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\cot(e+fx)}{af} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}f}$$

input
```
int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)),x)
```

output
```
- cot(e + f*x)/(a*f) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(a^(
3/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.06

$$\int \frac{\csc^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx+e)-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx+e)-\cos(f}{\sin(fx+e)\,a^2 f}$$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*si
n(e + f*x) - sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))
/sqrt(b))*sin(e + f*x) - cos(e + f*x)*a)/(sin(e + f*x)*a**2*f)
```

3.66 $\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(a-b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af}$$

output
```
-(a-b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2)/f-(a-b)*cot(f*x+
e)/a^2/f-1/3*cot(f*x+e)^3/a/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{3\sqrt{b}(-a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \sqrt{a}\cot(e+fx)\left(2a-3b+a\csc^2(e+fx)\right)}{3a^{5/2}f}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(3*Sqrt[b]*(-a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Cot[e
 + f*x]*(2*a - 3*b + a*Csc[e + f*x]^2))/(3*a^(5/2)*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4146, 359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(a-b)\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{(a-b)\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a}\right)}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a-b)\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(e+fx)}{a}\right)}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/a + ((a - b)*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a]])/a^(3/2)) - Cot[e + f*x]/a))/a)/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{1}{3a\tan(fx+e)^3}-\frac{a-b}{a^2\tan(fx+e)}-\frac{b(a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
default	$\frac{-\frac{1}{3a\tan(fx+e)^3}-\frac{a-b}{a^2\tan(fx+e)}-\frac{b(a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
risch	$\frac{2i\left(3b\,\mathrm{e}^{4i(fx+e)}+6a\,\mathrm{e}^{2i(fx+e)}-6b\,\mathrm{e}^{2i(fx+e)}-2a+3b\right)}{3f\,a^2\left(\mathrm{e}^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a^2f}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}\right.}{2a^3}$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/a/tan(f*x+e)^3-(a-b)/a^2/tan(f*x+e)-b*(a-b)/a^2/(a*b)^(1/2)*arct
an(b*tan(f*x+e)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(66) = 132.

Time = 0.14 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.91

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$=\left[-\frac{4\,(2\,a-3\,b)\cos\left(fx+e\right)^3+3\left((a-b)\cos\left(fx+e\right)^2-a+b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(fx+e)^4-2\,(3\,ab+b^2}{(a^2-2\,a}\right.}{12\left(a^2f\cos\left(fx+e\right)^2-\right.}\right.$$
$$-\frac{2\,(2\,a-3\,b)\cos\left(fx+e\right)^3-3\left((a-b)\cos\left(fx+e\right)^2-a+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(\right.}{6\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(4*(2*a - 3*b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 - a + b)*
sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f
*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*s
in(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos
(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(a - b)*cos(f*x + e))/((a^2*f*cos(f*
x + e)^2 - a^2*f)*sin(f*x + e)), -1/6*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*((
a - b)*cos(f*x + e)^2 - a + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^
2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a - b)*c
os(f*x + e))/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \int \frac{\csc^4{(e+fx)}}{a+b\tan^2{(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{3\,(ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,(a-b)\tan(fx+e)^2+a}{a^2\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*(a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*
(a - b)*tan(f*x + e)^2 + a)/(a^2*tan(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{(ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2f} - \frac{3\,a\tan{(fx+e)}^2 - 3\,b\tan{(fx+e)}^2 + a}{3\,a^2f\tan{(fx+e)}^3}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-(a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - 1/3*(3*a
*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 + a)/(a^2*f*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{1}{3\,a}+\frac{{\tan(e+f\,x)}^2\,(a-b)}{a^2}}{f\,{\tan\left(e+f\,x\right)}^3} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(e+f\,x)}{\sqrt{a}}\right)\,(a-b)}{a^{5/2}\,f}$$

input
```
int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)),x)
```

output
```
- (1/(3*a) + (tan(e + f*x)^2*(a - b))/a^2)/(f*tan(e + f*x)^3) - (b^(1/2)*a
tan((b^(1/2)*tan(e + f*x))/a^(1/2))*(a - b))/(a^(5/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.99

$$\int \frac{\csc^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)^3 a-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)^3 b -}{}$$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
sin(e + f*x)**3*a - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e +
 f*x)/2))/sqrt(b))*sin(e + f*x)**3*b - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
 + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a + 3*sqrt(b)*sqrt(a
)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*b
 - 2*cos(e + f*x)*sin(e + f*x)**2*a**2 + 3*cos(e + f*x)*sin(e + f*x)**2*a*
b - cos(e + f*x)*a**2)/(3*sin(e + f*x)**3*a**3*f)
```

3.67 $\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{(a-b)^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}f} - \frac{(a-b)^2\cot(e+fx)}{a^3f} - \frac{(2a-b)\cot^3(e+fx)}{3a^2f} - \frac{\cot^5(e+fx)}{5af}$$

output
```
-(a-b)^2*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/f-(a-b)^2*cot(
f*x+e)/a^3/f-1/3*(2*a-b)*cot(f*x+e)^3/a^2/f-1/5*cot(f*x+e)^5/a/f
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{-15(a-b)^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - \sqrt{a}\cot(e+fx)\left(8a^2-25ab+15b^2+a(4a-5b)\csc^2(e+fx)+\right.}{15a^{7/2}f}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-15*(a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*Co
t[e + f*x]*(8*a^2 - 25*a*b + 15*b^2 + a*(4*a - 5*b)*Csc[e + f*x]^2 + 3*a^2
*Csc[e + f*x]^4))/(15*a^(7/2)*f)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{b\tan^2(e+fx)+a}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\frac{\cot^6(e+fx)}{a}+\frac{(2a-b)\cot^4(e+fx)}{a^2}+\frac{(a-b)^2\cot^2(e+fx)}{a^3}-\frac{(a-b)^2b}{a^3\left(b\tan^2(e+fx)+a\right)}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt{b}(a-b)^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}}-\frac{(a-b)^2\cot(e+fx)}{a^3}-\frac{(2a-b)\cot^3(e+fx)}{3a^2}-\frac{\cot^5(e+fx)}{5a}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-(((a - b)^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(7/2)) - (
(a - b)^2*Cot[e + f*x])/a^3 - ((2*a - b)*Cot[e + f*x]^3)/(3*a^2) - Cot[e +
 f*x]^5/(5*a))/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{5a\tan(fx+e)^5}-\frac{2a-b}{3a^2\tan(fx+e)^3}-\frac{a^2-2ab+b^2}{a^3\tan(fx+e)}-\frac{b(a^2-2ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}}{f}$
default	$\frac{-\frac{1}{5a\tan(fx+e)^5}-\frac{2a-b}{3a^2\tan(fx+e)^3}-\frac{a^2-2ab+b^2}{a^3\tan(fx+e)}-\frac{b(a^2-2ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}}{f}$
risch	$\frac{2i(15ab\,e^{8i(fx+e)}-15b^2e^{8i(fx+e)}-90ab\,e^{6i(fx+e)}+60b^2e^{6i(fx+e)}-80a^2e^{4i(fx+e)}+160ab\,e^{4i(fx+e)}-90b^2e^{4i(fx+e)}+4}{15f\,a^3\left(e^{2i(fx+e)}-1\right)^5}$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5/a/tan(f*x+e)^5-1/3*(2*a-b)/a^2/tan(f*x+e)^3-(a^2-2*a*b+b^2)/a^3/
tan(f*x+e)-b*(a^2-2*a*b+b^2)/a^3/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(93) = 186$.

Time = 0.12 (sec) , antiderivative size = 543, normalized size of antiderivative = 5.17

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(8\,a^2-25\,ab+15\,b^2)\cos{(fx+e)}^5-20\,(4\,a^2-11\,ab+6\,b^2)\cos{(fx+e)}^3-15\,\left((a^2-2\,ab+b^2)\,\mathrm{co}\right.}{}\right.$$

$$-\frac{2\,(8\,a^2-25\,ab+15\,b^2)\cos{(fx+e)}^5-10\,(4\,a^2-11\,ab+6\,b^2)\cos{(fx+e)}^3-15\,\left((a^2-2\,ab+b^2)\,\mathrm{cos}\right.}{30\,\left(a^3f\,\mathrm{co}\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 11*a*b +
6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 -
2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(-b/a)*log(((a^2 + 6*
a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b
)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2
- 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f
*x + e) + 60*(a^2 - 2*a*b + b^2)*cos(f*x + e))/((a^3*f*cos(f*x + e)^4 - 2*
a^3*f*cos(f*x + e)^2 + a^3*f)*sin(f*x + e)), -1/30*(2*(8*a^2 - 25*a*b + 15
*b^2)*cos(f*x + e)^5 - 10*(4*a^2 - 11*a*b + 6*b^2)*cos(f*x + e)^3 - 15*((a
^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 +
a^2 - 2*a*b + b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(
b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^2 - 2*a*b + b^2)*
cos(f*x + e))/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f)*sin
(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{15\,(a^2b-2\,ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}+\frac{15\,(a^2-2\,ab+b^2)\tan(fx+e)^4+5\,(2\,a^2-ab)\tan(fx+e)^2+3\,a^2}{a^3\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*(a^2*b - 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a
*b)*a^3) + (15*(a^2 - 2*a*b + b^2)*tan(f*x + e)^4 + 5*(2*a^2 - a*b)*tan(f*
x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.22

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{(a^2b-2\,ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3f} - \frac{15\,a^2\tan\left(fx+e\right)^4-30\,ab\tan\left(fx+e\right)^4+15\,b^2\tan\left(fx+e\right)^4+10\,a^2\tan\left(fx+e\right)^2-5\,ab\tan\left(fx+\right.}{15\,a^3f\tan\left(fx+e\right)^5}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-(a^2*b - 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f
) - 1/15*(15*a^2*tan(f*x + e)^4 - 30*a*b*tan(f*x + e)^4 + 15*b^2*tan(f*x +
 e)^4 + 10*a^2*tan(f*x + e)^2 - 5*a*b*tan(f*x + e)^2 + 3*a^2)/(a^3*f*tan(f
*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{1}{5\,a}+\frac{\tan(e+f\,x)^2\,(2\,a-b)}{3\,a^2}+\frac{\tan(e+f\,x)^4\,(a^2-2\,a\,b+b^2)}{a^3}}{f\tan\left(e+f\,x\right)^5} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,(a-b)^2}{\sqrt{a}\,(a^2-2\,a\,b+b^2)}\right)\,(a-b)^2}{a^{7/2}\,f}$$

input
```
int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)),x)
```

output
```
- (1/(5*a) + (tan(e + f*x)^2*(2*a - b))/(3*a^2) + (tan(e + f*x)^4*(a^2 - 2
*a*b + b^2))/a^3)/(f*tan(e + f*x)^5) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)
*(a - b)^2)/(a^(1/2)*(a^2 - 2*a*b + b^2)))*(a - b)^2)/(a^(7/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.61

$$\int \frac{\csc^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)^5 a^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)^5}{}$$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**5*a**2 - 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a*b + 15*sqrt(b)*sqrt(a)*atan((sqr
t(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*b**2 - 15*sq
rt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
 + f*x)**5*a**2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e +
f*x)/2))/sqrt(b))*sin(e + f*x)**5*a*b - 15*sqrt(b)*sqrt(a)*atan((sqrt(a -
b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*b**2 - 8*cos(e + f
*x)*sin(e + f*x)**4*a**3 + 25*cos(e + f*x)*sin(e + f*x)**4*a**2*b - 15*cos
(e + f*x)*sin(e + f*x)**4*a*b**2 - 4*cos(e + f*x)*sin(e + f*x)**2*a**3 + 5
*cos(e + f*x)*sin(e + f*x)**2*a**2*b - 3*cos(e + f*x)*a**3)/(15*sin(e + f*
x)**5*a**4*f)
```

3.68 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{a\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{9/2}f} - \frac{a(a+2b)\cos(e+fx)}{(a-b)^4f} + \frac{2a\cos^3(e+fx)}{3(a-b)^3f} - \frac{\cos^5(e+fx)}{5(a-b)^2f} - \frac{a^2b\sec(e+fx)}{2(a-b)^4f\,(a-b+b\sec^2(e+fx))}$$

output
```
-1/2*a*b^(1/2)*(3*a+4*b)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(9/2
)/f-a*(a+2*b)*cos(f*x+e)/(a-b)^4/f+2/3*a*cos(f*x+e)^3/(a-b)^3/f-1/5*cos(f*
x+e)^5/(a-b)^2/f-1/2*a^2*b*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.33

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{\frac{120a\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}}+\frac{120a\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}}+\frac{-30\cos(e+fx)\left(18ab+b^2+a^2\left(5\right.\right.}{}}{240f}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2]
)/Sqrt[b]])/(a - b)^(9/2) + (120*a*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b]
 + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (-30*Cos[e + f*x]*(
18*a*b + b^2 + a^2*(5 + (8*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a -
b)*(5*(5*a + 3*b)*Cos[3*(e + f*x)] + 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)
^4)/(240*f)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4147, 365, 25, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(e+fx)^5}{(a+b\tan(e+fx)^2)^2}dx$$
$$\Big\downarrow \; 4147$$

$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{f}$$

$\downarrow$ 365

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(-5(a-b)\sec^2(e+fx)+10a-3b\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

$\downarrow$ 25

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(-5(a-b)\sec^2(e+fx)+10a-3b\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

$\downarrow$ 361

$$\frac{-\frac{\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{2(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}-\frac{1}{2}b\int -\frac{\cos^4(e+fx)\left(\frac{\left(5a^2+2b^2\right)\sec^4(e+fx)}{(a-b)^3}-\frac{2\left(5a^2+2b^2\right)\sec^2(e+fx)}{(a-b)^2b}+\frac{2(10a-3b)}{(a-b)b}\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+f}{5(a-b)(a+b\sec^2(}}{f}$$

$\downarrow$ 25

$$\frac{-\frac{\frac{1}{2}b\int \frac{\cos^4(e+fx)\left(\frac{\left(5a^2+2b^2\right)\sec^4(e+fx)}{(a-b)^3}-\frac{2\left(5a^2+2b^2\right)\sec^2(e+fx)}{(a-b)^2b}+\frac{2(10a-3b)}{(a-b)b}\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)+\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{2(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)(a+b\sec^2(e-}}{f}$$

$\downarrow$ 1584

$$\frac{-\frac{\frac{1}{2}b\int \left(\frac{2(10a-3b)\cos^4(e+fx)}{(a-b)^2b}-\frac{2\left(5a^2+10ba-b^2\right)\cos^2(e+fx)}{(a-b)^3b}+\frac{5a(3a+4b)}{(a-b)^3\left(b\sec^2(e+fx)+a-b\right)}\right)d\sec(e+fx)+\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{2(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}}{5(a-b)}-\frac{\text{co}}{5(a-b)(a-}}{f}$$

$\downarrow$ 2009

$$\frac{-\frac{\frac{1}{2}b\left(\frac{2\left(5a^2+10ab-b^2\right)\cos(e+fx)}{b(a-b)^3}+\frac{5a(3a+4b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{\sqrt{b}(a-b)^{7/2}}-\frac{2(10a-3b)\cos^3(e+fx)}{3b(a-b)^2}\right)+\frac{b\left(5a^2+2b^2\right)\sec(e+fx)}{2(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)(a+b\sec^2(e-}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/5*Cos[e + f*x]^5/((a - b)*(a - b + b*Sec[e + f*x]^2)) - ((b*((5*a*(3*a
 + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b)^(7/2)*Sqrt[b]
) + (2*(5*a^2 + 10*a*b - b^2)*Cos[e + f*x])/((a - b)^3*b) - (2*(10*a - 3*b
)*Cos[e + f*x]^3)/(3*(a - b)^2*b)))/2 + (b*(5*a^2 + 2*b^2)*Sec[e + f*x])/(
2*(a - b)^3*(a - b + b*Sec[e + f*x]^2)))/(5*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 62.51 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{\frac{a^2\cos(fx+e)^5}{5}-\frac{2ab\cos(fx+e)^5}{5}+\frac{b^2\cos(fx+e)^5}{5}-\frac{2a^2\cos(fx+e)^3}{3}+\frac{2ab\cos(fx+e)^3}{3}+\cos(fx+e)a^2+2ab\cos(fx+e)}{(a^2-2ab+b^2)(a-b)^2}+\frac{ab\left(-\frac{}{2(}\right.}{}}{f}$
default	$\frac{-\frac{\frac{a^2\cos(fx+e)^5}{5}-\frac{2ab\cos(fx+e)^5}{5}+\frac{b^2\cos(fx+e)^5}{5}-\frac{2a^2\cos(fx+e)^3}{3}+\frac{2ab\cos(fx+e)^3}{3}+\cos(fx+e)a^2+2ab\cos(fx+e)}{(a^2-2ab+b^2)(a-b)^2}+\frac{ab\left(-\frac{}{2(}\right.}{}}{f}$
risch	$-\frac{5\,e^{3i(fx+e)}a}{96(-a+b)^3f}-\frac{e^{3i(fx+e)}b}{32(-a+b)^3f}-\frac{5\,e^{i(fx+e)}a^2}{16f(a^2-2ab+b^2)(a-b)^2}-\frac{9\,e^{i(fx+e)}ab}{8f(a^2-2ab+b^2)(a-b)^2}-\frac{e^{i(fx+e)}b^2}{16f(a^2-2ab+b^2)(a-b)}$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a^2-2*a*b+b^2)/(a-b)^2*(1/5*a^2*cos(f*x+e)^5-2/5*a*b*cos(f*x+e)^5
+1/5*b^2*cos(f*x+e)^5-2/3*a^2*cos(f*x+e)^3+2/3*a*b*cos(f*x+e)^3+cos(f*x+e)
*a^2+2*a*b*cos(f*x+e))+a*b/(a-b)^4*(-1/2*a*cos(f*x+e)/(a*cos(f*x+e)^2-b*co
s(f*x+e)^2+b)+1/2*(3*a+4*b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-
b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.66

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$

$$= \left[-\frac{12\,(a^3-3\,a^2b+3\,ab^2-b^3)\cos(fx+e)^7-4\,(10\,a^3-23\,a^2b+16\,ab^2-3\,b^3)\cos(fx+e)^5+20\,(3\,a^3}{60\,((a^5-5\,a^4b+}\right.$$

$$-\frac{6\,(a^3-3\,a^2b+3\,ab^2-b^3)\cos(fx+e)^7-2\,(10\,a^3-23\,a^2b+16\,ab^2-3\,b^3)\cos(fx+e)^5+10\,(3\,a^3+}{30\,((a^5-5\,a^4b+10\,a^3b^2-}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/60*(12*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 4*(10*a^3 - 23
*a^2*b + 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 20*(3*a^3 + a^2*b - 4*a*b^2)*c
os(f*x + e)^3 - 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x
+ e)^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(
a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(3*a^2*b + 4*
a*b^2)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
 b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f
), -1/30*(6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(10*a^3 - 2
3*a^2*b + 16*a*b^2 - 3*b^3)*cos(f*x + e)^5 + 10*(3*a^3 + a^2*b - 4*a*b^2)*
cos(f*x + e)^3 + 15*(3*a^2*b + 4*a*b^2 + (3*a^3 + a^2*b - 4*a*b^2)*cos(f*x
 + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) +
 15*(3*a^2*b + 4*a*b^2)*cos(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^
2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 -
 4*a*b^4 + b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(146) = 292$.

Time = 0.67 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.27

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/30*(15*(3*a^2*b + 4*a*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b
)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^4 - 4*a^3*b + 6*a^
2*b^2 - 4*a*b^3 + b^4)*sqrt(a*b - b^2)) + 30*(a^2*b + a^2*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 2*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(
(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a + 2*a*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2)) - 4*(8*a^2 + 34*a*b + 3*b^2 - 40*a^2*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) - 140*a*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 30*b^2*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a
*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*(cos(f*x + e) - 1)^4
/(cos(f*x + e) + 1)^4)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*((cos(
f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 11.93 (sec) , antiderivative size = 1049, normalized size of antiderivative = 6.48

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
- ((6*a*b^2 + 83*a^2*b + 16*a^3)/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)) + (tan(e/2 + (f*x)/2)^8*(366*a*b^2 - 83*a^2*b + 32*a^3))/(3*(a - b)*(3
*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^4*(1336*a*b^2 + 223*a
^2*b + 16*a^3))/(15*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (2*tan(e/2
+ (f*x)/2)^10*(11*a*b^2 + 6*a^2*b + 4*b^3))/((a - b)*(3*a*b^2 - 3*a^2*b +
a^3 - b^3)) + (4*tan(e/2 + (f*x)/2)^6*(73*a*b^2 + 32*a^2*b - 12*a^3 + 12*b
^3))/(3*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (2*tan(e/2 + (f*x)/2)^2
*(145*a*b^2 + 134*a^2*b + 24*a^3 + 12*b^3))/(15*(a - b)*(3*a*b^2 - 3*a^2*b
 + a^3 - b^3)) + (a*tan(e/2 + (f*x)/2)^12*(3*a*b + 4*b^2))/((a - b)*(3*a*b
^2 - 3*a^2*b + a^3 - b^3)))/(f*(a + tan(e/2 + (f*x)/2)^4*(a + 20*b) + tan(
e/2 + (f*x)/2)^10*(a + 20*b) + tan(e/2 + (f*x)/2)^2*(3*a + 4*b) + tan(e/2
+ (f*x)/2)^12*(3*a + 4*b) - tan(e/2 + (f*x)/2)^6*(5*a - 40*b) - tan(e/2 +
(f*x)/2)^8*(5*a - 40*b) + a*tan(e/2 + (f*x)/2)^14)) - (a*b^(1/2)*atan(((a
- b)^9*(tan(e/2 + (f*x)/2)^2*((b^(1/2)*(3*a + 4*b)*(24*a^12*b + 32*a^3*b^1
0 - 232*a^4*b^9 + 704*a^5*b^8 - 1120*a^6*b^7 + 896*a^7*b^6 - 112*a^8*b^5 -
 448*a^9*b^4 + 416*a^10*b^3 - 160*a^11*b^2))/(4*(a - b)^(17/2)) - (a*b^(1/
2)*(a - 2*b)*(3*a + 4*b)^2*(224*a^14*b - 16*a^15 + 32*a^2*b^13 - 400*a^3*b
^12 + 2304*a^4*b^11 - 8096*a^5*b^10 + 19360*a^6*b^9 - 33264*a^7*b^8 + 4224
0*a^8*b^7 - 40128*a^9*b^6 + 28512*a^10*b^5 - 14960*a^11*b^4 + 5632*a^12*b^
3 - 1440*a^13*b^2))/(32*(a - b)^(27/2))) - (a*b^(1/2)*(a - 2*b)*(3*a + ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1032, normalized size of antiderivative = 6.37

$$\int \frac{\sin^5(e + fx)}{(a + b\tan^2(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(45*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**2*a**3 + 15*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqr
t(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2*b - 60*sqrt(b)*sqrt(a
 - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*
*2*a*b**2 - 45*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*
x)/2))/sqrt(b))*a**3 - 60*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*a**2*b + 45*sqrt(b)*sqrt(a - b)*atan((sqrt(a -
b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**3 + 15*sqrt(b)*
sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e +
 f*x)**2*a**2*b - 60*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((
e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b**2 - 45*sqrt(b)*sqrt(a - b)*atan
((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**3 - 60*sqrt(b)*sqrt(
a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**2*b - 6*c
os(e + f*x)*sin(e + f*x)**6*a**4 + 24*cos(e + f*x)*sin(e + f*x)**6*a**3*b
- 36*cos(e + f*x)*sin(e + f*x)**6*a**2*b**2 + 24*cos(e + f*x)*sin(e + f*x)
**6*a*b**3 - 6*cos(e + f*x)*sin(e + f*x)**6*b**4 - 2*cos(e + f*x)*sin(e +
f*x)**4*a**4 - 6*cos(e + f*x)*sin(e + f*x)**4*a**3*b + 30*cos(e + f*x)*sin
(e + f*x)**4*a**2*b**2 - 34*cos(e + f*x)*sin(e + f*x)**4*a*b**3 + 12*cos(e
 + f*x)*sin(e + f*x)**4*b**4 - 8*cos(e + f*x)*sin(e + f*x)**2*a**4 - 40*co
s(e + f*x)*sin(e + f*x)**2*a**3*b + 98*cos(e + f*x)*sin(e + f*x)**2*a**...
```

3.69 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} - \frac{(a+b)\cos(e+fx)}{(a-b)^3 f} + \frac{\cos^3(e+fx)}{3(a-b)^2 f} - \frac{ab\sec(e+fx)}{2(a-b)^3 f\left(a-b+b\sec^2(e+fx)\right)}$$

output
```
-1/2*b^(1/2)*(3*a+2*b)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(7/2)/
f-(a+b)*cos(f*x+e)/(a-b)^3/f+1/3*cos(f*x+e)^3/(a-b)^2/f-1/2*a*b*sec(f*x+e)
/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.37

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{6\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}} + \frac{6\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}} - \frac{\cos(e+fx)\left(9a+15b+\frac{12ab}{a+b+(a-b)\cos(2(e\cdots}\right.}{(a-b)^3}}{12f}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]])/(a - b)^(7/2) + (6*Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[a - b] + Sqrt[
a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(7/2) - (Cos[e + f*x]*(9*a + 15*b +
 (12*a*b)/(a + b + (a - b)*Cos[2*(e + f*x)])) + (-a + b)*Cos[3*(e + f*x)])
/(a - b)^3)/(12*f)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4147, 25, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{1}{2}b\int -\frac{\cos^4(e+fx)\left(\frac{a\sec^4(e+fx)}{(a-b)^3}-\frac{2a\sec^2(e+fx)}{(a-b)^2b}+\frac{2}{(a-b)b}\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)-\frac{ab\sec(e+fx)}{2(a-b)^3\left(a+b\sec^2(e+fx)-b\right)}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{1}{2}b\int \frac{\cos^4(e+fx)\left(\frac{a\sec^4(e+fx)}{(a-b)^3}-\frac{2a\sec^2(e+fx)}{(a-b)^2b}+\frac{2}{(a-b)b}\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)-\frac{ab\sec(e+fx)}{2(a-b)^3(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \ 1584$$

$$\frac{-\frac{1}{2}b\int \left(\frac{2\cos^4(e+fx)}{(a-b)^2b}+\frac{2(a+b)\cos^2(e+fx)}{b(b-a)^3}+\frac{3a+2b}{(a-b)^3(b\sec^2(e+fx)+a-b)}\right)d\sec(e+fx)-\frac{ab\sec(e+fx)}{2(a-b)^3(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{2}b\left(\frac{(3a+2b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{\sqrt{b}(a-b)^{7/2}}-\frac{2\cos^3(e+fx)}{3b(a-b)^2}+\frac{2(a+b)\cos(e+fx)}{b(a-b)^3}\right)-\frac{ab\sec(e+fx)}{2(a-b)^3(a+b\sec^2(e+fx)-b)}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/2*(b*(((3*a + 2*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/((a - b
)^(7/2)*Sqrt[b]) + (2*(a + b)*Cos[e + f*x])/((a - b)^3*b) - (2*Cos[e + f*x
]^3)/(3*(a - b)^2*b))) - (a*b*Sec[e + f*x])/(2*(a - b)^3*(a - b + b*Sec[e
+ f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 15.54 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\frac{b\cos(fx+e)^3}{3}-a\cos(fx+e)-\cos(fx+e)b}{\left(a^2-2ab+b^2\right)(a-b)}+\frac{b\left(-\frac{a\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{(3a+2b)\arctan\left(\frac{(a-b)\cos(fx}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{(a-b)^3}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\frac{b\cos(fx+e)^3}{3}-a\cos(fx+e)-\cos(fx+e)b}{\left(a^2-2ab+b^2\right)(a-b)}+\frac{b\left(-\frac{a\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{(3a+2b)\arctan\left(\frac{(a-b)\cos(fx}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{(a-b)^3}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24(a^2-2ab+b^2)f}-\frac{3\,e^{i(fx+e)}a}{8f(a^2-2ab+b^2)(a-b)}-\frac{5\,e^{i(fx+e)}b}{8f(a^2-2ab+b^2)(a-b)}-\frac{3\,e^{-i(fx+e)}a}{8(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{5\,e^{-i(f}}{8(a^3-3a^2b+}$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*cos(f*x+e)^3-1/3*b*cos(f*x+e)^3-a*cos(
f*x+e)-cos(f*x+e)*b)+b/(a-b)^3*(-1/2*a*cos(f*x+e)/(a*cos(f*x+e)^2-b*cos(f*
x+e)^2+b)+1/2*(3*a+2*b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^
(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.43

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{4\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-4\left(3\,a^2-ab-2\,b^2\right)\cos\left(fx+e\right)^3-3\left(\left(3\,a^2-ab-2\,b^2\right)\cos\left(fx+e\right)^2\right.}{12\left(\left(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4\right)f\cos\left(f\right.\right.}\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(3*a^2 - a*b - 2*b^2)*cos(
f*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos
(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a*b + 2*b^2)*cos(f*x +
 e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*
b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/6*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)
^5 - 2*(3*a^2 - a*b - 2*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - a*b - 2*b^2)*cos
(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b
))*cos(f*x + e)/b) - 3*(3*a*b + 2*b^2)*cos(f*x + e))/((a^4 - 4*a^3*b + 6*a
^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(119) = 238$.

Time = 0.59 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.66

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{a^4f^{11}\cos(fx+e)^3 - 4\,a^3bf^{11}\cos(fx+e)^3 + 6\,a^2b^2f^{11}\cos(fx+e)^3 - 4\,ab^3f^{11}\cos(fx+e)^3 + b^4f^{11}\mathrm{co}}{3\,(a^6f^{12} - 6\,a^5bf^{12} + 15\,a^4b^2f^{12} - 20\,a^3}$$
$$- \frac{ab\cos(fx+e)}{2\,(a^3 - 3\,a^2b + 3\,ab^2 - b^3)\big(a\cos(fx+e)^2 - b\cos(fx+e)^2 + b\big)f}$$
$$+ \frac{(3\,ab + 2\,b^2)\arctan\left(\frac{a\cos(fx+e) - b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{2\,(a^3 - 3\,a^2b + 3\,ab^2 - b^3)\sqrt{ab-b^2}\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(a^4*f^11*cos(f*x + e)^3 - 4*a^3*b*f^11*cos(f*x + e)^3 + 6*a^2*b^2*f^1
1*cos(f*x + e)^3 - 4*a*b^3*f^11*cos(f*x + e)^3 + b^4*f^11*cos(f*x + e)^3 -
 3*a^4*f^11*cos(f*x + e) + 6*a^3*b*f^11*cos(f*x + e) - 6*a*b^3*f^11*cos(f*
x + e) + 3*b^4*f^11*cos(f*x + e))/(a^6*f^12 - 6*a^5*b*f^12 + 15*a^4*b^2*f^
12 - 20*a^3*b^3*f^12 + 15*a^2*b^4*f^12 - 6*a*b^5*f^12 + b^6*f^12) - 1/2*a*
b*cos(f*x + e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a*cos(f*x + e)^2 - b*cos(
f*x + e)^2 + b)*f) + 1/2*(3*a*b + 2*b^2)*arctan((a*cos(f*x + e) - b*cos(f*
x + e))/sqrt(a*b - b^2))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b - b^2)*
f)
```

Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.54

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2*((b^(1/2)*(3*a + 2*b)*(24*a^9*b + 16*
a^2*b^8 - 72*a^3*b^7 + 96*a^4*b^6 + 40*a^5*b^5 - 240*a^6*b^4 + 264*a^7*b^3
 - 128*a^8*b^2))/(4*a*(a - b)^(13/2)) + (b^(1/2)*(a - 2*b)*(3*a + 2*b)^2*(
16*a^12 - 176*a^11*b + 32*a^2*b^10 - 304*a^3*b^9 + 1296*a^4*b^8 - 3264*a^5
*b^7 + 5376*a^6*b^6 - 6048*a^7*b^5 + 4704*a^8*b^4 - 2496*a^9*b^3 + 864*a^1
0*b^2))/(32*a*(a - b)^(21/2))) + (b^(1/2)*(a - 2*b)*(3*a + 2*b)^2*(144*a^1
1*b - 16*a^12 + 16*a^3*b^9 - 144*a^4*b^8 + 576*a^5*b^7 - 1344*a^6*b^6 + 20
16*a^7*b^5 - 2016*a^8*b^4 + 1344*a^9*b^3 - 576*a^10*b^2))/(32*a*(a - b)^(2
1/2)))*(a - b)^7)/(12*a^3*b^8 - 4*a^2*b^9 - 9*a^10*b + 3*a^4*b^7 - 46*a^5*
b^6 + 45*a^6*b^5 + 24*a^7*b^4 - 67*a^8*b^3 + 42*a^9*b^2))*(3*a + 2*b))/(2*
f*(a - b)^(7/2)) - ((11*a*b + 4*a^2)/(3*(a - b)*(a^2 - 2*a*b + b^2)) + (ta
n(e/2 + (f*x)/2)^8*(3*a*b + 2*b^2))/((a - b)*(a^2 - 2*a*b + b^2)) + (2*tan
(e/2 + (f*x)/2)^6*(2*a^2 - 3*a*b + 11*b^2))/((a - b)*(a^2 - 2*a*b + b^2))
 + (2*tan(e/2 + (f*x)/2)^2*(9*a*b + 2*a^2 + 19*b^2))/(3*(a - b)*(a^2 - 2*a*
b + b^2)) + (2*tan(e/2 + (f*x)/2)^4*(22*a*b - 10*a^2 + 33*b^2))/(3*(a - b)
*(a^2 - 2*a*b + b^2)))/(f*(a + tan(e/2 + (f*x)/2)^2*(a + 4*b) + tan(e/2 +
(f*x)/2)^8*(a + 4*b) - tan(e/2 + (f*x)/2)^4*(2*a - 12*b) - tan(e/2 + (f*x)
/2)^6*(2*a - 12*b) + a*tan(e/2 + (f*x)/2)^10))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 818, normalized size of antiderivative = 6.15

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(9*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**2*a**2 - 3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(
a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b - 6*sqrt(b)*sqrt(a - b)*
atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*b**
2 - 9*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sq
rt(b))*a**2 - 6*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f
*x)/2))/sqrt(b))*a*b + 9*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*t
an((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2 - 3*sqrt(b)*sqrt(a - b)*ata
n((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b -
6*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*sin(e + f*x)**2*b**2 - 9*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a
)*tan((e + f*x)/2))/sqrt(b))*a**2 - 6*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b
) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a*b - 2*cos(e + f*x)*sin(e + f*x)**
4*a**3 + 6*cos(e + f*x)*sin(e + f*x)**4*a**2*b - 6*cos(e + f*x)*sin(e + f*
x)**4*a*b**2 + 2*cos(e + f*x)*sin(e + f*x)**4*b**3 - 2*cos(e + f*x)*sin(e
+ f*x)**2*a**3 - 4*cos(e + f*x)*sin(e + f*x)**2*a**2*b + 14*cos(e + f*x)*s
in(e + f*x)**2*a*b**2 - 8*cos(e + f*x)*sin(e + f*x)**2*b**3 + 4*cos(e + f*
x)*a**3 + 7*cos(e + f*x)*a**2*b - 11*cos(e + f*x)*a*b**2 + 4*sin(e + f*x)*
*2*a**3 + 3*sin(e + f*x)**2*a**2*b - 18*sin(e + f*x)**2*a*b**2 + 11*sin(e
+ f*x)**2*b**3 - 4*a**3 - 7*a**2*b + 11*a*b**2)/(6*f*(sin(e + f*x)**2*a...
```

3.70 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} - \frac{3\cos(e+fx)}{2(a-b)^2f} + \frac{\cos(e+fx)}{2(a-b)f\left(a-b+b\sec^2(e+fx)\right)}$$

output
```
-3/2*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(5/2)/f-3/2*cos(
f*x+e)/(a-b)^2/f+1/2*cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{2\cos(e+fx)\left(-1-\frac{b}{a+b+(a-b)\cos(2(e+fx))}\right)}{(a-b)^2}}{2f}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((3*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a -
 b)^(5/2) + (3*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqr
t[b]])/(a - b)^(5/2) + (2*Cos[e + f*x]*(-1 - b/(a + b + (a - b)*Cos[2*(e +
 f*x)])))/(a - b)^2)/(2*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4147, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{3\int \frac{\cos^2(e+fx)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{2(a-b)}+\frac{\cos(e+fx)}{2(a-b)\left(a+b\sec^2(e+fx)-b\right)}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{3\left(-\frac{b\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{a-b}\right)}{2(a-b)}+\frac{\cos(e+fx)}{2(a-b)\left(a+b\sec^2(e+fx)-b\right)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{\cos(e+fx)}{a-b}\right)}{2(a-b)}+\frac{\cos(e+fx)}{2(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((3*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a - b)^(3/2))
 - Cos[e + f*x]/(a - b)))/(2*(a - b)) + Cos[e + f*x]/(2*(a - b)*(a - b + b
*Sec[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)}{a^2-2ab+b^2}+\frac{b\left(-\frac{\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{3\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{(a-b)^2}}{f}$
default	$\frac{-\frac{\cos(fx+e)}{a^2-2ab+b^2}+\frac{b\left(-\frac{\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{3\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{(a-b)^2}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2(a^2-2ab+b^2)f}-\frac{e^{-i(fx+e)}}{2(a^2-2ab+b^2)f}+\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{f(-a+b)^2\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-cos(f*x+e)/(a^2-2*a*b+b^2)+b/(a-b)^2*(-1/2*cos(f*x+e)/(a*cos(f*x+e)^
2-b*cos(f*x+e)^2+b)+3/2/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^
(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.04

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{4\,(a-b)\cos\left(fx+e\right)^3 - 3\,\left((a-b)\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos(fx+e)^2+2\,(a-b)\sqrt{-\frac{b}{a-b}}\cos(fx+e}{(a-b)\cos(fx+e)^2+b}\right.}{4\,\left((a^3-3\,a^2b+3\,ab^2-b^3)f\cos\left(fx+e\right)^2+(a^2b-2\,ab^2+b^3)f\right)}\right.$$

$$-\frac{2\,(a-b)\cos\left(fx+e\right)^3+3\,\left((a-b)\cos\left(fx+e\right)^2+b\right)\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right)+3\,b\cos\left(f\right.}{2\,\left((a^3-3\,a^2b+3\,ab^2-b^3)f\cos\left(fx+e\right)^2+(a^2b-2\,ab^2+b^3)f\right)}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(a - b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(-b/(
a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x +
 e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 6*b*cos(f*x + e))/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f), -1/2*(2*(
a - b)*cos(f*x + e)^3 + 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(b/(a - b))*arc
tan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 3*b*cos(f*x + e))/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f)]
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx &= -\frac{f^3\cos(fx+e)}{a^2f^4-2\,abf^4+b^2f^4}+\frac{3\,b\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{2\,(a^2-2\,ab+b^2)\sqrt{ab-b^2}f}\\&\quad-\frac{b\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)(a^2-2\,ab+b^2)f}\end{aligned}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-f^3*cos(f*x + e)/(a^2*f^4 - 2*a*b*f^4 + b^2*f^4) + 3/2*b*arctan((a*cos(f*
x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^2 - 2*a*b + b^2)*sqrt(a*b -
b^2)*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)*(a
^2 - 2*a*b + b^2)*f)
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 436, normalized size of antiderivative = 4.32

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= -\frac{\frac{2a+b}{(a-b)^2}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(2a^2-ab+2b^2\right)}{a\left(a^2-2ab+b^2\right)}+\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(-2a^2+4ab+b^2\right)}{a\,(a-b)^2}}{f\,\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+(4b-a)\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+(4b-a)\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a\right)}$$
$$-\frac{3\sqrt{b}\,\mathrm{atan}\left(\frac{(a-b)^5\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{\sqrt{b}\left(18a^6b-72a^5b^2+108a^4b^3-72a^3b^4+18a^2b^5\right)}{a\,(a-b)^{9/2}}-\frac{9\sqrt{b}\,(a-2b)\left(-16a^9+128a^8b-432a^7b^2+800a^6b^3-\right.}{32\,a\,(a-b)^{15/}}\right.\right.}{9a^6b-36a^5b^2+54a^4b^3}\right.}{2f\,(a-b)^{5/2}}$$

input
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
- ((2*a + b)/(a - b)^2 + (tan(e/2 + (f*x)/2)^4*(2*a^2 - a*b + 2*b^2))/(a*(
a^2 - 2*a*b + b^2)) + (2*tan(e/2 + (f*x)/2)^2*(4*a*b - 2*a^2 + b^2))/(a*(a
 - b)^2))/(f*(a - tan(e/2 + (f*x)/2)^2*(a - 4*b) - tan(e/2 + (f*x)/2)^4*(a
 - 4*b) + a*tan(e/2 + (f*x)/2)^6)) - (3*b^(1/2)*atan(((a - b)^5*(tan(e/2 +
 (f*x)/2)^2*((b^(1/2)*(18*a^6*b + 18*a^2*b^5 - 72*a^3*b^4 + 108*a^4*b^3 -
72*a^5*b^2))/(a*(a - b)^(9/2)) - (9*b^(1/2)*(a - 2*b)*(128*a^8*b - 16*a^9
+ 32*a^2*b^7 - 208*a^3*b^6 + 576*a^4*b^5 - 880*a^5*b^4 + 800*a^6*b^3 - 432
*a^7*b^2))/(32*a*(a - b)^(15/2))) - (9*b^(1/2)*(a - 2*b)*(16*a^9 - 96*a^8*
b + 16*a^3*b^6 - 96*a^4*b^5 + 240*a^5*b^4 - 320*a^6*b^3 + 240*a^7*b^2))/(3
2*a*(a - b)^(15/2))))/(9*a^6*b + 9*a^2*b^5 - 36*a^3*b^4 + 54*a^4*b^3 - 36*
a^5*b^2)))/(2*f*(a - b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 544, normalized size of antiderivative = 5.39

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx+e)^2\,a^3-3\sqrt{b}\,\sqrt{a-b}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin(fx}{}$$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**2*a**3 - 3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(
a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2*b - 3*sqrt(b)*sqrt(a -
b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**3 + 3*sqrt(b)
*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
+ f*x)**2*a**3 - 3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e
+ f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2*b - 3*sqrt(b)*sqrt(a - b)*atan((s
qrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**3 - 2*cos(e + f*x)*sin(
e + f*x)**2*a**4 + 4*cos(e + f*x)*sin(e + f*x)**2*a**3*b - 2*cos(e + f*x)*
sin(e + f*x)**2*a**2*b**2 + 2*cos(e + f*x)*a**4 - cos(e + f*x)*a**3*b - co
s(e + f*x)*a**2*b**2 + 5*sin(e + f*x)**2*a**3*b - 12*sin(e + f*x)**2*a**2*
b**2 + 9*sin(e + f*x)**2*a*b**3 - 2*sin(e + f*x)**2*b**4 - 5*a**3*b + 7*a*
*2*b**2 - 2*a*b**3)/(2*a**2*f*(sin(e + f*x)**2*a**4 - 4*sin(e + f*x)**2*a*
*3*b + 6*sin(e + f*x)**2*a**2*b**2 - 4*sin(e + f*x)**2*a*b**3 + sin(e + f*
x)**2*b**4 - a**4 + 3*a**3*b - 3*a**2*b**2 + a*b**3))
```

3.71 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{a^2f} - \frac{b\sec(e+fx)}{2a(a-b)f\left(a-b+b\sec^2(e+fx)\right)}$$

output
```
-1/2*(3*a-2*b)*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/a^2/(a-b)^(3
/2)/f-arctanh(cos(f*x+e))/a^2/f-1/2*b*sec(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+
e)^2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}}+\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}}-\frac{2ab\cos(e+fx)}{(a-b)(a+b+(a-b)\cos(2(e+fx)))}-2\mathrm{l}}{2a^2f}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt
[b]])/(a - b)^(3/2) + ((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*T
an[(e + f*x)/2])/Sqrt[b]])/(a - b)^(3/2) - (2*a*b*Cos[e + f*x])/((a - b)*(
a + b + (a - b)*Cos[2*(e + f*x)])) - 2*Log[Cos[(e + f*x)/2]] + 2*Log[Sin[(
e + f*x)/2]])/(2*a^2*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4147, 25, 316, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)\left(a+b\tan(e+fx)^2\right)^2}dx$$
$$\Big\downarrow \text{4147}$$
$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int -\frac{-b\sec^2(e+fx)+2a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}-\frac{b\sec(e+fx)}{2a(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{-b\sec^2(e+fx)+2a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}-\frac{b\sec(e+fx)}{2a(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{\frac{2(a-b)\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{b(3a-2b)\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}}{2a(a-b)}-\frac{b\sec(e+fx)}{2a(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\frac{2(a-b)\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{\sqrt{b}(3a-2b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a(a-b)}-\frac{b\sec(e+fx)}{2a(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\frac{\sqrt{b}(3a-2b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{2(a-b)\text{arctanh}(\sec(e+fx))}{a}}{2a(a-b)}-\frac{b\sec(e+fx)}{2a(a-b)(a+b\sec^2(e+fx)-b)}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/2*(((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a
*Sqrt[a - b]) + (2*(a - b)*ArcTanh[Sec[e + f*x]])/a)/(a*(a - b)) - (b*Sec[
e + f*x])/(2*a*(a - b)*(a - b + b*Sec[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a^2}+\frac{b\left(-\frac{a\cos(fx+e)}{2(a-b)\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{(3a-2b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2(a-b)\sqrt{b(a-b)}}\right)}{a^2}-\frac{\ln(\cos(fx+e)+1)}{2a^2}}{f}$
default	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a^2}+\frac{b\left(-\frac{a\cos(fx+e)}{2(a-b)\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{(3a-2b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2(a-b)\sqrt{b(a-b)}}\right)}{a^2}-\frac{\ln(\cos(fx+e)+1)}{2a^2}}{f}$
risch	$-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{a(-a+b)f\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{a^2 f}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{a^2 f}-$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/a^2*ln(cos(f*x+e)-1)+b/a^2*(-1/2*a/(a-b)*cos(f*x+e)/(a*cos(f*x+e)
^2-b*cos(f*x+e)^2+b)+1/2*(3*a-2*b)/(a-b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(
f*x+e)/(b*(a-b))^(1/2)))-1/2/a^2*ln(cos(f*x+e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(98) = 196.

Time = 0.16 (sec) , antiderivative size = 470, normalized size of antiderivative = 4.27

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{2\,ab\cos(fx+e)-\left((3\,a^2-5\,ab+2\,b^2)\cos(fx+e)^2+3\,ab-2\,b^2\right)\sqrt{-\frac{b}{a-b}}\log\left(\frac{(a-b)\cos(fx+e)^2+2\,(a-b}{(a-b)\cos(f}\right.}{}\right.$$

$$-\frac{ab\cos(fx+e)+\left((3\,a^2-5\,ab+2\,b^2)\cos(fx+e)^2+3\,ab-2\,b^2\right)\sqrt{\frac{b}{a-b}}\arctan\left(-\frac{(a-b)\sqrt{\frac{b}{a-b}}\cos(fx+e)}{b}\right.}{2\left((a^4-\right.}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*cos(f*x + e) - ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*
b - 2*b^2)*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-
b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*((a^2 - 2*a
*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) - 2*((a^
2 - 2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))
/((a^4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -1/2*
(a*b*cos(f*x + e) + ((3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 + 3*a*b - 2*b^
2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + ((a^2
 - 2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) -
((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*log(-1/2*cos(f*x + e) + 1
/2))/((a^4 - 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \int \frac{\csc{(e+fx)}}{\left(a+b\tan^2{(e+fx)}\right)^2}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(98) = 196$.

Time = 0.63 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.32

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{(3ab-2b^2)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{(a^3-a^2b)\sqrt{ab-b^2}} + \frac{2\left(ab+\frac{ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{(a^3-a^2b)\left(a+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)} - \frac{\log\left(\frac{|-\cos(}{|\cos(}\right.}{a}}{2f}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((3*a*b - 2*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(
a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^3 - a^2*b)*sqrt(a*b - b^2)
) + 2*(a*b + a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1))/((a^3 - a^2*b)*(a + 2*a*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2)) - log(abs(-cos(f*x + e) + 1)/abs(cos(f*x
 + e) + 1))/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 10.05 (sec) , antiderivative size = 1140, normalized size of antiderivative = 10.36

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^2),x)
```

output
```
log(tan(e/2 + (f*x)/2))/(a^2*f) - (b/(a*(a - b)) - (tan(e/2 + (f*x)/2)^2*(
a*b - 2*b^2))/(a^2*(a - b)))/(f*(a - tan(e/2 + (f*x)/2)^2*(2*a - 4*b) + a*
tan(e/2 + (f*x)/2)^4)) + (b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*
(3*a - 2*b)^3*(2*a^10 - 58*a^9*b + 96*a^4*b^6 - 432*a^5*b^5 + 772*a^6*b^4
- 686*a^7*b^3 + 306*a^8*b^2))/(8*a^6*(a - b)^(9/2)*(3*a^4*b - a^5 + a^2*b^
3 - 3*a^3*b^2)) + (2*b^(1/2)*(3*a - 2*b)*(108*a*b^5 + 9*a^5*b - 24*b^6 - 1
88*a^2*b^4 + 158*a^3*b^3 - 63*a^4*b^2))/(a^2*(a - b)^(3/2)*(3*a^4*b - a^5
+ a^2*b^3 - 3*a^3*b^2)))*(768*a*b^4 - 259*a^4*b + 27*a^5 - 192*b^5 - 1164*
a^2*b^3 + 820*a^3*b^2))/(2*a^5*(a - b)^(9/2)*(36*a*b^2 - 39*a^2*b + 16*a^3
 - 12*b^3)) - (((8*(4*b^4 - 12*a*b^3 + 9*a^2*b^2))/(3*a^4*b - a^5 + a^2*b^
3 - 3*a^3*b^2) - (b*(3*a - 2*b)^2*(2*a^8 - 35*a^7*b + 96*a^2*b^6 - 432*a^3
*b^5 + 746*a^4*b^4 - 611*a^5*b^3 + 234*a^6*b^2))/(2*a^4*(a - b)^3*(3*a^4*b
 - a^5 + a^2*b^3 - 3*a^3*b^2)))*(2*a^4 - 47*a^3*b - 240*a*b^3 + 96*b^4 + 1
86*a^2*b^2))/(a^5*b^(1/2)*(a - b)^3*(36*a*b^2 - 39*a^2*b + 16*a^3 - 12*b^3
))) + (((b^(1/2)*(3*a - 2*b)*(12*a^5*b - 20*a^2*b^4 + 60*a^3*b^3 - 53*a^4*
b^2))/(a^2*(a - b)^(3/2)*(a^5 - 2*a^4*b + a^3*b^2)) + (b^(3/2)*(3*a - 2*b)
^3*(4*a^10 - 24*a^9*b + 16*a^6*b^4 - 48*a^7*b^3 + 52*a^8*b^2))/(16*a^6*(a
- b)^(9/2)*(a^5 - 2*a^4*b + a^3*b^2)))*(768*a*b^4 - 259*a^4*b + 27*a^5 - 1
92*b^5 - 1164*a^2*b^3 + 820*a^3*b^2))/(2*a^5*(a - b)^(9/2)*(36*a*b^2 - 39*
a^2*b + 16*a^3 - 12*b^3)) - (((4*(4*b^4 - 12*a*b^3 + 9*a^2*b^2))/(a^5 -...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 725, normalized size of antiderivative = 6.59

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**2*a**2 - 5*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(
a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b + 2*sqrt(b)*sqrt(a - b)*
atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*b**
2 - 3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sq
rt(b))*a**2 + 2*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f
*x)/2))/sqrt(b))*a*b + 3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*t
an((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2 - 5*sqrt(b)*sqrt(a - b)*ata
n((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b +
2*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*sin(e + f*x)**2*b**2 - 3*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a
)*tan((e + f*x)/2))/sqrt(b))*a**2 + 2*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b
) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a*b + cos(e + f*x)*a**2*b - cos(e +
 f*x)*a*b**2 + 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**3 - 6*log(tan((e
 + f*x)/2))*sin(e + f*x)**2*a**2*b + 6*log(tan((e + f*x)/2))*sin(e + f*x)*
*2*a*b**2 - 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*b**3 - 2*log(tan((e +
f*x)/2))*a**3 + 4*log(tan((e + f*x)/2))*a**2*b - 2*log(tan((e + f*x)/2))*a
*b**2 + sin(e + f*x)**2*a**2*b - 2*sin(e + f*x)**2*a*b**2 + sin(e + f*x)**
2*b**3 - a**2*b + a*b**2)/(2*a**2*f*(sin(e + f*x)**2*a**3 - 3*sin(e + f*x)
**2*a**2*b + 3*sin(e + f*x)**2*a*b**2 - sin(e + f*x)**2*b**3 - a**3 + 2...
```

3.72 $\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^3\sqrt{a-b}f} - \frac{(a-4b)\text{arctanh}(\cos(e+fx))}{2a^3f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\,(a-b+b\sec^2(e+fx))} - \frac{b\sec(e+fx)}{a^2f\,(a-b+b\sec^2(e+fx))}$$

output
```
-1/2*(3*a-4*b)*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/a^3/(a-b)^(1
/2)/f-1/2*(a-4*b)*arctanh(cos(f*x+e))/a^3/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/
(a-b+b*sec(f*x+e)^2)-b*sec(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.48

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{4(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{\sqrt{a-b}}+\frac{4(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{\sqrt{a-b}}-\frac{8ab\cos(e+fx)}{a+b+(a-b)\cos(2(e+fx))}-a\csc^2}{8a^3}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((4*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sq
rt[b]])/Sqrt[a - b] + (4*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]
*Tan[(e + f*x)/2])/Sqrt[b]])/Sqrt[a - b] - (8*a*b*Cos[e + f*x])/(a + b + (
a - b)*Cos[2*(e + f*x)]) - a*Csc[(e + f*x)/2]^2 - 4*(a - 4*b)*Log[Cos[(e +
 f*x)/2]] + 4*(a - 4*b)*Log[Sin[(e + f*x)/2]] + a*Sec[(e + f*x)/2]^2)/(8*a
^3*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4147, 373, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)^2}dx$$
$$\Big\downarrow \ 4147$$

$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a-b)^2}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 373

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)}-\frac{\int \frac{-3b\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{2a}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)}-\frac{\frac{2b\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}-\frac{\int -\frac{2(a-b)\left(-2b\sec^2(e+fx)+a-2b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}}{2a}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)}-\frac{\frac{\int \frac{-2b\sec^2(e+fx)+a-2b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{a}+\frac{2b\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}}{2a}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)}-\frac{\frac{\frac{(a-4b)\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{b(3a-4b)\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}}{a}+\frac{2b\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}}{2a}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)}-\frac{\frac{\frac{(a-4b)\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{\sqrt{b}(3a-4b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{a}+\frac{2b\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}}{2a}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)}-\frac{\frac{\frac{\sqrt{b}(3a-4b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{(a-4b)\text{arctanh}(\sec(e+fx))}{a}}{a}+\frac{2b\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}}{2a}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(Sec[e + f*x]/(2*a*(1 - Sec[e + f*x]^2)*(a - b + b*Sec[e + f*x]^2)) - ((((
3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a*Sqrt[a -
 b]) + ((a - 4*b)*ArcTanh[Sec[e + f*x]])/a)/a + (2*b*Sec[e + f*x])/(a*(a -
 b + b*Sec[e + f*x]^2)))/(2*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{1}{4a^2(\cos(fx+e)-1)}+\frac{(a-4b)\ln(\cos(fx+e)-1)}{4a^3}+\frac{b\left(-\frac{a\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{(3a-4b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{a^3}+}{f}$
default	$\frac{\frac{1}{4a^2(\cos(fx+e)-1)}+\frac{(a-4b)\ln(\cos(fx+e)-1)}{4a^3}+\frac{b\left(-\frac{a\cos(fx+e)}{2\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)}+\frac{(3a-4b)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{a^3}+}{f}$
risch	$\frac{a\,e^{7i(fx+e)}-2b\,e^{7i(fx+e)}+3a\,e^{5i(fx+e)}+2b\,e^{5i(fx+e)}+3a\,e^{3i(fx+e)}+2b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}-2b\,e^{i(fx+e)}}{f\,a^2\left(e^{2i(fx+e)}-1\right)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)}-\frac{\ln\left(e^{i(f}\right.}{2a}$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4/a^2/(cos(f*x+e)-1)+1/4*(a-4*b)/a^3*ln(cos(f*x+e)-1)+b/a^3*(-1/2*a
*cos(f*x+e)/(a*cos(f*x+e)^2-b*cos(f*x+e)^2+b)+1/2*(3*a-4*b)/(b*(a-b))^(1/2
)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))+1/4/a^2/(cos(f*x+e)+1)+1/4/a^3
*(-a+4*b)*ln(cos(f*x+e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(133) = 266$.

Time = 0.15 (sec) , antiderivative size = 672, normalized size of antiderivative = 4.57

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) - ((3*a^2 - 7*a*
b + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f*x + e)^2 - 3*a*
b + 4*b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(
-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - ((a^2 - 5*a*
b + 4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4
*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4
- (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log(-1/2*cos(f*x + e
) + 1/2))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*co
s(f*x + e)^2), 1/4*(2*(a^2 - 2*a*b)*cos(f*x + e)^3 + 4*a*b*cos(f*x + e) -
2*((3*a^2 - 7*a*b + 4*b^2)*cos(f*x + e)^4 - (3*a^2 - 10*a*b + 8*b^2)*cos(f
*x + e)^2 - 3*a*b + 4*b^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))
*cos(f*x + e)/b) - ((a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b +
8*b^2)*cos(f*x + e)^2 - a*b + 4*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 -
 5*a*b + 4*b^2)*cos(f*x + e)^4 - (a^2 - 6*a*b + 8*b^2)*cos(f*x + e)^2 - a*
b + 4*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4 - a^3*b)*f*cos(f*x + e)^4 -
 a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx=\int \frac{\csc^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(133) = 266$.

Time = 0.63 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.65

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$=\frac{\frac{6\,(a-4\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3}-\frac{12\,\left(3\,ab-4\,b^2\right)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}a^3}-\frac{3\,(\cos(fx+e)-1)}{a^2(\cos(fx+e)+1)}+\frac{3\,a^2+\frac{4\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\cdots}{a^3\cdots}}{24\,f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/24*(6*(a - 4*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^3 -
12*(3*a*b - 4*b^2)*arctan(-(a*cos(f*x + e) - b*cos(f*x + e) - b)/(sqrt(a*b
 - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b - b^2)*a^3) - 3*(cos(f*
x + e) - 1)/(a^2*(cos(f*x + e) + 1)) + (3*a^2 + 4*a^2*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 28*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 16*b^2*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 - 2*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b*
(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/(a^3*(a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 4*b*(cos(
f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3)))/f
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 917, normalized size of antiderivative = 6.24

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^2),x)
```

output
```
tan(e/2 + (f*x)/2)^2/(8*a^2*f) - (a/2 - tan(e/2 + (f*x)/2)^2*(a - 6*b) + (
tan(e/2 + (f*x)/2)^4*(a^2 - 8*a*b + 16*b^2))/(2*a))/(f*(4*a^3*tan(e/2 + (f
*x)/2)^2 + 4*a^3*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^4*(16*a^2*b - 8
*a^3))) + (log(tan(e/2 + (f*x)/2))*(a - 4*b))/(2*a^3*f) + (b^(1/2)*atan((2
*a^2*((b^(1/2)*(3*a - 4*b)*(12*a^6*b - 160*a^3*b^4 + 240*a^4*b^3 - 106*a^5
*b^2))/(2*a^9*(a - b)^(1/2)) + (b^(3/2)*(3*a - 4*b)^3*(8*a^11 - 32*a^10*b
+ 32*a^9*b^2))/(32*a^15*(a - b)^(3/2)))*(a - b)*(15*a^4 - 182*a^3*b - 864*
a*b^3 + 384*b^4 + 648*a^2*b^2))/((9*a^2*b - 24*a*b^2 + 16*b^3)*(72*a*b^2 -
 27*a^2*b + 4*a^3 - 48*b^3)) - (4*a^7*tan(e/2 + (f*x)/2)^2*(a - b)^(3/2)*(
(((4*(16*b^4 - 24*a*b^3 + 9*a^2*b^2))/a^5 - (b*(3*a - 4*b)^2*(2*a^8 - 46*a
^7*b + 384*a^4*b^4 - 672*a^5*b^3 + 344*a^6*b^2))/(4*a^11*(a - b)))*(a^4 -
31*a^3*b - 336*a*b^3 + 192*b^4 + 180*a^2*b^2))/(b^(1/2)*(b*(27*a^7 + b*(48
*a^5*b - 72*a^6)) - 4*a^8)) + (((b^(1/2)*(3*a - 4*b)*(192*a*b^5 + 9*a^5*b
- 384*a^2*b^4 + 268*a^3*b^3 - 78*a^4*b^2))/(a^8*(a - b)^(1/2)) - (b^(3/2)*
(3*a - 4*b)^3*(104*a^9*b - 4*a^10 + 192*a^7*b^3 - 288*a^8*b^2))/(16*a^14*(
a - b)^(3/2)))*(15*a^4 - 182*a^3*b - 864*a*b^3 + 384*b^4 + 648*a^2*b^2))/(
2*a^5*(a - b)^(1/2)*(72*a*b^2 - 27*a^2*b + 4*a^3 - 48*b^3))))/(9*a^2*b - 2
4*a*b^2 + 16*b^3) + (4*a^7*(a - b)^(3/2)*((2*(112*a*b^4 - 64*b^5 - 60*a^2*
b^3 + 9*a^3*b^2))/a^6 + (b*(3*a - 4*b)^2*(56*a^8*b - 4*a^9 + 128*a^6*b^3 -
 160*a^7*b^2))/(8*a^12*(a - b)))*(a^4 - 31*a^3*b - 336*a*b^3 + 192*b^4 ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 866, normalized size of antiderivative = 5.89

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

output

```
(6*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**4*a**2 - 14*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt
(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b + 8*sqrt(b)*sqrt(a - b)
*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*b*
*2 - 6*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/s
qrt(b))*sin(e + f*x)**2*a**2 + 8*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - s
qrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b + 6*sqrt(b)*sqrt(a -
 b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4
*a**2 - 14*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2
))/sqrt(b))*sin(e + f*x)**4*a*b + 8*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b)
+ sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*b**2 - 6*sqrt(b)*sqrt
(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x
)**2*a**2 + 8*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x
)/2))/sqrt(b))*sin(e + f*x)**2*a*b - 2*cos(e + f*x)*sin(e + f*x)**2*a**3 +
 6*cos(e + f*x)*sin(e + f*x)**2*a**2*b - 4*cos(e + f*x)*sin(e + f*x)**2*a*
b**2 + 2*cos(e + f*x)*a**3 - 2*cos(e + f*x)*a**2*b + 2*log(tan((e + f*x)/2
))*sin(e + f*x)**4*a**3 - 12*log(tan((e + f*x)/2))*sin(e + f*x)**4*a**2*b
+ 18*log(tan((e + f*x)/2))*sin(e + f*x)**4*a*b**2 - 8*log(tan((e + f*x)/2)
)*sin(e + f*x)**4*b**3 - 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**3 + 10
*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**2*b - 8*log(tan((e + f*x)/2))...
```

3.73 $\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 210

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{3(a-2b)\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{2a^4f} - \frac{3(a^2-8ab+8b^2)\operatorname{arctanh}(\cos(e+fx))}{8a^4f} - \frac{(5a-6b)\cot(e+fx)\csc(e+fx)}{8a^2f\,(a-b+b\sec^2(e+fx))} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\,(a-b+b\sec^2(e+fx))} - \frac{3(3a-4b)b\sec(e+fx)}{8a^3f\,(a-b+b\sec^2(e+fx))}$$

output
```
-3/2*(a-2*b)*(a-b)^(1/2)*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/a^
4/f-3/8*(a^2-8*a*b+8*b^2)*arctanh(cos(f*x+e))/a^4/f-1/8*(5*a-6*b)*cot(f*x+
e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(
a-b+b*sec(f*x+e)^2)-3/8*(3*a-4*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 6.63 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.87

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{3(a-2b)\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sec(\frac{1}{2}(e+fx))(\sqrt{a-b}\cos(\frac{1}{2}(e+fx))-\sqrt{a}\sin(\frac{1}{2}(e+fx)))}{\sqrt{b}}\right)}{2a^4f}$$
$$+\frac{3(a-2b)\sqrt{a-b}\sqrt{b}\arctan\left(\frac{\sec(\frac{1}{2}(e+fx))(\sqrt{a-b}\cos(\frac{1}{2}(e+fx))+\sqrt{a}\sin(\frac{1}{2}(e+fx)))}{\sqrt{b}}\right)}{2a^4f}$$
$$+\frac{-ab\cos(e+fx)+b^2\cos(e+fx)}{a^3f(a+b+a\cos(2(e+fx))-b\cos(2(e+fx)))}$$
$$+\frac{(-3a+8b)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^3f}-\frac{\csc^4\left(\frac{1}{2}(e+fx)\right)}{64a^2f}$$
$$-\frac{3(a^2-8ab+8b^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8a^4f}+\frac{3(a^2-8ab+8b^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^4f}$$
$$+\frac{(3a-8b)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32a^3f}+\frac{\sec^4\left(\frac{1}{2}(e+fx)\right)}{64a^2f}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(3*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos
[(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (3*(a - 2
*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x
)/2] + Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(2*a^4*f) + (-(a*b*Cos[e + f*x
]) + b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f
*x)])) + ((-3*a + 8*b)*Csc[(e + f*x)/2]^2)/(32*a^3*f) - Csc[(e + f*x)/2]^4
/(64*a^2*f) - (3*(a^2 - 8*a*b + 8*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^4*f) +
(3*(a^2 - 8*a*b + 8*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^4*f) + ((3*a - 8*b)*S
ec[(e + f*x)/2]^2)/(32*a^3*f) + Sec[(e + f*x)/2]^4/(64*a^2*f)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4147, 25, 372, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a-b)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a-b)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(4a-5b)\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int -\frac{3\left((a-2b)(a-b)-(5a-6b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{2a}+\frac{(5a-6b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{(5a-6b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\int\frac{(a-2b)(a-b)-(5a-6b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(5a-6b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\left(\frac{b(3a-4b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}-\frac{\int-\frac{2(a-b)\left((a-4b)(a-b)-(3a-4b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}\right)}{2a}}{4a}-\frac{\sec}{4a(1-\sec^2(e+fx)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{(5a-6b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\left(\frac{\int\frac{(a-4b)(a-b)-(3a-4b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{a}+\frac{b(3a-4b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}\right)}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{(5a-6b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\left(\frac{\frac{\left(a^2-8ab+8b^2\right)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{4b(a-2b)(a-b)\int\frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}}{a}+\frac{b(3a-4b)\sec(}{a\left(a+b\sec^2(e+\right.}\right.}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(5a-6b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\left(\frac{\frac{\left(a^2-8ab+8b^2\right)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{4\sqrt{b}(a-2b)\sqrt{a-b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a}}{a}+\frac{b(3a-4b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}\right.}{2a}}{4a}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{(5a-6b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)}-\frac{3\left(\frac{\frac{\left(a^2-8ab+8b^2\right)\text{arctanh}(\sec(e+fx))}{a}+\frac{4\sqrt{b}(a-2b)\sqrt{a-b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a}}{a}+\frac{b(3a-4b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}\right)}{2a}}{4a} -}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/4*Sec[e + f*x]/(a*(1 - Sec[e + f*x]^2)^2*(a - b + b*Sec[e + f*x]^2)) +
 (((5*a - 6*b)*Sec[e + f*x])/(2*a*(1 - Sec[e + f*x]^2)*(a - b + b*Sec[e +
f*x]^2)) - (3*(((4*(a - 2*b)*Sqrt[a - b]*Sqrt[b]*ArcTan[(Sqrt[b]*Sec[e + f
*x])/Sqrt[a - b]])/a + ((a^2 - 8*a*b + 8*b^2)*ArcTanh[Sec[e + f*x]])/a)/a
+ ((3*a - 4*b)*b*Sec[e + f*x])/(a*(a - b + b*Sec[e + f*x]^2))))/(2*a))/(4*
a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\left(-\frac{1}{2}a^2+\frac{1}{2}ab\right)\cos(fx+e)}{a\cos(fx+e)^2-b\cos(fx+e)^2+b}+\frac{3\left(a^2-3ab+2b^2\right)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{a^4}-\frac{1}{16a^2(\cos(fx+e)-1)^2}-\frac{-3a+8b}{16a^3(\cos(fx+e)-1)}-}{f}$
default	$\frac{\frac{b\left(\frac{\left(-\frac{1}{2}a^2+\frac{1}{2}ab\right)\cos(fx+e)}{a\cos(fx+e)^2-b\cos(fx+e)^2+b}+\frac{3\left(a^2-3ab+2b^2\right)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{2\sqrt{b(a-b)}}\right)}{a^4}-\frac{1}{16a^2(\cos(fx+e)-1)^2}-\frac{-3a+8b}{16a^3(\cos(fx+e)-1)}-}{f}$
risch	$\frac{3a^2e^{11i(fx+e)}-15ab\,e^{11i(fx+e)}+12b^2e^{11i(fx+e)}-5a^2e^{9i(fx+e)}+21ab\,e^{9i(fx+e)}-36b^2e^{9i(fx+e)}-30a^2e^{7i(fx+e)}-6ab\,e}{4f\,a^3\left(e^{2i(fx+e)}-\right.}$

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/a^4*((-1/2*a^2+1/2*a*b)*cos(f*x+e)/(a*cos(f*x+e)^2-b*cos(f*x+e)^2+b
)+3/2*(a^2-3*a*b+2*b^2)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^
(1/2)))-1/16/a^2/(cos(f*x+e)-1)^2-1/16*(-3*a+8*b)/a^3/(cos(f*x+e)-1)+1/16/
a^4*(3*a^2-24*a*b+24*b^2)*ln(cos(f*x+e)-1)+1/16/a^2/(cos(f*x+e)+1)^2-1/16*
(-3*a+8*b)/a^3/(cos(f*x+e)+1)+1/16/a^4*(-3*a^2+24*a*b-24*b^2)*ln(cos(f*x+e
)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(192) = 384.

Time = 0.18 (sec) , antiderivative size = 1052, normalized size of antiderivative = 5.01

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/16*(6*(a^3 - 5*a^2*b + 4*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b +
24*a*b^2)*cos(f*x + e)^3 - 12*((a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a
^2 - 7*a*b + 6*b^2)*cos(f*x + e)^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2
+ a*b - 2*b^2)*sqrt(-a*b + b^2)*log(((a - b)*cos(f*x + e)^2 - 2*sqrt(-a*b
+ b^2)*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) - 6*(3*a^2*b - 4*a*
b^2)*cos(f*x + e) - 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 -
 (2*a^3 - 19*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 +
 8*b^3 + (a^3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos(
f*x + e) + 1/2) + 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (
2*a^3 - 19*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8
*b^3 + (a^3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4
*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), 1/16*(6*(a^3 - 5
*a^2*b + 4*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 24*a^2*b + 24*a*b^2)*cos(f*x
 + e)^3 + 24*((a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^6 - (2*a^2 - 7*a*b + 6*b^
2)*cos(f*x + e)^4 + (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 + a*b - 2*b^2)*sq
rt(a*b - b^2)*arctan(sqrt(a*b - b^2)*cos(f*x + e)/b) - 6*(3*a^2*b - 4*a*b^
2)*cos(f*x + e) - 3*((a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^6 - (
2*a^3 - 19*a^2*b + 40*a*b^2 - 24*b^3)*cos(f*x + e)^4 + a^2*b - 8*a*b^2 + 8
*b^3 + (a^3 - 11*a^2*b + 32*a*b^2 - 24*b^3)*cos(f*x + e)^2)*log(1/2*cos...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(192) = 384$.

Time = 0.71 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.46

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/64*(12*(a^2 - 8*a*b + 8*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e)
 + 1))/a^4 - 96*(a^2*b - 3*a*b^2 + 2*b^3)*arctan(-(a*cos(f*x + e) - b*cos(
f*x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqrt(a*b
- b^2)*a^4) - (8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2)/a^4 - (a^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 - 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 144*b^
2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(a^4*(co
s(f*x + e) - 1)^2) - 64*(a^2*b - a*b^2 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 3*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*b^3*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1))/((a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2)*a^4))/f
```

Mupad [B] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 1113, normalized size of antiderivative = 5.30

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^2),x)
```

output
```
tan(e/2 + (f*x)/2)^4/(64*a^2*f) - (a^2/4 - tan(e/2 + (f*x)/2)^4*((15*a^2)/
4 - 32*a*b + 32*b^2) + (3*a*tan(e/2 + (f*x)/2)^2*(a - 2*b))/2 + (2*tan(e/2
 + (f*x)/2)^6*(24*a*b^2 - 10*a^2*b + a^3 - 16*b^3))/a)/(f*(16*a^4*tan(e/2
+ (f*x)/2)^4 + 16*a^4*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^6*(64*a^3*
b - 32*a^4))) + (tan(e/2 + (f*x)/2)^2*(a - 2*b))/(8*a^3*f) + (log(tan(e/2
+ (f*x)/2))*(3*a^2 - 24*a*b + 24*b^2))/(8*a^4*f) + (3*atan((8*a^10*tan(e/2
 + (f*x)/2)^2*((((756*a*b^6 - 216*b^7 - 1026*a^2*b^5 + 675*a^3*b^4 - 216*a
^4*b^3 + 27*a^5*b^2)/a^8 + (9*(a - 2*b)^2*(a*b - b^2)*(180*a^10*b - 6*a^11
 + 2304*a^6*b^5 - 5760*a^7*b^4 + 4944*a^8*b^3 - 1656*a^9*b^2))/(16*a^16))*
(960*a*b^4 - 38*a^4*b + a^5 - 384*b^5 - 840*a^2*b^3 + 300*a^3*b^2))/(2*a^5
*(b*(a - b))^(3/2)*(a^4 - 12*a^3*b - 96*a*b^3 + 48*b^4 + 60*a^2*b^2)) + ((
(27*(a - 2*b)^3*(a*b - b^2)^(3/2)*(416*a^12*b - 16*a^13 + 768*a^10*b^3 - 1
152*a^11*b^2))/(64*a^20) - (3*(a - 2*b)*(a*b - b^2)^(1/2)*(27*a^8*b + 1728
*a^2*b^7 - 6048*a^3*b^6 + 8352*a^4*b^5 - 5760*a^5*b^4 + 2070*a^6*b^3 - 369
*a^7*b^2))/(4*a^12))*(4*a^4 - 60*a^3*b - 384*a*b^3 + 192*b^4 + 252*a^2*b^2
))/(a^5*b*(144*a*b^4 - 13*a^4*b + a^5 - 48*b^5 - 156*a^2*b^3 + 72*a^3*b^2)
)))/(27*a^2 - 108*a*b + 108*b^2) + (8*a^5*((27*(a - 2*b)^3*(a*b - b^2)^(3/
2)*(32*a^14 - 128*a^13*b + 128*a^12*b^2))/(128*a^21) + (3*(a - 2*b)*(a*b -
 b^2)^(1/2)*(36*a^9*b - 1440*a^4*b^6 + 4320*a^5*b^5 - 4824*a^6*b^4 + 2448*
a^7*b^3 - 540*a^8*b^2))/(8*a^13))*(4*a^4 - 60*a^3*b - 384*a*b^3 + 192*b...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.16

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

output

```
(96*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**6*a**2 - 288*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sq
rt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**6*a*b + 192*sqrt(b)*sqrt(a
- b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**
6*b**2 - 96*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/
2))/sqrt(b))*sin(e + f*x)**4*a**2 + 192*sqrt(b)*sqrt(a - b)*atan((sqrt(a -
 b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b + 96*sqrt(b)*
sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e +
 f*x)**6*a**2 - 288*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e
 + f*x)/2))/sqrt(b))*sin(e + f*x)**6*a*b + 192*sqrt(b)*sqrt(a - b)*atan((s
qrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**6*b**2 - 96*
sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**4*a**2 + 192*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a
)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b - 24*cos(e + f*x)*sin(e +
 f*x)**4*a**3 + 120*cos(e + f*x)*sin(e + f*x)**4*a**2*b - 96*cos(e + f*x)*
sin(e + f*x)**4*a*b**2 + 8*cos(e + f*x)*sin(e + f*x)**2*a**3 - 48*cos(e +
f*x)*sin(e + f*x)**2*a**2*b + 16*cos(e + f*x)*a**3 + 24*log(tan((e + f*x)/
2))*sin(e + f*x)**6*a**3 - 216*log(tan((e + f*x)/2))*sin(e + f*x)**6*a**2*
b + 384*log(tan((e + f*x)/2))*sin(e + f*x)**6*a*b**2 - 192*log(tan((e + f*
x)/2))*sin(e + f*x)**6*b**3 - 24*log(tan((e + f*x)/2))*sin(e + f*x)**4*...
```

3.74 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 196

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{3(a^2+6ab+b^2)\,x}{8(a-b)^4} - \frac{3\sqrt{a}\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^4f} - \frac{(5a+b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))} - \frac{3b(3a+b)\tan(e+fx)}{8(a-b)^3f\,(a+b\tan^2(e+fx))}$$

output
```
3/8*(a^2+6*a*b+b^2)*x/(a-b)^4-3/2*a^(1/2)*b^(1/2)*(a+b)*arctan(b^(1/2)*tan
(f*x+e)/a^(1/2))/(a-b)^4/f-1/8*(5*a+b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+
b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)-3/8
*b*(3*a+b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.69

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{12(a^2+6ab+b^2)(e+fx)-48\sqrt{a}\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)-8(a-b)(a+b)\sin(2(e+fx))-\frac{1}{a}}{32(a-b)^4f}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(12*(a^2 + 6*a*b + b^2)*(e + f*x) - 48*Sqrt[a]*Sqrt[b]*(a + b)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sin[2*(e + f*x)] - (16*a*(
a - b)*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]) + (a - b)^2*
Sin[4*(e + f*x)])/(32*(a - b)^4*f)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4146, 372, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^4}{(a+b\tan(e+fx)^2)^2}dx$$
$$\Big\downarrow \text{4146}$$
$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\int \frac{a-(4a+b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4(a-b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\frac{(5a+b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)}-\frac{\int \frac{3\left(a(a+b)-b(5a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{2(a-b)}}{4(a-b)}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\frac{(5a+b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)}-\frac{3\int \frac{a(a+b)-b(5a+b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{2(a-b)}}{4(a-b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\frac{(5a+b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)}-\frac{3\left(\frac{\int \frac{2a\left(a(a+3b)-b(3a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b(}{(a-b}\right.}{2(a-b)}}{4(a-b)}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\frac{(5a+b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)}-\frac{3\left(\frac{\int \frac{a(a+3b)-b(3a+b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a-b}-\frac{b(}{(a-b}\right.}{2(a-b)}}{4(a-b)}}{f}$$

$\downarrow$ 397

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\frac{(5a+b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)}-\frac{3\left(\frac{\frac{\left(a^2+6ab+b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{4ab(a+b)\int}{}}{a-b}\right.}{2(a-}}{4(a-b)}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\frac{(5a+b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)}-\frac{3\left(\frac{\frac{\left(a^2+6ab+b^2\right)\arctan(\tan(e+fx))}{a-b}-\frac{4ab(a+b)\int\frac{1}{b\tan^2(e+}}{a-}}{a-b}\right.}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))}-\frac{\frac{(5a+b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)}-\frac{3\left(\frac{\frac{\left(a^2+6ab+b^2\right)\arctan(\tan(e+fx))}{a-b}-\frac{4\sqrt{a}\sqrt{b}(a+b)\arctan\left(\right.}{a-b}}{a-b}\right.}{2(a-b)}}{4(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(4*(a - b)*(1 + Tan[e + f*x]^2)^2*(a + b*Tan[e + f*x]^2)) -
(((5*a + b)*Tan[e + f*x])/(2*(a - b)*(1 + Tan[e + f*x]^2)*(a + b*Tan[e + f
*x]^2)) - (3*((((a^2 + 6*a*b + b^2)*ArcTan[Tan[e + f*x]])/(a - b) - (4*Sqr
t[a]*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a - b))/(a -
 b) - (b*(3*a + b)*Tan[e + f*x])/((a - b)*(a + b*Tan[e + f*x]^2))))/(2*(a
- b)))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 30.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{ab\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{3(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^4}+\frac{\frac{\left(-\frac{5}{8}a^2+\frac{1}{4}ab+\frac{3}{8}b^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2+\frac{5}{8}b^2-\frac{1}{4}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}\cdots}{(a-b)^4}}{f}$
default	$\frac{-\frac{ab\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{3(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^4}+\frac{\frac{\left(-\frac{5}{8}a^2+\frac{1}{4}ab+\frac{3}{8}b^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2+\frac{5}{8}b^2-\frac{1}{4}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}\cdots}{(a-b)^4}}{f}$
risch	$\frac{3x\,a^2}{8(a^2-2ab+b^2)(a-b)^2}+\frac{9xab}{4(a^2-2ab+b^2)(a-b)^2}+\frac{3x\,b^2}{8(a^2-2ab+b^2)(a-b)^2}-\frac{i\mathrm{e}^{4i(fx+e)}}{64(a-b)^2f}+\frac{i\mathrm{e}^{2i(fx+e)}a}{8(a-b)^3f}+\cdots$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*b/(a-b)^4*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+3/2*(a+b)/(
a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^4*(((-5/8*a^2+1/4*a*b
+3/8*b^2)*tan(f*x+e)^3+(-3/8*a^2+5/8*b^2-1/4*a*b)*tan(f*x+e))/(1+tan(f*x+e
)^2)^2+3/8*(a^2+6*a*b+b^2)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.60

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(3*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*
a*b^2 + b^3)*f*x + 3*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a*b)*l
og(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 +
4*((a + b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b)*sin(f*x + e) + b^2)
/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)
) + (2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b +
 3*a*b^2 + b^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e))
*sin(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f), 1/8
*(3*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 6*a*b^
2 + b^3)*f*x + 6*((a^2 - b^2)*cos(f*x + e)^2 + a*b + b^2)*sqrt(a*b)*arctan
(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(a*b)/(a*b*cos(f*x + e)*sin(f*x + e)
)) + (2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^5 - (5*a^3 - 9*a^2*b
+ 3*a*b^2 + b^3)*cos(f*x + e)^3 - 3*(3*a^2*b - 2*a*b^2 - b^3)*cos(f*x + e)
)*sin(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)
*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.59

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{3\,(a^2+6\,ab+b^2)(fx+e)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} - \frac{12\,(a^2b+ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)\sqrt{ab}} - \frac{3\,(3\,ab+b^2)\tan(fx+e)^5+(5\,a^2+14\,a}{(a^3b-3\,a^2b^2+3\,ab^3-b^4)\tan(fx+e)^6+(a^4-a^3b-3\,a^2b^2+5\,ab^3-2\,b^4)\,\mathrm{ta}}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/8*(3*(a^2 + 6*a*b + b^2)*(f*x + e)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
+ b^4) - 12*(a^2*b + a*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 4*a^3
*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a*b)) - (3*(3*a*b + b^2)*tan(f*x + e)
^5 + (5*a^2 + 14*a*b + 5*b^2)*tan(f*x + e)^3 + 3*(a^2 + 3*a*b)*tan(f*x + e
))/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*tan(f*x + e)^6 + (a^4 - a^3*b - 3*
a^2*b^2 + 5*a*b^3 - 2*b^4)*tan(f*x + e)^4 + a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3 + (2*a^4 - 5*a^3*b + 3*a^2*b^2 + a*b^3 - b^4)*tan(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx &= \frac{3\,(a^2+6\,ab+b^2)(fx+e)}{8\,(a^4f-4\,a^3bf+6\,a^2b^2f-4\,ab^3f+b^4f)} \\ &\quad - \frac{ab\tan(fx+e)}{2\,(a^3f-3\,a^2bf+3\,ab^2f-b^3f)\left(b\tan(fx+e)^2+a\right)} \\ &\quad - \frac{3\,(a^2b+ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,(a^4f-4\,a^3bf+6\,a^2b^2f-4\,ab^3f+b^4f)\sqrt{ab}} \\ &\quad - \frac{5\,a\tan(fx+e)^3+3\,b\tan(fx+e)^3+3\,a\tan(fx+e)+5\,b\tan(fx+e)}{8\,(a^3f-3\,a^2bf+3\,ab^2f-b^3f)\left(\tan(fx+e)^2+1\right)^2}\end{aligned}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
3/8*(a^2 + 6*a*b + b^2)*(f*x + e)/(a^4*f - 4*a^3*b*f + 6*a^2*b^2*f - 4*a*b
^3*f + b^4*f) - 1/2*a*b*tan(f*x + e)/((a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3
*f)*(b*tan(f*x + e)^2 + a)) - 3/2*(a^2*b + a*b^2)*arctan(b*tan(f*x + e)/sq
rt(a*b))/((a^4*f - 4*a^3*b*f + 6*a^2*b^2*f - 4*a*b^3*f + b^4*f)*sqrt(a*b))
 - 1/8*(5*a*tan(f*x + e)^3 + 3*b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 5*b*t
an(f*x + e))/((a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3*f)*(tan(f*x + e)^2 + 1)
^2)
```

Mupad [B] (verification not implemented)

Time = 12.21 (sec) , antiderivative size = 4616, normalized size of antiderivative = 23.55

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(atan((((((tan(e + f*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4 + 15
3*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 +
 15*a^4*b^2)) - (3*(((9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 210*a^
4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*b^3)/
2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*a^3*b
^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) - (3*tan(e + f*x
)*(a*b*6i + a^2*1i + b^2*1i)*(256*b^11 - 1792*a*b^10 + 5120*a^2*b^9 - 7168
*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b^4 - 179
2*a^8*b^3 + 256*a^9*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)
*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*
(a*b*6i + a^2*1i + b^2*1i))/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2
)))*(a*b*6i + a^2*1i + b^2*1i)*3i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*
a^2*b^2)) + (((tan(e + f*x)*(108*a*b^6 + 9*b^7 + 486*a^2*b^5 + 396*a^3*b^4
 + 153*a^4*b^3))/(32*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*
b^3 + 15*a^4*b^2)) + (3*(((9*a*b^11)/2 - (69*a^2*b^10)/2 + 114*a^3*b^9 - 2
10*a^4*b^8 + 231*a^5*b^7 - 147*a^6*b^6 + 42*a^7*b^5 + 6*a^8*b^4 - (15*a^9*
b^3)/2 + (3*a^10*b^2)/2)/(9*a*b^8 - 9*a^8*b + a^9 - b^9 - 36*a^2*b^7 + 84*
a^3*b^6 - 126*a^4*b^5 + 126*a^5*b^4 - 84*a^6*b^3 + 36*a^7*b^2) + (3*tan(e
+ f*x)*(a*b*6i + a^2*1i + b^2*1i)*(256*b^11 - 1792*a*b^10 + 5120*a^2*b^9 -
 7168*a^3*b^8 + 3584*a^4*b^7 + 3584*a^5*b^6 - 7168*a^6*b^5 + 5120*a^7*b...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 805, normalized size of antiderivative = 4.11

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

output

```
(12*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**2*a**2 - 12*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*b**2 - 12*sqrt(b)*sqrt(a)*atan((sq
rt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**2 - 12*sqrt(b)*sqrt(a)*a
tan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a*b - 12*sqrt(b)*sqr
t(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**
2*a**2 + 12*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/
sqrt(b))*sin(e + f*x)**2*b**2 + 12*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqr
t(a)*tan((e + f*x)/2))/sqrt(b))*a**2 + 12*sqrt(b)*sqrt(a)*atan((sqrt(a - b
) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a*b - 2*cos(e + f*x)*sin(e + f*x)**
5*a**3 + 6*cos(e + f*x)*sin(e + f*x)**5*a**2*b - 6*cos(e + f*x)*sin(e + f*
x)**5*a*b**2 + 2*cos(e + f*x)*sin(e + f*x)**5*b**3 - cos(e + f*x)*sin(e +
f*x)**3*a**3 - 3*cos(e + f*x)*sin(e + f*x)**3*a**2*b + 9*cos(e + f*x)*sin(
e + f*x)**3*a*b**2 - 5*cos(e + f*x)*sin(e + f*x)**3*b**3 + 3*cos(e + f*x)*
sin(e + f*x)*a**3 + 6*cos(e + f*x)*sin(e + f*x)*a**2*b - 9*cos(e + f*x)*si
n(e + f*x)*a*b**2 + 3*sin(e + f*x)**2*a**3*e + 3*sin(e + f*x)**2*a**3*f*x
+ 15*sin(e + f*x)**2*a**2*b*e + 15*sin(e + f*x)**2*a**2*b*f*x - 15*sin(e +
 f*x)**2*a*b**2*e - 15*sin(e + f*x)**2*a*b**2*f*x - 3*sin(e + f*x)**2*b**3
*e - 3*sin(e + f*x)**2*b**3*f*x - 3*a**3*e - 3*a**3*f*x - 18*a**2*b*e - 18
*a**2*b*f*x - 3*a*b**2*e - 3*a*b**2*f*x)/(8*f*(sin(e + f*x)**2*a**5 - 5...
```

3.75 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{(a+3b)x}{2(a-b)^3} - \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}(a-b)^3f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{(a-b)^2f\left(a+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a+3*b)*x/(a-b)^3-1/2*b^(1/2)*(3*a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2
))/a^(1/2)/(a-b)^3/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)-
b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{-2(a+3b)(e+fx)+\frac{2\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}+(a-b)\sin(2(e+fx))+\frac{2(a-b)b\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}}{4(a-b)^3f}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-1/4*(-2*(a + 3*b)*(e + f*x) + (2*Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tan[e
+ f*x])/Sqrt[a]])/Sqrt[a] + (a - b)*Sin[2*(e + f*x)] + (2*(a - b)*b*Sin[2*
(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/((a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 373, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^2}{(a+b\tan(e+fx)^2)^2}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 373$$

$$\frac{\frac{\int \frac{a-3b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{2a\left(-2b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{2b\tan(e+fx)}{(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{-2b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a-b}-\frac{2b\tan(e+fx)}{(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{(a+3b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b(3a+b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a-b}-\frac{2b\tan(e+fx)}{(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{(a+3b)\arctan(\tan(e+fx))}{a-b}-\frac{b(3a+b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a-b}-\frac{2b\tan(e+fx)}{(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{(a+3b)\arctan(\tan(e+fx))}{a-b}-\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a-b}-\frac{2b\tan(e+fx)}{(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/2*Tan[e + f*x]/((a - b)*(1 + Tan[e + f*x]^2)*(a + b*Tan[e + f*x]^2)) +
 ((((a + 3*b)*ArcTan[Tan[e + f*x]])/(a - b) - (Sqrt[b]*(3*a + b)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(a - b) - (2*b*Tan[e + f
*x])/((a - b)*(a + b*Tan[e + f*x]^2)))/(2*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(3a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan(fx+e)^2}+\frac{(a+3b)\arctan(\tan(fx+e))}{2}}{(a-b)^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(3a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan(fx+e)^2}+\frac{(a+3b)\arctan(\tan(fx+e))}{2}}{(a-b)^3}}{f}$
risch	$\frac{xa}{2(a^2-2ab+b^2)(a-b)}+\frac{3xb}{2(a^2-2ab+b^2)(a-b)}+\frac{i e^{2i(fx+e)}}{8(a^2-2ab+b^2)f}-\frac{i e^{-2i(fx+e)}}{8(a^2-2ab+b^2)f}-\frac{ib(a e}{f(-a+b)^3(-a e^{4i(fx+e)}}$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)^3*b*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(3*a+b)
/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^3*((-1/2*a+1/2*b)*t
an(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+3*b)*arctan(tan(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 568, normalized size of antiderivative = 4.12

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \left[\frac{4\,(a^2+2\,ab-3\,b^2)fx\cos{(fx+e)}^2+4\,(ab+3\,b^2)fx-\left((3\,a^2-2\,ab-b^2)\cos{(fx+e)}^2+3\,ab+b^2\right)\sqrt{\ }}{8\left((a^4-\right.}\right.$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2 + 2*a*b - 3*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b + 3*b^2)*f*x - (
(3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(((a^2 +
 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 +
a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a
^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)) -
4*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 2*(a*b - b^2)*cos(f*x + e))*sin(f*
x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a
^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/4*(2*(a^2 + 2*a*b - 3*b^2)*f*x*cos
(f*x + e)^2 + 2*(a*b + 3*b^2)*f*x + ((3*a^2 - 2*a*b - b^2)*cos(f*x + e)^2
+ 3*a*b + b^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)
/(b*cos(f*x + e)*sin(f*x + e))) - 2*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 +
2*(a*b - b^2)*cos(f*x + e))*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.34

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\frac{(fx+e)(a+3b)}{a^3-3a^2b+3ab^2-b^3} - \frac{(3ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-3a^2b+3ab^2-b^3)\sqrt{ab}} - \frac{2b\tan(fx+e)^3+(a+b)\tan(fx+e)}{(a^2b-2ab^2+b^3)\tan(fx+e)^4+a^3-2a^2b+ab^2+(a^3-a^2b-ab^2+b^3)\tan(fx+e)^2}}{2f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a + 3*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*a*b + b^2)*a
rctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)
) - (2*b*tan(f*x + e)^3 + (a + b)*tan(f*x + e))/((a^2*b - 2*a*b^2 + b^3)*t
an(f*x + e)^4 + a^3 - 2*a^2*b + a*b^2 + (a^3 - a^2*b - a*b^2 + b^3)*tan(f*
x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.30

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{(fx+e)(a+3b)}{2(a^3f-3a^2bf+3ab^2f-b^3f)} - \frac{(3ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2(a^3f-3a^2bf+3ab^2f-b^3f)\sqrt{ab}} - \frac{2b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)}{2\left(b\tan(fx+e)^4+a\tan(fx+e)^2+b\tan(fx+e)^2+a\right)(a^2f-2abf+b^2f)}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(f*x + e)*(a + 3*b)/(a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3*f) - 1/2*(3*a
*b + b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*f - 3*a^2*b*f + 3*a*b^2*f
 - b^3*f)*sqrt(a*b)) - 1/2*(2*b*tan(f*x + e)^3 + a*tan(f*x + e) + b*tan(f*
x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + b*tan(f*x + e)^2 + a)*(a^2
*f - 2*a*b*f + b^2*f))
```

Mupad [B] (verification not implemented)

Time = 11.05 (sec) , antiderivative size = 3301, normalized size of antiderivative = 23.92

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(atan((((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*b^3))/(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) + ((-a*b)^(1/2)*(3*a + b)*((10*a*b^8
- 2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 - 18*a^5*b^4 + 10*a^6*b^3 -
 2*a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*
a^4*b^2) - (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(40*a*b^8 - 8*b^9 - 72*a^2
*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*a^6*b^3 - 8*a^7*b^2))/(4*
(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2
*b^2))))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))*(3*a + b)*1i)/(4*(a*b^3
+ 3*a^3*b - a^4 - 3*a^2*b^2)) + ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*
b^5 + 5*a^2*b^3))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) - ((-a*b)^(1
/2)*(3*a + b)*((10*a*b^8 - 2*b^9 - 18*a^2*b^7 + 10*a^3*b^6 + 10*a^4*b^5 -
18*a^5*b^4 + 10*a^6*b^3 - 2*a^7*b^2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a
^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2) + (tan(e + f*x)*(-a*b)^(1/2)*(3*a + b)*(
40*a*b^8 - 8*b^9 - 72*a^2*b^7 + 40*a^3*b^6 + 40*a^4*b^5 - 72*a^5*b^4 + 40*
a^6*b^3 - 8*a^7*b^2))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)*(a^4 - 4*a^3*
b - 4*a*b^3 + b^4 + 6*a^2*b^2))))/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))
*(3*a + b)*1i)/(4*(a*b^3 + 3*a^3*b - a^4 - 3*a^2*b^2)))/((5*a*b^4 + (3*b^5
)/2 + (3*a^2*b^3)/2)/(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*
b^3 + 15*a^4*b^2) - ((-a*b)^(1/2)*((tan(e + f*x)*(6*a*b^4 + 5*b^5 + 5*a^2*
b^3))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) + ((-a*b)^(1/2)*(3*a ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 701, normalized size of antiderivative = 5.08

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

output

```
(3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
sin(e + f*x)**2*a**2 - 2*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((
e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b - sqrt(b)*sqrt(a)*atan((sqrt(a -
 b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*b**2 - 3*sqrt(b)*
sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**2 - sqrt
(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a*b - 3
*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*si
n(e + f*x)**2*a**2 + 2*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e
+ f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b + sqrt(b)*sqrt(a)*atan((sqrt(a - b
) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*b**2 + 3*sqrt(b)*sq
rt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**2 + sqrt(b
)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a*b - cos
(e + f*x)*sin(e + f*x)**3*a**3 + 2*cos(e + f*x)*sin(e + f*x)**3*a**2*b - c
os(e + f*x)*sin(e + f*x)**3*a*b**2 + cos(e + f*x)*sin(e + f*x)*a**3 - cos(
e + f*x)*sin(e + f*x)*a*b**2 + sin(e + f*x)**2*a**3*e + sin(e + f*x)**2*a*
*3*f*x + 2*sin(e + f*x)**2*a**2*b*e + 2*sin(e + f*x)**2*a**2*b*f*x - 3*sin
(e + f*x)**2*a*b**2*e - 3*sin(e + f*x)**2*a*b**2*f*x - a**3*e - a**3*f*x -
 3*a**2*b*e - 3*a**2*b*f*x)/(2*a*f*(sin(e + f*x)**2*a**4 - 4*sin(e + f*x)*
*2*a**3*b + 6*sin(e + f*x)**2*a**2*b**2 - 4*sin(e + f*x)**2*a*b**3 + sin(e
 + f*x)**2*b**4 - a**4 + 3*a**3*b - 3*a**2*b**2 + a*b**3))
```

3.76 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2f} - \frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
x/(a-b)^2-1/2*(3*a-b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)/(
a-b)^2/f-1/2*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{2\arctan(\tan(e+fx)) + \frac{\sqrt{b}(-3a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(-a+b)\tan(e+fx)}{a(a+b\tan^2(e+fx))}}{2(a-b)^2f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

output
```
(2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2))
)/(2*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4144, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-b\tan^2(e+fx)+2a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2a\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b(3a-b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2a\arctan(\tan(e+fx))}{a-b}-\frac{b(3a-b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2a\arctan(\tan(e+fx))}{a-b}-\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```

output
```
(((2*a*ArcTan[Tan[e + f*x]])/(a - b) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - (b*Tan[e + f*x]
)/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\tan(fx+e)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
default	$\frac{-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\tan(fx+e)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{ib\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{fa(-a+b)^2\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\right.}{4a(a-b)^2}$

input
```
int(1/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a-b)^2*(1/2/a*(a-b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(3*a-b)/a/(
a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^2*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.02

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{8\,abfx\tan\left(fx+e\right)^2+8\,a^2fx-\left((3\,ab-b^2)\tan\left(fx+e\right)^2+3\,a^2-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(f}{b^2\tan}\right.}{8\left((a^3b-2\,a^2b^2+ab^3)f\tan\left(fx+e\right)^2+(a^4-2\right.}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2
 + 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2
+ a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x
 + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3
*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1
/4*(4*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 +
 3*a^2 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan
(f*x + e)) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*t
an(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(78) = 156$.

Time = 9.96 (sec) , antiderivative size = 2125, normalized size of antiderivative = 21.91

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b
, 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0))
, (3*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x
)**2 + 8*b**2*f) + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b*
*2*f*tan(e + f*x)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b*
*2*f*tan(e + f*x)**2 + 8*b**2*f) + 3*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x
)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 5*tan(e + f*x)/(8*b**2*f*ta
n(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x/(a +
b*tan(e)**2)**2, Eq(f, 0)), (4*a**2*f*x*sqrt(-a/b)/(4*a**4*f*sqrt(-a/b) +
4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**
2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqr
t(-a/b)*tan(e + f*x)**2) - 3*a**2*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**4*
f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/
b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) +
 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 3*a**2*log(sqrt(-a/b) + tan(e +
f*x))/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**
3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*
f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*a*b*f*x*sqrt(-a/
b)*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*
x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3-a^2b+(a^2b-ab^2)\tan(fx+e)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(f*x + e)^2) + (3*a
*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a
*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,(a^3f-2\,a^2bf+ab^2f)\sqrt{ab}}+\frac{fx+e}{a^2f-2\,abf+b^2f} - \frac{b\tan\left(fx+e\right)}{2\,(a^2f-abf)\left(b\tan\left(fx+e\right)^2+a\right)}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*f - 2*a^2*b*f +
a*b^2*f)*sqrt(a*b)) + (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) - 1/2*b*tan(f*x
+ e)/((a^2*f - a*b*f)*(b*tan(f*x + e)^2 + a))
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 2489, normalized size of antiderivative = 25.66

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*
a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))
/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2
*b^2) + (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^
2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 -
 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3
*b^2) + (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 -
 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^
3))/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7
- 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^
4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6
+ 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b +
a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e +
f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^
2 - 4*a*b + 2*b^2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 +
 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(
e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 +
 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i...
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^2\,ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^2\,b^2-3\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{2a^2f\left(\tan(fx+e)^2\,a^2b-2\tan(fx+\right.}$$

input
```
int(1/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x
)**2*a*b + sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e
+ f*x)**2*b**2 - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a))
)*a**2 + sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a*b + 2*
tan(e + f*x)**2*a**2*b*f*x - tan(e + f*x)*a**2*b + tan(e + f*x)*a*b**2 + 2
*a**3*f*x)/(2*a**2*f*(tan(e + f*x)**2*a**2*b - 2*tan(e + f*x)**2*a*b**2 +
tan(e + f*x)**2*b**3 + a**3 - 2*a**2*b + a*b**2))
```

3.77 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx=-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{3\cot(e+fx)}{2a^2f}+\frac{\cot(e+fx)}{2af\left(a+b\tan^2(e+fx)\right)}$$

output
```
-3/2*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2)/f-3/2*cot(f*x+e)/a
^2/f+1/2*cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx=\frac{-3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\left(-2\cot(e+fx)-\frac{b\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}\right)}{2a^{5/2}f}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-2*Cot[e + f
*x] - (b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(2*a^(5/2)
*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4146, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{3\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{2a}+\frac{\cot(e+fx)}{2a\left(a+b\tan^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{3\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a}\right)}{2a}+\frac{\cot(e+fx)}{2a\left(a+b\tan^2(e+fx)\right)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(e+fx)}{a}\right)}{2a}+\frac{\cot(e+fx)}{2a\left(a+b\tan^2(e+fx)\right)}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((3*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(3/2)) - Cot[e +
 f*x]/a))/(2*a) + Cot[e + f*x]/(2*a*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{1}{a^2\tan(fx+e)}-\frac{b\left(\frac{\tan(fx+e)}{2a+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}}{f}$
default	$\frac{-\frac{1}{a^2\tan(fx+e)}-\frac{b\left(\frac{\tan(fx+e)}{2a+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}}{f}$
risch	$-\frac{i\left(2a^2e^{4i(fx+e)}-3ab\,e^{4i(fx+e)}+3b^2e^{4i(fx+e)}+4a^2e^{2i(fx+e)}-6b^2e^{2i(fx+e)}+2a^2-5ab+3b^2\right)}{f(a-b)a^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)\left(e^{2i(fx+e)}-1\right)}-\frac{3\sqrt{-ab}\ln\left(e^{2i(fx+}\right.}{4a}$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2/tan(f*x+e)-1/a^2*b*(1/2*tan(f*x+e)/(a+b*tan(f*x+e)^2)+3/2/(a*b
)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(68) = 136.

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.55

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$

$$= \left[-\frac{4\,(2\,a-3\,b)\cos(fx+e)^3-3\,\left((a-b)\cos(fx+e)^2+b\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(fx+e)^4-2\,(3\,ab+b^2)\cos(}{(a^2-2\,ab+b^2)}\right.}{8\left(a^2bf+(a^3-a^2b)f\cos(fx+\right.}\right.$$

$$-\frac{2\,(2\,a-3\,b)\cos(fx+e)^3-3\,\left((a-b)\cos(fx+e)^2+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx-}{4\left(a^2bf+(a^3-a^2b)f\cos(fx+e)^2\right)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a - 3*b)*cos(f*x + e)^3 - 3*((a - b)*cos(f*x + e)^2 + b)*sqrt(
-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x +
e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*
x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x
+ e)^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/((a^2*b*f + (a^3 - a^2*b)
*f*cos(f*x + e)^2)*sin(f*x + e)), -1/4*(2*(2*a - 3*b)*cos(f*x + e)^3 - 3*(
(a - b)*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 -
 b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 6*b*cos(f*x +
e))/((a^2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \int \frac{\csc^2{(e+fx)}}{\left(a+b\tan^2{(e+fx)}\right)^2}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{3\,b\tan(fx+e)^2+2\,a}{a^2b\tan(fx+e)^3+a^3\tan(fx+e)} + \frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}}{2\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*b*tan(f*x + e)^2 + 2*a)/(a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))
 + 3*b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2))/f
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2f} - \frac{3\,b\tan\left(fx+e\right)^2+2\,a}{2\left(b\tan\left(fx+e\right)^3+a\tan\left(fx+e\right)\right)a^2f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-3/2*b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - 1/2*(3*b*tan(f
*x + e)^2 + 2*a)/((b*tan(f*x + e)^3 + a*tan(f*x + e))*a^2*f)
```

Mupad [B] (verification not implemented)

Time = 7.78 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{1}{a}+\frac{3\,b\tan(e+f\,x)^2}{2\,a^2}}{f\,\left(b\tan\left(e+f\,x\right)^3+a\tan\left(e+f\,x\right)\right)} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a}}\right)}{2\,a^{5/2}\,f}$$

input
```
int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^2),x)
```

output
```
- (1/a + (3*b*tan(e + f*x)^2)/(2*a^2))/(f*(a*tan(e + f*x) + b*tan(e + f*x)
^3)) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(2*a^(5/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 4.04

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)^3 a-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)^3 b-}{}$$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
sin(e + f*x)**3*a - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e +
 f*x)/2))/sqrt(b))*sin(e + f*x)**3*b - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
 - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*a - 3*sqrt(b)*sqrt(a)*a
tan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a +
3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*s
in(e + f*x)**3*b + 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e +
f*x)/2))/sqrt(b))*sin(e + f*x)*a - 2*cos(e + f*x)*sin(e + f*x)**2*a**2 + 3
*cos(e + f*x)*sin(e + f*x)**2*a*b + 2*cos(e + f*x)*a**2)/(2*sin(e + f*x)*a
**3*f*(sin(e + f*x)**2*a - sin(e + f*x)**2*b - a))
```

3.78 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} \, dx = -\frac{(3a-5b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{(a-2b)\cot(e+fx)}{a^3 f} - \frac{\cot^3(e+fx)}{3a^2 f} - \frac{(a-b)b\tan(e+fx)}{2a^3 f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-1/2*(3*a-5*b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/f-(a-2*b
)*cot(f*x+e)/a^3/f-1/3*cot(f*x+e)^3/a^2/f-1/2*(a-b)*b*tan(f*x+e)/a^3/f/(a+
b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{3\sqrt{b}(-3a+5b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\Big(-2\cot(e+fx)\left(2a-6b+a\csc^2(e+fx)\right)+\frac{3b(-a+b)\sin(2(e+}{a+b+(a-b)\cos(2(e}}{6a^{7/2}f}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(3*Sqrt[b]*(-3*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(
-2*Cot[e + f*x]*(2*a - 6*b + a*Csc[e + f*x]^2) + (3*b*(-a + b)*Sin[2*(e +
f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])))/(6*a^(7/2)*f)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4146, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \ 4146$$

$$\frac{\int \frac{\cot^4(e+fx)\left(\tan^2(e+fx)+1\right)}{\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 361$$

$$\frac{-\frac{1}{2}b\int -\frac{\cot^4(e+fx)\left(-\frac{(a-b)\tan^4(e+fx)}{a^3}+\frac{2(a-b)\tan^2(e+fx)}{a^2b}+\frac{2}{ab}\right)}{b\tan^2(e+fx)+a}d\tan(e+fx)-\frac{b(a-b)\tan(e+fx)}{2a^3(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{1}{2}b\int \frac{\cot^4(e+fx)\left(-\frac{(a-b)\tan^4(e+fx)}{a^3}+\frac{2(a-b)\tan^2(e+fx)}{a^2b}+\frac{2}{ab}\right)}{b\tan^2(e+fx)+a}d\tan(e+fx)-\frac{b(a-b)\tan(e+fx)}{2a^3(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \textbf{1584}$$

$$\frac{\frac{1}{2}b\int \left(\frac{2\cot^4(e+fx)}{a^2b}+\frac{2(a-2b)\cot^2(e+fx)}{a^3b}+\frac{5b-3a}{a^3(b\tan^2(e+fx)+a)}\right)d\tan(e+fx)-\frac{b(a-b)\tan(e+fx)}{2a^3(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{1}{2}b\left(-\frac{(3a-5b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}-\frac{2(a-2b)\cot(e+fx)}{a^3b}-\frac{2\cot^3(e+fx)}{3a^2b}\right)-\frac{b(a-b)\tan(e+fx)}{2a^3(a+b\tan^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((b*(-(((3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*Sqrt[
b])) - (2*(a - 2*b)*Cot[e + f*x])/(a^3*b) - (2*Cot[e + f*x]^3)/(3*a^2*b)))
/2 - ((a - b)*b*Tan[e + f*x])/(2*a^3*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))    Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{1}{3a^2\tan(fx+e)^3}-\frac{a-2b}{a^3\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
default	$\frac{-\frac{1}{3a^2\tan(fx+e)^3}-\frac{a-2b}{a^3\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
risch	$\frac{i\left(9ab\,e^{8i(fx+e)}-15b^2e^{8i(fx+e)}+12a^2e^{6i(fx+e)}-6ab\,e^{6i(fx+e)}+60b^2e^{6i(fx+e)}+20a^2e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}-90b^2e^{4}\cdots\right.}{3f\,a^3\left(e^{2i(fx+e)}-1\right)^3\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+\cdots\right.}$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/a^2/tan(f*x+e)^3-(a-2*b)/a^3/tan(f*x+e)-1/a^3*b*((1/2*a-1/2*b)*t
an(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(3*a-5*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)
/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(102) = 204$.

Time = 0.12 (sec) , antiderivative size = 587, normalized size of antiderivative = 5.06

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/24*(4*(4*a^2 - 19*a*b + 15*b^2)*cos(f*x + e)^5 - 8*(3*a^2 - 14*a*b + 1
5*b^2)*cos(f*x + e)^3 + 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2
 - 11*a*b + 10*b^2)*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(-b/a)*log(((a^2 +
 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 - 4*((a^2 +
a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a
^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*si
n(f*x + e) - 12*(3*a*b - 5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x +
e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/12*(2
*(4*a^2 - 19*a*b + 15*b^2)*cos(f*x + e)^5 - 4*(3*a^2 - 14*a*b + 15*b^2)*co
s(f*x + e)^3 - 3*((3*a^2 - 8*a*b + 5*b^2)*cos(f*x + e)^4 - (3*a^2 - 11*a*b
 + 10*b^2)*cos(f*x + e)^2 - 3*a*b + 5*b^2)*sqrt(b/a)*arctan(1/2*((a + b)*c
os(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) -
 6*(3*a*b - 5*b^2)*cos(f*x + e))/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*
f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{3\,(3\,ab-5\,b^2)\tan(fx+e)^4+2\,(3\,a^2-5\,ab)\tan(fx+e)^2+2\,a^2}{a^3b\tan(fx+e)^5+a^4\tan(fx+e)^3} + \frac{3\,(3\,ab-5\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}}{6\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*((3*(3*a*b - 5*b^2)*tan(f*x + e)^4 + 2*(3*a^2 - 5*a*b)*tan(f*x + e)^2
 + 2*a^2)/(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3) + 3*(3*a*b - 5*b^2)*
arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3))/f
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{(3\,ab-5\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3f} - \frac{ab\tan(fx+e)-b^2\tan(fx+e)}{2\,\left(b\tan(fx+e)^2+a\right)a^3f} - \frac{3\,a\tan(fx+e)^2-6\,b\tan(fx+e)^2+a}{3\,a^3f\tan(fx+e)^3}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b - 5*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) -
1/2*(a*b*tan(f*x + e) - b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a)*a^3*f) -
 1/3*(3*a*tan(f*x + e)^2 - 6*b*tan(f*x + e)^2 + a)/(a^3*f*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 7.83 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\frac{1}{3\,a}+\frac{\tan(e+f\,x)^2\,(3\,a-5\,b)}{3\,a^2}+\frac{b\tan(e+f\,x)^4\,(3\,a-5\,b)}{2\,a^3}}{f\,\left(b\tan\left(e+f\,x\right)^5+a\tan\left(e+f\,x\right)^3\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a}}\right)\,(3\,a-5\,b)}{2\,a^{7/2}\,f}$$

input
```
int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^2),x)
```

output
```
- (1/(3*a) + (tan(e + f*x)^2*(3*a - 5*b))/(3*a^2) + (b*tan(e + f*x)^4*(3*a
 - 5*b))/(2*a^3))/(f*(a*tan(e + f*x)^3 + b*tan(e + f*x)^5)) - (b^(1/2)*ata
n((b^(1/2)*tan(e + f*x))/a^(1/2))*(3*a - 5*b))/(2*a^(7/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.95

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(9*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
sin(e + f*x)**5*a**2 - 24*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan(
(e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a*b + 15*sqrt(b)*sqrt(a)*atan((sqrt
(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*b**2 - 9*sqrt
(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e +
 f*x)**3*a**2 + 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*
x)/2))/sqrt(b))*sin(e + f*x)**3*a*b - 9*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
+ sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**2 + 24*sqrt(b)*sqr
t(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**
5*a*b - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/s
qrt(b))*sin(e + f*x)**5*b**2 + 9*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(
a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a**2 - 15*sqrt(b)*sqrt(a)*at
an((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a*b -
 4*cos(e + f*x)*sin(e + f*x)**4*a**3 + 19*cos(e + f*x)*sin(e + f*x)**4*a**
2*b - 15*cos(e + f*x)*sin(e + f*x)**4*a*b**2 + 2*cos(e + f*x)*sin(e + f*x)
**2*a**3 - 10*cos(e + f*x)*sin(e + f*x)**2*a**2*b + 2*cos(e + f*x)*a**3)/(
6*sin(e + f*x)**3*a**4*f*(sin(e + f*x)**2*a - sin(e + f*x)**2*b - a))
```

3.79 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2} dx = -\frac{(3a-7b)(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}f} - \frac{(a-3b)(a-b)\cot(e+fx)}{a^4f} - \frac{2(a-b)\cot^3(e+fx)}{3a^3f} - \frac{\cot^5(e+fx)}{5a^2f} - \frac{(a-b)^2b\tan(e+fx)}{2a^4f\,(a+b\tan^2(e+fx))}$$

output
```
-1/2*(3*a-7*b)*(a-b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(9/2)/f-
(a-3*b)*(a-b)*cot(f*x+e)/a^4/f-2/3*(a-b)*cot(f*x+e)^3/a^3/f-1/5*cot(f*x+e)
^5/a^2/f-1/2*(a-b)^2*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{-15\sqrt{b}(3a^2-10ab+7b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\Big(-2\cot(e+fx)\,(8a^2-50ab+45b^2+2a(2a-5b)}{30a^{9/2}f}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-15*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
]] + Sqrt[a]*(-2*Cot[e + f*x]*(8*a^2 - 50*a*b + 45*b^2 + 2*a*(2*a - 5*b)*C
sc[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4) - (15*(a - b)^2*b*Sin[2*(e + f*x)])/
(a + b + (a - b)*Cos[2*(e + f*x)])))/(30*a^(9/2)*f)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4146, 365, 361, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^6\,(a+b\tan(e+fx)^2)^2}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\cot^6(e+fx)(\tan^2(e+fx)+1)^2}{(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5a\tan^2(e+fx)+10a-7b\right)}{\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)}{5a(a+b\tan^2(e+fx))}}{f}$$

$\downarrow$ 361

$$\frac{\frac{-\frac{1}{2}b\int \frac{\cot^4(e+fx)\left(\frac{\left(5a^2-10ba+7b^2\right)\tan^4(e+fx)}{a^3}+2\left(-\frac{7b}{a^2}+\frac{10}{a}-\frac{5}{b}\right)\tan^2(e+fx)+2\left(\frac{7}{a}-\frac{10}{b}\right)\right)}{b\tan^2(e+fx)+a}d\tan(e+fx)-\frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{2a^3\left(a+b\tan^2(e+fx)\right)}}{5a}-\frac{\cot^5(}{5a(a+b\,\mathrm{ta}}}{f}$$

$\downarrow$ 1584

$$\frac{\frac{-\frac{1}{2}b\int \left(-\frac{2(10a-7b)\cot^4(e+fx)}{a^2b}-\frac{2\left(5a^2-20ba+14b^2\right)\cot^2(e+fx)}{a^3b}+\frac{5(3a-7b)(a-b)}{a^3\left(b\tan^2(e+fx)+a\right)}\right)d\tan(e+fx)-\frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{2a^3\left(a+b\tan^2(e+fx)\right)}}{5a}-\frac{\cot^5(e+}{5a(a+b\tan^2}}{f}$$

$\downarrow$ 2009

$$\frac{\frac{-\frac{b\left(5a^2-10ab+7b^2\right)\tan(e+fx)}{2a^3\left(a+b\tan^2(e+fx)\right)}-\frac{1}{2}b\left(\frac{5(3a-7b)(a-b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}+\frac{2(10a-7b)\cot^3(e+fx)}{3a^2b}+\frac{2\left(5a^2-20ab+14b^2\right)\cot(e+fx)}{a^3b}\right)}{5a}-\frac{\cot^5(e+}{5a(a+b\tan^2(}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/5*Cot[e + f*x]^5/(a*(a + b*Tan[e + f*x]^2)) + (-1/2*(b*((5*(3*a - 7*b)
*(a - b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*Sqrt[b]) + (2*(5
*a^2 - 20*a*b + 14*b^2)*Cot[e + f*x])/(a^3*b) + (2*(10*a - 7*b)*Cot[e + f*
x]^3)/(3*a^2*b))) - (b*(5*a^2 - 10*a*b + 7*b^2)*Tan[e + f*x])/(2*a^3*(a +
b*Tan[e + f*x]^2)))/(5*a))/f
```

Defintions of rubi rules used

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\left(\frac{1}{2}a^2-ab+\frac{1}{2}b^2\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{\left(3a^2-10ab+7b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}-\frac{1}{5a^2\tan(fx+e)^5}-\frac{2a-2b}{3a^3\tan(fx+e)^3}-\frac{a^2-4ab+3b}{a^4\tan(fx+e}}{f}$
default	$\frac{-\frac{b\left(\frac{\left(\frac{1}{2}a^2-ab+\frac{1}{2}b^2\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{\left(3a^2-10ab+7b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}-\frac{1}{5a^2\tan(fx+e)^5}-\frac{2a-2b}{3a^3\tan(fx+e)^3}-\frac{a^2-4ab+3b}{a^4\tan(fx+e}}{f}$
risch	$i(105b^3-16a^3-220a\,b^2+131a^2b+1575b^3e^{8i(fx+e)}-16a^3e^{4i(fx+e)}+48a^3e^{2i(fx+e)}-240a^3e^{6i(fx+e)}+1575b^3e^{4i(fx+e)}$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^4*((1/2*a^2-a*b+1/2*b^2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(3*a^
2-10*a*b+7*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/5/a^2/tan(
f*x+e)^5-1/3*(2*a-2*b)/a^3/tan(f*x+e)^3-(a^2-4*a*b+3*b^2)/a^4/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(135) = 270.

Time = 0.15 (sec) , antiderivative size = 855, normalized size of antiderivative = 5.66

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/120*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(
40*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 47*
a^2*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*
b^2 - 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*b^3)*cos(f
*x + e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*a*b^2 - 21
*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 -
 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(
f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)
^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 10*
a*b^2 + 7*b^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f -
(2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)*sin
(f*x + e)), -1/60*(2*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x +
e)^7 - 2*(40*a^3 - 321*a^2*b + 590*a*b^2 - 315*b^3)*cos(f*x + e)^5 + 10*(6
*a^3 - 47*a^2*b + 104*a*b^2 - 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2
*b + 17*a*b^2 - 7*b^3)*cos(f*x + e)^6 - (6*a^3 - 29*a^2*b + 44*a*b^2 - 21*
b^3)*cos(f*x + e)^4 + 3*a^2*b - 10*a*b^2 + 7*b^3 + (3*a^3 - 19*a^2*b + 37*
a*b^2 - 21*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)
^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3*a^2*
b - 10*a*b^2 + 7*b^3)*cos(f*x + e))/(((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4
*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\frac{15\,(3\,a^2b-10\,ab^2+7\,b^3)\tan(fx+e)^6+10\,(3\,a^3-10\,a^2b+7\,ab^2)\tan(fx+e)^4+6\,a^3+2\,(10\,a^3-7\,a^2b)\tan(fx+e)^2}{a^4b\tan(fx+e)^7+a^5\tan(fx+e)^5}+\frac{15\,(3\,a^2b-10\,ab^2+7\,b^3)\,\mathrm{a}\cdots}{\sqrt{ab}a^{\cdots}}}{30\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/30*((15*(3*a^2*b - 10*a*b^2 + 7*b^3)*tan(f*x + e)^6 + 10*(3*a^3 - 10*a^
2*b + 7*a*b^2)*tan(f*x + e)^4 + 6*a^3 + 2*(10*a^3 - 7*a^2*b)*tan(f*x + e)^
2)/(a^4*b*tan(f*x + e)^7 + a^5*tan(f*x + e)^5) + 15*(3*a^2*b - 10*a*b^2 +
7*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4))/f
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.24

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{(3\,a^2b-10\,ab^2+7\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4f} - \frac{a^2b\tan\left(fx+e\right)-2\,ab^2\tan\left(fx+e\right)+b^3\tan\left(fx+e\right)}{2\left(b\tan\left(fx+e\right)^2+a\right)a^4f} - \frac{15\,a^2\tan\left(fx+e\right)^4-60\,ab\tan\left(fx+e\right)^4+45\,b^2\tan\left(fx+e\right)^4+10\,a^2\tan\left(fx+e\right)^2-10\,ab\tan\left(fx+\cdots\right.}{15\,a^4f\tan\left(fx+e\right)^5}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a^2*b - 10*a*b^2 + 7*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a
*b)*a^4*f) - 1/2*(a^2*b*tan(f*x + e) - 2*a*b^2*tan(f*x + e) + b^3*tan(f*x
+ e))/((b*tan(f*x + e)^2 + a)*a^4*f) - 1/15*(15*a^2*tan(f*x + e)^4 - 60*a*
b*tan(f*x + e)^4 + 45*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 - 10*a*b*
tan(f*x + e)^2 + 3*a^2)/(a^4*f*tan(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.18

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{1}{5a}+\frac{\tan(e+fx)^4(3a^2-10ab+7b^2)}{3a^3}+\frac{\tan(e+fx)^2(10a-7b)}{15a^2}+\frac{b\tan(e+fx)^6(3a^2-10ab+7b^2)}{2a^4}}{f\left(b\tan(e+fx)^7+a\tan(e+fx)^5\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+fx)(a-b)(3a-7b)}{\sqrt{a}(3a^2-10ab+7b^2)}\right)(a-b)(3a-7b)}{2a^{9/2}f}$$

input
```
int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^2),x)
```

output
```
- (1/(5*a) + (tan(e + f*x)^4*(3*a^2 - 10*a*b + 7*b^2))/(3*a^3) + (tan(e +
f*x)^2*(10*a - 7*b))/(15*a^2) + (b*tan(e + f*x)^6*(3*a^2 - 10*a*b + 7*b^2)
)/(2*a^4))/(f*(a*tan(e + f*x)^5 + b*tan(e + f*x)^7)) - (b^(1/2)*atan((b^(1
/2)*tan(e + f*x)*(a - b)*(3*a - 7*b))/(a^(1/2)*(3*a^2 - 10*a*b + 7*b^2)))*
(a - b)*(3*a - 7*b))/(2*a^(9/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 839, normalized size of antiderivative = 5.56

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**7*a**3 - 195*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*ta
n((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a**2*b + 255*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a*b**2
- 105*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*sin(e + f*x)**7*b**3 - 45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*t
an((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**3 + 150*sqrt(b)*sqrt(a)*atan(
(sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**2*b -
 105*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**5*a*b**2 - 45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a**3 + 195*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a**2*b
- 255*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*sin(e + f*x)**7*a*b**2 + 105*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a
)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*b**3 + 45*sqrt(b)*sqrt(a)*ata
n((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**3 -
 150*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**5*a**2*b + 105*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)
*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a*b**2 - 16*cos(e + f*x)*sin(e
 + f*x)**6*a**4 + 131*cos(e + f*x)*sin(e + f*x)**6*a**3*b - 220*cos(e +...
```

3.80 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{\sqrt{b}(15a^2+40ab+8b^2)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{11/2}f} - \frac{(a^2+4ab+b^2)\cos(e+fx)}{(a-b)^5f} + \frac{(2a+b)\cos^3(e+fx)}{3(a-b)^4f} - \frac{\cos^5(e+fx)}{5(a-b)^3f} - \frac{a^2b\sec(e+fx)}{4(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{ab(7a+8b)\sec(e+fx)}{8(a-b)^5f\left(a-b+b\sec^2(e+fx)\right)}$$

output

```
-1/8*b^(1/2)*(15*a^2+40*a*b+8*b^2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/
(a-b)^(11/2)/f-(a^2+4*a*b+b^2)*cos(f*x+e)/(a-b)^5/f+1/3*(2*a+b)*cos(f*x+e)
^3/(a-b)^4/f-1/5*cos(f*x+e)^5/(a-b)^3/f-1/4*a^2*b*sec(f*x+e)/(a-b)^4/f/(a-
b+b*sec(f*x+e)^2)^2-1/8*a*b*(7*a+8*b)*sec(f*x+e)/(a-b)^5/f/(a-b+b*sec(f*x+
e)^2)
```

Mathematica [A] (verified)

Time = 4.05 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.24

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\frac{30\sqrt{b}(15a^2+40ab+8b^2)\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{11/2}}+\frac{30\sqrt{b}(15a^2+40ab+8b^2)\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{11/2}}+\frac{-30\cos(e+fx)(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(
e + f*x)/2])/Sqrt[b]])/(a - b)^(11/2) + (30*Sqrt[b]*(15*a^2 + 40*a*b + 8*b
^2)*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(11/
2) + (-30*Cos[e + f*x]*(11*b^2 + 16*a*b*(2 + b/(a + b + (a - b)*Cos[2*(e +
 f*x)])) + a^2*(5 - (8*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 + (18*b)/
(a + b + (a - b)*Cos[2*(e + f*x)]))) + (a - b)*(5*(5*a + 7*b)*Cos[3*(e + f
*x)] + 3*(-a + b)*Cos[5*(e + f*x)]))/(a - b)^5)/(240*f)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4147, 365, 25, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^5}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 365

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(-5(a-b)\sec^2(e+fx)+10a-b\right)}{\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(-5(a-b)\sec^2(e+fx)+10a-b\right)}{\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$\Big\downarrow$ 361

$$\frac{-\frac{\frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{4(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{1}{4}b\int -\frac{\cos^4(e+fx)\left(\frac{3\left(5a^2+4b^2\right)\sec^4(e+fx)}{(a-b)^3}-\frac{4\left(5a^2+4b^2\right)\sec^2(e+fx)}{(a-b)^2b}+\frac{4(10a-b)}{(a-b)b}\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+}{5(a-b)(a+b\sec^2}}{f}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\frac{1}{4}b\int \frac{\cos^4(e+fx)\left(\frac{3\left(5a^2+4b^2\right)\sec^4(e+fx)}{(a-b)^3}-\frac{4\left(5a^2+4b^2\right)\sec^2(e+fx)}{(a-b)^2b}+\frac{4(10a-b)}{(a-b)b}\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)+\frac{b\left(5a^2+4b^2\right)\sec(e+fx)}{4(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}}{5(a-b)}-\frac{\cos^5(e+f}{5(a-b)(a+b\sec^2(e}}{f}$$

$\Big\downarrow$ 1582

$$\frac{-\frac{\frac{1}{4}b\left(\frac{\int \frac{\cos^4(e+fx)\left(\frac{b^2\left(35a^2+40ba+24b^2\right)\sec^4(e+fx)}{a-b}-8b\left(5a^2+10ba+3b^2\right)\sec^2(e+fx)+8(a-b)(10a-b)b\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{2b^2(a-b)^3}+\frac{\left(35a^2+40ab+24b^2\right)\sec(e+fx)}{2(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}\right)}{5(a-b)}}{f}$$

$\Big\downarrow$ 1584

$$-\frac{\frac{\frac{1}{4}b\left(\frac{\int \left(8(10a-b)b\cos^4(e+fx)-\frac{8b\left(5a^2+20ba+2b^2\right)\cos^2(e+fx)}{a-b}+\frac{5b^2\left(15a^2+40ba+8b^2\right)}{(a-b)\left(b\sec^2(e+fx)+a-b\right)}\right)d\sec(e+fx)}{2b^2(a-b)^3}+\frac{\left(35a^2+40ab+24b^2\right)\sec(e+fx)}{2(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}\right)+\frac{b\left(5a^2+\right.}{4(a-b)^3\left(a\right.}}{5(a-b)}}{f}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\frac{\frac{1}{4}b\left(\frac{\frac{5b^{3/2}\left(15a^2+40ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{8b\left(5a^2+20ab+2b^2\right)\cos(e+fx)}{a-b}-\frac{8}{3}b(10a-b)\cos^3(e+fx)}{2b^2(a-b)^3}+\frac{\left(35a^2+40ab+24b^2\right)\sec(e+fx)}{2(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}\right)+\frac{b\left(5\right.}{4(a-b)}}{5(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/5*Cos[e + f*x]^5/((a - b)*(a - b + b*Sec[e + f*x]^2)^2) - ((b*(5*a^2 +
 4*b^2)*Sec[e + f*x])/(4*(a - b)^3*(a - b + b*Sec[e + f*x]^2)^2) + (b*(((5
*b^(3/2)*(15*a^2 + 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a -
b]])/(a - b)^(3/2) + (8*b*(5*a^2 + 20*a*b + 2*b^2)*Cos[e + f*x])/(a - b) -
 (8*(10*a - b)*b*Cos[e + f*x]^3)/3)/(2*(a - b)^3*b^2) + ((35*a^2 + 40*a*b
+ 24*b^2)*Sec[e + f*x])/(2*(a - b)^4*(a - b + b*Sec[e + f*x]^2))))/4)/(5*(
a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
 - a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 158.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{\frac{a^{2} \cos(fx+e)^{5}}{5}-\frac{2ab\cos(fx+e)^{5}}{5}+\frac{b^{2} \cos(fx+e)^{5}}{5}-\frac{2a^{2} \cos(fx+e)^{3}}{3}+\frac{ab\cos(fx+e)^{3}}{3}+\frac{b^{2} \cos(fx+e)^{3}}{3}+\cos(fx+e)a^{2}+4ab\cos(fx}{\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)\left(a^{2}-2ab+b^{2}\right)}$
default	$-\frac{\frac{a^{2} \cos(fx+e)^{5}}{5}-\frac{2ab\cos(fx+e)^{5}}{5}+\frac{b^{2} \cos(fx+e)^{5}}{5}-\frac{2a^{2} \cos(fx+e)^{3}}{3}+\frac{ab\cos(fx+e)^{3}}{3}+\frac{b^{2} \cos(fx+e)^{3}}{3}+\cos(fx+e)a^{2}+4ab\cos(fx}{\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)\left(a^{2}-2ab+b^{2}\right)}$
risch	Expression too large to display

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a^3-3*a^2*b+3*a*b^2-b^3)/(a^2-2*a*b+b^2)*(1/5*a^2*cos(f*x+e)^5-2/
5*a*b*cos(f*x+e)^5+1/5*b^2*cos(f*x+e)^5-2/3*a^2*cos(f*x+e)^3+1/3*a*b*cos(f
*x+e)^3+1/3*b^2*cos(f*x+e)^3+cos(f*x+e)*a^2+4*a*b*cos(f*x+e)+b^2*cos(f*x+e
))+b/(a-b)^5*((-1/8*a*(9*a^2-a*b-8*b^2)*cos(f*x+e)^3+(-7/8*a^2*b-a*b^2)*co
s(f*x+e))/(a*cos(f*x+e)^2-b*cos(f*x+e)^2+b)^2+1/8*(15*a^2+40*a*b+8*b^2)/(b
*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(206) = 412$.

Time = 0.21 (sec) , antiderivative size = 1018, normalized size of antiderivative = 4.54

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/240*(48*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 1
6*(10*a^4 - 31*a^3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 16*(1
5*a^4 + 10*a^3*b - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 50*(15*
a^3*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a
^3*b - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b
^3 + 8*b^4 + 2*(15*a^3*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*
sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))
*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 40*a*b
^3 + 8*b^4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a^4*b^3 + 35*a
^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e)^4 + 2*(a^6*b - 6*a^5*b
^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)*f*cos(f*x + e)^
2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5 + 5*a*b^6 - b^7)*f), -1
/120*(24*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 8*(1
0*a^4 - 31*a^3*b + 33*a^2*b^2 - 13*a*b^3 + b^4)*cos(f*x + e)^7 + 8*(15*a^4
 + 10*a^3*b - 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^5 + 25*(15*a^3*b
 + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^3 + 15*((15*a^4 + 10*a^3*b
- 57*a^2*b^2 + 24*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
8*b^4 + 2*(15*a^3*b + 25*a^2*b^2 - 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(
b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) + 15*(15*a^2*b^
2 + 40*a*b^3 + 8*b^4)*cos(f*x + e))/((a^7 - 7*a^6*b + 21*a^5*b^2 - 35*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(206) = 412$.

Time = 0.82 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.72

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/120*(15*(15*a^2*b + 40*a*b^2 + 8*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f
*x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - 5*a
^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sqrt(a*b - b^2)) + 30*(9*a
^3*b + 6*a^2*b^2 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 32*a^2
*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 40*a*b^3*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 5
4*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 24*a*b^3*(cos(f*x +
e) - 1)^2/(cos(f*x + e) + 1)^2 + 48*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a^2*b^2*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 8*a*b^3*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) -
16*(8*a^2 + 59*a*b + 23*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 - 250*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 70*b^2*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 320*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*b^2*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e)
+ 1)^3 - 90*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a*b*(cos(f*
x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f...
```

Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 1536, normalized size of antiderivative = 6.86

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(b^(1/2)*atan(((a - b)^11*(2*tan(e/2 + (f*x)/2)^2*((b^(1/2)*(40*a*b + 15*a
^2 + 8*b^2)*(640*a^3*b^12 - 128*a^2*b^13 - 240*a^14*b + 400*a^4*b^11 - 110
40*a^5*b^10 + 39120*a^6*b^9 - 73344*a^7*b^8 + 84000*a^8*b^7 - 58560*a^9*b^
6 + 20640*a^10*b^5 + 1280*a^11*b^4 - 4528*a^12*b^3 + 1760*a^13*b^2))/(16*a
*(a - b)^(21/2)) - (b^(1/2)*(a - 2*b)*(40*a*b + 15*a^2 + 8*b^2)^2*(128*a^1
8 - 2176*a^17*b + 256*a^2*b^16 - 3968*a^3*b^15 + 28800*a^4*b^14 - 129920*a
^5*b^13 + 407680*a^6*b^12 - 943488*a^7*b^11 + 1665664*a^8*b^10 - 2288000*a
^9*b^9 + 2471040*a^10*b^8 - 2104960*a^11*b^7 + 1409408*a^12*b^6 - 733824*a
^13*b^5 + 291200*a^14*b^4 - 85120*a^15*b^3 + 17280*a^16*b^2))/(512*a*(a -
b)^(33/2))) - (b^(1/2)*(a - 2*b)*(40*a*b + 15*a^2 + 8*b^2)^2*(1920*a^17*b
- 128*a^18 + 128*a^3*b^15 - 1920*a^4*b^14 + 13440*a^5*b^13 - 58240*a^6*b^1
2 + 174720*a^7*b^11 - 384384*a^8*b^10 + 640640*a^9*b^9 - 823680*a^10*b^8 +
 823680*a^11*b^7 - 640640*a^12*b^6 + 384384*a^13*b^5 - 174720*a^14*b^4 + 5
8240*a^15*b^3 - 13440*a^16*b^2))/(256*a*(a - b)^(33/2))))/(225*a^16*b + 64
*a^2*b^15 - 1680*a^4*b^13 + 3920*a^5*b^12 + 7665*a^6*b^11 - 50778*a^7*b^10
 + 104685*a^8*b^9 - 111960*a^9*b^8 + 57330*a^10*b^7 + 2660*a^11*b^6 - 2028
6*a^12*b^5 + 9240*a^13*b^4 - 35*a^14*b^3 - 1050*a^15*b^2))*(40*a*b + 15*a^
2 + 8*b^2))/(8*f*(a - b)^(11/2)) - ((607*a^3*b + 64*a^4 + 274*a^2*b^2)/(60
*(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (tan(e/2 + (f*x)/2
)^14*(128*a*b^3 + 15*a^3*b + 24*b^4 + 85*a^2*b^2))/(2*(a - b)*(a^4 - 4*...
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 2112, normalized size of antiderivative = 9.43

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(225*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqr
t(b))*sin(e + f*x)**4*a**4 + 150*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - s
qrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b - 855*sqrt(b)*sqr
t(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*
x)**4*a**2*b**2 + 360*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan(
(e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b**3 + 120*sqrt(b)*sqrt(a - b)*at
an((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*b**4
- 450*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sq
rt(b))*sin(e + f*x)**2*a**4 - 750*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) -
sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**3*b + 960*sqrt(b)*sq
rt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f
*x)**2*a**2*b**2 + 240*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 225*sqrt(b)*sqrt(a - b)*a
tan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**4 + 600*sqrt(b)*s
qrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**3*b +
 120*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqr
t(b))*a**2*b**2 + 225*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan(
(e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**4 + 150*sqrt(b)*sqrt(a - b)*atan
((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b
- 855*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))...
```

3.81 $\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 180

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{5\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f} - \frac{(a+2b)\cos(e+fx)}{(a-b)^4f} + \frac{\cos^3(e+fx)}{3(a-b)^3f} - \frac{ab\sec(e+fx)}{4(a-b)^3f\,(a-b+b\sec^2(e+fx))^2} - \frac{b(7a+4b)\sec(e+fx)}{8(a-b)^4f\,(a-b+b\sec^2(e+fx))}$$

output

```
-5/8*b^(1/2)*(3*a+4*b)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(9/2)/
f-(a+2*b)*cos(f*x+e)/(a-b)^4/f+1/3*cos(f*x+e)^3/(a-b)^3/f-1/4*a*b*sec(f*x+
e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^2-1/8*b*(7*a+4*b)*sec(f*x+e)/(a-b)^4/f/(
a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 4.17 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{15\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}}+\frac{15\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{9/2}}+\frac{2\left(3\cos(e+fx)\left(a\left(-3+\frac{4\ldots}{(a+b+(a-b)\,c\ldots}\right.\right.\right.}{\ldots}}{24f}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/S
qrt[b]])/(a - b)^(9/2) + (15*Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[a - b] + Sqr
t[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(9/2) + (2*(3*Cos[e + f*x]*(a*(-3
 + (4*b^2)/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*C
os[2*(e + f*x)])) + b*(-9 - (4*b)/(a + b + (a - b)*Cos[2*(e + f*x)]))) + (
a - b)*Cos[3*(e + f*x)]))/(a - b)^4)/(24*f)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4147, 25, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{1}{4}b\int -\frac{\cos^4(e+fx)\left(\frac{3a\sec^4(e+fx)}{(a-b)^3}-\frac{4a\sec^2(e+fx)}{(a-b)^2b}+\frac{4}{(a-b)b}\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)-\frac{ab\sec(e+fx)}{4(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{4}b\int \frac{\cos^4(e+fx)\left(\frac{3a\sec^4(e+fx)}{(a-b)^3}-\frac{4a\sec^2(e+fx)}{(a-b)^2b}+\frac{4}{(a-b)b}\right)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)-\frac{ab\sec(e+fx)}{4(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$$\Big\downarrow \text{1582}$$

$$\frac{-\frac{1}{4}b\left(\frac{\int \frac{\cos^4(e+fx)\left(\frac{b^2(7a+4b)\sec^4(e+fx)}{a-b}-8b(a+b)\sec^2(e+fx)+8(a-b)b\right)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{2b^2(a-b)^3}+\frac{(7a+4b)\sec(e+fx)}{2(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}\right)-\frac{ab\sec(e+fx)}{4(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$$\Big\downarrow \text{1584}$$

$$\frac{-\frac{1}{4}b\left(\frac{\int \left(8b\cos^4(e+fx)-\frac{8b(a+2b)\cos^2(e+fx)}{a-b}+\frac{5b^2(3a+4b)}{(a-b)\left(b\sec^2(e+fx)+a-b\right)}\right)d\sec(e+fx)}{2b^2(a-b)^3}+\frac{(7a+4b)\sec(e+fx)}{2(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}\right)-\frac{ab\sec(e+fx)}{4(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}b\left(\frac{\frac{5b^{3/2}(3a+4b)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{8b(a+2b)\cos(e+fx)}{a-b}-\frac{8}{3}b\cos^3(e+fx)}{2b^2(a-b)^3}+\frac{(7a+4b)\sec(e+fx)}{2(a-b)^4\left(a+b\sec^2(e+fx)-b\right)}\right)-\frac{ab\sec(e+fx)}{4(a-b)^3\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*b*Sec[e + f*x])/((a - b)^3*(a - b + b*Sec[e + f*x]^2)^2) - (b*(((
5*b^(3/2)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a - b)^
(3/2) + (8*b*(a + 2*b)*Cos[e + f*x])/(a - b) - (8*b*Cos[e + f*x]^3)/3)/(2*
(a - b)^3*b^2) + ((7*a + 4*b)*Sec[e + f*x])/(2*(a - b)^4*(a - b + b*Sec[e
+ f*x]^2))))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 51.00 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\frac{b\cos(fx+e)^3}{3}-a\cos(fx+e)-2\cos(fx+e)b}{(a^3-3a^2b+3a\,b^2-b^3)(a-b)}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2+\frac{5}{8}ab+\frac{1}{2}b^2\right)\cos(fx+e)^3+\left(-\frac{7}{8}ab-\frac{1}{2}b^2\right)\cos(fx+e)}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{5(3a}{}\right.}{(a-b)^4}}{f}$
default	$\frac{\frac{\frac{a\cos(fx+e)^3}{3}-\frac{b\cos(fx+e)^3}{3}-a\cos(fx+e)-2\cos(fx+e)b}{(a^3-3a^2b+3a\,b^2-b^3)(a-b)}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2+\frac{5}{8}ab+\frac{1}{2}b^2\right)\cos(fx+e)^3+\left(-\frac{7}{8}ab-\frac{1}{2}b^2\right)\cos(fx+e)}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{5(3a}{}\right.}{(a-b)^4}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{3\,e^{i(fx+e)}a}{8(a^3-3a^2b+3a\,b^2-b^3)f(a-b)}-\frac{9\,e^{i(fx+e)}b}{8(a^3-3a^2b+3a\,b^2-b^3)f(a-b)}-\frac{3\,e^{-i(fx}}{8(a^4-4a^3b+6a^2b}$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*(1/3*a*cos(f*x+e)^3-1/3*b*cos(f*x+e
)^3-a*cos(f*x+e)-2*cos(f*x+e)*b)+b/(a-b)^4*(((-9/8*a^2+5/8*a*b+1/2*b^2)*co
s(f*x+e)^3+(-7/8*a*b-1/2*b^2)*cos(f*x+e))/(a*cos(f*x+e)^2-b*cos(f*x+e)^2+b
)^2+5/8*(3*a+4*b)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(164) = 328$.

Time = 0.17 (sec) , antiderivative size = 775, normalized size of antiderivative = 4.31

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 16*(3*a^3 - 2*a
^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^5 - 50*(3*a^2*b + a*b^2 - 4*b^3)*cos(
f*x + e)^3 + 15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*
b^2 + 4*b^3 + 2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(-b/(a - b))
*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b
)/((a - b)*cos(f*x + e)^2 + b)) - 30*(3*a*b^2 + 4*b^3)*cos(f*x + e))/((a^6
 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f
*x + e)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6
)*f*cos(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f),
 1/24*(8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 8*(3*a^3 - 2*a^2
*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^5 - 25*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*
x + e)^3 - 15*((3*a^3 - 2*a^2*b - 5*a*b^2 + 4*b^3)*cos(f*x + e)^4 + 3*a*b^
2 + 4*b^3 + 2*(3*a^2*b + a*b^2 - 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a - b))*ar
ctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b) - 15*(3*a*b^2 + 4*b^3)*cos(f
*x + e))/((a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5
+ b^6)*f*cos(f*x + e)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 +
 5*a*b^5 - b^6)*f*cos(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(164) = 328$.

Time = 0.81 (sec), antiderivative size = 541, normalized size of antiderivative = 3.01

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{a^6f^{17}\cos(fx+e)^3 - 6a^5bf^{17}\cos(fx+e)^3 + 15a^4b^2f^{17}\cos(fx+e)^3 - 20a^3b^3f^{17}\cos(fx+e)^3 + 15a}{3(a^9f^1}$$
$$+ \frac{5(3ab+4b^2)\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{8(a^4-4a^3b+6a^2b^2-4ab^3+b^4)\sqrt{ab-b^2}f}$$
$$- \frac{\frac{9a^2b\cos(fx+e)^3}{f} - \frac{5ab^2\cos(fx+e)^3}{f} - \frac{4b^3\cos(fx+e)^3}{f} + \frac{7ab^2\cos(fx+e)}{f} + \frac{4b^3\cos(fx+e)}{f}}{8(a^4-4a^3b+6a^2b^2-4ab^3+b^4)\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/3*(a^6*f^17*cos(f*x + e)^3 - 6*a^5*b*f^17*cos(f*x + e)^3 + 15*a^4*b^2*f^
17*cos(f*x + e)^3 - 20*a^3*b^3*f^17*cos(f*x + e)^3 + 15*a^2*b^4*f^17*cos(f
*x + e)^3 - 6*a*b^5*f^17*cos(f*x + e)^3 + b^6*f^17*cos(f*x + e)^3 - 3*a^6*
f^17*cos(f*x + e) + 9*a^5*b*f^17*cos(f*x + e) - 30*a^3*b^3*f^17*cos(f*x +
e) + 45*a^2*b^4*f^17*cos(f*x + e) - 27*a*b^5*f^17*cos(f*x + e) + 6*b^6*f^1
7*cos(f*x + e))/(a^9*f^18 - 9*a^8*b*f^18 + 36*a^7*b^2*f^18 - 84*a^6*b^3*f^
18 + 126*a^5*b^4*f^18 - 126*a^4*b^5*f^18 + 84*a^3*b^6*f^18 - 36*a^2*b^7*f^
18 + 9*a*b^8*f^18 - b^9*f^18) + 5/8*(3*a*b + 4*b^2)*arctan((a*cos(f*x + e)
 - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
+ b^4)*sqrt(a*b - b^2)*f) - 1/8*(9*a^2*b*cos(f*x + e)^3/f - 5*a*b^2*cos(f*
x + e)^3/f - 4*b^3*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x + e)/f + 4*b^3*cos(f
*x + e)/f)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a*cos(f*x + e)^2
- b*cos(f*x + e)^2 + b)^2)
```

Mupad [B] (verification not implemented)

Time = 11.38 (sec) , antiderivative size = 1154, normalized size of antiderivative = 6.41

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
- ((6*a*b^2 + 83*a^2*b + 16*a^3)/(12*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)) + (tan(e/2 + (f*x)/2)^2*(299*a*b^2 - 8*a^3 + 24*b^3))/(6*(a - b)*(3*a*
b^2 - 3*a^2*b + a^3 - b^3)) + (5*a*tan(e/2 + (f*x)/2)^12*(3*a*b + 4*b^2))/
(4*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^10*(28*a
*b^3 - 32*a^3*b + 8*a^4 + 8*b^4 + 93*a^2*b^2))/(2*a*(a - b)*(3*a*b^2 - 3*a
^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^6*(546*a*b^3 - 144*a^3*b + 56*a^4
 + 36*b^4 + 31*a^2*b^2))/(3*a*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (
tan(e/2 + (f*x)/2)^4*(1208*a*b^3 + 71*a^3*b - 96*a^4 + 48*b^4 + 344*a^2*b^
2))/(12*a*(a - b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) + (tan(e/2 + (f*x)/2)^8
*(1704*a*b^3 + 569*a^3*b - 176*a^4 + 144*b^4 - 666*a^2*b^2))/(12*a*(a - b)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(f*(tan(e/2 + (f*x)/2)^2*(8*a*b - a^2)
+ tan(e/2 + (f*x)/2)^12*(8*a*b - a^2) + a^2*tan(e/2 + (f*x)/2)^14 + tan(e/
2 + (f*x)/2)^4*(8*a*b - 3*a^2 + 16*b^2) + tan(e/2 + (f*x)/2)^10*(8*a*b - 3
*a^2 + 16*b^2) + tan(e/2 + (f*x)/2)^6*(3*a^2 - 16*a*b + 48*b^2) + tan(e/2
+ (f*x)/2)^8*(3*a^2 - 16*a*b + 48*b^2) + a^2)) - (5*b^(1/2)*atan((2*(tan(e
/2 + (f*x)/2)^2*((5*b^(1/2)*(3*a + 4*b)*(240*a^11*b + 320*a^2*b^10 - 2320*
a^3*b^9 + 7040*a^4*b^8 - 11200*a^5*b^7 + 8960*a^6*b^6 - 1120*a^7*b^5 - 448
0*a^8*b^4 + 4160*a^9*b^3 - 1600*a^10*b^2))/(16*a*(a - b)^(17/2)) - (25*b^(
1/2)*(a - 2*b)*(3*a + 4*b)^2*(1792*a^14*b - 128*a^15 + 256*a^2*b^13 - 3200
*a^3*b^12 + 18432*a^4*b^11 - 64768*a^5*b^10 + 154880*a^6*b^9 - 266112*a...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1627, normalized size of antiderivative = 9.04

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
(45*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**4*a**4 - 30*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqr
t(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b - 75*sqrt(b)*sqrt(a
 - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*
*4*a**2*b**2 + 60*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e +
 f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b**3 - 90*sqrt(b)*sqrt(a - b)*atan((s
qrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**4 - 30*
sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**2*a**3*b + 120*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt
(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2*b**2 + 45*sqrt(b)*sqrt
(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**4 + 60*s
qrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
a**3*b + 45*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/
2))/sqrt(b))*sin(e + f*x)**4*a**4 - 30*sqrt(b)*sqrt(a - b)*atan((sqrt(a -
b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b - 75*sqrt(b
)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
 + f*x)**4*a**2*b**2 + 60*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b**3 - 90*sqrt(b)*sqrt(a - b)
*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*
*4 - 30*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2...
```

3.82 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} - \frac{15\cos(e+fx)}{8(a-b)^3f} + \frac{\cos(e+fx)}{4(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} + \frac{5\cos(e+fx)}{8(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)}$$

output
```
-15/8*b^(1/2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/(a-b)^(7/2)/f-15/8*co
s(f*x+e)/(a-b)^3/f+1/4*cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^2+5/8*cos(f
*x+e)/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{7/2}}+\frac{2\cos(e+fx)\left(-4+\frac{4b^2}{(a+b+(a-b)\cos(2(e+fx)))^2}-\frac{\cdots}{a+b+(\cdots}\right.}{(a-b)^3}}{8f}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((15*Sqrt[b]*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a
- b)^(7/2) + (15*Sqrt[b]*ArcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/S
qrt[b]])/(a - b)^(7/2) + (2*Cos[e + f*x]*(-4 + (4*b^2)/(a + b + (a - b)*Co
s[2*(e + f*x)])^2 - (9*b)/(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^3)/
(8*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4147, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)}{\left(a+b\tan(e+fx)^2\right)^3}dx$$
$$\Big\downarrow \text{4147}$$
$$\frac{\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{5\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{4(a-b)}+\frac{\cos(e+fx)}{4(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{5\left(\frac{3\int \frac{\cos^2(e+fx)}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{2(a-b)}+\frac{\cos(e+fx)}{2(a-b)\left(a+b\sec^2(e+fx)-b\right)}\right)}{4(a-b)}+\frac{\cos(e+fx)}{4(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{a-b}\right)}{2(a-b)}+\frac{\cos(e+fx)}{2(a-b)\left(a+b\sec^2(e+fx)-b\right)}\right)}{4(a-b)}+\frac{\cos(e+fx)}{4(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{\cos(e+fx)}{a-b}\right)}{2(a-b)}+\frac{\cos(e+fx)}{2(a-b)\left(a+b\sec^2(e+fx)-b\right)}\right)}{4(a-b)}+\frac{\cos(e+fx)}{4(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(Cos[e + f*x]/(4*(a - b)*(a - b + b*Sec[e + f*x]^2)^2) + (5*((3*(-((Sqrt[b
]*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a - b)^(3/2)) - Cos[e + f*x
]/(a - b)))/(2*(a - b)) + Cos[e + f*x]/(2*(a - b)*(a - b + b*Sec[e + f*x]^
2))))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 6.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)}{a^3-3a^2b+3a\,b^2-b^3}+\frac{b\left(\frac{\left(-\frac{9a}{8}+\frac{9b}{8}\right)\cos(fx+e)^3-\frac{7\cos(fx+e)b}{8}}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{15\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8\sqrt{b(a-b)}}\right)}{(a-b)^3}}{f}$
default	$\frac{-\frac{\cos(fx+e)}{a^3-3a^2b+3a\,b^2-b^3}+\frac{b\left(\frac{\left(-\frac{9a}{8}+\frac{9b}{8}\right)\cos(fx+e)^3-\frac{7\cos(fx+e)b}{8}}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{15\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8\sqrt{b(a-b)}}\right)}{(a-b)^3}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{e^{-i(fx+e)}}{2(a^3-3a^2b+3a\,b^2-b^3)f}-\frac{b\left(-9a\,e^{7i(fx+e)}+9b\,e^{7i(fx+e)}-27a\,e^{5i(fx+e)}-b\,e^{5i(fx+e}\right.}{4(-a+b)(a^2-2ab+b^2)\left(-a\,e^{4i(fx+e)}+b\,e^{4i(\right.}}$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-cos(f*x+e)/(a^3-3*a^2*b+3*a*b^2-b^3)+b/(a-b)^3*(((-9/8*a+9/8*b)*cos(
f*x+e)^3-7/8*cos(f*x+e)*b)/(a*cos(f*x+e)^2-b*cos(f*x+e)^2+b)^2+15/8/(b*(a-
b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(122) = 244$.

Time = 0.16 (sec) , antiderivative size = 556, normalized size of antiderivative = 4.03

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$

$$= \left[-\frac{16\,(a^2-2\,ab+b^2)\cos{(fx+e)}^5+50\,(ab-b^2)\cos{(fx+e)}^3+30\,b^2\cos{(fx+e)}+15\,\big((a^2-2\,ab+}{16\,\big((a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)f\cos{(fx+e)}^4+2\,(a^4b-}\right.$$

$$-\frac{8\,(a^2-2\,ab+b^2)\cos{(fx+e)}^5+25\,(ab-b^2)\cos{(fx+e)}^3+15\,b^2\cos{(fx+e)}+15\,\big((a^2-2\,ab+b^2}{8\,\big((a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)f\cos{(fx+e)}^4+2\,(a^4b-4\,a^3b^2+6}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 50*(a*b - b^2)*cos(f*x + e
)^3 + 30*b^2*cos(f*x + e) + 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*
b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(-b/(a - b))*log(-((a - b)*cos(f*x + e)
^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2
+ b)))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*
x + e)^4 + 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e
)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f), -1/8*(8*(a^2 - 2*a*b + b^2
)*cos(f*x + e)^5 + 25*(a*b - b^2)*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) + 1
5*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2
)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b))/((a^5 -
 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4 + 2*(
a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^3*b^2
 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.56

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= -\frac{f^5\cos(fx+e)}{a^3f^6-3\,a^2bf^6+3\,ab^2f^6-b^3f^6}+\frac{15\,b\arctan\left(\frac{a\cos(fx+e)-b\cos(fx+e)}{\sqrt{ab-b^2}}\right)}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab-b^2}f}$$
$$-\frac{\frac{9\,ab\cos(fx+e)^3}{f}-\frac{9\,b^2\cos(fx+e)^3}{f}+\frac{7\,b^2\cos(fx+e)}{f}}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-f^5*cos(f*x + e)/(a^3*f^6 - 3*a^2*b*f^6 + 3*a*b^2*f^6 - b^3*f^6) + 15/8*b
*arctan((a*cos(f*x + e) - b*cos(f*x + e))/sqrt(a*b - b^2))/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*sqrt(a*b - b^2)*f) - 1/8*(9*a*b*cos(f*x + e)^3/f - 9*b^2
*cos(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*(a*cos(f*x + e)^2 - b*cos(f*x + e)^2 + b)^2)
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 780, normalized size of antiderivative = 5.65

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
- ((9*a*b + 8*a^2 - 2*b^2)/(4*(a - b)*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f
*x)/2)^6*(16*a^4 - 41*a^3*b - 40*a*b^3 + 8*b^4 + 27*a^2*b^2))/(2*a^2*(a -
b)*(a^2 - 2*a*b + b^2)) + (tan(e/2 + (f*x)/2)^4*(40*a*b^3 - 64*a^3*b + 24*
a^4 - 8*b^4 + 53*a^2*b^2))/(2*a^2*(a - b)*(a^2 - 2*a*b + b^2)) + (tan(e/2
+ (f*x)/2)^8*(24*a*b^2 - 9*a^2*b + 8*a^3 - 8*b^3))/(4*a*(a - b)*(a^2 - 2*a
*b + b^2)) + (tan(e/2 + (f*x)/2)^2*(27*a*b^2 + 23*a^2*b - 16*a^3 - 4*b^3))
/(2*a*(a - b)*(a^2 - 2*a*b + b^2)))/(f*(tan(e/2 + (f*x)/2)^2*(8*a*b - 3*a^
2) + tan(e/2 + (f*x)/2)^8*(8*a*b - 3*a^2) + a^2*tan(e/2 + (f*x)/2)^10 + ta
n(e/2 + (f*x)/2)^4*(2*a^2 - 8*a*b + 16*b^2) + tan(e/2 + (f*x)/2)^6*(2*a^2
- 8*a*b + 16*b^2) + a^2)) - (15*b^(1/2)*atan(((a - b)^7*(2*tan(e/2 + (f*x)
/2)^2*((b^(1/2)*(225*a^8*b + 225*a^2*b^7 - 1350*a^3*b^6 + 3375*a^4*b^5 - 4
500*a^5*b^4 + 3375*a^6*b^3 - 1350*a^7*b^2))/(a*(a - b)^(13/2)) + (225*b^(1
/2)*(a - 2*b)*(128*a^12 - 1408*a^11*b + 256*a^2*b^10 - 2432*a^3*b^9 + 1036
8*a^4*b^8 - 26112*a^5*b^7 + 43008*a^6*b^6 - 48384*a^7*b^5 + 37632*a^8*b^4
- 19968*a^9*b^3 + 6912*a^10*b^2))/(512*a*(a - b)^(21/2))) + (225*b^(1/2)*(
a - 2*b)*(1152*a^11*b - 128*a^12 + 128*a^3*b^9 - 1152*a^4*b^8 + 4608*a^5*b
^7 - 10752*a^6*b^6 + 16128*a^7*b^5 - 16128*a^8*b^4 + 10752*a^9*b^3 - 4608*
a^10*b^2))/(256*a*(a - b)^(21/2))))/(225*a^8*b + 225*a^2*b^7 - 1350*a^3*b^
6 + 3375*a^4*b^5 - 4500*a^5*b^4 + 3375*a^6*b^3 - 1350*a^7*b^2)))/(8*f*(a -
 b)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 8454, normalized size of antiderivative = 61.26

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(30*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**4*tan(e + f*x)**4*a**6*b**2 - 75*sqrt(b)*sqrt(a - b)*at
an((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*tan(e
 + f*x)**4*a**5*b**3 + 90*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*tan(e + f*x)**4*a**4*b**4 - 75*
sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**4*tan(e + f*x)**4*a**3*b**5 + 30*sqrt(b)*sqrt(a - b)*atan((
sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*tan(e + f
*x)**4*a**2*b**6 + 60*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan(
(e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*tan(e + f*x)**2*a**7*b - 150*sqrt(b
)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
 + f*x)**4*tan(e + f*x)**2*a**6*b**2 + 180*sqrt(b)*sqrt(a - b)*atan((sqrt(
a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*tan(e + f*x)**
2*a**5*b**3 - 150*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e +
 f*x)/2))/sqrt(b))*sin(e + f*x)**4*tan(e + f*x)**2*a**4*b**4 + 60*sqrt(b)*
sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e +
 f*x)**4*tan(e + f*x)**2*a**3*b**5 + 30*sqrt(b)*sqrt(a - b)*atan((sqrt(a -
 b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**8 - 75*sqrt(b)
*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
+ f*x)**4*a**7*b + 90*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*t...
```

3.83 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{a^3f} - \frac{b\sec(e+fx)}{4a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(7a-4b)b\sec(e+fx)}{8a^2(a-b)^2f\left(a-b+b\sec^2(e+fx)\right)}$$

output
```
-1/8*b^(1/2)*(15*a^2-20*a*b+8*b^2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/
a^3/(a-b)^(5/2)/f-arctanh(cos(f*x+e))/a^3/f-1/4*b*sec(f*x+e)/a/(a-b)/f/(a-
b+b*sec(f*x+e)^2)^2-1/8*(7*a-4*b)*b*sec(f*x+e)/a^2/(a-b)^2/f/(a-b+b*sec(f*
x+e)^2)
```

Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.49

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{\frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{5/2}} + \frac{8a^2b^2\cos(e}{(a-b)^2(a+b+(a-b)\,c}}{8a^3f}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e +
 f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*Arc
Tan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(5/2) + (8*
a^2*b^2*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - (
2*a*(9*a - 4*b)*b*Cos[e + f*x])/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x
)])) - 8*Log[Cos[(e + f*x)/2]] + 8*Log[Sin[(e + f*x)/2]])/(8*a^3*f)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4147, 25, 316, 25, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(e+fx)\left(a+b\tan(e+fx)^2\right)^3}dx$$
$$\Big\downarrow \; 4147$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int -\frac{-3b\sec^2(e+fx)+4a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{4a(a-b)}-\frac{b\sec(e+fx)}{4a(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{-3b\sec^2(e+fx)+4a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{4a(a-b)}-\frac{b\sec(e+fx)}{4a(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{-\frac{\frac{b(7a-4b)\sec(e+fx)}{2a(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{\int -\frac{8a^2-9ba+4b^2-(7a-4b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}}{4a(a-b)}-\frac{b\sec(e+fx)}{4a(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\frac{\int \frac{8a^2-9ba+4b^2-(7a-4b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}+\frac{b(7a-4b)\sec(e+fx)}{2a(a-b)\left(a+b\sec^2(e+fx)-b\right)}}{4a(a-b)}-\frac{b\sec(e+fx)}{4a(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{\frac{\frac{b\left(15a^2-20ab+8b^2\right)\int \frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}+\frac{8(a-b)^2\int \frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}}{2a(a-b)}+\frac{b(7a-4b)\sec(e+fx)}{2a(a-b)\left(a+b\sec^2(e+fx)-b\right)}}{4a(a-b)}-\frac{b\sec(e+fx)}{4a(a-b)(a+b\sec^2(e+f}}{f}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\frac{\frac{8(a-b)^2\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a(a-b)}+\frac{b(7a-4b)\sec(e+fx)}{2a(a-b)\left(a+b\sec^2(e+fx)-b\right)}}{4a(a-b)}-\frac{b\sec(e+fx)}{4a(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{\frac{\sqrt{b}(15a^2-20ab+8b^2)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{8(a-b)^2\text{arctanh}(\sec(e+fx))}{a}}{2a(a-b)}+\frac{b(7a-4b)\sec(e+fx)}{2a(a-b)\left(a+b\sec^2(e+fx)-b\right)}}{4a(a-b)}-\frac{b\sec(e+fx)}{4a(a-b)(a+b\sec^2(e+fx)-b)^2}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Sec[e + f*x])/(a*(a - b)*(a - b + b*Sec[e + f*x]^2)^2) - (((Sqrt[
b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(
a*Sqrt[a - b]) + (8*(a - b)^2*ArcTanh[Sec[e + f*x]])/a)/(2*a*(a - b)) + ((
7*a - 4*b)*b*Sec[e + f*x])/(2*a*(a - b)*(a - b + b*Sec[e + f*x]^2)))/(4*a*
(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a^3}-\frac{\ln(\cos(fx+e)+1)}{2a^3}+\frac{b\left(\frac{-\frac{(9a-4b)a\cos(fx+e)^3}{8(a-b)}-\frac{ab(7a-4b)\cos(fx+e)}{8\left(a^2-2ab+b^2\right)}}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{\left(15a^2-20ab+8b^2\right)\arctan\left(\frac{(a-b)\cos(}{\sqrt{b(a-}}\right.}{8\left(a^2-2ab+b^2\right)\sqrt{b(a-b)}}\right.}{a^3}}{f}$
default	$\frac{\frac{\ln(\cos(fx+e)-1)}{2a^3}-\frac{\ln(\cos(fx+e)+1)}{2a^3}+\frac{b\left(\frac{-\frac{(9a-4b)a\cos(fx+e)^3}{8(a-b)}-\frac{ab(7a-4b)\cos(fx+e)}{8\left(a^2-2ab+b^2\right)}}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{\left(15a^2-20ab+8b^2\right)\arctan\left(\frac{(a-b)\cos(}{\sqrt{b(a-}}\right.}{8\left(a^2-2ab+b^2\right)\sqrt{b(a-b)}}\right.}{a^3}}{f}$
risch	$-\frac{b\left(9a^2e^{7i(fx+e)}-13ab\,e^{7i(fx+e)}+4b^2e^{7i(fx+e)}+27a^2e^{5i(fx+e)}-11ab\,e^{5i(fx+e)}-4b^2e^{5i(fx+e)}+27a^2e^{3i(fx+e)}-11a\right.}{4(a^2-2ab+b^2)\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(\right.}}$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/a^3*ln(cos(f*x+e)-1)-1/2/a^3*ln(cos(f*x+e)+1)+b/a^3*((-1/8*(9*a-4
*b)*a/(a-b)*cos(f*x+e)^3-1/8*a*b*(7*a-4*b)/(a^2-2*a*b+b^2)*cos(f*x+e))/(a*
cos(f*x+e)^2-b*cos(f*x+e)^2+b)^2+1/8*(15*a^2-20*a*b+8*b^2)/(a^2-2*a*b+b^2)
/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(152) = 304.

Time = 0.29 (sec) , antiderivative size = 1050, normalized size of antiderivative = 6.33

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 50*
a^3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 20*a*
b^3 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(f*x + e)^2)
*sqrt(-b/(a - b))*log(((a - b)*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))
*cos(f*x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^2 - 4*a*b^3)
*cos(f*x + e) + 8*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e
)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(
f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) - 8*((a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2
*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7
- 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b -
 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3
+ a^3*b^4)*f), -1/8*((9*a^3*b - 13*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^3 + ((1
5*a^4 - 50*a^3*b + 63*a^2*b^2 - 36*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*
b^2 - 20*a*b^3 + 8*b^4 + 2*(15*a^3*b - 35*a^2*b^2 + 28*a*b^3 - 8*b^4)*cos(
f*x + e)^2)*sqrt(b/(a - b))*arctan(-(a - b)*sqrt(b/(a - b))*cos(f*x + e)/b
) + (7*a^2*b^2 - 4*a*b^3)*cos(f*x + e) + 4*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4
*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) - 4*((a^4
 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*b^2 - 2*a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(152) = 304$.

Time = 0.80 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.03

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b - 20*a*b^2 + 8*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x +
e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - 2*a^4*b
+ a^3*b^2)*sqrt(a*b - b^2)) + 2*(9*a^3*b - 6*a^2*b^2 + 27*a^3*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - 68*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + 120*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 48*b
^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^
3/(cos(f*x + e) + 1)^3 - 28*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1
)^3 + 16*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 - 2*a^4*b
+ a^3*b^2)*(a + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)
^2) - 4*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 1844, normalized size of antiderivative = 11.11

$$\int \frac{\csc(e + fx)}{(a + b\tan^2(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^3),x)
```

output
```
log(tan(e/2 + (f*x)/2))/(a^3*f) - ((3*(3*a*b - 2*b^2))/(4*a*(a^2 - 2*a*b +
 b^2)) + (3*tan(e/2 + (f*x)/2)^4*(40*a*b^3 + 9*a^3*b - 16*b^4 - 30*a^2*b^2
))/(4*a^3*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*x)/2)^6*(9*a^2*b - 28*a*b^2
 + 16*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)) - (tan(e/2 + (f*x)/2)^2*(27*a^2*b
- 68*a*b^2 + 32*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)))/(f*(tan(e/2 + (f*x)/2)^
2*(8*a*b - 4*a^2) + tan(e/2 + (f*x)/2)^6*(8*a*b - 4*a^2) + a^2*tan(e/2 + (
f*x)/2)^8 + tan(e/2 + (f*x)/2)^4*(6*a^2 - 16*a*b + 16*b^2) + a^2)) + (b^(1
/2)*atan(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(15*a^2 - 20*a*b + 8*b^2)^3*(4
096*a^15*b - 128*a^16 + 6144*a^7*b^9 - 46080*a^8*b^8 + 150784*a^9*b^7 - 28
1216*a^10*b^6 + 327168*a^11*b^5 - 243584*a^12*b^4 + 113920*a^13*b^3 - 3110
4*a^14*b^2))/(32768*a^9*(a - b)^(15/2)*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6*
b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)) - (b^(1/2)*(15*a^2 - 20*a*b +
 8*b^2)*(1536*a*b^9 + 720*a^9*b - 11520*a^2*b^8 + 37760*a^3*b^7 - 70400*a^
4*b^6 + 81384*a^5*b^5 - 59564*a^6*b^4 + 26864*a^7*b^3 - 6780*a^8*b^2))/(12
8*a^3*(a - b)^(5/2)*(a^11 - 6*a^10*b + a^5*b^6 - 6*a^6*b^5 + 15*a^7*b^4 -
20*a^8*b^3 + 15*a^9*b^2)))*(3072*a*b^4 - 1090*a^4*b + 111*a^5 - 768*b^5 -
4752*a^2*b^3 + 3424*a^3*b^2))/(2*a^5*(a - b)^(13/2)*(960*a*b^4 - 1055*a^4*
b + 256*a^5 - 192*b^5 - 1920*a^2*b^3 + 1960*a^3*b^2)) + (((576*a*b^6 - 64*
b^7 - 1920*a^2*b^5 + 3160*a^3*b^4 - 2625*a^4*b^3 + 900*a^5*b^2)/(8*(a^11 -
 6*a^10*b + a^5*b^6 - 6*a^6*b^5 + 15*a^7*b^4 - 20*a^8*b^3 + 15*a^9*b^2)...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1906, normalized size of antiderivative = 11.48

$$\int \frac{\csc(e + fx)}{(a + b\tan^2(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
(15*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**4*a**4 - 50*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqr
t(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b + 63*sqrt(b)*sqrt(a
 - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*
*4*a**2*b**2 - 36*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e +
 f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b**3 + 8*sqrt(b)*sqrt(a - b)*atan((sq
rt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*b**4 - 30*s
qrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
sin(e + f*x)**2*a**4 + 70*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**3*b - 56*sqrt(b)*sqrt(a - b)
*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*
*2*b**2 + 16*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)
/2))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 15*sqrt(b)*sqrt(a - b)*atan((sqrt(a
 - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**4 - 20*sqrt(b)*sqrt(a - b)*a
tan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**3*b + 8*sqrt(b)*s
qrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**2*b**
2 + 15*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/s
qrt(b))*sin(e + f*x)**4*a**4 - 50*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) +
sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b + 63*sqrt(b)*sqr
t(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e +...
```

3.84 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 205

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{\sqrt{b}(15a^2-40ab+24b^2)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{3/2}f} - \frac{(a-6b)\text{arctanh}(\cos(e+fx))}{2a^4f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{3b\sec(e+fx)}{4a^2f\left(a-b+b\sec^2(e+fx)\right)^2} - \frac{(11a-12b)b\sec(e+fx)}{8a^3(a-b)f\left(a-b+b\sec^2(e+fx)\right)}$$

output
```
-1/8*b^(1/2)*(15*a^2-40*a*b+24*b^2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))
/a^4/(a-b)^(3/2)/f-1/2*(a-6*b)*arctanh(cos(f*x+e))/a^4/f-1/2*cot(f*x+e)*cs
c(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^2-3/4*b*sec(f*x+e)/a^2/f/(a-b+b*sec(f*x+
e)^2)^2-1/8*(11*a-12*b)*b*sec(f*x+e)/a^3/(a-b)/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.40

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{\sqrt{b}(15a^2-40ab+24b^2)\arctan\left(\frac{\sqrt{a-b}-\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}(15a^2-40ab+24b^2)\arctan\left(\frac{\sqrt{a-b}+\sqrt{a}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{(a-b)^{3/2}} + \frac{8a^2b^2\cos}{(a-b)(a+b+(a-b)}}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[a - b] - Sqrt[a]*Tan[(e
+ f*x)/2])/Sqrt[b]])/(a - b)^(3/2) + (Sqrt[b]*(15*a^2 - 40*a*b + 24*b^2)*A
rcTan[(Sqrt[a - b] + Sqrt[a]*Tan[(e + f*x)/2])/Sqrt[b]])/(a - b)^(3/2) + (
8*a^2*b^2*Cos[e + f*x])/((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2) - (
2*a*(9*a - 8*b)*b*Cos[e + f*x])/((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]
)) - a*Csc[(e + f*x)/2]^2 - 4*(a - 6*b)*Log[Cos[(e + f*x)/2]] + 4*(a - 6*b
)*Log[Sin[(e + f*x)/2]] + a*Sec[(e + f*x)/2]^2)/(8*a^4*f)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4147, 373, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)^3}dx$$
$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a-b)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 373$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\int \frac{-5b\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{2a}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\frac{3b\sec(e+fx)}{2a\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\int -\frac{2(a-b)\left(-9b\sec^2(e+fx)+2a-3b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{4a(a-b)}}{2a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\frac{\int \frac{-9b\sec^2(e+fx)+2a-3b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{2a}+\frac{3b\sec(e+fx)}{2a\left(a+b\sec^2(e+fx)-b\right)^2}}{2a}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\frac{\frac{b(11a-12b)\sec(e+fx)}{2a(a-b)\left(a+b\sec^2(e+fx)-b\right)}-\frac{\int -\frac{4a^2-17ba+12b^2-(11a-12b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}}{2a}+\frac{3b\sec(e+f\ldots}{2a\left(a+b\sec^2(e+\ldots\right.}}{2a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\frac{\frac{\int \frac{4a^2-17ba+12b^2-(11a-12b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}+\frac{b(11a-12b)\sec(e+fx)}{2a(a-b)\left(a+b\sec^2(e+fx)-b\right)}}{2a}+\frac{3b\sec(e+fx)}{2a\left(a+b\sec^2(e+fx\ldots\right.}}{2a}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\frac{\frac{\frac{b\left(15a^2-40ab+24b^2\right)\int\frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}+\frac{4(a-6b)(a-b)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}}{2a(a-b)}+\frac{b(1}{2a(a-}}{2a}}{2a}}{f}$$

$\big\downarrow$ 218

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\frac{\frac{\frac{4(a-6b)(a-b)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{2a(a-b)}+\frac{b(11a-12b}{2a(a-b)\left(a+b\right.}}{2a}}{2a}}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^2}-\frac{\frac{\frac{\frac{\sqrt{b}\left(15a^2-40ab+24b^2\right)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{4(a-6b)(a-b)\text{arctanh}(\sec(e+fx))}{a}}{2a(a-b)}+\frac{b(11a-12b)\sec(}{2a(a-b)\left(a+b\sec^2(\right.}}{2a}}{2a}}{f}$$

input

```
Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output

```
(Sec[e + f*x]/(2*a*(1 - Sec[e + f*x]^2)*(a - b + b*Sec[e + f*x]^2)^2) - ((
3*b*Sec[e + f*x])/(2*a*(a - b + b*Sec[e + f*x]^2)^2) + (((Sqrt[b]*(15*a^2
 - 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b]])/(a*Sqrt[a -
 b]) + (4*(a - 6*b)*(a - b)*ArcTanh[Sec[e + f*x]])/a)/(2*a*(a - b)) + ((11
*a - 12*b)*b*Sec[e + f*x])/(2*a*(a - b)*(a - b + b*Sec[e + f*x]^2)))/(2*a)
)/(2*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{b\left(\frac{-\frac{(9a-8b)a\cos(fx+e)^3}{8}-\frac{ab(7a-8b)\cos(fx+e)}{8(a-b)}}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{\left(15a^2-40ab+24b^2\right)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8(a-b)\sqrt{b(a-b)}}\right)}{a^4}+\frac{1}{4a^3(\cos(fx+e)-1)}+\frac{(a-}{}}{f}$
default	$\frac{\frac{b\left(\frac{-\frac{(9a-8b)a\cos(fx+e)^3}{8}-\frac{ab(7a-8b)\cos(fx+e)}{8(a-b)}}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{\left(15a^2-40ab+24b^2\right)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8(a-b)\sqrt{b(a-b)}}\right)}{a^4}+\frac{1}{4a^3(\cos(fx+e)-1)}+\frac{(a-}{}}{f}$
risch	Expression too large to display

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/a^4*((-1/8*(9*a-8*b)*a*cos(f*x+e)^3-1/8*a*b*(7*a-8*b)/(a-b)*cos(f*x
+e))/(a*cos(f*x+e)^2-b*cos(f*x+e)^2+b)^2+1/8*(15*a^2-40*a*b+24*b^2)/(a-b)/
(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))+1/4/a^3/(cos(f*x
+e)-1)+1/4*(a-6*b)/a^4*ln(cos(f*x+e)-1)+1/4/a^3/(cos(f*x+e)+1)+1/4/a^4*(-a
+6*b)*ln(cos(f*x+e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(187) = 374$.

Time = 0.29 (sec) , antiderivative size = 1419, normalized size of antiderivative = 6.92

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(4*a^4 - 21*a^3*b + 29*a^2*b^2 - 12*a*b^3)*cos(f*x + e)^5 + 2*(17
*a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^3 - ((15*a^4 - 70*a^3*b + 119
*a^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x + e)^6 - (15*a^4 - 100*a^3*b + 229*a
^2*b^2 - 216*a*b^3 + 72*b^4)*cos(f*x + e)^4 - 15*a^2*b^2 + 40*a*b^3 - 24*b
^4 - (30*a^3*b - 125*a^2*b^2 + 168*a*b^3 - 72*b^4)*cos(f*x + e)^2)*sqrt(-b
/(a - b))*log(-((a - b)*cos(f*x + e)^2 - 2*(a - b)*sqrt(-b/(a - b))*cos(f*
x + e) - b)/((a - b)*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 - 12*a*b^3)*cos(
f*x + e) - 4*((a^4 - 9*a^3*b + 21*a^2*b^2 - 19*a*b^3 + 6*b^4)*cos(f*x + e)
^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3 + 18*b^4)*cos(f*x + e)^4 - a^
2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*b^2 + 33*a*b^3 - 18*b^4)*cos(f
*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 9*a^3*b + 21*a^2*b^2 -
19*a*b^3 + 6*b^4)*cos(f*x + e)^6 - (a^4 - 11*a^3*b + 37*a^2*b^2 - 45*a*b^3
 + 18*b^4)*cos(f*x + e)^4 - a^2*b^2 + 7*a*b^3 - 6*b^4 - (2*a^3*b - 17*a^2*
b^2 + 33*a*b^3 - 18*b^4)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a
^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*
a^5*b^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*
f*cos(f*x + e)^2 - (a^5*b^2 - a^4*b^3)*f), 1/8*((4*a^4 - 21*a^3*b + 29*a^2
*b^2 - 12*a*b^3)*cos(f*x + e)^5 + (17*a^3*b - 40*a^2*b^2 + 24*a*b^3)*cos(f
*x + e)^3 - ((15*a^4 - 70*a^3*b + 119*a^2*b^2 - 88*a*b^3 + 24*b^4)*cos(f*x
 + e)^6 - (15*a^4 - 100*a^3*b + 229*a^2*b^2 - 216*a*b^3 + 72*b^4)*cos(f...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(187) = 374$.

Time = 0.74 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.84

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b - 40*a*b^2 + 24*b^3)*arctan(-(a*cos(f*x + e) - b*cos(f*x +
 e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/((a^5 - a^4*b)*
sqrt(a*b - b^2)) + 2*(9*a^3*b - 10*a^2*b^2 + 27*a^3*b*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 80*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 56*
a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2 - 102*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
1)^2 + 152*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*b^4*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*
x + e) + 1)^3 - 32*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 24*
a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 - a^4*b)*(a + 2*a*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 2*(a - 6*b)*log
(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/a^4 - (a - 2*a*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 12*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(c
os(f*x + e) + 1)/(a^4*(cos(f*x + e) - 1)) + (cos(f*x + e) - 1)/(a^3*(cos(f
*x + e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 1652, normalized size of antiderivative = 8.06

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^3),x)
```

output
```
tan(e/2 + (f*x)/2)^2/(8*a^3*f) - (a^2/2 + (tan(e/2 + (f*x)/2)^4*(96*a*b^2
- 38*a^2*b + 3*a^3 - 64*b^3))/(a - b) + (tan(e/2 + (f*x)/2)^8*(64*a*b^2 -
19*a^2*b + a^3 - 48*b^3))/(2*(a - b)) - (tan(e/2 + (f*x)/2)^2*(14*a*b^2 -
15*a^2*b + 2*a^3))/(a - b) - (tan(e/2 + (f*x)/2)^6*(2*a^4 - 33*a^3*b - 152
*a*b^3 + 80*b^4 + 106*a^2*b^2))/(a*(a - b)))/(f*(4*a^5*tan(e/2 + (f*x)/2)^
2 + 4*a^5*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^6*(24*a^5 - 64*a^4*b
+ 64*a^3*b^2) + tan(e/2 + (f*x)/2)^4*(32*a^4*b - 16*a^5) + tan(e/2 + (f*x)
/2)^8*(32*a^4*b - 16*a^5))) + (log(tan(e/2 + (f*x)/2))*(a - 6*b))/(2*a^4*f
) + (b^(1/2)*atan(((tan(e/2 + (f*x)/2)^2*((((b^(3/2)*(15*a^2 - 40*a*b + 24
*b^2)^3*(128*a^16 - 3712*a^15*b + 6144*a^10*b^6 - 27648*a^11*b^5 + 49408*a
^12*b^4 - 43904*a^13*b^3 + 19584*a^14*b^2))/(32768*a^12*(a - b)^(9/2)*(3*a
^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)) + (b^(1/2)*(15*a^2 - 40*a*b + 24*b^2)
*(360*a^9*b - 13824*a^2*b^8 + 66816*a^3*b^7 - 132864*a^4*b^6 + 139776*a^5*
b^5 - 83240*a^6*b^4 + 27836*a^7*b^3 - 4860*a^8*b^2))/(128*a^4*(a - b)^(3/2
)*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))*(63*a^6 - 1013*a^5*b - 9600*a*
b^5 + 2304*b^6 + 15792*a^2*b^4 - 12888*a^3*b^3 + 5342*a^4*b^2))/(2*a^5*(a
- b)^(9/2)*(5760*a*b^4 - 735*a^4*b + 64*a^5 - 1728*b^5 - 6960*a^2*b^3 + 36
00*a^3*b^2)) - (((6912*a*b^6 - 1728*b^7 - 10800*a^2*b^5 + 8240*a^3*b^4 - 3
075*a^4*b^3 + 450*a^5*b^2)/(8*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)) + (
b*(15*a^2 - 40*a*b + 24*b^2)^2*(1936*a^12*b - 64*a^13 + 18432*a^6*b^7 -...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2114, normalized size of antiderivative = 10.31

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
(15*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**6*a**4 - 70*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqr
t(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**6*a**3*b + 119*sqrt(b)*sqrt(
a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)
**6*a**2*b**2 - 88*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e
+ f*x)/2))/sqrt(b))*sin(e + f*x)**6*a*b**3 + 24*sqrt(b)*sqrt(a - b)*atan((
sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**6*b**4 - 30
*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**4*a**4 + 110*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(
a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b - 128*sqrt(b)*sqrt(a
- b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**
4*a**2*b**2 + 48*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e +
f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b**3 + 15*sqrt(b)*sqrt(a - b)*atan((sq
rt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**4 - 40*s
qrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*
sin(e + f*x)**2*a**3*b + 24*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a
)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2*b**2 + 15*sqrt(b)*sqrt(a
 - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*
*6*a**4 - 70*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)
/2))/sqrt(b))*sin(e + f*x)**6*a**3*b + 119*sqrt(b)*sqrt(a - b)*atan((sq...
```

3.85 $\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 259

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{3\sqrt{b}(5a^2-20ab+16b^2)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{8a^5\sqrt{a-b}f} - \frac{3(a^2-12ab+16b^2)\operatorname{arctanh}(\cos(e+fx))}{8a^5f} - \frac{(5a-8b)\cot(e+fx)\csc(e+fx)}{8a^2f\,(a-b+b\sec^2(e+fx))^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\,(a-b+b\sec^2(e+fx))^2} - \frac{(7a-12b)b\sec(e+fx)}{8a^3f\,(a-b+b\sec^2(e+fx))^2} - \frac{3(a-2b)b\sec(e+fx)}{2a^4f\,(a-b+b\sec^2(e+fx))}$$

output
```
-3/8*b^(1/2)*(5*a^2-20*a*b+16*b^2)*arctan(b^(1/2)*sec(f*x+e)/(a-b)^(1/2))/
a^5/(a-b)^(1/2)/f-3/8*(a^2-12*a*b+16*b^2)*arctanh(cos(f*x+e))/a^5/f-1/8*(5
*a-8*b)*cot(f*x+e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^2-1/4*cot(f*x+e)^
3*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^2-1/8*(7*a-12*b)*b*sec(f*x+e)/a^3/f/
(a-b+b*sec(f*x+e)^2)^2-3/2*(a-2*b)*b*sec(f*x+e)/a^4/f/(a-b+b*sec(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 6.81 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.81

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= -\frac{3\sqrt{a-b}\sqrt{b}(5a^2-20ab+16b^2)\arctan\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{8a^5(-a+b)f}$$
$$-\frac{3\sqrt{a-b}\sqrt{b}(5a^2-20ab+16b^2)\arctan\left(\frac{\sec\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{a-b}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{b}}\right)}{8a^5(-a+b)f}$$
$$+\frac{b^2\cos(e+fx)}{a^3f(a+b+a\cos(2(e+fx))-b\cos(2(e+fx)))^2}$$
$$-\frac{3(3ab\cos(e+fx)-4b^2\cos(e+fx))}{4a^4f(a+b+a\cos(2(e+fx))-b\cos(2(e+fx)))}-\frac{3(a-4b)\csc^2\left(\frac{1}{2}(e+fx)\right)}{32a^4f}$$
$$-\frac{\csc^4\left(\frac{1}{2}(e+fx)\right)}{64a^3f}-\frac{3(a^2-12ab+16b^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{8a^5f}$$
$$+\frac{3(a^2-12ab+16b^2)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{8a^5f}$$
$$+\frac{3(a-4b)\sec^2\left(\frac{1}{2}(e+fx)\right)}{32a^4f}+\frac{\sec^4\left(\frac{1}{2}(e+fx)\right)}{64a^3f}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sec[(e + f*x)/2]
*(Sqrt[a - b]*Cos[(e + f*x)/2] - Sqrt[a]*Sin[(e + f*x)/2]))/Sqrt[b]])/(8*a
^5*(-a + b)*f) - (3*Sqrt[a - b]*Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(
Sec[(e + f*x)/2]*(Sqrt[a - b]*Cos[(e + f*x)/2] + Sqrt[a]*Sin[(e + f*x)/2])
)/Sqrt[b]])/(8*a^5*(-a + b)*f) + (b^2*Cos[e + f*x])/(a^3*f*(a + b + a*Cos[
2*(e + f*x)] - b*Cos[2*(e + f*x)])^2) - (3*(3*a*b*Cos[e + f*x] - 4*b^2*Cos
[e + f*x]))/(4*a^4*f*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) -
(3*(a - 4*b)*Csc[(e + f*x)/2]^2)/(32*a^4*f) - Csc[(e + f*x)/2]^4/(64*a^3*f
) - (3*(a^2 - 12*a*b + 16*b^2)*Log[Cos[(e + f*x)/2]])/(8*a^5*f) + (3*(a^2
- 12*a*b + 16*b^2)*Log[Sin[(e + f*x)/2]])/(8*a^5*f) + (3*(a - 4*b)*Sec[(e
+ f*x)/2]^2)/(32*a^4*f) + Sec[(e + f*x)/2]^4/(64*a^3*f)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4147, 25, 372, 402, 25, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \ 4147$$

$$\frac{\int -\frac{\sec^4(e+fx)}{\left(1-\sec^2(e+fx)\right)^3\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{\left(1-\sec^2(e+fx)\right)^3\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 372$$

$$\frac{\frac{\int \frac{(4a-7b)\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{4a}-\frac{\sec(e+fx)}{4a\left(1-\sec^2(e+fx)\right)^2\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\int -\frac{(3a-8b)(a-b)-5(5a-8b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{2a}+\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}}{4a}-\frac{\sec(e+fx)}{4a\left(1-\sec^2(e+fx)\right)^2\left(a+b\sec^2(e+fx)-b\right)^2}}{f}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\int \frac{(3a-8b)(a-b)-5(5a-8b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^3}d\sec(e+fx)}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^2}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\frac{b(7a-12b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\int -\frac{12(a-b)\left((a-4b)(a-b)-(7a-12b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{4a(a-b)}}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^2}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\frac{3\int \frac{(a-4b)(a-b)-(7a-12b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^2}d\sec(e+fx)}{a}+\frac{b(7a-12b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)^2}}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^2}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\frac{3\left(\frac{4b(a-2b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}-\frac{\int -\frac{2(a-b)\left(a^2-8ba+8b^2-4(a-2b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{2a(a-b)}\right)}{a}+\frac{b(7a-12b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)^2}}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^2}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\frac{3\left(\frac{\int \frac{a^2-8ba+8b^2-4(a-2b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)}d\sec(e+fx)}{a}+\frac{4b(a-2b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)}\right)}{a}+\frac{b(7a-12b)\sec(e+fx)}{a\left(a+b\sec^2(e+fx)-b\right)^2}}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^2}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\frac{3\left(\frac{\frac{\left(a^2-12ab+16b^2\right)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{b\left(5a^2-20ab+16b^2\right)\int\frac{1}{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{a}}{a}+\frac{4b(a\cdots}{a\left(a+b\cdots\right.}\right.}{a}\cdots}{2a}\cdots}{4a}\cdots}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\frac{3\left(\frac{\frac{\left(a^2-12ab+16b^2\right)\int\frac{1}{1-\sec^2(e+fx)}d\sec(e+fx)}{a}+\frac{\sqrt{b}\left(5a^2-20ab+16b^2\right)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}}{a}+\frac{4b(a-2b)\sec\cdots}{a\left(a+b\sec^2(e\cdots\right.}\right.}{a}\cdots}{2a}\cdots}{4a}\cdots}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{(5a-8b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^2}-\frac{\frac{3\left(\frac{\frac{\sqrt{b}\left(5a^2-20ab+16b^2\right)\arctan\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}}+\frac{\left(a^2-12ab+16b^2\right)\text{arctanh}(\sec(e+fx))}{a}}{a}+\frac{4b(a-2b)\sec(e+f\cdots}{a\left(a+b\sec^2(e+fx)-\cdots\right.}\right.}{a}\cdots}{2a}\cdots}{4a}\cdots}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*Sec[e + f*x]/(a*(1 - Sec[e + f*x]^2)^2*(a - b + b*Sec[e + f*x]^2)^2)
 + (((5*a - 8*b)*Sec[e + f*x])/(2*a*(1 - Sec[e + f*x]^2)*(a - b + b*Sec[e
+ f*x]^2)^2) - (((7*a - 12*b)*b*Sec[e + f*x])/(a*(a - b + b*Sec[e + f*x]^2
)^2) + (3*(((Sqrt[b]*(5*a^2 - 20*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Sec[e + f*x
])/Sqrt[a - b]])/(a*Sqrt[a - b]) + ((a^2 - 12*a*b + 16*b^2)*ArcTanh[Sec[e
+ f*x]])/a)/a + (4*(a - 2*b)*b*Sec[e + f*x])/(a*(a - b + b*Sec[e + f*x]^2)
)))/a)/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{b\left(\frac{-\frac{3a(3a^2-7ab+4b^2)\cos(fx+e)^3}{8}+\left(-\frac{7}{8}a^2b+\frac{3}{2}ab^2\right)\cos(fx+e)}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{3(5a^2-20ab+16b^2)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8\sqrt{b(a-b)}}\right)}{a^5}+\frac{}{16a^3(\cos}}{}$
default	$\frac{\frac{b\left(\frac{-\frac{3a(3a^2-7ab+4b^2)\cos(fx+e)^3}{8}+\left(-\frac{7}{8}a^2b+\frac{3}{2}ab^2\right)\cos(fx+e)}{\left(a\cos(fx+e)^2-b\cos(fx+e)^2+b\right)^2}+\frac{3(5a^2-20ab+16b^2)\arctan\left(\frac{(a-b)\cos(fx+e)}{\sqrt{b(a-b)}}\right)}{8\sqrt{b(a-b)}}\right)}{a^5}+\frac{}{16a^3(\cos}}{}$
risch	Expression too large to display

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b/a^5*((-3/8*a*(3*a^2-7*a*b+4*b^2)*cos(f*x+e)^3+(-7/8*a^2*b+3/2*a*b^2
)*cos(f*x+e))/(a*cos(f*x+e)^2-b*cos(f*x+e)^2+b)^2+3/8*(5*a^2-20*a*b+16*b^2
)/(b*(a-b))^(1/2)*arctan((a-b)*cos(f*x+e)/(b*(a-b))^(1/2)))+1/16/a^3/(cos(
f*x+e)+1)^2-1/16*(-3*a+12*b)/a^4/(cos(f*x+e)+1)+1/16/a^5*(-3*a^2+36*a*b-48
*b^2)*ln(cos(f*x+e)+1)-1/16/a^3/(cos(f*x+e)-1)^2-1/16*(-3*a+12*b)/a^4/(cos
(f*x+e)-1)+1/16/a^5*(3*a^2-36*a*b+48*b^2)*ln(cos(f*x+e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(239) = 478$.

Time = 0.32 (sec) , antiderivative size = 1693, normalized size of antiderivative = 6.54

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*(a^4 - 9*a^3*b + 16*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^7 - 2*(5*a^4
- 46*a^3*b + 108*a^2*b^2 - 72*a*b^3)*cos(f*x + e)^5 - 2*(19*a^3*b - 72*a^2
*b^2 + 72*a*b^3)*cos(f*x + e)^3 + 3*((5*a^4 - 30*a^3*b + 61*a^2*b^2 - 52*a
*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(5*a^4 - 35*a^3*b + 86*a^2*b^2 - 88*a*b^
3 + 32*b^4)*cos(f*x + e)^6 + (5*a^4 - 50*a^3*b + 166*a^2*b^2 - 216*a*b^3 +
 96*b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 20*a*b^3 + 16*b^4 + 2*(5*a^3*b - 30*
a^2*b^2 + 56*a*b^3 - 32*b^4)*cos(f*x + e)^2)*sqrt(-b/(a - b))*log(((a - b)
*cos(f*x + e)^2 + 2*(a - b)*sqrt(-b/(a - b))*cos(f*x + e) - b)/((a - b)*co
s(f*x + e)^2 + b)) - 24*(a^2*b^2 - 2*a*b^3)*cos(f*x + e) - 3*((a^4 - 14*a^
3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*x + e)^8 - 2*(a^4 - 15*a^3*b +
 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e)^6 + (a^4 - 18*a^3*b + 94*a^2
*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 + a^2*b^2 - 12*a*b^3 + 16*b^4 +
2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x
 + e) + 1/2) + 3*((a^4 - 14*a^3*b + 41*a^2*b^2 - 44*a*b^3 + 16*b^4)*cos(f*
x + e)^8 - 2*(a^4 - 15*a^3*b + 54*a^2*b^2 - 72*a*b^3 + 32*b^4)*cos(f*x + e
)^6 + (a^4 - 18*a^3*b + 94*a^2*b^2 - 168*a*b^3 + 96*b^4)*cos(f*x + e)^4 +
a^2*b^2 - 12*a*b^3 + 16*b^4 + 2*(a^3*b - 14*a^2*b^2 + 40*a*b^3 - 32*b^4)*c
os(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 - 2*a^6*b + a^5*b^2)*f*
cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^
6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b - 2*a^5*b^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(239) = 478$.

Time = 0.77 (sec) , antiderivative size = 861, normalized size of antiderivative = 3.32

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/64*(12*(a^2 - 12*a*b + 16*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x +
e) + 1))/a^5 - 24*(5*a^2*b - 20*a*b^2 + 16*b^3)*arctan(-(a*cos(f*x + e) -
b*cos(f*x + e) - b)/(sqrt(a*b - b^2)*cos(f*x + e) + sqrt(a*b - b^2)))/(sqr
t(a*b - b^2)*a^5) - (8*a^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a^2*
b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^3*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2)/a^6 - (a^4 - 4*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
16*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^4*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2 + 216*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 - 304*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 20*a^4*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 360*a^3*b*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 1024*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
+ 896*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 5*a^4*(cos(f*x + e
) - 1)^4/(cos(f*x + e) + 1)^4 + 64*a^3*b*(cos(f*x + e) - 1)^4/(cos(f*x + e
) + 1)^4 - 192*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 256*a*b
^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 256*b^4*(cos(f*x + e) - 1)^
4/(cos(f*x + e) + 1)^4 + 16*a^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5
- 168*a^3*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 384*a^2*b^2*(cos(f
*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 256*a*b^3*(cos(f*x + e) - 1)^5/(cos(
f*x + e) + 1)^5 + 6*a^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 - 72*a^3
*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 96*a^2*b^2*(cos(f*x + e)...
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 1357, normalized size of antiderivative = 5.24

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^3),x)
```

output
```
tan(e/2 + (f*x)/2)^4/(64*a^3*f) + (tan(e/2 + (f*x)/2)^2*((3*(a - 2*b))/(16
*a^4) - 1/(16*a^3)))/f + (tan(e/2 + (f*x)/2)^4*(100*a*b^2 - 72*a^2*b + (13
*a^3)/2) - tan(e/2 + (f*x)/2)^10*(144*a*b^2 - 42*a^2*b + 2*a^3 - 128*b^3)
- tan(e/2 + (f*x)/2)^6*(496*a*b^2 - 174*a^2*b + 11*a^3 - 416*b^3) + tan(e/
2 + (f*x)/2)^2*(4*a^2*b - a^3) - a^3/4 + (tan(e/2 + (f*x)/2)^8*(31*a^4 - 5
92*a^3*b - 2944*a*b^3 + 1792*b^4 + 2016*a^2*b^2))/(4*a))/(f*(16*a^6*tan(e/
2 + (f*x)/2)^4 + 16*a^6*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^8*(96*a
^6 - 256*a^5*b + 256*a^4*b^2) + tan(e/2 + (f*x)/2)^6*(128*a^5*b - 64*a^6)
+ tan(e/2 + (f*x)/2)^10*(128*a^5*b - 64*a^6))) + (log(tan(e/2 + (f*x)/2))*
(3*a^2 - 36*a*b + 48*b^2))/(8*a^5*f) + (3*b^(1/2)*atan((a^13*(a - b)^(3/2)
*((256*((3456*b^8 - 11232*a*b^7 + 14256*a^2*b^6 - 8910*a^3*b^5 + 2835*a^4*
b^4 - (3375*a^5*b^3)/8 + (675*a^6*b^2)/32)/a^12 - (9*b*(5*a^2 - 20*a*b + 1
6*b^2)^2*(192*a^14 - 4992*a^13*b + 24576*a^10*b^4 - 43008*a^11*b^3 + 24576
*a^12*b^2))/(8192*a^22*(a - b)))*(1728*a*b^4 - 45*a^4*b + a^5 - 768*b^5 -
1344*a^2*b^3 + 420*a^3*b^2))/(b^(1/2)*(b*(b*(b*(1680*a^7 + b*(768*a^5*b -
1920*a^6)) - 600*a^8) + 75*a^9) - 4*a^10)) - 256*tan(e/2 + (f*x)/2)^2*((((
4752*a*b^6 - 1728*b^7 - 4860*a^2*b^5 + 2295*a^3*b^4 - (2025*a^4*b^3)/4 + (
675*a^5*b^2)/16)/a^11 + (9*b*(5*a^2 - 20*a*b + 16*b^2)^2*(3552*a^12*b - 96
*a^13 + 73728*a^8*b^5 - 165888*a^9*b^4 + 125952*a^10*b^3 - 36480*a^11*b^2)
)/(4096*a^21*(a - b)))*(1728*a*b^4 - 45*a^4*b + a^5 - 768*b^5 - 1344*a^...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2123, normalized size of antiderivative = 8.20

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
(60*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**8*a**4 - 360*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sq
rt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**8*a**3*b + 732*sqrt(b)*sqrt
(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x
)**8*a**2*b**2 - 624*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((
e + f*x)/2))/sqrt(b))*sin(e + f*x)**8*a*b**3 + 192*sqrt(b)*sqrt(a - b)*ata
n((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**8*b**4 -
 120*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqr
t(b))*sin(e + f*x)**6*a**4 + 600*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - s
qrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**6*a**3*b - 864*sqrt(b)*sqr
t(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*
x)**6*a**2*b**2 + 384*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan(
(e + f*x)/2))/sqrt(b))*sin(e + f*x)**6*a*b**3 + 60*sqrt(b)*sqrt(a - b)*ata
n((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**4 -
 240*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqr
t(b))*sin(e + f*x)**4*a**3*b + 192*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) -
 sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 + 60*sqrt(b)
*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
+ f*x)**8*a**4 - 360*sqrt(b)*sqrt(a - b)*atan((sqrt(a - b) + sqrt(a)*tan((
e + f*x)/2))/sqrt(b))*sin(e + f*x)**8*a**3*b + 732*sqrt(b)*sqrt(a - b)*...
```

3.86 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 250

$$\begin{aligned}\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx &= \frac{3(a^2+10ab+5b^2)\,x}{8(a-b)^5} \\ &\quad - \frac{3\sqrt{b}(5a^2+10ab+b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^5 f} \\ &\quad - \frac{(5a+3b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2 f\left(a+b\tan^2(e+fx)\right)^2} \\ &\quad + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} \\ &\quad - \frac{b(7a+5b)\tan(e+fx)}{8(a-b)^3 f\left(a+b\tan^2(e+fx)\right)^2} \\ &\quad - \frac{3b(a+b)\tan(e+fx)}{2(a-b)^4 f\left(a+b\tan^2(e+fx)\right)}\end{aligned}$$

output
```
3/8*(a^2+10*a*b+5*b^2)*x/(a-b)^5-3/8*b^(1/2)*(5*a^2+10*a*b+b^2)*arctan(b^(
1/2)*tan(f*x+e)/a^(1/2))/a^(1/2)/(a-b)^5/f-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*
x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a
+b*tan(f*x+e)^2)^2-1/8*b*(7*a+5*b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)
^2-3/2*b*(a+b)*tan(f*x+e)/(a-b)^4/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.78

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{12(a^2+10ab+5b^2)(e+fx) - \frac{12\sqrt{b}(5a^2+10ab+b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}} - 8(a-b)(a+2b)\sin(2(e+fx)) + \frac{1}{(a}}{32(a-b)^5f}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(12*(a^2 + 10*a*b + 5*b^2)*(e + f*x) - (12*Sqrt[b]*(5*a^2 + 10*a*b + b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/Sqrt[a] - 8*(a - b)*(a + 2*b)*Sin[
2*(e + f*x)] + (16*a*(a - b)*b^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*
(e + f*x)])^2 - (4*(a - b)*b*(9*a + 5*b)*Sin[2*(e + f*x)])/(a + b + (a - b
)*Cos[2*(e + f*x)]) + (a - b)^2*Sin[4*(e + f*x)])/(32*(a - b)^5*f)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4146, 372, 402, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^4}{(a+b\tan(e+fx)^2)^3}dx$$
$$\Big\downarrow \text{4146}$$
$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$\downarrow$ 372

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\int \frac{a-(4a+3b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{4(a-b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\int \frac{a(3a+5b)-5b(5a+3b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{2(a-b)}}{4(a-b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\frac{\int \frac{12a\left(a(a+3b)-b(7a+5b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4a(a-b)}-\frac{b\cdots}{(a-\cdots}}{2(a-b)}}{4(a-b)}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{\frac{3\int \frac{a(a+3b)-b(7a+5b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{a-b}-\frac{\cdots}{(a\cdots}}{2(a-b)}}{4(a-b)}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{3\left(\frac{\int \frac{2a\left(a^2+6ba+b^2-4b(a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\cdots\right)}{a-b}\cdots}{2(a-\cdots}}{4(a-b)}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{3\left(\frac{\frac{\int\frac{a^2+6ba+b^2-4b(a+b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a-b}-\cdots}{a-b}\cdots\right)}{2(a-b)\cdots}}{4(a-b)}\cdots}{f}$$

$\downarrow$ 397

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{3\left(\frac{\frac{\left(a^2+10ab+5b^2\right)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b\left(5a^2\cdots\right.}{a-b}}{\cdots}\cdots\right)}{\cdots}}{4(a-\cdots}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{3\left(\frac{\frac{\left(a^2+10ab+5b^2\right)\arctan(\tan(e+fx))}{a-b}-\frac{b\left(5a^2+10ab+b^2\cdots\right.}{a-b}}{\cdots}\cdots\right)}{\cdots}}{4(a-b)}\cdots}{f}$$

$\downarrow$ 218

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^2}-\frac{\frac{(5a+3b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^2}-\frac{3\left(\frac{\frac{\left(a^2+10ab+5b^2\right)\arctan(\tan(e+fx))}{a-b}-\frac{\sqrt{b}\left(5a^2+10ab+\cdots\right.}{a-b}}{a-\cdots}\cdots\right)}{\cdots}}{4(a-b)}\cdots}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*(a - b)*(1 + Tan[e + f*x]^2)^2*(a + b*Tan[e + f*x]^2)^2)
- (((5*a + 3*b)*Tan[e + f*x])/(2*(a - b)*(1 + Tan[e + f*x]^2)*(a + b*Tan[e
 + f*x]^2)^2) - (-((b*(7*a + 5*b)*Tan[e + f*x])/((a - b)*(a + b*Tan[e + f*
x]^2)^2)) + (3*((((a^2 + 10*a*b + 5*b^2)*ArcTan[Tan[e + f*x]])/(a - b) - (
Sqrt[b]*(5*a^2 + 10*a*b + b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sq
rt[a]*(a - b)))/(a - b) - (4*b*(a + b)*Tan[e + f*x])/((a - b)*(a + b*Tan[e
 + f*x]^2))))/(a - b))/(2*(a - b)))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 50.74 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^{2}b-\frac{1}{4}a\,b^{2}-\frac{5}{8}b^{3}\right)\tan(fx+e)^{3}+\frac{3a\left(3a^{2}-2ab-b^{2}\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^{2}\right)^{2}}+\frac{3\left(5a^{2}+10ab+b^{2}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a-b)^{5}}+\frac{\left(-\frac{1}{4}ab+\frac{7}{8}b\cdots\right.}{\cdots}}{f}$
default	$\frac{-\frac{b\left(\frac{\left(\frac{7}{8}a^{2}b-\frac{1}{4}a\,b^{2}-\frac{5}{8}b^{3}\right)\tan(fx+e)^{3}+\frac{3a\left(3a^{2}-2ab-b^{2}\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^{2}\right)^{2}}+\frac{3\left(5a^{2}+10ab+b^{2}\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a-b)^{5}}+\frac{\left(-\frac{1}{4}ab+\frac{7}{8}b\cdots\right.}{\cdots}}{f}$
risch	$\frac{3x\,a^{2}}{8(a^{3}-3a^{2}b+3a\,b^{2}-b^{3})(a-b)^{2}}+\frac{15xab}{4(a^{3}-3a^{2}b+3a\,b^{2}-b^{3})(a-b)^{2}}+\frac{15x\,b^{2}}{8(a^{3}-3a^{2}b+3a\,b^{2}-b^{3})(a-b)^{2}}+\frac{i\mathrm{e}^{-4i(fx\cdots}}{64(a^{2}-2ab+b\cdots}$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a-b)^5*(((7/8*a^2*b-1/4*a*b^2-5/8*b^3)*tan(f*x+e)^3+3/8*a*(3*a^2-
2*a*b-b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+3/8*(5*a^2+10*a*b+b^2)/(a*b)^(
1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^5*(((-1/4*a*b+7/8*b^2-5/8*a
^2)*tan(f*x+e)^3+(-3/8*a^2-3/4*a*b+9/8*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)^2
+3/8*(a^2+10*a*b+5*b^2)*arctan(tan(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(230) = 460$.

Time = 0.23 (sec) , antiderivative size = 1191, normalized size of antiderivative = 4.76

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(12*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x*cos(f*x + e)^4 + 24*(a^
3*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 10*a
*b^3 + 5*b^4)*f*x - 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^4
 + 5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*b^2 - 9*a*b^3 - b^4)*co
s(f*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*
b + b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e)
)*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(
a*b - b^2)*cos(f*x + e)^2 + b^2)) + 4*(2*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*
b^3 + b^4)*cos(f*x + e)^7 - (5*a^4 - 12*a^3*b + 6*a^2*b^2 + 4*a*b^3 - 3*b^
4)*cos(f*x + e)^5 - (19*a^3*b - 21*a^2*b^2 - 15*a*b^3 + 17*b^4)*cos(f*x +
e)^3 - 12*(a^2*b^2 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 7*a^6*b + 21
*a^5*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x
 + e)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*
a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2
*b^5 + 5*a*b^6 - b^7)*f), 1/16*(6*(a^4 + 8*a^3*b - 14*a^2*b^2 + 5*b^4)*f*x
*cos(f*x + e)^4 + 12*(a^3*b + 9*a^2*b^2 - 5*a*b^3 - 5*b^4)*f*x*cos(f*x + e
)^2 + 6*(a^2*b^2 + 10*a*b^3 + 5*b^4)*f*x + 3*((5*a^4 - 14*a^2*b^2 + 8*a*b^
3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 + 10*a*b^3 + b^4 + 2*(5*a^3*b + 5*a^2*
b^2 - 9*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x
 + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^4 - 4*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.84

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{3\,(a^2+10\,ab+5\,b^2)(fx+e)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5} - \frac{3\,(5\,a^2b+10\,ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)\sqrt{ab}} - \frac{}{(a^4b^2-4\,a^3b^3+6\,a^2b^4-4\,ab^5+b^6)\tan(fx+e)^8+2\,(}}{}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(a^2 + 10*a*b + 5*b^2)*(f*x + e)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a
^2*b^3 + 5*a*b^4 - b^5) - 3*(5*a^2*b + 10*a*b^2 + b^3)*arctan(b*tan(f*x +
e)/sqrt(a*b))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*s
qrt(a*b)) - (12*(a*b^2 + b^3)*tan(f*x + e)^7 + (19*a^2*b + 34*a*b^2 + 19*b
^3)*tan(f*x + e)^5 + (5*a^3 + 31*a^2*b + 31*a*b^2 + 5*b^3)*tan(f*x + e)^3
+ 3*(a^3 + 6*a^2*b + a*b^2)*tan(f*x + e))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^
4 - 4*a*b^5 + b^6)*tan(f*x + e)^8 + 2*(a^5*b - 3*a^4*b^2 + 2*a^3*b^3 + 2*a
^2*b^4 - 3*a*b^5 + b^6)*tan(f*x + e)^6 + a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3
*b^3 + a^2*b^4 + (a^6 - 9*a^4*b^2 + 16*a^3*b^3 - 9*a^2*b^4 + b^6)*tan(f*x
+ e)^4 + 2*(a^6 - 3*a^5*b + 2*a^4*b^2 + 2*a^3*b^3 - 3*a^2*b^4 + a*b^5)*tan
(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.52

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{3\,(a^2+10\,ab+5\,b^2)(fx+e)}{8\,(a^5f-5\,a^4bf+10\,a^3b^2f-10\,a^2b^3f+5\,ab^4f-b^5f)} - \frac{3\,(5\,a^2b+10\,ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,(a^5f-5\,a^4bf+10\,a^3b^2f-10\,a^2b^3f+5\,ab^4f-b^5f)\sqrt{ab}} - \frac{12\,ab^2\tan(fx+e)^7+12\,b^3\tan(fx+e)^7+19\,a^2b\tan(fx+e)^5+34\,ab^2\tan(fx+e)^5+19\,b^3\tan(fx}{8\,(a^4f-4\,a^3bf+6\,a^2b}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
3/8*(a^2 + 10*a*b + 5*b^2)*(f*x + e)/(a^5*f - 5*a^4*b*f + 10*a^3*b^2*f - 1
0*a^2*b^3*f + 5*a*b^4*f - b^5*f) - 3/8*(5*a^2*b + 10*a*b^2 + b^3)*arctan(b
*tan(f*x + e)/sqrt(a*b))/((a^5*f - 5*a^4*b*f + 10*a^3*b^2*f - 10*a^2*b^3*f
 + 5*a*b^4*f - b^5*f)*sqrt(a*b)) - 1/8*(12*a*b^2*tan(f*x + e)^7 + 12*b^3*t
an(f*x + e)^7 + 19*a^2*b*tan(f*x + e)^5 + 34*a*b^2*tan(f*x + e)^5 + 19*b^3
*tan(f*x + e)^5 + 5*a^3*tan(f*x + e)^3 + 31*a^2*b*tan(f*x + e)^3 + 31*a*b^
2*tan(f*x + e)^3 + 5*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 18*a^2*b*ta
n(f*x + e) + 3*a*b^2*tan(f*x + e))/((a^4*f - 4*a^3*b*f + 6*a^2*b^2*f - 4*a
*b^3*f + b^4*f)*(b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + b*tan(f*x + e)^2 +
a)^2)
```

Mupad [B] (verification not implemented)

Time = 12.80 (sec) , antiderivative size = 5965, normalized size of antiderivative = 23.86

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(atan(((((tan(e + f*x)*(540*a*b^6 + 117*b^7 + 990*a^2*b^5 + 540*a^3*b^4 +
117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5
 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (3*((6*a*b^13 - (3*b^14)/2 + 2
1*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10)/2 - 1332*a^5*b^9 + 1638*a^6*b^
8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 210*a^9*b^5 + 21*a^10*b^4 + 6*a^11*b
^3 - (3*a^12*b^2)/2)/(a^12 - 12*a^11*b - 12*a*b^11 + b^12 + 66*a^2*b^10 -
220*a^3*b^9 + 495*a^4*b^8 - 792*a^5*b^7 + 924*a^6*b^6 - 792*a^7*b^5 + 495*
a^8*b^4 - 220*a^9*b^3 + 66*a^10*b^2) - (3*tan(e + f*x)*(10*a*b + a^2 + 5*b
^2)*(1152*a*b^12 - 128*b^13 - 4480*a^2*b^11 + 9600*a^3*b^10 - 11520*a^4*b^
9 + 5376*a^5*b^8 + 5376*a^6*b^7 - 11520*a^7*b^6 + 9600*a^8*b^5 - 4480*a^9*
b^4 + 1152*a^10*b^3 - 128*a^11*b^2))/(256*(a*b^4*5i - a^4*b*5i + a^5*1i -
b^5*1i - a^2*b^3*10i + a^3*b^2*10i)*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^
2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(10*a*b + a^2
 + 5*b^2))/(16*(a*b^4*5i - a^4*b*5i + a^5*1i - b^5*1i - a^2*b^3*10i + a^3*
b^2*10i)))*(10*a*b + a^2 + 5*b^2)*3i)/(16*(a*b^4*5i - a^4*b*5i + a^5*1i -
b^5*1i - a^2*b^3*10i + a^3*b^2*10i)) + (((tan(e + f*x)*(540*a*b^6 + 117*b^
7 + 990*a^2*b^5 + 540*a^3*b^4 + 117*a^4*b^3))/(16*(a^8 - 8*a^7*b - 8*a*b^7
 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2))
- (3*((6*a*b^13 - (3*b^14)/2 + 21*a^2*b^12 - 210*a^3*b^11 + (1395*a^4*b^10
)/2 - 1332*a^5*b^9 + 1638*a^6*b^8 - 1332*a^7*b^7 + (1395*a^8*b^6)/2 - 2...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1874, normalized size of antiderivative = 7.50

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**4*a**4 - 42*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 + 24*sqrt(b)*sqrt(a)*ata
n((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b**3
 + 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**4*b**4 - 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*ta
n((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**4 - 30*sqrt(b)*sqrt(a)*atan((s
qrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**3*b + 5
4*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*s
in(e + f*x)**2*a**2*b**2 + 6*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*t
an((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 15*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**4 + 30*sqrt(b)*sqrt(
a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**3*b + 3*sqrt(
b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a**2*b**
2 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**4*a**4 + 42*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**2*b**2 - 24*sqrt(b)*sqrt(a)*
atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a*b
**3 - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt
(b))*sin(e + f*x)**4*b**4 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt...
```

3.87 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 193

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{(a+5b)x}{2(a-b)^4} - \frac{\sqrt{b}(15a^2+10ab-b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^4f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4(a-b)^2f\left(a+b\tan^2(e+fx)\right)^2} - \frac{b(11a+b)\tan(e+fx)}{8a(a-b)^3f\left(a+b\tan^2(e+fx)\right)}$$

output

```
1/2*(a+5*b)*x/(a-b)^4-1/8*b^(1/2)*(15*a^2+10*a*b-b^2)*arctan(b^(1/2)*tan(f
*x+e)/a^(1/2))/a^(3/2)/(a-b)^4/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*ta
n(f*x+e)^2)^2-3/4*b*tan(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^2-1/8*b*(11*a+
b)*tan(f*x+e)/a/(a-b)^3/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{4(a+5b)(e+fx)+\frac{\sqrt{b}(-15a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}-2(a-b)\sin(2(e+fx))+\frac{4(a-b)b^2\sin(2(e+fx))}{(a+b+(a-b)\cos(2(e+fx)))^2}}{8(a-b)^4f}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(4*(a + 5*b)*(e + f*x) + (Sqrt[b]*(-15*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a]])/a^(3/2) - 2*(a - b)*Sin[2*(e + f*x)] + (4*(a - b)
*b^2*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2 - ((a - b)*b*(
9*a + b)*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*x)])))/(8*(a -
 b)^4*f)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4146, 373, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^2}{\left(a+b\tan(e+fx)^2\right)^3}dx$$
$$\Big\downarrow \text{4146}$$
$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\int \frac{a-5b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\frac{\int \frac{2a\left(-9b\tan^2(e+fx)+2a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4a(a-b)}-\frac{3b\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{\int \frac{-9b\tan^2(e+fx)+2a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{2(a-b)}-\frac{3b\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\frac{\frac{\int \frac{4a^2+9ba-b^2-b(11a+b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b(11a+b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{3b\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^2}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\frac{\frac{\frac{4a(a+5b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b\left(15a^2+10ab-b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b(11a+b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{3b\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^2}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\frac{\frac{\frac{4a(a+5b)\arctan(\tan(e+fx))}{a-b}-\frac{b\left(15a^2+10ab-b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b(11a+b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{3b\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^2}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{4a(a+5b)\arctan(\tan(e+fx))}{a-b}-\frac{\sqrt{b}\left(15a^2+10ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b(11a+b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{2(a-b)}-\frac{3b\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{2(a-b)}-\frac{}{2(a-b)(\tan^2(e+}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/2*Tan[e + f*x]/((a - b)*(1 + Tan[e + f*x]^2)*(a + b*Tan[e + f*x]^2)^2)
 + ((-3*b*Tan[e + f*x])/(2*(a - b)*(a + b*Tan[e + f*x]^2)^2) + (((4*a*(a +
 5*b)*ArcTan[Tan[e + f*x]])/(a - b) - (Sqrt[b]*(15*a^2 + 10*a*b - b^2)*Arc
Tan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - (b
*(11*a + b)*Tan[e + f*x])/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/(2*(a - b)
))/(2*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 12.49 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan(fx+e)^2}+\frac{(a+5b)\arctan(\tan(fx+e))}{2}}{(a-b)^4}-\frac{b\left(\frac{\frac{b\left(7a^2-6ab-b^2\right)\tan(fx+e)^3}{8a}+\left(\frac{9}{8}a^2-\frac{5}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+10a\right.}{}\right.}{(a-b)^4}}{f}$
default	$\frac{\frac{\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{1+\tan(fx+e)^2}+\frac{(a+5b)\arctan(\tan(fx+e))}{2}}{(a-b)^4}-\frac{b\left(\frac{\frac{b\left(7a^2-6ab-b^2\right)\tan(fx+e)^3}{8a}+\left(\frac{9}{8}a^2-\frac{5}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+10a\right.}{}\right.}{(a-b)^4}}{f}$
risch	$\frac{xa}{2(a^3-3a^2b+3a\,b^2-b^3)(a-b)}+\frac{5xb}{2(a^3-3a^2b+3a\,b^2-b^3)(a-b)}+\frac{i\,e^{2i(fx+e)}}{8f(a^3-3a^2b+3a\,b^2-b^3)}-\frac{i\,e^{-2i(fx+e)}}{8f(a^3-3a^2b+3a\,b^2-b}$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^4*((-1/2*a+1/2*b)*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+5*b)*arc
tan(tan(f*x+e)))-1/(a-b)^4*b*((1/8*b*(7*a^2-6*a*b-b^2)/a*tan(f*x+e)^3+(9/8
*a^2-5/4*a*b+1/8*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(15*a^2+10*a*b-
b^2)/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(175) = 350.

Time = 0.21 (sec) , antiderivative size = 1076, normalized size of antiderivative = 5.58

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(16*(a^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 32*(a
^3*b + 4*a^2*b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 5*a*b^3)*f*
x - ((15*a^4 - 20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*
a^2*b^2 + 10*a*b^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f
*x + e)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b +
 b^2)*cos(f*x + e)^2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*s
qrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b
 - b^2)*cos(f*x + e)^2 + b^2)) - 4*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*
cos(f*x + e)^5 + (17*a^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 +
 (11*a^2*b^2 - 10*a*b^3 - b^4)*cos(f*x + e))*sin(f*x + e))/((a^7 - 6*a^6*b
 + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e
)^4 + 2*(a^6*b - 5*a^5*b^2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*
f*cos(f*x + e)^2 + (a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f
), 1/16*(8*(a^4 + 3*a^3*b - 9*a^2*b^2 + 5*a*b^3)*f*x*cos(f*x + e)^4 + 16*(
a^3*b + 4*a^2*b^2 - 5*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 5*a*b^3)*f*
x + ((15*a^4 - 20*a^3*b - 6*a^2*b^2 + 12*a*b^3 - b^4)*cos(f*x + e)^4 + 15*
a^2*b^2 + 10*a*b^3 - b^4 + 2*(15*a^3*b - 5*a^2*b^2 - 11*a*b^3 + b^4)*cos(f
*x + e)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*
cos(f*x + e)*sin(f*x + e))) - 2*(4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*cos
(f*x + e)^5 + (17*a^3*b - 33*a^2*b^2 + 15*a*b^3 + b^4)*cos(f*x + e)^3 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e + fx)}{(a + b\tan^2(e + fx))^3}\, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.79

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{\frac{4\,(fx+e)(a+5\,b)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} - \frac{(15\,a^2b+10\,ab^2-b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sqrt{ab}} - \frac{(11\,ab^2+b^3)\tan(fx+e)^5+(}{(a^4b^2-3\,a^3b^3+3\,a^2b^4-ab^5)\tan(fx+e)^6+a^6-3\,a^5b+3\,a^4b^2-a^3b}}{8\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(4*(f*x + e)*(a + 5*b)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (
15*a^2*b + 10*a*b^2 - b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((11*a*b^2 + b^3)*tan(f*x
+ e)^5 + (17*a^2*b + 6*a*b^2 + b^3)*tan(f*x + e)^3 + (4*a^3 + 9*a^2*b - a*
b^2)*tan(f*x + e))/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*tan(f*x + e)
^6 + a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3 + (2*a^5*b - 5*a^4*b^2 + 3*a^3*b^
3 + a^2*b^4 - a*b^5)*tan(f*x + e)^4 + (a^6 - a^5*b - 3*a^4*b^2 + 5*a^3*b^3
 - 2*a^2*b^4)*tan(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.41

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{(fx+e)(a+5\,b)}{2\,(a^4f-4\,a^3bf+6\,a^2b^2f-4\,ab^3f+b^4f)} - \frac{(15\,a^2b+10\,ab^2-b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,(a^5f-4\,a^4bf+6\,a^3b^2f-4\,a^2b^3f+ab^4f)\sqrt{ab}} - \frac{\tan(fx+e)}{2\,(a^3f-3\,a^2bf+3\,ab^2f-b^3f)\left(\tan(fx+e)^2+1\right)} - \frac{7\,ab^2\tan(fx+e)^3+b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)-ab^2\tan(fx+e)}{8\,(a^4f-3\,a^3bf+3\,a^2b^2f-ab^3f)\left(b\tan(fx+e)^2+a\right)^2}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/2*(f*x + e)*(a + 5*b)/(a^4*f - 4*a^3*b*f + 6*a^2*b^2*f - 4*a*b^3*f + b^4
*f) - 1/8*(15*a^2*b + 10*a*b^2 - b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a
^5*f - 4*a^4*b*f + 6*a^3*b^2*f - 4*a^2*b^3*f + a*b^4*f)*sqrt(a*b)) - 1/2*t
an(f*x + e)/((a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3*f)*(tan(f*x + e)^2 + 1))
 - 1/8*(7*a*b^2*tan(f*x + e)^3 + b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e)
 - a*b^2*tan(f*x + e))/((a^4*f - 3*a^3*b*f + 3*a^2*b^2*f - a*b^3*f)*(b*tan
(f*x + e)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 4997, normalized size of antiderivative = 25.89

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
- ((tan(e + f*x)^5*(11*a*b^2 + b^3))/(8*a*(3*a*b^2 - 3*a^2*b + a^3 - b^3))
 + (tan(e + f*x)*(9*a*b + 4*a^2 - b^2))/(8*(a - b)*(a^2 - 2*a*b + b^2)) +
(b*tan(e + f*x)^3*(6*a*b + 17*a^2 + b^2))/(8*a*(a - b)*(a^2 - 2*a*b + b^2)
))/(f*(tan(e + f*x)^2*(2*a*b + a^2) + tan(e + f*x)^4*(2*a*b + b^2) + a^2 +
 b^2*tan(e + f*x)^6)) - (atan(((((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b
^5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5
+ 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) - ((((17*a^2*b^11)/2 - (a*b^12)/2
 - 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 + 1
8*a^8*b^5 + (39*a^9*b^4)/2 - (23*a^10*b^3)/2 + 2*a^11*b^2)/(9*a^10*b - a^1
1 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a^5*b^6 + 126*a^6*b^5 - 126*a^7*
b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(e + f*x)*(a*1i + b*5i)*(256*a^2*b^11
 - 1792*a^3*b^10 + 5120*a^4*b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b
^6 - 7168*a^8*b^5 + 5120*a^9*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(128*(a^
4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*
b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a*1i + b*5i))/(4*(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(a*1i + b*5i)*1i)/(4*(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2)) + (((tan(e + f*x)*(b^7 - 20*a*b^6 + 470*a^2*b^
5 + 460*a^3*b^4 + 241*a^4*b^3))/(32*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 +
 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + ((((17*a^2*b^11)/2 - (a*b^12)/2
- 48*a^3*b^10 + 138*a^4*b^9 - 231*a^5*b^8 + 231*a^6*b^7 - 126*a^7*b^6 +...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1744, normalized size of antiderivative = 9.04

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**4*a**4 - 20*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b - 6*sqrt(b)*sqrt(a)*atan((s
qrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**2*b**2
+ 12*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**4*a*b**3 - sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*b**4 - 30*sqrt(b)*sqrt(a)*atan((sq
rt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**4 + 10*s
qrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(
e + f*x)**2*a**3*b + 22*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e
 + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a**2*b**2 - 2*sqrt(b)*sqrt(a)*atan((s
qrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**2*a*b**3 + 1
5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*a
**4 + 10*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqr
t(b))*a**3*b - sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2
))/sqrt(b))*a**2*b**2 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**4 + 20*sqrt(b)*sqrt(a)*atan((sq
rt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**4*a**3*b + 6*
sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin
(e + f*x)**4*a**2*b**2 - 12*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)...
```

3.88 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{x}{(a-b)^3} - \frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)}$$

output
```
x/(a-b)^3-1/8*b^(1/2)*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1
/2))/a^(5/2)/(a-b)^3/f-1/4*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8
*(7*a-3*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{-8\arctan(\tan(e+fx))+\frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2(a-b)^2b\tan(e+fx)}{a(a+b\tan^2(e+fx))^2}+\frac{(7a-3b)(a-b)b\tan(e+fx)}{a^2(a+b\tan^2(e+fx))}}{8(a-b)^3f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

output
```
-1/8*(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[
(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a
*(a + b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a
+ b*Tan[e + f*x]^2)))/((a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\frac{\int \frac{-3b\tan^2(e+fx)+4a-3b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{8a^2-7ba+3b^2-(7a-3b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{8a^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{8a^2\arctan(\tan(e+fx))}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{8a^2\arctan(\tan(e+fx))}{a-b}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a - b)*(a + b*Tan[e + f*x]^2)^2) + (((8*a^2*Arc
Tan[Tan[e + f*x]])/(a - b) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - ((7*a - 3
*b)*b*Tan[e + f*x])/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/(4*a*(a - b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\tan(fx+e)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(fx+e)}{8a}}{(a+b\tan(fx+e)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(f\ldots}{(a-b)^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\tan(fx+e)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(fx+e)}{8a}}{(a+b\tan(fx+e)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(f\ldots}{(a-b)^3}}{f}$
risch	$\frac{x}{a^3-3a^2b+3ab^2-b^3}+\frac{ib(9a^3e^{6i(fx+e)}+a^2be^{6i(fx+e)}-13ab^2e^{6i(fx+e)}+3b^3e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+9a^2be^{4i(fx+\ldots}{4(-ae^{4i(fx+e)}+be^{4i(fx+e)}\ldots}$

input
```
int(1/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a-b)^3*((1/8*b*(7*a^2-10*a*b+3*b^2)/a^2*tan(f*x+e)^3+1/8*(9*a^2-1
4*a*b+5*b^2)/a*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(15*a^2-10*a*b+3*b^2)/
a^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^3*arctan(tan(f*x
+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(136) = 272$.

Time = 0.14 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.95

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^
4*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 1
0*a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^
3*b - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x +
e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x +
e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*
a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b
^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*
a^2*b^2*f*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*
(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 +
3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a
^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 -
 a)*sqrt(b/a)/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f
*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2
*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6
*b + 3*a^5*b^2 - a^4*b^3)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(129) = 258$.

Time = 47.80 (sec) , antiderivative size = 8964, normalized size of antiderivative = 59.76

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**3, Eq(b
, 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e
+ f*x)**5))/b**3, Eq(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*
x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*
f) + 45*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)*
*2/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*ta
n(e + f*x)**2 + 48*b**3*f) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*
f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 15*tan(e + f
*x)**5/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*
f*tan(e + f*x)**2 + 48*b**3*f) + 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x
)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f
) + 33*tan(e + f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**
4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x/(a + b*tan(e)**
2)**3, Eq(f, 0)), (16*a**4*f*x*sqrt(-a/b)/(16*a**7*f*sqrt(-a/b) + 32*a**6*
b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*s
qrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48
*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a
**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**
3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e +...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}+\frac{(7\,ab^2-3\,b^3)\tan(fx+e)^3+(9\,a^2b-5\,ab^2)\tan(fx+e)}{a^6-2\,a^5b+a^4b^2+(a^4b^2-2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b-2\,a^4b^2+a^3b^3)\tan(fx+e)^2}-}{8\,f}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(f*x
+ e)^3 + (9*a^2*b - 5*a*b^2)*tan(f*x + e))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4
*b^2 - 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^
3)*tan(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= -\frac{(15\,a^2b-10\,ab^2+3\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,(a^5f-3\,a^4bf+3\,a^3b^2f-a^2b^3f)\sqrt{ab}}+\frac{fx+e}{a^3f-3\,a^2bf+3\,ab^2f-b^3f}$$
$$-\frac{7\,ab^2\tan{(fx+e)}^3-3\,b^3\tan{(fx+e)}^3+9\,a^2b\tan{(fx+e)}-5\,ab^2\tan{(fx+e)}}{8\,(a^4f-2\,a^3bf+a^2b^2f)\left(b\tan{(fx+e)}^2+a\right)^2}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5*
f - 3*a^4*b*f + 3*a^3*b^2*f - a^2*b^3*f)*sqrt(a*b)) + (f*x + e)/(a^3*f - 3
*a^2*b*f + 3*a*b^2*f - b^3*f) - 1/8*(7*a*b^2*tan(f*x + e)^3 - 3*b^3*tan(f*
x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))/((a^4*f - 2*a^3*b*
f + a^2*b^2*f)*(b*tan(f*x + e)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 11.23 (sec) , antiderivative size = 3901, normalized size of antiderivative = 26.01

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(atan((((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 30
0*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6
*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760
*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*
(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b
^2)) - (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9
 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^
4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b
^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15
*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^
2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^
5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 +
289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((
96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9
056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^
9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(
e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*
b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^1
0*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4
*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.85

$$\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4\,a^2b^2+10\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4\,a\,b^3-3\sqrt{b}\,\sqrt{a}}{}$$

input
```
int(1/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*
x)**4*a**2*b**2 + 10*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a
)))*tan(e + f*x)**4*a*b**3 - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt
(b)*sqrt(a)))*tan(e + f*x)**4*b**4 - 30*sqrt(b)*sqrt(a)*atan((tan(e + f*x)
*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**3*b + 20*sqrt(b)*sqrt(a)*atan((t
an(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**2*b**2 - 6*sqrt(b)*sq
rt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a*b**3 - 15
*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 10*sqrt(b
)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b - 3*sqrt(b)*sqrt
(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*tan(e + f*x)**4
*a**3*b**2*f*x - 7*tan(e + f*x)**3*a**3*b**2 + 10*tan(e + f*x)**3*a**2*b**
3 - 3*tan(e + f*x)**3*a*b**4 + 16*tan(e + f*x)**2*a**4*b*f*x - 9*tan(e + f
*x)*a**4*b + 14*tan(e + f*x)*a**3*b**2 - 5*tan(e + f*x)*a**2*b**3 + 8*a**5
*f*x)/(8*a**3*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4*a**2*b**3 +
 3*tan(e + f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f*x)**2*a**4*
b - 6*tan(e + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3 - 2*tan(e +
f*x)**2*a*b**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.89 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}f} - \frac{15\cot(e+fx)}{8a^3f} + \frac{\cot(e+fx)}{4af\left(a+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8a^2f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-15/8*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/f-15/8*cot(f*x+e)
/a^3/f+1/4*cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)^2+5/8*cot(f*x+e)/a^2/f/(a+b*t
an(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3} dx$$
$$= \frac{-15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right) - 8\sqrt{a}\cot(e+fx) + \frac{4a^{3/2}b^2\sin(2(e+fx))}{(a-b)(a+b+(a-b)\cos(2(e+fx)))^2} - \frac{\sqrt{a}(9a-7b)b\sin(2(e+fx))}{(a-b)(a+b+(a-b)\cos(2(e+fx)))}}{8a^{7/2}f}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - 8*Sqrt[a]*Cot[e + f*
x] + (4*a^(3/2)*b^2*Sin[2*(e + f*x)])/((a - b)*(a + b + (a - b)*Cos[2*(e +
 f*x)])^2) - (Sqrt[a]*(9*a - 7*b)*b*Sin[2*(e + f*x)])/((a - b)*(a + b + (a
 - b)*Cos[2*(e + f*x)])))/(8*a^(7/2)*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4146, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 253$$

$$\frac{\frac{5\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4a}+\frac{\cot(e+fx)}{4a\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 253$$

$$\frac{\frac{5\left(\frac{3\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{2a}+\frac{\cot(e+fx)}{2a\left(a+b\tan^2(e+fx)\right)}\right)}{4a}+\frac{\cot(e+fx)}{4a\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a}\right)}{2a}+\frac{\cot(e+fx)}{2a\left(a+b\tan^2(e+fx)\right)}\right)}{4a}+\frac{\cot(e+fx)}{4a(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\cot(e+fx)}{a}\right)}{2a}+\frac{\cot(e+fx)}{2a\left(a+b\tan^2(e+fx)\right)}\right)}{4a}+\frac{\cot(e+fx)}{4a(a+b\tan^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(Cot[e + f*x]/(4*a*(a + b*Tan[e + f*x]^2)^2) + (5*((3*(-((Sqrt[b]*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(3/2)) - Cot[e + f*x]/a))/(2*a) + Cot[e
+ f*x]/(2*a*(a + b*Tan[e + f*x]^2))))/(4*a))/f
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{7b\tan(fx+e)^3}{8}+\frac{9\tan(fx+e)a}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{7b\tan(fx+e)^3}{8}+\frac{9\tan(fx+e)a}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\tan(fx+e)}}{f}$
risch	$-\frac{i\left(8a^4 e^{8i(fx+e)}-23a^3b\,e^{8i(fx+e)}+45a^2b^2e^{8i(fx+e)}-45a\,b^3e^{8i(fx+e)}+15b^4e^{8i(fx+e)}+32a^4e^{6i(fx+e)}-46a^3b\,e^{6i(fx-}\right.}{}$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*b*((7/8*b*tan(f*x+e)^3+9/8*tan(f*x+e)*a)/(a+b*tan(f*x+e)^2)^2+
15/8/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/a^3/tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(96) = 192$.

Time = 0.16 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.96

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$

$$= \left[-\frac{4\,(8\,a^2-25\,ab+15\,b^2)\cos{(fx+e)}^5+20\,(5\,ab-6\,b^2)\cos{(fx+e)}^3-15\,\left((a^2-2\,ab+b^2)\cos{(fx+\right.}{32\,\left(a^3b^2f+\right.} \right.$$

$$-\frac{2\,(8\,a^2-25\,ab+15\,b^2)\cos{(fx+e)}^5+10\,(5\,ab-6\,b^2)\cos{(fx+e)}^3-15\,\left((a^2-2\,ab+b^2)\cos{(fx+\right.}{16\,\left(a^3b^2f+(a^5-2\,a^4b+a^3b^2)f\cos{(fx+e}\right.}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(4*(8*a^2 - 25*a*b + 15*b^2)*cos(f*x + e)^5 + 20*(5*a*b - 6*b^2)*co
s(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(
f*x + e)^2 + b^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(
3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x
+ e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 +
 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/
((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b - a^3*
b^2)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/16*(2*(8*a^2 - 25*a*b + 15*b^2)*c
os(f*x + e)^5 + 10*(5*a*b - 6*b^2)*cos(f*x + e)^3 - 15*((a^2 - 2*a*b + b^2
)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(1/
2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*x + e)))*si
n(f*x + e) + 30*b^2*cos(f*x + e))/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*
f*cos(f*x + e)^4 + 2*(a^4*b - a^3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{15\,b^2\tan(fx+e)^4+25\,ab\tan(fx+e)^2+8\,a^2}{a^3b^2\tan(fx+e)^5+2\,a^4b\tan(fx+e)^3+a^5\tan(fx+e)} + \frac{15\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}}{8\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*b^2*tan(f*x + e)^4 + 25*a*b*tan(f*x + e)^2 + 8*a^2)/(a^3*b^2*tan
(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*tan(f*x + e)) + 15*b*arctan(b*t
an(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3))/f
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{15\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3f} - \frac{7\,b^2\tan\left(fx+e\right)^3+9\,ab\tan\left(fx+e\right)}{8\,\left(b\tan\left(fx+e\right)^2+a\right)^2a^3f} - \frac{1}{a^3f\tan\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-15/8*b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - 1/8*(7*b^2*ta
n(f*x + e)^3 + 9*a*b*tan(f*x + e))/((b*tan(f*x + e)^2 + a)^2*a^3*f) - 1/(a
^3*f*tan(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{1}{a}+\frac{25\,b\tan(e+f\,x)^2}{8\,a^2}+\frac{15\,b^2\tan(e+f\,x)^4}{8\,a^3}}{f\,\left(a^2\tan\left(e+f\,x\right)+2\,a\,b\tan\left(e+f\,x\right)^3+b^2\tan\left(e+f\,x\right)^5\right)} - \frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a}}\right)}{8\,a^{7/2}\,f}$$

input
```
int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^3),x)
```

output
```
- (1/a + (25*b*tan(e + f*x)^2)/(8*a^2) + (15*b^2*tan(e + f*x)^4)/(8*a^3))/
(f*(a^2*tan(e + f*x) + b^2*tan(e + f*x)^5 + 2*a*b*tan(e + f*x)^3)) - (15*b
^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2)))/(8*a^(7/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 695, normalized size of antiderivative = 6.21

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
(15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**5*a**2 - 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan
((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a*b + 15*sqrt(b)*sqrt(a)*atan((sqr
t(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*b**2 - 30*sq
rt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
 + f*x)**3*a**2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e +
f*x)/2))/sqrt(b))*sin(e + f*x)**3*a*b + 15*sqrt(b)*sqrt(a)*atan((sqrt(a -
b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*a**2 - 15*sqrt(b)*sqr
t(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**
5*a**2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/
sqrt(b))*sin(e + f*x)**5*a*b - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt
(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*b**2 + 30*sqrt(b)*sqrt(a)*a
tan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a**2
 - 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*sin(e + f*x)**3*a*b - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*ta
n((e + f*x)/2))/sqrt(b))*sin(e + f*x)*a**2 - 8*cos(e + f*x)*sin(e + f*x)**
4*a**3 + 25*cos(e + f*x)*sin(e + f*x)**4*a**2*b - 15*cos(e + f*x)*sin(e +
f*x)**4*a*b**2 + 16*cos(e + f*x)*sin(e + f*x)**2*a**3 - 25*cos(e + f*x)*si
n(e + f*x)**2*a**2*b - 8*cos(e + f*x)*a**3)/(8*sin(e + f*x)*a**4*f*(sin(e
+ f*x)**4*a**2 - 2*sin(e + f*x)**4*a*b + sin(e + f*x)**4*b**2 - 2*sin(e...
```

3.90 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{5(3a-7b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}f} - \frac{(a-3b)\cot(e+fx)}{a^4f} - \frac{\cot^3(e+fx)}{3a^3f} - \frac{(a-b)b\tan(e+fx)}{4a^3f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-11b)b\tan(e+fx)}{8a^4f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-5/8*(3*a-7*b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(9/2)/f-(a-3*b
)*cot(f*x+e)/a^4/f-1/3*cot(f*x+e)^3/a^3/f-1/4*(a-b)*b*tan(f*x+e)/a^3/f/(a+
b*tan(f*x+e)^2)^2-1/8*(7*a-11*b)*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{15\sqrt{b}(-3a+7b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}\left(-8\cot(e+fx)\left(2a-9b+a\csc^2(e+fx)\right)-\frac{3b(9a^2-6ab-11b^2}{}\right.}{24a^{9/2}f}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(15*Sqrt[b]*(-3*a + 7*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*
(-8*Cot[e + f*x]*(2*a - 9*b + a*Csc[e + f*x]^2) - (3*b*(9*a^2 - 6*a*b - 11
*b^2 + (9*a^2 - 20*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a +
b + (a - b)*Cos[2*(e + f*x)])^2))/(24*a^(9/2)*f)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4146, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)^3}dx$$
$$\Big\downarrow \text{4146}$$
$$\frac{\int \frac{\cot^4(e+fx)\left(\tan^2(e+fx)+1\right)}{\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{361}$$

$$\frac{-\frac{1}{4}b\int -\frac{\cot^4(e+fx)\left(-\frac{3(a-b)\tan^4(e+fx)}{a^3}+\frac{4(a-b)\tan^2(e+fx)}{a^2b}+\frac{4}{ab}\right)}{(b\tan^2(e+fx)+a)^2}d\tan(e+fx)-\frac{b(a-b)\tan(e+fx)}{4a^3(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}b\int \frac{\cot^4(e+fx)\left(-\frac{3(a-b)\tan^4(e+fx)}{a^3}+\frac{4(a-b)\tan^2(e+fx)}{a^2b}+\frac{4}{ab}\right)}{(b\tan^2(e+fx)+a)^2}d\tan(e+fx)-\frac{b(a-b)\tan(e+fx)}{4a^3(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{1}{4}b\left(\frac{\int \frac{\cot^4(e+fx)\left(-\frac{(7a-11b)b^2\tan^4(e+fx)}{a}+8(a-2b)b\tan^2(e+fx)+8ab\right)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{2a^3b^2}-\frac{(7a-11b)\tan(e+fx)}{2a^4(a+b\tan^2(e+fx))}\right)-\frac{b(a-b)\tan(e+fx)}{4a^3(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\frac{1}{4}b\left(\frac{\int \left(8b\cot^4(e+fx)+\frac{8(a-3b)b\cot^2(e+fx)}{a}-\frac{5(3a-7b)b^2}{a\left(b\tan^2(e+fx)+a\right)}\right)d\tan(e+fx)}{2a^3b^2}-\frac{(7a-11b)\tan(e+fx)}{2a^4(a+b\tan^2(e+fx))}\right)-\frac{b(a-b)\tan(e+fx)}{4a^3(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}(3a-7b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{8b(a-3b)\cot(e+fx)}{a}-\frac{8}{3}b\cot^3(e+fx)}{2a^3b^2}-\frac{(7a-11b)\tan(e+fx)}{2a^4(a+b\tan^2(e+fx))}\right)-\frac{b(a-b)\tan(e+fx)}{4a^3(a+b\tan^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*((a - b)*b*Tan[e + f*x])/(a^3*(a + b*Tan[e + f*x]^2)^2) + (b*(((-5*(
3*a - 7*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(3/2) - (8*(a
 - 3*b)*b*Cot[e + f*x])/a - (8*b*Cot[e + f*x]^3)/3)/(2*a^3*b^2) - ((7*a -
11*b)*Tan[e + f*x])/(2*a^4*(a + b*Tan[e + f*x]^2))))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-\frac{1}{3a^3\tan(fx+e)^3}-\frac{a-3b}{a^4\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{11}{8}b^2\right)\tan(fx+e)^3+\frac{a(9a-13b)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{5(3a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}}{f}$
default	$\frac{-\frac{1}{3a^3\tan(fx+e)^3}-\frac{a-3b}{a^4\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}ab-\frac{11}{8}b^2\right)\tan(fx+e)^3+\frac{a(9a-13b)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{5(3a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}}{f}$
risch	$i(-320a^3b\,e^{6i(fx+e)}-1600a\,b^3e^{6i(fx+e)}-313a^3b\,e^{4i(fx+e)}+19a^2b^2e^{4i(fx+e)}+1725a\,b^3e^{4i(fx+e)}+64a^3b\,e^{2i(fx+e)}-1$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/a^3/tan(f*x+e)^3-(a-3*b)/a^4/tan(f*x+e)-1/a^4*b*(((7/8*a*b-11/8*
b^2)*tan(f*x+e)^3+1/8*a*(9*a-13*b)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+5/8*(3
*a-7*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(138) = 276.

Time = 0.16 (sec) , antiderivative size = 857, normalized size of antiderivative = 5.56

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/96*(4*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 4*(2
4*a^3 - 206*a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 6
2*a*b^2 + 63*b^3)*cos(f*x + e)^3 + 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^
3)*cos(f*x + e)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4
- 3*a*b^2 + 7*b^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(-b/
a)*log(((a^2 + 6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^
2 - 4*((a^2 + a*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x +
 e) + b^2)/((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e
)^2 + b^2))*sin(f*x + e) - 60*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a
^5*b + a^4*b^2)*f*cos(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)
*f*cos(f*x + e)^4 - (2*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e)),
 -1/48*(2*(16*a^3 - 131*a^2*b + 220*a*b^2 - 105*b^3)*cos(f*x + e)^7 - 2*(2
4*a^3 - 206*a^2*b + 485*a*b^2 - 315*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b - 6
2*a*b^2 + 63*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 13*a^2*b + 17*a*b^2 - 7*b^
3)*cos(f*x + e)^6 - (3*a^3 - 19*a^2*b + 37*a*b^2 - 21*b^3)*cos(f*x + e)^4
- 3*a*b^2 + 7*b^3 - (6*a^2*b - 23*a*b^2 + 21*b^3)*cos(f*x + e)^2)*sqrt(b/a
)*arctan(1/2*((a + b)*cos(f*x + e)^2 - b)*sqrt(b/a)/(b*cos(f*x + e)*sin(f*
x + e)))*sin(f*x + e) - 30*(3*a*b^2 - 7*b^3)*cos(f*x + e))/(((a^6 - 2*a^5*
b + a^4*b^2)*f*cos(f*x + e)^6 - a^4*b^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*
cos(f*x + e)^4 - (2*a^5*b - 3*a^4*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{15\,(3\,ab^2-7\,b^3)\tan(fx+e)^6+25\,(3\,a^2b-7\,ab^2)\tan(fx+e)^4+8\,a^3+8\,(3\,a^3-7\,a^2b)\tan(fx+e)^2}{a^4b^2\tan(fx+e)^7+2\,a^5b\tan(fx+e)^5+a^6\tan(fx+e)^3}+\frac{15\,(3\,ab-7\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}}{24\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*((15*(3*a*b^2 - 7*b^3)*tan(f*x + e)^6 + 25*(3*a^2*b - 7*a*b^2)*tan(f
*x + e)^4 + 8*a^3 + 8*(3*a^3 - 7*a^2*b)*tan(f*x + e)^2)/(a^4*b^2*tan(f*x +
 e)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x + e)^3) + 15*(3*a*b - 7*b^2)*
arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4))/f
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{5\,(3\,ab-7\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4f} - \frac{7\,ab^2\tan\left(fx+e\right)^3-11\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)-13\,ab^2\tan\left(fx+e\right)}{8\,\left(b\tan\left(fx+e\right)^2+a\right)^2a^4f} - \frac{3\,a\tan\left(fx+e\right)^2-9\,b\tan\left(fx+e\right)^2+a}{3\,a^4f\tan\left(fx+e\right)^3}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-5/8*(3*a*b - 7*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4*f) -
1/8*(7*a*b^2*tan(f*x + e)^3 - 11*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e)
 - 13*a*b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a)^2*a^4*f) - 1/3*(3*a*tan(
f*x + e)^2 - 9*b*tan(f*x + e)^2 + a)/(a^4*f*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{1}{3a}+\frac{\tan(e+fx)^2(3a-7b)}{3a^2}+\frac{25b\tan(e+fx)^4(3a-7b)}{24a^3}+\frac{5b^2\tan(e+fx)^6(3a-7b)}{8a^4}}{f\left(a^2\tan(e+fx)^3+2ab\tan(e+fx)^5+b^2\tan(e+fx)^7\right)} - \frac{5\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)(3a-7b)}{8a^{9/2}f}$$

input
```
int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^3),x)
```

output
```
- (1/(3*a) + (tan(e + f*x)^2*(3*a - 7*b))/(3*a^2) + (25*b*tan(e + f*x)^4*(
3*a - 7*b))/(24*a^3) + (5*b^2*tan(e + f*x)^6*(3*a - 7*b))/(8*a^4))/(f*(a^2
*tan(e + f*x)^3 + b^2*tan(e + f*x)^7 + 2*a*b*tan(e + f*x)^5)) - (5*b^(1/2)
*atan((b^(1/2)*tan(e + f*x))/a^(1/2))*(3*a - 7*b))/(8*a^(9/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1032, normalized size of antiderivative = 6.70

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
(45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)**7*a**3 - 195*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*ta
n((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a**2*b + 255*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a*b**2
- 105*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*sin(e + f*x)**7*b**3 - 90*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*t
an((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**3 + 300*sqrt(b)*sqrt(a)*atan(
(sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**2*b -
 210*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**5*a*b**2 + 45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a**3 - 105*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a**2*b
- 45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**7*a**3 + 195*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*t
an((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a**2*b - 255*sqrt(b)*sqrt(a)*ata
n((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a*b**2
 + 105*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**7*b**3 + 90*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**3 - 300*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**2...
```

3.91 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 202

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{\sqrt{b}(15a^2-70ab+63b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}f} - \frac{(a^2-6ab+6b^2)\cot(e+fx)}{a^5f} - \frac{(2a-3b)\cot^3(e+fx)}{3a^4f} - \frac{\cot^5(e+fx)}{5a^3f} - \frac{(a-b)^2b\tan(e+fx)}{4a^4f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-15b)(a-b)b\tan(e+fx)}{8a^5f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-1/8*b^(1/2)*(15*a^2-70*a*b+63*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(
11/2)/f-(a^2-6*a*b+6*b^2)*cot(f*x+e)/a^5/f-1/3*(2*a-3*b)*cot(f*x+e)^3/a^4/
f-1/5*cot(f*x+e)^5/a^3/f-1/4*(a-b)^2*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)
^2-1/8*(7*a-15*b)*(a-b)*b*tan(f*x+e)/a^5/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.71

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{-960\sqrt{b}(15a^2-70ab+63b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)-\frac{2\sqrt{a}(1600a^4-165a^3b+637a^2b^2-28875ab^3+33075b^4+4(416a^4-447a}{}}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-960*Sqrt[b]*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqr
t[a]] - (2*Sqrt[a]*(1600*a^4 - 165*a^3*b + 637*a^2*b^2 - 28875*a*b^3 + 330
75*b^4 + 4*(416*a^4 - 447*a^3*b - 1400*a^2*b^2 + 13125*a*b^3 - 13230*b^4)*
Cos[2*(e + f*x)] - 4*(32*a^4 - 257*a^3*b - 2821*a^2*b^2 + 8925*a*b^3 - 661
5*b^4)*Cos[4*(e + f*x)] - 128*a^4*Cos[6*(e + f*x)] + 1788*a^3*b*Cos[6*(e +
 f*x)] - 8800*a^2*b^2*Cos[6*(e + f*x)] + 14700*a*b^3*Cos[6*(e + f*x)] - 75
60*b^4*Cos[6*(e + f*x)] + 64*a^4*Cos[8*(e + f*x)] - 863*a^3*b*Cos[8*(e + f
*x)] + 2479*a^2*b^2*Cos[8*(e + f*x)] - 2625*a*b^3*Cos[8*(e + f*x)] + 945*b
^4*Cos[8*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^4)/(a + b + (a - b)*Cos[2*(
e + f*x)])^2)/(7680*a^(11/2)*f)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 365, 361, 1582, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^6\,(a+b\tan(e+fx)^2)^3}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5a\tan^2(e+fx)+10a-9b\right)}{\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)}{5a(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 361$$

$$\frac{\frac{-\frac{1}{4}b\int \frac{\cot^4(e+fx)\left(\frac{3\left(5a^2-10ba+9b^2\right)\tan^4(e+fx)}{a^3}+4\left(-\frac{9b}{a^2}+\frac{10}{a}-\frac{5}{b}\right)\tan^2(e+fx)+4\left(\frac{9}{a}-\frac{10}{b}\right)\right)}{\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)-\frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{4a^3\left(a+b\tan^2(e+fx)\right)^2}}{5a}-\frac{\cot^5}{5a(a+b\,t}}{f}$$

$$\Big\downarrow \; 1582$$

$$\frac{\frac{-\frac{1}{4}b\left(\frac{\int -\frac{\cot^4(e+fx)\left(-\frac{b^2\left(35a^2-110ba+99b^2\right)\tan^4(e+fx)}{a}+8b\left(5a^2-20ba+18b^2\right)\tan^2(e+fx)+8a(10a-9b)b\right)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{2a^3b^2}+\frac{\left(35a^2-110ab+99b^2\right)\tan(e+fx}{2a^4\left(a+b\tan^2(e+fx)\right)}\right.}{5a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{-\frac{1}{4}b\left(\frac{\left(35a^2-110ab+99b^2\right)\tan(e+fx)}{2a^4\left(a+b\tan^2(e+fx)\right)}-\frac{\int \frac{\cot^4(e+fx)\left(-\frac{b^2\left(35a^2-110ba+99b^2\right)\tan^4(e+fx)}{a}+8b\left(5a^2-20ba+18b^2\right)\tan^2(e+fx)+8a(10a-9b)b\right)}{b\tan^2(e+fx)+a}d\tan(e+fx)}{2a^3b^2}\right.}{5a}}{f}$$

$$\Big\downarrow \; 1584$$

$$\frac{\frac{-\frac{1}{4}b\left(\frac{\left(35a^2-110ab+99b^2\right)\tan(e+fx)}{2a^4\left(a+b\tan^2(e+fx)\right)}-\frac{\int \left(8(10a-9b)b\cot^4(e+fx)+\frac{8b\left(5a^2-30ba+27b^2\right)\cot^2(e+fx)}{a}-\frac{5b^2\left(15a^2-70ba+63b^2\right)}{a\left(b\tan^2(e+fx)+a\right)}\right)d\tan(e+fx)}{2a^3b^2}\right)-\frac{b\left(5a^2-10a\right.}{4a^3\left(a+\right.}}{5a}}{f}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{b\left(5a^2-10ab+9b^2\right)\tan(e+fx)}{4a^3\left(a+b\tan^2(e+fx)\right)^2}-\frac{1}{4}b\left(\frac{\left(35a^2-110ab+99b^2\right)\tan(e+fx)}{2a^4\left(a+b\tan^2(e+fx)\right)}-\frac{-\frac{8b\left(5a^2-30ab+27b^2\right)\cot(e+fx)}{a}-\frac{5b^{3/2}\left(15a^2-70ab+63b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}}}{2a^3b^2}\right.}{5a}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/5*Cot[e + f*x]^5/(a*(a + b*Tan[e + f*x]^2)^2) + (-1/4*(b*(5*a^2 - 10*a
*b + 9*b^2)*Tan[e + f*x])/(a^3*(a + b*Tan[e + f*x]^2)^2) - (b*(-1/2*((-5*b
^(3/2)*(15*a^2 - 70*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/
a^(3/2) - (8*b*(5*a^2 - 30*a*b + 27*b^2)*Cot[e + f*x])/a - (8*(10*a - 9*b)
*b*Cot[e + f*x]^3)/3)/(a^3*b^2) + ((35*a^2 - 110*a*b + 99*b^2)*Tan[e + f*x
])/(2*a^4*(a + b*Tan[e + f*x]^2))))/4)/(5*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
 - a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
 - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
 + 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{1}{5a^3\tan(fx+e)^5}-\frac{2a-3b}{3a^4\tan(fx+e)^3}-\frac{a^2-6ab+6b^2}{a^5\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-\frac{11}{4}a\,b^2+\frac{15}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(9a^2-26ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+}{a^5}}{f}$
default	$\frac{-\frac{1}{5a^3\tan(fx+e)^5}-\frac{2a-3b}{3a^4\tan(fx+e)^3}-\frac{a^2-6ab+6b^2}{a^5\tan(fx+e)}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b-\frac{11}{4}a\,b^2+\frac{15}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(9a^2-26ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+}{a^5}}{f}$
risch	Expression too large to display

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5/a^3/tan(f*x+e)^5-1/3*(2*a-3*b)/a^4/tan(f*x+e)^3-(a^2-6*a*b+6*b^2
)/a^5/tan(f*x+e)-b/a^5*(((7/8*a^2*b-11/4*a*b^2+15/8*b^3)*tan(f*x+e)^3+1/8*
a*(9*a^2-26*a*b+17*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(15*a^2-70*a*
b+63*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(184) = 368.

Time = 0.19 (sec) , antiderivative size = 1199, normalized size of antiderivative = 5.94

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/480*(4*(64*a^4 - 863*a^3*b + 2479*a^2*b^2 - 2625*a*b^3 + 945*b^4)*cos(
f*x + e)^9 - 4*(160*a^4 - 2173*a^3*b + 7158*a^2*b^2 - 8925*a*b^3 + 3780*b^
4)*cos(f*x + e)^7 + 4*(120*a^4 - 1685*a^3*b + 7104*a^2*b^2 - 11025*a*b^3 +
 5670*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b - 530*a^2*b^2 + 1155*a*b^3 - 756*
b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 100*a^3*b + 218*a^2*b^2 - 196*a*b^3 +
63*b^4)*cos(f*x + e)^8 - 2*(15*a^4 - 115*a^3*b + 303*a^2*b^2 - 329*a*b^3 +
 126*b^4)*cos(f*x + e)^6 + (15*a^4 - 160*a^3*b + 573*a^2*b^2 - 798*a*b^3 +
 378*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 70*a*b^3 + 63*b^4 + 2*(15*a^3*b -
100*a^2*b^2 + 203*a*b^3 - 126*b^4)*cos(f*x + e)^2)*sqrt(-b/a)*log(((a^2 +
6*a*b + b^2)*cos(f*x + e)^4 - 2*(3*a*b + b^2)*cos(f*x + e)^2 + 4*((a^2 + a
*b)*cos(f*x + e)^3 - a*b*cos(f*x + e))*sqrt(-b/a)*sin(f*x + e) + b^2)/((a^
2 - 2*a*b + b^2)*cos(f*x + e)^4 + 2*(a*b - b^2)*cos(f*x + e)^2 + b^2))*sin
(f*x + e) + 60*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*cos(f*x + e))/(((a^7 - 2*a
^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b + 2*a^5*b^
2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^
6*b - 2*a^5*b^2)*f*cos(f*x + e)^2)*sin(f*x + e)), -1/240*(2*(64*a^4 - 863*
a^3*b + 2479*a^2*b^2 - 2625*a*b^3 + 945*b^4)*cos(f*x + e)^9 - 2*(160*a^4 -
 2173*a^3*b + 7158*a^2*b^2 - 8925*a*b^3 + 3780*b^4)*cos(f*x + e)^7 + 2*(12
0*a^4 - 1685*a^3*b + 7104*a^2*b^2 - 11025*a*b^3 + 5670*b^4)*cos(f*x + e)^5
 + 10*(75*a^3*b - 530*a^2*b^2 + 1155*a*b^3 - 756*b^4)*cos(f*x + e)^3 - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{15\,(15\,a^2b^2-70\,ab^3+63\,b^4)\tan(fx+e)^8+25\,(15\,a^3b-70\,a^2b^2+63\,ab^3)\tan(fx+e)^6+8\,(15\,a^4-70\,a^3b+63\,a^2b^2)\tan(fx+e)^4+24\,a^4+8\,(10\,a^4}{a^5b^2\tan(fx+e)^9+2\,a^6b\tan(fx+e)^7+a^7\tan(fx+e)^5}}{120\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/120*((15*(15*a^2*b^2 - 70*a*b^3 + 63*b^4)*tan(f*x + e)^8 + 25*(15*a^3*b
 - 70*a^2*b^2 + 63*a*b^3)*tan(f*x + e)^6 + 8*(15*a^4 - 70*a^3*b + 63*a^2*b
^2)*tan(f*x + e)^4 + 24*a^4 + 8*(10*a^4 - 9*a^3*b)*tan(f*x + e)^2)/(a^5*b^
2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(f*x + e)^5) + 15*(15*a
^2*b - 70*a*b^2 + 63*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^5)
)/f
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.16

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{(15\,a^2b-70\,ab^2+63\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5f}$$

$$-\frac{7\,a^2b^2\tan\left(fx+e\right)^3-22\,ab^3\tan\left(fx+e\right)^3+15\,b^4\tan\left(fx+e\right)^3+9\,a^3b\tan\left(fx+e\right)-26\,a^2b^2\tan\left(fx\right.}{8\left(b\tan\left(fx+e\right)^2+a\right)^2a^5f}$$

$$-\frac{15\,a^2\tan\left(fx+e\right)^4-90\,ab\tan\left(fx+e\right)^4+90\,b^2\tan\left(fx+e\right)^4+10\,a^2\tan\left(fx+e\right)^2-15\,ab\tan\left(fx+\right.}{15\,a^5f\tan\left(fx+e\right)^5}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(15*a^2*b - 70*a*b^2 + 63*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt
(a*b)*a^5*f) - 1/8*(7*a^2*b^2*tan(f*x + e)^3 - 22*a*b^3*tan(f*x + e)^3 + 1
5*b^4*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) - 26*a^2*b^2*tan(f*x + e) + 17
*a*b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a)^2*a^5*f) - 1/15*(15*a^2*tan(f
*x + e)^4 - 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x
 + e)^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(a^5*f*tan(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.99

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{1}{5\,a}+\frac{\tan(e+f\,x)^4\,(15\,a^2-70\,a\,b+63\,b^2)}{15\,a^3}+\frac{\tan(e+f\,x)^2\,(10\,a-9\,b)}{15\,a^2}+\frac{5\,b\tan(e+f\,x)^6\,(15\,a^2-70\,a\,b+63\,b^2)}{24\,a^4}+\frac{b^2\tan(e+f\,x)^8\,(15\,a^2-}{8\,a^5}}{f\,\left(a^2\tan\left(e+f\,x\right)^5+2\,a\,b\tan\left(e+f\,x\right)^7+b^2\tan\left(e+f\,x\right)^9\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a}}\right)\,(15\,a^2-70\,a\,b+63\,b^2)}{8\,a^{11/2}\,f}$$

input
```
int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^3),x)
```

output
```
- (1/(5*a) + (tan(e + f*x)^4*(15*a^2 - 70*a*b + 63*b^2))/(15*a^3) + (tan(e
 + f*x)^2*(10*a - 9*b))/(15*a^2) + (5*b*tan(e + f*x)^6*(15*a^2 - 70*a*b +
63*b^2))/(24*a^4) + (b^2*tan(e + f*x)^8*(15*a^2 - 70*a*b + 63*b^2))/(8*a^5
))/(f*(a^2*tan(e + f*x)^5 + b^2*tan(e + f*x)^9 + 2*a*b*tan(e + f*x)^7)) -
(b^(1/2)*atan((b^(1/2)*tan(e + f*x))/a^(1/2))*(15*a^2 - 70*a*b + 63*b^2))/
(8*a^(11/2)*f)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1429, normalized size of antiderivative = 7.07

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(225*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b)
)*sin(e + f*x)**9*a**4 - 1500*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**9*a**3*b + 3270*sqrt(b)*sqrt(a)*a
tan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**9*a**2
*b**2 - 2940*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))
/sqrt(b))*sin(e + f*x)**9*a*b**3 + 945*sqrt(b)*sqrt(a)*atan((sqrt(a - b) -
 sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**9*b**4 - 450*sqrt(b)*sqr
t(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**
7*a**4 + 2550*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2)
)/sqrt(b))*sin(e + f*x)**7*a**3*b - 3990*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
 - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**7*a**2*b**2 + 1890*sqr
t(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
+ f*x)**7*a*b**3 + 225*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e
+ f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**4 - 1050*sqrt(b)*sqrt(a)*atan((sqrt
(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**3*b + 945*
sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin
(e + f*x)**5*a**2*b**2 - 225*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*t
an((e + f*x)/2))/sqrt(b))*sin(e + f*x)**9*a**4 + 1500*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**9*a**3*b
- 3270*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sq...
```

3.92 $\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\begin{aligned}\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx &= \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}\\&\quad+\frac{2(5a-4b)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{15(a-b)^2f}\\&\quad-\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{5(a-b)f}\end{aligned}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f-cos(f*x+e
)*(a-b+b*sec(f*x+e)^2)^(1/2)/f+2/15*(5*a-4*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+
e)^2)^(3/2)/(a-b)^2/f-1/5*cos(f*x+e)^5*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f
```

Mathematica [A] (warning: unable to verify)

Time = 2.45 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\cos(e+fx)\left(120\sqrt{2}(a-b)^2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{a+b+(a-b)\cos(2(e+fx))}(-89\ldots\right.}{120\sqrt{2}(a\ldots}$$

input
```
Integrate[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(120*Sqrt[2]*(a - b)^2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*
Cos[2*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x
)]]*(-89*a^2 + 254*a*b - 149*b^2 + 4*(7*a^2 - 15*a*b + 8*b^2)*Cos[2*(e + f
*x)] - 3*(a - b)^2*Cos[4*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x
)])*Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^2*f*Sqrt[a + b + (a - b)*Cos[2*(
e + f*x)]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4147, 365, 25, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^5\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int -\cos^4(e+fx)\left(2(5a-4b)-5(a-b)\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5(a-b)}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \cos^4(e+fx)\left(2(5a-4b)-5(a-b)\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5(a-b)}}{f}$$

$$\Big\downarrow \; 358$$

$$\frac{-\frac{-5(a-b)\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)-\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}}{5(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{5(a-b)}}{f}$$

$$\Big\downarrow \; 247$$

$$\frac{-\frac{-5(a-b)\left(b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}}{5(a-b)}-\cos^5(e+fx)(}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{-\frac{-5(a-b)\left(b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}}{5(a-b)}-\text{co}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{-5(a-b)\left(\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\frac{2(5a-4b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}}{5(a-b)}-\cos^5(e+fx}{f}$$

input
```
Int[Sin[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/5*(Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(3/2))/(a - b) - ((-2*(5*
a - 4*b)*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(3/2))/(3*(a - b)) - 5*
(a - b)*(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x
]^2]] - Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2]))/(5*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(145) = 290$.

Time = 8.51 (sec) , antiderivative size = 645, normalized size of antiderivative = 4.01

method	result
default	$\frac{\left(15b^{\frac{5}{2}}\ln\left(-4\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}-4\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sec(fx+e)-4b\sec(fx+e)\right)-30b^{\frac{3}{2}}\ln\left(-4\sqrt{b}\sqrt{\frac{a\cos}{}}\right.\right.}{}$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/f/(a-b)^2*(15*b^(5/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/
(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*
x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))-30*b^(3/2)*ln(-4*b^(1/2)*((a*c
os(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*a+
15*b^(1/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
*sec(f*x+e)-4*b*sec(f*x+e))*a^2+(-3*cos(f*x+e)^5-3*cos(f*x+e)^4+10*cos(f*x
+e)^3+10*cos(f*x+e)^2-15*cos(f*x+e)-15)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(
cos(f*x+e)+1)^2)^(1/2)*a^2+(6*cos(f*x+e)^5+6*cos(f*x+e)^4-21*cos(f*x+e)^3-
21*cos(f*x+e)^2+40*cos(f*x+e)+40)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*
x+e)+1)^2)^(1/2)*a*b+(-3*cos(f*x+e)^5-3*cos(f*x+e)^4+11*cos(f*x+e)^3+11*co
s(f*x+e)^2-23*cos(f*x+e)-23)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*b^2)*cos(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(cos(f*x+e)+1)/((a*co
s(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.45

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \left[\frac{15\,(a^2-2\,ab+b^2)\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\,\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(3\,(a^2-2\,ab+b^2)\cos\right.}{30\,(a^2-}\right.$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*(a^2 - 2*a*b + b^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqr
t(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)
/cos(f*x + e)^2) - 2*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (10*a^2 - 21*
a*b + 11*b^2)*cos(f*x + e)^3 + (15*a^2 - 40*a*b + 23*b^2)*cos(f*x + e))*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 - 2*a*b + b^2)*f),
1/15*(15*(a^2 - 2*a*b + b^2)*sqrt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b))
- (3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (10*a^2 - 21*a*b + 11*b^2)*cos(f
*x + e)^3 + (15*a^2 - 40*a*b + 23*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.26

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \frac{\frac{20\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a-b} - 30\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - 15\sqrt{b}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{}}\right.}{30f}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/30*(20*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/(a - b) - 30*sqrt
(a - b + b/cos(f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a - b + b/c
os(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos
(f*x + e) + sqrt(b))) - 2*(3*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)
^5 - 5*(a - b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3)/(a^2 - 2*a*b + b
^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2554 vs. 2(145) = 290.

Time = 1.16 (sec) , antiderivative size = 2554, normalized size of antiderivative = 15.86

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*b*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
) - sqrt(a))/sqrt(-b))/sqrt(-b) - 2*(15*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^9*b + 165*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))^8*sqrt(a)*b - 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^7*a^2 + 540*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
7*a*b + 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^2 +
640*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2) - 2940
*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3/2)*b + 2960*
(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b^2 + 832*
(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 - 1246*(sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sin\left(e+f\,x\right)^5\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sin^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^5\,dx$$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**5,x)
```

3.93 $\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f-cos(f*x+e
)*(a-b+b*sec(f*x+e)^2)^(1/2)/f+1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)
/(a-b)/f
```

Mathematica [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\cos(e+fx)\left(6\sqrt{2}(a-b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right) + \sqrt{a+b+(a-b)\cos(2(e+fx))}(-5a+\right.}{6\sqrt{2}(a-b)f\sqrt{a+b+(a-b)\cos(2}$$

input
```
Integrate[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[
2*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*
(-5*a + 7*b + (a - b)*Cos[2*(e + f*x)]))*Sqrt[(a + b + (a - b)*Cos[2*(e +
f*x)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e
 + f*x)]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4147, 25, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{358}$$

$$\frac{\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{3(a-b)}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

input
```
Int[Sin[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]] -
Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2] + (Cos[e + f*x]^3*(a - b + b*S
ec[e + f*x]^2)^(3/2))/(3*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(101) = 202$.

Time = 6.44 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.92

method	result
default	$-\frac{\left(3b^{\frac{3}{2}}\ln\left(\frac{4b\cot(fx+e)^2-8b\cot(fx+e)\csc(fx+e)+4b\csc(fx+e)^2+8\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}+4b}{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-3\sqrt{b}\ln\left(\frac{4b\cot(fx+e)^2-8b\,\text{c}}{}\right.\right.}{}$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a-b)*(3*b^(3/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*c
sc(f*x+e)^2+2*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(
1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))-3*b^(1/2)*l
n(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^2-2*c
sc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a+(-cos(f*x+e)^3-cos(f*x+e)^2+3*cos(
f*x+e)+3)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a+(cos(
f*x+e)^3+cos(f*x+e)^2-4*cos(f*x+e)-4)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(co
s(f*x+e)+1)^2)^(1/2)*b)*cos(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(cos(f*x+e)+1)
/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.58

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{3\,(a-b)\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\,\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\left((a-b)\cos\left(fx+e\right)^3-(3\,a-\right.}{6\,(a-b)f}\right.$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a - b)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a
- b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^
2) + 2*((a - b)*cos(f*x + e)^3 - (3*a - 4*b)*cos(f*x + e))*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f), 1/3*(3*(a - b)*sqrt(-b)*a
rctan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e)/((a - b)*cos(f*x + e)^2 + b)) + ((a - b)*cos(f*x + e)^3 - (3*a - 4*b)
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)
*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\frac{2\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a-b} - 6\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - 3\sqrt{b}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{6f}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(2*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/(a - b) - 6*sqrt(a
- b + b/cos(f*x + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a - b + b/cos(f
*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x
 + e) + sqrt(b))))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(101) = 202$.

Time = 0.92 (sec) , antiderivative size = 1182, normalized size of antiderivative = 10.46

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*b*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
- sqrt(a))/sqrt(-b))/sqrt(-b) - 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^5*b - 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))^4*a^(3/2) + 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))
^4*sqrt(a)*b + 16*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*
a^2 - 50*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 40*
(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 24*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) - 54*(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b + 24*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^2 - 48*(sqrt(a)*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sin\left(e+f\,x\right)^3\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^3 dx$$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**3,x)
```

3.94 $\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f-cos(f*x+e
)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.94

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\left(-2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2}\sqrt{a+b+(a-b)\cos(2(e+fx))}\right)\csc(e+fx)\sqrt{(a+b-}{4f\sqrt{a+b+(a-b)\cos(2(e+fx))}}$$

input
```
Integrate[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/4*((-2*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]/(Sqrt[2]*
Sqrt[b])] + Sqrt[2]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Csc[e + f*x]*S
qrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(
f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4147, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

input
```
Int[Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]] -
Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(64) = 128$.

Time = 6.42 (sec) , antiderivative size = 404, normalized size of antiderivative = 5.61

method	result
default	$-\frac{\left(-\sqrt{b}\,\ln\left(\frac{4b\cot(fx+e)^2-8b\cot(fx+e)\csc(fx+e)+4b\csc(fx+e)^2+8\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}+4b}{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)a+b^{\frac{3}{2}}\ln\left(\frac{4b\cot(fx+e)^2-8b}{}\right.}{}$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a-b)*(-b^(1/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(
f*x+e)^2+2*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2
)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a+b^(3/2)*ln(4
*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((a*co
s(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(
f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))+(cos(f*x+e)+1)*((a*cos(f*x+e)^2+b*sin(f
*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a+(-cos(f*x+e)-1)*((a*cos(f*x+e)^2+b*sin(
f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b)*(a+b*tan(f*x+e)^2)^(1/2)*cos(f*x+e)/(
cos(f*x+e)+1)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.04

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[-\frac{2\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}\log\left(-\frac{(a-b)\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f}, \frac{\sqrt{-b}\ldots}{}\right.$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) -
sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, (sqrt(-b)
*arctan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)/((a - b)*cos(f*x + e)^2 + b)) - sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e))/f]
```

Sympy [F]

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\sin{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{2\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos{(fx+e)}+\sqrt{b}\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{2f}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a
- b + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x +
e)^2)*cos(f*x + e) + sqrt(b))))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(64) = 128$.

Time = 0.87 (sec) , antiderivative size = 389, normalized size of antiderivative = 5.40

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{2\left(\frac{b\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}-\sqrt{a}}{2\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{2\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\,\mathrm{ta}}\right.\right.}{\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan}\right.}\right.}{}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
2*(b*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sq
rt(a))/sqrt(-b))/sqrt(-b) + 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))*a - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b - a
^(3/2) + sqrt(a)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))
^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a
 + 4*b))*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sin(e+f\,x)\,\sqrt{b\tan(e+f\,x)^2+a}\,dx$$

input
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sin(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)\,dx$$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x),x)
```

3.95 $\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}$$

output
```
-a^(1/2)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f+b^(1/2)*
arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(84) = 168.

Time = 1.64 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.02

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sec(e+fx)\left(-\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{\sec^2(e+fx)}}{\sqrt{-a-b\tan^2(e+fx)}}\right)\sqrt{-a-b\tan^2(e+fx)}+\sqrt{a-b}\sqrt{b}\,\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{\sec^2(e+fx)}}{\sqrt{a-b}}\right)\right.}{f\sqrt{\sec^2(e+fx)}\sqrt{a+b\tan^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sec[e + f*x]*(-(Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[Sec[e + f*x]^2])/Sqrt[-a - b
*Tan[e + f*x]^2]]*Sqrt[-a - b*Tan[e + f*x]^2]) + Sqrt[a - b]*Sqrt[b]*ArcSi
nh[(Sqrt[b]*Sqrt[Sec[e + f*x]^2])/Sqrt[a - b]]*Sqrt[(a + b*Tan[e + f*x]^2)
/(a - b)]))/(f*Sqrt[Sec[e + f*x]^2]*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4147, 25, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\sqrt{b\sec^2(e+fx)+a-b}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sqrt{b\sec^2(e+fx)+a-b}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-a\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}-a\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-a\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-a\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{f}$$

input

```
Int[Csc[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output

```
(-(Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
 + Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])
/f
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(72) = 144$.

Time = 6.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 4.83

method	result
default	$-\frac{\sqrt{a+b\tan(fx+e)^2}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\left(2\sqrt{b}\,\ln\left(\frac{4(1-\cos(fx+e))^2 b\csc(fx+e)^2+8\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}+4}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right.\right.}{}$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/a^(1/2)*(a+b*tan(f*x+e)^2)^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*
(2*b^(1/2)*ln(4*((1-cos(f*x+e))^2*b*csc(f*x+e)^2+2*b^(1/2)*((a*cos(f*x+e)^
2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1))*a^(1/2)-ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(
f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)
+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a-a*ln(2
/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*
x+e)^2)))/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 579, normalized size of antiderivative = 6.89

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1
)) + sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, 1/2*
(2*sqrt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) + sqrt(a)*log(-2*((a - b)*
cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))/f, -1/2*(2*sqrt(-a)*arctan
(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
((a - b)*cos(f*x + e)^2 + b)) - sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*s
qrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*
b)/cos(f*x + e)^2))/f, -(sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) - sq
rt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)))/f]
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\csc{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^2+a}\csc{(fx+e)}\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionDegree mismatch inside factorisation over extensionDegree
mismatch
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)dx$$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x),x)
```

3.96 $\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f}$$

output
```
-1/2*(a+b)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(1/2)/
f+b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f-1/2*cot
(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 586 vs. 2(127) = 254.

Time = 3.77 (sec) , antiderivative size = 586, normalized size of antiderivative = 4.61

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx =$$

$$-\frac{\cot(e+fx)\csc(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-a\log\left(a-2b-a\tan^2\left(\tfrac{1}{2}(\right.\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/2*(Cot[e + f*x]*Csc[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Se
c[e + f*x]^2]*(-(a*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*T
an[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]) - b*Log[a - 2*b - a*T
an[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
 f*x)/2]^2)^2]] + a*Cos[e + f*x]*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt
[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + b*Cos[
e + f*x]*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*
x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + (Sqrt[a]*Sqrt[(a + b + (a - b)
*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])/Sqrt[2] - 16*Sqrt[a]*Sqrt[b]*ArcTa
nh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a
*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])]*Sin[(e + f*x)/2]^2 - 4*(a + b)
*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e
 + f*x)/2]^2)^2]/Sqrt[a]]*Sin[(e + f*x)/2]^2))/(Sqrt[a]*f*Sqrt[(a + b + (a
 - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4147, 369, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\sin(e+fx)^3}dx$$

$\Big\downarrow$ 4147

$$\frac{\int \frac{\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a-b}}{(1-\sec^2(e+fx))^2}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 369

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2(1-\sec^2(e+fx))}-\frac{1}{2}\int \frac{2b\sec^2(e+fx)+a-b}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{a+b\sec}}{2(1-\sec^2(e}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}-(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{}}{2(1-}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(}}{2(1-\sec^2(e+fx}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{2}\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-(a+b)\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2(1-\sec^2(e+fx))}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}\left(2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}}\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2(1-\sec^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-(((a + b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
])/Sqrt[a]) + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[
e + f*x]^2]])/2 + (Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*(1 - Se
c[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(109) = 218$.

Time = 6.49 (sec) , antiderivative size = 761, normalized size of antiderivative = 5.99

method	result
default	$-\frac{\left((1-\cos(fx+e))\ln\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\cos(fx+e)+2\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sqrt{a}-2a\cos(fx+e)+2\cos(fx+e)b+2b}{\sqrt{a}\,(\cos(fx+e)+1)}\right.\right.}{}$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/a^(3/2)*((1-cos(f*x+e))*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*si
n(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+
e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*
x+e)+1))*a*b+(1-cos(f*x+e))*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(
f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*
cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*a*b+(1
-cos(f*x+e))*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f
*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^2+(4*co
s(f*x+e)-4)*b^(1/2)*a^(3/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)
+b*csc(f*x+e)^2+2*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^
2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))+(1-cos(
f*x+e))*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos
(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*a^2+2*((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(3/2))*(a+b*tan(f*x+e)^2)^(1/2)/((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)*csc(f*x+e
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(109) = 218$.

Time = 0.58 (sec) , antiderivative size = 915, normalized size of antiderivative = 7.20

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 -
 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
+ a + b)/(cos(f*x + e)^2 - 1)) + 2*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(-((a
 - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*f),
-1/2*(((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a
- b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e
)^2 + b)) - a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) - (a*cos(f*x + e)^2 - a)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(
b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/c
os(f*x + e)^2))/(a*f*cos(f*x + e)^2 - a*f), 1/4*(4*(a*cos(f*x + e)^2 - a)*
sqrt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) + 2*a*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + ((a + b)*cos(f*x + e)^2 - a - b)
*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))/(
a*f*cos(f*x + e)^2 - a*f), -1/2*(((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)
*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)/((a - b)*cos(f*x + e)^2 + b)) - 2*(a*cos(f*x + e)^2 - a)*sqrt(-b...
```

Sympy [F]

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\csc^3{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \frac{\sqrt{b\tan(e+fx)^2+a}}{\sin(e+fx)^3}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^3\,dx$$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**3,x)
```

3.97 $\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$
\begin{aligned}
&\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx \\
&= -\frac{(3a^2+6ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} \\
&\quad -\frac{(3a+b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8af} \\
&\quad -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4f}
\end{aligned}
$$

output

```
-1/8*(3*a^2+6*a*b-b^2)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/
2))/a^(3/2)/f+b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2
))/f-1/8*(3*a+b)*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f-1/4*
cot(f*x+e)*csc(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1049 vs. $2(187) = 374$.

Time = 6.64 (sec) , antiderivative size = 1049, normalized size of antiderivative = 5.61

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*
x)])]*(((-3*a*Cos[e + f*x] - b*Cos[e + f*x])*Csc[e + f*x]^2)/(8*a) - (Cot[
e + f*x]*Csc[e + f*x]^3)/4))/f + (((3*a^2 - 2*a*b - b^2)*(1 + Cos[e + f*x]
)*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-
1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
f*x)/2]^2)^2])/(2*Sqrt[b])] - Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt
[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a
- 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1
 + Tan[(e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/
2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + T
an[(e + f*x)/2]^2)^2]])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e +
f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(
-1 + Tan[(e + f*x)/2]^2)^2]) - ((3*a^2 + 14*a*b - b^2)*(1 + Cos[e + f*x])*
Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Co
s[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1
+ Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*
x)/2]^2)^2])/(2*Sqrt[b])] + Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4
*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a -
2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-...
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4147, 25, 369, 440, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\sec^4(e+fx)\sqrt{b\sec^2(e+fx)+a-b}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(e+fx)\sqrt{b\sec^2(e+fx)+a-b}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{1}{4}\int \frac{\sec^2(e+fx)\left(4b\sec^2(e+fx)+3(a-b)\right)}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{440}$$

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{8ab\sec^2(e+fx)+(a-b)(3a+b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{2a}+\frac{(3a+b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a\left(1-\sec^2(e+fx)\right)}\right)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\left(\frac{(3a+b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\int \frac{8ab\sec^2(e+fx)+(a-b)(3a+b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{2a}\right)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{4}\left(\frac{(3a+b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{(3a^2+6ab-b^2)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-8ab\int\frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(}{2a}\right.}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{4}\left(\frac{(3a+b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{(3a^2+6ab-b^2)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-8ab\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{}{\sqrt{}}}{2a}\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{(3a+b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{(3a^2+6ab-b^2)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-8a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+}{\sqrt{a+b\sec^2(e+}}\right.}{2a}\right.}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{4}\left(\frac{(3a+b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{(3a^2+6ab-b^2)\int\frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}-8a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-}}\right.}{2a}\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{(3a+b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{\frac{(3a^2+6ab-b^2)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}}-8a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a}\right)-\text{s}}{f}$$

input
```
Int[Csc[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/4*(Sec[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(1 - Sec[e + f*x]^2)
^2 + (-1/2*(((3*a^2 + 6*a*b - b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a -
 b + b*Sec[e + f*x]^2]])/Sqrt[a] - 8*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*
x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/a + ((3*a + b)*Sec[e + f*x]*Sqrt[a -
b + b*Sec[e + f*x]^2])/(2*a*(1 - Sec[e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1149 vs. $2(165) = 330$.

Time = 6.30 (sec) , antiderivative size = 1150, normalized size of antiderivative = 6.15

method	result	size
default	Expression too large to display	1150

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/a^(7/2)*((-16*cos(f*x+e)+16)*sin(f*x+e)^2*a^(7/2)*b^(1/2)*ln(4*(b*c
ot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((a*cos(f*x
+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e
)*cot(f*x+e)+csc(f*x+e)^2-1))+(6*cos(f*x+e)^2-10)*((a*cos(f*x+e)^2+b*sin(f
*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)-2*b*((a*cos(f*x+e)^2+b*sin(f*x+e)
^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(5/2)*sin(f*x+e)^2+(3*cos(f*x+e)-3)*sin(f*x+
e)^2*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)
^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^4+(6*cos(f*x+e)
-6)*sin(f*x+e)^2*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(c
os(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x
+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^3*b
+(1-cos(f*x+e))*sin(f*x+e)^2*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(
f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)
^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+
e)+1))*a^2*b^2+(3*cos(f*x+e)-3)*sin(f*x+e)^2*ln(2*(2*((a*cos(f*x+e)^2+b*si
n(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*
cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*
x+e)-1)^2)*a^4+(6*cos(f*x+e)-6)*sin(f*x+e)^2*ln(2*(2*((a*cos(f*x+e)^2+b*si
n(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2...
```

Fricas [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 1340, normalized size of antiderivative = 7.17

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*co
s(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2
 + 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
) + a + b)/(cos(f*x + e)^2 - 1)) - 8*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x +
 e)^2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b
)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2)
- 2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a -
 b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*c
os(f*x + e)^2 + a^2*f), -1/8*(((3*a^2 + 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3
*a^2 + 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b - b^2)*sqrt(-a)*arctan(
-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(
(a - b)*cos(f*x + e)^2 + b)) - 4*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^
2 + a^2)*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - ((
3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x
 + e)^2 + a^2*f), 1/16*(16*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^
2)*sqrt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) - ((3*a^2 + 6*a*b - b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 + 6*a*b ...
```

Sympy [F]

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\csc^5{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**5, x)
```

Maxima [F]

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sin(e+f\,x)^5}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

Reduce [F]

$$\int \csc^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^5dx$$

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**5,x)
```

3.98 $\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{(3a^2-12ab+8b^2)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{3/2}f}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(3a-4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)f} - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

output
```
1/8*(3*a^2-12*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^
(1/2))/(a-b)^(3/2)/f+b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)
^(1/2))/f-1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-
b)/f-1/4*cos(f*x+e)*sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.59 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.75

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\left(-((a-b)\left(7a^2-11b^2+6(a^2-3ab+2b^2)\cos(2(e+fx))-(a-b)^2\cos(4(e+fx))\right))+2\sqrt{2}a(3a^2-7\right.}{}$$

input
```
Integrate[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-((a - b)*(7*a^2 - 11*b^2 + 6*(a^2 - 3*a*b + 2*b^2)*Cos[2*(e + f*x)] - (
a - b)^2*Cos[4*(e + f*x)])) + 2*Sqrt[2]*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[
((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*Sq
rt[2]*a*(3*a^2 - 12*a*b + 8*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*
Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e
 + f*x)])/(32*Sqrt[2]*(a - b)^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4146, 369, 440, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a}}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\frac{1}{4}\int \frac{\tan^2(e+fx)(4b\tan^2(e+fx)+3a)}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \text{ 440}$$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{8(a-b)b\tan^2(e+fx)+a(3a-4b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2(a-b)}-\frac{(3a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)(\tan^2(e+fx)+1)}\right)-\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{1}{4}\left(\frac{(3a^2-12ab+8b^2)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+8b(a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2(a-b)}-\frac{(3a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)(\tan^2(e+fx)+1)}\right)\ldots}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{4}\left(\frac{(3a^2-12ab+8b^2)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+8b(a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{2(a-b)}-\frac{(3a-4b)\tan(e+fx)\sqrt{\ldots}}{2(a-b)(\tan^2(e\ldots}\right)\ldots}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{4}\left(\frac{(3a^2-12ab+8b^2)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+8\sqrt{b}(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)}-\frac{(3a-4b)\tan(e+fx)\sqrt{a+b\,\ldots}}{2(a-b)(\tan^2(e+fx)\ldots}\right)\ldots}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{1}{4}\left(\frac{\left(3a^2-12ab+8b^2\right)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+8\sqrt{b}(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)}-\frac{(3a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+}}{2(a-b)(\tan^2(e+fx)+1)}\right.}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{1}{4}\left(\frac{\frac{\left(3a^2-12ab+8b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}+8\sqrt{b}(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)}-\frac{(3a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)(\tan^2(e+fx)+1)}\right)-\frac{\tan^3}{}}{f}$$

input
```
Int[Sin[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/4*(Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(1 + Tan[e + f*x]^2)^2 +
 ((((3*a^2 - 12*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*
Tan[e + f*x]^2]])/Sqrt[a - b] + 8*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*(a - b)) - ((3*a - 4*b)*Tan[e + f*x
]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a - b)*(1 + Tan[e + f*x]^2)))/4)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(167) = 334$.

Time = 23.13 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.97

method	result
default	$\frac{\left(3\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a^2-12\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)ab+8\arctan\left(\sqrt{\frac{a\cos(\ldots}{(\ldots}}\right.\right.}{}$

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(a-b)^(3/2)*(3*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)
/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a^2-12*arctan(1/(a-b)^
(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/
(cos(f*x+e)-1))*a*b+8*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*b^2+8*b^(3/2)*arctanh
(1/b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*
x+e)/(cos(f*x+e)-1))*(a-b)^(1/2)-8*b^(1/2)*arctanh(1/b^(1/2)*((a*cos(f*x+e
)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*(a-
b)^(1/2)*a+(2*cos(f*x+e)^3+2*cos(f*x+e)^2-5*cos(f*x+e)-5)*sin(f*x+e)*(a-b)
^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a+(-2*cos(
f*x+e)^3-2*cos(f*x+e)^2+6*cos(f*x+e)+6)*sin(f*x+e)*(a-b)^(1/2)*((a*cos(f*x
+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b)*cos(f*x+e)*(a+b*tan(f*x+e
)^2)^(1/2)/(cos(f*x+e)+1)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^
2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(167) = 334$.

Time = 7.13 (sec) , antiderivative size = 2068, normalized size of antiderivative = 10.94

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*((3*a^2 - 12*a*b + 8*b^2)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^
2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7
*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*
b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 +
128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e
)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 -
24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e)
)*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 16*(a^2 - 2*a*b + b^2)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x +
 e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*(2*(a^2 - 2*a*b + b^2)*cos(f*x +
e)^3 - (5*a^2 - 11*a*b + 6*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2 - 2*a*b + b^2)*f), -1/64*(32*(a^
2 - 2*a*b + b^2)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f
*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b
 - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - (3*a^2 - 12*a*b + 8*b^2)*sq
rt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e
)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 ...
```

Sympy [F]

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\sin^4{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan\left(fx+e\right)^2+a}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan\left(fx+e\right)^2+a}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sin\left(e+f\,x\right)^4\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sin^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^4 dx$$

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**4,x)
```

3.99 $\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{(a-2b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{a-b}f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*(a-2*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^
(1/2)/f+b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/2
*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.71 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.13

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx =$$
$$-\frac{\left((a-b)(a+b+(a-b)\cos(2(e+fx)))+\sqrt{2}a(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\operatorname{EllipticF}\left(\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/4*(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqr
t[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] +
 Sqrt[2]*a*(a - 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]
^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f
*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(S
qrt[2]*(a - b)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4146, 369, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^2\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{1}{2}\int \frac{2b\tan^2(e+fx)+a}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-2b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left((a-2b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+2b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}\right)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left((a-2b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{2}\left((a-2b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\left(\frac{(a-2b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}+2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((((a - 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]
)/Sqrt[a - b] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
+ f*x]^2]])/2 - (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(1 + Tan[e +
f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(110) = 220$.

Time = 14.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.54

method	result
default	$-\frac{\left(-\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a+2\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)b+2\sqrt{b}\,\operatorname{arctanh}\left(\sqrt{a \cdots}\right.\right.}{2f\sqrt{a-b}\,(\cos(fx+e)+\cdots}$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a-b)^(1/2)*(-arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)
/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a+2*arctan(1/(a-b)^(1/
2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(co
s(f*x+e)-1))*b+2*b^(1/2)*arctanh(1/b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*(a-b)^(1/2)+(cos(f*x+
e)+1)*sin(f*x+e)*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2))*cos(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(cos(f*x+e)+1)/((a*cos(f*
x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(110) = 220$.

Time = 0.73 (sec) , antiderivative size = 1847, normalized size of antiderivative = 14.43

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)*sin(f*x + e) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^
2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7
*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*
b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 +
128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e
)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 -
24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e)
)*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) - 4*(a - b)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*
b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
 8*b^2)/cos(f*x + e)^4))/((a - b)*f), -1/16*(8*(a - b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + 8*(a - b)*sqrt(
-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt
(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) - (a - 2*b)*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2
- 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 -...
```

Sympy [F]

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{a+b\tan^2{(e+fx)}}\sin^2{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{b\tan{(fx+e)}^2+a}\sin{(fx+e)}^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{b\tan{(fx+e)}^2+a}\sin{(fx+e)}^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sin\left(e+f\,x\right)^2\,\sqrt{b\,\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sin^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^2 dx$$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**2,x)
```

3.100 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx=\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

output
```
(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+b^(1
/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.27

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}\log\left(-\sqrt{b}\tan(e+fx)+\sqrt{a+b\tan^2(e\right.}{f}$$

input
```
Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-((Sqrt[a - b]*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*Sqr
t[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tan[e + f*x]
) + Sqrt[a + b*Tan[e + f*x]^2]])/f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4144, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{(a-b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]
 + Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(73) = 146$.

Time = 0.54 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{f}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{fb(a-b)}+\frac{a\sqrt{b^4(a-b)}\,\mathrm{arc}}{}$
default	$\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{f}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{fb(a-b)}+\frac{a\sqrt{b^4(a-b)}\,\mathrm{arc}}{}$

input
```
int((a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b)
)^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b
))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.64

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(f\right.}{2\,f}\right.$$
$$-\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2+2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2\,f},$$

input
```
integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b
)*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt
(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)
))/f, 1/2*(2*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x +
e)^2 + a)) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*tan(f*x +
e)/sqrt(b*tan(f*x + e)^2 + a)) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
+ e)^2 + 1)))/f, (sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f
*x + e)^2 + a)) - sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e
)^2 + a)))/f]
```

Sympy [F]

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2\left(e+fx\right)}\,dx$$

input
```
integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a + b\tan^2(e + fx)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \sqrt{a + b\tan^2(e + fx)}\, dx = \int \sqrt{b\tan{(fx + e)}^2 + a}\, dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2b+a}dx$$

input
```
int((a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a),x)
```

3.101 $\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

output
```
b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-cot(f*x+e)*
(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.36

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\left((a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx) - \sqrt{2}b\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\operatorname{EllipticF}\left(\operatorname{arcs}\right.\right.}{\sqrt{2}f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+f}}$$

input
```
Integrate[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-((((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - Sqrt[2]*b*Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[
((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e
 + f*x])/(Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2
]))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4146, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a}\,d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Csc[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] - Cot[
e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$-\frac{\left(a+b\tan(fx+e)^{2}\right)^{\frac{3}{2}}}{fa\tan(fx+e)}+\frac{b\tan(fx+e)\sqrt{a+b\tan(fx+e)^{2}}}{fa}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^{2}}\right)}{f}$	93
default	$-\frac{\left(a+b\tan(fx+e)^{2}\right)^{\frac{3}{2}}}{fa\tan(fx+e)}+\frac{b\tan(fx+e)\sqrt{a+b\tan(fx+e)^{2}}}{fa}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^{2}}\right)}{f}$	93

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+1/f/a*b*tan(f*x+e)*(a+b*tan(f*x
+e)^2)^(1/2)+1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(58) = 116$.

Time = 0.19 (sec) , antiderivative size = 331, normalized size of antiderivative = 5.02

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{b}\log\left(\frac{(a^2-8\,ab+8\,b^2)\cos(fx+e)^4+8\,(ab-2\,b^2)\cos(fx+e)^2+4\left((a-2\,b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+}{\cos(fx+e)^4}\right.}{4\,f\sin(fx+e)}\right.$$

$$-\frac{\sqrt{-b}\arctan\left(\frac{\left((a-2\,b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left((ab-b^2)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin(fx+e)+2\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\mathrm{co}}{2\,f\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*
cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4)*sin(f*x + e) - 4*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e))/(f*sin(f*x + e)), -1/2*(sqrt(-b)*arctan(1/2*((a - 2*b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*si
n(f*x + e) + 2*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e))/(f*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2(e+fx)}\csc^2(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right) - \frac{\sqrt{b\tan(fx+e)^2+a}}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
(sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - sqrt(b*tan(f*x + e)^2 + a)/ta
n(f*x + e))/f
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan\left(fx+e\right)^2+a}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^2dx$$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**2,x)
```

3.102 $\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af}$$

output
```
b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-cot(f*x+e)*
(a+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/a/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.79 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.04

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\left((6a^2+11ab+3b^2+4(a^2-3ab-b^2)\cos(2(e+fx))+(-2a^2+ab+b^2)\cos(4(e+fx))\right)\csc^4(e+f}{12\sqrt{2}af\sqrt{(a+b+(a}$$

input
```
Integrate[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/12*(((6*a^2 + 11*a*b + 3*b^2 + 4*(a^2 - 3*a*b - b^2)*Cos[2*(e + f*x)] +
 (-2*a^2 + a*b + b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^4 - 12*Sqrt[2]*a*b*Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin
[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])
*Tan[e + f*x])/(Sqrt[2]*a*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e
+ f*x]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4146, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}\,d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 358$$

$$\frac{\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a}\,d\tan(e+fx)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}}{f}$$

$$\Big\downarrow \; 247$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Csc[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] - Cot[
e + f*x]*Sqrt[a + b*Tan[e + f*x]^2] - (Cot[e + f*x]^3*(a + b*Tan[e + f*x]^
2)^(3/2))/(3*a))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.

Time = 12.43 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.41

method	result
default	$\frac{\left((3\cos(fx+e)-3)\sin(fx+e)a\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{b}\,(\cos(fx+e)-1)}\right)+\left(2\cos(fx+e)^2-3\right)\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+}{(\cos(fx+e)+1)^2}}\right.}{3fa\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}}$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a*((3*cos(f*x+e)-3)*sin(f*x+e)*a*b^(1/2)*arctanh(1/b^(1/2)*((a*cos(f
*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))
+(2*cos(f*x+e)^2-3)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/
2)*a-sin(f*x+e)^2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
*b)*(a+b*tan(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(88) = 176$.

Time = 0.24 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.35

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \left[\frac{3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{b}\log\left(\frac{(a^2-8ab+8b^2)\cos(fx+e)^4+8(ab-2b^2)\cos(fx+e)^2+4\left((a-2b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{\cdots}}{\cos(fx+e)^4}\right.}{12\left(af\cos\left(fx\cdots\right.\right.}\right.$$

$$-\frac{3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{-b}\arctan\left(\frac{\left((a-2b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left((ab-b^2)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sin\left(fx+e\right)+\cdots}{6\left(af\cos\left(fx+e\right)^2-af\right)\sin\left(fx+\cdots\right.}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x
 + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x
 + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e)), -1/6*(3*(a*cos(f*x +
 e)^2 - a)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2
)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((2*a + b)*cos(f*x
 + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/((a*f*cos(f*x + e)^2 - a*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2(e+fx)}\csc^4(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{3\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right) - \frac{3\sqrt{b\tan(fx+e)^2+a}}{\tan(fx+e)} - \frac{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{a\tan(fx+e)^3}}{3f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - 3*sqrt(b*tan(f*x + e)^2
 + a)/tan(f*x + e) - (b*tan(f*x + e)^2 + a)^(3/2)/(a*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^4dx$$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**4,x)
```

3.103 $\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{5af}$$

output
```
b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-cot(f*x+e)*
(a+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a-b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3
/2)/a^2/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2)/a/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.45 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.04

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\left((80a^3+198a^2b+98ab^2-20b^3+(40a^3-241a^2b-149ab^2+30b^3)\cos(2(e+fx))+(-32a^3+42a^2\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/240*(((80*a^3 + 198*a^2*b + 98*a*b^2 - 20*b^3 + (40*a^3 - 241*a^2*b - 1
49*a*b^2 + 30*b^3)*Cos[2*(e + f*x)] + (-32*a^3 + 42*a^2*b + 62*a*b^2 - 12*
b^3)*Cos[4*(e + f*x)] + 8*a^3*Cos[6*(e + f*x)] + a^2*b*Cos[6*(e + f*x)] -
11*a*b^2*Cos[6*(e + f*x)] + 2*b^3*Cos[6*(e + f*x)])*Csc[e + f*x]^6 - 240*S
qrt[2]*a^2*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*E
llipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
]/Sqrt[2]], 1])*Tan[e + f*x])/(Sqrt[2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4146, 365, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int \cot^4(e+fx)(5a\tan^2(e+fx)+2(5a-b))\sqrt{b\tan^2(e+fx)+a}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx))^{3/2}}{5a}}{f}$$

$$\Big\downarrow \; 358$$

$$\frac{\frac{5a\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a}d\tan(e+fx)-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx))^{3/2}}{5a}}{f}$$

$$\Big\downarrow \; 247$$

$$\frac{\frac{5a\left(b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx))^{3/2}}{5a}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{5a\left(b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx))^{3/2}}{5a}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{5a\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{2(5a-b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx))^{3/2}}{5a}}{f}$$

input
```
Int[Csc[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2))/a + ((-2*(5*a - b)*Cot
[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2))/(3*a) + 5*a*(Sqrt[b]*ArcTanh[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] - Cot[e + f*x]*Sqrt[a + b*
Tan[e + f*x]^2]))/(5*a))/f
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(125) = 250.

Time = 13.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.24

method	result
default	$-\frac{\left(\sin(fx+e)^3(-15\cos(fx+e)+15)a^2\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{b}\,(\cos(fx+e)-1)}\right)+\left(8\cos(fx+e)^4-20\cos(fx+e)^2+15\right.\right.}{}$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^2*(sin(f*x+e)^3*(-15*cos(f*x+e)+15)*a^2*b^(1/2)*arctanh(1/b^(1/2
)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos
(f*x+e)-1))+(8*cos(f*x+e)^4-20*cos(f*x+e)^2+15)*((a*cos(f*x+e)^2+b*sin(f*x
+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^2+(-9*cos(f*x+e)^2+10)*sin(f*x+e)^2*((a*c
os(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a*b-2*sin(f*x+e)^4*((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b^2)*(a+b*tan(f*x+e)
^2)^(1/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x
+e)*csc(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(125) = 250$.

Time = 0.67 (sec) , antiderivative size = 587, normalized size of antiderivative = 4.16

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/60*(15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b)*log(((
a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*(
(a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x
 + e) - 4*((8*a^2 + 9*a*b - 2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b
^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos
(f*x + e)^2 + a^2*f)*sin(f*x + e)), -1/30*(15*(a^2*cos(f*x + e)^4 - 2*a^2*
cos(f*x + e)^2 + a^2)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((8*a^2
 + 9*a*b - 2*b^2)*cos(f*x + e)^5 - (20*a^2 + 19*a*b - 4*b^2)*cos(f*x + e)^
3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/((a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2
*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\csc^6{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*csc(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{15\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right) - \frac{15\sqrt{b\tan(fx+e)^2+a}}{\tan(fx+e)} - \frac{10\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{a\tan(fx+e)^3} + \frac{2\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}b}{a^2\tan(fx+e)^3} - \frac{3\left(b\tan(fx+e)^2+a\right)}{a\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) - 15*sqrt(b*tan(f*x + e
)^2 + a)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a)^(3/2)/(a*tan(f*x + e)^3)
 + 2*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a^2*tan(f*x + e)^3) - 3*(b*tan(f*x +
e)^2 + a)^(3/2)/(a*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\,\csc(fx+e)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sin(e+f\,x)^6}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan(fx+e)^2 b + a}\,\csc(fx+e)^6\,dx$$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**6,x)
```

3.104 $\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 227

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{(3a-7b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-7b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f} - \frac{(3a-7b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f} + \frac{2\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f} - \frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{5(a-b)f}$$

output
```
1/2*(3*a-7*b)*b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2
))/f+1/2*(3*a-7*b)*b*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f-1/3*(3*
a-7*b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+2/3*cos(f*x+e)^3*(a-b
+b*sec(f*x+e)^2)^(5/2)/(a-b)/f-1/5*cos(f*x+e)^5*(a-b+b*sec(f*x+e)^2)^(5/2)
/(a-b)/f
```

Mathematica [A] (warning: unable to verify)

Time = 3.96 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\cos(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(120\sqrt{2}\sqrt{b}(3a^2-10ab+7b^2)\,\mathrm{a}\right.}{}$$

input
```
Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(120
*Sqrt[2]*Sqrt[b]*(3*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b + (a - b)*Cos
[2*(e + f*x)]]/(Sqrt[2]*Sqrt[b])] + 2*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)
]]*(-89*a^2 + 474*a*b - 409*b^2 + 4*(7*a^2 - 20*a*b + 13*b^2)*Cos[2*(e + f
*x)] - 3*(a - b)^2*Cos[4*(e + f*x)] + 60*(a - b)*b*Sec[e + f*x]^2)))/(240*
Sqrt[2]*(a - b)*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4147, 365, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^5\left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow 365$$

$$\frac{\frac{\int -5(a-b)\cos^4(e+fx)\left(2-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5(a-b)}}{f}$$

$$\Big\downarrow 27$$

$$\frac{-\int \cos^4(e+fx)\left(2-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}d\sec(e+fx)-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5(a-b)}}{f}$$

$$\Big\downarrow 359$$

$$\frac{\frac{(3a-7b)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^{3/2}d\sec(e+fx)}{3(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5(a-b)}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3(a-b)}}{f}$$

$$\Big\downarrow 247$$

$$\frac{\frac{(3a-7b)\left(3b\int \sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{5(a-b)}+\frac{2\cos^3(e+fx)}{}}{f}$$

$$\Big\downarrow 211$$

$$\frac{\frac{(3a-7b)\left(3b\left(\frac{1}{2}(a-b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}-\frac{\cos^5(e+}{}}{f}$$

$$\Big\downarrow 224$$

$$\frac{\frac{(3a-7b)\left(3b\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}}{f}$$

$$\Big\downarrow 219$$

$$\frac{\frac{(3a-7b)\left(3b\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}-\frac{\cos^5(e+f}{}}{f}$$

input
```
Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*(a - b)) - (Cos[e
+ f*x]^5*(a - b + b*Sec[e + f*x]^2)^(5/2))/(5*(a - b)) + ((3*a - 7*b)*(-(C
os[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2)) + 3*b*(((a - b)*ArcTanh[(Sqr
t[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec[e +
 f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/2)))/(3*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(203) = 406$.

Time = 10.86 (sec) , antiderivative size = 1637, normalized size of antiderivative = 7.21

method	result	size
default	Expression too large to display	1637

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/f/(a-b)^5/b*(-105*b^(15/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+
e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(
cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*cos(f*x+e)^2+570*b^(13/2
)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4
*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+
e)-4*b*sec(f*x+e))*a*cos(f*x+e)^2-1275*b^(11/2)*ln(-4*b^(1/2)*((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*
sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*a^2*cos(f
*x+e)^2+1500*b^(9/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f
*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1
)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*a^3*cos(f*x+e)^2-975*b^(7/2)*ln(-4*b
^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*
((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*se
c(f*x+e))*a^4*cos(f*x+e)^2+330*b^(5/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*si
n(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e
)^2)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*a^5*cos(f*x+e)^2-4
5*b^(3/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*
sec(f*x+e)-4*b*sec(f*x+e))*a^6*cos(f*x+e)^2+2*cos(f*x+e)^2*(3*cos(f*x+e)^5
+3*cos(f*x+e)^4-10*cos(f*x+e)^3-10*cos(f*x+e)^2+15*cos(f*x+e)+15)*((a*c...
```

Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.95

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{15\left(3a^2-10ab+7b^2\right)\sqrt{b}\cos(fx+e)\log\left(-\frac{(a-b)\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{\cos(fx+e)^2}\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/60*(15*(3*a^2 - 10*a*b + 7*b^2)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos
(f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(6*(a^2 - 2*a*b + b^2)*cos(f*x + e
)^6 - 4*(5*a^2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(15*a^2 - 70*a*b + 58*
b^2)*cos(f*x + e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2))/((a - b)*f*cos(f*x + e)), 1/30*(15*(3*a^2 - 10*a*b + 7*b^2
)*sqrt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b))*cos(f*x + e) - (6*(a^2 - 2*
a*b + b^2)*cos(f*x + e)^6 - 4*(5*a^2 - 13*a*b + 8*b^2)*cos(f*x + e)^4 + 2*
(15*a^2 - 70*a*b + 58*b^2)*cos(f*x + e)^2 - 15*a*b + 15*b^2)*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a - b)*f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.45

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} \, dx =$$

$$-\frac{\frac{12\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{a-b} - 40\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3 + 60\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}(a-b)\cos}{}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/60*(12*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/(a - b) - 40*(a
- b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a - b + b/cos(f*x +
 e)^2)*(a - b)*cos(f*x + e) - 120*sqrt(a - b + b/cos(f*x + e)^2)*b*cos(f*x
 + e) - 60*b^(3/2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt
(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 30*(a*b -
b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x + e)^
2)*cos(f*x + e)^2 - b) + 45*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^2
)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + s
qrt(b)))/sqrt(b))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4702 vs. $2(203) = 406$.

Time = 5.14 (sec) , antiderivative size = 4702, normalized size of antiderivative = 20.71

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/15*(15*(3*a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 7*b^2*sgn(tan(1/2*f*x +
1/2*e)^2 - 1))*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a) - sqrt(a))/sqrt(-b))/sqrt(-b) + 30*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^3*a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + (sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*
b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))^2*sqrt(a)*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 3*(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1) - 9*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^
2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^(5/2)*b*sgn(t...
```

Mupad [F(-1)]

Timed out.

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \sin\left(e+f\,x\right)^5\,{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

input
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^5\tan(fx+e)^2\,dx\right)b + \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^5\,dx\right)a$$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**5*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**5,x)*a
```

3.105 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{(3a-5b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{(3a-5b)b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2(a-b)f}-\frac{(3a-5b)\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{3(a-b)f}+\frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{5/2}}{3(a-b)f}$$

output
```
1/2*(3*a-5*b)*b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2
))/f+1/2*(3*a-5*b)*b*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f-1/3*(3*
a-5*b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(3/2)/(a-b)/f+1/3*cos(f*x+e)^3*(a-b
+b*sec(f*x+e)^2)^(5/2)/(a-b)/f
```

Mathematica [A] (warning: unable to verify)

Time = 1.67 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.01

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\left(12\sqrt{2}(3a-5b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)\cos^2(e+fx)+\sqrt{a+b+(a-b)\cos(2(e+fx))}\right.}{}$$

input
```
Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((12*Sqrt[2]*(3*a - 5*b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f
*x)]]/(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 + Sqrt[a + b + (a - b)*Cos[2*(e +
f*x)]]*(-9*a + 37*b - 8*(a - 3*b)*Cos[2*(e + f*x)] + (a - b)*Cos[4*(e + f*
x)]))*Sec[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]
)/(24*Sqrt[2]*f*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4147, 25, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4147}$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}d\sec(e+fx)}{f}$$

$\Big\downarrow$ 359

$$\frac{\frac{(3a-5b)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^{3/2}d\sec(e+fx)}{3(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3(a-b)}}{f}$$

$\Big\downarrow$ 247

$$\frac{\frac{(3a-5b)\left(3b\int \sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{5/2}}{3(a-b)}}{f}$$

$\Big\downarrow$ 211

$$\frac{\frac{(3a-5b)\left(3b\left(\frac{1}{2}(a-b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}+\frac{\cos^3(e+}{}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{(3a-5b)\left(3b\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{(3a-5b)\left(3b\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}\right)}{3(a-b)}+\frac{\cos^3(e+f}{}}{f}$$

input
```
Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(5/2))/(3*(a - b)) + ((3*a - 5
*b)*(-(Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2)) + 3*b*(((a - b)*ArcT
anh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*Sqrt[b]) +
(Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/2)))/(3*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(166) = 332$.

Time = 8.80 (sec) , antiderivative size = 888, normalized size of antiderivative = 4.77

method	result	size
default	Expression too large to display	888

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/(a-b)^2/b*(-15*b^(9/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(
f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*cos(f*x+e)^2+39*b^(7/2)*ln(-
4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/
2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b
*sec(f*x+e))*a*cos(f*x+e)^2-33*b^(5/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*si
n(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e
)^2)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)-4*b*sec(f*x+e))*a^2*cos(f*x+e)^2+9
*b^(3/2)*ln(-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^
(1/2)-4*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*s
ec(f*x+e)-4*b*sec(f*x+e))*a^3*cos(f*x+e)^2+cos(f*x+e)^2*(2*cos(f*x+e)^3+2*
cos(f*x+e)^2-6*cos(f*x+e)-6)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*a^3*b+(-6*cos(f*x+e)^5-6*cos(f*x+e)^4+26*cos(f*x+e)^3+26*cos(f
*x+e)^2+3*cos(f*x+e)+3)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)*a^2*b^2+(6*cos(f*x+e)^5+6*cos(f*x+e)^4-34*cos(f*x+e)^3-34*cos(f*x+e
)^2-6*cos(f*x+e)-6)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/
2)*a*b^3+(-2*cos(f*x+e)^5-2*cos(f*x+e)^4+14*cos(f*x+e)^3+14*cos(f*x+e)^2+3
*cos(f*x+e)+3)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b^
4)*cos(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(c
os(f*x+e)+1)^2)^(1/2)/(cos(f*x+e)+1)/(a*cos(f*x+e)^2+b*sin(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.73

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\,(3\,a-5\,b)\sqrt{b}\cos(fx+e)\log\left(-\frac{(a-b)\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\,(}{12\,f\cos(fx+}\right.$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(3*a - 5*b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 -
2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 2*b)/cos(f*x + e)^2) - 2*(2*(a - b)*cos(f*x + e)^4 - 2*(3*a - 7*b)*cos(f*
x + e)^2 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(
f*x + e)), 1/6*(3*(3*a - 5*b)*sqrt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b))
*cos(f*x + e) + (2*(a - b)*cos(f*x + e)^4 - 2*(3*a - 7*b)*cos(f*x + e)^2 +
 3*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.59

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{4\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-12\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}(a-b)\cos(fx+e)+12\sqrt{a}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(4*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a - b +
b/cos(f*x + e)^2)*(a - b)*cos(f*x + e) + 12*sqrt(a - b + b/cos(f*x + e)^2)
*b*cos(f*x + e) + 6*b^(3/2)*log((sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x +
e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6
*(a*b - b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f
*x + e)^2)*cos(f*x + e)^2 - b) - 9*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x
 + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x +
 e) + sqrt(b)))/sqrt(b))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2688 vs. $2(166) = 332$.

Time = 3.39 (sec) , antiderivative size = 2688, normalized size of antiderivative = 14.45

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*(3*(3*a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 5*b^2*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1))*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a) - sqrt(a))/sqrt(-b))/sqrt(-b) + 6*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))^3*a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + (sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 -
1) - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b*s
gn(tan(1/2*f*x + 1/2*e)^2 - 1) + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))^2*sqrt(a)*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 3*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b*sgn(tan(1/2*f*x + 1/2*e
)^2 - 1) - 9*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^2*s
gn(tan(1/2*f*x + 1/2*e)^2 - 1) + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^(5/2)*b*sgn(tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \sin\left(e+f\,x\right)^3\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^3\tan(fx+e)^2\,dx\right)b + \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^3\,dx\right)a$$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**3*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**3,x)*a
```

3.106 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{3(a-b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f}+\frac{3b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{f}$$

output
```
3/2*(a-b)*b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f
+3/2*b*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f-cos(f*x+e)*(a-b+b*sec(f*x+e
)^2)^(3/2)/f
```

Mathematica [A] (warning: unable to verify)

Time = 1.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\left(6\sqrt{2}(a-b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{a+b+(a-b)\cos(2(e+fx))}}{\sqrt{2}\sqrt{b}}\right)\cos^2(e+fx)-2(a-2b+(a-b)\cos(2(e-\cdots\right.}{4\sqrt{2}f\sqrt{a+b\cdots}}$$

input
```
Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((6*Sqrt[2]*(a - b)*Sqrt[b]*ArcTanh[Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]
/(Sqrt[2]*Sqrt[b])]*Cos[e + f*x]^2 - 2*(a - 2*b + (a - b)*Cos[2*(e + f*x)]
)*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])*Sec[e + f*x]*Sqrt[(a + b + (a -
b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(4*Sqrt[2]*f*Sqrt[a + b + (a - b)*Co
s[2*(e + f*x)]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4147, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx) \left(a+b\tan(e+fx)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4147}$$

$$\frac{\int \cos^2(e+fx) \left(b\sec^2(e+fx)+a-b\right)^{3/2} d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{3b\int \sqrt{b\sec^2(e+fx)+a-b}d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{f}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{3b\left(\frac{1}{2}(a-b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2\right.}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{3b\left(\frac{1}{2}(a-b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\,(a+}{f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3b\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right.}{f}$$

input
```
Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cos[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2)) + 3*b*(((a - b)*ArcTanh[
(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec
[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/2))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(99) = 198$.

Time = 8.50 (sec) , antiderivative size = 684, normalized size of antiderivative = 6.05

method	result
default	$-\frac{\left(-3b^{\frac{7}{2}}\ln\left(\frac{4b\cot(fx+e)^{2}-8b\cot(fx+e)\csc(fx+e)+4b\csc(fx+e)^{2}+8\sqrt{b}\sqrt{\frac{a\cos(fx+e)^{2}+b\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}}+4b}{\cot(fx+e)^{2}-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^{2}-1}\right)\cos(fx+e)^{2}+6b^{\frac{5}{2}}\ln\left(\frac{4b\,\mathrm{cc}}{}\right.\right.}{}$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a-b)/b*(-3*b^(7/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+
b*csc(f*x+e)^2+2*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*cos(f*x+
e)^2+6*b^(5/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)
^2+2*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(
cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a*cos(f*x+e)^2-3*b^(
3/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/
2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)
^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a^2*cos(f*x+e)^2+cos(f*x+e)^2*
(2*cos(f*x+e)+2)*a^2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1
/2)*b+(-4*cos(f*x+e)^3-4*cos(f*x+e)^2-cos(f*x+e)-1)*a*((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b^2+(2*cos(f*x+e)^3+2*cos(f*x+e)^2+co
s(f*x+e)+1)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b^3)*
cos(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(
f*x+e)+1)^2)^(1/2)/(cos(f*x+e)^2*(cos(f*x+e)+1)*a+(cos(f*x+e)+1)*sin(f*x+e
)^2*b)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.52

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\,(a-b)\sqrt{b}\cos\left(fx+e\right)\log\left(-\frac{(a-b)\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)+2\left(2\left(\right.\right.}{4\,f\cos\left(fx+e\right)}\right.$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(3*(a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x + e)^2 - 2*sqr
t(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)
/cos(f*x + e)^2) + 2*(2*(a - b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), 1/2*(3*(a - b)*sqrt(-b)*arc
tan(-sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/((a - b)*cos(f*x + e)^2 + b))*cos(f*x + e) - (2*(a - b)*cos(f*x + e)^2
- b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F]

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\sin\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.56

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=$$

$$-\frac{4\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}(a-b)\cos(fx+e)-\frac{2(ab-b^2)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2-b}+\frac{3(ab-b^2)\log\left(\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(}{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(}\right.}{\sqrt{b}}}{4f}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(a - b + b/cos(f*x + e)^2)*(a - b)*cos(f*x + e) - 2*(a*b - b^2
)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b + b/cos(f*x + e)^2)*
cos(f*x + e)^2 - b) + 3*(a*b - b^2)*log((sqrt(a - b + b/cos(f*x + e)^2)*co
s(f*x + e) - sqrt(b))/(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(
b)))/sqrt(b))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1366 vs. $2(99) = 198$.

Time = 1.98 (sec) , antiderivative size = 1366, normalized size of antiderivative = 12.09

$$\int \sin(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
(3*(a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - b^2*sgn(tan(1/2*f*x + 1/2*e)^2 -
 1))*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sq
rt(a))/sqrt(-b))/sqrt(-b) + 4*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))*a^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + (sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^2*sgn(tan(1/2*f*x + 1/2
*e)^2 - 1) - a^(5/2)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2*a^(3/2)*b*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1) - sqrt(a)*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/((
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a + 4*b) + 2*((sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b*sgn(tan(1/2*f*x + 1/2*e
)^2 - 1) + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \int \sin(e+f\,x)\, \left(b\tan(e+f\,x)^2+a\right)^{3/2} dx$$

input
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sin(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}\, \sin(fx+e)\tan(fx+e)^2 dx\right) b + \left(\int \sqrt{\tan(fx+e)^2 b+a}\, \sin(fx+e)\, dx\right) a$$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)*tan(e + f*x)**2,x)*b + int(sq
rt(tan(e + f*x)**2*b + a)*sin(e + f*x),x)*a
```

3.107 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2f}$$

output
```
-a^(3/2)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f+1/2*(3*a
-b)*b^(1/2)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/f+1/2*b
*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 492 vs. 2(127) = 254.

Time = 3.31 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.87

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-4\sqrt{b}(-3a+b)\operatorname{arctanh}\left(\frac{-\sqrt{a}\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\right.}}{2\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Sec[(e + f*x)/2]^2*(-4*Sqrt[b]*(-3*a + b)*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e
 + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)
^2])/(2*Sqrt[b])]*Cos[e + f*x]^2 + 4*a^(3/2)*ArcTanh[Tan[(e + f*x)/2]^2 -
Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]]*Cos[
e + f*x]^2 + a^(3/2)*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b
*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + a^(3/2)*Cos[2*(e +
 f*x)]*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)
/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] + (b*Sqrt[(a + b + (a - b)*Cos[2*(
e + f*x)])*Sec[(e + f*x)/2]^4])/Sqrt[2] + (b*Cos[e + f*x]*Sqrt[(a + b + (a
 - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])/Sqrt[2])*Sec[e + f*x]*Sqrt[(a
 + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(4*f*Sqrt[(a + b + (a -
b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4147, 25, 318, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$\downarrow$ 318

$$\frac{\frac{1}{2}\int -\frac{(3a-b)b\sec^2(e+fx)+(a-b)(2a-b)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}-\frac{1}{2}\int \frac{2a^2-3ba+b^2+(3a-b)b\sec^2(e+fx)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$\downarrow$ 398

$$\frac{\frac{1}{2}\left(b(3a-b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-2a^2\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{1}{2}b\sec(e+}{f}$$

$\downarrow$ 224

$$\frac{\frac{1}{2}\left(b(3a-b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}-2a^2\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{1}{2}b\sec}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-2a^2\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{1}{2}b\sec(e+fx)}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-2a^2\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}\right)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b}}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)\right)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e+}}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-2*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]
] + (3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
+ f*x]^2]])/2 + (b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(109) = 218$.

Time = 8.36 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.24

method	result
default	$-\frac{\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}\left(a^{\frac{3}{2}}\ln\left(\frac{4\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sqrt{a}\,\sin(fx+e)^2+2a\sin(fx+e)^2-2a\cos(fx+e)^2+4b\cos(fx+e)^2+4a\cos(fx+}{(\cos(fx+e)-1)^2}\right.\right.}{}$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/b*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*
x+e)+1)^2)^(1/2)/(cos(f*x+e)^2*(cos(f*x+e)+1)*a+(cos(f*x+e)+1)*sin(f*x+e)^
2*b)*(a^(3/2)*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2
*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*b*cos(f*x+e)^3+a^(3/
2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2
)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*b*cos(f*x+e)^3+b^(5
/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2
)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^
2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*cos(f*x+e)^3-3*b^(3/2)*ln(4*(b*
cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((a*cos(f*
x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+
e)*cot(f*x+e)+csc(f*x+e)^2-1))*a*cos(f*x+e)^3+b^2*((a*cos(f*x+e)^2+b*sin(f
*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*(-cos(f*x+e)^2-cos(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 809, normalized size of antiderivative = 6.37

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a^(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos
(f*x + e)^2 - 1)) - (3*a - b)*sqrt(b)*cos(f*x + e)*log(-((a - b)*cos(f*x +
 e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2))/(f*cos(f*x + e)), 1/2*((3*a - b)*sqrt(-b)*arctan(-sqrt(-b)*sq
rt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(
f*x + e)^2 + b))*cos(f*x + e) + a^(3/2)*cos(f*x + e)*log(-2*((a - b)*cos(f
*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + b*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/4*(4*sqrt(-a)*a*arctan(-sqrt(-
a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)
*cos(f*x + e)^2 + b))*cos(f*x + e) + (3*a - b)*sqrt(b)*cos(f*x + e)*log(-(
(a - b)*cos(f*x + e)^2 - 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*b*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*(2*sqrt(-a)*a*arctan(
-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(
(a - b)*cos(f*x + e)^2 + b))*cos(f*x + e) - (3*a - b)*sqrt(-b)*arctan(-sqr
t(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a -
 b)*cos(f*x + e)^2 + b))*cos(f*x + e) - b*sqrt(((a - b)*cos(f*x + e)^2 ...
```

Sympy [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\csc\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)\tan(fx+e)^2\,dx\right)b + \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)\,dx\right)a$$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)*tan(e + f*x)**2,x)*b + int(sq
rt(tan(e + f*x)**2*b + a)*csc(e + f*x),x)*a
```

3.108 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = -\frac{\sqrt{a}(a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{b\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

output
```
-1/2*a^(1/2)*(a+3*b)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2)
)/f+1/2*b^(1/2)*(3*a+b)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1
/2))/f+b*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f-1/2*cot(f*x+e)*csc(f*x+e)
*(a-b+b*sec(f*x+e)^2)^(3/2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1012 vs. $2(167) = 334$.

Time = 6.80 (sec) , antiderivative size = 1012, normalized size of antiderivative = 6.06

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*
x)])]*(-1/2*(a*Cot[e + f*x]*Csc[e + f*x]) + (b*Sec[e + f*x])/2))/f + (((a^
2 - b^2)*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])
^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqr
t[a]*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x
)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] - Sqrt[b]*(2*ArcTanh
[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/
2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*
Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)
/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan
[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a
 + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*
b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]) - ((a^2 + 6*a*b + b
^2)*(1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f*x])^2]*S
qrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*
ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^
2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] + Sqrt[b]*(2*ArcTanh[Tan[
(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)
^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(
e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]]))*(-1 + Tan[(e + f*x)/2...
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4147, 369, 403, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\sec^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{2\left(1-\sec^2(e+fx)\right)}-\frac{1}{2}\int \frac{\sqrt{b\sec^2(e+fx)+a-b}\left(4b\sec^2(e+fx)+a-b\right)}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int -\frac{2\left(a^2-b^2+b(3a+b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+2b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}\right)+\frac{\sec(e+fx)\left(a+b\sec\right.}{2\left(1-\sec^2(\right.}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\left(2b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}-\int \frac{a^2-b^2+b(3a+b)\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\left(a+b\sec^2(e+\right.}{2\left(1-\sec^2(e+f\right.}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(b(3a+b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)-a(a+3b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+2b\sec(\right.}{f}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{\frac{1}{2}\left(-a(a+3b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+b(3a+b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}+\right.}{f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{1}{2}\left(-a(a+3b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+2b\sec(e\right.}{f}$$

$$\Big\downarrow \textbf{291}$$

$$\frac{\frac{1}{2}\left(-a(a+3b)\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+2b\sec(e+fx)\sqrt{\right.}{f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{1}{2}\left(-\sqrt{a}(a+3b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+2b\sec(e+fx)\sqrt{a+b\,\text{se}\right.}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((Sec[e + f*x]*(a - b + b*Sec[e + f*x]^2)^(3/2))/(2*(1 - Sec[e + f*x]^2))
+ (-(Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
 + f*x]^2]]) + Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a - b
 + b*Sec[e + f*x]^2]] + 2*b*Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/2
)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(145) = 290$.

Time = 8.48 (sec) , antiderivative size = 1032, normalized size of antiderivative = 6.18

method	result	size
default	Expression too large to display	1032

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/b/a*(cos(f*x+e)^2*(cos(f*x+e)-1)*a^(5/2)*ln(2/a^(1/2)*(a^(1/2)*((a*c
os(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x
+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x
+e)*b+b)/(cos(f*x+e)+1))*b+cos(f*x+e)^2*(cos(f*x+e)-1)*a^(5/2)*ln(2*(2*((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2
+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e
)*b-a+2*b)/(cos(f*x+e)-1)^2)*b+cos(f*x+e)^2*(3*cos(f*x+e)-3)*a^(3/2)*ln(2/
a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*
cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2
)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*b^2+cos(f*x+e)^2*(3*cos(f*x
+e)-3)*a^(3/2)*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+
2*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*b^2+b^(5/2)*ln(4*(b
*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((a*cos(f
*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x
+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a*(-2*cos(f*x+e)^3+2*cos(f*x+e)^2)+b^(3/2)
*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*(
(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^2-2
*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*a^2*(-6*cos(f*x+e)^3+6*cos(f*x+e)^
2)-2*cos(f*x+e)^2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1...
```

Fricas [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 1059, normalized size of antiderivative = 6.34

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(a)*log(-2*(
(a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + ((3*a + b)*cos(f*
x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(-((a - b)*cos(f*x + e)^2 +
2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 2*b)/cos(f*x + e)^2) + 2*((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/4*(
2*((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(-b)*arctan(-sqr
t(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a -
 b)*cos(f*x + e)^2 + b)) + ((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos(f*x +
 e))*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)
) + 2*((a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((a + 3*b)*cos(f*
x + e)^3 - (a + 3*b)*cos(f*x + e))*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2
+ b)) - ((3*a + b)*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt(b)*log(-(
(a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*((a + b)*cos(f*x + e)^2
 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e...
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(145) = 290$.

Time = 2.59 (sec) , antiderivative size = 1467, normalized size of antiderivative = 8.78

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a)*a*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 4*(a^2*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1) + 3*a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*arctan(-
(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/sqrt(-a) +
 8*(3*a*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + b^2*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1))*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) -
sqrt(a))/sqrt(-b))/sqrt(-b) - 2*(a^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 3*a
*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*log(abs((sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))*sqrt(a) - a + 2*b))/sqrt(a) + 2*((sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) -
 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1) - a^(5/2)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/((sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a) + 16*((sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e...
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^3\tan(fx+e)^2\,dx\right)b+\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^3\,dx\right)a$$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**3*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**3,x)*a
```

3.109 $\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 223

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = -\frac{3(a^2+6ab+b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2f} + \frac{3(a+3b)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{3(a+b)\csc^2(e+fx)\sec(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{3/2}}{4f}$$

output
```
-3/8*(a^2+6*a*b+b^2)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2)
)/a^(1/2)/f+3/2*b^(1/2)*(a+b)*arctanh(b^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)
^2)^(1/2))/f+3/8*(a+3*b)*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f-3/8*(a+b)
*csc(f*x+e)^2*sec(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/f-1/4*cot(f*x+e)*csc(f
*x+e)^3*(a-b+b*sec(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (warning: unable to verify)

Time = 3.56 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.86

$$\int \csc^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{\cos(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)} \left(-2\csc^2(e+fx)(3a+5b+2a\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-2*
Csc[e + f*x]^2*(3*a + 5*b + 2*a*Csc[e + f*x]^2) + 8*b*Sec[e + f*x]^2 + (3*
(16*Sqrt[a]*Sqrt[b]*(a + b)*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2))
+ Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b]
)] + (a^2 + 6*a*b + b^2)*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e +
 f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan
[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f
*x)/2]^2)^2]]))*Sec[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]^2*Sec[(e + f*x)/2]^4]
)/(Sqrt[a]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 +
 a*(-1 + Tan[(e + f*x)/2]^2)^2])))/(16*Sqrt[2]*f)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4147, 25, 369, 27, 439, 25, 444, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\sin(e+fx)^5}dx$$

$\big\downarrow$ 4147

$$\frac{\int -\frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$\big\downarrow$ 25

$$-\frac{\int \frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$\big\downarrow$ 369

$$\frac{\frac{1}{4}\int \frac{3\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a-b}\left(2b\sec^2(e+fx)+a-b\right)}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{3}{4}\int \frac{\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a-b}\left(2b\sec^2(e+fx)+a-b\right)}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$\big\downarrow$ 439

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int -\frac{\sec^2(e+fx)\left(2b(a+3b)\sec^2(e+fx)+(a-b)(a+5b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+\frac{(a+b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2\left(1-\sec^2(e+fx)\right)}\right)-\frac{\sec^3(e+fx)(a+}{4(1-\mathrm{sec}}}{f}$$

$\big\downarrow$ 25

$$\frac{\frac{3}{4}\left(\frac{(a+b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2\left(1-\sec^2(e+fx)\right)}-\frac{1}{2}\int \frac{\sec^2(e+fx)\left(2b(a+3b)\sec^2(e+fx)+(a-b)(a+5b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)-\frac{\sec^3(e+fx)(a+b\,\mathrm{se}}{4(1-\sec^2(}}{f}$$

$\big\downarrow$ 444

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left((a+3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}-\frac{\int \frac{2b\left(4b(a+b)\sec^2(e+fx)+(a-b)(a+3b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{2b}\right)+\frac{(a+b)\sec^3(e+fx}{2(1-\mathrm{se}}}{f}$$

$\big\downarrow$ 27

$$\frac{3}{4}\left(\frac{1}{2}\left((a+3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}-\int\frac{4b(a+b)\sec^2(e+fx)+(a-b)(a+3b)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)\right)+\frac{(a+b)\sec^3(e\ldots}{2\ldots}\right.$$
$$f$$

$\downarrow$ 398

$$\frac{3}{4}\left(\frac{1}{2}\left(-(a^2+6ab+b^2)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+4b(a+b)\int\frac{1}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx\ldots\right.\right.$$

$\downarrow$ 224

$$\frac{3}{4}\left(\frac{1}{2}\left(-(a^2+6ab+b^2)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+4b(a+b)\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+f\ldots}{\sqrt{b\sec^2(e+f\ldots}}\right.\right.$$

$\downarrow$ 219

$$\frac{3}{4}\left(\frac{1}{2}\left(-(a^2+6ab+b^2)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)+4\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+\ldots\right.\right.$$

$\downarrow$ 291

$$\frac{3}{4}\left(\frac{1}{2}\left(-(a^2+6ab+b^2)\int\frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}+4\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+(a+3\ldots\right.\right.$$
$$f$$

$\downarrow$ 219

$$\frac{3}{4}\left(\frac{1}{2}\left(-\frac{(a^2+6ab+b^2)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}}+4\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)+(a+3b)\sec(e+fx)\sqrt{\ldots}\right.\right.$$
$$f$$

input
```
Int[Csc[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*(Sec[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(3/2))/(1 - Sec[e + f*x]^
2)^2 + (3*(((a + b)*Sec[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*(1 -
 Sec[e + f*x]^2)) + (-(((a^2 + 6*a*b + b^2)*ArcTanh[(Sqrt[a]*Sec[e + f*x])
/Sqrt[a - b + b*Sec[e + f*x]^2]])/Sqrt[a]) + 4*Sqrt[b]*(a + b)*ArcTanh[(Sq
rt[b]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]] + (a + 3*b)*Sec[e + f*
x]*Sqrt[a - b + b*Sec[e + f*x]^2])/2))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. $2(195) = 390$.

Time = 8.52 (sec) , antiderivative size = 1386, normalized size of antiderivative = 6.22

method	result	size
default	Expression too large to display	1386

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/a^(5/2)/b*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)
/(cos(f*x+e)+1)^2)^(1/2)/(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*((-24*cos(f*x+e)+
24)*b^(5/2)*a^(5/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(f*x+e)*csc(f*x+e)+b*csc(f
*x+e)^2+2*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1))*cos(f*x+e)*cot(
f*x+e)^2+(-24*cos(f*x+e)+24)*b^(3/2)*a^(7/2)*ln(4*(b*cot(f*x+e)^2-2*b*cot(
f*x+e)*csc(f*x+e)+b*csc(f*x+e)^2+2*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)+b)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x
+e)^2-1))*cos(f*x+e)*cot(f*x+e)^2+(6*cos(f*x+e)^2-10)*((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(7/2)*b*cot(f*x+e)^3*csc(f*x+e)+(-1
8*cos(f*x+e)^2+8)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
*a^(5/2)*b^2*cot(f*x+e)*csc(f*x+e)+(3*cos(f*x+e)-3)*ln(2/a^(1/2)*(a^(1/2)*
((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*co
s(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+co
s(f*x+e)*b+b)/(cos(f*x+e)+1))*a^4*b*cos(f*x+e)*cot(f*x+e)^2+(18*cos(f*x+e)
-18)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)
^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^3*b^2*cos(f*x+e
)*cot(f*x+e)^2+(3*cos(f*x+e)-3)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(...
```

Fricas [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 1431, normalized size of antiderivative = 6.42

$$\int \csc^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 + 6*a*b + b^2)*cos(f
*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*log(-2*((a - b)*cos(
f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*c
os(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + a*b)*cos(f*x + e)^
5 - 2*(a^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(b)*log(-
((a - b)*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 + 3*a*b)*cos(f
*x + e)^4 - (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 +
 a*f*cos(f*x + e)), -1/8*(3*((a^2 + 6*a*b + b^2)*cos(f*x + e)^5 - 2*(a^2 +
 6*a*b + b^2)*cos(f*x + e)^3 + (a^2 + 6*a*b + b^2)*cos(f*x + e))*sqrt(-a)*
arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e)/((a - b)*cos(f*x + e)^2 + b)) - 6*((a^2 + a*b)*cos(f*x + e)^5 - 2*(a
^2 + a*b)*cos(f*x + e)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(b)*log(-((a - b)
*cos(f*x + e)^2 + 2*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - (3*(a^2 + 3*a*b)*cos(f*x + e)^4
- (5*a^2 + 13*a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f
*x + e)), 1/16*(24*((a^2 + a*b)*cos(f*x + e)^5 - 2*(a^2 + a*b)*cos(f*x + e
)^3 + (a^2 + a*b)*cos(f*x + e))*sqrt(-b)*arctan(-sqrt(-b)*sqrt(((a - b)...
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

Reduce [F]

$$\int \csc^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^5 \tan(fx+e)^2 dx\right) b + \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^5 dx\right) a$$

input
```
int(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**5*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**5,x)*a
```

3.110 $\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{3(a^2-8ab+8b^2)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{a-b}f} + \frac{3(a-2b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} - \frac{3(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{3(a-2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/8*(a^2-8*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/
2))/(a-b)^(1/2)/f+3/2*(a-2*b)*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2))/f-3/8*(a-4*b)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f+3/8*(
a-2*b)*sin(f*x+e)^2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-1/4*cos(f*x+e)*s
in(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.55 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.25

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(\frac{3a\csc^2(e+fx)\left((a^2-5ab+4b^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{(a+\cdots}}{\cdots}\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((3*a*Csc[e + f*x
]^2*((a^2 - 5*a*b + 4*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - (a^2 - 8*a*b + 8*b^2)*Ellipti
cPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b]/Sqrt[2]], 1])*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)*b*Sqrt[((a + b
 + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]) + ((-8*a + 18*b)*Sin[2*(e
 + f*x)] + (a - b)*Sin[4*(e + f*x)] + 16*b*Tan[e + f*x])/4))/(8*Sqrt[2]*f)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4146, 369, 27, 439, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 369$$

$$\frac{\frac{1}{4}\int \frac{3\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a}\left(2b\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{4}\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a}\left(2b\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\Big\downarrow \; 439$$

$$\frac{\frac{3}{4}\left(\frac{(a-2b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2\left(\tan^2(e+fx)+1\right)}-\frac{1}{2}\int \frac{\tan^2(e+fx)\left(2(a-4b)b\tan^2(e+fx)+a(a-6b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)-\frac{\tan^3(e+fx)\left(a+b\tan^2(\ldots\right.}{4\left(\tan^2(e+fx)+\ldots\right.}}{f}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{2b\left(4(a-2b)b\tan^2(e+fx)+a(a-4b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}-(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)+\frac{(a-2b)\tan^3(e+fx)\sqrt{a\ldots}}{2\left(\tan^2(e+fx\ldots\right.}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\int \frac{4(a-2b)b\tan^2(e+fx)+a(a-4b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)+\frac{(a-2b)\tan^3(e+f\ldots}{2\left(\tan^2(\ldots\right.}\right.}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-8ab+8b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+4b(a-2b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\ldots\right.\right.}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-8ab+8b^2\right)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+4b(a-2b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)\cdots}}\right.\right.$$

$\downarrow$ 219

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-8ab+8b^2\right)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+4\sqrt{b}(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-(\cdots\right.\right.$$
$$f$$

$\downarrow$ 291

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-8ab+8b^2\right)\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+4\sqrt{b}(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-(a-4b)\text{ta}\cdots\right.\right.$$
$$f$$

$\downarrow$ 216

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{(a^2-8ab+8b^2)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}+4\sqrt{b}(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-(a-4b)\tan(e+fx)\sqrt{a+b\cdots}\right.\right.$$
$$f$$

input
```
Int[Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*(Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^2)^2
 + (3*(((a - 2*b)*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(2*(1 + Tan[e
 + f*x]^2)) + (((a^2 - 8*a*b + 8*b^2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sq
rt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b] + 4*(a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] - (a - 4*b)*Tan[e + f*x]*Sqr
t[a + b*Tan[e + f*x]^2])/2))/4)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 17.56 (sec) , antiderivative size = 2156, normalized size of antiderivative = 9.71

method	result	size
default	Expression too large to display	2156

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(a+b*tan(f*x+e)^2)^(3/2)/(
a^2*cos(f*x+e)^4+2*a*b*cos(f*x+e)^2*sin(f*x+e)^2+sin(f*x+e)^4*b^2)*((1/a*(
I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+a*cos(f*x+e)-cos(f*
x+e)*b+b)/(cos(f*x+e)+1))^(1/2)*(-1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b-b)/(cos(f*x+e)+1))^(1/2)*a^2
*EllipticPi(((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(csc(f*x+e)-cot(f*x+
e)),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)
/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*(12*cos(f*x+e)^4+12*c
os(f*x+e)^3)+(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+
a*cos(f*x+e)-cos(f*x+e)*b+b)/(cos(f*x+e)+1))^(1/2)*(-1/a*(I*cos(f*x+e)*b^(
1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b-b)/(cos(f
*x+e)+1))^(1/2)*a*b*EllipticPi(((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(
csc(f*x+e)-cot(f*x+e)),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)
*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*(-
96*cos(f*x+e)^4-96*cos(f*x+e)^3)+(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/(cos(f*x+e)+1))^(1/2)*(-1
/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-a*cos(f*x+e)+co
s(f*x+e)*b-b)/(cos(f*x+e)+1))^(1/2)*b^2*EllipticPi(((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)*(csc(f*x+e)-cot(f*x+e)),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2
*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(194) = 388$.

Time = 66.89 (sec) , antiderivative size = 2163, normalized size of antiderivative = 9.74

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2 - 8*a*b + 8*b^2)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4
*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b +
9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^
2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 -
 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4
)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 -
24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b
+ 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)
*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e)) + 24*(a^2 - 3*a*b + 2*b^2)*sqrt(b)*cos(f*x + e)*log(((a^
2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a
 - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 8*(2*(a
^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 5*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2
+ 4*a*b - 4*b^2)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a - b)*f*cos(f*x + e)), -1/64*(48*(a^2 - 3*a*b + 2*b^2)*sqrt(-b)*
arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a
 - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b
^2)*sin(f*x + e)))*cos(f*x + e) + 3*(a^2 - 8*a*b + 8*b^2)*sqrt(-a + b)*cos
(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \sin\left(e+f\,x\right)^4\,\left(b\,\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sin^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^4\tan(fx+e)^2 dx\right)b + \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^4 dx\right)a$$

input
```
int(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**4*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**4,x)*a
```

3.111 $\int \sin^2(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \sin^2(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{(a-4b)\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

output
```
1/2*(a-4*b)*(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(
1/2))/f+1/2*(3*a-4*b)*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2
)^(1/2))/f+b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-1/2*cos(f*x+e)*sin(f*x+
e)*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.75 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.96

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx =$$
$$-\frac{\sec^2(e+fx)\left(-4\sqrt{2}a(a-2b)\cot(e+fx)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\ \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b+(a-b}{}}}{}\right.\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/16*(Sec[e + f*x]^2*(-4*Sqrt[2]*a*(a - 2*b)*Cot[e + f*x]*Sqrt[((a + b +
(a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b
 + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 4*Sqrt[2]*a
*(a - 4*b)*Cot[e + f*x]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (3*a^2 - 6*a*b - 5*b^2 + 4*(a -
 b)^2*Cos[2*(e + f*x)] + (a - b)^2*Cos[4*(e + f*x)])*Csc[e + f*x]*Sec[e +
f*x])*Sin[2*(e + f*x)]*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4146, 369, 403, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(e+fx)^2 \left(a+b\tan(e+fx)^2\right)^{3/2} dx$$

$\big\downarrow$ 4146

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$\big\downarrow$ 369

$$\frac{\frac{1}{2}\int \frac{\sqrt{b\tan^2(e+fx)+a}\left(4b\tan^2(e+fx)+a\right)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$\big\downarrow$ 403

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{2\left((3a-4b)b\tan^2(e+fx)+a(a-2b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+2b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{\tan(e+fx)\left(a+b\tan^2(e+f\right.}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{2}\left(\int \frac{(3a-4b)b\tan^2(e+fx)+a(a-2b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+2b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$\big\downarrow$ 398

$$\frac{\frac{1}{2}\left(b(3a-4b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-4b)(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+2b\right.}{f}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{2}\left((a-4b)(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b(3a-4b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\right.}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{2}\left((a-4b)(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\sqrt{b}(3a-4b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+2b\tan\right.}{f}$$

$\big\downarrow$ 291

$$\frac{\frac{1}{2}\left((a-4b)(a-b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\sqrt{b}(3a-4b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+2b\tan(e+fx)\right.}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{2}\left((a-4b)\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+\sqrt{b}(3a-4b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+2b\tan(e+fx)\sqrt{a+b\tan\right.}{f}$$

input
```
Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*(Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^2) + (
(a - 4*b)*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e +
 f*x]^2]] + (3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*
Tan[e + f*x]^2]] + 2*b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(143) = 286$.

Time = 15.53 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.78

method	result
default	$\frac{\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}\left(4\cos(fx+e)^3 b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)-5\cos(fx+e)^3 b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2\cdots}{(\cos(fx\cdots}}}{\sqrt{a-b}\cdots}\right.\right.}{}$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/b^(1/2)/(a-b)^(1/2)*(a+b*tan(f*x+e)^2)^(3/2)/(cos(f*x+e)^2*(cos(f*x+
e)+1)*a+(cos(f*x+e)+1)*sin(f*x+e)^2*b)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(c
os(f*x+e)+1)^2)^(1/2)*(4*cos(f*x+e)^3*b^(5/2)*arctan(1/(a-b)^(1/2)*((a*cos
(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1
))-5*cos(f*x+e)^3*b^(3/2)*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+
e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a+cos(f*x+e)^3*b^
(1/2)*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)
^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a^2-3*cos(f*x+e)^3*(a-b)^(1/2)*arctan
h(1/b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f
*x+e)/(cos(f*x+e)-1))*a*b+4*cos(f*x+e)^3*(a-b)^(1/2)*arctanh(1/b^(1/2)*((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+
e)-1))*b^2+(cos(f*x+e)^3+cos(f*x+e)^2+cos(f*x+e)+1)*sin(f*x+e)*b^(3/2)*(a-
b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+
e)+sin(f*x+e)*cos(f*x+e)^3*(-cos(f*x+e)-1)*b^(1/2)*(a-b)^(1/2)*((a*cos(f*x
+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(143) = 286$.

Time = 4.08 (sec) , antiderivative size = 1931, normalized size of antiderivative = 11.70

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*((a - 4*b)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2
*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*
a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b
^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 1
28*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)
^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^
2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 2
4*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))
*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e)) + 2*(3*a - 4*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*
x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 - 4*((a - 2*b)*cos(f*x + e)^3 +
2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*((a - b)*cos(f*x + e)^2 - b)*s
qrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x
+ e)), -1/16*(4*(3*a - 4*b)*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + (a
- 4*b)*sqrt(-a + b)*cos(f*x + e)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*
b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b
^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*...
```

Sympy [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\sin^2\left(e+fx\right)dx$$

input
```
integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \sin\left(e+f\,x\right)^2\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^2\tan(fx+e)^2\,dx\right)b+\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^2\,dx\right)a$$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**2*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**2,x)*a
```

3.112 $\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

output
```
(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/2*
(3*a-2*b)*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1
/2*b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{-2(a-b)^{3/2}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}(-3a+2b)\log\left(-\sqrt{b}\tan\right.}{2f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a - b)^(3/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*(-3*a + 2*b)*Log[-(Sqrt
[b]*Tan[e + f*x]) + Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a +
b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4144, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{1}{2}\int \frac{(3a-2b)b\tan^2(e+fx)+a(2a-b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b(3a-2b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b(3a-2b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}\right)+\frac{1}{2}b\,\mathrm{t}}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}b\tan(e+}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a}}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{2}\left(2(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x
]^2]] + (3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(107) = 214.

Time = 0.34 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{b\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2f}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}}\right.}{f(a-b)}$
default	$\frac{b\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2f}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}}\right.}{f(a-b)}$

input
```
int((a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*ln(b^(1/2)*tan
(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(
a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*b^(3/2)*ln
(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-2/f*a/b*(b^4*(a-b))^(1/2)/(a
-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)
)+1/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.17

$$\int (a+b\tan^2(e+fx))^{3/2}\,dx = \left[-\frac{(3\,a-2\,b)\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2-2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+2}{\ldots}\right.$$

$$-\frac{(3\,a-2\,b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-(-a+b)^{\frac{3}{2}}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2-2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+}{\tan(fx+e)^2+1}\right.}{2\,f}$$

input
```
integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*
tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) -
a)/(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f,
 -1/2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x +
e)^2 + a)) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqr
t(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(sqr
t(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (3*a - 2*b)*sqrt(b)*lo
g(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) +
 a) + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)
*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (3*a - 2*b)
*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + sqrt(
b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

Sympy [F]

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \int \left(b\tan(e+f\,x)^2+a\right)^{3/2} dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}dx\right) a + \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^2 dx\right) b$$

input
```
int((a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a),x)*a + int(sqrt(tan(e + f*x)**2*b + a)*tan
(e + f*x)**2,x)*b
```

3.113 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{3a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f}+\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

output
```
3/2*a*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+3/2*b
*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)
/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.17 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.20

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\csc(e+fx)\sec^3(e+fx)\left(-6a^2-ab+3b^2-4(2a^2+b^2)\cos(2(e+fx))-2a^2\cos(4(e+f\right.}{}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Csc[e + f*x]*Sec[e + f*x]^3*(-6*a^2 - a*b + 3*b^2 - 4*(2*a^2 + b^2)*Cos[2
*(e + f*x)] - 2*a^2*Cos[4*(e + f*x)] + a*b*Cos[4*(e + f*x)] + b^2*Cos[4*(e
 + f*x)] + 3*Sqrt[2]*a*b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]^2)/b]/Sqrt[2]], 1]*Sin[2*(e + f*x)]^2))/(8*Sqrt[2]*f*Sqrt[(a + b +
 (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4146, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{247}$$

$$\frac{3b\int \sqrt{b\tan^2(e+fx)+a}d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

$$\Big\downarrow \text{211}$$

$$\frac{3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+f\right.}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*T
an[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt
[a + b*Tan[e + f*x]^2])/2))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21

method	result
derivativedivides	$-\frac{\left(a+b\tan\left(fx+e\right)^{2}\right)^{\frac{5}{2}}}{fa\tan\left(fx+e\right)}+\frac{b\tan\left(fx+e\right)\left(a+b\tan\left(fx+e\right)^{2}\right)^{\frac{3}{2}}}{fa}+\frac{3b\tan\left(fx+e\right)\sqrt{a+b\tan\left(fx+e\right)^{2}}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\right.}{}$
default	$-\frac{\left(a+b\tan\left(fx+e\right)^{2}\right)^{\frac{5}{2}}}{fa\tan\left(fx+e\right)}+\frac{b\tan\left(fx+e\right)\left(a+b\tan\left(fx+e\right)^{2}\right)^{\frac{3}{2}}}{fa}+\frac{3b\tan\left(fx+e\right)\sqrt{a+b\tan\left(fx+e\right)^{2}}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\right.}{}$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/tan(f*x+e)*(a+b*tan(f*x+e)^2)^(5/2)+1/f/a*b*tan(f*x+e)*(a+b*tan(f*x
+e)^2)^(3/2)+3/2*b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*l
n(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(86) = 172$.

Time = 0.32 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.87

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\,a\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-8\,ab+8\,b^2)\cos(fx+e)^4+8\,(ab-2\,b^2)\cos(fx+e)^2+4\left((a-2\,b)\cos(fx+e)^3+2\,b\cos(fx\right.}{\cos(fx+e)^4}\right.}{8\,f\cos(f}\right.$$

$$-\frac{3\,a\sqrt{-b}\arctan\left(\frac{\left((a-2\,b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left((ab-b^2)\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)\sin(fx+e)+2\left((2\,a+b\right.}{4\,f\cos(fx+e)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a*sqrt(b)*cos(f*x + e)*log(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 +
 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + b)*cos(f*x + e)^2 -
 b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin
(f*x + e)), -1/4*(3*a*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e
) + 2*((2*a + b)*cos(f*x + e)^2 - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos
(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x + e))]
```

Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\csc^2{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{3\,a\sqrt{b}\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+3\,\sqrt{b\tan(fx+e)^2+a}b\tan(fx+e)-\frac{2\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}{\tan(fx+e)}}{2\,f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 3*sqrt(b*tan(f*x + e)
^2 + a)*b*tan(f*x + e) - 2*(b*tan(f*x + e)^2 + a)^(3/2)/tan(f*x + e))/f
```

Giac [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^2\tan\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^2dx\right)a$$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**2*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**2,x)*a
```

3.114 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+2b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af}$$

output
```
1/2*b^(1/2)*(3*a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))
/f+1/2*b*(3*a+2*b)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/3*(3*a+2*b)*c
ot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/a/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)
^(5/2)/a/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-4(a+2b)\cot(e+fx)-2a\cot(e+fx)\right.}{}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-4*(a + 2*b)*Cot
[e + f*x] - 2*a*Cot[e + f*x]*Csc[e + f*x]^2 + (3*Sqrt[2]*(3*a + 2*b)*Cot[e
 + f*x]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b]/Sqrt[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b] + 3*b*Tan[e + f*x]))/(6*Sqrt[2]*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4146, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(3a+2b)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\Big\downarrow \; 247$$

$$\frac{\frac{(3a+2b)\left(3b\int \sqrt{b\tan^2(e+fx)+a}\,d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(3a+2b)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(3a+2b)\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(3a+2b)\left(3b\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3a}}{f}$$

input

```
Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/a + ((3*a + 2*b)*(-(Co
t[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*Tan[e
+ f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a +
b*Tan[e + f*x]^2])/2)))/(3*a))/f
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(142) = 284.

Time = 13.98 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.23

method	result
default	$\frac{\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}\left((9\cos(fx+e)-9)\operatorname{arctanh}\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sin(fx+e)}{\sqrt{b}\,(\cos(fx+e)-1)}\right)ab\cos(fx+e)\cot(fx+e)^2+(6\cos(fx+e\right.}{}$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/b^(1/2)*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(c
os(f*x+e)+1)^2)^(1/2)/(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*((9*cos(f*x+e)-9)*ar
ctanh(1/b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*s
in(f*x+e)/(cos(f*x+e)-1))*a*b*cos(f*x+e)*cot(f*x+e)^2+(6*cos(f*x+e)-6)*arc
tanh(1/b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*si
n(f*x+e)/(cos(f*x+e)-1))*b^2*cos(f*x+e)*cot(f*x+e)^2+(-11*cos(f*x+e)^2+3)*
b^(3/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e
)+(4*cos(f*x+e)^2-6)*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*a*cot(f*x+e)^3)
```

Fricas [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 497, normalized size of antiderivative = 3.07

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left[\frac{3\left((3a+2b)\cos(fx+e)^3-(3a+2b)\cos(fx+e)\right)\sqrt{b}\log\left(\frac{(a^2-8ab+8b^2)\cos(fx+e)^4+8(ab-2\cdots}{\cdots}\right)}{\cdots}\right.$$

$$-\frac{3\left((3a+2b)\cos(fx+e)^3-(3a+2b)\cos(fx+e)\right)\sqrt{-b}\arctan\left(\frac{\left((a-2b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{(a-b\cdots}{\cdots}}}{2\left((ab-b^2)\cos(fx+e)^2+b^2\right)\sin(fx+\cdots}\right)\cdots}{12\left(f\cos(fx+e)^3-\cdots\right)}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*((3*a + 2*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt(b)*l
og(((a^2 - 8*a*b + 8*b^2)*cos(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2
+ 4*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*si
n(f*x + e) - 4*((4*a + 11*b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2
 + 3*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e
)^3 - f*cos(f*x + e))*sin(f*x + e)), -1/12*(3*((3*a + 2*b)*cos(f*x + e)^3
- (3*a + 2*b)*cos(f*x + e))*sqrt(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(-b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(((a*b - b^2)*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*(
(4*a + 11*b)*cos(f*x + e)^4 - 2*(3*a + 7*b)*cos(f*x + e)^2 + 3*b)*sqrt(((a
 - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x
+ e))*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.08

$$\int \csc^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{9\,a\sqrt{b}\,\text{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right) + 6\,b^{\frac{3}{2}}\,\text{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right) + 9\,\sqrt{b\tan\left(fx+e\right)^2+a}\,b\tan\left(fx+e\right)}{6\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 6*b^(3/2)*arcsinh(b*t
an(f*x + e)/sqrt(a*b)) + 9*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e) + 6*s
qrt(b*tan(f*x + e)^2 + a)*b^2*tan(f*x + e)/a - 6*(b*tan(f*x + e)^2 + a)^(3
/2)/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a*tan(f*x + e)) - 2*(
b*tan(f*x + e)^2 + a)^(5/2)/(a*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^4\tan\left(fx+e\right)^2dx\right)b+\left(\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\csc\left(fx+e\right)^4dx\right)a$$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**4*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**4,x)*a
```

3.115 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af} - \frac{(3a+4b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3af} - \frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3af} - \frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5af}$$

output

```
1/2*b^(1/2)*(3*a+4*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))
/f+1/2*b*(3*a+4*b)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/3*(3*a+4*b)*c
ot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/a/f-2/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)
^(5/2)/a/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(5/2)/a/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\frac{2(8a^2+34ab+3b^2)\cot(e+fx)}{a}-4(2a+3b)\cot(\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((-2*(8*a^2 + 34*
a*b + 3*b^2)*Cot[e + f*x])/a - 4*(2*a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2 -
 6*a*Cot[e + f*x]*Csc[e + f*x]^4 + (15*Sqrt[2]*(3*a + 4*b)*Cot[e + f*x]*El
lipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]
/Sqrt[2]], 1])/Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]
 + 15*b*Tan[e + f*x]))/(30*Sqrt[2]*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4146, 365, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int 5a\cot^4(e+fx)\left(\tan^2(e+fx)+2\right)\left(b\tan^2(e+fx)+a\right)^{3/2}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+2\right)\left(b\tan^2(e+fx)+a\right)^{3/2}d\tan(e+fx)-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5a}}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(3a+4b)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}d\tan(e+fx)}{3a}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5a}-\frac{2\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\Big\downarrow \; 247$$

$$\frac{\frac{(3a+4b)\left(3b\int \sqrt{b\tan^2(e+fx)+a}d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5a}-\frac{2\cot^3(e+fx)(a+b\,\mathrm{ta}}{3a}}{f}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(3a+4b)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cot^5(e+fx)(a+b\,\mathrm{ta}}{5a}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(3a+4b)\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\cot^5(e+fx}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(3a+4b)\left(3b\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\right)}{3a}-\frac{\cot^5(e+fx)(a+b\tan}{5a}}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-2*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(5/2))/(3*a) - (Cot[e + f*x]^5*
(a + b*Tan[e + f*x]^2)^(5/2))/(5*a) + ((3*a + 4*b)*(-(Cot[e + f*x]*(a + b*
Tan[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])
/2)))/(3*a))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(172) = 344$.

Time = 15.48 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.34

method	result
default	$-\frac{\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}\left((-45\cos(fx+e)+45)\operatorname{arctanh}\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sin(fx+e)}{\sqrt{b}\,(\cos(fx+e)-1)}\right)a^2b\cos(fx+e)\cot(fx+e)^2+(-60$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/f/a/b^(1/2)*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)/(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*((-45*cos(f*x+e)
+45)*arctanh(1/b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^
(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a^2*b*cos(f*x+e)*cot(f*x+e)^2+(-60*cos(f*
x+e)+60)*arctanh(1/b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)
^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a*b^2*cos(f*x+e)*cot(f*x+e)^2+6*b^(5/
2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2*c
ot(f*x+e)+(-83*cos(f*x+e)^4+110*cos(f*x+e)^2-15)*b^(3/2)*((a*cos(f*x+e)^2+
b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a*cot(f*x+e)*csc(f*x+e)^2+(16*cos(
f*x+e)^4-40*cos(f*x+e)^2+30)*b^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos
(f*x+e)+1)^2)^(1/2)*a^2*cot(f*x+e)^3*csc(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 655, normalized size of antiderivative = 3.34

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/120*(15*((3*a^2 + 4*a*b)*cos(f*x + e)^5 - 2*(3*a^2 + 4*a*b)*cos(f*x + e
)^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt(b)*log(((a^2 - 8*a*b + 8*b^2)*cos
(f*x + e)^4 + 8*(a*b - 2*b^2)*cos(f*x + e)^2 + 4*((a - 2*b)*cos(f*x + e)^3
 + 2*b*cos(f*x + e))*sqrt(b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16*a^2 + 83*
a*b + 6*b^2)*cos(f*x + e)^6 - (40*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 +
 (30*a^2 + 125*a*b + 6*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/((a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3
+ a*f*cos(f*x + e))*sin(f*x + e)), -1/60*(15*((3*a^2 + 4*a*b)*cos(f*x + e)
^5 - 2*(3*a^2 + 4*a*b)*cos(f*x + e)^3 + (3*a^2 + 4*a*b)*cos(f*x + e))*sqrt
(-b)*arctan(1/2*((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a*b - b^2)*cos(f*x + e)^
2 + b^2)*sin(f*x + e)))*sin(f*x + e) + 2*((16*a^2 + 83*a*b + 6*b^2)*cos(f*
x + e)^6 - (40*a^2 + 193*a*b + 12*b^2)*cos(f*x + e)^4 + (30*a^2 + 125*a*b
+ 6*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/((a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))
*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{45\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+60\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)+45\sqrt{b\tan\left(fx+e\right)^2+ab}\tan\left(fx\right.}{}$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt(a*b)) + 60*b^(3/2)*arcsinh(
b*tan(f*x + e)/sqrt(a*b)) + 45*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e) +
 60*sqrt(b*tan(f*x + e)^2 + a)*b^2*tan(f*x + e)/a - 30*(b*tan(f*x + e)^2 +
 a)^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a)^(3/2)*b/(a*tan(f*x + e)
) - 20*(b*tan(f*x + e)^2 + a)^(5/2)/(a*tan(f*x + e)^3) - 6*(b*tan(f*x + e)
^2 + a)^(5/2)/(a*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^6\tan(fx+e)^2\,dx\right)b+\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^6\,dx\right)a$$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**6*tan(e + f*x)**2,x)*b + int
(sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**6,x)*a
```

3.116 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{(15a^2-10ab+3b^2)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^3f} + \frac{2(5a-3b)\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^2f} - \frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{5(a-b)f}$$

output

```
-1/15*(15*a^2-10*a*b+3*b^2)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^3/
f+2/15*(5*a-3*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^2/f-1/5*cos
(f*x+e)^5*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$=\frac{\cos(e+fx)\left(-89a^2+34ab-9b^2+4(7a^2-10ab+3b^2)\cos(2(e+fx))-3(a-b)^2\cos(4(e+fx))\right)\sqrt{(\cdots}}{120\sqrt{2}(a-b)^3f}$$

input
```
Integrate[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(-89*a^2 + 34*a*b - 9*b^2 + 4*(7*a^2 - 10*a*b + 3*b^2)*Cos[2
*(e + f*x)] - 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e
 + f*x)])*Sec[e + f*x]^2])/(120*Sqrt[2]*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4147, 365, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(e+fx)^5}{\sqrt{a+b\tan(e+fx)^2}}dx$$
$$\Big\downarrow \; 4147$$
$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$
$$\Big\downarrow \; 365$$

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(2(5a-3b)-5(a-b)\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5(a-b)}}{f}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a-3b)-5(a-b)\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5(a-b)}}{f}$$

$\Big\downarrow$ 359

$$\frac{-\frac{-\frac{\left(15a^2-10ab+3b^2\right)\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{3(a-b)}-\frac{2(5a-3b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3(a-b)}}{5(a-b)}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5(a-b)}}{f}$$

$\Big\downarrow$ 242

$$\frac{-\frac{\frac{\left(15a^2-10ab+3b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3(a-b)^2}-\frac{2(5a-3b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3(a-b)}}{5(a-b)}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{5(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/5*(Cos[e + f*x]^5*Sqrt[a - b + b*Sec[e + f*x]^2])/(a - b) - (((15*a^2
- 10*a*b + 3*b^2)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)^
2) - (2*(5*a - 3*b)*Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a -
 b)))/(5*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\left(a \cos(fx+e)^2+b \sin(fx+e)^2\right)\left(3 \sin(fx+e)^4 b^2+6ab \cos(fx+e)^2 \sin(fx+e)^2+3a^2 \cos(fx+e)^4-10ab \sin(fx+e)^2-10a^2 \cos(fx-\right.}{15f(a-b)^3 \sqrt{a+b \tan(fx+e)^2}}$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a-b)^3*(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*(3*sin(f*x+e)^4*b^2+6*a*b*
cos(f*x+e)^2*sin(f*x+e)^2+3*a^2*cos(f*x+e)^4-10*a*b*sin(f*x+e)^2-10*a^2*co
s(f*x+e)^2+15*a^2)/(a+b*tan(f*x+e)^2)^(1/2)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\left(3\left(a^2-2\,ab+b^2\right)\cos\left(fx+e\right)^5-2\left(5\,a^2-8\,ab+3\,b^2\right)\cos\left(fx+e\right)^3+\left(15\,a^2-10\,ab+3\,b^2\right)\cos\left(fx\right.\right.}{15\left(a^3-3\,a^2b+3\,ab^2-b^3\right)f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 2*(5*a^2 - 8*a*b + 3*b^2)*co
s(f*x + e)^3 + (15*a^2 - 10*a*b + 3*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.49

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\frac{15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a-b}+\frac{3\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-10\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}b^2\cdots}{a^3-3\,a^2b+3\,ab^2-b^3}}{15\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) + (3*(a - b
+ b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a - b + b/cos(f*x + e)^2)^(
3/2)*b*cos(f*x + e)^3 + 15*sqrt(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)
)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*c
os(f*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^2 - 2*
a*b + b^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(132) = 264$.

Time = 1.59 (sec) , antiderivative size = 1303, normalized size of antiderivative = 9.05

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
256/15*(5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a - 10*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3/2) + 15*(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b - 13*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2 - 14*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 12*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))^4*a^(5/2) - 55*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))^4*a^(3/2)*b - 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))^3*a^3 + 60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
3*a^2*b - 40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{{\sin\left(e+f\,x\right)}^5}{\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^5}{\tan\left(fx+e\right)^2 b+a}dx$$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**5)/(tan(e + f*x)**2*b + a),
x)
```

3.117 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{(3a-b)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^2f} + \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)f}$$

output
```
-1/3*(3*a-b)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^2/f+1/3*cos(f*x+e
)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\cos(e+fx)(-5a+b+(a-b)\cos(2(e+fx)))\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{6\sqrt{2}(a-b)^2f}$$

input
```
Integrate[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(-5*a + b + (a - b)*Cos[2*(e + f*x)])*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(6*Sqrt[2]*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4147, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^3}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a-b)\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{3(a-b)}+\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3(a-b)}}{f}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3(a-b)}-\frac{(3a-b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{3(a-b)^2}}{f}$$

input
```
Int[Sin[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/3*((3*a - b)*Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(a - b)^2 +
(Cos[e + f*x]^3*Sqrt[a - b + b*Sec[e + f*x]^2])/(3*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(a\cos(fx+e)^2+b\sin(fx+e)^2\right)\left(b\sin(fx+e)^2+a\cos(fx+e)^2-3a\right)\sec(fx+e)}{3f(a-b)^2\sqrt{a+b\tan(fx+e)^2}}$	78

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a-b)^2*(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*(b*sin(f*x+e)^2+a*cos(f*x+e)
^2-3*a)/(a+b*tan(f*x+e)^2)^(1/2)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\left((a-b)\cos\left(fx+e\right)^3-(3\,a-b)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,(a^2-2\,ab+b^2)f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((a - b)*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt(((a - b)*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((a^2 - 2*a*b + b^2)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\frac{3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a-b} - \frac{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^2-2\,ab+b^2}}{3\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a - b) - ((a - b + b/
cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a - b + b/cos(f*x + e)^2)*b*
cos(f*x + e))/(a^2 - 2*a*b + b^2))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(80) = 160.

Time = 1.53 (sec) , antiderivative size = 586, normalized size of antiderivative = 6.66

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
16/3*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a) -
4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a + 4*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b - 6*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 12*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))*a^2 - 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))*a*b - 5*a^(5/2) + 8*a^(3/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))*sqrt(a) - 3*a + 4*b)^3*f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{b\,\tan\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^3}{\tan(fx+e)^2 b+a}dx$$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**3)/(tan(e + f*x)**2*b + a),
x)
```

3.118 $\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 37

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{(a-b)f}$$

output
```
-cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\cos(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{\sqrt{2}(-a+b)f}$$

input
```
Integrate[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sq
rt[2]*(-a + b)*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4147, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4147}$$

$$\frac{\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{f(a-b)}$$

input
```
Int[Sin[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-((Cos[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/((a - b)*f))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{-\sin(fx+e) b \tan(fx+e)-a\cos(fx+e)}{f(a-b)\sqrt{a+b\tan(fx+e)^{2}}}$	51

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-sin(f*x+e)*b*tan(f*x+e)-a*cos(f*x+e))/(a-b)/(a+b*tan(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{(a-b)f}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f
)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sin\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos\left(fx+e\right)}{(a-b)f}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*f)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(35) = 70.

Time = 1.22 (sec) , antiderivative size = 231, normalized size of antiderivative = 6.24

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

$$= \frac{4\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a}\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-\right.}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/(((sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a + 4*b)*f*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)}{\sqrt{b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a}}\,dx$$

input
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^{2} b+a}\,\sin\left(fx+e\right)}{\tan\left(fx+e\right)^{2} b+a}dx$$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x))/(tan(e + f*x)**2*b + a),x)
```

3.119 $\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
-arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 221 vs. 2(42) = 84.

Time = 1.59 (sec) , antiderivative size = 221, normalized size of antiderivative = 5.26

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\cos(e+fx)\left(2\operatorname{arctanh}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\frac{\sqrt{4b\tan^2\left(\frac{1}{2}(e+fx)\right)+a\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2}}{\sqrt{a}}\right)+\log\left(a-2b-a\tan\right.\right.}{2\sqrt{a}f\sqrt{(\ldots}}$$

input
```
Integrate[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2
+ a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/
2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2
]])*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x
]^2])/(2*Sqrt[a]*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2
]^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4147, 25, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

input
```
Int[Csc[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/(Sqrt[a]*
f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(36) = 72$.

Time = 3.14 (sec), antiderivative size = 302, normalized size of antiderivative = 7.19

method	result
default	$\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\left(\ln\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\cos(fx+e)+2\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sqrt{a}-2a\cos(fx+e)+2\cos(f}{\sqrt{a}\,(\cos(fx+e)+1)}\right.\right.}{f\sqrt{a}\sqrt{a+b\tan(fx+e)^2}\,((1-\cos(fa}$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*(ln(2
/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/
2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))+ln(2/(1-cos(f*x+e))^2*(-a*
(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(
cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2)))/(a+b*tan(f*x
+e)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec), antiderivative size = 151, normalized size of antiderivative = 3.60

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \left[\frac{\log\left(-\frac{2\left((a-b)\cos(fx+e)^2-2\sqrt{a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a}f},\; -\frac{\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{(a-b)\cos(fx+e)^2+b}\right)}{af}\right]$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1))/(sqrt(
a)*f), -sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b))/(a*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc{\left(e+fx\right)}}{\sqrt{a+b\tan^2{\left(e+fx\right)}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{1}{\sin(e+fx)\sqrt{b\tan(e+fx)^2+a}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)}{\tan(fx+e)^2 b+a}dx$$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x))/(tan(e + f*x)**2*b + a),x)
```

3.120 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{2af}$$

output
```
-1/2*(a-b)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(3/2)/
f-1/2*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 367 vs. 2(91) = 182.

Time = 2.36 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.03

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\cot^2(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(2(-a+b)\log\left(a-2b-a\tan^2\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Cot[e + f*x]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(2
*(-a + b)*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f
*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] - (-2*(a - b)*Log[a - 2*b - a*T
an[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e +
 f*x)/2]^2)^2]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*
Sec[(e + f*x)/2]^4])*Sec[e + f*x] + 8*(a - b)*ArcTanh[Tan[(e + f*x)/2]^2 -
 Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]]*Sec
[e + f*x]*Sin[(e + f*x)/2]^2))/(4*a^(3/2)*f*Sqrt[(a + b + (a - b)*Cos[2*(e
 + f*x)])*Sec[(e + f*x)/2]^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4147, 373, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a\left(1-\sec^2(e+fx)\right)}-\frac{\int \frac{a-b}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{2a}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{(a-b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{2a}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{(a-b)\int\frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{2a}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a(1-\sec^2(e+fx))}-\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/2*((a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^
2]])/a^(3/2) + (Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(2*a*(1 - Sec
[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(79) = 158$.

Time = 3.33 (sec) , antiderivative size = 647, normalized size of antiderivative = 7.11

method	result
default	$\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\left(\ln\left(\frac{2\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\cos(fx+e)+2\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sqrt{a}-2a\cos(fx+e)+2\cos(f}{\sqrt{a}\,(\cos(fx+e)+1)}\right.\right.}{}$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^(5/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)/(a+
b*tan(f*x+e)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)/(1-cos(f*x+e))^2*(
ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(
1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a
^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^2*(1-cos(f*x+e))^2+l
n(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f
*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin
(f*x+e)^2))*a^2*(1-cos(f*x+e))^2-ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*
sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*
x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(
f*x+e)+1))*a*(1-cos(f*x+e))^2*b-ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2
+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*a*(1-cos(f*x+e))^2*b+(2-2*cos
(f*x+e))*a^(3/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.31

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

$$= \left[\frac{2\,a\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) - \left((a-b)\cos(fx+e)^2-a+b\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cos(fx+e)^2+2\sqrt{a}\sqrt{\frac{(a}{}}\right.}{\cos(f}\right.}{4\left(a^2f\cos(fx+e)^2-a^2f\right)}\right.$$

$$-\frac{\left((a-b)\cos(fx+e)^2-a+b\right)\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{(a-b)\cos(fx+e)^2+b}\right)-a\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,\mathrm{c}}{2\left(a^2f\cos(fx+e)^2-a^2f\right)}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) -
 ((a - b)*cos(f*x + e)^2 - a + b)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 +
 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
+ a + b)/(cos(f*x + e)^2 - 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), -1/2*(((a
- b)*cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b))
- a*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(a^2*f
*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\csc^3{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\csc{(fx+e)}^3}{\sqrt{b\tan{(fx+e)}^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(79) = 158$.

Time = 1.13 (sec) , antiderivative size = 464, normalized size of antiderivative = 5.10

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(4*(a - b)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))/sqrt(-a))/(sqrt(-a)*a) + 2*(a^(3/2) - sqrt(a)*b)*log(abs(-(sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a + a^(3/2) - 2*sqrt(a)*b))/a^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a)/a - 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*a - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b - a^
(3/2))/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*a))/
(f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{1}{\sin{(e+f\,x)}^3\,\sqrt{b\tan{(e+f\,x)}^2+a}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan{(fx+e)}^2\,b+a}\,\csc{(fx+e)}^3}{\tan{(fx+e)}^2\,b+a}dx$$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**3)/(tan(e + f*x)**2*b + a),
x)
```

3.121 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{3(a-b)^2\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a-3b)\cot(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{8a^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{4af}$$

output
```
-3/8*(a-b)^2*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2
)/f-1/8*(5*a-3*b)*cot(f*x+e)*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a^2/f-1
/4*cot(f*x+e)^3*csc(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.91

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \frac{\left(-\sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\left(3a-3b+2a\csc^2(e+fx)\right)+\frac{3(a-b)^2\cos(e+fx)\left(2\text{arctanh}\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-(Sqrt[2]*Sqrt[a]*Cot[e + f*x]*Csc[e + f*x]*(3*a - 3*b + 2*a*Csc[e + f*x
]^2)) + (3*(a - b)^2*Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b
*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*
b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + T
an[(e + f*x)/2]^2)^2]])*Sec[(e + f*x)/2]^2)/Sqrt[(a + b + (a - b)*Cos[2*(e
 + f*x)])*Sec[(e + f*x)/2]^4])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec
[e + f*x]^2])/(16*a^(5/2)*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4147, 25, 372, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(e+fx)^5\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \ 4147$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \ 372$$

$$\frac{\frac{\int \frac{2(2a-b)\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{4a}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4a(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\frac{\int -\frac{3(a-b)^2}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{2a}+\frac{(5a-3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a\left(1-\sec^2(e+fx)\right)}}{4a}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4a(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{(5a-3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a\left(1-\sec^2(e+fx)\right)}-\frac{3(a-b)^2\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{2a}}{4a}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4a(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{\frac{(5a-3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a\left(1-\sec^2(e+fx)\right)}-\frac{3(a-b)^2\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{2a}}{4a}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4a(1-\sec^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{(5a-3b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{2a\left(1-\sec^2(e+fx)\right)}-\frac{3(a-b)^2\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a^{3/2}}}{4a}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)-b}}{4a(1-\sec^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/4*(Sec[e + f*x]*Sqrt[a - b + b*Sec[e + f*x]^2])/(a*(1 - Sec[e + f*x]^2
)^2) + ((-3*(a - b)^2*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e
+ f*x]^2]])/(2*a^(3/2)) + ((5*a - 3*b)*Sec[e + f*x]*Sqrt[a - b + b*Sec[e +
 f*x]^2])/(2*a*(1 - Sec[e + f*x]^2)))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. $2(127) = 254$.

Time = 3.20 (sec) , antiderivative size = 1167, normalized size of antiderivative = 8.16

method	result	size
default	Expression too large to display	1167

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/a^(7/2)*((3*cos(f*x+e)+3)*sin(f*x+e)^4*((a*cos(f*x+e)^2+b*sin(f*x+
e)^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(co
s(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*
b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*a^3+
(-6*cos(f*x+e)-6)*sin(f*x+e)^4*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e
)+1)^2)^(1/2)*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(
1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2
*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*a^2*b+(3*cos(f*x+e)+
3)*sin(f*x+e)^4*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*l
n(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*si
n(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4
*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*a*b^2+(3*cos(f*x+e)+3)*sin(f*x+e)^4
*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^
(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+
((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x
+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^3+(-6*cos(f*x+e)-6)*sin(f*x+e)^4*((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(1/2
)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*
cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+
cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^2*b+(3*cos(f*x+e)+3)*sin(f*x+e)^4*((a...
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.17

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

$$= \left[\frac{3\left((a^2-2\,ab+b^2)\cos{(fx+e)}^4-2\,(a^2-2\,ab+b^2)\cos{(fx+e)}^2+a^2-2\,ab+b^2\right)\sqrt{a}\log\left(-\frac{2\left((a-b)\cdots\right.}{\cdots}\right.}{16\,\left(a^3f\cos{(fx}\cdots\right.}\right.$$

$$-\frac{3\left((a^2-2\,ab+b^2)\cos{(fx+e)}^4-2\,(a^2-2\,ab+b^2)\cos{(fx+e)}^2+a^2-2\,ab+b^2\right)\sqrt{-a}\arctan\left(-\cdots\right.}{8\,\left(a^3f\cos{(fx+e)}^4-\cdots\right.}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 - 2*a*b + b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*
sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
 + b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 - a*b)*cos(f*x + e)^3 - (5*a^2 - 3
*a*b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^
3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), -1/8*(3*((a^2 - 2*a*
b + b^2)*cos(f*x + e)^4 - 2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a^2 - 2*a
*b + b^2)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) - (3*(a^2 - a*b)*co
s(f*x + e)^3 - (5*a^2 - 3*a*b)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3
*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc^5{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^5}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(127) = 254$.

Time = 1.34 (sec) , antiderivative size = 861, normalized size of antiderivative = 6.02

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a)*(tan(1/2*f*x + 1/2*e)^2/a + 3*(3*a - 2*b)/a^2) -
24*(a^2 - 2*a*b + b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 12*(a^(5/2) - 2*a^(3/2)*b + sqrt(a)*
b^2)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a + a^
(3/2) - 2*sqrt(a)*b))/a^3 + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))^3*a^2 - 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))^3*a*b + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2
- 3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) - 6*(s
qrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 + 16*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b - 10*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^5\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\;\csc\left(fx+e\right)^5}{\tan\left(fx+e\right)^2 b+a}dx$$

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**5)/(tan(e + f*x)**2*b + a),
x)
```

3.122 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{3a^2\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{5/2}f} - \frac{(5a-2b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8(a-b)^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)f}$$

output
```
3/8*a^2*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2
)/f-1/8*(5*a-2*b)*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)^2/f
+1/4*cos(f*x+e)^3*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.98 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.15

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx =$$
$$-\frac{\left((a-b)\left(7a^2+8ab-3b^2+2(3a^2-5ab+2b^2)\cos(2(e+fx))-(a-b)^2\cos(4(e+fx))\right)+6\sqrt{2}a^2(-\right.}{}$$

input
```
Integrate[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/32*(((a - b)*(7*a^2 + 8*a*b - 3*b^2 + 2*(3*a^2 - 5*a*b + 2*b^2)*Cos[2*(
e + f*x)] - (a - b)^2*Cos[4*(e + f*x)]) + 6*Sqrt[2]*a^2*(-a + b)*Sqrt[((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[(
(a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 6*Sqr
t[2]*a^3*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Ellip
ticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e
+ f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]*(a -
b)^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4146, 372, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^4}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\int \frac{a-2(2a-b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{4(a-b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{3a^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{3a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{3a^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4(a-b)(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a-2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)\left(\tan^2(e+fx)+1\right)}-\frac{3a^2\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{3/2}}}{4(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(4*(a - b)*(1 + Tan[e + f*x]^2)
^2) - ((-3*a^2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/(2*(a - b)^(3/2)) + ((5*a - 2*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^
2])/(2*(a - b)*(1 + Tan[e + f*x]^2)))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 21.66 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.56

method	result
default	$\frac{\left((3\cos(fx+e)+3)\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,a^2\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)+\sin(fx+e)\cos(fx+e)^2\left(2\cos(\right.\right.}{8f(a-b)^{\frac{5}{2}}\sqrt{a+b\tan(fx+e)^2}}$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(a-b)^(5/2)*((3*cos(f*x+e)+3)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(
f*x+e)+1)^2)^(1/2)*a^2*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^
2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))+sin(f*x+e)*cos(f*x+e
)^2*(2*cos(f*x+e)^2-5)*(a-b)^(1/2)*a^2+sin(f*x+e)^3*(4*cos(f*x+e)^2-5)*(a-
b)^(1/2)*a*b+2*sin(f*x+e)^5*(a-b)^(1/2)*b^2)/(a+b*tan(f*x+e)^2)^(1/2)*sec(
f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(130) = 260$.

Time = 2.06 (sec) , antiderivative size = 788, normalized size of antiderivative = 5.40

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*a^2*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*co
s(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(
f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4
- 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x +
e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqr
t(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*(a^2 -
 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*b^2)*cos(f*x + e))*sqrt(
((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*f), 1/32*(3*sqrt(a - b)*a^2*arctan(-1/4*(8*(a^2 - 2*a*b
+ b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*
a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b
+ 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(
f*x + e))) + 4*(2*(a^2 - 2*a*b + b^2)*cos(f*x + e)^3 - (5*a^2 - 7*a*b + 2*
b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)]
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sin^4{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sin(fx+e)^4}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sin(fx+e)^4}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sin(e+f\,x)^4}{\sqrt{b\tan(e+f\,x)^2+a}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sqrt{\tan(fx+e)^2\,b+a}\,\sin(fx+e)^4}{\tan(fx+e)^2\,b+a}dx$$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**4)/(tan(e + f*x)**2*b + a),
x)
```

3.123 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{a\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)f}$$

output
```
1/2*a*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/
f-1/2*cos(f*x+e)*sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/(a-b)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.55 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.90

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\left((a-b)(a+b+(a-b)\cos(2(e+fx))) + \sqrt{2}a(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\text{EllipticF}\left(\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/4*(((a - b)*(a + b + (a - b)*Cos[2*(e + f*x)]) + Sqrt[2]*a*(-a + b)*Sqr
t[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] +
 Sqrt[2]*a^2*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*E
llipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Cs
c[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]*(
a - b)^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4146, 373, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\int \frac{a}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2(a-b)}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2(a-b)}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{a\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{2(a-b)}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{a\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{3/2}}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((a*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(2*(a -
 b)^(3/2)) - (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*(a - b)*(1 + Tan
[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(81) = 162$.

Time = 12.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\, a\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)(-1-\sec(fx+e))+\sqrt{a-b}\,\sin(fx+e)\cos(fx+e)a-}{2f(a-b)^{\frac{3}{2}}\sqrt{a+b\tan(fx+e)^2}}$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a-b)^(3/2)/(a+b*tan(f*x+e)^2)^(1/2)*(((a*cos(f*x+e)^2+b*sin(f*x+e)
^2)/(cos(f*x+e)+1)^2)^(1/2)*a*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(
f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*(-1-sec(f*x+e
))+(a-b)^(1/2)*sin(f*x+e)*cos(f*x+e)*a+(a-b)^(1/2)*b*sin(f*x+e)^2*tan(f*x+
e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(81) = 162$.

Time = 0.29 (sec) , antiderivative size = 696, normalized size of antiderivative = 7.48

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)*sin(f*x + e) - a*sqrt(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 +
 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24
*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4
- 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*
(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*
cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-
a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((
a^2 - 2*a*b + b^2)*f), -1/8*(4*(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - sqrt(a - b)*a*arctan(-1/4*(8*(a
^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3
+ (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)
^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x +
 e)^2)*sin(f*x + e))))/((a^2 - 2*a*b + b^2)*f)]
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sin^2{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^2}{\tan(fx+e)^2 b+a}dx$$

input

```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output

```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**2)/(tan(e + f*x)**2*b + a),
x)
```

3.124 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

input
```
Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]
*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4144, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

input
```
Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]
*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

method	result	size
derivativedivides	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{f\,b^2(a-b)}$	67
default	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{f\,b^2(a-b)}$	67

input
```
int(1/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \left[-\frac{\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2(a-b)f}, \frac{\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)}{\sqrt{a-b}f}\right]$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), a
rctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a))/(sqrt(a - b)*f)
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,\sqrt{a-b}}{\sqrt{b\tan(e+f\,x)^2+a}}\right)}{f\,\sqrt{a-b}}$$

input
```
int(1/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
atan((tan(e + f*x)*(a - b)^(1/2))/(a + b*tan(e + f*x)^2)^(1/2))/(f*(a - b)
^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a + b\tan^2(e + fx)}}\, dx = \int \frac{\sqrt{\tan(fx + e)^2 b + a}}{\tan(fx + e)^2 b + a} dx$$

input
```
int(1/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)/(tan(e + f*x)**2*b + a),x)
```

3.125 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 30

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

output
```
-cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{\sqrt{2}af}$$

input
```
Integrate[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(
Sqrt[2]*a*f))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3042, 4146, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{242}$$

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

input
```
Int[Csc[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a*f))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\tan(fx+e)^2}}{fa\tan(fx+e)}$	31
default	$-\frac{\sqrt{a+b\tan(fx+e)^2}}{fa\tan(fx+e)}$	31

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{af\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f*sin(f
*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\sqrt{b\tan(fx+e)^2+a}}{af\tan(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*tan(f*x + e)^2 + a)/(a*f*tan(f*x + e))
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=-\frac{\cot(e+f\,x)\,\sqrt{a+\frac{b\sin(e+f\,x)^2}{\cos(e+f\,x)^2}}}{a\,f}$$

input
```
int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

output
```
-(cot(e + f*x)*(a + (b*sin(e + f*x)^2)/cos(e + f*x)^2)^(1/2))/(a*f)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^2}{\tan\left(fx+e\right)^2 b+a}dx$$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**2)/(tan(e + f*x)**2*b + a),
x)
```

3.126 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

output
```
-1/3*(3*a-2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)^3*
(a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=-\frac{\cot(e+fx)\left(2a-2b+a\csc^2(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}{3\sqrt{2}a^2f}$$

input
```
Integrate[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/3*(Cot[e + f*x]*(2*a - 2*b + a*Csc[e + f*x]^2)*Sqrt[(a + b + (a - b)*Co
s[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*a^2*f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4146, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^4\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a-2b)\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

$$\Big\downarrow \text{242}$$

$$\frac{-\frac{(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

input
```
Int[Csc[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/3*((3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a^2 - (Cot[e +
 f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a))/f
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\left(a\cos\left(fx+e\right)^{2}+b\sin\left(fx+e\right)^{2}\right)\left(2b\sin\left(fx+e\right)^{2}+2a\cos\left(fx+e\right)^{2}-3a\right)\sec\left(fx+e\right)\csc\left(fx+e\right)^{3}}{3f\,a^{2}\sqrt{a+b\tan\left(fx+e\right)^{2}}}$	84

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a^2*(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*(2*b*sin(f*x+e)^2+2*a*cos(f*x+e)
^2-3*a)/(a+b*tan(f*x+e)^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\left(2\,(a-b)\cos\left(fx+e\right)^3-\left(3\,a-2\,b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a - b)*cos(f*x + e)^3 - (3*a - 2*b)*cos(f*x + e))*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^2 - a^2*f)*sin(f*x
 + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\frac{3\sqrt{b\tan(fx+e)^2+a}}{a\tan(fx+e)}-\frac{2\sqrt{b\tan(fx+e)^2+a}b}{a^2\tan(fx+e)}+\frac{\sqrt{b\tan(fx+e)^2+a}}{a\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)) - 2*sqrt(b*tan(f*x + e
)^2 + a)*b/(a^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)
^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.96

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{2\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)^2}{\left(e^{e\,2i+f\,x\,2i}+1\right)^2}}\left(a\,1i-b\,1i-a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i+b\,e^{e\,2i+f\,x\,2i}\,2i-b\,e\right.}{3\,a^2\,f\left(e^{e\,2i+f\,x\,2i}-1\right)^3}$$

input
```
int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

output
```
-(2*(exp(e*2i + f*x*2i) + 1)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(
e*2i + f*x*2i) + 1)^2)^(1/2)*(a*1i - b*1i - a*exp(e*2i + f*x*2i)*4i + a*ex
p(e*4i + f*x*4i)*1i + b*exp(e*2i + f*x*2i)*2i - b*exp(e*4i + f*x*4i)*1i))/
(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^3)
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^4}{\tan\left(fx+e\right)^2 b+a}dx$$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**4)/(tan(e + f*x)**2*b + a),
x)
```

3.127 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{(15a^2-20ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} - \frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af}$$

output
```
-1/15*(15*a^2-20*a*b+8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f-2/15
*(5*a-2*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/5*cot(f*x+e)^5*(a
+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\cot(e+fx)\left(8(a-b)^2+4a(a-b)\csc^2(e+fx)+3a^2\csc^4(e+fx)\right)\sqrt{(a+b+(a-b)\cos(2(e+fx)}}{15\sqrt{2}a^3f}$$

input
```
Integrate[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/15*(Cot[e + f*x]*(8*(a - b)^2 + 4*a*(a - b)*Csc[e + f*x]^2 + 3*a^2*Csc[
e + f*x]^4)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt
[2]*a^3*f)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4146, 365, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^6\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{365}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5a\tan^2(e+fx)+2(5a-2b)\right)}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\frac{\frac{\left(15a^2-20ab+8b^2\right)\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{\frac{-\frac{\left(15a^2-20ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2}-\frac{2(5a-2b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}}{f}$$

input
```
Int[Csc[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/a + (-1/3*((15*a^2 - 20*
a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a^2 - (2*(5*a - 2*b)
*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a))/(5*a))/f
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10

method	result
default	$-\frac{\left(a\cos(fx+e)^2+b\sin(fx+e)^2\right)\left(8\sin(fx+e)^4b^2+16ab\cos(fx+e)^2\sin(fx+e)^2+8a^2\cos(fx+e)^4-20ab\sin(fx+e)^2-20a^2\cos(f\right.}{15fa^3\sqrt{a+b\tan(fx+e)^2}}$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^3*(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*(8*sin(f*x+e)^4*b^2+16*a*b*cos
(f*x+e)^2*sin(f*x+e)^2+8*a^2*cos(f*x+e)^4-20*a*b*sin(f*x+e)^2-20*a^2*cos(f
*x+e)^2+15*a^2)/(a+b*tan(f*x+e)^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^5
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\left(8\left(a^2-2ab+b^2\right)\cos\left(fx+e\right)^5-4\left(5a^2-9ab+4b^2\right)\cos\left(fx+e\right)^3+\left(15a^2-20ab+8b^2\right)\cos\left(fx\right.}{15\left(a^3f\cos\left(fx+e\right)^4-2a^3f\cos\left(fx+e\right)^2+a^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 4*(5*a^2 - 9*a*b + 4*b^2)*co
s(f*x + e)^3 + (15*a^2 - 20*a*b + 8*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x +
e)^2 + a^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.41

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\frac{15\sqrt{b\tan(fx+e)^2+a}}{a\tan(fx+e)}-\frac{20\sqrt{b\tan(fx+e)^2+a}b}{a^2\tan(fx+e)}+\frac{8\sqrt{b\tan(fx+e)^2+a}b^2}{a^3\tan(fx+e)}+\frac{10\sqrt{b\tan(fx+e)^2+a}}{a\tan(fx+e)^3}-\frac{4\sqrt{b\tan(fx+e)^2+a}b}{a^2\tan(fx+e)^3}+\frac{3\sqrt{b}}{a\,t}}{15f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)) - 20*sqrt(b*tan(f*x
+ e)^2 + a)*b/(a^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x + e)^2 + a)*b^2/(a^3*t
an(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a)/(a*tan(f*x + e)^3) - 4*sqrt(b
*tan(f*x + e)^2 + a)*b/(a^2*tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a)
/(a*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^6}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 18.12 (sec) , antiderivative size = 761, normalized size of antiderivative = 6.19

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2)),x)
```

output
```
((((a - b)*(32*a*b - 64*a^2 + 32*b^2))/(120*a^3*f*(a*1i - b*1i)) - ((a - b
)*(64*a^2 - 96*a*b + 32*b^2))/(120*a^3*f*(a*1i - b*1i)))*(a + (b*(exp(e*2i
 + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x
*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^2*(exp(e*2i + f*
x*2i) + 1)) + ((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i)
 + 1)^2)^(1/2)*((2*(3*a - 3*b))/(3*a*f*(a*1i - b*1i)) + ((3*a - 3*b)*(96*a
 - 64*b))/(240*a^2*f*(a*1i - b*1i)) + ((3*a - 3*b)*(256*a + 64*b))/(240*a^
2*f*(a*1i - b*1i)))*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp
(e*2i + f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) + ((((a - b)*(32*a - 16*b
))/(30*a^2*f*(a*1i - b*1i)) + ((a - b)*(32*a + 48*b))/(30*a^2*f*(a*1i - b*
1i)))*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^
(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i
) - 1)^3*(exp(e*2i + f*x*2i) + 1)) - ((a - b)^2*(a + (b*(exp(e*2i + f*x*2i
)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + ex
p(e*4i + f*x*4i) + 1)*8i)/(15*a^3*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f
*x*2i) + 1)) + (8*(2*a - 2*b)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp
(e*2i + f*x*2i) + 1)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) +
 1))/(5*a*f*(exp(e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*2i) + 1)*(a*1i - b*
1i))
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^6}{\tan(fx+e)^2 b+a}dx$$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**6)/(tan(e + f*x)**2*b + a),
x)
```

3.128 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 185

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{a^2b\sec(e+fx)}{(a-b)^4f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(15a^2+20ab-2b^2)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^4f} + \frac{(10a-b)\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^3f} - \frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{5(a-b)^2f}$$

output

```
-a^2*b*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/15*(15*a^2+20*a*b
-2*b^2)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^4/f+1/15*(10*a-b)*cos(
f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^3/f-1/5*cos(f*x+e)^5*(a-b+b*sec(
f*x+e)^2)^(1/2)/(a-b)^2/f
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.01

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx =$$
$$-\frac{(150a^3+1078a^2b+338ab^2-30b^3+(125a^3+169a^2b-329ab^2+35b^3)\cos(2(e+fx))-2(a-b)^2(11a}{240\sqrt{2}(a-b)^4f\sqrt{(a+}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/240*((150*a^3 + 1078*a^2*b + 338*a*b^2 - 30*b^3 + (125*a^3 + 169*a^2*b
- 329*a*b^2 + 35*b^3)*Cos[2*(e + f*x)] - 2*(a - b)^2*(11*a + b)*Cos[4*(e +
 f*x)] + 3*a^3*Cos[6*(e + f*x)] - 9*a^2*b*Cos[6*(e + f*x)] + 9*a*b^2*Cos[6
*(e + f*x)] - 3*b^3*Cos[6*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^4*f*S
qrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4147, 365, 25, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^5}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{4147}$$
$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(2(5a-2b)-5(a-b)\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a-2b)-5(a-b)\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{-\frac{-\frac{\left(15a^2+10ab-b^2\right)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3(a-b)}-\frac{2(5a-2b)\cos^3(e+fx)}{3(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 245$$

$$\frac{-\frac{-\frac{\left(15a^2+10ab-b^2\right)\left(-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{(a-b)\sqrt{a+b\sec^2(e+fx)-b}}\right)}{3(a-b)}-\frac{2(5a-2b)\cos^3(e+fx)}{3(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 208$$

$$\frac{-\frac{-\frac{\left(15a^2+10ab-b^2\right)\left(-\frac{2b\sec(e+fx)}{(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{(a-b)\sqrt{a+b\sec^2(e+fx)-b}}\right)}{3(a-b)}-\frac{2(5a-2b)\cos^3(e+fx)}{3(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*Cos[e + f*x]^5/((a - b)*Sqrt[a - b + b*Sec[e + f*x]^2]) - ((-2*(5*a
- 2*b)*Cos[e + f*x]^3)/(3*(a - b)*Sqrt[a - b + b*Sec[e + f*x]^2]) - ((15*a
^2 + 10*a*b - b^2)*(-(Cos[e + f*x]/((a - b)*Sqrt[a - b + b*Sec[e + f*x]^2]
)) - (2*b*Sec[e + f*x])/((a - b)^2*Sqrt[a - b + b*Sec[e + f*x]^2])))/(3*(a
 - b)))/(5*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 198.66 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.62

method	result
default	$\frac{\left(-\frac{\cos(fx+e)^4\left(3\cos(fx+e)^4-10\cos(fx+e)^2+15\right)a^4}{15}-\frac{\cos(fx+e)^2\left(-12\cos(fx+e)^6+37\cos(fx+e)^4-30\cos(fx+e)^2+45\right)a^3b}{15}-\frac{\left(-18\cos(fx+e\right.}{}}{f(a-b}$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/15*cos(f*x+e)^4*(3*cos(f*x+e)^4-10*cos(f*x+e)^2+15)*a^4-1/15*cos(f
*x+e)^2*(-12*cos(f*x+e)^6+37*cos(f*x+e)^4-30*cos(f*x+e)^2+45)*a^3*b-1/15*(
-18*cos(f*x+e)^6+33*cos(f*x+e)^4+15*cos(f*x+e)^2+30)*sin(f*x+e)^2*a^2*b^2-
1/15*(-12*cos(f*x+e)^4+7*cos(f*x+e)^2+20)*sin(f*x+e)^4*a*b^3-1/15*(-3*cos(
f*x+e)^2-2)*sin(f*x+e)^6*b^4)*a^6/(a-b)^3/((-b*(a-b))^(1/2)-a+b)^6/((-b*(a
-b))^(1/2)+a-b)^6*(a^5-5*a^4*b+10*a^3*b^2-10*a^2*b^3+5*a*b^4-b^5)/(a+b*tan
(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.26

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left(3\left(a^3-3a^2b+3ab^2-b^3\right)\cos(fx+e)^7-2\left(5a^3-12a^2b+9ab^2-2b^3\right)\cos(fx+e)^5+\left(15a^3-5a^2b\right.\right.}{15\left(\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)f\cos(fx+e\right.}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 2*(5*a^3 - 12*a^
2*b + 9*a*b^2 - 2*b^3)*cos(f*x + e)^5 + (15*a^3 - 5*a^2*b - 11*a*b^2 + b^3
)*cos(f*x + e)^3 + 2*(15*a^2*b + 10*a*b^2 - b^3)*cos(f*x + e))*sqrt(((a -
b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a
^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3
- 4*a*b^4 + b^5)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(171) = 342$.

Time = 0.04 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.10

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{15\,b^3}{(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)} + \frac{15\,\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2-2\,ab+b^2} + \frac{3\left(\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-5}{}}{}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*b^3/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(a - b + b/
cos(f*x + e)^2)*cos(f*x + e)) + 15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x
+ e)/(a^2 - 2*a*b + b^2) + 3*((a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e
)^5 - 5*(a - b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a - b
+ b/cos(f*x + e)^2)*b^2*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a -
 b + b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3
0*b^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(
f*x + e)) + 15*b/((a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f
*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3260 vs. $2(171) = 342$.

Time = 3.07 (sec) , antiderivative size = 3260, normalized size of antiderivative = 17.62

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/15*(15*((a^5*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3*a^4*b^3*sgn(tan(1/2
*f*x + 1/2*e)^2 - 1) + 3*a^3*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^2*b^5
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/(a^7*b - 7*a^6*b^
2 + 21*a^5*b^3 - 35*a^4*b^4 + 35*a^3*b^5 - 21*a^2*b^6 + 7*a*b^7 - b^8) + (
a^5*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3*a^4*b^3*sgn(tan(1/2*f*x + 1/2*
e)^2 - 1) + 3*a^3*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^2*b^5*sgn(tan(1/
2*f*x + 1/2*e)^2 - 1))/(a^7*b - 7*a^6*b^2 + 21*a^5*b^3 - 35*a^4*b^4 + 35*a
^3*b^5 - 21*a^2*b^6 + 7*a*b^7 - b^8))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + 4*(15*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^9*a*b + 165*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^8*a^(3/2)*b - 60*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(a)*b^2 + 320*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^3 - 420*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))^7*a^2*b + 480*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^5}{\tan\left(fx+e\right)^4 b^2+2\tan\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**5)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.129 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{ab\sec(e+fx)}{(a-b)^3f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(3a+2b)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^3f} + \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^2f}$$

output

```
-a*b*sec(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/3*(3*a+2*b)*cos(f*x
+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^3/f+1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e
)^2)^(1/2)/(a-b)^2/f
```

Mathematica [A] (verified)

Time = 3.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{(9a^2+46ab+9b^2+8(a^2-b^2)\cos(2(e+fx))-(a-b)^2\cos(4(e+fx)))\sec(e+fx)}{12\sqrt{2}(a-b)^3f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/12*((9*a^2 + 46*a*b + 9*b^2 + 8*(a^2 - b^2)*Cos[2*(e + f*x)] - (a - b)^
2*Cos[4*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a -
b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4147, 25, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a-b)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a+b)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3(a-b)}+\frac{\cos^3(e+fx)}{3(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(3a+b)\left(-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{(a-b)\sqrt{a+b\sec^2(e+fx)-b}}\right)}{3(a-b)}+\frac{\cos^3(e+fx)}{3(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\cos^3(e+fx)}{3(a-b)\sqrt{a+b\sec^2(e+fx)-b}}+\frac{(3a+b)\left(-\frac{2b\sec(e+fx)}{(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{(a-b)\sqrt{a+b\sec^2(e+fx)-b}}\right)}{3(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]^3/(3*(a - b)*Sqrt[a - b + b*Sec[e + f*x]^2]) + ((3*a + b)*(-
(Cos[e + f*x]/((a - b)*Sqrt[a - b + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x
])/((a - b)^2*Sqrt[a - b + b*Sec[e + f*x]^2])))/(3*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 6.77 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

method	result
default	$\frac{(a-b)a^4\left(a^3\left(\cos(fx+e)^3-3\cos(fx+e)\right)+a^2b\left(-3\cos(fx+e)^3+6\cos(fx+e)-9\sec(fx+e)\right)+\left(-3\cos(fx+e)^4-6\right)a\,b^2\tan(fx+e)}{3f\left(\sqrt{-b(a-b)}-a+b\right)^4\left(\sqrt{-b(a-b)}+a-b\right)^4\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*(a-b)/((-b*(a-b))^(1/2)-a+b)^4/((-b*(a-b))^(1/2)+a-b)^4*a^4/(a+b*tan
(f*x+e)^2)^(3/2)*(a^3*(cos(f*x+e)^3-3*cos(f*x+e))+a^2*b*(-3*cos(f*x+e)^3+6
*cos(f*x+e)-9*sec(f*x+e))+(-3*cos(f*x+e)^4-6)*a*b^2*tan(f*x+e)^2*sec(f*x+e
)+(-cos(f*x+e)^2-2)*b^3*sin(f*x+e)*tan(f*x+e)^3)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.24

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\left((a^2-2\,ab+b^2)\cos\left(fx+e\right)^5-(3\,a^2-2\,ab-b^2)\cos\left(fx+e\right)^3-2\,(3\,ab}{3\,\left((a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)f\cos\left(fx+e\right)^2+(a^3b-}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - (3*a^2 - 2*a*b - b^2)*cos(f*x +
e)^3 - 2*(3*a*b + b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x + e)^2
 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.70

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2-2\,ab+b^2}-\frac{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{3\,b^2}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{a-b+\frac{}{\cos}}}}{3\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) -
((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a - b + b/cos(f*
x + e)^2)*b*cos(f*x + e))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 3*b^2/((a^3 -
3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)) + 3*
b/((a^2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(117) = 234$.

Time = 2.86 (sec) , antiderivative size = 1414, normalized size of antiderivative = 11.13

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*(3*((a^3*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^2*b^3*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1) + a*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x +
1/2*e)^2/(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6) + (
a^3*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^2*b^3*sgn(tan(1/2*f*x + 1/2*
e)^2 - 1) + a*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^5*b - 5*a^4*b^2 + 10
*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + 4*(3*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b + 12*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2) - 3*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b - 16*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))^3*a^2 + 14*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^3*a*b + 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^3*b^2 - 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\int \frac{\sin\left(e+f\,x\right)^3}{{\left(b\,\tan\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^3}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**3)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.130 $\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{(a-b)f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

output
```
-cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/(a-b)^2/f/(a
-b+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{(a+3b+(a-b)\cos(2(e+fx)))\sec(e+fx)}{\sqrt{2}(a-b)^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-(((a + 3*b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x])/(Sqrt[2]*(a - b)^2*f
*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4147, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{2b\sec(e+fx)}{(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\cos(e+fx)}{(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cos[e + f*x]/((a - b)*Sqrt[a - b + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f
*x])/((a - b)^2*Sqrt[a - b + b*Sec[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 6.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{\left(-\cos(fx+e)a^2+\left(\cos(fx+e)^2-2\right)\tan(fx+e)^2\sec(fx+e)b^2+\left(2\cos(fx+e)-3\sec(fx+e)\right)ab\right)a^2}{f\left(\sqrt{-b(a-b)}-a+b\right)^2\left(\sqrt{-b(a-b)}+a-b\right)^2\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}$	116

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-cos(f*x+e)*a^2+(cos(f*x+e)^2-2)*tan(f*x+e)^2*sec(f*x+e)*b^2+(2*cos(f
*x+e)-3*sec(f*x+e))*a*b)*a^2/((-b*(a-b))^(1/2)-a+b)^2/((-b*(a-b))^(1/2)+a-
b)^2/(a+b*tan(f*x+e)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\left((a-b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{(a^3-3\,a^2b+3\,ab^2-b^3)f\cos\left(fx+e\right)^2+(a^2b-2\,ab^2+b^3)f}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cos(f*x + e)^2 + (
a^2*b - 2*a*b^2 + b^3)*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\frac{\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2-2\,ab+b^2}+\frac{b}{(a^2-2\,ab+b^2)\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}}{f}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^2 - 2*a*b + b^2) + b/((a^
2 - 2*a*b + b^2)*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(72) = 144$.

Time = 2.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 5.86

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\frac{\frac{\left(ab^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)-b^3\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^3b-3a^2b^2+3ab^3-b^4}+\frac{ab^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right.}{a^3b-3a^2b}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}}{}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
(((a*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - b^3*sgn(tan(1/2*f*x + 1/2*e)^2
- 1))*tan(1/2*f*x + 1/2*e)^2/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4) + (a*b^2*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^
3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + 4*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/(((sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))*sqrt(a) - 3*a + 4*b)*(a*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - b
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)}{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x))/(tan(e + f*x)**4*b**2 + 2*t
an(e + f*x)**2*a*b + a**2),x)
```

3.131 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\sec(e+fx)}{a(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}$$

output
```
-arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(3/2)/f-b*sec(f*
x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 505 vs. 2(84) = 168.

Time = 5.57 (sec) , antiderivative size = 505, normalized size of antiderivative = 6.01

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\cos(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)\left(2(a-b)\operatorname{arctanh}\left(\tan^2\left(\frac{1}{2}(e+fx)\right) - \frac{\sqrt{4b\tan \cdots}}{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*Sec[(e + f*x)/2]^6*(2*(a - b)*ArcTanh[Tan[(e + f*x)/2]^2 - S
qrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]]*(a +
b + (a - b)*Cos[2*(e + f*x)]) + a^2*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + S
qrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]] - b^2
*Log[a - 2*b - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2
+ a*(-1 + Tan[(e + f*x)/2]^2)^2]] + (a - b)^2*Cos[2*(e + f*x)]*Log[a - 2*b
 - a*Tan[(e + f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Ta
n[(e + f*x)/2]^2)^2]] - Sqrt[2]*Sqrt[a]*b*Sqrt[(a + b + (a - b)*Cos[2*(e +
 f*x)])*Sec[(e + f*x)/2]^4] - Sqrt[2]*Sqrt[a]*b*Cos[e + f*x]*Sqrt[(a + b +
 (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])*Sqrt[(a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[e + f*x]^2])/(2*a^(3/2)*(a - b)*f*((a + b + (a - b)*Cos[
2*(e + f*x)])*Sec[(e + f*x)/2]^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4147, 25, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{-\frac{\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a}-\frac{b\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{-\frac{\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{a}-\frac{b\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}}-\frac{b\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]/a^(3/2))
 - (b*Sec[e + f*x])/(a*(a - b)*Sqrt[a - b + b*Sec[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(76) = 152$.

Time = 5.94 (sec) , antiderivative size = 752, normalized size of antiderivative = 8.95

method	result
default	$\frac{\left(a(1-\cos(fx+e))^4 \csc(fx+e)^4-2a(1-\cos(fx+e))^2 \csc(fx+e)^2+4(1-\cos(fx+e))^2 b \csc(fx+e)^2+a\right)\left(2(1-\cos(fx+e))^2 b\, a^{\frac{3}{2}} \csc(\right.}{}$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^(5/2)/(a-b)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+4*(1-cos(f*x+e))^2*b*csc(f*x+e)^2+a)*(2*(1-cos(f*x+e))^2*b*a^
(3/2)*csc(f*x+e)^2+2*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos
(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a
^2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)-2*ln(2/a^(1/2)
*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x
+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos
(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)
/(cos(f*x+e)+1)^2)^(1/2)*b+2*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*
(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1
/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*a^2*((a*cos(f*x+e)^2+b*sin(f*x+e
)^2)/(cos(f*x+e)+1)^2)^(1/2)-2*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+
2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*a*((a*cos(f*x+e)^2+b*sin(f*x+e
)^2)/(cos(f*x+e)+1)^2)^(1/2)*b+2*b*a^(3/2))/(a+b*tan(f*x+e)^2)^(3/2)/((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(76) = 152$.

Time = 0.21 (sec) , antiderivative size = 370, normalized size of antiderivative = 4.40

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[-\frac{2\,ab\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) - \left((a^2-2\,ab+b^2)\cos(fx+e)^2+\right.}{2\left((a^4-2\,a^3b+a^2b^2)f\cos\right.}\right.$$

$$-\frac{ab\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + \left((a^2-2\,ab+b^2)\cos(fx+e)^2+ab-b^2\right)\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}\sqrt{\frac{(a-b)\,c}{co}}}{(a-b)}\right.}{(a^4-2\,a^3b+a^2b^2)f\cos(fx+e)^2+(a^3b-a^2b^2)f}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*b*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
) - ((a^2 - 2*a*b + b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a)*log(-2*((a -
b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)))/((a^4 - 2*a^3*b + a^2*b
^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f), -(a*b*sqrt(((a - b)*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + ((a^2 - 2*a*b + b^2)*cos(f*x + e
)^2 + a*b - b^2)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)))/((a^4 - 2*a
^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b - a^2*b^2)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)}{(b\tan(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,{\left(b\,{\mathrm{tan}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

input
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^{2} b+a}\,\csc\left(fx+e\right)}{\tan\left(fx+e\right)^{4} b^{2}+2\tan\left(fx+e\right)^{2} a b+a^{2}}dx$$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x))/(tan(e + f*x)**4*b**2 + 2*t
an(e + f*x)**2*a*b + a**2),x)
```

3.132 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{(a-3b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2a^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

output
```
-1/2*(a-3*b)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2
)/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(1/2)-3/2*b*sec(f*x
+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. $2(127) = 254$.

Time = 2.74 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.39

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{-\frac{(a+3b+(a-3b)\cos(2(e+fx)))\csc^2(e+fx)\sec(e+fx)}{\sqrt{2}a^2\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}} + \frac{(a-3b)\cos(e+fx)\left(2\text{arctanh}\left(\tan^2\left(\frac{1}{2}\right.\right.\right.}{}}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(((a + 3*b + (a - 3*b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2*Sec[e + f*x])/(S
qrt[2]*a^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])) + ((a
 - 3*b)*Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x
)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e
+ f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/
2]^2)^2]])*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[
e + f*x]^2])/(2*a^(5/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f
*x)/2]^4]))/(2*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4147, 373, 402, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\int \frac{-2b\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{2a}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\frac{3b\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\int -\frac{(a-3b)(a-b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a(a-b)}}{2a}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\frac{\int \frac{(a-3b)(a-b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a(a-b)}+\frac{3b\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}}{2a}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\frac{(a-3b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a}+\frac{3b\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}}{2a}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\frac{(a-3b)\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{a}+\frac{3b\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}}{2a}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\frac{(a-3b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}}+\frac{3b\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}}{2a}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Sec[e + f*x]/(2*a*(1 - Sec[e + f*x]^2)*Sqrt[a - b + b*Sec[e + f*x]^2]) -
(((a - 3*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]]
)/a^(3/2) + (3*b*Sec[e + f*x])/(a*Sqrt[a - b + b*Sec[e + f*x]^2]))/(2*a))/
f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(111) = 222$.

Time = 6.21 (sec) , antiderivative size = 934, normalized size of antiderivative = 7.35

method	result	size
default	Expression too large to display	934

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a^(7/2)*(((1-cos(f*x+e))^6*csc(f*x+e)^6-(1-cos(f*x+e))^4*csc(f*x+e)^
4-(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*a^(5/2)+(12*(1-cos(f*x+e))^4*csc(f*x+e)
^4+12*(1-cos(f*x+e))^2*csc(f*x+e)^2)*b*a^(3/2)+4*ln(2/(1-cos(f*x+e))^2*(-a
*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^2*(1-cos(f*x+e))^2*csc(f*x+
e)^2-12*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*
((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e
)^2+a*sin(f*x+e)^2))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1
/2)*a*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+4*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b
*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b
)/(cos(f*x+e)+1))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
*a^2*(1-cos(f*x+e))^2*csc(f*x+e)^2-12*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)
^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/
(cos(f*x+e)+1))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a
*b*(1-cos(f*x+e))^2*csc(f*x+e)^2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-
cos(f*x+e))^2*csc(f*x+e)^2+4*(1-cos(f*x+e))^2*b*csc(f*x+e)^2+a)/(1-cos(f*x
+e))^2*sin(f*x+e)^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^3/(a+b*tan(f*x+e)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(111) = 222.

Time = 0.26 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.72

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[-\frac{((a^2-4\,ab+3\,b^2)\cos(fx+e)^4-(a^2-5\,ab+6\,b^2)\cos(fx+e)^2-ab\right.$$

$$-\frac{((a^2-4\,ab+3\,b^2)\cos(fx+e)^4-(a^2-5\,ab+6\,b^2)\cos(fx+e)^2-ab+3\,b^2)\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}\sqrt{\frac{(a-b}{}}}{(a-}\right.}{2\,((a^4-a^3b)f\cos(fx+e)^4-a^3bf-(a^4}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f
*x + e)^2 - a*b + 3*b^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a
)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/
(cos(f*x + e)^2 - 1)) - 2*((a^2 - 3*a*b)*cos(f*x + e)^3 + 3*a*b*cos(f*x +
e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 - a^3*b)*f*co
s(f*x + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*cos(f*x + e)^2), -1/2*(((a^2 -
4*a*b + 3*b^2)*cos(f*x + e)^4 - (a^2 - 5*a*b + 6*b^2)*cos(f*x + e)^2 - a*b
 + 3*b^2)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) - ((a^2 - 3*a*b)*co
s(f*x + e)^3 + 3*a*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4 - 2*a^3*b)*f*
cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(111) = 222$.

Time = 1.71 (sec) , antiderivative size = 731, normalized size of antiderivative = 5.76

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*((((a^5*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^4*b^2*sgn(tan(1/2*f*x +
1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/(a^6*b - a^5*b^2) - 2*(a^5*b*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1) - 7*a^4*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 6*a
^3*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^6*b - a^5*b^2))*tan(1/2*f*x + 1
/2*e)^2 + (a^5*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 7*a^4*b^2*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1) - 8*a^3*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^6*b - a
^5*b^2))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a) - 4*(a - 3*b)*arctan(-(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1)) - 2*(a - 3*b)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))*sqrt(a) + a - 2*b))/(a^(5/2)*sgn(tan(1/2*f*x + 1/2*e)^2 -
1)) + 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - 2*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b - a^(3/2))/(((sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*a^2*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(b\,{\tan\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\csc\left(fx+e\right)^3}{\tan\left(fx+e\right)^4 b^2+2\tan\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**3)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.133 $\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{3(a-5b)(a-b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a-b)\cot(e+fx)\csc(e+fx)}{8a^2f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4af\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(13a-15b)b\sec(e+fx)}{8a^3f\sqrt{a-b+b\sec^2(e+fx)}}$$

output
```
-3/8*(a-5*b)*(a-b)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/
a^(7/2)/f-5/8*(a-b)*cot(f*x+e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)
-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-15*b
)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.84

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\frac{((-8a^2+52ab-60b^2)\cos(2(e+fx))+(a-b)(-11a-45b+3(a-5b)\cos(4(e+fx))))\csc^4(e+fx)\sec(e+f}{4\sqrt{2}a^3\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}}{}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((((-8*a^2 + 52*a*b - 60*b^2)*Cos[2*(e + f*x)] + (a - b)*(-11*a - 45*b + 3
*(a - 5*b)*Cos[4*(e + f*x)]))*Csc[e + f*x]^4*Sec[e + f*x])/(4*Sqrt[2]*a^3*
Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]) + (3*(a - 5*b)*(a
 - b)*Cos[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/
2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e +
f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]
^2)^2]])*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e
+ f*x]^2])/(2*a^(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x
)/2]^4]))/(8*f)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4147, 25, 372, 402, 25, 27, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^5\,(a+b\tan(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a-b)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a-b)^{3/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{4(a-b)\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int -\frac{(a-b)\left(-10b\sec^2(e+fx)+3a-5b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{2a}+\frac{5(a-b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{5(a-b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\int \frac{(a-b)\left(-10b\sec^2(e+fx)+3a-5b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{5(a-b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-b)\int \frac{-10b\sec^2(e+fx)+3a-5b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)-b}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{5(a-b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-b)\left(\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\int -\frac{3(a-5b)(a-b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a(a-b)}\right)}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+f}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{5(a-b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-b)\left(\frac{3(a-5b)\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a}+\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a}}{4a}-\frac{}{4a(1-\sec^2}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{5(a-b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-b)\left(\frac{3(a-5b)\int\frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{a}+\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a}}{4a}-\frac{}{4a(1-\sec^2(e+}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{5(a-b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{(a-b)\left(\frac{3(a-5b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}}+\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}\right)}{2a}}{4a}-\frac{\sec(e+f}{4a(1-\sec^2(e+fx))^2\sqrt{a}}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*Sec[e + f*x]/(a*(1 - Sec[e + f*x]^2)^2*Sqrt[a - b + b*Sec[e + f*x]^2
]) + ((5*(a - b)*Sec[e + f*x])/(2*a*(1 - Sec[e + f*x]^2)*Sqrt[a - b + b*Se
c[e + f*x]^2]) - ((a - b)*((3*(a - 5*b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqr
t[a - b + b*Sec[e + f*x]^2]])/a^(3/2) + ((13*a - 15*b)*b*Sec[e + f*x])/(a*
(a - b)*Sqrt[a - b + b*Sec[e + f*x]^2])))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1413 vs. $2(167) = 334$.

Time = 6.31 (sec) , antiderivative size = 1414, normalized size of antiderivative = 7.56

method	result	size
default	Expression too large to display	1414

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64/f/a^(9/2)*(((1-cos(f*x+e))^10*csc(f*x+e)^10+7*(cos(f*x+e)-1)^8*csc(f*
x+e)^8-8*(1-cos(f*x+e))^6*csc(f*x+e)^6-8*(1-cos(f*x+e))^4*csc(f*x+e)^4+7*(
1-cos(f*x+e))^2*csc(f*x+e)^2+1)*a^(7/2)+(-10*(cos(f*x+e)-1)^8*csc(f*x+e)^8
+114*(1-cos(f*x+e))^6*csc(f*x+e)^6+114*(1-cos(f*x+e))^4*csc(f*x+e)^4-10*(1
-cos(f*x+e))^2*csc(f*x+e)^2)*b*a^(5/2)+(-120*(1-cos(f*x+e))^6*csc(f*x+e)^6
-120*(1-cos(f*x+e))^4*csc(f*x+e)^4)*b^2*a^(3/2)+24*ln(2/a^(1/2)*(a^(1/2)*(
(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos
(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos
(f*x+e)*b+b)/(cos(f*x+e)+1))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*a^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-144*ln(2/a^(1/2)*(a^(1/2)*((
a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(
f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(
f*x+e)*b+b)/(cos(f*x+e)+1))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1
)^2)^(1/2)*a^2*b*(1-cos(f*x+e))^4*csc(f*x+e)^4+120*ln(2/a^(1/2)*(a^(1/2)*(
(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos
(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos
(f*x+e)*b+b)/(cos(f*x+e)+1))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*a*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+24*ln(2/(1-cos(f*x+e))^2*(
-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*((a*cos...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(167) = 334$.

Time = 0.30 (sec) , antiderivative size = 722, normalized size of antiderivative = 3.86

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*((a^3 - 7*a^2*b + 11*a*b^2 - 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*
a^2*b + 28*a*b^2 - 15*b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3
 - 9*a^2*b + 23*a*b^2 - 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*co
s(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^3 - 6*a^2*b + 5*a*b
^2)*cos(f*x + e)^5 - (5*a^3 - 31*a^2*b + 30*a*b^2)*cos(f*x + e)^3 - (13*a^
2*b - 15*a*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos
(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2), -1/8*(3*((a^3 - 7*a^2*b +
 11*a*b^2 - 5*b^3)*cos(f*x + e)^6 - (2*a^3 - 15*a^2*b + 28*a*b^2 - 15*b^3)
*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + 5*b^3 + (a^3 - 9*a^2*b + 23*a*b^2 - 15
*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) - (3*(a
^3 - 6*a^2*b + 5*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 31*a^2*b + 30*a*b^2)*cos
(f*x + e)^3 - (13*a^2*b - 15*a*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/((a^5 - a^4*b)*f*cos(f*x + e)^6 + a^4*b*f - (2*
a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4*b)*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(167) = 334$.

Time = 1.87 (sec) , antiderivative size = 1250, normalized size of antiderivative = 6.68

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
1/64*(((((a^8*b - a^7*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^9*b*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1) - a^8*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) + (7*a^8*b - 17*
a^7*b^2 + 10*a^6*b^3)/(a^9*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^8*b^2*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 - (17*a^8*b - 145*a^
7*b^2 + 248*a^6*b^3 - 120*a^5*b^4)/(a^9*b*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
- a^8*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 + (9*a^
8*b + 41*a^7*b^2 - 114*a^6*b^3 + 64*a^5*b^4)/(a^9*b*sgn(tan(1/2*f*x + 1/2*
e)^2 - 1) - a^8*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))/sqrt(a*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) -
 24*(a^2 - 6*a*b + 5*b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) - 1
2*(a^2 - 6*a*b + 5*b^2)*log(abs((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))*sqrt(a) - a + 2*b))/(a^(7/2)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))
+ 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 - 16*(s
qrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 14*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^5}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**5)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.134 $\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{3a(a+4b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{7/2}f} - \frac{5a\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{b(13a+2b)\tan(e+fx)}{8(a-b)^3f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
3/8*a*(a+4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b
)^(7/2)/f-5/8*a*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1
/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+2*
b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.95 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.74

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\left(-((a-b)\,(7a^2+48ab+5b^2+(6a^2-2ab-4b^2)\cos(2(e+fx))-(a-\right.}{}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-((a - b)*(7*a^2 + 48*a*b + 5*b^2 + (6*a^2 - 2*a*b - 4*b^2)*Cos[2*(e + f
*x)] - (a - b)^2*Cos[4*(e + f*x)])) + 6*Sqrt[2]*a*(a^2 + 3*a*b - 4*b^2)*Sq
rt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin
[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]
- 6*Sqrt[2]*a^2*(a + 4*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)
])/(32*Sqrt[2]*(a - b)^4*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e +
 f*x]^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4146, 372, 27, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^4}{(a+b\tan(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\int \frac{a\left(1-4\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{4(a-b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\int \frac{1-4\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{4(a-b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\left(\frac{5\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)}}-\frac{\int \frac{-10b\tan^2(e+fx)+3a+2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{2(a-b)}\right)}{4(a-b)}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\left(\frac{5\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)}}-\frac{\frac{\int \frac{3a(a+4b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(}{a(a-}}{2(a-b)}\right.}{4(a-b)}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\left(\frac{5\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)}}-\frac{\frac{3(a+4b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+f}{a-b}}{2(a-b)}\right.}{4(a-b)}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\left(\frac{5\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)}}-\frac{\frac{3(a+4b)\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{}{a}}{2(a-b)}\right)}{4(a-b)}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)}}-\frac{a\left(\frac{5\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)}}-\frac{\frac{3(a+4b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b(13a+2b)\tan(}{a(a-b)\sqrt{a+b\tan}}}{2(a-b)}\right)}{4(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(4*(a - b)*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b*Tan[e + f*x]^2]
) - (a*((5*Tan[e + f*x])/(2*(a - b)*(1 + Tan[e + f*x]^2)*Sqrt[a + b*Tan[e
+ f*x]^2]) - ((3*(a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Ta
n[e + f*x]^2]])/(a - b)^(3/2) - (b*(13*a + 2*b)*Tan[e + f*x])/(a*(a - b)*S
qrt[a + b*Tan[e + f*x]^2]))/(2*(a - b))))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10275 vs. 2(167) = 334.

Time = 47.73 (sec) , antiderivative size = 10276, normalized size of antiderivative = 54.95

method	result	size
default	Expression too large to display	10276

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(167) = 334.

Time = 85.88 (sec) , antiderivative size = 1046, normalized size of antiderivative = 5.59

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/64*(3*(a^2*b + 4*a*b^2 + (a^3 + 3*a^2*b - 4*a*b^2)*cos(f*x + e)^2)*sqrt
(-a + b)*log(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^
8 - 256*(a^4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*
(5*a^4 - 34*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 -
 32*a^3*b + 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^
2*b^2 - 40*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2
 - b^3)*cos(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)
^5 + 2*(5*a^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a
^2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(a^3 - 3*a^2*b + 3*a*b^2
 - b^3)*cos(f*x + e)^5 - 5*(a^3 - 2*a^2*b + a*b^2)*cos(f*x + e)^3 - (13*a^
2*b - 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e))/((a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*
a*b^4 - b^5)*f*cos(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 +
 b^5)*f), 1/32*(3*(a^2*b + 4*a*b^2 + (a^3 + 3*a^2*b - 4*a*b^2)*cos(f*x + e
)^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^
2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b + 8*b^2)*cos(f*x + e))*sq
rt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*(a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*a*b^2 - 2*b^3 - (a^3 - 6*
a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(a^3 - 3...
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^4{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^4}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**4)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.135 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{(a+2b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
1/2*(a+2*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^
(5/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-3/2*b*t
an(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.78 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.10

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\left((a-b)(a+5b+(a-b)\cos(2(e+fx))) - \sqrt{2}(a^2+ab-2b^2)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\text{Elliptic}\right.}{}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*(((a - b)*(a + 5*b + (a - b)*Cos[2*(e + f*x)]) - Sqrt[2]*(a^2 + a*b -
 2*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Ellipt
icF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqr
t[2]], 1] + Sqrt[2]*a*(a + 2*b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*C
sc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*C
os[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e
+ f*x)])/(Sqrt[2]*(a - b)^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[
e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4146, 373, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{(a+b\tan(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\int \frac{a-2b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{a(a+2b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{3b\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{(a+2b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}-\frac{3b\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{(a+2b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{3b\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{(a+2b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{3b\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*Tan[e + f*x]/((a - b)*(1 + Tan[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2
]) + (((a + 2*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/(a - b)^(3/2) - (3*b*Tan[e + f*x])/((a - b)*Sqrt[a + b*Tan[e + f*x]^
2]))/(2*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(118) = 236$.

Time = 14.12 (sec) , antiderivative size = 1597, normalized size of antiderivative = 11.92

method	result	size
default	Expression too large to display	1597

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^2*(b^4*(a-b))^(
1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f
*x+e))-1/2/f/(a-b)^2*a/((1-cos(2*f*x+2*e))^2*b*csc(2*f*x+2*e)^2+a)^(1/2)/(
1/(csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b
+b*csc(2*f*x+2*e)^2+a)*a*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2-2/(csc(2*f*x+2*
e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*csc(2*f*x+2*
e)^2+a)*a*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)+1/(csc(2*f*x+2*e)^2*cos(2*f*x+2*
e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*csc(2*f*x+2*e)^2+a)*a*csc(2*f
*x+2*e)^2-1/(csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*
f*x+2*e)*b+b*csc(2*f*x+2*e)^2+a)*b*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2+2/(cs
c(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*cs
c(2*f*x+2*e)^2+a)*b*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)-1/(csc(2*f*x+2*e)^2*co
s(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*csc(2*f*x+2*e)^2+a)
*b*csc(2*f*x+2*e)^2+1)*csc(2*f*x+2*e)+1/2/f/(a-b)^2*a/((1-cos(2*f*x+2*e))^
2*b*csc(2*f*x+2*e)^2+a)^(1/2)/(1/(csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*cs
c(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*csc(2*f*x+2*e)^2+a)*a*csc(2*f*x+2*e)^2*c
os(2*f*x+2*e)^2-2/(csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*
cos(2*f*x+2*e)*b+b*csc(2*f*x+2*e)^2+a)*a*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)+1
/(csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+
b*csc(2*f*x+2*e)^2+a)*a*csc(2*f*x+2*e)^2-1/(csc(2*f*x+2*e)^2*cos(2*f*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(118) = 236$.

Time = 4.14 (sec) , antiderivative size = 908, normalized size of antiderivative = 6.78

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(((a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a + b)*lo
g(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^
4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34
*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b +
 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*
a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos
(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a
^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a
*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*sin(f*x + e)) + 8*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 +
3*(a*b - b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*f*cos(f*x +
 e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f), 1/8*(((a^2 + a*b - 2*b^2)*
cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 - 2*a*b + b^
2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 - 8*a*b
+ 8*b^2)*cos(f*x + e))*sqrt(a - b)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - a^2*b + 3*
a*b^2 - 2*b^3 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(f*x + e)^2)*sin(f*x
+ e))) - 4*((a^2 - 2*a*b + b^2)*cos(f*x + e)^3 + 3*(a*b - b^2)*cos(f*x + e
))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^...
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\int \frac{\sin^2{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\int \frac{\sin{(fx+e)}^2}{(b\tan{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\int \frac{\sin{(fx+e)}^2}{(b\tan{(fx+e)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sin\left(e+f\,x\right)^2}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^2}{\tan\left(fx+e\right)^4 b^2+2\tan\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**2)/(tan(e + f*x)**4*b**2 + 
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.136 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*ta
n(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.61 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.73

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\frac{\cos(e+fx)\sin(e+fx)\left(\frac{4(a-b)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)}{a^2}\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

output
```
(Cos[e + f*x]*Sin[e + f*x]*((4*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a -
b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2 - (15*(2*
b + 3*a*Cot[e + f*x]^2)*(-(a*Sec[e + f*x]^2*Sqrt[((a - b)*(b + a*Cot[e + f
*x]^2)*Sin[e + f*x]^4)/a^2]) + ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a
 + b*Tan[e + f*x]^2)))/(a*(a - b)*Sqrt[((a - b)*(b + a*Cot[e + f*x]^2)*Sin
[e + f*x]^4)/a^2])))/(15*a*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4144, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}-\frac{b\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{b\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(a - b)^(3/
2) - (b*Tan[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^2 b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}}{f}$	102
default	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^2 b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}}{f}$	102

input
```
int(1/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-b/(a-b)*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.61

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2+2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{}}{\tan(fx+e)^2+1}\right.}{2\left(\left(a^3b-2\,a^2b^2+ab^3\right)f\tan\left(fx+\right.\right.}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
+ e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^
3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f),
((a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sq
rt(b*tan(f*x + e)^2 + a)) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x
 + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^
2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2b+a}}{\tan\left(fx+e\right)^4b^2+2\tan\left(fx+e\right)^2ab+a^2}dx$$

input
```
int(1/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)/(tan(e + f*x)**4*b**2 + 2*tan(e + f*x)**2*
a*b + a**2),x)
```

3.137 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=-\frac{\cot(e+fx)}{af\sqrt{a+b\tan^2(e+fx)}}-\frac{2b\tan(e+fx)}{a^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/a^2/f/(a+b*tan(f*x
+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{(a+2b+(a-2b)\cos(2(e+fx)))\csc(e+fx)\sec(e+fx)}{\sqrt{2}a^2f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-(((a + 2*b + (a - 2*b)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/(Sqrt
[2]*a^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4146, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{2b\tan(e+fx)}{a^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cot[e + f*x]/(a*Sqrt[a + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f*x])/(a^2*
Sqrt[a + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{-\frac{1}{a\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}-\frac{2b\tan(fx+e)}{a^2\sqrt{a+b\tan(fx+e)^2}}}{f}$	59
default	$\frac{-\frac{1}{a\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}-\frac{2b\tan(fx+e)}{a^2\sqrt{a+b\tan(fx+e)^2}}}{f}$	59

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)-2*b/a^2*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\left((a-2\,b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{\left(a^2bf+(a^3-a^2b)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-((a - 2*b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((a^2*b*f + (a^3 - a^2*b)*f*cos(f*x + e)^2)*sin(f*x
 + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\frac{2\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^2}+\frac{1}{\sqrt{b\tan(fx+e)^2+a}a\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) + 1/(sqrt(b*tan(f*x +
e)^2 + a)*a*tan(f*x + e)))/f
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 2978, normalized size of antiderivative = 48.03

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

output
```
((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)
*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*2i + f*x*2i)*(((a
+ 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*
b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*
b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i))
- (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))
+ ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i -
 b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f*(a*b^2 - a^2*b)*(a*b - a^
2)*(a*1i - b*1i)) - (3*((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
2*b)^2)/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*1i)
) + (((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b)
- (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*(a - b))/(8*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + ((a - b)^3*(a + 2*b)*(a + 3*b))/(8*f*(a*b^2 -
a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (3*(a - b)^4*(a + 2*b))/(8*f
*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a +
 2*b)^2)*(a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*
(a + 3*b))*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b))/(4*f*(a*b^2 - a^2*b)
*(a + 2*b)*(a*1i - b*1i)) + (3*((a + 2*b)^3 + (((a - b)*(a - 2*b) - (a + 2
*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b))/(a*b - a^2))*...
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^2}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**2)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.138 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{(a-b)b\tan(e+fx)}{a^3 f\sqrt{a+b\tan^2(e+fx)}} - \frac{(3a-5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3 f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2 f}$$

output
```
-(a-b)*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*(3*a-5*b)*cot(f*x+e
)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)
/a^2/f
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(-3a^2-7ab+12b^2-2\left(a^2-6ab+8b^2\right)\cos(2(e+fx))+\left(a^2-5ab+4b^2\right.\right.}{6\sqrt{2}a^3 f\sqrt{(a+b+(a-b)\cos(2(e+fx)))\,\mathrm{s}}}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-3*a^2 - 7*a*b + 12*b^2 - 2*(a^2 - 6*a*b + 8*b^2)*Cos[2*(e + f*x)] + (a^
2 - 5*a*b + 4*b^2)*Cos[4*(e + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*Sqrt[
2]*a^3*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4146, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(\tan^2(e+fx)+1\right)}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a-4b)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)}{3a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(3a-4b)\left(-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}\right)}{3a}-\frac{\cot^3(e+fx)}{3a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{(3a-4b)\left(-\frac{2b\tan(e+fx)}{a^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}\right)}{3a}-\frac{\cot^3(e+fx)}{3a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/(a*Sqrt[a + b*Tan[e + f*x]^2]) + ((3*a - 4*b)*(-(Cot[
e + f*x]/(a*Sqrt[a + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f*x])/(a^2*Sqrt[a
+ b*Tan[e + f*x]^2])))/(3*a))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 11.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.20

method	result
default	$\frac{\left(a\cos(fx+e)^2+b\sin(fx+e)^2\right)\left(8\sin(fx+e)^4b^2+10ab\cos(fx+e)^2\sin(fx+e)^2+2a^2\cos(fx+e)^4-6ab\sin(fx+e)^2-3a^2\cos(fx+e)^2\right.}{3fa^3\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a^3*(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*(8*sin(f*x+e)^4*b^2+10*a*b*cos(f
*x+e)^2*sin(f*x+e)^2+2*a^2*cos(f*x+e)^4-6*a*b*sin(f*x+e)^2-3*a^2*cos(f*x+e
)^2)/(a+b*tan(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.41

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx =$$
$$-\frac{\left(2\left(a^2-5ab+4b^2\right)\cos\left(fx+e\right)^5-\left(3a^2-16ab+16b^2\right)\cos\left(fx+e\right)^3-2\left(3ab-4b^2\right)\cos\left(fx+e\right)\right)\sqrt{}}{3\left(\left(a^4-a^3b\right)f\cos\left(fx+e\right)^4-a^3bf-\left(a^4-2a^3b\right)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2 - 5*a*b + 4*b^2)*cos(f*x + e)^5 - (3*a^2 - 16*a*b + 16*b^2)*c
os(f*x + e)^3 - 2*(3*a*b - 4*b^2)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)/(((a^4 - a^3*b)*f*cos(f*x + e)^4 - a^3*b*f - (a^4
- 2*a^3*b)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^4{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\frac{6\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^2} - \frac{8\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^3} + \frac{3}{\sqrt{b\tan(fx+e)^2+a}a\tan(fx+e)} - \frac{4\,b}{\sqrt{b\tan(fx+e)^2+a}a^2\tan(fx+e)} + \frac{1}{\sqrt{b\tan(fx+e)^2+\cdots}}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) - 8*b^2*tan(f*x +
e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 3/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(
f*x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)) + 1/(sqrt(b*
tan(f*x + e)^2 + a)*a*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 30.16 (sec) , antiderivative size = 269040, normalized size of antiderivative = 2445.82

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

output
```
((a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*2i + f*x*2i) + 1)^2)^(1/2)
*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(((a + 3*b)*(((a + 3*b)*(
((a + 3*b)*(((a + 3*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(
a - b)^2*(a + 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))
*(a - b)*(a + 2*b)^2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*
(a + 2*b)*(a*1i - b*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*
b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/
(1024*a^4*f*(a*b^2 - a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + ((a + 3
*b)*(((a + 3*b)*((((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a + 2*b))
/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a + 2*b)^
2)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b
*1i)) + ((a - b)^7*(a + 2*b)*(a + 3*b))/(3072*a^4*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - ((a - b)^7*(a + 2*b)*(3*a + b))/(1024*a^4*f*(a*b^2
- a^2*b)*(a*b - a^2)*(a*1i - b*1i))))/(a - b) + (((a + 2*b)^3 + (((a - b)*
(a - 2*b) - (a + 2*b)^2)*((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a + 2*b)
)/(a*b - a^2))*(a - b)^5)/(3072*a^4*f*(a*b^2 - a^2*b)*(a + 2*b)*(a*1i - b*
1i)) + ((a - b)^6*(a + 2*b)*(9*a + 4*b))/(768*a^3*f*(a*b^2 - a^2*b)*(a*b -
 a^2)*(a*1i - b*1i)) - (((((a - b)*(a - 2*b) - (a + 2*b)^2)*(a - b)^2*(a +
 2*b))/(a*b - a^2) + (((a - b)*(a + 2*b) - (a - b)*(a + 3*b))*(a - b)*(a +
 2*b)^2)/(a*b - a^2))*(a - b)^4*(3*a + b))/(1024*a^4*f*(a*b^2 - a^2*b)*...
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^4}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**4)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.139 $\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{(a-b)^2b\tan(e+fx)}{a^4f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15a^2-50ab+33b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4f} - \frac{(10a-9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^2f}$$

output
```
-(a-b)^2*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^2-50*a*b+3
3*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^4/f-1/15*(10*a-9*b)*cot(f*x+e
)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/
2)/a^2/f
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(\cot(e+fx)\left(8a^2-41ab+33b^2+a(4a-9b)\csc^2(e+fx)\right.\right.}{15\sqrt{2}a^4f}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/15*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(Cot[e + f*
x]*(8*a^2 - 41*a*b + 33*b^2 + a*(4*a - 9*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e +
 f*x]^4) + (15*(a - b)^2*b*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f
*x)])))/(Sqrt[2]*a^4*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4146, 365, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 4146$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{365}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5a\tan^2(e+fx)+2(5a-3b)\right)}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)}{5a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{\frac{\left(15a^2-8b(5a-3b)\right)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a}-\frac{2(5a-3b)\cot^3(e+fx)}{3a\sqrt{a+b\tan^2(e+fx)}}}{5a}-\frac{\cot^5(e+fx)}{5a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{\frac{\left(15a^2-8b(5a-3b)\right)\left(-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}\right)}{3a}-\frac{2(5a-3b)\cot^3(e+fx)}{3a\sqrt{a+b\tan^2(e+fx)}}}{5a}-\frac{\cot^5(e+fx)}{5a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\frac{\left(15a^2-8b(5a-3b)\right)\left(-\frac{2b\tan(e+fx)}{a^2\sqrt{a+b\tan^2(e+fx)}}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}\right)}{3a}-\frac{2(5a-3b)\cot^3(e+fx)}{3a\sqrt{a+b\tan^2(e+fx)}}}{5a}-\frac{\cot^5(e+fx)}{5a\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*Cot[e + f*x]^5/(a*Sqrt[a + b*Tan[e + f*x]^2]) + ((-2*(5*a - 3*b)*Cot
[e + f*x]^3)/(3*a*Sqrt[a + b*Tan[e + f*x]^2]) + ((15*a^2 - 8*(5*a - 3*b)*b
)*(-(Cot[e + f*x]/(a*Sqrt[a + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f*x])/(a^
2*Sqrt[a + b*Tan[e + f*x]^2])))/(3*a))/(5*a))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 14.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\left(\cos(fx+e)^{4}\left(8\cos(fx+e)^{4}-20\cos(fx+e)^{2}+15\right)a^{4}+\cos(fx+e)^{2}\left(72\cos(fx+e)^{4}-120\cos(fx+e)^{2}+45\right)\sin(fx+e)^{2}a^{3}b+\left(168\right.\right.}{15f\,a^{4}\left(a\right.}$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^4*(cos(f*x+e)^4*(8*cos(f*x+e)^4-20*cos(f*x+e)^2+15)*a^4+cos(f*x+
e)^2*(72*cos(f*x+e)^4-120*cos(f*x+e)^2+45)*sin(f*x+e)^2*a^3*b+(168*cos(f*x
+e)^4-180*cos(f*x+e)^2+30)*sin(f*x+e)^4*a^2*b^2+(152*cos(f*x+e)^2-80)*sin(
f*x+e)^6*a*b^3+48*sin(f*x+e)^8*b^4)/(a+b*tan(f*x+e)^2)^(3/2)*sec(f*x+e)^3*
csc(f*x+e)^5
```

Fricas [A] (verification not implemented)

Time = 20.97 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.44

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\left(8\left(a^{3}-8a^{2}b+13ab^{2}-6b^{3}\right)\cos\left(fx+e\right)^{7}-4\left(5a^{3}-41a^{2}b+72ab^{2}-36b^{3}\right)\cos\left(fx+e\right)^{5}+\left(15a^{3}-\right.\right.}{15\left(\left(a^{5}-a^{4}b\right)f\cos\left(fx+e\right)^{6}+a^{4}bf-\left(2a^{5}-3a^{4}b\right)f\cos\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a^3 - 8*a^2*b + 13*a*b^2 - 6*b^3)*cos(f*x + e)^7 - 4*(5*a^3 - 41
*a^2*b + 72*a*b^2 - 36*b^3)*cos(f*x + e)^5 + (15*a^3 - 130*a^2*b + 264*a*b
^2 - 144*b^3)*cos(f*x + e)^3 + 2*(15*a^2*b - 40*a*b^2 + 24*b^3)*cos(f*x +
e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5 - a^4*b)*f*co
s(f*x + e)^6 + a^4*b*f - (2*a^5 - 3*a^4*b)*f*cos(f*x + e)^4 + (a^5 - 3*a^4
*b)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.58

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{30\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^2} - \frac{80\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^3} + \frac{48\,b^3\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^4} + \frac{15}{\sqrt{b\tan(fx+e)^2+a}a\tan(fx+e)} - \frac{40\,b}{\sqrt{b\tan(fx+e)^2+a}a^2\tan(fx\cdots}}{}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^2) - 80*b^2*tan(f*x
 + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 48*b^3*tan(f*x + e)/(sqrt(b*tan(f
*x + e)^2 + a)*a^4) + 15/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*x + e)) - 40*
b/(sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x +
 e)^2 + a)*a^3*tan(f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a)*a*tan(f*x +
e)^3) - 6*b/(sqrt(b*tan(f*x + e)^2 + a)*a^2*tan(f*x + e)^3) + 3/(sqrt(b*ta
n(f*x + e)^2 + a)*a*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\tan(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^6}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**6)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.140 $\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 235

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{a^2b\sec(e+fx)}{3(a-b)^4f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{ab(5a+6b)\sec(e+fx)}{3(a-b)^5f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(15a^2+50ab+8b^2)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^5f} + \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{15(a-b)^4f} - \frac{\cos^5(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{5(a-b)^3f}$$

output
```
-1/3*a^2*b*sec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/3*a*b*(5*a+6*
b)*sec(f*x+e)/(a-b)^5/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/15*(15*a^2+50*a*b+8*b
^2)*cos(f*x+e)*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^5/f+2/15*(5*a+2*b)*cos(f*x
+e)^3*(a-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^4/f-1/5*cos(f*x+e)^5*(a-b+b*sec(f*x
+e)^2)^(1/2)/(a-b)^3/f
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.25

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx =$$
$$-\frac{\cos(e+fx)\left(425a^4+4700a^3b+6134a^2b^2+4700ab^3+425b^4+48(11a^4+106a^3b-106ab^3-11b^4)\cos(2\right.}{}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-1/480*(Cos[e + f*x]*(425*a^4 + 4700*a^3*b + 6134*a^2*b^2 + 4700*a*b^3 + 4
25*b^4 + 48*(11*a^4 + 106*a^3*b - 106*a*b^3 - 11*b^4)*Cos[2*(e + f*x)] + 1
2*(a - b)^2*(7*a^2 + 50*a*b + 7*b^2)*Cos[4*(e + f*x)] - 16*a^4*Cos[6*(e +
f*x)] + 32*a^3*b*Cos[6*(e + f*x)] - 32*a*b^3*Cos[6*(e + f*x)] + 16*b^4*Cos
[6*(e + f*x)] + 3*a^4*Cos[8*(e + f*x)] - 12*a^3*b*Cos[8*(e + f*x)] + 18*a^
2*b^2*Cos[8*(e + f*x)] - 12*a*b^3*Cos[8*(e + f*x)] + 3*b^4*Cos[8*(e + f*x)
])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*(a -
b)^5*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4147, 365, 25, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(e+fx)^5}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{f}$$

$\downarrow$ 365

$$\frac{\frac{\int -\frac{\cos^4(e+fx)\left(2(5a-b)-5(a-b)\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{f}$$

$\downarrow$ 25

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a-b)-5(a-b)\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{f}$$

$\downarrow$ 359

$$\frac{-\frac{-\frac{\left(5a^2+10ab+b^2\right)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{a-b}-\frac{2(5a-b)\cos^3(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{5(a-b)}-\frac{\cos^5(e+fx)}{5(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{f}$$

$\downarrow$ 245

$$\frac{-\frac{-\frac{\left(5a^2+10ab+b^2\right)\left(-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{2(5a-b)\cos^3(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{5(a-b)}-\frac{\cos^5(e}{5(a-b)(a+b\sec^2}}{f}$$

$\downarrow$ 209

$$\frac{-\frac{-\frac{\left(5a^2+10ab+b^2\right)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3(a-b)}+\frac{\sec(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{2(5a-b)\,c}{3(a-b)\left(a+b\,\mathrm{sec}\right.}}{5(a-b)}}{f}$$

$\downarrow$ 208

$$\frac{-\frac{-\frac{\left(5a^2+10ab+b^2\right)\left(-\frac{4b\left(\frac{2\sec(e+fx)}{3(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\sec(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{2(5a-b)\cos^3(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right.}}{5(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/5*Cos[e + f*x]^5/((a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((-2*(5*
a - b)*Cos[e + f*x]^3)/(3*(a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((5*
a^2 + 10*a*b + b^2)*(-(Cos[e + f*x]/((a - b)*(a - b + b*Sec[e + f*x]^2)^(3
/2))) - (4*b*(Sec[e + f*x]/(3*(a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2)) +
(2*Sec[e + f*x])/(3*(a - b)^2*Sqrt[a - b + b*Sec[e + f*x]^2])))/(a - b)))/
(a - b))/(5*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.42

$$\left(\frac{\cos(fx+e)^6\left(3\cos(fx+e)^4-10\cos(fx+e)^2+15\right)a^5}{15}+\frac{\cos(fx+e)^4\left(-3\cos(fx+e)^6+9\cos(fx+e)^4-5\cos(fx+e)^2+15\right)a^4b}{3}+\frac{2\cos(fx+e)^2\left(3\right.}{}\text{[illegible]}\right.$$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
1/f*(1/15*cos(f*x+e)^6*(3*cos(f*x+e)^4-10*cos(f*x+e)^2+15)*a^5+1/3*cos(f*x
+e)^4*(-3*cos(f*x+e)^6+9*cos(f*x+e)^4-5*cos(f*x+e)^2+15)*a^4*b+2/3*cos(f*x
+e)^2*(3*cos(f*x+e)^8-8*cos(f*x+e)^6-cos(f*x+e)^4+10)*a^3*b^2+2/3*(3*cos(f
*x+e)^8-4*cos(f*x+e)^6-7*cos(f*x+e)^4+14*cos(f*x+e)^2+4)*sin(f*x+e)^2*a^2*
b^3+1/3*(3*cos(f*x+e)^6-4*cos(f*x+e)^2+16)*sin(f*x+e)^4*a*b^4+1/15*(3*cos(
f*x+e)^4+4*cos(f*x+e)^2+8)*sin(f*x+e)^6*b^5)*a^7/((-b*(a-b))^(1/2)+a-b)^7/
((-b*(a-b))^(1/2)-a+b)^7*(a^2-2*a*b+b^2)/(a+b*tan(f*x+e)^2)^(5/2)*sec(f*x+
e)^5
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.57

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\left(3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\cos\left(fx+e\right)^9-2\left(5a^4-16a^3b+18a^2b^2-8ab^3+b^4\right)\cos\left(fx+e\right)^7\right.}{15\left(\left(a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-21a^2b^5+7ab^6-b^7\right)f\cos\left(fx+e\right)\right.}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^9 - 2*(5
*a^4 - 16*a^3*b + 18*a^2*b^2 - 8*a*b^3 + b^4)*cos(f*x + e)^7 + 3*(5*a^4 -
14*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)^5 + 12*(5*a^3*b + 5*a^2*b^2 - 9*a
*b^3 - b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 10*a*b^3 + b^4)*cos(f*x + e))*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^7 - 7*a^6*b + 21*a^5
*b^2 - 35*a^4*b^3 + 35*a^3*b^4 - 21*a^2*b^5 + 7*a*b^6 - b^7)*f*cos(f*x + e
)^4 + 2*(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^
6 + b^7)*f*cos(f*x + e)^2 + (a^5*b^2 - 5*a^4*b^3 + 10*a^3*b^4 - 10*a^2*b^5
 + 5*a*b^6 - b^7)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(215) = 430.

Time = 0.04 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.27

$$\int \frac{\sin^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\frac{15\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{3\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5-20\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}b\cos(fx+e)^3+90\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}b^2\cos(}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}}{}$$

input
```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output

```
-1/15*(15*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a
*b^2 - b^3) + (3*(a - b + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a -
 b + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a - b + b/cos(f*x
+ e)^2)*b^2*cos(f*x + e))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5) - 10*((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a
 - b + b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*
b^3 + b^4) + 5*(12*(a - b + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((
a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(a - b + b/cos(f*
x + e)^2)^(3/2)*cos(f*x + e)^3) + 10*(9*(a - b + b/cos(f*x + e)^2)*b^2*cos
(f*x + e)^2 - b^3)/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b
/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3) + 5*(6*(a - b + b/cos(f*x + e)^2)*b
*cos(f*x + e)^2 - b^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5261 vs. $2(215) = 430$.

Time = 5.91 (sec) , antiderivative size = 5261, normalized size of antiderivative = 22.39

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/15*(5*((((6*a^16*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 73*a^15*b^4*sgn(t
an(1/2*f*x + 1/2*e)^2 - 1) + 403*a^14*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
- 1326*a^13*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2860*a^12*b^7*sgn(tan(1/
2*f*x + 1/2*e)^2 - 1) - 4147*a^11*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 38
61*a^10*b^9*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1716*a^9*b^10*sgn(tan(1/2*f*
x + 1/2*e)^2 - 1) - 858*a^8*b^11*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2145*a^
7*b^12*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1859*a^6*b^13*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1) + 962*a^5*b^14*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 312*a^4*b^15
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 59*a^3*b^16*sgn(tan(1/2*f*x + 1/2*e)^2
- 1) - 5*a^2*b^17*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/
(a^18*b^2 - 18*a^17*b^3 + 153*a^16*b^4 - 816*a^15*b^5 + 3060*a^14*b^6 - 85
68*a^13*b^7 + 18564*a^12*b^8 - 31824*a^11*b^9 + 43758*a^10*b^10 - 48620*a^
9*b^11 + 43758*a^8*b^12 - 31824*a^7*b^13 + 18564*a^6*b^14 - 8568*a^5*b^15
+ 3060*a^4*b^16 - 816*a^3*b^17 + 153*a^2*b^18 - 18*a*b^19 + b^20) - 3*(2*a
^16*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 31*a^15*b^4*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1) + 213*a^14*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 858*a^13*b^6*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2236*a^12*b^7*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) - 3861*a^11*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 4147*a^10*b^9*sgn(
tan(1/2*f*x + 1/2*e)^2 - 1) - 1716*a^9*b^10*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
) - 2574*a^8*b^11*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 5863*a^7*b^12*sgn(t...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx=\int \frac{\sin(e+f\,x)^5}{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^5}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**5)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.141 $\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{ab\sec(e+fx)}{3(a-b)^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\sec(e+fx)}{3(a-b)^4f\sqrt{a-b+b\sec^2(e+fx)}} - \frac{(3a+5b)\cos(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^4f} + \frac{\cos^3(e+fx)\sqrt{a-b+b\sec^2(e+fx)}}{3(a-b)^3f}$$

output

```
-1/3*a*b*sec(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/3*b*(5*a+3*b)*s
ec(f*x+e)/(a-b)^4/f/(a-b+b*sec(f*x+e)^2)^(1/2)-1/3*(3*a+5*b)*cos(f*x+e)*(a
-b+b*sec(f*x+e)^2)^(1/2)/(a-b)^4/f+1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(
1/2)/(a-b)^3/f
```

Mathematica [A] (verified)

Time = 6.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx =$$
$$-\frac{\cos(e+fx)\left(26a^3+186a^2b+190ab^2+110b^3+3(11a^3+63a^2b-31ab^2-43b^3)\cos(2(e+fx))+6(a-\right.}{}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-1/24*(Cos[e + f*x]*(26*a^3 + 186*a^2*b + 190*a*b^2 + 110*b^3 + 3*(11*a^3
+ 63*a^2*b - 31*a*b^2 - 43*b^3)*Cos[2*(e + f*x)] + 6*(a - b)^2*(a + 3*b)*C
os[4*(e + f*x)] - a^3*Cos[6*(e + f*x)] + 3*a^2*b*Cos[6*(e + f*x)] - 3*a*b^
2*Cos[6*(e + f*x)] + b^3*Cos[6*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e
+ f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e +
 f*x)])^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4147, 25, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$
$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a-b)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a-b)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(a+b)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{a-b}+\frac{\cos^3(e+fx)}{3(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \; 245$$

$$\frac{\frac{(a+b)\left(-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}+\frac{\cos^3(e+fx)}{3(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \; 209$$

$$\frac{\frac{(a+b)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3(a-b)}+\frac{\sec(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}+\frac{\cos^3(e+fx)}{3(a-b)(a+b\sec^2(e+fx)}}{f}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{\cos^3(e+fx)}{3(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}+\frac{(a+b)\left(-\frac{4b\left(\frac{2\sec(e+fx)}{3(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\sec(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Cos[e + f*x]^3/(3*(a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2)) + ((a + b)*(-
(Cos[e + f*x]/((a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e +
f*x]/(3*(a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*(a
 - b)^2*Sqrt[a - b + b*Sec[e + f*x]^2])))/(a - b)))/(a - b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 170.36 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.43

method	result
default	$-\frac{a^5(a-b)\left(a^4\left(\cos(fx+e)^3-3\cos(fx+e)\right)+a^3b\left(-4\cos(fx+e)^3+7\cos(fx+e)-15\sec(fx+e)\right)+a^2b^2\left(6\cos(fx+e)^3-3\cos(fx+e\right.\right.}{3f\left(\sqrt{-b\right.}}$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*a^5/((-b*(a-b))^(1/2)+a-b)^5/((-b*(a-b))^(1/2)-a+b)^5*(a-b)/(a+b*ta
n(f*x+e)^2)^(5/2)*(a^4*(cos(f*x+e)^3-3*cos(f*x+e))+a^3*b*(-4*cos(f*x+e)^3+
7*cos(f*x+e)-15*sec(f*x+e))+a^2*b^2*(6*cos(f*x+e)^3-3*cos(f*x+e)+15*sec(f*
x+e)-20*sec(f*x+e)^3)+(4*cos(f*x+e)^6+7*cos(f*x+e)^4-8*cos(f*x+e)^2-8)*a*b
^3*tan(f*x+e)^2*sec(f*x+e)^3+(cos(f*x+e)^4+4*cos(f*x+e)^2-8)*b^4*tan(f*x+e
)^4*sec(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.53

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\left((a^3-3a^2b+3ab^2-b^3)\cos(fx+e)^7-3(a^3-a^2b-ab^2+\right.}{3\left((a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6)f\cos(fx+e)^4+\right.}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^7 - 3*(a^3 - a^2*b - a*b
^2 + b^3)*cos(f*x + e)^5 - 12*(a^2*b - b^3)*cos(f*x + e)^3 - 8*(a*b^2 + b^
3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^6 -
 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*f*cos(f*x
 + e)^4 + 2*(a^5*b - 5*a^4*b^2 + 10*a^3*b^3 - 10*a^2*b^4 + 5*a*b^5 - b^6)*
f*cos(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.75

$$\int \frac{\sin^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\frac{3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3} - \frac{\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-9\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} + \frac{9\left(a-b+\frac{b}{\cos(fx+e)^2}\right)}{\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)\left(a-\right.}}{3f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - ((a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a -
b + b/cos(f*x + e)^2)*b*cos(f*x + e))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4) + (9*(a - b + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 - b^3)/((a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(
f*x + e)^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3
- 3*a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3
))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2800 vs. $2(160) = 320$.

Time = 4.38 (sec) , antiderivative size = 2800, normalized size of antiderivative = 15.91

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*(((((3*a^12*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 29*a^11*b^4*sgn(tan(
1/2*f*x + 1/2*e)^2 - 1) + 125*a^10*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3
15*a^9*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 510*a^8*b^7*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1) - 546*a^7*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 378*a^6*b^9
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 150*a^5*b^10*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) + 15*a^4*b^11*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 15*a^3*b^12*sgn(tan(
1/2*f*x + 1/2*e)^2 - 1) - 7*a^2*b^13*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a*b
^14*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/(a^14*b^2 - 14
*a^13*b^3 + 91*a^12*b^4 - 364*a^11*b^5 + 1001*a^10*b^6 - 2002*a^9*b^7 + 30
03*a^8*b^8 - 3432*a^7*b^9 + 3003*a^6*b^10 - 2002*a^5*b^11 + 1001*a^4*b^12
- 364*a^3*b^13 + 91*a^2*b^14 - 14*a*b^15 + b^16) - 3*(a^12*b^3*sgn(tan(1/2
*f*x + 1/2*e)^2 - 1) - 13*a^11*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 73*a^
10*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 235*a^9*b^6*sgn(tan(1/2*f*x + 1/2
*e)^2 - 1) + 480*a^8*b^7*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 642*a^7*b^8*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1) + 546*a^6*b^9*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
 - 246*a^5*b^10*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 15*a^4*b^11*sgn(tan(1/2*
f*x + 1/2*e)^2 - 1) + 95*a^3*b^12*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 59*a^2
*b^13*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 17*a*b^14*sgn(tan(1/2*f*x + 1/2*e)
^2 - 1) - 2*b^15*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^14*b^2 - 14*a^13*b^3
+ 91*a^12*b^4 - 364*a^11*b^5 + 1001*a^10*b^6 - 2002*a^9*b^7 + 3003*a^8*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^3}{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^3}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2b+a^3}dx$$

input
```
int(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**3)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.142 $\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cos(e+fx)}{(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3(a-b)^2 f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b\sec(e+fx)}{3(a-b)^3 f\sqrt{a-b+b\sec^2(e+fx)}}$$

output
```
-cos(f*x+e)/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/(a-b)^2/f/
(a-b+b*sec(f*x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/(a-b)^3/f/(a-b+b*sec(f*x+e)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 3.90 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cos(e+fx)\left((3a+5b)^2+12(a^2+2ab-3b^2)\cos(2(e+fx))+3(a-b)^2\cos(4(e+fx))\right)\sqrt{(a+b+(a-}}{6\sqrt{2}(a-b)^3 f(a+b+(a-b)\cos(2(e+fx)))^2}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-1/6*(Cos[e + f*x]*((3*a + 5*b)^2 + 12*(a^2 + 2*a*b - 3*b^2)*Cos[2*(e + f*
x)] + 3*(a - b)^2*Cos[4*(e + f*x)])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)]
)*Sec[e + f*x]^2])/(Sqrt[2]*(a - b)^3*f*(a + b + (a - b)*Cos[2*(e + f*x)])
^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4147, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3(a-b)}+\frac{\sec(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{\cos(e+fx)}{(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{4b\left(\frac{2\sec(e+fx)}{3(a-b)^2\sqrt{a+b\sec^2(e+fx)-b}}+\frac{\sec(e+fx)}{3(a-b)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}\right)}{a-b}-\frac{\cos(e+fx)}{(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-(Cos[e + f*x]/((a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e
+ f*x]/(3*(a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*
(a - b)^2*Sqrt[a - b + b*Sec[e + f*x]^2])))/(a - b))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 68.84 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

method	result
default	$\frac{\left(\cos(fx+e)a^3+\left(\cos(fx+e)^4-4\cos(fx+e)^2+\frac{8}{3}\right)\tan(fx+e)^2\sec(fx+e)^3b^3+(-3\cos(fx+e)+5\sec(fx+e))b\,a^2+\left(\frac{20\sec(fx+e)^3}{3}-\right.\right.}{f\left(\sqrt{-b(a-b)}+a-b\right)^3\left(\sqrt{-b(a-b)}-a+b\right)^3\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(cos(f*x+e)*a^3+(cos(f*x+e)^4-4*cos(f*x+e)^2+8/3)*tan(f*x+e)^2*sec(f*x
+e)^3*b^3+(-3*cos(f*x+e)+5*sec(f*x+e))*b*a^2+(20/3*sec(f*x+e)^3-10*sec(f*x
+e)+3*cos(f*x+e))*b^2*a)*a^3/((-b*(a-b))^(1/2)+a-b)^3/((-b*(a-b))^(1/2)-a+
b)^3/(a+b*tan(f*x+e)^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.71

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{(3\,(a^2-2\,ab+b^2)\cos{(fx+e)}^5+12\,(ab-b^2)\cos{(fx+e)}^3+8\,b^2\cos{(fx+e)}}{3\,((a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)f\cos{(fx+e)}^4+2\,(a^4b-4\,a^3b^2+6\,a^2b^3-4\,ab^4+b^5)f\,c}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*(a^2 - 2*a*b + b^2)*cos(f*x + e)^5 + 12*(a*b - b^2)*cos(f*x + e)^3
 + 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*f*cos(f*x + e)^4
 + 2*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\frac{3\sqrt{a-b+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3-3a^2b+3ab^2-b^3}+\frac{6\left(a-b+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{(a^3-3a^2b+3ab^2-b^3)\left(a-b+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}}{3f}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a - b + b/cos(f*x + e)^2)*cos(f*x + e)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + (6*(a - b + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a^3 -
3*a^2*b + 3*a*b^2 - b^3)*(a - b + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3))
/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1471 vs. $2(108) = 216$.

Time = 3.74 (sec) , antiderivative size = 1471, normalized size of antiderivative = 12.47

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*(((((6*a^8*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 43*a^7*b^4*sgn(tan(1/
2*f*x + 1/2*e)^2 - 1) + 133*a^6*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 231*
a^5*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 245*a^4*b^7*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1) - 161*a^3*b^8*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 63*a^2*b^9*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1) - 13*a*b^10*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) +
 b^11*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/(a^10*b^2 -
10*a^9*b^3 + 45*a^8*b^4 - 120*a^7*b^5 + 210*a^6*b^6 - 252*a^5*b^7 + 210*a^
4*b^8 - 120*a^3*b^9 + 45*a^2*b^10 - 10*a*b^11 + b^12) - 3*(2*a^8*b^3*sgn(t
an(1/2*f*x + 1/2*e)^2 - 1) - 21*a^7*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) +
91*a^6*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 217*a^5*b^6*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1) + 315*a^4*b^7*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 287*a^3*b^8
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 161*a^2*b^9*sgn(tan(1/2*f*x + 1/2*e)^2
- 1) - 51*a*b^10*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 7*b^11*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1))/(a^10*b^2 - 10*a^9*b^3 + 45*a^8*b^4 - 120*a^7*b^5 + 210*a
^6*b^6 - 252*a^5*b^7 + 210*a^4*b^8 - 120*a^3*b^9 + 45*a^2*b^10 - 10*a*b^11
 + b^12))*tan(1/2*f*x + 1/2*e)^2 - 3*(2*a^8*b^3*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) - 21*a^7*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 91*a^6*b^5*sgn(tan(1/
2*f*x + 1/2*e)^2 - 1) - 217*a^5*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 315*
a^4*b^7*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 287*a^3*b^8*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1) + 161*a^2*b^9*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 51*a*b^10*s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)}{\tan\left(fx+e\right)^6 b^3+3\tan\left(fx+e\right)^4 a\,b^2+3\tan\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x))/(tan(e + f*x)**6*b**3 + 3*t
an(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.143 $\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\sec(e+fx)}{3a(a-b)f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\sec(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\sec^2(e+fx)}}$$

output
```
-arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*se
c(f*x+e)/a/(a-b)/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/3*(5*a-3*b)*b*sec(f*x+e)/a
^2/(a-b)^2/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(136) = 272.

Time = 6.63 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.21

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\cos(e+fx)\left(-\frac{2\sqrt{2}\sqrt{ab}\left(6a^2+ab-3b^2+3\left(2a^2-3ab+b^2\right)\cos(2(e+fx))\right)}{(a-b)^2(a+b+(a-b)\cos(2(e+fx)))^2}+\frac{3\left(2\operatorname{arctanh}\left(\operatorname{ta}\right.\right.}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Cos[e + f*x]*((-2*Sqrt[2]*Sqrt[a]*b*(6*a^2 + a*b - 3*b^2 + 3*(2*a^2 - 3*a
*b + b^2)*Cos[2*(e + f*x)]))/((a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])
^2) + (3*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(
-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^2
+ Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]])*S
ec[(e + f*x)/2]^2)/Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2
]^4])*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(6*a^(5/2)*
f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4147, 25, 316, 25, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a-b)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int -\frac{-2b\sec^2(e+fx)+3a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a(a-b)}-\frac{b\sec(e+fx)}{3a(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{-2b\sec^2(e+fx)+3a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a(a-b)}-\frac{b\sec(e+fx)}{3a(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{-\frac{\frac{b(5a-3b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\int -\frac{3(a-b)^2}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a(a-b)}}{3a(a-b)}-\frac{b\sec(e+fx)}{3a(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{3(a-b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a}+\frac{b(5a-3b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{3a(a-b)}-\frac{b\sec(e+fx)}{3a(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{-\frac{\frac{3(a-b)\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{a}+\frac{b(5a-3b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{3a(a-b)}-\frac{b\sec(e+fx)}{3a(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\frac{3(a-b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}}+\frac{b(5a-3b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{3a(a-b)}-\frac{b\sec(e+fx)}{3a(a-b)(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Sec[e + f*x])/(a*(a - b)*(a - b + b*Sec[e + f*x]^2)^(3/2)) - ((3*
(a - b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/a^
(3/2) + ((5*a - 3*b)*b*Sec[e + f*x])/(a*(a - b)*Sqrt[a - b + b*Sec[e + f*x
]^2]))/(3*a*(a - b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3061 vs. $2(122) = 244$.

Time $= 171.67$ (sec) , antiderivative size $= 3062$, normalized size of antiderivative $=$ 22.51

method	result	size
default	Expression too large to display	3062

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/a^(13/2)/(2*(-b*(a-b))^(1/2)-a+2*b)^2/(a^2-2*a*b+b^2)/(2*(-b*(a-b))
^(1/2)+a-2*b)^2*(12*cos(f*x+e)^8*a^(27/2)*b+cos(f*x+e)^6*(-54*cos(f*x+e)^2
+46)*a^(25/2)*b^2+cos(f*x+e)^4*(-96*cos(f*x+e)^2+66)*sin(f*x+e)^2*a^(23/2)
*b^3+cos(f*x+e)^2*(-84*cos(f*x+e)^2+42)*sin(f*x+e)^4*a^(21/2)*b^4+(-36*cos
(f*x+e)^2+10)*sin(f*x+e)^6*a^(19/2)*b^5-6*sin(f*x+e)^8*a^(17/2)*b^6+cos(f*
x+e)^8*(3*cos(f*x+e)+3)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+
1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^
(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^14+cos(f*x+e)
^6*(-18*cos(f*x+e)^3-18*cos(f*x+e)^2+12*cos(f*x+e)+12)*((a*cos(f*x+e)^2+b*
sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)
^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/
(cos(f*x+e)+1))*a^13*b+cos(f*x+e)^4*(45*cos(f*x+e)^5+45*cos(f*x+e)^4-60*co
s(f*x+e)^3-60*cos(f*x+e)^2+18*cos(f*x+e)+18)*((a*cos(f*x+e)^2+b*sin(f*x+e)
^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f
*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^
2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e
)+1))*a^12*b^2+cos(f*x+e)^2*(60*cos(f*x+e)^5+60*cos(f*x+e)^4-60*cos(f*x+e)
^3-60*cos(f*x+e)^2+12*cos(f*x+e)+12)*sin(f*x+e)^2*((a*cos(f*x+e)^2+b*si...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(122) = 244$.

Time = 0.30 (sec) , antiderivative size = 711, normalized size of antiderivative = 5.23

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^4 + a^2*
b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(f*x + e)^2
)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*sqrt(((a - b)*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 - 1)) -
 2*(3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 - 3*a*b^3)
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 -
4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3
*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 +
a^3*b^4)*f), -1/3*(3*((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x
+ e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 2*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*c
os(f*x + e)^2)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x + e)^2 + b)) + (3*(2*a^3*b
- 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 - 3*a*b^3)*cos(f*x + e))*
sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 4*a^6*b + 6*a^5
*b^2 - 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 - a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\csc\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b*tan(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a\,b^2+3\tan(fx+e)^2 a^2b+a^3}dx$$

input
```
int(csc(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x))/(tan(e + f*x)**6*b**3 + 3*t
an(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.144 $\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\csc^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a-5b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-15b)b\sec(e+fx)}{6a^3(a-b)f\sqrt{a-b+b\sec^2(e+fx)}}$$

output
```
-1/2*(a-5*b)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)^(1/2))/a^(7/2
)/f-1/2*cot(f*x+e)*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(3/2)-5/6*b*sec(f*x
+e)/a^2/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-15*b)*b*sec(f*x+e)/a^3/(a-b
)/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 380 vs. $2(177) = 354$.

Time = 3.53 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.15

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\frac{\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{1+\cos(2(e+fx))}}\left(8ab^2\cos(e+fx)-24(a-b)b\cos(e+fx)(a+b+(a-b)\cos(2(e+fx)))-3(a-b)(\cdots\right.}{3a^3(a-b)(a+b+(a-b)\cos(2(e+fx)))^2}\cdots}{\cdots}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
((Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(8*a*b^2
*Cos[e + f*x] - 24*(a - b)*b*Cos[e + f*x]*(a + b + (a - b)*Cos[2*(e + f*x)
]) - 3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f
*x]))/(3*a^3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2) + ((a - 5*b)*Co
s[e + f*x]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/2]^2 + a
*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e + f*x)/2]^
2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]])
*Sec[(e + f*x)/2]^2*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2
])/(2*a^(7/2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[(e + f*x)/2]^4])
)/(2*f)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4147, 373, 402, 25, 27, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^3 \left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2 (b\sec^2(e+fx)+a-b)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 373$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\int \frac{-4b\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{2a}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\frac{5b\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\int -\frac{(a-b)\left(-10b\sec^2(e+fx)+3a-5b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a(a-b)}}{2a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\frac{\int \frac{(a-b)\left(-10b\sec^2(e+fx)+3a-5b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a(a-b)}+\frac{5b\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{2a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\frac{\int \frac{-10b\sec^2(e+fx)+3a-5b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{5b\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{2a}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\frac{\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\int -\frac{3(a-5b)(a-b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a(a-b)}}{3a}+\frac{5b\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right.}}{2a}}{f}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\frac{\frac{3(a-5b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a}+\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{3a}+\frac{5b\sec(}{3a\left(a+b\sec^2(\right.}}{2a}}{f}$$

$$\Big\downarrow \ \textbf{291}$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\frac{\frac{3(a-5b)\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{a}+\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{3a}+\frac{5b\sec(e+f}{3a\left(a+b\sec^2(e+f\right.}}{2a}}{f}$$

$$\Big\downarrow \ \textbf{219}$$

$$\frac{\frac{\sec(e+fx)}{2a(1-\sec^2(e+fx))(a+b\sec^2(e+fx)-b)^{3/2}}-\frac{\frac{\frac{3(a-5b)\text{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}}+\frac{b(13a-15b)\sec(e+fx)}{a(a-b)\sqrt{a+b\sec^2(e+fx)-b}}}{3a}+\frac{5b\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{2a}}{f}$$

input

```
Int[Csc[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output

```
(Sec[e + f*x]/(2*a*(1 - Sec[e + f*x]^2)*(a - b + b*Sec[e + f*x]^2)^(3/2))
- ((5*b*Sec[e + f*x])/(3*a*(a - b + b*Sec[e + f*x]^2)^(3/2)) + ((3*(a - 5*
b)*ArcTanh[(Sqrt[a]*Sec[e + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/a^(3/2)
 + ((13*a - 15*b)*b*Sec[e + f*x])/(a*(a - b)*Sqrt[a - b + b*Sec[e + f*x]^2
]))/(3*a))/(2*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3761 vs. $2(157) = 314$.

Time = 174.00 (sec) , antiderivative size = 3762, normalized size of antiderivative = 21.25

method	result	size
default	Expression too large to display	3762

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f/a^(17/2)/(a^2-2*a*b+b^2)/(2*(-b*(a-b))^(1/2)+a-2*b)^3/(2*(-b*(a-b)
)^(1/2)-a+2*b)^3*(6*a^(37/2)*cos(f*x+e)^10+cos(f*x+e)^8*(-66*cos(f*x+e)^2+
54)*a^(35/2)*b+cos(f*x+e)^6*(270*cos(f*x+e)^4-416*cos(f*x+e)^2+152)*a^(33/
2)*b^2+cos(f*x+e)^4*(570*cos(f*x+e)^4-704*cos(f*x+e)^2+192)*sin(f*x+e)^2*a
^(31/2)*b^3+cos(f*x+e)^2*(690*cos(f*x+e)^4-636*cos(f*x+e)^2+114)*sin(f*x+e
)^4*a^(29/2)*b^4+(486*cos(f*x+e)^4-296*cos(f*x+e)^2+26)*sin(f*x+e)^6*a^(27
/2)*b^5+(186*cos(f*x+e)^2-56)*sin(f*x+e)^8*a^(25/2)*b^6+30*sin(f*x+e)^10*a
^(23/2)*b^7+cos(f*x+e)^8*(3*cos(f*x+e)+3)*sin(f*x+e)^2*((a*cos(f*x+e)^2+b*
sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)
^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/
(cos(f*x+e)+1))*a^18+cos(f*x+e)^6*(-33*cos(f*x+e)^3-33*cos(f*x+e)^2+12*cos
(f*x+e)+12)*sin(f*x+e)^2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)
+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^17*b+cos(f*x
+e)^4*(135*cos(f*x+e)^5+135*cos(f*x+e)^4-120*cos(f*x+e)^3-120*cos(f*x+e)^2
+18*cos(f*x+e)+18)*sin(f*x+e)^2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+
e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos
(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(157) = 314$.

Time = 0.30 (sec) , antiderivative size = 906, normalized size of antiderivative = 5.12

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 16*a*b^3 + 5*b^4)*cos(f*x + e)^6
- (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3 + 15*b^4)*cos(f*x + e)^4 - a^2*b
^2 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b^2 + 28*a*b^3 - 15*b^4)*cos(f*x
+ e)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 + 2*sqrt(a)*sqrt(((a - b)*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(cos(f*x + e)^2 -
 1)) - 2*(3*(a^4 - 7*a^3*b + 11*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^5 + 2*(9*a
^3*b - 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e)^3 + (13*a^2*b^2 - 15*a*b^3)*cos
(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 - 3*a^
6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b^2 -
 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*cos(f*x
 + e)^2 - (a^5*b^2 - a^4*b^3)*f), -1/6*(3*((a^4 - 8*a^3*b + 18*a^2*b^2 - 1
6*a*b^3 + 5*b^4)*cos(f*x + e)^6 - (a^4 - 10*a^3*b + 32*a^2*b^2 - 38*a*b^3
+ 15*b^4)*cos(f*x + e)^4 - a^2*b^2 + 6*a*b^3 - 5*b^4 - (2*a^3*b - 15*a^2*b
^2 + 28*a*b^3 - 15*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(-sqrt(-a)*sqrt(((a
 - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a - b)*cos(f*x +
e)^2 + b)) - (3*(a^4 - 7*a^3*b + 11*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^5 + 2*
(9*a^3*b - 23*a^2*b^2 + 15*a*b^3)*cos(f*x + e)^3 + (13*a^2*b^2 - 15*a*b^3)
*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 -
3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f*cos(f*x + e)^6 - (a^7 - 5*a^6*b + 7*a^5*b
^2 - 3*a^4*b^3)*f*cos(f*x + e)^4 - (2*a^6*b - 5*a^5*b^2 + 3*a^4*b^3)*f*...
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(157) = 314$.

Time = 2.16 (sec) , antiderivative size = 1111, normalized size of antiderivative = 6.28

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/24*(((((3*(a^12*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^11*b^3*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1) + a^10*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1
/2*f*x + 1/2*e)^2/(a^13*b^2 - 2*a^12*b^3 + a^11*b^4) - 4*(3*a^12*b^2*sgn(t
an(1/2*f*x + 1/2*e)^2 - 1) - 24*a^11*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) +
 41*a^10*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 20*a^9*b^5*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1))/(a^13*b^2 - 2*a^12*b^3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^
2 + 6*(3*a^12*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 22*a^11*b^3*sgn(tan(1/
2*f*x + 1/2*e)^2 - 1) + 71*a^10*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 92*a
^9*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 40*a^8*b^6*sgn(tan(1/2*f*x + 1/2*
e)^2 - 1))/(a^13*b^2 - 2*a^12*b^3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^2 - 12
*(a^12*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 17*a^10*b^4*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1) + 32*a^9*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 16*a^8*b^6*s
gn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^13*b^2 - 2*a^12*b^3 + a^11*b^4))*tan(1/
2*f*x + 1/2*e)^2 + (3*a^12*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 42*a^11*b
^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 101*a^10*b^4*sgn(tan(1/2*f*x + 1/2*e)
^2 - 1) + 56*a^9*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^13*b^2 - 2*a^12*b
^3 + a^11*b^4))/(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 12*(a - 5*b)*arctan(-(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a^3*sgn(tan...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\,\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b + a}\,\csc(fx+e)^3}{\tan(fx+e)^6 b^3 + 3\tan(fx+e)^4 a b^2 + 3\tan(fx+e)^2 a^2 b + a^3}dx$$

input
```
int(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**3)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.145 $\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx &= -\frac{(3a^2-30ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a-b+b\sec^2(e+fx)}}\right)}{8a^{9/2}f} \\ &\quad -\frac{(5a-7b)\cot(e+fx)\csc(e+fx)}{8a^2f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4af\left(a-b+b\sec^2(e+fx)\right)^{3/2}} \\ &\quad -\frac{(23a-35b)b\sec(e+fx)}{24a^3f\left(a-b+b\sec^2(e+fx)\right)^{3/2}}-\frac{5(11a-21b)b\sec(e+fx)}{24a^4f\sqrt{a-b+b\sec^2(e+fx)}}\end{aligned}$$

output
```
-1/8*(3*a^2-30*a*b+35*b^2)*arctanh(a^(1/2)*sec(f*x+e)/(a-b+b*sec(f*x+e)^2)
^(1/2))/a^(9/2)/f-1/8*(5*a-7*b)*cot(f*x+e)*csc(f*x+e)/a^2/f/(a-b+b*sec(f*x
+e)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/a/f/(a-b+b*sec(f*x+e)^2)^(3/2)-1/
24*(23*a-35*b)*b*sec(f*x+e)/a^3/f/(a-b+b*sec(f*x+e)^2)^(3/2)-5/24*(11*a-21
*b)*b*sec(f*x+e)/a^4/f/(a-b+b*sec(f*x+e)^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1132 vs. $2(237) = 474$.

Time = 7.42 (sec) , antiderivative size = 1132, normalized size of antiderivative = 4.78

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*
x)])]*((4*b^2*Cos[e + f*x])/(3*a^3*(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(
e + f*x)])^2) - (2*(2*a*b*Cos[e + f*x] - 3*b^2*Cos[e + f*x]))/(a^4*(a + b
+ a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])) + ((-3*a*Cos[e + f*x] + 11*b*C
os[e + f*x])*Csc[e + f*x]^2)/(8*a^4) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4*a^
3)))/f + ((3*a^2 - 30*a*b + 35*b^2)*(((1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(
e + f*x)])/(1 + Cos[e + f*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(
1 + Cos[2*(e + f*x)])]*(4*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2
]^2)) + Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*S
qrt[b])] - Sqrt[b]*(2*ArcTanh[Tan[(e + f*x)/2]^2 - Sqrt[4*b*Tan[(e + f*x)/
2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2]/Sqrt[a]] + Log[a - 2*b - a*Tan[(e +
f*x)/2]^2 + Sqrt[a]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]
^2)^2]]))*(-1 + Tan[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(4*b*Tan
[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2)/(1 + Tan[(e + f*x)/2]^2)^
2])/(4*Sqrt[a]*Sqrt[b]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]*Sqrt[(-1 + T
an[(e + f*x)/2]^2)^2]*Sqrt[4*b*Tan[(e + f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/
2]^2)^2]) - ((1 + Cos[e + f*x])*Sqrt[(1 + Cos[2*(e + f*x)])/(1 + Cos[e + f
*x])^2]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(4
*Sqrt[a]*ArcTanh[(-(Sqrt[a]*(-1 + Tan[(e + f*x)/2]^2)) + Sqrt[4*b*Tan[(e +
 f*x)/2]^2 + a*(-1 + Tan[(e + f*x)/2]^2)^2])/(2*Sqrt[b])] + Sqrt[b]*(2*...
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 4147, 25, 372, 402, 25, 402, 25, 27, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a-b)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a-b)^{5/2}}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{2(2a-3b)\sec^2(e+fx)+a-b}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int -\frac{(3a-7b)(a-b)-4(5a-7b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{2a}+\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\int \frac{(3a-7b)(a-b)-4(5a-7b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{5/2}}d\sec(e+fx)}{2a}}{4a}-\frac{\sec(e+fx)}{4a(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx)-b)^{3/2}}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\frac{b(23a-35b)\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\int -\frac{(a-b)\left((9a-35b)(a-b)-2(23a-35b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a(a-b)}}{2a}}{4a}-\frac{\cdots}{4a(1-\sec^2\cdots}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\frac{\int \frac{(a-b)\left((9a-35b)(a-b)-2(23a-35b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a(a-b)}+\frac{b(23a-35b)\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{2a}}{4a}-\frac{\cdots}{4a(1-\sec^2(\cdots}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\frac{\int \frac{(9a-35b)(a-b)-2(23a-35b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{b(23a-35b)\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{2a}}{4a}-\frac{\sec\cdots}{4a(1-\sec^2(e+fx))\cdots}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\frac{\frac{5b(11a-21b)\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}-\frac{\int -\frac{3(a-b)\left(3a^2-30ba+35b^2\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a(a-b)}}{3a}+\frac{b(23a-35b)\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{2a}}{4a}-\cdots}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-30ab+35b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a-b}}d\sec(e+fx)}{a}+\frac{5b(11a-21b)\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}}{3a}+\frac{b(23a-35}{3a\left(a+b\sec^2\right.}}{2a}}{4a}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-30ab+35b^2\right)\int \frac{1}{1-\frac{a\sec^2(e+fx)}{b\sec^2(e+fx)+a-b}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a-b}}}{a}+\frac{5b(11a-21b)\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}}{3a}+\frac{b(23a-35b)\,\mathrm{se}}{3a\left(a+b\sec^2(e+\right.}}{2a}}{4a}}{f}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{(5a-7b)\sec(e+fx)}{2a\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)-b\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-30ab+35b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)-b}}\right)}{a^{3/2}}+\frac{5b(11a-21b)\sec(e+fx)}{a\sqrt{a+b\sec^2(e+fx)-b}}}{3a}+\frac{b(23a-35b)\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)-b\right)^{3/2}}}{2a}}{4a}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/4*Sec[e + f*x]/(a*(1 - Sec[e + f*x]^2)^2*(a - b + b*Sec[e + f*x]^2)^(3
/2)) + (((5*a - 7*b)*Sec[e + f*x])/(2*a*(1 - Sec[e + f*x]^2)*(a - b + b*Se
c[e + f*x]^2)^(3/2)) - (((23*a - 35*b)*b*Sec[e + f*x])/(3*a*(a - b + b*Sec
[e + f*x]^2)^(3/2)) + ((3*(3*a^2 - 30*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sec[e
 + f*x])/Sqrt[a - b + b*Sec[e + f*x]^2]])/a^(3/2) + (5*(11*a - 21*b)*b*Sec
[e + f*x])/(a*Sqrt[a - b + b*Sec[e + f*x]^2]))/(3*a))/(2*a))/(4*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4401 vs. 2(213) = 426.

Time = 162.72 (sec) , antiderivative size = 4402, normalized size of antiderivative = 18.57

method	result	size
default	Expression too large to display	4402

input
```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/f/a^(21/2)/(a^2-2*a*b+b^2)/(2*(-b*(a-b))^(1/2)-a+2*b)^4/(2*(-b*(a-b)
)^(1/2)+a-2*b)^4*((3510*cos(f*x+e)^9+3510*cos(f*x+e)^8-6060*cos(f*x+e)^7-6
060*cos(f*x+e)^6+3114*cos(f*x+e)^5+3114*cos(f*x+e)^4-468*cos(f*x+e)^3-468*
cos(f*x+e)^2+9*cos(f*x+e)+9)*sin(f*x+e)^4*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)
/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+
e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1
))*a^18*b^4+cos(f*x+e)^6*(-1560*cos(f*x+e)^6+3770*cos(f*x+e)^4-2866*cos(f*
x+e)^2+668)*a^(41/2)*b^2+(-7008*cos(f*x+e)^6+7946*cos(f*x+e)^4-2212*cos(f*
x+e)^2+110)*sin(f*x+e)^6*a^(35/2)*b^5+(-4248*cos(f*x+e)^4+3166*cos(f*x+e)^
2-430)*sin(f*x+e)^8*a^(33/2)*b^6+(-1440*cos(f*x+e)^2+530)*sin(f*x+e)^10*a^
(31/2)*b^7+cos(f*x+e)^8*(288*cos(f*x+e)^4-558*cos(f*x+e)^2+246)*a^(43/2)*b
+(3504*cos(f*x+e)^7+3504*cos(f*x+e)^6-4476*cos(f*x+e)^5-4476*cos(f*x+e)^4+
1500*cos(f*x+e)^3+1500*cos(f*x+e)^2-108*cos(f*x+e)-108)*sin(f*x+e)^6*((a*c
os(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(1/2)*
((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*co
s(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+co
s(f*x+e)*b+b)/(cos(f*x+e)+1))*a^17*b^5+(2124*cos(f*x+e)^5+2124*cos(f*x+e)^
4-1788*cos(f*x+e)^3-1788*cos(f*x+e)^2+294*cos(f*x+e)+294)*sin(f*x+e)^8*((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*ln(2/a^(1/2)*(a^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(213) = 426$.

Time = 0.36 (sec) , antiderivative size = 1054, normalized size of antiderivative = 4.45

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((3*a^4 - 36*a^3*b + 98*a^2*b^2 - 100*a*b^3 + 35*b^4)*cos(f*x + e
)^8 - 2*(3*a^4 - 39*a^3*b + 131*a^2*b^2 - 165*a*b^3 + 70*b^4)*cos(f*x + e)
^6 + (3*a^4 - 48*a^3*b + 233*a^2*b^2 - 390*a*b^3 + 210*b^4)*cos(f*x + e)^4
 + 3*a^2*b^2 - 30*a*b^3 + 35*b^4 + 2*(3*a^3*b - 36*a^2*b^2 + 95*a*b^3 - 70
*b^4)*cos(f*x + e)^2)*sqrt(a)*log(-2*((a - b)*cos(f*x + e)^2 - 2*sqrt(a)*s
qrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + b)/(co
s(f*x + e)^2 - 1)) + 2*(3*(3*a^4 - 33*a^3*b + 65*a^2*b^2 - 35*a*b^3)*cos(f
*x + e)^7 - (15*a^4 - 177*a^3*b + 445*a^2*b^2 - 315*a*b^3)*cos(f*x + e)^5
- (78*a^3*b - 305*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^3 - 5*(11*a^2*b^2 - 21
*a*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(
(a^7 - 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^8 + a^5*b^2*f - 2*(a^7 - 3*a^6*b
+ 2*a^5*b^2)*f*cos(f*x + e)^6 + (a^7 - 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)
^4 + 2*(a^6*b - 2*a^5*b^2)*f*cos(f*x + e)^2), -1/24*(3*((3*a^4 - 36*a^3*b
+ 98*a^2*b^2 - 100*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(3*a^4 - 39*a^3*b +
131*a^2*b^2 - 165*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (3*a^4 - 48*a^3*b + 233
*a^2*b^2 - 390*a*b^3 + 210*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 30*a*b^3 + 35
*b^4 + 2*(3*a^3*b - 36*a^2*b^2 + 95*a*b^3 - 70*b^4)*cos(f*x + e)^2)*sqrt(-
a)*arctan(-sqrt(-a)*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/((a - b)*cos(f*x + e)^2 + b)) - (3*(3*a^4 - 33*a^3*b + 65*a^2*b^2
 - 35*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 177*a^3*b + 445*a^2*b^2 - 315*a...
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1648 vs. $2(213) = 426$.

Time = 2.49 (sec) , antiderivative size = 1648, normalized size of antiderivative = 6.95

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
1/192*(((((3*((a^17*b^2 - 2*a^16*b^3 + a^15*b^4)*tan(1/2*f*x + 1/2*e)^2/(a
^18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2
*e)^2 - 1) + a^16*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) + (5*a^17*b^2 - 24*
a^16*b^3 + 33*a^15*b^4 - 14*a^14*b^5)/(a^18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4*sgn(tan(1/2*
f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 - 2*(45*a^17*b^2 - 498*a^16*b
^3 + 1421*a^15*b^4 - 1528*a^14*b^5 + 560*a^13*b^6)/(a^18*b^2*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4
*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 + 6*(25*a^17*b^2
 - 248*a^16*b^3 + 989*a^15*b^4 - 1894*a^14*b^5 + 1688*a^13*b^6 - 560*a^12*
b^7)/(a^18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*
x + 1/2*e)^2 - 1) + a^16*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))*tan(1/2*f*x
 + 1/2*e)^2 - 3*(35*a^17*b^2 - 102*a^16*b^3 - 365*a^15*b^4 + 1664*a^14*b^5
 - 2000*a^13*b^6 + 768*a^12*b^7)/(a^18*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
 - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1)))*tan(1/2*f*x + 1/2*e)^2 + (27*a^17*b^2 + 264*a^16*b^3 - 12
49*a^15*b^4 + 1598*a^14*b^5 - 640*a^13*b^6)/(a^18*b^2*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1) - 2*a^17*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^16*b^4*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1)))/(a*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 24*(3*a^2 - 30*a*b +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\text{Hanged}$$

input
```
int(1/(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^5}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2b+a^3}dx$$

input

```
int(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output

```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**5)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.146 $\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{(3a^2+24ab+8b^2)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8(a-b)^{9/2}f} - \frac{(5a+2b)\cos(e+fx)\sin(e+fx)}{8(a-b)^2f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{b(23a+12b)\tan(e+fx)}{24(a-b)^3f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{5b(11a+10b)\tan(e+fx)}{24(a-b)^4f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
1/8*(3*a^2+24*a*b+8*b^2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^
(1/2))/(a-b)^(9/2)/f-1/8*(5*a+2*b)*cos(f*x+e)*sin(f*x+e)/(a-b)^2/f/(a+b*ta
n(f*x+e)^2)^(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(
3/2)-1/24*b*(23*a+12*b)*tan(f*x+e)/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(3/2)-5/24
*b*(11*a+10*b)*tan(f*x+e)/(a-b)^4/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 4.13 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.54

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-3\sqrt{2}ab(3a^2+24ab+8b^2)\left(\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e\text{-}}{b}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-1/96*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-3*Sqrt[2]
*a*b*(3*a^2 + 24*a*b + 8*b^2)*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b)^(3/2)*(2*(a - b)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2
*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] - 2*a*EllipticPi[-(b/(a - b))
, ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[
2]], 1])*Sin[e + f*x]^2*Sin[2*(e + f*x)] - a*(a - b)*(64*a*b^2*Sin[2*(e +
f*x)] - 64*b*(3*a + 2*b)*(a + b + (a - b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x
)] - 6*(4*a + 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[2*(e + f*x)] +
 3*(a - b)*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Sin[4*(e + f*x)])))/(Sqrt[
2]*a*(a - b)^5*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4146, 372, 402, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^4}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\int \frac{a-2(2a+b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{4(a-b)}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{(5a+2b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\int \frac{a(3a+4b)-4b(5a+2b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{(5a+2b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\frac{\int \frac{a\left(a(9a+26b)-2b(23a+12b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a(a-b)}}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{(5a+2b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\frac{\int \frac{a(9a+26b)-2b(23a+12b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3(a-b)}}{2(a-b)}}{4(a-b)}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{(5a+2b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{\int\frac{3a\left(3a^2+24ba+8b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\cdots}{3(a-b)}}{2(\cdots}}{4(a-b)}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{(5a+2b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2+24ab+8b^2\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx\cdots}}}{a-b}\cdots}{3(a-b)}\cdots}{4(a-b)}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{(5a+2b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2+24ab+8b^2\right)\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(\cdots}{\sqrt{b\tan^2\cdots}}}{a-b}\cdots}{3(a-b)}\cdots}{4(a-b)}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4(a-b)(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{(5a+2b)\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2+24ab+8b^2\right)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{5b\cdots}{(a\cdots}}{3(a-b)}}{2(a\cdots}}{4(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(4*(a - b)*(1 + Tan[e + f*x]^2)^2*(a + b*Tan[e + f*x]^2)^(3/
2)) - (((5*a + 2*b)*Tan[e + f*x])/(2*(a - b)*(1 + Tan[e + f*x]^2)*(a + b*T
an[e + f*x]^2)^(3/2)) - (-1/3*(b*(23*a + 12*b)*Tan[e + f*x])/((a - b)*(a +
 b*Tan[e + f*x]^2)^(3/2)) + ((3*(3*a^2 + 24*a*b + 8*b^2)*ArcTan[(Sqrt[a -
b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(a - b)^(3/2) - (5*b*(11*a +
 10*b)*Tan[e + f*x])/((a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(3*(a - b)))/(2
*(a - b)))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10809 vs. $2(222) = 444$.

Time = 67.44 (sec) , antiderivative size = 10810, normalized size of antiderivative = 43.94

method	result	size
default	Expression too large to display	10810

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sin^4\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sin\left(fx+e\right)^4}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sin\left(e+f\,x\right)^4}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\sin\left(fx+e\right)^4}{\tan\left(fx+e\right)^6 b^3+3\tan\left(fx+e\right)^4 a\,b^2+3\tan\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(sin(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**4)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.147 $\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 181

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\frac{(a+4b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2(a-b)^{7/2}f}-\frac{\cos(e+fx)\sin(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{5b\tan(e+fx)}{6(a-b)^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(13a+2b)\tan(e+fx)}{6a(a-b)^3f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
1/2*(a+4*b)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^
(7/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-5/6*b*t
an(f*x+e)/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/6*b*(13*a+2*b)*tan(f*x+e)/a
/(a-b)^3/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 4.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.71

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-((a-b)\left(8ab^2-4b(6a+b)(a+b+(a-b)\cos(2(e+\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-1/12*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-((a - b)*
(8*a*b^2 - 4*b*(6*a + b)*(a + b + (a - b)*Cos[2*(e + f*x)]) - 3*a*(a + b +
 (a - b)*Cos[2*(e + f*x)])^2)*Sin[2*(e + f*x)]) - (3*a*b*(a + 4*b)*(((a +
b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*(2*(a - b)*Elliptic
F[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[
2]], 1] - 2*a*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2
*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Sin[2*(e + f*
x)])/Sqrt[2]))/(Sqrt[2]*a*(a - b)^4*f*(a + b + (a - b)*Cos[2*(e + f*x)])^2
)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4146, 373, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$$

$$\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^2}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow \text{373}$$

$$\frac{\frac{\int \frac{a-4b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{f}$$

$$\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{a\left(-10b\tan^2(e+fx)+3a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a(a-b)}-\frac{5b\tan(e+fx)}{3(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{f}$$

$$\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{-10b\tan^2(e+fx)+3a+2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3(a-b)}-\frac{5b\tan(e+fx)}{3(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{f}$$

$$\downarrow \text{402}$$

$$\frac{\frac{\frac{\frac{\int \frac{3a(a+4b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(13a+2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}-\frac{5b\tan(e+fx)}{3(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+f\right.}}{f}$$

$$\downarrow \text{27}$$

$$\frac{\frac{\frac{\frac{3(a+4b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}-\frac{b(13a+2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}-\frac{5b\tan(e+fx)}{3(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{\frac{3(a+4b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{b(13a+2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}-\frac{5b\tan(e+fx)}{3(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{3(a+4b)\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b(13a+2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}-\frac{5b\tan(e+fx)}{3(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2(a-b)}-\frac{\tan(e+fx)}{2(a-b)(\tan^2(e+fx)+1)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/2*Tan[e + f*x]/((a - b)*(1 + Tan[e + f*x]^2)*(a + b*Tan[e + f*x]^2)^(3
/2)) + ((-5*b*Tan[e + f*x])/(3*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + ((3
*(a + 4*b)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/
(a - b)^(3/2) - (b*(13*a + 2*b)*Tan[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e
+ f*x]^2]))/(3*(a - b)))/(2*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1894 vs. 2(161) = 322.

Time = 20.87 (sec) , antiderivative size = 1895, normalized size of antiderivative = 10.47

method	result	size
default	Expression too large to display	1895

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x
+e)/(a+b*tan(f*x+e)^2)^(1/2))-b/(a-b)^2*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1
/2)+1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-1/3/f*b^2/(a-b)^2/a/((1-cos(2*f*x+2*e))
^2*b*csc(2*f*x+2*e)^2+a)^(3/2)*csc(2*f*x+2*e)+1/3/f*b^2/(a-b)^2/a/((1-cos(
2*f*x+2*e))^2*b*csc(2*f*x+2*e)^2+a)^(3/2)*cot(2*f*x+2*e)-2/3/f*b^2/(a-b)^2
/a^2/((1-cos(2*f*x+2*e))^2*b*csc(2*f*x+2*e)^2+a)^(1/2)*csc(2*f*x+2*e)+2/3/
f*b^2/(a-b)^2/a^2/((1-cos(2*f*x+2*e))^2*b*csc(2*f*x+2*e)^2+a)^(1/2)*cot(2*
f*x+2*e)-1/2/f/(a-b)^3*a/((1-cos(2*f*x+2*e))^2*b*csc(2*f*x+2*e)^2+a)^(1/2)
/(1/(csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)
*b+b*csc(2*f*x+2*e)^2+a)*a*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2-2/(csc(2*f*x+
2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*csc(2*f*x+
2*e)^2+a)*a*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)+1/(csc(2*f*x+2*e)^2*cos(2*f*x+
2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*csc(2*f*x+2*e)^2+a)*a*csc(2
*f*x+2*e)^2-1/(csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(
2*f*x+2*e)*b+b*csc(2*f*x+2*e)^2+a)*b*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2+2/(
csc(2*f*x+2*e)^2*cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*
csc(2*f*x+2*e)^2+a)*b*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)-1/(csc(2*f*x+2*e)^2*
cos(2*f*x+2*e)^2*b-2*csc(2*f*x+2*e)^2*cos(2*f*x+2*e)*b+b*csc(2*f*x+2*e)^2+
a)*b*csc(2*f*x+2*e)^2+1)*csc(2*f*x+2*e)+1/2/f/(a-b)^3*a/((1-cos(2*f*x+2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(161) = 322$.

Time = 130.35 (sec) , antiderivative size = 1294, normalized size of antiderivative = 7.15

$$\int \frac{\sin^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[1/48*(3*((a^4 + 2*a^3*b - 7*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^4 + a^2*b^2 +
 4*a*b^3 + 2*(a^3*b + 3*a^2*b^2 - 4*a*b^3)*cos(f*x + e)^2)*sqrt(-a + b)*lo
g(128*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^8 - 256*(a^
4 - 5*a^3*b + 9*a^2*b^2 - 7*a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^4 - 34
*a^3*b + 77*a^2*b^2 - 72*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 - 32*a^3*b +
 160*a^2*b^2 - 256*a*b^3 + 128*b^4 - 32*(a^4 - 11*a^3*b + 34*a^2*b^2 - 40*
a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos
(f*x + e)^7 - 24*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos(f*x + e)^5 + 2*(5*a
^3 - 29*a^2*b + 48*a*b^2 - 24*b^3)*cos(f*x + e)^3 - (a^3 - 10*a^2*b + 24*a
*b^2 - 16*b^3)*cos(f*x + e))*sqrt(-a + b)*sqrt(((a - b)*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
*cos(f*x + e)^5 + 2*(9*a^3*b - 17*a^2*b^2 + 7*a*b^3 + b^4)*cos(f*x + e)^3
+ (13*a^2*b^2 - 11*a*b^3 - 2*b^4)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^
4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*f*cos(f*x + e)^4 + 2*(a^6*b - 5*a^
5*b^2 + 10*a^4*b^3 - 10*a^3*b^4 + 5*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 + (a
^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6)*f), 1/24*(3*((a^4 + 2*
a^3*b - 7*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^4 + a^2*b^2 + 4*a*b^3 + 2*(a^3*b
 + 3*a^2*b^2 - 4*a*b^3)*cos(f*x + e)^2)*sqrt(a - b)*arctan(-1/4*(8*(a^2 -
2*a*b + b^2)*cos(f*x + e)^5 - 8*(a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^3 + ...
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sin^2{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\tan{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\tan{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^2}{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\sin(fx+e)^2}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(sin(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*sin(e + f*x)**2)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.148 $\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*(5*a-2*b)*b*tan(f*x+e)
/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.02 (sec) , antiderivative size = 1331, normalized size of antiderivative = 9.93

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

output
```
(Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]
- (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a
 + (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4
)/a^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a
 - (4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2
*Tan[e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]
^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)
*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[(
(a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a -
 b)^2*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e +
f*x]^4)/a^4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2
*(a + b*Tan[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin
[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a
 + b*Tan[e + f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a -
 b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*
x]^2*(a + b*Tan[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/
2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (16
8*b*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin
[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f
*x]^2))/a])/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*S...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4144, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 316

$$\frac{\frac{\int \frac{-2b\tan^2(e+fx)+3a-2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{3a^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3a\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\downarrow$ 291

$$\frac{\frac{\frac{3a\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\frac{3a\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

output
```
(-1/3*(b*Tan[e + f*x])/(a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + ((3*a*Ar
cTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(a - b)^(3/2)
 - ((5*a - 2*b)*b*Tan[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(3
*a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a-b}-\frac{b\tan(fx+e)}{(a-b)^2 a\sqrt{a+b\tan(fx+e)^2}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(f}}\right.}{(a-b)^3 b^2}}{f}$
default	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a-b}-\frac{b\tan(fx+e)}{(a-b)^2 a\sqrt{a+b\tan(fx+e)^2}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(f}}\right.}{(a-b)^3 b^2}}{f}$

input
```
int(1/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x
+e)/(a+b*tan(f*x+e)^2)^(1/2))-b/(a-b)^2*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1
/2)+1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(120) = 240.

Time = 0.17 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.16

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \left[-\frac{3\left(a^2b^2\tan\left(fx+e\right)^4+2\,a^3b\tan\left(fx+e\right)^2+a^4\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)}{}\right.}{6\left(\left(a^5b^2-3\,a^4b^3+3\,a^3b^4-\right.\right.}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
 b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2
*b^4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(
f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2
+ (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4
 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x +
e)/sqrt(b*tan(f*x + e)^2 + a)) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x +
e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2
 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(
a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b
 + 3*a^5*b^2 - a^4*b^3)*f)]
```

Sympy [F]

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(a+b\tan^2{\left(e+fx\right)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}}{\tan\left(fx+e\right)^6 b^3+3\tan\left(fx+e\right)^4 a\,b^2+3\tan\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(1/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x)**4*
a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.149 $\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cot(e+fx)}{af\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3a^2f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan(e+fx)}{3a^3f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-cot(f*x+e)/a/f/(a+b*tan(f*x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/a^2/f/(a+b*tan(f
*x+e)^2)^(3/2)-8/3*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{(3(3a^2+4ab+8b^2)+4(3a^2-8b^2)\cos(2(e+fx))+(3a^2-12ab+8b^2)\cos(4(e+fx)))\cot(e+fx)\sqrt{(}}{6\sqrt{2}a^3f(a+b+(a-b)\cos(2(e+fx)))^2}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-1/6*((3*(3*a^2 + 4*a*b + 8*b^2) + 4*(3*a^2 - 8*b^2)*Cos[2*(e + f*x)] + (3
*a^2 - 12*a*b + 8*b^2)*Cos[4*(e + f*x)])*Cot[e + f*x]*Sqrt[(a + b + (a - b
)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])/(Sqrt[2]*a^3*f*(a + b + (a - b)*Cos[2
*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4146, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a}+\frac{\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{-\frac{4b\left(\frac{2\tan(e+fx)}{3a^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot(e+fx)}{a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-(Cot[e + f*x]/(a*(a + b*Tan[e + f*x]^2)^(3/2))) - (4*b*(Tan[e + f*x]/(3*
a*(a + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*x])/(3*a^2*Sqrt[a + b*Tan[e
 + f*x]^2])))/a)/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{1}{a\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a}}{f}$	90
default	$\frac{-\frac{1}{a\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a}}{f}$	90

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2)-4*b/a*(1/3*tan(f*x+e)/a/(a+b
*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\left((3\,a^2-12\,ab+8\,b^2)\cos\left(fx+e\right)^5+4\,(3\,ab-4\,b^2)\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{(a-b)\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(a^3b^2f+(a^5-2\,a^4b+a^3b^2)f\cos\left(fx+e\right)^4+2\,(a^4b-a^3b^2)f\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((3*a^2 - 12*a*b + 8*b^2)*cos(f*x + e)^5 + 4*(3*a*b - 4*b^2)*cos(f*x
+ e)^3 + 8*b^2*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((a^3*b^2*f + (a^5 - 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^4 + 2*(a^4*b -
 a^3*b^2)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^2{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^3} + \frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a^2} + \frac{3}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 4*b*tan(f*x + e)
/((b*tan(f*x + e)^2 + a)^(3/2)*a^2) + 3/((b*tan(f*x + e)^2 + a)^(3/2)*a*ta
n(f*x + e)))/f
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\csc{(fx+e)}^2}{(b\tan{(fx+e)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 22.15 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.34

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{a+\frac{b\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)^2}{\left(e^{e\,2i+f\,x\,2i}+1\right)^2}}\left(-a\,b\,12i+a^2\,3i+b^2\,8i+a^2\,e^{e\,2i+f\,x\,2i}\,12i+a^2\,e^{e\,4i+f\,x\,4i}\,18i+a^2\,e^{e}\right.}{3\,a^3\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)\,(a-b}$$

input
```
int(1/(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

output
```
-((exp(e*2i + f*x*2i) + 1)*(a + (b*(exp(e*2i + f*x*2i)*1i - 1i)^2)/(exp(e*
2i + f*x*2i) + 1)^2)^(1/2)*(a^2*3i - a*b*12i + b^2*8i + a^2*exp(e*2i + f*x
*2i)*12i + a^2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*e
xp(e*8i + f*x*8i)*3i - b^2*exp(e*2i + f*x*2i)*32i + b^2*exp(e*4i + f*x*4i)
*48i - b^2*exp(e*6i + f*x*6i)*32i + b^2*exp(e*8i + f*x*8i)*8i + a*b*exp(e*
4i + f*x*4i)*24i - a*b*exp(e*8i + f*x*8i)*12i))/(3*a^3*f*(exp(e*2i + f*x*2
i) - 1)*(a - b + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 2*b*exp(e
*2i + f*x*2i) - b*exp(e*4i + f*x*4i))^2)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^2}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a\,b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(csc(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**2)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.150 $\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{(a-b)b\tan(e+fx)}{3a^3f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-8b)b\tan(e+fx)}{3a^4f\sqrt{a+b\tan^2(e+fx)}} - \frac{(3a-8b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^4f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3f}$$

output
```
-1/3*(a-b)*b*tan(f*x+e)/a^3/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*(5*a-8*b)*b*tan
(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*(3*a-8*b)*cot(f*x+e)*(a+b*tan(f
*x+e)^2)^(1/2)/a^4/f-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\cot(e+fx)\left(2a-8b+\right.\right.}{3\sqrt{2}a}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(
2*a - 8*b + a*Csc[e + f*x]^2)) + (2*b*(-3*a^2 + 2*a*b + 4*b^2 + (-3*a^2 +
7*a*b - 4*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*
(e + f*x)])^2))/(3*Sqrt[2]*a^4*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4146, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\cot^4(e+fx)\left(\tan^2(e+fx)+1\right)}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(a-2b)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{a}-\frac{\cot^3(e+fx)}{3a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(a-2b)\left(-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot^3(e+fx)}{3a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{(a-2b)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a}+\frac{\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot^3(e+fx)}{3a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{(a-2b)\left(-\frac{4b\left(\frac{2\tan(e+fx)}{3a^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot^3(e+fx)}{3a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/(a*(a + b*Tan[e + f*x]^2)^(3/2)) + ((a - 2*b)*(-(Cot[
e + f*x]/(a*(a + b*Tan[e + f*x]^2)^(3/2))) - (4*b*(Tan[e + f*x]/(3*a*(a +
b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*x])/(3*a^2*Sqrt[a + b*Tan[e + f*x]
^2])))/a))/a)/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 15.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

method	result
default	$\frac{a^4\left(2\cot(fx+e)^3-3\cot(fx+e)\csc(fx+e)^2\right)+a^3b(20\cot(fx+e)-15\sec(fx+e)\csc(fx+e))+a^2b^2\left(50\tan(fx+e)-20\tan(fx+e)\operatorname{se}\right.}{3f\,a^4\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}$

input

```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/3/f/a^4/(a+b*tan(f*x+e)^2)^(5/2)*(a^4*(2*cot(f*x+e)^3-3*cot(f*x+e)*csc(f
*x+e)^2)+a^3*b*(20*cot(f*x+e)-15*sec(f*x+e)*csc(f*x+e))+a^2*b^2*(50*tan(f*
x+e)-20*tan(f*x+e)*sec(f*x+e)^2)+a*b^3*(48*tan(f*x+e)^3-8*tan(f*x+e)^3*sec
(f*x+e)^2)+16*b^4*tan(f*x+e)^5)
```

Fricas [A] (verification not implemented)

Time = 52.85 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.58

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(2\left(a^3-9a^2b+16ab^2-8b^3\right)\cos\left(fx+e\right)^7-3\left(a^3-10a^2b+24ab^2-16b^3\right)\cos\left(fx+e\right)^5-12\left(a^2b-\right.\right.}{3\left(\left(a^6-2a^5b+a^4b^2\right)f\cos\left(fx+e\right)^6-a^4b^2f-\left(a^6-4a^5b+3a^4b^2\right)f\cos\left(fx+\right.\right.}$$

input

```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
-1/3*(2*(a^3 - 9*a^2*b + 16*a*b^2 - 8*b^3)*cos(f*x + e)^7 - 3*(a^3 - 10*a^
2*b + 24*a*b^2 - 16*b^3)*cos(f*x + e)^5 - 12*(a^2*b - 4*a*b^2 + 4*b^3)*cos
(f*x + e)^3 - 8*(a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(((a - b)*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/(((a^6 - 2*a^5*b + a^4*b^2)*f*cos(f*x + e)^6 - a^4*b
^2*f - (a^6 - 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 - (2*a^5*b - 3*a^4*b^2
)*f*cos(f*x + e)^2)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx=\int \frac{\csc^4{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.28

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx= -\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^3}+\frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a^2}-\frac{16\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}a^4}-\frac{8\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a^3}+\frac{3}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 4*b*tan(f*x + e)
/((b*tan(f*x + e)^2 + a)^(3/2)*a^2) - 16*b^2*tan(f*x + e)/(sqrt(b*tan(f*x
+ e)^2 + a)*a^4) - 8*b^2*tan(f*x + e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^3) +
 3/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2
+ a)^(3/2)*a^2*tan(f*x + e)) + 1/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x +
 e)^3))/f
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^4}{\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^4*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^4}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(csc(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**4)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.151 $\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\begin{aligned}\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}} \, dx = & -\frac{(a-b)^2 b\tan(e+fx)}{3a^4 f\left(a+b\tan^2(e+fx)\right)^{3/2}} \\ & -\frac{(5a-11b)(a-b)b\tan(e+fx)}{3a^5 f\sqrt{a+b\tan^2(e+fx)}} \\ & -\frac{(15a^2-80ab+73b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^5 f} \\ & -\frac{2(5a-7b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4 f} \\ & -\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^3 f}\end{aligned}$$

output
```
-1/3*(a-b)^2*b*tan(f*x+e)/a^4/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*(5*a-11*b)*(a
-b)*b*tan(f*x+e)/a^5/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^2-80*a*b+73*b^2
)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^5/f-2/15*(5*a-7*b)*cot(f*x+e)^3*(a
+b*tan(f*x+e)^2)^(1/2)/a^4/f-1/5*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2)/a^3
/f
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\cot(e+fx)\left(8a^2-66a\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(-(Cot[e + f*x]*(
8*a^2 - 66*a*b + 73*b^2 + 2*a*(2*a - 7*b)*Csc[e + f*x]^2 + 3*a^2*Csc[e + f
*x]^4)) + (5*b*(-a + b)*(6*a^2 - 7*a*b - 11*b^2 + (6*a^2 - 17*a*b + 11*b^2
)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)])^2
))/(15*Sqrt[2]*a^5*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4146, 365, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 365$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(5a\tan^2(e+fx)+2(5a-4b)\right)}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)}{5a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{\frac{\left(5a^2-4b(5a-4b)\right)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{a}-\frac{2(5a-4b)\cot^3(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cot^5(e+fx)}{5a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 245$$

$$\frac{\frac{\frac{\left(5a^2-4b(5a-4b)\right)\left(-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a-4b)\cot^3(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cot^5(e+fx)}{5a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 209$$

$$\frac{\frac{\frac{\left(5a^2-4b(5a-4b)\right)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a}+\frac{\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a-4b)\cot^3(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cot^5(e+fx)}{5a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{\frac{\left(5a^2-4b(5a-4b)\right)\left(-\frac{4b\left(\frac{2\tan(e+fx)}{3a^2\sqrt{a+b\tan^2(e+fx)}}+\frac{\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a-4b)\cot^3(e+fx)}{3a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cot^5(e+fx)}{5a(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/5*Cot[e + f*x]^5/(a*(a + b*Tan[e + f*x]^2)^(3/2)) + ((-2*(5*a - 4*b)*C
ot[e + f*x]^3)/(3*a*(a + b*Tan[e + f*x]^2)^(3/2)) + ((5*a^2 - 4*(5*a - 4*b
)*b)*(-(Cot[e + f*x]/(a*(a + b*Tan[e + f*x]^2)^(3/2))) - (4*b*(Tan[e + f*x
]/(3*a*(a + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*x])/(3*a^2*Sqrt[a + b*
Tan[e + f*x]^2])))/a))/a)/(5*a))/f
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 14.72 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\left(\cos(fx+e)^6\left(8\cos(fx+e)^4-20\cos(fx+e)^2+15\right)a^5+\cos(fx+e)^4\left(120\cos(fx+e)^4-200\cos(fx+e)^2+75\right)\sin(fx+e)^2a^4b+\cos(\right.}{}$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^5*(cos(f*x+e)^6*(8*cos(f*x+e)^4-20*cos(f*x+e)^2+15)*a^5+cos(f*x+
e)^4*(120*cos(f*x+e)^4-200*cos(f*x+e)^2+75)*sin(f*x+e)^2*a^4*b+cos(f*x+e)^
2*(440*cos(f*x+e)^4-500*cos(f*x+e)^2+100)*sin(f*x+e)^4*a^3*b^2+(680*cos(f*
x+e)^4-480*cos(f*x+e)^2+40)*sin(f*x+e)^6*a^2*b^3+(480*cos(f*x+e)^2-160)*si
n(f*x+e)^8*a*b^4+128*b^5*sin(f*x+e)^10)/(a+b*tan(f*x+e)^2)^(5/2)*sec(f*x+e
)^5*csc(f*x+e)^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^6{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.62

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\frac{40\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^3} + \frac{20\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a^2} - \frac{160\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^4} - \frac{80\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}a^3} + \frac{128\,b^3\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}\,a^5} + \frac{64\,b^3 \cdots}{\left(b\tan(f \cdots\right.}}{\cdots}$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^3) + 20*b*tan(f*x +
 e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^2) - 160*b^2*tan(f*x + e)/(sqrt(b*tan(
f*x + e)^2 + a)*a^4) - 80*b^2*tan(f*x + e)/((b*tan(f*x + e)^2 + a)^(3/2)*a
^3) + 128*b^3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a)*a^5) + 64*b^3*tan(f
*x + e)/((b*tan(f*x + e)^2 + a)^(3/2)*a^4) + 15/((b*tan(f*x + e)^2 + a)^(3
/2)*a*tan(f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a)^(3/2)*a^2*tan(f*x + e))
 + 48*b^2/((b*tan(f*x + e)^2 + a)^(3/2)*a^3*tan(f*x + e)) + 10/((b*tan(f*x
 + e)^2 + a)^(3/2)*a*tan(f*x + e)^3) - 8*b/((b*tan(f*x + e)^2 + a)^(3/2)*a
^2*tan(f*x + e)^3) + 3/((b*tan(f*x + e)^2 + a)^(3/2)*a*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\tan(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^6*(a + b*tan(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\csc(fx+e)^6}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2b+a^3}dx$$

input
```
int(csc(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*csc(e + f*x)**6)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.152 $\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \frac{\cos^2(e+fx)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+2p), \frac{1}{2}(1+m+2p), \frac{1}{2}(3+m+2p), \sin^2(e+fx)\right) (d\sin(e+ \cdots}{f(1+m+2p)}$$

output
```
(cos(f*x+e)^2)^(1/2+p)*hypergeom([1/2+p, 1/2+1/2*m+p],[3/2+1/2*m+p],sin(f*
x+e)^2)*(d*sin(f*x+e))^m*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+m+2*p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.97 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.17

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \frac{(3+m+2p) \cdots}{f(1+m+2p)\left((3+m+2p)\text{AppellF1}\left(\frac{1}{2}+\frac{m}{2}+p, 2p, 1+m, \frac{3}{2}+\frac{m}{2}+p, \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e \cdots\right.\right.\right.}$$

input
```
Integrate[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p, 1 + m, 3/2 + m/2 + p, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*Tan[e
+ f*x]^2)^p)/(f*(1 + m + 2*p)*((3 + m + 2*p)*AppellF1[1/2 + m/2 + p, 2*p,
1 + m, 3/2 + m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m
)*AppellF1[3/2 + m/2 + p, 2*p, 2 + m, 5/2 + m/2 + p, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] - 2*p*AppellF1[3/2 + m/2 + p, 1 + 2*p, 1 + m, 5/2 + m/
2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4141, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b \tan^2(e+fx)\right)^p (d \sin(e+fx))^m \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b \tan(e+fx)^2\right)^p (d \sin(e+fx))^m dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2p}(e+fx) \left(b \tan^2(e+fx)\right)^p \int (d \sin(e+fx))^m \tan^{2p}(e+fx) dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx) \left(b \tan^2(e+fx)\right)^p \int (d \sin(e+fx))^m \tan(e+fx)^{2p} dx$$

$$\Big\downarrow \text{3082}$$

$$d \sin(e+fx) \cos^{2p}(e+fx) \left(b \tan^2(e+fx)\right)^p (d \sin(e+fx))^{-2p-1} \int \cos^{-2p}(e+fx)(d \sin(e+fx))^{m+2p} dx$$

$$\Big\downarrow \text{3042}$$

$$d \sin(e+fx) \cos^{2p}(e+fx) \left(b \tan^2(e+fx)\right)^p (d \sin(e+fx))^{-2p-1} \int \cos(e+fx)^{-2p}(d \sin(e+fx))^{m+2p} dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\sin(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2p+1),\right.}{f(m+2p+1)}$$

input
```
Int[(d*Sin[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
((Cos[e + f*x]^2)^(1/2 + p)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m + 2*p)/2
, (3 + m + 2*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*Tan[
e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\sin(fx+e))^m \left(b\tan(fx+e)^2\right)^p dx$$

input
```
int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \int \left(b\tan(fx+e)^2\right)^p (d\sin(fx+e))^m dx$$

input
```
integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \int \left(b\tan^2(e+fx)\right)^p (d\sin(e+fx))^m dx$$

input
```
integrate((d*sin(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((b*tan(e + f*x)**2)**p*(d*sin(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan\left(fx+e\right)^2\right)^p (d\sin\left(fx+e\right))^m \,dx$$

input
```
integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan\left(fx+e\right)^2\right)^p (d\sin\left(fx+e\right))^m \,dx$$

input
```
integrate((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int (d\,\sin\left(e+f\,x\right))^m \left(b\tan(e+f\,x)^2\right)^p dx$$

input
```
int((d*sin(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*sin(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\sin(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = d^m b^p \left(\int \tan(fx+e)^{2p}\sin(fx+e)^m \, dx\right)$$

input
```
int((d*sin(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*b**p*int(tan(e + f*x)**(2*p)*sin(e + f*x)**m,x)
```

3.153 $\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2},\frac{2+m}{2},-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m\tan(e+f}{f(1+m)}$$

output
```
AppellF1(1/2+1/2*m,1+1/2*m,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(
sec(f*x+e)^2)^(1/2*m)*(d*sin(f*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(
1+m)/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 275 vs. 2(122) = 244.

Time = 2.46 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.25

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{a(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},\frac{2+m}{2},-p,\frac{3+m}{2},-\tan^2(\right.}{f(1+m)\left(a(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},\frac{2+m}{2},-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)+\left(2bp\,\text{AppellF1}\left(\frac{3+}{2}\right.\right.\right.}$$

input
```
Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(a +
 b*Tan[e + f*x]^2)^p)/(f*(1 + m)*(a*(3 + m)*AppellF1[(1 + m)/2, (2 + m)/2,
 -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF
1[(3 + m)/2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/a)] - a*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4150, 393, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sin(e+fx))^m \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4150}$$

$$\frac{\tan^{-m}(e+fx)\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \int \tan^m(e+fx)\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}\left(b\tan^2(e+fx)+a\right)^p}{f}$$

$$\Big\downarrow \text{393}$$

$$\frac{\cot(e+fx)\tan^2(e+fx)^{\frac{1-m}{2}}\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m \int \tan^2(e+fx)^{\frac{m-1}{2}}\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}\left(b\tan\right.}{2f}$$

$$\Big\downarrow \text{152}$$

$$\frac{\cot(e+fx)\tan^2(e+fx)^{\frac{1-m}{2}}\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int\tan^2}{2f}$$

$$\Big\downarrow \text{150}$$

$$\frac{\cot(e+fx)\tan^2(e+fx)^{\frac{1-m}{2}+\frac{m+1}{2}}\sec^2(e+fx)^{m/2}(d\sin(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}A}{f(m+1)}$$

input
```
Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/a)]*Cot[e + f*x]*(Sec[e + f*x]^2)^(m/2)*(d*Sin[e + f*x])^m*(Tan
[e + f*x]^2)^((1 - m)/2 + (1 + m)/2)*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + m)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4150
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff
*(d*Sin[e + f*x])^m*((Sec[e + f*x]^2)^(m/2)/(f*Tan[e + f*x]^m))   Subst[Int
[(ff*x)^m*((a + b*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x
]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]
```

Maple [F]

$$\int \left(d \sin \left(f x +e \right)\right)^{m} \left(a +b \tan \left(f x +e \right)^{2}\right)^{p}d x$$

input
```
int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \int \left(b \tan \left(f x + e\right)^{2} + a\right)^{p} \left(d \sin \left(f x + e\right)\right)^{m} \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\sin\left(fx+e\right))^m \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \int (d\,\sin\left(e+f\,x\right))^m \left(b\tan(e+f\,x)^2+a\right)^p dx$$

input
```
int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\sin(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= d^m\left(\int \sin\left(fx+e\right)^m \left(\tan\left(fx+e\right)^2 b+a\right)^p dx\right)$$

input
```
int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*int(sin(e + f*x)**m*(tan(e + f*x)**2*b + a)**p,x)
```

3.154 $\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 212

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$=\frac{(10a-7b-2bp)\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{15(a-b)^2f}$$
$$-\frac{\cos^5(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{5(a-b)f}$$
$$-\frac{(15a^2-20ab(1+p)+4b^2(2+3p+p^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)(a}{15(a-b)^2f}$$

output
```
1/15*(-2*b*p+10*a-7*b)*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(p+1)/(a-b)^2/f-1
/5*cos(f*x+e)^5*(a-b+b*sec(f*x+e)^2)^(p+1)/(a-b)/f-1/15*(15*a^2-20*a*b*(p+
1)+4*b^2*(p^2+3*p+2))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^
2/(a-b))*(a-b+b*sec(f*x+e)^2)^p/(a-b)^2/f/(((a-b+b*sec(f*x+e)^2)/(a-b))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 5.47 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.33

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx =$$

$$-\frac{2^{3+p}\cos(e+fx)\sin^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\left(15a^2-20ab(1+p)+4b^2(2+3p+p^2)\right)\text{Hyperg}\right.}{15(a-b)^2f\left(3\left(\frac{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+}{a-b}\right.\right.}$$

input
```
Integrate[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-1/15*(2^(3 + p)*Cos[e + f*x]*Sin[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p*((15
*a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Hypergeometric2F1[-1/2, -p,
 1/2, -((b*Sec[e + f*x]^2)/(a - b))] + ((a + b + (a - b)*Cos[2*(e + f*x)])
*(-17*a + b*(11 + 4*p) + 3*(a - b)*Cos[2*(e + f*x)])*((a + b*Tan[e + f*x]^
2)/(a - b))^p)/4))/((a - b)^2*f*(3*(((a + b + (a - b)*Cos[2*(e + f*x)])*Se
c[e + f*x]^2)/(a - b))^p - 2^(2 + p)*Cos[2*(e + f*x)]*((a + b*Tan[e + f*x]
^2)/(a - b))^p + 2^p*Cos[4*(e + f*x)]*((a + b*Tan[e + f*x]^2)/(a - b))^p))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4147, 365, 25, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^5\left(a+b\tan(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{f}$$

$\Big\downarrow$ 365

$$\frac{\frac{\int -\cos^4(e+fx)\left(-5(a-b)\sec^2(e+fx)+10a-b(2p+7)\right)\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{5(a-b)}}{f}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \cos^4(e+fx)\left(-5(a-b)\sec^2(e+fx)+10a-7b-2bp\right)\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{5(a-b)}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{5(a-b)}}{f}$$

$\Big\downarrow$ 359

$$\frac{-\frac{-\frac{\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{3(a-b)}-\frac{(10a-2bp-7b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{3(a-b)}}{5(a-b)}-\cos^5(e+f\ldots}{f}$$

$\Big\downarrow$ 279

$$\frac{-\frac{-\frac{\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\int \cos^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^p d\sec(e+fx)}{3(a-b)}-\frac{(10a-2bp-7b)\cos^3(e+f\ldots}{3\ldots}}{5(a-b)}}{f}$$

$\Big\downarrow$ 278

$$\frac{-\frac{\frac{\left(15a^2-20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)}{3(a-b)}-\frac{(10a-2bp-7\ldots}{\ldots}}{5(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(-1/5*(Cos[e + f*x]^5*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(a - b) - (-1/3*
((10*a - 7*b - 2*b*p)*Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(
a - b) + ((15*a^2 - 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cos[e + f*x]*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))]*(a - b + b*
Sec[e + f*x]^2)^p)/(3*(a - b)*(1 + (b*Sec[e + f*x]^2)/(a - b))^p))/(5*(a -
 b)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [F]

$$\int \sin (fx+e)^5 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan (fx+e)^2+a\right)^p \sin (fx+e)^5 \, dx$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e)^2 + a)^p*
sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5 \, dx$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Giac [F]

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5 \, dx$$

input
```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \sin\left(e+f\,x\right)^5 \left(b\tan(e+f\,x)^2+a\right)^p \, dx$$

input
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sin^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(\tan\left(fx+e\right)^2 b+a\right)^p \sin\left(fx+e\right)^5 dx$$

input
```
int(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*sin(e + f*x)**5,x)
```

3.155 $\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\frac{\cos^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^{1+p}}{3(a-b)f}-\frac{(3a-2b(1+p))\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)\left(a-b+b\sec^2(e+fx)\right)^p}{3(a-b)f}$$

output
```
1/3*cos(f*x+e)^3*(a-b+b*sec(f*x+e)^2)^(p+1)/(a-b)/f-1/3*(3*a-2*b*(p+1))*co
s(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(a-b))*(a-b+b*sec(f*x+
e)^2)^p/(a-b)/f/(((a-b+b*sec(f*x+e)^2)/(a-b))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 2.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.28

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$=\frac{\sin(e+fx)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left((-3a+2b(1+p))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\,\text{se}}{}\right.\right.}{f\left(3a\sec^2(e+fx)\left(\frac{a-b+b\sec^2(e+fx)}{a-b}\right)^p-3(a-b)\left(\frac{a+b\,\text{ta}}{}\right.\right.}$$

input
```
Integrate[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(Sin[e + f*x]*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((-3*a + 2*b*(1 + p))*
Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a - b))] + (a*Cos[e
 + f*x]^2 + b*Sin[e + f*x]^2)*((a + b*Tan[e + f*x]^2)/(a - b))^p))/(f*(3*a
*Sec[e + f*x]^2*((a - b + b*Sec[e + f*x]^2)/(a - b))^p - 3*(a - b)*((a + b
*Tan[e + f*x]^2)/(a - b))^(1 + p)))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4147, 25, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(e+fx)^3\left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{ 4147}$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\frac{(3a-2b(p+1))\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{3(a-b)}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^{p+1}}{3(a-b)}}{f}$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\frac{(3a-2b(p+1))(a+b\sec^2(e+fx)-b)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\int\cos^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^p d\sec(e+fx)}{3(a-b)}+\frac{\cos^3(e+fx)(a+b\sec^2(e+fx)-b)^{p+}}{3(a-b)}}{f}$$

$\Big\downarrow$ 278

$$\frac{\frac{\cos^3(e+fx)(a+b\sec^2(e+fx)-b)^{p+1}}{3(a-b)}-\frac{(3a-2b(p+1))\cos(e+fx)(a+b\sec^2(e+fx)-b)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\right.}{3(a-b)}}{f}$$

input
```
Int[Sin[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
((Cos[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^(1 + p))/(3*(a - b)) - ((3*a -
 2*b*(1 + p))*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f
*x]^2)/(a - b))]*(a - b + b*Sec[e + f*x]^2)^p)/(3*(a - b)*(1 + (b*Sec[e +
f*x]^2)/(a - b))^p))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [F]

$$\int \sin \left(f x+e\right)^{3} \left(a+b \tan \left(f x+e\right)^{2}\right)^{p}d x$$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sin^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b \tan \left(f x+e\right)^{2}+a\right)^{p} \sin \left(f x+e\right)^{3} \, dx$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Giac [F]

$$\int \sin^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \sin\left(e+f\,x\right)^3 \left(b\tan(e+f\,x)^2+a\right)^p \, dx$$

input
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sin^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(\tan\left(fx+e\right)^2 b+a\right)^p \sin\left(fx+e\right)^3 dx$$

input
```
int(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*sin(e + f*x)**3,x)
```

3.156 $\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)\left(a-b+b\sec^2(e+fx)\right)^p\left(\frac{a-b+b\sec^2(e+fx)}{a-b}\right)}{f}$$

output
```
-cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/(a-b))*(a-b+b*sec(f
*x+e)^2)^p/f/(((a-b+b*sec(f*x+e)^2)/(a-b))^p)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)\left(\frac{a-b+b\sec^2(e+fx)}{a-b}\right)^{-p}\left(a+b\tan^2(e+fx)\right)^p}{f}$$

input
```
Integrate[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
 b))]*(a + b*Tan[e + f*x]^2)^p)/(f*((a - b + b*Sec[e + f*x]^2)/(a - b))^p)
)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4147, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\int \cos^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^p d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

input
```
Int[Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/(a -
 b))]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a - b))^p)
)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [F]

$$\int \sin (fx+e) \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Giac [F]

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \sin\left(e+f\,x\right)\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sin(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\sin\left(fx+e\right)dx$$

input
```
int(sin(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*sin(e + f*x),x)
```

3.157 $\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx =$$

$$-\frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec(e+fx)\left(a-b+b\sec^2(e+fx)\right)^p\left(\frac{a-b+b\sec^2(e+f\cdots}{a-b}\right.}{f}$$

output
```
-AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/(a-b))*sec(f*x+e)*(a-b
+b*sec(f*x+e)^2)^p/f/(((a-b+b*sec(f*x+e)^2)/(a-b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1215 vs. 2(92) = 184.

Time = 14.20 (sec) , antiderivative size = 1215, normalized size of antiderivative = 13.21

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^(2*p)*((2*AppellF1[-1/2 - p, -1/2, -p
, 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])
/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1
[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/
(1 + (b*Tan[e + f*x]^2)/a)^p))/(2*f*(b*p*Sec[e + f*x]^2*Tan[e + f*x]*(a +
b*Tan[e + f*x]^2)^(-1 + p)*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[
e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 +
 (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)/(1 + (b*Tan[e +
f*x]^2)/a)^p) + ((a + b*Tan[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p,
 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[
e + f*x]^2])/((1 + 2*p)*(1 + (a*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2])
 + (4*a*p*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -((a*Cot[
e + f*x]^2)/b)]*Cot[e + f*x]*(1 + (a*Cot[e + f*x]^2)/b)^(-1 - p)*Sqrt[Csc[
e + f*x]^2]*Sqrt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*((-2*a*(-1/2 - p)*p*A
ppellF1[1/2 - p, -1/2, 1 - p, 3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^
2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(b*(1/2 - p)) - ((-1/2 - p)*AppellF1[1
/2 - p, 1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)]*Cot[e
+ f*x]*Csc[e + f*x]^2)/(1/2 - p))*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + (a
*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*AppellF1[-1/2 - p, -1/...
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4147, 25, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\left(b\sec^2(e+fx)+a-b\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\left(b\sec^2(e+fx)+a-b\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 334$$

$$-\frac{\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\int \frac{\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 333$$

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))
]*Sec[e + f*x]*(a - b + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/(a
 - b))^p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [F]

$$\int \csc \left(f x+e\right) \left(a+b \tan \left(f x+e\right)^{2}\right)^{p}dx$$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc(e+fx) \left(a+b\tan^2(e+fx)\right)^p\,dx=\int { {\left(b \tan \left(f x + e\right)^{2} + a\right)}^{p} \csc \left(f x + e\right)\,dx}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x),x)
```

output
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)dx$$

input
```
int(csc(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*csc(e + f*x),x)
```

3.158 $\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)\sec^3(e+fx)\left(a-b+b\sec^2(e+fx)\right)^p\left(\frac{a-b+b\sec^2(e+fx)}{a-b}\right.}{3f}$$

output
```
1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/(a-b))*sec(f*x+e)^3
*(a-b+b*sec(f*x+e)^2)^p/f/(((a-b+b*sec(f*x+e)^2)/(a-b))^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 252 vs. 2(96) = 192.

Time = 15.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.62

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{b(-3+2p)\,\text{AppellF1}\left(\frac{1}{2}-p,-\frac{1}{2},-p,\frac{3}{2}-p\right.}{f(-1+2p)\left(b(-3+2p)\,\text{AppellF1}\left(\frac{1}{2}-p,-\frac{1}{2},-p,\frac{3}{2}-p,-\cot^2(e+fx),-\frac{a\cot^2(e+fx)}{b}\right)-\left(2ap\,\text{AppellF}\right.\right.}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -((a*C
ot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*
(-1 + 2*p)*(b*(-3 + 2*p)*AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x
]^2, -((a*Cot[e + f*x]^2)/b)] - (2*a*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2
- p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2,
-p, 5/2 - p, -Cot[e + f*x]^2, -((a*Cot[e + f*x]^2)/b)])*Cot[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4147, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int \frac{\sec^2(e+fx)\left(b\sec^2(e+fx)+a-b\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\int \frac{\sec^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)-b\right)^p\left(\frac{b\sec^2(e+fx)}{a-b}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a-b}\right)}{3f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/(a - b))]*
Sec[e + f*x]^3*(a - b + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/
(a - b))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [F]

$$\int \csc(fx+e)^3 \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan(fx+e)^2+a\right)^p \csc(fx+e)^3 \, dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^3 \, dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^3 \, dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^3} \, dx$$

input
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^3,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p \,dx = \int \left(\tan \left(fx+e\right)^2 b+a\right)^p \csc \left(fx+e\right)^3 dx$$

input

```
int(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output

```
int((tan(e + f*x)**2*b + a)**p*csc(e + f*x)**3,x)
```

3.159 $\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\operatorname{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)}{3f}$$

output
```
1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^3*(a
+b*tan(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.17 (sec) , antiderivative size = 3698, normalized size of antiderivative = 44.02

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(3*a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Tan[e + f*x]^2)^p*(AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2
, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[
3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2
) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*S
ec[e + f*x]^2)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Ta
n[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a
), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a),
 -Tan[e + f*x]^2])*Tan[e + f*x]^2))*(-1/4*(Cos[2*(e + f*x)]^3*(a + b*Tan[e
 + f*x]^2)^p) + (I/4)*Sin[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p + (Sin[2*(
e + f*x)]^2*(a + b*Tan[e + f*x]^2)^p)/2 - (I/4)*Sin[2*(e + f*x)]^3*(a + b*
Tan[e + f*x]^2)^p + Cos[2*(e + f*x)]^2*((a + b*Tan[e + f*x]^2)^p/2 - (I/4)
*Sin[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p) + Cos[2*(e + f*x)]*(-1/4*(a +
b*Tan[e + f*x]^2)^p - (Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2)^p)/4)))/(
f*(6*a*b*p*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p)*(AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]/(-3*a*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*(b*p*AppellF1[3/2
, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - 2*a*AppellF1[
3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x...
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2\left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a)^p}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 395}$$

$$\frac{(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \frac{\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 394}$$

$$\frac{\tan^3(e+fx)\,(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

input
```
Int[Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
 + f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [F]

$$\int \sin (fx+e)^2 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \sin^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan (fx+e)^2+a\right)^p \sin (fx+e)^2 \, dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \sin\left(e+f\,x\right)^2\,\left(b\,\mathrm{tan}\left(e+f\,x\right)^2+a\right)^p\,dx$$

input
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \sin^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\sin\left(fx+e\right)^2dx$$

input

```
int(sin(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output

```
int((tan(e + f*x)**2*b + a)**p*sin(e + f*x)**2,x)
```

3.160 $\int (a+b\tan^2(e+fx))^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int (a+b\tan^2(e+fx))^p\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)^{-}}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)*(a+b*tan
(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 192 vs. 2(79) = 158.

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.43

$$\int (a+b\tan^2(e+fx))^p\,dx$$
$$= \frac{3a\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx\right.}{6af\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+4f\left(bp\operatorname{AppellF1}\left(\frac{3}{2},1-p,1,\frac{5}{2},-\frac{b\tan^2(e+fx)}{a}\right.\right.}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*S
in[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p,
 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2
, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \frac{\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
 + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [F]

$$\int \left(a + b\tan\left(fx+e\right)^{2}\right)^{p} dx$$

input
```
int((a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \left(a + b\tan^{2}(e+fx)\right)^{p} dx = \int \left(b\tan\left(fx+e\right)^{2} + a\right)^{p} dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a + b\tan^{2}(e+fx)\right)^{p} dx = \int \left(a + b\tan^{2}\left(e+fx\right)\right)^{p} dx$$

input
```
integrate((a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^p,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p dx$$

input
```
int((a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p,x)
```

3.161 $\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
-cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^
2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*
(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{3042, 4146, 279, 278\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*
(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [F]

$$\int \csc (fx+e)^2 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)^2dx$$

input
```
int(csc(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*csc(e + f*x)**2,x)
```

3.162 $\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{3af} - \frac{(3a-b(1-2p))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+\cdots}{\cdots}\right.}{3af}$$

output
```
-1/3*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(p+1)/a/f-1/3*(3*a-b*(1-2*p))*cot(f*x
+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*x+e)^2)^p/a/f
/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(-a-b\tan^2(e+fx)-(3a+b(-1+2p))\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\cdots\right.\right.}{3af}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p*(-a - b*Tan[e + f*x]^2 - ((3*a +
b*(-1 + 2*p))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Ta
n[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/a)^p))/(3*a*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4146, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3a-b(1-2p))\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^p d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{3a}}{f}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{(3a-b(1-2p))\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{3a}}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\frac{(3a-b(1-2p))\cot(e+fx)(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)}{3a}-\frac{\cot^3(e+fx)(a+b\tan^2}{3a}}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(-1/3*(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/a - ((3*a - b*(1 - 2
*p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)
]*(a + b*Tan[e + f*x]^2)^p)/(3*a*(1 + (b*Tan[e + f*x]^2)/a)^p))/f
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [F]

$$\int \csc(fx+e)^4 \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\, dx = \int \left(b\tan(fx+e)^2+a\right)^p \csc(fx+e)^4\, dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\, dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^4,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)^4dx$$

input
```
int(csc(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*csc(e + f*x)**4,x)
```

3.163 $\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\begin{aligned}\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx &= -\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{15a^2f} \\ &\quad-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{5af} \\ &\quad-\frac{(15a^2-b(10a-b(3-2p))(1-2p))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)(a+b}{15a^2f}\end{aligned}$$

output
```
-1/15*(10*a-b*(3-2*p))*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(p+1)/a^2/f-1/5*cot
(f*x+e)^5*(a+b*tan(f*x+e)^2)^(p+1)/a/f-1/15*(15*a^2-b*(10*a-b*(3-2*p))*(1-
2*p))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/a)*(a+b*tan(f*
x+e)^2)^p/a^2/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx =$$
$$-\frac{\cot(e+fx)\left(3\cot^4(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-p,-\frac{3}{2},-\frac{b\tan^2(e+fx)}{a}\right)+10\cot^2(e+fx)\,\text{Hyper}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -(
(b*Tan[e + f*x]^2)/a)] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p, -1/
2, -((b*Tan[e + f*x]^2)/a)] + 15*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan
[e + f*x]^2)/a)])*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^
p)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4146, 365, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a\right)^p\,d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{365}$$

$$\frac{\frac{\int \cot^4(e+fx)\left(5a\tan^2(e+fx)+10a-b(3-2p)\right)\left(b\tan^2(e+fx)+a\right)^p d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{5a}}{f}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{\frac{\left(15a^2-b(1-2p)(10a-b(3-2p))\right)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^p d\tan(e+fx)}{3a}-\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{3a}}{5a}-\frac{\cot^5(e+fx)\left(a+b\tan^2\right.}{5a}}{f}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{\frac{\left(15a^2-b(1-2p)(10a-b(3-2p))\right)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p d\tan(e+fx)}{3a}-\frac{(10a-b(3-2p))\cot^3(e+fx)\left(a+b\tan\right.}{3a}}{5a}}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{-\frac{\left(15a^2-b(1-2p)(10a-b(3-2p))\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a}\right)}{3a}-\frac{(10a-b(3-2p))\cot^3}{}}{5a}}{f}$$

input

```
Int[Csc[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

output

```
(-1/5*(Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(1 + p))/a + (-1/3*((10*a - b
*(3 - 2*p))*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(1 + p))/a - ((15*a^2 -
b*(10*a - b*(3 - 2*p))*(1 - 2*p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p,
 1/2, -((b*Tan[e + f*x]^2)/a)]*(a + b*Tan[e + f*x]^2)^p)/(3*a*(1 + (b*Tan[
e + f*x]^2)/a)^p))/(5*a))/f
```

Defintions of rubi rules used

rule 278

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [F]

$$\int \csc(fx+e)^{6} \left(a+b\tan(fx+e)^{2}\right)^{p} dx$$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Giac [F]

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \frac{\left(b\tan(e+f\,x)^2+a\right)^p}{\sin\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^6,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\csc\left(fx+e\right)^6\,dx$$

input
```
int(csc(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*csc(e + f*x)**6,x)
```

3.164 $\int (d\sin(e+fx))^m (b(c\tan(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int (d\sin(e+fx))^m (b(c\tan(e+fx))^n)^p\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+np),\frac{1}{2}(1+m+np),\frac{1}{2}(3+m+np),\sin^2(e+fx)\right)(d\sin}{f(1+m+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+1/2, 1/2*n*p+1/2*m+1/2],[1
/2*n*p+1/2*m+3/2],sin(f*x+e)^2)*(d*sin(f*x+e))^m*tan(f*x+e)*(b*(c*tan(f*x+
e))^n)^p/f/(n*p+m+1)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.03 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.01

$$\int (d\sin(e+fx))^m (b(c\tan(e+fx))^n)^p\,dx = \frac{(3+}{f(1+m+np)\left((3+m+np)\,\text{AppellF1}\left(\frac{1}{2}(1+m+np),np,1+m,\frac{1}{2}(3+m+np),\tan^2\left(\frac{1}{2}(e+fx)\right),-\right.\right.}$$

input
```
Integrate[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((3 + m + n*p)*AppellF1[(1 + m + n*p)/2, n*p, 1 + m, (3 + m + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(d*Sin[e + f*x])^m*(b*(c
*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p)*((3 + m + n*p)*AppellF1[(1 + m + n*p
)/2, n*p, 1 + m, (3 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - 2*((1 + m)*AppellF1[(3 + m + n*p)/2, n*p, 2 + m, (5 + m + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*p*AppellF1[(3 + m + n*p)/2, 1 + n*
p, 1 + m, (5 + m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4142, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sin(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sin(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (d\sin(e+fx))^m (c\tan(e+fx))^{np} dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (d\sin(e+fx))^m (c\tan(e+fx))^{np} dx$$

$$\Big\downarrow \text{3082}$$

$$d\sin(e+fx)\cos^{np}(e+fx)(d\sin(e+fx))^{-np-1} \left(b(c\tan(e+fx))^n\right)^p \int \cos^{-np}(e+fx)(d\sin(e+fx))^{m+np} dx$$

$$\Big\downarrow \text{3042}$$

$$d\sin(e+fx)\cos^{np}(e+fx)(d\sin(e+fx))^{-np-1}\left(b(c\tan(e+fx))^n\right)^p\int\cos(e+fx)^{-np}(d\sin(e+fx))^{m+np}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\tan(e+fx)(d\sin(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(np+1)}\left(b(c\tan(e+fx))^n\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(m+np\right.}{f(m+np+1)}$$

input
```
Int[(d*Sin[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m + n*
p)/2, (3 + m + n*p)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^m*Tan[e + f*x]*(b*
(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\sin (fx+e))^{m} (b(c\tan (fx+e))^{n})^{p} dx$$

input
```
int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (d\sin(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^{n} b)^{p} (d\sin (fx+e))^{m} \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sin(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \int (b(c\tan (e+fx))^{n})^{p} (d\sin (e+fx))^{m} \, dx$$

input
```
integrate((d*sin(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*(d*sin(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\sin(e+fx))^m\,(b(c\tan(e+fx))^n)^p\;dx=\int ((c\tan{(fx+e)})^n\,b)^p\,(d\sin{(fx+e)})^m\;dx$$

input
```
integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sin(e+fx))^m\,(b(c\tan(e+fx))^n)^p\;dx=\int ((c\tan{(fx+e)})^n\,b)^p\,(d\sin{(fx+e)})^m\;dx$$

input
```
integrate((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^m\,(b(c\tan(e+fx))^n)^p\;dx=\int (d\,\sin{(e+f\,x)})^m\,\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int((d*sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d*sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (d\sin(e+fx))^m\,(b(c\tan(e+fx))^n)^p\,dx = d^m c^{np} b^p \left(\int \tan(fx+e)^{np}\sin(fx+e)^m\,dx\right)$$

input
```
int((d*sin(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sin(e + f*x)**m,x)
```

3.165 $\int \sin^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \sin^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}(3+np),\frac{1}{2}(5+np),-\tan^2(e+fx)\right)\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3+np)}$$

output
```
hypergeom([2, 1/2*n*p+3/2],[1/2*n*p+5/2],-tan(f*x+e)^2)*tan(f*x+e)^3*(b*(c
*tan(f*x+e))^n)^p/f/(n*p+3)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \sin^2(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}(3+np),\frac{1}{2}(5+np),-\tan^2(e+fx)\right)\tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(3+np)}$$

input
```
Integrate[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[2, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4142, 3042, 3071, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2\left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sin^2(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sin(e+fx)^2(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np+2}}{(\tan^2(e+fx)c^2+c^2)^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan^3(e+fx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}(np+3),\frac{1}{2}(np+5),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

input
```
Int[Sin[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[2, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sin (fx+e)^2 (b(c\tan (fx+e))^n)^p \,dx$$

input
```
int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sin^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan\left(fx+e\right))^n b\right)^p \sin\left(fx+e\right)^2 \, dx$$

input
```
integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]

$$\int \sin^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(b(c\tan\left(e+fx\right))^n\right)^p \sin^2\left(e+fx\right) dx$$

input
```
integrate(sin(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sin^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan\left(fx+e\right))^n b\right)^p \sin\left(fx+e\right)^2 \, dx$$

input
```
integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b)^p \sin(fx+e)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \sin(e+f\,x)^2\,\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int(sin(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(sin(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \sin^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=c^{np}b^p\left(\int \tan(fx+e)^{np}\sin(fx+e)^2\,dx\right)$$

input
```
int(sin(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sin(e + f*x)**2,x)
```

3.166 $\int (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+np), \frac{1}{2}(3+np), -\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*t
an(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+np), \frac{1}{2}(3+np), -\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

input
```
Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b(c\tan(e+fx))^n)^p\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b(c\tan(e+fx))^n)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

input
```
Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b(c\tan (fx+e))^n)^p \,dx$$

input
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n b)^p\,dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]

$$\int (b(c\tan(e+fx))^n)^p\,dx = \int (b(c\tan(e+fx))^n)^p\,dx$$

input
```
integrate((b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Maxima [F]

$$\int (b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n b)^p\,dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Giac [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int (b\,(c\tan(e+f\,x))^n)^p \, dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = c^{np} b^p \left(\int \tan(fx+e)^{np} \, dx \right)$$

input
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p),x)
```

3.167 $\int \csc^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \csc^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx = -\frac{\cot(e+fx) (b(c\tan(e+fx))^n)^p}{f(1-np)}$$

output
```
-cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \csc^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \frac{\cot(e+fx) (b(c\tan(e+fx))^n)^p}{f(-1+np)}$$

input
```
Integrate[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4142, 3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \csc^2(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np-2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

input
```
Int[Csc[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
-((Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 13.12 (sec) , antiderivative size = 9186, normalized size of antiderivative = 278.36

method	result	size
risch	Expression too large to display	9186

input
```
int(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{\cos\left(fx+e\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}}{(fnp-f)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
cos(f*x + e)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n*p -
 f)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left(b(c\tan\left(e+fx\right))^n\right)^p\csc^2\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \csc^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{b^pc^{np}(\tan\left(fx+e\right)^n)^p}{(np-1)f\tan\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*f*tan(f*x + e))
```

Giac [F]

$$\int \csc^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int ((c\tan(fx+e))^n b)^p \csc(fx+e)^2 \, dx$$

input
```
integrate(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \frac{\left(b\left(c\tan(e+f\,x)\right)^n\right)^p}{\sin\left(e+f\,x\right)^2} \, dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^2,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = c^{np} b^p \left(\int \tan(fx+e)^{np} \csc(fx+e)^2 \, dx \right)$$

input
```
int(csc(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*csc(e + f*x)**2,x)
```

3.168 $\int \csc^4(e+fx) (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \csc^4(e+fx) (b(c\tan(e+fx))^n)^p \, dx = -\frac{\cot(e+fx) (b(c\tan(e+fx))^n)^p}{f(1-np)} - \frac{\cot^3(e+fx) (b(c\tan(e+fx))^n)^p}{f(3-np)}$$

output
```
-cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)-cot(f*x+e)^3*(b*(c*tan(f*x+e
))^n)^p/f/(-n*p+3)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \csc^4(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \frac{(-2+np+\cos(2(e+fx)))\cot(e+fx)\csc^2(e+fx) (b(c\tan(e+fx))^n)^p}{f(-3+np)(-1+np)}$$

input
```
Integrate[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((-2 + n*p + Cos[2*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2*(b*(c*Tan[e + f
*x])^n)^p)/(f*(-3 + n*p)*(-1 + n*p))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \csc^4(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \frac{(c\tan(e+fx))^{np}}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{c(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (c\tan(e+fx))^{np-4} \left(\tan^2(e+fx)c^2+c^2\right) d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{c(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \left(c^2(c\tan(e+fx))^{np-4}+(c\tan(e+fx))^{np-2}\right) d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(c\tan(e+fx))^{-np} \left(-\frac{c^2(c\tan(e+fx))^{np-3}}{3-np}-\frac{(c\tan(e+fx))^{np-1}}{1-np}\right) \left(b(c\tan(e+fx))^n\right)^p}{f}$$

input
```
Int[Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(c*(b*(c*Tan[e + f*x])^n)^p*(-((c^2*(c*Tan[e + f*x])^(-3 + n*p))/(3 - n*p)
) - (c*Tan[e + f*x])^(-1 + n*p)/(1 - n*p)))/(f*(c*Tan[e + f*x])^(n*p))
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 97.88 (sec) , antiderivative size = 29777, normalized size of antiderivative = 431.55

method	result	size
risch	Expression too large to display	29777

input
```
int(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.51

$$\int \csc^4(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{\left(2\,\cos\left(fx+e\right)^3+(np-3)\cos\left(fx+e\right)\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}}{\left(fn^2p^2-4\,fnp-\left(fn^2p^2-4\,fnp+3\,f\right)\cos\left(fx+e\right)^2+3\,f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
(2*cos(f*x + e)^3 + (n*p - 3)*cos(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(
f*x + e)) + p*log(b))/((f*n^2*p^2 - 4*f*n*p - (f*n^2*p^2 - 4*f*n*p + 3*f)*
cos(f*x + e)^2 + 3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{\frac{b^pc^{np}(\tan(fx+e)^n)^p}{(np-1)\tan(fx+e)}+\frac{b^pc^{np}(\tan(fx+e)^n)^p}{(np-3)\tan(fx+e)^3}}{f}$$

input
```
integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
(b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + b^p*c^(n*p)*(ta
n(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3))/f
```

Giac [F]

$$\int \csc^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan(fx+e))^n b\right)^p\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \frac{\left(b\left(c\tan(e+f\,x)\right)^n\right)^p}{\sin\left(e+f\,x\right)^4} \, dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^4,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = c^{np} b^p \left(\int \tan\left(fx+e\right)^{np} \csc\left(fx+e\right)^4 dx\right)$$

input
```
int(csc(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*csc(e + f*x)**4,x)
```

3.169 $\int \csc^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \csc^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = -\frac{\cot(e+fx) \left(b(c\tan(e+fx))^n\right)^p}{f(1-np)} - \frac{2\cot^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p}{f(3-np)} - \frac{\cot^5(e+fx) \left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

output
```
-cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+1)-2*cot(f*x+e)^3*(b*(c*tan(f*x
+e))^n)^p/f/(-n*p+3)-cot(f*x+e)^5*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+5)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \csc^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{(8-6np+n^2p^2+2(-3+np)\cos(2(e+fx))+\cos(4(e+fx)))\cot(e+fx)\csc^4(e+fx)\left(b(c\tan(e+f\right.}{f(-5+np)(-3+np)(-1+np)}$$

input
```
Integrate[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((8 - 6*n*p + n^2*p^2 + 2*(-3 + n*p)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*
Cot[e + f*x]*Csc[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p)*(-3 +
n*p)*(-1 + n*p))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b(c\tan(e+fx))^n)^p}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \csc^6(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np-6}\left(\tan^2(e+fx)c^2+c^2\right)^2d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \left(c^4(c\tan(e+fx))^{np-6}+2c^2(c\tan(e+fx))^{np-4}+(c\tan(e+fx))^{np}\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(c\tan(e+fx))^{-np}\left(-\frac{c^4(c\tan(e+fx))^{np-5}}{5-np}-\frac{2c^2(c\tan(e+fx))^{np-3}}{3-np}-\frac{(c\tan(e+fx))^{np-1}}{1-np}\right)(b(c\tan(e+fx))^n)^p}{f}$$

input
```
Int[Csc[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(c*(b*(c*Tan[e + f*x])^n)^p*(-((c^4*(c*Tan[e + f*x])^(-5 + n*p))/(5 - n*p)
) - (2*c^2*(c*Tan[e + f*x])^(-3 + n*p))/(3 - n*p) - (c*Tan[e + f*x])^(-1 +
 n*p)/(1 - n*p)))/(f*(c*Tan[e + f*x])^(n*p))
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 60670, normalized size of antiderivative = 583.37

output too large to display

input
```
int(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.73

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\left(8\cos\left(fx+e\right)^5+4\left(np-5\right)\cos\left(fx+e\right)^3+\left(n^2p^2-8\,np+15\right)\cos\left(fx+e\right)\right)e}{\left(fn^3p^3-9\,fn^2p^2+\left(fn^3p^3-9\,fn^2p^2+23\,fnp-15\,f\right)\cos\left(fx+e\right)^4+23\,fnp-2\left(fn^3p^3-9\,fn^2p^2+2\right.\right.}$$

input
```
integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
(8*cos(f*x + e)^5 + 4*(n*p - 5)*cos(f*x + e)^3 + (n^2*p^2 - 8*n*p + 15)*co
s(f*x + e))*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))/((f*n^3*p^
3 - 9*f*n^2*p^2 + (f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)
^4 + 23*f*n*p - 2*(f*n^3*p^3 - 9*f*n^2*p^2 + 23*f*n*p - 15*f)*cos(f*x + e)
^2 - 15*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\frac{b^pc^{np}(\tan(fx+e)^n)^p}{(np-1)\tan(fx+e)}+\frac{2\,b^pc^{np}(\tan(fx+e)^n)^p}{(np-3)\tan(fx+e)^3}+\frac{b^pc^{np}(\tan(fx+e)^n)^p}{(np-5)\tan(fx+e)^5}}{f}$$

input
```
integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
(b^p*c^(n*p)*(tan(f*x + e)^n)^p/((n*p - 1)*tan(f*x + e)) + 2*b^p*c^(n*p)*(
tan(f*x + e)^n)^p/((n*p - 3)*tan(f*x + e)^3) + b^p*c^(n*p)*(tan(f*x + e)^n
)^p/((n*p - 5)*tan(f*x + e)^5))/f
```

Giac [F]

$$\int \csc^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan(fx+e))^n\,b\right)^p\csc(fx+e)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \frac{\left(b\,(c\tan(e+f\,x))^n\right)^p}{\sin\,(e+f\,x)^6} \, dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^6,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = c^{np} b^p \left(\int \tan\,(fx+e)^{np} \csc\,(fx+e)^6 \, dx\right)$$

input
```
int(csc(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*csc(e + f*x)**6,x)
```

3.170 $\int \sin^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \sin^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+np), \frac{1}{2}(4+np), \frac{1}{2}(6+np), \sin^2(e+fx)\right) \sin^3(e+fx) \tan}{f(4+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+2, 1/2*n*p+1/2],[1/2*n*p+3
],sin(f*x+e)^2)*sin(f*x+e)^3*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.22 (sec) , antiderivative size = 506, normalized size of antiderivative = 5.44

$$\int \sin^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{}{f(2+np) \left(2(4+np) \text{AppellF1}\left(1+\frac{np}{2}, np, 3, 2+\frac{np}{2}, \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) \cos^2\left(\frac{1}{2}(e+\right.\right.}$$

input
```
Integrate[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(4*(4 + n*p)*(AppellF1[1 + (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*S
in[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1
[1 + (n*p)/2, n*p, 3, 2 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Cos[(e + f*x)/2]^2 - 2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 4, 2 + (n*p)
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*App
ellF1[2 + (n*p)/2, n*p, 4, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] - 4*AppellF1[2 + (n*p)/2, n*p, 5, 3 + (n*p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + n*p*(-AppellF1[2 + (n*p)/2, 1 + n*p, 3, 3 + (n*p)/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[2 + (n*p)/2, 1 + n*p
, 4, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e +
 f*x])))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3 \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \sin^3(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \sin(e+fx)^3(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3082}$$

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int\cos^{-np}(e+fx)\sin^{np+3}(e+fx)dx$$

$\Big\downarrow$ 3042

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int\cos(e+fx)^{-np}\sin(e+fx)^{np+3}dx$$

$\Big\downarrow$ 3057

$$\frac{\sin^3(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4),\frac{1}{2}(np+6),\sin^2(e+fx\right.}{f(np+4)}$$

input
```
Int[Sin[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2
, (6 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f
*x])^n)^p)/(f*(4 + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sin \left(f x+e\right)^{3} \left(b \left(c \tan \left(f x+e\right)\right)^{n}\right)^{p}d x$$

input
```
int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sin^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(\left(c \tan \left(f x+e\right)\right)^{n} b\right)^{p} \sin \left(f x+e\right)^{3} \, dx$$

input
```
integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\sin\left(fx+e\right)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

Giac [F]

$$\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\sin\left(fx+e\right)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \sin\left(e+f\,x\right)^3\,{\left(b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

input
```
int(sin(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(sin(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \sin^3(e+fx) (b(c\tan(e+fx))^n)^p \, dx = c^{np}b^p\left(\int \tan(fx+e)^{np}\sin(fx+e)^3 \, dx\right)$$

input
```
int(sin(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sin(e + f*x)**3,x)
```

3.171 $\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np),\frac{1}{2}(4+np),\sin^2(e+fx)\right)\sin(e+fx)\tan}{f(2+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p+1, 1/2*n*p+1/2],[1/2*n*p+2
],sin(f*x+e)^2)*sin(f*x+e)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.12

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{8(4+np)\,\text{AppellF1}\left(1+\frac{n}{}\right.}{f(2+np)\left(2(4+np)\,\text{AppellF1}\left(1+\frac{np}{2},np,2,2+\frac{np}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+\right.\right.}$$

input
```
Integrate[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(8*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*(c*Tan[e +
 f*x])^n)^p)/(f*(2 + n*p)*(2*(4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 2, 2 + (
n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2
*AppellF1[2 + (n*p)/2, n*p, 3, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] - n*p*AppellF1[2 + (n*p)/2, 1 + n*p, 2, 3 + (n*p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4142, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sin(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sin(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3082}$$

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int \cos^{-np}(e+fx)\sin^{np+1}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int \cos(e+fx)^{-np}\sin(e+fx)^{np+1}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin(e+fx)\tan(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2),\frac{1}{2}(np+4),\sin^2(e+fx)\right)}{f(np+2)}$$

input
```
Int[Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2
, (4 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(2 + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sin(fx+e)\left(b(c\tan(fx+e))^n\right)^p dx$$

input
```
int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left((c\tan(fx+e))^n b\right)^p \sin(fx+e)\,dx$$

input
```
integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Sympy [F]

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left(b(c\tan(e+fx))^n\right)^p \sin(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*sin(e + f*x), x)
```

Maxima [F]

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\sin\left(fx+e\right)\,dx$$

input
```
integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Giac [F]

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\sin\left(fx+e\right)\,dx$$

input
```
integrate(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sin(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \sin\left(e+f\,x\right)\,\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)^p\,dx$$

input
```
int(sin(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(sin(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \sin(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = c^{np}b^p \left(\int \tan(fx+e)^{np}\sin(fx+e)\,dx\right)$$

input
```
int(sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sin(e + f*x),x)
```

3.172 $\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\,\text{Hypergeometric2F1}\left(\frac{np}{2},\frac{1}{2}(1+np),\frac{1}{2}(2+np),\sin^2(e+fx)\right)\sec(e+fx)\,(b(c\tan(e\ \cdots}{fnp}$$

output

```
(cos(f*x+e)^2)^(1/2*n*p+1/2)*hypergeom([1/2*n*p, 1/2*n*p+1/2],[1/2*n*p+1],
sin(f*x+e)^2)*sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/n/p
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{np}{2},np,1+\frac{np}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^{np}(b(c\tan(e+fx\ \cdots}{fnp}$$

input

```
Integrate[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/2]^2]*(Cos[e +
f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4142, 3042, 3081, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\sin(e+fx)}dx$$

$$\Big\downarrow \ 4142$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \csc(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \ 3042$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\sin(e+fx)}dx$$

$$\Big\downarrow \ 3081$$

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int \cos^{-np}(e+fx)\sin^{np-1}(e+fx)dx$$

$$\Big\downarrow \ 3042$$

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int \cos(e+fx)^{-np}\sin(e+fx)^{np-1}dx$$

$$\Big\downarrow \ 3057$$

$$\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\,\text{Hypergeometric2F1}\left(\frac{np}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+2),\sin^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right.}{fnp}$$

input
```
Int[Csc[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (2
 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \csc(fx+e)\left(b(c\tan(fx+e))^{n}\right)^{p}dx$$

input
```
int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^{n}\right)^{p}\,dx=\int \left((c\tan(fx+e))^{n}b\right)^{p}\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Sympy [F]

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^{n}\right)^{p}\,dx=\int \left(b(c\tan(e+fx))^{n}\right)^{p}\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p \csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p \csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(b\left(c\tan(e+f\,x)\right)^n\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x),x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\csc\left(fx+e\right)dx\right)$$

input
```
int(csc(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*csc(e + f*x),x)
```

3.173 $\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}\csc^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np),\frac{np}{2},\sin^2(e+fx)\right)\sec(e}{f(2-np)}$$

output
```
-(cos(f*x+e)^2)^(1/2*n*p+1/2)*csc(f*x+e)^2*hypergeom([1/2*n*p-1, 1/2*n*p+1
/2],[1/2*n*p],sin(f*x+e)^2)*sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(-n*p+2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 13.65 (sec) , antiderivative size = 1399, normalized size of antiderivative = 15.21

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[(e + f*x)/2]^2*Hypergeometric2F1[n*p, -1 + (n*p)/2, (n*p)/2, Tan[(e +
 f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n*p)*(b*(c*Tan[e + f*x])^n)
^p)/(f*(-8 + 4*n*p)) + ((4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]^2*(b*(c*Tan[e + f
*x])^n)^p)/(8*f*(2 + n*p)*((4 + n*p)*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + Appell
F1[2 + (n*p)/2, n*p, 2, 3 + (n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(-1 + Cos[e + f*x]) + 2*n*p*AppellF1[2 + (n*p)/2, 1 + n*p, 1, 3 + (n*p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^2)) + (Hype
rgeometric2F1[n*p, 1 + (n*p)/2, 2 + (n*p)/2, Tan[(e + f*x)/2]^2]*(Cos[e +
f*x]*Sec[(e + f*x)/2]^2)^(n*p)*Tan[(e + f*x)/2]^2*(b*(c*Tan[e + f*x])^n)^p
)/(f*(8 + 4*n*p)) + (Cot[(e + f*x)/2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(n
*p)*((2 + n*p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2, Tan[(e + f*x)/
2]^2] - n*p*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(n*p)*(b*(c*Tan[e +
 f*x])^n)^p)/(8*f*n*p*(2 + n*p)*(((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 +
n*p)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2])*((2 + n*p)*Hypergeometric2F1[(n*p)/2, n*p, 1 + (n*p)/2,
 Tan[(e + f*x)/2]^2] - n*p*AppellF1[1 + (n*p)/2, n*p, 1, 2 + (n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^(...
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 3042, 3081, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \csc^3(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{3081}$$

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int \cos^{-np}(e+fx)\sin^{np-3}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-np}(e+fx)\cos^{np}(e+fx)\left(b(c\tan(e+fx))^n\right)^p\int \cos(e+fx)^{-np}\sin(e+fx)^{np-3}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{\csc^2(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1),\frac{np}{2},\sin^2(e+fx)\right)(b(c}{f(2-np)}$$

input
```
Int[Csc[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
-(((Cos[e + f*x]^2)^((1 + n*p)/2)*Csc[e + f*x]^2*Hypergeometric2F1[(-2 + n
*p)/2, (1 + n*p)/2, (n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(b*(c*Tan[e + f*
x])^n)^p)/(f*(2 - n*p)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \csc(fx+e)^3 \left(b(c\tan(fx+e))^n\right)^p dx$$

input
```
int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \csc^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan(fx+e))^n b\right)^p \csc(fx+e)^3 \, dx$$

input
```
integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Sympy [F]

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left(b(c\tan{(e+fx)})^n\right)^p\csc^3{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan{(fx+e)})^n b\right)^p\csc{(fx+e)}^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan{(fx+e)})^n b\right)^p\csc{(fx+e)}^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(b\left(c\tan(e+f\,x)\right)^n\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^3,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\csc\left(fx+e\right)^3dx\right)$$

input
```
int(csc(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*csc(e + f*x)**3,x)
```

3.174 $\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \text{Int}((d\sin(e+fx))^m (a+b\tan^n(e+fx))^p, x)$$

output
```
Defer(Int)((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx$$

input
```
Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
```

output
```
Integrate[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (d\sin(e+fx))^m \left(a+b\tan(e+fx)^n\right)^p dx$$

$$\Big\downarrow \text{ 4151}$$

$$\int (d\sin(e+fx))^m \left(a+b\tan^n(e+fx)\right)^p dx$$

input
```
Int[(d*Sin[e + f*x])^m*(a + b*Tan[e + f*x]^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4151
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sin[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sin (fx+e))^m (a+b\tan (fx+e)^n)^p \, dx$$

input
```
int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

output
```
int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \int (b\tan (fx+e)^n+a)^p (d\sin (fx+e))^m \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**m*(a+b*tan(f*x+e)**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 6.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \int (b\tan(fx+e)^n + a)^p (d\sin(fx+e))^m \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 6.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \int (b\tan(fx+e)^n + a)^p (d\sin(fx+e))^m \, dx$$

input
```
integrate((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^n + a)^p*(d*sin(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 8.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sin(e+fx))^m (a+b\tan^n(e+fx))^p \, dx = \int (d\sin(e+fx))^m (a+b\tan(e+fx)^n)^p \, dx$$

input
```
int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^n)^p,x)
```

output
```
int((d*sin(e + f*x))^m*(a + b*tan(e + f*x)^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (d\sin(e+fx))^m\,(a+b\tan^n(e+fx))^p\,dx$$
$$= d^m\left(\int \sin\left(fx+e\right)^m\left(\tan\left(fx+e\right)^n b+a\right)^p dx\right)$$

input
```
int((d*sin(f*x+e))^m*(a+b*tan(f*x+e)^n)^p,x)
```

output
```
d**m*int(sin(e + f*x)**m*(tan(e + f*x)**n*b + a)**p,x)
```

3.175 $\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \frac{(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+2p)} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+2p), \frac{1}{2}(1-m+2p), \frac{1}{2}(3+2p), \sin^2(e\right.}{f(1+2p)}$$

output
```
(d*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2-1/2*m+p)*hypergeom([1/2+p, 1/2-1/2*m+
p],[3/2+p],sin(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \frac{(d\cos(e+fx))^m \text{Hypergeometric2F1}\left(1+\frac{m}{2}, \frac{1}{2}+p, \frac{3}{2}+p, -\tan^2(e+fx)\right) \sec^2(e+fx)^{m/2} \tan(e+fx}{f(1+2p)}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
((d*Cos[e + f*x])^m*Hypergeometric2F1[1 + m/2, 1/2 + p, 3/2 + p, -Tan[e +
f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 +
2*p))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4141, 3042, 3083, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(e+fx)\right)^p (d\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^2\right)^p (d\cos(e+fx))^m dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\int (d\cos(e+fx))^m\tan^{2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\int (d\cos(e+fx))^m\tan(e+fx)^{2p}dx$$

$$\Big\downarrow \text{3083}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p(d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m\int\left(\frac{\sec(e+fx)}{d}\right)^{-m}\tan^{2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p(d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m\int\left(\frac{\sec(e+fx)}{d}\right)^{-m}\tan(e+fx)^{2p}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\cos(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(-m+2p+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m\right.}{f(2p+1)}$$

input
```
Int[(d*Cos[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + 2*p)/2)*Hypergeometric2F1[(
1 + 2*p)/2, (1 - m + 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*
Tan[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3083
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(d\cos\left(fx+e\right)\right)^{m} \left(b\tan\left(fx+e\right)^{2}\right)^{p} dx$$

input
```
int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^{2}\right)^{p} \left(d\cos\left(fx+e\right)\right)^{m} \, dx$$

input
```
integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan^{2}\left(e+fx\right)\right)^{p} \left(d\cos\left(e+fx\right)\right)^{m} \, dx$$

input
```
integrate((d*cos(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((b*tan(e + f*x)**2)**p*(d*cos(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cos\left(fx+e\right)\right)^m \,dx$$

input
```
integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\cos\left(fx+e\right)\right)^m \,dx$$

input
```
integrate((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(d\,\cos\left(e+f\,x\right)\right)^m \left(b\tan(e+f\,x)^2\right)^p \,dx$$

input
```
int((d*cos(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*cos(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx = d^m b^p \left(\int \tan(fx+e)^{2p}\cos(fx+e)^m\,dx\right)$$

input
```
int((d*cos(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*b**p*int(tan(e + f*x)**(2*p)*cos(e + f*x)**m,x)
```

3.176 $\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{\operatorname{AppellF1}\left(\frac{1}{2},\frac{2+m}{2},-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)\,(a}{f}$$

output
```
AppellF1(1/2,1+1/2*m,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*cos(f*x+e)
)^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(((a+b*tan(f*
x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2033 vs. 2(109) = 218.

Time = 15.37 (sec) , antiderivative size = 2033, normalized size of antiderivative = 18.65

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(-1 - m/2)*Tan[e + f*x]*(a + b*
Tan[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (
4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]
^2)*((6*a*b*p*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/a)]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/((Sec[e +
f*x]^2)^(m/2)*(3*a*AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)) + (3*a*
AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*(a + b*Tan[e + f*x]^2)^p)/((Sec[e + f*x]^2)^(m/2)*(3*a*AppellF1[1/2, (2
+ m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*Appell
F1[3/2, (2 + m)/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] -
 a*(2 + m)*AppellF1[3/2, (4 + m)/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/a)])*Tan[e + f*x]^2)) + (6*a*(-1 - m/2)*AppellF1[1/2, (2 + m)/2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 -
m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, (2 + m...
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4152, 3042, 4162, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (d\cos(e+fx))^m \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{ 4152}$$

$$(d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \int \left(\frac{\sec(e+fx)}{d}\right)^{-m} \left(b\tan^2(e+fx)+a\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \int \left(\frac{\sec(e+fx)}{d}\right)^{-m} \left(b\tan(e+fx)^2+a\right)^p dx$$

$$\Big\downarrow \text{4162}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m \int \left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1} \left(b\tan^2(e+fx)+a\right)^p d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \int \left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1} \left(\frac{b\tan^2(e-}{a}\right.}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},\frac{m+2}{2},-p,\right.}{f}$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, (2 + m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a
)]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(a + b*Tan[e + f
*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[(d*Cos[e + f*x])^FracPart[m]*(Sec
[e + f*x]/d)^FracPart[m]   Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sec[e + f*x]/d
)^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

rule 4162
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff
*((d*Sec[e + f*x])^m/(f*(Sec[e + f*x]^2)^(m/2)))   Subst[Int[(1 + ff^2*x^2)
^(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\cos(fx+e))^m \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\cos\left(fx+e\right))^m dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\cos\left(fx+e\right))^m dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\cos\left(fx+e\right))^m \, dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \int (d\,\cos\left(e+f\,x\right))^m \left(b\tan(e+f\,x)^2+a\right)^p dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= d^m \left(\int \left(\tan\left(fx+e\right)^2 b+a\right)^p \cos\left(fx+e\right)^m dx\right)$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*int((tan(e + f*x)**2*b + a)**p*cos(e + f*x)**m,x)
```

3.177 $\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \,dx$$
$$= \frac{(d\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+np)} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+np), \frac{1}{2}(1-m+np), \frac{1}{2}(3+np), \sin^2(\right.}{f(1+np)}$$

output
```
(d*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2*n*p-1/2*m+1/2)*hypergeom([1/2*n*p+1/2
, 1/2*n*p-1/2*m+1/2],[1/2*n*p+3/2],sin(f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+
e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \,dx$$
$$= \frac{(d\cos(e+fx))^m \text{Hypergeometric2F1}\left(\frac{2+m}{2}, \frac{1}{2}(1+np), \frac{1}{2}(3+np), -\tan^2(e+fx)\right) \sec^2(e+fx)^{m/2} \tan(}{f(1+np)}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((d*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n*p)/2, (3 + n*p)/2,
 -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*(c*Tan[e + f*x])^
n)^p)/(f*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4142, 3042, 3083, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cos(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (d\cos(e+fx))^m(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (d\cos(e+fx))^m(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3083}$$

$$(d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int\left(\frac{\sec(e+fx)}{d}\right)^{-m}(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int\left(\frac{\sec(e+fx)}{d}\right)^{-m}(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan(e+fx)(d\cos(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(-m+np+1)}\left(b(c\tan(e+fx))^n\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(-\right.}{f(np+1)}$$

input
```
Int[(d*Cos[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((d*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n*p)/2)*Hypergeometric2F1[(
1 + n*p)/2, (1 - m + n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*
(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3083
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\cos(fx+e))^m (b(c\tan(fx+e))^n)^p \, dx$$

input
```
int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p (d\cos(fx+e))^m \, dx$$

input
```
integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \int (b(c\tan(e+fx))^n)^p (d\cos(e+fx))^m \, dx$$

input
```
integrate((d*cos(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*(d*cos(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \,dx = \int ((c\tan(fx+e))^n b)^p (d\cos(fx+e))^m \,dx$$

input
```
integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \,dx = \int ((c\tan(fx+e))^n b)^p (d\cos(fx+e))^m \,dx$$

input
```
integrate((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \,dx = \int (d\cos(e+f\,x))^m (b\,(c\tan(e+f\,x))^n)^p \,dx$$

input
```
int((d*cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d*cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = d^m c^{np} b^p \left(\int \tan(fx+e)^{np} \cos(fx+e)^m \, dx\right)$$

input
```
int((d*cos(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cos(e + f*x)**m,x)
```

3.178 $\int (d\cos(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (d\cos(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx = (d\cos(e+fx))^m\left(\frac{\sec(e+fx)}{d}\right)^m \text{Int}\left(\left(\frac{\sec(e+fx)}{d}\right)^{-m}\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

output
```
(d*cos(f*x+e))^m*(sec(f*x+e)/d)^m*Defer(Int)((a+b*(c*tan(f*x+e))^n)^p/((se
c(f*x+e)/d)^m),x)
```

Mathematica [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cos(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int (d\cos(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4152, 3042, 4163}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(e+fx))^m (a+b(c\tan(e+fx))^n)^p\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cos(e+fx))^m (a+b(c\tan(e+fx))^n)^p dx$$

$$\Big\downarrow \text{4152}$$

$$(d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \int \left(\frac{\sec(e+fx)}{d}\right)^{-m} (b(c\tan(e+fx))^n+a)^p dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \int \left(\frac{\sec(e+fx)}{d}\right)^{-m} (b(c\tan(e+fx))^n+a)^p dx$$

$$\Big\downarrow \text{4163}$$

$$(d\cos(e+fx))^m \left(\frac{\sec(e+fx)}{d}\right)^m \int \left(\frac{\sec(e+fx)}{d}\right)^{-m} (b(c\tan(e+fx))^n+a)^p dx$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[(d*Cos[e + f*x])^FracPart[m]*(Sec
[e + f*x]/d)^FracPart[m]   Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sec[e + f*x]/d
)^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

rule 4163
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(d \cos \left(f x +e \right)\right)^{m} \left(a +b \left(c \tan \left(f x +e \right)\right)^{n}\right)^{p}d x$$

input
```
int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cos(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx = \int \left(\left(c \tan \left(f x + e\right)\right)^{n} b + a\right)^{p} \left(d \cos \left(f x + e\right)\right)^{m} \,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx = \text{Timed out}$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 6.61 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cos(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n b + a)^p (d\cos(fx+e))^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 45.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cos(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx$$
$$=\int ((c\tan{(fx+e)})^n\,b+a)^p(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cos(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 8.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cos(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx$$
$$=\int (d\,\cos{(e+f\,x)})^m\,(a+b\,(c\tan(e+f\,x))^n)^p\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int (d\cos(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$
$$= d^m \left(\int (c^n \tan(fx+e)^n b + a)^p \cos(fx+e)^m \, dx \right)$$

input
```
int((d*cos(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*int((c**n*tan(e + f*x)**n*b + a)**p*cos(e + f*x)**m,x)
```

3.179 $\int (a+a\tan^2(c+dx))^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int (a+a\tan^2(c+dx))^4\,dx=\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{d}+\frac{3a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}$$

output
```
a^4*tan(d*x+c)/d+a^4*tan(d*x+c)^3/d+3/5*a^4*tan(d*x+c)^5/d+1/7*a^4*tan(d*x
+c)^7/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int (a+a\tan^2(c+dx))^4\,dx=\frac{a^4\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^4,x]
```

output
```
(a^4*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^
7/7))/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4140, 27, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \tan^2(c+dx)+a\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \tan(c+dx)^2+a\right)^4 dx$$

$$\Big\downarrow \text{4140}$$

$$\int a^4 \sec^8(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a^4 \int \sec^8(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^4 \int \csc\left(c+dx+\frac{\pi}{2}\right)^8 dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a^4 \int \left(\tan^6(c+dx)+3\tan^4(c+dx)+3\tan^2(c+dx)+1\right) d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\left(-\frac{1}{7}\tan^7(c+dx)-\frac{3}{5}\tan^5(c+dx)-\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^4,x]
```

output
```
-((a^4*(-Tan[c + d*x] - Tan[c + d*x]^3 - (3*Tan[c + d*x]^5)/5 - Tan[c + d*
x]^7/7))/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{a^{4}\left(\frac{\tan(dx+c)^{7}}{7}+\frac{3\tan(dx+c)^{5}}{5}+\tan(dx+c)^{3}+\tan(dx+c)\right)}{d}$
default	$\frac{a^{4}\left(\frac{\tan(dx+c)^{7}}{7}+\frac{3\tan(dx+c)^{5}}{5}+\tan(dx+c)^{3}+\tan(dx+c)\right)}{d}$
parallelrisch	$\frac{5a^{4}\tan(dx+c)^{7}+21a^{4}\tan(dx+c)^{5}+35a^{4}\tan(dx+c)^{3}+35a^{4}\tan(dx+c)}{35d}$
risch	$\frac{32ia^{4}\left(35\,\mathrm{e}^{6i(dx+c)}+21\,\mathrm{e}^{4i(dx+c)}+7\,\mathrm{e}^{2i(dx+c)}+1\right)}{35d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{7}}$
norman	$\frac{a^{4}\tan(dx+c)}{d}+\frac{a^{4}\tan(dx+c)^{3}}{d}+\frac{3a^{4}\tan(dx+c)^{5}}{5d}+\frac{a^{4}\tan(dx+c)^{7}}{7d}$
parts	$x\,a^{4}+\frac{a^{4}\left(\frac{\tan(dx+c)^{7}}{7}-\frac{\tan(dx+c)^{5}}{5}+\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{4a^{4}(\tan(dx+c)-\arctan(\mathrm{ta}}{d}$

input
```
int((a+a*tan(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^4*(1/7*tan(d*x+c)^7+3/5*tan(d*x+c)^5+tan(d*x+c)^3+tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \left(a+a\tan^{2}(c+dx)\right)^{4}\,dx = \frac{5\,a^{4}\tan\left(dx+c\right)^{7}+21\,a^{4}\tan\left(dx+c\right)^{5}+35\,a^{4}\tan\left(dx+c\right)^{3}+35\,a^{4}\tan\left(dx+c\right)}{35\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/35*(5*a^4*tan(d*x + c)^7 + 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^3
 + 35*a^4*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \left(a + a\tan^2(c + dx)\right)^4 \, dx = \begin{cases} \frac{a^4 \tan^7(c+dx)}{7d} + \frac{3a^4 \tan^5(c+dx)}{5d} + \frac{a^4 \tan^3(c+dx)}{d} + \frac{a^4 \tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\tan^2(c) + a)^4 & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*tan(d*x+c)**2)**4,x)
```

output
```
Piecewise((a**4*tan(c + d*x)**7/(7*d) + 3*a**4*tan(c + d*x)**5/(5*d) + a**
4*tan(c + d*x)**3/d + a**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)
**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.42

$$\int \left(a + a\tan^2(c + dx)\right)^4 \, dx = a^4x + \frac{\left(15 \tan(dx+c)^7 - 21 \tan(dx+c)^5 + 35 \tan(dx+c)^3 + 105\,dx + 105\,c - 105 \tan(dx+c)\right)a^4}{105\,d} + \frac{4\left(3 \tan(dx+c)^5 - 5 \tan(dx+c)^3 - 15\,dx - 15\,c + 15 \tan(dx+c)\right)a^4}{15\,d} + \frac{2\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3 \tan(dx+c)\right)a^4}{d} - \frac{4\left(dx + c - \tan(dx+c)\right)a^4}{d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 +
 105*d*x + 105*c - 105*tan(d*x + c))*a^4/d + 4/15*(3*tan(d*x + c)^5 - 5*ta
n(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^4/d + 2*(tan(d*x + c)^3
+ 3*d*x + 3*c - 3*tan(d*x + c))*a^4/d - 4*(d*x + c - tan(d*x + c))*a^4/d
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \left(a+a\tan^2(c+dx)\right)^4\,dx = \frac{5\,a^4\tan\left(dx+c\right)^7+21\,a^4\tan\left(dx+c\right)^5+35\,a^4\tan\left(dx+c\right)^3+35\,a^4\tan\left(dx+c\right)}{35\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/35*(5*a^4*tan(d*x + c)^7 + 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^3
 + 35*a^4*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(a+a\tan^2(c+dx)\right)^4\,dx = \frac{\frac{a^4\tan(c+d\,x)^7}{7}+\frac{3\,a^4\tan(c+d\,x)^5}{5}+a^4\tan(c+d\,x)^3+a^4\tan(c+d\,x)}{d}$$

input
```
int((a + a*tan(c + d*x)^2)^4,x)
```

output
```
(a^4*tan(c + d*x) + a^4*tan(c + d*x)^3 + (3*a^4*tan(c + d*x)^5)/5 + (a^4*t
an(c + d*x)^7)/7)/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int (a+a\tan^2(c+dx))^4\,dx$$
$$=\frac{\tan(dx+c)\,a^4\left(5\tan(dx+c)^6+21\tan(dx+c)^4+35\tan(dx+c)^2+35\right)}{35d}$$

input
```
int((a+a*tan(d*x+c)^2)^4,x)
```

output
```
(tan(c + d*x)*a**4*(5*tan(c + d*x)**6 + 21*tan(c + d*x)**4 + 35*tan(c + d*
x)**2 + 35))/(35*d)
```

3.180 $\int (a+a\tan^2(c+dx))^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int (a+a\tan^2(c+dx))^3 \, dx = \frac{a^3\tan(c+dx)}{d}+\frac{2a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan^5(c+dx)}{5d}$$

output
```
a^3*tan(d*x+c)/d+2/3*a^3*tan(d*x+c)^3/d+1/5*a^3*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int (a+a\tan^2(c+dx))^3 \, dx = \frac{a^3\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^3,x]
```

output
```
(a^3*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4140, 27, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\tan^2(c+dx)+a\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\tan(c+dx)^2+a\right)^3dx$$

$$\Big\downarrow \text{4140}$$

$$\int a^3\sec^6(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a^3\int \sec^6(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a^3\int \left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^3,x]
```

output
```
-((a^3*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Tan[c + d*x]^5/5))/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{a^{3}\left(\frac{\tan(dx+c)^{5}}{5}+\frac{2\tan(dx+c)^{3}}{3}+\tan(dx+c)\right)}{d}$
default	$\frac{a^{3}\left(\frac{\tan(dx+c)^{5}}{5}+\frac{2\tan(dx+c)^{3}}{3}+\tan(dx+c)\right)}{d}$
parallelrisch	$\frac{3a^{3}\tan(dx+c)^{5}+10a^{3}\tan(dx+c)^{3}+15a^{3}\tan(dx+c)}{15d}$
norman	$\frac{a^{3}\tan(dx+c)}{d}+\frac{2a^{3}\tan(dx+c)^{3}}{3d}+\frac{a^{3}\tan(dx+c)^{5}}{5d}$
risch	$\frac{16ia^{3}\left(10\,\mathrm{e}^{4i(dx+c)}+5\,\mathrm{e}^{2i(dx+c)}+1\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$
parts	$x\,a^{3}+\frac{a^{3}\left(\frac{\tan(dx+c)^{5}}{5}-\frac{\tan(dx+c)^{3}}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{3a^{3}(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+$

input
```
int((a+a*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^3*(1/5*tan(d*x+c)^5+2/3*tan(d*x+c)^3+tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \left(a+a\tan^2(c+dx)\right)^3\,dx=\frac{3\,a^3\tan\left(dx+c\right)^5+10\,a^3\tan\left(dx+c\right)^3+15\,a^3\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(3*a^3*tan(d*x + c)^5 + 10*a^3*tan(d*x + c)^3 + 15*a^3*tan(d*x + c))/
d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \left(a+a\tan^2(c+dx)\right)^3\,dx=\begin{cases}\frac{a^3\tan^5\left(c+dx\right)}{5d}+\frac{2a^3\tan^3\left(c+dx\right)}{3d}+\frac{a^3\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\left(a\tan^2\left(c\right)+a\right)^3 & \text{otherwise}\end{cases}$$

input
```
integrate((a+a*tan(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*tan(c + d*x)**5/(5*d) + 2*a**3*tan(c + d*x)**3/(3*d) + a**
3*tan(c + d*x)/d, Ne(d, 0)), (x*(a*tan(c)**2 + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(46) = 92$.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.04

$$\int \left(a + a\tan^2(c + dx)\right)^3 \, dx$$
$$= a^3x + \frac{\left(3\,\tan\left(dx + c\right)^5 - 5\,\tan\left(dx + c\right)^3 - 15\,dx - 15\,c + 15\,\tan\left(dx + c\right)\right)a^3}{15\,d}$$
$$+ \frac{\left(\tan\left(dx + c\right)^3 + 3\,dx + 3\,c - 3\,\tan\left(dx + c\right)\right)a^3}{d} - \frac{3\left(dx + c - \tan\left(dx + c\right)\right)a^3}{d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan
(d*x + c))*a^3/d + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3/d -
 3*(d*x + c - tan(d*x + c))*a^3/d
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \left(a+a\tan^2(c+dx)\right)^3 \, dx = \frac{3\,a^3\tan\left(dx + c\right)^5 + 10\,a^3\tan\left(dx + c\right)^3 + 15\,a^3\tan\left(dx + c\right)}{15\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(3*a^3*tan(d*x + c)^5 + 10*a^3*tan(d*x + c)^3 + 15*a^3*tan(d*x + c))/
d
```

Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \left(a+a\tan^2(c+dx)\right)^3\,dx=\frac{a^3\tan(c+d\,x)\,\left(3\tan(c+d\,x)^4+10\tan(c+d\,x)^2+15\right)}{15\,d}$$

input
```
int((a + a*tan(c + d*x)^2)^3,x)
```

output
```
(a^3*tan(c + d*x)*(10*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + 15))/(15*d)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \left(a+a\tan^2(c+dx)\right)^3\,dx=\frac{\tan\left(dx+c\right)a^3\left(3\tan\left(dx+c\right)^4+10\tan\left(dx+c\right)^2+15\right)}{15d}$$

input
```
int((a+a*tan(d*x+c)^2)^3,x)
```

output
```
(tan(c + d*x)*a**3*(3*tan(c + d*x)**4 + 10*tan(c + d*x)**2 + 15))/(15*d)
```

3.181 $\int (a+a\tan^2(c+dx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int (a+a\tan^2(c+dx))^2\,dx=\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}$$

output
```
a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int (a+a\tan^2(c+dx))^2\,dx=\frac{a^2\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4140, 27, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\tan^2(c+dx)+a\right)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\tan(c+dx)^2+a\right)^2dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int a^2\sec^4(c+dx)dx$$

$$\Big\downarrow \text{ 27}$$

$$a^2\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{a^2\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^2,x]
```

output
```
-((a^2*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{a^{2}\left(\frac{\tan\left(dx+c\right)^{3}}{3}+\tan\left(dx+c\right)\right)}{d}$	25
default	$\frac{a^{2}\left(\frac{\tan\left(dx+c\right)^{3}}{3}+\tan\left(dx+c\right)\right)}{d}$	25
parallelrisch	$\frac{a^{2}\tan\left(dx+c\right)^{3}+3a^{2}\tan\left(dx+c\right)}{3d}$	30
norman	$\frac{a^{2}\tan\left(dx+c\right)}{d}+\frac{a^{2}\tan\left(dx+c\right)^{3}}{3d}$	31
risch	$\frac{4ia^{2}\left(3\,{\mathrm e}^{2i\left(dx+c\right)}+1\right)}{3d\left({\mathrm e}^{2i\left(dx+c\right)}+1\right)^{3}}$	36
parts	$x\,a^{2}+\frac{a^{2}\left(\frac{\tan\left(dx+c\right)^{3}}{3}-\tan\left(dx+c\right)+\arctan\left(\tan\left(dx+c\right)\right)\right)}{d}+\frac{2a^{2}\left(\tan\left(dx+c\right)-\arctan\left(\tan\left(dx+c\right)\right)\right)}{d}$	64

input
```
int((a+a*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(a+a\tan^2(c+dx)\right)^2\,dx=\frac{a^2\tan\left(dx+c\right)^3+3\,a^2\tan\left(dx+c\right)}{3\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \left(a+a\tan^2(c+dx)\right)^2\,dx=\begin{cases}\frac{a^2\tan^3\left(c+dx\right)}{3d}+\frac{a^2\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\left(a\tan^2\left(c\right)+a\right)^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a+a*tan(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*tan(c + d*x)**3/(3*d) + a**2*tan(c + d*x)/d, Ne(d, 0)), (x
*(a*tan(c)**2 + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \left(a+a\tan^2(c+dx)\right)^2\,dx=a^2x+\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2}{3\,d}-\frac{2\left(dx+c-\tan\left(dx+c\right)\right)a^2}{d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2/d - 2*(d*x
 + c - tan(d*x + c))*a^2/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(a+a\tan^2(c+dx)\right)^2\,dx=\frac{a^2\tan\left(dx+c\right)^3+3\,a^2\tan\left(dx+c\right)}{3\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+a\tan^2(c+dx)\right)^2\,dx=\frac{a^2\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+3\right)}{3\,d}$$

input
```
int((a + a*tan(c + d*x)^2)^2,x)
```

output
```
(a^2*tan(c + d*x)*(tan(c + d*x)^2 + 3))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a + a\tan^2(c + dx)\right)^2 \, dx = \frac{\tan(dx + c)\, a^2 \left(\tan(dx + c)^2 + 3\right)}{3d}$$

input
```
int((a+a*tan(d*x+c)^2)^2,x)
```

output
```
(tan(c + d*x)*a**2*(tan(c + d*x)**2 + 3))/(3*d)
```

3.182 $\int \frac{1}{a+a\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{a+a\tan^2(c+dx)}\,dx = \frac{x}{2a}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

output
```
1/2*x/a+1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+a\tan^2(c+dx)}\,dx = \frac{2(c+dx)+\sin(2(c+dx))}{4ad}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(-1),x]
```

output
```
(2*(c + d*x) + Sin[2*(c + d*x)])/(4*a*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4140, 27, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\tan^2(c+dx)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a\tan(c+dx)^2+a}dx$$

$$\Big\downarrow \ 4140$$

$$\int \frac{\cos^2(c+dx)}{a}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \cos^2(c+dx)dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}}{a}$$

$$\Big\downarrow \ 24$$

$$\frac{\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}}{a}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(-1),x]
```

output
```
(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{2a}+\frac{\sin(2dx+2c)}{4ad}$	25
derivativedivides	$\frac{\frac{\tan(dx+c)}{2+2\tan(dx+c)^2}+\frac{\arctan(\tan(dx+c))}{2}}{da}$	38
default	$\frac{\frac{\tan(dx+c)}{2+2\tan(dx+c)^2}+\frac{\arctan(\tan(dx+c))}{2}}{da}$	38
parallelrisch	$\frac{\tan(dx+c)^2xd+dx+\tan(dx+c)}{2da\left(1+\tan(dx+c)^2\right)}$	42
norman	$\frac{\frac{x}{2a}+\frac{\tan(dx+c)}{2ad}+\frac{x\tan(dx+c)^2}{2a}}{1+\tan(dx+c)^2}$	49

input
```
int(1/(a+a*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/4/a/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{a + a\tan^2(c + dx)}\,dx = \frac{dx\tan{(dx + c)}^2 + dx + \tan{(dx + c)}}{2\left(ad\tan{(dx + c)}^2 + ad\right)}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(d*x*tan(d*x + c)^2 + d*x + tan(d*x + c))/(a*d*tan(d*x + c)^2 + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(22) = 44.

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.81

$$\int \frac{1}{a + a\tan^2(c + dx)}\,dx = \begin{cases} \frac{dx\tan^2{(c+dx)}}{2ad\tan^2{(c+dx)}+2ad} + \frac{dx}{2ad\tan^2{(c+dx)}+2ad} + \frac{\tan{(c+dx)}}{2ad\tan^2{(c+dx)}+2ad} & \text{for } d \neq 0 \\ \frac{x}{a\tan^2{(c)}+a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+a*tan(d*x+c)**2),x)
```

output
```
Piecewise(((d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*a*d) + d*x/(2*a*
d*tan(c + d*x)**2 + 2*a*d) + tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*a*d),
 Ne(d, 0)), (x/(a*tan(c)**2 + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{a + a\tan^2(c + dx)}\,dx = \frac{\frac{dx+c}{a} + \frac{\tan(dx+c)}{a\tan(dx+c)^2+a}}{2\,d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)/a + tan(d*x + c)/(a*tan(d*x + c)^2 + a))/d
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{a + a\tan^2(c + dx)}\,dx = \frac{dx + c}{2\,ad} + \frac{\tan(dx + c)}{2\left(\tan(dx + c)^2 + 1\right)ad}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(d*x + c)/(a*d) + 1/2*tan(d*x + c)/((tan(d*x + c)^2 + 1)*a*d)
```

Mupad [B] (verification not implemented)

Time = 7.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{a + a\tan^2(c + dx)}\,dx = \frac{\frac{\sin(2\,c+2\,d\,x)}{4\,a} + \frac{d\,x}{2\,a}}{d}$$

input
```
int(1/(a + a*tan(c + d*x)^2),x)
```

output
```
(sin(2*c + 2*d*x)/(4*a) + (d*x)/(2*a))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{a+a\tan^2(c+dx)}\,dx=\frac{\tan(dx+c)^2\,dx+\tan(dx+c)+dx}{2ad\left(\tan(dx+c)^2+1\right)}$$

input
```
int(1/(a+a*tan(d*x+c)^2),x)
```

output
```
(tan(c + d*x)**2*d*x + tan(c + d*x) + d*x)/(2*a*d*(tan(c + d*x)**2 + 1))
```

3.183 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^2} dx = \frac{3x}{8a^2}+\frac{3\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}$$

output
```
3/8*x/a^2+3/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/
d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^2} dx = \frac{12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx))}{32a^2d}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(-2),x]
```

output
```
(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])/(32*a^2*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4140, 27, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\tan^2(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\tan(c+dx)^2+a)^2}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\cos^4(c+dx)}{a^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cos^4(c+dx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}}{a^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}}{a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(-2),x]
```

output
```
((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x
])/(2*d)))/4)/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{3x}{8a^2}+\frac{\sin(4dx+4c)}{32a^2d}+\frac{\sin(2dx+2c)}{4a^2d}$	42
derivativedivides	$\frac{\frac{\tan(dx+c)}{4\left(1+\tan(dx+c)^2\right)^2}+\frac{3\tan(dx+c)}{8\left(1+\tan(dx+c)^2\right)}+\frac{3\arctan(\tan(dx+c))}{8}}{d\,a^2}$	58
default	$\frac{\frac{\tan(dx+c)}{4\left(1+\tan(dx+c)^2\right)^2}+\frac{3\tan(dx+c)}{8\left(1+\tan(dx+c)^2\right)}+\frac{3\arctan(\tan(dx+c))}{8}}{d\,a^2}$	58
parallelrisch	$\frac{3\tan(dx+c)^4xd+6\tan(dx+c)^2xd+3\tan(dx+c)^3+3dx+5\tan(dx+c)}{8d\,a^2\left(1+\tan(dx+c)^2\right)^2}$	68
norman	$\frac{\frac{3x}{8a}+\frac{5\tan(dx+c)}{8ad}+\frac{3\tan(dx+c)^3}{8ad}+\frac{3x\tan(dx+c)^2}{4a}+\frac{3x\tan(dx+c)^4}{8a}}{a\left(1+\tan(dx+c)^2\right)^2}$	82

input
```
int(1/(a+a*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8*x/a^2+1/32/a^2/d*sin(4*d*x+4*c)+1/4/a^2/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a+a\tan^2(c+dx))^2}\,dx = \frac{3\,dx\tan\left(dx+c\right)^4+6\,dx\tan\left(dx+c\right)^2+3\,\tan\left(dx+c\right)^3+3\,dx+5\,\tan\left(dx+c\right)}{8\,\left(a^2d\tan\left(dx+c\right)^4+2\,a^2d\tan\left(dx+c\right)^2+a^2d\right)}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(3*d*x*tan(d*x + c)^4 + 6*d*x*tan(d*x + c)^2 + 3*tan(d*x + c)^3 + 3*d*
x + 5*tan(d*x + c))/(a^2*d*tan(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(51) = 102$.

Time = 0.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.51

$$\int \frac{1}{(a + a\tan^2(c + dx))^2}\,dx$$
$$= \begin{cases} \frac{3dx\tan^4(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d} + \frac{6dx\tan^2(c+dx)}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a^2d} + \frac{3dx}{8a^2d\tan^4(c+dx)+16a^2d\tan^2(c+dx)+8a} \\ \frac{x}{(a\tan^2(c)+a)^2} \end{cases}$$

input
```
integrate(1/(a+a*tan(d*x+c)**2)**2,x)
```

output
```
Piecewise((3*d*x*tan(c + d*x)**4/(8*a**2*d*tan(c + d*x)**4 + 16*a**2*d*tan
(c + d*x)**2 + 8*a**2*d) + 6*d*x*tan(c + d*x)**2/(8*a**2*d*tan(c + d*x)**4
 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*d*x/(8*a**2*d*tan(c + d*x)**4
 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 3*tan(c + d*x)**3/(8*a**2*d*tan
(c + d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d) + 5*tan(c + d*x)/(8*a
**2*d*tan(c + d*x)**4 + 16*a**2*d*tan(c + d*x)**2 + 8*a**2*d), Ne(d, 0)),
(x/(a*tan(c)**2 + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a + a\tan^2(c + dx))^2}\,dx = \frac{\frac{3\tan(dx+c)^3+5\tan(dx+c)}{a^2\tan(dx+c)^4+2a^2\tan(dx+c)^2+a^2} + \frac{3(dx+c)}{a^2}}{8d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*((3*tan(d*x + c)^3 + 5*tan(d*x + c))/(a^2*tan(d*x + c)^4 + 2*a^2*tan(d
*x + c)^2 + a^2) + 3*(d*x + c)/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+a\tan^2(c+dx))^2}\,dx = \frac{3\,(dx+c)}{8\,a^2d} + \frac{3\,\tan(dx+c)^3 + 5\,\tan(dx+c)}{8\,\left(\tan(dx+c)^2+1\right)^2a^2d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
3/8*(d*x + c)/(a^2*d) + 1/8*(3*tan(d*x + c)^3 + 5*tan(d*x + c))/((tan(d*x
+ c)^2 + 1)^2*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+a\tan^2(c+dx))^2}\,dx = \frac{2\,\sin(2\,c+2\,d\,x) + \frac{\sin(4\,c+4\,d\,x)}{4} + 3\,d\,x}{8\,a^2\,d}$$

input
```
int(1/(a + a*tan(c + d*x)^2)^2,x)
```

output
```
(2*sin(2*c + 2*d*x) + sin(4*c + 4*d*x)/4 + 3*d*x)/(8*a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+a\tan^2(c+dx))^2}\,dx = \frac{3\tan(dx+c)^4\,dx + 3\tan(dx+c)^3 + 6\tan(dx+c)^2\,dx + 5\tan(dx+c) + 3dx}{8a^2d\left(\tan(dx+c)^4 + 2\tan(dx+c)^2 + 1\right)}$$

input
```
int(1/(a+a*tan(d*x+c)^2)^2,x)
```

output
```
(3*tan(c + d*x)**4*d*x + 3*tan(c + d*x)**3 + 6*tan(c + d*x)**2*d*x + 5*tan
(c + d*x) + 3*d*x)/(8*a**2*d*(tan(c + d*x)**4 + 2*tan(c + d*x)**2 + 1))
```

3.184 $\int \frac{1}{(a+a\tan^2(c+dx))^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{1}{(a+a\tan^2(c+dx))^3} dx = \frac{5x}{16a^3} + \frac{5\cos(c+dx)\sin(c+dx)}{16a^3d} + \frac{5\cos^3(c+dx)\sin(c+dx)}{24a^3d} + \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}$$

output
```
5/16*x/a^3+5/16*cos(d*x+c)*sin(d*x+c)/a^3/d+5/24*cos(d*x+c)^3*sin(d*x+c)/a
^3/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a^3/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a+a\tan^2(c+dx))^3} dx = \frac{60c+60dx+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))}{192a^3d}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(-3),x]
```

output
```
(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x
)])/(192*a^3*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4140, 27, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\tan^2(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\tan(c+dx)^2+a)^3}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\cos^6(c+dx)}{a^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cos^6(c+dx)dx}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx}{a^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}}{a^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}}{a^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}}{a^3}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^3}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(-3),x]
```

output
```
((Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*Sin[c + d*x])/(
4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6)/a^3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{5x}{16a^3}+\frac{\sin(6dx+6c)}{192a^3d}+\frac{3\sin(4dx+4c)}{64a^3d}+\frac{15\sin(2dx+2c)}{64a^3d}$	59
derivativedivides	$\frac{\frac{\tan(dx+c)}{6\left(1+\tan(dx+c)^2\right)^3}+\frac{5\tan(dx+c)}{24\left(1+\tan(dx+c)^2\right)^2}+\frac{5\tan(dx+c)}{16\left(1+\tan(dx+c)^2\right)}+\frac{5\arctan(\tan(dx+c))}{16}}{d\,a^3}$	78
default	$\frac{\frac{\tan(dx+c)}{6\left(1+\tan(dx+c)^2\right)^3}+\frac{5\tan(dx+c)}{24\left(1+\tan(dx+c)^2\right)^2}+\frac{5\tan(dx+c)}{16\left(1+\tan(dx+c)^2\right)}+\frac{5\arctan(\tan(dx+c))}{16}}{d\,a^3}$	78
parallelrisch	$\frac{15\tan(dx+c)^6xd+45\tan(dx+c)^4xd+15\tan(dx+c)^5+45\tan(dx+c)^2xd+40\tan(dx+c)^3+15dx+33\tan(dx+c)}{48d\,a^3\left(1+\tan(dx+c)^2\right)^3}$	90
norman	$\frac{\frac{5x}{16a}+\frac{11\tan(dx+c)}{16ad}+\frac{5\tan(dx+c)^3}{6ad}+\frac{5\tan(dx+c)^5}{16ad}+\frac{15x\tan(dx+c)^2}{16a}+\frac{15x\tan(dx+c)^4}{16a}+\frac{5x\tan(dx+c)^6}{16a}}{a^2\left(1+\tan(dx+c)^2\right)^3}$	112

input
```
int(1/(a+a*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
5/16*x/a^3+1/192/a^3/d*sin(6*d*x+6*c)+3/64/a^3/d*sin(4*d*x+4*c)+15/64/a^3/
d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a + a\tan^2(c + dx)\right)^3}\,dx$$
$$= \frac{15\,dx\tan\left(dx+c\right)^6 + 45\,dx\tan\left(dx+c\right)^4 + 15\,\tan\left(dx+c\right)^5 + 45\,dx\tan\left(dx+c\right)^2 + 40\,\tan\left(dx+c\right)^3 +}{48\left(a^3d\tan\left(dx+c\right)^6 + 3\,a^3d\tan\left(dx+c\right)^4 + 3\,a^3d\tan\left(dx+c\right)^2 + a^3d\right)}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/48*(15*d*x*tan(d*x + c)^6 + 45*d*x*tan(d*x + c)^4 + 15*tan(d*x + c)^5 +
45*d*x*tan(d*x + c)^2 + 40*tan(d*x + c)^3 + 15*d*x + 33*tan(d*x + c))/(a^3
*d*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*tan(d*x + c)^2 + a^3*
d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(75) = 150.

Time = 0.48 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.75

$$\int \frac{1}{\left(a + a\tan^2(c + dx)\right)^3}\,dx$$
$$= \begin{cases} \frac{15dx\tan^6(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48a^3d} + \frac{45dx\tan^4(c+dx)}{48a^3d\tan^6(c+dx)+144a^3d\tan^4(c+dx)+144a^3d\tan^2(c+dx)+48} \\ \frac{x}{\left(a\tan^2(c)+a\right)^3} \end{cases}$$

input
```
integrate(1/(a+a*tan(d*x+c)**2)**3,x)
```

output
```
Piecewise((15*d*x*tan(c + d*x)**6/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*
tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c +
 d*x)**4/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**
3*d*tan(c + d*x)**2 + 48*a**3*d) + 45*d*x*tan(c + d*x)**2/(48*a**3*d*tan(c
 + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*
a**3*d) + 15*d*x/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 +
 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 15*tan(c + d*x)**5/(48*a**3*d*t
an(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 +
 48*a**3*d) + 40*tan(c + d*x)**3/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*t
an(c + d*x)**4 + 144*a**3*d*tan(c + d*x)**2 + 48*a**3*d) + 33*tan(c + d*x)
/(48*a**3*d*tan(c + d*x)**6 + 144*a**3*d*tan(c + d*x)**4 + 144*a**3*d*tan(
c + d*x)**2 + 48*a**3*d), Ne(d, 0)), (x/(a*tan(c)**2 + a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+a\tan^2(c+dx))^3}\,dx = \frac{\frac{15\tan(dx+c)^5+40\tan(dx+c)^3+33\tan(dx+c)}{a^3\tan(dx+c)^6+3a^3\tan(dx+c)^4+3a^3\tan(dx+c)^2+a^3}+\frac{15(dx+c)}{a^3}}{48d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/48*((15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*tan(d*x + c))/(a^3*tan(d
*x + c)^6 + 3*a^3*tan(d*x + c)^4 + 3*a^3*tan(d*x + c)^2 + a^3) + 15*(d*x +
 c)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+a\tan^2(c+dx))^3}\,dx = \frac{5(dx+c)}{16a^3d} + \frac{15\tan(dx+c)^5+40\tan(dx+c)^3+33\tan(dx+c)}{48(\tan(dx+c)^2+1)^3a^3d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
5/16*(d*x + c)/(a^3*d) + 1/48*(15*tan(d*x + c)^5 + 40*tan(d*x + c)^3 + 33*
tan(d*x + c))/((tan(d*x + c)^2 + 1)^3*a^3*d)
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^3}\,dx=\frac{5\,x}{16\,a^3}+\frac{\cos\left(c+d\,x\right)^6\,\left(\frac{5\,\tan\left(c+d\,x\right)^5}{16}+\frac{5\,\tan\left(c+d\,x\right)^3}{6}+\frac{11\,\tan\left(c+d\,x\right)}{16}\right)}{a^3\,d}$$

input
```
int(1/(a + a*tan(c + d*x)^2)^3,x)
```

output
```
(5*x)/(16*a^3) + (cos(c + d*x)^6*((11*tan(c + d*x))/16 + (5*tan(c + d*x)^3
)/6 + (5*tan(c + d*x)^5)/16))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^3}\,dx$$
$$=\frac{15\tan\left(dx+c\right)^6dx+15\tan\left(dx+c\right)^5+45\tan\left(dx+c\right)^4dx+40\tan\left(dx+c\right)^3+45\tan\left(dx+c\right)^2dx+3}{48a^3d\left(\tan\left(dx+c\right)^6+3\tan\left(dx+c\right)^4+3\tan\left(dx+c\right)^2+1\right)}$$

input
```
int(1/(a+a*tan(d*x+c)^2)^3,x)
```

output
```
(15*tan(c + d*x)**6*d*x + 15*tan(c + d*x)**5 + 45*tan(c + d*x)**4*d*x + 40
*tan(c + d*x)**3 + 45*tan(c + d*x)**2*d*x + 33*tan(c + d*x) + 15*d*x)/(48*
a**3*d*(tan(c + d*x)**6 + 3*tan(c + d*x)**4 + 3*tan(c + d*x)**2 + 1))
```

3.185 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a-b)\log(\cos(e+fx))}{f}-\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{(a-b)\tan^4(e+fx)}{4f}+\frac{b\tan^6(e+fx)}{6f}$$

output
```
-(a-b)*ln(cos(f*x+e))/f-1/2*(a-b)*tan(f*x+e)^2/f+1/4*(a-b)*tan(f*x+e)^4/f+
1/6*b*tan(f*x+e)^6/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{12(-a+b)\log(\cos(e+fx))-6(2a-3b)\sec^2(e+fx)+3(a-3b)\sec^4(e+fx)+2b\sec^6(e+fx)}{12f}$$

input
```
Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
(12*(-a + b)*Log[Cos[e + f*x]] - 6*(2*a - 3*b)*Sec[e + f*x]^2 + 3*(a - 3*b
)*Sec[e + f*x]^4 + 2*b*Sec[e + f*x]^6)/(12*f)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4114, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^5\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{ 4114}$$

$$(a-b)\int \tan^5(e+fx)dx+\frac{b\tan^6(e+fx)}{6f}$$

$$\Big\downarrow \text{ 3042}$$

$$(a-b)\int \tan(e+fx)^5dx+\frac{b\tan^6(e+fx)}{6f}$$

$$\Big\downarrow \text{ 3954}$$

$$(a-b)\left(\frac{\tan^4(e+fx)}{4f}-\int \tan^3(e+fx)dx\right)+\frac{b\tan^6(e+fx)}{6f}$$

$$\Big\downarrow \text{ 3042}$$

$$(a-b)\left(\frac{\tan^4(e+fx)}{4f}-\int \tan(e+fx)^3dx\right)+\frac{b\tan^6(e+fx)}{6f}$$

$$\Big\downarrow \text{ 3954}$$

$$(a-b)\left(\int \tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)+\frac{b\tan^6(e+fx)}{6f}$$

$$\Big\downarrow \text{ 3042}$$

$$(a-b)\left(\int \tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)+\frac{b\tan^6(e+fx)}{6f}$$

$$\Big\downarrow \text{ 3956}$$

$$(a-b)\left(\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}-\frac{\log(\cos(e+fx))}{f}\right)+\frac{b\tan^6(e+fx)}{6f}$$

input
```
Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
(b*Tan[e + f*x]^6)/(6*f) + (a - b)*(-(Log[Cos[e + f*x]]/f) - Tan[e + f*x]^
2/(2*f) + Tan[e + f*x]^4/(4*f))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b\tan(fx+e)^6}{6f}-\frac{(a-b)\tan(fx+e)^2}{2f}+\frac{(a-b)\tan(fx+e)^4}{4f}+\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2f}$
derivativedivides	$\frac{\frac{b\tan(fx+e)^6}{6}+\frac{a\tan(fx+e)^4}{4}-\frac{b\tan(fx+e)^4}{4}-\frac{a\tan(fx+e)^2}{2}+\frac{b\tan(fx+e)^2}{2}+\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$
default	$\frac{\frac{b\tan(fx+e)^6}{6}+\frac{a\tan(fx+e)^4}{4}-\frac{b\tan(fx+e)^4}{4}-\frac{a\tan(fx+e)^2}{2}+\frac{b\tan(fx+e)^2}{2}+\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$
parallelrisch	$\frac{2b\tan(fx+e)^6+3a\tan(fx+e)^4-3b\tan(fx+e)^4-6a\tan(fx+e)^2+6b\tan(fx+e)^2+6\ln\left(1+\tan(fx+e)^2\right)a-6\ln\left(1+\text{ta}\right.}{12f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\tan(fx+e)^6}{6}-\frac{\tan(fx+e)^4}{4}+\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}$
risch	$ixa-ixb+\frac{2iae}{f}-\frac{2ibe}{f}-\frac{2\left(6a\,e^{10i(fx+e)}-9b\,e^{10i(fx+e)}+18a\,e^{8i(fx+e)}-18b\,e^{8i(fx+e)}+24a\,e^{6i(fx+e)}-34b\,\text{e}\right.}{3f\left(e^{2i(fx+e)}+1\right.}$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*tan(f*x+e)^6/f-1/2*(a-b)*tan(f*x+e)^2/f+1/4*(a-b)*tan(f*x+e)^4/f+1/2
*(a-b)/f*ln(1+tan(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{2\,b\tan\left(fx+e\right)^6+3\,(a-b)\tan\left(fx+e\right)^4-6\,(a-b)\tan\left(fx+e\right)^2-6\,(a-b)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{12\,f}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/12*(2*b*tan(f*x + e)^6 + 3*(a - b)*tan(f*x + e)^4 - 6*(a - b)*tan(f*x +
e)^2 - 6*(a - b)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\begin{cases}\frac{a\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{a\tan^4(e+fx)}{4f}-\frac{a\tan^2(e+fx)}{2f}-\frac{b\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{b\tan^6(e+fx)}{6f}-\frac{b\tan^4(e+fx)}{4f}+\frac{b\tan^2(e+}{2f}\\ x(a+b\tan^2(e))\tan^5(e)\end{cases}$$

input
```
integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*
tan(e + f*x)**2/(2*f) - b*log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)*
*6/(6*f) - b*tan(e + f*x)**4/(4*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (
x*(a + b*tan(e)**2)*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=-\frac{6\,(a-b)\log\left(\sin(fx+e)^2-1\right)-\frac{6\,(2\,a-3\,b)\sin(fx+e)^4-3\,(7\,a-9\,b)\sin(fx+e)^2+9\,a-11\,b}{\sin(fx+e)^6-3\,\sin(fx+e)^4+3\,\sin(fx+e)^2-1}}{12\,f}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/12*(6*(a - b)*log(sin(f*x + e)^2 - 1) - (6*(2*a - 3*b)*sin(f*x + e)^4 -
 3*(7*a - 9*b)*sin(f*x + e)^2 + 9*a - 11*b)/(sin(f*x + e)^6 - 3*sin(f*x +
e)^4 + 3*sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{(a-b)\log\left(\tan(fx+e)^2+1\right)}{2f} + \frac{2\,bf^2\tan(fx+e)^6+3\,af^2\tan(fx+e)^4-3\,bf^2\tan(fx+e)^4-6\,af^2\tan(fx+e)^2+6\,bf^2\tan(fx-}{12\,f^3}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(a - b)*log(tan(f*x + e)^2 + 1)/f + 1/12*(2*b*f^2*tan(f*x + e)^6 + 3*a
*f^2*tan(f*x + e)^4 - 3*b*f^2*tan(f*x + e)^4 - 6*a*f^2*tan(f*x + e)^2 + 6*
b*f^2*tan(f*x + e)^2)/f^3
```

Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\tan(e+f\,x)^4\left(\frac{a}{4}-\frac{b}{4}\right)-\tan(e+f\,x)^2\left(\frac{a}{2}-\frac{b}{2}\right)+\frac{b\tan(e+f\,x)^6}{6}+\ln\left(\tan(e+f\,x)^2+1\right)\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

input
```
int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

output
```
(tan(e + f*x)^4*(a/4 - b/4) - tan(e + f*x)^2*(a/2 - b/2) + (b*tan(e + f*x)
^6)/6 + log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{6\log\left(\tan\left(fx+e\right)^2+1\right)a-6\log\left(\tan\left(fx+e\right)^2+1\right)b+2\tan\left(fx+e\right)^6b+3\tan\left(fx+e\right)^4a-3\tan\left(f\right.}{12f}$$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

output
```
(6*log(tan(e + f*x)**2 + 1)*a - 6*log(tan(e + f*x)**2 + 1)*b + 2*tan(e + f
*x)**6*b + 3*tan(e + f*x)**4*a - 3*tan(e + f*x)**4*b - 6*tan(e + f*x)**2*a
 + 6*tan(e + f*x)**2*b)/(12*f)
```

3.186 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{(a-b)\tan^2(e+fx)}{2f}+\frac{b\tan^4(e+fx)}{4f}$$

output
```
(a-b)*ln(cos(f*x+e))/f+1/2*(a-b)*tan(f*x+e)^2/f+1/4*b*tan(f*x+e)^4/f
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{4(a-b)\log(\cos(e+fx))+2(a-2b)\sec^2(e+fx)+b\sec^4(e+fx)}{4f}$$

input
```
Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
(4*(a - b)*Log[Cos[e + f*x]] + 2*(a - 2*b)*Sec[e + f*x]^2 + b*Sec[e + f*x]
^4)/(4*f)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4114, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4114}$$

$$(a-b)\int \tan^3(e+fx)dx+\frac{b\tan^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\int \tan(e+fx)^3dx+\frac{b\tan^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3954}$$

$$(a-b)\left(\frac{\tan^2(e+fx)}{2f}-\int \tan(e+fx)dx\right)+\frac{b\tan^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\left(\frac{\tan^2(e+fx)}{2f}-\int \tan(e+fx)dx\right)+\frac{b\tan^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3956}$$

$$(a-b)\left(\frac{\tan^2(e+fx)}{2f}+\frac{\log(\cos(e+fx))}{f}\right)+\frac{b\tan^4(e+fx)}{4f}$$

input
```
Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
(b*Tan[e + f*x]^4)/(4*f) + (a - b)*(Log[Cos[e + f*x]]/f + Tan[e + f*x]^2/(
2*f))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result
norman	$\frac{b\tan(fx+e)^4}{4f}+\frac{(a-b)\tan(fx+e)^2}{2f}-\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2f}$
derivativedivides	$\frac{\frac{b\tan(fx+e)^4}{4}+\frac{a\tan(fx+e)^2}{2}-\frac{b\tan(fx+e)^2}{2}+\frac{(-a+b)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$
default	$\frac{\frac{b\tan(fx+e)^4}{4}+\frac{a\tan(fx+e)^2}{2}-\frac{b\tan(fx+e)^2}{2}+\frac{(-a+b)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$
parallelrisch	$-\frac{-b\tan(fx+e)^4-2a\tan(fx+e)^2+2b\tan(fx+e)^2+2\ln\left(1+\tan(fx+e)^2\right)a-2\ln\left(1+\tan(fx+e)^2\right)b}{4f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}$
risch	$-ixa+ixb-\frac{2iae}{f}+\frac{2ibe}{f}+\frac{2a\,e^{6i(fx+e)}-4b\,e^{6i(fx+e)}+4a\,e^{4i(fx+e)}-4b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-4b\,e^{2i(fx+}}{f\left(e^{2i(fx+e)}+1\right)^4}$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*tan(f*x+e)^4/f+1/2*(a-b)*tan(f*x+e)^2/f-1/2*(a-b)/f*ln(1+tan(f*x+e)^
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan\left(fx+e\right)^4+2\,(a-b)\tan\left(fx+e\right)^2+2\,(a-b)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{4\,f}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(b*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 + 2*(a - b)*log(1/(tan(f*
x + e)^2 + 1)))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(42) = 84.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \begin{cases} -\frac{a\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{a\tan^2(e+fx)}{2f} + \frac{b\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{b\tan^4(e+fx)}{4f} - \frac{b\tan^2(e+fx)}{2f} & \text{for } f \neq 0 \\ x(a+b\tan^2(e))\tan^3(e) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b
*log(tan(e + f*x)**2 + 1)/(2*f) + b*tan(e + f*x)**4/(4*f) - b*tan(e + f*x)
**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{2\,(a-b)\log\left(\sin\left(fx+e\right)^2-1\right) - \frac{2\,(a-2\,b)\sin(fx+e)^2-2\,a+3\,b}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(2*(a - b)*log(sin(f*x + e)^2 - 1) - (2*(a - 2*b)*sin(f*x + e)^2 - 2*a
 + 3*b)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(a-b)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f} + \frac{bf\tan\left(fx+e\right)^4+2\,af\tan\left(fx+e\right)^2-2\,bf\tan\left(fx+e\right)^2}{4\,f^2}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(a - b)*log(tan(f*x + e)^2 + 1)/f + 1/4*(b*f*tan(f*x + e)^4 + 2*a*f*t
an(f*x + e)^2 - 2*b*f*tan(f*x + e)^2)/f^2
```

Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)^4}{4\,f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f} + \frac{\tan(e+f\,x)^2\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

input
```
int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

output
```
(b*tan(e + f*x)^4)/(4*f) - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f + (tan(
e + f*x)^2*(a/2 - b/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{-2\log\left(\tan\left(fx+e\right)^2+1\right)a+2\log\left(\tan\left(fx+e\right)^2+1\right)b+\tan\left(fx+e\right)^4b+2\tan\left(fx+e\right)^2a-2\tan\left(\right.}{4f}$$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

output
```
( - 2*log(tan(e + f*x)**2 + 1)*a + 2*log(tan(e + f*x)**2 + 1)*b + tan(e +
f*x)**4*b + 2*tan(e + f*x)**2*a - 2*tan(e + f*x)**2*b)/(4*f)
```

3.187 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a-b)\log(\cos(e+fx))}{f}+\frac{b\tan^2(e+fx)}{2f}$$

output
```
-(a-b)*ln(cos(f*x+e))/f+1/2*b*tan(f*x+e)^2/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\log(\cos(e+fx))}{f}+\frac{b(2\log(\cos(e+fx))+\sec^2(e+fx))}{2f}$$

input
```
Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((a*Log[Cos[e + f*x]])/f) + (b*(2*Log[Cos[e + f*x]] + Sec[e + f*x]^2))/(2
*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4114, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(e+fx)\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \ 4114$$

$$(a-b)\int \tan(e+fx)dx+\frac{b\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \ 3042$$

$$(a-b)\int \tan(e+fx)dx+\frac{b\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \ 3956$$

$$\frac{b\tan^2(e+fx)}{2f}-\frac{(a-b)\log(\cos(e+fx))}{f}$$

input
```
Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
-(((a - b)*Log[Cos[e + f*x]])/f) + (b*Tan[e + f*x]^2)/(2*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\frac{b\tan(fx+e)^2}{2}+\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$	35
default	$\frac{\frac{b\tan(fx+e)^2}{2}+\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$	35
norman	$\frac{b\tan(fx+e)^2}{2f}+\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2f}$	37
parallelrisch	$\frac{b\tan(fx+e)^2+\ln\left(1+\tan(fx+e)^2\right)a-\ln\left(1+\tan(fx+e)^2\right)b}{2f}$	44
parts	$\frac{a\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}$	48
risch	$ixa-ixb+\frac{2iae}{f}-\frac{2ibe}{f}+\frac{2b\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}+1\right)a}{f}+\frac{\ln\left(e^{2i(fx+e)}+1\right)b}{f}$	91

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b*tan(f*x+e)^2+1/2*(a-b)*ln(1+tan(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\tan\left(fx+e\right)^2-(a-b)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(b*tan(f*x + e)^2 - (a - b)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\begin{cases}\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-\frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{b\tan^2\left(e+fx\right)}{2f} & \text{for } f\neq 0\\ x(a+b\tan^2\left(e\right))\tan\left(e\right) & \text{otherwise}\end{cases}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x)**2 + 1)/(
2*f) + b*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(a-b)\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{b}{\sin(fx+e)^2-1}}{2\,f}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*((a - b)*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan\left(fx+e\right)^2}{2\,f}+\frac{(a-b)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*b*tan(f*x + e)^2/f + 1/2*(a - b)*log(tan(f*x + e)^2 + 1)/f
```

Mupad [B] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)^2}{2\,f}+\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

input
```
int(tan(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

output
```
(b*tan(e + f*x)^2)/(2*f) + (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\log\left(\tan\left(fx+e\right)^2+1\right)a-\log\left(\tan\left(fx+e\right)^2+1\right)b+\tan\left(fx+e\right)^2b}{2f}$$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

output
```
(log(tan(e + f*x)**2 + 1)*a - log(tan(e + f*x)**2 + 1)*b + tan(e + f*x)**2
*b)/(2*f)
```

3.188 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{b\log(\cos(e+fx))}{f}+\frac{a\log(\sin(e+fx))}{f}$$

output
```
-b*ln(cos(f*x+e))/f+a*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{b\log(\cos(e+fx))}{f}+\frac{a\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((b*Log[Cos[e + f*x]])/f) + (a*Log[Sin[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4108, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 4108}$$

$$a\int \cot(e+fx)dx+b\int \tan(e+fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$a\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+b\int \tan(e+fx)dx$$

$$\Big\downarrow \text{ 25}$$

$$b\int \tan(e+fx)dx-a\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{a\log(-\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

input
```
Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((b*Log[Cos[e + f*x]])/f) + (a*Log[-Sin[e + f*x]])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4108
```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Simp[A   Int[1/Tan[e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x]
, x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-b\ln(\cos(fx+e))+a\ln(\sin(fx+e))}{f}$	25
default	$\frac{-b\ln(\cos(fx+e))+a\ln(\sin(fx+e))}{f}$	25
parallelrisch	$\frac{(-a+b)\ln\left(\sec(fx+e)^2\right)+2a\ln(\tan(fx+e))}{2f}$	32
norman	$\frac{a\ln(\tan(fx+e))}{f}-\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2f}$	35
risch	$-ixa+ixb-\frac{2iae}{f}+\frac{2ibe}{f}+\frac{a\ln(e^{2i(fx+e)}-1)}{f}-\frac{\ln(e^{2i(fx+e)}+1)b}{f}$	63

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b*ln(cos(f*x+e))+a*ln(sin(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{a\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)-b\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2f}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(a*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - b*log(1/(tan(f*x + e)^2
+ 1)))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx =\begin{cases}-\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\log\left(\tan\left(e+fx\right)\right)}{f}+\frac{b\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} & \text{for } f\neq 0\\ x(a+b\tan^2\left(e\right))\cot\left(e\right) & \text{otherwise}\end{cases}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*log(tan(e + f*x))/f + b*l
og(tan(e + f*x)**2 + 1)/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)*cot(e), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \cot(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx = -\frac{b\log\left(\sin\left(fx+e\right)^2-1\right)-a\log\left(\sin\left(fx+e\right)^2\right)}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(b*log(sin(f*x + e)^2 - 1) - a*log(sin(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \cot(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx = -\frac{(a-b)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f} + \frac{a\log\left(\tan\left(fx+e\right)^2\right)}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(a - b)*log(tan(f*x + e)^2 + 1)/f + 1/2*a*log(tan(f*x + e)^2)/f
```

Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \cot(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx = \frac{a\,\ln\left(\tan(e+f\,x)\right)}{f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

input
```
int(cot(e + f*x)*(a + b*tan(e + f*x)^2),x)
```

output
```
(a*log(tan(e + f*x)))/f - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.08

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a+\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)b-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}{f}$$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2),x)
```

output
```
( - log(tan((e + f*x)/2)**2 + 1)*a + log(tan((e + f*x)/2)**2 + 1)*b - log(
tan((e + f*x)/2) - 1)*b - log(tan((e + f*x)/2) + 1)*b + log(tan((e + f*x)/
2))*a)/f
```

3.189 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\cot^2(e+fx)}{2f}-\frac{(a-b)\log(\sin(e+fx))}{f}$$

output
```
-1/2*a*cot(f*x+e)^2/f-(a-b)*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}+\frac{b\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/2*(a*Csc[e + f*x]^2)/f - (a*Log[Sin[e + f*x]])/f + (b*Log[Sin[e + f*x]]
)/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4112, 25, 27, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4112}$$

$$\int -((a-b)\cot(e+fx))dx-\frac{a\cot^2(e+fx)}{2f}$$

$$\Big\downarrow \text{25}$$

$$-\int (a-b)\cot(e+fx)dx-\frac{a\cot^2(e+fx)}{2f}$$

$$\Big\downarrow \text{27}$$

$$-(a-b)\int \cot(e+fx)dx-\frac{a\cot^2(e+fx)}{2f}$$

$$\Big\downarrow \text{3042}$$

$$-(a-b)\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx-\frac{a\cot^2(e+fx)}{2f}$$

$$\Big\downarrow \text{25}$$

$$(a-b)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx-\frac{a\cot^2(e+fx)}{2f}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(a-b)\log(-\sin(e+fx))}{f}-\frac{a\cot^2(e+fx)}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/2*(a*Cot[e + f*x]^2)/f - ((a - b)*Log[-Sin[e + f*x]])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{b\ln(\sin(fx+e))+a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)}{f}$	37
default	$\frac{b\ln(\sin(fx+e))+a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)}{f}$	37
parallelrisch	$\frac{(a-b)\ln\left(\sec(fx+e)^2\right)+(-2a+2b)\ln(\tan(fx+e))-a\cot(fx+e)^2}{2f}$	48
norman	$-\frac{a}{2f\tan(fx+e)^2}-\frac{(a-b)\ln(\tan(fx+e))}{f}+\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2f}$	54
risch	$ixa-ixb+\frac{2iae}{f}-\frac{2ibe}{f}+\frac{2a\,e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{a\ln\left(e^{2i(fx+e)}-1\right)}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)b}{f}$	91

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*ln(sin(f*x+e))+a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(a-b)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+a\tan\left(fx+e\right)^2+a}{2\,f\tan\left(fx+e\right)^2}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a*
tan(f*x + e)^2 + a)/(f*tan(f*x + e)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(27) = 54$.

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.85

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \begin{cases} \tilde{\infty}ax & \text{for } e=0 \wedge f= \\ x(a+b\tan^2(e))\cot^3(e) & \text{for } f=0 \\ \tilde{\infty}ax & \text{for } e=-fx \\ \frac{a\log(\tan^2(e+fx)+1)}{2f} - \frac{a\log(\tan(e+fx))}{f} - \frac{a}{2f\tan^2(e+fx)} - \frac{b\log(\tan^2(e+fx)+1)}{2f} + \frac{b\log(\tan(e+fx))}{f} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**3,
Eq(f, 0)), (zoo*a*x, Eq(e, -f*x)), (a*log(tan(e + f*x)**2 + 1)/(2*f) - a*l
og(tan(e + f*x))/f - a/(2*f*tan(e + f*x)**2) - b*log(tan(e + f*x)**2 + 1)/
(2*f) + b*log(tan(e + f*x))/f, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(a-b)\log\left(\sin(fx+e)^2\right)+\frac{a}{\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*((a - b)*log(sin(f*x + e)^2) + a/sin(f*x + e)^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

Time = 0.70 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{(a-b)\log\left(\tan(fx+e)^2+1\right)}{2f} - \frac{(a-b)\log\left(\tan(fx+e)^2\right)}{2f} + \frac{a\tan(fx+e)^2 - b\tan(fx+e)^2 - a}{2f\tan(fx+e)^2}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(a - b)*log(tan(f*x + e)^2 + 1)/f - 1/2*(a - b)*log(tan(f*x + e)^2)/f
+ 1/2*(a*tan(f*x + e)^2 - b*tan(f*x + e)^2 - a)/(f*tan(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 8.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a-b)}{f} - \frac{a\cot(e+f\,x)^2}{2f}$$

input
```
int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2),x)
```

output
```
(log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f - (log(tan(e + f*x))*(a - b))/f -
(a*cot(e + f*x)^2)/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \frac{4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2 a-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2 b-4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{4\sin\left(fx+e\right)^2 f}$$

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x)
```

output
```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a - 4*log(tan((e + f*x)/2)
**2 + 1)*sin(e + f*x)**2*b - 4*log(tan((e + f*x)/2))*sin(e + f*x)**2*a + 4
*log(tan((e + f*x)/2))*sin(e + f*x)**2*b + sin(e + f*x)**2*a - 2*a)/(4*sin
(e + f*x)**2*f)
```

3.190 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{(a-b)\cot^2(e+fx)}{2f}-\frac{a\cot^4(e+fx)}{4f}+\frac{(a-b)\log(\sin(e+fx))}{f}$$

output
```
1/2*(a-b)*cot(f*x+e)^2/f-1/4*a*cot(f*x+e)^4/f+(a-b)*ln(sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{a\csc^2(e+fx)}{f}-\frac{b\csc^2(e+fx)}{2f}-\frac{a\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}-\frac{b\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
(a*Csc[e + f*x]^2)/f - (b*Csc[e + f*x]^2)/(2*f) - (a*Csc[e + f*x]^4)/(4*f)
 + (a*Log[Sin[e + f*x]])/f - (b*Log[Sin[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4112, 25, 27, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4112}$$

$$\int -\left((a-b)\cot^3(e+fx)\right)dx-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{25}$$

$$-\int (a-b)\cot^3(e+fx)dx-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{27}$$

$$-(a-b)\int \cot^3(e+fx)dx-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3042}$$

$$-(a-b)\int -\tan\left(e+fx+\frac{\pi}{2}\right)^3dx-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{25}$$

$$(a-b)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3954}$$

$$(a-b)\left(\frac{\cot^2(e+fx)}{2f}-\int -\cot(e+fx)dx\right)-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{25}$$

$$(a-b)\left(\int \cot(e+fx)dx+\frac{\cot^2(e+fx)}{2f}\right)-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\left(\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\cot^2(e+fx)}{2f}\right)-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{25}$$

$$(a-b)\left(\frac{\cot^2(e+fx)}{2f}-\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx\right)-\frac{a\cot^4(e+fx)}{4f}$$

$$\Big\downarrow \text{3956}$$

$$(a-b)\left(\frac{\cot^2(e+fx)}{2f}+\frac{\log(-\sin(e+fx))}{f}\right)-\frac{a\cot^4(e+fx)}{4f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/4*(a*Cot[e + f*x]^4)/f + (a - b)*(Cot[e + f*x]^2/(2*f) + Log[-Sin[e + f
*x]]/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{b\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)}{f}$
default	$\frac{b\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)}{f}$
parallelrisch	$\frac{(-2a+2b)\ln\left(\sec(fx+e)^2\right)+(4a-4b)\ln(\tan(fx+e))-\cot(fx+e)^2\left(a\cot(fx+e)^2-2a+2b\right)}{4f}$
norman	$\frac{-\frac{a}{4f}+\frac{(a-b)\tan(fx+e)^2}{2f}}{\tan(fx+e)^4}+\frac{(a-b)\ln(\tan(fx+e))}{f}-\frac{(a-b)\ln\left(1+\tan(fx+e)^2\right)}{2f}$
risch	$-ixa+ixb-\frac{2iae}{f}+\frac{2ibe}{f}-\frac{2\left(2a\,e^{6i(fx+e)}-b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}+2b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-b\,e^{2i(fx+}\right.}{f\left(e^{2i(fx+e)}-1\right)^4}$

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))+a*(-1/4*cot(f*x+e)^4+1/2*cot(f*x
+e)^2+ln(sin(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.60

$$\int \cot^5(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx$$
$$= \frac{2(a-b)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan(fx+e)^4 + (3a-2b)\tan(fx+e)^4 + 2(a-b)\tan(fx+e)^2 - a}{4f\tan(fx+e)^4}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(2*(a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (
3*a - 2*b)*tan(f*x + e)^4 + 2*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f*x + e)^
4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(42) = 84.

Time = 1.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int \cot^5(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx$$
$$= \begin{cases} \tilde{\infty}ax \\ x(a+b\tan^2(e))\cot^5(e) \\ \tilde{\infty}ax \\ -\frac{a\log(\tan^2(e+fx)+1)}{2f} + \frac{a\log(\tan(e+fx))}{f} + \frac{a}{2f\tan^2(e+fx)} - \frac{a}{4f\tan^4(e+fx)} + \frac{b\log(\tan^2(e+fx)+1)}{2f} - \frac{b\log(\tan(e+fx))}{f} - \end{cases}$$

input
```
integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**5,
Eq(f, 0)), (zoo*a*x, Eq(e, -f*x)), (-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*
log(tan(e + f*x))/f + a/(2*f*tan(e + f*x)**2) - a/(4*f*tan(e + f*x)**4) +
b*log(tan(e + f*x)**2 + 1)/(2*f) - b*log(tan(e + f*x))/f - b/(2*f*tan(e +
f*x)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{2\,(a-b)\log\left(\sin\left(fx+e\right)^2\right)+\frac{2\,(2\,a-b)\sin(fx+e)^2-a}{\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(2*(a - b)*log(sin(f*x + e)^2) + (2*(2*a - b)*sin(f*x + e)^2 - a)/sin(
f*x + e)^4)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.

Time = 1.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(a-b)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}+\frac{(a-b)\log\left(\tan\left(fx+e\right)^2\right)}{2\,f} - \frac{3\,a\tan\left(fx+e\right)^4-3\,b\tan\left(fx+e\right)^4-2\,a\tan\left(fx+e\right)^2+2\,b\tan\left(fx+e\right)^2+a}{4\,f\tan\left(fx+e\right)^4}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(a - b)*log(tan(f*x + e)^2 + 1)/f + 1/2*(a - b)*log(tan(f*x + e)^2)/f
 - 1/4*(3*a*tan(f*x + e)^4 - 3*b*tan(f*x + e)^4 - 2*a*tan(f*x + e)^2 + 2*b
*tan(f*x + e)^2 + a)/(f*tan(f*x + e)^4)
```

Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{\ln\left(\tan(e+f\,x)\right)\,(a-b)}{f}-\frac{\frac{a}{4}-\tan(e+f\,x)^2\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f\tan\left(e+f\,x\right)^4}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2),x)
```

output
```
(log(tan(e + f*x))*(a - b))/f - (a/4 - tan(e + f*x)^2*(a/2 - b/2))/(f*tan(
e + f*x)^4) - (log(tan(e + f*x)^2 + 1)*(a/2 - b/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.89

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{-32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^4a+32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^4b+32\log\left(\tan\left(\frac{f}{2}\right.\right.}{}$$

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x)
```

output
```
( - 32*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a + 32*log(tan((e + f*
x)/2)**2 + 1)*sin(e + f*x)**4*b + 32*log(tan((e + f*x)/2))*sin(e + f*x)**4
*a - 32*log(tan((e + f*x)/2))*sin(e + f*x)**4*b - 13*sin(e + f*x)**4*a + 8
*sin(e + f*x)**4*b + 32*sin(e + f*x)**2*a - 16*sin(e + f*x)**2*b - 8*a)/(3
2*sin(e + f*x)**4*f)
```

3.191 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -((a-b)x)+\frac{(a-b)\tan(e+fx)}{f}-\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{(a-b)\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}$$

output
```
-(a-b)*x+(a-b)*tan(f*x+e)/f-1/3*(a-b)*tan(f*x+e)^3/f+1/5*(a-b)*tan(f*x+e)^
5/f+1/7*b*tan(f*x+e)^7/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.61

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{a\arctan(\tan(e+fx))}{f} + \frac{b\arctan(\tan(e+fx))}{f} + \frac{a\tan(e+fx)}{f} - \frac{b\tan(e+fx)}{f} - \frac{a\tan^3(e+fx)}{3f} + \frac{b\tan^3(e+fx)}{3f} + \frac{a\tan^5(e+fx)}{5f} - \frac{b\tan^5(e+fx)}{5f} + \frac{b\tan^7(e+fx)}{7f}$$

input
```
Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*
x])/f - (b*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^3)
/(3*f) + (a*Tan[e + f*x]^5)/(5*f) - (b*Tan[e + f*x]^5)/(5*f) + (b*Tan[e +
f*x]^7)/(7*f)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4114, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4114}$$

$$(a-b)\int \tan^6(e+fx)dx+\frac{b\tan^7(e+fx)}{7f}$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\int \tan(e+fx)^6dx+\frac{b\tan^7(e+fx)}{7f}$$

$$\Big\downarrow \text{3954}$$

$$(a-b)\left(\frac{\tan^5(e+fx)}{5f}-\int \tan^4(e+fx)dx\right)+\frac{b\tan^7(e+fx)}{7f}$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\left(\frac{\tan^5(e+fx)}{5f}-\int \tan(e+fx)^4dx\right)+\frac{b\tan^7(e+fx)}{7f}$$

$$\Big\downarrow \text{3954}$$

$$(a-b)\left(\int \tan^2(e+fx)dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)+\frac{b\tan^7(e+fx)}{7f}$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\left(\int \tan(e+fx)^2dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)+\frac{b\tan^7(e+fx)}{7f}$$

$$\Big\downarrow \text{3954}$$

$$(a-b)\left(-\int 1dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}\right)+\frac{b\tan^7(e+fx)}{7f}$$

$$\Big\downarrow \text{24}$$

$$(a-b)\left(\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}-x\right)+\frac{b\tan^7(e+fx)}{7f}$$

input
```
Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
(b*Tan[e + f*x]^7)/(7*f) + (a - b)*(-x + Tan[e + f*x]/f - Tan[e + f*x]^3/(
3*f) + Tan[e + f*x]^5/(5*f))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result
norman	$(-a+b)\,x+\frac{(a-b)\tan(fx+e)}{f}+\frac{b\tan(fx+e)^{7}}{7f}-\frac{(a-b)\tan(fx+e)^{3}}{3f}+\frac{(a-b)\tan(fx+e)^{5}}{5f}$
parallelrisch	$-\frac{-15b\tan(fx+e)^{7}-21a\tan(fx+e)^{5}+21\tan(fx+e)^{5}b+35\tan(fx+e)^{3}a-35b\tan(fx+e)^{3}+105afx-105bfx-105a\,\mathrm{t}}{105f}$
derivativedivides	$\frac{\frac{b\tan(fx+e)^{7}}{7}+\frac{a\tan(fx+e)^{5}}{5}-\frac{\tan(fx+e)^{5}b}{5}-\frac{\tan(fx+e)^{3}a}{3}+\frac{b\tan(fx+e)^{3}}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\arctan(\mathrm{ta}}{f}$
default	$\frac{\frac{b\tan(fx+e)^{7}}{7}+\frac{a\tan(fx+e)^{5}}{5}-\frac{\tan(fx+e)^{5}b}{5}-\frac{\tan(fx+e)^{3}a}{3}+\frac{b\tan(fx+e)^{3}}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\arctan(\mathrm{ta}}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^{5}}{5}-\frac{\tan(fx+e)^{3}}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{b\left(\frac{\tan(fx+e)^{7}}{7}-\frac{\tan(fx+e)^{5}}{5}+\frac{\tan(fx+e)^{3}}{3}-\tan(fx\text{-}\right.}{f}$
risch	$-ax+bx+\frac{2i\left(315a\,\mathrm{e}^{12i(fx+e)}-420b\,\mathrm{e}^{12i(fx+e)}+1260a\,\mathrm{e}^{10i(fx+e)}-1260b\,\mathrm{e}^{10i(fx+e)}+2555a\,\mathrm{e}^{8i(fx+e)}-3080b\,\mathrm{e}\right.}{}$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
(-a+b)*x+(a-b)*tan(f*x+e)/f+1/7*b*tan(f*x+e)^7/f-1/3*(a-b)*tan(f*x+e)^3/f+
1/5*(a-b)*tan(f*x+e)^5/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \frac{15\,b\tan\left(fx+e\right)^7+21\left(a-b\right)\tan\left(fx+e\right)^5-35\left(a-b\right)\tan\left(fx+e\right)^3-105\left(a-b\right)fx+105\left(a-b\right)\text{ta}}{105\,f}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*
x + e)^3 - 105*(a - b)*f*x + 105*(a - b)*tan(f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \begin{cases} -ax+\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}+bx+\frac{b\tan^7(e+fx)}{7f}-\frac{b\tan^5(e+fx)}{5f}+\frac{b\tan^3(e+fx)}{3f}-\frac{b\tan(e+fx}{f} \\ x(a+b\tan^2(e))\tan^6(e) \end{cases}$$

input
```
integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((-a*x + a*tan(e + f*x)**5/(5*f) - a*tan(e + f*x)**3/(3*f) + a*ta
n(e + f*x)/f + b*x + b*tan(e + f*x)**7/(7*f) - b*tan(e + f*x)**5/(5*f) + b
*tan(e + f*x)**3/(3*f) - b*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)
*tan(e)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{15\,b\tan\left(fx+e\right)^7+21\,(a-b)\tan\left(fx+e\right)^5-35\,(a-b)\tan\left(fx+e\right)^3-105\,(fx+e)(a-b)+105\,(a-}{105\,f}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*(a - b)*tan(f*x + e)^5 - 35*(a - b)*tan(f*
x + e)^3 - 105*(f*x + e)*(a - b) + 105*(a - b)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(fx+e)(a-b)}{f}$$
$$+\frac{15\,bf^6\tan\left(fx+e\right)^7+21\,af^6\tan\left(fx+e\right)^5-21\,bf^6\tan\left(fx+e\right)^5-35\,af^6\tan\left(fx+e\right)^3+35\,bf^6\tan}{105\,f^7}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-(f*x + e)*(a - b)/f + 1/105*(15*b*f^6*tan(f*x + e)^7 + 21*a*f^6*tan(f*x +
 e)^5 - 21*b*f^6*tan(f*x + e)^5 - 35*a*f^6*tan(f*x + e)^3 + 35*b*f^6*tan(f
*x + e)^3 + 105*a*f^6*tan(f*x + e) - 105*b*f^6*tan(f*x + e))/f^7
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{\frac{b\tan(e+f\,x)^7}{7}+\left(\frac{a}{5}-\frac{b}{5}\right)\,\tan(e+f\,x)^5+\left(\frac{b}{3}-\frac{a}{3}\right)\,\tan(e+f\,x)^3+(a-b)\,\tan(e+f\,x)-f\,x\,(a-b)}{f}$$

input
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

output
```
(tan(e + f*x)^5*(a/5 - b/5) - tan(e + f*x)^3*(a/3 - b/3) + tan(e + f*x)*(a
 - b) + (b*tan(e + f*x)^7)/7 - f*x*(a - b))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{15\tan\left(fx+e\right)^7b+21\tan\left(fx+e\right)^5a-21\tan\left(fx+e\right)^5b-35\tan\left(fx+e\right)^3a+35\tan\left(fx+e\right)^3b+1}{105f}$$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

output
```
(15*tan(e + f*x)**7*b + 21*tan(e + f*x)**5*a - 21*tan(e + f*x)**5*b - 35*t
an(e + f*x)**3*a + 35*tan(e + f*x)**3*b + 105*tan(e + f*x)*a - 105*tan(e +
 f*x)*b - 105*a*f*x + 105*b*f*x)/(105*f)
```

3.192 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=(a-b)x-\frac{(a-b)\tan(e+fx)}{f}+\frac{(a-b)\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

output
```
(a-b)*x-(a-b)*tan(f*x+e)/f+1/3*(a-b)*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{a\arctan(\tan(e+fx))}{f}-\frac{b\arctan(\tan(e+fx))}{f}-\frac{a\tan(e+fx)}{f}+\frac{b\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}-\frac{b\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

input
```
Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
(a*ArcTan[Tan[e + f*x]])/f - (b*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])
/f + (b*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) - (b*Tan[e + f*x]^3)/(3
*f) + (b*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4114, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^4\left(a+b\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{ 4114}$$

$$(a-b)\int \tan^4(e+fx)dx+\frac{b\tan^5(e+fx)}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$(a-b)\int \tan(e+fx)^4dx+\frac{b\tan^5(e+fx)}{5f}$$

$$\Big\downarrow \text{ 3954}$$

$$(a-b)\left(\frac{\tan^3(e+fx)}{3f}-\int \tan^2(e+fx)dx\right)+\frac{b\tan^5(e+fx)}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$(a-b)\left(\frac{\tan^3(e+fx)}{3f}-\int \tan(e+fx)^2dx\right)+\frac{b\tan^5(e+fx)}{5f}$$

$$\Big\downarrow \text{ 3954}$$

$$(a-b)\left(\int 1dx+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}\right)+\frac{b\tan^5(e+fx)}{5f}$$

$$\Big\downarrow \ 24$$

$$(a-b)\left(\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}+x\right)+\frac{b\tan^5(e+fx)}{5f}$$

input
```
Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
(b*Tan[e + f*x]^5)/(5*f) + (a - b)*(x - Tan[e + f*x]/f + Tan[e + f*x]^3/(3
*f))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result
norman	$(a-b)x-\frac{(a-b)\tan(fx+e)}{f}+\frac{(a-b)\tan(fx+e)^3}{3f}+\frac{b\tan(fx+e)^5}{5f}$
parallelrisch	$\frac{3\tan(fx+e)^5 b+5\tan(fx+e)^3 a-5b\tan(fx+e)^3+15afx-15bfx-15a\tan(fx+e)+15b\tan(fx+e)}{15f}$
derivativedivides	$\frac{\frac{\tan(fx+e)^5 b}{5}+\frac{\tan(fx+e)^3 a}{3}-\frac{b\tan(fx+e)^3}{3}-a\tan(fx+e)+b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$
default	$\frac{\frac{\tan(fx+e)^5 b}{5}+\frac{\tan(fx+e)^3 a}{3}-\frac{b\tan(fx+e)^3}{3}-a\tan(fx+e)+b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}$
risch	$ax-bx-\frac{2i(30a\,e^{8i(fx+e)}-45b\,e^{8i(fx+e)}+90a\,e^{6i(fx+e)}-90b\,e^{6i(fx+e)}+110a\,e^{4i(fx+e)}-140b\,e^{4i(fx+e)}+70a\,e^{2}}{15f\left(e^{2i(fx+e)}+1\right)^5}$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
(a-b)*x-(a-b)*tan(f*x+e)/f+1/3*(a-b)*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\,b\tan\left(fx+e\right)^5+5\,(a-b)\tan\left(fx+e\right)^3+15\,(a-b)fx-15\,(a-b)\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(a - b)*f*x - 15*
(a - b)*tan(f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\begin{cases} ax+\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}-bx+\frac{b\tan^5(e+fx)}{5f}-\frac{b\tan^3(e+fx)}{3f}+\frac{b\tan(e+fx)}{f} & \text{for } f\neq 0\\ x(a+b\tan^2(e))\tan^4(e) & \text{otherwise}\end{cases}$$

input
```
integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((a*x + a*tan(e + f*x)**3/(3*f) - a*tan(e + f*x)/f - b*x + b*tan(
e + f*x)**5/(5*f) - b*tan(e + f*x)**3/(3*f) + b*tan(e + f*x)/f, Ne(f, 0)),
 (x*(a + b*tan(e)**2)*tan(e)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\,b\tan(fx+e)^5+5\,(a-b)\tan(fx+e)^3+15\,(fx+e)(a-b)-15\,(a-b)\tan(fx+e)}{15\,f}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*(a - b)*tan(f*x + e)^3 + 15*(f*x + e)*(a - b)
 - 15*(a - b)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{(fx+e)(a-b)}{f} + \frac{3\,bf^4\tan\left(fx+e\right)^5+5\,af^4\tan\left(fx+e\right)^3-5\,bf^4\tan\left(fx+e\right)^3-15\,af^4\tan\left(fx+e\right)+15\,bf^4\tan\left(fx\right.}{15\,f^5}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
(f*x + e)*(a - b)/f + 1/15*(3*b*f^4*tan(f*x + e)^5 + 5*a*f^4*tan(f*x + e)^
3 - 5*b*f^4*tan(f*x + e)^3 - 15*a*f^4*tan(f*x + e) + 15*b*f^4*tan(f*x + e)
)/f^5
```

Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\frac{b\tan(e+f\,x)^5}{5}+\left(\frac{a}{3}-\frac{b}{3}\right)\,\tan(e+f\,x)^3+(b-a)\,\tan(e+f\,x)+f\,x\,(a-b)}{f}$$

input
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

output
```
(tan(e + f*x)^3*(a/3 - b/3) - tan(e + f*x)*(a - b) + (b*tan(e + f*x)^5)/5
+ f*x*(a - b))/f
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\tan(fx+e)^5 b+5\tan(fx+e)^3 a-5\tan(fx+e)^3 b-15\tan(fx+e)a+15\tan(fx+e)b+15afx}{15f}$$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

output
```
(3*tan(e + f*x)**5*b + 5*tan(e + f*x)**3*a - 5*tan(e + f*x)**3*b - 15*tan(
e + f*x)*a + 15*tan(e + f*x)*b + 15*a*f*x - 15*b*f*x)/(15*f)
```

3.193 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-((a-b)x)+\frac{(a-b)\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

output
```
-(a-b)*x+(a-b)*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\arctan(\tan(e+fx))}{f}+\frac{b\arctan(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}-\frac{b\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

input
```
Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((a*ArcTan[Tan[e + f*x]])/f) + (b*ArcTan[Tan[e + f*x]])/f + (a*Tan[e + f*
x])/f - (b*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4114, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2 \left(a+b\tan(e+fx)^2\right) dx$$

$$\Big\downarrow \text{4114}$$

$$(a-b)\int \tan^2(e+fx)dx+\frac{b\tan^3(e+fx)}{3f}$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\int \tan(e+fx)^2dx+\frac{b\tan^3(e+fx)}{3f}$$

$$\Big\downarrow \text{3954}$$

$$(a-b)\left(\frac{\tan(e+fx)}{f}-\int 1dx\right)+\frac{b\tan^3(e+fx)}{3f}$$

$$\Big\downarrow \text{24}$$

$$(a-b)\left(\frac{\tan(e+fx)}{f}-x\right)+\frac{b\tan^3(e+fx)}{3f}$$

input
```
Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
(b*Tan[e + f*x]^3)/(3*f) + (a - b)*(-x + Tan[e + f*x]/f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result	size
norman	$(-a+b)\,x+\frac{(a-b)\tan(fx+e)}{f}+\frac{b\tan(fx+e)^{3}}{3f}$	38
parallelrisch	$-\frac{-b\tan(fx+e)^{3}+3afx-3bfx-3a\tan(fx+e)+3b\tan(fx+e)}{3f}$	46
derivativedivides	$\frac{\frac{b\tan(fx+e)^{3}}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\arctan(\tan(fx+e))}{f}$	47
default	$\frac{\frac{b\tan(fx+e)^{3}}{3}+a\tan(fx+e)-b\tan(fx+e)+(-a+b)\arctan(\tan(fx+e))}{f}$	47
parts	$\frac{a(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b\left(\frac{\tan(fx+e)^{3}}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}$	54
risch	$-ax+bx+\frac{2i\left(3a\,e^{4i(fx+e)}-6b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+3a-4b\right)}{3f\left(e^{2i(fx+e)}+1\right)^{3}}$	83

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
(-a+b)*x+(a-b)*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan\left(fx+e\right)^3-3\left(a-b\right)fx+3\left(a-b\right)\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*(b*tan(f*x + e)^3 - 3*(a - b)*f*x + 3*(a - b)*tan(f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \begin{cases} -ax+\frac{a\tan\left(e+fx\right)}{f}+bx+\frac{b\tan^3\left(e+fx\right)}{3f}-\frac{b\tan\left(e+fx\right)}{f} & \text{for } f\neq 0 \\ x(a+b\tan^2\left(e\right))\tan^2\left(e\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((-a*x + a*tan(e + f*x)/f + b*x + b*tan(e + f*x)**3/(3*f) - b*tan
(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)*tan(e)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{b\tan\left(fx+e\right)^3-3\left(fx+e\right)(a-b)+3\left(a-b\right)\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*(a - b) + 3*(a - b)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(fx+e)(a-b)}{f}+\frac{bf^2\tan\left(fx+e\right)^3+3\,af^2\tan\left(fx+e\right)-3\,bf^2\tan\left(fx+e\right)}{3\,f^3}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-(f*x + e)*(a - b)/f + 1/3*(b*f^2*tan(f*x + e)^3 + 3*a*f^2*tan(f*x + e) -
3*b*f^2*tan(f*x + e))/f^3
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\frac{b\tan(e+f\,x)^3}{3}+(a-b)\,\tan(e+f\,x)-f\,x\,(a-b)}{f}$$

input
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

output
```
(tan(e + f*x)*(a - b) + (b*tan(e + f*x)^3)/3 - f*x*(a - b))/f
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{\tan(fx+e)^3 b+3\tan(fx+e)\,a-3\tan(fx+e)\,b-3afx+3bfx}{3f}$$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

output
```
(tan(e + f*x)**3*b + 3*tan(e + f*x)*a - 3*tan(e + f*x)*b - 3*a*f*x + 3*b*f
*x)/(3*f)
```

3.194 $\int (a + b\tan^2(e + fx))\, dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int (a + b\tan^2(e + fx))\, dx = ax - bx + \frac{b\tan(e + fx)}{f}$$

output
```
a*x-b*x+b*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int (a + b\tan^2(e + fx))\, dx = ax - \frac{b\arctan(\tan(e + fx))}{f} + \frac{b\tan(e + fx)}{f}$$

input
```
Integrate[a + b*Tan[e + f*x]^2,x]
```

output
```
a*x - (b*ArcTan[Tan[e + f*x]])/f + (b*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b\tan(e+fx)}{f}-bx$$

input
```
Int[a + b*Tan[e + f*x]^2,x]
```

output
```
a*x - b*x + (b*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
norman	$(a-b)\,x+\frac{b\tan(fx+e)}{f}$	20
parallelrisch	$-\frac{b(fx-\tan(fx+e))}{f}+ax$	23
default	$ax+\frac{b(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}$	26
parts	$ax+\frac{b(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}$	26
derivativedivides	$\frac{b\tan(fx+e)+(a-b)\arctan(\tan(fx+e))}{f}$	27
risch	$ax-bx+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$	29

input
```
int(a+b*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(a-b)*x+b*tan(f*x+e)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=\frac{(a-b)fx+b\tan\left(fx+e\right)}{f}$$

input
```
integrate(a+b*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
((a - b)*f*x + b*tan(f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=ax+b\left(\begin{cases}-x+\frac{\tan{(e+fx)}}{f} & \text{for}\ f\neq 0\\ x\tan^2{(e)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*tan(f*x+e)**2,x)
```

output
```
a*x + b*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=ax-\frac{(fx+e-\tan{(fx+e)})b}{f}$$

input
```
integrate(a+b*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x - (f*x + e - tan(f*x + e))*b/f
```

Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=ax-b\left(\frac{fx+e}{f}-\frac{\tan{(fx+e)}}{f}\right)$$

input
```
integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x - b*((f*x + e)/f - tan(f*x + e)/f)
```

Mupad [B] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=\frac{b\tan(e+f\,x)+f\,x\,(a-b)}{f}$$

input
```
int(a + b*tan(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + f*x*(a - b))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \left(a+b\tan^2(e+fx)\right)\,dx=\frac{\tan\left(fx+e\right)b+afx-bfx}{f}$$

input
```
int(a+b*tan(f*x+e)^2,x)
```

output
```
(tan(e + f*x)*b + a*f*x - b*f*x)/f
```

3.195 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-((a-b)x)-\frac{a\cot(e+fx)}{f}$$

output
```
-(a-b)*x-a*cot(f*x+e)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx\\=bx-\frac{a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
b*x - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4112, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4112}$$

$$\int (b-a)dx-\frac{a\cot(e+fx)}{f}$$

$$\Big\downarrow \text{24}$$

$$-(x(a-b))-\frac{a\cot(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2),x]
```

output
```
-((a - b)*x) - (a*Cot[e + f*x])/f
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

method	result	size
parallelrisch	$\frac{-a\cot(fx+e)-fx(a-b)}{f}$	24
risch	$-ax+bx-\frac{2ia}{f\left(e^{2i(fx+e)}-1\right)}$	29
derivativedivides	$\frac{-\frac{a}{\tan(fx+e)}+(-a+b)\arctan(\tan(fx+e))}{f}$	30
default	$\frac{-\frac{a}{\tan(fx+e)}+(-a+b)\arctan(\tan(fx+e))}{f}$	30
norman	$\frac{(-a+b)x\tan(fx+e)-\frac{a}{f}}{\tan(fx+e)}$	30

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
(-a*cot(f*x+e)-f*x*(a-b))/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{(a-b)fx\tan(fx+e)+a}{f\tan(fx+e)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-((a - b)*f*x*tan(f*x + e) + a)/(f*tan(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \begin{cases} \tilde{\infty}ax & \text{for } e=0 \wedge f=0 \\ x(a+b\tan^2(e))\cot^2(e) & \text{for } f=0 \\ \tilde{\infty}ax & \text{for } e=-fx \\ -ax - \frac{a}{f\tan(e+fx)} + bx & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**2,
Eq(f, 0)), (zoo*a*x, Eq(e, -f*x)), (-a*x - a/(f*tan(e + f*x)) + b*x, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(fx+e)(a-b)+\frac{a}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-((f*x + e)*(a - b) + a/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(fx+e)(a-b)}{f} - \frac{a}{f\tan(fx+e)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-(f*x + e)*(a - b)/f - a/(f*tan(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -x\,(a-b) - \frac{a\cot(e+f\,x)}{f}$$

input
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2),x)
```

output
```
- x*(a - b) - (a*cot(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{-\cos(fx+e)\,a-\sin(fx+e)\,afx+\sin(fx+e)\,bfx}{\sin(fx+e)\,f}$$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2),x)
```

output
```
( - cos(e + f*x)*a - sin(e + f*x)*a*f*x + sin(e + f*x)*b*f*x)/(sin(e + f*x
)*f)
```

3.196 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=(a-b)x+\frac{(a-b)\cot(e+fx)}{f}-\frac{a\cot^3(e+fx)}{3f}$$

output
```
(a-b)*x+(a-b)*cot(f*x+e)/f-1/3*a*cot(f*x+e)^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\begin{aligned}&\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx\\&=-\frac{a\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f}\\&\quad-\frac{b\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}\end{aligned}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/3*(a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/
f - (b*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4112, 25, 27, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4112}$$

$$\int -\left((a-b)\cot^2(e+fx)\right)dx-\frac{a\cot^3(e+fx)}{3f}$$

$$\Big\downarrow \text{25}$$

$$-\int (a-b)\cot^2(e+fx)dx-\frac{a\cot^3(e+fx)}{3f}$$

$$\Big\downarrow \text{27}$$

$$-(a-b)\int \cot^2(e+fx)dx-\frac{a\cot^3(e+fx)}{3f}$$

$$\Big\downarrow \text{3042}$$

$$-(a-b)\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{a\cot^3(e+fx)}{3f}$$

$$\Big\downarrow \text{3954}$$

$$-(a-b)\left(-\int 1dx-\frac{\cot(e+fx)}{f}\right)-\frac{a\cot^3(e+fx)}{3f}$$

$$\Big\downarrow \text{24}$$

$$-(a-b)\left(-\frac{\cot(e+fx)}{f}-x\right)-\frac{a\cot^3(e+fx)}{3f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/3*(a*Cot[e + f*x]^3)/f - (a - b)*(-x - Cot[e + f*x]/f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{-\cot(fx+e)^3 a+(3a-3b)\cot(fx+e)+3fx(a-b)}{3f}$	41
derivativedivides	$\frac{b(-\cot(fx+e)-fx-e)+a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)}{f}$	47
default	$\frac{b(-\cot(fx+e)-fx-e)+a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)}{f}$	47
norman	$\frac{(a-b)x\tan(fx+e)^3+\frac{(a-b)\tan(fx+e)^2}{f}-\frac{a}{3f}}{\tan(fx+e)^3}$	49
risch	$ax-bx+\frac{2i(6a\,e^{4i(fx+e)}-3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+4a-3b)}{3f(e^{2i(fx+e)}-1)^3}$	83

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-cot(f*x+e)^3*a+(3*a-3*b)*cot(f*x+e)+3*f*x*(a-b))/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{3\,(a-b)fx\tan\left(fx+e\right)^3+3\,(a-b)\tan\left(fx+e\right)^2-a}{3\,f\tan\left(fx+e\right)^3}$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*(3*(a - b)*f*x*tan(f*x + e)^3 + 3*(a - b)*tan(f*x + e)^2 - a)/(f*tan(f
*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.75 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.69

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \begin{cases} \tilde{\infty}ax & \text{for } e=0 \wedge f=0 \\ x(a+b\tan^2(e))\cot^4(e) & \text{for } f=0 \\ \tilde{\infty}ax & \text{for } e=-fx \\ ax+\frac{a}{f\tan(e+fx)}-\frac{a}{3f\tan^3(e+fx)}-bx-\frac{b}{f\tan(e+fx)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**4,
Eq(f, 0)), (zoo*a*x, Eq(e, -f*x)), (a*x + a/(f*tan(e + f*x)) - a/(3*f*tan(
e + f*x)**3) - b*x - b/(f*tan(e + f*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{3\,(fx+e)(a-b)+\frac{3\,(a-b)\tan(fx+e)^2-a}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*(a - b) + (3*(a - b)*tan(f*x + e)^2 - a)/tan(f*x + e)^3)/
f
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=\frac{(fx+e)(a-b)}{f}+\frac{3\,a\tan\left(fx+e\right)^2-3\,b\tan\left(fx+e\right)^2-a}{3\,f\tan\left(fx+e\right)^3}$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
(f*x + e)*(a - b)/f + 1/3*(3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 - a)/(f
*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=x\,(a-b)-\frac{\frac{a}{3}-\tan(e+f\,x)^2\,(a-b)}{f\tan\left(e+f\,x\right)^3}$$

input
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2),x)
```

output
```
x*(a - b) - (a/3 - tan(e + f*x)^2*(a - b))/(f*tan(e + f*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.13

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$=\frac{4\cos\left(fx+e\right)\sin\left(fx+e\right)^2a-3\cos\left(fx+e\right)\sin\left(fx+e\right)^2b-\cos\left(fx+e\right)a+3\sin\left(fx+e\right)^3afx-3\,\mathrm{s}}{3\sin\left(fx+e\right)^3f}$$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2),x)
```

output
```
(4*cos(e + f*x)*sin(e + f*x)**2*a - 3*cos(e + f*x)*sin(e + f*x)**2*b - cos
(e + f*x)*a + 3*sin(e + f*x)**3*a*f*x - 3*sin(e + f*x)**3*b*f*x)/(3*sin(e
+ f*x)**3*f)
```

3.197 $\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-((a-b)x)-\frac{(a-b)\cot(e+fx)}{f}+\frac{(a-b)\cot^3(e+fx)}{3f}-\frac{a\cot^5(e+fx)}{5f}$$

output
```
-(a-b)*x-(a-b)*cot(f*x+e)/f+1/3*(a-b)*cot(f*x+e)^3/f-1/5*a*cot(f*x+e)^5/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx=-\frac{a\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f}-\frac{b\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/5*(a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/
f - (b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(
3*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4112, 25, 27, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)^2}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{4112}$$

$$\int -\left((a-b)\cot^4(e+fx)\right)dx-\frac{a\cot^5(e+fx)}{5f}$$

$$\Big\downarrow \text{25}$$

$$-\int (a-b)\cot^4(e+fx)dx-\frac{a\cot^5(e+fx)}{5f}$$

$$\Big\downarrow \text{27}$$

$$-(a-b)\int \cot^4(e+fx)dx-\frac{a\cot^5(e+fx)}{5f}$$

$$\Big\downarrow \text{3042}$$

$$-(a-b)\int \tan\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{a\cot^5(e+fx)}{5f}$$

$$\Big\downarrow \text{3954}$$

$$-(a-b)\left(-\int \cot^2(e+fx)dx-\frac{\cot^3(e+fx)}{3f}\right)-\frac{a\cot^5(e+fx)}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$-(a-b)\left(-\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{\cot^3(e+fx)}{3f}\right)-\frac{a\cot^5(e+fx)}{5f}$$

$$\Big\downarrow \text{ 3954}$$

$$-(a-b)\left(\int 1dx-\frac{\cot^3(e+fx)}{3f}+\frac{\cot(e+fx)}{f}\right)-\frac{a\cot^5(e+fx)}{5f}$$

$$\Big\downarrow \text{ 24}$$

$$-(a-b)\left(-\frac{\cot^3(e+fx)}{3f}+\frac{\cot(e+fx)}{f}+x\right)-\frac{a\cot^5(e+fx)}{5f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/5*(a*Cot[e + f*x]^5)/f - (a - b)*(x + Cot[e + f*x]/f - Cot[e + f*x]^3/(
3*f))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{-3\cot(fx+e)^5 a+5\cot(fx+e)^3(a-b)+15(-a+b)\cot(fx+e)-15fx(a-b)}{15f}$
derivativedivides	$\frac{b\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)+a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)}{f}$
default	$\frac{b\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)+a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)}{f}$
norman	$\frac{(-a+b)x\tan(fx+e)^5-\frac{a}{5f}+\frac{(a-b)\tan(fx+e)^2}{3f}-\frac{(a-b)\tan(fx+e)^4}{f}}{\tan(fx+e)^5}$
risch	$-ax+bx-\frac{2i\left(45a\,e^{8i(fx+e)}-30b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}+90b\,e^{6i(fx+e)}+140a\,e^{4i(fx+e)}-110b\,e^{4i(fx+e)}-70a\right.}{15f\left(e^{2i(fx+e)}-1\right)^5}$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*cot(f*x+e)^5*a+5*cot(f*x+e)^3*(a-b)+15*(-a+b)*cot(f*x+e)-15*f*x*(
a-b))/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= -\frac{15\,(a-b)fx\tan\left(fx+e\right)^5+15\,(a-b)\tan\left(fx+e\right)^4-5\,(a-b)\tan\left(fx+e\right)^2+3\,a}{15\,f\tan\left(fx+e\right)^5}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/15*(15*(a - b)*f*x*tan(f*x + e)^5 + 15*(a - b)*tan(f*x + e)^4 - 5*(a -
b)*tan(f*x + e)^2 + 3*a)/(f*tan(f*x + e)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.

Time = 1.68 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx$$
$$= \begin{cases} \tilde{\infty}ax & \text{for } e=0 \wedge f=0 \\ x(a+b\tan^2(e))\cot^6(e) & \text{for } f=0 \\ \tilde{\infty}ax & \text{for } e=-fx \\ -ax-\frac{a}{f\tan(e+fx)}+\frac{a}{3f\tan^3(e+fx)}-\frac{a}{5f\tan^5(e+fx)}+bx+\frac{b}{f\tan(e+fx)}-\frac{b}{3f\tan^3(e+fx)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*a*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)*cot(e)**6,
Eq(f, 0)), (zoo*a*x, Eq(e, -f*x)), (-a*x - a/(f*tan(e + f*x)) + a/(3*f*tan
(e + f*x)**3) - a/(5*f*tan(e + f*x)**5) + b*x + b/(f*tan(e + f*x)) - b/(3*
f*tan(e + f*x)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{15\,(fx+e)(a-b)+\frac{15\,(a-b)\tan(fx+e)^4-5\,(a-b)\tan(fx+e)^2+3\,a}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*(f*x + e)*(a - b) + (15*(a - b)*tan(f*x + e)^4 - 5*(a - b)*tan(f
*x + e)^2 + 3*a)/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -\frac{(fx+e)(a-b)}{f} - \frac{15\,a\tan\left(fx+e\right)^4-15\,b\tan\left(fx+e\right)^4-5\,a\tan\left(fx+e\right)^2+5\,b\tan\left(fx+e\right)^2+3\,a}{15\,f\tan\left(fx+e\right)^5}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-(f*x + e)*(a - b)/f - 1/15*(15*a*tan(f*x + e)^4 - 15*b*tan(f*x + e)^4 - 5
*a*tan(f*x + e)^2 + 5*b*tan(f*x + e)^2 + 3*a)/(f*tan(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = -x\,(a-b)-\frac{(a-b)\,\tan(e+f\,x)^4+\left(\frac{b}{3}-\frac{a}{3}\right)\,\tan(e+f\,x)^2+\frac{a}{5}}{f\tan\left(e+f\,x\right)^5}$$

input
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2),x)
```

output
```
- x*(a - b) - (a/5 - tan(e + f*x)^2*(a/3 - b/3) + tan(e + f*x)^4*(a - b))/
(f*tan(e + f*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.92

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)\,dx = \frac{-23\cos\left(fx+e\right)\sin\left(fx+e\right)^4a+20\cos\left(fx+e\right)\sin\left(fx+e\right)^4b+11\cos\left(fx+e\right)\sin\left(fx+e\right)^2a-5\cos}{15\sin\left(fx+e\right)^5}$$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2),x)
```

output
```
( - 23*cos(e + f*x)*sin(e + f*x)**4*a + 20*cos(e + f*x)*sin(e + f*x)**4*b
+ 11*cos(e + f*x)*sin(e + f*x)**2*a - 5*cos(e + f*x)*sin(e + f*x)**2*b - 3
*cos(e + f*x)*a - 15*sin(e + f*x)**5*a*f*x + 15*sin(e + f*x)**5*b*f*x)/(15
*sin(e + f*x)**5*f)
```

3.198 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{(a-b)^2\log(\cos(e+fx))}{f} - \frac{(a-b)^2\tan^2(e+fx)}{2f} + \frac{(a-b)^2\tan^4(e+fx)}{4f} + \frac{(2a-b)b\tan^6(e+fx)}{6f} + \frac{b^2\tan^8(e+fx)}{8f}$$

output
```
-(a-b)^2*ln(cos(f*x+e))/f-1/2*(a-b)^2*tan(f*x+e)^2/f+1/4*(a-b)^2*tan(f*x+e
)^4/f+1/6*(2*a-b)*b*tan(f*x+e)^6/f+1/8*b^2*tan(f*x+e)^8/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{-24(a-b)^2\log(\cos(e+fx))-24(a^2-3ab+2b^2)\sec^2(e+fx)+6(a^2-6ab+6b^2)\sec^4(e+fx)+8(a}{24f}$$

input
```
Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-24*(a - b)^2*Log[Cos[e + f*x]] - 24*(a^2 - 3*a*b + 2*b^2)*Sec[e + f*x]^2
 + 6*(a^2 - 6*a*b + 6*b^2)*Sec[e + f*x]^4 + 8*(a - 2*b)*b*Sec[e + f*x]^6 +
 3*b^2*Sec[e + f*x]^8)/(24*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\left(a+b\tan(e+fx)^2\right)^2dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^5(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a)^2}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(b^2\tan^6(e+fx)+(2a-b)b\tan^4(e+fx)+(a-b)^2\tan^2(e+fx)-(a-b)^2+\frac{(a-b)^2}{\tan^2(e+fx)+1}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3}b(2a-b)\tan^6(e+fx)+\frac{1}{2}(a-b)^2\tan^4(e+fx)-(a-b)^2\tan^2(e+fx)+(a-b)^2\log\left(\tan^2(e+fx)+1\right)+\frac{1}{4}}{2f}$$

input
```
Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((a - b)^2*Log[1 + Tan[e + f*x]^2] - (a - b)^2*Tan[e + f*x]^2 + ((a - b)^2
*Tan[e + f*x]^4)/2 + ((2*a - b)*b*Tan[e + f*x]^6)/3 + (b^2*Tan[e + f*x]^8)
/4)/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

method	result
norman	$\frac{b^2\tan(fx+e)^8}{8f}-\frac{(a^2-2ab+b^2)\tan(fx+e)^2}{2f}+\frac{(a^2-2ab+b^2)\tan(fx+e)^4}{4f}+\frac{(2a-b)b\tan(fx+e)^6}{6f}+\frac{(a^2-2ab+}{}$
derivativedivides	$\frac{\frac{b^2\tan(fx+e)^8}{8}+\frac{ab\tan(fx+e)^6}{3}-\frac{b^2\tan(fx+e)^6}{6}+\frac{a^2\tan(fx+e)^4}{4}-\frac{ab\tan(fx+e)^4}{2}+\frac{b^2\tan(fx+e)^4}{4}-\frac{a^2\tan(fx+e)^2}{2}+ab\tan(f}{f}$
default	$\frac{\frac{b^2\tan(fx+e)^8}{8}+\frac{ab\tan(fx+e)^6}{3}-\frac{b^2\tan(fx+e)^6}{6}+\frac{a^2\tan(fx+e)^4}{4}-\frac{ab\tan(fx+e)^4}{2}+\frac{b^2\tan(fx+e)^4}{4}-\frac{a^2\tan(fx+e)^2}{2}+ab\tan(f}{f}$
parts	$\frac{a^2\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^8}{8}-\frac{\tan(fx+e)^6}{6}+\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\right.}{}\right.}{f}$
parallelrisch	$\frac{3b^2\tan(fx+e)^8+8ab\tan(fx+e)^6-4b^2\tan(fx+e)^6+6a^2\tan(fx+e)^4-12ab\tan(fx+e)^4+6b^2\tan(fx+e)^4-12a^2\tan(}{}$
risch	$ia^2x-2iabx+ib^2x+\frac{2ia^2e}{f}-\frac{4iabe}{f}+\frac{2ib^2e}{f}-\frac{4\left(3a^2e^{14i(fx+e)}-9ab\,e^{14i(fx+e)}+6b^2e^{14i(fx+e)}+15a^2e\right.}{}$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*tan(f*x+e)^8/f-1/2*(a^2-2*a*b+b^2)/f*tan(f*x+e)^2+1/4*(a^2-2*a*b+b
^2)/f*tan(f*x+e)^4+1/6*(2*a-b)*b*tan(f*x+e)^6/f+1/2*(a^2-2*a*b+b^2)/f*ln(1
+tan(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,b^2\tan\left(fx+e\right)^8+4\,(2\,ab-b^2)\tan\left(fx+e\right)^6+6\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)^4-12\,(a^2-2\,ab+b^2)\,\mathrm{ta}}{24\,f}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*b^2*tan(f*x + e)^8 + 4*(2*a*b - b^2)*tan(f*x + e)^6 + 6*(a^2 - 2*a
*b + b^2)*tan(f*x + e)^4 - 12*(a^2 - 2*a*b + b^2)*tan(f*x + e)^2 - 12*(a^2
 - 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(82) = 164.

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.96

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a^2\tan^4\left(e+fx\right)}{4f}-\frac{a^2\tan^2\left(e+fx\right)}{2f}-\frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{ab\tan^6\left(e+fx\right)}{3f}-\frac{ab\tan^4\left(e+fx\right)}{2f}+\frac{ab\,\mathrm{t}}{} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^5\left(e\right)\end{cases}$$

input
```
integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f
) - a**2*tan(e + f*x)**2/(2*f) - a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(
e + f*x)**6/(3*f) - a*b*tan(e + f*x)**4/(2*f) + a*b*tan(e + f*x)**2/f + b*
*2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**8/(8*f) - b**2*tan(
e + f*x)**6/(6*f) + b**2*tan(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f
), Ne(f, 0)), (x*(a + b*tan(e)**2)**2*tan(e)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.54

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx =$$

$$-\frac{12\,(a^2-2\,ab+b^2)\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{24\,(a^2-3\,ab+2\,b^2)\sin(fx+e)^6-6\,(11\,a^2-30\,ab+18\,b^2)\sin(fx+e)^4+4\,(15\,a^2-}{\sin(fx+e)^8-4\,\sin(fx+e)^6+6\,\sin(fx+e)^4-4\,\mathrm{s}}}{24\,f}$$

input

```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output

```
-1/24*(12*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (24*(a^2 - 3*a*b +
 2*b^2)*sin(f*x + e)^6 - 6*(11*a^2 - 30*a*b + 18*b^2)*sin(f*x + e)^4 + 4*(
15*a^2 - 38*a*b + 22*b^2)*sin(f*x + e)^2 - 18*a^2 + 44*a*b - 25*b^2)/(sin(
f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1))/
f
```

Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{(a^2-2\,ab+b^2)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

$$+\frac{3\,b^2f^3\tan\left(fx+e\right)^8+8\,abf^3\tan\left(fx+e\right)^6-4\,b^2f^3\tan\left(fx+e\right)^6+6\,a^2f^3\tan\left(fx+e\right)^4-12\,abf^3\tan}{2}$$

input

```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output

```
1/2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2 + 1)/f + 1/24*(3*b^2*f^3*tan(f*
x + e)^8 + 8*a*b*f^3*tan(f*x + e)^6 - 4*b^2*f^3*tan(f*x + e)^6 + 6*a^2*f^3
*tan(f*x + e)^4 - 12*a*b*f^3*tan(f*x + e)^4 + 6*b^2*f^3*tan(f*x + e)^4 - 1
2*a^2*f^3*tan(f*x + e)^2 + 24*a*b*f^3*tan(f*x + e)^2 - 12*b^2*f^3*tan(f*x
+ e)^2)/f^4
```

Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)+\tan(e+f\,x)^6\,\left(\frac{a\,b}{3}-\frac{b^2}{6}\right)+\frac{b^2\tan(e+f\,x)^8}{8}-\tan(e+f\,x)^2\,\left(\frac{a^2}{2}-c\right.}{f}$$

input
```
int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2) + tan(e + f*x)^6*((a*b)/3 -
 b^2/6) + (b^2*tan(e + f*x)^8)/8 - tan(e + f*x)^2*(a^2/2 - a*b + b^2/2) +
tan(e + f*x)^4*(a^2/4 - (a*b)/2 + b^2/4))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.59

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{12\log\left(\tan(fx+e)^2+1\right)a^2-24\log\left(\tan(fx+e)^2+1\right)ab+12\log\left(\tan(fx+e)^2+1\right)b^2+3\tan(fx+}{}$$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(12*log(tan(e + f*x)**2 + 1)*a**2 - 24*log(tan(e + f*x)**2 + 1)*a*b + 12*l
og(tan(e + f*x)**2 + 1)*b**2 + 3*tan(e + f*x)**8*b**2 + 8*tan(e + f*x)**6*
a*b - 4*tan(e + f*x)**6*b**2 + 6*tan(e + f*x)**4*a**2 - 12*tan(e + f*x)**4
*a*b + 6*tan(e + f*x)**4*b**2 - 12*tan(e + f*x)**2*a**2 + 24*tan(e + f*x)*
*2*a*b - 12*tan(e + f*x)**2*b**2)/(24*f)
```

3.199 $\int \tan^3(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \tan^3(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \frac{(a-b)^2\log(\cos(e+fx))}{f} + \frac{(a-b)^2\tan^2(e+fx)}{2f} + \frac{(2a-b)b\tan^4(e+fx)}{4f} + \frac{b^2\tan^6(e+fx)}{6f}$$

output
```
(a-b)^2*ln(cos(f*x+e))/f+1/2*(a-b)^2*tan(f*x+e)^2/f+1/4*(2*a-b)*b*tan(f*x+
e)^4/f+1/6*b^2*tan(f*x+e)^6/f
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \tan^3(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = \frac{12(a-b)^2\log(\cos(e+fx)) + 6(a^2-4ab+3b^2)\sec^2(e+fx) + 3(2a-3b)b\sec^4(e+fx) + 2b^2\sec^6(e+}{12f}$$

input
```
Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(12*(a - b)^2*Log[Cos[e + f*x]] + 6*(a^2 - 4*a*b + 3*b^2)*Sec[e + f*x]^2 +
 3*(2*a - 3*b)*b*Sec[e + f*x]^4 + 2*b^2*Sec[e + f*x]^6)/(12*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^3 \left(a+b\tan(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\tan^3(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 86$$

$$\frac{\int \left(b^2\tan^4(e+fx)+(2a-b)b\tan^2(e+fx)+(a-b)^2-\frac{(a-b)^2}{\tan^2(e+fx)+1}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}b(2a-b)\tan^4(e+fx)+(a-b)^2\tan^2(e+fx)-(a-b)^2\log\left(\tan^2(e+fx)+1\right)+\frac{1}{3}b^2\tan^6(e+fx)}{2f}$$

input
```
Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a - b)^2*Log[1 + Tan[e + f*x]^2]) + (a - b)^2*Tan[e + f*x]^2 + ((2*a -
 b)*b*Tan[e + f*x]^4)/2 + (b^2*Tan[e + f*x]^6)/3)/(2*f)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

method	result
norman	$\frac{b^2\tan(fx+e)^6}{6f}+\frac{(a^2-2ab+b^2)\tan(fx+e)^2}{2f}+\frac{(2a-b)b\tan(fx+e)^4}{4f}-\frac{(a^2-2ab+b^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}$
derivativedivides	$\frac{\frac{b^2\tan(fx+e)^6}{6}+\frac{ab\tan(fx+e)^4}{2}-\frac{b^2\tan(fx+e)^4}{4}+\frac{a^2\tan(fx+e)^2}{2}-ab\tan(fx+e)^2+\frac{b^2\tan(fx+e)^2}{2}+\frac{(-a^2+2ab-b^2)\ln\left(1+\tan\right.}{2}}{f}$
default	$\frac{\frac{b^2\tan(fx+e)^6}{6}+\frac{ab\tan(fx+e)^4}{2}-\frac{b^2\tan(fx+e)^4}{4}+\frac{a^2\tan(fx+e)^2}{2}-ab\tan(fx+e)^2+\frac{b^2\tan(fx+e)^2}{2}+\frac{(-a^2+2ab-b^2)\ln\left(1+\tan\right.}{2}}{f}$
parts	$\frac{a^2\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^6}{6}-\frac{\tan(fx+e)^4}{4}+\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{2ab\left(\tan\right.}{}$
parallelrisch	$-\frac{-2b^2\tan(fx+e)^6-6ab\tan(fx+e)^4+3b^2\tan(fx+e)^4-6a^2\tan(fx+e)^2+12ab\tan(fx+e)^2-6b^2\tan(fx+e)^2+6\ln\left(1\right.}{12f}$
risch	$-ia^2x+2iabx-ib^2x-\frac{2ia^2e}{f}+\frac{4iabe}{f}-\frac{2ib^2e}{f}+\frac{2a^2e^{10i(fx+e)}-8ab\,e^{10i(fx+e)}+6b^2e^{10i(fx+e)}+8a^2e^{8}}{}$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*tan(f*x+e)^6/f+1/2*(a^2-2*a*b+b^2)/f*tan(f*x+e)^2+1/4*(2*a-b)*b*ta
n(f*x+e)^4/f-1/2*(a^2-2*a*b+b^2)/f*ln(1+tan(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{2\,b^2\tan\left(fx+e\right)^6+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)^4+6\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^2+6\left(a^2-2\,ab+b^2\right)\log}{12\,f}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/12*(2*b^2*tan(f*x + e)^6 + 3*(2*a*b - b^2)*tan(f*x + e)^4 + 6*(a^2 - 2*a
*b + b^2)*tan(f*x + e)^2 + 6*(a^2 - 2*a*b + b^2)*log(1/(tan(f*x + e)^2 + 1
)))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{a^2\tan^2\left(e+fx\right)}{2f} + \frac{ab\log\left(\tan^2\left(e+fx\right)+1\right)}{f} + \frac{ab\tan^4\left(e+fx\right)}{2f} - \frac{ab\tan^2\left(e+fx\right)}{f} - \frac{b^2\log\left(\tan^2\left(e+fx\right)+\right.}{2f} \\ x\left(a+b\tan^2\left(e\right)\right)^2\tan^3\left(e\right) \end{cases}$$

input
```
integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*
f) + a*b*log(tan(e + f*x)**2 + 1)/f + a*b*tan(e + f*x)**4/(2*f) - a*b*tan(
e + f*x)**2/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2*tan(e + f*x)**6
/(6*f) - b**2*tan(e + f*x)**4/(4*f) + b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)
), (x*(a + b*tan(e)**2)**2*tan(e)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.55

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{6\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(fx+e\right)^2-1\right) - \frac{6\left(a^2-4\,ab+3\,b^2\right)\sin(fx+e)^4-3\left(4\,a^2-14\,ab+9\,b^2\right)\sin(fx+e)^2+6\,a^2-18\,ab+11\,b^2}{\sin(fx+e)^6-3\,\sin(fx+e)^4+3\,\sin(fx+e)^2-1}}{12\,f}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/12*(6*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 4*a*b + 3*
b^2)*sin(f*x + e)^4 - 3*(4*a^2 - 14*a*b + 9*b^2)*sin(f*x + e)^2 + 6*a^2 -
18*a*b + 11*b^2)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1
))/f
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.56

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{(a^2-2\,ab+b^2)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f} + \frac{2\,b^2f^2\tan\left(fx+e\right)^6+6\,abf^2\tan\left(fx+e\right)^4-3\,b^2f^2\tan\left(fx+e\right)^4+6\,a^2f^2\tan\left(fx+e\right)^2-12\,abf^2\tan}{12\,f^3}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2 + 1)/f + 1/12*(2*b^2*f^2*tan(f
*x + e)^6 + 6*a*b*f^2*tan(f*x + e)^4 - 3*b^2*f^2*tan(f*x + e)^4 + 6*a^2*f^
2*tan(f*x + e)^2 - 12*a*b*f^2*tan(f*x + e)^2 + 6*b^2*f^2*tan(f*x + e)^2)/f
^3
```

Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\tan(e+f\,x)^4\left(\frac{a\,b}{2}-\frac{b^2}{4}\right)}{f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f} + \frac{b^2\tan(e+f\,x)^6}{6\,f} + \frac{\tan(e+f\,x)^2\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}$$

input
```
int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^4*((a*b)/2 - b^2/4))/f - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a
*b + b^2/2))/f + (b^2*tan(e + f*x)^6)/(6*f) + (tan(e + f*x)^2*(a^2/2 - a*b
 + b^2/2))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.57

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{-6\log\left(\tan(fx+e)^2+1\right)a^2+12\log\left(\tan(fx+e)^2+1\right)ab-6\log\left(\tan(fx+e)^2+1\right)b^2+2\tan(fx+$$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 6*log(tan(e + f*x)**2 + 1)*a**2 + 12*log(tan(e + f*x)**2 + 1)*a*b - 6*
log(tan(e + f*x)**2 + 1)*b**2 + 2*tan(e + f*x)**6*b**2 + 6*tan(e + f*x)**4
*a*b - 3*tan(e + f*x)**4*b**2 + 6*tan(e + f*x)**2*a**2 - 12*tan(e + f*x)**
2*a*b + 6*tan(e + f*x)**2*b**2)/(12*f)
```

3.200 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{(a-b)^2\log(\cos(e+fx))}{f} + \frac{(a-b)b\tan^2(e+fx)}{2f} + \frac{\left(a+b\tan^2(e+fx)\right)^2}{4f}$$

output
```
-(a-b)^2*ln(cos(f*x+e))/f+1/2*(a-b)*b*tan(f*x+e)^2/f+1/4*(a+b*tan(f*x+e)^2
)^2/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{-4(a-b)^2\log(\cos(e+fx))+4(a-b)b\sec^2(e+fx)+b^2\sec^4(e+fx)}{4f}$$

input
```
Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-4*(a - b)^2*Log[Cos[e + f*x]] + 4*(a - b)*b*Sec[e + f*x]^2 + b^2*Sec[e +
 f*x]^4)/(4*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4153, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(a+b\tan(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(a-b)^2}{\tan^2(e+fx)+1}+b(a-b)+b\left(b\tan^2(e+fx)+a\right)\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(a-b)\tan^2(e+fx)+\frac{1}{2}\left(a+b\tan^2(e+fx)\right)^2+(a-b)^2\log\left(\tan^2(e+fx)+1\right)}{2f}$$

input
```
Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((a - b)^2*Log[1 + Tan[e + f*x]^2] + (a - b)*b*Tan[e + f*x]^2 + (a + b*Tan
[e + f*x]^2)^2/2)/(2*f)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

method	result
norman	$\frac{b^2\tan(fx+e)^4}{4f}+\frac{b(2a-b)\tan(fx+e)^2}{2f}+\frac{(a^2-2ab+b^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}$
derivativedivides	$\frac{\frac{b^2\tan(fx+e)^4}{4}+ab\tan(fx+e)^2-\frac{b^2\tan(fx+e)^2}{2}+\frac{(a^2-2ab+b^2)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$
default	$\frac{\frac{b^2\tan(fx+e)^4}{4}+ab\tan(fx+e)^2-\frac{b^2\tan(fx+e)^2}{2}+\frac{(a^2-2ab+b^2)\ln\left(1+\tan(fx+e)^2\right)}{2}}{f}$
parallelrisch	$\frac{b^2\tan(fx+e)^4+4ab\tan(fx+e)^2-2b^2\tan(fx+e)^2+2\ln\left(1+\tan(fx+e)^2\right)a^2-4\ln\left(1+\tan(fx+e)^2\right)ab+2\ln\left(1+\tan(f\right.}{4f}$
parts	$\frac{a^2\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b^2\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{ab\tan(fx+e)^2}{f}-\frac{ab\ln\left(1+\tan(fx\right.}{f}$
risch	$ia^2x-2iabx+ib^2x+\frac{2ia^2e}{f}-\frac{4iabe}{f}+\frac{2ib^2e}{f}+\frac{4b\left(a\,e^{6i(fx+e)}-b\,e^{6i(fx+e)}+2a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+\right.}{f\left(e^{2i(fx+e)}+1\right)^4}$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2/f*tan(f*x+e)^4+1/2*b*(2*a-b)/f*tan(f*x+e)^2+1/2*(a^2-2*a*b+b^2)/f*
ln(1+tan(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan\left(fx+e\right)^4+2\left(2\,ab-b^2\right)\tan\left(fx+e\right)^2-2\left(a^2-2\,ab+b^2\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{4\,f}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*tan(f*x + e)^4 + 2*(2*a*b - b^2)*tan(f*x + e)^2 - 2*(a^2 - 2*a*b
+ b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\begin{cases} \frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f}-\frac{ab\log\left(\tan^2(e+fx)+1\right)}{f}+\frac{ab\tan^2(e+fx)}{f}+\frac{b^2\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{b^2\tan^4(e+fx)}{4f}-\frac{b^2\tan^2(e+fx)}{2f} \\ x\left(a+b\tan^2(e)\right)^2\tan(e) \end{cases}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) - a*b*log(tan(e + f*x)**2 +
 1)/f + a*b*tan(e + f*x)**2/f + b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b**2
*tan(e + f*x)**4/(4*f) - b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*
tan(e)**2)**2*tan(e), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=-\frac{2\left(a^2-2ab+b^2\right)\log\left(\sin(fx+e)^2-1\right)+\frac{4\left(ab-b^2\right)\sin(fx+e)^2-4ab+3b^2}{\sin(fx+e)^4-2\sin(fx+e)^2+1}}{4f}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2 - 1) + (4*(a*b - b^2)*sin(f
*x + e)^2 - 4*a*b + 3*b^2)/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \tan(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{(a^2-2ab+b^2)\log\left(\tan\left(fx+e\right)^2+1\right)}{2f} + \frac{b^2f\tan\left(fx+e\right)^4+4abf\tan\left(fx+e\right)^2-2b^2f\tan\left(fx+e\right)^2}{4f^2}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2 + 1)/f + 1/4*(b^2*f*tan(f*x + e
)^4 + 4*a*b*f*tan(f*x + e)^2 - 2*b^2*f*tan(f*x + e)^2)/f^2
```

Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \tan(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f} + \frac{\tan(e+f\,x)^2\,\left(a\,b-\frac{b^2}{2}\right)}{f} + \frac{b^2\tan(e+f\,x)^4}{4\,f}$$

input
```
int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2))/f + (tan(e + f*x)^2*(a*b -
 b^2/2))/f + (b^2*tan(e + f*x)^4)/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{2\log\left(\tan\left(fx+e\right)^2+1\right)a^2-4\log\left(\tan\left(fx+e\right)^2+1\right)ab+2\log\left(\tan\left(fx+e\right)^2+1\right)b^2+\tan\left(fx+e\right)^4 b}{4f}$$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(2*log(tan(e + f*x)**2 + 1)*a**2 - 4*log(tan(e + f*x)**2 + 1)*a*b + 2*log(
tan(e + f*x)**2 + 1)*b**2 + tan(e + f*x)**4*b**2 + 4*tan(e + f*x)**2*a*b -
 2*tan(e + f*x)**2*b**2)/(4*f)
```

3.201 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{(a-b)^2\log(\cos(e+fx))}{f} + \frac{a^2\log(\tan(e+fx))}{f} + \frac{b^2\tan^2(e+fx)}{2f}$$

output
```
(a-b)^2*ln(cos(f*x+e))/f+a^2*ln(tan(f*x+e))/f+1/2*b^2*tan(f*x+e)^2/f
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{2ab\log(\cos(e+fx))}{f} + \frac{a^2\log(\sin(e+fx))}{f} + \frac{b^2(2\log(\cos(e+fx))+\sec^2(e+fx))}{2f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-2*a*b*Log[Cos[e + f*x]])/f + (a^2*Log[Sin[e + f*x]])/f + (b^2*(2*Log[Cos
[e + f*x]] + Sec[e + f*x]^2))/(2*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4153, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\cot(e+fx)a^2+b^2-\frac{(a-b)^2}{\tan^2(e+fx)+1}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\log\left(\tan^2(e+fx)\right)-(a-b)^2\log\left(\tan^2(e+fx)+1\right)+b^2\tan^2(e+fx)}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(a^2*Log[Tan[e + f*x]^2] - (a - b)^2*Log[1 + Tan[e + f*x]^2] + b^2*Tan[e +
 f*x]^2)/(2*f)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{-(a-b)^2 \ln\left(\sec(fx+e)^2\right)+b^2 \tan(fx+e)^2+2a^2 \ln(\tan(fx+e))}{2f}$
derivativedivides	$\frac{\frac{b^2 \tan(fx+e)^2}{2}+\frac{\left(-a^2+2ab-b^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+a^2 \ln(\tan(fx+e))}{f}$
default	$\frac{\frac{b^2 \tan(fx+e)^2}{2}+\frac{\left(-a^2+2ab-b^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+a^2 \ln(\tan(fx+e))}{f}$
norman	$\frac{b^2 \tan(fx+e)^2}{2f}+\frac{a^2 \ln(\tan(fx+e))}{f}-\frac{(a^2-2ab+b^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}$
risch	$-ia^2x+2iabx-ib^2x+\frac{4iabe}{f}-\frac{2ib^2e}{f}-\frac{2ia^2e}{f}+\frac{2b^2 e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2\ln\left(e^{2i(fx+e)}+1\right)ab}{f}+\frac{\ln(e^{2i}}{}$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(a-b)^2*ln(sec(f*x+e)^2)+b^2*tan(f*x+e)^2+2*a^2*ln(tan(f*x+e)))/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{b^2\tan\left(fx+e\right)^2+a^2\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)-\left(2\,ab-b^2\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*tan(f*x + e)^2 + a^2*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) - (
2*a*b - b^2)*log(1/(tan(f*x + e)^2 + 1)))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \begin{cases} -\frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{a^2\log\left(\tan(e+fx)\right)}{f} + \frac{ab\log\left(\tan^2(e+fx)+1\right)}{f} - \frac{b^2\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{b^2\tan^2(e+fx)}{2f} & \text{for } f \ne \\ x(a+b\tan^2(e))^2\cot(e) & \text{otherw} \end{cases}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*log(tan(e + f*x))/f
 + a*b*log(tan(e + f*x)**2 + 1)/f - b**2*log(tan(e + f*x)**2 + 1)/(2*f) +
b**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{a^2\log\left(\sin(fx+e)^2\right) - (2\,ab-b^2)\log\left(\sin(fx+e)^2-1\right) - \frac{b^2}{\sin(fx+e)^2-1}}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a^2*log(sin(f*x + e)^2) - (2*a*b - b^2)*log(sin(f*x + e)^2 - 1) - b^2
/(sin(f*x + e)^2 - 1))/f
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{b^2\tan\left(fx+e\right)^2}{2\,f}+\frac{a^2\log\left(\tan\left(fx+e\right)^2\right)}{2\,f}-\frac{\left(a^2-2\,ab+b^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*b^2*tan(f*x + e)^2/f + 1/2*a^2*log(tan(f*x + e)^2)/f - 1/2*(a^2 - 2*a*
b + b^2)*log(tan(f*x + e)^2 + 1)/f
```

Mupad [B] (verification not implemented)

Time = 7.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{b^2\tan(e+f\,x)^2}{2\,f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}+\frac{a^2\,\ln\left(\tan(e+f\,x)\right)}{f}$$

input
```
int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x)^2)/(2*f) - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2
))/f + (a^2*log(tan(e + f*x)))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 369, normalized size of antiderivative = 7.24

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2a^2+4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2ab-2\log\left(\tan\left(\frac{fx}{2}\right.\right.}{}$$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**2 + 4*log(tan((e + f
*x)/2)**2 + 1)*sin(e + f*x)**2*a*b - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e
+ f*x)**2*b**2 + 2*log(tan((e + f*x)/2)**2 + 1)*a**2 - 4*log(tan((e + f*x)
/2)**2 + 1)*a*b + 2*log(tan((e + f*x)/2)**2 + 1)*b**2 - 4*log(tan((e + f*x
)/2) - 1)*sin(e + f*x)**2*a*b + 2*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**
2*b**2 + 4*log(tan((e + f*x)/2) - 1)*a*b - 2*log(tan((e + f*x)/2) - 1)*b**
2 - 4*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2*a*b + 2*log(tan((e + f*x)/
2) + 1)*sin(e + f*x)**2*b**2 + 4*log(tan((e + f*x)/2) + 1)*a*b - 2*log(tan
((e + f*x)/2) + 1)*b**2 + 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**2 - 2
*log(tan((e + f*x)/2))*a**2 - sin(e + f*x)**2*b**2)/(2*f*(sin(e + f*x)**2
- 1))
```

3.202 $\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = -\frac{a^2\cot^2(e+fx)}{2f} - \frac{(a-b)^2\log(\cos(e+fx))}{f} - \frac{a(a-2b)\log(\tan(e+fx))}{f}$$

output
```
-1/2*a^2*cot(f*x+e)^2/f-(a-b)^2*ln(cos(f*x+e))/f-a*(a-2*b)*ln(tan(f*x+e))/
f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx = -\frac{a^2\csc^2(e+fx)}{2f} - \frac{b^2\log(\cos(e+fx))}{f} - \frac{a^2\log(\sin(e+fx))}{f} + \frac{2ab\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-1/2*(a^2*Csc[e + f*x]^2)/f - (b^2*Log[Cos[e + f*x]])/f - (a^2*Log[Sin[e +
 f*x]])/f + (2*a*b*Log[Sin[e + f*x]])/f
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^3(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{(a-b)^2}{\tan^2(e+fx)+1}+a^2\cot^2(e+fx)-a(a-2b)\cot(e+fx)\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(-\cot(e+fx))-a(a-2b)\log\left(\tan^2(e+fx)\right)+(a-b)^2\log\left(\tan^2(e+fx)+1\right)}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(a^2*Cot[e + f*x]) - a*(a - 2*b)*Log[Tan[e + f*x]^2] + (a - b)^2*Log[1 +
 Tan[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{(a-b)^2\ln\left(\sec(fx+e)^2\right)-a\left((2a-4b)\ln(\tan(fx+e))+a\cot(fx+e)^2\right)}{2f}$
derivativedivides	$\frac{\frac{\left(a^2-2ab+b^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}-\frac{a^2}{2\tan(fx+e)^2}-a(a-2b)\ln(\tan(fx+e))}{f}$
default	$\frac{\frac{\left(a^2-2ab+b^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}-\frac{a^2}{2\tan(fx+e)^2}-a(a-2b)\ln(\tan(fx+e))}{f}$
norman	$-\frac{a^2}{2f\tan(fx+e)^2}+\frac{(a^2-2ab+b^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}-\frac{a(a-2b)\ln(\tan(fx+e))}{f}$
risch	$ia^2x-2iabx+ib^2x+\frac{2ib^2e}{f}+\frac{2ia^2e}{f}-\frac{4iabe}{f}+\frac{2a^2e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln(e^{2i(fx+e)}+1)b^2}{f}-\frac{a^2\ln(e^{2i(}}{f}$

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a-b)^2*ln(sec(f*x+e)^2)-a*((2*a-4*b)*ln(tan(f*x+e))+a*cot(f*x+e)^2))
/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx =$$
$$-\frac{b^2\log\left(\frac{1}{\tan(fx+e)^2+1}\right)\tan(fx+e)^2+a^2\tan(fx+e)^2+(a^2-2\,ab)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan(fx+e)^2+}{2\,f\tan(fx+e)^2}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*log(1/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + a^2*tan(f*x + e)^2
+ (a^2 - 2*a*b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 +
a^2)/(f*tan(f*x + e)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(48) = 96$.

Time = 1.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.30

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \begin{cases} \tilde{\infty}a^2x \\ x(a+b\tan^2(e))^2\cot^3(e) \\ \tilde{\infty}a^2x \\ \frac{a^2\log(\tan^2(e+fx)+1)}{2f} - \frac{a^2\log(\tan(e+fx))}{f} - \frac{a^2}{2f\tan^2(e+fx)} - \frac{ab\log(\tan^2(e+fx)+1)}{f} + \frac{2ab\log(\tan(e+fx))}{f} + \frac{b^2\log(\tan^2(e}{2f} \end{cases}$$

input
```
integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e
)**3, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (a**2*log(tan(e + f*x)**2 + 1)
/(2*f) - a**2*log(tan(e + f*x))/f - a**2/(2*f*tan(e + f*x)**2) - a*b*log(t
an(e + f*x)**2 + 1)/f + 2*a*b*log(tan(e + f*x))/f + b**2*log(tan(e + f*x)*
*2 + 1)/(2*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{b^2\log\left(\sin(fx+e)^2-1\right)+(a^2-2\,ab)\log\left(\sin(fx+e)^2\right)+\frac{a^2}{\sin(fx+e)^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - 2*a*b)*log(sin(f*x + e)^2) + a^
2/sin(f*x + e)^2)/f
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{(a^2-2\,ab+b^2)\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,f} - \frac{(a^2-2\,ab)\log\left(\tan\left(fx+e\right)^2\right)}{2\,f} + \frac{a^2\tan\left(fx+e\right)^2-2\,ab\tan\left(fx+e\right)^2-a^2}{2\,f\tan\left(fx+e\right)^2}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2 + 1)/f - 1/2*(a^2 - 2*a*b)*log(
tan(f*x + e)^2)/f + 1/2*(a^2*tan(f*x + e)^2 - 2*a*b*tan(f*x + e)^2 - a^2)/
(f*tan(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f} + \frac{\ln\left(\tan(e+f\,x)\right)\,\left(2\,a\,b-a^2\right)}{f} - \frac{a^2\cot(e+f\,x)^2}{2\,f}$$

input
```
int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/2))/f + (log(tan(e + f*x))*(2*
a*b - a^2))/f - (a^2*cot(e + f*x)^2)/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.68

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2a^2-8\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2ab+4\log\left(\tan\left(\frac{fx}{2}+\right.\right.}{}$$

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**2 - 8*log(tan((e + f*x)
/2)**2 + 1)*sin(e + f*x)**2*a*b + 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f
*x)**2*b**2 - 4*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2*b**2 - 4*log(tan
((e + f*x)/2) + 1)*sin(e + f*x)**2*b**2 - 4*log(tan((e + f*x)/2))*sin(e +
f*x)**2*a**2 + 8*log(tan((e + f*x)/2))*sin(e + f*x)**2*a*b + sin(e + f*x)*
*2*a**2 - 2*a**2)/(4*sin(e + f*x)**2*f)
```

3.203 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{a(a-2b)\cot^2(e+fx)}{2f}-\frac{a^2\cot^4(e+fx)}{4f}+\frac{(a-b)^2\log(\cos(e+fx))}{f}+\frac{(a-b)^2\log(\tan(e+fx))}{f}$$

output
```
1/2*a*(a-2*b)*cot(f*x+e)^2/f-1/4*a^2*cot(f*x+e)^4/f+(a-b)^2*ln(cos(f*x+e))
/f+(a-b)^2*ln(tan(f*x+e))/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{a^2\csc^2(e+fx)}{f}-\frac{ab\csc^2(e+fx)}{f}-\frac{a^2\csc^4(e+fx)}{4f}+\frac{a^2\log(\sin(e+fx))}{f}-\frac{2ab\log(\sin(e+fx))}{f}+\frac{b^2\log(\sin(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(a^2*Csc[e + f*x]^2)/f - (a*b*Csc[e + f*x]^2)/f - (a^2*Csc[e + f*x]^4)/(4*
f) + (a^2*Log[Sin[e + f*x]])/f - (2*a*b*Log[Sin[e + f*x]])/f + (b^2*Log[Si
n[e + f*x]])/f
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^5(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^3(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(a^2\cot^3(e+fx)-a(a-2b)\cot^2(e+fx)+(a-b)^2\cot(e+fx)-\frac{(a-b)^2}{\tan^2(e+fx)+1}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^2\cot^2(e+fx)+a(a-2b)\cot(e+fx)+(a-b)^2\log\left(\tan^2(e+fx)\right)-(a-b)^2\log\left(\tan^2(e+fx)+1\right)}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(a*(a - 2*b)*Cot[e + f*x] - (a^2*Cot[e + f*x]^2)/2 + (a - b)^2*Log[Tan[e +
 f*x]^2] - (a - b)^2*Log[1 + Tan[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{-2(a-b)^{2} \ln\left(\sec(fx+e)^{2}\right)+4(a-b)^{2} \ln(\tan(fx+e))-\cot(fx+e)^{2} a\left(a\cot(fx+e)^{2}-2a+4b\right)}{4f}$
derivativedivides	$\frac{-\frac{a^{2}}{4\tan(fx+e)^{4}}+\left(a^{2}-2ab+b^{2}\right)\ln(\tan(fx+e))+\frac{a(a-2b)}{2\tan(fx+e)^{2}}+\frac{\left(-a^{2}+2ab-b^{2}\right)\ln\left(1+\tan(fx+e)^{2}\right)}{2}}{f}$
default	$\frac{-\frac{a^{2}}{4\tan(fx+e)^{4}}+\left(a^{2}-2ab+b^{2}\right)\ln(\tan(fx+e))+\frac{a(a-2b)}{2\tan(fx+e)^{2}}+\frac{\left(-a^{2}+2ab-b^{2}\right)\ln\left(1+\tan(fx+e)^{2}\right)}{2}}{f}$
norman	$\frac{-\frac{a^{2}}{4f}+\frac{a(a-2b)\tan(fx+e)^{2}}{2f}}{\tan(fx+e)^{4}}+\frac{\left(a^{2}-2ab+b^{2}\right)\ln(\tan(fx+e))}{f}-\frac{\left(a^{2}-2ab+b^{2}\right)\ln\left(1+\tan(fx+e)^{2}\right)}{2f}$
risch	$-ia^{2}x+2iabx-ib^{2}x-\frac{2ia^{2}e}{f}+\frac{4iabe}{f}-\frac{2ib^{2}e}{f}-\frac{4a\left(a\,e^{6i(fx+e)}-b\,e^{6i(fx+e)}-a\,e^{4i(fx+e)}+2b\,e^{4i(fx+e}\right.}{f\left(e^{2i(fx+e)}-1\right)^{4}}$

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*(a-b)^2*ln(sec(f*x+e)^2)+4*(a-b)^2*ln(tan(f*x+e))-cot(f*x+e)^2*a*(
a*cot(f*x+e)^2-2*a+4*b))/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int \cot^{5}(e+fx)\left(a+b\tan^{2}(e+fx)\right)^{2}\,dx$$
$$=\frac{2\left(a^{2}-2\,ab+b^{2}\right)\log\left(\frac{\tan(fx+e)^{2}}{\tan(fx+e)^{2}+1}\right)\tan\left(fx+e\right)^{4}+\left(3\,a^{2}-4\,ab\right)\tan\left(fx+e\right)^{4}+2\left(a^{2}-2\,ab\right)\tan\left(fx+\right.}{4\,f\tan\left(fx+e\right)^{4}}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*
x + e)^4 + (3*a^2 - 4*a*b)*tan(f*x + e)^4 + 2*(a^2 - 2*a*b)*tan(f*x + e)^2
 - a^2)/(f*tan(f*x + e)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(63) = 126$.

Time = 3.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.26

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \begin{cases} \tilde{\infty}a^2x \\ x(a+b\tan^2(e))^2\cot^5(e) \\ \tilde{\infty}a^2x \\ -\frac{a^2\log(\tan^2(e+fx)+1)}{2f}+\frac{a^2\log(\tan(e+fx))}{f}+\frac{a^2}{2f\tan^2(e+fx)}-\frac{a^2}{4f\tan^4(e+fx)}+\frac{ab\log(\tan^2(e+fx)+1)}{f}-\frac{2ab\log(\tan(e+}{f} \end{cases}$$

input
```
integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e
)**5, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (-a**2*log(tan(e + f*x)**2 + 1
)/(2*f) + a**2*log(tan(e + f*x))/f + a**2/(2*f*tan(e + f*x)**2) - a**2/(4*
f*tan(e + f*x)**4) + a*b*log(tan(e + f*x)**2 + 1)/f - 2*a*b*log(tan(e + f*
x))/f - a*b/(f*tan(e + f*x)**2) - b**2*log(tan(e + f*x)**2 + 1)/(2*f) + b*
*2*log(tan(e + f*x))/f, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{2\,(a^2-2\,ab+b^2)\log\left(\sin(fx+e)^2\right)+\frac{4\,(a^2-ab)\sin(fx+e)^2-a^2}{\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*(a^2 - 2*a*b + b^2)*log(sin(f*x + e)^2) + (4*(a^2 - a*b)*sin(f*x +
e)^2 - a^2)/sin(f*x + e)^4)/f
```

Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.75

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= -\frac{(a^2-2ab+b^2)\log\left(\tan(fx+e)^2+1\right)}{2f}+\frac{(a^2-2ab+b^2)\log\left(\tan(fx+e)^2\right)}{2f}$$
$$-\frac{3a^2\tan(fx+e)^4-6ab\tan(fx+e)^4+3b^2\tan(fx+e)^4-2a^2\tan(fx+e)^2+4ab\tan(fx+e)^2+}{4f\tan(fx+e)^4}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(a^2 - 2*a*b + b^2)*log(tan(f*x + e)^2 + 1)/f + 1/2*(a^2 - 2*a*b + b^
2)*log(tan(f*x + e)^2)/f - 1/4*(3*a^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^
4 + 3*b^2*tan(f*x + e)^4 - 2*a^2*tan(f*x + e)^2 + 4*a*b*tan(f*x + e)^2 + a
^2)/(f*tan(f*x + e)^4)
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2-2\,a\,b+b^2\right)}{f} - \frac{\frac{a^2}{4}+\tan(e+f\,x)^2\,\left(a\,b-\frac{a^2}{2}\right)}{f\,\tan\left(e+f\,x\right)^4} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(log(tan(e + f*x))*(a^2 - 2*a*b + b^2))/f - (a^2/4 + tan(e + f*x)^2*(a*b -
 a^2/2))/(f*tan(e + f*x)^4) - (log(tan(e + f*x)^2 + 1)*(a^2/2 - a*b + b^2/
2))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.86

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{-32\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin(fx+e)^4a^2+64\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin(fx+e)^4ab-32\log\left(\tan\right.}{}$$

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 32*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**2 + 64*log(tan((e +
 f*x)/2)**2 + 1)*sin(e + f*x)**4*a*b - 32*log(tan((e + f*x)/2)**2 + 1)*sin
(e + f*x)**4*b**2 + 32*log(tan((e + f*x)/2))*sin(e + f*x)**4*a**2 - 64*log
(tan((e + f*x)/2))*sin(e + f*x)**4*a*b + 32*log(tan((e + f*x)/2))*sin(e +
f*x)**4*b**2 - 13*sin(e + f*x)**4*a**2 + 16*sin(e + f*x)**4*a*b + 32*sin(e
 + f*x)**2*a**2 - 32*sin(e + f*x)**2*a*b - 8*a**2)/(32*sin(e + f*x)**4*f)
```

3.204 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -(a-b)^2x+\frac{(a-b)^2\tan(e+fx)}{f} - \frac{(a-b)^2\tan^3(e+fx)}{3f} + \frac{(a-b)^2\tan^5(e+fx)}{5f} + \frac{(2a-b)b\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

output
```
-(a-b)^2*x+(a-b)^2*tan(f*x+e)/f-1/3*(a-b)^2*tan(f*x+e)^3/f+1/5*(a-b)^2*tan
(f*x+e)^5/f+1/7*(2*a-b)*b*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. $2(113) = 226$.

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.15

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{a^2\arctan(\tan(e+fx))}{f} + \frac{2ab\arctan(\tan(e+fx))}{f} - \frac{b^2\arctan(\tan(e+fx))}{f} + \frac{a^2\tan(e+fx)}{f} - \frac{2ab\tan(e+fx)}{f} + \frac{b^2\tan(e+fx)}{f} - \frac{a^2\tan^3(e+fx)}{3f} + \frac{2ab\tan^3(e+fx)}{3f} - \frac{b^2\tan^3(e+fx)}{3f} + \frac{a^2\tan^5(e+fx)}{5f} - \frac{2ab\tan^5(e+fx)}{5f} + \frac{b^2\tan^5(e+fx)}{5f} + \frac{2ab\tan^7(e+fx)}{7f} - \frac{b^2\tan^7(e+fx)}{7f} + \frac{b^2\tan^9(e+fx)}{9f}$$

input
```
Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Ar
cTan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b
^2*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^3)/(
3*f) - (b^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e + f*x]^5)/(5*f) - (2*a*b*Ta
n[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^5)/(5*f) + (2*a*b*Tan[e + f*x]^7)/
(7*f) - (b^2*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(b^2\tan^8(e+fx)+(2a-b)b\tan^6(e+fx)+(a-b)^2\tan^4(e+fx)-(a-b)^2\tan^2(e+fx)+(a-b)^2+\frac{-a^2+}{\tan^2(e}\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-(a-b)^2\arctan(\tan(e+fx))+\frac{1}{7}b(2a-b)\tan^7(e+fx)+\frac{1}{5}(a-b)^2\tan^5(e+fx)-\frac{1}{3}(a-b)^2\tan^3(e+fx)+}{f}$$

input
```
Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a - b)^2*ArcTan[Tan[e + f*x]]) + (a - b)^2*Tan[e + f*x] - ((a - b)^2*T
an[e + f*x]^3)/3 + ((a - b)^2*Tan[e + f*x]^5)/5 + ((2*a - b)*b*Tan[e + f*x
]^7)/7 + (b^2*Tan[e + f*x]^9)/9)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11

method	result
norman	$\left(-a^{2}+2ab-b^{2}\right)x+\frac{\left(a^{2}-2ab+b^{2}\right)\tan\left(fx+e\right)}{f}+\frac{b^{2}\tan\left(fx+e\right)^{9}}{9f}-\frac{\left(a^{2}-2ab+b^{2}\right)\tan\left(fx+e\right)^{3}}{3f}+\frac{\left(a^{2}-2a\right.}{}$
parts	$\frac{a^{2}\left(\frac{\tan\left(fx+e\right)^{5}}{5}-\frac{\tan\left(fx+e\right)^{3}}{3}+\tan\left(fx+e\right)-\arctan\left(\tan\left(fx+e\right)\right)\right)}{f}+\frac{b^{2}\left(\frac{\tan\left(fx+e\right)^{9}}{9}-\frac{\tan\left(fx+e\right)^{7}}{7}+\frac{\tan\left(fx+e\right)^{5}}{5}-\frac{\tan(f}{}\right.}{f}$
derivativedivides	$\frac{\frac{b^{2}\tan\left(fx+e\right)^{9}}{9}+\frac{2ab\tan\left(fx+e\right)^{7}}{7}-\frac{b^{2}\tan\left(fx+e\right)^{7}}{7}+\frac{a^{2}\tan\left(fx+e\right)^{5}}{5}-\frac{2ab\tan\left(fx+e\right)^{5}}{5}+\frac{b^{2}\tan\left(fx+e\right)^{5}}{5}-\frac{a^{2}\tan\left(fx+e\right)^{3}}{3}+\frac{2ab\tan}{}}{f}$
default	$\frac{\frac{b^{2}\tan\left(fx+e\right)^{9}}{9}+\frac{2ab\tan\left(fx+e\right)^{7}}{7}-\frac{b^{2}\tan\left(fx+e\right)^{7}}{7}+\frac{a^{2}\tan\left(fx+e\right)^{5}}{5}-\frac{2ab\tan\left(fx+e\right)^{5}}{5}+\frac{b^{2}\tan\left(fx+e\right)^{5}}{5}-\frac{a^{2}\tan\left(fx+e\right)^{3}}{3}+\frac{2ab\tan}{}}{f}$
parallelrisch	$-\frac{-35b^{2}\tan\left(fx+e\right)^{9}-90ab\tan\left(fx+e\right)^{7}+45b^{2}\tan\left(fx+e\right)^{7}-63a^{2}\tan\left(fx+e\right)^{5}+126ab\tan\left(fx+e\right)^{5}-63b^{2}\tan\left(fx+e\right)^{5}}{}$
risch	$-x\,a^{2}+2xab-x\,b^{2}+\frac{2i\left(483a^{2}+563b^{2}-1056ab+945a^{2}\mathrm{e}^{16i(fx+e)}+21000b^{2}\mathrm{e}^{12i(fx+e)}+11718a^{2}\mathrm{e}^{4i(fx+e)}+\right.}{}$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-a^2+2*a*b-b^2)*x+(a^2-2*a*b+b^2)/f*tan(f*x+e)+1/9*b^2*tan(f*x+e)^9/f-1/3
*(a^2-2*a*b+b^2)/f*tan(f*x+e)^3+1/5*(a^2-2*a*b+b^2)/f*tan(f*x+e)^5+1/7*(2*
a-b)*b*tan(f*x+e)^7/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{35\,b^2\tan\left(fx+e\right)^9+45\left(2\,ab-b^2\right)\tan\left(fx+e\right)^7+63\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^5-105\left(a^2-2\,ab+\right.}{315\,f}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 -
 2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 31
5*(a^2 - 2*a*b + b^2)*f*x + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(88) = 176.

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.88

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\begin{cases}-a^2x+\frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}+2abx+\frac{2ab\tan^7(e+fx)}{7f}-\frac{2ab\tan^5(e+fx)}{5f}+\frac{2ab\tan^3(e+fx)}{3f} \\ x(a+b\tan^2(e))^2\tan^6(e)\end{cases}$$

input
```
integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((-a**2*x + a**2*tan(e + f*x)**5/(5*f) - a**2*tan(e + f*x)**3/(3*
f) + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**7/(7*f) - 2*a*b*t
an(e + f*x)**5/(5*f) + 2*a*b*tan(e + f*x)**3/(3*f) - 2*a*b*tan(e + f*x)/f
- b**2*x + b**2*tan(e + f*x)**9/(9*f) - b**2*tan(e + f*x)**7/(7*f) + b**2*
tan(e + f*x)**5/(5*f) - b**2*tan(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f,
Ne(f, 0)), (x*(a + b*tan(e)**2)**2*tan(e)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{35\,b^2\tan\left(fx+e\right)^9+45\,(2\,ab-b^2)\tan\left(fx+e\right)^7+63\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)^5-105\,(a^2-2\,ab+}{315\,f}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b - b^2)*tan(f*x + e)^7 + 63*(a^2 -
 2*a*b + b^2)*tan(f*x + e)^5 - 105*(a^2 - 2*a*b + b^2)*tan(f*x + e)^3 - 31
5*(a^2 - 2*a*b + b^2)*(f*x + e) + 315*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.86

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{(a^2-2\,ab+b^2)(fx+e)}{f}$$
$$+\frac{35\,b^2f^8\tan\left(fx+e\right)^9+90\,abf^8\tan\left(fx+e\right)^7-45\,b^2f^8\tan\left(fx+e\right)^7+63\,a^2f^8\tan\left(fx+e\right)^5-126\,a}{}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-(a^2 - 2*a*b + b^2)*(f*x + e)/f + 1/315*(35*b^2*f^8*tan(f*x + e)^9 + 90*a
*b*f^8*tan(f*x + e)^7 - 45*b^2*f^8*tan(f*x + e)^7 + 63*a^2*f^8*tan(f*x + e
)^5 - 126*a*b*f^8*tan(f*x + e)^5 + 63*b^2*f^8*tan(f*x + e)^5 - 105*a^2*f^8
*tan(f*x + e)^3 + 210*a*b*f^8*tan(f*x + e)^3 - 105*b^2*f^8*tan(f*x + e)^3
+ 315*a^2*f^8*tan(f*x + e) - 630*a*b*f^8*tan(f*x + e) + 315*b^2*f^8*tan(f*
x + e))/f^9
```

Mupad [B] (verification not implemented)

Time = 7.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.37

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{\tan(e+f\,x)^7\,\left(\frac{2\,a\,b}{7}-\frac{b^2}{7}\right)}{f} - \frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f} + \frac{\tan(e+f\,x)\,\left(a^2-2\,a\,b+b^2\right)}{f} + \frac{b^2\tan(e+f\,x)^9}{9\,f} - \frac{\tan(e+f\,x)^3\,\left(\frac{a^2}{3}-\frac{2\,a\,b}{3}+\frac{b^2}{3}\right)}{f} + \frac{\tan(e+f\,x)^5\,\left(\frac{a^2}{5}-\frac{2\,a\,b}{5}+\frac{b^2}{5}\right)}{f}$$

input
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^7*((2*a*b)/7 - b^2/7))/f - (atan((tan(e + f*x)*(a - b)^2)/(a
^2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (
b^2*tan(e + f*x)^9)/(9*f) - (tan(e + f*x)^3*(a^2/3 - (2*a*b)/3 + b^2/3))/f
 + (tan(e + f*x)^5*(a^2/5 - (2*a*b)/5 + b^2/5))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.52

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{35\tan(fx+e)^9 b^2+90\tan(fx+e)^7 ab-45\tan(fx+e)^7 b^2+63\tan(fx+e)^5 a^2-126\tan(fx+e)^5}{}$$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(35*tan(e + f*x)**9*b**2 + 90*tan(e + f*x)**7*a*b - 45*tan(e + f*x)**7*b**
2 + 63*tan(e + f*x)**5*a**2 - 126*tan(e + f*x)**5*a*b + 63*tan(e + f*x)**5
*b**2 - 105*tan(e + f*x)**3*a**2 + 210*tan(e + f*x)**3*a*b - 105*tan(e + f
*x)**3*b**2 + 315*tan(e + f*x)*a**2 - 630*tan(e + f*x)*a*b + 315*tan(e + f
*x)*b**2 - 315*a**2*f*x + 630*a*b*f*x - 315*b**2*f*x)/(315*f)
```

3.205 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = (a-b)^2x-\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(a-b)^2\tan^3(e+fx)}{3f}+\frac{(2a-b)b\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

output
```
(a-b)^2*x-(a-b)^2*tan(f*x+e)/f+1/3*(a-b)^2*tan(f*x+e)^3/f+1/5*(2*a-b)*b*ta
n(f*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(91) = 182.

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.09

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{a^2\arctan(\tan(e+fx))}{f}-\frac{2ab\arctan(\tan(e+fx))}{f}+\frac{b^2\arctan(\tan(e+fx))}{f}-\frac{a^2\tan(e+fx)}{f}+\frac{2ab\tan(e+fx)}{f}-\frac{b^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}-\frac{2ab\tan^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{2ab\tan^5(e+fx)}{5f}-\frac{b^2\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

input
```
Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]])/f - (2*a*b*ArcTan[Tan[e + f*x]])/f + (b^2*ArcTa
n[Tan[e + f*x]])/f - (a^2*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x])/f - (b^2*
Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) - (2*a*b*Tan[e + f*x]^3)/(3*f
) + (b^2*Tan[e + f*x]^3)/(3*f) + (2*a*b*Tan[e + f*x]^5)/(5*f) - (b^2*Tan[e
 + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4 \left(a+b\tan(e+fx)^2\right)^2 dx$$

$\Big\downarrow$ 4153

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 364

$$\frac{\int \left(b^2\tan^6(e+fx)+(2a-b)b\tan^4(e+fx)+(a-b)^2\tan^2(e+fx)-(a-b)^2+\frac{a^2-2ba+b^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$\Big\downarrow$ 2009

$$\frac{(a-b)^2\arctan(\tan(e+fx))+\frac{1}{5}b(2a-b)\tan^5(e+fx)+\frac{1}{3}(a-b)^2\tan^3(e+fx)-(a-b)^2\tan(e+fx)+\frac{1}{7}b^2 t}{f}$$

input
```
Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((a - b)^2*ArcTan[Tan[e + f*x]] - (a - b)^2*Tan[e + f*x] + ((a - b)^2*Tan[
e + f*x]^3)/3 + ((2*a - b)*b*Tan[e + f*x]^5)/5 + (b^2*Tan[e + f*x]^7)/7)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

method	result
norman	$(a^2-2ab+b^2)\,x+\frac{b^2\tan(fx+e)^7}{7f}-\frac{(a^2-2ab+b^2)\tan(fx+e)}{f}+\frac{(a^2-2ab+b^2)\tan(fx+e)^3}{3f}+\frac{(2a-b)b\,\mathrm{t}}{5}$
parts	$\frac{a^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^7}{7}-\frac{\tan(fx+e)^5}{5}+\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\mathrm{arctan}\right.}{f}$
derivativedivides	$\frac{\frac{b^2\tan(fx+e)^7}{7}+\frac{2ab\tan(fx+e)^5}{5}-\frac{b^2\tan(fx+e)^5}{5}+\frac{a^2\tan(fx+e)^3}{3}-\frac{2ab\tan(fx+e)^3}{3}+\frac{\tan(fx+e)^3b^2}{3}-a^2\tan(fx+e)+2\tan(f}{f}$
default	$\frac{\frac{b^2\tan(fx+e)^7}{7}+\frac{2ab\tan(fx+e)^5}{5}-\frac{b^2\tan(fx+e)^5}{5}+\frac{a^2\tan(fx+e)^3}{3}-\frac{2ab\tan(fx+e)^3}{3}+\frac{\tan(fx+e)^3b^2}{3}-a^2\tan(fx+e)+2\tan(f}{f}$
parallelrisch	$\frac{15b^2\tan(fx+e)^7+42ab\tan(fx+e)^5-21b^2\tan(fx+e)^5+35a^2\tan(fx+e)^3-70ab\tan(fx+e)^3+35\tan(fx+e)^3b^2+105a}{105f}$
risch	$x\,a^2-2xab+x\,b^2-\frac{4i\left(105a^2\mathrm{e}^{12i(fx+e)}-315ab\,\mathrm{e}^{12i(fx+e)}+210b^2\mathrm{e}^{12i(fx+e)}+525a^2\mathrm{e}^{10i(fx+e)}-1260ab\,\mathrm{e}^{10i(}\right.}{}$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(a^2-2*a*b+b^2)*x+1/7*b^2*tan(f*x+e)^7/f-(a^2-2*a*b+b^2)/f*tan(f*x+e)+1/3*
(a^2-2*a*b+b^2)/f*tan(f*x+e)^3+1/5*(2*a-b)*b*tan(f*x+e)^5/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\left(a^2-2\,ab+\right.}{105\,f}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
 2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*f*x - 105*(a^2 - 2*
a*b + b^2)*tan(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(71) = 142.

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.81

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \begin{cases} a^2x+\frac{a^2\tan^3(e+fx)}{3f}-\frac{a^2\tan(e+fx)}{f}-2abx+\frac{2ab\tan^5(e+fx)}{5f}-\frac{2ab\tan^3(e+fx)}{3f}+\frac{2ab\tan(e+fx)}{f}+b^2x+\frac{b^2\tan^7(e+}{7f} \\ x\left(a+b\tan^2(e)\right)^2\tan^4(e) \end{cases}$$

input
```
integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((a**2*x + a**2*tan(e + f*x)**3/(3*f) - a**2*tan(e + f*x)/f - 2*a
*b*x + 2*a*b*tan(e + f*x)**5/(5*f) - 2*a*b*tan(e + f*x)**3/(3*f) + 2*a*b*t
an(e + f*x)/f + b**2*x + b**2*tan(e + f*x)**7/(7*f) - b**2*tan(e + f*x)**5
/(5*f) + b**2*tan(e + f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(
a + b*tan(e)**2)**2*tan(e)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \tan^4(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab-b^2\right)\tan\left(fx+e\right)^5+35\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^3+105\left(a^2-2\,ab+b^2\right)\ldots}{105\,f}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b - b^2)*tan(f*x + e)^5 + 35*(a^2 -
 2*a*b + b^2)*tan(f*x + e)^3 + 105*(a^2 - 2*a*b + b^2)*(f*x + e) - 105*(a^
2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.78

$$\int \tan^4(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{(a^2-2\,ab+b^2)(fx+e)}{f} + \frac{15\,b^2f^6\tan\left(fx+e\right)^7+42\,abf^6\tan\left(fx+e\right)^5-21\,b^2f^6\tan\left(fx+e\right)^5+35\,a^2f^6\tan\left(fx+e\right)^3-70\,ab\ldots}{}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
(a^2 - 2*a*b + b^2)*(f*x + e)/f + 1/105*(15*b^2*f^6*tan(f*x + e)^7 + 42*a*
b*f^6*tan(f*x + e)^5 - 21*b^2*f^6*tan(f*x + e)^5 + 35*a^2*f^6*tan(f*x + e)
^3 - 70*a*b*f^6*tan(f*x + e)^3 + 35*b^2*f^6*tan(f*x + e)^3 - 105*a^2*f^6*t
an(f*x + e) + 210*a*b*f^6*tan(f*x + e) - 105*b^2*f^6*tan(f*x + e))/f^7
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.40

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f}+\frac{\tan(e+f\,x)^5\,\left(\frac{2\,a\,b}{5}-\frac{b^2}{5}\right)}{f}-\frac{\tan(e+f\,x)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\tan(e+f\,x)^7}{7\,f}+\frac{\tan(e+f\,x)^3\,\left(\frac{a^2}{3}-\frac{2\,a\,b}{3}+\frac{b^2}{3}\right)}{f}$$

input
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e
+ f*x)^5*((2*a*b)/5 - b^2/5))/f - (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (
b^2*tan(e + f*x)^7)/(7*f) + (tan(e + f*x)^3*(a^2/3 - (2*a*b)/3 + b^2/3))/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{15\tan\left(fx+e\right)^7b^2+42\tan\left(fx+e\right)^5ab-21\tan\left(fx+e\right)^5b^2+35\tan\left(fx+e\right)^3a^2-70\tan\left(fx+e\right)^3a}{}$$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(15*tan(e + f*x)**7*b**2 + 42*tan(e + f*x)**5*a*b - 21*tan(e + f*x)**5*b**
2 + 35*tan(e + f*x)**3*a**2 - 70*tan(e + f*x)**3*a*b + 35*tan(e + f*x)**3*
b**2 - 105*tan(e + f*x)*a**2 + 210*tan(e + f*x)*a*b - 105*tan(e + f*x)*b**
2 + 105*a**2*f*x - 210*a*b*f*x + 105*b**2*f*x)/(105*f)
```

3.206 $\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-(a-b)^2x+\frac{(a-b)^2\tan(e+fx)}{f}+\frac{(2a-b)b\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

output
```
-(a-b)^2*x+(a-b)^2*tan(f*x+e)/f+1/3*(2*a-b)*b*tan(f*x+e)^3/f+1/5*b^2*tan(f
*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.99

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{a^2\arctan(\tan(e+fx))}{f}+\frac{2ab\arctan(\tan(e+fx))}{f}-\frac{b^2\arctan(\tan(e+fx))}{f}+\frac{a^2\tan(e+fx)}{f}-\frac{2ab\tan(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}+\frac{2ab\tan^3(e+fx)}{3f}-\frac{b^2\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

input
```
Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-((a^2*ArcTan[Tan[e + f*x]])/f) + (2*a*b*ArcTan[Tan[e + f*x]])/f - (b^2*Ar
cTan[Tan[e + f*x]])/f + (a^2*Tan[e + f*x])/f - (2*a*b*Tan[e + f*x])/f + (b
^2*Tan[e + f*x])/f + (2*a*b*Tan[e + f*x]^3)/(3*f) - (b^2*Tan[e + f*x]^3)/(
3*f) + (b^2*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^2 \left(a+b\tan(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(b^2\tan^4(e+fx)+(2a-b)b\tan^2(e+fx)+(a-b)^2+\frac{-a^2+2ba-b^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-(a-b)^2\arctan(\tan(e+fx))+\frac{1}{3}b(2a-b)\tan^3(e+fx)+(a-b)^2\tan(e+fx)+\frac{1}{5}b^2\tan^5(e+fx)}{f}$$

input
```
Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a - b)^2*ArcTan[Tan[e + f*x]]) + (a - b)^2*Tan[e + f*x] + ((2*a - b)*b
*Tan[e + f*x]^3)/3 + (b^2*Tan[e + f*x]^5)/5)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

method	result
norman	$(-a^2+2ab-b^2)\,x+\frac{(a^2-2ab+b^2)\tan(fx+e)}{f}+\frac{b^2\tan(fx+e)^5}{5f}+\frac{(2a-b)b\tan(fx+e)^3}{3f}$
derivativedivides	$\frac{\frac{b^2\tan(fx+e)^5}{5}+\frac{2ab\tan(fx+e)^3}{3}-\frac{\tan(fx+e)^3b^2}{3}+a^2\tan(fx+e)-2\tan(fx+e)ab+\tan(fx+e)b^2+(-a^2+2ab-b^2)\arctan(}{f}$
default	$\frac{\frac{b^2\tan(fx+e)^5}{5}+\frac{2ab\tan(fx+e)^3}{3}-\frac{\tan(fx+e)^3b^2}{3}+a^2\tan(fx+e)-2\tan(fx+e)ab+\tan(fx+e)b^2+(-a^2+2ab-b^2)\arctan(}{f}$
parallelrisch	$-\frac{-3b^2\tan(fx+e)^5-10ab\tan(fx+e)^3+5\tan(fx+e)^3b^2+15a^2fx-30abfx+15b^2fx-15a^2\tan(fx+e)+30\tan(fx+e)}{15f}$
parts	$\frac{a^2(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b^2\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{2ab\left(\tan\right.}{}$
risch	$-x\,a^2+2xab-x\,b^2+\frac{2i\left(15a^2e^{8i(fx+e)}-60ab\,e^{8i(fx+e)}+45b^2e^{8i(fx+e)}+60a^2e^{6i(fx+e)}-180ab\,e^{6i(fx+e)}+9\right.}{}$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-a^2+2*a*b-b^2)*x+(a^2-2*a*b+b^2)/f*tan(f*x+e)+1/5*b^2*tan(f*x+e)^5/f+1/3
*(2*a-b)*b*tan(f*x+e)^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,b^2\tan\left(fx+e\right)^5+5\,(2\,ab-b^2)\tan\left(fx+e\right)^3-15\,(a^2-2\,ab+b^2)fx+15\,(a^2-2\,ab+b^2)\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*
a*b + b^2)*f*x + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(54) = 108$.

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\begin{cases}-a^2x+\frac{a^2\tan\left(e+fx\right)}{f}+2abx+\frac{2ab\tan^3\left(e+fx\right)}{3f}-\frac{2ab\tan\left(e+fx\right)}{f}-b^2x+\frac{b^2\tan^5\left(e+fx\right)}{5f}-\frac{b^2\tan^3\left(e+fx\right)}{3f}+\frac{b^2\tan\left(e+fx\right)}{f} \\ x(a+b\tan^2\left(e\right))^2\tan^2\left(e\right)\end{cases}$$

input
```
integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((-a**2*x + a**2*tan(e + f*x)/f + 2*a*b*x + 2*a*b*tan(e + f*x)**3
/(3*f) - 2*a*b*tan(e + f*x)/f - b**2*x + b**2*tan(e + f*x)**5/(5*f) - b**2
*tan(e + f*x)**3/(3*f) + b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)*
*2)**2*tan(e)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{3\,b^2\tan\left(fx+e\right)^5+5\,(2\,ab-b^2)\tan\left(fx+e\right)^3-15\,(a^2-2\,ab+b^2)(fx+e)+15\,(a^2-2\,ab+b^2)\tan\left(f\right.}{15\,f}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b - b^2)*tan(f*x + e)^3 - 15*(a^2 - 2*
a*b + b^2)*(f*x + e) + 15*(a^2 - 2*a*b + b^2)*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{(a^2-2\,ab+b^2)(fx+e)}{f} + \frac{3\,b^2f^4\tan\left(fx+e\right)^5+10\,abf^4\tan\left(fx+e\right)^3-5\,b^2f^4\tan\left(fx+e\right)^3+15\,a^2f^4\tan\left(fx+e\right)-30\,abf^4\,\mathrm{t}}{15\,f^5}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-(a^2 - 2*a*b + b^2)*(f*x + e)/f + 1/15*(3*b^2*f^4*tan(f*x + e)^5 + 10*a*b
*f^4*tan(f*x + e)^3 - 5*b^2*f^4*tan(f*x + e)^3 + 15*a^2*f^4*tan(f*x + e) -
 30*a*b*f^4*tan(f*x + e) + 15*b^2*f^4*tan(f*x + e))/f^5
```

Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\tan(e+f\,x)^3\,\left(\frac{2\,a\,b}{3}-\frac{b^2}{3}\right)}{f}-\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f}+\frac{\tan(e+f\,x)\,\left(a^2-2\,a\,b+b^2\right)}{f}+\frac{b^2\,\tan(e+f\,x)^5}{5\,f}$$

input
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^3*((2*a*b)/3 - b^2/3))/f - (atan((tan(e + f*x)*(a - b)^2)/(a
^2 - 2*a*b + b^2))*(a - b)^2)/f + (tan(e + f*x)*(a^2 - 2*a*b + b^2))/f + (
b^2*tan(e + f*x)^5)/(5*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.39

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$

$$=\frac{3\tan(fx+e)^5b^2+10\tan(fx+e)^3ab-5\tan(fx+e)^3b^2+15\tan(fx+e)a^2-30\tan(fx+e)ab+}{15f}$$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(3*tan(e + f*x)**5*b**2 + 10*tan(e + f*x)**3*a*b - 5*tan(e + f*x)**3*b**2
+ 15*tan(e + f*x)*a**2 - 30*tan(e + f*x)*a*b + 15*tan(e + f*x)*b**2 - 15*a
**2*f*x + 30*a*b*f*x - 15*b**2*f*x)/(15*f)
```

3.207 $\int \left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \left(a+b\tan^2(e+fx)\right)^2 dx = (a-b)^2x+\frac{(2a-b)b\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

output
```
(a-b)^2*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{\tan(e+fx)\left(\frac{3(a-b)^2\operatorname{arctanh}\left(\sqrt{-\tan^2(e+fx)}\right)}{\sqrt{-\tan^2(e+fx)}}+b(6a-b(3-\tan^2(e+fx)))\right)}{3f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[e + f*x]^2]])/Sqrt[-Tan[e +
f*x]^2] + b*(6*a - b*(3 - Tan[e + f*x]^2))))/(3*f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{(a-b)^2}{\tan^2(e+fx)+1}+b^2\tan^2(e+fx)+(2a-b)b\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a-b)^2\arctan(\tan(e+fx))+b(2a-b)\tan(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((a - b)^2*ArcTan[Tan[e + f*x]] + (2*a - b)*b*Tan[e + f*x] + (b^2*Tan[e +
f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
norman	$(a^2-2ab+b^2)\,x+\frac{(2a-b)b\tan(fx+e)}{f}+\frac{b^2\tan(fx+e)^3}{3f}$	49
derivativedivides	$\frac{\frac{\tan(fx+e)^3b^2}{3}+2\tan(fx+e)ab-\tan(fx+e)b^2+(a^2-2ab+b^2)\arctan(\tan(fx+e))}{f}$	59
default	$\frac{\frac{\tan(fx+e)^3b^2}{3}+2\tan(fx+e)ab-\tan(fx+e)b^2+(a^2-2ab+b^2)\arctan(\tan(fx+e))}{f}$	59
parallelrisch	$\frac{\tan(fx+e)^3b^2+3a^2fx-6abfx+3b^2fx+6\tan(fx+e)ab-3\tan(fx+e)b^2}{3f}$	60
parts	$x\,a^2+\frac{b^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{2ab(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}$	63
risch	$x\,a^2-2xab+x\,b^2-\frac{4ib\left(-3a\,\mathrm{e}^{4i(fx+e)}+3b\,\mathrm{e}^{4i(fx+e)}-6a\,\mathrm{e}^{2i(fx+e)}+3b\,\mathrm{e}^{2i(fx+e)}-3a+2b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$	92

input
```
int((a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(a^2-2*a*b+b^2)*x+(2*a-b)*b*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{b^2\tan\left(fx+e\right)^3+3\left(a^2-2\,ab+b^2\right)fx+3\left(2\,ab-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*x + 3*(2*a*b - b^2)*tan(
f*x + e))/f
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx = \begin{cases} a^2x-2abx+\frac{2ab\tan\left(e+fx\right)}{f}+b^2x+\frac{b^2\tan^3\left(e+fx\right)}{3f}-\frac{b^2\tan\left(e+fx\right)}{f} & \text{for } f\neq 0 \\ x\left(a+b\tan^2\left(e\right)\right)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(e + f*x)/f + b**2*x + b**2*tan(e +
 f*x)**3/(3*f) - b**2*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e)**2)**2,
True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=a^2x-\frac{2\,(fx+e-\tan\left(fx+e\right))ab}{f}+\frac{\left(\tan\left(fx+e\right)^3+3\,fx+3\,e-3\,\tan\left(fx+e\right)\right)b^2}{3\,f}$$

input
```
integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x - 2*(f*x + e - tan(f*x + e))*a*b/f + 1/3*(tan(f*x + e)^3 + 3*f*x + 3
*e - 3*tan(f*x + e))*b^2/f
```

Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{(a^2-2\,ab+b^2)(fx+e)}{f}+\frac{b^2f^2\tan\left(fx+e\right)^3+6\,abf^2\tan\left(fx+e\right)-3\,b^2f^2\tan\left(fx+e\right)}{3\,f^3}$$

input
```
integrate((a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
(a^2 - 2*a*b + b^2)*(f*x + e)/f + 1/3*(b^2*f^2*tan(f*x + e)^3 + 6*a*b*f^2*
tan(f*x + e) - 3*b^2*f^2*tan(f*x + e))/f^3
```

Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\tan(e+f\,x)\,\left(2\,a\,b-b^2\right)}{f}+\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f}+\frac{b^2\tan(e+f\,x)^3}{3\,f}$$

input
```
int((a + b*tan(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)*(2*a*b - b^2))/f + (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a
*b + b^2))*(a - b)^2)/f + (b^2*tan(e + f*x)^3)/(3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \left(a+b\tan^2(e+fx)\right)^2\,dx=\frac{\tan(fx+e)^3b^2+6\tan(fx+e)ab-3\tan(fx+e)b^2+3a^2fx-6abfx+3b^2fx}{3f}$$

input
```
int((a+b*tan(f*x+e)^2)^2,x)
```

output
```
(tan(e + f*x)**3*b**2 + 6*tan(e + f*x)*a*b - 3*tan(e + f*x)*b**2 + 3*a**2*
f*x - 6*a*b*f*x + 3*b**2*f*x)/(3*f)
```

3.208 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-(a-b)^2x-\frac{a^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

output
```
-(a-b)^2*x-a^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\begin{aligned}&\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx\\&=2abx-\frac{b^2\arctan(\tan(e+fx))}{f}\\&\quad-\frac{a^2\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}+\frac{b^2\tan(e+fx)}{f}\end{aligned}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
2*a*b*x - (b^2*ArcTan[Tan[e + f*x]])/f - (a^2*Cot[e + f*x]*Hypergeometric2
F1[-1/2, 1, 1/2, -Tan[e + f*x]^2])/f + (b^2*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(-\frac{(a-b)^2}{\tan^2(e+fx)+1}+b^2+a^2\cot^2(e+fx)\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(-\cot(e+fx))-(a-b)^2\arctan(\tan(e+fx))+b^2\tan(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a - b)^2*ArcTan[Tan[e + f*x]]) - a^2*Cot[e + f*x] + b^2*Tan[e + f*x])/
f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-a^{2} \cot \left(f x +e \right)+\tan \left(f x +e \right) b^{2}-f x \left(a -b \right)^{2}}{f}$	38
derivativedivides	$\frac{b^{2} \left(\tan \left(f x +e \right)-f x -e \right)+2 a b \left(f x +e \right)+a^{2} \left(-\cot \left(f x +e \right)-f x -e \right)}{f}$	53
default	$\frac{b^{2} \left(\tan \left(f x +e \right)-f x -e \right)+2 a b \left(f x +e \right)+a^{2} \left(-\cot \left(f x +e \right)-f x -e \right)}{f}$	53
norman	$\frac{\frac{b^{2} \tan \left(f x +e \right)^{2}}{f}+\left(-a^{2}+2 a b -b^{2}\right) x \tan \left(f x +e \right)-\frac{a^{2}}{f}}{\tan \left(f x +e \right)}$	57
risch	$-x \,a^{2}+2 x a b -x \,b^{2}-\frac{2 i \left(a^{2} {\mathrm e}^{2 i \left(f x +e \right)}-b^{2} {\mathrm e}^{2 i \left(f x +e \right)}+a^{2}+b^{2}\right)}{f \left({\mathrm e}^{2 i \left(f x +e \right)}-1\right) \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)}$	85

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-a^2*cot(f*x+e)+tan(f*x+e)*b^2-f*x*(a-b)^2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{(a^2-2\,ab+b^2)fx\tan\left(fx+e\right)-b^2\tan\left(fx+e\right)^2+a^2}{f\tan\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 - 2*a*b + b^2)*f*x*tan(f*x + e) - b^2*tan(f*x + e)^2 + a^2)/(f*tan(
f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(29) = 58.

Time = 0.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \begin{cases} \tilde{\infty}a^2x & \text{for } e=0\wedge f=0 \\ x\left(a+b\tan^2\left(e\right)\right)^2\cot^2\left(e\right) & \text{for } f=0 \\ \tilde{\infty}a^2x & \text{for } e=-fx \\ -a^2x-\frac{a^2}{f\tan\left(e+fx\right)}+2abx-b^2x+\frac{b^2\tan\left(e+fx\right)}{f} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e
)**2, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (-a**2*x - a**2/(f*tan(e + f*x
)) + 2*a*b*x - b**2*x + b**2*tan(e + f*x)/f, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{b^2\tan(fx+e)-(a^2-2\,ab+b^2)(fx+e)-\frac{a^2}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
(b^2*tan(f*x + e) - (a^2 - 2*a*b + b^2)*(f*x + e) - a^2/tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{b^2\tan(fx+e)}{f}-\frac{(a^2-2\,ab+b^2)(fx+e)}{f}-\frac{a^2}{f\tan(fx+e)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
b^2*tan(f*x + e)/f - (a^2 - 2*a*b + b^2)*(f*x + e)/f - a^2/(f*tan(f*x + e)
)
```

Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{b^2\tan(e+f\,x)}{f}-\frac{\mathrm{atan}\left(\frac{\tan(e+f\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{f}-\frac{a^2}{f\tan(e+f\,x)}$$

input
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x))/f - (atan((tan(e + f*x)*(a - b)^2)/(a^2 - 2*a*b + b^2))
*(a - b)^2)/f - a^2/(f*tan(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.79

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$=\frac{-\cos(fx+e)\sin(fx+e)\,a^2fx+2\cos(fx+e)\sin(fx+e)\,abfx-\cos(fx+e)\sin(fx+e)\,b^2fx+\mathrm{si}}{\cos(fx+e)\sin(fx+e)\,f}$$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - cos(e + f*x)*sin(e + f*x)*a**2*f*x + 2*cos(e + f*x)*sin(e + f*x)*a*b*f
*x - cos(e + f*x)*sin(e + f*x)*b**2*f*x + sin(e + f*x)**2*a**2 + sin(e + f
*x)**2*b**2 - a**2)/(cos(e + f*x)*sin(e + f*x)*f)
```

3.209 $\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 44

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=(a-b)^2x+\frac{a(a-2b)\cot(e+fx)}{f}-\frac{a^2\cot^3(e+fx)}{3f}$$

output
```
(a-b)^2*x+a*(a-2*b)*cot(f*x+e)/f-1/3*a^2*cot(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx=-\frac{\cot(e+fx)\left(a(-3a+6b+a\cot^2(e+fx))+3(a-b)^2\operatorname{arctanh}\left(\sqrt{-\tan^2(e+fx)}\right)\sqrt{-\tan^2(e+fx}\right.}{3f}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-1/3*(Cot[e + f*x]*(a*(-3*a + 6*b + a*Cot[e + f*x]^2) + 3*(a - b)^2*ArcTan
h[Sqrt[-Tan[e + f*x]^2]]*Sqrt[-Tan[e + f*x]^2]))/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(a^2\cot^4(e+fx)-a(a-2b)\cot^2(e+fx)+\frac{(a-b)^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^2\cot^3(e+fx)+(a-b)^2\arctan(\tan(e+fx))+a(a-2b)\cot(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((a - b)^2*ArcTan[Tan[e + f*x]] + a*(a - 2*b)*Cot[e + f*x] - (a^2*Cot[e +
f*x]^3)/3)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

method	result	size
parallelrisch	$\frac{-\cot(fx+e)^3 a^2+3a(a-2b)\cot(fx+e)+3fx(a-b)^2}{3f}$	45
derivativedivides	$\frac{-\frac{a^2}{3\tan(fx+e)^3}+\frac{a(a-2b)}{\tan(fx+e)}+(a^2-2ab+b^2)\arctan(\tan(fx+e))}{f}$	53
default	$\frac{-\frac{a^2}{3\tan(fx+e)^3}+\frac{a(a-2b)}{\tan(fx+e)}+(a^2-2ab+b^2)\arctan(\tan(fx+e))}{f}$	53
norman	$\frac{(a^2-2ab+b^2)x\tan(fx+e)^3+\frac{a(a-2b)\tan(fx+e)^2}{f}-\frac{a^2}{3f}}{\tan(fx+e)^3}$	58
risch	$x\,a^2-2xab+x\,b^2+\frac{4ia\left(3a\,e^{4i(fx+e)}-3b\,e^{4i(fx+e)}-3a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+2a-3b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}$	92

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-cot(f*x+e)^3*a^2+3*a*(a-2*b)*cot(f*x+e)+3*f*x*(a-b)^2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = \frac{3 (a^2-2 ab+b^2) f x \tan(fx+e)^3+3 (a^2-2 ab) \tan(fx+e)^2-a^2}{3 f \tan(fx+e)^3}$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^3 + 3*(a^2 - 2*a*b)*tan(f*x +
e)^2 - a^2)/(f*tan(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.

Time = 1.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = \begin{cases} \tilde{\infty} a^2 x & \text{for } e=0 \wedge f=0 \\ x(a+b\tan^2(e))^2 \cot^4(e) & \text{for } f=0 \\ \tilde{\infty} a^2 x & \text{for } e=-fx \\ a^2 x+\frac{a^2}{f\tan(e+fx)}-\frac{a^2}{3f\tan^3(e+fx)}-2abx-\frac{2ab}{f\tan(e+fx)}+b^2 x & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e
)**4, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (a**2*x + a**2/(f*tan(e + f*x)
) - a**2/(3*f*tan(e + f*x)**3) - 2*a*b*x - 2*a*b/(f*tan(e + f*x)) + b**2*x
, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{3\,(a^2-2\,ab+b^2)(fx+e)+\frac{3\,(a^2-2\,ab)\tan(fx+e)^2-a^2}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(3*(a^2 - 2*a*b + b^2)*(f*x + e) + (3*(a^2 - 2*a*b)*tan(f*x + e)^2 - a
^2)/tan(f*x + e)^3)/f
```

Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{(a^2-2\,ab+b^2)(fx+e)}{f}+\frac{3\,a^2\tan\left(fx+e\right)^2-6\,ab\tan\left(fx+e\right)^2-a^2}{3\,f\tan\left(fx+e\right)^3}$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
(a^2 - 2*a*b + b^2)*(f*x + e)/f + 1/3*(3*a^2*tan(f*x + e)^2 - 6*a*b*tan(f*
x + e)^2 - a^2)/(f*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 7.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = a^2\,x+b^2\,x+\frac{a^2\cot(e+f\,x)}{f}-2\,a\,b\,x-\frac{a^2\cot(e+f\,x)^3}{3\,f}-\frac{2\,a\,b\cot(e+f\,x)}{f}$$

input
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
a^2*x + b^2*x + (a^2*cot(e + f*x))/f - 2*a*b*x - (a^2*cot(e + f*x)^3)/(3*f
) - (2*a*b*cot(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.41

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = \frac{4\cos(fx+e)\sin(fx+e)^2a^2-6\cos(fx+e)\sin(fx+e)^2ab-\cos(fx+e)a^2+3\sin(fx+e)^3a^2fx}{3\sin(fx+e)^3f}$$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(4*cos(e + f*x)*sin(e + f*x)**2*a**2 - 6*cos(e + f*x)*sin(e + f*x)**2*a*b
- cos(e + f*x)*a**2 + 3*sin(e + f*x)**3*a**2*f*x - 6*sin(e + f*x)**3*a*b*f
*x + 3*sin(e + f*x)**3*b**2*f*x)/(3*sin(e + f*x)**3*f)
```

3.210 $\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = -(a-b)^2x - \frac{(a-b)^2\cot(e+fx)}{f} + \frac{a(a-2b)\cot^3(e+fx)}{3f} - \frac{a^2\cot^5(e+fx)}{5f}$$

output
```
-(a-b)^2*x-(a-b)^2*cot(f*x+e)/f+1/3*a*(a-2*b)*cot(f*x+e)^3/f-1/5*a^2*cot(f
*x+e)^5/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx = -\frac{a^2\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f} - \frac{2ab\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f} - \frac{b^2\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-1/5*(a^2*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2]
)/f - (2*a*b*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]
^2])/(3*f) - (b^2*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[e + f*
x]^2])/f
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^2}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(b\tan^2(e+fx)+a\right)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(a^2\cot^6(e+fx)-a(a-2b)\cot^4(e+fx)+(a-b)^2\cot^2(e+fx)-\frac{(a-b)^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{5}a^2\cot^5(e+fx)-(a-b)^2\arctan(\tan(e+fx))+\frac{1}{3}a(a-2b)\cot^3(e+fx)-(a-b)^2\cot(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-((a - b)^2*ArcTan[Tan[e + f*x]]) - (a - b)^2*Cot[e + f*x] + (a*(a - 2*b)
*Cot[e + f*x]^3)/3 - (a^2*Cot[e + f*x]^5)/5)/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{-3\cot(fx+e)^5a^2+5a\cot(fx+e)^3(a-2b)-15(a-b)^2\cot(fx+e)-15fx(a-b)^2}{15f}$
derivativedivides	$\frac{-\frac{a^2}{5\tan(fx+e)^5}-\frac{a^2-2ab+b^2}{\tan(fx+e)}+\frac{a(a-2b)}{3\tan(fx+e)^3}+(-a^2+2ab-b^2)\arctan(\tan(fx+e))}{f}$
default	$\frac{-\frac{a^2}{5\tan(fx+e)^5}-\frac{a^2-2ab+b^2}{\tan(fx+e)}+\frac{a(a-2b)}{3\tan(fx+e)^3}+(-a^2+2ab-b^2)\arctan(\tan(fx+e))}{f}$
norman	$\frac{(-a^2+2ab-b^2)x\tan(fx+e)^5-\frac{a^2}{5f}-\frac{(a^2-2ab+b^2)\tan(fx+e)^4}{f}+\frac{a(a-2b)\tan(fx+e)^2}{3f}}{\tan(fx+e)^5}$
risch	$-x\,a^2+2xab-x\,b^2-\frac{2i(45a^2\mathrm{e}^{8i(fx+e)}-60ab\,\mathrm{e}^{8i(fx+e)}+15b^2\mathrm{e}^{8i(fx+e)}-90a^2\mathrm{e}^{6i(fx+e)}+180ab\,\mathrm{e}^{6i(fx+e)}-6}{}$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*cot(f*x+e)^5*a^2+5*a*cot(f*x+e)^3*(a-2*b)-15*(a-b)^2*cot(f*x+e)-1
5*f*x*(a-b)^2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{15\left(a^2-2\,ab+b^2\right)fx\tan\left(fx+e\right)^5+15\left(a^2-2\,ab+b^2\right)\tan\left(fx+e\right)^4-5\left(a^2-2\,ab\right)\tan\left(fx+e\right)^2+}{15\,f\tan\left(fx+e\right)^5}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*(15*(a^2 - 2*a*b + b^2)*f*x*tan(f*x + e)^5 + 15*(a^2 - 2*a*b + b^2)*
tan(f*x + e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/(f*tan(f*x + e)^5
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(53) = 106$.

Time = 3.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.96

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= \begin{cases} \tilde{\infty}a^2x \\ x(a+b\tan^2(e))^2\cot^6(e) \\ \tilde{\infty}a^2x \\ -a^2x - \frac{a^2}{f\tan(e+fx)} + \frac{a^2}{3f\tan^3(e+fx)} - \frac{a^2}{5f\tan^5(e+fx)} + 2abx + \frac{2ab}{f\tan(e+fx)} - \frac{2ab}{3f\tan^3(e+fx)} - b^2x - \frac{b^2}{f\tan(e+fx)} \end{cases}$$

input
```
integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*a**2*x, Eq(e, 0) & Eq(f, 0)), (x*(a + b*tan(e)**2)**2*cot(e
)**6, Eq(f, 0)), (zoo*a**2*x, Eq(e, -f*x)), (-a**2*x - a**2/(f*tan(e + f*x
)) + a**2/(3*f*tan(e + f*x)**3) - a**2/(5*f*tan(e + f*x)**5) + 2*a*b*x + 2
*a*b/(f*tan(e + f*x)) - 2*a*b/(3*f*tan(e + f*x)**3) - b**2*x - b**2/(f*tan
(e + f*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= -\frac{15\,(a^2-2\,ab+b^2)(fx+e)+\frac{15\,(a^2-2\,ab+b^2)\tan(fx+e)^4-5\,(a^2-2\,ab)\tan(fx+e)^2+3\,a^2}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/15*(15*(a^2 - 2*a*b + b^2)*(f*x + e) + (15*(a^2 - 2*a*b + b^2)*tan(f*x
+ e)^4 - 5*(a^2 - 2*a*b)*tan(f*x + e)^2 + 3*a^2)/tan(f*x + e)^5)/f
```

Giac [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx = -\frac{(a^2-2\,ab+b^2)(fx+e)}{f}$$
$$-\frac{15\,a^2\tan\left(fx+e\right)^4-30\,ab\tan\left(fx+e\right)^4+15\,b^2\tan\left(fx+e\right)^4-5\,a^2\tan\left(fx+e\right)^2+10\,ab\tan\left(fx+\right.}{15\,f\tan\left(fx+e\right)^5}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-(a^2 - 2*a*b + b^2)*(f*x + e)/f - 1/15*(15*a^2*tan(f*x + e)^4 - 30*a*b*ta
n(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 - 5*a^2*tan(f*x + e)^2 + 10*a*b*tan(f
*x + e)^2 + 3*a^2)/(f*tan(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^2\,dx$$
$$= 2\,a\,b\,x-b^2\,x - \frac{\cot(e+f\,x)^5\left(\tan(e+f\,x)^4\,(a^2-2\,a\,b+b^2)+\frac{a^2}{5}+\tan(e+f\,x)^2\left(\frac{2\,a\,b}{3}-\frac{a^2}{3}\right)\right)}{f} - a^2\,x$$

input
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^2,x)
```

output
```
2*a*b*x - b^2*x - (cot(e + f*x)^5*(tan(e + f*x)^4*(a^2 - 2*a*b + b^2) + a^
2/5 + tan(e + f*x)^2*((2*a*b)/3 - a^2/3)))/f - a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.38

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^2 \, dx$$
$$= \frac{-23\cos(fx+e)\sin(fx+e)^4 a^2 + 40\cos(fx+e)\sin(fx+e)^4 ab - 15\cos(fx+e)\sin(fx+e)^4 b^2 + 1}{}$$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 23*cos(e + f*x)*sin(e + f*x)**4*a**2 + 40*cos(e + f*x)*sin(e + f*x)**4
*a*b - 15*cos(e + f*x)*sin(e + f*x)**4*b**2 + 11*cos(e + f*x)*sin(e + f*x)
**2*a**2 - 10*cos(e + f*x)*sin(e + f*x)**2*a*b - 3*cos(e + f*x)*a**2 - 15*
sin(e + f*x)**5*a**2*f*x + 30*sin(e + f*x)**5*a*b*f*x - 15*sin(e + f*x)**5
*b**2*f*x)/(15*sin(e + f*x)**5*f)
```

3.211 $\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx=-\frac{\log(\cos(e+fx))}{(a-b)f}-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)b^2f}+\frac{\tan^2(e+fx)}{2bf}$$

output
```
-ln(cos(f*x+e))/(a-b)/f-1/2*a^2*ln(a+b*tan(f*x+e)^2)/(a-b)/b^2/f+1/2*tan(f
*x+e)^2/b/f
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx=\frac{-\frac{2\log(\cos(e+fx))}{a-b}-\frac{a^2\log\left(a+b\tan^2(e+fx)\right)}{(a-b)b^2}+\frac{\tan^2(e+fx)}{b}}{2f}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
((-2*Log[Cos[e + f*x]])/(a - b) - (a^2*Log[a + b*Tan[e + f*x]^2])/((a - b)
*b^2) + Tan[e + f*x]^2/b)/(2*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^5(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(-\frac{a^2}{(a-b)b(b\tan^2(e+fx)+a)}+\frac{1}{b}+\frac{1}{(a-b)(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2\log(a+b\tan^2(e+fx))}{b^2(a-b)}+\frac{\log(\tan^2(e+fx)+1)}{a-b}+\frac{\tan^2(e+fx)}{b}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Log[1 + Tan[e + f*x]^2]/(a - b) - (a^2*Log[a + b*Tan[e + f*x]^2])/((a - b
)*b^2) + Tan[e + f*x]^2/b)/(2*f)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)^2}{2b}-\frac{a^2\ln\left(a+b\tan(fx+e)^2\right)}{2b^2(a-b)}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2a-2b}}{f}$
default	$\frac{\frac{\tan(fx+e)^2}{2b}-\frac{a^2\ln\left(a+b\tan(fx+e)^2\right)}{2b^2(a-b)}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2a-2b}}{f}$
norman	$\frac{\tan(fx+e)^2}{2bf}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a-b)}-\frac{a^2\ln\left(a+b\tan(fx+e)^2\right)}{2(a-b)b^2f}$
parallelrisch	$-\frac{-ab\tan(fx+e)^2+b^2\tan(fx+e)^2+a^2\ln\left(a+b\tan(fx+e)^2\right)-\ln\left(1+\tan(fx+e)^2\right)b^2}{2(a-b)b^2f}$
risch	$-\frac{ix}{a-b}-\frac{2iax}{b^2}-\frac{2iae}{b^2f}-\frac{2ix}{b}-\frac{2ie}{bf}+\frac{2ia^2x}{(a-b)b^2}+\frac{2ia^2e}{(a-b)b^2f}+\frac{2\,\mathrm{e}^{2i(fx+e)}}{fb\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)a}{b^2f}+$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*tan(f*x+e)^2/b-1/2*a^2/b^2/(a-b)*ln(a+b*tan(f*x+e)^2)+1/2/(a-b)*l
n(1+tan(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{a^2\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)-(ab-b^2)\tan(fx+e)^2-(a^2-b^2)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2(ab^2-b^3)f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*(a^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a*b - b^2)*t
an(f*x + e)^2 - (a^2 - b^2)*log(1/(tan(f*x + e)^2 + 1)))/((a*b^2 - b^3)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(53) = 106$.

Time = 3.26 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.76

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \begin{cases} \tilde{\infty} x\tan^3(e) & \text{fo} \\ \frac{\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}}{a} & \text{fo} \\ -\frac{2\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{2\log(\tan^2(e+fx)+1)}{2bf\tan^2(e+fx)+2bf}+\frac{\tan^4(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{2}{2bf\tan^2(e+fx)+2bf} & \text{fo} \\ \frac{x\tan^5(e)}{a+b\tan^2(e)} & \text{fo} \\ -\frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2ab^2f-2b^3f}-\frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2ab^2f-2b^3f}+\frac{ab\tan^2(e+fx)}{2ab^2f-2b^3f}+\frac{b^2\log(\tan^2(e+fx)+1)}{2ab^2f-2b^3f}-\frac{b^2\tan^2(e+fx)}{2ab^2f-2b^3f} & \text{ot} \end{cases}$$

input
```
integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x*tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(tan(e +
 f*x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*f))/a, Eq
(b, 0)), (-2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)*
*2 + 2*b*f) - 2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) +
 tan(e + f*x)**4/(2*b*f*tan(e + f*x)**2 + 2*b*f) - 2/(2*b*f*tan(e + f*x)**
2 + 2*b*f), Eq(a, b)), (x*tan(e)**5/(a + b*tan(e)**2), Eq(f, 0)), (-a**2*l
og(-sqrt(-a/b) + tan(e + f*x))/(2*a*b**2*f - 2*b**3*f) - a**2*log(sqrt(-a/
b) + tan(e + f*x))/(2*a*b**2*f - 2*b**3*f) + a*b*tan(e + f*x)**2/(2*a*b**2
*f - 2*b**3*f) + b**2*log(tan(e + f*x)**2 + 1)/(2*a*b**2*f - 2*b**3*f) - b
**2*tan(e + f*x)**2/(2*a*b**2*f - 2*b**3*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= -\frac{\frac{a^2\log\left(-(a-b)\sin(fx+e)^2+a\right)}{ab^2-b^3}-\frac{(a+b)\log\left(\sin(fx+e)^2-1\right)}{b^2}+\frac{1}{b\sin(fx+e)^2-b}}{2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(a^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b^2 - b^3) - (a + b)*log(sin
(f*x + e)^2 - 1)/b^2 + 1/(b*sin(f*x + e)^2 - b))/f
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{a^2\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(ab^2f-b^3f\right)} + \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(af-bf\right)} + \frac{\tan\left(fx+e\right)^2}{2\,bf}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*a^2*log(abs(b*tan(f*x + e)^2 + a))/(a*b^2*f - b^3*f) + 1/2*log(tan(f*
x + e)^2 + 1)/(a*f - b*f) + 1/2*tan(f*x + e)^2/(b*f)
```

Mupad [B] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,\left(a-b\right)} + \frac{\tan(e+f\,x)^2}{2\,b\,f} - \frac{a^2\,\ln\left(b\tan(e+f\,x)^2+a\right)}{2\,f\,\left(a\,b^2-b^3\right)}$$

input
```
int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) + tan(e + f*x)^2/(2*b*f) - (a^2*log(
a + b*tan(e + f*x)^2))/(2*f*(a*b^2 - b^3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{\tan^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\log\left(\tan\left(fx+e\right)^2+1\right)b^2-\log\left(\tan\left(fx+e\right)^2b+a\right)a^2+\tan\left(fx+e\right)^2ab-\tan\left(fx+e\right)^2b^2}{2b^2f\left(a-b\right)}$$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

output
```
(log(tan(e + f*x)**2 + 1)*b**2 - log(tan(e + f*x)**2*b + a)*a**2 + tan(e +
 f*x)**2*a*b - tan(e + f*x)**2*b**2)/(2*b**2*f*(a - b))
```

3.212 $\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{a\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)bf}$$

output
```
ln(cos(f*x+e))/(a-b)/f+1/2*a*ln(a+b*tan(f*x+e)^2)/(a-b)/b/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{2b\log(\cos(e+fx)) + a\log\left(a+b\tan^2(e+fx)\right)}{2abf-2b^2f}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
(2*b*Log[Cos[e + f*x]] + a*Log[a + b*Tan[e + f*x]^2])/(2*a*b*f - 2*b^2*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^3}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{\int \left(\frac{a}{(a-b)(b\tan^2(e+fx)+a)}-\frac{1}{(a-b)(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a\log(a+b\tan^2(e+fx))}{b(a-b)}-\frac{\log(\tan^2(e+fx)+1)}{a-b}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-(Log[1 + Tan[e + f*x]^2]/(a - b)) + (a*Log[a + b*Tan[e + f*x]^2])/((a -
b)*b))/(2*f)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$-\frac{\ln\left(1+\tan(fx+e)^2\right)b-a\ln\left(a+b\tan(fx+e)^2\right)}{2bf(a-b)}$	46
derivativedivides	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)}+\frac{a\ln\left(a+b\tan(fx+e)^2\right)}{2(a-b)b}}{f}$	52
default	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)}+\frac{a\ln\left(a+b\tan(fx+e)^2\right)}{2(a-b)b}}{f}$	52
norman	$-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a-b)}+\frac{a\ln\left(a+b\tan(fx+e)^2\right)}{2(a-b)bf}$	54
risch	$\frac{ix}{a-b}+\frac{2ix}{b}+\frac{2ie}{bf}-\frac{2iax}{b(a-b)}-\frac{2iae}{bf(a-b)}-\frac{\ln(e^{2i(fx+e)}+1)}{bf}+\frac{a\ln\left(e^{4i(fx+e)}+\frac{2(a+b)e^{2i(fx+e)}}{a-b}+1\right)}{2bf(a-b)}$	132

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(1+tan(f*x+e)^2)*b-a*ln(a+b*tan(f*x+e)^2))/b/f/(a-b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{a\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)-(a-b)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,(ab-b^2)f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(a*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) - (a - b)*log(1/(t
an(f*x + e)^2 + 1)))/((a*b - b^2)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(36) = 72$.

Time = 1.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.60

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \begin{cases} \tilde{\infty}x\tan(e) & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{-\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^2(e+fx)}{2f}}{a} & \text{for } b=0 \\ \frac{\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{\log(\tan^2(e+fx)+1)}{2bf\tan^2(e+fx)+2bf}+\frac{1}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x\tan^3(e)}{a+b\tan^2(e)} & \text{for } f=0 \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf-2b^2f}+\frac{a\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf-2b^2f}-\frac{b\log(\tan^2(e+fx)+1)}{2abf-2b^2f} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e + f
*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a, Eq(b, 0)), (log(tan(e + f*x)
**2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) + log(tan(e + f*x
)**2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) + 1/(2*b*f*tan(e + f*x)**2 + 2*b
*f), Eq(a, b)), (x*tan(e)**3/(a + b*tan(e)**2), Eq(f, 0)), (a*log(-sqrt(-a
/b) + tan(e + f*x))/(2*a*b*f - 2*b**2*f) + a*log(sqrt(-a/b) + tan(e + f*x)
)/(2*a*b*f - 2*b**2*f) - b*log(tan(e + f*x)**2 + 1)/(2*a*b*f - 2*b**2*f),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{a\log\left(-(a-b)\sin(fx+e)^2+a\right)}{ab-b^2}-\frac{\log\left(\sin(fx+e)^2-1\right)}{b}}{2\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(a*log(-(a - b)*sin(f*x + e)^2 + a)/(a*b - b^2) - log(sin(f*x + e)^2 -
 1)/b)/f
```

Giac [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{a\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(abf-b^2f\right)} - \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(af-bf\right)}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*a*log(abs(b*tan(f*x + e)^2 + a))/(a*b*f - b^2*f) - 1/2*log(tan(f*x + e
)^2 + 1)/(a*f - b*f)
```

Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{a\ln\left(b\tan(e+f\,x)^2+a\right)}{2\,f\,\left(a\,b-b^2\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,\left(a-b\right)}$$

input
```
int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

output
```
(a*log(a + b*tan(e + f*x)^2))/(2*f*(a*b - b^2)) - log(tan(e + f*x)^2 + 1)/
(2*f*(a - b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{\tan^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{-\log\left(\tan\left(fx+e\right)^2+1\right)b+\log\left(\tan\left(fx+e\right)^2b+a\right)a}{2bf\,(a-b)}$$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

output
```
( - log(tan(e + f*x)**2 + 1)*b + log(tan(e + f*x)**2*b + a)*a)/(2*b*f*(a -
 b))
```

3.213 $\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)f}$$

output
```
-1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{2\log(\cos(e+fx))+\log\left(a+b\tan^2(e+fx)\right)}{2(a-b)f}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/2*(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2])/((a - b)*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4153, 353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{\int \frac{1}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{a-b}-\frac{b\int \frac{1}{b\tan^2(e+fx)+a}d\tan^2(e+fx)}{a-b}}{2f}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{\log(\tan^2(e+fx)+1)}{a-b}-\frac{\log(a+b\tan^2(e+fx))}{a-b}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Log[1 + Tan[e + f*x]^2]/(a - b) - Log[a + b*Tan[e + f*x]^2]/(a - b))/(2*f
)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{\ln\left(1+\tan(fx+e)^2\right)-\ln\left(a+b\tan(fx+e)^2\right)}{2f(a-b)}$	40
derivativedivides	$\frac{\frac{\ln\left(1+\tan(fx+e)^2\right)}{2a-2b}-\frac{\ln\left(a+b\tan(fx+e)^2\right)}{2(a-b)}}{f}$	48
default	$\frac{\frac{\ln\left(1+\tan(fx+e)^2\right)}{2a-2b}-\frac{\ln\left(a+b\tan(fx+e)^2\right)}{2(a-b)}}{f}$	48
norman	$\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a-b)}-\frac{\ln\left(a+b\tan(fx+e)^2\right)}{2f(a-b)}$	50
risch	$\frac{ix}{a-b}+\frac{2ie}{f(a-b)}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+b)e^{2i(fx+e)}}{a-b}+1\right)}{2f(a-b)}$	72

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(1+tan(f*x+e)^2)-ln(a+b*tan(f*x+e)^2))/f/(a-b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{2(a-b)f}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))/((a - b)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(29) = 58$.

Time = 1.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.69

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\tan{(e)}} & \text{for}\ a=0 \wedge b=0 \wedge f=0 \\ \frac{\log{(\tan^2{(e+fx)}+1)}}{2af} & \text{for}\ b=0 \\ -\frac{1}{2bf\tan^2{(e+fx)}+2bf} & \text{for}\ a=b \\ \frac{x\tan{(e)}}{a+b\tan^2{(e)}} & \text{for}\ f=0 \\ -\frac{\log{\left(-\sqrt{-\frac{a}{b}}+\tan{(e+fx)}\right)}}{2af-2bf} - \frac{\log{\left(\sqrt{-\frac{a}{b}}+\tan{(e+fx)}\right)}}{2af-2bf} + \frac{\log{(\tan^2{(e+fx)}+1)}}{2af-2bf} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e + f*x
)**2 + 1)/(2*a*f), Eq(b, 0)), (-1/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b
)), (x*tan(e)/(a + b*tan(e)**2), Eq(f, 0)), (-log(-sqrt(-a/b) + tan(e + f*
x))/(2*a*f - 2*b*f) - log(sqrt(-a/b) + tan(e + f*x))/(2*a*f - 2*b*f) + log
(tan(e + f*x)**2 + 1)/(2*a*f - 2*b*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\log\left(-(a-b)\sin{(fx+e)}^2+a\right)}{2\,(a-b)f}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*log(-(a - b)*sin(f*x + e)^2 + a)/((a - b)*f)
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{b\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(abf-b^2f\right)}+\frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(af-bf\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*b*log(abs(b*tan(f*x + e)^2 + a))/(a*b*f - b^2*f) + 1/2*log(tan(f*x +
e)^2 + 1)/(a*f - b*f)
```

Mupad [B] (verification not implemented)

Time = 7.83 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right)\,1\mathrm{i}}{f\,\left(a-b\right)}$$

input
```
int(tan(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

output
```
-(atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2
 + b*tan(e + f*x)^2))*1i)/(f*(a - b))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\tan(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\log\left(\tan\left(fx+e\right)^2+1\right)-\log\left(\tan\left(fx+e\right)^2b+a\right)}{2f\left(a-b\right)}$$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

output
```
(log(tan(e + f*x)**2 + 1) - log(tan(e + f*x)**2*b + a))/(2*f*(a - b))
```

3.214 $\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{\log(\tan(e+fx))}{af} + \frac{b\log\left(a+b\tan^2(e+fx)\right)}{2a(a-b)f}$$

output
```
ln(cos(f*x+e))/(a-b)/f+ln(tan(f*x+e))/a/f+1/2*b*ln(a+b*tan(f*x+e)^2)/a/(a-
b)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{\log(\cos(e+fx))}{a-b} + \frac{\log(\tan(e+fx))}{a} + \frac{b\log\left(a+b\tan^2(e+fx)\right)}{2a(a-b)}}{f}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Log[Cos[e + f*x]]/(a - b) + Log[Tan[e + f*x]]/a + (b*Log[a + b*Tan[e + f*
x]^2])/(2*a*(a - b)))/f
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4153, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{b^2}{a(a-b)(b\tan^2(e+fx)+a)}+\frac{\cot(e+fx)}{a}-\frac{1}{(a-b)(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log(\tan^2(e+fx)+1)}{a-b}+\frac{b\log(a+b\tan^2(e+fx))}{a(a-b)}+\frac{\log(\tan^2(e+fx))}{a}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Log[Tan[e + f*x]^2]/a - Log[1 + Tan[e + f*x]^2]/(a - b) + (b*Log[a + b*Ta
n[e + f*x]^2])/(a*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

method	result	size
parallelrisch	$\frac{b\ln\left(a+b\tan(fx+e)^2\right)-\ln\left(\sec(fx+e)^2\right)a+2\ln(\tan(fx+e))(a-b)}{2af(a-b)}$	58
derivativedivides	$\frac{\frac{b\ln\left(a+b\tan(fx+e)^2\right)}{2a(a-b)}+\frac{\ln(\tan(fx+e))}{a}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)}}{f}$	63
default	$\frac{\frac{b\ln\left(a+b\tan(fx+e)^2\right)}{2a(a-b)}+\frac{\ln(\tan(fx+e))}{a}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)}}{f}$	63
norman	$\frac{\ln(\tan(fx+e))}{af}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a-b)}+\frac{b\ln\left(a+b\tan(fx+e)^2\right)}{2a(a-b)f}$	68
risch	$\frac{ix}{a-b}-\frac{2ix}{a}-\frac{2ie}{af}-\frac{2ibx}{a(a-b)}-\frac{2ibe}{af(a-b)}+\frac{\ln(e^{2i(fx+e)}-1)}{af}+\frac{b\ln\left(e^{4i(fx+e)}+\frac{2(a+b)e^{2i(fx+e)}}{a-b}+1\right)}{2af(a-b)}$	131

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*ln(a+b*tan(f*x+e)^2)-ln(sec(f*x+e)^2)*a+2*ln(tan(f*x+e))*(a-b))/a/f
/(a-b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx=\frac{(a-b)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)+b\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)}{2\,(a^2-ab)f}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*((a - b)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + b*log((b*tan(f*x +
 e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^2 - a*b)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(48) = 96$.

Time = 3.27 (sec) , antiderivative size = 388, normalized size of antiderivative = 6.06

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty}x\cot(e)}{\tan^2(e)} \\ \frac{-\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\log(\tan(e+fx))}{f}}{a} \\ \frac{\frac{\log(\tan^2(e+fx)+1)}{2f}-\frac{\log(\tan(e+fx))}{f}-\frac{1}{2f\tan^2(e+fx)}}{b} \\ -\frac{\log(\tan^2(e+fx)+1)\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{\log(\tan^2(e+fx)+1)}{2bf\tan^2(e+fx)+2bf}+\frac{2\log(\tan(e+fx))\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{2\log(\tan(e+fx))}{2bf\tan^2(e+fx)+2bf}+\frac{}{2bf\tan^2(e} \\ \frac{x\cot(e)}{a+b\tan^2(e)} \\ -\frac{a\log(\tan^2(e+fx)+1)}{2a^2f-2abf}+\frac{2a\log(\tan(e+fx))}{2a^2f-2abf}+\frac{b\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2a^2f-2abf}+\frac{b\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2a^2f-2abf}-\frac{2b\log(\tan(e+fx))}{2a^2f-2abf} \end{cases}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x*cot(e)/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log
(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f)/a, Eq(b, 0)), ((log(tan
(e + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/b
, Eq(a, 0)), (-log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*b*f*tan(e + f*x
)**2 + 2*b*f) - log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x)**2 + 2*b*f) +
 2*log(tan(e + f*x))*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) + 2*l
og(tan(e + f*x))/(2*b*f*tan(e + f*x)**2 + 2*b*f) + 1/(2*b*f*tan(e + f*x)**
2 + 2*b*f), Eq(a, b)), (x*cot(e)/(a + b*tan(e)**2), Eq(f, 0)), (-a*log(tan
(e + f*x)**2 + 1)/(2*a**2*f - 2*a*b*f) + 2*a*log(tan(e + f*x))/(2*a**2*f -
 2*a*b*f) + b*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) + b*log
(sqrt(-a/b) + tan(e + f*x))/(2*a**2*f - 2*a*b*f) - 2*b*log(tan(e + f*x))/(
2*a**2*f - 2*a*b*f), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{b\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2-ab}+\frac{\log\left(\sin(fx+e)^2\right)}{a}}{2\,f}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(b*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - a*b) + log(sin(f*x + e)^2)/
a)/f
```

Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{b^2\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(a^2bf-ab^2f\right)} - \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(af-bf\right)} + \frac{\log\left(\tan\left(fx+e\right)^2\right)}{2\,af}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*b^2*log(abs(b*tan(f*x + e)^2 + a))/(a^2*b*f - a*b^2*f) - 1/2*log(tan(f
*x + e)^2 + 1)/(a*f - b*f) + 1/2*log(tan(f*x + e)^2)/(a*f)
```

Mupad [B] (verification not implemented)

Time = 7.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)}{a\,f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,\left(a-b\right)} - \frac{b\,\ln\left(b\tan(e+f\,x)^2+a\right)}{2\,f\,\left(a\,b-a^2\right)}$$

input
```
int(cot(e + f*x)/(a + b*tan(e + f*x)^2),x)
```

output
```
log(tan(e + f*x))/(a*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) - (b*log(a
 + b*tan(e + f*x)^2))/(2*f*(a*b - a^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.08

$$\int \frac{\cot(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{-2\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a+\log\left(-2\sqrt{a-b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a}\right)b+\log\left(2\sqrt{a-\right.}{2af\,(a-b)}$$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2),x)
```

output
```
( - 2*log(tan((e + f*x)/2)**2 + 1)*a + log( - 2*sqrt(a - b)*tan((e + f*x)/
2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*b + log(2*sqrt(a - b)*tan((e +
 f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*b + 2*log(tan((e + f*x)/
2))*a - 2*log(tan((e + f*x)/2))*b)/(2*a*f*(a - b))
```

3.215 $\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\cot^2(e+fx)}{2af}-\frac{\log(\cos(e+fx))}{(a-b)f}-\frac{(a+b)\log(\tan(e+fx))}{a^2f}-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^2(a-b)f}$$

output
```
-1/2*cot(f*x+e)^2/a/f-ln(cos(f*x+e))/(a-b)/f-(a+b)*ln(tan(f*x+e))/a^2/f-1/
2*b^2*ln(a+b*tan(f*x+e)^2)/a^2/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{\cot^2(e+fx)}{a}+\frac{b^2\log\left(b+a\cot^2(e+fx)\right)}{a^2(a-b)}+\frac{2\log(\sin(e+fx))}{a-b}}{2f}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/2*(Cot[e + f*x]^2/a + (b^2*Log[b + a*Cot[e + f*x]^2])/(a^2*(a - b)) + (
2*Log[Sin[e + f*x]])/(a - b))/f
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(-\frac{b^3}{a^2(a-b)(b\tan^2(e+fx)+a)}+\frac{\cot^2(e+fx)}{a}+\frac{(-a-b)\cot(e+fx)}{a^2}+\frac{1}{(a-b)(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2\log\left(a+b\tan^2(e+fx)\right)}{a^2(a-b)}-\frac{(a+b)\log\left(\tan^2(e+fx)\right)}{a^2}+\frac{\log\left(\tan^2(e+fx)+1\right)}{a-b}-\frac{\cot(e+fx)}{a}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-(Cot[e + f*x]/a) - ((a + b)*Log[Tan[e + f*x]^2])/a^2 + Log[1 + Tan[e + f
*x]^2]/(a - b) - (b^2*Log[a + b*Tan[e + f*x]^2])/(a^2*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{-b^2\ln\left(a+b\tan(fx+e)^2\right)+\ln\left(\sec(fx+e)^2\right)a^2-\left((2a+2b)\ln(\tan(fx+e))+a\cot(fx+e)^2\right)(a-b)}{2a^2f(a-b)}$
derivativedivides	$\frac{-\frac{b^2\ln\left(a+b\tan(fx+e)^2\right)}{2a^2(a-b)}-\frac{1}{2a\tan(fx+e)^2}+\frac{(-b-a)\ln(\tan(fx+e))}{a^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2a-2b}}{f}$
default	$\frac{-\frac{b^2\ln\left(a+b\tan(fx+e)^2\right)}{2a^2(a-b)}-\frac{1}{2a\tan(fx+e)^2}+\frac{(-b-a)\ln(\tan(fx+e))}{a^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2a-2b}}{f}$
norman	$-\frac{1}{2af\tan(fx+e)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a-b)}-\frac{(a+b)\ln(\tan(fx+e))}{a^2f}-\frac{b^2\ln\left(a+b\tan(fx+e)^2\right)}{2a^2(a-b)f}$
risch	$-\frac{ix}{a-b}+\frac{2ix}{a}+\frac{2ie}{af}+\frac{2ibx}{a^2}+\frac{2ibe}{a^2f}+\frac{2ib^2x}{a^2(a-b)}+\frac{2ib^2e}{a^2f(a-b)}+\frac{2\,\mathrm{e}^{2i(fx+e)}}{fa\left(\mathrm{e}^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(\mathrm{e}^{2i(fx+e)}-1\right)}{af}-$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b^2*ln(a+b*tan(f*x+e)^2)+ln(sec(f*x+e)^2)*a^2-((2*a+2*b)*ln(tan(f*x+
e))+a*cot(f*x+e)^2)*(a-b))/a^2/f/(a-b)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx =$$
$$-\frac{b^2\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+(a^2-b^2)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+(a^2-ab)\tan\left(fx+\right.}{2\left(a^3-a^2b\right)f\tan\left(fx+e\right)^2}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*(b^2*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2
+ (a^2 - b^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (a
^2 - a*b)*tan(f*x + e)^2 + a^2 - a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(71) = 142$.

Time = 8.42 (sec) , antiderivative size = 733, normalized size of antiderivative = 8.24

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((log(tan(e
+ f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2))/a, E
q(b, 0)), ((-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*f
*tan(e + f*x)**2) - 1/(4*f*tan(e + f*x)**4))/b, Eq(a, 0)), (2*log(tan(e +
f*x)**2 + 1)*tan(e + f*x)**4/(2*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**
2) + 2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a*f*tan(e + f*x)**4 + 2
*a*f*tan(e + f*x)**2) - 4*log(tan(e + f*x))*tan(e + f*x)**4/(2*a*f*tan(e +
 f*x)**4 + 2*a*f*tan(e + f*x)**2) - 4*log(tan(e + f*x))*tan(e + f*x)**2/(2
*a*f*tan(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) - 2*tan(e + f*x)**2/(2*a*f*t
an(e + f*x)**4 + 2*a*f*tan(e + f*x)**2) - 1/(2*a*f*tan(e + f*x)**4 + 2*a*f
*tan(e + f*x)**2), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**3/(a + b*
tan(e)**2), Eq(f, 0)), (a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a
**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - 2*a**2*log(tan(e + f
*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**
2) - a**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) + a*b/(2
*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) - b**2*log(-sqrt(-a/
b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*
tan(e + f*x)**2) - b**2*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*
a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*x)**2) + 2*b**2*log(tan(e +
f*x))*tan(e + f*x)**2/(2*a**3*f*tan(e + f*x)**2 - 2*a**2*b*f*tan(e + f*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{b^2\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^3-a^2b}+\frac{(a+b)\log\left(\sin(fx+e)^2\right)}{a^2}+\frac{1}{a\sin(fx+e)^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(b^2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - a^2*b) + (a + b)*log(sin
(f*x + e)^2)/a^2 + 1/(a*sin(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = &-\frac{b^3\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(a^3bf-a^2b^2f\right)}\\ &+\frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(af-bf\right)}-\frac{(a+b)\log\left(\tan\left(fx+e\right)^2\right)}{2\,a^2f}\\ &+\frac{a\tan\left(fx+e\right)^2+b\tan\left(fx+e\right)^2-a}{2\,a^2f\tan\left(fx+e\right)^2}\end{aligned}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*b^3*log(abs(b*tan(f*x + e)^2 + a))/(a^3*b*f - a^2*b^2*f) + 1/2*log(ta
n(f*x + e)^2 + 1)/(a*f - b*f) - 1/2*(a + b)*log(tan(f*x + e)^2)/(a^2*f) +
1/2*(a*tan(f*x + e)^2 + b*tan(f*x + e)^2 - a)/(a^2*f*tan(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)} - \frac{\cot(e+f\,x)^2}{2\,a\,f} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+b)}{a^2\,f} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a\right)}{2\,a^2\,f\,(a-b)}$$

input
```
int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2),x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*f*(a - b)) - cot(e + f*x)^2/(2*a*f) - (log(tan(
e + f*x))*(a + b))/(a^2*f) - (b^2*log(a + b*tan(e + f*x)^2))/(2*a^2*f*(a -
 b))
```

Reduce [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.54

$$\int \frac{\cot^3(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{4\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sin\left(fx+e\right)^2a^2 - 2\log\left(-2\sqrt{a-b}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\sqrt{a}\right)\mathrm{s}}{}$$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x)
```

output
```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**2 - 2*log( - 2*sqrt(a -
 b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)
**2*b**2 - 2*log(2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)
**2 + sqrt(a))*sin(e + f*x)**2*b**2 - 4*log(tan((e + f*x)/2))*sin(e + f*x)
**2*a**2 + 4*log(tan((e + f*x)/2))*sin(e + f*x)**2*b**2 + sin(e + f*x)**2*
a**2 - sin(e + f*x)**2*a*b - 2*a**2 + 2*a*b)/(4*sin(e + f*x)**2*a**2*f*(a
- b))
```

3.216 $\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{(a+b)\cot^2(e+fx)}{2a^2f} - \frac{\cot^4(e+fx)}{4af} + \frac{\log(\cos(e+fx))}{(a-b)f} + \frac{(a^2+ab+b^2)\log(\tan(e+fx))}{a^3f} + \frac{b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^3(a-b)f}$$

output
```
1/2*(a+b)*cot(f*x+e)^2/a^2/f-1/4*cot(f*x+e)^4/a/f+ln(cos(f*x+e))/(a-b)/f+(
a^2+a*b+b^2)*ln(tan(f*x+e))/a^3/f+1/2*b^3*ln(a+b*tan(f*x+e)^2)/a^3/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{-\frac{(a+b)\cot^2(e+fx)}{a^2} + \frac{\cot^4(e+fx)}{2a} - \frac{b^3\log\left(b+a\cot^2(e+fx)\right)}{a^3(a-b)} - \frac{2\log(\sin(e+fx))}{a-b}}{2f}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
-1/2*(-(((a + b)*Cot[e + f*x]^2)/a^2) + Cot[e + f*x]^4/(2*a) - (b^3*Log[b
+ a*Cot[e + f*x]^2])/(a^3*(a - b)) - (2*Log[Sin[e + f*x]])/(a - b))/f
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^5(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{b^4}{a^3(a-b)(b\tan^2(e+fx)+a)}+\frac{\cot^3(e+fx)}{a}+\frac{(-a-b)\cot^2(e+fx)}{a^2}+\frac{(a^2+ba+b^2)\cot(e+fx)}{a^3}-\frac{1}{(a-b)(\tan^2(e+fx)+1)}\right)d\tan^2(e+ \ldots}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^3\log(a+b\tan^2(e+fx))}{a^3(a-b)}+\frac{(a+b)\cot(e+fx)}{a^2}+\frac{(a^2+ab+b^2)\log(\tan^2(e+fx))}{a^3}-\frac{\log(\tan^2(e+fx)+1)}{a-b}-\frac{\cot^2(e+fx)}{2a}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2),x]
```

output
```
(((a + b)*Cot[e + f*x])/a^2 - Cot[e + f*x]^2/(2*a) + ((a^2 + a*b + b^2)*Lo
g[Tan[e + f*x]^2])/a^3 - Log[1 + Tan[e + f*x]^2]/(a - b) + (b^3*Log[a + b*
Tan[e + f*x]^2])/(a^3*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{2\ln\left(a+b\tan(fx+e)^2\right)b^3-2\ln\left(\sec(fx+e)^2\right)a^3+(4a^3-4b^3)\ln(\tan(fx+e))-a\cot(fx+e)^2(a-b)\left(a\cot(fx+e)^2-2a-\right.}{4(a-b)a^3f}$
derivativedivides	$\frac{\frac{b^3\ln\left(a+b\tan(fx+e)^2\right)}{2a^3(a-b)}-\frac{1}{4a\tan(fx+e)^4}-\frac{-b-a}{2a^2\tan(fx+e)^2}+\frac{\left(a^2+ab+b^2\right)\ln(\tan(fx+e))}{a^3}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)}}{f}$
default	$\frac{\frac{b^3\ln\left(a+b\tan(fx+e)^2\right)}{2a^3(a-b)}-\frac{1}{4a\tan(fx+e)^4}-\frac{-b-a}{2a^2\tan(fx+e)^2}+\frac{\left(a^2+ab+b^2\right)\ln(\tan(fx+e))}{a^3}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)}}{f}$
norman	$\frac{-\frac{1}{4af}+\frac{(a+b)\tan(fx+e)^2}{2a^2f}}{\tan(fx+e)^4}+\frac{\left(a^2+ab+b^2\right)\ln(\tan(fx+e))}{a^3f}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a-b)}+\frac{b^3\ln\left(a+b\tan(fx+e)^2\right)}{2a^3(a-b)f}$
risch	$\frac{ix}{a-b}-\frac{2ix}{a}-\frac{2ie}{af}-\frac{2ibx}{a^2}-\frac{2ibe}{a^2f}-\frac{2ib^2x}{a^3}-\frac{2ib^2e}{a^3f}-\frac{2ib^3x}{(a-b)a^3}-\frac{2ib^3e}{(a-b)a^3f}-\frac{2\left(2a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}\right.}{}$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*ln(a+b*tan(f*x+e)^2)*b^3-2*ln(sec(f*x+e)^2)*a^3+(4*a^3-4*b^3)*ln(ta
n(f*x+e))-a*cot(f*x+e)^2*(a-b)*(a*cot(f*x+e)^2-2*a-2*b))/(a-b)/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{2\,b^3\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^4+2\left(a^3-b^3\right)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^4+\left(3\,a^3-a^2b-2\,ab^2\right.}{4\left(a^4-a^3b\right)f\tan\left(fx+e\right)^4}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(2*b^3*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4
 + 2*(a^3 - b^3)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 +
 (3*a^3 - a^2*b - 2*a*b^2)*tan(f*x + e)^4 - a^3 + a^2*b + 2*(a^3 - a*b^2)*
tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(97) = 194$.

Time = 36.78 (sec) , antiderivative size = 898, normalized size of antiderivative = 7.81

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-log(tan(e
 + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*f*tan(e + f*x)**2) - 1/
(4*f*tan(e + f*x)**4))/a, Eq(b, 0)), ((log(tan(e + f*x)**2 + 1)/(2*f) - lo
g(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2) + 1/(4*f*tan(e + f*x)**4) - 1/
(6*f*tan(e + f*x)**6))/b, Eq(a, 0)), (-6*log(tan(e + f*x)**2 + 1)*tan(e +
f*x)**6/(4*a*f*tan(e + f*x)**6 + 4*a*f*tan(e + f*x)**4) - 6*log(tan(e + f*
x)**2 + 1)*tan(e + f*x)**4/(4*a*f*tan(e + f*x)**6 + 4*a*f*tan(e + f*x)**4)
 + 12*log(tan(e + f*x))*tan(e + f*x)**6/(4*a*f*tan(e + f*x)**6 + 4*a*f*tan
(e + f*x)**4) + 12*log(tan(e + f*x))*tan(e + f*x)**4/(4*a*f*tan(e + f*x)**
6 + 4*a*f*tan(e + f*x)**4) + 6*tan(e + f*x)**4/(4*a*f*tan(e + f*x)**6 + 4*
a*f*tan(e + f*x)**4) + 3*tan(e + f*x)**2/(4*a*f*tan(e + f*x)**6 + 4*a*f*ta
n(e + f*x)**4) - 1/(4*a*f*tan(e + f*x)**6 + 4*a*f*tan(e + f*x)**4), Eq(a,
b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**5/(a + b*tan(e)**2), Eq(f, 0)), (-
2*a**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**4*f*tan(e + f*x)**4
- 4*a**3*b*f*tan(e + f*x)**4) + 4*a**3*log(tan(e + f*x))*tan(e + f*x)**4/(
4*a**4*f*tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) + 2*a**3*tan(e + f*
x)**2/(4*a**4*f*tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) - a**3/(4*a*
*4*f*tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) + a**2*b/(4*a**4*f*tan(
e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) - 2*a*b**2*tan(e + f*x)**2/(4*a*
*4*f*tan(e + f*x)**4 - 4*a**3*b*f*tan(e + f*x)**4) + 2*b**3*log(-sqrt(-...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{2\,b^3\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^4-a^3b}+\frac{2\,(a^2+ab+b^2)\log\left(\sin(fx+e)^2\right)}{a^3}+\frac{2\,(2\,a+b)\sin(fx+e)^2-a}{a^2\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(2*b^3*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - a^3*b) + 2*(a^2 + a*b +
 b^2)*log(sin(f*x + e)^2)/a^3 + (2*(2*a + b)*sin(f*x + e)^2 - a)/(a^2*sin(
f*x + e)^4))/f
```

Giac [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{b^4\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(a^4bf-a^3b^2f\right)} - \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(af-bf\right)} + \frac{\left(a^2+ab+b^2\right)\log\left(\tan\left(fx+e\right)^2\right)}{2\,a^3f} - \frac{3\,a^2\tan\left(fx+e\right)^4+3\,ab\tan\left(fx+e\right)^4+3\,b^2\tan\left(fx+e\right)^4-2\,a^2\tan\left(fx+e\right)^2-2\,ab\tan\left(fx+e\right)^2+}{4\,a^3f\tan\left(fx+e\right)^4}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*b^4*log(abs(b*tan(f*x + e)^2 + a))/(a^4*b*f - a^3*b^2*f) - 1/2*log(tan
(f*x + e)^2 + 1)/(a*f - b*f) + 1/2*(a^2 + a*b + b^2)*log(tan(f*x + e)^2)/(
a^3*f) - 1/4*(3*a^2*tan(f*x + e)^4 + 3*a*b*tan(f*x + e)^4 + 3*b^2*tan(f*x
+ e)^4 - 2*a^2*tan(f*x + e)^2 - 2*a*b*tan(f*x + e)^2 + a^2)/(a^3*f*tan(f*x
 + e)^4)
```

Mupad [B] (verification not implemented)

Time = 7.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\ln(\tan(e+f\,x))\,(a^2+a\,b+b^2)}{a^3\,f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)} - \frac{b^3\,\ln\left(b\tan(e+f\,x)^2+a\right)}{f\,(2\,a^3\,b-2\,a^4)} - \frac{\cot(e+f\,x)^4\left(\frac{1}{4\,a}-\frac{\tan(e+f\,x)^2\,(a+b)}{2\,a^2}\right)}{f}$$

input
```
int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2),x)
```

output
```
(log(tan(e + f*x))*(a*b + a^2 + b^2))/(a^3*f) - log(tan(e + f*x)^2 + 1)/(2
*f*(a - b)) - (b^3*log(a + b*tan(e + f*x)^2))/(f*(2*a^3*b - 2*a^4)) - (cot
(e + f*x)^4*(1/(4*a) - (tan(e + f*x)^2*(a + b))/(2*a^2)))/f
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{a+b\tan^2(e+fx)}\,dx = \int \frac{\cot(fx+e)^5}{\tan(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

output
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x)
```

3.217 $\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{x}{a-b}+\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{5/2}f} -\frac{(a+b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

output
```
-x/(a-b)+a^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)/b^(5/2)/f-(a+b)*
tan(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{-3a^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{b}(3b^2(e+fx)+(a-b)\left(3a+4b-b\sec^2(e+fx)\right)\tan(e+fx))}{3b^{5/2}(-a+b)f}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-3*a^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[b]*(3*b^2*(e + f
*x) + (a - b)*(3*a + 4*b - b*Sec[e + f*x]^2)*Tan[e + f*x]))/(3*b^(5/2)*(-a
 + b)*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4153, 381, 27, 444, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^6}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{381}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\int \frac{3\tan^2(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{3b}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\int \frac{\tan^2(e+fx)\left((a+b)\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+b)\tan(e+fx)}{b}-\frac{\int \frac{\left(a^2+ba+b^2\right)\tan^2(e+fx)+a(a+b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{b}}{b}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+b)\tan(e+fx)}{b}-\frac{\frac{a^3\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{b^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}}{b}}{b}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+b)\tan(e+fx)}{b}-\frac{\frac{a^3\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{b^2\arctan(\tan(e+fx))}{a-b}}{b}}{b}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+b)\tan(e+fx)}{b}-\frac{\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}-\frac{b^2\arctan(\tan(e+fx))}{a-b}}{b}}{b}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]^3/(3*b) - (-((-((b^2*ArcTan[Tan[e + f*x]])/(a - b)) + (a^(5/
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*Sqrt[b]))/b) + ((a + b
)*Tan[e + f*x])/b)/b)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+b\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b^2(a-b)\sqrt{ab}}}{f}$
default	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+b\tan(fx+e)}{b^2}-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b^2(a-b)\sqrt{ab}}}{f}$
risch	$-\frac{x}{a-b}-\frac{2i(3a\,e^{4i(fx+e)}+6b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+6b\,e^{2i(fx+e)}+3a+4b)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{\sqrt{-ab}\,a^2\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a-}{a-b}\right.}{2b^3(a-b)f}$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)+b*tan(f*x+e))-1/(a-b)*arctan
(tan(f*x+e))+1/b^2*a^3/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.27

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \left[-\frac{12\,b^2fx-4\,(ab-b^2)\tan\left(fx+e\right)^3+3\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(b^2\tan(fx+e)^3-ab\tan(\right.}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right.}{12\,(ab^2-b^3)f}\right.$$

$$\left.-\frac{6\,b^2fx-2\,(ab-b^2)\tan\left(fx+e\right)^3-3\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{a}{b}}}{2\,a\tan(fx+e)}\right)+6\,(a^2-b^2)\tan\left(fx+e\right)}{6\,(ab^2-b^3)f}\right]$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(12*b^2*f*x - 4*(a*b - b^2)*tan(f*x + e)^3 + 3*a^2*sqrt(-a/b)*log((
b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 -
a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 +
 a^2)) + 12*(a^2 - b^2)*tan(f*x + e))/((a*b^2 - b^3)*f), -1/6*(6*b^2*f*x -
 2*(a*b - b^2)*tan(f*x + e)^3 - 3*a^2*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)
^2 - a)*sqrt(a/b)/(a*tan(f*x + e))) + 6*(a^2 - b^2)*tan(f*x + e))/((a*b^2
- b^3)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(66) = 132$.

Time = 7.84 (sec) , antiderivative size = 595, normalized size of antiderivative = 7.00

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \begin{cases} \tilde{\infty} x \tan^4(e) \\ \frac{-x+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}}{a} \\ \frac{x+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}}{b} \\ \frac{15fx\tan^2(e+fx)}{6bf\tan^2(e+fx)+6bf}+\frac{15fx}{6bf\tan^2(e+fx)+6bf}+\frac{2\tan^5(e+fx)}{6bf\tan^2(e+fx)+6bf}-\frac{10\tan^3(e+fx)}{6bf\tan^2(e+fx)+6bf}-\frac{15\tan(e+fx)}{6bf\tan^2(e+fx)+6bf} \\ \frac{x\tan^6(e)}{a+b\tan^2(e)} \\ \frac{3a^3\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{6ab^3f\sqrt{-\frac{a}{b}}-6b^4f\sqrt{-\frac{a}{b}}}-\frac{3a^3\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{6ab^3f\sqrt{-\frac{a}{b}}-6b^4f\sqrt{-\frac{a}{b}}}-\frac{6a^2b\sqrt{-\frac{a}{b}}\tan(e+fx)}{6ab^3f\sqrt{-\frac{a}{b}}-6b^4f\sqrt{-\frac{a}{b}}}+\frac{2ab^2\sqrt{-\frac{a}{b}}\tan^3(e+fx)}{6ab^3f\sqrt{-\frac{a}{b}}-6b^4f\sqrt{-\frac{a}{b}}}-\frac{6b^3f\cdots}{6ab^3f\sqrt{-\frac{a}{b}}\cdots} \end{cases}$$

input
```
integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x*tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e
+ f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a, Eq(b, 0)), ((
x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f)/b, Eq(a, 0)), (15*f*x*tan(e +
f*x)**2/(6*b*f*tan(e + f*x)**2 + 6*b*f) + 15*f*x/(6*b*f*tan(e + f*x)**2 +
6*b*f) + 2*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 10*tan(e + f*
x)**3/(6*b*f*tan(e + f*x)**2 + 6*b*f) - 15*tan(e + f*x)/(6*b*f*tan(e + f*x
)**2 + 6*b*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2), Eq(f, 0)), (3*a*
*3*log(-sqrt(-a/b) + tan(e + f*x))/(6*a*b**3*f*sqrt(-a/b) - 6*b**4*f*sqrt(
-a/b)) - 3*a**3*log(sqrt(-a/b) + tan(e + f*x))/(6*a*b**3*f*sqrt(-a/b) - 6*
b**4*f*sqrt(-a/b)) - 6*a**2*b*sqrt(-a/b)*tan(e + f*x)/(6*a*b**3*f*sqrt(-a/
b) - 6*b**4*f*sqrt(-a/b)) + 2*a*b**2*sqrt(-a/b)*tan(e + f*x)**3/(6*a*b**3*
f*sqrt(-a/b) - 6*b**4*f*sqrt(-a/b)) - 6*b**3*f*x*sqrt(-a/b)/(6*a*b**3*f*sq
rt(-a/b) - 6*b**4*f*sqrt(-a/b)) - 2*b**3*sqrt(-a/b)*tan(e + f*x)**3/(6*a*b
**3*f*sqrt(-a/b) - 6*b**4*f*sqrt(-a/b)) + 6*b**3*sqrt(-a/b)*tan(e + f*x)/(
6*a*b**3*f*sqrt(-a/b) - 6*b**4*f*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{3\,a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(ab^2-b^3)\sqrt{ab}} - \frac{3\,(fx+e)}{a-b} + \frac{b\tan(fx+e)^3-3\,(a+b)\tan(fx+e)}{b^2}}{3\,f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*a^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b^2 - b^3)*sqrt(a*b)) - 3*
(f*x + e)/(a - b) + (b*tan(f*x + e)^3 - 3*(a + b)*tan(f*x + e))/b^2)/f
```

Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{a^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(ab^2f-b^3f)\sqrt{ab}} - \frac{fx+e}{af-bf} + \frac{b^2f^2\tan{(fx+e)}^3 - 3\,abf^2\tan{(fx+e)} - 3\,b^2f^2\tan{(fx+e)}}{3\,b^3f^3}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
a^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b^2*f - b^3*f)*sqrt(a*b)) - (f*x
+ e)/(a*f - b*f) + 1/3*(b^2*f^2*tan(f*x + e)^3 - 3*a*b*f^2*tan(f*x + e) -
3*b^2*f^2*tan(f*x + e))/(b^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 1310, normalized size of antiderivative = 15.41

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

output
```
tan(e + f*x)^3/(3*b*f) + (2*atan((((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*
a^4*b^3)/b^3 - (tan(e + f*x)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)
/(b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) + (2*tan(e + f*x)*(a^6 + b^6))/b^3)/(2
*a - 2*b) - ((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e
+ f*x)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i
)/(2*a - 2*b) - (2*tan(e + f*x)*(a^6 + b^6))/b^3)/(2*a - 2*b))/((((((4*a*b
^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e + f*x)*(4*a*b^7 - 4*b
^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2*a - 2*b) + (2*ta
n(e + f*x)*(a^6 + b^6))/b^3)*1i)/(2*a - 2*b) - (2*(a^4*b + a^5 + a^3*b^2))
/b^3 + ((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*
x)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5)*2i)/(b^3*(2*a - 2*b)))*1i)/(2
*a - 2*b) - (2*tan(e + f*x)*(a^6 + b^6))/b^3)*1i)/(2*a - 2*b))))/(f*(2*a -
 2*b)) - (tan(e + f*x)*(a + b))/(b^2*f) - (atan((((((4*a*b^6 - 4*a^2*b^5
- 4*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4
*b^8 + 4*a^2*b^6 - 4*a^3*b^5))/(b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(
a*b^5 - b^6)) - (2*tan(e + f*x)*(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2)*1i)/(2*
(a*b^5 - b^6)) - (((((4*a*b^6 - 4*a^2*b^5 - 4*a^3*b^4 + 4*a^4*b^3)/b^3 - (
tan(e + f*x)*(-a^5*b^5)^(1/2)*(4*a*b^7 - 4*b^8 + 4*a^2*b^6 - 4*a^3*b^5))/(
b^3*(a*b^5 - b^6)))*(-a^5*b^5)^(1/2))/(2*(a*b^5 - b^6)) + (2*tan(e + f*x)*
(a^6 + b^6))/b^3)*(-a^5*b^5)^(1/2)*1i)/(2*(a*b^5 - b^6)))/((((((4*a*b^6...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

$$\int \frac{\tan^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)a^2+\tan(fx+e)^3\,a\,b^2-\tan(fx+e)^3\,b^3-3\tan(fx+e)\,a^2b+3\tan(fx+e)}{3b^3f\,(a-b)}$$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**2 + tan(e +
 f*x)**3*a*b**2 - tan(e + f*x)**3*b**3 - 3*tan(e + f*x)*a**2*b + 3*tan(e +
 f*x)*b**3 - 3*b**3*f*x)/(3*b**3*f*(a - b))
```

3.218 $\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{x}{a-b} - \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

output
```
x/(a-b)-a^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)/b^(3/2)/f+tan(f*x
+e)/b/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{e+fx}{(a-b)f} - \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)b^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(e + f*x)/((a - b)*f) - (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(
(a - b)*b^(3/2)*f) + Tan[e + f*x]/(b*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4153, 381, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^4}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 381}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\int \frac{(a+b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{b}}{f}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\frac{a^2\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{b\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}}{b}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\frac{a^2\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{b\arctan(\tan(e+fx))}{a-b}}{b}}{f}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\tan(e+fx)}{b}-\frac{\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}-\frac{b\arctan(\tan(e+fx))}{a-b}}{b}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-((-((b*ArcTan[Tan[e + f*x]])/(a - b)) + (a^(3/2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/((a - b)*Sqrt[b]))/b) + Tan[e + f*x]/b)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}-\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b(a-b)\sqrt{ab}}+\frac{\arctan(\tan(fx+e))}{a-b}}{f}$	65
default	$\frac{\frac{\tan(fx+e)}{b}-\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{b(a-b)\sqrt{ab}}+\frac{\arctan(\tan(fx+e))}{a-b}}{f}$	65
risch	$\frac{x}{a-b}+\frac{2i}{fb(e^{2i(fx+e)}+1)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2b^2(a-b)f}-\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2b^2(a-b)f}$	14

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b-1/b*a^2/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2
))+1/(a-b)*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.49

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$=\left[\frac{4\,bfx-a\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(b^2\tan(fx+e)^3-ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)+4\,(a-b)\tan\left(fx+\right.}{4\,(ab-b^2)f}\right.$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*b*f*x - a*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^
2 + a^2 + 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f
*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*(a - b)*tan(f*x + e))/((a*b -
 b^2)*f), 1/2*(2*b*f*x - a*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqr
t(a/b)/(a*tan(f*x + e))) + 2*(a - b)*tan(f*x + e))/((a*b - b^2)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(48) = 96$.

Time = 2.11 (sec) , antiderivative size = 427, normalized size of antiderivative = 6.78

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \begin{cases} \tilde{\infty} x\tan^2(e) \\ \frac{x+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}}{a} \\ \frac{-x+\frac{\tan(e+fx)}{f}}{b} \\ -\frac{3fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}-\frac{3fx}{2bf\tan^2(e+fx)+2bf}+\frac{2\tan^3(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{3\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} \\ \frac{x\tan^4(e)}{a+b\tan^2(e)} \\ -\frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2ab^2f\sqrt{-\frac{a}{b}}-2b^3f\sqrt{-\frac{a}{b}}}+\frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2ab^2f\sqrt{-\frac{a}{b}}-2b^3f\sqrt{-\frac{a}{b}}}+\frac{2ab\sqrt{-\frac{a}{b}}\tan(e+fx)}{2ab^2f\sqrt{-\frac{a}{b}}-2b^3f\sqrt{-\frac{a}{b}}}+\frac{2b^2fx\sqrt{-\frac{a}{b}}}{2ab^2f\sqrt{-\frac{a}{b}}-2b^3f\sqrt{-\frac{a}{b}}}-\frac{2b^2\sqrt{-\frac{a}{b}}}{2ab^2f\sqrt{-\frac{a}{b}}} \end{cases}$$

input
```
integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

output

```
Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e +
 f*x)**3/(3*f) - tan(e + f*x)/f)/a, Eq(b, 0)), ((-x + tan(e + f*x)/f)/b, E
q(a, 0)), (-3*f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2 + 2*b*f) - 3*f*x/
(2*b*f*tan(e + f*x)**2 + 2*b*f) + 2*tan(e + f*x)**3/(2*b*f*tan(e + f*x)**2
 + 2*b*f) + 3*tan(e + f*x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x*
tan(e)**4/(a + b*tan(e)**2), Eq(f, 0)), (-a**2*log(-sqrt(-a/b) + tan(e + f
*x))/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) + a**2*log(sqrt(-a/b) +
 tan(e + f*x))/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) + 2*a*b*sqrt(
-a/b)*tan(e + f*x)/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) + 2*b**2*
f*x*sqrt(-a/b)/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)) - 2*b**2*sqrt
(-a/b)*tan(e + f*x)/(2*a*b**2*f*sqrt(-a/b) - 2*b**3*f*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(ab-b^2)\sqrt{ab}} - \frac{fx+e}{a-b} - \frac{\tan(fx+e)}{b}}{f}$$

input

```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output

```
-(a^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b - b^2)*sqrt(a*b)) - (f*x + e)
/(a - b) - tan(f*x + e)/b)/f
```

Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(abf-b^2f)\sqrt{ab}} + \frac{fx+e}{af-bf} + \frac{\tan(fx+e)}{bf}$$

input

```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-a^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a*b*f - b^2*f)*sqrt(a*b)) + (f*x +
 e)/(a*f - b*f) + tan(f*x + e)/(b*f)
```

Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 1212, normalized size of antiderivative = 19.24

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

output
```
tan(e + f*x)/(b*f) - (2*atan((((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (ta
n(e + f*x)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*
1i)/(2*a - 2*b) + (2*tan(e + f*x)*(a^4 + b^4))/b)/(2*a - 2*b) - ((((4*a*b^
4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4
- 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2*b) - (2*tan(e + f*x)*(a^4 +
 b^4))/b)/(2*a - 2*b))/((((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b - (tan(e +
 f*x)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(
2*a - 2*b) + (2*tan(e + f*x)*(a^4 + b^4))/b)*1i)/(2*a - 2*b) - (2*(a^2*b +
 a^3))/b + (((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(4*a*b^
5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3)*2i)/(b*(2*a - 2*b)))*1i)/(2*a - 2*b) -
(2*tan(e + f*x)*(a^4 + b^4))/b)*1i)/(2*a - 2*b))))/(f*(2*a - 2*b)) + (atan
((((-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)
*(-a^3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a*b^3 - b
^4)))*(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) - (2*tan(e + f*x)*(a^4 + b^4))/b
)*1i)/(2*(a*b^3 - b^4)) - ((-a^3*b^3)^(1/2)*((((4*a*b^4 - 8*a^2*b^3 + 4*a^
3*b^2)/b - (tan(e + f*x)*(-a^3*b^3)^(1/2)*(4*a*b^5 - 4*b^6 + 4*a^2*b^4 - 4
*a^3*b^3))/(b*(a*b^3 - b^4)))*(-a^3*b^3)^(1/2))/(2*(a*b^3 - b^4)) + (2*tan
(e + f*x)*(a^4 + b^4))/b)*1i)/(2*(a*b^3 - b^4)))/(((-a^3*b^3)^(1/2)*((((4*
a*b^4 - 8*a^2*b^3 + 4*a^3*b^2)/b + (tan(e + f*x)*(-a^3*b^3)^(1/2)*(4*a*b^5
 - 4*b^6 + 4*a^2*b^4 - 4*a^3*b^3))/(b*(a*b^3 - b^4)))*(-a^3*b^3)^(1/2))...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\tan^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)a+\tan\left(fx+e\right)ab-\tan\left(fx+e\right)b^2+b^2fx}{b^2f\left(a-b\right)}$$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a + tan(e + f
*x)*a*b - tan(e + f*x)*b**2 + b**2*f*x)/(b**2*f*(a - b))
```

3.219 $\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx=-\frac{x}{a-b}+\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}f}$$

output
```
-x/(a-b)+a^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)/b^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx=\frac{\arctan(\tan(e+fx))-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}}}{-af+bf}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
(ArcTan[Tan[e + f*x]] - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/S
qrt[b])/(-(a*f) + b*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4153, 383, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{383}$$

$$\frac{\frac{a\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{a\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{\arctan(\tan(e+fx))}{a-b}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}-\frac{\arctan(\tan(e+fx))}{a-b}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-(ArcTan[Tan[e + f*x]]/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a]])/((a - b)*Sqrt[b]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
risch	$-\frac{x}{a-b}-\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2b(a-b)f}+\frac{\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2b(a-b)f}$	121

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)*arctan(tan(f*x+e))+a/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(
a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.62

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \left[-\frac{4\,fx+\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(b^2\tan(fx+e)^3-ab\tan(fx+e)\right)\sqrt{-\frac{a}{b}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{4\,(a-b)f},\ -\frac{2\,fx-\sqrt{\frac{a}{b}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{a}{b}}}{2\,a\tan(fx+e)}\right)}{2\,(a-b)f}\right]$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(4*f*x + sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
 a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x
+ e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), -1/2*(2*f*x - sqrt(a/b
)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))))/((a - b)*
f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(37) = 74$.

Time = 1.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 5.04

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{-x+\frac{\tan(e+fx)}{f}}{a} & \text{for } b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf} + \frac{fx}{2bf\tan^2(e+fx)+2bf} - \frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for } a=b \\ \frac{x\tan^2(e)}{a+b\tan^2(e)} & \text{for } f=0 \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf\sqrt{-\frac{a}{b}}-2b^2f\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2abf\sqrt{-\frac{a}{b}}-2b^2f\sqrt{-\frac{a}{b}}} - \frac{2bfx\sqrt{-\frac{a}{b}}}{2abf\sqrt{-\frac{a}{b}}-2b^2f\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e + f*x)/f)/
a, Eq(b, 0)), (x/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**2
 + 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) - tan(e + f*x)/(2*b*f*tan(
e + f*x)**2 + 2*b*f), Eq(a, b)), (x*tan(e)**2/(a + b*tan(e)**2), Eq(f, 0))
, (a*log(-sqrt(-a/b) + tan(e + f*x))/(2*a*b*f*sqrt(-a/b) - 2*b**2*f*sqrt(-
a/b)) - a*log(sqrt(-a/b) + tan(e + f*x))/(2*a*b*f*sqrt(-a/b) - 2*b**2*f*sq
rt(-a/b)) - 2*b*f*x*sqrt(-a/b)/(2*a*b*f*sqrt(-a/b) - 2*b**2*f*sqrt(-a/b)),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)} - \frac{fx+e}{a-b}}{f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
(a*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a - b
))/f
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(af-bf)} - \frac{fx+e}{af-bf}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
a*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a*f - b*f)) - (f*x + e)/(a*
f - b*f)
```

Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.70

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\tan(e+f\,x)\,\left(2\,a^2\,b+2\,b^3\right)+\frac{\tan(e+f\,x)\,\left(-8\,a^3\,b^2+8\,a^2\,b^3+8\,a\,b^4-8\,b^5\right)}{(2\,a-2\,b)^2}}{a\,b\,(2\,a-2\,b)}\right)}{f\,\left(2\,a-2\,b\right)} - \frac{\mathrm{atanh}\left(\frac{\tan(e+f\,x)\,\sqrt{-a\,b}}{a}\right)\,\sqrt{-a\,b}}{f\,\left(a\,b-b^2\right)}$$

input
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

output
```
- (2*atan((tan(e + f*x)*(2*a^2*b + 2*b^3) + (tan(e + f*x)*(8*a*b^4 - 8*b^5
 + 8*a^2*b^3 - 8*a^3*b^2))/(2*a - 2*b)^2)/(a*b*(2*a - 2*b))))/(f*(2*a - 2*
b)) - (atanh((tan(e + f*x)*(-a*b)^(1/2))/a)*(-a*b)^(1/2))/(f*(a*b - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\tan^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)-bfx}{bf\,(a-b)}$$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a))) - b*f*x)/(b*f*(a
 - b))
```

3.220 $\int \frac{1}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{x}{a-b} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)f}$$

output
```
x/(a-b)-b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(1/2)/(a-b)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{\arctan(\tan(e+fx)) - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}}}{af-bf}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-1),x]
```

output
```
(ArcTan[Tan[e + f*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/S
qrt[a])/(a*f - b*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4143, 3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4143}$$

$$\frac{x}{a-b}-\frac{b\int \frac{\sec^2(e+fx)}{b\tan^2(e+fx)+a}dx}{a-b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{a-b}-\frac{b\int \frac{\sec(e+fx)^2}{b\tan(e+fx)^2+a}dx}{a-b}$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{x}{a-b}-\frac{b\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{f(a-b)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x}{a-b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}f(a-b)}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-1),x]
```

output
```
x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a -
 b)*f)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4143
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Simp[b/(a - b)   Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x
] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
default	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{f}$	50
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)f}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)f}$	120

input
```
int(1/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)*arctan(tan(f*x+e))-b/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.64

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{4\,fx-\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{4\,(a-b)f}, \frac{2\,fx-\sqrt{\frac{b}{a}}\arctan\left(\left(\cdots\right.\right.}{2\,(a-\cdots}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x - sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 +
a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
 e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a - b)*f), 1/2*(2*f*x - sqrt(b/a)*
arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a - b)*f)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.

Time = 1.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.80

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\tan^2(e)} & \text{for}\ a=0\wedge b=0\wedge f=0 \\ \frac{x}{a} & \text{for}\ b=0 \\ \frac{-x-\frac{1}{f\tan(e+fx)}}{b} & \text{for}\ a=0 \\ \frac{fx\tan^2(e+fx)}{2bf\tan^2(e+fx)+2bf}+\frac{fx}{2bf\tan^2(e+fx)+2bf}+\frac{\tan(e+fx)}{2bf\tan^2(e+fx)+2bf} & \text{for}\ a=b \\ \frac{x}{a+b\tan^2(e)} & \text{for}\ f=0 \\ \frac{2fx\sqrt{-\frac{a}{b}}}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}-\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{-\frac{a}{b}}+\tan(e+fx)\right)}{2af\sqrt{-\frac{a}{b}}-2bf\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a, Eq(b, 0
)), ((-x - 1/(f*tan(e + f*x)))/b, Eq(a, 0)), (f*x*tan(e + f*x)**2/(2*b*f*t
an(e + f*x)**2 + 2*b*f) + f*x/(2*b*f*tan(e + f*x)**2 + 2*b*f) + tan(e + f*
x)/(2*b*f*tan(e + f*x)**2 + 2*b*f), Eq(a, b)), (x/(a + b*tan(e)**2), Eq(f,
 0)), (2*f*x*sqrt(-a/b)/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) - log(-sqrt(
-a/b) + tan(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)) + log(sqrt(-a/
b) + tan(e + f*x))/(2*a*f*sqrt(-a/b) - 2*b*f*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)}-\frac{fx+e}{a-b}}{f}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (f*x + e)/(a -
b))/f
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx=-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(af-bf)}+\frac{fx+e}{af-bf}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-b*arctan(b*tan(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a*f - b*f)) + (f*x + e)/(a
*f - b*f)
```

Mupad [B] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 948, normalized size of antiderivative = 18.96

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx=\text{Too large to display}$$

input
```
int(1/(a + b*tan(e + f*x)^2),x)
```

output
```
(atan((((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3
+ 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*
a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/
2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan
(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b
- a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(e +
f*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(
1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b
- a^2))))/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(e + f*x) - ((-a*b)^(1/2)*
(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(e + f*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5
 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2)
))*(-a*b)^(1/2)*1i)/(a*f*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 +
 (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*
1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4
*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*
a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))/(2*a - 2*b))/(((((4*b^4 -
8*a*b^3 + 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b
^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x))*1i)/(2*a - 2*b)
 - ((((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(e + f*x)*(8*a*b^4 - 8*b^5 + 8*a^
2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(e + f*x...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+b\tan^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)+afx}{af\,(a-b)}$$

input
```
int(1/(a+b*tan(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a))) + a*f*x)/(a*f
*(a - b))
```

3.221 $\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{x}{a-b}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)f}-\frac{\cot(e+fx)}{af}$$

output
```
-x/(a-b)+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)/(a-b)/f-cot(f*
x+e)/a/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)-\sqrt{a}(a(e+fx)+(a-b)\cot(e+fx))}{a^{3/2}(a-b)f}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
(b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] - Sqrt[a]*(a*(e + f*x) + (
a - b)*Cot[e + f*x]))/(a^(3/2)*(a - b)*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 382, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\cot(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{-\frac{\frac{a\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^2\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\cot(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{-\frac{\frac{a\arctan(\tan(e+fx))}{a-b}-\frac{b^2\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\cot(e+fx)}{a}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{a\arctan(\tan(e+fx))}{a-b}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}-\frac{\cot(e+fx)}{a}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-(((a*ArcTan[Tan[e + f*x]])/(a - b) - (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/a) - Cot[e + f*x]/a)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

method	result	si
derivativedivides	$\frac{-\frac{1}{a\tan(fx+e)}+\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a(a-b)\sqrt{ab}}-\frac{\arctan(\tan(fx+e))}{a-b}}{f}$	6
default	$\frac{-\frac{1}{a\tan(fx+e)}+\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a(a-b)\sqrt{ab}}-\frac{\arctan(\tan(fx+e))}{a-b}}{f}$	6
risch	$-\frac{x}{a-b}-\frac{2i}{fa\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-ab}\,b\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a^2(a-b)f}+\frac{\sqrt{-ab}\,b\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a^2(a-b)f}$	1

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/tan(f*x+e)+1/a*b^2/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(
1/2))-1/(a-b)*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.80

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,afx\tan(fx+e) + b\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4 - 6\,ab\tan(fx+e)^2 + a^2 - 4\left(ab\tan(fx+e)^3 - a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4 + 2\,ab\tan(fx+e)^2 + a^2}\right)\tan(f}{4\,(a^2-ab)f\tan(fx+e)}\right.$$

$$\left. -\frac{2\,afx\tan(fx+e) - b\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2 - a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)\tan(fx+e) + 2\,a - 2\,b}{2\,(a^2-ab)f\tan(fx+e)}\right]$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(4*a*f*x*tan(f*x + e) + b*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b
*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/
a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x + e) + 4*a
- 4*b)/((a^2 - a*b)*f*tan(f*x + e)), -1/2*(2*a*f*x*tan(f*x + e) - b*sqrt(b
/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x
+ e) + 2*a - 2*b)/((a^2 - a*b)*f*tan(f*x + e))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(48) = 96$.

Time = 6.26 (sec) , antiderivative size = 522, normalized size of antiderivative = 8.16

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \begin{cases} \tilde{\infty} x \\ \frac{-x-\frac{\cot(e+fx)}{f}}{a} \\ \frac{x+\frac{1}{f\tan(e+fx)}-\frac{1}{3f\tan^3(e+fx)}}{b} \\ -\frac{3fx\tan^3(e+fx)}{2bf\tan^3(e+fx)+2bf\tan(e+fx)} - \frac{3fx\tan(e+fx)}{2bf\tan^3(e+fx)+2bf\tan(e+fx)} - \frac{3\tan^2(e+fx)}{2bf\tan^3(e+fx)+2bf\tan(e+fx)} - \frac{2}{2bf\tan^3(e+fx)+2bf\,t} \\ \frac{\tilde{\infty} x}{a} \\ \frac{x\cot^2(e)}{a+b\tan^2(e)} \\ -\frac{2afx\sqrt{-\frac{a}{b}}\tan(e+fx)}{2a^2f\sqrt{-\frac{a}{b}}\tan(e+fx)-2abf\sqrt{-\frac{a}{b}}\tan(e+fx)} - \frac{2a\sqrt{-\frac{a}{b}}}{2a^2f\sqrt{-\frac{a}{b}}\tan(e+fx)-2abf\sqrt{-\frac{a}{b}}\tan(e+fx)} + \frac{2b\sqrt{-\frac{a}{b}}}{2a^2f\sqrt{-\frac{a}{b}}\tan(e+fx)-2abf\sqrt{-\frac{}{}}} \end{cases}$$

input
```
integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-x - cot(e
 + f*x)/f)/a, Eq(b, 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3
))/b, Eq(a, 0)), (-3*f*x*tan(e + f*x)**3/(2*b*f*tan(e + f*x)**3 + 2*b*f*ta
n(e + f*x)) - 3*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*
x)) - 3*tan(e + f*x)**2/(2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)) - 2/(
2*b*f*tan(e + f*x)**3 + 2*b*f*tan(e + f*x)), Eq(a, b)), (zoo*x/a, Eq(e, -f
*x)), (x*cot(e)**2/(a + b*tan(e)**2), Eq(f, 0)), (-2*a*f*x*sqrt(-a/b)*tan(
e + f*x)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b*f*sqrt(-a/b)*tan(e + f*
x)) - 2*a*sqrt(-a/b)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b*f*sqrt(-a/b
)*tan(e + f*x)) + 2*b*sqrt(-a/b)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b
*f*sqrt(-a/b)*tan(e + f*x)) + b*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*
x)/(2*a**2*f*sqrt(-a/b)*tan(e + f*x) - 2*a*b*f*sqrt(-a/b)*tan(e + f*x)) -
b*log(sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)/(2*a**2*f*sqrt(-a/b)*tan(e +
 f*x) - 2*a*b*f*sqrt(-a/b)*tan(e + f*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-ab)\sqrt{ab}} - \frac{fx+e}{a-b} - \frac{1}{a\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - a*b)*sqrt(a*b)) - (f*x + e)/
(a - b) - 1/(a*tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2f-abf)\sqrt{ab}} - \frac{fx+e}{af-bf} - \frac{1}{af\tan(fx+e)}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
b^2*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*f - a*b*f)*sqrt(a*b)) - (f*x +
e)/(a*f - b*f) - 1/(a*f*tan(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 7.82 (sec) , antiderivative size = 438, normalized size of antiderivative = 6.84

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{a^2\,b-a^3}{f\,\left(a^4\tan\left(e+f\,x\right)-a^3\,b\tan\left(e+f\,x\right)\right)}$$

$$+ \frac{\mathrm{atan}\left(\frac{a^6\,b\tan(e+f\,x)\,\sqrt{-a^3\,b^3}\,1\mathrm{i}-a^3\,b^4\tan(e+f\,x)\,\sqrt{-a^3\,b^3}\,1\mathrm{i}}{a^5\,b^5-a^8\,b^2}\right)\,\sqrt{-a^3\,b^3}\,1\mathrm{i}-a^3\,\mathrm{atan}\left(\frac{\tan(e+f\,x)\,\left(2\,a^5\,b^3+2\,a^3\,b^5\right)+\frac{\left(4\,a^5\,b^4-4\right.}{}}{}\right.}{}$$

input
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2),x)
```

output
```
(a^2*b - a^3)/(f*(a^4*tan(e + f*x) - a^3*b*tan(e + f*x))) + (atan((a^6*b*t
an(e + f*x)*(-a^3*b^3)^(1/2)*1i - a^3*b^4*tan(e + f*x)*(-a^3*b^3)^(1/2)*1i
)/(a^5*b^5 - a^8*b^2))*(-a^3*b^3)^(1/2)*1i - a^3*atan(((((4*a^5*b^4 - 4*a^
4*b^5 + 4*a^6*b^3 - 4*a^7*b^2 + (tan(e + f*x)*(8*a^5*b^5 - 8*a^6*b^4 - 8*a
^7*b^3 + 8*a^8*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^3
*b^5 + 2*a^5*b^3))/(2*a - 2*b) + (((4*a^4*b^5 - 4*a^5*b^4 - 4*a^6*b^3 + 4*
a^7*b^2 + (tan(e + f*x)*(8*a^5*b^5 - 8*a^6*b^4 - 8*a^7*b^3 + 8*a^8*b^2)*1i
)/(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^3*b^5 + 2*a^5*b^3))/(2*
a - 2*b))/(2*a^3*b^4 + 2*a^4*b^3 + 2*a^5*b^2)))/(f*(a^3*b - a^4))
```

Reduce [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int \frac{\cot^2(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)b+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{b}}\right)\sin\left(fx+e\right)b-c}{\sin\left(fx+e\right)a^2 f\left(a-b\right)}$$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))
*sin(e + f*x)*b + sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x
)/2))/sqrt(b))*sin(e + f*x)*b - cos(e + f*x)*a**2 + cos(e + f*x)*a*b - sin
(e + f*x)*a**2*f*x)/(sin(e + f*x)*a**2*f*(a - b))
```

3.222 $\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{x}{a-b} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)f} + \frac{(a+b)\cot(e+fx)}{a^2f} - \frac{\cot^3(e+fx)}{3af}$$

output
```
x/(a-b)-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2)/(a-b)/f+(a+b)*c
ot(f*x+e)/a^2/f-1/3*cot(f*x+e)^3/a/f
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{-3b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}(3a^2(e+fx)-(a-b)\cot(e+fx)\left(-4a-3b+a\csc^2(e+fx)\right))}{3a^{5/2}(a-b)f}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-3*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(3*a^2*(e + f
*x) - (a - b)*Cot[e + f*x]*(-4*a - 3*b + a*Csc[e + f*x]^2)))/(3*a^(5/2)*(a
 - b)*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4153, 382, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \ 4153$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 382$$

$$\frac{\frac{\int -\frac{3\cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{-\frac{-\frac{\int \frac{a^2+ba+b^2+b(a+b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{(a+b)\cot(e+fx)}{a}}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{-\frac{\frac{a^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^3\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{(a+b)\cot(e+fx)}{a}}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{-\frac{-\frac{\frac{a^2\arctan(\tan(e+fx))}{a-b}-\frac{b^3\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{(a+b)\cot(e+fx)}{a}}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{\frac{a^2\arctan(\tan(e+fx))}{a-b}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}-\frac{(a+b)\cot(e+fx)}{a}}{a}-\frac{\cot^3(e+fx)}{3a}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/a - (-(((a^2*ArcTan[Tan[e + f*x]])/(a - b) - (b^(5/2)
*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/a) - ((a + b)*
Cot[e + f*x])/a)/a)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{1}{3a\tan(fx+e)^3}-\frac{-b-a}{a^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2(a-b)\sqrt{ab}}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a-b}-\frac{1}{3a\tan(fx+e)^3}-\frac{-b-a}{a^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^2(a-b)\sqrt{ab}}}{f}$
risch	$\frac{x}{a-b}+\frac{2i(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+4a+3b)}{3f\,a^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-ab}\,b^2\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right.}{2a^3(a-b)f}$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)*arctan(tan(f*x+e))-1/3/a/tan(f*x+e)^3-(-b-a)/a^2/tan(f*x+e)-1
/a^2*b^3/(a-b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.67

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{12\,a^2fx\tan(fx+e)^3-3\,b^2\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2+4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)\tan}{12\,(a^3-a^2b)f\tan(fx+e)^3}\right.$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/12*(12*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(-b/a)*log((b^2*tan(f*x + e)^
4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))
*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x +
e)^3 + 12*(a^2 - b^2)*tan(f*x + e)^2 - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*f*tan
(f*x + e)^3), 1/6*(6*a^2*f*x*tan(f*x + e)^3 - 3*b^2*sqrt(b/a)*arctan(1/2*(
b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e)))*tan(f*x + e)^3 + 6*(a^2
- b^2)*tan(f*x + e)^2 - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*f*tan(f*x + e)^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(66) = 132$.

Time = 17.43 (sec) , antiderivative size = 775, normalized size of antiderivative = 9.23

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((x - cot(e
+ f*x)**3/(3*f) + cot(e + f*x)/f)/a, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x))
+ 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e + f*x)**5))/b, Eq(a, 0)), (15*f*x
*tan(e + f*x)**5/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 15*f*x*
tan(e + f*x)**3/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 15*tan(e
 + f*x)**4/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) + 10*tan(e + f*
x)**2/(6*b*f*tan(e + f*x)**5 + 6*b*f*tan(e + f*x)**3) - 2/(6*b*f*tan(e + f
*x)**5 + 6*b*f*tan(e + f*x)**3), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot
(e)**4/(a + b*tan(e)**2), Eq(f, 0)), (6*a**2*f*x*sqrt(-a/b)*tan(e + f*x)**
3/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x
)**3) + 6*a**2*sqrt(-a/b)*tan(e + f*x)**2/(6*a**3*f*sqrt(-a/b)*tan(e + f*x
)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) - 2*a**2*sqrt(-a/b)/(6*a**3*
f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) + 2*
a*b*sqrt(-a/b)/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b
)*tan(e + f*x)**3) - 6*b**2*sqrt(-a/b)*tan(e + f*x)**2/(6*a**3*f*sqrt(-a/b
)*tan(e + f*x)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) - 3*b**2*log(-s
qrt(-a/b) + tan(e + f*x))*tan(e + f*x)**3/(6*a**3*f*sqrt(-a/b)*tan(e + f*x
)**3 - 6*a**2*b*f*sqrt(-a/b)*tan(e + f*x)**3) + 3*b**2*log(sqrt(-a/b) + ta
n(e + f*x))*tan(e + f*x)**3/(6*a**3*f*sqrt(-a/b)*tan(e + f*x)**3 - 6*a**2*
b*f*sqrt(-a/b)*tan(e + f*x)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{\frac{3\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-a^2b)\sqrt{ab}} - \frac{3\,(fx+e)}{a-b} - \frac{3\,(a+b)\tan(fx+e)^2-a}{a^2\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - a^2*b)*sqrt(a*b)) - 3
*(f*x + e)/(a - b) - (3*(a + b)*tan(f*x + e)^2 - a)/(a^2*tan(f*x + e)^3))/
f
```

Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.17

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3f-a^2bf)\sqrt{ab}} + \frac{fx+e}{af-bf} + \frac{3\,a\tan\left(fx+e\right)^2+3\,b\tan\left(fx+e\right)^2-a}{3\,a^2f\tan\left(fx+e\right)^3}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
-b^3*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*f - a^2*b*f)*sqrt(a*b)) + (f*x
 + e)/(a*f - b*f) + 1/3*(3*a*tan(f*x + e)^2 + 3*b*tan(f*x + e)^2 - a)/(a^2
*f*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 484, normalized size of antiderivative = 5.76

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{a^4\,b+\tan(e+f\,x)^2\,(3\,a^5-3\,a^3\,b^2)-a^5}{f\,\left(3\,a^6\tan\left(e+f\,x\right)^3-3\,a^5\,b\tan\left(e+f\,x\right)^3\right)} - \frac{\mathrm{atan}\left(\frac{a^{10}\,b\tan(e+f\,x)\,\sqrt{-a^5\,b^5}\,1\mathrm{i}-a^5\,b^6\tan(e+f\,x)\,\sqrt{-a^5\,b^5}\,1\mathrm{i}}{a^8\,b^8-a^{13}\,b^3}\right)\sqrt{-a^5\,b^5}\,3\mathrm{i}-3\,a^5\,\mathrm{atan}\left(\frac{\tan(e+f\,x)\,\left(2\,a^{10}\,b^3+2\,a^6\,b^7\right)+\frac{\left(4\,a^9\,b \cdots\right.}{\cdots}}{\cdots}\right.\cdots}{\cdots}$$

input
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2),x)
```

output
```
(a^4*b + tan(e + f*x)^2*(3*a^5 - 3*a^3*b^2) - a^5)/(f*(3*a^6*tan(e + f*x)^
3 - 3*a^5*b*tan(e + f*x)^3)) - (atan((a^10*b*tan(e + f*x)*(-a^5*b^5)^(1/2)
*1i - a^5*b^6*tan(e + f*x)*(-a^5*b^5)^(1/2)*1i)/(a^8*b^8 - a^13*b^3))*(-a^
5*b^5)^(1/2)*3i - 3*a^5*atan(((((4*a^9*b^5 - 4*a^8*b^6 + 4*a^11*b^3 - 4*a^
12*b^2 + (tan(e + f*x)*(8*a^10*b^5 - 8*a^11*b^4 - 8*a^12*b^3 + 8*a^13*b^2)
*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^6*b^7 + 2*a^10*b^3))
/(2*a - 2*b) + (((4*a^8*b^6 - 4*a^9*b^5 - 4*a^11*b^3 + 4*a^12*b^2 + (tan(e
 + f*x)*(8*a^10*b^5 - 8*a^11*b^4 - 8*a^12*b^3 + 8*a^13*b^2)*1i)/(2*a - 2*b
))*1i)/(2*a - 2*b) + tan(e + f*x)*(2*a^6*b^7 + 2*a^10*b^3))/(2*a - 2*b))/(
2*a^6*b^6 + 2*a^7*b^5 + 2*a^8*b^4 + 2*a^9*b^3 + 2*a^10*b^2)))/(f*(3*a^5*b
- 3*a^6))
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{a+b\tan^2(e+fx)}\,dx=\int \frac{\cot(fx+e)^4}{\tan(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

output
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2),x)
```

3.223 $\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = -\frac{x}{a-b}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)f} - \frac{(a^2+ab+b^2)\cot(e+fx)}{a^3f} + \frac{(a+b)\cot^3(e+fx)}{3a^2f}-\frac{\cot^5(e+fx)}{5af}$$

output
```
-x/(a-b)+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)/(a-b)/f-(a^2+a
*b+b^2)*cot(f*x+e)/a^3/f+1/3*(a+b)*cot(f*x+e)^3/a^2/f-1/5*cot(f*x+e)^5/a/f
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{15b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)+\sqrt{a}(-15a^3(e+fx)-(a-b)\cot(e+fx)\left(23a^2+20ab+15b^2-a(11a+\right.}{15a^{7/2}(a-b)f}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
(15*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]] + Sqrt[a]*(-15*a^3*(e +
 f*x) - (a - b)*Cot[e + f*x]*(23*a^2 + 20*a*b + 15*b^2 - a*(11*a + 5*b)*Cs
c[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4)))/(15*a^(7/2)*(a - b)*f)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4153, 382, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{5\cot^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(a^2+ba+b^2+b(a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{3a}-\frac{(a+b)\cot^3(e+fx)}{3a}}{a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(a^2+ba+b^2+b(a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{(a+b)\cot^3(e+fx)}{3a}}{a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{b\left(a^2+ba+b^2\right)\tan^2(e+fx)+(a+b)\left(a^2+b^2\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a}}{a}-\frac{(a+b)\cot^3(e+fx)}{3a}}{a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{-\frac{-\frac{\frac{a^3\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^4\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a}}{a}-\frac{(a+b)\cot^3(e+fx)}{3a}}{a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{-\frac{-\frac{-\frac{\frac{a^3\arctan(\tan(e+fx))}{a-b}-\frac{b^4\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a}}{a}-\frac{(a+b)\cot^3(e+fx)}{3a}}{a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{-\frac{\frac{a^3\arctan(\tan(e+fx))}{a-b}-\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}-\frac{\left(a^2+ab+b^2\right)\cot(e+fx)}{a}}{a}-\frac{(a+b)\cot^3(e+fx)}{3a}}{a}-\frac{\cot^5(e+fx)}{5a}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2),x]
```

output
```
(-1/5*Cot[e + f*x]^5/a - (-1/3*((a + b)*Cot[e + f*x]^3)/a - (-(((a^3*ArcTa
n[Tan[e + f*x]])/(a - b) - (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]]
)/(Sqrt[a]*(a - b)))/a) - ((a^2 + a*b + b^2)*Cot[e + f*x])/a)/a)/a)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3(a-b)\sqrt{ab}}-\frac{1}{5a\tan(fx+e)^5}-\frac{-b-a}{3a^2\tan(fx+e)^3}-\frac{a^2+ab+b^2}{a^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a-b}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{a^3(a-b)\sqrt{ab}}-\frac{1}{5a\tan(fx+e)^5}-\frac{-b-a}{3a^2\tan(fx+e)^3}-\frac{a^2+ab+b^2}{a^3\tan(fx+e)}}{f}$
risch	$-\frac{x}{a-b}-\frac{2i(45a^2e^{8i(fx+e)}+30ab\,e^{8i(fx+e)}+15b^2e^{8i(fx+e)}-90a^2e^{6i(fx+e)}-90ab\,e^{6i(fx+e)}-60b^2e^{6i(fx+e)}+140a^2e}{15f\,a^3(e}$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)*arctan(tan(f*x+e))+1/a^3*b^4/(a-b)/(a*b)^(1/2)*arctan(b*tan(
f*x+e)/(a*b)^(1/2))-1/5/a/tan(f*x+e)^5-1/3*(-b-a)/a^2/tan(f*x+e)^3-(a^2+a*
b+b^2)/a^3/tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.12

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$
$$= \left[-\frac{60\,a^3fx\tan\left(fx+e\right)^5+15\,b^3\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(fx+e)^2+a^2-4\left(ab\tan(fx+e)^3-a^2\tan(fx+e)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(fx+e)^4+2\,ab\tan(fx+e)^2+a^2}\right)}{60\,(a^4-a^3b)f\tan(\cdots}\right.$$
$$-\frac{30\,a^3fx\tan\left(fx+e\right)^5-15\,b^3\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\tan(fx+e)^2-a\right)\sqrt{\frac{b}{a}}}{2\,b\tan(fx+e)}\right)\tan\left(fx+e\right)^5+30\left(a^3-b^3\right)\tan\left(fx+\cdots\right.}{30\,(a^4-a^3b)f\tan\left(fx+e\right)^5}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/60*(60*a^3*f*x*tan(f*x + e)^5 + 15*b^3*sqrt(-b/a)*log((b^2*tan(f*x + e
)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e
))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2))*tan(f*x
+ e)^5 + 60*(a^3 - b^3)*tan(f*x + e)^4 + 12*a^3 - 12*a^2*b - 20*(a^3 - a*b
^2)*tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*(30*a^3*f*x*ta
n(f*x + e)^5 - 15*b^3*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a
)/(b*tan(f*x + e)))*tan(f*x + e)^5 + 30*(a^3 - b^3)*tan(f*x + e)^4 + 6*a^3
 - 6*a^2*b - 10*(a^3 - a*b^2)*tan(f*x + e)^2)/((a^4 - a^3*b)*f*tan(f*x + e
)^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. 2(94) = 188.

Time = 86.16 (sec) , antiderivative size = 984, normalized size of antiderivative = 8.71

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-x - cot(e
 + f*x)**5/(5*f) + cot(e + f*x)**3/(3*f) - cot(e + f*x)/f)/a, Eq(b, 0)), (
(x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3) + 1/(5*f*tan(e + f*x)**5
) - 1/(7*f*tan(e + f*x)**7))/b, Eq(a, 0)), (-105*f*x*tan(e + f*x)**7/(30*b
*f*tan(e + f*x)**7 + 30*b*f*tan(e + f*x)**5) - 105*f*x*tan(e + f*x)**5/(30
*b*f*tan(e + f*x)**7 + 30*b*f*tan(e + f*x)**5) - 105*tan(e + f*x)**6/(30*b
*f*tan(e + f*x)**7 + 30*b*f*tan(e + f*x)**5) - 70*tan(e + f*x)**4/(30*b*f*
tan(e + f*x)**7 + 30*b*f*tan(e + f*x)**5) + 14*tan(e + f*x)**2/(30*b*f*tan
(e + f*x)**7 + 30*b*f*tan(e + f*x)**5) - 6/(30*b*f*tan(e + f*x)**7 + 30*b*
f*tan(e + f*x)**5), Eq(a, b)), (zoo*x/a, Eq(e, -f*x)), (x*cot(e)**6/(a + b
*tan(e)**2), Eq(f, 0)), (-30*a**3*f*x*sqrt(-a/b)*tan(e + f*x)**5/(30*a**4*
f*sqrt(-a/b)*tan(e + f*x)**5 - 30*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**5) - 3
0*a**3*sqrt(-a/b)*tan(e + f*x)**4/(30*a**4*f*sqrt(-a/b)*tan(e + f*x)**5 -
30*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**5) + 10*a**3*sqrt(-a/b)*tan(e + f*x)*
*2/(30*a**4*f*sqrt(-a/b)*tan(e + f*x)**5 - 30*a**3*b*f*sqrt(-a/b)*tan(e +
f*x)**5) - 6*a**3*sqrt(-a/b)/(30*a**4*f*sqrt(-a/b)*tan(e + f*x)**5 - 30*a*
*3*b*f*sqrt(-a/b)*tan(e + f*x)**5) + 6*a**2*b*sqrt(-a/b)/(30*a**4*f*sqrt(-
a/b)*tan(e + f*x)**5 - 30*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**5) - 10*a*b**2
*sqrt(-a/b)*tan(e + f*x)**2/(30*a**4*f*sqrt(-a/b)*tan(e + f*x)**5 - 30*a**
3*b*f*sqrt(-a/b)*tan(e + f*x)**5) + 30*b**3*sqrt(-a/b)*tan(e + f*x)**4/...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{\frac{15\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4-a^3b)\sqrt{ab}} - \frac{15\,(fx+e)}{a-b} - \frac{15\,(a^2+ab+b^2)\tan(fx+e)^4-5\,(a^2+ab)\tan(fx+e)^2+3\,a^2}{a^3\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - a^3*b)*sqrt(a*b)) -
15*(f*x + e)/(a - b) - (15*(a^2 + a*b + b^2)*tan(f*x + e)^4 - 5*(a^2 + a*b
)*tan(f*x + e)^2 + 3*a^2)/(a^3*tan(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx = \frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4f-a^3bf)\sqrt{ab}} - \frac{fx+e}{af-bf} - \frac{15\,a^2\tan(fx+e)^4+15\,ab\tan(fx+e)^4+15\,b^2\tan(fx+e)^4-5\,a^2\tan(fx+e)^2-5\,ab\tan(fx+e}{15\,a^3f\tan(fx+e)^5}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

output
```
b^4*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4*f - a^3*b*f)*sqrt(a*b)) - (f*x
+ e)/(a*f - b*f) - 1/15*(15*a^2*tan(f*x + e)^4 + 15*a*b*tan(f*x + e)^4 + 1
5*b^2*tan(f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 5*a*b*tan(f*x + e)^2 + 3*a^2
)/(a^3*f*tan(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 524, normalized size of antiderivative = 4.64

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx$$

$$= \frac{\mathrm{atan}\left(\frac{a^{14}\,b\tan(e+f\,x)\,\sqrt{-a^7\,b^7}\,1\mathrm{i}-a^7\,b^8\tan(e+f\,x)\,\sqrt{-a^7\,b^7}\,1\mathrm{i}}{a^{11}\,b^{11}-a^{18}\,b^4}\right)\sqrt{-a^7\,b^7}\,15\mathrm{i}-15\,a^7\,\mathrm{atan}\left(\frac{\tan(e+f\,x)\left(2\,a^{15}\,b^3+2\,a^9\,b^9\right)+\frac{\left(4\,a^{13}\right.}{}}{}\right.}{}$$

$$+ \frac{3\,a^6\,b+\tan(e+f\,x)^2\,(5\,a^7-5\,a^5\,b^2)-\tan(e+f\,x)^4\,(15\,a^7-15\,a^4\,b^3)-3\,a^7}{f\,\left(15\,a^8\tan(e+f\,x)^5-15\,a^7\,b\tan(e+f\,x)^5\right)}$$

input
```
int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2),x)
```

output
```
(atan((a^14*b*tan(e + f*x)*(-a^7*b^7)^(1/2)*1i - a^7*b^8*tan(e + f*x)*(-a^
7*b^7)^(1/2)*1i)/(a^11*b^11 - a^18*b^4))*(-a^7*b^7)^(1/2)*15i - 15*a^7*ata
n(((((4*a^13*b^6 - 4*a^12*b^7 + 4*a^16*b^3 - 4*a^17*b^2 + (tan(e + f*x)*(8
*a^15*b^5 - 8*a^16*b^4 - 8*a^17*b^3 + 8*a^18*b^2)*1i)/(2*a - 2*b))*1i)/(2*
a - 2*b) + tan(e + f*x)*(2*a^9*b^9 + 2*a^15*b^3))/(2*a - 2*b) + (((4*a^12*
b^7 - 4*a^13*b^6 - 4*a^16*b^3 + 4*a^17*b^2 + (tan(e + f*x)*(8*a^15*b^5 - 8
*a^16*b^4 - 8*a^17*b^3 + 8*a^18*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) + ta
n(e + f*x)*(2*a^9*b^9 + 2*a^15*b^3))/(2*a - 2*b))/(2*a^9*b^8 + 2*a^10*b^7
+ 2*a^11*b^6 + 2*a^12*b^5 + 2*a^13*b^4 + 2*a^14*b^3 + 2*a^15*b^2)))/(f*(15
*a^7*b - 15*a^8)) + (3*a^6*b + tan(e + f*x)^2*(5*a^7 - 5*a^5*b^2) - tan(e
+ f*x)^4*(15*a^7 - 15*a^4*b^3) - 3*a^7)/(f*(15*a^8*tan(e + f*x)^5 - 15*a^7
*b*tan(e + f*x)^5))
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{a+b\tan^2(e+fx)}\,dx=\int \frac{\cot(fx+e)^6}{\tan(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

output
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2),x)
```

3.224 $\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^2} \, dx = -\frac{\log(\cos(e+fx))}{(a-b)^2 f} + \frac{a(a-2b)\log(a+b\tan^2(e+fx))}{2(a-b)^2 b^2 f} + \frac{a^2}{2(a-b)b^2 f\,(a+b\tan^2(e+fx))}$$

output
```
-ln(cos(f*x+e))/(a-b)^2/f+1/2*a*(a-2*b)*ln(a+b*tan(f*x+e)^2)/(a-b)^2/b^2/f
+1/2*a^2/(a-b)/b^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^2} \, dx = \frac{-2\log(\cos(e+fx)) + \frac{a(a-2b)\log(a+b\tan^2(e+fx))}{b^2} + \frac{a^2(a-b)}{b^2(a+b\tan^2(e+fx))}}{2(a-b)^2 f}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-2*Log[Cos[e + f*x]] + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2])/b^2 + (a^2
*(a - b))/(b^2*(a + b*Tan[e + f*x]^2)))/(2*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^5(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(-\frac{a^2}{(a-b)b(b\tan^2(e+fx)+a)^2}+\frac{(a-2b)a}{(a-b)^2b(b\tan^2(e+fx)+a)}+\frac{1}{(a-b)^2(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2}{b^2(a-b)(a+b\tan^2(e+fx))}+\frac{a(a-2b)\log(a+b\tan^2(e+fx))}{b^2(a-b)^2}+\frac{\log(\tan^2(e+fx)+1)}{(a-b)^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(Log[1 + Tan[e + f*x]^2]/(a - b)^2 + (a*(a - 2*b)*Log[a + b*Tan[e + f*x]^2
])/((a - b)^2*b^2) + a^2/((a - b)*b^2*(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{a\left(\frac{(a-2b)\ln\left(a+b\tan(fx+e)^2\right)}{b^2}+\frac{a(a-b)}{b^2\left(a+b\tan(fx+e)^2\right)}\right)}{2(a-b)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}}{f}$
default	$\frac{\frac{a\left(\frac{(a-2b)\ln\left(a+b\tan(fx+e)^2\right)}{b^2}+\frac{a(a-b)}{b^2\left(a+b\tan(fx+e)^2\right)}\right)}{2(a-b)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}}{f}$
norman	$\frac{a^2}{2(a-b)b^2f\left(a+b\tan(fx+e)^2\right)}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}+\frac{a(a-2b)\ln\left(a+b\tan(fx+e)^2\right)}{2b^2f(a^2-2ab+b^2)}$
parallelrisch	$\frac{\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^2b^3+\ln\left(a+b\tan(fx+e)^2\right)\tan(fx+e)^2a^2b-2\ln\left(a+b\tan(fx+e)^2\right)\tan(fx+e)^2ab^2+\mathrm{l}}{2(a^2-2ab+b^2)\left(a+b\tan(fx+e)^2\right)}$
risch	$-\frac{ix}{a^2-2ab+b^2}+\frac{2ix}{b^2}+\frac{2ie}{b^2f}-\frac{2ia^2x}{b^2(a^2-2ab+b^2)}-\frac{2ia^2e}{b^2f(a^2-2ab+b^2)}+\frac{4iax}{b(a^2-2ab+b^2)}+\frac{4iae}{bf(a^2-2ab+b^2)}-$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*a/(a-b)^2*((a-2*b)/b^2*ln(a+b*tan(f*x+e)^2)+a*(a-b)/b^2/(a+b*tan(
f*x+e)^2))+1/2/(a-b)^2*ln(1+tan(f*x+e)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(86) = 172$.

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.07

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{a^2b\tan(fx+e)^2+a^2b-\left(a^3-2a^2b+(a^2b-2ab^2)\tan(fx+e)^2\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)+\left(a^3-2a^2b\right.}{2\left((a^2b^3-2ab^4+b^5)f\tan(fx+e)^2+(a^3b^2-2a^2b\right.}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*b*tan(f*x + e)^2 + a^2*b - (a^3 - 2*a^2*b + (a^2*b - 2*a*b^2)*ta
n(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)) + (a^3 - 2*
a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*log(1/(tan(f*x + e
)^2 + 1)))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*
b^3 + a*b^4)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(71) = 142$.

Time = 14.42 (sec) , antiderivative size = 1542, normalized size of antiderivative = 17.13

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x*tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(tan(e + f*
x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*f))/a**2, Eq
(b, 0)), (2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*b**2*f*tan(e + f*x
)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 4*log(tan(e + f*x)**2 + 1)*t
an(e + f*x)**2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b*
*2*f) + 2*log(tan(e + f*x)**2 + 1)/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*ta
n(e + f*x)**2 + 4*b**2*f) + 4*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**4 +
8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) + 3/(4*b**2*f*tan(e + f*x)**4 + 8*b**
2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)), (x*tan(e)**5/(a + b*tan(e)**2)
**2, Eq(f, 0)), (a**3*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**3*b**2*f + 2*a
**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 +
2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**3*log(sqrt(-a/b) + tan(e + f*x
))/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b*
*4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**3/(2*a*
*3*b**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan
(e + f*x)**2 + 2*a*b**4*f + 2*b**5*f*tan(e + f*x)**2) + a**2*b*log(-sqrt(-
a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b**2*f + 2*a**2*b**3*f*tan(e
+ f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(e + f*x)**2 + 2*a*b**4*f + 2*b*
*5*f*tan(e + f*x)**2) - 2*a**2*b*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**3*b
**2*f + 2*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f - 4*a*b**4*f*tan(...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.42

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\frac{a^2}{a^3b-2\,a^2b^2+ab^3-(a^3b-3\,a^2b^2+3\,ab^3-b^4)\sin(fx+e)^2}-\frac{(a^2-2\,ab)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2b^2-2\,ab^3+b^4}+\frac{\log\left(\sin(fx+e)^2-1\right)}{b^2}}{2\,f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2/(a^3*b - 2*a^2*b^2 + a*b^3 - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)
*sin(f*x + e)^2) - (a^2 - 2*a*b)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^2*b^2
 - 2*a*b^3 + b^4) + log(sin(f*x + e)^2 - 1)/b^2)/f
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{(a^2-2\,ab)\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\,(a^2b^2f-2\,ab^3f+b^4f)} + \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,(a^2f-2\,abf+b^2f)} - \frac{a^2\tan\left(fx+e\right)^2-2\,ab\tan\left(fx+e\right)^2-a^2}{2\,(a^2bf-2\,ab^2f+b^3f)\left(b\tan\left(fx+e\right)^2+a\right)}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2 - 2*a*b)*log(abs(b*tan(f*x + e)^2 + a))/(a^2*b^2*f - 2*a*b^3*f +
b^4*f) + 1/2*log(tan(f*x + e)^2 + 1)/(a^2*f - 2*a*b*f + b^2*f) - 1/2*(a^2*
tan(f*x + e)^2 - 2*a*b*tan(f*x + e)^2 - a^2)/((a^2*b*f - 2*a*b^2*f + b^3*f
)*(b*tan(f*x + e)^2 + a))
```

Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^2} + \frac{a^2}{2\,b^2\,f\,\left(b\tan\left(e+f\,x\right)^2+a\right)\,(a-b)} + \frac{a\,\ln\left(b\tan(e+f\,x)^2+a\right)\,(a-2\,b)}{2\,b^2\,f\,(a-b)^2}$$

input
```
int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) + a^2/(2*b^2*f*(a + b*tan(e + f*x)
^2)*(a - b)) + (a*log(a + b*tan(e + f*x)^2)*(a - 2*b))/(2*b^2*f*(a - b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.44

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\log\left(\tan\left(fx+e\right)^2+1\right)\tan\left(fx+e\right)^2b^3+\log\left(\tan\left(fx+e\right)^2+1\right)a\,b^2+\log\left(\tan\left(fx+e\right)^2b+a\right)\tan\left(fx\right.}{2b^2f\left(\tan\left(fx\right.\right.}$$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2*b**3 + log(tan(e + f*x)**2 + 1)*
a*b**2 + log(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*a**2*b - 2*log(tan(e +
 f*x)**2*b + a)*tan(e + f*x)**2*a*b**2 + log(tan(e + f*x)**2*b + a)*a**3 -
 2*log(tan(e + f*x)**2*b + a)*a**2*b - tan(e + f*x)**2*a**2*b + tan(e + f*
x)**2*a*b**2)/(2*b**2*f*(tan(e + f*x)**2*a**2*b - 2*tan(e + f*x)**2*a*b**2
 + tan(e + f*x)**2*b**3 + a**3 - 2*a**2*b + a*b**2))
```

3.225 $\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^2} dx = \frac{\log(a\cos^2(e+fx)+b\sin^2(e+fx))}{2(a-b)^2f} - \frac{a}{2(a-b)bf(a+b\tan^2(e+fx))}$$

output
```
1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^2/f-1/2*a/(a-b)/b/f/(a+b*tan(f
*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^2} dx = \frac{2\log(\cos(e+fx))+\log(a+b\tan^2(e+fx))+\frac{a(-a+b)}{b(a+b\tan^2(e+fx))}}{2(a-b)^2f}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(b*(a + b*
Tan[e + f*x]^2)))/(2*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4153, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{a}{(a-b)(b\tan^2(e+fx)+a)^2}-\frac{1}{(a-b)^2(\tan^2(e+fx)+1)}+\frac{b}{(a-b)^2(b\tan^2(e+fx)+a)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a}{b(a-b)(a+b\tan^2(e+fx))}-\frac{\log(\tan^2(e+fx)+1)}{(a-b)^2}+\frac{\log(a+b\tan^2(e+fx))}{(a-b)^2}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(Log[1 + Tan[e + f*x]^2]/(a - b)^2) + Log[a + b*Tan[e + f*x]^2]/(a - b)^
2 - a/((a - b)*b*(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}+\frac{\ln\left(a+b\tan(fx+e)^2\right)-\frac{a(a-b)}{b\left(a+b\tan(fx+e)^2\right)}}{2(a-b)^2}}{f}$
default	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}+\frac{\ln\left(a+b\tan(fx+e)^2\right)-\frac{a(a-b)}{b\left(a+b\tan(fx+e)^2\right)}}{2(a-b)^2}}{f}$
norman	$\frac{\tan(fx+e)^2}{2f(a-b)\left(a+b\tan(fx+e)^2\right)}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}+\frac{\ln\left(a+b\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}$
parallelrisch	$-\frac{\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^2b^2-b^2\ln\left(a+b\tan(fx+e)^2\right)\tan(fx+e)^2+\ln\left(1+\tan(fx+e)^2\right)ab-\ln\left(a+b\tan(fx+\right.}{2(a^2-2ab+b^2)\left(a+b\tan(fx+e)^2\right)bf}$
risch	$-\frac{ix}{a^2-2ab+b^2}-\frac{2ie}{f(a^2-2ab+b^2)}+\frac{2a\,\mathrm{e}^{2i(fx+e)}}{f(a-b)^2\left(a\,\mathrm{e}^{4i(fx+e)}-b\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a-b\right)}+\frac{\ln\left(\mathrm{e}^{4}\right.}{}$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(a-b)^2*ln(1+tan(f*x+e)^2)+1/2/(a-b)^2*(ln(a+b*tan(f*x+e)^2)-a*(
a-b)/b/(a+b*tan(f*x+e)^2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{a\tan(fx+e)^2+\left(b\tan(fx+e)^2+a\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)+a}{2\left((a^2b-2\,ab^2+b^3)f\tan(fx+e)^2+(a^3-2\,a^2b+ab^2)f\right)}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)/
(tan(f*x + e)^2 + 1)) + a)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^
3 - 2*a^2*b + a*b^2)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(51) = 102$.

Time = 9.12 (sec) , antiderivative size = 910, normalized size of antiderivative = 13.19

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e + f
*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**2, Eq(b, 0)), (-2*tan(e + f*
x)**2/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f) - 1
/(4*b**2*f*tan(e + f*x)**4 + 8*b**2*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b
)), (x*tan(e)**3/(a + b*tan(e)**2)**2, Eq(f, 0)), (-a**2/(2*a**3*b*f + 2*a
**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 +
2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(-sqrt(-a/b) + tan(e + f*x
))/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*
f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + a*b*log(sqrt(
-a/b) + tan(e + f*x))/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2
*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)*
*2) - a*b*log(tan(e + f*x)**2 + 1)/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x
)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*
tan(e + f*x)**2) + a*b/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**
2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)
**2) + b**2*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*b*f +
2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2*b**2*f - 4*a*b**3*f*tan(e + f*x)**2
 + 2*a*b**3*f + 2*b**4*f*tan(e + f*x)**2) + b**2*log(sqrt(-a/b) + tan(e +
f*x))*tan(e + f*x)**2/(2*a**3*b*f + 2*a**2*b**2*f*tan(e + f*x)**2 - 4*a**2
*b**2*f - 4*a*b**3*f*tan(e + f*x)**2 + 2*a*b**3*f + 2*b**4*f*tan(e + f*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\frac{a}{a^3-2\,a^2b+ab^2-(a^3-3\,a^2b+3\,ab^2-b^3)\sin(fx+e)^2}+\frac{\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2-2\,ab+b^2}}{2\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x +
e)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{b\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(a^2bf-2\,ab^2f+b^3f\right)} - \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(a^2f-2\,abf+b^2f\right)} - \frac{a^2-ab}{2\left(b\tan\left(fx+e\right)^2+a\right)(a-b)^2bf}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*b*log(abs(b*tan(f*x + e)^2 + a))/(a^2*b*f - 2*a*b^2*f + b^3*f) - 1/2*l
og(tan(f*x + e)^2 + 1)/(a^2*f - 2*a*b*f + b^2*f) - 1/2*(a^2 - a*b)/((b*tan
(f*x + e)^2 + a)*(a - b)^2*b*f)
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.91

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\frac{a^2\cos(e+f\,x)^2}{2}+b^2\sin\left(e+f\,x\right)^2\operatorname{atan}\left(\frac{a\sin(e+f\,x)^2-b\sin(e+f\,x)^2}{a\cos(e+f\,x)^2\,2\mathrm{i}+a\sin(e+f\,x)^2\,1\mathrm{i}+b\sin(e+f\,x)^2\,1\mathrm{i}}\right)\,1\mathrm{i}-\frac{a\,b\cos(e+f\,x)^2}{2}+a\,b\cos}{f\,\left(a^3\,b\cos\left(e+f\,x\right)^2-2\,a^2\,b^2\cos\left(e+f\,x\right)^2+a^2\,b^2\sin\left(e+f\,x\right)^2+a\,b^3\cos\left(e+f\,x\right)^2\right.}$$

input
```
int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
-((a^2*cos(e + f*x)^2)/2 + b^2*sin(e + f*x)^2*atan((a*sin(e + f*x)^2 - b*s
in(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)
^2*1i))*1i - (a*b*cos(e + f*x)^2)/2 + a*b*cos(e + f*x)^2*atan((a*sin(e + f
*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*s
in(e + f*x)^2*1i))*1i)/(f*(b^4*sin(e + f*x)^2 + a*b^3*cos(e + f*x)^2 + a^3
*b*cos(e + f*x)^2 - 2*a*b^3*sin(e + f*x)^2 - 2*a^2*b^2*cos(e + f*x)^2 + a^
2*b^2*sin(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.28

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{-\log\left(\tan\left(fx+e\right)^2+1\right)\tan\left(fx+e\right)^2b-\log\left(\tan\left(fx+e\right)^2+1\right)a+\log\left(\tan\left(fx+e\right)^2b+a\right)\tan\left(fx+\right.}{2f\left(\tan\left(fx+e\right)^2a^2b-2\tan\left(fx+e\right)^2a\,b^2+\tan\left(fx+e\right.\right.}$$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2*b - log(tan(e + f*x)**2 + 1)*
a + log(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*b + log(tan(e + f*x)**2*b +
 a)*a + tan(e + f*x)**2*a - tan(e + f*x)**2*b)/(2*f*(tan(e + f*x)**2*a**2*
b - 2*tan(e + f*x)**2*a*b**2 + tan(e + f*x)**2*b**3 + a**3 - 2*a**2*b + a*
b**2))
```

3.226 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^2f} + \frac{1}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^2/f+1/2/(a-b)/f/(a+b*tan(f*x+
e)^2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{2\log(\cos(e+fx))+\log\left(a+b\tan^2(e+fx)\right)+\frac{-a+b}{a+b\tan^2(e+fx)}}{2(a-b)^2f}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-1/2*(2*Log[Cos[e + f*x]] + Log[a + b*Tan[e + f*x]^2] + (-a + b)/(a + b*Ta
n[e + f*x]^2))/((a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4153, 353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(-\frac{b}{(a-b)^2(b\tan^2(e+fx)+a)}-\frac{b}{(a-b)(b\tan^2(e+fx)+a)^2}+\frac{1}{(a-b)^2(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{(a-b)(a+b\tan^2(e+fx))}+\frac{\log(\tan^2(e+fx)+1)}{(a-b)^2}-\frac{\log(a+b\tan^2(e+fx))}{(a-b)^2}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(Log[1 + Tan[e + f*x]^2]/(a - b)^2 - Log[a + b*Tan[e + f*x]^2]/(a - b)^2 +
 1/((a - b)*(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{-\frac{b\left(-\frac{a-b}{b\left(a+b\tan(fx+e)^2\right)}+\frac{\ln\left(a+b\tan(fx+e)^2\right)}{b}\right)}{2(a-b)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}}{f}$
default	$\frac{-\frac{b\left(-\frac{a-b}{b\left(a+b\tan(fx+e)^2\right)}+\frac{\ln\left(a+b\tan(fx+e)^2\right)}{b}\right)}{2(a-b)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}}{f}$
norman	$-\frac{b\tan(fx+e)^2}{2af(a-b)\left(a+b\tan(fx+e)^2\right)}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}-\frac{\ln\left(a+b\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}$
parallelrisch	$\frac{\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^2b^2-b^2\ln\left(a+b\tan(fx+e)^2\right)\tan(fx+e)^2+\ln\left(1+\tan(fx+e)^2\right)ab-\ln\left(a+b\tan(fx+e)\right.}{2(a^2-2ab+b^2)\left(a+b\tan(fx+e)^2\right)bf}$
risch	$\frac{ix}{a^2-2ab+b^2}+\frac{2ie}{f(a^2-2ab+b^2)}+\frac{2b\,e^{2i(fx+e)}}{f(-a+b)^2\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}-\frac{\ln\left(e\right.}{}$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(a-b)^2*b*(-(a-b)/b/(a+b*tan(f*x+e)^2)+1/b*ln(a+b*tan(f*x+e)^2))
+1/2/(a-b)^2*ln(1+tan(f*x+e)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$=-\frac{b\tan\left(fx+e\right)^2+\left(b\tan\left(fx+e\right)^2+a\right)\log\left(\frac{b\tan(fx+e)^2+a}{\tan(fx+e)^2+1}\right)+b}{2\left(\left(a^2b-2\,ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2\,a^2b+ab^2\right)f\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(b*tan(f*x + e)^2 + (b*tan(f*x + e)^2 + a)*log((b*tan(f*x + e)^2 + a)
/(tan(f*x + e)^2 + 1)) + b)/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x + e)^2 + (a
^3 - 2*a^2*b + a*b^2)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(49) = 98.

Time = 9.15 (sec) , antiderivative size = 796, normalized size of antiderivative = 12.25

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e +
f*x)**2 + 1)/(2*a**2*f), Eq(b, 0)), (-1/(4*b**2*f*tan(e + f*x)**4 + 8*b**2
*f*tan(e + f*x)**2 + 4*b**2*f), Eq(a, b)), (x*tan(e)/(a + b*tan(e)**2)**2,
 Eq(f, 0)), (-a*log(-sqrt(-a/b) + tan(e + f*x))/(2*a**3*f + 2*a**2*b*f*tan
(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b*
*3*f*tan(e + f*x)**2) - a*log(sqrt(-a/b) + tan(e + f*x))/(2*a**3*f + 2*a**
2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2
*f + 2*b**3*f*tan(e + f*x)**2) + a*log(tan(e + f*x)**2 + 1)/(2*a**3*f + 2*
a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b
**2*f + 2*b**3*f*tan(e + f*x)**2) + a/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)*
*2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e
 + f*x)**2) - b*log(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(2*a**3*f
+ 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2
*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) - b*log(sqrt(-a/b) + tan(e + f*x))*t
an(e + f*x)**2/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b
**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2) + b*log(tan
(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 -
 4*a**2*b*f - 4*a*b**2*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f
*x)**2) - b/(2*a**3*f + 2*a**2*b*f*tan(e + f*x)**2 - 4*a**2*b*f - 4*a*b**2
*f*tan(e + f*x)**2 + 2*a*b**2*f + 2*b**3*f*tan(e + f*x)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\frac{b}{a^3-2\,a^2b+ab^2-(a^3-3\,a^2b+3\,ab^2-b^3)\sin(fx+e)^2}+\frac{\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^2-2\,ab+b^2}}{2\,f}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b/(a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sin(f*x +
 e)^2) + log(-(a - b)*sin(f*x + e)^2 + a)/(a^2 - 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{b\log\left(\left|b\tan(fx+e)^2+a\right|\right)}{2\,(a^2bf-2\,ab^2f+b^3f)}+\frac{\log\left(\tan(fx+e)^2+1\right)}{2\,(a^2f-2\,abf+b^2f)} + \frac{1}{2\left(b\tan(fx+e)^2+a\right)(a-b)f}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*b*log(abs(b*tan(f*x + e)^2 + a))/(a^2*b*f - 2*a*b^2*f + b^3*f) + 1/2*
log(tan(f*x + e)^2 + 1)/(a^2*f - 2*a*b*f + b^2*f) + 1/2/((b*tan(f*x + e)^2
 + a)*(a - b)*f)
```

Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.00

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{b\left(1+\tan(e+f\,x)^2\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right)\,2\mathrm{i}\right)+a\left(-1+\mathrm{atan}\left(\frac{a\tan(e+f\,x)^2\,1\mathrm{i}-b\tan(e+f\,x)^2\,1\mathrm{i}}{2\,a+a\tan(e+f\,x)^2+b\tan(e+f\,x)^2}\right.\right.}{f\,\left(2\,a^3+2\,a^2\,b\tan\left(e+f\,x\right)^2-4\,a^2\,b-4\,a\,b^2\tan\left(e+f\,x\right)^2+2\,a\,b^2+2\,b^3\tan\left(e+f\,x\right)^2\right)}$$

input
```
int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
-(b*(tan(e + f*x)^2*atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a
+ a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*2i + 1) + a*(atan((a*tan(e + f*x)^
2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*2
i - 1))/(f*(2*a*b^2 - 4*a^2*b + 2*a^3 + 2*b^3*tan(e + f*x)^2 - 4*a*b^2*tan
(e + f*x)^2 + 2*a^2*b*tan(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.60

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\log\left(\tan\left(fx+e\right)^2+1\right)\tan\left(fx+e\right)^2ab+\log\left(\tan\left(fx+e\right)^2+1\right)a^2-\log\left(\tan\left(fx+e\right)^2b+a\right)\tan\left(fx+\right.}{2af\left(\tan\left(fx+e\right)^2a^2b-2\tan\left(fx+e\right)^2ab^2+\tan\left(fx+\right.\right.}$$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2*a*b + log(tan(e + f*x)**2 + 1)*a
**2 - log(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*a*b - log(tan(e + f*x)**2
*b + a)*a**2 - tan(e + f*x)**2*a*b + tan(e + f*x)**2*b**2)/(2*a*f*(tan(e +
 f*x)**2*a**2*b - 2*tan(e + f*x)**2*a*b**2 + tan(e + f*x)**2*b**3 + a**3 -
 2*a**2*b + a*b**2))
```

3.227 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\log(\cos(e+fx))}{(a-b)^2f}+\frac{\log(\tan(e+fx))}{a^2f}+\frac{(2a-b)b\log\left(a+b\tan^2(e+fx)\right)}{2a^2(a-b)^2f}-\frac{b}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
ln(cos(f*x+e))/(a-b)^2/f+ln(tan(f*x+e))/a^2/f+1/2*(2*a-b)*b*ln(a+b*tan(f*x
+e)^2)/a^2/(a-b)^2/f-1/2*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\frac{2\log(\cos(e+fx))}{(a-b)^2}+\frac{2\log(\tan(e+fx))+\frac{b\left((2a-b)\log\left(a+b\tan^2(e+fx)\right)+\frac{a(-a+b)}{a+b\tan^2(e+fx)}\right)}{(a-b)^2}}{a^2}}{2f}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((2*Log[Cos[e + f*x]])/(a - b)^2 + (2*Log[Tan[e + f*x]] + (b*((2*a - b)*Lo
g[a + b*Tan[e + f*x]^2] + (a*(-a + b))/(a + b*Tan[e + f*x]^2)))/(a - b)^2)
/a^2)/(2*f)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4153, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{(2a-b)b^2}{a^2(a-b)^2(b\tan^2(e+fx)+a)}+\frac{b^2}{a(a-b)(b\tan^2(e+fx)+a)^2}+\frac{\cot(e+fx)}{a^2}-\frac{1}{(a-b)^2(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b(2a-b)\log(a+b\tan^2(e+fx))}{a^2(a-b)^2}+\frac{\log(\tan^2(e+fx))}{a^2}-\frac{b}{a(a-b)(a+b\tan^2(e+fx))}-\frac{\log(\tan^2(e+fx)+1)}{(a-b)^2}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(Log[Tan[e + f*x]^2]/a^2 - Log[1 + Tan[e + f*x]^2]/(a - b)^2 + ((2*a - b)*
b*Log[a + b*Tan[e + f*x]^2])/(a^2*(a - b)^2) - b/(a*(a - b)*(a + b*Tan[e +
 f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{b^2\left(-\frac{a(a-b)}{b\left(a+b\tan(fx+e)^2\right)}+\frac{(2a-b)\ln\left(a+b\tan(fx+e)^2\right)}{b}\right)}{2a^2(a-b)^2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}+\frac{\ln(\tan(fx+e))}{a^2}}{f}$
default	$\frac{\frac{b^2\left(-\frac{a(a-b)}{b\left(a+b\tan(fx+e)^2\right)}+\frac{(2a-b)\ln\left(a+b\tan(fx+e)^2\right)}{b}\right)}{2a^2(a-b)^2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}+\frac{\ln(\tan(fx+e))}{a^2}}{f}$
norman	$\frac{b^2\tan(fx+e)^2}{2a^2f(a-b)\left(a+b\tan(fx+e)^2\right)}+\frac{\ln(\tan(fx+e))}{a^2f}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}+\frac{b(2a-b)\ln\left(a+b\tan(fx+e)^2\right)}{2a^2f(a^2-2ab+b^2)}$
parallelrisch	$\frac{2\left(a+b\tan(fx+e)^2\right)\left(a-\frac{b}{2}\right)b\ln\left(a+b\tan(fx+e)^2\right)+\left(-\tan(fx+e)^2a^2b-a^3\right)\ln\left(\sec(fx+e)^2\right)+2\left((a-b)\left(a+b\tan(fx\right.\right.}{2(a-b)^2\left(a+b\tan(fx+e)^2\right)a^2f}$
risch	$\frac{ix}{a^2-2ab+b^2}-\frac{2ix}{a^2}-\frac{2ie}{a^2f}-\frac{4ibx}{a(a^2-2ab+b^2)}-\frac{4ibe}{af(a^2-2ab+b^2)}+\frac{2ib^2x}{a^2(a^2-2ab+b^2)}+\frac{2ib^2e}{a^2f(a^2-2ab+b^2)}-$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b^2/a^2/(a-b)^2*(-a*(a-b)/b/(a+b*tan(f*x+e)^2)+(2*a-b)/b*ln(a+b*t
an(f*x+e)^2))-1/2/(a-b)^2*ln(1+tan(f*x+e)^2)+1/a^2*ln(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.91

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$=\frac{ab^2\tan(fx+e)^2+ab^2+\left(a^3-2a^2b+ab^2+(a^2b-2ab^2+b^3)\tan(fx+e)^2\right)\log\left(\frac{\tan(fx+e)^2}{\tan(fx+e)^2+1}\right)+(2a}{2\left((a^4b-2a^3b^2+a^2b^3)f\tan(fx+e)^2+(a^5-2a^4b+\right.}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a*b^2*tan(f*x + e)^2 + a*b^2 + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*
b^2 + b^3)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + (2*a
^2*b - a*b^2 + (2*a*b^2 - b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/
(tan(f*x + e)^2 + 1)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (
a^5 - 2*a^4*b + a^3*b^2)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2377 vs. $2(80) = 160$.

Time = 69.11 (sec) , antiderivative size = 2377, normalized size of antiderivative = 23.08

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x*cot(e)/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log
(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f)/a**2, Eq(b, 0)), ((-log
(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f + 1/(2*f*tan(e + f*x)**2
) - 1/(4*f*tan(e + f*x)**4))/b**2, Eq(a, 0)), (-2*log(tan(e + f*x)**2 + 1)
*tan(e + f*x)**4/(4*a**2*f*tan(e + f*x)**4 + 8*a**2*f*tan(e + f*x)**2 + 4*
a**2*f) - 4*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*a**2*f*tan(e + f*x
)**4 + 8*a**2*f*tan(e + f*x)**2 + 4*a**2*f) - 2*log(tan(e + f*x)**2 + 1)/(
4*a**2*f*tan(e + f*x)**4 + 8*a**2*f*tan(e + f*x)**2 + 4*a**2*f) + 4*log(ta
n(e + f*x))*tan(e + f*x)**4/(4*a**2*f*tan(e + f*x)**4 + 8*a**2*f*tan(e + f
*x)**2 + 4*a**2*f) + 8*log(tan(e + f*x))*tan(e + f*x)**2/(4*a**2*f*tan(e +
 f*x)**4 + 8*a**2*f*tan(e + f*x)**2 + 4*a**2*f) + 4*log(tan(e + f*x))/(4*a
**2*f*tan(e + f*x)**4 + 8*a**2*f*tan(e + f*x)**2 + 4*a**2*f) + 2*tan(e + f
*x)**2/(4*a**2*f*tan(e + f*x)**4 + 8*a**2*f*tan(e + f*x)**2 + 4*a**2*f) +
3/(4*a**2*f*tan(e + f*x)**4 + 8*a**2*f*tan(e + f*x)**2 + 4*a**2*f), Eq(a,
b)), (zoo*(-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f), Eq(b, -
a/tan(e + f*x)**2)), (x*cot(e)/(a + b*tan(e)**2)**2, Eq(f, 0)), (-a**3*log
(tan(e + f*x)**2 + 1)/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 - 4*a**4*b*f
- 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b**3*f*tan(e + f*
x)**2) + 2*a**3*log(tan(e + f*x))/(2*a**5*f + 2*a**4*b*f*tan(e + f*x)**2 -
 4*a**4*b*f - 4*a**3*b**2*f*tan(e + f*x)**2 + 2*a**3*b**2*f + 2*a**2*b*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\frac{b^2}{a^4-2a^3b+a^2b^2-(a^4-3a^3b+3a^2b^2-ab^3)\sin(fx+e)^2}+\frac{(2ab-b^2)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^4-2a^3b+a^2b^2}+\frac{\log\left(\sin(fx+e)^2\right)}{a^2}}{2f}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2/(a^4 - 2*a^3*b + a^2*b^2 - (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*si
n(f*x + e)^2) + (2*a*b - b^2)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^4 - 2*a^
3*b + a^2*b^2) + log(sin(f*x + e)^2)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.77

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{(2ab^2-b^3)\log\left(\left|b\tan(fx+e)^2+a\right|\right)}{2(a^4bf-2a^3b^2f+a^2b^3f)} - \frac{\log\left(\tan(fx+e)^2+1\right)}{2(a^2f-2abf+b^2f)} - \frac{2ab^2\tan(fx+e)^2-b^3\tan(fx+e)^2+3a^2b-2ab^2}{2(a^4f-2a^3bf+a^2b^2f)\left(b\tan(fx+e)^2+a\right)} + \frac{\log\left(\tan(fx+e)^2\right)}{2a^2f}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*a*b^2 - b^3)*log(abs(b*tan(f*x + e)^2 + a))/(a^4*b*f - 2*a^3*b^2*f
+ a^2*b^3*f) - 1/2*log(tan(f*x + e)^2 + 1)/(a^2*f - 2*a*b*f + b^2*f) - 1/2
*(2*a*b^2*tan(f*x + e)^2 - b^3*tan(f*x + e)^2 + 3*a^2*b - 2*a*b^2)/((a^4*f
 - 2*a^3*b*f + a^2*b^2*f)*(b*tan(f*x + e)^2 + a)) + 1/2*log(tan(f*x + e)^2
)/(a^2*f)
```

Mupad [B] (verification not implemented)

Time = 8.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\ln(\tan(e+fx))}{a^2 f} - \frac{\ln\left(\tan(e+fx)^2+1\right)}{2f(a-b)^2} - \frac{b}{2af\left(b\tan(e+fx)^2+a\right)(a-b)} + \frac{b\ln\left(b\tan(e+fx)^2+a\right)(2a-b)}{2a^2f(a-b)^2}$$

input
```
int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
log(tan(e + f*x))/(a^2*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) - b/(2
*a*f*(a + b*tan(e + f*x)^2)*(a - b)) + (b*log(a + b*tan(e + f*x)^2)*(2*a -
 b))/(2*a^2*f*(a - b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 775, normalized size of antiderivative = 7.52

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**3 + 2*log(tan((e + f
*x)/2)**2 + 1)*sin(e + f*x)**2*a**2*b + 2*log(tan((e + f*x)/2)**2 + 1)*a**
3 + 2*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2
+ sqrt(a))*sin(e + f*x)**2*a**2*b - 3*log( - 2*sqrt(a - b)*tan((e + f*x)/2
) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**2*a*b**2 + log( -
 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*s
in(e + f*x)**2*b**3 - 2*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*ta
n((e + f*x)/2)**2 + sqrt(a))*a**2*b + log( - 2*sqrt(a - b)*tan((e + f*x)/2
) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*a*b**2 + 2*log(2*sqrt(a - b)*ta
n((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**2*a*
*2*b - 3*log(2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2
+ sqrt(a))*sin(e + f*x)**2*a*b**2 + log(2*sqrt(a - b)*tan((e + f*x)/2) + s
qrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**2*b**3 - 2*log(2*sqrt(
a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*a**2*b +
log(2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a)
)*a*b**2 + 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*a**3 - 6*log(tan((e + f
*x)/2))*sin(e + f*x)**2*a**2*b + 6*log(tan((e + f*x)/2))*sin(e + f*x)**2*a
*b**2 - 2*log(tan((e + f*x)/2))*sin(e + f*x)**2*b**3 - 2*log(tan((e + f*x)
/2))*a**3 + 4*log(tan((e + f*x)/2))*a**2*b - 2*log(tan((e + f*x)/2))*a*b**
2 - sin(e + f*x)**2*a*b**2 + sin(e + f*x)**2*b**3)/(2*a**2*f*(sin(e + f...
```

3.228 $\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\cot^2(e+fx)}{2a^2f} - \frac{\log(\cos(e+fx))}{(a-b)^2f} - \frac{(a+2b)\log(\tan(e+fx))}{a^3f} - \frac{(3a-2b)b^2\log\left(a+b\tan^2(e+fx)\right)}{2a^3(a-b)^2f} + \frac{b^2}{2a^2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-1/2*cot(f*x+e)^2/a^2/f-ln(cos(f*x+e))/(a-b)^2/f-(a+2*b)*ln(tan(f*x+e))/a^
3/f-1/2*(3*a-2*b)*b^2*ln(a+b*tan(f*x+e)^2)/a^3/(a-b)^2/f+1/2*b^2/a^2/(a-b)
/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{\cot^2(e+fx)}{a^2}+\frac{b^3}{a^3(a-b)(b+a\cot^2(e+fx))}+\frac{(3a-2b)b^2\log(b+a\cot^2(e+fx))}{a^3(a-b)^2}+\frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-1/2*(Cot[e + f*x]^2/a^2 + b^3/(a^3*(a - b)*(b + a*Cot[e + f*x]^2)) + ((3*
a - 2*b)*b^2*Log[b + a*Cot[e + f*x]^2])/(a^3*(a - b)^2) + (2*Log[Sin[e + f
*x]])/(a - b)^2)/f
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\tan(e+fx)^2)^2}dx$$

$$\Big\downarrow \ 4153$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 354$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(-\frac{(3a-2b)b^3}{a^3(a-b)^2(b\tan^2(e+fx)+a)}-\frac{b^3}{a^2(a-b)(b\tan^2(e+fx)+a)^2}+\frac{\cot^2(e+fx)}{a^2}+\frac{(-a-2b)\cot(e+fx)}{a^3}+\frac{1}{(a-b)^2(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b^2(3a-2b)\log(a+b\tan^2(e+fx))}{a^3(a-b)^2}-\frac{(a+2b)\log(\tan^2(e+fx))}{a^3}+\frac{b^2}{a^2(a-b)(a+b\tan^2(e+fx))}-\frac{\cot(e+fx)}{a^2}+\frac{\log(\tan^2(e+fx)+1)}{(a-b)^2}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-(Cot[e + f*x]/a^2) - ((a + 2*b)*Log[Tan[e + f*x]^2])/a^3 + Log[1 + Tan[e
 + f*x]^2]/(a - b)^2 - ((3*a - 2*b)*b^2*Log[a + b*Tan[e + f*x]^2])/(a^3*(a
 - b)^2) + b^2/(a^2*(a - b)*(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{b^3\left(-\frac{a(a-b)}{b\left(a+b\tan(fx+e)^2\right)}+\frac{(3a-2b)\ln\left(a+b\tan(fx+e)^2\right)}{b}\right)}{2a^3(a-b)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}-\frac{1}{2a^2\tan(fx+e)^2}+\frac{(-2b-a)\ln(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b^3\left(-\frac{a(a-b)}{b\left(a+b\tan(fx+e)^2\right)}+\frac{(3a-2b)\ln\left(a+b\tan(fx+e)^2\right)}{b}\right)}{2a^3(a-b)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}-\frac{1}{2a^2\tan(fx+e)^2}+\frac{(-2b-a)\ln(\tan(fx+e))}{a^3}}{f}$
parallelrisch	$\frac{-3b^2\left(a+b\tan(fx+e)^2\right)\left(a-\frac{2b}{3}\right)\ln\left(a+b\tan(fx+e)^2\right)+a^3\left(a+b\tan(fx+e)^2\right)\ln\left(\sec(fx+e)^2\right)-\left(2(a+2b)(a-b)\left(a+\right.\right.}{2(a-b)^2a^3f\left(a+b\tan(fx+e)^2\right)}$
norman	$\frac{-\frac{1}{2af}+\frac{\left(-a\,b^2+2b^3\right)\tan(fx+e)^2}{2a^2fb(a-b)}}{\tan(fx+e)^2\left(a+b\tan(fx+e)^2\right)}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}-\frac{(a+2b)\ln(\tan(fx+e))}{a^3f}-\frac{b^2(3a-2b)\ln\left(a+b\tan(fx+e\right.}{2a^3f(a^2-2ab+b^2)}$
risch	$-\frac{ix}{a^2-2ab+b^2}+\frac{2ix}{a^2}+\frac{2ie}{a^2f}+\frac{4ibx}{a^3}+\frac{4ibe}{a^3f}+\frac{6ib^2x}{a^2(a^2-2ab+b^2)}+\frac{6ib^2e}{a^2f(a^2-2ab+b^2)}-\frac{4ib^3x}{a^3(a^2-2ab+b^2)}-$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*b^3/a^3/(a-b)^2*(-a*(a-b)/b/(a+b*tan(f*x+e)^2)+(3*a-2*b)/b*ln(a+
b*tan(f*x+e)^2))+1/2/(a-b)^2*ln(1+tan(f*x+e)^2)-1/2/a^2/tan(f*x+e)^2+(-2*b
-a)/a^3*ln(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(126) = 252.

Time = 0.13 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.21

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{(a^3b-2a^2b^2+2ab^3)\tan(fx+e)^4+a^4-2a^3b+a^2b^2+(a^4-a^3b-a^2b^2+2ab^3)\tan(fx+e)^2+(\cdots}{2(\cdots}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((a^3*b - 2*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^4 + a^4 - 2*a^3*b + a^2*b
^2 + (a^4 - a^3*b - a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2 + ((a^3*b - 3*a*b^3
+ 2*b^4)*tan(f*x + e)^4 + (a^4 - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*log(
tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + ((3*a*b^3 - 2*b^4)*tan(f*x + e)^4 +
 (3*a^2*b^2 - 2*a*b^3)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(tan(f*x
 + e)^2 + 1)))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4 + (a^6 - 2*
a^5*b + a^4*b^2)*f*tan(f*x + e)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3568 vs. 2(107) = 214.

Time = 97.68 (sec) , antiderivative size = 3568, normalized size of antiderivative = 27.03

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise(((log(tan(e + f*x)**2 + 1)/(2*f) - log(tan(e + f*x))/f - 1/(2*f*
tan(e + f*x)**2))/a**2, Eq(b, 0)), ((log(tan(e + f*x)**2 + 1)/(2*f) - log(
tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2) + 1/(4*f*tan(e + f*x)**4) - 1/(6
*f*tan(e + f*x)**6))/b**2, Eq(a, 0)), (6*log(tan(e + f*x)**2 + 1)*tan(e +
f*x)**6/(4*a**2*f*tan(e + f*x)**6 + 8*a**2*f*tan(e + f*x)**4 + 4*a**2*f*ta
n(e + f*x)**2) + 12*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4/(4*a**2*f*tan
(e + f*x)**6 + 8*a**2*f*tan(e + f*x)**4 + 4*a**2*f*tan(e + f*x)**2) + 6*lo
g(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*a**2*f*tan(e + f*x)**6 + 8*a**2*
f*tan(e + f*x)**4 + 4*a**2*f*tan(e + f*x)**2) - 12*log(tan(e + f*x))*tan(e
 + f*x)**6/(4*a**2*f*tan(e + f*x)**6 + 8*a**2*f*tan(e + f*x)**4 + 4*a**2*f
*tan(e + f*x)**2) - 24*log(tan(e + f*x))*tan(e + f*x)**4/(4*a**2*f*tan(e +
 f*x)**6 + 8*a**2*f*tan(e + f*x)**4 + 4*a**2*f*tan(e + f*x)**2) - 12*log(t
an(e + f*x))*tan(e + f*x)**2/(4*a**2*f*tan(e + f*x)**6 + 8*a**2*f*tan(e +
f*x)**4 + 4*a**2*f*tan(e + f*x)**2) - 6*tan(e + f*x)**4/(4*a**2*f*tan(e +
f*x)**6 + 8*a**2*f*tan(e + f*x)**4 + 4*a**2*f*tan(e + f*x)**2) - 9*tan(e +
 f*x)**2/(4*a**2*f*tan(e + f*x)**6 + 8*a**2*f*tan(e + f*x)**4 + 4*a**2*f*t
an(e + f*x)**2) - 2/(4*a**2*f*tan(e + f*x)**6 + 8*a**2*f*tan(e + f*x)**4 +
 4*a**2*f*tan(e + f*x)**2), Eq(a, b)), (zoo*(log(tan(e + f*x)**2 + 1)/(2*f
) - log(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2)), Eq(b, -a/tan(e + f*x)*
*2)), (zoo*x/a**2, Eq(e, -f*x)), (x*cot(e)**3/(a + b*tan(e)**2)**2, Eq(...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.42

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx =$$
$$-\frac{\frac{(3\,ab^2-2\,b^3)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^5-2\,a^4b+a^3b^2} - \frac{a^3-2\,a^2b+ab^2-(a^3-3\,a^2b+3\,ab^2-2\,b^3)\sin(fx+e)^2}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sin(fx+e)^4-(a^5-2\,a^4b+a^3b^2)\sin(fx+e)^2} + \frac{(a+2\,b)\log\left(\sin(fx-\right.}{a^3}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*a*b^2 - 2*b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^5 - 2*a^4*b +
a^3*b^2) - (a^3 - 2*a^2*b + a*b^2 - (a^3 - 3*a^2*b + 3*a*b^2 - 2*b^3)*sin(
f*x + e)^2)/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(f*x + e)^4 - (a^5 -
 2*a^4*b + a^3*b^2)*sin(f*x + e)^2) + (a + 2*b)*log(sin(f*x + e)^2)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.82

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= -\frac{(3\,ab^3-2\,b^4)\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\,(a^5bf-2\,a^4b^2f+a^3b^3f)} + \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,(a^2f-2\,abf+b^2f)}$$
$$+ \frac{a^2b\tan\left(fx+e\right)^4+a^3\tan\left(fx+e\right)^2-2\,a^2b\tan\left(fx+e\right)^2+6\,ab^2\tan\left(fx+e\right)^2-4\,b^3\tan\left(fx+e\right)^2-}{4\,(a^4f-2\,a^3bf+a^2b^2f)\left(b\tan\left(fx+e\right)^4+a\tan\left(fx+e\right)^2\right)}$$
$$- \frac{(a+2\,b)\log\left(\tan\left(fx+e\right)^2\right)}{2\,a^3f}$$

input

```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output

```
-1/2*(3*a*b^3 - 2*b^4)*log(abs(b*tan(f*x + e)^2 + a))/(a^5*b*f - 2*a^4*b^2
*f + a^3*b^3*f) + 1/2*log(tan(f*x + e)^2 + 1)/(a^2*f - 2*a*b*f + b^2*f) +
1/4*(a^2*b*tan(f*x + e)^4 + a^3*tan(f*x + e)^2 - 2*a^2*b*tan(f*x + e)^2 +
6*a*b^2*tan(f*x + e)^2 - 4*b^3*tan(f*x + e)^2 - 2*a^3 + 4*a^2*b - 2*a*b^2)
/((a^4*f - 2*a^3*b*f + a^2*b^2*f)*(b*tan(f*x + e)^4 + a*tan(f*x + e)^2)) -
 1/2*(a + 2*b)*log(tan(f*x + e)^2)/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(\frac{b}{a^3}+\frac{1}{2\,a^2}-\frac{1}{2\,(a-b)^2}\right)}{f}$$
$$- \frac{\frac{1}{2\,a}+\frac{\tan(e+f\,x)^2\,(a\,b-2\,b^2)}{2\,a^2\,(a-b)}}{f\,\left(b\tan\left(e+f\,x\right)^4+a\tan\left(e+f\,x\right)^2\right)}$$
$$+ \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^2} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+2\,b)}{a^3\,f}$$

input

```
int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(log(a + b*tan(e + f*x)^2)*(b/a^3 + 1/(2*a^2) - 1/(2*(a - b)^2)))/f - (1/(
2*a) + (tan(e + f*x)^2*(a*b - 2*b^2))/(2*a^2*(a - b)))/(f*(a*tan(e + f*x)^
2 + b*tan(e + f*x)^4)) + log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) - (log(ta
n(e + f*x))*(a + 2*b))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 976, normalized size of antiderivative = 7.39

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(2*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**4 - 2*log(tan((e + f*x)
/2)**2 + 1)*sin(e + f*x)**4*a**3*b - 2*log(tan((e + f*x)/2)**2 + 1)*sin(e
+ f*x)**2*a**4 - 3*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e
+ f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*a**2*b**2 + 5*log( - 2*sqrt(a - b)
*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4
*a*b**3 - 2*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/
2)**2 + sqrt(a))*sin(e + f*x)**4*b**4 + 3*log( - 2*sqrt(a - b)*tan((e + f*
x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**2*a**2*b**2 -
 2*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + s
qrt(a))*sin(e + f*x)**2*a*b**3 - 3*log(2*sqrt(a - b)*tan((e + f*x)/2) + sq
rt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*a**2*b**2 + 5*log(2*s
qrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e
 + f*x)**4*a*b**3 - 2*log(2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e
+ f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*b**4 + 3*log(2*sqrt(a - b)*tan((e
+ f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**2*a**2*b*
*2 - 2*log(2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 +
sqrt(a))*sin(e + f*x)**2*a*b**3 - 2*log(tan((e + f*x)/2))*sin(e + f*x)**4*
a**4 + 2*log(tan((e + f*x)/2))*sin(e + f*x)**4*a**3*b + 6*log(tan((e + f*x
)/2))*sin(e + f*x)**4*a**2*b**2 - 10*log(tan((e + f*x)/2))*sin(e + f*x)**4
*a*b**3 + 4*log(tan((e + f*x)/2))*sin(e + f*x)**4*b**4 + 2*log(tan((e +...
```

3.229 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{(a+2b)\cot^2(e+fx)}{2a^3f} - \frac{\cot^4(e+fx)}{4a^2f} + \frac{\log(\cos(e+fx))}{(a-b)^2f} + \frac{(a^2+2ab+3b^2)\log(\tan(e+fx))}{a^4f} + \frac{(4a-3b)b^3\log\left(a+b\tan^2(e+fx)\right)}{2a^4(a-b)^2f} - \frac{b^3}{2a^3(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a+2*b)*cot(f*x+e)^2/a^3/f-1/4*cot(f*x+e)^4/a^2/f+ln(cos(f*x+e))/(a-b)
^2/f+(a^2+2*a*b+3*b^2)*ln(tan(f*x+e))/a^4/f+1/2*(4*a-3*b)*b^3*ln(a+b*tan(f
*x+e)^2)/a^4/(a-b)^2/f-1/2*b^3/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{-\frac{(a+2b)\cot^2(e+fx)}{a^3}+\frac{\cot^4(e+fx)}{2a^2}-\frac{b^4}{a^4(a-b)(b+a\cot^2(e+fx))}-\frac{(4a-3b)b^3\log(b+a\cot^2(e+fx))}{a^4(a-b)^2}-\frac{2\log(\sin(e+fx))}{(a-b)^2}}{2f}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
-1/2*(-(((a + 2*b)*Cot[e + f*x]^2)/a^3) + Cot[e + f*x]^4/(2*a^2) - b^4/(a^
4*(a - b)*(b + a*Cot[e + f*x]^2)) - ((4*a - 3*b)*b^3*Log[b + a*Cot[e + f*x
]^2])/(a^4*(a - b)^2) - (2*Log[Sin[e + f*x]])/(a - b)^2)/f
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\tan(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^5(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan^2(e+fx)}{2f}$$

$\Big\downarrow$ 99

$$\frac{\int \left(\frac{(4a-3b)b^4}{a^4(a-b)^2(b\tan^2(e+fx)+a)}+\frac{b^4}{a^3(a-b)(b\tan^2(e+fx)+a)^2}+\frac{\cot^3(e+fx)}{a^2}+\frac{(-a-2b)\cot^2(e+fx)}{a^3}+\frac{(a^2+2ba+3b^2)\cot(e+fx)}{a^4}-\frac{(a-}{}\right.}{2f}$$

$\Big\downarrow$ 2009

$$\frac{\frac{b^3(4a-3b)\log(a+b\tan^2(e+fx))}{a^4(a-b)^2}-\frac{b^3}{a^3(a-b)(a+b\tan^2(e+fx))}+\frac{(a+2b)\cot(e+fx)}{a^3}-\frac{\cot^2(e+fx)}{2a^2}+\frac{(a^2+2ab+3b^2)\log(\tan^2(e+fx))}{a^4}-}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(((a + 2*b)*Cot[e + f*x])/a^3 - Cot[e + f*x]^2/(2*a^2) + ((a^2 + 2*a*b + 3
*b^2)*Log[Tan[e + f*x]^2])/a^4 - Log[1 + Tan[e + f*x]^2]/(a - b)^2 + ((4*a
 - 3*b)*b^3*Log[a + b*Tan[e + f*x]^2])/(a^4*(a - b)^2) - b^3/(a^3*(a - b)*
(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}-\frac{1}{4a^2\tan(fx+e)^4}-\frac{-2b-a}{2a^3\tan(fx+e)^2}+\frac{\left(a^2+2ab+3b^2\right)\ln(\tan(fx+e))}{a^4}+\frac{b^4\left(\frac{(4a-3b)\ln\left(a+b\tan(fx+e)^2\right)}{b}-\right.}{2a^4(a-b)^2}}{f}$
default	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^2}-\frac{1}{4a^2\tan(fx+e)^4}-\frac{-2b-a}{2a^3\tan(fx+e)^2}+\frac{\left(a^2+2ab+3b^2\right)\ln(\tan(fx+e))}{a^4}+\frac{b^4\left(\frac{(4a-3b)\ln\left(a+b\tan(fx+e)^2\right)}{b}-\right.}{2a^4(a-b)^2}}{f}$
norman	$\frac{-\frac{1}{4af}+\frac{(2a+3b)\tan(fx+e)^2}{4a^2f}+\frac{\left(-a^2b-a\,b^2+3b^3\right)b\tan(fx+e)^6}{2a^4f(a-b)}}{\tan(fx+e)^4\left(a+b\tan(fx+e)^2\right)}+\frac{(a^2+2ab+3b^2)\ln(\tan(fx+e))}{a^4f}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^2-2ab+b^2)}$
parallelrisch	$\frac{8b^3\left(a-\frac{3b}{4}\right)\left(a+b\tan(fx+e)^2\right)\ln\left(a+b\tan(fx+e)^2\right)+\left(-2\tan(fx+e)^2a^4b-2a^5\right)\ln\left(\sec(fx+e)^2\right)-\left(-4(a-b)(a^2+2\right.}{4(a-b)^2}$
risch	$\frac{ix}{a^2-2ab+b^2}+\frac{6ib^4e}{a^4f(a^2-2ab+b^2)}-\frac{2ix}{a^2}-\frac{4ibx}{a^3}-\frac{6ib^2e}{a^4f}-\frac{4ibe}{a^3f}-\frac{2ie}{a^2f}-\frac{8ib^3x}{a^3(a^2-2ab+b^2)}-\frac{6ib^2x}{a^4}-$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(a-b)^2*ln(1+tan(f*x+e)^2)-1/4/a^2/tan(f*x+e)^4-1/2*(-2*b-a)/a^3
/tan(f*x+e)^2+(a^2+2*a*b+3*b^2)/a^4*ln(tan(f*x+e))+1/2*b^4/a^4/(a-b)^2*((4
*a-3*b)/b*ln(a+b*tan(f*x+e)^2)-a*(a-b)/b/(a+b*tan(f*x+e)^2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(153) = 306.

Time = 0.13 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.16

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{(3\,a^4b-2\,a^3b^2-5\,a^2b^3+6\,ab^4)\tan\left(fx+e\right)^6-a^5+2\,a^4b-a^3b^2+(3\,a^5-5\,a^3b^2-2\,a^2b^3+6\,ab^4)\tan\left(\right.}{}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/4*((3*a^4*b - 2*a^3*b^2 - 5*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^6 - a^5 + 2*
a^4*b - a^3*b^2 + (3*a^5 - 5*a^3*b^2 - 2*a^2*b^3 + 6*a*b^4)*tan(f*x + e)^4
 + (2*a^5 - a^4*b - 4*a^3*b^2 + 3*a^2*b^3)*tan(f*x + e)^2 + 2*((a^4*b - 4*
a*b^4 + 3*b^5)*tan(f*x + e)^6 + (a^5 - 4*a^2*b^3 + 3*a*b^4)*tan(f*x + e)^4
)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((4*a*b^4 - 3*b^5)*tan(f*x
+ e)^6 + (4*a^2*b^3 - 3*a*b^4)*tan(f*x + e)^4)*log((b*tan(f*x + e)^2 + a)/
(tan(f*x + e)^2 + 1)))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (
a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.47

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{\frac{2\,(4\,ab^3-3\,b^4)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^6-2\,a^5b+a^4b^2} + \frac{2\,(2\,a^4-4\,a^3b+4\,ab^3-3\,b^4)\sin(fx+e)^4+a^4-2\,a^3b+a^2b^2-(5\,a^4-7\,a^3b-a^2b^2+3\,ab^3)\sin(fx+e)}{(a^6-3\,a^5b+3\,a^4b^2-a^3b^3)\sin(fx+e)^6-(a^6-2\,a^5b+a^4b^2)\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*(4*a*b^3 - 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 2*a^5*b +
 a^4*b^2) + (2*(2*a^4 - 4*a^3*b + 4*a*b^3 - 3*b^4)*sin(f*x + e)^4 + a^4 -
2*a^3*b + a^2*b^2 - (5*a^4 - 7*a^3*b - a^2*b^2 + 3*a*b^3)*sin(f*x + e)^2)/
((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sin(f*x + e)^6 - (a^6 - 2*a^5*b + a
^4*b^2)*sin(f*x + e)^4) + 2*(a^2 + 2*a*b + 3*b^2)*log(sin(f*x + e)^2)/a^4)
/f
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.74

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{(4\,ab^4-3\,b^5)\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\,(a^6bf-2\,a^5b^2f+a^4b^3f)}$$
$$-\frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,(a^2f-2\,abf+b^2f)} - \frac{4\,ab^4\tan\left(fx+e\right)^2-3\,b^5\tan\left(fx+e\right)^2+5\,a^2b^3-4\,ab^4}{2\,(a^6f-2\,a^5bf+a^4b^2f)\left(b\tan\left(fx+e\right)^2+a\right)}$$
$$+\frac{(a^2+2\,ab+3\,b^2)\log\left(\tan\left(fx+e\right)^2\right)}{2\,a^4f}$$
$$-\frac{3\,a^2\tan\left(fx+e\right)^4+6\,ab\tan\left(fx+e\right)^4+9\,b^2\tan\left(fx+e\right)^4-2\,a^2\tan\left(fx+e\right)^2-4\,ab\tan\left(fx+e\right)^2+}{4\,a^4f\tan\left(fx+e\right)^4}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(4*a*b^4 - 3*b^5)*log(abs(b*tan(f*x + e)^2 + a))/(a^6*b*f - 2*a^5*b^2*
f + a^4*b^3*f) - 1/2*log(tan(f*x + e)^2 + 1)/(a^2*f - 2*a*b*f + b^2*f) - 1
/2*(4*a*b^4*tan(f*x + e)^2 - 3*b^5*tan(f*x + e)^2 + 5*a^2*b^3 - 4*a*b^4)/(
(a^6*f - 2*a^5*b*f + a^4*b^2*f)*(b*tan(f*x + e)^2 + a)) + 1/2*(a^2 + 2*a*b
 + 3*b^2)*log(tan(f*x + e)^2)/(a^4*f) - 1/4*(3*a^2*tan(f*x + e)^4 + 6*a*b*
tan(f*x + e)^4 + 9*b^2*tan(f*x + e)^4 - 2*a^2*tan(f*x + e)^2 - 4*a*b*tan(f
*x + e)^2 + a^2)/(a^4*f*tan(f*x + e)^4)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\frac{\tan(e+f\,x)^2\,(2\,a+3\,b)}{4\,a^2}-\frac{1}{4\,a}+\frac{\tan(e+f\,x)^4\,(a^2\,b+a\,b^2-3\,b^3)}{2\,a^3\,(a-b)}}{f\,\left(b\tan\left(e+f\,x\right)^6+a\tan\left(e+f\,x\right)^4\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^2} + \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+2\,a\,b+3\,b^2\right)}{a^4\,f} + \frac{\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(4\,a\,b^3-3\,b^4\right)}{f\,\left(2\,a^6-4\,a^5\,b+2\,a^4\,b^2\right)}$$

input
```
int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)^2*(2*a + 3*b))/(4*a^2) - 1/(4*a) + (tan(e + f*x)^4*(a*b^2 +
 a^2*b - 3*b^3))/(2*a^3*(a - b)))/(f*(a*tan(e + f*x)^4 + b*tan(e + f*x)^6)
) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^2) + (log(tan(e + f*x))*(2*a*b +
a^2 + 3*b^2))/(a^4*f) + (log(a + b*tan(e + f*x)^2)*(4*a*b^3 - 3*b^4))/(f*(
2*a^6 - 4*a^5*b + 2*a^4*b^2))
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \int \frac{\cot\left(fx+e\right)^5}{\left(\tan\left(fx+e\right)^2 b+a\right)^2}dx$$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x)
```

3.230 $\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{x}{(a-b)^2} - \frac{a^{3/2}(3a-5b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{5/2}f} + \frac{(3a-2b)\tan(e+fx)}{2(a-b)b^2f} - \frac{a\tan^3(e+fx)}{2(a-b)bf\,(a+b\tan^2(e+fx))}$$

output
```
-x/(a-b)^2-1/2*a^(3/2)*(3*a-5*b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^
2/b^(5/2)/f+1/2*(3*a-2*b)*tan(f*x+e)/(a-b)/b^2/f-1/2*a*tan(f*x+e)^3/(a-b)/
b/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{-\frac{2(e+fx)}{(a-b)^2}-\frac{a^{3/2}(3a-5b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{(a-b)^2b^{5/2}}+\frac{a^2\sin(2(e+fx))}{(a-b)b^2(a+b+(a-b)\cos(2(e+fx)))}+\frac{2\tan(e+fx)}{b^2}}{2f}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((-2*(e + f*x))/(a - b)^2 - (a^(3/2)*(3*a - 5*b)*ArcTan[(Sqrt[b]*Tan[e + f
*x])/Sqrt[a]])/((a - b)^2*b^(5/2)) + (a^2*Sin[2*(e + f*x)])/((a - b)*b^2*(
a + b + (a - b)*Cos[2*(e + f*x)])) + (2*Tan[e + f*x])/b^2)/(2*f)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 372, 444, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^6}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((3a-2b)\tan^2(e+fx)+3a\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2b(a-b)}-\frac{a\tan^3(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \ 444$$

$$\frac{\frac{\frac{(3a-2b)\tan(e+fx)}{b}-\frac{\int \frac{\left(3a^2-2ba-2b^2\right)\tan^2(e+fx)+a(3a-2b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{b}}{2b(a-b)}-\frac{a\tan^3(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\frac{(3a-2b)\tan(e+fx)}{b}-\frac{\frac{a^2(3a-5b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}+\frac{2b^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}}{b}}{2b(a-b)}-\frac{a\tan^3(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{\frac{(3a-2b)\tan(e+fx)}{b}-\frac{\frac{a^2(3a-5b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}+\frac{2b^2\arctan(\tan(e+fx))}{a-b}}{b}}{2b(a-b)}-\frac{a\tan^3(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{(3a-2b)\tan(e+fx)}{b}-\frac{\frac{a^{3/2}(3a-5b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}+\frac{2b^2\arctan(\tan(e+fx))}{a-b}}{b}}{2b(a-b)}-\frac{a\tan^3(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((-(((2*b^2*ArcTan[Tan[e + f*x]])/(a - b) + (a^(3/2)*(3*a - 5*b)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*Sqrt[b]))/b) + ((3*a - 2*b)*Tan[e
+ f*x])/b)/(2*(a - b)*b) - (a*Tan[e + f*x]^3)/(2*(a - b)*b*(a + b*Tan[e +
f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a^2\left(\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^2b^2}-\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a^2\left(\frac{\left(-\frac{a}{2}+\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(3a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a-b)^2b^2}-\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
risch	$-\frac{x}{a^2-2ab+b^2}+\frac{i\left(3a^3e^{4i(fx+e)}-5a^2b\,e^{4i(fx+e)}+6a\,b^2e^{4i(fx+e)}-2b^3e^{4i(fx+e)}+6a^3e^{2i(fx+e)}-4a^2b\,e^{2i(fx+e)}-4a\right.}{f\,b^2(a-b)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right.}$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b^2-a^2/(a-b)^2/b^2*((-1/2*a+1/2*b)*tan(f*x+e)/(a+b*tan(f*
x+e)^2)+1/2*(3*a-5*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/(a-b
)^2*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.65

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \left[-\frac{8\,b^3fx\tan(fx+e)^2+8\,ab^2fx-8\,(a^2b-2\,ab^2+b^3)\tan(fx+e)^3+(3\,a^3-5\,a^2b+(3\,a^2b-5\,ab^2)\,\mathrm{t}}{8\,((a^2b^3-2\,ab^4+b^5}\right.$$

$$-\frac{4\,b^3fx\tan(fx+e)^2+4\,ab^2fx-4\,(a^2b-2\,ab^2+b^3)\tan(fx+e)^3+(3\,a^3-5\,a^2b+(3\,a^2b-5\,ab^2)\,\mathrm{ta}}{4\,((a^2b^3-2\,ab^4+b^5)f\tan(fx+e)^2+(}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(8*b^3*f*x*tan(f*x + e)^2 + 8*a*b^2*f*x - 8*(a^2*b - 2*a*b^2 + b^3)*
tan(f*x + e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sq
rt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(b^2*tan
(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*ta
n(f*x + e)^2 + a^2)) - 4*(3*a^3 - 5*a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b
^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), -
1/4*(4*b^3*f*x*tan(f*x + e)^2 + 4*a*b^2*f*x - 4*(a^2*b - 2*a*b^2 + b^3)*ta
n(f*x + e)^3 + (3*a^3 - 5*a^2*b + (3*a^2*b - 5*a*b^2)*tan(f*x + e)^2)*sqrt
(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan(f*x + e))) - 2*(3
*a^3 - 5*a^2*b + 2*a*b^2)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f
*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2859 vs. 2(102) = 204.

Time = 23.93 (sec) , antiderivative size = 2859, normalized size of antiderivative = 21.99

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x*tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e
+ f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a**2, Eq(b, 0)),
 ((-x + tan(e + f*x)/f)/b**2, Eq(a, 0)), (-15*f*x*tan(e + f*x)**4/(8*b**2*
f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) - 30*f*x*tan(e +
 f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f)
 - 15*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f
) + 8*tan(e + f*x)**5/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**
2 + 8*b**2*f) + 25*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*t
an(e + f*x)**2 + 8*b**2*f) + 15*tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4 + 1
6*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e
)**2)**2, Eq(f, 0)), (-3*a**4*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b**3
*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**4*f*s
qrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**5*f*sqrt(-a/b)
+ 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) + 3*a**4*log(sqrt(-a/b) + tan(e + f
*x))/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(e + f*x)**2
- 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b
**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) + 6*a**3*b*sqrt(-a
/b)*tan(e + f*x)/(4*a**3*b**3*f*sqrt(-a/b) + 4*a**2*b**4*f*sqrt(-a/b)*tan(
e + f*x)**2 - 8*a**2*b**4*f*sqrt(-a/b) - 8*a*b**5*f*sqrt(-a/b)*tan(e + f*x
)**2 + 4*a*b**5*f*sqrt(-a/b) + 4*b**6*f*sqrt(-a/b)*tan(e + f*x)**2) - 3...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\frac{a^2\tan(fx+e)}{a^2b^2-ab^3+(ab^3-b^4)\tan(fx+e)^2} - \frac{(3\,a^3-5\,a^2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2b^2-2\,ab^3+b^4)\sqrt{ab}} - \frac{2\,(fx+e)}{a^2-2\,ab+b^2} + \frac{2\,\tan(fx+e)}{b^2}}{2\,f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a^2*tan(f*x + e)/(a^2*b^2 - a*b^3 + (a*b^3 - b^4)*tan(f*x + e)^2) - (
3*a^3 - 5*a^2*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b^2 - 2*a*b^3 + b^
4)*sqrt(a*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2) + 2*tan(f*x + e)/b^2)/f
```

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{a^2\tan\left(fx+e\right)}{2\left(ab^2f-b^3f\right)\left(b\tan\left(fx+e\right)^2+a\right)} - \frac{\left(3\,a^3-5\,a^2b\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\left(a^2b^2f-2\,ab^3f+b^4f\right)\sqrt{ab}} - \frac{fx+e}{a^2f-2\,abf+b^2f} + \frac{\tan\left(fx+e\right)}{b^2f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*a^2*tan(f*x + e)/((a*b^2*f - b^3*f)*(b*tan(f*x + e)^2 + a)) - 1/2*(3*a
^3 - 5*a^2*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b^2*f - 2*a*b^3*f + b
^4*f)*sqrt(a*b)) - (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) + tan(f*x + e)/(b^2
*f)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 2581, normalized size of antiderivative = 19.85

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(2*atan((((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 -
 6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(16*
b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(2*(
b^5 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2)
 + (tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^
4 + a^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2) - ((((4*a*b^8 - 22*a^2*b^7 + 48*a^3
*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4
 + a^3*b^3) + (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7
- 48*a^4*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a*b^4 + a^2*b^3)*(2*a^2 - 4*a*b +
2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6
 + 25*a^4*b^2))/(2*(b^5 - 2*a*b^4 + a^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2))/((
((((4*a*b^8 - 22*a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^
3)*1i)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) - (tan(e + f*x)*(16*b^10 - 48
*a*b^9 + 32*a^2*b^8 + 32*a^3*b^7 - 48*a^4*b^6 + 16*a^5*b^5))/(2*(b^5 - 2*a
*b^4 + a^2*b^3)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (t
an(e + f*x)*(9*a^6 - 30*a^5*b + 4*b^6 + 25*a^4*b^2)*1i)/(2*(b^5 - 2*a*b^4
+ a^2*b^3)))/(2*a^2 - 4*a*b + 2*b^2) - ((9*a^5)/2 - (21*a^4*b)/2 + 5*a^2*b
^3 + 2*a^3*b^2)/(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3) + (((((4*a*b^8 - 22*
a^2*b^7 + 48*a^3*b^6 - 52*a^4*b^5 + 28*a^5*b^4 - 6*a^6*b^3)*1i)/(3*a*b^5 -
 b^6 - 3*a^2*b^4 + a^3*b^3) + (tan(e + f*x)*(16*b^10 - 48*a*b^9 + 32*a^...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.25

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^2\,a^2b+5\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^2\,a\,b^2-3\sqrt{b}\,\sqrt{a}\,ata}{}$$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x
)**2*a**2*b + 5*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*t
an(e + f*x)**2*a*b**2 - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*s
qrt(a)))*a**3 + 5*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))
*a**2*b + 2*tan(e + f*x)**3*a**2*b**2 - 4*tan(e + f*x)**3*a*b**3 + 2*tan(e
 + f*x)**3*b**4 - 2*tan(e + f*x)**2*b**4*f*x + 3*tan(e + f*x)*a**3*b - 5*t
an(e + f*x)*a**2*b**2 + 2*tan(e + f*x)*a*b**3 - 2*a*b**3*f*x)/(2*b**3*f*(t
an(e + f*x)**2*a**2*b - 2*tan(e + f*x)**2*a*b**2 + tan(e + f*x)**2*b**3 +
a**3 - 2*a**2*b + a*b**2))
```

3.231 $\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{x}{(a-b)^2} + \frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2(a-b)^2b^{3/2}f} - \frac{a\tan(e+fx)}{2(a-b)bf\,(a+b\tan^2(e+fx))}$$

output
```
x/(a-b)^2+1/2*a^(1/2)*(a-3*b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/(a-b)^2/b
^(3/2)/f-1/2*a*tan(f*x+e)/(a-b)/b/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{2(e+fx) + \frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{3/2}} - \frac{a(a-b)\sin(2(e+fx))}{b(a+b+(a-b)\cos(2(e+fx)))}}{2(a-b)^2f}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(2*(e + f*x) + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/
b^(3/2) - (a*(a - b)*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x)
])))/(2*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4153, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(a-2b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2b(a-b)}-\frac{a\tan(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2b\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}+\frac{a(a-3b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2b(a-b)}-\frac{a\tan(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{a(a-3b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}+\frac{2b\arctan(\tan(e+fx))}{a-b}}{2b(a-b)}-\frac{a\tan(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2b\arctan(\tan(e+fx))}{a-b}+\frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}}{2b(a-b)}-\frac{a\tan(e+fx)}{2b(a-b)(a+b\tan^2(e+fx))}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(((2*b*ArcTan[Tan[e + f*x]])/(a - b) + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/((a - b)*Sqrt[b]))/(2*(a - b)*b) - (a*Tan[e + f*x]
)/(2*(a - b)*b*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{(a-b)\tan(fx+e)}{2b\left(a+b\tan(fx+e)^2\right)}+\frac{(a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
default	$\frac{\frac{a\left(-\frac{(a-b)\tan(fx+e)}{2b\left(a+b\tan(fx+e)^2\right)}+\frac{(a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}-\frac{ia\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{f(a-b)^2b\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{}}{}\right.}{4b^2(a-b)^2f}$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a/(a-b)^2*(-1/2*(a-b)/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(a-3*b)/b/(
a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^2*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.01

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{8\,b^2 f x \tan\left(fx+e\right)^2 + 8\,abfx - \left((ab-3\,b^2)\tan\left(fx+e\right)^2 + a^2 - 3\,ab\right)\sqrt{-\frac{a}{b}}\log\left(\frac{b^2\tan(fx+e)^4 - 6\,ab\tan(f\cdots}{b^2\tan\cdots}\right.}{8\left((a^2b^2 - 2\,ab^3 + b^4)f\tan\left(fx+e\right)^2 + (a^3b - 2\,a\cdots\right.}\right.$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*b^2*f*x*tan(f*x + e)^2 + 8*a*b*f*x - ((a*b - 3*b^2)*tan(f*x + e)^2
 + a^2 - 3*a*b)*sqrt(-a/b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2
+ a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x + e))*sqrt(-a/b))/(b^2*tan(f*x
 + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2 - a*b)*tan(f*x + e))/((a^2
*b^2 - 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), 1
/4*(4*b^2*f*x*tan(f*x + e)^2 + 4*a*b*f*x + ((a*b - 3*b^2)*tan(f*x + e)^2 +
 a^2 - 3*a*b)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a/b)/(a*tan
(f*x + e))) - 2*(a^2 - a*b)*tan(f*x + e))/((a^2*b^2 - 2*a*b^3 + b^4)*f*tan
(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2157 vs. 2(76) = 152.

Time = 10.55 (sec) , antiderivative size = 2157, normalized size of antiderivative = 22.71

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e + f*x)**3/(
3*f) - tan(e + f*x)/f)/a**2, Eq(b, 0)), (x/b**2, Eq(a, 0)), (3*f*x*tan(e +
 f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f)
 + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x
)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x
)**2 + 8*b**2*f) - 5*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f
*tan(e + f*x)**2 + 8*b**2*f) - 3*tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4 +
16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**4/(a + b*tan(
e)**2)**2, Eq(f, 0)), (a**3*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b**2*f
*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqr
t(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*f*sqrt(-a/b) +
4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) - a**3*log(sqrt(-a/b) + tan(e + f*x))
/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*
a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**4*
f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) - 2*a**2*b*sqrt(-a/b)*
tan(e + f*x)/(4*a**3*b**2*f*sqrt(-a/b) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e +
f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - 8*a*b**4*f*sqrt(-a/b)*tan(e + f*x)**2
 + 4*a*b**4*f*sqrt(-a/b) + 4*b**5*f*sqrt(-a/b)*tan(e + f*x)**2) + a**2*b*l
og(-sqrt(-a/b) + tan(e + f*x))*tan(e + f*x)**2/(4*a**3*b**2*f*sqrt(-a/b) +
 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**3*f*sqrt(-a/b) - ...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{a\tan(fx+e)}{a^2b-ab^2+(ab^2-b^3)\tan(fx+e)^2} - \frac{(a^2-3\,ab)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2b-2\,ab^2+b^3)\sqrt{ab}} - \frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*tan(f*x + e)/(a^2*b - a*b^2 + (a*b^2 - b^3)*tan(f*x + e)^2) - (a^2
 - 3*a*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b - 2*a*b^2 + b^3)*sqrt(a
*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{(a^2-3\,ab)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,(a^2bf-2\,ab^2f+b^3f)\sqrt{ab}} + \frac{fx+e}{a^2f-2\,abf+b^2f} - \frac{a\tan(fx+e)}{2\,(abf-b^2f)\big(b\tan(fx+e)^2+a\big)}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2 - 3*a*b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*b*f - 2*a*b^2*f +
 b^3*f)*sqrt(a*b)) + (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) - 1/2*a*tan(f*x +
 e)/((a*b*f - b^2*f)*(b*tan(f*x + e)^2 + a))
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 2358, normalized size of antiderivative = 24.82

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(2*atan((((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^2)*1i)
/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(16*b^8 - 48*a*b^7 +
32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*b^2 +
b^3)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(a^
4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^2 - 4*
a*b + 2*b^2) - ((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2*a^5*b^
2)*1i)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e + f*x)*(16*b^8 - 48*a*
b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b - 2*a*
b^2 + b^3)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*
x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2))/(2*(a^2*b - 2*a*b^2 + b^3)))/(2*a^
2 - 4*a*b + 2*b^2))/((((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 - 8*a^4*b^3 + 2
*a^5*b^2)*1i)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) - (tan(e + f*x)*(16*b^8
- 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b^3))/(2*(a^2*b
 - 2*a*b^2 + b^3)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) +
(tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2)*1i)/(2*(a^2*b - 2*a*b^2
+ b^3)))/(2*a^2 - 4*a*b + 2*b^2) + (((((2*a*b^6 - 8*a^2*b^5 + 12*a^3*b^4 -
 8*a^4*b^3 + 2*a^5*b^2)*1i)/(3*a*b^3 + a^3*b - b^4 - 3*a^2*b^2) + (tan(e +
 f*x)*(16*b^8 - 48*a*b^7 + 32*a^2*b^6 + 32*a^3*b^5 - 48*a^4*b^4 + 16*a^5*b
^3))/(2*(a^2*b - 2*a*b^2 + b^3)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a
*b + 2*b^2) - (tan(e + f*x)*(a^4 - 6*a^3*b + 4*b^4 + 9*a^2*b^2)*1i)/(2*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.39

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan\left(fx+e\right)^2 ab - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan\left(fx+e\right)^2 b^2 + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(}{\sqrt{}}\right.}{2b^2 f\left(\tan\left(fx+e\right)^2 a^2 b - 2\tan\left(fx+\right.\right.}$$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*
a*b - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f
*x)**2*b**2 + sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**
2 - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a*b + 2*tan
(e + f*x)**2*b**3*f*x - tan(e + f*x)*a**2*b + tan(e + f*x)*a*b**2 + 2*a*b*
*2*f*x)/(2*b**2*f*(tan(e + f*x)**2*a**2*b - 2*tan(e + f*x)**2*a*b**2 + tan
(e + f*x)**2*b**3 + a**3 - 2*a**2*b + a*b**2))
```

3.232 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{x}{(a-b)^2}+\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2\sqrt{a}(a-b)^2\sqrt{b}f}+\frac{\tan(e+fx)}{2(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-x/(a-b)^2+1/2*(a+b)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(1/2)/(a-b)^2/b^
(1/2)/f+1/2*tan(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{-2(e+fx)+\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{(a-b)\sin(2(e+fx))}{a+b+(a-b)\cos(2(e+fx))}}{2(a-b)^2f}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-2*(e + f*x) + ((a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*
Sqrt[b]) + ((a - b)*Sin[2*(e + f*x)])/(a + b + (a - b)*Cos[2*(e + f*x)]))/
(2*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4153, 373, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\tan(e+fx)}{2(a-b)(a+b\tan^2(e+fx))}-\frac{\int \frac{1-\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2(a-b)}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tan(e+fx)}{2(a-b)(a+b\tan^2(e+fx))}-\frac{\frac{2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{(a+b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)}{2(a-b)(a+b\tan^2(e+fx))}-\frac{\frac{2\arctan(\tan(e+fx))}{a-b}-\frac{(a+b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2(a-b)}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\tan(e+fx)}{2(a-b)(a+b\tan^2(e+fx))}-\frac{\frac{2\arctan(\tan(e+fx))}{a-b}-\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a-b)}}{2(a-b)}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(-1/2*((2*ArcTan[Tan[e + f*x]])/(a - b) - ((a + b)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)*Sqrt[b]))/(a - b) + Tan[e + f*x]/(2*(a -
 b)*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a-b)^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a-b)^2}}{f}$
risch	$-\frac{x}{a^2-2ab+b^2}+\frac{i\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{f(a-b)^2\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{-2iab+\sqrt{-}}{(a-b}\right.}{4\sqrt{-ab}\,(a-b)}$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)^2*arctan(tan(f*x+e))+1/(a-b)^2*((1/2*a-1/2*b)*tan(f*x+e)/(a+
b*tan(f*x+e)^2)+1/2*(a+b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.

Time = 0.12 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.37

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \left[-\frac{8\,ab^2fx\tan(fx+e)^2 + 8\,a^2bfx + \left((ab+b^2)\tan(fx+e)^2 + a^2 + ab\right)\sqrt{-ab}\log\left(\frac{b^2\tan(fx+e)^4 - 6\,ab\tan(}{b^2\tan}\right.}{8\left((a^3b^2 - 2\,a^2b^3 + ab^4)f\tan(fx+e)^2 + (a^4b - \right.}\right.$$
$$-\frac{4\,ab^2fx\tan(fx+e)^2 + 4\,a^2bfx - \left((ab+b^2)\tan(fx+e)^2 + a^2 + ab\right)\sqrt{ab}\arctan\left(\frac{\left(b\tan(fx+e)^2 - a\right)\sqrt{ab}}{2\,ab\tan(fx+e)}\right.}{4\left((a^3b^2 - 2\,a^2b^3 + ab^4)f\tan(fx+e)^2 + (a^4b - 2\,a^3b^2 + a^2b^3)f\right)}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b^2*f*x*tan(f*x + e)^2 + 8*a^2*b*f*x + ((a*b + b^2)*tan(f*x + e
)^2 + a^2 + a*b)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2
 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a*b))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^2*b - a*b^2)*tan(f*x + e))/((a^
3*b^2 - 2*a^2*b^3 + a*b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3
)*f), -1/4*(4*a*b^2*f*x*tan(f*x + e)^2 + 4*a^2*b*f*x - ((a*b + b^2)*tan(f*
x + e)^2 + a^2 + a*b)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b
)/(a*b*tan(f*x + e))) - 2*(a^2*b - a*b^2)*tan(f*x + e))/((a^3*b^2 - 2*a^2*
b^3 + a*b^4)*f*tan(f*x + e)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2113 vs. 2(73) = 146.

Time = 10.72 (sec) , antiderivative size = 2113, normalized size of antiderivative = 23.48

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e
+ f*x)/f)/a**2, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x)))/b**2, Eq(a, 0)), (f*
x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 +
8*b**2*f) + 2*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*ta
n(e + f*x)**2 + 8*b**2*f) + f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(
e + f*x)**2 + 8*b**2*f) + tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**4 + 16*b
**2*f*tan(e + f*x)**2 + 8*b**2*f) - tan(e + f*x)/(8*b**2*f*tan(e + f*x)**4
 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x*tan(e)**2/(a + b*t
an(e)**2)**2, Eq(f, 0)), (a**2*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**3*b*f
*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b**2*f*sqr
t(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt(-a/b) +
4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) - a**2*log(sqrt(-a/b) + tan(e + f*x))
/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**
2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*s
qrt(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) - 4*a*b*f*x*sqrt(-a/b)/(4
*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**2*b
**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b**3*f*sqrt
(-a/b) + 4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2) + 2*a*b*sqrt(-a/b)*tan(e + f
*x)/(4*a**3*b*f*sqrt(-a/b) + 4*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*
a**2*b**2*f*sqrt(-a/b) - 8*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a*b*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}} - \frac{2\,(fx+e)}{a^2-2\,ab+b^2} + \frac{\tan(fx+e)}{(ab-b^2)\tan(fx+e)^2+a^2-ab}}{2\,f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a + b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*
b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2) + tan(f*x + e)/((a*b - b^2)*tan(f*x
+ e)^2 + a^2 - a*b))/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,(a^2f-2\,abf+b^2f)\sqrt{ab}} - \frac{fx+e}{a^2f-2\,abf+b^2f} + \frac{\tan(fx+e)}{2\left(b\tan(fx+e)^2+a\right)(af-bf)}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a + b)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^2*f - 2*a*b*f + b^2*f)*sq
rt(a*b)) - (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) + 1/2*tan(f*x + e)/((b*tan(
f*x + e)^2 + a)*(a*f - b*f))
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 2136, normalized size of antiderivative = 23.73

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
tan(e + f*x)/(2*f*(a + b*tan(e + f*x)^2)*(a - b)) - (2*atan((((((2*b^6 - 8
*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3
- b^3) - (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a
^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*
a^2 - 4*a*b + 2*b^2) + (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 -
2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*b^6 - 8*a*b^5 + 12*a^2*b^4
- 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*
x)*(16*b^7 - 48*a*b^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2)
)/(2*(a^2 - 2*a*b + b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2*b^2)
 - (tan(e + f*x)*(2*a*b^2 + a^2*b + 5*b^3))/(2*(a^2 - 2*a*b + b^2)))/(2*a^
2 - 4*a*b + 2*b^2))/((((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4
*b^2)*1i)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (tan(e + f*x)*(16*b^7 - 48*a*b
^6 + 32*a^2*b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b +
 b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e + f*x
)*(2*a*b^2 + a^2*b + 5*b^3)*1i)/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b +
2*b^2) + (((((2*b^6 - 8*a*b^5 + 12*a^2*b^4 - 8*a^3*b^3 + 2*a^4*b^2)*1i)/(3
*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(e + f*x)*(16*b^7 - 48*a*b^6 + 32*a^2*
b^5 + 32*a^3*b^4 - 48*a^4*b^3 + 16*a^5*b^2))/(2*(a^2 - 2*a*b + b^2)*(2*a^2
 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(2*a*b^2 +
 a^2*b + 5*b^3)*1i)/(2*(a^2 - 2*a*b + b^2)))/(2*a^2 - 4*a*b + 2*b^2) + ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.54

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^2\,ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^2\,b^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(f}{\sqrt{b}}\right.}{2abf\left(\tan(fx+e)^2\,a^2b-2\tan(fx+\right.}$$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*
a*b + sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x
)**2*b**2 + sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**2
+ sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a*b - 2*tan(e +
 f*x)**2*a*b**2*f*x + tan(e + f*x)*a**2*b - tan(e + f*x)*a*b**2 - 2*a**2*b
*f*x)/(2*a*b*f*(tan(e + f*x)**2*a**2*b - 2*tan(e + f*x)**2*a*b**2 + tan(e
+ f*x)**2*b**3 + a**3 - 2*a**2*b + a*b**2))
```

3.233 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2} dx = \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2f} - \frac{b\tan(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
x/(a-b)^2-1/2*(3*a-b)*b^(1/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2)/(
a-b)^2/f-1/2*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2} dx = \frac{2\arctan(\tan(e+fx)) + \frac{\sqrt{b}(-3a+b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(-a+b)\tan(e+fx)}{a(a+b\tan^2(e+fx))}}{2(a-b)^2f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-2),x]
```

output
```
(2*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[e + f*x])/(a*(a + b*Tan[e + f*x]^2))
)/(2*(a - b)^2*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4144, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-b\tan^2(e+fx)+2a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2a\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b(3a-b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2a\arctan(\tan(e+fx))}{a-b}-\frac{b(3a-b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2a\arctan(\tan(e+fx))}{a-b}-\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-2),x]
```

output
```
(((2*a*ArcTan[Tan[e + f*x]])/(a - b) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - (b*Tan[e + f*x]
)/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\tan(fx+e)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
default	$\frac{-\frac{b\left(\frac{(a-b)\tan(fx+e)}{2a\left(a+b\tan(fx+e)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{ib\left(a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}+a-b\right)}{fa(-a+b)^2\left(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2i(fx+e)}+\right.}{4a(a-b)^2}$

input
```
int(1/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a-b)^2*(1/2/a*(a-b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(3*a-b)/a/(
a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^2*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.02

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{8\,abfx\tan\left(fx+e\right)^2+8\,a^2fx-\left((3\,ab-b^2)\tan\left(fx+e\right)^2+3\,a^2-ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(fx+e)^4-6\,ab\tan(f}{b^2\tan}\right.}{8\left((a^3b-2\,a^2b^2+ab^3)f\tan\left(fx+e\right)^2+(a^4-2\,\right.}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*a*b*f*x*tan(f*x + e)^2 + 8*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2
 + 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2
+ a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x
 + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a*b - b^2)*tan(f*x + e))/((a^3
*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f), 1
/4*(4*a*b*f*x*tan(f*x + e)^2 + 4*a^2*f*x - ((3*a*b - b^2)*tan(f*x + e)^2 +
 3*a^2 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan
(f*x + e)) - 2*(a*b - b^2)*tan(f*x + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*t
an(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. $2(78) = 156$.

Time = 9.99 (sec) , antiderivative size = 2125, normalized size of antiderivative = 21.91

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**2, Eq(b
, 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)**3))/b**2, Eq(a, 0))
, (3*f*x*tan(e + f*x)**4/(8*b**2*f*tan(e + f*x)**4 + 16*b**2*f*tan(e + f*x
)**2 + 8*b**2*f) + 6*f*x*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**4 + 16*b*
*2*f*tan(e + f*x)**2 + 8*b**2*f) + 3*f*x/(8*b**2*f*tan(e + f*x)**4 + 16*b*
*2*f*tan(e + f*x)**2 + 8*b**2*f) + 3*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x
)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f) + 5*tan(e + f*x)/(8*b**2*f*ta
n(e + f*x)**4 + 16*b**2*f*tan(e + f*x)**2 + 8*b**2*f), Eq(a, b)), (x/(a +
b*tan(e)**2)**2, Eq(f, 0)), (4*a**2*f*x*sqrt(-a/b)/(4*a**4*f*sqrt(-a/b) +
4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**
2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) + 4*a*b**3*f*sqr
t(-a/b)*tan(e + f*x)**2) - 3*a**2*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**4*
f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**3*b*f*sqrt(-a/
b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*f*sqrt(-a/b) +
 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 3*a**2*log(sqrt(-a/b) + tan(e +
f*x))/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*x)**2 - 8*a**
3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 4*a**2*b**2*
f*sqrt(-a/b) + 4*a*b**3*f*sqrt(-a/b)*tan(e + f*x)**2) + 4*a*b*f*x*sqrt(-a/
b)*tan(e + f*x)**2/(4*a**4*f*sqrt(-a/b) + 4*a**3*b*f*sqrt(-a/b)*tan(e + f*
x)**2 - 8*a**3*b*f*sqrt(-a/b) - 8*a**2*b**2*f*sqrt(-a/b)*tan(e + f*x)**...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{\frac{b\tan(fx+e)}{a^3-a^2b+(a^2b-ab^2)\tan(fx+e)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(f*x + e)^2) + (3*a
*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a
*b)) - 2*(f*x + e)/(a^2 - 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,(a^3f-2\,a^2bf+ab^2f)\sqrt{ab}} + \frac{fx+e}{a^2f-2\,abf+b^2f} - \frac{b\tan\left(fx+e\right)}{2\,(a^2f-abf)\left(b\tan\left(fx+e\right)^2+a\right)}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*f - 2*a^2*b*f +
a*b^2*f)*sqrt(a*b)) + (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) - 1/2*b*tan(f*x
+ e)/((a^2*f - a*b*f)*(b*tan(f*x + e)^2 + a))
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 2489, normalized size of antiderivative = 25.66

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*
a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))
/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2
*b^2) + (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^
2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 -
 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3
*b^2) + (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 -
 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(e + f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^
3))/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7
- 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^
4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(e + f*x)*(16*a^2*b^7 - 48*a^3*b^6
+ 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b +
a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(e +
f*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^
2 - 4*a*b + 2*b^2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 +
 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(
e + f*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 +
 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan\left(fx+e\right)^2 ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan\left(fx+e\right)^2 b^2-3\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{2a^2f\left(\tan\left(fx+e\right)^2 a^2b-2\tan\left(fx+\right.\right.}$$

input
```
int(1/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x
)**2*a*b + sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e
+ f*x)**2*b**2 - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a))
)*a**2 + sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a*b + 2*
tan(e + f*x)**2*a**2*b*f*x - tan(e + f*x)*a**2*b + tan(e + f*x)*a*b**2 + 2
*a**3*f*x)/(2*a**2*f*(tan(e + f*x)**2*a**2*b - 2*tan(e + f*x)**2*a*b**2 +
tan(e + f*x)**2*b**3 + a**3 - 2*a**2*b + a*b**2))
```

3.234 $\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = -\frac{x}{(a-b)^2}+\frac{(5a-3b)b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^2f} - \frac{(2a-3b)\cot(e+fx)}{2a^2(a-b)f} - \frac{b\cot(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))}$$

output
```
-x/(a-b)^2+1/2*(5*a-3*b)*b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(5/2
)/(a-b)^2/f-1/2*(2*a-3*b)*cot(f*x+e)/a^2/(a-b)/f-1/2*b*cot(f*x+e)/a/(a-b)/
f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\frac{(5a-3b)b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^2} - \frac{2\cot(e+fx)}{a^2} + \frac{-2(e+fx)+\frac{(a-b)b^2\sin(2(e+fx))}{a^2(a+b+(a-b)\cos(2(e+fx)))}}{(a-b)^2}}{2f}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
(((5*a - 3*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(5/2)*(a
- b)^2) - (2*Cot[e + f*x])/a^2 + (-2*(e + f*x) + ((a - b)*b^2*Sin[2*(e + f
*x)])/(a^2*(a + b + (a - b)*Cos[2*(e + f*x)])))/(a - b)^2)/(2*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 374, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\,(a+b\tan(e+fx)^2)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-3b\tan^2(e+fx)+2a-3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b\cot(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{2a^2+2ba-3b^2+(2a-3b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{(2a-3b)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b\cot(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{-\frac{\frac{2a^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^2(5a-3b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{(2a-3b)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b\cot(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{-\frac{\frac{2a^2\arctan(\tan(e+fx))}{a-b}-\frac{b^2(5a-3b)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{(2a-3b)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b\cot(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{-\frac{\frac{2a^2\arctan(\tan(e+fx))}{a-b}-\frac{b^{3/2}(5a-3b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}-\frac{(2a-3b)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b\cot(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

input

```
Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^2,x]
```

output

```
((-(((2*a^2*ArcTan[Tan[e + f*x]])/(a - b) - ((5*a - 3*b)*b^(3/2)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/a) - ((2*a - 3*b)*Cot[e
+ f*x])/a)/(2*a*(a - b)) - (b*Cot[e + f*x])/(2*a*(a - b)*(a + b*Tan[e + f*
x]^2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{-\frac{1}{a^{2}\tan(fx+e)}+\frac{b^{2}\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^{2}}+\frac{(5a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}(a-b)^{2}}-\frac{\arctan(\tan(fx+e))}{(a-b)^{2}}}{f}$
default	$\frac{-\frac{1}{a^{2}\tan(fx+e)}+\frac{b^{2}\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^{2}}+\frac{(5a-3b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}(a-b)^{2}}-\frac{\arctan(\tan(fx+e))}{(a-b)^{2}}}{f}$
risch	$-\frac{x}{a^{2}-2ab+b^{2}}-\frac{i\left(2a^{3}e^{4i(fx+e)}-6a^{2}b\,e^{4i(fx+e)}+5a\,b^{2}e^{4i(fx+e)}-3b^{3}e^{4i(fx+e)}+4a^{3}e^{2i(fx+e)}-4a^{2}b\,e^{2i(fx+e)}-4a\right.}{f\,a^{2}(a-b)^{2}\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+\right.}$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2/tan(f*x+e)+b^2/a^2/(a-b)^2*((1/2*a-1/2*b)*tan(f*x+e)/(a+b*tan(
f*x+e)^2)+1/2*(5*a-3*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/(a
-b)^2*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.93

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{8\,a^2bfx\tan\left(fx+e\right)^3+8\,a^3fx\tan\left(fx+e\right)+8\,a^3-16\,a^2b+8\,ab^2+4\,(2\,a^2b-5\,ab^2+3\,b^3)\tan\left(fx\right.}{8\left((a^4b-2\,a^3b^2+\right.}\right.$$

$$-\frac{4\,a^2bfx\tan\left(fx+e\right)^3+4\,a^3fx\tan\left(fx+e\right)+4\,a^3-8\,a^2b+4\,ab^2+2\,(2\,a^2b-5\,ab^2+3\,b^3)\tan\left(fx+\right.}{4\left((a^4b-2\,a^3b^2+a^2b^3)f\tan\left(fx+e\right)^3\right.}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(8*a^2*b*f*x*tan(f*x + e)^3 + 8*a^3*f*x*tan(f*x + e) + 8*a^3 - 16*a^
2*b + 8*a*b^2 + 4*(2*a^2*b - 5*a*b^2 + 3*b^3)*tan(f*x + e)^2 + ((5*a*b^2 -
 3*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan(f*x + e))*sqrt(-b/a)*log(
(b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 -
 a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2
+ a^2)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b
+ a^3*b^2)*f*tan(f*x + e)), -1/4*(4*a^2*b*f*x*tan(f*x + e)^3 + 4*a^3*f*x*t
an(f*x + e) + 4*a^3 - 8*a^2*b + 4*a*b^2 + 2*(2*a^2*b - 5*a*b^2 + 3*b^3)*ta
n(f*x + e)^2 - ((5*a*b^2 - 3*b^3)*tan(f*x + e)^3 + (5*a^2*b - 3*a*b^2)*tan
(f*x + e))*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*
x + e))))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b
 + a^3*b^2)*f*tan(f*x + e))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3245 vs. $2(102) = 204$.

Time = 83.40 (sec) , antiderivative size = 3245, normalized size of antiderivative = 25.35

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(e, 0) & Eq(f, 0)), ((-x - cot(e
 + f*x)/f)/a**2, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x
)**3) - 1/(5*f*tan(e + f*x)**5))/b**2, Eq(a, 0)), (-15*f*x*tan(e + f*x)**5
/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*tan(e + f*x)**3 + 8*b**2*f*tan(e +
f*x)) - 30*f*x*tan(e + f*x)**3/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*tan(e
 + f*x)**3 + 8*b**2*f*tan(e + f*x)) - 15*f*x*tan(e + f*x)/(8*b**2*f*tan(e
+ f*x)**5 + 16*b**2*f*tan(e + f*x)**3 + 8*b**2*f*tan(e + f*x)) - 15*tan(e
+ f*x)**4/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*tan(e + f*x)**3 + 8*b**2*f
*tan(e + f*x)) - 25*tan(e + f*x)**2/(8*b**2*f*tan(e + f*x)**5 + 16*b**2*f*
tan(e + f*x)**3 + 8*b**2*f*tan(e + f*x)) - 8/(8*b**2*f*tan(e + f*x)**5 + 1
6*b**2*f*tan(e + f*x)**3 + 8*b**2*f*tan(e + f*x)), Eq(a, b)), (zoo*x/a**2,
 Eq(e, -f*x)), (x*cot(e)**2/(a + b*tan(e)**2)**2, Eq(f, 0)), (-4*a**3*f*x*
sqrt(-a/b)*tan(e + f*x)/(4*a**5*f*sqrt(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqr
t(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*sqrt(-a/b)*tan(e + f*x) - 8*a**3*b**2
*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*
a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3) - 4*a**3*sqrt(-a/b)/(4*a**5*f*sqrt
(-a/b)*tan(e + f*x) + 4*a**4*b*f*sqrt(-a/b)*tan(e + f*x)**3 - 8*a**4*b*f*s
qrt(-a/b)*tan(e + f*x) - 8*a**3*b**2*f*sqrt(-a/b)*tan(e + f*x)**3 + 4*a**3
*b**2*f*sqrt(-a/b)*tan(e + f*x) + 4*a**2*b**3*f*sqrt(-a/b)*tan(e + f*x)**3
) - 4*a**2*b*f*x*sqrt(-a/b)*tan(e + f*x)**3/(4*a**5*f*sqrt(-a/b)*tan(e ...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\frac{(5\,ab^2-3\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4-2\,a^3b+a^2b^2)\sqrt{ab}} - \frac{(2\,ab-3\,b^2)\tan(fx+e)^2+2\,a^2-2\,ab}{(a^3b-a^2b^2)\tan(fx+e)^3+(a^4-a^3b)\tan(fx+e)} - \frac{2\,(fx+e)}{a^2-2\,ab+b^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((5*a*b^2 - 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 2*a^3*b +
a^2*b^2)*sqrt(a*b)) - ((2*a*b - 3*b^2)*tan(f*x + e)^2 + 2*a^2 - 2*a*b)/((a
^3*b - a^2*b^2)*tan(f*x + e)^3 + (a^4 - a^3*b)*tan(f*x + e)) - 2*(f*x + e)
/(a^2 - 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.21

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{(5\,ab^2-3\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,(a^4f-2\,a^3bf+a^2b^2f)\sqrt{ab}} - \frac{fx+e}{a^2f-2\,abf+b^2f} - \frac{2\,ab\tan(fx+e)^2-3\,b^2\tan(fx+e)^2+2\,a^2-2\,ab}{2\,(a^3f-a^2bf)\left(b\tan(fx+e)^3+a\tan(fx+e)\right)}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(5*a*b^2 - 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4*f - 2*a^3*b*f
 + a^2*b^2*f)*sqrt(a*b)) - (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) - 1/2*(2*a*
b*tan(f*x + e)^2 - 3*b^2*tan(f*x + e)^2 + 2*a^2 - 2*a*b)/((a^3*f - a^2*b*f
)*(b*tan(f*x + e)^3 + a*tan(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 2674, normalized size of antiderivative = 20.89

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
- (1/a + (tan(e + f*x)^2*(2*a*b - 3*b^2))/(2*a^2*(a - b)))/(f*(a*tan(e + f
*x) + b*tan(e + f*x)^3)) - (2*atan(((((1280*a^9*b^9 - 192*a^8*b^10 - 3520*
a^10*b^8 + 4992*a^11*b^7 - 3520*a^12*b^6 + 512*a^13*b^5 + 960*a^14*b^4 - 6
40*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^9
+ 3584*a^12*b^8 - 3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^1
7*b^3 - 256*a^18*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*
b^2) + tan(e + f*x)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*
b^7 + 1744*a^10*b^6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3))/(2*a^2 -
 4*a*b + 2*b^2) + (((192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^
11*b^7 + 3520*a^12*b^6 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*
a^16*b^2 + (tan(e + f*x)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 -
3584*a^13*b^7 + 3584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b
^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) + tan(e + f*x
)*(144*a^6*b^10 - 912*a^7*b^9 + 2272*a^8*b^8 - 2784*a^9*b^7 + 1744*a^10*b^
6 - 592*a^11*b^5 + 192*a^12*b^4 - 64*a^13*b^3))/(2*a^2 - 4*a*b + 2*b^2))/(
((((192*a^8*b^10 - 1280*a^9*b^9 + 3520*a^10*b^8 - 4992*a^11*b^7 + 3520*a^1
2*b^6 - 512*a^13*b^5 - 960*a^14*b^4 + 640*a^15*b^3 - 128*a^16*b^2 + (tan(e
 + f*x)*(256*a^10*b^10 - 1536*a^11*b^9 + 3584*a^12*b^8 - 3584*a^13*b^7 + 3
584*a^15*b^5 - 3584*a^16*b^4 + 1536*a^17*b^3 - 256*a^18*b^2)*1i)/(2*a^2 -
4*a*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2) + tan(e + f*x)*(144*a^6*b^1...
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 677, normalized size of antiderivative = 5.29

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
( - 5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b
))*sin(e + f*x)**3*a**2*b + 8*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*
tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a*b**2 - 3*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*b**3 +
5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*s
in(e + f*x)*a**2*b - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e
+ f*x)/2))/sqrt(b))*sin(e + f*x)*a*b**2 + 5*sqrt(b)*sqrt(a)*atan((sqrt(a -
 b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a**2*b - 8*sqrt(b
)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f
*x)**3*a*b**2 + 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x
)/2))/sqrt(b))*sin(e + f*x)**3*b**3 - 5*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
+ sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*a**2*b + 3*sqrt(b)*sqrt(
a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*a*b
**2 - 2*cos(e + f*x)*sin(e + f*x)**2*a**4 + 6*cos(e + f*x)*sin(e + f*x)**2
*a**3*b - 7*cos(e + f*x)*sin(e + f*x)**2*a**2*b**2 + 3*cos(e + f*x)*sin(e
+ f*x)**2*a*b**3 + 2*cos(e + f*x)*a**4 - 4*cos(e + f*x)*a**3*b + 2*cos(e +
 f*x)*a**2*b**2 - 2*sin(e + f*x)**3*a**4*f*x + 2*sin(e + f*x)**3*a**3*b*f*
x + 2*sin(e + f*x)*a**4*f*x)/(2*sin(e + f*x)*a**3*f*(sin(e + f*x)**2*a**3
- 3*sin(e + f*x)**2*a**2*b + 3*sin(e + f*x)**2*a*b**2 - sin(e + f*x)**2*b*
*3 - a**3 + 2*a**2*b - a*b**2))
```

3.235 $\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{x}{(a-b)^2} - \frac{(7a-5b)b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^2f} + \frac{(2a^2+2ab-5b^2)\cot(e+fx)}{2a^3(a-b)f} - \frac{(2a-5b)\cot^3(e+fx)}{6a^2(a-b)f} - \frac{b\cot^3(e+fx)}{2a(a-b)f\,(a+b\tan^2(e+fx))}$$

output
```
x/(a-b)^2-1/2*(7*a-5*b)*b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(7/2)
/(a-b)^2/f+1/2*(2*a^2+2*a*b-5*b^2)*cot(f*x+e)/a^3/(a-b)/f-1/6*(2*a-5*b)*co
t(f*x+e)^3/a^2/(a-b)/f-1/2*b*cot(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \frac{\frac{3b^{5/2}(-7a+5b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^2}-\frac{2\cot(e+fx)\left(-4a-6b+a\csc^2(e+fx)\right)}{a^3}+\frac{3\left(2(e+fx)-\frac{(a-b)b^3\sin(2(e+fx))}{a^3(a+b+(a-b)\cos(2(e+fx)))}\right)}{(a-b)^2}}{6f}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((3*b^(5/2)*(-7*a + 5*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*
(a - b)^2) - (2*Cot[e + f*x]*(-4*a - 6*b + a*Csc[e + f*x]^2))/a^3 + (3*(2*
(e + f*x) - ((a - b)*b^3*Sin[2*(e + f*x)])/(a^3*(a + b + (a - b)*Cos[2*(e
+ f*x)]))))/(a - b)^2)/(6*f)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4153, 374, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^4(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 374$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-5b\tan^2(e+fx)+2a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b\cot^3(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(2a^2+2ba-5b^2+(2a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{3a}-\frac{(2a-5b)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b\cot^3(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2a^2+2ba-5b^2+(2a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{(2a-5b)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b\cot^3(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{-\frac{-\frac{\int \frac{2a^3+2ba^2+2b^2a-5b^3+b\left(2a^2+2ba-5b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{a}}{a}-\frac{(2a-5b)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b\cot^3(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{-\frac{-\frac{\frac{2a^3\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^3(7a-5b)\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{a}}{a}-\frac{(2a-5b)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b\cot^3(e+f}{2a(a-b)(a+b\tan}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{-\frac{-\frac{\frac{2a^3\arctan(\tan(e+fx))}{a-b}-\frac{b^3(7a-5b)\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{a}}{a}-\frac{(2a-5b)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b\cot^3(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{-\frac{-\frac{\frac{2a^3\arctan(\tan(e+fx))}{a-b}-\frac{b^{5/2}(7a-5b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}-\frac{\left(2a^2+2ab-5b^2\right)\cot(e+fx)}{a}}{a}-\frac{(2a-5b)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b\cot^3(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((-1/3*((2*a - 5*b)*Cot[e + f*x]^3)/a - (-(((2*a^3*ArcTan[Tan[e + f*x]])/(
a - b) - ((7*a - 5*b)*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqr
t[a]*(a - b)))/a) - ((2*a^2 + 2*a*b - 5*b^2)*Cot[e + f*x])/a)/a)/(2*a*(a -
 b)) - (b*Cot[e + f*x]^3)/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{1}{3a^2\tan(fx+e)^3}-\frac{-2b-a}{a^3\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(7a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
default	$\frac{-\frac{1}{3a^2\tan(fx+e)^3}-\frac{-2b-a}{a^3\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(7a-5b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(a-b)^2}+\frac{\arctan(\tan(fx+e))}{(a-b)^2}}{f}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{i\left(12a^4e^{8i(fx+e)}-24a^3b\,e^{8i(fx+e)}+21a\,b^3e^{8i(fx+e)}-15b^4e^{8i(fx+e)}+12a^4e^{6i(fx+e)}+12a^3b\,e^{6i(fx+e)}\right.}{}$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/a^2/tan(f*x+e)^3-(-2*b-a)/a^3/tan(f*x+e)-b^3/a^3/(a-b)^2*((1/2*a
-1/2*b)*tan(f*x+e)/(a+b*tan(f*x+e)^2)+1/2*(7*a-5*b)/(a*b)^(1/2)*arctan(b*t
an(f*x+e)/(a*b)^(1/2)))+1/(a-b)^2*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 596, normalized size of antiderivative = 3.53

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \left[\frac{24\,a^3bfx\tan\left(fx+e\right)^5+24\,a^4fx\tan\left(fx+e\right)^3+12\left(2\,a^3b-7\,ab^3+5\,b^4\right)\tan\left(fx+e\right)^4-8\,a^4+16\,a^3}{}\right.$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/24*(24*a^3*b*f*x*tan(f*x + e)^5 + 24*a^4*f*x*tan(f*x + e)^3 + 12*(2*a^3
*b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 8*a^4 + 16*a^3*b - 8*a^2*b^2 + 8*(3
*a^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2 - 3*((7*a*b^3 - 5*b^4)*
tan(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)^3)*sqrt(-b/a)*log((b^2
*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2
*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^
2)))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^
4*b^2)*f*tan(f*x + e)^3), 1/12*(12*a^3*b*f*x*tan(f*x + e)^5 + 12*a^4*f*x*t
an(f*x + e)^3 + 6*(2*a^3*b - 7*a*b^3 + 5*b^4)*tan(f*x + e)^4 - 4*a^4 + 8*a
^3*b - 4*a^2*b^2 + 4*(3*a^4 - a^3*b - 7*a^2*b^2 + 5*a*b^3)*tan(f*x + e)^2
- 3*((7*a*b^3 - 5*b^4)*tan(f*x + e)^5 + (7*a^2*b^2 - 5*a*b^3)*tan(f*x + e)
^3)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))
))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*
b^2)*f*tan(f*x + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{\frac{3\,(7\,ab^3-5\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-2\,a^4b+a^3b^2)\sqrt{ab}} - \frac{3\,(2\,a^2b+2\,ab^2-5\,b^3)\tan(fx+e)^4-2\,a^3+2\,a^2b+2\,(3\,a^3+2\,a^2b-5\,ab^2)\tan(fx+e)^2}{(a^4b-a^3b^2)\tan(fx+e)^5+(a^5-a^4b)\tan(fx+e)^3} - \frac{6\,(fx+e)}{a^2-2\,ab+b^2}}{6\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(7*a*b^3 - 5*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5 - 2*a^4*b
 + a^3*b^2)*sqrt(a*b)) - (3*(2*a^2*b + 2*a*b^2 - 5*b^3)*tan(f*x + e)^4 - 2
*a^3 + 2*a^2*b + 2*(3*a^3 + 2*a^2*b - 5*a*b^2)*tan(f*x + e)^2)/((a^4*b - a
^3*b^2)*tan(f*x + e)^5 + (a^5 - a^4*b)*tan(f*x + e)^3) - 6*(f*x + e)/(a^2
- 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{b^3\tan\left(fx+e\right)}{2\left(a^4f-a^3bf\right)\left(b\tan\left(fx+e\right)^2+a\right)} - \frac{\left(7\,ab^3-5\,b^4\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\left(a^5f-2\,a^4bf+a^3b^2f\right)\sqrt{ab}} + \frac{fx+e}{a^2f-2\,abf+b^2f} + \frac{3\,a\tan\left(fx+e\right)^2+6\,b\tan\left(fx+e\right)^2-a}{3\,a^3f\tan\left(fx+e\right)^3}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*b^3*tan(f*x + e)/((a^4*f - a^3*b*f)*(b*tan(f*x + e)^2 + a)) - 1/2*(7*
a*b^3 - 5*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5*f - 2*a^4*b*f + a^3*
b^2*f)*sqrt(a*b)) + (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) + 1/3*(3*a*tan(f*x
 + e)^2 + 6*b*tan(f*x + e)^2 - a)/(a^3*f*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 2000, normalized size of antiderivative = 11.83

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
(2*atan((2*tan(e + f*x)*(2320*a^10*b^11 - 400*a^9*b^12 - 5344*a^11*b^10 +
6112*a^12*b^9 - 3472*a^13*b^8 + 784*a^14*b^7 - 64*a^15*b^6 + 192*a^16*b^5
- 192*a^17*b^4 + 64*a^18*b^3 + (256*a^15*b^10 - 1536*a^16*b^9 + 3584*a^17*
b^8 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21*b^4 + 1536*a^22*b^3 - 256*
a^23*b^2)/(2*a^2 - 4*a*b + 2*b^2)^2))/((2*a^2 - 4*a*b + 2*b^2)*((2*(320*a^
12*b^11 - 2048*a^13*b^10 + 5440*a^14*b^9 - 7680*a^15*b^8 + 6208*a^16*b^7 -
 3200*a^17*b^6 + 1728*a^18*b^5 - 1280*a^19*b^4 + 640*a^20*b^3 - 128*a^21*b
^2))/(2*a^2 - 4*a*b + 2*b^2)^2 - 400*a^9*b^10 + 1520*a^10*b^9 - 1904*a^11*
b^8 + 624*a^12*b^7 + 384*a^13*b^6 - 224*a^14*b^5))))/(f*(2*a^2 - 4*a*b + 2
*b^2)) + ((tan(e + f*x)^2*(3*a + 5*b))/(3*a^2) - 1/(3*a) + (tan(e + f*x)^4
*(2*a*b^2 + 2*a^2*b - 5*b^3))/(2*a^3*(a - b)))/(f*(a*tan(e + f*x)^3 + b*ta
n(e + f*x)^5)) + (atan((((tan(e + f*x)*(400*a^9*b^12 - 2320*a^10*b^11 + 53
44*a^11*b^10 - 6112*a^12*b^9 + 3472*a^13*b^8 - 784*a^14*b^7 + 64*a^15*b^6
- 192*a^16*b^5 + 192*a^17*b^4 - 64*a^18*b^3) + ((7*a - 5*b)*(-a^7*b^5)^(1/
2)*(2048*a^13*b^10 - 320*a^12*b^11 - 5440*a^14*b^9 + 7680*a^15*b^8 - 6208*
a^16*b^7 + 3200*a^17*b^6 - 1728*a^18*b^5 + 1280*a^19*b^4 - 640*a^20*b^3 +
128*a^21*b^2 + (tan(e + f*x)*(7*a - 5*b)*(-a^7*b^5)^(1/2)*(256*a^15*b^10 -
 1536*a^16*b^9 + 3584*a^17*b^8 - 3584*a^18*b^7 + 3584*a^20*b^5 - 3584*a^21
*b^4 + 1536*a^22*b^3 - 256*a^23*b^2))/(4*(a^9 - 2*a^8*b + a^7*b^2))))/(4*(
a^9 - 2*a^8*b + a^7*b^2)))*(7*a - 5*b)*(-a^7*b^5)^(1/2)*1i)/(4*(a^9 - 2...
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \int \frac{\cot(fx+e)^4}{(\tan(fx+e)^2 b+a)^2}dx$$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^2,x)
```

3.236 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 218

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = -\frac{x}{(a-b)^2}+\frac{(9a-7b)b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{2a^{9/2}(a-b)^2f} - \frac{(2a^3+2a^2b+2ab^2-7b^3)\cot(e+fx)}{2a^4(a-b)f} + \frac{(2a^2+2ab-7b^2)\cot^3(e+fx)}{6a^3(a-b)f} - \frac{(2a-7b)\cot^5(e+fx)}{10a^2(a-b)f} - \frac{b\cot^5(e+fx)}{2a(a-b)f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-x/(a-b)^2+1/2*(9*a-7*b)*b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(9/2
)/(a-b)^2/f-1/2*(2*a^3+2*a^2*b+2*a*b^2-7*b^3)*cot(f*x+e)/a^4/(a-b)/f+1/6*(
2*a^2+2*a*b-7*b^2)*cot(f*x+e)^3/a^3/(a-b)/f-1/10*(2*a-7*b)*cot(f*x+e)^5/a^
2/(a-b)/f-1/2*b*cot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 3.50 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.76

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{15(9a-7b)b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{9/2}(a-b)^2} - \frac{2\cot(e+fx)(23a^2+40ab+45b^2-a(11a+10b)\csc^2(e+fx)+3a^2\csc^4(e+fx))}{a^4} + \frac{15\left(-2(e+fx)+\frac{}{a^4(}\right.}{(}}{30f}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((15*(9*a - 7*b)*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(9/2)*
(a - b)^2) - (2*Cot[e + f*x]*(23*a^2 + 40*a*b + 45*b^2 - a*(11*a + 10*b)*C
sc[e + f*x]^2 + 3*a^2*Csc[e + f*x]^4))/a^4 + (15*(-2*(e + f*x) + ((a - b)*
b^4*Sin[2*(e + f*x)])/(a^4*(a + b + (a - b)*Cos[2*(e + f*x)]))))/(a - b)^2
)/(30*f)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4153, 374, 445, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 374

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-7b\tan^2(e+fx)+2a-7b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b\cot^5(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 445

$$\frac{\frac{-\frac{\int \frac{5\cot^4(e+fx)\left(2a^2+2ba-7b^2+(2a-7b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{5a}-\frac{(2a-7b)\cot^5(e+fx)}{5a}}{2a(a-b)}-\frac{b\cot^5(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{\int \frac{\cot^4(e+fx)\left(2a^2+2ba-7b^2+(2a-7b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{(2a-7b)\cot^5(e+fx)}{5a}}{2a(a-b)}-\frac{b\cot^5(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{f}$$

$\Big\downarrow$ 445

$$\frac{\frac{-\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(2a^3+2ba^2+2b^2a-7b^3+b\left(2a^2+2ba-7b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{3a}-\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{3a}}{a}-\frac{(2a-7b)\cot^5(e+fx)}{5a}}{2a(a-b)}-\frac{b\cot}{2a(a-b)(a+}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2a^3+2ba^2+2b^2a-7b^3+b\left(2a^2+2ba-7b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{3a}}{a}-\frac{(2a-7b)\cot^5(e+fx)}{5a}}{2a(a-b)}-\frac{b\cot^5(}{2a(a-b)(a+b}}{f}$$

$\Big\downarrow$ 445

$$\frac{\frac{-\frac{-\frac{-\frac{\int \frac{2a^4+2ba^3+2b^2a^2+2b^3a-7b^4+b\left(2a^3+2ba^2+2b^2a-7b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{a}}{a}-\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{3a}-}{a}}{2a(a-b)}}{f}$$

$\Big\downarrow$ 397

$$-\frac{-\frac{-\frac{\frac{2a^4\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^4(9a-7b)\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}}{a}-\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{a}}{a}-\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{3a}-\frac{(2a}{}}{2a(a-b)}}{f}$$

$\Big\downarrow$ 216

$$-\frac{-\frac{-\frac{\frac{2a^4\arctan(\tan(e+fx))}{a-b}-\frac{b^4(9a-7b)\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}}{a}-\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{a}}{a}-\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{3a}-\frac{(2a-7b)\cot^5(e}{5a}}{2a(a-b)}}{f}$$

$\Big\downarrow$ 218

$$-\frac{-\frac{\left(2a^2+2ab-7b^2\right)\cot^3(e+fx)}{3a}-\frac{-\frac{\frac{2a^4\arctan(\tan(e+fx))}{a-b}-\frac{b^{7/2}(9a-7b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}}{a}-\frac{\left(2a^3+2a^2b+2ab^2-7b^3\right)\cot(e+fx)}{a}}{a}-\frac{(2a-7b)\cot^5(e+fx}{5a}}{2a(a-b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^2,x]
```

output
```
((-1/5*((2*a - 7*b)*Cot[e + f*x]^5)/a - (-1/3*((2*a^2 + 2*a*b - 7*b^2)*Cot
[e + f*x]^3)/a - (-(((2*a^4*ArcTan[Tan[e + f*x]])/(a - b) - ((9*a - 7*b)*b
^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/a) - ((2
*a^3 + 2*a^2*b + 2*a*b^2 - 7*b^3)*Cot[e + f*x])/a)/a)/a)/(2*a*(a - b)) - (
b*Cot[e + f*x]^5)/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{b^4\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(9a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4(a-b)^2}-\frac{1}{5a^2\tan(fx+e)^5}-\frac{-2b-a}{3a^3\tan(fx+e)^3}-\frac{a^2+2ab}{a^4\tan(}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^2}+\frac{b^4\left(\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(fx+e)}{a+b\tan(fx+e)^2}+\frac{(9a-7b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4(a-b)^2}-\frac{1}{5a^2\tan(fx+e)^5}-\frac{-2b-a}{3a^3\tan(fx+e)^3}-\frac{a^2+2ab}{a^4\tan(}}{f}$
risch	$-\frac{x}{a^2-2ab+b^2}-\frac{i(-58a^4b+205a\,b^4+46a^5-105b^5+90a^5e^{12i(fx+e)}-1575b^5e^{8i(fx+e)}-105b^5e^{12i(fx+e)}+2100b^5e^{6i(}}{}$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)^2*arctan(tan(f*x+e))+b^4/a^4/(a-b)^2*((1/2*a-1/2*b)*tan(f*x+
e)/(a+b*tan(f*x+e)^2)+1/2*(9*a-7*b)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^
(1/2)))-1/5/a^2/tan(f*x+e)^5-1/3*(-2*b-a)/a^3/tan(f*x+e)^3-(a^2+2*a*b+3*b^
2)/a^4/tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 672, normalized size of antiderivative = 3.08

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/120*(120*a^4*b*f*x*tan(f*x + e)^7 + 120*a^5*f*x*tan(f*x + e)^5 + 60*(2
*a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 24*a^5 - 48*a^4*b + 24*a^3*b^2
+ 40*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4 - 8*(5*a^5 - 3*a
^4*b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 + 15*((9*a*b^4 - 7*b^5)*tan(f
*x + e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(
f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(
f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/
((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2
)*f*tan(f*x + e)^5), -1/60*(60*a^4*b*f*x*tan(f*x + e)^7 + 60*a^5*f*x*tan(f
*x + e)^5 + 30*(2*a^4*b - 9*a*b^4 + 7*b^5)*tan(f*x + e)^6 + 12*a^5 - 24*a^
4*b + 12*a^3*b^2 + 20*(3*a^5 - a^4*b - 9*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^4
 - 4*(5*a^5 - 3*a^4*b - 9*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2 - 15*((9*a*b
^4 - 7*b^5)*tan(f*x + e)^7 + (9*a^2*b^3 - 7*a*b^4)*tan(f*x + e)^5)*sqrt(b/
a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f*x + e))))/((a^6*b
- 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(
f*x + e)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{\frac{15\,(9\,ab^4-7\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^6-2\,a^5b+a^4b^2)\sqrt{ab}} - \frac{15\,(2\,a^3b+2\,a^2b^2+2\,ab^3-7\,b^4)\tan(fx+e)^6+10\,(3\,a^4+2\,a^3b+2\,a^2b^2-7\,ab^3)\tan(fx+e)^4+6\,a^4-6\,a^3b-}{(a^5b-a^4b^2)\tan(fx+e)^7+(a^6-a^5b)\tan(fx+e)^5}}{30\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/30*(15*(9*a*b^4 - 7*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^6 - 2*a^5*
b + a^4*b^2)*sqrt(a*b)) - (15*(2*a^3*b + 2*a^2*b^2 + 2*a*b^3 - 7*b^4)*tan(
f*x + e)^6 + 10*(3*a^4 + 2*a^3*b + 2*a^2*b^2 - 7*a*b^3)*tan(f*x + e)^4 + 6
*a^4 - 6*a^3*b - 2*(5*a^4 + 2*a^3*b - 7*a^2*b^2)*tan(f*x + e)^2)/((a^5*b -
 a^4*b^2)*tan(f*x + e)^7 + (a^6 - a^5*b)*tan(f*x + e)^5) - 30*(f*x + e)/(a
^2 - 2*a*b + b^2))/f
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^2}\,dx = \frac{b^4\tan(fx+e)}{2\,(a^5f-a^4bf)\left(b\tan(fx+e)^2+a\right)} + \frac{(9\,ab^4-7\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{2\,(a^6f-2\,a^5bf+a^4b^2f)\sqrt{ab}} - \frac{fx+e}{a^2f-2\,abf+b^2f} - \frac{15\,a^2\tan(fx+e)^4+30\,ab\tan(fx+e)^4+45\,b^2\tan(fx+e)^4-5\,a^2\tan(fx+e)^2-10\,ab\tan(fx+}{15\,a^4f\tan(fx+e)^5}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*b^4*tan(f*x + e)/((a^5*f - a^4*b*f)*(b*tan(f*x + e)^2 + a)) + 1/2*(9*a
*b^4 - 7*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^6*f - 2*a^5*b*f + a^4*b
^2*f)*sqrt(a*b)) - (f*x + e)/(a^2*f - 2*a*b*f + b^2*f) - 1/15*(15*a^2*tan(
f*x + e)^4 + 30*a*b*tan(f*x + e)^4 + 45*b^2*tan(f*x + e)^4 - 5*a^2*tan(f*x
 + e)^2 - 10*a*b*tan(f*x + e)^2 + 3*a^2)/(a^4*f*tan(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 11.52 (sec) , antiderivative size = 3030, normalized size of antiderivative = 13.90

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^2,x)
```

output
```
- (1/(5*a) + (tan(e + f*x)^4*(5*a*b + 3*a^2 + 7*b^2))/(3*a^3) - (tan(e + f
*x)^2*(5*a + 7*b))/(15*a^2) + (tan(e + f*x)^6*(2*a*b^3 + 2*a^3*b - 7*b^4 +
 2*a^2*b^2))/(2*a^4*(a - b)))/(f*(a*tan(e + f*x)^5 + b*tan(e + f*x)^7)) -
(2*atan(((tan(e + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 -
10720*a^15*b^11 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*
b^5 + 192*a^22*b^4 - 64*a^23*b^3) + ((2816*a^17*b^11 - 448*a^16*b^12 - 736
0*a^18*b^10 + 10240*a^19*b^9 - 8000*a^20*b^8 + 3200*a^21*b^7 + 64*a^22*b^6
 - 1280*a^23*b^5 + 1280*a^24*b^4 - 640*a^25*b^3 + 128*a^26*b^2 + (tan(e +
f*x)*(256*a^20*b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584
*a^25*b^5 - 3584*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4*a
*b + 2*b^2))*1i)/(2*a^2 - 4*a*b + 2*b^2))/(2*a^2 - 4*a*b + 2*b^2) + (tan(e
 + f*x)*(784*a^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11
 + 5904*a^16*b^10 - 1296*a^17*b^9 + 64*a^20*b^6 - 192*a^21*b^5 + 192*a^22*
b^4 - 64*a^23*b^3) + ((448*a^16*b^12 - 2816*a^17*b^11 + 7360*a^18*b^10 - 1
0240*a^19*b^9 + 8000*a^20*b^8 - 3200*a^21*b^7 - 64*a^22*b^6 + 1280*a^23*b^
5 - 1280*a^24*b^4 + 640*a^25*b^3 - 128*a^26*b^2 + (tan(e + f*x)*(256*a^20*
b^10 - 1536*a^21*b^9 + 3584*a^22*b^8 - 3584*a^23*b^7 + 3584*a^25*b^5 - 358
4*a^26*b^4 + 1536*a^27*b^3 - 256*a^28*b^2)*1i)/(2*a^2 - 4*a*b + 2*b^2))*1i
)/(2*a^2 - 4*a*b + 2*b^2))/(2*a^2 - 4*a*b + 2*b^2))/(((tan(e + f*x)*(784*a
^12*b^14 - 4368*a^13*b^13 + 9696*a^14*b^12 - 10720*a^15*b^11 + 5904*a^1...
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^2}\,dx = \int \frac{\cot\left(fx+e\right)^6}{\left(\tan\left(fx+e\right)^2 b+a\right)^2}dx$$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

output
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^2,x)
```

3.237 $\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3} dx = -\frac{\log(a\cos^2(e+fx)+b\sin^2(e+fx))}{2(a-b)^3f} + \frac{a^2}{4(a-b)b^2f(a+b\tan^2(e+fx))^2} - \frac{a(a-2b)}{2(a-b)^2b^2f(a+b\tan^2(e+fx))}$$

output
```
-1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f+1/4*a^2/(a-b)/b^2/f/(a+b*
tan(f*x+e)^2)^2-1/2*a*(a-2*b)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3} dx = \frac{-4\log(\cos(e+fx)) - 2\log(a+b\tan^2(e+fx)) + \frac{a^2(a-b)^2}{b^2(a+b\tan^2(e+fx))^2} - \frac{2a(a-2b)(a-b)}{b^2(a+b\tan^2(e+fx))}}{4(a-b)^3f}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a^2*(a - b)^2)/(b^2
*(a + b*Tan[e + f*x]^2)^2) - (2*a*(a - 2*b)*(a - b))/(b^2*(a + b*Tan[e + f
*x]^2)))/(4*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^5(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{a^2}{(a-b)b(b\tan^2(e+fx)+a)^3}+\frac{(a-2b)a}{(a-b)^2b(b\tan^2(e+fx)+a)^2}+\frac{1}{(a-b)^3(\tan^2(e+fx)+1)}+\frac{b}{(b-a)^3(b\tan^2(e+fx)+a)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2}{2b^2(a-b)(a+b\tan^2(e+fx))^2}-\frac{a(a-2b)}{b^2(a-b)^2(a+b\tan^2(e+fx))}+\frac{\log(\tan^2(e+fx)+1)}{(a-b)^3}-\frac{\log(a+b\tan^2(e+fx))}{(a-b)^3}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(Log[1 + Tan[e + f*x]^2]/(a - b)^3 - Log[a + b*Tan[e + f*x]^2]/(a - b)^3 +
 a^2/(2*(a - b)*b^2*(a + b*Tan[e + f*x]^2)^2) - (a*(a - 2*b))/((a - b)^2*b
^2*(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{-\ln\left(a+b\tan(fx+e)^{2}\right)-\frac{a\left(a^{2}-3ab+2b^{2}\right)}{b^{2}\left(a+b\tan(fx+e)^{2}\right)}+\frac{a^{2}\left(a^{2}-2ab+b^{2}\right)}{2b^{2}\left(a+b\tan(fx+e)^{2}\right)^{2}}}{2(a-b)^{3}}+\frac{\ln\left(1+\tan(fx+e)^{2}\right)}{2(a-b)^{3}}}{f}$
default	$\frac{\frac{-\ln\left(a+b\tan(fx+e)^{2}\right)-\frac{a\left(a^{2}-3ab+2b^{2}\right)}{b^{2}\left(a+b\tan(fx+e)^{2}\right)}+\frac{a^{2}\left(a^{2}-2ab+b^{2}\right)}{2b^{2}\left(a+b\tan(fx+e)^{2}\right)^{2}}}{2(a-b)^{3}}+\frac{\ln\left(1+\tan(fx+e)^{2}\right)}{2(a-b)^{3}}}{f}$
norman	$\frac{\frac{(-a+3b)a^{2}}{4b^{2}\left(a^{2}-2ab+b^{2}\right)f}+\frac{a(-a+2b)\tan(fx+e)^{2}}{2b\left(a^{2}-2ab+b^{2}\right)f}}{\left(a+b\tan(fx+e)^{2}\right)^{2}}+\frac{\ln\left(1+\tan(fx+e)^{2}\right)}{2f(a^{3}-3a^{2}b+3ab^{2}-b^{3})}-\frac{\ln\left(a+b\tan(fx+e)^{2}\right)}{2f(a^{3}-3a^{2}b+3ab^{2}-b^{3})}$
parallelrisch	$\frac{2\ln\left(1+\tan(fx+e)^{2}\right)\tan(fx+e)^{4}b^{4}-2\ln\left(a+b\tan(fx+e)^{2}\right)\tan(fx+e)^{4}b^{4}+4\ln\left(1+\tan(fx+e)^{2}\right)\tan(fx+e)^{2}ab^{3}-4}{}$
risch	$\frac{ix}{a^{3}-3a^{2}b+3ab^{2}-b^{3}}+\frac{2ie}{f(a^{3}-3a^{2}b+3ab^{2}-b^{3})}-\frac{4a(a\,e^{6i(fx+e)}-b\,e^{6i(fx+e)}+a\,e^{4i(fx+e)}+2b\,e^{4i(fx+e)}+a\,e^{2i(fx+}}{(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a-b)}$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a-b)^3*(-ln(a+b*tan(f*x+e)^2)-a*(a^2-3*a*b+2*b^2)/b^2/(a+b*tan(f
*x+e)^2)+1/2*a^2*(a^2-2*a*b+b^2)/b^2/(a+b*tan(f*x+e)^2)^2)+1/2/(a-b)^3*ln(
1+tan(f*x+e)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(102) = 204.

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.91

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{(a^2-4\,ab)\tan\left(fx+e\right)^4-2\left(a^2+2\,ab\right)\tan\left(fx+e\right)^2-3\,a^2-2\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2-\right.}{4\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan\left(fx+e\right)^4+2\left(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4\right)f\tan\left(fx+e\right)^2+\left(a^5-3\right.\right.}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*((a^2 - 4*a*b)*tan(f*x + e)^4 - 2*(a^2 + 2*a*b)*tan(f*x + e)^2 - 3*a^2
 - 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)
^2 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*ta
n(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2
+ (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3315 vs. $2(87) = 174$.

Time = 47.18 (sec) , antiderivative size = 3315, normalized size of antiderivative = 30.69

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(e), Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((log(tan(e + f*
x)**2 + 1)/(2*f) + tan(e + f*x)**4/(4*f) - tan(e + f*x)**2/(2*f))/a**3, Eq
(b, 0)), (-3*tan(e + f*x)**4/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*f*tan(e +
 f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f) - 3*tan(e + f*x)**2/(6*b*
*3*f*tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)*
*2 + 6*b**3*f) - 1/(6*b**3*f*tan(e + f*x)**6 + 18*b**3*f*tan(e + f*x)**4 +
 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f), Eq(a, b)), (x*tan(e)**5/(a + b*tan
(e)**2)**3, Eq(f, 0)), (-a**4/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)*
*2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e
 + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**
5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b*
*6*f*tan(e + f*x)**2 - 4*b**7*f*tan(e + f*x)**4) - 2*a**3*b*tan(e + f*x)**
2/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x)**2 - 12*a**4*b**3*f + 4*a**3
*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan(e + f*x)**2 + 12*a**3*b**4*f
- 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b**5*f*tan(e + f*x)**2 - 4*a**2
*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8*a*b**6*f*tan(e + f*x)**2 - 4*b**
7*f*tan(e + f*x)**4) + 4*a**3*b/(4*a**5*b**2*f + 8*a**4*b**3*f*tan(e + f*x
)**2 - 12*a**4*b**3*f + 4*a**3*b**4*f*tan(e + f*x)**4 - 24*a**3*b**4*f*tan
(e + f*x)**2 + 12*a**3*b**4*f - 12*a**2*b**5*f*tan(e + f*x)**4 + 24*a**2*b
**5*f*tan(e + f*x)**2 - 4*a**2*b**5*f + 12*a*b**6*f*tan(e + f*x)**4 - 8...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.75

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\frac{4\left(a^2-ab\right)\sin(fx+e)^2-3a^2}{a^5-3a^4b+3a^3b^2-a^2b^3+\left(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5\right)\sin(fx+e)^4-2\left(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4\right)\sin(fx+e)^2}-\frac{2\log\left(-(a-b\right.}{a^3-3a^2}}{4f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*(a^2 - a*b)*sin(f*x + e)^2 - 3*a^2)/(a^5 - 3*a^4*b + 3*a^3*b^2 - a
^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin(f*x
 + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x + e)^2
) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(102) = 204$.

Time = 0.68 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.93

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{b\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(a^3bf-3\,a^2b^2f+3\,ab^3f-b^4f\right)}+\frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(a^3f-3\,a^2bf+3\,ab^2f-b^3f\right)} + \frac{3\,b^4\tan\left(fx+e\right)^4-2\,a^3b\tan\left(fx+e\right)^2+6\,a^2b^2\tan\left(fx+e\right)^2+2\,ab^3\tan\left(fx+e\right)^2-a^4+4\,a^3b}{4\left(a^3b^2f-3\,a^2b^3f+3\,ab^4f-b^5f\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/2*b*log(abs(b*tan(f*x + e)^2 + a))/(a^3*b*f - 3*a^2*b^2*f + 3*a*b^3*f -
 b^4*f) + 1/2*log(tan(f*x + e)^2 + 1)/(a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3
*f) + 1/4*(3*b^4*tan(f*x + e)^4 - 2*a^3*b*tan(f*x + e)^2 + 6*a^2*b^2*tan(f
*x + e)^2 + 2*a*b^3*tan(f*x + e)^2 - a^4 + 4*a^3*b)/((a^3*b^2*f - 3*a^2*b^
3*f + 3*a*b^4*f - b^5*f)*(b*tan(f*x + e)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 577, normalized size of antiderivative = 5.34

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{a^3\,b\cos\left(e+f\,x\right)^4-\frac{a^4\cos(e+f\,x)^4}{4}-\frac{3\,a^2\,b^2\cos(e+f\,x)^4}{4}+b^4\sin\left(e+f\,x\right)^4\mathrm{atan}\left(\frac{a\sin(e+f\,x)^2-b\sin(e+f\,x}{a\cos(e+f\,x)^2\,2\mathrm{i}+a\sin(e+f\,x)^2\,1\mathrm{i}+b}\right.}{f\,\left(-a^5\,b^2\cos\left(e+f\,x\right)^4+3\,a^4\,b^3\cos\left(e+f\,x\right)^4-2\,a^4\,b^3\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-\right.}$$

input
```
int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
-(a^3*b*cos(e + f*x)^4 - (a^4*cos(e + f*x)^4)/4 - (3*a^2*b^2*cos(e + f*x)^
4)/4 + b^4*sin(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*co
s(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i - a*b^3*c
os(e + f*x)^2*sin(e + f*x)^2 - (a^3*b*cos(e + f*x)^2*sin(e + f*x)^2)/2 + a
^2*b^2*cos(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e
+ f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i + (3*a^2*b^2*
cos(e + f*x)^2*sin(e + f*x)^2)/2 + a*b^3*cos(e + f*x)^2*sin(e + f*x)^2*ata
n((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f
*x)^2*1i + b*sin(e + f*x)^2*1i))*2i)/(f*(b^7*sin(e + f*x)^4 - 3*a*b^6*sin(
e + f*x)^4 + a^2*b^5*cos(e + f*x)^4 - 3*a^3*b^4*cos(e + f*x)^4 + 3*a^4*b^3
*cos(e + f*x)^4 - a^5*b^2*cos(e + f*x)^4 + 3*a^2*b^5*sin(e + f*x)^4 - a^3*
b^4*sin(e + f*x)^4 + 2*a*b^6*cos(e + f*x)^2*sin(e + f*x)^2 - 6*a^2*b^5*cos
(e + f*x)^2*sin(e + f*x)^2 + 6*a^3*b^4*cos(e + f*x)^2*sin(e + f*x)^2 - 2*a
^4*b^3*cos(e + f*x)^2*sin(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.20

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{2\log\left(\tan(fx+e)^2+1\right)\tan(fx+e)^4 b^2+4\log\left(\tan(fx+e)^2+1\right)\tan(fx+e)^2 ab+2\log\left(\tan(fx+ \right.\cdots}{4f\left(\tan(fx+e)^4 a^3b^2-3\tan(fx+ \cdots\right.}$$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4*b**2 + 4*log(tan(e + f*x)**2 +
 1)*tan(e + f*x)**2*a*b + 2*log(tan(e + f*x)**2 + 1)*a**2 - 2*log(tan(e +
f*x)**2*b + a)*tan(e + f*x)**4*b**2 - 4*log(tan(e + f*x)**2*b + a)*tan(e +
 f*x)**2*a*b - 2*log(tan(e + f*x)**2*b + a)*a**2 + tan(e + f*x)**4*a**2 -
4*tan(e + f*x)**4*a*b + 3*tan(e + f*x)**4*b**2 - 2*tan(e + f*x)**2*a**2 +
2*tan(e + f*x)**2*a*b)/(4*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4
*a**2*b**3 + 3*tan(e + f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f
*x)**2*a**4*b - 6*tan(e + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3
- 2*tan(e + f*x)**2*a*b**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.238 $\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} - \frac{a}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{1}{2(a-b)^2f\left(a+b\tan^2(e+fx)\right)}$$

output
```
1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f-1/4*a/(a-b)/b/f/(a+b*tan(f
*x+e)^2)^2-1/2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{4\log(\cos(e+fx))+2\log\left(a+b\tan^2(e+fx)\right)-\frac{a(a-b)^2}{b(a+b\tan^2(e+fx))^2}-\frac{2(a-b)}{a+b\tan^2(e+fx)}}{4(a-b)^3f}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(4*Log[Cos[e + f*x]] + 2*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)^2)/(b*(a +
 b*Tan[e + f*x]^2)^2) - (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f
)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^3(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^3}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{a}{(a-b)\left(b\tan^2(e+fx)+a\right)^3}-\frac{1}{(a-b)^3\left(\tan^2(e+fx)+1\right)}+\frac{b}{(a-b)^3\left(b\tan^2(e+fx)+a\right)}+\frac{b}{(a-b)^2\left(b\tan^2(e+fx)+a\right)^2}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a}{2b(a-b)(a+b\tan^2(e+fx))^2}-\frac{1}{(a-b)^2(a+b\tan^2(e+fx))}-\frac{\log(\tan^2(e+fx)+1)}{(a-b)^3}+\frac{\log(a+b\tan^2(e+fx))}{(a-b)^3}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-(Log[1 + Tan[e + f*x]^2]/(a - b)^3) + Log[a + b*Tan[e + f*x]^2]/(a - b)^
3 - a/(2*(a - b)*b*(a + b*Tan[e + f*x]^2)^2) - 1/((a - b)^2*(a + b*Tan[e +
 f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\ln\left(a+b\tan(fx+e)^2\right)-\frac{a-b}{a+b\tan(fx+e)^2}-\frac{a\left(a^2-2ab+b^2\right)}{2b\left(a+b\tan(fx+e)^2\right)^2}}{2(a-b)^3}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}}{f}$
default	$\frac{\frac{\ln\left(a+b\tan(fx+e)^2\right)-\frac{a-b}{a+b\tan(fx+e)^2}-\frac{a\left(a^2-2ab+b^2\right)}{2b\left(a+b\tan(fx+e)^2\right)^2}}{2(a-b)^3}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}}{f}$
norman	$\frac{-\frac{b\tan(fx+e)^2}{2\left(a^2-2ab+b^2\right)f}+\frac{\left(-ab-b^2\right)a}{4b^2\left(a^2-2ab+b^2\right)f}}{\left(a+b\tan(fx+e)^2\right)^2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^3-3a^2b+3ab^2-b^3)}+\frac{\ln\left(a+b\tan(fx+e)^2\right)}{2f(a^3-3a^2b+3ab^2-b^3)}$
parallelrisch	$-\frac{2\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^4b^4-2\ln\left(a+b\tan(fx+e)^2\right)\tan(fx+e)^4b^4+4\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^2ab^3}{4(a^3-3}$
risch	$-\frac{ix}{a^3-3a^2b+3ab^2-b^3}-\frac{2ie}{f(a^3-3a^2b+3ab^2-b^3)}+\frac{2a^2e^{6i(fx+e)}-2b^2e^{6i(fx+e)}+4a^2e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}+4b^2}{\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}\right.}$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a-b)^3*(ln(a+b*tan(f*x+e)^2)-(a-b)/(a+b*tan(f*x+e)^2)-1/2*a*(a^2
-2*a*b+b^2)/b/(a+b*tan(f*x+e)^2)^2)-1/2/(a-b)^3*ln(1+tan(f*x+e)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(91) = 182.

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.19

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$=\frac{(ab+2b^2)\tan(fx+e)^4+2(a^2+ab+b^2)\tan(fx+e)^2+2a^2+ab+2\left(b^2\tan(fx+e)^4+2ab\tan(f\right.}{4\left((a^3b^2-3a^2b^3+3ab^4-b^5)f\tan(fx+e)^4+2(a^4b-3a^3b^2+3a^2b^3-ab^4)f\tan(fx+e)^2+(a^5\right.}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*((a*b + 2*b^2)*tan(f*x + e)^4 + 2*(a^2 + a*b + b^2)*tan(f*x + e)^2 + 2
*a^2 + a*b + 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*ta
n(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 -
 b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f
*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2819 vs. $2(75) = 150$.

Time = 46.78 (sec) , antiderivative size = 2819, normalized size of antiderivative = 29.06

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-log(tan(e
+ f*x)**2 + 1)/(2*f) + tan(e + f*x)**2/(2*f))/a**3, Eq(b, 0)), (-3*tan(e +
 f*x)**2/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*tan(e + f*x)**4 + 36*b**3*
f*tan(e + f*x)**2 + 12*b**3*f) - 1/(12*b**3*f*tan(e + f*x)**6 + 36*b**3*f*
tan(e + f*x)**4 + 36*b**3*f*tan(e + f*x)**2 + 12*b**3*f), Eq(a, b)), (x*ta
n(e)**3/(a + b*tan(e)**2)**3, Eq(f, 0)), (-a**3/(4*a**5*b*f + 8*a**4*b**2*
f*tan(e + f*x)**2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a*
*3*b**3*f*tan(e + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**
4 + 24*a**2*b**4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f
*x)**4 - 8*a*b**5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b
*log(-sqrt(-a/b) + tan(e + f*x))/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)*
*2 - 12*a**4*b**2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e
 + f*x)**2 + 12*a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**
4*f*tan(e + f*x)**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b*
*5*f*tan(e + f*x)**2 - 4*b**6*f*tan(e + f*x)**4) + 2*a**2*b*log(sqrt(-a/b)
 + tan(e + f*x))/(4*a**5*b*f + 8*a**4*b**2*f*tan(e + f*x)**2 - 12*a**4*b**
2*f + 4*a**3*b**3*f*tan(e + f*x)**4 - 24*a**3*b**3*f*tan(e + f*x)**2 + 12*
a**3*b**3*f - 12*a**2*b**4*f*tan(e + f*x)**4 + 24*a**2*b**4*f*tan(e + f*x)
**2 - 4*a**2*b**4*f + 12*a*b**5*f*tan(e + f*x)**4 - 8*a*b**5*f*tan(e + f*x
)**2 - 4*b**6*f*tan(e + f*x)**4) - 2*a**2*b*log(tan(e + f*x)**2 + 1)/(4...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(91) = 182$.

Time = 0.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.00

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{2\,(a^2-b^2)\sin(fx+e)^2-2\,a^2-ab}{a^5-3\,a^4b+3\,a^3b^2-a^2b^3+(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)\sin(fx+e)^4-2\,(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sin(fx+e)^2}-\frac{2\,\log\left(-(\cdots\right.}{a^3-\cdots}}{4\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((2*(a^2 - b^2)*sin(f*x + e)^2 - 2*a^2 - a*b)/(a^5 - 3*a^4*b + 3*a^3*
b^2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*
sin(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x
 + e)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3))/f
```

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{b\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\,(a^3bf-3\,a^2b^2f+3\,ab^3f-b^4f)} - \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\,(a^3f-3\,a^2bf+3\,ab^2f-b^3f)} - \frac{a^3-ab^2+2\,(ab^2-b^3)\tan\left(fx+e\right)^2}{4\,\left(b\tan\left(fx+e\right)^2+a\right)^2(a-b)^3bf}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/2*b*log(abs(b*tan(f*x + e)^2 + a))/(a^3*b*f - 3*a^2*b^2*f + 3*a*b^3*f -
b^4*f) - 1/2*log(tan(f*x + e)^2 + 1)/(a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3*
f) - 1/4*(a^3 - a*b^2 + 2*(a*b^2 - b^3)*tan(f*x + e)^2)/((b*tan(f*x + e)^2
 + a)^2*(a - b)^3*b*f)
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 532, normalized size of antiderivative = 5.48

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{a^3\cos(e+fx)^4}{4} - \frac{a\,b^2\cos(e+fx)^4}{4} + b^3\sin\left(e+f\,x\right)^4\,\mathrm{atan}\left(\frac{a\sin(e+fx)^2-b\sin(e+fx)^2}{a\cos(e+fx)^2\,2\mathrm{i}+a\sin(e+fx)^2\,1\mathrm{i}+b\sin(e+fx)^2\,1}\right.}{f\,\left(-a^5\,b\cos\left(e+f\,x\right)^4+3\,a^4\,b^2\cos\left(e+f\,x\right)^4-2\,a^4\,b^2\cos\left(e+f\,x\right)^2\sin\left(e+f\,x\right)^2-3\,a^3\,b^3\cos\left(e+f\,x\right.\right.}$$

input
```
int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
((a^3*cos(e + f*x)^4)/4 - (a*b^2*cos(e + f*x)^4)/4 + b^3*sin(e + f*x)^4*at
an((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e +
f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i - (b^3*cos(e + f*x)^2*sin(e + f*x)^2)
/2 + a^2*b*cos(e + f*x)^4*atan((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*co
s(e + f*x)^2*2i + a*sin(e + f*x)^2*1i + b*sin(e + f*x)^2*1i))*1i + (a*b^2*
cos(e + f*x)^2*sin(e + f*x)^2)/2 + a*b^2*cos(e + f*x)^2*sin(e + f*x)^2*ata
n((a*sin(e + f*x)^2 - b*sin(e + f*x)^2)/(a*cos(e + f*x)^2*2i + a*sin(e + f
*x)^2*1i + b*sin(e + f*x)^2*1i))*2i)/(f*(b^6*sin(e + f*x)^4 - a^5*b*cos(e
+ f*x)^4 - 3*a*b^5*sin(e + f*x)^4 + a^2*b^4*cos(e + f*x)^4 - 3*a^3*b^3*cos
(e + f*x)^4 + 3*a^4*b^2*cos(e + f*x)^4 + 3*a^2*b^4*sin(e + f*x)^4 - a^3*b^
3*sin(e + f*x)^4 + 2*a*b^5*cos(e + f*x)^2*sin(e + f*x)^2 - 6*a^2*b^4*cos(e
 + f*x)^2*sin(e + f*x)^2 + 6*a^3*b^3*cos(e + f*x)^2*sin(e + f*x)^2 - 2*a^4
*b^2*cos(e + f*x)^2*sin(e + f*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.57

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{-2\log\left(\tan\left(fx+e\right)^2+1\right)\tan\left(fx+e\right)^4a\,b^2-4\log\left(\tan\left(fx+e\right)^2+1\right)\tan\left(fx+e\right)^2a^2b-2\log\left(\tan\left(f\right.\right.}{4af\left(\tan\left(fx+e\right)^4a^3b^2-3\tan\left(fx+e\right)^4a^2\right.}$$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
( - 2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4*a*b**2 - 4*log(tan(e + f*x)
**2 + 1)*tan(e + f*x)**2*a**2*b - 2*log(tan(e + f*x)**2 + 1)*a**3 + 2*log(
tan(e + f*x)**2*b + a)*tan(e + f*x)**4*a*b**2 + 4*log(tan(e + f*x)**2*b +
a)*tan(e + f*x)**2*a**2*b + 2*log(tan(e + f*x)**2*b + a)*a**3 + tan(e + f*
x)**4*a**2*b - tan(e + f*x)**4*b**3 + 2*tan(e + f*x)**2*a**3 - 2*tan(e + f
*x)**2*a**2*b)/(4*a*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4*a**2*
b**3 + 3*tan(e + f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f*x)**2
*a**4*b - 6*tan(e + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3 - 2*ta
n(e + f*x)**2*a*b**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.239 $\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\log\left(a\cos^2(e+fx)+b\sin^2(e+fx)\right)}{2(a-b)^3f} + \frac{1}{4(a-b)f\left(a+b\tan^2(e+fx)\right)^2} + \frac{1}{2(a-b)^2f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-1/2*ln(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(a-b)^3/f+1/4/(a-b)/f/(a+b*tan(f*x+
e)^2)^2+1/2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{-4\log(\cos(e+fx))-2\log\left(a+b\tan^2(e+fx)\right)+\frac{(a-b)^2}{(a+b\tan^2(e+fx))^2}+\frac{2(a-b)}{a+b\tan^2(e+fx)}}{4(a-b)^3f}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-4*Log[Cos[e + f*x]] - 2*Log[a + b*Tan[e + f*x]^2] + (a - b)^2/(a + b*Tan
[e + f*x]^2)^2 + (2*(a - b))/(a + b*Tan[e + f*x]^2))/(4*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4153, 353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{b}{(b-a)^3(b\tan^2(e+fx)+a)}-\frac{b}{(a-b)^2(b\tan^2(e+fx)+a)^2}-\frac{b}{(a-b)(b\tan^2(e+fx)+a)^3}+\frac{1}{(a-b)^3(\tan^2(e+fx)+1)}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{(a-b)^2(a+b\tan^2(e+fx))}+\frac{1}{2(a-b)(a+b\tan^2(e+fx))^2}+\frac{\log\left(\tan^2(e+fx)+1\right)}{(a-b)^3}-\frac{\log\left(a+b\tan^2(e+fx)\right)}{(a-b)^3}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(Log[1 + Tan[e + f*x]^2]/(a - b)^3 - Log[a + b*Tan[e + f*x]^2]/(a - b)^3 +
 1/(2*(a - b)*(a + b*Tan[e + f*x]^2)^2) + 1/((a - b)^2*(a + b*Tan[e + f*x]
^2)))/(2*f)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\ln\left(a+b\tan(fx+e)^2\right)}{b}-\frac{a-b}{b\left(a+b\tan(fx+e)^2\right)}-\frac{a^2-2ab+b^2}{2b\left(a+b\tan(fx+e)^2\right)^2}\right)}{2(a-b)^3}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\ln\left(a+b\tan(fx+e)^2\right)}{b}-\frac{a-b}{b\left(a+b\tan(fx+e)^2\right)}-\frac{a^2-2ab+b^2}{2b\left(a+b\tan(fx+e)^2\right)^2}\right)}{2(a-b)^3}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}}{f}$
norman	$\frac{\frac{3a\,b^2-b^3}{4b^2\left(a^2-2ab+b^2\right)f}+\frac{b\tan(fx+e)^2}{2\left(a^2-2ab+b^2\right)f}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^3-3a^2b+3a\,b^2-b^3)}-\frac{\ln\left(a+b\tan(fx+e)^2\right)}{2f(a^3-3a^2b+3a\,b^2-b^3)}$
parallelrisch	$\frac{2\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^4b^4-2\ln\left(a+b\tan(fx+e)^2\right)\tan(fx+e)^4b^4+4\ln\left(1+\tan(fx+e)^2\right)\tan(fx+e)^2a\,b^3-4}{4(a^3-}$
risch	$\frac{ix}{a^3-3a^2b+3a\,b^2-b^3}+\frac{2ie}{f(a^3-3a^2b+3a\,b^2-b^3)}-\frac{4b(-a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}-a\,e^{2i(fx}}{(-a\,e^{4i(fx+e)}+b\,e^{4i(fx+e)}-2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}-a+b}$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*b/(a-b)^3*(1/b*ln(a+b*tan(f*x+e)^2)-(a-b)/b/(a+b*tan(f*x+e)^2)-1
/2*(a^2-2*a*b+b^2)/b/(a+b*tan(f*x+e)^2)^2)+1/2/(a-b)^3*ln(1+tan(f*x+e)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(87) = 174$.

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.22

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx =$$

$$-\frac{3\,b^2\tan\left(fx+e\right)^4+2\left(2\,ab+b^2\right)\tan\left(fx+e\right)^2+4\,ab-b^2+2\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2\right.}{4\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan\left(fx+e\right)^4+2\left(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4\right)f\tan\left(fx+e\right)^2+\left(a^5\right.\right.}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(3*b^2*tan(f*x + e)^4 + 2*(2*a*b + b^2)*tan(f*x + e)^2 + 4*a*b - b^2
+ 2*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*log((b*tan(f*x + e)^
2 + a)/(tan(f*x + e)^2 + 1)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan
(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 +
 (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2846 vs. $2(73) = 146$.

Time = 46.45 (sec) , antiderivative size = 2846, normalized size of antiderivative = 30.60

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(e)**5, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(tan(e +
f*x)**2 + 1)/(2*a**3*f), Eq(b, 0)), (-1/(6*b**3*f*tan(e + f*x)**6 + 18*b**
3*f*tan(e + f*x)**4 + 18*b**3*f*tan(e + f*x)**2 + 6*b**3*f), Eq(a, b)), (x
*tan(e)/(a + b*tan(e)**2)**3, Eq(f, 0)), (-2*a**2*log(-sqrt(-a/b) + tan(e
+ f*x))/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2
*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*
a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3
*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*t
an(e + f*x)**4) - 2*a**2*log(sqrt(-a/b) + tan(e + f*x))/(4*a**5*f + 8*a**4
*b*f*tan(e + f*x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a*
*3*b**2*f*tan(e + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**
4 + 24*a**2*b**3*f*tan(e + f*x)**2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f
*x)**4 - 8*a*b**4*f*tan(e + f*x)**2 - 4*b**5*f*tan(e + f*x)**4) + 2*a**2*l
og(tan(e + f*x)**2 + 1)/(4*a**5*f + 8*a**4*b*f*tan(e + f*x)**2 - 12*a**4*b
*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e + f*x)**2 + 12*a
**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*b**3*f*tan(e + f*x)*
*2 - 4*a**2*b**3*f + 12*a*b**4*f*tan(e + f*x)**4 - 8*a*b**4*f*tan(e + f*x)
**2 - 4*b**5*f*tan(e + f*x)**4) + 3*a**2/(4*a**5*f + 8*a**4*b*f*tan(e + f*
x)**2 - 12*a**4*b*f + 4*a**3*b**2*f*tan(e + f*x)**4 - 24*a**3*b**2*f*tan(e
 + f*x)**2 + 12*a**3*b**2*f - 12*a**2*b**3*f*tan(e + f*x)**4 + 24*a**2*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(87) = 174$.

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.06

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{\frac{4\,(ab-b^2)\sin(fx+e)^2-4\,ab+b^2}{a^5-3\,a^4b+3\,a^3b^2-a^2b^3+(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5)\sin(fx+e)^4-2\,(a^5-4\,a^4b+6\,a^3b^2-4\,a^2b^3+ab^4)\sin(fx+e)^2} - \frac{2\,\log\left(-(a-b\right.}{a^3-3\,a^2}}{4\,f}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*(a*b - b^2)*sin(f*x + e)^2 - 4*a*b + b^2)/(a^5 - 3*a^4*b + 3*a^3*b
^2 - a^2*b^3 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*s
in(f*x + e)^4 - 2*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sin(f*x
+ e)^2) - 2*log(-(a - b)*sin(f*x + e)^2 + a)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3))/f
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.59

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{b\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(a^3bf-3\,a^2b^2f+3\,ab^3f-b^4f\right)} + \frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(a^3f-3\,a^2bf+3\,ab^2f-b^3f\right)} + \frac{2\left(ab-b^2\right)\tan\left(fx+e\right)^2+3\,a^2-4\,ab+b^2}{4\left(b\tan\left(fx+e\right)^2+a\right)^2(a-b)^3f}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/2*b*log(abs(b*tan(f*x + e)^2 + a))/(a^3*b*f - 3*a^2*b^2*f + 3*a*b^3*f -
 b^4*f) + 1/2*log(tan(f*x + e)^2 + 1)/(a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3
*f) + 1/4*(2*(a*b - b^2)*tan(f*x + e)^2 + 3*a^2 - 4*a*b + b^2)/((b*tan(f*x
 + e)^2 + a)^2*(a - b)^3*f)
```

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 375, normalized size of antiderivative = 4.03

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{a^2\left(-3+\operatorname{atan}\left(\frac{a\tan(e+fx)^2\,1\mathrm{i}-b\tan(e+fx)^2\,1\mathrm{i}}{2a+a\tan(e+fx)^2+b\tan(e+fx)^2}\right)4\mathrm{i}\right)+b^2\left(2\tan(e+fx)^2-1+\tan(e+fx)^4\operatorname{atan}\left(\right.\right.}{f\left(-4a^5-8a^4b\tan(e+fx)^2+12a^4b-4a^3b^2\tan(e+fx)^4+24a^3b^2\tan(e+fx)^2-12a^3b^2+12\right.}$$

input
```
int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(a^2*(atan((a*tan(e + f*x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*
x)^2 + b*tan(e + f*x)^2))*4i - 3) + b^2*(tan(e + f*x)^4*atan((a*tan(e + f*
x)^2*1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2)
)*4i + 2*tan(e + f*x)^2 - 1) + a*b*(tan(e + f*x)^2*atan((a*tan(e + f*x)^2*
1i - b*tan(e + f*x)^2*1i)/(2*a + a*tan(e + f*x)^2 + b*tan(e + f*x)^2))*8i
- 2*tan(e + f*x)^2 + 4))/(f*(12*a^4*b - 4*a^5 + 4*a^2*b^3 - 12*a^3*b^2 + 4
*b^5*tan(e + f*x)^4 + 8*a*b^4*tan(e + f*x)^2 - 8*a^4*b*tan(e + f*x)^2 - 12
*a*b^4*tan(e + f*x)^4 - 24*a^2*b^3*tan(e + f*x)^2 + 24*a^3*b^2*tan(e + f*x
)^2 + 12*a^2*b^3*tan(e + f*x)^4 - 4*a^3*b^2*tan(e + f*x)^4))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.13

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{2\log\left(\tan(fx+e)^2+1\right)\tan(fx+e)^4a^2b^2+4\log\left(\tan(fx+e)^2+1\right)\tan(fx+e)^2a^3b+2\log\left(\tan(fx\right.}{4a^2f\left(\tan(fx+e)^4\right.}$$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
(2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**4*a**2*b**2 + 4*log(tan(e + f*x)
**2 + 1)*tan(e + f*x)**2*a**3*b + 2*log(tan(e + f*x)**2 + 1)*a**4 - 2*log(
tan(e + f*x)**2*b + a)*tan(e + f*x)**4*a**2*b**2 - 4*log(tan(e + f*x)**2*b
 + a)*tan(e + f*x)**2*a**3*b - 2*log(tan(e + f*x)**2*b + a)*a**4 - 3*tan(e
 + f*x)**4*a**2*b**2 + 4*tan(e + f*x)**4*a*b**3 - tan(e + f*x)**4*b**4 - 4
*tan(e + f*x)**2*a**3*b + 6*tan(e + f*x)**2*a**2*b**2 - 2*tan(e + f*x)**2*
a*b**3)/(4*a**2*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4*a**2*b**3
 + 3*tan(e + f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f*x)**2*a**
4*b - 6*tan(e + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3 - 2*tan(e
+ f*x)**2*a*b**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.240 $\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\log(\cos(e+fx))}{(a-b)^3f}+\frac{\log(\tan(e+fx))}{a^3f}+\frac{b(3a^2-3ab+b^2)\log(a+b\tan^2(e+fx))}{2a^3(a-b)^3f}-\frac{b}{4a(a-b)f(a+b\tan^2(e+fx))^2}-\frac{(2a-b)b}{2a^2(a-b)^2f(a+b\tan^2(e+fx))}$$

output
```
ln(cos(f*x+e))/(a-b)^3/f+ln(tan(f*x+e))/a^3/f+1/2*b*(3*a^2-3*a*b+b^2)*ln(a
+b*tan(f*x+e)^2)/a^3/(a-b)^3/f-1/4*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/2*(2
*a-b)*b/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\frac{4\log(\cos(e+fx))}{(a-b)^3}+\frac{4\log(\tan(e+fx))+\frac{b\left(2\left(3a^2-3ab+b^2\right)\log\left(a+b\tan^2(e+fx)\right)-\frac{a(a-b)\left(a(5a-3b)+2(2a-b)b\tan^2(e+fx)\right)}{\left(a+b\tan^2(e+fx)\right)^2}\right)}{(a-b)^3}}{a^3}}{4f}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((4*Log[Cos[e + f*x]])/(a - b)^3 + (4*Log[Tan[e + f*x]] + (b*(2*(3*a^2 - 3
*a*b + b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(5*a - 3*b) + 2*(2*a
 - b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^3)/a^3)/(4*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4153, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 93$$

$$\frac{\int \left(\frac{(3a^2-3ba+b^2)b^2}{a^3(a-b)^3(b\tan^2(e+fx)+a)}+\frac{(2a-b)b^2}{a^2(a-b)^2(b\tan^2(e+fx)+a)^2}+\frac{b^2}{a(a-b)(b\tan^2(e+fx)+a)^3}+\frac{\cot(e+fx)}{a^3}-\frac{1}{(a-b)^3(\tan^2(e+fx)+1)}\right)d}{2f}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\log(\tan^2(e+fx))}{a^3}-\frac{b(2a-b)}{a^2(a-b)^2(a+b\tan^2(e+fx))}+\frac{b(3a^2-3ab+b^2)\log(a+b\tan^2(e+fx))}{a^3(a-b)^3}-\frac{b}{2a(a-b)(a+b\tan^2(e+fx))^2}-\frac{\log(\tan^2(e+fx}{(a-b)^3}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(Log[Tan[e + f*x]^2]/a^3 - Log[1 + Tan[e + f*x]^2]/(a - b)^3 + (b*(3*a^2 -
 3*a*b + b^2)*Log[a + b*Tan[e + f*x]^2])/(a^3*(a - b)^3) - b/(2*a*(a - b)*
(a + b*Tan[e + f*x]^2)^2) - ((2*a - b)*b)/(a^2*(a - b)^2*(a + b*Tan[e + f*
x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b^2\left(-\frac{a^2(a^2-2ab+b^2)}{2b(a+b\tan(fx+e)^2)^2}-\frac{a(2a^2-3ab+b^2)}{b(a+b\tan(fx+e)^2)}+\frac{(3a^2-3ab+b^2)\ln(a+b\tan(fx+e)^2)}{b}\right)}{2a^3(a-b)^3}+\frac{\ln(\tan(fx+e))}{a^3}-\frac{\ln(1+\tan(fx+e)^2)}{2(a-b)^3}}{f}$
default	$\frac{\frac{b^2\left(-\frac{a^2(a^2-2ab+b^2)}{2b(a+b\tan(fx+e)^2)^2}-\frac{a(2a^2-3ab+b^2)}{b(a+b\tan(fx+e)^2)}+\frac{(3a^2-3ab+b^2)\ln(a+b\tan(fx+e)^2)}{b}\right)}{2a^3(a-b)^3}+\frac{\ln(\tan(fx+e))}{a^3}-\frac{\ln(1+\tan(fx+e)^2)}{2(a-b)^3}}{f}$
norman	$\frac{\frac{(3ab-2b^2)b\tan(fx+e)^2}{2a^2f(a^2-2ab+b^2)}+\frac{(5ab-3b^2)b^2\tan(fx+e)^4}{4a^3f(a^2-2ab+b^2)}}{(a+b\tan(fx+e)^2)^2}+\frac{\ln(\tan(fx+e))}{a^3f}-\frac{\ln(1+\tan(fx+e)^2)}{2f(a^3-3a^2b+3ab^2-b^3)}+\frac{b(3a^2-3ab+b^2)}{2a^3f(a^3-3}$
parallelrisch	$12(a^2-ab+\frac{1}{3}b^2)b\left(\frac{(a-b)^2\cos(4fx+4e)}{4}+(a^2-b^2)\cos(2fx+2e)+\frac{3a^2}{4}+\frac{ab}{2}+\frac{3b^2}{4}\right)\ln(a+b\tan(fx+e)^2)+2\left(-\frac{\ln(\sec(fx}{2}\right.$
risch	$\frac{ix}{a^3-3a^2b+3ab^2-b^3}-\frac{2ix}{a^3}-\frac{2ie}{a^3f}-\frac{6ibx}{a(a^3-3a^2b+3ab^2-b^3)}-\frac{6ibe}{af(a^3-3a^2b+3ab^2-b^3)}+\frac{6ib^2x}{a^2(a^3-3a^2b+3ab^2-}$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b^2/a^3/(a-b)^3*(-1/2*a^2*(a^2-2*a*b+b^2)/b/(a+b*tan(f*x+e)^2)^2-
a*(2*a^2-3*a*b+b^2)/b/(a+b*tan(f*x+e)^2)+(3*a^2-3*a*b+b^2)/b*ln(a+b*tan(f*
x+e)^2))+1/a^3*ln(tan(f*x+e))-1/2/(a-b)^3*ln(1+tan(f*x+e)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(142) = 284.

Time = 0.14 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.85

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{6\,a^3b^2 - 3\,a^2b^3 + (5\,a^2b^3 - 2\,ab^4)\tan(fx+e)^4 + 2\,(3\,a^3b^2 + a^2b^3 - ab^4)\tan(fx+e)^2 + 2\,(a^5 - 3\,a^4b +}{}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(6*a^3*b^2 - 3*a^2*b^3 + (5*a^2*b^3 - 2*a*b^4)*tan(f*x + e)^4 + 2*(3*a
^3*b^2 + a^2*b^3 - a*b^4)*tan(f*x + e)^2 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*log(tan(f*x + e)^2/(tan(f
*x + e)^2 + 1)) + 2*(3*a^4*b - 3*a^3*b^2 + a^2*b^3 + (3*a^2*b^3 - 3*a*b^4
+ b^5)*tan(f*x + e)^4 + 2*(3*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*
log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^6*b^2 - 3*a^5*b^3 +
3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 -
 a^4*b^4)*f*tan(f*x + e)^2 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.69

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\frac{2\left(3a^2b-3ab^2+b^3\right)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^6-3a^5b+3a^4b^2-a^3b^3}+\frac{6a^2b^2-3ab^3-2\left(3a^2b^2-4ab^3+b^4\right)\sin(fx+e)^2}{a^7-3a^6b+3a^5b^2-a^4b^3+\left(a^7-5a^6b+10a^5b^2-10a^4b^3+5a^3b^4-a^2b^5\right)\sin(fx+e)^4-2\left(a^7-\dots\right.}}{4f}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(3*a^2*b - 3*a*b^2 + b^3)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3) + (6*a^2*b^2 - 3*a*b^3 - 2*(3*a^2*b^2 - 4*a*
b^3 + b^4)*sin(f*x + e)^2)/(a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3 + (a^7 - 5
*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*sin(f*x + e)^4 - 2
*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*sin(f*x + e)^2) + 2*log
(sin(f*x + e)^2)/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(142) = 284.

Time = 0.72 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.96

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\left(3a^2b^2-3ab^3+b^4\right)\log\left(\left|b\tan\left(fx+e\right)^2+a\right|\right)}{2\left(a^6bf-3a^5b^2f+3a^4b^3f-a^3b^4f\right)}-\frac{\log\left(\tan\left(fx+e\right)^2+1\right)}{2\left(a^3f-3a^2bf+3ab^2f-b^3f\right)} - \frac{9a^2b^3\tan\left(fx+e\right)^4-9ab^4\tan\left(fx+e\right)^4+3b^5\tan\left(fx+e\right)^4+22a^3b^2\tan\left(fx+e\right)^2-24a^2b^3\tan\left(f\dots\right.}{4\left(a^6f-3a^5bf+3a^4b^2f-a^3b^3f\right)\left(b\tan\left(fx+e\right)^2\dots\right.} + \frac{\log\left(\tan\left(fx+e\right)^2\right)}{2a^3f}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/2*(3*a^2*b^2 - 3*a*b^3 + b^4)*log(abs(b*tan(f*x + e)^2 + a))/(a^6*b*f -
3*a^5*b^2*f + 3*a^4*b^3*f - a^3*b^4*f) - 1/2*log(tan(f*x + e)^2 + 1)/(a^3*
f - 3*a^2*b*f + 3*a*b^2*f - b^3*f) - 1/4*(9*a^2*b^3*tan(f*x + e)^4 - 9*a*b
^4*tan(f*x + e)^4 + 3*b^5*tan(f*x + e)^4 + 22*a^3*b^2*tan(f*x + e)^2 - 24*
a^2*b^3*tan(f*x + e)^2 + 8*a*b^4*tan(f*x + e)^2 + 14*a^4*b - 17*a^3*b^2 +
6*a^2*b^3)/((a^6*f - 3*a^5*b*f + 3*a^4*b^2*f - a^3*b^3*f)*(b*tan(f*x + e)^
2 + a)^2) + 1/2*log(tan(f*x + e)^2)/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.22

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\ln(\tan(e+f\,x))}{a^3\,f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^3} - \frac{\frac{5\,a\,b-3\,b^2}{4\,a\,(a^2-2\,a\,b+b^2)} + \frac{b\tan(e+f\,x)^2\,(2\,a\,b-b^2)}{2\,a^2\,(a^2-2\,a\,b+b^2)}}{f\,\left(a^2+2\,a\,b\tan(e+f\,x)^2+b^2\tan(e+f\,x)^4\right)} + \frac{b\ln\left(b\tan(e+f\,x)^2+a\right)\,(3\,a^2-3\,a\,b+b^2)}{2\,a^3\,f\,(a-b)^3}$$

input
```
int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
log(tan(e + f*x))/(a^3*f) - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) - ((5*
a*b - 3*b^2)/(4*a*(a^2 - 2*a*b + b^2)) + (b*tan(e + f*x)^2*(2*a*b - b^2))/
(2*a^2*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*tan(e +
f*x)^2)) + (b*log(a + b*tan(e + f*x)^2)*(3*a^2 - 3*a*b + b^2))/(2*a^3*f*(a
 - b)^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1945, normalized size of antiderivative = 13.14

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
( - 4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**5 + 8*log(tan((e + f
*x)/2)**2 + 1)*sin(e + f*x)**4*a**4*b - 4*log(tan((e + f*x)/2)**2 + 1)*sin
(e + f*x)**4*a**3*b**2 + 8*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a*
*5 - 8*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**2*a**4*b - 4*log(tan((e
+ f*x)/2)**2 + 1)*a**5 + 6*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)
*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*a**4*b - 18*log( - 2*sqrt(
a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f
*x)**4*a**3*b**2 + 20*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan(
(e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*a**2*b**3 - 10*log( - 2*sqrt(a
- b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x
)**4*a*b**4 + 2*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f
*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*b**5 - 12*log( - 2*sqrt(a - b)*tan((e
 + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**2*a**4*b
 + 24*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2
+ sqrt(a))*sin(e + f*x)**2*a**3*b**2 - 16*log( - 2*sqrt(a - b)*tan((e + f*
x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**2*a**2*b**3 +
 4*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + s
qrt(a))*sin(e + f*x)**2*a*b**4 + 6*log( - 2*sqrt(a - b)*tan((e + f*x)/2) +
 sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*a**4*b - 6*log( - 2*sqrt(a - b)*ta
n((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*a**3*b**2 + 2*l...
```

3.241 $\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\cot^2(e+fx)}{2a^3f} - \frac{\log(\cos(e+fx))}{(a-b)^3f} - \frac{(a+3b)\log(\tan(e+fx))}{a^4f} - \frac{b^2(6a^2-8ab+3b^2)\log(a+b\tan^2(e+fx))}{2a^4(a-b)^3f} + \frac{b^2}{4a^2(a-b)f(a+b\tan^2(e+fx))^2} + \frac{(3a-2b)b^2}{2a^3(a-b)^2f(a+b\tan^2(e+fx))}$$

output
```
-1/2*cot(f*x+e)^2/a^3/f-ln(cos(f*x+e))/(a-b)^3/f-(a+3*b)*ln(tan(f*x+e))/a^
4/f-1/2*b^2*(6*a^2-8*a*b+3*b^2)*ln(a+b*tan(f*x+e)^2)/a^4/(a-b)^3/f+1/4*b^2
/a^2/(a-b)/f/(a+b*tan(f*x+e)^2)^2+1/2*(3*a-2*b)*b^2/a^3/(a-b)^2/f/(a+b*tan
(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.80

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{\frac{\cot^2(e+fx)}{a^3}-\frac{b^4}{2a^4(a-b)(b+a\cot^2(e+fx))^2}+\frac{(4a-3b)b^3}{a^4(a-b)^2(b+a\cot^2(e+fx))}+\frac{b^2(6a^2-8ab+3b^2)\log(b+a\cot^2(e+fx))}{a^4(a-b)^3}+\frac{2\log(\sin(e+}{(a-b)^3}}{2f}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
-1/2*(Cot[e + f*x]^2/a^3 - b^4/(2*a^4*(a - b)*(b + a*Cot[e + f*x]^2)^2) +
((4*a - 3*b)*b^3)/(a^4*(a - b)^2*(b + a*Cot[e + f*x]^2)) + (b^2*(6*a^2 - 8
*a*b + 3*b^2)*Log[b + a*Cot[e + f*x]^2])/(a^4*(a - b)^3) + (2*Log[Sin[e +
f*x]])/(a - b)^3)/f
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan^2(e+fx)}{2f}$$

$\Big\downarrow$ 99

$$\frac{\int \left(-\frac{(6a^2-8ba+3b^2)b^3}{a^4(a-b)^3(b\tan^2(e+fx)+a)}-\frac{(3a-2b)b^3}{a^3(a-b)^2(b\tan^2(e+fx)+a)^2}-\frac{b^3}{a^2(a-b)(b\tan^2(e+fx)+a)^3}+\frac{\cot^2(e+fx)}{a^3}+\frac{(-a-3b)\cot(e+fx)}{a^4}+\right.}{2f}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{(a+3b)\log(\tan^2(e+fx))}{a^4}+\frac{b^2(3a-2b)}{a^3(a-b)^2(a+b\tan^2(e+fx))}-\frac{\cot(e+fx)}{a^3}+\frac{b^2}{2a^2(a-b)(a+b\tan^2(e+fx))^2}-\frac{b^2(6a^2-8ab+3b^2)\log(a+b\tan^2}{a^4(a-b)^3}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-(Cot[e + f*x]/a^3) - ((a + 3*b)*Log[Tan[e + f*x]^2])/a^4 + Log[1 + Tan[e
 + f*x]^2]/(a - b)^3 - (b^2*(6*a^2 - 8*a*b + 3*b^2)*Log[a + b*Tan[e + f*x]
^2])/(a^4*(a - b)^3) + b^2/(2*a^2*(a - b)*(a + b*Tan[e + f*x]^2)^2) + ((3*
a - 2*b)*b^2)/(a^3*(a - b)^2*(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{2a^3\tan(fx+e)^2}+\frac{(-3b-a)\ln(\tan(fx+e))}{a^4}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}-\frac{b^3\left(-\frac{a\left(3a^2-5ab+2b^2\right)}{b\left(a+b\tan(fx+e)^2\right)}-\frac{a^2\left(a^2-2ab+b^2\right)}{2b\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(6a^2-8\right.}{}\right.}{2a^4(a-b)^3}}{f}$
default	$\frac{-\frac{1}{2a^3\tan(fx+e)^2}+\frac{(-3b-a)\ln(\tan(fx+e))}{a^4}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}-\frac{b^3\left(-\frac{a\left(3a^2-5ab+2b^2\right)}{b\left(a+b\tan(fx+e)^2\right)}-\frac{a^2\left(a^2-2ab+b^2\right)}{2b\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(6a^2-8\right.}{}\right.}{2a^4(a-b)^3}}{f}$
norman	$\frac{-\frac{1}{2af}+\frac{\left(3a^2b-10a\,b^2+6b^3\right)b\tan(fx+e)^4}{2a^3f\left(a^2-2ab+b^2\right)}+\frac{\left(4a^2b-15a\,b^2+9b^3\right)b^2\tan(fx+e)^6}{4a^4f\left(a^2-2ab+b^2\right)}}{\tan(fx+e)^2\left(a+b\tan(fx+e)^2\right)^2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2f(a^3-3a^2b+3a\,b^2-b^3)}-\frac{(a+3b)\ln(\mathrm{t}}{a^4}$
parallelrisch	$\frac{-48b^2\left(a^2-\frac{4}{3}ab+\frac{1}{2}b^2\right)\left(\frac{(a-b)^2\cos(4fx+4e)}{4}+\left(a^2-b^2\right)\cos(2fx+2e)+\frac{3a^2}{4}+\frac{ab}{2}+\frac{3b^2}{4}\right)\ln\left(a+b\tan(fx+e)^2\right)+\left(2a^4(a-\right.}{}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/a^3/tan(f*x+e)^2+(-3*b-a)/a^4*ln(tan(f*x+e))+1/2/(a-b)^3*ln(1+ta
n(f*x+e)^2)-1/2*b^3/a^4/(a-b)^3*(-a*(3*a^2-5*a*b+2*b^2)/b/(a+b*tan(f*x+e)^
2)-1/2*a^2*(a^2-2*a*b+b^2)/b/(a+b*tan(f*x+e)^2)^2+(6*a^2-8*a*b+3*b^2)/b*ln
(a+b*tan(f*x+e)^2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. 2(173) = 346.

Time = 0.15 (sec) , antiderivative size = 545, normalized size of antiderivative = 3.01

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{(2\,a^4b^2-6\,a^3b^3+13\,a^2b^4-6\,ab^5)\tan{(fx+e)}^6+2\,a^6-6\,a^5b+6\,a^4b^2-2\,a^3b^3+2\,(2\,a^5b-5\,a^4b^2+}{}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
-1/4*((2*a^4*b^2 - 6*a^3*b^3 + 13*a^2*b^4 - 6*a*b^5)*tan(f*x + e)^6 + 2*a^
6 - 6*a^5*b + 6*a^4*b^2 - 2*a^3*b^3 + 2*(2*a^5*b - 5*a^4*b^2 + 7*a^3*b^3 +
 2*a^2*b^4 - 3*a*b^5)*tan(f*x + e)^4 + (2*a^6 - 2*a^5*b - 6*a^4*b^2 + 18*a
^3*b^3 - 9*a^2*b^4)*tan(f*x + e)^2 + 2*((a^4*b^2 - 6*a^2*b^4 + 8*a*b^5 - 3
*b^6)*tan(f*x + e)^6 + 2*(a^5*b - 6*a^3*b^3 + 8*a^2*b^4 - 3*a*b^5)*tan(f*x
 + e)^4 + (a^6 - 6*a^4*b^2 + 8*a^3*b^3 - 3*a^2*b^4)*tan(f*x + e)^2)*log(ta
n(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((6*a^2*b^4 - 8*a*b^5 + 3*b^6)*tan(
f*x + e)^6 + 2*(6*a^3*b^3 - 8*a^2*b^4 + 3*a*b^5)*tan(f*x + e)^4 + (6*a^4*b
^2 - 8*a^3*b^3 + 3*a^2*b^4)*tan(f*x + e)^2)*log((b*tan(f*x + e)^2 + a)/(ta
n(f*x + e)^2 + 1)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x
 + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^4 + (
a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.91

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{2\,(6\,a^2b^2-8\,ab^3+3\,b^4)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^7-3\,a^6b+3\,a^5b^2-a^4b^3}+\frac{2\,a^5-6\,a^4b+6\,a^3b^2-2\,a^2b^3+2\,(a^5-5\,a^4b+10\,a^3b^2-14\,a^2b^3+11\,ab^4-3\,b^5)\sin(fx}{(a^8-5\,a^7b+10\,a^6b^2-10\,a^5b^3+5\,a^4b^4-a^3b^5)\sin(fx+e)^6-2\,(a^8-4\,a^7b+6\,a^6b^2-4\,a^5}}{4\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(6*a^2*b^2 - 8*a*b^3 + 3*b^4)*log(-(a - b)*sin(f*x + e)^2 + a)/(a^
7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) + (2*a^5 - 6*a^4*b + 6*a^3*b^2 - 2*a^2*
b^3 + 2*(a^5 - 5*a^4*b + 10*a^3*b^2 - 14*a^2*b^3 + 11*a*b^4 - 3*b^5)*sin(f
*x + e)^4 - (4*a^5 - 16*a^4*b + 24*a^3*b^2 - 24*a^2*b^3 + 9*a*b^4)*sin(f*x
 + e)^2)/((a^8 - 5*a^7*b + 10*a^6*b^2 - 10*a^5*b^3 + 5*a^4*b^4 - a^3*b^5)*
sin(f*x + e)^6 - 2*(a^8 - 4*a^7*b + 6*a^6*b^2 - 4*a^5*b^3 + a^4*b^4)*sin(f
*x + e)^4 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*sin(f*x + e)^2) + 2*(a +
 3*b)*log(sin(f*x + e)^2)/a^4)/f
```

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.88

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= -\frac{(6\,a^2b^3-8\,ab^4+3\,b^5)\log\left(\left|b\tan(fx+e)^2+a\right|\right)}{2\,(a^7bf-3\,a^6b^2f+3\,a^5b^3f-a^4b^4f)}+\frac{\log\left(\tan(fx+e)^2+1\right)}{2\,(a^3f-3\,a^2bf+3\,ab^2f-b^3f)}$$
$$+\frac{18\,a^2b^4\tan(fx+e)^4-24\,ab^5\tan(fx+e)^4+9\,b^6\tan(fx+e)^4+42\,a^3b^3\tan(fx+e)^2-58\,a^2b^4\tan}{4\,(a^7f-3\,a^6bf+3\,a^5b^2f-a^4b^3f)\left(b\tan(fx+e\right.}$$
$$-\frac{(a+3\,b)\log\left(\tan(fx+e)^2\right)}{2\,a^4f}+\frac{a\tan(fx+e)^2+3\,b\tan(fx+e)^2-a}{2\,a^4f\tan(fx+e)^2}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/2*(6*a^2*b^3 - 8*a*b^4 + 3*b^5)*log(abs(b*tan(f*x + e)^2 + a))/(a^7*b*f
 - 3*a^6*b^2*f + 3*a^5*b^3*f - a^4*b^4*f) + 1/2*log(tan(f*x + e)^2 + 1)/(a
^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3*f) + 1/4*(18*a^2*b^4*tan(f*x + e)^4 - 2
4*a*b^5*tan(f*x + e)^4 + 9*b^6*tan(f*x + e)^4 + 42*a^3*b^3*tan(f*x + e)^2
- 58*a^2*b^4*tan(f*x + e)^2 + 22*a*b^5*tan(f*x + e)^2 + 25*a^4*b^2 - 36*a^
3*b^3 + 14*a^2*b^4)/((a^7*f - 3*a^6*b*f + 3*a^5*b^2*f - a^4*b^3*f)*(b*tan(
f*x + e)^2 + a)^2) - 1/2*(a + 3*b)*log(tan(f*x + e)^2)/(a^4*f) + 1/2*(a*ta
n(f*x + e)^2 + 3*b*tan(f*x + e)^2 - a)/(a^4*f*tan(f*x + e)^2)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.27

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^3} - \frac{\frac{1}{2\,a}+\frac{\tan(e+f\,x)^4\,\left(a^2\,b^2-5\,a\,b^3+3\,b^4\right)}{2\,a^3\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan(e+f\,x)^2\,\left(4\,a^2\,b-15\,a\,b^2+9\,b^3\right)}{4\,a^2\,\left(a^2-2\,a\,b+b^2\right)}}{f\,\left(a^2\tan\left(e+f\,x\right)^2+2\,a\,b\tan\left(e+f\,x\right)^4+b^2\tan\left(e+f\,x\right)^6\right)} - \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a+3\,b\right)}{a^4\,f} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(6\,a^2-8\,a\,b+3\,b^2\right)}{2\,a^4\,f\,(a-b)^3}$$

input
```
int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) - (1/(2*a) + (tan(e + f*x)^4*(3*b^
4 - 5*a*b^3 + a^2*b^2))/(2*a^3*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^2*(4*a
^2*b - 15*a*b^2 + 9*b^3))/(4*a^2*(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x
)^2 + b^2*tan(e + f*x)^6 + 2*a*b*tan(e + f*x)^4)) - (log(tan(e + f*x))*(a
+ 3*b))/(a^4*f) - (b^2*log(a + b*tan(e + f*x)^2)*(6*a^2 - 8*a*b + 3*b^2))/
(2*a^4*f*(a - b)^3)
```

Reduce [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 2266, normalized size of antiderivative = 12.52

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(4*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**6*a**6 - 8*log(tan((e + f*x)
/2)**2 + 1)*sin(e + f*x)**6*a**5*b + 4*log(tan((e + f*x)/2)**2 + 1)*sin(e
+ f*x)**6*a**4*b**2 - 8*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**6
+ 8*log(tan((e + f*x)/2)**2 + 1)*sin(e + f*x)**4*a**5*b + 4*log(tan((e + f
*x)/2)**2 + 1)*sin(e + f*x)**2*a**6 - 12*log( - 2*sqrt(a - b)*tan((e + f*x
)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**6*a**4*b**2 +
40*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + s
qrt(a))*sin(e + f*x)**6*a**3*b**3 - 50*log( - 2*sqrt(a - b)*tan((e + f*x)/
2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**6*a**2*b**4 + 28
*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqr
t(a))*sin(e + f*x)**6*a*b**5 - 6*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + s
qrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**6*b**6 + 24*log( - 2*s
qrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e
 + f*x)**4*a**4*b**2 - 56*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*
tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*a**3*b**3 + 44*log( - 2*sqr
t(a - b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e +
 f*x)**4*a**2*b**4 - 12*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*ta
n((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)**4*a*b**5 - 12*log( - 2*sqrt(a -
 b)*tan((e + f*x)/2) + sqrt(a)*tan((e + f*x)/2)**2 + sqrt(a))*sin(e + f*x)
**2*a**4*b**2 + 16*log( - 2*sqrt(a - b)*tan((e + f*x)/2) + sqrt(a)*tan(...
```

3.242 $\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 210

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{(a+3b)\cot^2(e+fx)}{2a^4f} - \frac{\cot^4(e+fx)}{4a^3f} + \frac{\log(\cos(e+fx))}{(a-b)^3f} + \frac{(a^2+3ab+6b^2)\log(\tan(e+fx))}{a^5f} + \frac{b^3(10a^2-15ab+6b^2)\log(a+b\tan^2(e+fx))}{2a^5(a-b)^3f} - \frac{b^3}{4a^3(a-b)f(a+b\tan^2(e+fx))^2} - \frac{(4a-3b)b^3}{2a^4(a-b)^2f(a+b\tan^2(e+fx))}$$

output
```
1/2*(a+3*b)*cot(f*x+e)^2/a^4/f-1/4*cot(f*x+e)^4/a^3/f+ln(cos(f*x+e))/(a-b)
^3/f+(a^2+3*a*b+6*b^2)*ln(tan(f*x+e))/a^5/f+1/2*b^3*(10*a^2-15*a*b+6*b^2)*
ln(a+b*tan(f*x+e)^2)/a^5/(a-b)^3/f-1/4*b^3/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^
2-1/2*(4*a-3*b)*b^3/a^4/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{(a+3b)\cot^2(e+fx)}{a^4} - \frac{\cot^4(e+fx)}{2a^3} + \frac{2\log(\cos(e+fx))}{(a-b)^3} + \frac{4(a^2+3ab+6b^2)\log(\tan(e+fx)) + \frac{b^3\left(2\left(10a^2-15ab+6b^2\right)\log\left(a+b\tan^2(e+fx)\right) - \frac{a(\cdots}{\cdots}\right.}{(a-b)^3}}{2a^5}}{2f}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(((a + 3*b)*Cot[e + f*x]^2)/a^4 - Cot[e + f*x]^4/(2*a^3) + (2*Log[Cos[e +
f*x]])/(a - b)^3 + (4*(a^2 + 3*a*b + 6*b^2)*Log[Tan[e + f*x]] + (b^3*(2*(1
0*a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2] - (a*(a - b)*(a*(9*a - 7
*b) + 2*(4*a - 3*b)*b*Tan[e + f*x]^2))/(a + b*Tan[e + f*x]^2)^2))/(a - b)^
3)/(2*a^5))/(2*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^5\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\cot^5(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$\big\downarrow$ 354

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan^2(e+fx)}{2f}$$

$\big\downarrow$ 99

$$\frac{\int \left(\frac{(10a^2-15ba+6b^2)b^4}{a^5(a-b)^3(b\tan^2(e+fx)+a)}+\frac{(4a-3b)b^4}{a^4(a-b)^2(b\tan^2(e+fx)+a)^2}+\frac{b^4}{a^3(a-b)(b\tan^2(e+fx)+a)^3}+\frac{\cot^3(e+fx)}{a^3}+\frac{(-a-3b)\cot^2(e+fx)}{a^4}+\right.}{2f}$$

$\big\downarrow$ 2009

$$\frac{-\frac{b^3(4a-3b)}{a^4(a-b)^2(a+b\tan^2(e+fx))}+\frac{(a+3b)\cot(e+fx)}{a^4}-\frac{b^3}{2a^3(a-b)(a+b\tan^2(e+fx))^2}-\frac{\cot^2(e+fx)}{2a^3}+\frac{(a^2+3ab+6b^2)\log(\tan^2(e+fx))}{a^5}+}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(((a + 3*b)*Cot[e + f*x])/a^4 - Cot[e + f*x]^2/(2*a^3) + ((a^2 + 3*a*b + 6
*b^2)*Log[Tan[e + f*x]^2])/a^5 - Log[1 + Tan[e + f*x]^2]/(a - b)^3 + (b^3*
(10*a^2 - 15*a*b + 6*b^2)*Log[a + b*Tan[e + f*x]^2])/(a^5*(a - b)^3) - b^3
/(2*a^3*(a - b)*(a + b*Tan[e + f*x]^2)^2) - ((4*a - 3*b)*b^3)/(a^4*(a - b)
^2*(a + b*Tan[e + f*x]^2)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}-\frac{1}{4a^3\tan(fx+e)^4}-\frac{-3b-a}{2a^4\tan(fx+e)^2}+\frac{\left(a^2+3ab+6b^2\right)\ln(\tan(fx+e))}{a^5}+\frac{b^4\left(-\frac{a\left(4a^2-7ab+3b^2\right)}{b\left(a+b\tan(fx+e)^2\right)}+\frac{\left(10a^2-\cdots\right.}{\cdots}\right.}{\cdots}}{f}$
default	$\frac{-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2(a-b)^3}-\frac{1}{4a^3\tan(fx+e)^4}-\frac{-3b-a}{2a^4\tan(fx+e)^2}+\frac{\left(a^2+3ab+6b^2\right)\ln(\tan(fx+e))}{a^5}+\frac{b^4\left(-\frac{a\left(4a^2-7ab+3b^2\right)}{b\left(a+b\tan(fx+e)^2\right)}+\frac{\left(10a^2-\cdots\right.}{\cdots}\right.}{\cdots}}{f}$
parallelrisch	$\frac{20b^3\left(a^2-\frac{3}{2}ab+\frac{3}{5}b^2\right)\left(a+b\tan(fx+e)^2\right)^2\ln\left(a+b\tan(fx+e)^2\right)-2a^5\left(a+b\tan(fx+e)^2\right)^2\ln\left(\sec(fx+e)^2\right)-\left(-4(a^2+\cdots\right.}{\cdots}$
norman	$\frac{-\frac{1}{4af}+\frac{(a+2b)\tan(fx+e)^2}{2a^2f}+\frac{\left(-4a^3b-3a^2b^2+27a\,b^3-18b^4\right)b^2\tan(fx+e)^8}{4f\,a^5\left(a^2-2ab+b^2\right)}+\frac{\left(-3a^3b-2a^2b^2+18a\,b^3-12b^4\right)b\tan(fx+e)^6}{2a^4f\left(a^2-2ab+b^2\right)}}{\tan(fx+e)^4\left(a+b\tan(fx+e)^2\right)^2}+\cdots$
risch	Expression too large to display

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(a-b)^3*ln(1+tan(f*x+e)^2)-1/4/a^3/tan(f*x+e)^4-1/2*(-3*b-a)/a^4
/tan(f*x+e)^2+(a^2+3*a*b+6*b^2)/a^5*ln(tan(f*x+e))+1/2*b^4/a^5/(a-b)^3*(-a
*(4*a^2-7*a*b+3*b^2)/b/(a+b*tan(f*x+e)^2)+(10*a^2-15*a*b+6*b^2)/b*ln(a+b*t
an(f*x+e)^2)-1/2*a^2*(a^2-2*a*b+b^2)/b/(a+b*tan(f*x+e)^2)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(200) = 400$.

Time = 0.21 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.91

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{3\,(a^5b^2 - a^4b^3 - 3\,a^3b^4 + 8\,a^2b^5 - 4\,ab^6)\tan{(fx+e)}^8 - a^7 + 3\,a^6b - 3\,a^5b^2 + a^4b^3 + 2\,(3\,a^6b - 2\,a^5b^2 -}{}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(3*(a^5*b^2 - a^4*b^3 - 3*a^3*b^4 + 8*a^2*b^5 - 4*a*b^6)*tan(f*x + e)^
8 - a^7 + 3*a^6*b - 3*a^5*b^2 + a^4*b^3 + 2*(3*a^6*b - 2*a^5*b^2 - 9*a^4*b
^3 + 14*a^3*b^4 + 3*a^2*b^5 - 6*a*b^6)*tan(f*x + e)^6 + (3*a^7 + a^6*b - 1
0*a^5*b^2 - 6*a^4*b^3 + 33*a^3*b^4 - 18*a^2*b^5)*tan(f*x + e)^4 + 2*(a^7 -
 a^6*b - 3*a^5*b^2 + 5*a^4*b^3 - 2*a^3*b^4)*tan(f*x + e)^2 + 2*((a^5*b^2 -
 10*a^2*b^5 + 15*a*b^6 - 6*b^7)*tan(f*x + e)^8 + 2*(a^6*b - 10*a^3*b^4 + 1
5*a^2*b^5 - 6*a*b^6)*tan(f*x + e)^6 + (a^7 - 10*a^4*b^3 + 15*a^3*b^4 - 6*a
^2*b^5)*tan(f*x + e)^4)*log(tan(f*x + e)^2/(tan(f*x + e)^2 + 1)) + 2*((10*
a^2*b^5 - 15*a*b^6 + 6*b^7)*tan(f*x + e)^8 + 2*(10*a^3*b^4 - 15*a^2*b^5 +
6*a*b^6)*tan(f*x + e)^6 + (10*a^4*b^3 - 15*a^3*b^4 + 6*a^2*b^5)*tan(f*x +
e)^4)*log((b*tan(f*x + e)^2 + a)/(tan(f*x + e)^2 + 1)))/((a^8*b^2 - 3*a^7*
b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7
*b^3 - a^6*b^4)*f*tan(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*
f*tan(f*x + e)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(200) = 400.

Time = 0.05 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.98

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\frac{2\left(10a^2b^3-15ab^4+6b^5\right)\log\left(-(a-b)\sin(fx+e)^2+a\right)}{a^8-3a^7b+3a^6b^2-a^5b^3}+\frac{2\left(2a^6-7a^5b+5a^4b^2+10a^3b^3-25a^2b^4+21ab^5-6b^6\right)\sin(fx+e)^6-a^6+3a^5b-3a^4b}{\left(a^9-5a^8b+10a^7b^2-10a^6b^3+5a^5b^4-a^4b^5\right)\sin(fx+}}{}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(10*a^2*b^3 - 15*a*b^4 + 6*b^5)*log(-(a - b)*sin(f*x + e)^2 + a)/(a
^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3) + (2*(2*a^6 - 7*a^5*b + 5*a^4*b^2 + 10
*a^3*b^3 - 25*a^2*b^4 + 21*a*b^5 - 6*b^6)*sin(f*x + e)^6 - a^6 + 3*a^5*b -
 3*a^4*b^2 + a^3*b^3 - (9*a^6 - 25*a^5*b + 10*a^4*b^2 + 30*a^3*b^3 - 45*a^
2*b^4 + 18*a*b^5)*sin(f*x + e)^4 + 2*(3*a^6 - 7*a^5*b + 3*a^4*b^2 + 3*a^3*
b^3 - 2*a^2*b^4)*sin(f*x + e)^2)/((a^9 - 5*a^8*b + 10*a^7*b^2 - 10*a^6*b^3
 + 5*a^5*b^4 - a^4*b^5)*sin(f*x + e)^8 - 2*(a^9 - 4*a^8*b + 6*a^7*b^2 - 4*
a^6*b^3 + a^5*b^4)*sin(f*x + e)^6 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*
sin(f*x + e)^4) + 2*(a^2 + 3*a*b + 6*b^2)*log(sin(f*x + e)^2)/a^5)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(200) = 400$.

Time = 0.65 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.30

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{(10\,a^2b^4 - 15\,ab^5 + 6\,b^6)\log\left(\left|b\tan(fx+e)^2 + a\right|\right)}{2\,(a^8bf - 3\,a^7b^2f + 3\,a^6b^3f - a^5b^4f)} - \frac{\log\left(\tan(fx+e)^2 + 1\right)}{2\,(a^3f - 3\,a^2bf + 3\,ab^2f - b^3f)}$$
$$- \frac{3\,a^4b^2\tan(fx+e)^8 + 6\,a^5b\tan(fx+e)^6 - 4\,a^4b^2\tan(fx+e)^6 + 40\,a^2b^4\tan(fx+e)^6 - 60\,ab^5\tan(}{}$$
$$+ \frac{(a^2 + 3\,ab + 6\,b^2)\log\left(\tan(fx+e)^2\right)}{2\,a^5f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/2*(10*a^2*b^4 - 15*a*b^5 + 6*b^6)*log(abs(b*tan(f*x + e)^2 + a))/(a^8*b*
f - 3*a^7*b^2*f + 3*a^6*b^3*f - a^5*b^4*f) - 1/2*log(tan(f*x + e)^2 + 1)/(
a^3*f - 3*a^2*b*f + 3*a*b^2*f - b^3*f) - 1/8*(3*a^4*b^2*tan(f*x + e)^8 + 6
*a^5*b*tan(f*x + e)^6 - 4*a^4*b^2*tan(f*x + e)^6 + 40*a^2*b^4*tan(f*x + e)
^6 - 60*a*b^5*tan(f*x + e)^6 + 24*b^6*tan(f*x + e)^6 + 3*a^6*tan(f*x + e)^
4 - 8*a^5*b*tan(f*x + e)^4 + 2*a^4*b^2*tan(f*x + e)^4 + 60*a^3*b^3*tan(f*x
 + e)^4 - 90*a^2*b^4*tan(f*x + e)^4 + 36*a*b^5*tan(f*x + e)^4 - 4*a^6*tan(
f*x + e)^2 + 4*a^5*b*tan(f*x + e)^2 + 12*a^4*b^2*tan(f*x + e)^2 - 20*a^3*b
^3*tan(f*x + e)^2 + 8*a^2*b^4*tan(f*x + e)^2 + 2*a^6 - 6*a^5*b + 6*a^4*b^2
 - 2*a^3*b^3)/((a^7*f - 3*a^6*b*f + 3*a^5*b^2*f - a^4*b^3*f)*(b*tan(f*x +
e)^4 + a*tan(f*x + e)^2)^2) + 1/2*(a^2 + 3*a*b + 6*b^2)*log(tan(f*x + e)^2
)/(a^5*f)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.28

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$=\frac{\frac{\tan(e+f\,x)^2\,(a+2\,b)}{2\,a^2}-\frac{1}{4\,a}+\frac{\tan(e+f\,x)^6\,(a^3\,b^2+a^2\,b^3-9\,a\,b^4+6\,b^5)}{2\,a^4\,(a^2-2\,a\,b+b^2)}+\frac{\tan(e+f\,x)^4\,(4\,a^3\,b+3\,a^2\,b^2-27\,a\,b^3+18\,b^4)}{4\,a^3\,(a^2-2\,a\,b+b^2)}}{f\,\left(a^2\tan\left(e+f\,x\right)^4+2\,a\,b\tan\left(e+f\,x\right)^6+b^2\tan\left(e+f\,x\right)^8\right)}$$
$$-\frac{\ln\left(b\tan(e+f\,x)^2+a\right)\,\left(\frac{3\,b}{2\,a^4}+\frac{1}{2\,a^3}-\frac{1}{2\,(a-b)^3}+\frac{3\,b^2}{a^5}\right)}{f}$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f\,(a-b)^3}+\frac{\ln\left(\tan(e+f\,x)\right)\,(a^2+3\,a\,b+6\,b^2)}{a^5\,f}$$

input
```
int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^2*(a + 2*b))/(2*a^2) - 1/(4*a) + (tan(e + f*x)^6*(6*b^5 - 9
*a*b^4 + a^2*b^3 + a^3*b^2))/(2*a^4*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^4
*(4*a^3*b - 27*a*b^3 + 18*b^4 + 3*a^2*b^2))/(4*a^3*(a^2 - 2*a*b + b^2)))/(
f*(a^2*tan(e + f*x)^4 + b^2*tan(e + f*x)^8 + 2*a*b*tan(e + f*x)^6)) - (log
(a + b*tan(e + f*x)^2)*((3*b)/(2*a^4) + 1/(2*a^3) - 1/(2*(a - b)^3) + (3*b
^2)/a^5))/f - log(tan(e + f*x)^2 + 1)/(2*f*(a - b)^3) + (log(tan(e + f*x))
*(3*a*b + a^2 + 6*b^2))/(a^5*f)
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx=\int \frac{\cot\left(fx+e\right)^5}{\left(\tan\left(fx+e\right)^2b+a\right)^3}dx$$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x)
```

3.243 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{x}{(a-b)^3}+\frac{\sqrt{a}(3a^2-10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8(a-b)^3b^{5/2}f} - \frac{a\tan^3(e+fx)}{4(a-b)bf\left(a+b\tan^2(e+fx)\right)^2} - \frac{a(3a-7b)\tan(e+fx)}{8(a-b)^2b^2f\left(a+b\tan^2(e+fx)\right)}$$

output
```
-x/(a-b)^3+1/8*a^(1/2)*(3*a^2-10*a*b+15*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(
1/2))/(a-b)^3/b^(5/2)/f-1/4*a*tan(f*x+e)^3/(a-b)/b/f/(a+b*tan(f*x+e)^2)^2-
1/8*a*(3*a-7*b)*tan(f*x+e)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{-8(e+fx)+\frac{\sqrt{a}(3a^2-10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a(a-b)(3a^2-2ab-9b^2+3(a^2-4ab+3b^2)\cos(2(e+fx)))\sin(2(e+fx))}{b^2(a+b+(a-b)\cos(2(e+fx)))^2}}{8(a-b)^3f}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-8*(e + f*x) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a]])/b^(5/2) - (a*(a - b)*(3*a^2 - 2*a*b - 9*b^2 + 3*(a^2 - 4*
a*b + 3*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b^2*(a + b + (a - b)*Cos
[2*(e + f*x)])^2))/(8*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4153, 372, 440, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^6}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((3a-4b)\tan^2(e+fx)+3a\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4b(a-b)}-\frac{a\tan^3(e+fx)}{4b(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 440$$

$$\frac{-\frac{\frac{\int -\frac{\left(3a^2-7ba+8b^2\right)\tan^2(e+fx)+a(3a-7b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2b(a-b)}-\frac{a(3a-7b)\tan(e+fx)}{2b(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan^3(e+fx)}{4b(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{\left(3a^2-7ba+8b^2\right)\tan^2(e+fx)+a(3a-7b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2b(a-b)}-\frac{a(3a-7b)\tan(e+fx)}{2b(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan^3(e+fx)}{4b(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{a\left(3a^2-10ab+15b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{8b^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}}{2b(a-b)}-\frac{a(3a-7b)\tan(e+fx)}{2b(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan^3(e+fx)}{4b(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{a\left(3a^2-10ab+15b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}-\frac{8b^2\arctan(\tan(e+fx))}{a-b}}{2b(a-b)}-\frac{a(3a-7b)\tan(e+fx)}{2b(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan^3(e+fx)}{4b(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{\sqrt{a}\left(3a^2-10ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}-\frac{8b^2\arctan(\tan(e+fx))}{a-b}}{2b(a-b)}-\frac{a(3a-7b)\tan(e+fx)}{2b(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan^3(e+fx)}{4b(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

input

```
Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*Tan[e + f*x]^3)/((a - b)*b*(a + b*Tan[e + f*x]^2)^2) + (((-8*b^2*
ArcTan[Tan[e + f*x]])/(a - b) + (Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[
(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/((a - b)*Sqrt[b]))/(2*(a - b)*b) - (a*(3*
a - 7*b)*Tan[e + f*x])/(2*(a - b)*b*(a + b*Tan[e + f*x]^2)))/(4*(a - b)*b)
)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{a\left(\frac{-\frac{\left(5a^2-14ab+9b^2\right)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-10ab+7b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-10ab+15b^2\right)\arctan\left(\frac{b\tan(fx}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{a\left(\frac{-\frac{\left(5a^2-14ab+9b^2\right)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-10ab+7b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-10ab+15b^2\right)\arctan\left(\frac{b\tan(fx}{\sqrt{ab}}\right.}{8b^2\sqrt{ab}}\right.}{(a-b)^3}}{f}$
risch	$-\frac{x}{a^3-3a^2b+3ab^2-b^3}-\frac{i\left(3a^3e^{6i(fx+e)}-13a^2b\,e^{6i(fx+e)}+a\,b^2e^{6i(fx+e)}+9b^3e^{6i(fx+e)}+9a^3e^{4i(fx+e)}-21a^2b\,e^{4i(f\right.}{4\left(a\,e^{4i(fx+e)}-b\right.}$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)^3*arctan(tan(f*x+e))+a/(a-b)^3*((-1/8*(5*a^2-14*a*b+9*b^2)/b
*tan(f*x+e)^3-1/8*a*(3*a^2-10*a*b+7*b^2)/b^2*tan(f*x+e))/(a+b*tan(f*x+e)^2
)^2+1/8*(3*a^2-10*a*b+15*b^2)/b^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1
/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(139) = 278$.

Time = 0.13 (sec) , antiderivative size = 743, normalized size of antiderivative = 4.86

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(32*b^4*f*x*tan(f*x + e)^4 + 64*a*b^3*f*x*tan(f*x + e)^2 + 32*a^2*b
^2*f*x + 4*(5*a^3*b - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 + ((3*a^2*b^2 -
 10*a*b^3 + 15*b^4)*tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*
a^3*b - 10*a^2*b^2 + 15*a*b^3)*tan(f*x + e)^2)*sqrt(-a/b)*log((b^2*tan(f*x
 + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b^2*tan(f*x + e)^3 - a*b*tan(f*x
 + e))*sqrt(-a/b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) + 4*
(3*a^4 - 10*a^3*b + 7*a^2*b^2)*tan(f*x + e))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b
^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f
*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f), -1/16*(1
6*b^4*f*x*tan(f*x + e)^4 + 32*a*b^3*f*x*tan(f*x + e)^2 + 16*a^2*b^2*f*x +
2*(5*a^3*b - 14*a^2*b^2 + 9*a*b^3)*tan(f*x + e)^3 - ((3*a^2*b^2 - 10*a*b^3
 + 15*b^4)*tan(f*x + e)^4 + 3*a^4 - 10*a^3*b + 15*a^2*b^2 + 2*(3*a^3*b - 1
0*a^2*b^2 + 15*a*b^3)*tan(f*x + e)^2)*sqrt(a/b)*arctan(1/2*(b*tan(f*x + e)
^2 - a)*sqrt(a/b)/(a*tan(f*x + e))) + 2*(3*a^4 - 10*a^3*b + 7*a^2*b^2)*tan
(f*x + e))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^
4*b^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4
*b^3 + 3*a^3*b^4 - a^2*b^5)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8974 vs. 2(129) = 258.

Time = 48.85 (sec) , antiderivative size = 8974, normalized size of antiderivative = 58.65

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e + f*x)**5/
(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f*x)/f)/a**3, Eq(b, 0)), (x/b**3,
Eq(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f
*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e
+ f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b*
*3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)**2/(48*b**3*f*tan(
e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48
*b**3*f) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4
+ 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 33*tan(e + f*x)**5/(48*b**3*f*
tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2
+ 48*b**3*f) - 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*
tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 15*tan(e + f*x
)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan
(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x*tan(e)**6/(a + b*tan(e)**2)**3, E
q(f, 0)), (3*a**5*log(-sqrt(-a/b) + tan(e + f*x))/(16*a**5*b**3*f*sqrt(-a/
b) + 32*a**4*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**4*f*sqrt(-a/b)
 + 16*a**3*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**5*f*sqrt(-a/b)*t
an(e + f*x)**2 + 48*a**3*b**5*f*sqrt(-a/b) - 48*a**2*b**6*f*sqrt(-a/b)*tan
(e + f*x)**4 + 96*a**2*b**6*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**6*f*
sqrt(-a/b) + 48*a*b**7*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**7*f*sqrt(...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.50

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$=\frac{\frac{(3a^3-10a^2b+15ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3b^2-3a^2b^3+3ab^4-b^5)\sqrt{ab}}-\frac{(5a^2b-9ab^2)\tan(fx+e)^3+(3a^3-7a^2b)\tan(fx+e)}{a^4b^2-2a^3b^3+a^2b^4+(a^2b^4-2ab^5+b^6)\tan(fx+e)^4+2(a^3b^3-2a^2b^4+ab^5)\tan(fx+e)^2}-\frac{}{a^3-}}{8f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^3 - 10*a^2*b + 15*a*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*
b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sqrt(a*b)) - ((5*a^2*b - 9*a*b^2)*tan(f*x
 + e)^3 + (3*a^3 - 7*a^2*b)*tan(f*x + e))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 +
 (a^2*b^4 - 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5
)*tan(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.30

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$=\frac{(3a^3-10a^2b+15ab^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8(a^3b^2f-3a^2b^3f+3ab^4f-b^5f)\sqrt{ab}}-\frac{fx+e}{a^3f-3a^2bf+3ab^2f-b^3f}$$
$$-\frac{5a^2b\tan(fx+e)^3-9ab^2\tan(fx+e)^3+3a^3\tan(fx+e)-7a^2b\tan(fx+e)}{8(a^2b^2f-2ab^3f+b^4f)\left(b\tan(fx+e)^2+a\right)^2}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*a^3 - 10*a^2*b + 15*a*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*b
^2*f - 3*a^2*b^3*f + 3*a*b^4*f - b^5*f)*sqrt(a*b)) - (f*x + e)/(a^3*f - 3*
a^2*b*f + 3*a*b^2*f - b^3*f) - 1/8*(5*a^2*b*tan(f*x + e)^3 - 9*a*b^2*tan(f
*x + e)^3 + 3*a^3*tan(f*x + e) - 7*a^2*b*tan(f*x + e))/((a^2*b^2*f - 2*a*b
^3*f + b^4*f)*(b*tan(f*x + e)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 11.78 (sec) , antiderivative size = 3838, normalized size of antiderivative = 25.08

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^3*(9*a*b - 5*a^2))/(8*(a^2*b - 2*a*b^2 + b^3)) + (a*tan(e +
 f*x)*(7*a*b - 3*a^2))/(8*b*(a^2*b - 2*a*b^2 + b^3)))/(f*(a^2 + b^2*tan(e
+ f*x)^4 + 2*a*b*tan(e + f*x)^2)) - (2*atan((((((224*a*b^10 - 1440*a^2*b^9
 + 3936*a^3*b^8 - 5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4
 - 96*a^8*b^3)/(64*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 -
 6*a^5*b^4 + a^6*b^3)) - (tan(e + f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*
b^10 + 1280*a^3*b^9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7
*b^5)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b
^5 - 4*a^3*b^4 + a^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan
(e + f*x)*(9*a^6 - 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4
*b^2))/(32*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))/(6*a*b^2 -
6*a^2*b + 2*a^3 - 2*b^3) - ((((224*a*b^10 - 1440*a^2*b^9 + 3936*a^3*b^8 -
5920*a^4*b^7 + 5280*a^5*b^6 - 2784*a^6*b^5 + 800*a^7*b^4 - 96*a^8*b^3)/(64
*(b^9 - 6*a*b^8 + 15*a^2*b^7 - 20*a^3*b^6 + 15*a^4*b^5 - 6*a^5*b^4 + a^6*b
^3)) + (tan(e + f*x)*(1280*a*b^11 - 256*b^12 - 2304*a^2*b^10 + 1280*a^3*b^
9 + 1280*a^4*b^8 - 2304*a^5*b^7 + 1280*a^6*b^6 - 256*a^7*b^5)*1i)/(32*(6*a
*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a
^4*b^3)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^6 -
 60*a^5*b + 64*b^6 + 225*a^2*b^4 - 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7 -
4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)))/(6*a*b^2 - 6*a^2*b + 2*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.77

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**
4*a**2*b**2 - 10*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*
tan(e + f*x)**4*a*b**3 + 15*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)
*sqrt(a)))*tan(e + f*x)**4*b**4 + 6*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/
(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**3*b - 20*sqrt(b)*sqrt(a)*atan((tan(e
 + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**2*b**2 + 30*sqrt(b)*sqrt(
a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a*b**3 + 3*sqr
t(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**4 - 10*sqrt(b)*sq
rt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b + 15*sqrt(b)*sqrt(a)
*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 - 8*tan(e + f*x)**4*b*
*5*f*x - 5*tan(e + f*x)**3*a**3*b**2 + 14*tan(e + f*x)**3*a**2*b**3 - 9*ta
n(e + f*x)**3*a*b**4 - 16*tan(e + f*x)**2*a*b**4*f*x - 3*tan(e + f*x)*a**4
*b + 10*tan(e + f*x)*a**3*b**2 - 7*tan(e + f*x)*a**2*b**3 - 8*a**2*b**3*f*
x)/(8*b**3*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4*a**2*b**3 + 3*
tan(e + f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f*x)**2*a**4*b -
 6*tan(e + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3 - 2*tan(e + f*x
)**2*a*b**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.244 $\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{x}{(a-b)^3}+\frac{(a^2-6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^3b^{3/2}f} - \frac{a\tan(e+fx)}{4(a-b)bf\,(a+b\tan^2(e+fx))^2} + \frac{(a-5b)\tan(e+fx)}{8(a-b)^2bf\,(a+b\tan^2(e+fx))}$$

output
```
x/(a-b)^3+1/8*(a^2-6*a*b-3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(1/2)
/(a-b)^3/b^(3/2)/f-1/4*a*tan(f*x+e)/(a-b)/b/f/(a+b*tan(f*x+e)^2)^2+1/8*(a-
5*b)*tan(f*x+e)/(a-b)^2/b/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{8(e+fx)+\frac{(a^2-6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{(a-b)(a^2+2ab+5b^2+(a^2+4ab-5b^2)\cos(2(e+fx)))\sin(2(e+fx))}{b(a+b+(a-b)\cos(2(e+fx)))^2}}{8(a-b)^3f}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(8*(e + f*x) + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
]])/(Sqrt[a]*b^(3/2)) - ((a - b)*(a^2 + 2*a*b + 5*b^2 + (a^2 + 4*a*b - 5*b
^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b*(a + b + (a - b)*Cos[2*(e + f*x
)])^2))/(8*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4153, 372, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 372$$

$$\frac{\frac{\int \frac{(a-4b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4b(a-b)}-\frac{a\tan(e+fx)}{4b(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{a\left((a-5b)\tan^2(e+fx)+a+3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}+\frac{(a-5b)\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan(e+fx)}{4b(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{(a-5b)\tan^2(e+fx)+a+3b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2(a-b)}+\frac{(a-5b)\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan(e+fx)}{4b(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{\left(a^2-6ab-3b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}+\frac{8b\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}}{2(a-b)}+\frac{(a-5b)\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan(e+fx)}{4b(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{\left(a^2-6ab-3b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}+\frac{8b\arctan(\tan(e+fx))}{a-b}}{2(a-b)}+\frac{(a-5b)\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan(e+fx)}{4b(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{\left(a^2-6ab-3b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a-b)}+\frac{8b\arctan(\tan(e+fx))}{a-b}}{2(a-b)}+\frac{(a-5b)\tan(e+fx)}{2(a-b)\left(a+b\tan^2(e+fx)\right)}}{4b(a-b)}-\frac{a\tan(e+fx)}{4b(a-b)\left(a+b\tan^2(e+fx)\right)^2}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*Tan[e + f*x])/((a - b)*b*(a + b*Tan[e + f*x]^2)^2) + (((8*b*ArcTa
n[Tan[e + f*x]])/(a - b) + ((a^2 - 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)*Sqrt[b]))/(2*(a - b)) + ((a - 5*b)*Tan[e
+ f*x])/(2*(a - b)*(a + b*Tan[e + f*x]^2)))/(4*(a - b)*b))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{3}{4}ab+\frac{5}{8}b^2\right)\tan(fx+e)^3-\frac{a\left(a^2+2ab-3b^2\right)\tan(fx+e)}{8b}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-6ab-3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a-b)^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{\frac{\left(\frac{1}{8}a^2-\frac{3}{4}ab+\frac{5}{8}b^2\right)\tan(fx+e)^3-\frac{a\left(a^2+2ab-3b^2\right)\tan(fx+e)}{8b}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-6ab-3b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a-b)^3}}{f}$
risch	$\frac{x}{a^3-3a^2b+3ab^2-b^3}-\frac{i\left(a^3e^{6i(fx+e)}+9a^2b\,e^{6i(fx+e)}-5a\,b^2e^{6i(fx+e)}-5b^3e^{6i(fx+e)}+3a^3e^{4i(fx+e)}+17a^2b\,e^{4i(fx+e)}\right.}{4\left(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}+\right.}$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^3*arctan(tan(f*x+e))+1/(a-b)^3*(((1/8*a^2-3/4*a*b+5/8*b^2)*ta
n(f*x+e)^3-1/8*a*(a^2+2*a*b-3*b^2)/b*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*
(a^2-6*a*b-3*b^2)/b/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(131) = 262$.

Time = 0.12 (sec) , antiderivative size = 749, normalized size of antiderivative = 5.17

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*a*b^4*f*x*tan(f*x + e)^4 + 64*a^2*b^3*f*x*tan(f*x + e)^2 + 32*a^
3*b^2*f*x + 4*(a^3*b^2 - 6*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 - ((a^2*b^2 -
 6*a*b^3 - 3*b^4)*tan(f*x + e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b -
6*a^2*b^2 - 3*a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 -
6*a*b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-a
*b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(a^4*b + 2*a^3
*b^2 - 3*a^2*b^3)*tan(f*x + e))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)
*f*tan(f*x + e)^4 + 2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*
x + e)^2 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f), 1/16*(16*a*b^4*
f*x*tan(f*x + e)^4 + 32*a^2*b^3*f*x*tan(f*x + e)^2 + 16*a^3*b^2*f*x + 2*(a
^3*b^2 - 6*a^2*b^3 + 5*a*b^4)*tan(f*x + e)^3 + ((a^2*b^2 - 6*a*b^3 - 3*b^4
)*tan(f*x + e)^4 + a^4 - 6*a^3*b - 3*a^2*b^2 + 2*(a^3*b - 6*a^2*b^2 - 3*a*
b^3)*tan(f*x + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*b)
/(a*b*tan(f*x + e))) - 2*(a^4*b + 2*a^3*b^2 - 3*a^2*b^3)*tan(f*x + e))/((a
^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^4 + 2*(a^5*b^3 - 3*
a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^2 + (a^6*b^2 - 3*a^5*b^3 + 3
*a^4*b^4 - a^3*b^5)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8957 vs. $2(126) = 252$.

Time = 47.80 (sec) , antiderivative size = 8957, normalized size of antiderivative = 61.77

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((x + tan(e +
 f*x)**3/(3*f) - tan(e + f*x)/f)/a**3, Eq(b, 0)), ((-x - 1/(f*tan(e + f*x)
))/b**3, Eq(a, 0)), (3*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)**6 + 14
4*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x
*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 +
 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x*tan(e + f*x)**2/(48*b**3*
f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**
2 + 48*b**3*f) + 3*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x
)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 3*tan(e + f*x)**5/(48*b**
3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)
**2 + 48*b**3*f) - 8*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b**3
*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 3*tan(e + f
*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*t
an(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x*tan(e)**4/(a + b*tan(e)**2)**3,
 Eq(f, 0)), (a**4*log(-sqrt(-a/b) + tan(e + f*x))/(16*a**5*b**2*f*sqrt(-a/
b) + 32*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**4*b**3*f*sqrt(-a/b)
 + 16*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**3*b**4*f*sqrt(-a/b)*t
an(e + f*x)**2 + 48*a**3*b**4*f*sqrt(-a/b) - 48*a**2*b**5*f*sqrt(-a/b)*tan
(e + f*x)**4 + 96*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**2*b**5*f*
sqrt(-a/b) + 48*a*b**6*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a*b**6*f*sqrt(...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.46

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{\frac{(a^2-6\,ab-3\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^3b-3\,a^2b^2+3\,ab^3-b^4)\sqrt{ab}} + \frac{(ab-5\,b^2)\tan(fx+e)^3-(a^2+3\,ab)\tan(fx+e)}{a^4b-2\,a^3b^2+a^2b^3+(a^2b^3-2\,ab^4+b^5)\tan(fx+e)^4+2\,(a^3b^2-2\,a^2b^3+ab^4)\tan(fx+e)^2} + \frac{8\,(fx+}{a^3-3\,a^2b+3}}{8\,f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((a^2 - 6*a*b - 3*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*sqrt(a*b)) + ((a*b - 5*b^2)*tan(f*x + e)^3 - (a^2 +
 3*a*b)*tan(f*x + e))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^2*b^3 - 2*a*b^4 +
b^5)*tan(f*x + e)^4 + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*tan(f*x + e)^2) + 8*
(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.26

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{(a^2-6\,ab-3\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,(a^3bf-3\,a^2b^2f+3\,ab^3f-b^4f)\sqrt{ab}} + \frac{fx+e}{a^3f-3\,a^2bf+3\,ab^2f-b^3f} + \frac{ab\tan(fx+e)^3-5\,b^2\tan(fx+e)^3-a^2\tan(fx+e)-3\,ab\tan(fx+e)}{8\,(a^2bf-2\,ab^2f+b^3f)\left(b\tan(fx+e)^2+a\right)^2}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(a^2 - 6*a*b - 3*b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^3*b*f - 3*a
^2*b^2*f + 3*a*b^3*f - b^4*f)*sqrt(a*b)) + (f*x + e)/(a^3*f - 3*a^2*b*f +
3*a*b^2*f - b^3*f) + 1/8*(a*b*tan(f*x + e)^3 - 5*b^2*tan(f*x + e)^3 - a^2*
tan(f*x + e) - 3*a*b*tan(f*x + e))/((a^2*b*f - 2*a*b^2*f + b^3*f)*(b*tan(f
*x + e)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 11.53 (sec) , antiderivative size = 3667, normalized size of antiderivative = 25.29

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^3*(a - 5*b))/(8*(a^2 - 2*a*b + b^2)) - (a*tan(e + f*x)*(a +
 3*b))/(8*(a^2*b - 2*a*b^2 + b^3)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*t
an(e + f*x)^2)) - (2*atan((((((544*a*b^8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^
3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^2)/(64*(a^6*b - 6
*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a^5*b^2)) - (tan(e
 + f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*b^7 + 1280*a^4*b^
6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6*a*b^2 - 6*a^2*b
+ 2*a^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*1i)/(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + (tan(e + f*x)*(36*a*b^3 - 12*a^3*b + a^
4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b
^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((544*a*b^8 - 96*b^9 - 1248*
a^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 96*a^6*b^3 - 32*a^7*b^
2)/(64*(a^6*b - 6*a*b^6 + b^7 + 15*a^2*b^5 - 20*a^3*b^4 + 15*a^4*b^3 - 6*a
^5*b^2)) + (tan(e + f*x)*(1280*a*b^9 - 256*b^10 - 2304*a^2*b^8 + 1280*a^3*
b^7 + 1280*a^4*b^6 - 2304*a^5*b^5 + 1280*a^6*b^4 - 256*a^7*b^3)*1i)/(32*(6
*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a
^3*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(36*a*b^
3 - 12*a^3*b + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(a^4*b - 4*a*b^4 + b^5 + 6*
a^2*b^3 - 4*a^3*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((((((544*a*b^
8 - 96*b^9 - 1248*a^2*b^7 + 1440*a^3*b^6 - 800*a^4*b^5 + 96*a^5*b^4 + 9...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.00

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4 a^2b^2 - 6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4 a\,b^3 - 3\sqrt{b}\,\sqrt{a}\,atan}{}$$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**4*
a**2*b**2 - 6*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan
(e + f*x)**4*a*b**3 - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqr
t(a)))*tan(e + f*x)**4*b**4 + 2*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqr
t(b)*sqrt(a)))*tan(e + f*x)**2*a**3*b - 12*sqrt(b)*sqrt(a)*atan((tan(e + f
*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**2*b**2 - 6*sqrt(b)*sqrt(a)*at
an((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a*b**3 + sqrt(b)*sq
rt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**4 - 6*sqrt(b)*sqrt(a)*at
an((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b - 3*sqrt(b)*sqrt(a)*atan((ta
n(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*tan(e + f*x)**4*a*b**4*f*x
+ tan(e + f*x)**3*a**3*b**2 - 6*tan(e + f*x)**3*a**2*b**3 + 5*tan(e + f*x)
**3*a*b**4 + 16*tan(e + f*x)**2*a**2*b**3*f*x - tan(e + f*x)*a**4*b - 2*ta
n(e + f*x)*a**3*b**2 + 3*tan(e + f*x)*a**2*b**3 + 8*a**3*b**2*f*x)/(8*a*b*
*2*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4*a**2*b**3 + 3*tan(e +
f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f*x)**2*a**4*b - 6*tan(e
 + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3 - 2*tan(e + f*x)**2*a*b
**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.245 $\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3} dx = -\frac{x}{(a-b)^3} + \frac{(3a^2+6ab-b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^3\sqrt{b}f} + \frac{\tan(e+fx)}{4(a-b)f(a+b\tan^2(e+fx))^2} + \frac{(3a+b)\tan(e+fx)}{8a(a-b)^2f(a+b\tan^2(e+fx))}$$

output
```
-x/(a-b)^3+1/8*(3*a^2+6*a*b-b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1/2))/a^(3/2
)/(a-b)^3/b^(1/2)/f+1/4*tan(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^2+1/8*(3*a+b
)*tan(f*x+e)/a/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{-8(e+fx)+\frac{(3a^2+6ab-b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{(a-b)(5a^2+2ab+b^2+(5a^2-4ab-b^2)\cos(2(e+fx)))\sin(2(e+fx))}{a(a+b+(a-b)\cos(2(e+fx)))^2}}{8(a-b)^3f}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-8*(e + f*x) + ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a]])/(a^(3/2)*Sqrt[b]) + ((a - b)*(5*a^2 + 2*a*b + b^2 + (5*a^2 - 4*a*b -
b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a*(a + b + (a - b)*Cos[2*(e + f*
x)])^2))/(8*(a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 373, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^3}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 373

$$\frac{\frac{\tan(e+fx)}{4(a-b)(a+b\tan^2(e+fx))^2}-\frac{\int\frac{1-3\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{4(a-b)}}{f}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tan(e+fx)}{4(a-b)(a+b\tan^2(e+fx))^2}-\frac{\frac{\int\frac{-((3a+b)\tan^2(e+fx))+5a-b}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{2a(a-b)}-\frac{(3a+b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4(a-b)}}{f}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tan(e+fx)}{4(a-b)(a+b\tan^2(e+fx))^2}-\frac{\frac{\frac{8a\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{(3a^2+6ab-b^2)\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{(3a+b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4(a-b)}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{4(a-b)(a+b\tan^2(e+fx))^2}-\frac{\frac{\frac{8a\arctan(\tan(e+fx))}{a-b}-\frac{(3a^2+6ab-b^2)\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{(3a+b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4(a-b)}}{f}$$

$\Big\downarrow$ 218

$$\frac{\frac{\tan(e+fx)}{4(a-b)(a+b\tan^2(e+fx))^2}-\frac{\frac{\frac{8a\arctan(\tan(e+fx))}{a-b}-\frac{(3a^2+6ab-b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a-b)}}{2a(a-b)}-\frac{(3a+b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4(a-b)}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*(a - b)*(a + b*Tan[e + f*x]^2)^2) - (((8*a*ArcTan[Tan[e +
 f*x]])/(a - b) - ((3*a^2 + 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqr
t[a]])/(Sqrt[a]*(a - b)*Sqrt[b]))/(2*a*(a - b)) - ((3*a + b)*Tan[e + f*x])
/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/(4*(a - b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{\frac{\frac{b(3a^2-2ab-b^2)\tan(fx+e)^3}{8a}+\left(\frac{5}{8}a^2-\frac{3}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{(3a^2+6ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a-b)^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{\frac{\frac{b(3a^2-2ab-b^2)\tan(fx+e)^3}{8a}+\left(\frac{5}{8}a^2-\frac{3}{4}ab+\frac{1}{8}b^2\right)\tan(fx+e)}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{(3a^2+6ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a-b)^3}}{f}$
risch	$-\frac{x}{a^3-3a^2b+3a\,b^2-b^3}+\frac{i(5a^3e^{6i(fx+e)}+5a^2b\,e^{6i(fx+e)}-9a\,b^2e^{6i(fx+e)}-b^3e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+13a^2b\,e^{4i(f}}{4(a\,e^{4i(fx+e)}-b\,e^{4i(fx+e)}}$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)^3*arctan(tan(f*x+e))+1/(a-b)^3*((1/8*b*(3*a^2-2*a*b-b^2)/a*t
an(f*x+e)^3+(5/8*a^2-3/4*a*b+1/8*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8
*(3*a^2+6*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(130) = 260.

Time = 0.14 (sec) , antiderivative size = 759, normalized size of antiderivative = 5.27

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(32*a^2*b^3*f*x*tan(f*x + e)^4 + 64*a^3*b^2*f*x*tan(f*x + e)^2 + 32
*a^4*b*f*x - 4*(3*a^3*b^2 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 + ((3*a^2*b^
2 + 6*a*b^3 - b^4)*tan(f*x + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b
 + 6*a^2*b^2 - a*b^3)*tan(f*x + e)^2)*sqrt(-a*b)*log((b^2*tan(f*x + e)^4 -
 6*a*b*tan(f*x + e)^2 + a^2 - 4*(b*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(-
a*b))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(5*a^4*b - 6*
a^3*b^2 + a^2*b^3)*tan(f*x + e))/((a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b
^6)*f*tan(f*x + e)^4 + 2*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan
(f*x + e)^2 + (a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f), -1/16*(16*a^2*
b^3*f*x*tan(f*x + e)^4 + 32*a^3*b^2*f*x*tan(f*x + e)^2 + 16*a^4*b*f*x - 2*
(3*a^3*b^2 - 2*a^2*b^3 - a*b^4)*tan(f*x + e)^3 - ((3*a^2*b^2 + 6*a*b^3 - b
^4)*tan(f*x + e)^4 + 3*a^4 + 6*a^3*b - a^2*b^2 + 2*(3*a^3*b + 6*a^2*b^2 -
a*b^3)*tan(f*x + e)^2)*sqrt(a*b)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(a*
b)/(a*b*tan(f*x + e))) - 2*(5*a^4*b - 6*a^3*b^2 + a^2*b^3)*tan(f*x + e))/(
(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f*tan(f*x + e)^4 + 2*(a^6*b^2
- 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^2 + (a^7*b - 3*a^6*b^2 +
 3*a^5*b^3 - a^4*b^4)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9051 vs. $2(122) = 244$.

Time = 47.07 (sec) , antiderivative size = 9051, normalized size of antiderivative = 62.85

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(e)**4, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((-x + tan(e
+ f*x)/f)/a**3, Eq(b, 0)), ((x + 1/(f*tan(e + f*x)) - 1/(3*f*tan(e + f*x)*
*3))/b**3, Eq(a, 0)), (3*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*x)**6 +
144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f
*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4
 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 9*f*x*tan(e + f*x)**2/(48*b**
3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)
**2 + 48*b**3*f) + 3*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f
*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 3*tan(e + f*x)**5/(48*b
**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*
x)**2 + 48*b**3*f) + 8*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x)**6 + 144*b*
*3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) - 3*tan(e +
 f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f
*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x*tan(e)**2/(a + b*tan(e)**2)**
3, Eq(f, 0)), (3*a**4*log(-sqrt(-a/b) + tan(e + f*x))/(16*a**6*b*f*sqrt(-a
/b) + 32*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**5*b**2*f*sqrt(-a/b
) + 16*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 - 96*a**4*b**3*f*sqrt(-a/b)*
tan(e + f*x)**2 + 48*a**4*b**3*f*sqrt(-a/b) - 48*a**3*b**4*f*sqrt(-a/b)*ta
n(e + f*x)**4 + 96*a**3*b**4*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**3*b**4*f
*sqrt(-a/b) + 48*a**2*b**5*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**2*b**5*...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.48

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\frac{(3a^2+6ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^4-3a^3b+3a^2b^2-ab^3)\sqrt{ab}} + \frac{(3ab+b^2)\tan(fx+e)^3+(5a^2-ab)\tan(fx+e)}{a^5-2a^4b+a^3b^2+(a^3b^2-2a^2b^3+ab^4)\tan(fx+e)^4+2(a^4b-2a^3b^2+a^2b^3)\tan(fx+e)^2} - \frac{8(fx\cdots}{a^3-3a^2b+\cdots}}{8f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 6*a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4 - 3*a^3*
b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)) + ((3*a*b + b^2)*tan(f*x + e)^3 + (5*a^2
 - a*b)*tan(f*x + e))/(a^5 - 2*a^4*b + a^3*b^2 + (a^3*b^2 - 2*a^2*b^3 + a*
b^4)*tan(f*x + e)^4 + 2*(a^4*b - 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^2) - 8*
(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.29

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{(3a^2+6ab-b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8(a^4f-3a^3bf+3a^2b^2f-ab^3f)\sqrt{ab}} - \frac{fx+e}{a^3f-3a^2bf+3ab^2f-b^3f} + \frac{3ab\tan(fx+e)^3+b^2\tan(fx+e)^3+5a^2\tan(fx+e)-ab\tan(fx+e)}{8(a^3f-2a^2bf+ab^2f)\left(b\tan(fx+e)^2+a\right)^2}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*a^2 + 6*a*b - b^2)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^4*f - 3*a^3
*b*f + 3*a^2*b^2*f - a*b^3*f)*sqrt(a*b)) - (f*x + e)/(a^3*f - 3*a^2*b*f +
3*a*b^2*f - b^3*f) + 1/8*(3*a*b*tan(f*x + e)^3 + b^2*tan(f*x + e)^3 + 5*a^
2*tan(f*x + e) - a*b*tan(f*x + e))/((a^3*f - 2*a^2*b*f + a*b^2*f)*(b*tan(f
*x + e)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 11.73 (sec) , antiderivative size = 3817, normalized size of antiderivative = 26.51

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)*(5*a - b))/(8*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^3*(3*a*b
 + b^2))/(8*a*(a^2 - 2*a*b + b^2)))/(f*(a^2 + b^2*tan(e + f*x)^4 + 2*a*b*t
an(e + f*x)^2)) - (2*atan((((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 304
0*a^4*b^6 + 3680*a^5*b^5 - 2592*a^6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(
a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2
)) - (tan(e + f*x)*(256*a^2*b^9 - 1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b
^6 - 1280*a^6*b^5 + 2304*a^7*b^4 - 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*
a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*
a^4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - (tan(e + f*x)*(9*a^4*
b - 12*a*b^4 + b^5 + 94*a^2*b^3 + 36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^
4 - 4*a^3*b^3 + 6*a^4*b^2)))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - ((((32*
a*b^9 - 352*a^2*b^8 + 1440*a^3*b^7 - 3040*a^4*b^6 + 3680*a^5*b^5 - 2592*a^
6*b^4 + 992*a^7*b^3 - 160*a^8*b^2)/(64*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^
5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)) + (tan(e + f*x)*(256*a^2*b^9 -
1280*a^3*b^8 + 2304*a^4*b^7 - 1280*a^5*b^6 - 1280*a^6*b^5 + 2304*a^7*b^4 -
 1280*a^8*b^3 + 256*a^9*b^2)*1i)/(32*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(
a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*1i)/(6*a*b^2 - 6*a^2*b
+ 2*a^3 - 2*b^3) + (tan(e + f*x)*(9*a^4*b - 12*a*b^4 + b^5 + 94*a^2*b^3 +
36*a^3*b^2))/(32*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))/(6*a*
b^2 - 6*a^2*b + 2*a^3 - 2*b^3))/((((((32*a*b^9 - 352*a^2*b^8 + 1440*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.04

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4 a^2b^2 + 6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4 a\,b^3 - \sqrt{b}\,\sqrt{a}\,atan}{}$$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**
4*a**2*b**2 + 6*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*t
an(e + f*x)**4*a*b**3 - sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqr
t(a)))*tan(e + f*x)**4*b**4 + 6*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqr
t(b)*sqrt(a)))*tan(e + f*x)**2*a**3*b + 12*sqrt(b)*sqrt(a)*atan((tan(e + f
*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**2*b**2 - 2*sqrt(b)*sqrt(a)*at
an((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a*b**3 + 3*sqrt(b)*
sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 6*sqrt(b)*sqrt(a)*
atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b - sqrt(b)*sqrt(a)*atan((ta
n(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 - 8*tan(e + f*x)**4*a**2*b**3*f
*x + 3*tan(e + f*x)**3*a**3*b**2 - 2*tan(e + f*x)**3*a**2*b**3 - tan(e + f
*x)**3*a*b**4 - 16*tan(e + f*x)**2*a**3*b**2*f*x + 5*tan(e + f*x)*a**4*b -
 6*tan(e + f*x)*a**3*b**2 + tan(e + f*x)*a**2*b**3 - 8*a**4*b*f*x)/(8*a**2
*b*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4*a**2*b**3 + 3*tan(e +
f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f*x)**2*a**4*b - 6*tan(e
 + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3 - 2*tan(e + f*x)**2*a*b
**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.246 $\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \frac{x}{(a-b)^3} - \frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3f} - \frac{b\tan(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(7a-3b)b\tan(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)}$$

output
```
x/(a-b)^3-1/8*b^(1/2)*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(1
/2))/a^(5/2)/(a-b)^3/f-1/4*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8
*(7*a-3*b)*b*tan(f*x+e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{-8\arctan(\tan(e+fx))+\frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2(a-b)^2b\tan(e+fx)}{a(a+b\tan^2(e+fx))^2}+\frac{(7a-3b)(a-b)b\tan(e+fx)}{a^2(a+b\tan^2(e+fx))}}{8(a-b)^3f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-3),x]
```

output
```
-1/8*(-8*ArcTan[Tan[e + f*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[
(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[e + f*x])/(a
*(a + b*Tan[e + f*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[e + f*x])/(a^2*(a
+ b*Tan[e + f*x]^2)))/((a - b)^3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4144, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\frac{\int \frac{-3b\tan^2(e+fx)+4a-3b}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^2}d\tan(e+fx)}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{8a^2-7ba+3b^2-(7a-3b)b\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)}d\tan(e+fx)}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{\frac{8a^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b(15a^2-10ab+3b^2)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{\frac{8a^2\arctan(\tan(e+fx))}{a-b}-\frac{b(15a^2-10ab+3b^2)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{8a^2\arctan(\tan(e+fx))}{a-b}-\frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b(7a-3b)\tan(e+fx)}{2a(a-b)(a+b\tan^2(e+fx))}}{4a(a-b)}-\frac{b\tan(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a - b)*(a + b*Tan[e + f*x]^2)^2) + (((8*a^2*Arc
Tan[Tan[e + f*x]])/(a - b) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - ((7*a - 3
*b)*b*Tan[e + f*x])/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/(4*a*(a - b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\tan(fx+e)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(fx+e)}{8a}}{(a+b\tan(fx+e)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(f}{(a-b)^3}}{f}$
default	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\tan(fx+e)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(fx+e)}{8a}}{(a+b\tan(fx+e)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(f}{(a-b)^3}}{f}$
risch	$\frac{x}{a^3-3a^2b+3ab^2-b^3}+\frac{ib(9a^3e^{6i(fx+e)}+a^2be^{6i(fx+e)}-13ab^2e^{6i(fx+e)}+3b^3e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+9a^2be^{4i(fx+}{4(-ae^{4i(fx+e)}+be^{4i(fx+e)}}$

input
```
int(1/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a-b)^3*((1/8*b*(7*a^2-10*a*b+3*b^2)/a^2*tan(f*x+e)^3+1/8*(9*a^2-1
4*a*b+5*b^2)/a*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(15*a^2-10*a*b+3*b^2)/
a^2/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))+1/(a-b)^3*arctan(tan(f*x
+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(136) = 272$.

Time = 0.12 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.95

$$\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*a^2*b^2*f*x*tan(f*x + e)^4 + 64*a^3*b*f*x*tan(f*x + e)^2 + 32*a^
4*f*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 1
0*a*b^3 + 3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^
3*b - 10*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(-b/a)*log((b^2*tan(f*x +
e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x +
e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)) - 4*(9*
a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b
^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/16*(16*
a^2*b^2*f*x*tan(f*x + e)^4 + 32*a^3*b*f*x*tan(f*x + e)^2 + 16*a^4*f*x - 2*
(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(f*x + e)^3 - ((15*a^2*b^2 - 10*a*b^3 +
3*b^4)*tan(f*x + e)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a
^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 -
 a)*sqrt(b/a)/(b*tan(f*x + e))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(f
*x + e))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2
*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6
*b + 3*a^5*b^2 - a^4*b^3)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(129) = 258$.

Time = 46.71 (sec) , antiderivative size = 8964, normalized size of antiderivative = 59.76

$$\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(e)**6, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (x/a**3, Eq(b
, 0)), ((-x - 1/(f*tan(e + f*x)) + 1/(3*f*tan(e + f*x)**3) - 1/(5*f*tan(e
+ f*x)**5))/b**3, Eq(a, 0)), (15*f*x*tan(e + f*x)**6/(48*b**3*f*tan(e + f*
x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*
f) + 45*f*x*tan(e + f*x)**4/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e
+ f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 45*f*x*tan(e + f*x)*
*2/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*ta
n(e + f*x)**2 + 48*b**3*f) + 15*f*x/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*
f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f) + 15*tan(e + f
*x)**5/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*
f*tan(e + f*x)**2 + 48*b**3*f) + 40*tan(e + f*x)**3/(48*b**3*f*tan(e + f*x
)**6 + 144*b**3*f*tan(e + f*x)**4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f
) + 33*tan(e + f*x)/(48*b**3*f*tan(e + f*x)**6 + 144*b**3*f*tan(e + f*x)**
4 + 144*b**3*f*tan(e + f*x)**2 + 48*b**3*f), Eq(a, b)), (x/(a + b*tan(e)**
2)**3, Eq(f, 0)), (16*a**4*f*x*sqrt(-a/b)/(16*a**7*f*sqrt(-a/b) + 32*a**6*
b*f*sqrt(-a/b)*tan(e + f*x)**2 - 48*a**6*b*f*sqrt(-a/b) + 16*a**5*b**2*f*s
qrt(-a/b)*tan(e + f*x)**4 - 96*a**5*b**2*f*sqrt(-a/b)*tan(e + f*x)**2 + 48
*a**5*b**2*f*sqrt(-a/b) - 48*a**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**4 + 96*a
**4*b**3*f*sqrt(-a/b)*tan(e + f*x)**2 - 16*a**4*b**3*f*sqrt(-a/b) + 48*a**
3*b**4*f*sqrt(-a/b)*tan(e + f*x)**4 - 32*a**3*b**4*f*sqrt(-a/b)*tan(e +...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx = -\frac{\frac{(15a^2b-10ab^2+3b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^5-3a^4b+3a^3b^2-a^2b^3)\sqrt{ab}} + \frac{(7ab^2-3b^3)\tan(fx+e)^3+(9a^2b-5ab^2)\tan(fx+e)}{a^6-2a^5b+a^4b^2+(a^4b^2-2a^3b^3+a^2b^4)\tan(fx+e)^4+2(a^5b-2a^4b^2+a^3b^3)\tan(fx+e)^2} - \cdots}{8f}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(f*x
+ e)^3 + (9*a^2*b - 5*a*b^2)*tan(f*x + e))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4
*b^2 - 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^
3)*tan(f*x + e)^2) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\left(a^5f-3\,a^4bf+3\,a^3b^2f-a^2b^3f\right)\sqrt{ab}}+\frac{fx+e}{a^3f-3\,a^2bf+3\,ab^2f-b^3f} - \frac{7\,ab^2\tan\left(fx+e\right)^3-3\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)-5\,ab^2\tan\left(fx+e\right)}{8\left(a^4f-2\,a^3bf+a^2b^2f\right)\left(b\tan\left(fx+e\right)^2+a\right)^2}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^5*
f - 3*a^4*b*f + 3*a^3*b^2*f - a^2*b^3*f)*sqrt(a*b)) + (f*x + e)/(a^3*f - 3
*a^2*b*f + 3*a*b^2*f - b^3*f) - 1/8*(7*a*b^2*tan(f*x + e)^3 - 3*b^3*tan(f*
x + e)^3 + 9*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e))/((a^4*f - 2*a^3*b*
f + a^2*b^2*f)*(b*tan(f*x + e)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 11.95 (sec) , antiderivative size = 3901, normalized size of antiderivative = 26.01

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(atan((((-a^5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 30
0*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6
*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760
*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*
(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b
^2)) - (tan(e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9
 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^
4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b
^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15
*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^
2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^
5*b)^(1/2)*((tan(e + f*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 +
289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((
96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9
056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^
9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(
e + f*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*
b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^1
0*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4
*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.85

$$\int \frac{1}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4\,a^2b^2+10\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(fx+e)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(fx+e)^4\,a\,b^3-3\sqrt{b}\,\sqrt{a}}{}$$

input
```
int(1/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*
x)**4*a**2*b**2 + 10*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a
)))*tan(e + f*x)**4*a*b**3 - 3*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt
(b)*sqrt(a)))*tan(e + f*x)**4*b**4 - 30*sqrt(b)*sqrt(a)*atan((tan(e + f*x)
*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**3*b + 20*sqrt(b)*sqrt(a)*atan((t
an(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a**2*b**2 - 6*sqrt(b)*sq
rt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*tan(e + f*x)**2*a*b**3 - 15
*sqrt(b)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 10*sqrt(b
)*sqrt(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b - 3*sqrt(b)*sqrt
(a)*atan((tan(e + f*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*tan(e + f*x)**4
*a**3*b**2*f*x - 7*tan(e + f*x)**3*a**3*b**2 + 10*tan(e + f*x)**3*a**2*b**
3 - 3*tan(e + f*x)**3*a*b**4 + 16*tan(e + f*x)**2*a**4*b*f*x - 9*tan(e + f
*x)*a**4*b + 14*tan(e + f*x)*a**3*b**2 - 5*tan(e + f*x)*a**2*b**3 + 8*a**5
*f*x)/(8*a**3*f*(tan(e + f*x)**4*a**3*b**2 - 3*tan(e + f*x)**4*a**2*b**3 +
 3*tan(e + f*x)**4*a*b**4 - tan(e + f*x)**4*b**5 + 2*tan(e + f*x)**2*a**4*
b - 6*tan(e + f*x)**2*a**3*b**2 + 6*tan(e + f*x)**2*a**2*b**3 - 2*tan(e +
f*x)**2*a*b**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.247 $\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 189

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^3} \, dx = -\frac{x}{(a-b)^3} + \frac{b^{3/2}(35a^2-42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^3 f} - \frac{(8a^2-27ab+15b^2)\cot(e+fx)}{8a^3(a-b)^2 f} - \frac{b\cot(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} - \frac{(9a-5b)b\cot(e+fx)}{8a^2(a-b)^2 f\left(a+b\tan^2(e+fx)\right)}$$

output

```
-x/(a-b)^3+1/8*b^(3/2)*(35*a^2-42*a*b+15*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^
(1/2))/a^(7/2)/(a-b)^3/f-1/8*(8*a^2-27*a*b+15*b^2)*cot(f*x+e)/a^3/(a-b)^2/
f-1/4*b*cot(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(9*a-5*b)*b*cot(f*x+
e)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{8(e+fx)}{(-a+b)^3}+\frac{b^{3/2}(35a^2-42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{7/2}(a-b)^3}-\frac{8\cot(e+fx)}{a^3}-\frac{4b^3\sin(2(e+fx))}{a^2(a-b)^2(a+b+(a-b)\cos(2(e+fx)))^2}+\frac{(13a-7b)b^2}{a^3(a-b)^2(a+b+(}}{8f}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((8*(e + f*x))/(-a + b)^3 + (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a]])/(a^(7/2)*(a - b)^3) - (8*Cot[e + f*x])/a^3 -
 (4*b^3*Sin[2*(e + f*x)])/(a^2*(a - b)^2*(a + b + (a - b)*Cos[2*(e + f*x)]
)^2) + ((13*a - 7*b)*b^2*Sin[2*(e + f*x)])/(a^3*(a - b)^2*(a + b + (a - b)
*Cos[2*(e + f*x)])))/(8*f)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4153, 374, 441, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \ 4153$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 374$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-5b\tan^2(e+fx)+4a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4a(a-b)}-\frac{b\cot(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 441$$

$$\frac{\frac{\frac{\int \frac{\cot^2(e+fx)\left(8a^2-27ba+15b^2-3(9a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b(9a-5b)\cot(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\cot(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{\frac{-\frac{\int \frac{8a^3+8ba^2-27b^2a+15b^3+b\left(8a^2-27ba+15b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b(9a-5b)\cot(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\cot(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{-\frac{\frac{8a^3\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^2\left(35a^2-42ab+15b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b(9a-5b)\cot(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\cot(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{-\frac{\frac{8a^3\arctan(\tan(e+fx))}{a-b}-\frac{b^2\left(35a^2-42ab+15b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b(9a-5b)\cot(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\cot(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{-\frac{\left(8a^2-27ab+15b^2\right)\cot(e+fx)}{a}-\frac{\frac{8a^3\arctan(\tan(e+fx))}{a-b}-\frac{b^{3/2}\left(35a^2-42ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}}{2a(a-b)}-\frac{b(9a-5b)\cot(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\cot(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Cot[e + f*x])/(a*(a - b)*(a + b*Tan[e + f*x]^2)^2) + ((-(((8*a^3*
ArcTan[Tan[e + f*x]])/(a - b) - (b^(3/2)*(35*a^2 - 42*a*b + 15*b^2)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/a) - ((8*a^2 - 27*a*b
 + 15*b^2)*Cot[e + f*x])/a)/(2*a*(a - b)) - ((9*a - 5*b)*b*Cot[e + f*x])/(
2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/(4*a*(a - b)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{1}{a^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b-\frac{9}{4}a\,b^2+\frac{7}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(13a^2-22ab+9b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2-42ab+15b^2\right)\arctan\left(\frac{b\tan(fx}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^3(a-b)^3}}{f}$
default	$\frac{-\frac{1}{a^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b-\frac{9}{4}a\,b^2+\frac{7}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(13a^2-22ab+9b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2-42ab+15b^2\right)\arctan\left(\frac{b\tan(fx}{\sqrt{ab}}\right.}{8\sqrt{ab}}\right.}{a^3(a-b)^3}}{f}$
risch	$-\frac{x}{a^3-3a^2b+3a\,b^2-b^3}-\frac{i\left(-40a^4b+67a\,b^4+8a^5-15b^5-15b^5\mathrm{e}^{8i(fx+e)}+60b^5\mathrm{e}^{6i(fx+e)}+32a^5\mathrm{e}^{6i(fx+e)}+8a^5\mathrm{e}^{8i(fx+e}\right.}{}$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3/tan(f*x+e)+b^2/a^3/(a-b)^3*(((11/8*a^2*b-9/4*a*b^2+7/8*b^3)*ta
n(f*x+e)^3+1/8*a*(13*a^2-22*a*b+9*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/
8*(35*a^2-42*a*b+15*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/(
a-b)^3*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(173) = 346$.

Time = 0.14 (sec) , antiderivative size = 881, normalized size of antiderivative = 4.66

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(32*a^3*b^2*f*x*tan(f*x + e)^5 + 64*a^4*b*f*x*tan(f*x + e)^3 + 32*a
^5*f*x*tan(f*x + e) + 32*a^5 - 96*a^4*b + 96*a^3*b^2 - 32*a^2*b^3 + 4*(8*a
^3*b^2 - 35*a^2*b^3 + 42*a*b^4 - 15*b^5)*tan(f*x + e)^4 + 4*(16*a^4*b - 61
*a^3*b^2 + 70*a^2*b^3 - 25*a*b^4)*tan(f*x + e)^2 + ((35*a^2*b^3 - 42*a*b^4
 + 15*b^5)*tan(f*x + e)^5 + 2*(35*a^3*b^2 - 42*a^2*b^3 + 15*a*b^4)*tan(f*x
 + e)^3 + (35*a^4*b - 42*a^3*b^2 + 15*a^2*b^3)*tan(f*x + e))*sqrt(-b/a)*lo
g((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3
 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^
2 + a^2)))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 +
 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a
^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e)), -1/16*(16*a^3*b^2*f*x*tan(f*x
 + e)^5 + 32*a^4*b*f*x*tan(f*x + e)^3 + 16*a^5*f*x*tan(f*x + e) + 16*a^5 -
 48*a^4*b + 48*a^3*b^2 - 16*a^2*b^3 + 2*(8*a^3*b^2 - 35*a^2*b^3 + 42*a*b^4
 - 15*b^5)*tan(f*x + e)^4 + 2*(16*a^4*b - 61*a^3*b^2 + 70*a^2*b^3 - 25*a*b
^4)*tan(f*x + e)^2 - ((35*a^2*b^3 - 42*a*b^4 + 15*b^5)*tan(f*x + e)^5 + 2*
(35*a^3*b^2 - 42*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^3 + (35*a^4*b - 42*a^3*b
^2 + 15*a^2*b^3)*tan(f*x + e))*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)
*sqrt(b/a)/(b*tan(f*x + e))))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)
*f*tan(f*x + e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x
+ e)^3 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.46

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{(35\,a^2b^2-42\,ab^3+15\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^6-3\,a^5b+3\,a^4b^2-a^3b^3)\sqrt{ab}} - \frac{(8\,a^2b^2-27\,ab^3+15\,b^4)\tan(fx+e)^4+8\,a^4-16\,a^3b+8\,a^2b^2+(16\,a^3b-45\,a^2b^2+25\,ab^3)\tan(fx}{(a^5b^2-2\,a^4b^3+a^3b^4)\tan(fx+e)^5+2\,(a^6b-2\,a^5b^2+a^4b^3)\tan(fx+e)^3+(a^7-2\,a^6b+a^5b^2)\tan(}}{8\,f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((35*a^2*b^2 - 42*a*b^3 + 15*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a
^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*sqrt(a*b)) - ((8*a^2*b^2 - 27*a*b^3 +
15*b^4)*tan(f*x + e)^4 + 8*a^4 - 16*a^3*b + 8*a^2*b^2 + (16*a^3*b - 45*a^2
*b^2 + 25*a*b^3)*tan(f*x + e)^2)/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*tan(f*x
+ e)^5 + 2*(a^6*b - 2*a^5*b^2 + a^4*b^3)*tan(f*x + e)^3 + (a^7 - 2*a^6*b +
 a^5*b^2)*tan(f*x + e)) - 8*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.16

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$
$$= \frac{(35\,a^2b^2-42\,ab^3+15\,b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,(a^6f-3\,a^5bf+3\,a^4b^2f-a^3b^3f)\sqrt{ab}} - \frac{fx+e}{a^3f-3\,a^2bf+3\,ab^2f-b^3f}$$
$$+ \frac{11\,ab^3\tan(fx+e)^3-7\,b^4\tan(fx+e)^3+13\,a^2b^2\tan(fx+e)-9\,ab^3\tan(fx+e)}{8\,(a^5f-2\,a^4bf+a^3b^2f)\left(b\tan(fx+e)^2+a\right)^2}$$
$$- \frac{1}{a^3f\tan(fx+e)}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(35*a^2*b^2 - 42*a*b^3 + 15*b^4)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a^
6*f - 3*a^5*b*f + 3*a^4*b^2*f - a^3*b^3*f)*sqrt(a*b)) - (f*x + e)/(a^3*f -
 3*a^2*b*f + 3*a*b^2*f - b^3*f) + 1/8*(11*a*b^3*tan(f*x + e)^3 - 7*b^4*tan
(f*x + e)^3 + 13*a^2*b^2*tan(f*x + e) - 9*a*b^3*tan(f*x + e))/((a^5*f - 2*
a^4*b*f + a^3*b^2*f)*(b*tan(f*x + e)^2 + a)^2) - 1/(a^3*f*tan(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 11.96 (sec) , antiderivative size = 915, normalized size of antiderivative = 4.84

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(atan((b^5*tan(e + f*x)*(-a^7*b^3)^(3/2)*225i - a*b^4*tan(e + f*x)*(-a^7*b
^3)^(3/2)*1260i + a^4*b*tan(e + f*x)*(-a^7*b^3)^(3/2)*1225i + a^14*b*tan(e
 + f*x)*(-a^7*b^3)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^7*b^3)^(3/2)*2814i
 - a^3*b^2*tan(e + f*x)*(-a^7*b^3)^(3/2)*2940i)/(225*a^11*b^9 - 1260*a^12*
b^8 + 2814*a^13*b^7 - 2940*a^14*b^6 + 1225*a^15*b^5 - 64*a^18*b^2))*(-a^7*
b^3)^(1/2)*(35*a^2 - 42*a*b + 15*b^2)*1i)/(8*f*(3*a^9*b - a^10 + a^7*b^3 -
 3*a^8*b^2)) - (1/a + (tan(e + f*x)^4*(15*b^4 - 27*a*b^3 + 8*a^2*b^2))/(8*
a^3*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^2*(16*a^2*b - 45*a*b^2 + 25*b^3))
/(8*a^2*(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e + f*x) + b^2*tan(e + f*x)^5 +
2*a*b*tan(e + f*x)^3)) - (2*atan((2*tan(e + f*x)*((262144*a^15*b^15 - 2883
584*a^16*b^14 + 14155776*a^17*b^13 - 40370176*a^18*b^12 + 72089600*a^19*b^
11 - 77856768*a^20*b^10 + 34603008*a^21*b^9 + 34603008*a^22*b^8 - 77856768
*a^23*b^7 + 72089600*a^24*b^6 - 40370176*a^25*b^5 + 14155776*a^26*b^4 - 28
83584*a^27*b^3 + 262144*a^28*b^2)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 -
230400*a^9*b^15 + 2672640*a^10*b^14 - 14078976*a^11*b^13 + 44261376*a^12*b
^12 - 91801600*a^13*b^11 + 131051520*a^14*b^10 - 130287616*a^15*b^9 + 8921
9072*a^16*b^8 - 40743936*a^17*b^7 + 11847680*a^18*b^6 - 2237440*a^19*b^5 +
 393216*a^20*b^4 - 65536*a^21*b^3))/((6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)*(
(2*(245760*a^12*b^15 - 2899968*a^13*b^14 + 15613952*a^14*b^13 - 50577408*a
^15*b^12 + 109281280*a^16*b^11 - 164659200*a^17*b^10 + 174882816*a^18*b...
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1631, normalized size of antiderivative = 8.63

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input

```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^3,x)
```

output

```
( - 35*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(
b))*sin(e + f*x)**5*a**4*b + 112*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(
a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**3*b**2 - 134*sqrt(b)*sqrt
(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5
*a**2*b**3 + 72*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/
2))/sqrt(b))*sin(e + f*x)**5*a*b**4 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
 - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*b**5 + 70*sqrt(b)*sq
rt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*
*3*a**4*b - 154*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/
2))/sqrt(b))*sin(e + f*x)**3*a**3*b**2 + 114*sqrt(b)*sqrt(a)*atan((sqrt(a
- b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**3*a**2*b**3 - 30*s
qrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(
e + f*x)**3*a*b**4 - 35*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e
 + f*x)/2))/sqrt(b))*sin(e + f*x)*a**4*b + 42*sqrt(b)*sqrt(a)*atan((sqrt(a
 - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)*a**3*b**2 - 15*sqr
t(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e
+ f*x)*a**2*b**3 + 35*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e +
 f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**4*b - 112*sqrt(b)*sqrt(a)*atan((sqrt
(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b))*sin(e + f*x)**5*a**3*b**2 + 1
34*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((e + f*x)/2))/sqrt(b...
```

3.248 $\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{x}{(a-b)^3} - \frac{b^{5/2}(63a^2-90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^3f} + \frac{(8a^3+8a^2b-55ab^2+35b^3)\cot(e+fx)}{8a^4(a-b)^2f} - \frac{(8a^2-55ab+35b^2)\cot^3(e+fx)}{24a^3(a-b)^2f} - \frac{b\cot^3(e+fx)}{4a(a-b)f\,(a+b\tan^2(e+fx))^2} - \frac{(11a-7b)b\cot^3(e+fx)}{8a^2(a-b)^2f\,(a+b\tan^2(e+fx))}$$

output
```
x/(a-b)^3-1/8*b^(5/2)*(63*a^2-90*a*b+35*b^2)*arctan(b^(1/2)*tan(f*x+e)/a^(
1/2))/a^(9/2)/(a-b)^3/f+1/8*(8*a^3+8*a^2*b-55*a*b^2+35*b^3)*cot(f*x+e)/a^4
/(a-b)^2/f-1/24*(8*a^2-55*a*b+35*b^2)*cot(f*x+e)^3/a^3/(a-b)^2/f-1/4*b*cot
(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(11*a-7*b)*b*cot(f*x+e)^3/a^2
/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$
$$= \frac{-\frac{3b^{5/2}(63a^2-90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{a^{9/2}(a-b)^3} - \frac{8\cot(e+fx)(-4a-9b+a\csc^2(e+fx))}{a^4} + \frac{3\left(8(e+fx)-\frac{(a-b)b^3(17a^2+2ab-11b^2+(17a^2-}{a^4(a+b+(a-}\right.}{(a-b)^3}}{24f}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
((-3*b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a]])/(a^(9/2)*(a - b)^3) - (8*Cot[e + f*x]*(-4*a - 9*b + a*Csc[e + f*x]^2
))/a^4 + (3*(8*(e + f*x) - ((a - b)*b^3*(17*a^2 + 2*a*b - 11*b^2 + (17*a^2
 - 28*a*b + 11*b^2)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(a^4*(a + b + (a -
 b)*Cos[2*(e + f*x)])^2)))/(a - b)^3)/(24*f)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4153, 374, 441, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(e+fx)^4\,(a+b\tan(e+fx)^2)^3}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 374$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-7b\tan^2(e+fx)+4a-7b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4a(a-b)}-\frac{b\cot^3(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 441$$

$$\frac{\frac{\frac{\int \frac{\cot^4(e+fx)\left(8a^2-55ba+35b^2-5(11a-7b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b(11a-7b)\cot^3(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\cot^3(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(8a^3+8ba^2-55b^2a+35b^3+b\left(8a^2-55ba+35b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{3a}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b(11a-7b)\cot^3(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{}{4}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{-\frac{\int \frac{\cot^2(e+fx)\left(8a^3+8ba^2-55b^2a+35b^3+b\left(8a^2-55ba+35b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{b(11a-7b)\cot^3(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{}{4a}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{\frac{-\frac{-\frac{\int \frac{8a^4+8ba^3+8b^2a^2-55b^3a+35b^4+b\left(8a^3+8ba^2-55b^2a+35b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(8a^3+8a^2b-55ab^2+35b^3\right)\cot(e+fx)}{a}}{a}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3(e}{3a}}{2a(a-b)}}{4a(a-b)}}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{-\frac{-\frac{\frac{8a^4\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^3\left(63a^2-90ab+35b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}-\frac{\left(8a^3+8a^2b-55ab^2+35b^3\right)\cot(e+fx)}{a}}{a}-\frac{\left(8a^2-55ab+35b^2\right)\cot^3}{3a}}{2a(a-b)}}{4a(a-b)}}{f}$$

$\Big\downarrow$ 216

$$\frac{-\frac{-\frac{\frac{8a^4\arctan(\tan(e+fx))}{a-b}-\frac{b^3(63a^2-90ab+35b^2)\int\frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}}{a}-\frac{(8a^3+8a^2b-55ab^2+35b^3)\cot(e+fx)}{a}}{2a(a-b)}-\frac{(8a^2-55ab+35b^2)\cot^3(e+fx)}{3a}-}{4a(a-b)}}{f}$$

$\Big\downarrow$ 218

$$\frac{-\frac{(8a^2-55ab+35b^2)\cot^3(e+fx)}{3a}-\frac{-\frac{\frac{8a^4\arctan(\tan(e+fx))}{a-b}-\frac{b^{5/2}(63a^2-90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}}{a}-\frac{(8a^3+8a^2b-55ab^2+35b^3)\cot(e+fx)}{a}}{2a(a-b)}-\frac{b(}{2a(a}}{4a(a-b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Cot[e + f*x]^3)/(a*(a - b)*(a + b*Tan[e + f*x]^2)^2) + ((-1/3*((8
*a^2 - 55*a*b + 35*b^2)*Cot[e + f*x]^3)/a - (-(((8*a^4*ArcTan[Tan[e + f*x]
])/(a - b) - (b^(5/2)*(63*a^2 - 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tan[e + f
*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/a) - ((8*a^3 + 8*a^2*b - 55*a*b^2 + 35*b
^3)*Cot[e + f*x])/a)/a)/(2*a*(a - b)) - ((11*a - 7*b)*b*Cot[e + f*x]^3)/(2
*a*(a - b)*(a + b*Tan[e + f*x]^2)))/(4*a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b-\frac{13}{4}a\,b^2+\frac{11}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(17a^2-30ab+13b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(63a^2-90ab+35b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4(a-b)^3}-\frac{}{3a}}{f}$
default	$\frac{-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b-\frac{13}{4}a\,b^2+\frac{11}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(17a^2-30ab+13b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(63a^2-90ab+35b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4(a-b)^3}-\frac{}{3a}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b^3/a^4/(a-b)^3*(((15/8*a^2*b-13/4*a*b^2+11/8*b^3)*tan(f*x+e)^3+1/8*
a*(17*a^2-30*a*b+13*b^2)*tan(f*x+e))/(a+b*tan(f*x+e)^2)^2+1/8*(63*a^2-90*a
*b+35*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(a*b)^(1/2)))-1/3/a^3/tan(f*x+e
)^3-(-3*b-a)/a^4/tan(f*x+e)+1/(a-b)^3*arctan(tan(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(222) = 444.

Time = 0.16 (sec) , antiderivative size = 1006, normalized size of antiderivative = 4.19

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/96*(96*a^4*b^2*f*x*tan(f*x + e)^7 + 192*a^5*b*f*x*tan(f*x + e)^5 + 96*a
^6*f*x*tan(f*x + e)^3 + 12*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*ta
n(f*x + e)^6 - 32*a^6 + 96*a^5*b - 96*a^4*b^2 + 32*a^3*b^3 + 4*(48*a^5*b -
 8*a^4*b^2 - 315*a^3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 32*(3
*a^6 - 2*a^5*b - 12*a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*(
(63*a^2*b^4 - 90*a*b^5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b
^4 + 35*a*b^5)*tan(f*x + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan
(f*x + e)^3)*sqrt(-b/a)*log((b^2*tan(f*x + e)^4 - 6*a*b*tan(f*x + e)^2 + a
^2 + 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e))*sqrt(-b/a))/(b^2*tan(f*x +
e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a
^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*t
an(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^3), 1
/48*(48*a^4*b^2*f*x*tan(f*x + e)^7 + 96*a^5*b*f*x*tan(f*x + e)^5 + 48*a^6*
f*x*tan(f*x + e)^3 + 6*(8*a^4*b^2 - 63*a^2*b^4 + 90*a*b^5 - 35*b^6)*tan(f*
x + e)^6 - 16*a^6 + 48*a^5*b - 48*a^4*b^2 + 16*a^3*b^3 + 2*(48*a^5*b - 8*a
^4*b^2 - 315*a^3*b^3 + 450*a^2*b^4 - 175*a*b^5)*tan(f*x + e)^4 + 16*(3*a^6
 - 2*a^5*b - 12*a^4*b^2 + 18*a^3*b^3 - 7*a^2*b^4)*tan(f*x + e)^2 - 3*((63*
a^2*b^4 - 90*a*b^5 + 35*b^6)*tan(f*x + e)^7 + 2*(63*a^3*b^3 - 90*a^2*b^4 +
 35*a*b^5)*tan(f*x + e)^5 + (63*a^4*b^2 - 90*a^3*b^3 + 35*a^2*b^4)*tan(f*x
 + e)^3)*sqrt(b/a)*arctan(1/2*(b*tan(f*x + e)^2 - a)*sqrt(b/a)/(b*tan(f...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.38

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = -\frac{\frac{3\,(63\,a^2b^3-90\,ab^4+35\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^7-3\,a^6b+3\,a^5b^2-a^4b^3)\sqrt{ab}} - \frac{3\,(8\,a^3b^2+8\,a^2b^3-55\,ab^4+35\,b^5)\tan(fx+e)^6-8\,a^5+16\,a^4b-8\,a^3b^2+(48\,a^4b+40\,a^3b^2-2}{(a^6b^2-2\,a^5b^3+a^4b^4)\tan(fx+e)^7+2\,(a^7b-2\,a^6b^2+a^5b^3)}}{24\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(3*(63*a^2*b^3 - 90*a*b^4 + 35*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))
/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*sqrt(a*b)) - (3*(8*a^3*b^2 + 8*a^2
*b^3 - 55*a*b^4 + 35*b^5)*tan(f*x + e)^6 - 8*a^5 + 16*a^4*b - 8*a^3*b^2 +
(48*a^4*b + 40*a^3*b^2 - 275*a^2*b^3 + 175*a*b^4)*tan(f*x + e)^4 + 8*(3*a^
5 + a^4*b - 11*a^3*b^2 + 7*a^2*b^3)*tan(f*x + e)^2)/((a^6*b^2 - 2*a^5*b^3
+ a^4*b^4)*tan(f*x + e)^7 + 2*(a^7*b - 2*a^6*b^2 + a^5*b^3)*tan(f*x + e)^5
 + (a^8 - 2*a^7*b + a^6*b^2)*tan(f*x + e)^3) - 24*(f*x + e)/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx &= -\frac{(63\,a^2b^3-90\,ab^4+35\,b^5)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,(a^7f-3\,a^6bf+3\,a^5b^2f-a^4b^3f)\sqrt{ab}} + \frac{fx+e}{a^3f-3\,a^2bf+3\,ab^2f-b^3f} \\ &\quad - \frac{15\,ab^4\tan(fx+e)^3-11\,b^5\tan(fx+e)^3+17\,a^2b^3\tan(fx+e)-13\,ab^4\tan(fx+e)}{8\,(a^6f-2\,a^5bf+a^4b^2f)\left(b\tan(fx+e)^2+a\right)^2} \\ &\quad + \frac{3\,a\tan(fx+e)^2+9\,b\tan(fx+e)^2-a}{3\,a^4f\tan(fx+e)^3}\end{aligned}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(63*a^2*b^3 - 90*a*b^4 + 35*b^5)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a
^7*f - 3*a^6*b*f + 3*a^5*b^2*f - a^4*b^3*f)*sqrt(a*b)) + (f*x + e)/(a^3*f
- 3*a^2*b*f + 3*a*b^2*f - b^3*f) - 1/8*(15*a*b^4*tan(f*x + e)^3 - 11*b^5*t
an(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) - 13*a*b^4*tan(f*x + e))/((a^6*f -
 2*a^5*b*f + a^4*b^2*f)*(b*tan(f*x + e)^2 + a)^2) + 1/3*(3*a*tan(f*x + e)^
2 + 9*b*tan(f*x + e)^2 - a)/(a^4*f*tan(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 986, normalized size of antiderivative = 4.11

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(2*atan((2*tan(e + f*x)*((262144*a^20*b^15 - 2883584*a^21*b^14 + 14155776*
a^22*b^13 - 40370176*a^23*b^12 + 72089600*a^24*b^11 - 77856768*a^25*b^10 +
 34603008*a^26*b^9 + 34603008*a^27*b^8 - 77856768*a^28*b^7 + 72089600*a^29
*b^6 - 40370176*a^30*b^5 + 14155776*a^31*b^4 - 2883584*a^32*b^3 + 262144*a
^33*b^2)/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 - 1254400*a^12*b^17 + 13977
600*a^13*b^16 - 70333440*a^14*b^15 + 210329600*a^15*b^14 - 413730816*a^16*
b^13 + 559067136*a^17*b^12 - 525322240*a^18*b^11 + 338780160*a^19*b^10 - 1
43512576*a^20*b^9 + 36390912*a^21*b^8 - 5047296*a^22*b^7 + 1310720*a^23*b^
6 - 983040*a^24*b^5 + 393216*a^25*b^4 - 65536*a^26*b^3))/((6*a*b^2 - 6*a^2
*b + 2*a^3 - 2*b^3)*((2*(573440*a^16*b^16 - 6635520*a^17*b^15 + 34947072*a
^18*b^14 - 110542848*a^19*b^13 + 233275392*a^20*b^12 - 344883200*a^21*b^11
 + 365199360*a^22*b^10 - 279281664*a^23*b^9 + 155959296*a^24*b^8 - 6751846
4*a^25*b^7 + 27279360*a^26*b^6 - 12042240*a^27*b^5 + 4718592*a^28*b^4 - 11
79648*a^29*b^3 + 131072*a^30*b^2))/(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3)^2 +
 1254400*a^12*b^14 - 10214400*a^13*b^13 + 35927040*a^14*b^12 - 70650880*a^
15*b^11 + 83495936*a^16*b^10 - 58242048*a^17*b^9 + 20216832*a^18*b^8 - 174
08*a^19*b^7 - 2285568*a^20*b^6 + 516096*a^21*b^5))))/(f*(6*a*b^2 - 6*a^2*b
 + 2*a^3 - 2*b^3)) + ((tan(e + f*x)^2*(3*a + 7*b))/(3*a^2) - 1/(3*a) + (ta
n(e + f*x)^6*(35*b^5 - 55*a*b^4 + 8*a^2*b^3 + 8*a^3*b^2))/(8*a^4*(a^2 - 2*
a*b + b^2)) + (tan(e + f*x)^4*(48*a^3*b - 275*a*b^3 + 175*b^4 + 40*a^2*...
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \int \frac{\cot(fx+e)^4}{\left(\tan(fx+e)^2 b+a\right)^3}dx$$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^3,x)
```

3.249 $\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 297

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = & -\frac{x}{(a-b)^3} \\ & + \frac{b^{7/2}(99a^2-154ab+63b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}(a-b)^3f} \\ & - \frac{(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4)\cot(e+fx)}{8a^5(a-b)^2f} \\ & + \frac{(8a^3+8a^2b-91ab^2+63b^3)\cot^3(e+fx)}{24a^4(a-b)^2f} \\ & - \frac{(8a^2-91ab+63b^2)\cot^5(e+fx)}{40a^3(a-b)^2f} \\ & - \frac{b\cot^5(e+fx)}{4a(a-b)f\left(a+b\tan^2(e+fx)\right)^2} \\ & - \frac{(13a-9b)b\cot^5(e+fx)}{8a^2(a-b)^2f\left(a+b\tan^2(e+fx)\right)}\end{aligned}$$

output
```
-x/(a-b)^3+1/8*b^(7/2)*(99*a^2-154*a*b+63*b^2)*arctan(b^(1/2)*tan(f*x+e)/a
^(1/2))/a^(11/2)/(a-b)^3/f-1/8*(8*a^4+8*a^3*b+8*a^2*b^2-91*a*b^3+63*b^4)*c
ot(f*x+e)/a^5/(a-b)^2/f+1/24*(8*a^3+8*a^2*b-91*a*b^2+63*b^3)*cot(f*x+e)^3/
a^4/(a-b)^2/f-1/40*(8*a^2-91*a*b+63*b^2)*cot(f*x+e)^5/a^3/(a-b)^2/f-1/4*b*
cot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)^2-1/8*(13*a-9*b)*b*cot(f*x+e)^5/
a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 949 vs. $2(297) = 594$.

Time = 6.22 (sec) , antiderivative size = 949, normalized size of antiderivative = 3.20

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{b^{7/2}(99a^2-154ab+63b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{8a^{11/2}(a-b)^3f}$$
$$+ \frac{\csc^5(e+fx)\left(-3184a^7\cos(e+fx)+7440a^6b\cos(e+fx)-12000a^5b^2\cos(e+fx)+10240a^4b^3\cos(e\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a]
])/(8*a^(11/2)*(a - b)^3*f) + (Csc[e + f*x]^5*(-3184*a^7*Cos[e + f*x] + 74
40*a^6*b*Cos[e + f*x] - 12000*a^5*b^2*Cos[e + f*x] + 10240*a^4*b^3*Cos[e +
 f*x] + 6450*a^3*b^4*Cos[e + f*x] + 714*a^2*b^5*Cos[e + f*x] - 22890*a*b^6
*Cos[e + f*x] + 13230*b^7*Cos[e + f*x] - 1536*a^7*Cos[3*(e + f*x)] + 7648*
a^6*b*Cos[3*(e + f*x)] - 2912*a^5*b^2*Cos[3*(e + f*x)] - 1152*a^4*b^3*Cos[
3*(e + f*x)] - 14872*a^3*b^4*Cos[3*(e + f*x)] - 12796*a^2*b^5*Cos[3*(e + f
*x)] + 52080*a*b^6*Cos[3*(e + f*x)] - 26460*b^7*Cos[3*(e + f*x)] - 704*a^7
*Cos[5*(e + f*x)] + 2656*a^6*b*Cos[5*(e + f*x)] - 4128*a^5*b^2*Cos[5*(e +
f*x)] - 3712*a^4*b^3*Cos[5*(e + f*x)] + 5504*a^3*b^4*Cos[5*(e + f*x)] + 27
684*a^2*b^5*Cos[5*(e + f*x)] - 46200*a*b^6*Cos[5*(e + f*x)] + 18900*b^7*Co
s[5*(e + f*x)] - 536*a^7*Cos[7*(e + f*x)] + 248*a^6*b*Cos[7*(e + f*x)] + 7
68*a^5*b^2*Cos[7*(e + f*x)] + 128*a^4*b^3*Cos[7*(e + f*x)] + 6553*a^3*b^4*
Cos[7*(e + f*x)] - 21441*a^2*b^5*Cos[7*(e + f*x)] + 20895*a*b^6*Cos[7*(e +
 f*x)] - 6615*b^7*Cos[7*(e + f*x)] - 184*a^7*Cos[9*(e + f*x)] + 440*a^6*b*
Cos[9*(e + f*x)] - 160*a^5*b^2*Cos[9*(e + f*x)] + 640*a^4*b^3*Cos[9*(e + f
*x)] - 3635*a^3*b^4*Cos[9*(e + f*x)] + 5839*a^2*b^5*Cos[9*(e + f*x)] - 388
5*a*b^6*Cos[9*(e + f*x)] + 945*b^7*Cos[9*(e + f*x)] - 720*a^7*(e + f*x)*Si
n[e + f*x] - 3360*a^6*b*(e + f*x)*Sin[e + f*x] - 15120*a^5*b^2*(e + f*x)*S
in[e + f*x] - 480*a^7*(e + f*x)*Sin[3*(e + f*x)] + 10080*a^5*b^2*(e + f...
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4153, 374, 441, 445, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 374$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-9b\tan^2(e+fx)+4a-9b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^2}d\tan(e+fx)}{4a(a-b)}-\frac{b\cot^5(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 441$$

$$\frac{\frac{\frac{\int \frac{\cot^6(e+fx)\left(8a^2-91ba+63b^2-7(13a-9b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{2a(a-b)}-\frac{b(13a-9b)\cot^5(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\frac{b\cot^5(e+fx)}{4a(a-b)(a+b\tan^2(e+fx))^2}}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{\frac{\frac{-\frac{\int \frac{5\cot^4(e+fx)\left(8a^3+8ba^2-91b^2a+63b^3+b\left(8a^2-91ba+63b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{5a}-\frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{5a}}{2a(a-b)}-\frac{b(13a-9b)\cot^5(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\cdots}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{-\frac{\int \frac{\cot^4(e+fx)\left(8a^3+8ba^2-91b^2a+63b^3+b\left(8a^2-91ba+63b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}-\frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{5a}}{2a(a-b)}-\frac{b(13a-9b)\cot^5(e+fx)}{2a(a-b)\left(a+b\tan^2(e+fx)\right)}}{4a(a-b)}-\cdots}{f}$$

$$\Big\downarrow \ 445$$

$$\frac{\frac{\frac{-\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(8a^4+8ba^3+8b^2a^2-91b^3a+63b^4+b\left(8a^3+8ba^2-91b^2a+63b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{3a}-\frac{\left(8a^3+8a^2b-91ab^2+63b^3\right)\cot^3(e+fx)}{3a}}{a}-\frac{\left(8a^2-9\cdots\right.}{\cdots}}{2a(a-b)}\cdots}{4a(a-b)}\cdots}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(8a^4+8ba^3+8b^2a^2-91b^3a+63b^4+b\left(8a^3+8ba^2-91b^2a+63b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}}{a}-\frac{\left(8a^3+8a^2b-91ab^2+63b^3\right)\cot^3(e+fx)}{3a}}{2a(a-b)}-\frac{\left(8a^2-91a\cdots\right.}{\cdots}}{4a(a-b)}\cdots}{f}$$

$\big\downarrow$ 445

$$\frac{-\frac{-\frac{-\frac{-\frac{\int \frac{8a^5+8ba^4+8b^2a^3+8b^3a^2-91b^4a+63b^5+b\left(8a^4+8ba^3+8b^2a^2-91b^3a+63b^4\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)}d\tan(e+fx)}{a}}{a}-\frac{\left(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4\right)\cot(e+fx)}{a}}{a}-\cdots}{2a(a-b)}\cdots}{4a(a-b)}\cdots}{f}$$

$\big\downarrow$ 397

$$\frac{-\frac{-\frac{-\frac{\frac{8a^5\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a-b}-\frac{b^4\left(99a^2-154ab+63b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}}{a}-\frac{\left(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4\right)\cot(e+fx)}{a}}{a}-\frac{\left(8a^3+8a^2b\cdots\right.}{\cdots}}{2a(a-b)}\cdots}{4a(a-b)}\cdots}{f}$$

$\big\downarrow$ 216

$$\frac{-\frac{-\frac{-\frac{\frac{8a^5\arctan(\tan(e+fx))}{a-b}-\frac{b^4\left(99a^2-154ab+63b^2\right)\int \frac{1}{b\tan^2(e+fx)+a}d\tan(e+fx)}{a-b}}{a}}{a}-\frac{\left(8a^4+8a^3b+8a^2b^2-91ab^3+63b^4\right)\cot(e+fx)}{a}}{a}-\frac{\left(8a^3+8a^2b-91ab^2+63\cdots\right.}{3a}}{2a(a-b)}\cdots}{4a(a-b)}\cdots}{f}$$

$\big\downarrow$ 218

$$\frac{-\frac{\left(8a^2-91ab+63b^2\right)\cot^5(e+fx)}{5a}-\frac{-\frac{\left(8a^3+8a^2b-91ab^2+63b^3\right)\cot^3(e+fx)}{3a}-\frac{-\frac{-\frac{\frac{8a^5\arctan(\tan(e+fx))}{a-b}-\frac{b^{7/2}\left(99a^2-154ab+63b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a}}{a}-\cdots}{a}}{2a(a-b)}}{4a(a-b)}}{f}$$

input

```
Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Cot[e + f*x]^5)/(a*(a - b)*(a + b*Tan[e + f*x]^2)^2) + ((-1/5*((8
*a^2 - 91*a*b + 63*b^2)*Cot[e + f*x]^5)/a - (-1/3*((8*a^3 + 8*a^2*b - 91*a
*b^2 + 63*b^3)*Cot[e + f*x]^3)/a - (-(((8*a^5*ArcTan[Tan[e + f*x]])/(a - b
) - (b^(7/2)*(99*a^2 - 154*a*b + 63*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqr
t[a]])/(Sqrt[a]*(a - b)))/a) - ((8*a^4 + 8*a^3*b + 8*a^2*b^2 - 91*a*b^3 +
63*b^4)*Cot[e + f*x])/a)/a)/a)/(2*a*(a - b)) - ((13*a - 9*b)*b*Cot[e + f*x
]^5)/(2*a*(a - b)*(a + b*Tan[e + f*x]^2)))/(4*a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b-\frac{17}{4}a\,b^2+\frac{15}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(21a^2-38ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(99a^2-154ab+63b^2\right)\arctan}{8\sqrt{ab}}\right.}{a^5(a-b)^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{(a-b)^3}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b-\frac{17}{4}a\,b^2+\frac{15}{8}b^3\right)\tan(fx+e)^3+\frac{a\left(21a^2-38ab+17b^2\right)\tan(fx+e)}{8}}{\left(a+b\tan(fx+e)^2\right)^2}+\frac{\left(99a^2-154ab+63b^2\right)\arctan}{8\sqrt{ab}}\right.}{a^5(a-b)^3}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)^3*arctan(tan(f*x+e))+b^4/a^5/(a-b)^3*(((19/8*a^2*b-17/4*a*b^
2+15/8*b^3)*tan(f*x+e)^3+1/8*a*(21*a^2-38*a*b+17*b^2)*tan(f*x+e))/(a+b*tan
(f*x+e)^2)^2+1/8*(99*a^2-154*a*b+63*b^2)/(a*b)^(1/2)*arctan(b*tan(f*x+e)/(
a*b)^(1/2)))-1/5/a^3/tan(f*x+e)^5-1/3*(-3*b-a)/a^4/tan(f*x+e)^3-(a^2+3*a*b
+6*b^2)/a^5/tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1114, normalized size of antiderivative = 3.75

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/480*(480*a^5*b^2*f*x*tan(f*x + e)^9 + 960*a^6*b*f*x*tan(f*x + e)^7 + 4
80*a^7*f*x*tan(f*x + e)^5 + 60*(8*a^5*b^2 - 99*a^2*b^5 + 154*a*b^6 - 63*b^
7)*tan(f*x + e)^8 + 96*a^7 - 288*a^6*b + 288*a^5*b^2 - 96*a^4*b^3 + 20*(48
*a^6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^5 - 315*a*b^6)*tan(f*x + e)^6
 + 32*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4*b^3 + 154*a^3*b^4 - 63*a^2*b
^5)*tan(f*x + e)^4 - 32*(5*a^7 - 6*a^6*b - 12*a^5*b^2 + 22*a^4*b^3 - 9*a^3
*b^4)*tan(f*x + e)^2 + 15*((99*a^2*b^5 - 154*a*b^6 + 63*b^7)*tan(f*x + e)^
9 + 2*(99*a^3*b^4 - 154*a^2*b^5 + 63*a*b^6)*tan(f*x + e)^7 + (99*a^4*b^3 -
 154*a^3*b^4 + 63*a^2*b^5)*tan(f*x + e)^5)*sqrt(-b/a)*log((b^2*tan(f*x + e
)^4 - 6*a*b*tan(f*x + e)^2 + a^2 - 4*(a*b*tan(f*x + e)^3 - a^2*tan(f*x + e
))*sqrt(-b/a))/(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)))/((a^8*b
^2 - 3*a^7*b^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*
b^2 + 3*a^7*b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2
- a^7*b^3)*f*tan(f*x + e)^5), -1/240*(240*a^5*b^2*f*x*tan(f*x + e)^9 + 480
*a^6*b*f*x*tan(f*x + e)^7 + 240*a^7*f*x*tan(f*x + e)^5 + 30*(8*a^5*b^2 - 9
9*a^2*b^5 + 154*a*b^6 - 63*b^7)*tan(f*x + e)^8 + 48*a^7 - 144*a^6*b + 144*
a^5*b^2 - 48*a^4*b^3 + 10*(48*a^6*b - 8*a^5*b^2 - 495*a^3*b^4 + 770*a^2*b^
5 - 315*a*b^6)*tan(f*x + e)^6 + 16*(15*a^7 - 10*a^6*b + 3*a^5*b^2 - 99*a^4
*b^3 + 154*a^3*b^4 - 63*a^2*b^5)*tan(f*x + e)^4 - 16*(5*a^7 - 6*a^6*b - 12
*a^5*b^2 + 22*a^4*b^3 - 9*a^3*b^4)*tan(f*x + e)^2 - 15*((99*a^2*b^5 - 1...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.33

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{\frac{15\,(99\,a^2b^4-154\,ab^5+63\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{(a^8-3\,a^7b+3\,a^6b^2-a^5b^3)\sqrt{ab}} - \frac{15\,(8\,a^4b^2+8\,a^3b^3+8\,a^2b^4-91\,ab^5+63\,b^6)\tan(fx+e)^8+5\,(48\,a^5b+40\,a^4b^2+40\,a^3b^3-455}{(a^7b^2-2\,a^6b^3+a^5b}}{}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/120*(15*(99*a^2*b^4 - 154*a*b^5 + 63*b^6)*arctan(b*tan(f*x + e)/sqrt(a*b
))/((a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*sqrt(a*b)) - (15*(8*a^4*b^2 + 8*
a^3*b^3 + 8*a^2*b^4 - 91*a*b^5 + 63*b^6)*tan(f*x + e)^8 + 5*(48*a^5*b + 40
*a^4*b^2 + 40*a^3*b^3 - 455*a^2*b^4 + 315*a*b^5)*tan(f*x + e)^6 + 24*a^6 -
 48*a^5*b + 24*a^4*b^2 + 8*(15*a^6 + 5*a^5*b + 8*a^4*b^2 - 91*a^3*b^3 + 63
*a^2*b^4)*tan(f*x + e)^4 - 8*(5*a^6 - a^5*b - 13*a^4*b^2 + 9*a^3*b^3)*tan(
f*x + e)^2)/((a^7*b^2 - 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^9 + 2*(a^8*b - 2
*a^7*b^2 + a^6*b^3)*tan(f*x + e)^7 + (a^9 - 2*a^8*b + a^7*b^2)*tan(f*x + e
)^5) - 120*(f*x + e)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/f
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.97

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \frac{(99\,a^2b^4-154\,ab^5+63\,b^6)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab}}\right)}{8\,(a^8f-3\,a^7bf+3\,a^6b^2f-a^5b^3f)\sqrt{ab}} - \frac{fx+e}{a^3f-3\,a^2bf+3\,ab^2f-b^3f} + \frac{19\,ab^5\tan(fx+e)^3-15\,b^6\tan(fx+e)^3+21\,a^2b^4\tan(fx+e)-17\,ab^5\tan(fx+e)}{8\,(a^7f-2\,a^6bf+a^5b^2f)\left(b\tan(fx+e)^2+a\right)^2} - \frac{15\,a^2\tan(fx+e)^4+45\,ab\tan(fx+e)^4+90\,b^2\tan(fx+e)^4-5\,a^2\tan(fx+e)^2-15\,ab\tan(fx+}{15\,a^5f\tan(fx+e)^5}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(99*a^2*b^4 - 154*a*b^5 + 63*b^6)*arctan(b*tan(f*x + e)/sqrt(a*b))/((a
^8*f - 3*a^7*b*f + 3*a^6*b^2*f - a^5*b^3*f)*sqrt(a*b)) - (f*x + e)/(a^3*f
- 3*a^2*b*f + 3*a*b^2*f - b^3*f) + 1/8*(19*a*b^5*tan(f*x + e)^3 - 15*b^6*t
an(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) - 17*a*b^5*tan(f*x + e))/((a^7*f -
 2*a^6*b*f + a^5*b^2*f)*(b*tan(f*x + e)^2 + a)^2) - 1/15*(15*a^2*tan(f*x +
 e)^4 + 45*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 - 5*a^2*tan(f*x + e)
^2 - 15*a*b*tan(f*x + e)^2 + 3*a^2)/(a^5*f*tan(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 2507, normalized size of antiderivative = 8.44

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^3,x)
```

output
```
(atan((b^5*tan(e + f*x)*(-a^11*b^7)^(3/2)*3969i - a*b^4*tan(e + f*x)*(-a^1
1*b^7)^(3/2)*19404i + a^4*b*tan(e + f*x)*(-a^11*b^7)^(3/2)*9801i + a^22*b*
tan(e + f*x)*(-a^11*b^7)^(1/2)*64i + a^2*b^3*tan(e + f*x)*(-a^11*b^7)^(3/2
)*36190i - a^3*b^2*tan(e + f*x)*(-a^11*b^7)^(3/2)*30492i)/(3969*a^17*b^15
- 19404*a^18*b^14 + 36190*a^19*b^13 - 30492*a^20*b^12 + 9801*a^21*b^11 - 6
4*a^28*b^4))*(-a^11*b^7)^(1/2)*(99*a^2 - 154*a*b + 63*b^2)*1i)/(8*f*(3*a^1
3*b - a^14 + a^11*b^3 - 3*a^12*b^2)) - (1/(5*a) + (tan(e + f*x)^4*(35*a*b
+ 15*a^2 + 63*b^2))/(15*a^3) - (tan(e + f*x)^2*(5*a + 9*b))/(15*a^2) + (ta
n(e + f*x)^6*(48*a^4*b - 455*a*b^4 + 315*b^5 + 40*a^2*b^3 + 40*a^3*b^2))/(
24*a^4*(a^2 - 2*a*b + b^2)) + (tan(e + f*x)^8*(63*b^6 - 91*a*b^5 + 8*a^2*b
^4 + 8*a^3*b^3 + 8*a^4*b^2))/(8*a^5*(a^2 - 2*a*b + b^2)))/(f*(a^2*tan(e +
f*x)^5 + b^2*tan(e + f*x)^9 + 2*a*b*tan(e + f*x)^7)) - (2*atan((((1032192
*a^20*b^17 - 11812864*a^21*b^16 + 61489152*a^22*b^15 - 192135168*a^23*b^14
 + 400392192*a^24*b^13 - 584220672*a^25*b^12 + 608862208*a^26*b^11 - 45229
6704*a^27*b^10 + 231653376*a^28*b^9 - 71122944*a^29*b^8 + 606208*a^30*b^7
+ 14893056*a^31*b^6 - 11010048*a^32*b^5 + 4718592*a^33*b^4 - 1179648*a^34*
b^3 + 131072*a^35*b^2 + (tan(e + f*x)*(262144*a^25*b^15 - 2883584*a^26*b^1
4 + 14155776*a^27*b^13 - 40370176*a^28*b^12 + 72089600*a^29*b^11 - 7785676
8*a^30*b^10 + 34603008*a^31*b^9 + 34603008*a^32*b^8 - 77856768*a^33*b^7 +
72089600*a^34*b^6 - 40370176*a^35*b^5 + 14155776*a^36*b^4 - 2883584*a^3...
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^3}\,dx = \int \frac{\cot(fx+e)^6}{\left(\tan(fx+e)^2 b+a\right)^3}dx$$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

output
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^3,x)
```

3.250 $\int \left(a+b\tan^2(c+dx)\right)^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \left(a+b\tan^2(c+dx)\right)^4\,dx=(a-b)^4x+\frac{(2a-b)b(2a^2-2ab+b^2)\tan(c+dx)}{d}+\frac{b^2(6a^2-4ab+b^2)\tan^3(c+dx)}{3d}+\frac{(4a-b)b^3\tan^5(c+dx)}{5d}+\frac{b^4\tan^7(c+dx)}{7d}$$

output
```
(a-b)^4*x+(2*a-b)*b*(2*a^2-2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2-4*a*b+b^
2)*tan(d*x+c)^3/d+1/5*(4*a-b)*b^3*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

$$\int \left(a+b\tan^2(c+dx)\right)^4\,dx$$

$$=\frac{\tan(c+dx)\left(\frac{105(a-b)^4\text{arctanh}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}+b(105(4a^3-6a^2b+4ab^2-b^3)+35b(6a^2-4ab+b^2)\tan\right.}{105d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^2)^4,x]
```

output
```
(Tan[c + d*x]*((105*(a - b)^4*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c
+ d*x]^2] + b*(105*(4*a^3 - 6*a^2*b + 4*a*b^2 - b^3) + 35*b*(6*a^2 - 4*a*b
 + b^2)*Tan[c + d*x]^2 + 21*(4*a - b)*b^2*Tan[c + d*x]^4 + 15*b^3*Tan[c +
d*x]^6)))/(105*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(c+dx)\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(c+dx)^2\right)^4 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a\right)^4}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^4\tan^6(c+dx)+(4a-b)b^3\tan^4(c+dx)+b^2\left(6a^2-4ba+b^2\right)\tan^2(c+dx)+(2a-b)b\left(2a^2-2ba+b^2\right)+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}b^2\left(6a^2-4ab+b^2\right)\tan^3(c+dx)+b(2a-b)\left(2a^2-2ab+b^2\right)\tan(c+dx)+(a-b)^4\arctan(\tan(c+dx))+\frac{1}{5}b^3}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^2)^4,x]
```

output
```
((a - b)^4*ArcTan[Tan[c + d*x]] + (2*a - b)*b*(2*a^2 - 2*a*b + b^2)*Tan[c
+ d*x] + (b^2*(6*a^2 - 4*a*b + b^2)*Tan[c + d*x]^3)/3 + ((4*a - b)*b^3*Tan
[c + d*x]^5)/5 + (b^4*Tan[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

method	result
norman	$\left(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4\right)x+\frac{b\left(4a^3-6a^2b+4a\,b^2-b^3\right)\tan(dx+c)}{d}+\frac{b^4\tan(dx+c)^7}{7d}+\frac{b^2\left(6a^2-\right.}{}$
parts	$a^4x+\frac{b^4\left(\frac{\tan(dx+c)^7}{7}-\frac{\tan(dx+c)^5}{5}+\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{4a\,b^3\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+}{3}\right.}{}$
derivativedivides	$\frac{\frac{b^4\tan(dx+c)^7}{7}+\frac{4a\,b^3\tan(dx+c)^5}{5}-\frac{b^4\tan(dx+c)^5}{5}+2a^2b^2\tan(dx+c)^3-\frac{4a\,b^3\tan(dx+c)^3}{3}+\frac{b^4\tan(dx+c)^3}{3}+4a^3b\tan(dx+c)}{d}$
default	$\frac{\frac{b^4\tan(dx+c)^7}{7}+\frac{4a\,b^3\tan(dx+c)^5}{5}-\frac{b^4\tan(dx+c)^5}{5}+2a^2b^2\tan(dx+c)^3-\frac{4a\,b^3\tan(dx+c)^3}{3}+\frac{b^4\tan(dx+c)^3}{3}+4a^3b\tan(dx+c)}{d}$
parallelrisch	$\frac{15b^4\tan(dx+c)^7+84a\,b^3\tan(dx+c)^5-21b^4\tan(dx+c)^5+210a^2b^2\tan(dx+c)^3-140a\,b^3\tan(dx+c)^3+35b^4\tan(dx+c)^3}{}$
risch	$a^4x-4a^3bx+6a^2b^2x-4a\,b^3x+b^4x-\frac{8ib\left(44b^3-105a^3-161a\,b^2+210a^2b+315b^3\mathrm{e}^{10i(dx+c)}-2100a^3\mathrm{e}\right.}{}$

input
```
int((a+b*tan(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
(a^4-4*a^3*b+6*a^2*b^2-4*a*b^3+b^4)*x+b*(4*a^3-6*a^2*b+4*a*b^2-b^3)/d*tan(
d*x+c)+1/7*b^4*tan(d*x+c)^7/d+1/3*b^2*(6*a^2-4*a*b+b^2)*tan(d*x+c)^3/d+1/5
*(4*a-b)*b^3*tan(d*x+c)^5/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \left(a+b\tan^2(c+dx)\right)^4\,dx$$
$$=\frac{15\,b^4\tan\left(dx+c\right)^7+21\left(4\,ab^3-b^4\right)\tan\left(dx+c\right)^5+35\left(6\,a^2b^2-4\,ab^3+b^4\right)\tan\left(dx+c\right)^3+105\left(a^4-4\,a\right.}{105\,d}$$

input
```
integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*(15*b^4*tan(d*x + c)^7 + 21*(4*a*b^3 - b^4)*tan(d*x + c)^5 + 35*(6*a
^2*b^2 - 4*a*b^3 + b^4)*tan(d*x + c)^3 + 105*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x + 105*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*tan(d*x + c
))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(100) = 200$.

Time = 0.22 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.82

$$\int (a+b\tan^2(c+dx))^4\,dx$$
$$= \begin{cases} a^4x - 4a^3bx + \frac{4a^3b\tan(c+dx)}{d} + 6a^2b^2x + \frac{2a^2b^2\tan^3(c+dx)}{d} - \frac{6a^2b^2\tan(c+dx)}{d} - 4ab^3x + \frac{4ab^3\tan^5(c+dx)}{5d} - \frac{4ab^3\tan}{} \\ x(a+b\tan^2(c))^4 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**2)**4,x)
```

output
```
Piecewise((a**4*x - 4*a**3*b*x + 4*a**3*b*tan(c + d*x)/d + 6*a**2*b**2*x +
 2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**2*tan(c + d*x)/d - 4*a*b**3*x +
 4*a*b**3*tan(c + d*x)**5/(5*d) - 4*a*b**3*tan(c + d*x)**3/(3*d) + 4*a*b**
3*tan(c + d*x)/d + b**4*x + b**4*tan(c + d*x)**7/(7*d) - b**4*tan(c + d*x)
**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4*tan(c + d*x)/d, Ne(d, 0)), (
x*(a + b*tan(c)**2)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.41

$$\int (a+b\tan^2(c+dx))^4\,dx = a^4x - \frac{4\,(dx+c-\tan(dx+c))a^3b}{d}$$
$$+ \frac{2\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^2b^2}{d}$$
$$+ \frac{4\left(3\,\tan(dx+c)^5 - 5\,\tan(dx+c)^3 - 15\,dx - 15\,c + 15\,\tan(dx+c)\right)ab^3}{15\,d}$$
$$+ \frac{\left(15\,\tan(dx+c)^7 - 21\,\tan(dx+c)^5 + 35\,\tan(dx+c)^3 + 105\,dx + 105\,c - 105\,\tan(dx+c)\right)b^4}{105\,d}$$

input
```
integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
a^4*x - 4*(d*x + c - tan(d*x + c))*a^3*b/d + 2*(tan(d*x + c)^3 + 3*d*x + 3
*c - 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3
 - 15*d*x - 15*c + 15*tan(d*x + c))*a*b^3/d + 1/105*(15*tan(d*x + c)^7 - 2
1*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c - 105*tan(d*x + c))
*b^4/d
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \left(a+b\tan^2(c+dx)\right)^4\,dx=\frac{(a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4)(dx+c)}{d}+\frac{15\,b^4d^6\tan\left(dx+c\right)^7+84\,ab^3d^6\tan\left(dx+c\right)^5-21\,b^4d^6\tan\left(dx+c\right)^5+210\,a^2b^2d^6\tan\left(dx+c\right)^3-140$$

input
```
integrate((a+b*tan(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c)/d + 1/105*(15*b^4*d^
6*tan(d*x + c)^7 + 84*a*b^3*d^6*tan(d*x + c)^5 - 21*b^4*d^6*tan(d*x + c)^5
 + 210*a^2*b^2*d^6*tan(d*x + c)^3 - 140*a*b^3*d^6*tan(d*x + c)^3 + 35*b^4*
d^6*tan(d*x + c)^3 + 420*a^3*b*d^6*tan(d*x + c) - 630*a^2*b^2*d^6*tan(d*x
+ c) + 420*a*b^3*d^6*tan(d*x + c) - 105*b^4*d^6*tan(d*x + c))/d^7
```

Mupad [B] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.43

$$\int \left(a+b\tan^2(c+dx)\right)^4\,dx=\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)^4}{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}\right)\,(a-b)^4}{d}+\frac{b^4\tan(c+d\,x)^7}{7\,d}+\frac{\tan(c+d\,x)^3\,\left(2\,a^2\,b^2-\frac{4\,a\,b^3}{3}+\frac{b^4}{3}\right)}{d}+\frac{\tan(c+d\,x)^5\,\left(\frac{4\,a\,b^3}{5}-\frac{b^4}{5}\right)}{d}+\frac{\tan(c+d\,x)\,(4\,a^3\,b-6\,a^2\,b^2+4\,a\,b^3-b^4)}{d}$$

input
```
int((a + b*tan(c + d*x)^2)^4,x)
```

output
```
(atan((tan(c + d*x)*(a - b)^4)/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)
)*(a - b)^4)/d + (b^4*tan(c + d*x)^7)/(7*d) + (tan(c + d*x)^3*(b^4/3 - (4*
a*b^3)/3 + 2*a^2*b^2))/d + (tan(c + d*x)^5*((4*a*b^3)/5 - b^4/5))/d + (tan
(c + d*x)*(4*a*b^3 + 4*a^3*b - b^4 - 6*a^2*b^2))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.55

$$\int \left(a+b\tan^2(c+dx)\right)^4 \, dx$$
$$= \frac{15\tan\left(dx+c\right)^7 b^4+84\tan\left(dx+c\right)^5 a\,b^3-21\tan\left(dx+c\right)^5 b^4+210\tan\left(dx+c\right)^3 a^2b^2-140\tan\left(dx+\right.}{}$$

input
```
int((a+b*tan(d*x+c)^2)^4,x)
```

output
```
(15*tan(c + d*x)**7*b**4 + 84*tan(c + d*x)**5*a*b**3 - 21*tan(c + d*x)**5*
b**4 + 210*tan(c + d*x)**3*a**2*b**2 - 140*tan(c + d*x)**3*a*b**3 + 35*tan
(c + d*x)**3*b**4 + 420*tan(c + d*x)*a**3*b - 630*tan(c + d*x)*a**2*b**2 +
 420*tan(c + d*x)*a*b**3 - 105*tan(c + d*x)*b**4 + 105*a**4*d*x - 420*a**3
*b*d*x + 630*a**2*b**2*d*x - 420*a*b**3*d*x + 105*b**4*d*x)/(105*d)
```

3.251 $\int \left(a+b\tan^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \left(a+b\tan^2(c+dx)\right)^3\,dx=(a-b)^3x+\frac{b(3a^2-3ab+b^2)\tan(c+dx)}{d}+\frac{(3a-b)b^2\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

output
```
(a-b)^3*x+b*(3*a^2-3*a*b+b^2)*tan(d*x+c)/d+1/3*(3*a-b)*b^2*tan(d*x+c)^3/d+
1/5*b^3*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

$$\int \left(a+b\tan^2(c+dx)\right)^3\,dx$$

$$=\frac{\tan(c+dx)\left(\frac{15(a-b)^3\operatorname{arctanh}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}+b(45a^2-15ab(3-\tan^2(c+dx))+b^2(15-5\tan^2(c+dx)\right.}{15d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^2)^3,x]
```

output
```
(Tan[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
 d*x]^2] + b*(45*a^2 - 15*a*b*(3 - Tan[c + d*x]^2) + b^2*(15 - 5*Tan[c + d
*x]^2 + 3*Tan[c + d*x]^4))))/(15*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(c+dx)^2\right)^3 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a\right)^3}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^3\tan^4(c+dx)+(3a-b)b^2\tan^2(c+dx)+b\left(3a^2-3ba+b^2\right)+\frac{(a-b)^3}{\tan^2(c+dx)+1}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(3a^2-3ab+b^2\right)\tan(c+dx)+(a-b)^3\arctan(\tan(c+dx))+\frac{1}{3}b^2(3a-b)\tan^3(c+dx)+\frac{1}{5}b^3\tan^5(c+dx)}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^2)^3,x]
```

output
```
((a - b)^3*ArcTan[Tan[c + d*x]] + b*(3*a^2 - 3*a*b + b^2)*Tan[c + d*x] + (
(3*a - b)*b^2*Tan[c + d*x]^3)/3 + (b^3*Tan[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

method	result
norman	$\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)x+\frac{b\left(3a^{2}-3ab+b^{2}\right)\tan(dx+c)}{d}+\frac{b^{3}\tan(dx+c)^{5}}{5d}+\frac{(3a-b)b^{2}\tan(dx+c)^{3}}{3d}$
derivativedivides	$\frac{\frac{b^{3}\tan(dx+c)^{5}}{5}+a\,b^{2}\tan(dx+c)^{3}-\frac{b^{3}\tan(dx+c)^{3}}{3}+3a^{2}b\tan(dx+c)-3\tan(dx+c)a\,b^{2}+\tan(dx+c)b^{3}+\left(a^{3}-3a^{2}b+3a\,b^{2}-\right.}{d}$
default	$\frac{\frac{b^{3}\tan(dx+c)^{5}}{5}+a\,b^{2}\tan(dx+c)^{3}-\frac{b^{3}\tan(dx+c)^{3}}{3}+3a^{2}b\tan(dx+c)-3\tan(dx+c)a\,b^{2}+\tan(dx+c)b^{3}+\left(a^{3}-3a^{2}b+3a\,b^{2}-\right.}{d}$
parts	$a^{3}x+\frac{b^{3}\left(\frac{\tan(dx+c)^{5}}{5}-\frac{\tan(dx+c)^{3}}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{3a\,b^{2}\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\mathrm{t}\right.}{d}$
parallelrisch	$\frac{3b^{3}\tan(dx+c)^{5}+15a\,b^{2}\tan(dx+c)^{3}-5b^{3}\tan(dx+c)^{3}+15a^{3}dx-45a^{2}bdx+45a\,b^{2}dx-15b^{3}dx+45a^{2}b\tan(dx+c)-45\,\mathrm{ta}}{15d}$
risch	$a^{3}x-3a^{2}bx+3a\,b^{2}x-b^{3}x+\frac{2ib\left(45a^{2}\mathrm{e}^{8i(dx+c)}-90ab\,\mathrm{e}^{8i(dx+c)}+45b^{2}\mathrm{e}^{8i(dx+c)}+180a^{2}\mathrm{e}^{6i(dx+c)}-270ab\,\mathrm{e}\right.}{}$

input
```
int((a+b*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(a^3-3*a^2*b+3*a*b^2-b^3)*x+b*(3*a^2-3*a*b+b^2)*tan(d*x+c)/d+1/5*b^3*tan(d
*x+c)^5/d+1/3*(3*a-b)*b^2*tan(d*x+c)^3/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \left(a+b\tan^2(c+dx)\right)^3\,dx$$
$$=\frac{3\,b^{3}\tan\left(dx+c\right)^{5}+5\left(3\,ab^{2}-b^{3}\right)\tan\left(dx+c\right)^{3}+15\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)dx+15\left(3\,a^{2}b-3\,ab^{2}+b^{3}\right)}{15\,d}$$

input
```
integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(3*b^3*tan(d*x + c)^5 + 5*(3*a*b^2 - b^3)*tan(d*x + c)^3 + 15*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*x + 15*(3*a^2*b - 3*a*b^2 + b^3)*tan(d*x + c))/
d
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.64

$$\int \left(a+b\tan^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} a^3x-3a^2bx+\frac{3a^2b\tan{(c+dx)}}{d}+3ab^2x+\frac{ab^2\tan^3{(c+dx)}}{d}-\frac{3ab^2\tan{(c+dx)}}{d}-b^3x+\frac{b^3\tan^5{(c+dx)}}{5d}-\frac{b^3\tan^3{(c+dx)}}{3d} \\ x\left(a+b\tan^2{(c)}\right)^3 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - 3*a**2*b*x + 3*a**2*b*tan(c + d*x)/d + 3*a*b**2*x + a*
b**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d - b**3*x + b**3*tan(c + d
*x)**5/(5*d) - b**3*tan(c + d*x)**3/(3*d) + b**3*tan(c + d*x)/d, Ne(d, 0))
, (x*(a + b*tan(c)**2)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \left(a+b\tan^2(c+dx)\right)^3\,dx$$
$$= a^3x-\frac{3\,(dx+c-\tan{(dx+c)})a^2b}{d}+\frac{\left(\tan{(dx+c)}^3+3\,dx+3\,c-3\,\tan{(dx+c)}\right)ab^2}{d}+\frac{\left(3\,\tan{(dx+c)}^5-5\,\tan{(dx+c)}^3-15\,dx-15\,c+15\,\tan{(dx+c)}\right)b^3}{15\,d}$$

input
```
integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x - 3*(d*x + c - tan(d*x + c))*a^2*b/d + (tan(d*x + c)^3 + 3*d*x + 3*c
 - 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 1
5*d*x - 15*c + 15*tan(d*x + c))*b^3/d
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int \left(a + b\tan^2(c + dx)\right)^3 \, dx = \frac{(a^3 - 3a^2b + 3ab^2 - b^3)(dx + c)}{d} + \frac{3b^3d^4\tan(dx + c)^5 + 15ab^2d^4\tan(dx + c)^3 - 5b^3d^4\tan(dx + c)^3 + 45a^2bd^4\tan(dx + c) - 45ab^2d^4}{15d^5}$$

input
```
integrate((a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c)/d + 1/15*(3*b^3*d^4*tan(d*x + c)
^5 + 15*a*b^2*d^4*tan(d*x + c)^3 - 5*b^3*d^4*tan(d*x + c)^3 + 45*a^2*b*d^4
*tan(d*x + c) - 45*a*b^2*d^4*tan(d*x + c) + 15*b^3*d^4*tan(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int \left(a + b\tan^2(c + dx)\right)^3 \, dx = \frac{b^3\tan(c + d\,x)^5}{5\,d} + \frac{\tan(c + d\,x)\,\left(3\,a^2\,b - 3\,a\,b^2 + b^3\right)}{d} + \frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)^3}{a^3-3\,a^2\,b+3\,a\,b^2-b^3}\right)\,(a - b)^3}{d} + \frac{\tan(c + d\,x)^3\,\left(a\,b^2 - \frac{b^3}{3}\right)}{d}$$

input
```
int((a + b*tan(c + d*x)^2)^3,x)
```

output
```
(b^3*tan(c + d*x)^5)/(5*d) + (tan(c + d*x)*(3*a^2*b - 3*a*b^2 + b^3))/d +
(atan((tan(c + d*x)*(a - b)^3)/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(a - b)^3)
/d + (tan(c + d*x)^3*(a*b^2 - b^3/3))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.44

$$\int \left(a+b\tan^2(c+dx)\right)^3\,dx$$
$$=\frac{3\tan\left(dx+c\right)^5 b^3+15\tan\left(dx+c\right)^3 a\,b^2-5\tan\left(dx+c\right)^3 b^3+45\tan\left(dx+c\right)a^2 b-45\tan\left(dx+c\right)a\,b^2}{15d}$$

input
```
int((a+b*tan(d*x+c)^2)^3,x)
```

output
```
(3*tan(c + d*x)**5*b**3 + 15*tan(c + d*x)**3*a*b**2 - 5*tan(c + d*x)**3*b*
*3 + 45*tan(c + d*x)*a**2*b - 45*tan(c + d*x)*a*b**2 + 15*tan(c + d*x)*b**
3 + 15*a**3*d*x - 45*a**2*b*d*x + 45*a*b**2*d*x - 15*b**3*d*x)/(15*d)
```

3.252 $\int (a+b\tan^2(c+dx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int (a+b\tan^2(c+dx))^2\,dx=(a-b)^2x+\frac{(2a-b)b\tan(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

output
```
(a-b)^2*x+(2*a-b)*b*tan(d*x+c)/d+1/3*b^2*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int (a+b\tan^2(c+dx))^2\,dx$$
$$=\frac{\tan(c+dx)\left(\frac{3(a-b)^2\operatorname{arctanh}\left(\sqrt{-\tan^2(c+dx)}\right)}{\sqrt{-\tan^2(c+dx)}}+b(6a-b(3-\tan^2(c+dx)))\right)}{3d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(Tan[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Tan[c + d*x]^2]])/Sqrt[-Tan[c +
d*x]^2] + b*(6*a - b*(3 - Tan[c + d*x]^2))))/(3*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(c+dx)^2\right)^2 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a\right)^2}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{(a-b)^2}{\tan^2(c+dx)+1}+b^2\tan^2(c+dx)+(2a-b)b\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a-b)^2\arctan(\tan(c+dx))+b(2a-b)\tan(c+dx)+\frac{1}{3}b^2\tan^3(c+dx)}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((a - b)^2*ArcTan[Tan[c + d*x]] + (2*a - b)*b*Tan[c + d*x] + (b^2*Tan[c + 
d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
norman	$\left(a^{2}-2ab+b^{2}\right)x+\frac{(2a-b)b\tan(dx+c)}{d}+\frac{b^{2}\tan(dx+c)^{3}}{3d}$	49
derivativedivides	$\frac{\frac{b^{2}\tan(dx+c)^{3}}{3}+2ab\tan(dx+c)-\tan(dx+c)b^{2}+\left(a^{2}-2ab+b^{2}\right)\arctan(\tan(dx+c))}{d}$	59
default	$\frac{\frac{b^{2}\tan(dx+c)^{3}}{3}+2ab\tan(dx+c)-\tan(dx+c)b^{2}+\left(a^{2}-2ab+b^{2}\right)\arctan(\tan(dx+c))}{d}$	59
parallelrisch	$\frac{b^{2}\tan(dx+c)^{3}+3a^{2}dx-6abdx+3b^{2}dx+6ab\tan(dx+c)-3\tan(dx+c)b^{2}}{3d}$	60
parts	$x\,a^{2}+\frac{b^{2}\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{2ab(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	63
risch	$x\,a^{2}-2xab+x\,b^{2}-\frac{4ib\left(-3a\,{\rm e}^{4i(dx+c)}+3b\,{\rm e}^{4i(dx+c)}-6a\,{\rm e}^{2i(dx+c)}+3b\,{\rm e}^{2i(dx+c)}-3a+2b\right)}{3d\left({\rm e}^{2i(dx+c)}+1\right)^{3}}$	92

input
```
int((a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(a^2-2*a*b+b^2)*x+(2*a-b)*b*tan(d*x+c)/d+1/3*b^2*tan(d*x+c)^3/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int (a+b\tan^2(c+dx))^2\,dx = \frac{b^2\tan(dx+c)^3+3\,(a^2-2\,ab+b^2)dx+3\,(2\,ab-b^2)\tan(dx+c)}{3\,d}$$

input
```
integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*tan(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*d*x + 3*(2*a*b - b^2)*tan(
d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int (a+b\tan^2(c+dx))^2\,dx = \begin{cases} a^2x-2abx+\frac{2ab\tan(c+dx)}{d}+b^2x+\frac{b^2\tan^3(c+dx)}{3d}-\frac{b^2\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x(a+b\tan^2(c))^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - 2*a*b*x + 2*a*b*tan(c + d*x)/d + b**2*x + b**2*tan(c +
 d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**2)**2,
True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \left(a+b\tan^2(c+dx)\right)^2\,dx = a^2x - \frac{2\,(dx+c-\tan(dx+c))ab}{d} + \frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)b^2}{3\,d}$$

input
```
integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x - 2*(d*x + c - tan(d*x + c))*a*b/d + 1/3*(tan(d*x + c)^3 + 3*d*x + 3
*c - 3*tan(d*x + c))*b^2/d
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{(a^2-2\,ab+b^2)(dx+c)}{d} + \frac{b^2d^2\tan(dx+c)^3+6\,abd^2\tan(dx+c)-3\,b^2d^2\tan(dx+c)}{3\,d^3}$$

input
```
integrate((a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
(a^2 - 2*a*b + b^2)*(d*x + c)/d + 1/3*(b^2*d^2*tan(d*x + c)^3 + 6*a*b*d^2*
tan(d*x + c) - 3*b^2*d^2*tan(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int \left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{\tan(c+d\,x)\,\left(2\,a\,b-b^2\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)^2}{a^2-2\,a\,b+b^2}\right)\,(a-b)^2}{d}+\frac{b^2\,\tan(c+d\,x)^3}{3\,d}$$

input
```
int((a + b*tan(c + d*x)^2)^2,x)
```

output
```
(tan(c + d*x)*(2*a*b - b^2))/d + (atan((tan(c + d*x)*(a - b)^2)/(a^2 - 2*a
*b + b^2))*(a - b)^2)/d + (b^2*tan(c + d*x)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{\tan(dx+c)^3 b^2+6\tan(dx+c)\,ab-3\tan(dx+c)\,b^2+3a^2dx-6abdx+3b^2dx}{3d}$$

input
```
int((a+b*tan(d*x+c)^2)^2,x)
```

output
```
(tan(c + d*x)**3*b**2 + 6*tan(c + d*x)*a*b - 3*tan(c + d*x)*b**2 + 3*a**2*
d*x - 6*a*b*d*x + 3*b**2*d*x)/(3*d)
```

3.253 $\int (a+b\tan^2(c+dx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int (a+b\tan^2(c+dx))\,dx = ax-bx+\frac{b\tan(c+dx)}{d}$$

output
```
a*x-b*x+b*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int (a+b\tan^2(c+dx))\,dx = ax-\frac{b\arctan(\tan(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

input
```
Integrate[a + b*Tan[c + d*x]^2,x]
```

output
```
a*x - (b*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(c+dx)\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$ax+\frac{b\tan(c+dx)}{d}-bx$$

input
```
Int[a + b*Tan[c + d*x]^2,x]
```

output
```
a*x - b*x + (b*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
norman	$(a-b)\,x+\frac{b\tan(dx+c)}{d}$	20
parallelrisch	$-\frac{b(dx-\tan(dx+c))}{d}+ax$	23
default	$ax+\frac{b(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	26
parts	$ax+\frac{b(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	26
derivativedivides	$\frac{b\tan(dx+c)+(a-b)\arctan(\tan(dx+c))}{d}$	27
risch	$ax-bx+\frac{2ib}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	29

input
```
int(a+b*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(a-b)*x+b*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(c+dx)\right)\,dx=\frac{(a-b)dx+b\tan\left(dx+c\right)}{d}$$

input
```
integrate(a+b*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
((a - b)*d*x + b*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \left(a+b\tan^2(c+dx)\right)\,dx=ax+b\left(\begin{cases}-x+\frac{\tan{(c+dx)}}{d} & \text{for}\ d\neq 0\\ x\tan^2{(c)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*tan(d*x+c)**2,x)
```

output
```
a*x + b*Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \left(a+b\tan^2(c+dx)\right)\,dx=ax-\frac{(dx+c-\tan{(dx+c)})b}{d}$$

input
```
integrate(a+b*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
a*x - (d*x + c - tan(d*x + c))*b/d
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \left(a+b\tan^2(c+dx)\right)\,dx=ax-b\left(\frac{dx+c}{d}-\frac{\tan{(dx+c)}}{d}\right)$$

input
```
integrate(a+b*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
a*x - b*((d*x + c)/d - tan(d*x + c)/d)
```

Mupad [B] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^2(c+dx)\right) \, dx = \frac{b\tan(c+d\,x)+d\,x\,(a-b)}{d}$$

input
```
int(a + b*tan(c + d*x)^2,x)
```

output
```
(b*tan(c + d*x) + d*x*(a - b))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\tan\left(dx+c\right)b+adx-bdx}{d}$$

input
```
int(a+b*tan(d*x+c)^2,x)
```

output
```
(tan(c + d*x)*b + a*d*x - b*d*x)/d
```

3.254 $\int \frac{1}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx=\frac{x}{a-b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)d}$$

output
```
x/(a-b)-b^(1/2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(1/2)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx=\frac{\arctan(\tan(c+dx))-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{ad-bd}$$

input
```
Integrate[(a + b*Tan[c + d*x]^2)^(-1),x]
```

output
```
(ArcTan[Tan[c + d*x]] - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/S
qrt[a])/(a*d - b*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4143, 3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4143}$$

$$\frac{x}{a-b}-\frac{b\int \frac{\sec^2(c+dx)}{b\tan^2(c+dx)+a}dx}{a-b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a-b}-\frac{b\int \frac{\sec(c+dx)^2}{b\tan(c+dx)^2+a}dx}{a-b}$$

$$\Big\downarrow \text{4158}$$

$$\frac{x}{a-b}-\frac{b\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{d(a-b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{a-b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)}$$

input
```
Int[(a + b*Tan[c + d*x]^2)^(-1),x]
```

output
```
x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a -
 b)*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4143
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Simp[b/(a - b)   Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x
] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\arctan(\tan(dx+c))}{a-b}-\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{d}$	50
default	$\frac{\frac{\arctan(\tan(dx+c))}{a-b}-\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}}{d}$	50
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)d}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)d}$	120

input
```
int(1/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)*arctan(tan(d*x+c))-b/(a-b)/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.64

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx = \left[\frac{4\,dx-\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4-6\,ab\tan(dx+c)^2+a^2+4\left(ab\tan(dx+c)^3-a^2\tan(dx+c)\right)\sqrt{-\frac{b}{a}}}{b^2\tan(dx+c)^4+2\,ab\tan(dx+c)^2+a^2}\right)}{4\,(a-b)d}, \frac{2\,dx-\sqrt{\frac{b}{a}}\arctan\left(\frac{(b\ldots}{}\right.}{2\,(a-b\ldots}\right.$$

input
```
integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*d*x - sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2 +
a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x +
 c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)))/((a - b)*d), 1/2*(2*d*x - sqrt(b/a)*
arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan(d*x + c))))/((a - b)*d)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.

Time = 1.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.80

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\tan^2(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{-x-\frac{1}{d\tan(c+dx)}}{b} & \text{for } a=0 \\ \frac{dx\tan^2(c+dx)}{2bd\tan^2(c+dx)+2bd} + \frac{dx}{2bd\tan^2(c+dx)+2bd} + \frac{\tan(c+dx)}{2bd\tan^2(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\tan^2(c)} & \text{for } d=0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}} - \frac{\log\left(-\sqrt{-\frac{a}{b}}+\tan(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}} + \frac{\log\left(\sqrt{-\frac{a}{b}}+\tan(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}-2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*tan(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0
)), ((-x - 1/(d*tan(c + d*x)))/b, Eq(a, 0)), (d*x*tan(c + d*x)**2/(2*b*d*t
an(c + d*x)**2 + 2*b*d) + d*x/(2*b*d*tan(c + d*x)**2 + 2*b*d) + tan(c + d*
x)/(2*b*d*tan(c + d*x)**2 + 2*b*d), Eq(a, b)), (x/(a + b*tan(c)**2), Eq(d,
 0)), (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) - log(-sqrt(
-a/b) + tan(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)) + log(sqrt(-a/
b) + tan(c + d*x))/(2*a*d*sqrt(-a/b) - 2*b*d*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)} - \frac{dx+c}{a-b}}{d}$$

input
```
integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - (d*x + c)/(a -
b))/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx=-\frac{b\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}(ad-bd)}+\frac{dx+c}{ad-bd}$$

input
```
integrate(1/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-b*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*(a*d - b*d)) + (d*x + c)/(a
*d - b*d)
```

Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 948, normalized size of antiderivative = 18.96

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx=\text{Too large to display}$$

input
```
int(1/(a + b*tan(c + d*x)^2),x)
```

output
```
(atan((((-a*b)^(1/2)*(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3
+ 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*
a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2) + ((-a*b)^(1/
2)*(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan
(c + d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b
- a^2))))/(2*(a*b - a^2)))*1i)/(a*b - a^2))/(((-a*b)^(1/2)*(2*b^3*tan(c +
d*x) - ((-a*b)^(1/2)*(2*b^4 - 4*a*b^3 + 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(
1/2)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b
- a^2))))/(a*b - a^2) - ((-a*b)^(1/2)*(2*b^3*tan(c + d*x) - ((-a*b)^(1/2)*
(4*a*b^3 - 2*b^4 - 2*a^2*b^2 + (tan(c + d*x)*(-a*b)^(1/2)*(8*a*b^4 - 8*b^5
 + 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b - a^2))))/(2*(a*b - a^2))))/(a*b - a^2)
))*(-a*b)^(1/2)*1i)/(a*d*(a - b)) - atan(((((4*b^4 - 8*a*b^3 + 4*a^2*b^2 +
 (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*
1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x))/(2*a - 2*b) + (((8*a*b^3 - 4*b^4 - 4
*a^2*b^2 + (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b^2)*1i)/(2*
a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x))/(2*a - 2*b))/(((((4*b^4 -
8*a*b^3 + 4*a^2*b^2 + (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^2*b^3 - 8*a^3*b
^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x))*1i)/(2*a - 2*b)
 - ((((8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(c + d*x)*(8*a*b^4 - 8*b^5 + 8*a^
2*b^3 - 8*a^3*b^2)*1i)/(2*a - 2*b))*1i)/(2*a - 2*b) - 4*b^3*tan(c + d*x...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+b\tan^2(c+dx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)+adx}{ad\,(a-b)}$$

input
```
int(1/(a+b*tan(d*x+c)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a))) + a*d*x)/(a*d
*(a - b))
```

3.255 $\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{x}{(a-b)^2} - \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} - \frac{b\tan(c+dx)}{2a(a-b)d\left(a+b\tan^2(c+dx)\right)}$$

output
```
x/(a-b)^2-1/2*(3*a-b)*b^(1/2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/(
a-b)^2/d-1/2*b*tan(d*x+c)/a/(a-b)/d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{2\arctan(\tan(c+dx)) + \frac{\sqrt{b}(-3a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(-a+b)\tan(c+dx)}{a(a+b\tan^2(c+dx))}}{2(a-b)^2d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^2)^(-2),x]
```

output
```
(2*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x]
)/Sqrt[a]])/a^(3/2) + (b*(-a + b)*Tan[c + d*x])/(a*(a + b*Tan[c + d*x]^2))
)/(2*(a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4144, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^2(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-b\tan^2(c+dx)+2a-b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2a(a-b)}-\frac{b\tan(c+dx)}{2a(a-b)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2a\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a-b}-\frac{b(3a-b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{2a(a-b)}-\frac{b\tan(c+dx)}{2a(a-b)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2a\arctan(\tan(c+dx))}{a-b}-\frac{b(3a-b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{2a(a-b)}-\frac{b\tan(c+dx)}{2a(a-b)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2a\arctan(\tan(c+dx))}{a-b}-\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b\tan(c+dx)}{2a(a-b)(a+b\tan^2(c+dx))}}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^2)^(-2),x]
```

output
```
(((2*a*ArcTan[Tan[c + d*x]])/(a - b) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*
Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - (b*Tan[c + d*x]
)/(2*a*(a - b)*(a + b*Tan[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(dx+c))}{(a-b)^2}}{d}$
default	$\frac{-\frac{b\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{(3a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^2}+\frac{\arctan(\tan(dx+c))}{(a-b)^2}}{d}$
risch	$\frac{x}{a^2-2ab+b^2}+\frac{ib\left(a\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+a-b\right)}{da(-a+b)^2\left(-a\,\mathrm{e}^{4i(dx+c)}+b\,\mathrm{e}^{4i(dx+c)}-2a\,\mathrm{e}^{2i(dx+c)}-2b\,\mathrm{e}^{2i(dx+c)}-a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(\mathrm{e}^{2i(dx+c)}+2\right.}{4a(a-b)^2d}$

input
```
int(1/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b/(a-b)^2*(1/2/a*(a-b)*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(3*a-b)/a/(
a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))+1/(a-b)^2*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.02

$$\int \frac{1}{(a+b\tan^2(c+dx))^2}\,dx = \left[\frac{8\,abdx\tan(dx+c)^2 + 8\,a^2dx - \left((3\,ab-b^2)\tan(dx+c)^2 + 3\,a^2 - ab\right)\sqrt{-\frac{b}{a}}\log\left(\frac{b^2\tan(dx+c)^4 - 6\,ab\tan(dx\cdots}{b^2\tan(\cdots}\right.}{8\left((a^3b-2\,a^2b^2+ab^3)d\tan(dx+c)^2 + (a^4-2\,a\cdots\right.}\right.$$

input
```
integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*a*b*d*x*tan(d*x + c)^2 + 8*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2
 + 3*a^2 - a*b)*sqrt(-b/a)*log((b^2*tan(d*x + c)^4 - 6*a*b*tan(d*x + c)^2
+ a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x + c))*sqrt(-b/a))/(b^2*tan(d*x
 + c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(a*b - b^2)*tan(d*x + c))/((a^3
*b - 2*a^2*b^2 + a*b^3)*d*tan(d*x + c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d), 1
/4*(4*a*b*d*x*tan(d*x + c)^2 + 4*a^2*d*x - ((3*a*b - b^2)*tan(d*x + c)^2 +
 3*a^2 - a*b)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 - a)*sqrt(b/a)/(b*tan
(d*x + c))) - 2*(a*b - b^2)*tan(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*t
an(d*x + c)^2 + (a^4 - 2*a^3*b + a^2*b^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. 2(78) = 156.

Time = 10.00 (sec) , antiderivative size = 2125, normalized size of antiderivative = 21.91

$$\int \frac{1}{(a+b\tan^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Piecewise((zoo*x/tan(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**2, Eq(b
, 0)), ((x + 1/(d*tan(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b**2, Eq(a, 0))
, (3*d*x*tan(c + d*x)**4/(8*b**2*d*tan(c + d*x)**4 + 16*b**2*d*tan(c + d*x
)**2 + 8*b**2*d) + 6*d*x*tan(c + d*x)**2/(8*b**2*d*tan(c + d*x)**4 + 16*b*
*2*d*tan(c + d*x)**2 + 8*b**2*d) + 3*d*x/(8*b**2*d*tan(c + d*x)**4 + 16*b*
*2*d*tan(c + d*x)**2 + 8*b**2*d) + 3*tan(c + d*x)**3/(8*b**2*d*tan(c + d*x
)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d) + 5*tan(c + d*x)/(8*b**2*d*ta
n(c + d*x)**4 + 16*b**2*d*tan(c + d*x)**2 + 8*b**2*d), Eq(a, b)), (x/(a +
b*tan(c)**2)**2, Eq(d, 0)), (4*a**2*d*x*sqrt(-a/b)/(4*a**4*d*sqrt(-a/b) +
4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**
2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) + 4*a*b**3*d*sqr
t(-a/b)*tan(c + d*x)**2) - 3*a**2*log(-sqrt(-a/b) + tan(c + d*x))/(4*a**4*
d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**3*b*d*sqrt(-a/
b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*d*sqrt(-a/b) +
 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) + 3*a**2*log(sqrt(-a/b) + tan(c +
d*x))/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*x)**2 - 8*a**
3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 4*a**2*b**2*
d*sqrt(-a/b) + 4*a*b**3*d*sqrt(-a/b)*tan(c + d*x)**2) + 4*a*b*d*x*sqrt(-a/
b)*tan(c + d*x)**2/(4*a**4*d*sqrt(-a/b) + 4*a**3*b*d*sqrt(-a/b)*tan(c + d*
x)**2 - 8*a**3*b*d*sqrt(-a/b) - 8*a**2*b**2*d*sqrt(-a/b)*tan(c + d*x)**...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\tan^2(c+dx))^2}\,dx = -\frac{\frac{b\tan(dx+c)}{a^3-a^2b+(a^2b-ab^2)\tan(dx+c)^2}+\frac{(3\,ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-2\,a^2b+ab^2)\sqrt{ab}}-\frac{2\,(dx+c)}{a^2-2\,ab+b^2}}{2\,d}$$

input
```
integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(d*x + c)/(a^3 - a^2*b + (a^2*b - a*b^2)*tan(d*x + c)^2) + (3*a
*b - b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(a
*b)) - 2*(d*x + c)/(a^2 - 2*a*b + b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = -\frac{\left(3\,ab-b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2\,\left(a^3d-2\,a^2bd+ab^2d\right)\sqrt{ab}}+\frac{dx+c}{a^2d-2\,abd+b^2d}-\frac{b\tan\left(dx+c\right)}{2\,\left(a^2d-abd\right)\left(b\tan\left(dx+c\right)^2+a\right)}$$

input
```
integrate(1/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b - b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3*d - 2*a^2*b*d +
a*b^2*d)*sqrt(a*b)) + (d*x + c)/(a^2*d - 2*a*b*d + b^2*d) - 1/2*b*tan(d*x
+ c)/((a^2*d - a*b*d)*(b*tan(d*x + c)^2 + a))
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 2489, normalized size of antiderivative = 25.66

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(c + d*x)^2)^2,x)
```

output
```
(2*atan((((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 -
 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(16*
a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))
/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))/(2*a^2 - 4*a*b + 2
*b^2) + (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3))/(2*(a^4 - 2*a^3*b + a^
2*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - ((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 -
 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3
*b^2) + (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 -
 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2
*b^2)))/(2*a^2 - 4*a*b + 2*b^2) - (tan(c + d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^
3))/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^2 - 4*a*b + 2*b^2))/((((((2*a*b^7
- 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 + 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^
4*b - a^5 + a^2*b^3 - 3*a^3*b^2) - (tan(c + d*x)*(16*a^2*b^7 - 48*a^3*b^6
+ 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 + 16*a^7*b^2))/(2*(a^4 - 2*a^3*b +
a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i)/(2*a^2 - 4*a*b + 2*b^2) + (tan(c +
d*x)*(b^5 - 6*a*b^4 + 13*a^2*b^3)*1i)/(2*(a^4 - 2*a^3*b + a^2*b^2)))/(2*a^
2 - 4*a*b + 2*b^2) + (((((2*a*b^7 - 12*a^2*b^6 + 28*a^3*b^5 - 32*a^4*b^4 +
 18*a^5*b^3 - 4*a^6*b^2)*1i)/(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2) + (tan(
c + d*x)*(16*a^2*b^7 - 48*a^3*b^6 + 32*a^4*b^5 + 32*a^5*b^4 - 48*a^6*b^3 +
 16*a^7*b^2))/(2*(a^4 - 2*a^3*b + a^2*b^2)*(2*a^2 - 4*a*b + 2*b^2)))*1i...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tan\left(dx+c\right)^2 ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tan\left(dx+c\right)^2 b^2-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\text{ta}}{}\right.}{2a^2d\left(\tan\left(dx+c\right)^2a^2b-2\tan\left(dx+\right.\right.}$$

input
```
int(1/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tan(c + d*x
)**2*a*b + sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tan(c
+ d*x)**2*b**2 - 3*sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a))
)*a**2 + sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a*b + 2*
tan(c + d*x)**2*a**2*b*d*x - tan(c + d*x)*a**2*b + tan(c + d*x)*a*b**2 + 2
*a**3*d*x)/(2*a**2*d*(tan(c + d*x)**2*a**2*b - 2*tan(c + d*x)**2*a*b**2 +
tan(c + d*x)**2*b**3 + a**3 - 2*a**2*b + a*b**2))
```

3.256 $\int \frac{1}{(a+b\tan^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{(a+b\tan^2(c+dx))^3}\,dx = \frac{x}{(a-b)^3} - \frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{b\tan(c+dx)}{4a(a-b)d\,(a+b\tan^2(c+dx))^2} - \frac{(7a-3b)b\tan(c+dx)}{8a^2(a-b)^2d\,(a+b\tan^2(c+dx))}$$

output
```
x/(a-b)^3-1/8*b^(1/2)*(15*a^2-10*a*b+3*b^2)*arctan(b^(1/2)*tan(d*x+c)/a^(1
/2))/a^(5/2)/(a-b)^3/d-1/4*b*tan(d*x+c)/a/(a-b)/d/(a+b*tan(d*x+c)^2)^2-1/8
*(7*a-3*b)*b*tan(d*x+c)/a^2/(a-b)^2/d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx = -\frac{-8\arctan(\tan(c+dx))+\frac{\sqrt{b}(15a^2-10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2(a-b)^2b\tan(c+dx)}{a(a+b\tan^2(c+dx))^2}+\frac{(7a-3b)(a-b)b\tan(c+dx)}{a^2(a+b\tan^2(c+dx))}}{8(a-b)^3d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^2)^(-3),x]
```

output
```
-1/8*(-8*ArcTan[Tan[c + d*x]] + (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[
(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(5/2) + (2*(a - b)^2*b*Tan[c + d*x])/(a
*(a + b*Tan[c + d*x]^2)^2) + ((7*a - 3*b)*(a - b)*b*Tan[c + d*x])/(a^2*(a
+ b*Tan[c + d*x]^2)))/((a - b)^3*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(c+dx)^2\right)^3}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)\left(b\tan^2(c+dx)+a\right)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-3b\tan^2(c+dx)+4a-3b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{4a(a-b)}-\frac{b\tan(c+dx)}{4a(a-b)(a+b\tan^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{8a^2-7ba+3b^2-(7a-3b)b\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2a(a-b)}-\frac{b(7a-3b)\tan(c+dx)}{2a(a-b)\left(a+b\tan^2(c+dx)\right)}}{4a(a-b)}-\frac{b\tan(c+dx)}{4a(a-b)(a+b\tan^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{8a^2\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\tan(c+dx)}{2a(a-b)\left(a+b\tan^2(c+dx)\right)}}{4a(a-b)}-\frac{b\tan(c+dx)}{4a(a-b)(a+b\tan^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{8a^2\arctan(\tan(c+dx))}{a-b}-\frac{b\left(15a^2-10ab+3b^2\right)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{2a(a-b)}-\frac{b(7a-3b)\tan(c+dx)}{2a(a-b)\left(a+b\tan^2(c+dx)\right)}}{4a(a-b)}-\frac{b\tan(c+dx)}{4a(a-b)(a+b\tan^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{8a^2\arctan(\tan(c+dx))}{a-b}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2a(a-b)}-\frac{b(7a-3b)\tan(c+dx)}{2a(a-b)\left(a+b\tan^2(c+dx)\right)}}{4a(a-b)}-\frac{b\tan(c+dx)}{4a(a-b)(a+b\tan^2(c+dx))^2}}{d}$$

input

```
Int[(a + b*Tan[c + d*x]^2)^(-3),x]
```

output

```
(-1/4*(b*Tan[c + d*x])/(a*(a - b)*(a + b*Tan[c + d*x]^2)^2) + (((8*a^2*Arc
Tan[Tan[c + d*x]])/(a - b) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sq
rt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*a*(a - b)) - ((7*a - 3
*b)*b*Tan[c + d*x])/(2*a*(a - b)*(a + b*Tan[c + d*x]^2)))/(4*a*(a - b)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\tan(dx+c)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(dx+c)}{8a}}{(a+b\tan(dx+c)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(dx}{(a-b)^3}}{d}$
default	$\frac{-\frac{b\left(\frac{\frac{b(7a^2-10ab+3b^2)\tan(dx+c)^3}{8a^2}+\frac{(9a^2-14ab+5b^2)\tan(dx+c)}{8a}}{(a+b\tan(dx+c)^2)^2}+\frac{(15a^2-10ab+3b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a-b)^3}+\frac{\arctan(\tan(dx}{(a-b)^3}}{d}$
risch	$\frac{x}{a^3-3a^2b+3a\,b^2-b^3}+\frac{i(9a^3e^{6i(dx+c)}+a^2b\,e^{6i(dx+c)}-13a\,b^2e^{6i(dx+c)}+3b^3e^{6i(dx+c)}+27a^3e^{4i(dx+c)}+9a^2b\,e^{4i(dx+c)}}{4(-a\,e^{4i(dx+c)}+b\,e^{4i(}}$

input
```
int(1/(a+b*tan(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b/(a-b)^3*((1/8*b*(7*a^2-10*a*b+3*b^2)/a^2*tan(d*x+c)^3+1/8*(9*a^2-1
4*a*b+5*b^2)/a*tan(d*x+c))/(a+b*tan(d*x+c)^2)^2+1/8*(15*a^2-10*a*b+3*b^2)/
a^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))+1/(a-b)^3*arctan(tan(d*x
+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(136) = 272.

Time = 0.12 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.95

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*a^2*b^2*d*x*tan(d*x + c)^4 + 64*a^3*b*d*x*tan(d*x + c)^2 + 32*a^
4*d*x - 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 1
0*a*b^3 + 3*b^4)*tan(d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^
3*b - 10*a^2*b^2 + 3*a*b^3)*tan(d*x + c)^2)*sqrt(-b/a)*log((b^2*tan(d*x +
c)^4 - 6*a*b*tan(d*x + c)^2 + a^2 + 4*(a*b*tan(d*x + c)^3 - a^2*tan(d*x +
c))*sqrt(-b/a))/(b^2*tan(d*x + c)^4 + 2*a*b*tan(d*x + c)^2 + a^2)) - 4*(9*
a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*d*tan(d*x + c)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b
^4)*d*tan(d*x + c)^2 + (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d), 1/16*(16*
a^2*b^2*d*x*tan(d*x + c)^4 + 32*a^3*b*d*x*tan(d*x + c)^2 + 16*a^4*d*x - 2*
(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*tan(d*x + c)^3 - ((15*a^2*b^2 - 10*a*b^3 +
3*b^4)*tan(d*x + c)^4 + 15*a^4 - 10*a^3*b + 3*a^2*b^2 + 2*(15*a^3*b - 10*a
^2*b^2 + 3*a*b^3)*tan(d*x + c)^2)*sqrt(b/a)*arctan(1/2*(b*tan(d*x + c)^2 -
 a)*sqrt(b/a)/(b*tan(d*x + c))) - 2*(9*a^3*b - 14*a^2*b^2 + 5*a*b^3)*tan(d
*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*tan(d*x + c)^4 + 2
*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*tan(d*x + c)^2 + (a^7 - 3*a^6
*b + 3*a^5*b^2 - a^4*b^3)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8964 vs. $2(129) = 258$.

Time = 49.24 (sec) , antiderivative size = 8964, normalized size of antiderivative = 59.76

$$\int \frac{1}{\left(a + b\tan^2(c + dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)**2)**3,x)
```

output
```
Piecewise((zoo*x/tan(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**3, Eq(b
, 0)), ((-x - 1/(d*tan(c + d*x)) + 1/(3*d*tan(c + d*x)**3) - 1/(5*d*tan(c
+ d*x)**5))/b**3, Eq(a, 0)), (15*d*x*tan(c + d*x)**6/(48*b**3*d*tan(c + d*
x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*
d) + 45*d*x*tan(c + d*x)**4/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c
+ d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) + 45*d*x*tan(c + d*x)*
*2/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*ta
n(c + d*x)**2 + 48*b**3*d) + 15*d*x/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*
d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d) + 15*tan(c + d
*x)**5/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*
d*tan(c + d*x)**2 + 48*b**3*d) + 40*tan(c + d*x)**3/(48*b**3*d*tan(c + d*x
)**6 + 144*b**3*d*tan(c + d*x)**4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d
) + 33*tan(c + d*x)/(48*b**3*d*tan(c + d*x)**6 + 144*b**3*d*tan(c + d*x)**
4 + 144*b**3*d*tan(c + d*x)**2 + 48*b**3*d), Eq(a, b)), (x/(a + b*tan(c)**
2)**3, Eq(d, 0)), (16*a**4*d*x*sqrt(-a/b)/(16*a**7*d*sqrt(-a/b) + 32*a**6*
b*d*sqrt(-a/b)*tan(c + d*x)**2 - 48*a**6*b*d*sqrt(-a/b) + 16*a**5*b**2*d*s
qrt(-a/b)*tan(c + d*x)**4 - 96*a**5*b**2*d*sqrt(-a/b)*tan(c + d*x)**2 + 48
*a**5*b**2*d*sqrt(-a/b) - 48*a**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**4 + 96*a
**4*b**3*d*sqrt(-a/b)*tan(c + d*x)**2 - 16*a**4*b**3*d*sqrt(-a/b) + 48*a**
3*b**4*d*sqrt(-a/b)*tan(c + d*x)**4 - 32*a**3*b**4*d*sqrt(-a/b)*tan(c +...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx = -\frac{\frac{(15\,a^2b-10\,ab^2+3\,b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sqrt{ab}}+\frac{(7\,ab^2-3\,b^3)\tan(dx+c)^3+(9\,a^2b-5\,ab^2)\tan(dx+c)}{a^6-2\,a^5b+a^4b^2+(a^4b^2-2\,a^3b^3+a^2b^4)\tan(dx+c)^4+2\,(a^5b-2\,a^4b^2+a^3b^3)\tan(dx+c)^2}-\cdots}{8\,d}$$

input
```
integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(a*b)) + ((7*a*b^2 - 3*b^3)*tan(d*x
+ c)^3 + (9*a^2*b - 5*a*b^2)*tan(d*x + c))/(a^6 - 2*a^5*b + a^4*b^2 + (a^4
*b^2 - 2*a^3*b^3 + a^2*b^4)*tan(d*x + c)^4 + 2*(a^5*b - 2*a^4*b^2 + a^3*b^
3)*tan(d*x + c)^2) - 8*(d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx = -\frac{\left(15\,a^2b-10\,ab^2+3\,b^3\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{8\left(a^5d-3\,a^4bd+3\,a^3b^2d-a^2b^3d\right)\sqrt{ab}}+\frac{dx+c}{a^3d-3\,a^2bd+3\,ab^2d-b^3d} - \frac{7\,ab^2\tan\left(dx+c\right)^3-3\,b^3\tan\left(dx+c\right)^3+9\,a^2b\tan\left(dx+c\right)-5\,ab^2\tan\left(dx+c\right)}{8\left(a^4d-2\,a^3bd+a^2b^2d\right)\left(b\tan\left(dx+c\right)^2+a\right)^2}$$

input
```
integrate(1/(a+b*tan(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(15*a^2*b - 10*a*b^2 + 3*b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^5*
d - 3*a^4*b*d + 3*a^3*b^2*d - a^2*b^3*d)*sqrt(a*b)) + (d*x + c)/(a^3*d - 3
*a^2*b*d + 3*a*b^2*d - b^3*d) - 1/8*(7*a*b^2*tan(d*x + c)^3 - 3*b^3*tan(d*
x + c)^3 + 9*a^2*b*tan(d*x + c) - 5*a*b^2*tan(d*x + c))/((a^4*d - 2*a^3*b*
d + a^2*b^2*d)*(b*tan(d*x + c)^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 3901, normalized size of antiderivative = 26.01

$$\int \frac{1}{\left(a+b\tan^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(c + d*x)^2)^3,x)
```

output
```
(atan((((-a^5*b)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 30
0*a^3*b^4 + 289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6
*b^2)) - (((96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760
*a^6*b^6 - 9056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*
(a^10 - 6*a^9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b
^2)) - (tan(c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9
 - 1280*a^5*b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^
4 - 1280*a^10*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b
^2)*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15
*a^2 - 10*a*b + 3*b^2))/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)))*(15*a^
2 - 10*a*b + 3*b^2)*1i)/(16*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + ((-a^
5*b)^(1/2)*((tan(c + d*x)*(9*b^7 - 60*a*b^6 + 190*a^2*b^5 - 300*a^3*b^4 +
289*a^4*b^3))/(32*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)) + (((
96*a^2*b^10 - 800*a^3*b^9 + 3040*a^4*b^8 - 6816*a^5*b^7 + 9760*a^6*b^6 - 9
056*a^7*b^5 + 5280*a^8*b^4 - 1760*a^9*b^3 + 256*a^10*b^2)/(64*(a^10 - 6*a^
9*b + a^4*b^6 - 6*a^5*b^5 + 15*a^6*b^4 - 20*a^7*b^3 + 15*a^8*b^2)) + (tan(
c + d*x)*(-a^5*b)^(1/2)*(15*a^2 - 10*a*b + 3*b^2)*(256*a^4*b^9 - 1280*a^5*
b^8 + 2304*a^6*b^7 - 1280*a^7*b^6 - 1280*a^8*b^5 + 2304*a^9*b^4 - 1280*a^1
0*b^3 + 256*a^11*b^2))/(512*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)*(a^8 - 4
*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-a^5*b)^(1/2)*(15*a^2 - 10...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.85

$$\int \frac{1}{(a+b\tan^2(c+dx))^3}\,dx$$
$$= \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(dx+c)^4\,a^2b^2 + 10\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tan(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tan(dx+c)^4\,a\,b^3 - 3\sqrt{b}\,\sqrt{a}}{}$$

input
```
int(1/(a+b*tan(d*x+c)^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tan(c + d*
x)**4*a**2*b**2 + 10*sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a
)))*tan(c + d*x)**4*a*b**3 - 3*sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt
(b)*sqrt(a)))*tan(c + d*x)**4*b**4 - 30*sqrt(b)*sqrt(a)*atan((tan(c + d*x)
*b)/(sqrt(b)*sqrt(a)))*tan(c + d*x)**2*a**3*b + 20*sqrt(b)*sqrt(a)*atan((t
an(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tan(c + d*x)**2*a**2*b**2 - 6*sqrt(b)*sq
rt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tan(c + d*x)**2*a*b**3 - 15
*sqrt(b)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 10*sqrt(b
)*sqrt(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b - 3*sqrt(b)*sqrt
(a)*atan((tan(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*tan(c + d*x)**4
*a**3*b**2*d*x - 7*tan(c + d*x)**3*a**3*b**2 + 10*tan(c + d*x)**3*a**2*b**
3 - 3*tan(c + d*x)**3*a*b**4 + 16*tan(c + d*x)**2*a**4*b*d*x - 9*tan(c + d
*x)*a**4*b + 14*tan(c + d*x)*a**3*b**2 - 5*tan(c + d*x)*a**2*b**3 + 8*a**5
*d*x)/(8*a**3*d*(tan(c + d*x)**4*a**3*b**2 - 3*tan(c + d*x)**4*a**2*b**3 +
 3*tan(c + d*x)**4*a*b**4 - tan(c + d*x)**4*b**5 + 2*tan(c + d*x)**2*a**4*
b - 6*tan(c + d*x)**2*a**3*b**2 + 6*tan(c + d*x)**2*a**2*b**3 - 2*tan(c +
d*x)**2*a*b**4 + a**5 - 3*a**4*b + 3*a**3*b**2 - a**2*b**3))
```

3.257 $\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{3}{8}\text{arctanh}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} - \frac{3}{8}\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{4}\sqrt{a\sec^2(x)}\tan^3(x)$$

output
```
3/8*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-3/8*(a*sec(x)^2)^(1/2)*tan(x
)+1/4*(a*sec(x)^2)^(1/2)*tan(x)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{1}{8}\sqrt{a\sec^2(x)}\big(3\text{arctanh}(\sin(x))\cos(x) - 3\tan(x) + 2\tan^3(x)\big)$$

input
```
Integrate[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

output
```
(Sqrt[a*Sec[x]^2]*(3*ArcTanh[Sin[x]]*Cos[x] - 3*Tan[x] + 2*Tan[x]^3))/8
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 4140, 3042, 4613, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(x)\sqrt{a\tan^2(x)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(x)^4\sqrt{a\tan(x)^2+a}dx$$

$$\Big\downarrow \ 4140$$

$$\int \tan^4(x)\sqrt{a\sec^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(x)^4\sqrt{a\sec(x)^2}dx$$

$$\Big\downarrow \ 4613$$

$$\cos(x)\sqrt{a\sec^2(x)}\int \sec(x)\tan^4(x)dx$$

$$\Big\downarrow \ 3042$$

$$\cos(x)\sqrt{a\sec^2(x)}\int \sec(x)\tan(x)^4dx$$

$$\Big\downarrow \ 3091$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\tan^3(x)\sec(x)-\frac{3}{4}\int \sec(x)\tan^2(x)dx\right)$$

$$\Big\downarrow \ 3042$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\tan^3(x)\sec(x)-\frac{3}{4}\int \sec(x)\tan(x)^2dx\right)$$

$$\Big\downarrow \ 3091$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\tan^3(x)\sec(x)-\frac{3}{4}\left(\frac{1}{2}\tan(x)\sec(x)-\frac{\int \sec(x)dx}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\tan^3(x)\sec(x)-\frac{3}{4}\left(\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\int\csc\left(x+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{4257}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\tan^3(x)\sec(x)-\frac{3}{4}\left(\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\text{arctanh}(\sin(x))\right)\right)$$

input
```
Int[Tan[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

output
```
Cos[x]*Sqrt[a*Sec[x]^2]*((Sec[x]*Tan[x]^3)/4 - (3*(-1/2*ArcTanh[Sin[x]] +
(Sec[x]*Tan[x])/2))/4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\tan(x)\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}{4a}-\frac{5\tan(x)\sqrt{a+a\tan(x)^2}}{8}+\frac{3\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^2}\right)}{8}$
default	$\frac{\tan(x)\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}{4a}-\frac{5\tan(x)\sqrt{a+a\tan(x)^2}}{8}+\frac{3\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^2}\right)}{8}$
risch	$\frac{i\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(5\,e^{6ix}-3\,e^{4ix}+3\,e^{2ix}-5\right)}{4\left(e^{2ix}+1\right)^3}-\frac{3\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}-i\right)\cos(x)}{4}+\frac{3\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}+i\right)\cos(x)}{4}$

input
```
int(tan(x)^4*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)*(a+a*tan(x)^2)^(3/2)/a-5/8*tan(x)*(a+a*tan(x)^2)^(1/2)+3/8*a^(1
/2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{1}{8}\,\sqrt{a\tan\left(x\right)^2+a}\left(2\,\tan\left(x\right)^3-3\,\tan\left(x\right)\right) + \frac{3}{16}\,\sqrt{a}\log\left(2\,a\tan\left(x\right)^2+2\,\sqrt{a\tan\left(x\right)^2+a}\sqrt{a}\tan\left(x\right) + a\right)$$

input
```
integrate(tan(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^3 - 3*tan(x)) + 3/16*sqrt(a)*log(2*a*ta
n(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
```

Sympy [F]

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\left(\tan^2\left(x\right)+1\right)}\tan^4\left(x\right)dx$$

input
```
integrate(tan(x)**4*(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(42) = 84$.

Time = 0.30 (sec) , antiderivative size = 860, normalized size of antiderivative = 15.93

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(4*(5*sin(7*x) - 3*sin(5*x) + 3*sin(3*x) - 5*sin(x))*cos(8*x) - 40*(
2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) - 16*(3*sin(5*x) - 3*sin(3*
x) + 5*sin(x))*cos(6*x) + 24*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) + 24*(3*si
n(3*x) - 5*sin(x))*cos(4*x) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) +
 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*
cos(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2
+ 4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*s
in(4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*
x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*
sin(x) + 1) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + c
os(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*
(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x)
+ 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(
2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16
*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*
(5*cos(7*x) - 3*cos(5*x) + 3*cos(3*x) - 5*cos(x))*sin(8*x) + 20*(4*cos(6*x
) + 6*cos(4*x) + 4*cos(2*x) + 1)*sin(7*x) + 16*(3*cos(5*x) - 3*cos(3*x) +
5*cos(x))*sin(6*x) - 12*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) - 24*(3*cos
(3*x) - 5*cos(x))*sin(4*x) + 12*(4*cos(2*x) + 1)*sin(3*x) - 48*cos(3*x)*si
n(2*x) + 80*cos(x)*sin(2*x) - 80*cos(2*x)*sin(x) - 20*sin(x))*sqrt(a)/(...
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{1}{8}\sqrt{a\tan{(x)}^2+a}\big(2\tan{(x)}^2-3\big)\tan{(x)} - \frac{3}{8}\sqrt{a}\log\left(\left|-\sqrt{a}\tan{(x)}+\sqrt{a\tan{(x)}^2+a}\right|\right)$$

input
```
integrate(tan(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 3)*tan(x) - 3/8*sqrt(a)*log(abs(-sq
rt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \int \tan(x)^4\,\sqrt{a\tan{(x)}^2+a}\,dx$$

input
```
int(tan(x)^4*(a + a*tan(x)^2)^(1/2),x)
```

output
```
int(tan(x)^4*(a + a*tan(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \tan^4(x)\sqrt{a+a\tan^2(x)}\,dx = \sqrt{a}\left(\int \sqrt{\tan{(x)}^2+1}\,\tan{(x)}^4\,dx\right)$$

input
```
int(tan(x)^4*(a+a*tan(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(x)**2 + 1)*tan(x)**4,x)
```

3.258 $\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx = -\sqrt{a\sec^2(x)}+\frac{(a\sec^2(x))^{3/2}}{3a}$$

output
```
-(a*sec(x)^2)^(1/2)+1/3*(a*sec(x)^2)^(3/2)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{1}{3}\sqrt{a\sec^2(x)}\left(-3+\sec^2(x)\right)$$

input
```
Integrate[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

output
```
(Sqrt[a*Sec[x]^2]*(-3 + Sec[x]^2))/3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4140, 3042, 4612, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(x)\sqrt{a\tan^2(x)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^3\sqrt{a\tan(x)^2+a}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \tan^3(x)\sqrt{a\sec^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^3\sqrt{a\sec(x)^2}dx$$

$$\Big\downarrow \text{ 4612}$$

$$\frac{1}{2}a\int -\frac{1-\sec^2(x)}{\sqrt{a\sec^2(x)}}d\sec^2(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}a\int \frac{1-\sec^2(x)}{\sqrt{a\sec^2(x)}}d\sec^2(x)$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{1}{2}a\int \left(\frac{1}{\sqrt{a\sec^2(x)}}-\frac{\sqrt{a\sec^2(x)}}{a}\right)d\sec^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}a\left(\frac{2\left(a\sec^2(x)\right)^{3/2}}{3a^2}-\frac{2\sqrt{a\sec^2(x)}}{a}\right)$$

input
```
Int[Tan[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

output
```
(a*((-2*Sqrt[a*Sec[x]^2])/a + (2*(a*Sec[x]^2)^(3/2))/(3*a^2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}{3a}-\sqrt{a+a\tan\left(x\right)^2}$	29
default	$\frac{\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}{3a}-\sqrt{a+a\tan\left(x\right)^2}$	29
risch	$-\frac{2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(3\,e^{4ix}+2\,e^{2ix}+3\right)}{3\left(e^{2ix}+1\right)^2}$	46

input
```
int(tan(x)^3*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*(a+a*tan(x)^2)^(3/2)-(a+a*tan(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx=\frac{1}{3}\sqrt{a\tan\left(x\right)^2+a}\left(\tan\left(x\right)^2-2\right)$$

input
```
integrate(tan(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(a*tan(x)^2 + a)*(tan(x)^2 - 2)
```

Sympy [F]

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx=\int \sqrt{a\left(\tan^2\left(x\right)+1\right)}\tan^3\left(x\right)dx$$

input
```
integrate(tan(x)**3*(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(24) = 48$.

Time = 0.18 (sec) , antiderivative size = 276, normalized size of antiderivative = 9.20

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx = -\frac{2\left(\left(3\cos\left(5x\right)+2\cos\left(3x\right)+3\cos\left(x\right)\right)\cos\left(6x\right)+3\left(3\cos\left(4x\right)+3\cos\left(2x\right)+1\right)\cos\left(5x\right)+3\left(2\cos\right.\right.}{3\left(2\left(3\cos\left(4x\right)+3\cos\left(2x\right)+1\right)\cos\left(\right.\right.}$$

input
```
integrate(tan(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*((3*cos(5*x) + 2*cos(3*x) + 3*cos(x))*cos(6*x) + 3*(3*cos(4*x) + 3*co
s(2*x) + 1)*cos(5*x) + 3*(2*cos(3*x) + 3*cos(x))*cos(4*x) + 2*(3*cos(2*x)
 + 1)*cos(3*x) + 9*cos(2*x)*cos(x) + (3*sin(5*x) + 2*sin(3*x) + 3*sin(x))*s
in(6*x) + 9*(sin(4*x) + sin(2*x))*sin(5*x) + 3*(2*sin(3*x) + 3*sin(x))*sin
(4*x) + 6*sin(3*x)*sin(2*x) + 9*sin(2*x)*sin(x) + 3*cos(x))*sqrt(a)/(2*(3*
cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(
4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + si
n(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x)
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{\left(a\tan\left(x\right)^2+a\right)^{\frac{3}{2}}-3\sqrt{a\tan\left(x\right)^2+a}a}{3\,a}$$

input
```
integrate(tan(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*((a*tan(x)^2 + a)^(3/2) - 3*sqrt(a*tan(x)^2 + a)*a)/a
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx = -\frac{\sqrt{2}\,\sqrt{a}\,\left(6\cos\left(x\right)^2-2\right)}{3\,{\left(2\cos\left(x\right)^2\right)}^{3/2}}$$

input
```
int(tan(x)^3*(a + a*tan(x)^2)^(1/2),x)
```

output
```
-(2^(1/2)*a^(1/2)*(6*cos(x)^2 - 2))/(3*(2*cos(x)^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \tan^3(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{\sqrt{a}\,\sqrt{\tan\left(x\right)^2+1}\,\left(\tan\left(x\right)^2-2\right)}{3}$$

input
```
int(tan(x)^3*(a+a*tan(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*sqrt(tan(x)**2 + 1)*(tan(x)**2 - 2))/3
```

3.259 $\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = -\frac{1}{2}\text{arctanh}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}+\frac{1}{2}\sqrt{a\sec^2(x)}\tan(x)$$

output
```
-1/2*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)+1/2*(a*sec(x)^2)^(1/2)*tan(
x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{1}{2}\sqrt{a\sec^2(x)}(-\text{arctanh}(\sin(x))\cos(x)+\tan(x))$$

input
```
Integrate[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

output
```
(Sqrt[a*Sec[x]^2]*(-(ArcTanh[Sin[x]]*Cos[x]) + Tan[x]))/2
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4140, 3042, 4613, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\sqrt{a\tan^2(x)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^2\sqrt{a\tan(x)^2+a}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \tan^2(x)\sqrt{a\sec^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^2\sqrt{a\sec(x)^2}dx$$

$$\Big\downarrow \text{ 4613}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int \sec(x)\tan^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int \sec(x)\tan(x)^2dx$$

$$\Big\downarrow \text{ 3091}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{2}\tan(x)\sec(x)-\frac{\int \sec(x)dx}{2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \text{ 4257}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\text{arctanh}(\sin(x))\right)$$

input
```
Int[Tan[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

output
```
Cos[x]*Sqrt[a*Sec[x]^2]*(-1/2*ArcTanh[Sin[x]] + (Sec[x]*Tan[x])/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\tan(x)\sqrt{a+a\tan(x)^2}}{2}-\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^2}\right)}{2}$
default	$\frac{\tan(x)\sqrt{a+a\tan(x)^2}}{2}-\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^2}\right)}{2}$
risch	$-\frac{i\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,(e^{2ix}-1)}{e^{2ix}+1}+\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,\ln\left(e^{ix}-i\right)\cos\left(x\right)-\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,\ln\left(e^{ix}+i\right)\cos\left(x\right)$

input
```
int(tan(x)^2*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)*(a+a*tan(x)^2)^(1/2)-1/2*a^(1/2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^
2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{1}{4}\sqrt{a}\log\left(2\,a\tan\left(x\right)^2-2\,\sqrt{a\tan\left(x\right)^2+a}\sqrt{a}\tan\left(x\right)+a\right)+\frac{1}{2}\,\sqrt{a\tan\left(x\right)^2+a}\tan\left(x\right)$$

input
```
integrate(tan(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(a)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
+ 1/2*sqrt(a*tan(x)^2 + a)*tan(x)
```

Sympy [F]

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\left(\tan^2{(x)}+1\right)}\tan^2{(x)}\,dx$$

input
```
integrate(tan(x)**2*(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(tan(x)**2 + 1))*tan(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(28) = 56$.

Time = 0.17 (sec) , antiderivative size = 295, normalized size of antiderivative = 8.19

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{(4\left(\sin{(3x)}-\sin{(x)}\right)\cos{(4x)}-\left(2\left(2\cos{(2x)}+1\right)\cos{(4x)}+\cos{(4x)}^2+4\cos{(2x)}^2+\sin{(4x)}^2+4\right.}{}$$

input
```
integrate(tan(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(3*x) - sin(x))*cos(4*x) - (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4
*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4
*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (2*(2*cos(2*x) +
1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
 + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
- 4*(cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x
)*sin(2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))*sqrt(a)/(2*
(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin
(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{1}{2}\sqrt{a}\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right) + \frac{1}{2}\sqrt{a\tan(x)^2+a}\tan(x)$$

input
```
integrate(tan(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a)*log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a))) + 1/2*sqrt(a*
tan(x)^2 + a)*tan(x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \int \tan(x)^2\sqrt{a\tan(x)^2+a}\,dx$$

input
```
int(tan(x)^2*(a + a*tan(x)^2)^(1/2),x)
```

output
```
int(tan(x)^2*(a + a*tan(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \tan^2(x)\sqrt{a+a\tan^2(x)}\,dx = \sqrt{a}\left(\int\sqrt{\tan(x)^2+1}\,\tan(x)^2\,dx\right)$$

input
```
int(tan(x)^2*(a+a*tan(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(x)**2 + 1)*tan(x)**2,x)
```

3.260 $\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx=\sqrt{a\sec^2(x)}$$

output
```
(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx=\sqrt{a\sec^2(x)}$$

input
```
Integrate[Tan[x]*Sqrt[a + a*Tan[x]^2],x]
```

output
```
Sqrt[a*Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4140, 3042, 4612, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sqrt{a\tan^2(x)+a}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \tan(x)\sqrt{a\tan(x)^2+a}dx$$
$$\Big\downarrow \ 4140$$
$$\int \tan(x)\sqrt{a\sec^2(x)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \tan(x)\sqrt{a\sec(x)^2}dx$$
$$\Big\downarrow \ 4612$$
$$\frac{1}{2}a\int \frac{1}{\sqrt{a\sec^2(x)}}d\sec^2(x)$$
$$\Big\downarrow \ 17$$
$$\sqrt{a\sec^2(x)}$$

input
```
Int[Tan[x]*Sqrt[a + a*Tan[x]^2],x]
```

output
```
Sqrt[a*Sec[x]^2]
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\sqrt{a+a\tan\left(x\right)^{2}}$	11
default	$\sqrt{a+a\tan\left(x\right)^{2}}$	11
risch	$2\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}$	21

input
```
int(tan(x)*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a+a*tan(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx=\sqrt{a\tan{(x)}^2+a}$$

input
```
integrate(tan(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*tan(x)^2 + a)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx=\sqrt{a\tan^2{(x)}+a}$$

input
```
integrate(tan(x)*(a+a*tan(x)**2)**(1/2),x)
```

output
```
sqrt(a*tan(x)**2 + a)
```

Maxima [F]

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx=\int \sqrt{a\tan{(x)}^2+a}\tan{(x)}\,dx$$

input
```
integrate(tan(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*tan(x)^2 + a)*tan(x), x)
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx = \sqrt{a\tan{(x)}^2 + a}$$

input
```
integrate(tan(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(a*tan(x)^2 + a)
```

Mupad [B] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{\sqrt{a}}{\sqrt{\cos{(x)}^2}}$$

input
```
int(tan(x)*(a + a*tan(x)^2)^(1/2),x)
```

output
```
a^(1/2)/(cos(x)^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \tan(x)\sqrt{a+a\tan^2(x)}\,dx = \sqrt{a}\,\sqrt{\tan{(x)}^2 + 1}$$

input
```
int(tan(x)*(a+a*tan(x)^2)^(1/2),x)
```

output
```
sqrt(a)*sqrt(tan(x)**2 + 1)
```

3.261 $\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = -\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

output
```
-a^(1/2)*arctanh((a*sec(x)^2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = -\text{arctanh}(\cos(x))\cos(x)\sqrt{a\sec^2(x)}$$

input
```
Integrate[Cot[x]*Sqrt[a + a*Tan[x]^2],x]
```

output
```
-(ArcTanh[Cos[x]]*Cos[x]*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4140, 3042, 4612, 25, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\sqrt{a\tan^2(x)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a\tan(x)^2+a}}{\tan(x)}dx$$

$$\Big\downarrow \ 4140$$

$$\int \cot(x)\sqrt{a\sec^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a\sec(x)^2}}{\tan(x)}dx$$

$$\Big\downarrow \ 4612$$

$$\frac{1}{2}a\int -\frac{1}{\sqrt{a\sec^2(x)}\,(1-\sec^2(x))}d\sec^2(x)$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}a\int \frac{1}{\sqrt{a\sec^2(x)}\,(1-\sec^2(x))}d\sec^2(x)$$

$$\Big\downarrow \ 73$$

$$-\int \frac{1}{1-\frac{\sec^4(x)}{a}}d\sqrt{a\sec^2(x)}$$

$$\Big\downarrow \ 219$$

$$-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)$$

input
```
Int[Cot[x]*Sqrt[a + a*Tan[x]^2],x]
```

output
```
-(Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{a\sec\left(x\right)^{2}}\, \ln\left(-\cot\left(x\right)+\csc\left(x\right)\right) \cos\left(x\right)$	20
risch	$-2\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \ln\left({\mathrm e}^{i x}+1\right) \cos\left(x\right)+2\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \ln\left({\mathrm e}^{i x}-1\right) \cos\left(x\right)$	62

input
```
int(cot(x)*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*sec(x)^2)^(1/2)*ln(-cot(x)+csc(x))*cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx$$
$$= \left[\frac{1}{2}\,\sqrt{a}\log\left(\frac{a\tan\left(x\right)^{2}-2\,\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}+2\,a}{\tan\left(x\right)^{2}}\right),\ \sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a\tan\left(x\right)^{2}+a}}\right)\right]$$

input
```
integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x
)^2), sqrt(-a)*arctan(sqrt(-a)/sqrt(a*tan(x)^2 + a))]
```

Sympy [F]

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\left(\tan^2{(x)}+1\right)}\cot{(x)}\,dx$$

input
```
integrate(cot(x)*(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(tan(x)**2 + 1))*cot(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = -\frac{1}{2}\sqrt{a}\left(\log\left(\cos{(x)}^2+\sin{(x)}^2+2\cos{(x)}+1\right)-\log\left(\cos{(x)}^2+\sin{(x)}^2-2\cos{(x)}+1\right)\right)$$

input
```
integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin
(x)^2 - 2*cos(x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{a\arctan\left(\frac{\sqrt{a\tan{(x)}^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(cot(x)*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
a*arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.50

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = -\sqrt{a}\,\mathrm{atanh}\left(\sqrt{\frac{1}{\cos(x)^2}}\right)$$

input
```
int(cot(x)*(a + a*tan(x)^2)^(1/2),x)
```

output
```
-a^(1/2)*atanh((1/cos(x)^2)^(1/2))
```

Reduce [F]

$$\int \cot(x)\sqrt{a+a\tan^2(x)}\,dx = \sqrt{a}\left(\int \sqrt{\tan(x)^2+1}\,\cot(x)\,dx\right)$$

input
```
int(cot(x)*(a+a*tan(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(x)**2 + 1)*cot(x),x)
```

3.262 $\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx = -\cot(x)\sqrt{a\sec^2(x)}$$

output
```
-cot(x)*(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx = -\cot(x)\sqrt{a\sec^2(x)}$$

input
```
Integrate[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4140, 3042, 4613, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\sqrt{a\tan^2(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\tan(x)^2+a}}{\tan(x)^2}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \cot^2(x)\sqrt{a\sec^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sec(x)^2}}{\tan(x)^2}dx$$

$$\Big\downarrow \text{4613}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int \cot(x)\csc(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int -\sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\cos(x)\sqrt{a\sec^2(x)}\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\cos(x)\sqrt{a\sec^2(x)}\int 1d\csc(x)$$

$$\Big\downarrow \text{24}$$

$$-\cot(x)\sqrt{a\sec^2(x)}$$

input
```
Int[Cot[x]^2*Sqrt[a + a*Tan[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sec[x]^2])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\cot\left(x\right)\sqrt{a\sec\left(x\right)^{2}}$	13
risch	$-\frac{2i\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\left({\mathrm e}^{2 i x}+1\right)}{{\mathrm e}^{2 i x}-1}$	38

input
```
int(cot(x)^2*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cot(x)*(a*sec(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx=-\frac{\sqrt{a\tan\left(x\right)^{2}+a}}{\tan\left(x\right)}$$

input
```
integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*tan(x)^2 + a)/tan(x)
```

Sympy [F]

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx=\int\sqrt{a\left(\tan^{2}\left(x\right)+1\right)}\cot^{2}\left(x\right)dx$$

input
```
integrate(cot(x)**2*(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx = -\frac{\sqrt{\tan(x)^2+1}\sqrt{a}}{\tan(x)}$$

input
```
integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(tan(x)^2 + 1)*sqrt(a)/tan(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{2\,a^{\frac{3}{2}}}{\left(\sqrt{a}\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a}$$

input
```
integrate(cot(x)^2*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*a^(3/2)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 7.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{2\sqrt{a}\,\cos(x)\,\sin(x)}{\sqrt{\cos(x)^2}\,\left(2\cos(x)^2-2\right)}$$

input
```
int(cot(x)^2*(a + a*tan(x)^2)^(1/2),x)
```

output
```
(2*a^(1/2)*cos(x)*sin(x))/((cos(x)^2)^(1/2)*(2*cos(x)^2 - 2))
```

Reduce [F]

$$\int \cot^2(x)\sqrt{a+a\tan^2(x)}\,dx = \sqrt{a}\left(\int \sqrt{\tan(x)^2+1}\,\cot(x)^2\,dx\right)$$

input
```
int(cot(x)^2*(a+a*tan(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(x)**2 + 1)*cot(x)**2,x)
```

3.263 $\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx=\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)-\frac{1}{2}\cot^2(x)\sqrt{a\sec^2(x)}$$

output
```
1/2*a^(1/2)*arctanh((a*sec(x)^2)^(1/2)/a^(1/2))-1/2*cot(x)^2*(a*sec(x)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx=\frac{\frac{a\text{arctanh}\left(\sqrt{\cos^2(x)}\right)}{\sqrt{\cos^2(x)}}-a\csc^2(x)}{2\sqrt{a\sec^2(x)}}$$

input
```
Integrate[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

output
```
((a*ArcTanh[Sqrt[Cos[x]^2]])/Sqrt[Cos[x]^2] - a*Csc[x]^2)/(2*Sqrt[a*Sec[x]
^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 4140, 3042, 4612, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\sqrt{a\tan^2(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\tan(x)^2+a}}{\tan(x)^3}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \cot^3(x)\sqrt{a\sec^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sec(x)^2}}{\tan(x)^3}dx$$

$$\Big\downarrow \text{4612}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{a\sec^2(x)}\,(1-\sec^2(x))^2}d\sec^2(x)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(\frac{1}{2}\int \frac{1}{\sqrt{a\sec^2(x)}\,(1-\sec^2(x))}d\sec^2(x)+\frac{\sqrt{a\sec^2(x)}}{a\,(1-\sec^2(x))}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{1-\frac{\sec^4(x)}{a}}d\sqrt{a\sec^2(x)}}{a}+\frac{\sqrt{a\sec^2(x)}}{a\,(1-\sec^2(x))}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{a\sec^2(x)}}{a\,(1-\sec^2(x))}\right)$$

input
```
Int[Cot[x]^3*Sqrt[a + a*Tan[x]^2],x]
```

output
```
(a*(ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a*Sec[x]^2]/(a*(1 - S
ec[x]^2))))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$\left(-\frac{\ln(-\cot(x)+\csc(x))\cos(x)}{2}-\frac{\cot(x)^{2}}{2}\right)\sqrt{a\sec\left(x\right)^{2}}$	29
risch	$\frac{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}+\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}+1\right)\cos\left(x\right)-\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}-1\right)\cos\left(x\right)$	98

input
```
int(cot(x)^3*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*ln(-cot(x)+csc(x))*cos(x)-1/2*cot(x)^2)*(a*sec(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{\sqrt{a}\log\left(\frac{a\tan(x)^{2}+2\sqrt{a\tan(x)^{2}+a}\sqrt{a}+2\,a}{\tan(x)^{2}}\right)\tan\left(x\right)^{2}-2\sqrt{a\tan\left(x\right)^{2}+a}}{4\,\tan\left(x\right)^{2}}$$

input
```
integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(a)*log((a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x
)^2)*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a))/tan(x)^2
```

Sympy [F]

$$\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\left(\tan^2\left(x\right)+1\right)}\cot^3\left(x\right)dx$$

input
```
integrate(cot(x)**3*(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(33) = 66.

Time = 0.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 6.73

$$\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx = -\frac{\left(4\left(\cos\left(3x\right)+\cos\left(x\right)\right)\cos\left(4x\right)-4\left(2\cos\left(2x\right)-1\right)\cos\left(3x\right)-8\cos\left(2x\right)\cos\left(x\right)-\left(2\left(2\cos\left(2x\right)-\right.\right.\right.}{}$$

input
```
integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos
(2*x)*cos(x) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 -
sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(
x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^
2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos
(2*x) - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))
*sin(4*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))*sqrt(a)/(2
*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*si
n(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \cot^3(x)\sqrt{a + a\tan^2(x)}\,dx = -\frac{1}{2}\,a^2\left(\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{a\tan(x)^2 + a}}{a^2\tan(x)^2}\right)$$

input
```
integrate(cot(x)^3*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a^2*(arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(a*tan(
x)^2 + a)/(a^2*tan(x)^2))
```

Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\sqrt{a + a\tan^2(x)}\,dx = \frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)}{2} - \frac{\sqrt{a\tan(x)^2 + a}}{2\tan(x)^2}$$

input
```
int(cot(x)^3*(a + a*tan(x)^2)^(1/2),x)
```

output
```
(a^(1/2)*atanh((a + a*tan(x)^2)^(1/2)/a^(1/2)))/2 - (a + a*tan(x)^2)^(1/2)
/(2*tan(x)^2)
```

Reduce [F]

$$\int \cot^3(x)\sqrt{a+a\tan^2(x)}\,dx=\sqrt{a}\left(\int\sqrt{\tan(x)^2+1}\,\cot(x)^3\,dx\right)$$

input
```
int(cot(x)^3*(a+a*tan(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(x)**2 + 1)*cot(x)**3,x)
```

3.264 $\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx=\cot(x)\sqrt{a\sec^2(x)}-\frac{1}{3}\cot(x)\csc^2(x)\sqrt{a\sec^2(x)}$$

output
```
cot(x)*(a*sec(x)^2)^(1/2)-1/3*cot(x)*csc(x)^2*(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx=-\frac{1}{3}\cot(x)\left(-3+\csc^2(x)\right)\sqrt{a\sec^2(x)}$$

input
```
Integrate[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

output
```
-1/3*(Cot[x]*(-3 + Csc[x]^2)*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4140, 3042, 4613, 3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(x)\sqrt{a\tan^2(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\tan(x)^2+a}}{\tan(x)^4}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \cot^4(x)\sqrt{a\sec^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sec(x)^2}}{\tan(x)^4}dx$$

$$\Big\downarrow \text{4613}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int \cot^3(x)\csc(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int -\sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\cos(x)\sqrt{a\sec^2(x)}\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\cos(x)\sqrt{a\sec^2(x)}\int \left(\csc^2(x)-1\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$-\cos(x)\left(\frac{\csc^3(x)}{3}-\csc(x)\right)\sqrt{a\sec^2(x)}$$

input
```
Int[Cot[x]^4*Sqrt[a + a*Tan[x]^2],x]
```

output
```
-(Cos[x]*(-Csc[x] + Csc[x]^3/3)*Sqrt[a*Sec[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\sqrt{a\sec(x)^2}\left(3\cot(x)^3-2\cot(x)\csc(x)^2\right)}{3}$	26
risch	$\frac{2i\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,(e^{2ix}+1)\,(3\,e^{4ix}-2\,e^{2ix}+3)}{3(e^{2ix}-1)^3}$	54

input
```
int(cot(x)^4*(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*sec(x)^2)^(1/2)*(3*cot(x)^3-2*cot(x)*csc(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2-1\right)}{3\tan(x)^3}$$

input
```
integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 - 1)/tan(x)^3
```

Sympy [F]

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx = \int \sqrt{a\left(\tan^2(x)+1\right)}\cot^4(x)\,dx$$

input
```
integrate(cot(x)**4*(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*(tan(x)**2 + 1))*cot(x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{\left(2\sqrt{a}\tan\left(x\right)^2-\sqrt{a}\right)\sqrt{\tan\left(x\right)^2+1}}{3\,\tan\left(x\right)^3}$$

input
```
integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(2*sqrt(a)*tan(x)^2 - sqrt(a))*sqrt(tan(x)^2 + 1)/tan(x)^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.55 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{4\left(3\left(\sqrt{a}\tan\left(x\right)-\sqrt{a\tan\left(x\right)^2+a}\right)^2-a\right)a^{\frac{5}{2}}}{3\left(\left(\sqrt{a}\tan\left(x\right)-\sqrt{a\tan\left(x\right)^2+a}\right)^2-a\right)^3}$$

input
```
integrate(cot(x)^4*(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a)*a^(5/2)/((sqrt(a)*ta
n(x) - sqrt(a*tan(x)^2 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx = \frac{\sqrt{2}\,\sqrt{a}\,\left(2\,\sin\left(2\,x\right)-6\,\sin\left(2\,x\right)\,\left(2\cos\left(x\right)^2-1\right)\right)}{24\,\sqrt{2\cos\left(x\right)^2}\,\left(\cos\left(x\right)^2-1\right)^2}$$

input
```
int(cot(x)^4*(a + a*tan(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*a^(1/2)*(2*sin(2*x) - 6*sin(2*x)*(2*cos(x)^2 - 1)))/(24*(2*cos(x)
^2)^(1/2)*(cos(x)^2 - 1)^2)
```

Reduce [F]

$$\int \cot^4(x)\sqrt{a+a\tan^2(x)}\,dx = \sqrt{a}\left(\int \sqrt{\tan\left(x\right)^2+1}\,\cot\left(x\right)^4dx\right)$$

input
```
int(cot(x)^4*(a+a*tan(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(x)**2 + 1)*cot(x)**4,x)
```

3.265 $\int \sqrt{a+a\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx=\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{d}$$

output
```
a^(1/2)*arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx=\frac{\coth^{-1}(\sin(c+dx))\cos(c+dx)\sqrt{a\sec^2(c+dx)}}{d}$$

input
```
Integrate[Sqrt[a + a*Tan[c + d*x]^2],x]
```

output
```
(ArcCoth[Sin[c + d*x]]*Cos[c + d*x]*Sqrt[a*Sec[c + d*x]^2])/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4140, 3042, 4610, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \tan^2(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \tan(c+dx)^2+a}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \sqrt{a \sec^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sec(c+dx)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{a \int \frac{1}{\sqrt{a \tan^2(c+dx)+a}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{a \int \frac{1}{1-\frac{a \tan^2(c+dx)}{a \tan^2(c+dx)+a}}d\frac{\tan(c+dx)}{\sqrt{a \tan^2(c+dx)+a}}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a \tan^2(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Tan[c + d*x]^2],x]
```

output
```
(Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Tan[c + d*x]^2]])/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\tan(dx+c)^{2}}\right)}{d}$	34
default	$\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\tan(dx+c)^{2}}\right)}{d}$	34
risch	$-\frac{2\ln\left(\mathrm{e}^{idx}-i\mathrm{e}^{-ic}\right)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}}\,\cos(dx+c)}{d}+\frac{2\ln\left(\mathrm{e}^{idx}+i\mathrm{e}^{-ic}\right)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}}\,\cos(dx+c)}{d}$	108

input
```
int((a+a*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^(1/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx = \left[\frac{\sqrt{a}\log\left(2\,a\tan\left(dx+c\right)^2+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\sqrt{a}\tan\left(dx+c\right)+a\right)}{2\,d}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\tan(dx+c)}{\sqrt{a\tan(dx+c)^2+a}}\right)}{d}\right]$$

input
```
integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)
*tan(d*x + c) + a)/d, -sqrt(-a)*arctan(sqrt(-a)*tan(d*x + c)/sqrt(a*tan(d*
x + c)^2 + a))/d]
```

Sympy [F]

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx = \int \sqrt{a\tan^2\left(c+dx\right)+a}\,dx$$

input
```
integrate((a+a*tan(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*tan(c + d*x)**2 + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \sqrt{a + a\tan^2(c + dx)}\,dx = \frac{\sqrt{a}\left(\log\left(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\sin(dx + c) + 1\right) - \log\left(\cos(dx + c)^2 + \sin(dx + c)^2 - 2\sin(\right.\right.}{2d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.

Time = 0.80 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \sqrt{a + a\tan^2(c + dx)}\,dx = -\frac{\left(\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 1\right) - \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx\right.\right.\right.}{d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-(log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1) - log
(abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(tan(1/2*d*x + 1/2*c)^4 - 1))*sqrt(a)/d
```

Mupad [B] (verification not implemented)

Time = 7.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx = \begin{cases} 0 & \text{if } a=0 \\ \frac{\sqrt{a}\ln\left(\sqrt{a}\tan(c+dx)+\sqrt{a\tan(c+dx)^2+a}\right)}{d} & \text{if } a\neq 0 \end{cases}$$

input
```
int((a + a*tan(c + d*x)^2)^(1/2),x)
```

output
```
piecewise(a == 0, 0, a ~= 0, (a^(1/2)*log(a^(1/2)*tan(c + d*x) + (a + a*ta
n(c + d*x)^2)^(1/2)))/d)
```

Reduce [F]

$$\int \sqrt{a+a\tan^2(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\tan(dx+c)^2+1}dx\right)$$

input
```
int((a+a*tan(d*x+c)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(tan(c + d*x)**2 + 1),x)
```

3.266 $\int \tan^3(x) \left(a+a\tan^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \tan^3(x) \left(a+a\tan^2(x)\right)^{3/2} dx = -\frac{1}{3}\left(a\sec^2(x)\right)^{3/2}+\frac{\left(a\sec^2(x)\right)^{5/2}}{5a}$$

output
```
-1/3*(a*sec(x)^2)^(3/2)+1/5*(a*sec(x)^2)^(5/2)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \tan^3(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{1}{15}\left(a\sec^2(x)\right)^{3/2}\left(-5+3\sec^2(x)\right)$$

input
```
Integrate[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
((a*Sec[x]^2)^(3/2)*(-5 + 3*Sec[x]^2))/15
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4140, 3042, 4612, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(x) \left(a \tan^2(x)+a\right)^{3/2} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(x)^3 \left(a \tan(x)^2+a\right)^{3/2} dx$$

$$\Big\downarrow \ 4140$$

$$\int \tan^3(x) \left(a \sec^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(x)^3 \left(a \sec(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \ 4612$$

$$\frac{1}{2} a \int -\sqrt{a \sec^2(x)} \left(1-\sec^2(x)\right) d\sec^2(x)$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2} a \int \sqrt{a \sec^2(x)} \left(1-\sec^2(x)\right) d\sec^2(x)$$

$$\Big\downarrow \ 53$$

$$-\frac{1}{2} a \int \left(\sqrt{a \sec^2(x)}-\frac{\left(a \sec^2(x)\right)^{3/2}}{a}\right) d\sec^2(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} a \left(\frac{2 \left(a \sec^2(x)\right)^{5/2}}{5 a^2}-\frac{2 \left(a \sec^2(x)\right)^{3/2}}{3 a}\right)$$

input
```
Int[Tan[x]^3*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(a*((-2*(a*Sec[x]^2)^(3/2))/(3*a) + (2*(a*Sec[x]^2)^(5/2))/(5*a^2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\left(a+a\tan(x)^{2}\right)^{\frac{5}{2}}}{5a}-\frac{\left(a+a\tan(x)^{2}\right)^{\frac{3}{2}}}{3}$	29
default	$\frac{\left(a+a\tan(x)^{2}\right)^{\frac{5}{2}}}{5a}-\frac{\left(a+a\tan(x)^{2}\right)^{\frac{3}{2}}}{3}$	29
risch	$-\frac{8a\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\left(5\,e^{6ix}-2\,e^{4ix}+5\,e^{2ix}\right)}{15\left(e^{2ix}+1\right)^{4}}$	53

input
```
int(tan(x)^3*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/a*(a+a*tan(x)^2)^(5/2)-1/3*(a+a*tan(x)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx=\frac{1}{15}\left(3\,a\tan\left(x\right)^{4}+a\tan\left(x\right)^{2}-2\,a\right)\sqrt{a\tan\left(x\right)^{2}+a}$$

input
```
integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*a*tan(x)^4 + a*tan(x)^2 - 2*a)*sqrt(a*tan(x)^2 + a)
```

Sympy [F]

$$\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx=\int\left(a\left(\tan^2\left(x\right)+1\right)\right)^{\frac{3}{2}}\tan^3\left(x\right)dx$$

input
```
integrate(tan(x)**3*(a+a*tan(x)**2)**(3/2),x)
```

output
```
Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(24) = 48$.

Time = 0.18 (sec) , antiderivative size = 559, normalized size of antiderivative = 17.47

$$\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-8/15*(50*a*cos(4*x)*cos(3*x) + 50*a*sin(4*x)*sin(3*x) + 25*a*sin(3*x)*sin
(2*x) + (5*a*cos(7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(10*x) + 5*(5*a*co
s(7*x) - 2*a*cos(5*x) + 5*a*cos(3*x))*cos(8*x) + 5*(10*a*cos(6*x) + 10*a*c
os(4*x) + 5*a*cos(2*x) + a)*cos(7*x) - 10*(2*a*cos(5*x) - 5*a*cos(3*x))*co
s(6*x) - 2*(10*a*cos(4*x) + 5*a*cos(2*x) + a)*cos(5*x) + 5*(5*a*cos(2*x) +
 a)*cos(3*x) + (5*a*sin(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(10*x) + 5*
(5*a*sin(7*x) - 2*a*sin(5*x) + 5*a*sin(3*x))*sin(8*x) + 25*(2*a*sin(6*x) +
 2*a*sin(4*x) + a*sin(2*x))*sin(7*x) - 10*(2*a*sin(5*x) - 5*a*sin(3*x))*si
n(6*x) - 10*(2*a*sin(4*x) + a*sin(2*x))*sin(5*x))*sqrt(a)/(2*(5*cos(8*x) +
 10*cos(6*x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(10*x) + cos(10*x)^2 + 10*
(10*cos(6*x) + 10*cos(4*x) + 5*cos(2*x) + 1)*cos(8*x) + 25*cos(8*x)^2 + 20
*(10*cos(4*x) + 5*cos(2*x) + 1)*cos(6*x) + 100*cos(6*x)^2 + 20*(5*cos(2*x)
 + 1)*cos(4*x) + 100*cos(4*x)^2 + 25*cos(2*x)^2 + 10*(sin(8*x) + 2*sin(6*x
) + 2*sin(4*x) + sin(2*x))*sin(10*x) + sin(10*x)^2 + 50*(2*sin(6*x) + 2*si
n(4*x) + sin(2*x))*sin(8*x) + 25*sin(8*x)^2 + 100*(2*sin(4*x) + sin(2*x))*
sin(6*x) + 100*sin(6*x)^2 + 100*sin(4*x)^2 + 100*sin(4*x)*sin(2*x) + 25*si
n(2*x)^2 + 10*cos(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(24) = 48$.

Time = 0.51 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \tan^3(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx = \frac{1}{3} \left(a\tan\left(x\right)^2 + a\right)^{\frac{3}{2}} - \sqrt{a\tan\left(x\right)^2 + a}a + \frac{3 \left(a\tan\left(x\right)^2 + a\right)^{\frac{5}{2}} - 10 \left(a\tan\left(x\right)^2 + a\right)^{\frac{3}{2}}a + 15 \sqrt{a\tan\left(x\right)^2 + a}a^2}{15\,a}$$

input
```
integrate(tan(x)^3*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*(a*tan(x)^2 + a)^(3/2) - sqrt(a*tan(x)^2 + a)*a + 1/15*(3*(a*tan(x)^2
+ a)^(5/2) - 10*(a*tan(x)^2 + a)^(3/2)*a + 15*sqrt(a*tan(x)^2 + a)*a^2)/a
```

Mupad [B] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \tan^3(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx = -\frac{2\,\sqrt{2}\,a^{3/2}\,\left(10\cos\left(x\right)^2 - 6\right)}{15\,{\left(2\cos\left(x\right)^2\right)}^{5/2}}$$

input
```
int(tan(x)^3*(a + a*tan(x)^2)^(3/2),x)
```

output
```
-(2*2^(1/2)*a^(3/2)*(10*cos(x)^2 - 6))/(15*(2*cos(x)^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \tan^3(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx=\frac{\sqrt{a}\,\sqrt{\tan(x)^2+1}\,a\left(3\tan(x)^4+\tan(x)^2-2\right)}{15}$$

input
```
int(tan(x)^3*(a+a*tan(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(tan(x)**2 + 1)*a*(3*tan(x)**4 + tan(x)**2 - 2))/15
```

3.267 $\int \tan^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \tan^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx = -\frac{1}{8}a\text{arctanh}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} - \frac{1}{8}a\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{4}a\sec^2(x)\sqrt{a\sec^2(x)}\tan(x)$$

output
```
-1/8*a*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-1/8*a*(a*sec(x)^2)^(1/2)*
tan(x)+1/4*a*sec(x)^2*(a*sec(x)^2)^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

$$\int \tan^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx = \frac{1}{8}\left(a\sec^2(x)\right)^{3/2}\left(-\text{arctanh}(\sin(x))\cos^3(x)-\cos(x)\sin(x)+2\tan(x)\right)$$

input
```
Integrate[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
((a*Sec[x]^2)^(3/2)*(-(ArcTanh[Sin[x]]*Cos[x]^3) - Cos[x]*Sin[x] + 2*Tan[x
]))/8
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 4140, 3042, 4613, 3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x) \left(a\tan^2(x)+a\right)^{3/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2 \left(a\tan(x)^2+a\right)^{3/2} dx$$

$$\Big\downarrow \text{4140}$$

$$\int \tan^2(x) \left(a\sec^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^2 \left(a\sec(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4613}$$

$$a\cos(x)\sqrt{a\sec^2(x)}\int \sec^3(x)\tan^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\cos(x)\sqrt{a\sec^2(x)}\int \sec(x)^3\tan(x)^2dx$$

$$\Big\downarrow \text{3091}$$

$$a\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\tan(x)\sec^3(x)-\frac{1}{4}\int \sec^3(x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\tan(x)\sec^3(x)-\frac{1}{4}\int\csc\left(x+\frac{\pi}{2}\right)^3dx\right)$$

$$\Big\downarrow \textbf{4255}$$

$$a\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\left(-\frac{\int\sec(x)dx}{2}-\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\left(-\frac{1}{2}\int\csc\left(x+\frac{\pi}{2}\right)dx-\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)\right)$$

$$\Big\downarrow \textbf{4257}$$

$$a\cos(x)\sqrt{a\sec^2(x)}\left(\frac{1}{4}\left(-\frac{1}{2}\text{arctanh}(\sin(x))-\frac{1}{2}\tan(x)\sec(x)\right)+\frac{1}{4}\tan(x)\sec^3(x)\right)$$

input
```
Int[Tan[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
a*Cos[x]*Sqrt[a*Sec[x]^2]*((Sec[x]^3*Tan[x])/4 + (-1/2*ArcTanh[Sin[x]] - (
Sec[x]*Tan[x])/2)/4)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	s
derivativedivides	$\frac{\tan(x)\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}{4}-\frac{a\tan(x)\sqrt{a+a\tan(x)^2}}{8}-\frac{a^{\frac{3}{2}}\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^2}\right)}{8}$	5
default	$\frac{\tan(x)\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}{4}-\frac{a\tan(x)\sqrt{a+a\tan(x)^2}}{8}-\frac{a^{\frac{3}{2}}\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^2}\right)}{8}$	5
risch	$\frac{ia\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^2}}\,\left(\mathrm{e}^{6ix}-7\,\mathrm{e}^{4ix}+7\,\mathrm{e}^{2ix}-1\right)}{4\left(\mathrm{e}^{2ix}+1\right)^3}-\frac{a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^2}}\,\ln\left(\mathrm{e}^{ix}+i\right)\cos(x)}{4}+\frac{a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^2}}\,\ln\left(\mathrm{e}^{ix}-i\right)\cos(x)}{4}$	1

input
```
int(tan(x)^2*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)*(a+a*tan(x)^2)^(3/2)-1/8*a*tan(x)*(a+a*tan(x)^2)^(1/2)-1/8*a^(3
/2)*ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \tan^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx = \frac{1}{16} a^{\frac{3}{2}} \log \left(2 a \tan(x)^2 - 2 \sqrt{a \tan(x)^2 + a}\sqrt{a} \tan(x) + a\right) + \frac{1}{8} \left(2 a \tan(x)^3 + a \tan(x)\right) \sqrt{a \tan(x)^2 + a}$$

input
```
integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/16*a^(3/2)*log(2*a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
 + 1/8*(2*a*tan(x)^3 + a*tan(x))*sqrt(a*tan(x)^2 + a)
```

Sympy [F]

$$\int \tan^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx = \int \left(a\left(\tan^2(x) + 1\right)\right)^{\frac{3}{2}} \tan^2(x)\, dx$$

input
```
integrate(tan(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

output
```
Integral((a*(tan(x)**2 + 1))**(3/2)*tan(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 934 vs. 2(47) = 94.

Time = 0.32 (sec) , antiderivative size = 934, normalized size of antiderivative = 15.83

$$\int \tan^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/16*(112*a*cos(3*x)*sin(2*x) - 16*a*cos(x)*sin(2*x) + 16*a*cos(2*x)*sin(x
) - 4*(a*sin(7*x) - 7*a*sin(5*x) + 7*a*sin(3*x) - a*sin(x))*cos(8*x) + 8*(
2*a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*cos(7*x) + 16*(7*a*sin(5*x) -
7*a*sin(3*x) + a*sin(x))*cos(6*x) - 56*(3*a*sin(4*x) + 2*a*sin(2*x))*cos(5
*x) - 24*(7*a*sin(3*x) - a*sin(x))*cos(4*x) - (a*cos(8*x)^2 + 16*a*cos(6*x
)^2 + 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*x)^2 + 16*a*sin(6*x)^2 +
 36*a*sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*sin(2*x)^2 + 2*(4*a*cos(6
*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) + 8*(6*a*cos(4*x) + 4*a*co
s(2*x) + a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4*x) + 8*a*cos(2*x) + 4*(
2*a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*x) + 16*(3*a*sin(4*x) +
2*a*sin(2*x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*c
os(8*x)^2 + 16*a*cos(6*x)^2 + 36*a*cos(4*x)^2 + 16*a*cos(2*x)^2 + a*sin(8*
x)^2 + 16*a*sin(6*x)^2 + 36*a*sin(4*x)^2 + 48*a*sin(4*x)*sin(2*x) + 16*a*s
in(2*x)^2 + 2*(4*a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(8*x) +
8*(6*a*cos(4*x) + 4*a*cos(2*x) + a)*cos(6*x) + 12*(4*a*cos(2*x) + a)*cos(4
*x) + 8*a*cos(2*x) + 4*(2*a*sin(6*x) + 3*a*sin(4*x) + 2*a*sin(2*x))*sin(8*
x) + 16*(3*a*sin(4*x) + 2*a*sin(2*x))*sin(6*x) + a)*log(cos(x)^2 + sin(x)^
2 - 2*sin(x) + 1) + 4*(a*cos(7*x) - 7*a*cos(5*x) + 7*a*cos(3*x) - a*cos(x)
)*sin(8*x) - 4*(4*a*cos(6*x) + 6*a*cos(4*x) + 4*a*cos(2*x) + a)*sin(7*x) -
 16*(7*a*cos(5*x) - 7*a*cos(3*x) + a*cos(x))*sin(6*x) + 28*(6*a*cos(4*x...
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \tan^2(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{1}{8} \left(\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2+1\right)\tan(x)+\sqrt{a}\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+\dots}\right.\right.\right.$$

input
```
integrate(tan(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)*tan(x) + sqrt(a)*log(abs(-sqrt(
a)*tan(x) + sqrt(a*tan(x)^2 + a))))*a
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx=\int \tan(x)^2 \left(a\tan(x)^2+a\right)^{3/2} dx$$

input
```
int(tan(x)^2*(a + a*tan(x)^2)^(3/2),x)
```

output
```
int(tan(x)^2*(a + a*tan(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \tan^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx=\sqrt{a}\, a\left(\int \sqrt{\tan(x)^2+1}\,\tan(x)^4dx+\int \sqrt{\tan(x)^2+1}\,\tan(x)^2dx\right)$$

input
```
int(tan(x)^2*(a+a*tan(x)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(x)**2 + 1)*tan(x)**4,x) + int(sqrt(tan(x)**2 + 1)*
tan(x)**2,x))
```

3.268 $\int \tan(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \tan(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx = \frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

output
```
1/3*(a*sec(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \tan(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx = \frac{1}{3}\left(a\sec^2(x)\right)^{3/2}$$

input
```
Integrate[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(a*Sec[x]^2)^(3/2)/3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4140, 3042, 4612, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x) \left(a \tan^2(x)+a\right)^{3/2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(x) \left(a \tan(x)^2+a\right)^{3/2} dx$$

$$\Big\downarrow \; 4140$$

$$\int \tan(x) \left(a \sec^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(x) \left(a \sec(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \; 4612$$

$$\frac{1}{2} a \int \sqrt{a \sec^2(x)} d \sec^2(x)$$

$$\Big\downarrow \; 17$$

$$\frac{1}{3} \left(a \sec^2(x)\right)^{3/2}$$

input
```
Int[Tan[x]*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(a*Sec[x]^2)^(3/2)/3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\left(a+a\tan\left(x\right)^{2}\right)^{\frac{3}{2}}}{3}$	13
default	$\frac{\left(a+a\tan\left(x\right)^{2}\right)^{\frac{3}{2}}}{3}$	13
risch	$\frac{8a\,{\rm e}^{2ix}\sqrt{\frac{a\,{\rm e}^{2ix}}{\left({\rm e}^{2ix}+1\right)^{2}}}}{3\left({\rm e}^{2ix}+1\right)^{2}}$	36

input
```
int(tan(x)*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a+a*tan(x)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{1}{3} \left(a\tan\left(x\right)^2 + a\right)^{\frac{3}{2}}$$

input
```
integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*tan(x)^2 + a)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{\left(a\tan^2\left(x\right) + a\right)^{\frac{3}{2}}}{3}$$

input
```
integrate(tan(x)*(a+a*tan(x)**2)**(3/2),x)
```

output
```
(a*tan(x)**2 + a)**(3/2)/3
```

Maxima [F]

$$\int \tan(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \int \left(a\tan\left(x\right)^2 + a\right)^{\frac{3}{2}} \tan\left(x\right)\, dx$$

input
```
integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*tan(x)^2 + a)^(3/2)*tan(x), x)
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{1}{3} \left(a\tan{(x)}^2+a\right)^{\frac{3}{2}}$$

input
```
integrate(tan(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*(a*tan(x)^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \tan(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{a^{3/2}}{3\left(\cos{(x)}^2\right)^{3/2}}$$

input
```
int(tan(x)*(a + a*tan(x)^2)^(3/2),x)
```

output
```
a^(3/2)/(3*(cos(x)^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \tan(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{\sqrt{a}\,\sqrt{\tan{(x)}^2+1}\,a\left(\tan{(x)}^2+1\right)}{3}$$

input
```
int(tan(x)*(a+a*tan(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(tan(x)**2 + 1)*a*(tan(x)**2 + 1))/3
```

3.269 $\int \cot(x) \left(a+a\tan^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \cot(x) \left(a+a\tan^2(x)\right)^{3/2} dx = -a^{3/2}\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)+a\sqrt{a\sec^2(x)}$$

output
```
-a^(3/2)*arctanh((a*sec(x)^2)^(1/2)/a^(1/2))+a*(a*sec(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \cot(x) \left(a+a\tan^2(x)\right)^{3/2} dx = a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\cos^2(x)\right)\sqrt{a\sec^2(x)}$$

input
```
Integrate[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
a*Hypergeometric2F1[-1/2, 1, 1/2, Cos[x]^2]*Sqrt[a*Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 4140, 3042, 4612, 25, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x) \left(a \tan^2(x)+a\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a \tan(x)^2+a\right)^{3/2}}{\tan(x)}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \cot(x) \left(a \sec^2(x)\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a \sec(x)^2\right)^{3/2}}{\tan(x)}dx$$

$$\Big\downarrow \text{ 4612}$$

$$\frac{1}{2} a \int -\frac{\sqrt{a \sec^2(x)}}{1-\sec^2(x)}d\sec^2(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2} a \int \frac{\sqrt{a \sec^2(x)}}{1-\sec^2(x)}d\sec^2(x)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2} a \left(2 \sqrt{a \sec^2(x)}-a \int \frac{1}{\sqrt{a \sec^2(x)} \left(1-\sec^2(x)\right)}d\sec^2(x)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2} a \left(2 \sqrt{a \sec^2(x)}-2 \int \frac{1}{1-\frac{\sec^4(x)}{a}}d\sqrt{a \sec^2(x)}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}a\left(2\sqrt{a\sec^2(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)\right)$$

input
```
Int[Cot[x]*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(a*(-2*Sqrt[a]*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]] + 2*Sqrt[a*Sec[x]^2]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

method	result	s
derivativedivides	$-a^{\frac{3}{2}} \ln \left(\frac{2a+2\sqrt{a}\,\sqrt{a+a\tan(x)^{2}}}{\tan(x)}\right)+a\sqrt{a+a\tan\left(x\right)^{2}}$	4
default	$-a^{\frac{3}{2}} \ln \left(\frac{2a+2\sqrt{a}\,\sqrt{a+a\tan(x)^{2}}}{\tan(x)}\right)+a\sqrt{a+a\tan\left(x\right)^{2}}$	4
risch	$2a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}+2a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}\,\ln\left(\mathrm{e}^{ix}-1\right)\cos\left(x\right)-2a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}\,\ln\left(\mathrm{e}^{ix}+1\right)\cos\left(x\right)$	8

input
```
int(cot(x)*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a^(3/2)*ln((2*a+2*a^(1/2)*(a+a*tan(x)^2)^(1/2))/tan(x))+a*(a+a*tan(x)^2)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \cot(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx = \frac{1}{2}\,a^{\frac{3}{2}}\log\left(\frac{a\tan\left(x\right)^{2}-2\,\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}+2\,a}{\tan\left(x\right)^{2}}\right)+\sqrt{a\tan\left(x\right)^{2}+a}a$$

input
```
integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*a^(3/2)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(a) + 2*a)/tan(x)
^2) + sqrt(a*tan(x)^2 + a)*a
```

Sympy [F]

$$\int \cot(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx = \int \left(a\left(\tan^{2}\left(x\right)+1\right)\right)^{\frac{3}{2}}\cot\left(x\right)dx$$

input
```
integrate(cot(x)*(a+a*tan(x)**2)**(3/2),x)
```

output
```
Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(29) = 58.

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.62

$$\int \cot(x)\left(a+a\tan^2(x)\right)^{3/2}\,dx = \frac{\left(4\,a\cos\left(2\,x\right)\cos\left(x\right)+4\,a\sin\left(2\,x\right)\sin\left(x\right)+4\,a\cos\left(x\right)-\left(a\cos\left(2\,x\right)^{2}+a\sin\left(2\,x\right)^{2}+\right.\right.}{}$$

input
```
integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(4*a*cos(2*x)*cos(x) + 4*a*sin(2*x)*sin(x) + 4*a*cos(x) - (a*cos(2*x)^
2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) +
1) + (a*cos(2*x)^2 + a*sin(2*x)^2 + 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x
)^2 - 2*cos(x) + 1))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \cot(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx = a^2 \left(\frac{\arctan\left(\frac{\sqrt{a\tan(x)^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{\sqrt{a\tan(x)^2 + a}}{a} \right)$$

input
```
integrate(cot(x)*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
a^2*(arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(a*tan(x)^2 + a)
/a)
```

Mupad [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \cot(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx = a\sqrt{a\tan(x)^2 + a} - a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a\tan(x)^2 + a}}{\sqrt{a}}\right)$$

input
```
int(cot(x)*(a + a*tan(x)^2)^(3/2),x)
```

output
```
a*(a + a*tan(x)^2)^(1/2) - a^(3/2)*atanh((a + a*tan(x)^2)^(1/2)/a^(1/2))
```

Reduce [F]

$$\int \cot(x) \left(a + a\tan^2(x)\right)^{3/2} \, dx = \sqrt{a}\, a \left(\int \sqrt{\tan(x)^2 + 1}\, \cot(x) \tan(x)^2 \, dx + \int \sqrt{\tan(x)^2 + 1}\, \cot(x) \, dx \right)$$

input
```
int(cot(x)*(a+a*tan(x)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(x)**2 + 1)*cot(x)*tan(x)**2,x) + int(sqrt(tan(x)**
2 + 1)*cot(x),x))
```

3.270 $\int \cot^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \cot^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx = a\text{arctanh}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} - a\cot(x)\sqrt{a\sec^2(x)}$$

output
```
a*arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)-a*cot(x)*(a*sec(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cot^2(x) \left(a+a\tan^2(x)\right)^{3/2} \, dx = -a\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(x)\right)\sqrt{a\sec^2(x)}$$

input
```
Integrate[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
-(a*Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[x]^2]*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 4140, 3042, 4613, 3042, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x) \left(a \tan^2(x)+a\right)^{3/2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a \tan(x)^2+a\right)^{3/2}}{\tan(x)^2}dx$$

$$\Big\downarrow \; 4140$$

$$\int \cot^2(x) \left(a \sec^2(x)\right)^{3/2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a \sec(x)^2\right)^{3/2}}{\tan(x)^2}dx$$

$$\Big\downarrow \; 4613$$

$$a \cos(x) \sqrt{a \sec^2(x)} \int \csc^2(x) \sec(x)dx$$

$$\Big\downarrow \; 3042$$

$$a \cos(x) \sqrt{a \sec^2(x)} \int \csc(x)^2 \sec(x)dx$$

$$\Big\downarrow \; 3101$$

$$-a \cos(x) \sqrt{a \sec^2(x)} \int -\frac{\csc^2(x)}{1-\csc^2(x)}d\csc(x)$$

$$\Big\downarrow \; 25$$

$$a \cos(x) \sqrt{a \sec^2(x)} \int \frac{\csc^2(x)}{1-\csc^2(x)}d\csc(x)$$

$$\Big\downarrow \; 262$$

$$-a\cos(x)\sqrt{a\sec^2(x)}\left(\csc(x)-\int\frac{1}{1-\csc^2(x)}d\csc(x)\right)$$

$$\Big\downarrow \text{219}$$

$$-a\cos(x)\sqrt{a\sec^2(x)}(\csc(x)-\operatorname{arctanh}(\csc(x)))$$

input
```
Int[Cot[x]^2*(a + a*Tan[x]^2)^(3/2),x]
```

output
```
-(a*Cos[x]*(-ArcTanh[Csc[x]] + Csc[x])*Sqrt[a*Sec[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

method	result
derivativedivides	$-\frac{\left(a+a\tan\left(x\right)^{2}\right)^{\frac{3}{2}}}{\tan\left(x\right)}+a\tan\left(x\right)\sqrt{a+a\tan\left(x\right)^{2}}+a^{\frac{3}{2}}\ln\left(\sqrt{a}\,\tan\left(x\right)+\sqrt{a+a\tan\left(x\right)^{2}}\right)$
default	$-\frac{\left(a+a\tan\left(x\right)^{2}\right)^{\frac{3}{2}}}{\tan\left(x\right)}+a\tan\left(x\right)\sqrt{a+a\tan\left(x\right)^{2}}+a^{\frac{3}{2}}\ln\left(\sqrt{a}\,\tan\left(x\right)+\sqrt{a+a\tan\left(x\right)^{2}}\right)$
risch	$-\frac{2ia\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}}{e^{2ix}-1}+2a\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}+i\right)\cos\left(x\right)-2a\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}-i\right)\cos$

input
```
int(cot(x)^2*(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/tan(x)*(a+a*tan(x)^2)^(3/2)+a*tan(x)*(a+a*tan(x)^2)^(1/2)+a^(3/2)*ln(a^
(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \cot^2(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \frac{a^{\frac{3}{2}} \log \left(2\,a\tan(x)^2 + 2\,\sqrt{a\tan(x)^2+a}\sqrt{a}\tan(x)+a\right)\tan(x) - 2\,\sqrt{a\tan(x)^2+a}\,a}{2\,\tan(x)}$$

input
```
integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(a^(3/2)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt(a)*tan(x) + a)
*tan(x) - 2*sqrt(a*tan(x)^2 + a)*a)/tan(x)
```

Sympy [F]

$$\int \cot^2(x) \left(a+a\tan^2(x)\right)^{3/2} dx = \int \left(a\left(\tan^2(x)+1\right)\right)^{\frac{3}{2}} \cot^2(x)\, dx$$

input
```
integrate(cot(x)**2*(a+a*tan(x)**2)**(3/2),x)
```

output
```
Integral((a*(tan(x)**2 + 1))**(3/2)*cot(x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(29) = 58.

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.06

$$\int \cot^2(x) \left(a+a\tan^2(x)\right)^{3/2} dx = -\frac{\left(4\,a\cos(x)\sin(2\,x) - 4\,a\cos(2\,x)\sin(x) - \left(a\cos(2\,x)^2 + a\sin(2\,x)^2 - 2\,a\cos(2\,x) + a\right)\log\left(\cos(x)^2 + \cdots\right.}{2\left(\cos(2 \cdots\right.}$$

input
```
integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(4*a*cos(x)*sin(2*x) - 4*a*cos(2*x)*sin(x) - (a*cos(2*x)^2 + a*sin(2*
x)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(
2*x)^2 + a*sin(2*x)^2 - 2*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 - 2*sin(
x) + 1) + 4*a*sin(x))*sqrt(a)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.

Time = 0.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int \cot^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx =$$
$$-\frac{1}{2}\left(\sqrt{a}\log\left(\left(\sqrt{a}\tan(x) - \sqrt{a\tan(x)^2 + a}\right)^2\right) - \frac{4\,a^{\frac{3}{2}}}{\left(\sqrt{a}\tan(x) - \sqrt{a\tan(x)^2 + a}\right)^2 - a}\right)a$$

input
```
integrate(cot(x)^2*(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(a)*log((sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2) - 4*a^(3/2)/(
(sqrt(a)*tan(x) - sqrt(a*tan(x)^2 + a))^2 - a))*a
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx = \int \cot(x)^2 \left(a\tan(x)^2 + a\right)^{3/2} dx$$

input
```
int(cot(x)^2*(a + a*tan(x)^2)^(3/2),x)
```

output
```
int(cot(x)^2*(a + a*tan(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^2(x) \left(a + a\tan^2(x)\right)^{3/2} dx = \sqrt{a}\, a\left(\int \sqrt{\tan(x)^2 + 1}\cot(x)^2 \tan(x)^2 dx + \int \sqrt{\tan(x)^2 + 1}\cot(x)^2 dx\right)$$

input
```
int(cot(x)^2*(a+a*tan(x)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(x)**2 + 1)*cot(x)**2*tan(x)**2,x) + int(sqrt(tan(x
)**2 + 1)*cot(x)**2,x))
```

3.271 $\int (a + a\tan^2(c + dx))^{3/2}\, dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int (a+a\tan^2(c+dx))^{3/2}\, dx = \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{2d} + \frac{a\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{2d}$$

output
```
1/2*a^(3/2)*arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))/d+1/2*a*(a*
sec(d*x+c)^2)^(1/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int (a + a\tan^2(c + dx))^{3/2}\, dx = \frac{a\sqrt{a\sec^2(c+dx)}(\text{arctanh}(\sin(c+dx))\cos(c+dx) + \tan(c+dx))}{2d}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

output
```
(a*Sqrt[a*Sec[c + d*x]^2]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + Tan[c + d*
x]))/(2*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4140, 3042, 4610, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\tan^2(c+dx)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\tan(c+dx)^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \left(a\sec^2(c+dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(c+dx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{a\int \sqrt{a\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{211}$$

$$\frac{a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a\tan^2(c+dx)+a}}d\tan(c+dx)+\frac{1}{2}\tan(c+dx)\sqrt{a\tan^2(c+dx)+a}\right)}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{a\left(\frac{1}{2}a\int \frac{1}{1-\frac{a\tan^2(c+dx)}{a\tan^2(c+dx)+a}}d\frac{\tan(c+dx)}{\sqrt{a\tan^2(c+dx)+a}}+\frac{1}{2}\tan(c+dx)\sqrt{a\tan^2(c+dx)+a}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\left(\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\tan^2(c+dx)+a}}\right)+\frac{1}{2}\tan(c+dx)\sqrt{a\tan^2(c+dx)+a}\right)}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(3/2),x]
```

output
```
(a*((Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Tan[c + d*x]^2]])/2
 + (Tan[c + d*x]*Sqrt[a + a*Tan[c + d*x]^2])/2))/d
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)\sqrt{a+a\tan(dx+c)^2}}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\tan(dx+c)^2}\right)}{2}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)\sqrt{a+a\tan(dx+c)^2}}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\tan(dx+c)^2}\right)}{2}\right)}{d}$
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)d}-\frac{\ln\left(e^{idx}-ie^{-ic}\right)\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\,a\cos(dx+c)}{d}+\frac{\ln\left(e^{idx}+ie^{-ic}\right)\sqrt{\frac{a\,e^{2i}}{\left(e^{2i(d}\right.}}}{d}$

input
```
int((a+a*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(1/2*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(1/2)+1/2*a^(1/2)*ln(a^(1/2)*tan(
d*x+c)+(a+a*tan(d*x+c)^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int (a+a\tan^2(c+dx))^{3/2}\,dx=\frac{a^{\frac{3}{2}}\log\left(2\,a\tan\left(dx+c\right)^2+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\sqrt{a}\tan\left(dx+c\right)+a\right)+2\,\sqrt{a\tan\left(dx+c\right.}}{4\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(a^(3/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt(a)
*tan(d*x + c) + a) + 2*sqrt(a*tan(d*x + c)^2 + a)*a*tan(d*x + c))/d
```

Sympy [F]

$$\int \left(a+a\tan^2(c+dx)\right)^{3/2} dx = \int \left(a\tan^2(c+dx)+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+a*tan(d*x+c)**2)**(3/2),x)
```

output
```
Integral((a*tan(c + d*x)**2 + a)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(56) = 112$.

Time = 0.16 (sec) , antiderivative size = 556, normalized size of antiderivative = 8.18

$$\int \left(a+a\tan^2(c+dx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(8*a*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 8*a*cos(d*x + c)*sin(2*d*x +
 2*c) + 8*a*cos(2*d*x + 2*c)*sin(d*x + c) - 4*(a*sin(3*d*x + 3*c) - a*sin(
d*x + c))*cos(4*d*x + 4*c) - (a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^
2 + a*sin(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin
(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(
2*d*x + 2*c) + a)*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1
) + (a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2
 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*
cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*(a*cos(3*d*x + 3*c)
 - a*cos(d*x + c))*sin(4*d*x + 4*c) - 4*(2*a*cos(2*d*x + 2*c) + a)*sin(3*d
*x + 3*c) + 4*a*sin(d*x + c))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*
x + 2*c) + 1)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(56) = 112$.

Time = 0.96 (sec) , antiderivative size = 533, normalized size of antiderivative = 7.84

$$\int \left(a + a\tan^2(c + dx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
(a*tan(c)^2 + a)*(a*arctan(-((sqrt(a*tan(c)^2 + a)*tan(d*x) - sqrt(a*tan(d
*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a))*tan(c) - sqrt(a*tan(c)^2
+ a))/(sqrt(-a)*tan(c)^2 + sqrt(-a)))*sgn(tan(d*x)*tan(c) - 1)/((tan(c)^2
+ 1)*sqrt(-a)) + ((a*tan(c)^3 + 2*a*tan(c))*(sqrt(a*tan(c)^2 + a)*tan(d*x)
 - sqrt(a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a))^3*sgn(tan(
d*x)*tan(c) - 1) + sqrt(a*tan(c)^2 + a)*(a*tan(c)^2 - 2*a)*(sqrt(a*tan(c)^
2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 +
 a))^2*sgn(tan(d*x)*tan(c) - 1) + (a^2*tan(c)^5 - a^2*tan(c)^3 - 2*a^2*tan
(c))*(sqrt(a*tan(c)^2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*tan(c)^2 + a*tan(d
*x)^2 + a*tan(c)^2 + a))*sgn(tan(d*x)*tan(c) - 1) - (a^2*tan(c)^4 + a^2*ta
n(c)^2)*sqrt(a*tan(c)^2 + a)*sgn(tan(d*x)*tan(c) - 1))/((a*tan(c)^3 - (sqr
t(a*tan(c)^2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a
*tan(c)^2 + a))^2*tan(c) + a*tan(c) + 2*sqrt(a*tan(c)^2 + a)*(sqrt(a*tan(c
)^2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2
 + a)))^2*tan(c)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + a\tan^2(c + dx)\right)^{3/2} dx = \int \left(a\tan(c + d\,x)^2 + a\right)^{3/2} dx$$

input
```
int((a + a*tan(c + d*x)^2)^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)^2)^(3/2), x)
```

Reduce [F]

$$\int (a + a\tan^2(c + dx))^{3/2}\, dx = \sqrt{a}\, a\left(\int \sqrt{\tan (dx + c)^2 + 1}dx + \int \sqrt{\tan (dx + c)^2 + 1}\, \tan (dx + c)^2\, dx\right)$$

input
```
int((a+a*tan(d*x+c)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)**2 + 1),x) + int(sqrt(tan(c + d*x)**2 + 1
)*tan(c + d*x)**2,x))
```

3.272 $\int (a+a\tan^2(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 98

$$\int (a+a\tan^2(c+dx))^{5/2}\,dx=\frac{3a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec^2(c+dx)}}\right)}{8d}+\frac{3a^2\sqrt{a\sec^2(c+dx)}\tan(c+dx)}{8d}+\frac{a(a\sec^2(c+dx))^{3/2}\tan(c+dx)}{4d}$$

output
```
3/8*a^(5/2)*arctanh(a^(1/2)*tan(d*x+c)/(a*sec(d*x+c)^2)^(1/2))/d+3/8*a^2*(
a*sec(d*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a*(a*sec(d*x+c)^2)^(3/2)*tan(d*x+c)
/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

$$\int (a+a\tan^2(c+dx))^{5/2}\,dx=\frac{a^2\sqrt{a\sec^2(c+dx)}(3\text{arctanh}(\sin(c+dx))\cos(c+dx)+(3+2\sec^2(c+dx))\tan(c+dx))}{8d}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Sec[c + d*x]^2]*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + (3 + 2
*Sec[c + d*x]^2)*Tan[c + d*x]))/(8*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4140, 3042, 4610, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\tan^2(c+dx)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\tan(c+dx)^2+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \left(a\sec^2(c+dx)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(c+dx)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{a\int \left(a\tan^2(c+dx)+a\right)^{3/2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{211}$$

$$\frac{a\left(\frac{3}{4}a\int \sqrt{a\tan^2(c+dx)+a}d\tan(c+dx)+\frac{1}{4}\tan(c+dx)\left(a\tan^2(c+dx)+a\right)^{3/2}\right)}{d}$$

$$\Big\downarrow \text{211}$$

$$\frac{a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a\tan^2(c+dx)+a}}d\tan(c+dx)+\frac{1}{2}\tan(c+dx)\sqrt{a\tan^2(c+dx)+a}\right)+\frac{1}{4}\tan(c+dx)\left(a\tan^2(c+dx)\right.}{d}$$

$$\Big\downarrow \ 224$$

$$\frac{a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{a\tan^2(c+dx)}{a\tan^2(c+dx)+a}}d\frac{\tan(c+dx)}{\sqrt{a\tan^2(c+dx)+a}}+\frac{1}{2}\tan(c+dx)\sqrt{a\tan^2(c+dx)+a}\right)+\frac{1}{4}\tan(c+dx)\left(a\tan^2(c+\right.}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{a\left(\frac{3}{4}a\left(\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\tan^2(c+dx)+a}}\right)+\frac{1}{2}\tan(c+dx)\sqrt{a\tan^2(c+dx)+a}\right)+\frac{1}{4}\tan(c+dx)\left(a\tan^2(c+dx)+a\right.}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(5/2),x]
```

output
```
(a*((Tan[c + d*x]*(a + a*Tan[c + d*x]^2)^(3/2))/4 + (3*a*((Sqrt[a]*ArcTanh
[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Tan[c + d*x]^2]])/2 + (Tan[c + d*x]*Sqr
t[a + a*Tan[c + d*x]^2])/2))/4))/d
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a\tan(dx+c)\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}{4d}+\frac{3a^2\tan(dx+c)\sqrt{a+a\tan(dx+c)^2}}{8d}+\frac{3a^{\frac{5}{2}}\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\tan(dx+}\right.}{8d}$
default	$\frac{a\tan(dx+c)\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}{4d}+\frac{3a^2\tan(dx+c)\sqrt{a+a\tan(dx+c)^2}}{8d}+\frac{3a^{\frac{5}{2}}\ln\left(\sqrt{a}\,\tan(dx+c)+\sqrt{a+a\tan(dx+}\right.}{8d}$
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3\right)}{4\left(e^{2i(dx+c)}+1\right)^3d}-\frac{3\ln\left(e^{idx}-ie^{-ic}\right)\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\,a^2\cos(dx}{4d}$

input
```
int((a+a*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*a*tan(d*x+c)*(a+a*tan(d*x+c)^2)^(3/2)+3/8/d*a^2*tan(d*x+c)*(a+a*tan(
d*x+c)^2)^(1/2)+3/8/d*a^(5/2)*ln(a^(1/2)*tan(d*x+c)+(a+a*tan(d*x+c)^2)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int (a+a\tan^2(c+dx))^{5/2}\,dx=\frac{3\,a^{\frac{5}{2}}\log\left(2\,a\tan\left(dx+c\right)^2+2\,\sqrt{a\tan\left(dx+c\right)^2+a}\sqrt{a}\tan\left(dx+c\right)+a\right)+2\,\left(2\,a^2\tan\left(dx\right.\right.}{16\,d}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*(3*a^(5/2)*log(2*a*tan(d*x + c)^2 + 2*sqrt(a*tan(d*x + c)^2 + a)*sqrt
(a)*tan(d*x + c) + a) + 2*(2*a^2*tan(d*x + c)^3 + 5*a^2*tan(d*x + c))*sqrt
(a*tan(d*x + c)^2 + a))/d
```

Sympy [F]

$$\int \left(a + a\tan^2(c + dx)\right)^{5/2} dx = \int \left(a\tan^2(c + dx) + a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+a*tan(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a*tan(c + d*x)**2 + a)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(82) = 164$.

Time = 0.31 (sec) , antiderivative size = 1769, normalized size of antiderivative = 18.05

$$\int \left(a + a\tan^2(c + dx)\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*(176*a^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 48*a^2*cos(d*x + c)*sin(
2*d*x + 2*c) - 48*a^2*cos(2*d*x + 2*c)*sin(d*x + c) - 12*a^2*sin(d*x + c)
+ 4*(3*a^2*sin(7*d*x + 7*c) + 11*a^2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x +
 3*c) - 3*a^2*sin(d*x + c))*cos(8*d*x + 8*c) - 24*(2*a^2*sin(6*d*x + 6*c)
+ 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 16*(
11*a^2*sin(5*d*x + 5*c) - 11*a^2*sin(3*d*x + 3*c) - 3*a^2*sin(d*x + c))*co
s(6*d*x + 6*c) - 88*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*cos(
5*d*x + 5*c) - 24*(11*a^2*sin(3*d*x + 3*c) + 3*a^2*sin(d*x + c))*cos(4*d*x
 + 4*c) + 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*c
os(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 1
6*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c)
 + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*
d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos
(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*
cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*
a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^
2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2
+ 2*sin(d*x + c) + 1) - 3*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c
)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. $2(82) = 164$.

Time = 1.55 (sec) , antiderivative size = 1153, normalized size of antiderivative = 11.77

$$\int \left(a + a\tan^2(c + dx)\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
1/4*(3*(a^3*tan(c)^2 + a^3)*arctan(-((sqrt(a*tan(c)^2 + a)*tan(d*x) - sqrt
(a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a))*tan(c) - sqrt(a*t
an(c)^2 + a))/(sqrt(-a)*tan(c)^2 + sqrt(-a)))*sgn(tan(d*x)*tan(c) - 1)/((t
an(c)^2 + 1)*sqrt(-a)) + ((5*a^3*tan(c)^9 + 21*a^3*tan(c)^7 + 24*a^3*tan(c
)^5 + 8*a^3*tan(c)^3)*(sqrt(a*tan(c)^2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*t
an(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a))^7*sgn(tan(d*x)*tan(c) - 1) - 3*(
a^3*tan(c)^8 + 17*a^3*tan(c)^6 + 24*a^3*tan(c)^4 + 8*a^3*tan(c)^2)*sqrt(a*
tan(c)^2 + a)*(sqrt(a*tan(c)^2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*tan(c)^2
+ a*tan(d*x)^2 + a*tan(c)^2 + a))^6*sgn(tan(d*x)*tan(c) - 1) + (3*a^4*tan(
c)^11 - 30*a^4*tan(c)^9 - 13*a^4*tan(c)^7 + 108*a^4*tan(c)^5 + 120*a^4*tan
(c)^3 + 32*a^4*tan(c))*(sqrt(a*tan(c)^2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*
tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a))^5*sgn(tan(d*x)*tan(c) - 1) - (1
5*a^4*tan(c)^10 - 84*a^4*tan(c)^8 - 221*a^4*tan(c)^6 - 114*a^4*tan(c)^4 +
24*a^4*tan(c)^2 + 16*a^4)*sqrt(a*tan(c)^2 + a)*(sqrt(a*tan(c)^2 + a)*tan(d
*x) - sqrt(a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a))^4*sgn(t
an(d*x)*tan(c) - 1) + (3*a^5*tan(c)^13 + 93*a^5*tan(c)^11 + 205*a^5*tan(c)
^9 + 55*a^5*tan(c)^7 - 180*a^5*tan(c)^5 - 152*a^5*tan(c)^3 - 32*a^5*tan(c)
)*(sqrt(a*tan(c)^2 + a)*tan(d*x) - sqrt(a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)
^2 + a*tan(c)^2 + a))^3*sgn(tan(d*x)*tan(c) - 1) + (23*a^5*tan(c)^12 + 5*a
^5*tan(c)^10 - 147*a^5*tan(c)^8 - 241*a^5*tan(c)^6 - 136*a^5*tan(c)^4 -...
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+a\tan^2(c+dx)\right)^{5/2}\,dx=\int \left(a\tan(c+d\,x)^2+a\right)^{5/2}\,dx$$

input
```
int((a + a*tan(c + d*x)^2)^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a + a\tan^2(c + dx)\right)^{5/2} dx = \sqrt{a}\,a^2\left(\int \sqrt{\tan(dx + c)^2 + 1}dx + \int \sqrt{\tan(dx + c)^2 + 1}\,\tan(dx + c)^4\,dx + 2\left(\int \sqrt{\tan(dx + c)^2 + 1}\,\tan(dx + c)^2\,dx\right)\right)$$

input
```
int((a+a*tan(d*x+c)^2)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(tan(c + d*x)**2 + 1),x) + int(sqrt(tan(c + d*x)**2
+ 1)*tan(c + d*x)**4,x) + 2*int(sqrt(tan(c + d*x)**2 + 1)*tan(c + d*x)**2,
x))
```

3.273 $\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx=\frac{1}{\sqrt{a\sec^2(x)}}+\frac{\sqrt{a\sec^2(x)}}{a}$$

output
```
1/(a*sec(x)^2)^(1/2)+(a*sec(x)^2)^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx=\frac{(3+\cos(2x))\sqrt{a\sec^2(x)}}{2a}$$

input
```
Integrate[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
```

output
```
((3 + Cos[2*x])*Sqrt[a*Sec[x]^2])/(2*a)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4140, 3042, 4612, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(x)}{\sqrt{a\tan^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^3}{\sqrt{a\tan(x)^2+a}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\tan^3(x)}{\sqrt{a\sec^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^3}{\sqrt{a\sec(x)^2}}dx$$

$$\Big\downarrow \text{4612}$$

$$\frac{1}{2}a\int -\frac{1-\sec^2(x)}{(a\sec^2(x))^{3/2}}d\sec^2(x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}a\int \frac{1-\sec^2(x)}{(a\sec^2(x))^{3/2}}d\sec^2(x)$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}a\int \left(\frac{1}{(a\sec^2(x))^{3/2}}-\frac{1}{a\sqrt{a\sec^2(x)}}\right)d\sec^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a\left(\frac{2\sqrt{a\sec^2(x)}}{a^2}+\frac{2}{a\sqrt{a\sec^2(x)}}\right)$$

input
```
Int[Tan[x]^3/Sqrt[a + a*Tan[x]^2],x]
```

output
```
(a*(2/(a*Sqrt[a*Sec[x]^2]) + (2*Sqrt[a*Sec[x]^2])/a^2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\sqrt{a+a\tan(x)^2}}{a}+\frac{1}{\sqrt{a+a\tan(x)^2}}$	26
default	$\frac{\sqrt{a+a\tan(x)^2}}{a}+\frac{1}{\sqrt{a+a\tan(x)^2}}$	26
risch	$\frac{e^{4ix}+6\,e^{2ix}+1}{2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)^2}$	44

input
```
int(tan(x)^3/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a+a*tan(x)^2)^(1/2)+1/(a+a*tan(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx=\frac{\tan{(x)}^2+2}{\sqrt{a\tan{(x)}^2+a}}$$

input
```
integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(tan(x)^2 + 2)/sqrt(a*tan(x)^2 + a)
```

Sympy [F]

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\tan^3(x)}{\sqrt{a(\tan^2(x)+1)}}\,dx$$

input
```
integrate(tan(x)**3/(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)**3/sqrt(a*(tan(x)**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{(\sin(x)^2-2)\sqrt{\sin(x)+1}\sqrt{-\sin(x)+1}}{\sqrt{a}\sin(x)^2-\sqrt{a}}$$

input
```
integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
(sin(x)^2 - 2)*sqrt(sin(x) + 1)*sqrt(-sin(x) + 1)/(sqrt(a)*sin(x)^2 - sqrt
(a))
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\sqrt{a\tan(x)^2+a}+\frac{a}{\sqrt{a\tan(x)^2+a}}}{a}$$

input
```
integrate(tan(x)^3/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(sqrt(a*tan(x)^2 + a) + a/sqrt(a*tan(x)^2 + a))/a
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\sqrt{2}\,(\cos(2\,x)+3)}{2\,\sqrt{a}\,\sqrt{\cos(2\,x)+1}}$$

input
```
int(tan(x)^3/(a + a*tan(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*(cos(2*x) + 3))/(2*a^(1/2)*(cos(2*x) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\tan^3(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\sqrt{a}\,\sqrt{\tan(x)^2+1}\,\left(\tan(x)^2+2\right)}{a\left(\tan(x)^2+1\right)}$$

input
```
int(tan(x)^3/(a+a*tan(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*sqrt(tan(x)**2 + 1)*(tan(x)**2 + 2))/(a*(tan(x)**2 + 1))
```

3.274 $\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx=\frac{\operatorname{arctanh}(\sin(x))\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

output
```
arctanh(sin(x))*sec(x)/(a*sec(x)^2)^(1/2)-tan(x)/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx=\frac{\operatorname{arctanh}(\sin(x))\sec(x)-\tan(x)}{\sqrt{a\sec^2(x)}}$$

input
```
Integrate[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

output
```
(ArcTanh[Sin[x]]*Sec[x] - Tan[x])/Sqrt[a*Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4140, 3042, 4613, 3042, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)}{\sqrt{a\tan^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^2}{\sqrt{a\tan(x)^2+a}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\tan^2(x)}{\sqrt{a\sec^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^2}{\sqrt{a\sec(x)^2}}dx$$

$$\Big\downarrow \text{4613}$$

$$\frac{\sec(x)\int \sin(x)\tan(x)dx}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(x)\int \sin(x)\tan(x)dx}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{3072}$$

$$\frac{\sec(x)\int \frac{\sin^2(x)}{1-\sin^2(x)}d\sin(x)}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sec(x)\left(\int \frac{1}{1-\sin^2(x)}d\sin(x)-\sin(x)\right)}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow 219$$

$$\frac{\sec(x)(\operatorname{arctanh}(\sin(x))-\sin(x))}{\sqrt{a\sec^2(x)}}$$

input
```
Int[Tan[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

output
```
(Sec[x]*(ArcTanh[Sin[x]] - Sin[x]))/Sqrt[a*Sec[x]^2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^{2}}\right)}{\sqrt{a}}-\frac{\tan(x)}{\sqrt{a+a\tan(x)^{2}}}$	38
default	$\frac{\ln\left(\sqrt{a}\,\tan(x)+\sqrt{a+a\tan(x)^{2}}\right)}{\sqrt{a}}-\frac{\tan(x)}{\sqrt{a+a\tan(x)^{2}}}$	38
risch	$\frac{i\,{\mathrm e}^{2 i x}}{2\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\left({\mathrm e}^{2 i x}+1\right)}-\frac{i}{2\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\left({\mathrm e}^{2 i x}+1\right)}-\frac{{\mathrm e}^{i x}\ln\left({\mathrm e}^{i x}-i\right)}{\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\left({\mathrm e}^{2 i x}+1\right)}+\frac{{\mathrm e}^{i x}\ln\left({\mathrm e}^{i x}+i\right)}{\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\left({\mathrm e}^{2 i x}+1\right)}$	152

input
```
int(tan(x)^2/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(a^(1/2)*tan(x)+(a+a*tan(x)^2)^(1/2))/a^(1/2)-tan(x)/(a+a*tan(x)^2)^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(27) = 54.

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\left(\tan\left(x\right)^{2}+1\right)\sqrt{a}\log\left(2\,a\tan\left(x\right)^{2}+2\,\sqrt{a\tan\left(x\right)^{2}+a}\sqrt{a}\tan\left(x\right)+a\right)-2\,\sqrt{a\tan\left(x\right)^{2}+a}\tan\left(x\right)}{2\,\left(a\tan\left(x\right)^{2}+a\right)}$$

input
```
integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((tan(x)^2 + 1)*sqrt(a)*log(2*a*tan(x)^2 + 2*sqrt(a*tan(x)^2 + a)*sqrt
(a)*tan(x) + a) - 2*sqrt(a*tan(x)^2 + a)*tan(x))/(a*tan(x)^2 + a)
```

Sympy [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\tan^2{(x)}}{\sqrt{a\left(\tan^2{(x)}+1\right)}}\,dx$$

input
```
integrate(tan(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)-2\sin(x)}{2\sqrt{a}}$$

input
```
integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2
*sin(x) + 1) - 2*sin(x))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)}{\sqrt{a}} - \frac{\tan(x)}{\sqrt{a\tan(x)^2+a}}$$

input
```
integrate(tan(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a)))/sqrt(a) - tan(x)/sqrt(a*
tan(x)^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\tan(x)^2}{\sqrt{a\tan(x)^2+a}}\,dx$$

input
```
int(tan(x)^2/(a + a*tan(x)^2)^(1/2),x)
```

output
```
int(tan(x)^2/(a + a*tan(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\tan(x)^2+1}\,\tan(x)^2}{\tan(x)^2+1}dx\right)}{a}$$

input
```
int(tan(x)^2/(a+a*tan(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(tan(x)**2 + 1)*tan(x)**2)/(tan(x)**2 + 1),x))/a
```

3.275 $\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{1}{\sqrt{a\sec^2(x)}}$$

output
```
-1/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{1}{\sqrt{a\sec^2(x)}}$$

input
```
Integrate[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
```

output
```
-(1/Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4140, 3042, 4612, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a\tan^2(x)+a}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(x)}{\sqrt{a\tan(x)^2+a}}dx$$

$$\Big\downarrow \ 4140$$

$$\int \frac{\tan(x)}{\sqrt{a\sec^2(x)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(x)}{\sqrt{a\sec(x)^2}}dx$$

$$\Big\downarrow \ 4612$$

$$\frac{1}{2}a\int \frac{1}{(a\sec^2(x))^{3/2}}d\sec^2(x)$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{\sqrt{a\sec^2(x)}}$$

input
```
Int[Tan[x]/Sqrt[a + a*Tan[x]^2],x]
```

output
```
-(1/Sqrt[a*Sec[x]^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{1}{\sqrt{a+a\tan\left(x\right)^{2}}}$	13
default	$-\frac{1}{\sqrt{a+a\tan\left(x\right)^{2}}}$	13
risch	$-\frac{{\mathrm e}^{2 i x}}{2 \sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \left({\mathrm e}^{2 i x}+1\right)}-\frac{1}{2 \left({\mathrm e}^{2 i x}+1\right) \sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}}$	65

input
```
int(tan(x)/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a+a*tan(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{1}{\sqrt{a\tan(x)^2+a}}$$

input
```
integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/sqrt(a*tan(x)^2 + a)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{1}{\sqrt{a\tan^2(x)+a}}$$

input
```
integrate(tan(x)/(a+a*tan(x)**2)**(1/2),x)
```

output
```
-1/sqrt(a*tan(x)**2 + a)
```

Maxima [F]

$$\int \frac{\tan(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a\tan(x)^2+a}}\,dx$$

input
```
integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/sqrt(a*tan(x)^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a + a\tan^2(x)}}\, dx = -\frac{1}{\sqrt{a\tan{(x)}^2 + a}}$$

input
```
integrate(tan(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/sqrt(a*tan(x)^2 + a)
```

Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\tan(x)}{\sqrt{a + a\tan^2(x)}}\, dx = -\frac{\sqrt{\cos{(x)}^2}}{\sqrt{a}}$$

input
```
int(tan(x)/(a + a*tan(x)^2)^(1/2),x)
```

output
```
-(cos(x)^2)^(1/2)/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\tan(x)}{\sqrt{a + a\tan^2(x)}}\, dx = -\frac{\sqrt{a}\,\sqrt{\tan{(x)}^2 + 1}}{a\left(\tan{(x)}^2 + 1\right)}$$

input
```
int(tan(x)/(a+a*tan(x)^2)^(1/2),x)
```

output
```
( - sqrt(a)*sqrt(tan(x)**2 + 1))/(a*(tan(x)**2 + 1))
```

3.276 $\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{1}{\sqrt{a\sec^2(x)}}$$

output
```
-arctanh((a*sec(x)^2)^(1/2)/a^(1/2))/a^(1/2)+1/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{1-\frac{\arctan\left(\sqrt{-\cos^2(x)}\right)}{\sqrt{-\cos^2(x)}}}{\sqrt{a\sec^2(x)}}$$

input
```
Integrate[Cot[x]/Sqrt[a + a*Tan[x]^2],x]
```

output
```
(1 - ArcTan[Sqrt[-Cos[x]^2]]/Sqrt[-Cos[x]^2])/Sqrt[a*Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 4140, 3042, 4612, 25, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{a\tan^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)\sqrt{a\tan(x)^2+a}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\cot(x)}{\sqrt{a\sec^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)\sqrt{a\sec(x)^2}}dx$$

$$\Big\downarrow \text{4612}$$

$$\frac{1}{2}a\int -\frac{1}{\left(a\sec^2(x)\right)^{3/2}\left(1-\sec^2(x)\right)}d\sec^2(x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}a\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}\left(1-\sec^2(x)\right)}d\sec^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{2}{a\sqrt{a\sec^2(x)}}-\frac{\int \frac{1}{\sqrt{a\sec^2(x)}(1-\sec^2(x))}d\sec^2(x)}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{2}{a\sqrt{a\sec^2(x)}}-\frac{2\int \frac{1}{1-\frac{\sec^4(x)}{a}}d\sqrt{a\sec^2(x)}}{a^2}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{2}a\left(\frac{2}{a\sqrt{a\sec^2(x)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[Cot[x]/Sqrt[a + a*Tan[x]^2],x]
```

output
```
(a*((-2*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a*Sec[x]^2]
)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{1+\ln(-\cot(x)+\csc(x))\sec(x)+\sec(x)}{\sqrt{a\sec(x)^2}}$	25
risch	$\frac{e^{2ix}}{2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,(e^{2ix}+1)}+\frac{1}{2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}-\frac{e^{ix}\ln(e^{ix}+1)}{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,(e^{2ix}+1)}+\frac{e^{ix}\ln(e^{ix}-1)}{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,(e^{2ix}+1)}$	148

input
```
int(cot(x)/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*sec(x)^2)^(1/2)*(1+ln(-cot(x)+csc(x))*sec(x)+sec(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{(\tan(x)^2+1)\sqrt{a}\log\left(\frac{a\tan(x)^2-2\sqrt{a\tan(x)^2+a}\sqrt{a}+2a}{\tan(x)^2}\right)+2\sqrt{a\tan(x)^2+a}}{2\left(a\tan(x)^2+a\right)}$$

input
```
integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(x)^2 + a)*sqrt(
a) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a))/(a*tan(x)^2 + a)
```

Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{a(\tan^2(x)+1)}}\,dx$$

input
```
integrate(cot(x)/(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)/sqrt(a*(tan(x)**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\cot(x)}{\sqrt{a + a\tan^2(x)}}\,dx = \frac{2\,\cos\left(x\right) - \log\left(\cos\left(x\right)^{2} + \sin\left(x\right)^{2} + 2\,\cos\left(x\right) + 1\right) + \log\left(\cos\left(x\right)^{2} + \sin\left(x\right)^{2} - 2\,\cos\left(x\right) + 1\right)}{2\,\sqrt{a}}$$

input
```
integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*cos(x) - log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\cot(x)}{\sqrt{a + a\tan^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a\tan(x)^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{\sqrt{a\tan\left(x\right)^{2} + a}}$$

input
```
integrate(cot(x)/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/sqrt(-a) + 1/sqrt(a*tan(x)^2 + a)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\cot(x)}{\sqrt{a + a\tan^2(x)}}\,dx = \frac{1}{\sqrt{a\tan\left(x\right)^{2} + a}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{a\tan(x)^2 + a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(cot(x)/(a + a*tan(x)^2)^(1/2),x)
```

output
```
1/(a + a*tan(x)^2)^(1/2) - atanh((a + a*tan(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

Reduce [F]

$$\int \frac{\cot(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\tan(x)^2+1}\,\cot(x)}{\tan(x)^2+1}dx\right)}{a}$$

input
```
int(cot(x)/(a+a*tan(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(tan(x)**2 + 1)*cot(x))/(tan(x)**2 + 1),x))/a
```

3.277 $\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

output
```
-csc(x)*sec(x)/(a*sec(x)^2)^(1/2)-tan(x)/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{-\csc(x)\sec(x)-\tan(x)}{\sqrt{a\sec^2(x)}}$$

input
```
Integrate[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

output
```
(-(Csc[x]*Sec[x]) - Tan[x])/Sqrt[a*Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4140, 3042, 4613, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{\sqrt{a\tan^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)^2\sqrt{a\tan(x)^2+a}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\cot^2(x)}{\sqrt{a\sec^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)^2\sqrt{a\sec(x)^2}}dx$$

$$\Big\downarrow \text{4613}$$

$$\frac{\sec(x)\int \cos(x)\cot^2(x)dx}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(x)\int \sin\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)^2dx}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\sec(x)\int \csc^2(x)\left(1-\sin^2(x)\right)d(-\sin(x))}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\sec(x)\int \left(\csc^2(x)-1\right)d(-\sin(x))}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sec(x)(\sin(x)+\csc(x))}{\sqrt{a\sec^2(x)}}$$

input
```
Int[Cot[x]^2/Sqrt[a + a*Tan[x]^2],x]
```

output
```
-((Sec[x]*(Csc[x] + Sin[x]))/Sqrt[a*Sec[x]^2])
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\cot(x)-2\sec(x)\csc(x)}{\sqrt{a\sec(x)^2}}$	19
risch	$\frac{i(\mathrm{e}^{4ix}-6\,\mathrm{e}^{2ix}+1)}{2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)(\mathrm{e}^{2ix}-1)}$	54

input
```
int(cot(x)^2/(a+a*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*sec(x)^2)^(1/2)*(cot(x)-2*sec(x)*csc(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{\sqrt{a\tan(x)^2+a}\left(2\tan(x)^2+1\right)}{a\tan(x)^3+a\tan(x)}$$

input
```
integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*tan(x)^2 + a)*(2*tan(x)^2 + 1)/(a*tan(x)^3 + a*tan(x))
```

Sympy [F]

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \int \frac{\cot^2(x)}{\sqrt{a\left(\tan^2(x)+1\right)}}\,dx$$

input
```
integrate(cot(x)**2/(a+a*tan(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)**2/sqrt(a*(tan(x)**2 + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(27) = 54$.

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.13

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx$$
$$= \frac{((\sin(3x)-\sin(x))\cos(4x)-(\cos(3x)-\cos(x))\sin(4x)-(6\cos(2x)-1)\sin(3x)+6\cos(3x)\sin}{2\left(a\cos(3x)^2-2a\cos(3x)\cos(x)+a\cos(x)^2+a\sin(3x)^2-2a\sin\right.}$$

input
```
integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x) - (6*cos(
2*x) - 1)*sin(3*x) + 6*cos(3*x)*sin(2*x) - 6*cos(x)*sin(2*x) + 6*cos(2*x)*
sin(x) - sin(x))*sqrt(a)/(a*cos(3*x)^2 - 2*a*cos(3*x)*cos(x) + a*cos(x)^2
+ a*sin(3*x)^2 - 2*a*sin(3*x)*sin(x) + a*sin(x)^2)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = -\frac{\tan(x)}{\sqrt{a\tan(x)^2+a}}+\frac{2\sqrt{a}}{\left(\sqrt{a}\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a}$$

input
```
integrate(cot(x)^2/(a+a*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-tan(x)/sqrt(a*tan(x)^2 + a) + 2*sqrt(a)/((sqrt(a)*tan(x) - sqrt(a*tan(x)^
2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 7.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\sqrt{2}\left(6\sin(2x)-2\sin(2x)\left(2\cos(x)^2-1\right)\right)}{8\sqrt{a}\sqrt{2\cos(x)^2}\left(\cos(x)^2-1\right)}$$

input
```
int(cot(x)^2/(a + a*tan(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*(6*sin(2*x) - 2*sin(2*x)*(2*cos(x)^2 - 1)))/(8*a^(1/2)*(2*cos(x)^
2)^(1/2)*(cos(x)^2 - 1))
```

Reduce [F]

$$\int \frac{\cot^2(x)}{\sqrt{a+a\tan^2(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\tan(x)^2+1}\,\cot(x)^2}{\tan(x)^2+1}dx\right)}{a}$$

input
```
int(cot(x)^2/(a+a*tan(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(tan(x)**2 + 1)*cot(x)**2)/(tan(x)**2 + 1),x))/a
```

3.278 $\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{1}{3\left(a\sec^2(x)\right)^{3/2}} - \frac{1}{a\sqrt{a\sec^2(x)}}$$

output
```
1/3/(a*sec(x)^2)^(3/2)-1/a/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{-3+\cos^2(x)}{3a\sqrt{a\sec^2(x)}}$$

input
```
Integrate[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(-3 + Cos[x]^2)/(3*a*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 4140, 3042, 4612, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(x)}{\left(a\tan^2(x)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^3}{\left(a\tan(x)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\tan^3(x)}{\left(a\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^3}{\left(a\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4612}$$

$$\frac{1}{2}a\int -\frac{1-\sec^2(x)}{\left(a\sec^2(x)\right)^{5/2}}d\sec^2(x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}a\int \frac{1-\sec^2(x)}{\left(a\sec^2(x)\right)^{5/2}}d\sec^2(x)$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}a\int \left(\frac{1}{\left(a\sec^2(x)\right)^{5/2}}-\frac{1}{a\left(a\sec^2(x)\right)^{3/2}}\right)d\sec^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a\left(\frac{2}{3a\left(a\sec^2(x)\right)^{3/2}}-\frac{2}{a^2\sqrt{a\sec^2(x)}}\right)$$

input
```
Int[Tan[x]^3/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(a*(2/(3*a*(a*Sec[x]^2)^(3/2)) - 2/(a^2*Sqrt[a*Sec[x]^2])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{1}{a\sqrt{a+a\tan(x)^{2}}}+\frac{1}{3\left(a+a\tan(x)^{2}\right)^{\frac{3}{2}}}$
default	$-\frac{1}{a\sqrt{a+a\tan(x)^{2}}}+\frac{1}{3\left(a+a\tan(x)^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{e^{4ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}}-\frac{3\,e^{2ix}}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}}-\frac{3}{8\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,(e^{2ix}+1)a}+\frac{e^{-2ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}}$

input
```
int(tan(x)^3/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/(a+a*tan(x)^2)^(1/2)+1/3/(a+a*tan(x)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{\tan^3(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx=-\frac{\sqrt{a\tan\left(x\right)^{2}+a}\left(3\,\tan\left(x\right)^{2}+2\right)}{3\,\left(a^{2}\tan\left(x\right)^{4}+2\,a^{2}\tan\left(x\right)^{2}+a^{2}\right)}$$

input
```
integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 + 2)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2
+ a^2)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{a^2}{6\left(a\tan^2(x)+a\right)^{\frac{3}{2}}}-\frac{a}{2\sqrt{a\tan^2(x)+a}}\right)}{a^2} & \text{for } a \neq 0 \\ \tilde{\infty}\tan^4(x) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(x)**3/(a+a*tan(x)**2)**(3/2),x)
```

output
```
Piecewise((2*(a**2/(6*(a*tan(x)**2 + a)**(3/2)) - a/(2*sqrt(a*tan(x)**2 +
a)))/a**2, Ne(a, 0)), (zoo*tan(x)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\left(\sin(x)^2+2\right)\left(\sin(x)+1\right)^{\frac{3}{2}}\left(-\sin(x)+1\right)^{\frac{3}{2}}}{3\left(a^{\frac{3}{2}}\sin(x)^2-a^{\frac{3}{2}}\right)}$$

input
```
integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*(sin(x)^2 + 2)*(sin(x) + 1)^(3/2)*(-sin(x) + 1)^(3/2)/(a^(3/2)*sin(x)^
2 - a^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{3\,a\tan(x)^2+2\,a}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}a}$$

input
```
integrate(tan(x)^3/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(3*a*tan(x)^2 + 2*a)/((a*tan(x)^2 + a)^(3/2)*a)
```

Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\left(\tan(x)^2+\frac{2}{3}\right)\sqrt{a\tan(x)^2+a}}{a^2\left(\tan(x)^2+1\right)^2}$$

input
```
int(tan(x)^3/(a + a*tan(x)^2)^(3/2),x)
```

output
```
-((tan(x)^2 + 2/3)*(a + a*tan(x)^2)^(1/2))/(a^2*(tan(x)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{\tan^3(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{\tan(x)^2+1}\left(-3\tan(x)^2-2\right)}{3a^2\left(\tan(x)^4+2\tan(x)^2+1\right)}$$

input
```
int(tan(x)^3/(a+a*tan(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(tan(x)**2 + 1)*( - 3*tan(x)**2 - 2))/(3*a**2*(tan(x)**4 + 2*
tan(x)**2 + 1))
```

3.279 $\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

output
```
1/3*sin(x)^2*tan(x)/a/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\tan^3(x)}{3\,(a\sec^2(x))^{3/2}}$$

input
```
Integrate[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
Tan[x]^3/(3*(a*Sec[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4140, 3042, 4613, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)}{\left(a\tan^2(x)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(x)^2}{\left(a\tan(x)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \frac{\tan^2(x)}{\left(a\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(x)^2}{\left(a\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4613}$$

$$\frac{\sec(x)\int \cos(x)\sin^2(x)dx}{a\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sec(x)\int \cos(x)\sin(x)^2dx}{a\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{\sec(x)\int \sin^2(x)d\sin(x)}{a\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

input
```
Int[Tan[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.

Time = 0.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{\tan(x)}{a\sqrt{a+a\tan(x)^2}}-a\left(\frac{\tan(x)}{3a\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}+\frac{2\tan(x)}{3a^2\sqrt{a+a\tan(x)^2}}\right)$
default	$\frac{\tan(x)}{a\sqrt{a+a\tan(x)^2}}-a\left(\frac{\tan(x)}{3a\left(a+a\tan(x)^2\right)^{\frac{3}{2}}}+\frac{2\tan(x)}{3a^2\sqrt{a+a\tan(x)^2}}\right)$
risch	$\frac{i\,e^{4ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{i\,e^{2ix}}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{i}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{i\,e^{-2ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}$

input
```
int(tan(x)^2/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*tan(x)/(a+a*tan(x)^2)^(1/2)-a*(1/3/a*tan(x)/(a+a*tan(x)^2)^(3/2)+2/3/a
^2*tan(x)/(a+a*tan(x)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{\tan^2(x)}{\left(a+a\tan^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{a\tan{(x)}^2+a}\tan{(x)}^3}{3\left(a^2\tan{(x)}^4+2\,a^2\tan{(x)}^2+a^2\right)}$$

input
```
integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(a*tan(x)^2 + a)*tan(x)^3/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [F]

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \int \frac{\tan^2(x)}{(a(\tan^2(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)**2/(a+a*tan(x)**2)**(3/2),x)
```

output
```
Integral(tan(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\sin(3x)-3\sin(x)}{12a^{\frac{3}{2}}}$$

input
```
integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/12*(sin(3*x) - 3*sin(x))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\tan(x)^3}{3\left(a\tan(x)^2+a\right)^{\frac{3}{2}}}$$

input
```
integrate(tan(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/3*tan(x)^3/(a*tan(x)^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 7.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\tan(x)^3}{\left(3\,a\tan\left(x\right)^2+3\,a\right)\,\sqrt{a\tan\left(x\right)^2+a}}$$

input
```
int(tan(x)^2/(a + a*tan(x)^2)^(3/2),x)
```

output
```
tan(x)^3/((3*a + 3*a*tan(x)^2)*(a + a*tan(x)^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\tan^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\sqrt{a}\,\sqrt{\tan\left(x\right)^2+1}\,\tan\left(x\right)^3}{3a^2\left(\tan\left(x\right)^4+2\tan\left(x\right)^2+1\right)}$$

input
```
int(tan(x)^2/(a+a*tan(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(tan(x)**2 + 1)*tan(x)**3)/(3*a**2*(tan(x)**4 + 2*tan(x)**2 +
 1))
```

3.280 $\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{1}{3(a\sec^2(x))^{3/2}}$$

output
```
-1/3/(a*sec(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{1}{3(a\sec^2(x))^{3/2}}$$

input
```
Integrate[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
-1/3*1/(a*Sec[x]^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4140, 3042, 4612, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(a\tan^2(x)+a\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(a\tan(x)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\tan(x)}{\left(a\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(a\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4612}$$

$$\frac{1}{2}a\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}d\sec^2(x)$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{3\left(a\sec^2(x)\right)^{3/2}}$$

input
```
Int[Tan[x]/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
-1/3*1/(a*Sec[x]^2)^(3/2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
derivativedivides	$-\frac{1}{3\left(a+a\tan\left(x\right)^{2}\right)^{\frac{3}{2}}}$
default	$-\frac{1}{3\left(a+a\tan\left(x\right)^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{e^{4ix}}{24a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}}-\frac{e^{2ix}}{8a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}}-\frac{1}{8\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\left(e^{2ix}+1\right)a}-\frac{e^{-2ix}}{24a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}}$

input
```
int(tan(x)/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a+a*tan(x)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\sqrt{a\tan{(x)}^2+a}}{3\left(a^2\tan{(x)}^4+2\,a^2\tan{(x)}^2+a^2\right)}$$

input
```
integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{1}{3\left(a\tan^2{(x)}+a\right)^{\frac{3}{2}}}$$

input
```
integrate(tan(x)/(a+a*tan(x)**2)**(3/2),x)
```

output
```
-1/(3*(a*tan(x)**2 + a)**(3/2))
```

Maxima [F]

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \int \frac{\tan{(x)}}{\left(a\tan{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(a*tan(x)^2 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{1}{3\left(a\tan\left(x\right)^2+a\right)^{\frac{3}{2}}}$$

input
```
integrate(tan(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3/(a*tan(x)^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\sqrt{a\tan\left(x\right)^2+a}}{3\,a^2\left(\tan\left(x\right)^2+1\right)^2}$$

input
```
int(tan(x)/(a + a*tan(x)^2)^(3/2),x)
```

output
```
-(a + a*tan(x)^2)^(1/2)/(3*a^2*(tan(x)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\tan(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\sqrt{a}\,\sqrt{\tan\left(x\right)^2+1}}{3a^2\left(\tan\left(x\right)^4+2\tan\left(x\right)^2+1\right)}$$

input
```
int(tan(x)/(a+a*tan(x)^2)^(3/2),x)
```

output
```
( - sqrt(a)*sqrt(tan(x)**2 + 1))/(3*a**2*(tan(x)**4 + 2*tan(x)**2 + 1))
```

3.281 $\int \frac{\cot(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{\cot(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{1}{3(a\sec^2(x))^{3/2}} + \frac{1}{a\sqrt{a\sec^2(x)}}$$

output
```
-arctanh((a*sec(x)^2)^(1/2)/a^(1/2))/a^(3/2)+1/3/(a*sec(x)^2)^(3/2)+1/a/(a
*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{\cot(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{-3\arctan\left(\sqrt{-\cos^2(x)}\right) - \sqrt{-\cos^2(x)}(-4+\sin^2(x))}{3a\sqrt{-\cos^2(x)}\sqrt{a\sec^2(x)}}$$

input
```
Integrate[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(-3*ArcTan[Sqrt[-Cos[x]^2]] - Sqrt[-Cos[x]^2]*(-4 + Sin[x]^2))/(3*a*Sqrt[-
Cos[x]^2]*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4140, 3042, 4612, 25, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\left(a\tan^2(x)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)\left(a\tan(x)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\cot(x)}{\left(a\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)\left(a\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4612}$$

$$\frac{1}{2}a\int -\frac{1}{\left(a\sec^2(x)\right)^{5/2}\left(1-\sec^2(x)\right)}d\sec^2(x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}a\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}\left(1-\sec^2(x)\right)}d\sec^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{2}{3a\left(a\sec^2(x)\right)^{3/2}}-\frac{\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}\left(1-\sec^2(x)\right)}d\sec^2(x)}{a}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{2}{3a\left(a\sec^2(x)\right)^{3/2}}-\frac{\frac{\int\frac{1}{\sqrt{a\sec^2(x)}\left(1-\sec^2(x)\right)}d\sec^2(x)}{a}-\frac{2}{a\sqrt{a\sec^2(x)}}}{a}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}a\left(\frac{2}{3a\left(a\sec^2(x)\right)^{3/2}}-\frac{\frac{2\int\frac{1}{1-\frac{\sec^4(x)}{a}}d\sqrt{a\sec^2(x)}}{a^2}-\frac{2}{a\sqrt{a\sec^2(x)}}}{a}\right)$$

$$\Big\downarrow \textbf{219}$$

$$\frac{1}{2}a\left(\frac{2}{3a\left(a\sec^2(x)\right)^{3/2}}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a\sec^2(x)}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a\sec^2(x)}}}{a}\right)$$

input
```
Int[Cot[x]/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(a*(2/(3*a*(a*Sec[x]^2)^(3/2)) - ((2*ArcTanh[Sqrt[a*Sec[x]^2]/Sqrt[a]])/a^
(3/2) - 2/(a*Sqrt[a*Sec[x]^2]))/a))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4612
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.),
x_Symbol] :> Simp[b/(2*f)   Subst[Int[(-1 + x)^((m - 1)/2)*(b*x)^(p - 1), x
], x, Sec[e + f*x]^2], x] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p] && Int
egerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result
default	$\frac{\frac{\cos(x)^2}{3}+\ln(-\cot(x)+\csc(x))\sec(x)+1+\frac{4\sec(x)}{3}}{\sqrt{a\sec(x)^2}\,a}$
risch	$\frac{e^{4ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{5\,e^{2ix}}{8a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{5}{8\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}(e^{2ix}+1)a}+\frac{e^{-2ix}}{24a(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{e^{ix}\ln(}{a(e^{2ix}+1)}$

input
```
int(cot(x)/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*cos(x)^2+ln(-cot(x)+csc(x))*sec(x)+1+4/3*sec(x))/(a*sec(x)^2)^(1/2)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(41) = 82$.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.77

$$\int \frac{\cot(x)}{(a + a\tan^2(x))^{3/2}}\,dx = \frac{3\left(\tan(x)^4 + 2\,\tan(x)^2 + 1\right)\sqrt{a}\log\left(\frac{a\tan(x)^2 - 2\sqrt{a\tan(x)^2 + a}\sqrt{a} + 2\,a}{\tan(x)^2}\right) + 2\sqrt{a\tan(}}{6\left(a^2\tan(x)^4 + 2\,a^2\tan(x)^2 + a^2\right)}$$

input
```
integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(3*(tan(x)^4 + 2*tan(x)^2 + 1)*sqrt(a)*log((a*tan(x)^2 - 2*sqrt(a*tan(
x)^2 + a)*sqrt(a) + 2*a)/tan(x)^2) + 2*sqrt(a*tan(x)^2 + a)*(3*tan(x)^2 +
4))/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2)
```

Sympy [F]

$$\int \frac{\cot(x)}{(a + a\tan^2(x))^{3/2}}\,dx = \int \frac{\cot(x)}{(a(\tan^2(x) + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)/(a+a*tan(x)**2)**(3/2),x)
```

output
```
Integral(cot(x)/(a*(tan(x)**2 + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{\cot(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\cos(3x) + 15\cos(x) - 6\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + 6\log\left(\cos\left(\right.\right.}{12\,a^{\frac{3}{2}}}$$

input
```
integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(cos(3*x) + 15*cos(x) - 6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 6
*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{3\,a\tan(x)^2 + 4\,a}{3\left(a\tan(x)^2 + a\right)^{\frac{3}{2}}a}$$

input
```
integrate(cot(x)/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
arctan(sqrt(a*tan(x)^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/3*(3*a*tan(x)^2 + 4
*a)/((a*tan(x)^2 + a)^(3/2)*a)
```

Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{\cot(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\frac{a\tan(x)^2+a}{a} + \frac{1}{3}}{\left(a\tan(x)^2 + a\right)^{3/2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a\tan(x)^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(cot(x)/(a + a*tan(x)^2)^(3/2),x)
```

output
```
((a + a*tan(x)^2)/a + 1/3)/(a + a*tan(x)^2)^(3/2) - atanh((a + a*tan(x)^2)
^(1/2)/a^(1/2))/a^(3/2)
```

Reduce [F]

$$\int \frac{\cot(x)}{(a + a\tan^2(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\tan(x)^2+1}\,\cot(x)}{\tan(x)^4+2\tan(x)^2+1}dx\right)}{a^2}$$

input
```
int(cot(x)/(a+a*tan(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(tan(x)**2 + 1)*cot(x))/(tan(x)**4 + 2*tan(x)**2 + 1),x)
)/a**2
```

3.282 $\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\csc(x)\sec(x)}{a\sqrt{a\sec^2(x)}} - \frac{2\tan(x)}{a\sqrt{a\sec^2(x)}} + \frac{\sin^2(x)\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

output
```
-csc(x)*sec(x)/a/(a*sec(x)^2)^(1/2)-2*tan(x)/a/(a*sec(x)^2)^(1/2)+1/3*sin(
x)^2*tan(x)/a/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\sec^3(x)\left(-3\csc(x)-6\sin(x)+\sin^3(x)\right)}{3\left(a\sec^2(x)\right)^{3/2}}$$

input
```
Integrate[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
(Sec[x]^3*(-3*Csc[x] - 6*Sin[x] + Sin[x]^3))/(3*(a*Sec[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4140, 3042, 4613, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{\left(a\tan^2(x)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)^2\left(a\tan(x)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{\cot^2(x)}{\left(a\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)^2\left(a\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4613}$$

$$\frac{\sec(x)\int \cos^3(x)\cot^2(x)dx}{a\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(x)\int \sin\left(x+\frac{\pi}{2}\right)^3\tan\left(x+\frac{\pi}{2}\right)^2dx}{a\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\sec(x)\int \csc^2(x)\left(1-\sin^2(x)\right)^2d(-\sin(x))}{a\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\sec(x)\int \left(\csc^2(x)+\sin^2(x)-2\right)d(-\sin(x))}{a\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sec(x)\left(-\frac{1}{3}\sin^3(x)+2\sin(x)+\csc(x)\right)}{a\sqrt{a\sec^2(x)}}$$

input
```
Int[Cot[x]^2/(a + a*Tan[x]^2)^(3/2),x]
```

output
```
-((Sec[x]*(Csc[x] + 2*Sin[x] - Sin[x]^3/3))/(a*Sqrt[a*Sec[x]^2]))
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4613
```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Se
c[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{\frac{\cos(x)^2\cot(x)}{3}+\frac{4\cot(x)}{3}-\frac{8\sec(x)\csc(x)}{3}}{\sqrt{a\sec(x)^2}\,a}$	32
risch	$\frac{i\left(\mathrm{e}^{6ix}+20+20\,\mathrm{e}^{4ix}-89\cos(2x)-91i\sin(2x)\right)}{24a\left(\mathrm{e}^{2ix}+1\right)\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^2}}\,\left(\mathrm{e}^{2ix}-1\right)}$	70

input
```
int(cot(x)^2/(a+a*tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*cos(x)^2*cot(x)+4/3*cot(x)-8/3*sec(x)*csc(x))/(a*sec(x)^2)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{\left(8\,\tan\left(x\right)^4+12\,\tan\left(x\right)^2+3\right)\sqrt{a\tan\left(x\right)^2+a}}{3\,\left(a^2\tan\left(x\right)^5+2\,a^2\tan\left(x\right)^3+a^2\tan\left(x\right)\right)}$$

input
```
integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(8*tan(x)^4 + 12*tan(x)^2 + 3)*sqrt(a*tan(x)^2 + a)/(a^2*tan(x)^5 + 2
*a^2*tan(x)^3 + a^2*tan(x))
```

Sympy [F]

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \int \frac{\cot^2(x)}{(a(\tan^2(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)**2/(a+a*tan(x)**2)**(3/2),x)
```

output
```
Integral(cot(x)**2/(a*(tan(x)**2 + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(52) = 104$.

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.75

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{((\sin(5x)-\sin(3x))\cos(8x)+20(\sin(5x)-\sin(3x))\cos(6x)+10(9\sin(4$$

input
```
integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/24*((sin(5*x) - sin(3*x))*cos(8*x) + 20*(sin(5*x) - sin(3*x))*cos(6*x) +
 10*(9*sin(4*x) - 2*sin(2*x))*cos(5*x) - (cos(5*x) - cos(3*x))*sin(8*x) -
20*(cos(5*x) - cos(3*x))*sin(6*x) - (90*cos(4*x) - 20*cos(2*x) - 1)*sin(5*
x) - 90*cos(3*x)*sin(4*x) - (20*cos(2*x) + 1)*sin(3*x) + 90*cos(4*x)*sin(3
*x) + 20*cos(3*x)*sin(2*x))*sqrt(a)/(a^2*cos(5*x)^2 - 2*a^2*cos(5*x)*cos(3
*x) + a^2*cos(3*x)^2 + a^2*sin(5*x)^2 - 2*a^2*sin(5*x)*sin(3*x) + a^2*sin(
3*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = -\frac{(5\tan(x)^2+6)\tan(x)}{3\,(a\tan(x)^2+a)^{\frac{3}{2}}} + \frac{2}{\left(\left(\sqrt{a}\tan(x)-\sqrt{a\tan(x)^2+a}\right)^2-a\right)\sqrt{a}}$$

input
```
integrate(cot(x)^2/(a+a*tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(5*tan(x)^2 + 6)*tan(x)/(a*tan(x)^2 + a)^(3/2) + 2/(((sqrt(a)*tan(x)
- sqrt(a*tan(x)^2 + a))^2 - a)*sqrt(a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \int \frac{\cot(x)^2}{(a\tan(x)^2+a)^{3/2}}\,dx$$

input
```
int(cot(x)^2/(a + a*tan(x)^2)^(3/2),x)
```

output
```
int(cot(x)^2/(a + a*tan(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot^2(x)}{(a+a\tan^2(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\tan(x)^2+1}\,\cot(x)^2}{\tan(x)^4+2\tan(x)^2+1}dx\right)}{a^2}$$

input
```
int(cot(x)^2/(a+a*tan(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(tan(x)**2 + 1)*cot(x)**2)/(tan(x)**4 + 2*tan(x)**2 + 1)
,x))/a**2
```

3.283 $\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx = \frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

output
```
tan(d*x+c)/d/(a*sec(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx = \frac{\tan(c+dx)}{d\sqrt{a\sec^2(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + a*Tan[c + d*x]^2],x]
```

output
```
Tan[c + d*x]/(d*Sqrt[a*Sec[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\tan^2(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a\tan(c+dx)^2+a}}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \frac{1}{\sqrt{a\sec^2(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a\sec(c+dx)^2}}dx$$

$$\Big\downarrow \text{ 4610}$$

$$\frac{a\int \frac{1}{(a\tan^2(c+dx)+a)^{3/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\tan(c+dx)}{d\sqrt{a\tan^2(c+dx)+a}}$$

input
```
Int[1/Sqrt[a + a*Tan[c + d*x]^2],x]
```

output
```
Tan[c + d*x]/(d*Sqrt[a + a*Tan[c + d*x]^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{d\sqrt{a+a\tan(dx+c)^2}}$	25
default	$\frac{\tan(dx+c)}{d\sqrt{a+a\tan(dx+c)^2}}$	25
risch	$-\frac{i\mathrm{e}^{2i(dx+c)}}{2d\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}}}+\frac{i}{2\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}}\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$	101

input
```
int(1/(a+a*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a + a\tan^2(c + dx)}} dx = \frac{\sqrt{a\tan(dx + c)^2 + a}\tan(dx + c)}{ad\tan(dx + c)^2 + ad}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*tan(d*x + c)^2 + a)*tan(d*x + c)/(a*d*tan(d*x + c)^2 + a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + a\tan^2(c + dx)}} dx = \int \frac{1}{\sqrt{a\tan^2(c + dx) + a}} dx$$

input
```
integrate(1/(a+a*tan(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*tan(c + d*x)**2 + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a + a\tan^2(c + dx)}} dx = \frac{\sin(dx + c)}{\sqrt{a}d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(22) = 44$.

Time = 0.62 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{a + a\tan^2(c + dx)}}\,dx$$
$$= -\frac{2}{\sqrt{a}d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 - 1\right)}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-2/(sqrt(a)*d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))*sgn(tan(1/2*
d*x + 1/2*c)^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \frac{1}{\sqrt{a + a\tan^2(c + dx)}}\,dx = \frac{\sin(2\,c + 2\,d\,x)\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1)}{4\,\cos(2\,c+2\,d\,x)+\cos(4\,c+4\,d\,x)+3}}}{a\,d}$$

input
```
int(1/(a + a*tan(c + d*x)^2)^(1/2),x)
```

output
```
(sin(2*c + 2*d*x)*((a*(cos(2*c + 2*d*x) + 1))/(4*cos(2*c + 2*d*x) + cos(4*
c + 4*d*x) + 3))^(1/2))/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a+a\tan^2(c+dx)}}\,dx = \frac{\sqrt{a}\,\sqrt{\tan (dx+c)^2+1}\,\tan (dx+c)}{ad\left(\tan (dx+c)^2+1\right)}$$

input
```
int(1/(a+a*tan(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(a)*sqrt(tan(c + d*x)**2 + 1)*tan(c + d*x))/(a*d*(tan(c + d*x)**2 + 1
))
```

3.284 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\frac{\tan(c+dx)}{3d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{2\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}$$

output
```
1/3*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(3/2)+2/3*tan(d*x+c)/a/d/(a*sec(d*x+c)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=-\frac{\left(-3+\sin^2(c+dx)\right)\tan(c+dx)}{3ad\sqrt{a\sec^2(c+dx)}}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(-3/2),x]
```

output
```
-1/3*((-3 + Sin[c + d*x]^2)*Tan[c + d*x])/(a*d*Sqrt[a*Sec[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4140, 3042, 4610, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\tan(c+dx)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{1}{\left(a\sec^2(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(c+dx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{a\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{5/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{209}$$

$$\frac{a\left(\frac{2\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{3/2}}d\tan(c+dx)}{3a}+\frac{\tan(c+dx)}{3a\left(a\tan^2(c+dx)+a\right)^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{208}$$

$$\frac{a\left(\frac{2\tan(c+dx)}{3a^2\sqrt{a\tan^2(c+dx)+a}}+\frac{\tan(c+dx)}{3a\left(a\tan^2(c+dx)+a\right)^{3/2}}\right)}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(-3/2),x]
```

output
```
(a*(Tan[c + d*x]/(3*a*(a + a*Tan[c + d*x]^2)^(3/2)) + (2*Tan[c + d*x])/(3*
a^2*Sqrt[a + a*Tan[c + d*x]^2])))/d
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)}{3a\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}+\frac{2\tan(dx+c)}{3a^2\sqrt{a+a\tan(dx+c)^2}}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)}{3a\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}+\frac{2\tan(dx+c)}{3a^2\sqrt{a+a\tan(dx+c)^2}}\right)}{d}$
risch	$-\frac{i e^{4i(dx+c)}}{24d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a}-\frac{3i e^{2i(dx+c)}}{8d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a}+\frac{3i}{8a\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\,e^{2i(dx+}}{\left(e^{2i(dx+c)}\right.}}}$

input
```
int(1/(a+a*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*t
an(d*x+c)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\frac{\sqrt{a\tan\left(dx+c\right)^2+a}\left(2\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)}{3\left(a^2d\tan\left(dx+c\right)^4+2\,a^2d\tan\left(dx+c\right)^2+a^2d\right)}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(a*tan(d*x + c)^2 + a)*(2*tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d*
tan(d*x + c)^4 + 2*a^2*d*tan(d*x + c)^2 + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(a\tan^2\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*tan(d*x+c)**2)**(3/2),x)
```

output
```
Integral((a*tan(c + d*x)**2 + a)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(dx+c\right)}{12\,a^{\frac{3}{2}}d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*d*x + 3*c) + 9*sin(d*x + c))/(a^(3/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(50) = 100.

Time = 1.73 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.48

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=$$
$$-\frac{2\,\mathrm{sgn}(\tan\left(dx\right)\tan\left(c\right)-1)\tan\left(c\right)^{3}+\left(\left(\left(3\,\mathrm{sgn}(\tan\left(dx\right)\tan\left(c\right)-1\right)\tan\left(c\right)^{2}+2\,\mathrm{sgn}(\tan\left(dx\right)\tan\left(c\right)-1}{3\left(a\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+a}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(2*sgn(tan(d*x)*tan(c) - 1)*tan(c)^3 + (((3*sgn(tan(d*x)*tan(c) - 1)*
tan(c)^2 + 2*sgn(tan(d*x)*tan(c) - 1))*tan(d*x) + 3*tan(c)^3/sgn(tan(d*x)*
tan(c) - 1))*tan(d*x) + 3/sgn(tan(d*x)*tan(c) - 1))*tan(d*x) + 3*sgn(tan(d
*x)*tan(c) - 1)*tan(c))/((a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^
2 + a)^(3/2)*d)
```

Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\frac{\frac{2\tan(c+d\,x)^3}{3}+\tan(c+d\,x)}{d\,\left(a\tan\left(c+d\,x\right)^2+a\right)^{3/2}}$$

input
```
int(1/(a + a*tan(c + d*x)^2)^(3/2),x)
```

output
```
(tan(c + d*x) + (2*tan(c + d*x)^3)/3)/(d*(a + a*tan(c + d*x)^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{3/2}}\,dx=\frac{\sqrt{a}\,\sqrt{\tan\left(dx+c\right)^2+1}\,\tan\left(dx+c\right)\left(2\tan\left(dx+c\right)^2+3\right)}{3a^2d\left(\tan\left(dx+c\right)^4+2\tan\left(dx+c\right)^2+1\right)}$$

input
```
int(1/(a+a*tan(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(tan(c + d*x)**2 + 1)*tan(c + d*x)*(2*tan(c + d*x)**2 + 3))/(
3*a**2*d*(tan(c + d*x)**4 + 2*tan(c + d*x)**2 + 1))
```

3.285 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx=\frac{\tan(c+dx)}{5d\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{4\tan(c+dx)}{15ad\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{8\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

output
```
1/5*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(5/2)+4/15*tan(d*x+c)/a/d/(a*sec(d*x+c)^
2)^(3/2)+8/15*tan(d*x+c)/a^2/d/(a*sec(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx=\frac{\left(15-10\sin^2(c+dx)+3\sin^4(c+dx)\right)\tan(c+dx)}{15a^2d\sqrt{a\sec^2(c+dx)}}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(-5/2),x]
```

output
```
((15 - 10*Sin[c + d*x]^2 + 3*Sin[c + d*x]^4)*Tan[c + d*x])/(15*a^2*d*Sqrt[
a*Sec[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4140, 3042, 4610, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\tan(c+dx)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{1}{\left(a\sec^2(c+dx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(c+dx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{a\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{7/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{209}$$

$$\frac{a\left(\frac{4\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{5/2}}d\tan(c+dx)}{5a}+\frac{\tan(c+dx)}{5a\left(a\tan^2(c+dx)+a\right)^{5/2}}\right)}{d}$$

$$\Big\downarrow \text{209}$$

$$\frac{a\left(\frac{4\left(\frac{2\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{3/2}}d\tan(c+dx)}{3a}+\frac{\tan(c+dx)}{3a\left(a\tan^2(c+dx)+a\right)^{3/2}}\right)}{5a}+\frac{\tan(c+dx)}{5a(a\tan^2(c+dx)+a)^{5/2}}\right)}{d}$$

$$\Big\downarrow \text{208}$$

$$\frac{a\left(\frac{4\left(\frac{2\tan(c+dx)}{3a^2\sqrt{a\tan^2(c+dx)+a}}+\frac{\tan(c+dx)}{3a\left(a\tan^2(c+dx)+a\right)^{3/2}}\right)}{5a}+\frac{\tan(c+dx)}{5a(a\tan^2(c+dx)+a)^{5/2}}\right)}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(-5/2),x]
```

output
```
(a*(Tan[c + d*x]/(5*a*(a + a*Tan[c + d*x]^2)^(5/2)) + (4*(Tan[c + d*x]/(3*
a*(a + a*Tan[c + d*x]^2)^(3/2)) + (2*Tan[c + d*x])/(3*a^2*Sqrt[a + a*Tan[c
 + d*x]^2])))/(5*a)))/d
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)}{5a\left(a+a\tan(dx+c)^2\right)^{\frac{5}{2}}}+\frac{\frac{4\tan(dx+c)}{15a\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\tan(dx+c)^2}}}{a}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)}{5a\left(a+a\tan(dx+c)^2\right)^{\frac{5}{2}}}+\frac{\frac{4\tan(dx+c)}{15a\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\tan(dx+c)^2}}}{a}\right)}{d}$
risch	$-\frac{i e^{6i(dx+c)}}{160d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a^2}-\frac{5i e^{2i(dx+c)}}{16d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a^2}+\frac{5i}{16a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\,e}{\left(e^{2i}\right.}}}$

input
```
int(1/(a+a*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(1/5/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(
a+a*tan(d*x+c)^2)^(3/2)+2/3/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{\left(8\,\tan\left(dx+c\right)^{5}+20\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)\sqrt{a\tan\left(dx+c\right)^{2}+}}{15\,\left(a^{3}d\tan\left(dx+c\right)^{6}+3\,a^{3}d\tan\left(dx+c\right)^{4}+3\,a^{3}d\tan\left(dx+c\right)^{2}+a^{3}d\right)}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(8*tan(d*x + c)^5 + 20*tan(d*x + c)^3 + 15*tan(d*x + c))*sqrt(a*tan(d
*x + c)^2 + a)/(a^3*d*tan(d*x + c)^6 + 3*a^3*d*tan(d*x + c)^4 + 3*a^3*d*ta
n(d*x + c)^2 + a^3*d)
```

Sympy [F]

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\tan^{2}\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*tan(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a*tan(c + d*x)**2 + a)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{3\,\sin\left(5\,dx+5\,c\right)+25\,\sin\left(3\,dx+3\,c\right)+150\,\sin\left(dx+c\right)}{240\,a^{\frac{5}{2}}d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/240*(3*sin(5*d*x + 5*c) + 25*sin(3*d*x + 3*c) + 150*sin(d*x + c))/(a^(5/
2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(76) = 152$.

Time = 2.55 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+a\tan^2(c+dx))^{5/2}}\,dx =$$
$$-\frac{8\,\mathrm{sgn}(\tan(dx)\tan(c)-1)\tan(c)^5+20\,\mathrm{sgn}(\tan(dx)\tan(c)-1)\tan(c)^3+\left(\left(20\,\mathrm{sgn}(\tan(dx)\tan(c)-\right.\right.}{}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/15*(8*sgn(tan(d*x)*tan(c) - 1)*tan(c)^5 + 20*sgn(tan(d*x)*tan(c) - 1)*t
an(c)^3 + ((20*sgn(tan(d*x)*tan(c) - 1)*tan(c)^5 + 50*sgn(tan(d*x)*tan(c)
- 1)*tan(c)^3 + (50*sgn(tan(d*x)*tan(c) - 1)*tan(c)^2 + (15*tan(c)^5/sgn(t
an(d*x)*tan(c) - 1) + (15*sgn(tan(d*x)*tan(c) - 1)*tan(c)^4 + 20*sgn(tan(d
*x)*tan(c) - 1)*tan(c)^2 + 8*sgn(tan(d*x)*tan(c) - 1))*tan(d*x))*tan(d*x)
+ 20*sgn(tan(d*x)*tan(c) - 1))*tan(d*x))*tan(d*x) + 15/sgn(tan(d*x)*tan(c)
 - 1))*tan(d*x) + 15*sgn(tan(d*x)*tan(c) - 1)*tan(c))/((a*tan(d*x)^2*tan(c
)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a)^(5/2)*d)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a+a\tan^2(c+dx))^{5/2}}\,dx = \frac{\tan(c+d\,x)\,\left(8\tan(c+d\,x)^4+20\tan(c+d\,x)^2+15\right)}{15\,d\,\left(a\tan(c+d\,x)^2+a\right)^{5/2}}$$

input
```
int(1/(a + a*tan(c + d*x)^2)^(5/2),x)
```

output
```
(tan(c + d*x)*(20*tan(c + d*x)^2 + 8*tan(c + d*x)^4 + 15))/(15*d*(a + a*ta
n(c + d*x)^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,\sqrt{\tan\left(dx+c\right)^{2}+1}\,\tan\left(dx+c\right)\left(8\tan\left(dx+c\right)^{4}+20\tan\left(dx+c\right)^{2}+\right.}{15a^{3}d\left(\tan\left(dx+c\right)^{6}+3\tan\left(dx+c\right)^{4}+3\tan\left(dx+c\right)^{2}+1\right)}$$

input
```
int(1/(a+a*tan(d*x+c)^2)^(5/2),x)
```

output
```
(sqrt(a)*sqrt(tan(c + d*x)**2 + 1)*tan(c + d*x)*(8*tan(c + d*x)**4 + 20*ta
n(c + d*x)**2 + 15))/(15*a**3*d*(tan(c + d*x)**6 + 3*tan(c + d*x)**4 + 3*t
an(c + d*x)**2 + 1))
```

3.286 $\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx=\frac{\tan(c+dx)}{7d\left(a\sec^2(c+dx)\right)^{7/2}}+\frac{6\tan(c+dx)}{35ad\left(a\sec^2(c+dx)\right)^{5/2}}+\frac{8\tan(c+dx)}{35a^2d\left(a\sec^2(c+dx)\right)^{3/2}}+\frac{16\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}$$

output
```
1/7*tan(d*x+c)/d/(a*sec(d*x+c)^2)^(7/2)+6/35*tan(d*x+c)/a/d/(a*sec(d*x+c)^
2)^(5/2)+8/35*tan(d*x+c)/a^2/d/(a*sec(d*x+c)^2)^(3/2)+16/35*tan(d*x+c)/a^3
/d/(a*sec(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx=\frac{\left(35-35\sin^2(c+dx)+21\sin^4(c+dx)-5\sin^6(c+dx)\right)\tan(c+dx)}{35a^3d\sqrt{a\sec^2(c+dx)}}$$

input
```
Integrate[(a + a*Tan[c + d*x]^2)^(-7/2),x]
```

output
```
((35 - 35*Sin[c + d*x]^2 + 21*Sin[c + d*x]^4 - 5*Sin[c + d*x]^6)*Tan[c + d
*x])/(35*a^3*d*Sqrt[a*Sec[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4140, 3042, 4610, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\tan^2(c+dx)+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\tan(c+dx)^2+a)^{7/2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{1}{(a\sec^2(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sec(c+dx)^2)^{7/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\frac{a\int \frac{1}{(a\tan^2(c+dx)+a)^{9/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{209}$$

$$\frac{a\left(\frac{6\int \frac{1}{\left(a\tan^2(c+dx)+a\right)^{7/2}}d\tan(c+dx)}{7a}+\frac{\tan(c+dx)}{7a(a\tan^2(c+dx)+a)^{7/2}}\right)}{d}$$

$$\Big\downarrow \text{209}$$

$$\frac{a\left(\frac{6\left(\frac{4\int\frac{1}{\left(a\tan^2(c+dx)+a\right)^{5/2}}d\tan(c+dx)}{5a}+\frac{\tan(c+dx)}{5a\left(a\tan^2(c+dx)+a\right)^{5/2}}\right)}{7a}+\frac{\tan(c+dx)}{7a(a\tan^2(c+dx)+a)^{7/2}}\right)}{d}$$

$\downarrow$ 209

$$\frac{a\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(a\tan^2(c+dx)+a\right)^{3/2}}d\tan(c+dx)}{3a}+\frac{\tan(c+dx)}{3a\left(a\tan^2(c+dx)+a\right)^{3/2}}\right)}{5a}+\frac{\tan(c+dx)}{5a\left(a\tan^2(c+dx)+a\right)^{5/2}}\right)}{7a}+\frac{\tan(c+dx)}{7a(a\tan^2(c+dx)+a)^{7/2}}\right)}{d}$$

$\downarrow$ 208

$$\frac{a\left(\frac{6\left(\frac{4\left(\frac{2\tan(c+dx)}{3a^2\sqrt{a\tan^2(c+dx)+a}}+\frac{\tan(c+dx)}{3a\left(a\tan^2(c+dx)+a\right)^{3/2}}\right)}{5a}+\frac{\tan(c+dx)}{5a\left(a\tan^2(c+dx)+a\right)^{5/2}}\right)}{7a}+\frac{\tan(c+dx)}{7a(a\tan^2(c+dx)+a)^{7/2}}\right)}{d}$$

input
```
Int[(a + a*Tan[c + d*x]^2)^(-7/2),x]
```

output
```
(a*(Tan[c + d*x]/(7*a*(a + a*Tan[c + d*x]^2)^(7/2)) + (6*(Tan[c + d*x]/(5*
a*(a + a*Tan[c + d*x]^2)^(5/2)) + (4*(Tan[c + d*x]/(3*a*(a + a*Tan[c + d*x
]^2)^(3/2)) + (2*Tan[c + d*x])/(3*a^2*Sqrt[a + a*Tan[c + d*x]^2])))/(5*a))
)/(7*a)))/d
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a\left(\frac{\tan(dx+c)}{7a\left(a+a\tan(dx+c)^2\right)^{\frac{7}{2}}}+\frac{\frac{6\tan(dx+c)}{35a\left(a+a\tan(dx+c)^2\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4\tan(dx+c)}{15a\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\tan(dx+c)^2}}\right)}{7a}}{a}\right)}{d}$
default	$\frac{a\left(\frac{\tan(dx+c)}{7a\left(a+a\tan(dx+c)^2\right)^{\frac{7}{2}}}+\frac{\frac{6\tan(dx+c)}{35a\left(a+a\tan(dx+c)^2\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4\tan(dx+c)}{15a\left(a+a\tan(dx+c)^2\right)^{\frac{3}{2}}}+\frac{8\tan(dx+c)}{15a^2\sqrt{a+a\tan(dx+c)^2}}\right)}{7a}}{a}\right)}{d}$
risch	$-\frac{i e^{8i(dx+c)}}{896d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a^3}-\frac{35i e^{2i(dx+c)}}{128d\sqrt{\frac{a\,e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)a^3}+\frac{35i}{128a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{}{(e}}}$

input
```
int(1/(a+a*tan(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(1/7/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(7/2)+6/7/a*(1/5/a*tan(d*x+c)/(
a+a*tan(d*x+c)^2)^(5/2)+4/5/a*(1/3/a*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(3/2)+2
/3/a^2*tan(d*x+c)/(a+a*tan(d*x+c)^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+a\tan^2(c+dx)\right)^{7/2}}\,dx = \frac{\left(16\,\tan\left(dx+c\right)^7+56\,\tan\left(dx+c\right)^5+70\,\tan\left(dx+c\right)^3+35\,\tan\left(dx+c\right)\right.}{35\left(a^4d\tan\left(dx+c\right)^8+4\,a^4d\tan\left(dx+c\right)^6+6\,a^4d\tan\left(dx+c\right)^4+4\,a^4d\right.}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/35*(16*tan(d*x + c)^7 + 56*tan(d*x + c)^5 + 70*tan(d*x + c)^3 + 35*tan(d
*x + c))*sqrt(a*tan(d*x + c)^2 + a)/(a^4*d*tan(d*x + c)^8 + 4*a^4*d*tan(d*
x + c)^6 + 6*a^4*d*tan(d*x + c)^4 + 4*a^4*d*tan(d*x + c)^2 + a^4*d)
```

Sympy [F]

$$\int \frac{1}{(a+a\tan^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a\tan^2\left(c+dx\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+a*tan(d*x+c)**2)**(7/2),x)
```

output
```
Integral((a*tan(c + d*x)**2 + a)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{1}{(a+a\tan^2(c+dx))^{7/2}}\,dx = \frac{5\sin\left(7\,dx+7\,c\right)+49\sin\left(5\,dx+5\,c\right)+245\sin\left(3\,dx+3\,c\right)+1225\sin\left(d\right.}{2240\,a^{\frac{7}{2}}d}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2240*(5*sin(7*d*x + 7*c) + 49*sin(5*d*x + 5*c) + 245*sin(3*d*x + 3*c) +
1225*sin(d*x + c))/(a^(7/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(102) = 204$.

Time = 2.31 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.25

$$\int \frac{1}{(a+a\tan^2(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*tan(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/35*(16*sgn(tan(d*x)*tan(c) - 1)*tan(c)^7 + 56*sgn(tan(d*x)*tan(c) - 1)*
tan(c)^5 + 70*sgn(tan(d*x)*tan(c) - 1)*tan(c)^3 + ((56*sgn(tan(d*x)*tan(c)
 - 1)*tan(c)^7 + 196*sgn(tan(d*x)*tan(c) - 1)*tan(c)^5 + 245*sgn(tan(d*x)*
tan(c) - 1)*tan(c)^3 + (245*sgn(tan(d*x)*tan(c) - 1)*tan(c)^2 + (70*sgn(ta
n(d*x)*tan(c) - 1)*tan(c)^7 + 245*sgn(tan(d*x)*tan(c) - 1)*tan(c)^5 + (245
*sgn(tan(d*x)*tan(c) - 1)*tan(c)^4 + 196*sgn(tan(d*x)*tan(c) - 1)*tan(c)^2
 + (35*tan(c)^7/sgn(tan(d*x)*tan(c) - 1) + (35*sgn(tan(d*x)*tan(c) - 1)*ta
n(c)^6 + 70*sgn(tan(d*x)*tan(c) - 1)*tan(c)^4 + 56*sgn(tan(d*x)*tan(c) - 1
)*tan(c)^2 + 16*sgn(tan(d*x)*tan(c) - 1))*tan(d*x))*tan(d*x) + 56*sgn(tan(
d*x)*tan(c) - 1))*tan(d*x))*tan(d*x) + 70*sgn(tan(d*x)*tan(c) - 1))*tan(d*
x))*tan(d*x) + 35/sgn(tan(d*x)*tan(c) - 1))*tan(d*x) + 35*sgn(tan(d*x)*tan
(c) - 1)*tan(c))/((a*tan(d*x)^2*tan(c)^2 + a*tan(d*x)^2 + a*tan(c)^2 + a)^
(7/2)*d)
```

Mupad [B] (verification not implemented)

Time = 7.40 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+a\tan^2(c+dx))^{7/2}}\,dx = \frac{16\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{35\,a^4\,d\,\left(\tan\left(c+d\,x\right)^2+1\right)} + \frac{8\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{35\,a^4\,d\,{\left(\tan\left(c+d\,x\right)^2+1\right)}^2} + \frac{6\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{35\,a^4\,d\,{\left(\tan\left(c+d\,x\right)^2+1\right)}^3} + \frac{\tan(c+d\,x)\,\sqrt{a\tan\left(c+d\,x\right)^2+a}}{7\,a^4\,d\,{\left(\tan\left(c+d\,x\right)^2+1\right)}^4}$$

input
```
int(1/(a + a*tan(c + d*x)^2)^(7/2),x)
```

output
```
(16*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c + d*x)^2 +
 1)) + (8*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(tan(c + d*
x)^2 + 1)^2) + (6*tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(35*a^4*d*(ta
n(c + d*x)^2 + 1)^3) + (tan(c + d*x)*(a + a*tan(c + d*x)^2)^(1/2))/(7*a^4*
d*(tan(c + d*x)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+a\tan^2(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\,\sqrt{\tan(dx+c)^2+1}\,\tan(dx+c)\left(16\tan(dx+c)^6+56\tan(dx+c)^4+\cdots\right.}{35a^4d\left(\tan(dx+c)^8+4\tan(dx+c)^6+6\tan(dx+c)^4+4\tan\cdots\right.}$$

input
```
int(1/(a+a*tan(d*x+c)^2)^(7/2),x)
```

output
```
(sqrt(a)*sqrt(tan(c + d*x)**2 + 1)*tan(c + d*x)*(16*tan(c + d*x)**6 + 56*t
an(c + d*x)**4 + 70*tan(c + d*x)**2 + 35))/(35*a**4*d*(tan(c + d*x)**8 + 4
*tan(c + d*x)**6 + 6*tan(c + d*x)**4 + 4*tan(c + d*x)**2 + 1))
```

3.287 $\int \left(1+\tan^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 22

$$\int \left(1+\tan^2(x)\right)^{3/2} dx = \frac{1}{2}\text{arcsinh}(\tan(x)) + \frac{1}{2}\sqrt{\sec^2(x)}\tan(x)$$

output
```
1/2*arcsinh(tan(x))+1/2*(sec(x)^2)^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \left(1+\tan^2(x)\right)^{3/2} dx = \frac{\sec(x)(\text{arctanh}(\sin(x)) + \sec(x)\tan(x))}{2\sqrt{\sec^2(x)}}$$

input
```
Integrate[(1 + Tan[x]^2)^(3/2),x]
```

output
```
(Sec[x]*(ArcTanh[Sin[x]] + Sec[x]*Tan[x]))/(2*Sqrt[Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4140, 3042, 4610, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tan^2(x)+1\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\tan(x)^2+1\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \sec^2(x)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\sec(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4610}$$

$$\int \sqrt{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\tan^2(x)+1}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+1}\tan(x)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\text{arcsinh}(\tan(x))+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+1}$$

input
```
Int[(1 + Tan[x]^2)^(3/2),x]
```

output
```
ArcSinh[Tan[x]]/2 + (Tan[x]*Sqrt[1 + Tan[x]^2])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\tan(x)\sqrt{1+\tan(x)^2}}{2}+\frac{\operatorname{arcsinh}(\tan(x))}{2}$
default	$\frac{\tan(x)\sqrt{1+\tan(x)^2}}{2}+\frac{\operatorname{arcsinh}(\tan(x))}{2}$
risch	$-\frac{i\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}-1)}{e^{2ix}+1}-\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)+\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)$

input
```
int((1+tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x)*(1+tan(x)^2)^(1/2)+1/2*arcsinh(tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int \left(1+\tan^2(x)\right)^{3/2} dx = \frac{1}{2}\sqrt{\tan{(x)}^2+1}\tan{(x)} + \frac{1}{4}\log\left(\frac{\tan{(x)}^2+\sqrt{\tan{(x)}^2+1}\tan{(x)}+1}{\tan{(x)}^2+1}\right) - \frac{1}{4}\log\left(\frac{\tan{(x)}^2-\sqrt{\tan{(x)}^2+1}\tan{(x)}+1}{\tan{(x)}^2+1}\right)$$

input
```
integrate((1+tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/4*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan
(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) +
 1)/(tan(x)^2 + 1))
```

Sympy [F]

$$\int \left(1+\tan^2(x)\right)^{3/2} dx = \int \left(\tan^2{(x)}+1\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((1+tan(x)**2)**(3/2),x)
```

output
```
Integral((tan(x)**2 + 1)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1+\tan^2(x)\right)^{3/2}\,dx=\frac{1}{2}\sqrt{\tan\left(x\right)^{2}+1}\tan\left(x\right)+\frac{1}{2}\,\operatorname{arsinh}\left(\tan\left(x\right)\right)$$

input
```
integrate((1+tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*arcsinh(tan(x))
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \left(1+\tan^2(x)\right)^{3/2}\,dx=\frac{1}{2}\sqrt{\tan\left(x\right)^{2}+1}\tan\left(x\right)-\frac{1}{2}\log\left(\sqrt{\tan\left(x\right)^{2}+1}-\tan\left(x\right)\right)$$

input
```
integrate((1+tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(tan(x)^2 + 1)*tan(x) - 1/2*log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1+\tan^2(x)\right)^{3/2}\,dx=\frac{\operatorname{asinh}(\tan(x))}{2}+\frac{\tan(x)\,\sqrt{\tan\left(x\right)^{2}+1}}{2}$$

input
```
int((tan(x)^2 + 1)^(3/2),x)
```

output
```
asinh(tan(x))/2 + (tan(x)*(tan(x)^2 + 1)^(1/2))/2
```

Reduce [F]

$$\int \left(1+\tan^2(x)\right)^{3/2} dx = \int \sqrt{\tan (x)^2+1}dx + \int \sqrt{\tan (x)^2+1}\, \tan (x)^2 \, dx$$

input
```
int((1+tan(x)^2)^(3/2),x)
```

output
```
int(sqrt(tan(x)**2 + 1),x) + int(sqrt(tan(x)**2 + 1)*tan(x)**2,x)
```

3.288 $\int \sqrt{1+\tan^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 3

$$\int \sqrt{1+\tan^2(x)}\,dx = \operatorname{arcsinh}(\tan(x))$$

output
```
arcsinh(tan(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 14 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 4.67

$$\int \sqrt{1+\tan^2(x)}\,dx = \coth^{-1}(\sin(x))\cos(x)\sqrt{\sec^2(x)}$$

input
```
Integrate[Sqrt[1 + Tan[x]^2],x]
```

output
```
ArcCoth[Sin[x]]*Cos[x]*Sqrt[Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4140, 3042, 4610, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(x)^2+1}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \sqrt{\sec^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sec(x)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\sqrt{\tan^2(x)+1}}d\tan(x)$$

$$\Big\downarrow \text{222}$$

$$\text{arcsinh}(\tan(x))$$

input
```
Int[Sqrt[1 + Tan[x]^2],x]
```

output
```
ArcSinh[Tan[x]]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\operatorname{arcsinh}(\tan(x))$	4
default	$\operatorname{arcsinh}(\tan(x))$	4
risch	$2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)-2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)$	62

input
```
int((1+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(tan(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(3) = 6$.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 20.00

$$\int \sqrt{1+\tan^2(x)}\,dx = \frac{1}{2}\log\left(\frac{\tan(x)^2 + \sqrt{\tan(x)^2+1}\tan(x)+1}{\tan(x)^2+1}\right) - \frac{1}{2}\log\left(\frac{\tan(x)^2 - \sqrt{\tan(x)^2+1}\tan(x)+1}{\tan(x)^2+1}\right)$$

input
```
integrate((1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log((tan(x)^2 + sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1)) - 1/2*l
og((tan(x)^2 - sqrt(tan(x)^2 + 1)*tan(x) + 1)/(tan(x)^2 + 1))
```

Sympy [F]

$$\int \sqrt{1+\tan^2(x)}\,dx = \int \sqrt{\tan^2(x)+1}\,dx$$

input
```
integrate((1+tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(tan(x)**2 + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sqrt{1+\tan^2(x)}\,dx = \operatorname{arsinh}(\tan(x))$$

input
```
integrate((1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(tan(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.

Time = 0.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 5.33

$$\int \sqrt{1+\tan^2(x)}\,dx = -\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

input
```
integrate((1+tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sqrt{1+\tan^2(x)}\,dx = \operatorname{asinh}(\tan(x))$$

input
```
int((tan(x)^2 + 1)^(1/2),x)
```

output
```
asinh(tan(x))
```

Reduce [F]

$$\int \sqrt{1+\tan^2(x)}\,dx = \int \sqrt{\tan(x)^2+1}dx$$

input

```
int((1+tan(x)^2)^(1/2),x)
```

output

```
int(sqrt(tan(x)**2 + 1),x)
```

3.289 $\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

output
```
tan(x)/(sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

input
```
Integrate[1/Sqrt[1 + Tan[x]^2],x]
```

output
```
Tan[x]/Sqrt[Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\tan^2(x)+1}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{\tan(x)^2+1}}dx$$
$$\Big\downarrow \text{ 4140}$$
$$\int \frac{1}{\sqrt{\sec^2(x)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{\sec(x)^2}}dx$$
$$\Big\downarrow \text{ 4610}$$
$$\int \frac{1}{\left(\tan^2(x)+1\right)^{3/2}}d\tan(x)$$
$$\Big\downarrow \text{ 208}$$
$$\frac{\tan(x)}{\sqrt{\tan^2(x)+1}}$$

input
```
Int[1/Sqrt[1 + Tan[x]^2],x]
```

output
```
Tan[x]/Sqrt[1 + Tan[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\tan(x)}{\sqrt{1+\tan(x)^2}}$	12
default	$\frac{\tan(x)}{\sqrt{1+\tan(x)^2}}$	12
parallelrisch	$\frac{\tan(x)}{\sqrt{1+\tan(x)^2}}$	12
risch	$-\frac{i e^{2ix}}{2\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)}+\frac{i}{2\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)}$	65

input
```
int(1/(1+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1+tan(x)^2)^(1/2)*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
tan(x)/sqrt(tan(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{\tan^2(x)+1}}$$

input
```
integrate(1/(1+tan(x)**2)**(1/2),x)
```

output
```
tan(x)/sqrt(tan(x)**2 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
tan(x)/sqrt(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate(1/(1+tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
tan(x)/sqrt(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \tan(x)\ \sqrt{\cos(x)^2}$$

input
```
int(1/(tan(x)^2 + 1)^(1/2),x)
```

output
```
tan(x)*(cos(x)^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{1+\tan^2(x)}}\,dx = \frac{\sqrt{\tan(x)^2+1}\ \tan(x)}{\tan(x)^2+1}$$

input
```
int(1/(1+tan(x)^2)^(1/2),x)
```

output
```
(sqrt(tan(x)**2 + 1)*tan(x))/(tan(x)**2 + 1)
```

3.290 $\int (-1-\tan^2(x))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int (-1-\tan^2(x))^{3/2}\,dx = \frac{1}{2}\arctan\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right) - \frac{1}{2}\sqrt{-\sec^2(x)}\tan(x)$$

output
```
1/2*arctan(tan(x)/(-sec(x)^2)^(1/2))-1/2*(-sec(x)^2)^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int (-1-\tan^2(x))^{3/2}\,dx = -\frac{1}{2}\sqrt{-\sec^2(x)}(\operatorname{arctanh}(\sin(x))\cos(x)+\tan(x))$$

input
```
Integrate[(-1 - Tan[x]^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[-Sec[x]^2]*(ArcTanh[Sin[x]]*Cos[x] + Tan[x]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4140, 3042, 4610, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\tan^2(x)-1\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\tan(x)^2-1\right)^{3/2}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \left(-\sec^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\sec(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4610}$$

$$-\int \sqrt{-\tan^2(x)-1}d\tan(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-\tan^2(x)-1}}d\tan(x)-\frac{1}{2}\tan(x)\sqrt{-\tan^2(x)-1}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\int \frac{1}{\frac{\tan^2(x)}{-\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-\tan^2(x)-1}}-\frac{1}{2}\tan(x)\sqrt{-\tan^2(x)-1}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\frac{\tan(x)}{\sqrt{-\tan^2(x)-1}}\right)-\frac{1}{2}\tan(x)\sqrt{-\tan^2(x)-1}$$

input
```
Int[(-1 - Tan[x]^2)^(3/2),x]
```

output
```
ArcTan[Tan[x]/Sqrt[-1 - Tan[x]^2]]/2 - (Tan[x]*Sqrt[-1 - Tan[x]^2])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{\tan(x)\sqrt{-1-\tan(x)^2}}{2}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{-1-\tan(x)^2}}\right)}{2}$
default	$-\frac{\tan(x)\sqrt{-1-\tan(x)^2}}{2}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{-1-\tan(x)^2}}\right)}{2}$
risch	$\frac{i\sqrt{-\frac{e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,(e^{2ix}-1)}{e^{2ix}+1}-\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)+\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x$

input
```
int((-1-tan(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tan(x)*(-1-tan(x)^2)^(1/2)+1/2*arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (-1-\tan^2(x))^{3/2}\,dx=\frac{\left(-i\,e^{(4i\,x)}-2i\,e^{(2i\,x)}-i\right)\log\left(e^{(i\,x)}+i\right)+\left(i\,e^{(4i\,x)}+2i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}-i\right)-2\,e^{(3i}}{2\left(e^{(4i\,x)}+2\,e^{(2i\,x)}+1\right)}$$

input
```
integrate((-1-tan(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((-I*e^(4*I*x) - 2*I*e^(2*I*x) - I)*log(e^(I*x) + I) + (I*e^(4*I*x) +
2*I*e^(2*I*x) + I)*log(e^(I*x) - I) - 2*e^(3*I*x) + 2*e^(I*x))/(e^(4*I*x)
+ 2*e^(2*I*x) + 1)
```

Sympy [F]

$$\int \left(-1-\tan^2(x)\right)^{3/2} dx = \int \left(-\tan^2(x)-1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-tan(x)**2)**(3/2),x)
```

output
```
Integral((-tan(x)**2 - 1)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \left(-1-\tan^2(x)\right)^{3/2} dx = -\frac{1}{2}\sqrt{-\tan(x)^2-1}\tan(x) - \frac{1}{2}i \operatorname{arsinh}(\tan(x))$$

input
```
integrate((-1-tan(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-tan(x)^2 - 1)*tan(x) - 1/2*I*arcsinh(tan(x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \left(-1-\tan^2(x)\right)^{3/2} dx = -\frac{1}{2}i\sqrt{\tan(x)^2+1}\tan(x) + \frac{1}{2}i \log\left(\sqrt{\tan(x)^2+1} - \tan(x)\right)$$

input
```
integrate((-1-tan(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*I*log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(-1-\tan^2(x)\right)^{3/2} dx = \frac{\operatorname{atan}\left(\frac{\tan(x)}{\sqrt{-\tan(x)^2-1}}\right)}{2} - \frac{\tan(x)\,\sqrt{-\tan\left(x\right)^2-1}}{2}$$

input
```
int((- tan(x)^2 - 1)^(3/2),x)
```

output
```
atan(tan(x)/(- tan(x)^2 - 1)^(1/2))/2 - (tan(x)*(- tan(x)^2 - 1)^(1/2))/2
```

Reduce [F]

$$\int \left(-1-\tan^2(x)\right)^{3/2} dx = -i\left(\int \sqrt{\tan\left(x\right)^2+1}dx + \int \sqrt{\tan\left(x\right)^2+1}\,\tan\left(x\right)^2 dx\right)$$

input
```
int((-1-tan(x)^2)^(3/2),x)
```

output
```
- i*(int(sqrt(tan(x)**2 + 1),x) + int(sqrt(tan(x)**2 + 1)*tan(x)**2,x))
```

3.291 $\int \sqrt{-1-\tan^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 16

$$\int \sqrt{-1-\tan^2(x)}\,dx=-\arctan\left(\frac{\tan(x)}{\sqrt{-\sec^2(x)}}\right)$$

output
```
-arctan(tan(x)/(-sec(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{-1-\tan^2(x)}\,dx=\coth^{-1}(\sin(x))\cos(x)\sqrt{-\sec^2(x)}$$

input
```
Integrate[Sqrt[-1 - Tan[x]^2],x]
```

output
```
ArcCoth[Sin[x]]*Cos[x]*Sqrt[-Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4140, 3042, 4610, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\tan^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan(x)^2-1}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \sqrt{-\sec^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\sec(x)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$-\int \frac{1}{\sqrt{-\tan^2(x)-1}}d\tan(x)$$

$$\Big\downarrow \text{224}$$

$$-\int \frac{1}{\frac{\tan^2(x)}{-\tan^2(x)-1}+1}d\frac{\tan(x)}{\sqrt{-\tan^2(x)-1}}$$

$$\Big\downarrow \text{216}$$

$$-\arctan\left(\frac{\tan(x)}{\sqrt{-\tan^2(x)-1}}\right)$$

input
```
Int[Sqrt[-1 - Tan[x]^2],x]
```

output
```
-ArcTan[Tan[x]/Sqrt[-1 - Tan[x]^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\arctan\left(\frac{\tan(x)}{\sqrt{-1-\tan(x)^{2}}}\right)$	17
default	$-\arctan\left(\frac{\tan(x)}{\sqrt{-1-\tan(x)^{2}}}\right)$	17
risch	$2\sqrt{-\frac{e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}+i\right)\cos\left(x\right)-2\sqrt{-\frac{e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}-i\right)\cos\left(x\right)$	64

input
```
int((-1-tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(tan(x)/(-1-tan(x)^2)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \sqrt{-1-\tan^2(x)}\,dx = i\log\left(e^{(ix)}+i\right) - i\log\left(e^{(ix)}-i\right)$$

input
```
integrate((-1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
I*log(e^(I*x) + I) - I*log(e^(I*x) - I)
```

Sympy [F]

$$\int \sqrt{-1-\tan^2(x)}\,dx = \int \sqrt{-\tan^2(x)-1}\,dx$$

input
```
integrate((-1-tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-tan(x)**2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \sqrt{-1-\tan^2(x)}\,dx = \arctan\left(\cos(x),\sin(x)+1\right) + \arctan\left(\cos(x),-\sin(x)+1\right)$$

input
```
integrate((-1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arctan2(cos(x), sin(x) + 1) + arctan2(cos(x), -sin(x) + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{-1-\tan^2(x)}\,dx = -i\log\left(\sqrt{\tan(x)^2+1}-\tan(x)\right)$$

input
```
integrate((-1-tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-I*log(sqrt(tan(x)^2 + 1) - tan(x))
```

Mupad [B] (verification not implemented)

Time = 7.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{-1-\tan^2(x)}\,dx = -\mathrm{atan}\left(\frac{\tan(x)}{\sqrt{-\tan(x)^2-1}}\right)$$

input
```
int((- tan(x)^2 - 1)^(1/2),x)
```

output
```
-atan(tan(x)/(- tan(x)^2 - 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{-1-\tan^2(x)}\,dx = \left(\int \sqrt{\tan(x)^2+1}dx\right)i$$

input
```
int((-1-tan(x)^2)^(1/2),x)
```

output
```
int(sqrt(tan(x)**2 + 1),x)*i
```

3.292 $\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

output
```
tan(x)/(-sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{-\sec^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 - Tan[x]^2],x]
```

output
```
Tan[x]/Sqrt[-Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\tan^2(x)-1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-\tan(x)^2-1}}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \frac{1}{\sqrt{-\sec^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-\sec(x)^2}}dx$$

$$\Big\downarrow \text{ 4610}$$

$$-\int \frac{1}{\left(-\tan^2(x)-1\right)^{3/2}}d\tan(x)$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\tan(x)}{\sqrt{-\tan^2(x)-1}}$$

input
```
Int[1/Sqrt[-1 - Tan[x]^2],x]
```

output
```
Tan[x]/Sqrt[-1 - Tan[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\tan(x)}{\sqrt{-1-\tan(x)^2}}$	14
default	$\frac{\tan(x)}{\sqrt{-1-\tan(x)^2}}$	14
risch	$-\frac{i e^{2ix}}{2\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}+\frac{i}{2\sqrt{-\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}$	67

input
```
int(1/(-1-tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
tan(x)/(-1-tan(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = -\frac{1}{2}\left(e^{(2i\,x)}-1\right)e^{(-i\,x)}$$

input
```
integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(e^(2*I*x) - 1)*e^(-I*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tan^2{(x)}-1}}\,dx$$

input
```
integrate(1/(-1-tan(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-tan(x)**2 - 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tan(x)^2-1}}\,dx$$

input
```
integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-tan(x)^2 - 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = -\frac{i\tan(x)}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate(1/(-1-tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-I*tan(x)/sqrt(tan(x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = -\frac{\sqrt{2}\sin(2x)\ 1\mathrm{i}}{2\sqrt{2\cos(x)^2}}$$

input
```
int(1/(- tan(x)^2 - 1)^(1/2),x)
```

output
```
-(2^(1/2)*sin(2*x)*1i)/(2*(2*cos(x)^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{-1-\tan^2(x)}}\,dx = -\frac{\sqrt{\tan(x)^2+1}\,\tan(x)\,i}{\tan(x)^2+1}$$

input
```
int(1/(-1-tan(x)^2)^(1/2),x)
```

output
```
( - sqrt(tan(x)**2 + 1)*tan(x)*i)/(tan(x)**2 + 1)
```

3.293 $\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}$$

output
```
-(a-b)^(1/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a+b*tan(f*x+
e)^2)^(1/2)/f-1/3*(a+b)*(a+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*tan(f*x+e)
^2)^(5/2)/b^2/f
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{-15\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\frac{\sqrt{a+b\tan^2(e+fx)}(-2a^2-5ab+15b^2+(a-5b)b\tan^2(e+fx)+3b^2\tan^4(e+fx))}{b^2}}{15f}$$

input
```
Integrate[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-15*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (Sqrt[a
 + b*Tan[e + f*x]^2]*(-2*a^2 - 5*a*b + 15*b^2 + (a - 5*b)*b*Tan[e + f*x]^2
 + 3*b^2*Tan[e + f*x]^4))/b^2)/(15*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(e+fx)^5\sqrt{a+b\tan(e+fx)^2}dx$$
$$\Big\downarrow \text{4153}$$
$$\frac{\int \frac{\tan^5(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \ 99$$

$$\frac{\int \left(\frac{(b\tan^2(e+fx)+a)^{3/2}}{b}+\frac{(-a-b)\sqrt{b\tan^2(e+fx)+a}}{b}+\frac{\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \ 2009$$

$$\frac{-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\frac{2(a+b\tan^2(e+fx))^{5/2}}{5b^2}-\frac{2(a+b)(a+b\tan^2(e+fx))^{3/2}}{3b^2}+2\sqrt{a+b\tan^2(e+fx)}}{2f}$$

input
```
Int[Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + 2*Sqrt[a
 + b*Tan[e + f*x]^2] - (2*(a + b)*(a + b*Tan[e + f*x]^2)^(3/2))/(3*b^2) +
(2*(a + b*Tan[e + f*x]^2)^(5/2))/(5*b^2))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)^2\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{15b^2}+b\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)+\frac{a\arctan\left(\right.}{}}{f}$
default	$\frac{\frac{\tan(fx+e)^2\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{15b^2}+b\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}\right)+\frac{a\arctan\left(\right.}{}}{f}$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/5*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2)/b-2/15*a/b^2*(a+b*tan(f*x+e
)^2)^(3/2)+b*(1/b*(a+b*tan(f*x+e)^2)^(1/2)-1/(-a+b)^(1/2)*arctan((a+b*tan(
f*x+e)^2)^(1/2)/(-a+b)^(1/2)))+a/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1
/2)/(-a+b)^(1/2))-1/3*(a+b*tan(f*x+e)^2)^(3/2)/b)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.96

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{15\sqrt{a-b}b^2\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)}{60\,b^2f}\right.$$
$$-\frac{15\sqrt{-a+b}b^2\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right)-2\left(3\,b^2\tan\left(fx+e\right)^4+(ab-5\,b^2)\right.}{30\,b^2f}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/60*(15*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan
(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))
+ 4*(3*b^2*tan(f*x + e)^4 + (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b +
 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), -1/30*(15*sqrt(-a + b)*b^2*a
rctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
 + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) - 2*(3*b^2*tan(f*x + e)^4
+ (a*b - 5*b^2)*tan(f*x + e)^2 - 2*a^2 - 5*a*b + 15*b^2)*sqrt(b*tan(f*x +
e)^2 + a))/(b^2*f)]
```

Sympy [F]

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2\left(e+fx\right)}\tan^5\left(e+fx\right)dx$$

input
```
integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**5, x)
```

Maxima [F]

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan\left(fx+e\right)^2+a}\tan\left(fx+e\right)^5\,dx$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 16.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}{5\,b^2\,f} - \left(\frac{2\,a}{3\,b^2\,f} - \frac{a-b}{3\,b^2\,f}\right)\left(b\tan(e+f\,x)^2+a\right)^{3/2}$$
$$- \sqrt{b\tan\left(e+f\,x\right)^2+a}\left(\left(\frac{2\,a}{b^2\,f} - \frac{a-b}{b^2\,f}\right)(a-b) - \frac{a^2}{b^2\,f}\right)$$
$$+ \frac{\operatorname{atan}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}}{\sqrt{a-b}}\right)\sqrt{a-b}\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
(atan(((a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(1/2))*(a - b)^(1/2)*1i)/f
 - ((2*a)/(3*b^2*f) - (a - b)/(3*b^2*f))*(a + b*tan(e + f*x)^2)^(3/2) - (a
 + b*tan(e + f*x)^2)^(1/2)*(((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b) - a^
2/(b^2*f)) + (a + b*tan(e + f*x)^2)^(5/2)/(5*b^2*f)
```

Reduce [F]

$$\int \tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{3\sqrt{\tan\left(fx+e\right)^2b+a}\,\tan\left(fx+e\right)^4b^2 + \sqrt{\tan\left(fx+e\right)^2b+a}\,\tan\left(fx+e\right)^2ab - 5\sqrt{\tan\left(fx+e\right)^2b+}}{}$$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
(3*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**4*b**2 + sqrt(tan(e + f*x)**2
*b + a)*tan(e + f*x)**2*a*b - 5*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**
2*b**2 - 2*sqrt(tan(e + f*x)**2*b + a)*a**2 + 10*sqrt(tan(e + f*x)**2*b +
a)*a*b - 15*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x
)**2*b + a),x)*a*b**2*f + 15*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)
**3)/(tan(e + f*x)**2*b + a),x)*b**3*f)/(15*b**2*f)
```

3.294 $\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{a-b}\,\text{barctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3bf}$$

output
```
(a-b)^(1/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-(a+b*tan(f*x+e
)^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{3\sqrt{a-b}\,b\,\text{barctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a+b\tan^2(e+fx)}\left(a-3b+b\tan^2(e+fx)\right)}{3bf}$$

input
```
Integrate[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(3*Sqrt[a - b]*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a
+ b*Tan[e + f*x]^2]*(a - 3*b + b*Tan[e + f*x]^2))/(3*b*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4153, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^3(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{2\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b}-\int \frac{\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{-(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)+\frac{2\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b}-2\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{2(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}+\frac{2\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b}-2\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\frac{2\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b}-2\sqrt{a+b\tan^2(e+fx)}}{2f}$$

input
```
Int[Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - 2*Sqrt[a
+ b*Tan[e + f*x]^2] + (2*(a + b*Tan[e + f*x]^2)^(3/2))/(3*b))/(2*f)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

method	result	s
derivativedivides	$\frac{\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{3bf}-\frac{\sqrt{a+b\tan(fx+e)^2}}{f}+\frac{b\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}-\frac{a\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	1
default	$\frac{\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{3bf}-\frac{\sqrt{a+b\tan(fx+e)^2}}{f}+\frac{b\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}-\frac{a\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	1

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a+b*tan(f*x+e)^2)^(3/2)/b/f-(a+b*tan(f*x+e)^2)^(1/2)/f+1/f*b/(-a+b)^(
1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/f*a/(-a+b)^(1/2)*arct
an((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.18

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{3\sqrt{a-b}b\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)+}{12\,bf}\right.$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*
x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4
*(b*tan(f*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), 1/6*(3*sq
rt(-a + b)*b*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) + 2*(b*tan(f
*x + e)^2 + a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/(b*f)]
```

Sympy [F]

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\tan^3{(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\tan(fx+e)^3\,dx$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sqrt{a-b}}\right)\sqrt{a-b}}{f} - \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{f} + \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{3\,b\,f}$$

input
```
int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
(atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))*(a - b)^(1/2))/f - (a +
 b*tan(e + f*x)^2)^(1/2)/f + (a + b*tan(e + f*x)^2)^(3/2)/(3*b*f)
```

Reduce [F]

$$\int \tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^2 b-2\sqrt{\tan(fx+e)^2 b+a}\,a+3\left(\int\frac{\sqrt{\tan(fx+e)^2b+a}\,\tan(fx+e)^3}{\tan(fx+e)^2b+a}dx\right)abf}{3bf}$$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*b - 2*sqrt(tan(e + f*x)**2*b
+ a)*a + 3*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x)
**2*b + a),x)*a*b*f - 3*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/
(tan(e + f*x)**2*b + a),x)*b**2*f)/(3*b*f)
```

3.295 $\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\tan^2(e+fx)}}{f}$$

output
```
-(a-b)^(1/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a+b*tan(f*x+
e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{-\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Integrate[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a +
 b*Tan[e + f*x]^2])/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4153, 353, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\tan(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 353$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 60$$

$$\frac{(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)+2\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}+2\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{a+b\tan^2(e+fx)}-2\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}$$

input
```
Int[Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + 2*Sqrt[a
 + b*Tan[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$\frac{\sqrt{a+b\tan(fx+e)^2}}{f}-\frac{b\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}+\frac{a\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	91
default	$\frac{\sqrt{a+b\tan(fx+e)^2}}{f}-\frac{b\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}+\frac{a\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	91

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a+b*tan(f*x+e)^2)^(1/2)/f-1/f*b/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1
/2)/(-a+b)^(1/2))+1/f*a/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.87

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4\,ab-3\,b^2\right)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)+4\,\sqrt{}}{4\,f}\right.$$
$$\left. -\frac{\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left(\left(ab-b^2\right)\tan(fx+e)^2+a^2-ab\right)}\right)-2\sqrt{b\tan\left(fx+e\right)^2+a}}{2\,f}\right]$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e
)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b
) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt
(b*tan(f*x + e)^2 + a))/f, -1/2*(sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^
2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x
+ e)^2 + a^2 - a*b)) - 2*sqrt(b*tan(f*x + e)^2 + a))/f]
```

Sympy [F]

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2\left(e+fx\right)}\tan\left(e+fx\right)dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x), x)
```

Maxima [F]

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^2+a}\tan{(fx+e)}\,dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{b\tan{(e+f\,x)}^2+a}}{f} - \frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan{(e+f\,x)}^2+a}}{\sqrt{a-b}}\right)\sqrt{a-b}}{f}$$

input
```
int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
(a + b*tan(e + f*x)^2)^(1/2)/f - (atanh((a + b*tan(e + f*x)^2)^(1/2)/(a -
b)^(1/2))*(a - b)^(1/2))/f
```

Reduce [F]

$$\int \tan(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{\sqrt{\tan (fx+e)^2 b+a}\,a-\left(\int \frac{\sqrt{\tan(fx+e)^2b+a}\,\tan(fx+e)^3}{\tan(fx+e)^2b+a}dx\right)abf+\left(\int \frac{\sqrt{\tan(fx+e)^2b+a}\,\tan(fx+e)^3}{\tan(fx+e)^2b+a}dx\right)b^2f}{bf}$$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(tan(e + f*x)**2*b + a)*a - int((sqrt(tan(e + f*x)**2*b + a)*tan(e +
f*x)**3)/(tan(e + f*x)**2*b + a),x)*a*b*f + int((sqrt(tan(e + f*x)**2*b +
a)*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*b**2*f)/(b*f)
```

3.296 $\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

output
```
-a^(1/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f+(a-b)^(1/2)*arctanh((
a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

input
```
Integrate[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/f
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4153, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{94}$$

$$\frac{a\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{2(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

input
```
Int[Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a - b]*Ar
cTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/(2*f)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(62) = 124$.

Time = 6.56 (sec) , antiderivative size = 569, normalized size of antiderivative = 7.69

method	result
default	$\frac{\left(2\ln\left(4\sqrt{a-b}\,\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\cos(fx+e)+4\sqrt{a-b}\,\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}+4a\cos(fx+e)-4\cos(fx+e)b\right)a^{\frac{3}{2}}+\ln \cdots\right.}{\cdots}$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^(1/2)/(a-b)^(1/2)*(2*ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e
)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*a^(3/2
)+ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*(a-b)^(1/2)*a-a*ln(2
*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f
*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*co
s(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*(a-b)^(1/2)-2*ln(4*(a-b)^(1/2)*((a*cos
(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)
*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4
*cos(f*x+e)*b)*a^(1/2)*b)*(a+b*tan(f*x+e)^2)^(1/2)*cos(f*x+e)/(cos(f*x+e)+
1)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 362, normalized size of antiderivative = 4.89

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a-b}\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)+\sqrt{a}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a}+2a}{\tan(fx+e)^2}\right)}{2f},\right.$$
$$-\frac{2\sqrt{-a+b}\arctan\left(\frac{\sqrt{-a+b}}{\sqrt{b\tan(fx+e)^2+a}}\right)-\sqrt{a}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a}+2a}{\tan(fx+e)^2}\right)}{2f}, \frac{2\sqrt{-a}\arctan\left(\frac{}{\sqrt{}}\right.}{}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqr
t(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + sqrt(a)*log((b*tan(f*x + e)^2
- 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/f, -1/2*(2*
sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a)) - sqrt(a)*log
((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x +
 e)^2))/f, 1/2*(2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a)) + s
qrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b
) + 2*a - b)/(tan(f*x + e)^2 + 1)))/f, (sqrt(-a)*arctan(sqrt(-a)/sqrt(b*ta
n(f*x + e)^2 + a)) - sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^
2 + a)))/f]
```

Sympy [F]

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2(e+fx)}\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionDegree mismatch inside factorisation over extensionDegree
mismatch
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sqrt{a}}\right)}{f} - \frac{\mathrm{atanh}\left(\frac{a\,b^3\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a-b}}{a\,b^4-a^2\,b^3}\right)\,\sqrt{a-b}}{f}$$

input
```
int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
- (a^(1/2)*atanh((a + b*tan(e + f*x)^2)^(1/2)/a^(1/2)))/f - (atanh((a*b^3*
(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(a*b^4 - a^2*b^3))*(a - b)^(1/
2))/f
```

Reduce [F]

$$\int \cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan(fx+e)^2 b+a}\,\cot(fx+e)\,dx$$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x),x)
```

3.297 $\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*(2*a-b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f-(a-b)^(1/2
)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-1/2*cot(f*x+e)^2*(a+b*ta
n(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a}\left(2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)}{2\sqrt{a}f}$$

input
```
Integrate[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(2*Sqrt[a
 - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Cot[e + f*x]^2*Sqr
t[a + b*Tan[e + f*x]^2]))/(2*Sqrt[a]*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4153, 354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^3(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{110}$$

$$\frac{\int -\frac{\cot(e+fx)(b\tan^2(e+fx)+2a-b)}{2(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{2}\int \frac{\cot(e+fx)(b\tan^2(e+fx)+2a-b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{1}{2}\left(2(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-(2a-b)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)\right)-\cot(e+}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{2}\left(\frac{4(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{2(2a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{2}\left(\frac{2(2a-b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-4\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(((2*(2*a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] - 4*Sq
rt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/2 - Cot[e + f*x
]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(97) = 194$.

Time = 6.86 (sec) , antiderivative size = 934, normalized size of antiderivative = 8.12

method	result	size
default	Expression too large to display	934

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/a^(5/2)/(a-b)^(1/2)*((cos(f*x+e)-1)*(a-b)^(1/2)*ln(2/a^(1/2)*(a^(1/
2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)
+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^2*b+(4*cos(f*x+e)-4)*a^(5/2)*ln(4*(a-b)
^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)
+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+4*
a*cos(f*x+e)-4*cos(f*x+e)*b)*b+(1-cos(f*x+e))*(a-b)^(1/2)*ln(2*(2*((a*cos(
f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*si
n(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*b-a
+2*b)/(cos(f*x+e)-1)^2)*a^2*b+(2-2*cos(f*x+e))*(a-b)^(1/2)*ln(2/a^(1/2)*(a
^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)
+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*
x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^3+(-4*cos(f*x+e)+4)*a^(7/2)*ln(4*(a
-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x
+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
+4*a*cos(f*x+e)-4*cos(f*x+e)*b)+(2*cos(f*x+e)-2)*(a-b)^(1/2)*ln(2*(2*((a*c
os(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a
*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*
b-a+2*b)/(cos(f*x+e)-1)^2)*a^3+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x
+e)+1)^2)^(1/2)*(a-b)^(1/2)*a^(5/2)*cos(f*x+e))*(a+b*tan(f*x+e)^2)^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.97

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{2\sqrt{a-b}a\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)\tan(fx+e)^2-(2a-b)\sqrt{a}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{\cdots}}{\tan\cdots}\right.}{4af\tan(fx+e)^2}\right.$$
$$-\frac{\sqrt{-a}(2a-b)\arctan\left(\frac{\sqrt{-a}}{\sqrt{b\tan(fx+e)^2+a}}\right)\tan(fx+e)^2-\sqrt{a-b}a\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+\cdots}{\tan(fx+e)^2+1}\right.}{2af\tan(fx+e)^2}$$
$$-\frac{\sqrt{-a}(2a-b)\arctan\left(\frac{\sqrt{-a}}{\sqrt{b\tan(fx+e)^2+a}}\right)\tan(fx+e)^2-2a\sqrt{-a+b}\arctan\left(\frac{\sqrt{-a+b}}{\sqrt{b\tan(fx+e)^2+a}}\right)\tan(fx\cdots}{2af\tan(fx+e)^2}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(a - b)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a - b)*s
qrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)
/tan(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan
(f*x + e)^2), 1/4*(4*a*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e
)^2 + a))*tan(f*x + e)^2 - (2*a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 - 2*
sqrt(b*tan(f*x + e)^2 + a)*a)/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a -
b)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 - sqrt(a - b
)*a*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a
 - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*a)
/(a*f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - b)*arctan(sqrt(-a)/sqrt(b*tan
(f*x + e)^2 + a))*tan(f*x + e)^2 - 2*a*sqrt(-a + b)*arctan(sqrt(-a + b)/sq
rt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^2 + a)*a)/(
a*f*tan(f*x + e)^2)]
```

Sympy [F]

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\cot^3{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**3, x)
```

Maxima [F]

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^2+a}\cot{(fx+e)}^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.07

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,b^4\sqrt{b\tan(e+f\,x)^2+a}}{2\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}-\frac{3\,b^5\sqrt{b\tan(e+f\,x)^2+a}}{4\sqrt{a}\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}+\frac{b^6\sqrt{b\tan(e+f\,x)^2+a}}{4\,a^{3/2}\left(\frac{a\,b^4}{2}-\frac{3\,b^5}{4}+\frac{b^6}{4\,a}\right)}\right)(2\,a-b)}{2\sqrt{a}\,f}$$
$$-\frac{\operatorname{atanh}\left(\frac{b^4\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a-b}}{2\left(\frac{a\,b^4}{2}-\frac{b^5}{2}\right)}\right)\sqrt{a-b}}{f}-\frac{b\sqrt{b\tan\left(e+f\,x\right)^2+a}}{2\,\left(f\,\left(b\tan\left(e+f\,x\right)^2+a\right)-a\,f\right)}$$

input
```
int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
(atanh((a^(1/2)*b^4*(a + b*tan(e + f*x)^2)^(1/2))/(2*((a*b^4)/2 - (3*b^5)/
4 + b^6/(4*a))) - (3*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*a^(1/2)*((a*b^4)
/2 - (3*b^5)/4 + b^6/(4*a))) + (b^6*(a + b*tan(e + f*x)^2)^(1/2))/(4*a^(3/
2)*((a*b^4)/2 - (3*b^5)/4 + b^6/(4*a))))*(2*a - b))/(2*a^(1/2)*f) - (atanh
((b^4*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(2*((a*b^4)/2 - b^5/2)))
*(a - b)^(1/2))/f - (b*(a + b*tan(e + f*x)^2)^(1/2))/(2*(f*(a + b*tan(e +
f*x)^2) - a*f))
```

Reduce [F]

$$\int \cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\cot\left(fx+e\right)^3dx$$

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x)**3,x)
```

3.298 $\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{(8a^2-4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8af} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

output

```
-1/8*(8*a^2-4*a*b-b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f
+(a-b)^(1/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+1/8*(4*a-b)*c
ot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*cot(f*x+e)^4*(a+b*tan(f*x+e)^
2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{(-8a^2+4ab+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-\cot^2(e+fx)\,(-\right.}{8a^{3/2}f}$$

input
```
Integrate[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-8*a^2 + 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt
[a]*(8*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - Cot
[e + f*x]^2*(-4*a + b + 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]))/(
8*a^(3/2)*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4153, 354, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^5(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^3(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{110}$$

$$\frac{\frac{1}{2}\int -\frac{\cot^2(e+fx)(3b\tan^2(e+fx)+4a-b)}{2(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-\frac{1}{2}\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{4}\int \frac{\cot^2(e+fx)(3b\tan^2(e+fx)+4a-b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-\frac{1}{2}\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{\cot(e+fx)\left(8a^2-4ba-b^2+(4a-b)b\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a}+\frac{(4a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)-\frac{1}{2}\cot^2(e+fx)\sqrt{a+b\tan}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{\cot(e+fx)\left(8a^2-4ba-b^2+(4a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2a}+\frac{(4a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)-\frac{1}{2}\cot^2(e+fx)\sqrt{a+b\tan}}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{1}{4}\left(\frac{(8a^2-4ab-b^2)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-8a(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2a}+\frac{(4a-b)\cot(e+fx)\sqrt{a+b\tan}}{a}\right.}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{4}\left(\frac{\frac{2\left(8a^2-4ab-b^2\right)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{16a(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{2a}+\frac{(4a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right.}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{4}\left(\frac{16a\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-\frac{2(8a^2-4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}+\frac{(4a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)-\frac{1}{2}c}{2f}$$

input
```
Int[Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/2*(Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2]) + (((-2*(8*a^2 - 4*a*b -
 b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + 16*a*Sqrt[a -
 b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/(2*a) + ((4*a - b)*Co
t[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/4)/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(141) = 282$.

Time = 7.27 (sec) , antiderivative size = 1354, normalized size of antiderivative = 8.31

method	result	size
default	Expression too large to display	1354

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/a^(7/2)/(a-b)^(1/2)*((-8*cos(f*x+e)+8)*sin(f*x+e)^2*ln(2/a^(1/2)*(a
^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)
+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*
x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*(a-b)^(1/2)*a^4+(4*cos(f*x+e)-4)*sin(
f*x+e)^2*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e
)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*(a-b)^(1/2)*a
^3*b+(cos(f*x+e)-1)*sin(f*x+e)^2*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*
sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*
x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(
f*x+e)+1))*(a-b)^(1/2)*a^2*b^2+(8*cos(f*x+e)-8)*sin(f*x+e)^2*ln(2*(2*((a*c
os(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a
*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*
b-a+2*b)/(cos(f*x+e)-1)^2)*(a-b)^(1/2)*a^4+(-4*cos(f*x+e)+4)*sin(f*x+e)^2*
ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*s
in(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-
4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*(a-b)^(1/2)*a^3*b+(1-cos(f*x+e))*s
in(f*x+e)^2*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/
2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a
*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*(a-b)^(1/2)*a^2*b^2...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 708, normalized size of antiderivative = 4.34

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 - (8*a^2 -
 4*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*((4*a^2 - a*b)*tan(f*x
+ e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^4), -1/16*
(16*a^2*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a))*tan(f
*x + e)^4 + (8*a^2 - 4*a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*((4
*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan
(f*x + e)^4), 1/8*(4*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^
4 + (8*a^2 - 4*a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 +
 a))*tan(f*x + e)^4 + ((4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*
x + e)^2 + a))/(a^2*f*tan(f*x + e)^4), -1/8*(8*a^2*sqrt(-a + b)*arctan(sqr
t(-a + b)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^4 - (8*a^2 - 4*a*b - b^
2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^4 - (
(4*a^2 - a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*t
an(f*x + e)^4)]
```

Sympy [F]

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\cot^5{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**5, x)
```

Maxima [F]

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\cot(fx+e)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.33

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{3b^7\sqrt{b\tan(e+fx)^2+a}}{64\sqrt{a^3}\left(\frac{ab^5}{4}-\frac{11b^6}{32}+\frac{3b^7}{64a}+\frac{11b^8}{256a^2}+\frac{b^9}{256a^3}\right)}-\frac{11b^6\sqrt{b\tan(e+fx)^2+a}}{32\sqrt{a^3}\left(\frac{b^5}{4}-\frac{11b^6}{32a}+\frac{3b^7}{64a^2}+\frac{11b^8}{256a^3}+\frac{b^9}{256a^4}\right)}+\frac{11b^8\sqrt{b\tan(e+fx)^2+a}}{256\sqrt{a^3}\left(\frac{3b^7}{64}-\frac{11ab^6}{32}+\frac{a^2b^5}{4}+\frac{11b}{256}\right.}\right.}{8f\sqrt{}}$$
$$-\frac{\operatorname{atanh}\left(\frac{b^5\sqrt{b\tan(e+fx)^2+a}\sqrt{a-b}}{4\left(\frac{7b^6}{32}-\frac{ab^5}{4}+\frac{b^7}{32a}\right)}+\frac{b^6\sqrt{b\tan(e+fx)^2+a}\sqrt{a-b}}{32\left(-\frac{a^2b^5}{4}+\frac{7ab^6}{32}+\frac{b^7}{32}\right)}\right)\sqrt{a-b}}{f}$$
$$-\frac{\sqrt{b\tan(e+fx)^2+a}\left(\frac{b^2}{8}+\frac{ab}{2}\right)-\frac{b\left(b\tan(e+fx)^2+a\right)^{3/2}(4a-b)}{8a}}{f\left(b\tan(e+fx)^2+a\right)^2+a^2f-2af\left(b\tan(e+fx)^2+a\right)}$$

input
```
int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
(atanh((3*b^7*(a + b*tan(e + f*x)^2)^(1/2))/(64*(a^3)^(1/2)*((a*b^5)/4 - (
11*b^6)/32 + (3*b^7)/(64*a) + (11*b^8)/(256*a^2) + b^9/(256*a^3))) - (11*b
^6*(a + b*tan(e + f*x)^2)^(1/2))/(32*(a^3)^(1/2)*(b^5/4 - (11*b^6)/(32*a)
+ (3*b^7)/(64*a^2) + (11*b^8)/(256*a^3) + b^9/(256*a^4))) + (11*b^8*(a + b
*tan(e + f*x)^2)^(1/2))/(256*(a^3)^(1/2)*((3*b^7)/64 - (11*a*b^6)/32 + (a^
2*b^5)/4 + (11*b^8)/(256*a) + b^9/(256*a^2))) + (b^9*(a + b*tan(e + f*x)^2
)^(1/2))/(256*(a^3)^(1/2)*((3*a*b^7)/64 + (11*b^8)/256 - (11*a^2*b^6)/32 +
 (a^3*b^5)/4 + b^9/(256*a))) + (a*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^
3)^(1/2)*(b^5/4 - (11*b^6)/(32*a) + (3*b^7)/(64*a^2) + (11*b^8)/(256*a^3)
+ b^9/(256*a^4))))*(4*a*b - 8*a^2 + b^2))/(8*f*(a^3)^(1/2)) - (atanh((b^5*
(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(4*((7*b^6)/32 - (a*b^5)/4 + b
^7/(32*a))) + (b^6*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(1/2))/(32*((7*a*b
^6)/32 + b^7/32 - (a^2*b^5)/4)))*(a - b)^(1/2))/f - ((a + b*tan(e + f*x)^2
)^(1/2)*((a*b)/2 + b^2/8) - (b*(a + b*tan(e + f*x)^2)^(3/2)*(4*a - b))/(8*
a))/(f*(a + b*tan(e + f*x)^2)^2 + a^2*f - 2*a*f*(a + b*tan(e + f*x)^2))
```

Reduce [F]

$$\int \cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \cot(fx+e)^5\sqrt{\tan(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x)
```

3.299 $\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+2a^2b+8ab^2-16b^3)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} - \frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f}$$

output
```
-(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/1
6*(a^3+2*a^2*b+8*a*b^2-16*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^
2)^(1/2))/b^(5/2)/f-1/16*(a-2*b)*(a+4*b)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/
2)/b^2/f+1/24*(a-6*b)*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/b/f+1/6*tan(f*
x+e)^5*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 5.79 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.89

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \frac{3\sqrt{2}a(a^2+2ab-8b^2)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right)\right.}{}$$

input
```
Integrate[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(3*Sqrt[2]*a*(a^2 + 2*a*b - 8*b^2)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)]
)*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Tan[e + f*x] + 48*Sqrt[2]*a*b^2*Sqrt[
((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a -
 b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/S
qrt[2]], 1]*Tan[e + f*x] - (30*a^3 + 62*a^2*b - 224*a*b^2 - 104*b^3 + (45*
a^3 + 91*a^2*b - 332*a*b^2 + 84*b^3)*Cos[2*(e + f*x)] + 2*(9*a^3 + 17*a^2*
b - 80*a*b^2 - 12*b^3)*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)] + 5*a^2*b
*Cos[6*(e + f*x)] - 52*a*b^2*Cos[6*(e + f*x)] + 44*b^3*Cos[6*(e + f*x)])*C
sc[2*(e + f*x)]^4*Sin[e + f*x]^2*Tan[e + f*x]^3)/(48*Sqrt[2]*b^2*f*Sqrt[(a
 + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4153, 380, 444, 27, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\downarrow \text{3042}$$

$$\int \tan(e+fx)^6\sqrt{a+b\tan(e+fx)^2}dx$$

$$\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^6(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow \text{380}$$

$$\frac{\frac{1}{6}\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{1}{6}\int \frac{\tan^4(e+fx)(5a-(a-6b)\tan^2(e+fx))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\downarrow \text{444}$$

$$\frac{\frac{1}{6}\left(\frac{\int -\frac{3\tan^2(e+fx)\left((a-2b)(a+4b)\tan^2(e+fx)+a(a-6b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{4b}+\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}\right)+\frac{1}{6}\tan^5(e+fx)\sqrt{a+}}{f}$$

$$\downarrow \text{27}$$

$$\frac{\frac{1}{6}\left(\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\int \frac{\tan^2(e+fx)\left((a-2b)(a+4b)\tan^2(e+fx)+a(a-6b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{4b}\right)+\frac{1}{6}\tan^5(e+fx)\sqrt{a+b}}{f}$$

$$\downarrow \text{444}$$

$$\frac{\frac{1}{6}\left(\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\int \frac{\left(a^3+2ba^2+8b^2a-16b^3\right)\tan^2(e+fx)+a(a-2b)(a+4b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d}{2b}\right.}{4b}\right.}{f}$$

$$\downarrow \text{398}$$

$$\frac{\frac{1}{6}\left(\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\left(a^3+2a^2b+8ab^2-16b^3\right)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{}\right.}{4b}\right.}{f}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{6}\left(\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\left(a^3+2a^2b+8ab^2-16b^3\right)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e}{\sqrt{b\tan^2(e}}}{}\right.}{4b}\right.}{f}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{6}\left(\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{\left(a^3+2a^2b+8ab^2-16b^3\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{}\right.}{4b}\right.}{f}$$

$\big\downarrow$ 291

$$\frac{\frac{1}{6}\left(\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{\left(a^3+2a^2b+8ab^2-16b^3\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{2\cdots}\right.}{4b}\right.}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{(a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(a-2b)(a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{\left(a^3+2a^2b+8ab^2-16b^3\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{2b}\right.}{4b}\right.}{f}$$

input
```
Int[Tan[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/6 + (((a - 6*b)*Tan[e + f*x]^
3*Sqrt[a + b*Tan[e + f*x]^2])/(4*b) - (3*(-1/2*(-16*Sqrt[a - b]*b^2*ArcTan
[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] + ((a^3 + 2*a^2*b
+ 8*a*b^2 - 16*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]])/Sqrt[b])/b + ((a - 2*b)*(a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*
x]^2])/(2*b)))/(4*b))/6)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(196) = 392$.

Time = 0.84 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.93

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2\sqrt{b}}+\frac{\tan(fx+e)^3\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{\tan(fx+e)\left(a+b \cdots}{4 \cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2\sqrt{b}}+\frac{\tan(fx+e)^3\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{\tan(fx+e)\left(a+b \cdots}{4 \cdots}\right.}{\cdots}}{\cdots}$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(
f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/6*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2)
/b-1/2*a/b*(1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/b-1/4*a/b*(1/2*tan(f*x
+e)*(a+b*tan(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(
f*x+e)^2)^(1/2))))-1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/b+1/4*a/b*(1/2*
tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a
+b*tan(f*x+e)^2)^(1/2)))-b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)
)/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(
b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.65

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(48*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 3*(a
^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(8*b^3*tan(f*x + e)^5 +
 2*(a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x +
e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/96*(96*sqrt(a - b)*b^3*arctan(
sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + 3*(a^3 + 2*a^2*b +
8*a*b^2 - 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b
^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(
f*x + e)^2 + a))/(b^3*f), 1/48*(24*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*
x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(ta
n(f*x + e)^2 + 1)) - 3*(a^3 + 2*a^2*b + 8*a*b^2 - 16*b^3)*sqrt(-b)*arctan(
sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (8*b^3*tan(f*x + e)^5
+ 2*(a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x +
 e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/48*(48*sqrt(a - b)*b^3*arctan
(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + 3*(a^3 + 2*a^2*b +
 8*a*b^2 - 16*b^3)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x +
e)^2 + a)) - (8*b^3*tan(f*x + e)^5 + 2*(a*b^2 - 6*b^3)*tan(f*x + e)^3 - 3*
(a^2*b + 2*a*b^2 - 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^...
```

Sympy [F]

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{a+b\tan^2{(e+fx)}}\tan^6{(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**6, x)
```

Maxima [F]

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^2+a}\tan{(fx+e)}^6\,dx$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int {\tan\left(e+f\,x\right)}^6\,\sqrt{b\,{\tan\left(e+f\,x\right)}^2+a}\,dx$$

input
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \tan^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{\tan{(fx+e)}^2\,b+a}\,\tan{(fx+e)}^6\,dx$$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**6,x)
```

3.300 $\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+4ab-8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

output
```
(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/8*
(a^2+4*a*b-8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(
3/2)/f+1/8*(a-4*b)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/b/f+1/4*tan(f*x+e)^
3*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.16 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.54

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx =$$

$$-\frac{-\frac{b(a^2-4b^2)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{1+\cos(2(e+fx))}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(1+\cos(2(e+fx)))\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+fx)}{a(a+b+(a-b)\cos(2(e+fx)))}}{}$$

$$+\frac{\sqrt{\frac{a+b+a\cos(2(e+fx))-b\cos(2(e+fx))}{1+\cos(2(e+fx))}}\left(\frac{\sec(e+fx)(a\sin(e+fx)-6b\sin(e+fx))}{8b}+\frac{1}{4}\sec^2(e+fx)\tan(e+fx)\right)}{f}$$

input
```
Integrate[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/4*(-((b*(a^2 - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[
2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a
- b)*Cos[2*(e + f*x)]))) - (4*b*(-4*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e + f*x)]
]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-
((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e +
f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt
[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[
-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos
[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b))
, ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[
2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b +
 (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(b*f
) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e
+ f*x)])]*((Sec[e + f*x]*(a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/(8*b) + (Sec
[e + f*x]^2*Tan[e + f*x])/4))/f
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4153, 380, 444, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{380}$$

$$\frac{\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{1}{4}\int \frac{\tan^2(e+fx)(3a-(a-4b)\tan^2(e+fx))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{(a^2+4ba-8b^2)\tan^2(e+fx)+a(a-4b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}+\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}\right)+\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(e+}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\left(\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\int \frac{(a^2+4ba-8b^2)\tan^2(e+fx)+a(a-4b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}\right)+\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(e+}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{4}\left(\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(a^2+4ab-8b^2)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-8b(a-b)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(}{2b}\right.}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{4}\left(\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(a^2+4ab-8b^2)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-8b(a-b)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+}}}{2b}\right.}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}\left(\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{(a^2+4ab-8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8b(a-b)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+}{2b}\right.}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{1}{4}\left(\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{(a^2+4ab-8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8b(a-b)\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{2b}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}\left(\frac{(a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{(a^2+4ab-8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8b\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b}\right)+\frac{1}{4}\tan^3}{f}$$

input

```
Int[Tan[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/4 + (-1/2*(-8*Sqrt[a - b]*b*A
rcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] + ((a^2 + 4*a
*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sq
rt[b])/b + ((a - 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*b))/4)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(147) = 294$.

Time = 0.90 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2\sqrt{b}}\right)}{4b}+b\left(\frac{\ln\left(\sqrt{b}\,\tan(\right.}{}\right.}{}$
default	$\frac{\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2\sqrt{b}}\right)}{4b}+b\left(\frac{\ln\left(\sqrt{b}\,\tan(\right.}{}\right.}{}$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)/b-1/4*a/b*(1/2*tan(f*x+e)*(a+
b*tan(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^
2)^(1/2)))+b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4
*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e)))+a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4
*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/2*tan(f*x+e)*(a+b*tan
(f*x+e)^2)^(1/2)-1/2*a/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 655, normalized size of antiderivative = 3.88

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2
+ 4*a*b - 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*t
an(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/16*(16*sqrt(a - b)*b^2
*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (a^2 + 4*a*
b - 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(b)*tan(f*x + e) + a) + 2*(2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x
 + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(4*sqrt(-a + b)*b^2*log(-(
(a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f
*x + e) - a)/(tan(f*x + e)^2 + 1)) + (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan
(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (2*b^2*tan(f*x + e)^3
 + (a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/8*(8
*sqrt(a - b)*b^2*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a
)) + (a^2 + 4*a*b - 8*b^2)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*ta
n(f*x + e)^2 + a)) + (2*b^2*tan(f*x + e)^3 + (a*b - 4*b^2)*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F]

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\tan^4{(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\tan(fx+e)^4\,dx$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \tan(e+f\,x)^4\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \tan^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2b+a}\,\tan\left(fx+e\right)^4dx$$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**4,x)
```

3.301 $\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

output
```
-(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/2
*(a-2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)/f+1/
2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.04

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$=\frac{\left(-\sqrt{2}a\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right),1\right)+2\sqrt{2}a\sqrt{\cdots}\right.}{2\cdots}$$

input
```
Integrate[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-(Sqrt[2]*a*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*
EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/
b]/Sqrt[2]], 1]) + 2*Sqrt[2]*a*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Cs
c[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (a + b + (a - b)*Cos[2*(
e + f*x)])*Sec[e + f*x]^2)*Tan[e + f*x])/(2*Sqrt[2]*f*Sqrt[(a + b + (a - b
)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4153, 380, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^2\sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 380$$

$$\frac{\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{1}{2}\int \frac{a-(a-2b)\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{1}{2}\left((a-2b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-2(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e+}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{2}\left((a-2b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-2(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-2(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e+fx)\sqrt{a}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{1}{2}\left(\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-2(a-b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{2}\left(\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-2\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Tan[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-2*Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]] + ((a - 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/Sqrt[b])/2 + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(105) = 210$.

Time = 0.82 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2\sqrt{b}}-b\left(\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b}}{}\right.}{f}$
default	$\frac{\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2\sqrt{b}}-b\left(\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{\sqrt{b}}-\frac{\sqrt{b^4(a-b}}{}\right.}{f}$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*tan(
f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-b*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^
2)^(1/2))/b^(1/2)-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))
^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))-a*(b^4*(a-b))^(1/2)/b^2/(a-b)
*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 523, normalized size of antiderivative = 4.25

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \left[-\frac{(a-2b)\sqrt{b}\log\left(2b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{b}\tan(fx+e)+a\right)-2\sqrt{-a+b}b\log}{4bf}\right.$$

$$-\frac{4\sqrt{a-b}b\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)+(a-2b)\sqrt{b}\log\left(2b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{b}\right.}{4bf}$$

$$-\frac{(a-2b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-\sqrt{-a+b}b\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx}{\tan(fx+e)^2+1}\right.}{2bf}$$

$$-\frac{2\sqrt{a-b}b\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)+(a-2b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-\sqrt{b\tan(fx+e)^2+a}}{2bf}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x
+ e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(
f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/
4*(4*sqrt(a - b)*b*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 +
 a)) + (a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x +
e))/(b*f), -1/2*((a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*ta
n(f*x + e)^2 + a)) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1
)) - sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f), -1/2*(2*sqrt(a - b)
*b*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (a - 2*b)
*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - sqrt(
b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/(b*f)]
```

Sympy [F]

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\tan^2{(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)^2+a}\tan(fx+e)^2\,dx$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \tan(e+f\,x)^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \tan^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^2 dx$$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2,x)
```

3.302 $\int \sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

output
```
(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+b^(1
/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.27

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}\log\left(-\sqrt{b}\tan(e+fx)+\sqrt{a+b\tan^2(e}\right.}{f}$$

input
```
Integrate[Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-((Sqrt[a - b]*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*Sqr
t[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tan[e + f*x]
) + Sqrt[a + b*Tan[e + f*x]^2]])/f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4144, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{301}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{(a-b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]
 + Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(73) = 146$.

Time = 0.82 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{f}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{fb(a-b)}+\frac{a\sqrt{b^4(a-b)}\,\text{arc}}{}$
default	$\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{f}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{fb(a-b)}+\frac{a\sqrt{b^4(a-b)}\,\text{arc}}{}$

input
```
int((a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b)
)^(1/2)/b/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e))+1/f*a*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b
))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.64

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)+\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(f\right.}{2\,f}\right.$$

$$-\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2+2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2\,f},$$

input
```
integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b
)*tan(f*x + e) + a) + sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt
(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)
))/f, 1/2*(2*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x +
e)^2 + a)) + sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)
*sqrt(b)*tan(f*x + e) + a))/f, -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*tan(f*x +
e)/sqrt(b*tan(f*x + e)^2 + a)) - sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
+ e)^2 + 1)))/f, (sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f
*x + e)^2 + a)) - sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e
)^2 + a)))/f]
```

Sympy [F]

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2\left(e+fx\right)}\,dx$$

input
```
integrate((a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a + b\tan^2(e + fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \sqrt{a + b\tan^2(e + fx)}\,dx = \int \sqrt{b\tan(fx + e)^2 + a}\,dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{\tan\left(fx+e\right)^2b+a}dx$$

input
```
int((a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a),x)
```

3.303 $\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

output
```
-(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-cot
(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{(a-b)\tan^2(e+fx)}{a+b\tan^2(e+fx)}\right)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -(((a - b)*Tan[e + f*x]^2)
/(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*x]^2])/f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4153, 377, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{377}$$

$$\frac{\int -\frac{a-b}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)}\right)-\int \frac{a-b}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)}\right)-(a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)}\right)-(a-b)\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{-\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-(Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]]) - Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(67) = 134$.

Time = 22.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.51

method	result
default	$-\frac{\left(\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a\sin(fx+e)-\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)b\sin(fx+e)+\sqrt{\cdots}}{f\sqrt{a-b}\,(\cos(fx+e)+1)\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e\cdots}{(\cos(fx+e)+1)^2}}}$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a-b)^(1/2)*(arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a*sin(f*x+e)-arctan(1/(a-
b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+
e)/(cos(f*x+e)-1))*b*sin(f*x+e)+(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^
2)/(cos(f*x+e)+1)^2)^(1/2)*(cos(f*x+e)+1))*(a+b*tan(f*x+e)^2)^(1/2)/(cos(f
*x+e)+1)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+
e)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.43

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2-4\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+e)^2+\cdots}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right.\cdots}{4\,f\tan(fx+e)}\right.$$

$$\left.-\frac{\sqrt{a-b}\arctan\left(-\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan(fx+e)+2\sqrt{b\tan(fx+e)^2+a}}{2\,f\tan(fx+e)}\right]$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e
))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x +
e)^2 + 1))*tan(f*x + e) - 4*sqrt(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)),
-1/2*(sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x
 + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)
^2 + a))/(f*tan(f*x + e))]
```

Sympy [F]

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\cot^2{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^2+a}\cot{(fx+e)}^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^2+a}\cot{(fx+e)}^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \cot(e+f\,x)^2\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{\tan\left(fx+e\right)^2b+a}\,\cot\left(fx+e\right)^2dx$$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x)**2,x)
```

3.304 $\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

output
```
(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/3*
(3*a-b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)^3*(a+b*tan(
f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.06

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx =$$

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(\frac{\sec^2(e+fx)\left(\arcsin\left(\sqrt{\frac{(a-b)\sin^2(e+fx)}{a}}\right)\sqrt{\frac{(a-}{}}\right.}{\sqrt{\cos^2(e+fx)-}}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/3*(Cos[e + f*x]^2*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2]*(1 + (b*Tan
[e + f*x]^2)/a)*((Sec[e + f*x]^2*(ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]
*Sqrt[((a - b)*Sin[e + f*x]^2)/a] + Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^
2)/a])*(a - 2*b*Tan[e + f*x]^2))/(Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)
/a]*(a + b*Tan[e + f*x]^2)) - (4*(a - b)*Hypergeometric2F1[2, 2, 3/2, ((a
- b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2))/f
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4153, 377, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^4(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 377$$

$$\frac{\frac{1}{3}\int -\frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a-b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{1}{3}\int \frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a-b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{1}{3}\left(\frac{\int \frac{3a(a-b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}\left(3(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{1}{3}\left(3(a-b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{3}\left(3\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Cot[e + f*x]^4*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/3*(Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2]) + (3*Sqrt[a - b]*ArcTan[
(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] + ((3*a - b)*Cot[e
+ f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/3)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(103) = 206$.

Time = 24.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.12

method	result
default	$-\frac{\left((3\cos(fx+e)-3)\sin(fx+e)\sqrt{a-b}\,\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a+\left(4\cos(fx+e)^2-3\right)\sqrt{\frac{a\cos(fx+e)^2+b\sin}{(\cos(fx+e)+1}}}{3fa\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}}$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/a*((3*cos(f*x+e)-3)*sin(f*x+e)*(a-b)^(1/2)*arctan(1/(a-b)^(1/2)*((a
*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+
e)-1))*a+(4*cos(f*x+e)^2-3)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1
)^2)^(1/2)*a+sin(f*x+e)^2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^
2)^(1/2)*b)*(a+b*tan(f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos
(f*x+e)+1)^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.66

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$
$$= \left[\frac{3\,a\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2+4\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+e)}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right.}{12\,af\tan(fx+e)^3}\right.$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*
a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*
x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f
*x + e)^2 + 1))*tan(f*x + e)^3 + 4*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*t
an(f*x + e)^2 + a))/(a*f*tan(f*x + e)^3), 1/6*(3*sqrt(a - b)*a*arctan(-2*s
qrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)
^2 - a))*tan(f*x + e)^3 + 2*((3*a - b)*tan(f*x + e)^2 - a)*sqrt(b*tan(f*x
+ e)^2 + a))/(a*f*tan(f*x + e)^3)]
```

Sympy [F]

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2(e+fx)}\cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{b\tan(fx+e)^2+a}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{b\tan(fx+e)^2+a}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \cot(e+f\,x)^4\,\sqrt{b\tan(e+f\,x)^2+a}\,dx$$

input
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \cot(fx+e)^4\sqrt{\tan(fx+e)^2 b + a}dx$$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(1/2),x)
```

3.305 $\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\begin{aligned}\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx &= -\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}\\ &\quad -\frac{(15a^2-5ab-2b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f}\\ &\quad +\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}\end{aligned}$$

output
```
-(a-b)^(1/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/1
5*(15*a^2-5*a*b-2*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/15*(5*a
-b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a/f-1/5*cot(f*x+e)^5*(a+b*tan(f*
x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.44 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.92

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx =$$

$$-\frac{\cos^4(e+fx)\cot^5(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(8(a-b)\,{}_3F_2\left(2,2,2;1,\frac{3}{2};\frac{(a-b)\sin^2(e+fx)}{a}\right)\tan^2(e+fx)\,(a\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/15*(Cos[e + f*x]^4*Cot[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*(8*(a - b)
*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e
+ f*x]^2*(a + b*Tan[e + f*x]^2)^3 + 8*(a - b)*Hypergeometric2F1[2, 2, 3/2,
 ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^2*(-2*a
 + 3*b*Tan[e + f*x]^2) + (a^2*Sec[e + f*x]^4*(ArcSin[Sqrt[((a - b)*Sin[e +
 f*x]^2)/a]]*Sqrt[((a - b)*Sin[e + f*x]^2)/a] + Sqrt[Cos[e + f*x]^2 + (b*S
in[e + f*x]^2)/a])*(3*a^2 - 4*a*b*Tan[e + f*x]^2 + 8*b^2*Tan[e + f*x]^4))/
Sqrt[Cos[e + f*x]^2 + (b*Sin[e + f*x]^2)/a]))/(a^3*f*Sqrt[a + b*Tan[e + f*
x]^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4153, 377, 25, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)^2}}{\tan(e+fx)^6}dx$$

$\Big\downarrow$ 4153

$$\frac{\int \frac{\cot^6(e+fx)\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 377

$$\frac{\frac{1}{5}\int -\frac{\cot^4(e+fx)(4b\tan^2(e+fx)+5a-b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{5}\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$\Big\downarrow$ 25

$$\frac{-\frac{1}{5}\int \frac{\cot^4(e+fx)(4b\tan^2(e+fx)+5a-b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{5}\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$\Big\downarrow$ 445

$$\frac{\frac{1}{5}\left(\frac{\int \frac{\cot^2(e+fx)\left(15a^2-5ba-2b^2+2(5a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}\right)-\frac{1}{5}\cot^5(e+fx)\sqrt{a+b}}{f}$$

$\Big\downarrow$ 445

$$\frac{\frac{1}{5}\left(\frac{-\frac{\int \frac{15a^2(a-b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}\right)-}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{5}\left(\frac{-15a(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(}}{3a}\right.}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{1}{5}\left(\frac{-15a(a-b)\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}\right.}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{5}\left(\frac{-\frac{\left(15a^2-5ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}-15a\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{3a}+\frac{(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}\right)-\frac{1}{5}\cot}{f}$$

input
```
Int[Cot[e + f*x]^6*Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2]) + (((5*a - b)*Cot[e + f*
x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(3*a) + (-15*a*Sqrt[a - b]*ArcTan[(Sqrt[a
 - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] - ((15*a^2 - 5*a*b - 2*b^2
)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(3*a))/5)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(149) = 298$.

Time = 25.00 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(\sin(fx+e)^3(-15\cos(fx+e)+15)\sqrt{a-b}\,\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a^2+\left(23\cos(fx+e)^4-35\cos(fx+e)^2-\right.}{}$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^2*(sin(f*x+e)^3*(-15*cos(f*x+e)+15)*(a-b)^(1/2)*arctan(1/(a-b)^(
1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(
cos(f*x+e)-1))*a^2+(23*cos(f*x+e)^4-35*cos(f*x+e)^2+15)*((a*cos(f*x+e)^2+b
*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^2+(6*cos(f*x+e)^2-5)*sin(f*x+e)^2
*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a*b-2*sin(f*x+e)
^4*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b^2)*(a+b*tan(
f*x+e)^2)^(1/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*c
ot(f*x+e)*csc(f*x+e)^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.25

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx$$

$$= \left[\frac{15\,a^2\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2-4\,\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(f}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right.}{}\right.$$

$$-\frac{15\,\sqrt{a-b}a^2\arctan\left(-\frac{2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan(fx+e)^5+2\,\left((15\,a^2-5\,ab-2\,b^2)\tan(fx\right.}{30\,a^2f\tan(fx+e)^5}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/60*(15*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*
(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan
(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*ta
n(f*x + e)^2 + 1))*tan(f*x + e)^5 - 4*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x +
e)^4 - (5*a^2 - a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(
a^2*f*tan(f*x + e)^5), -1/30*(15*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x
+ e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f
*x + e)^5 + 2*((15*a^2 - 5*a*b - 2*b^2)*tan(f*x + e)^4 - (5*a^2 - a*b)*tan
(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a^2*f*tan(f*x + e)^5)]
```

Sympy [F]

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{a+b\tan^2{(e+fx)}}\cot^6{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x)**2)*cot(e + f*x)**6, x)
```

Maxima [F]

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx = \int \sqrt{b\tan{(fx+e)}^2+a}\cot{(fx+e)}^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Giac [F]

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \sqrt{b\tan\left(fx+e\right)^2+a}\cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \cot(e+f\,x)^6\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,dx$$

input
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \cot^6(e+fx)\sqrt{a+b\tan^2(e+fx)}\,dx=\int \cot\left(fx+e\right)^6\sqrt{\tan\left(fx+e\right)^2b+a}\,dx$$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(1/2),x)
```

3.306 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=-\frac{(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f}+\frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f}+\frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2f}$$

output
```
-(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a-b)*(a+b*ta
n(f*x+e)^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/f-1/5*(a+b)*(a+b*tan(f*x+
e)^2)^(5/2)/b^2/f+1/7*(a+b*tan(f*x+e)^2)^(7/2)/b^2/f
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2}-\frac{2(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2}+\frac{2\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2}+2(a-b)\left(-\sqrt{a-b}\right.}{2f}$$

input
```
Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a + b*Tan[e + f*x]^2)^(3/2))/3 - (2*(a + b)*(a + b*Tan[e + f*x]^2)^(5
/2))/(5*b^2) + (2*(a + b*Tan[e + f*x]^2)^(7/2))/(7*b^2) + 2*(a - b)*(-(Sqr
t[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]) + Sqrt[a + b*Tan
[e + f*x]^2]))/(2*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5 \left(a+b\tan(e+fx)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^5(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\left(b\tan^2(e+fx)+a\right)^{5/2}}{b}+\frac{(-a-b)\left(b\tan^2(e+fx)+a\right)^{3/2}}{b}+\frac{\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-2(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\frac{2\left(a+b\tan^2(e+fx)\right)^{7/2}}{7b^2}-\frac{2(a+b)\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b^2}+\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3}}{2f}$$

input
```
Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + 2*(a -
 b)*Sqrt[a + b*Tan[e + f*x]^2] + (2*(a + b*Tan[e + f*x]^2)^(3/2))/3 - (2*(
a + b)*(a + b*Tan[e + f*x]^2)^(5/2))/(5*b^2) + (2*(a + b*Tan[e + f*x]^2)^(
7/2))/(7*b^2))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(125) = 250$.

Time = 0.68 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{\tan(fx+e)^2\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}{7fb} - \frac{2a\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}{35f\,b^2} + \frac{b\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3f} + \frac{4a\sqrt{a+b\tan(}}{3f}$
default	$\frac{\tan(fx+e)^2\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}{7fb} - \frac{2a\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}{35f\,b^2} + \frac{b\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3f} + \frac{4a\sqrt{a+b\tan(}}{3f}$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/f*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(5/2)/b-2/35/f*a/b^2*(a+b*tan(f*x+e)
^2)^(5/2)+1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)+4/3/f*a*(a+b*tan(f
*x+e)^2)^(1/2)+1/f*b^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)
^(1/2))-b*(a+b*tan(f*x+e)^2)^(1/2)/f-2/f*a*b/(-a+b)^(1/2)*arctan((a+b*tan(
f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/f*a^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^
2)^(1/2)/(-a+b)^(1/2))-1/5*(a+b*tan(f*x+e)^2)^(5/2)/b/f
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.03

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{105\,(ab^2-b^3)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx-}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right.}{}\right.$$

$$-\frac{105\,(ab^2-b^3)\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right)-2\left(15\,b^3\tan\left(fx+e\right)^6+3\,(8\right.}{}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/420*(105*(a*b^2 - b^3)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b
 - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x +
 e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1)) - 4*(15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x + e)
^4 - 6*a^3 - 21*a^2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b^3
)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), -1/210*(105*(a*b^2
- b^3)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) - 2*(
15*b^3*tan(f*x + e)^6 + 3*(8*a*b^2 - 7*b^3)*tan(f*x + e)^4 - 6*a^3 - 21*a^
2*b + 140*a*b^2 - 105*b^3 + (3*a^2*b - 42*a*b^2 + 35*b^3)*tan(f*x + e)^2)*
sqrt(b*tan(f*x + e)^2 + a))/(b^2*f)]
```

Sympy [F]

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}\tan^5(e+fx)\,dx$$

input
```
integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

Maxima [F]

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^5\,dx$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 36.16 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.61

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\left(b\tan(e+f\,x)^2+a\right)^{7/2}}{7\,b^2\,f}$$
$$-\left(\frac{2\,a}{5\,b^2\,f}-\frac{a-b}{5\,b^2\,f}\right)\left(b\tan(e+f\,x)^2+a\right)^{5/2}$$
$$-\sqrt{b\tan\left(e+f\,x\right)^2+a}\,(a-b)\left(\left(\frac{2\,a}{b^2\,f}-\frac{a-b}{b^2\,f}\right)(a-b)-\frac{a^2}{b^2\,f}\right)-\left(b\tan(e+f\,x)^2+a\right)^{3/2}\left(\frac{\left(\frac{2\,a}{b^2\,f}-\frac{a-b}{b^2\,f}\right)}{3}\right.$$

input
```
int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(a + b*tan(e + f*x)^2)^(7/2)/(7*b^2*f) - ((2*a)/(5*b^2*f) - (a - b)/(5*b^2
*f))*(a + b*tan(e + f*x)^2)^(5/2) - (a + b*tan(e + f*x)^2)^(1/2)*(a - b)*(
((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b) - a^2/(b^2*f)) - (a + b*tan(e +
f*x)^2)^(3/2)*((((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a - b))/3 - a^2/(3*b^2*
f)) + (atan(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2)*1i)/(a^2 - 2*a*b +
 b^2))*(a - b)^(3/2)*1i)/f
```

Reduce [F]

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{15\sqrt{\tan\left(fx+e\right)^2b+a}\,\tan\left(fx+e\right)^6b^3+24\sqrt{\tan\left(fx+e\right)^2b+a}\,\tan\left(fx+e\right)^4a\,b^2-21}{}$$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
(15*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**6*b**3 + 24*sqrt(tan(e + f*x
)**2*b + a)*tan(e + f*x)**4*a*b**2 - 21*sqrt(tan(e + f*x)**2*b + a)*tan(e
+ f*x)**4*b**3 + 3*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*a**2*b - 42
*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*a*b**2 + 35*sqrt(tan(e + f*x)
**2*b + a)*tan(e + f*x)**2*b**3 - 6*sqrt(tan(e + f*x)**2*b + a)*a**3 + 84*
sqrt(tan(e + f*x)**2*b + a)*a**2*b - 70*sqrt(tan(e + f*x)**2*b + a)*a*b**2
 - 105*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x)**2*
b + a),x)*a**2*b**2*f + 210*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)*
*3)/(tan(e + f*x)**2*b + a),x)*a*b**3*f - 105*int((sqrt(tan(e + f*x)**2*b
+ a)*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*b**4*f)/(105*b**2*f)
```

3.307 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} - \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\tan^2(e+fx)\right)^{5/2}}{5bf}$$

output
```
(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-(a-b)*(a+b*tan
(f*x+e)^2)^(1/2)/f-1/3*(a+b*tan(f*x+e)^2)^(3/2)/f+1/5*(a+b*tan(f*x+e)^2)^(
5/2)/b/f
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{15(a-b)^{3/2}b\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + \sqrt{a+b\tan^2(e+fx)}(3a^2-20ab+15b^2+(6a-5}{15bf}$$

input
```
Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(15*(a - b)^(3/2)*b*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt
[a + b*Tan[e + f*x]^2]*(3*a^2 - 20*a*b + 15*b^2 + (6*a - 5*b)*b*Tan[e + f*
x]^2 + 3*b^2*Tan[e + f*x]^4))/(15*b*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4153, 354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3\left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^3(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{2\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b}-\int \frac{\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{-(a-b)\int \frac{\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)+\frac{2\left(a+b\tan^2(e+fx)\right)^{5/2}}{5b}-\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{-(a-b)\left((a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)+2\sqrt{a+b\tan^2(e+fx)}\right)+\frac{2(a+b\tan^2(e+fx))^{5/2}}{5b}-}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{-(a-b)\left(\frac{2(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}+2\sqrt{a+b\tan^2(e+fx)}\right)+\frac{2(a+b\tan^2(e+fx))^{5/2}}{5b}-\frac{2}{3}(a+b\tan^2}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{-(a-b)\left(2\sqrt{a+b\tan^2(e+fx)}-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)\right)+\frac{2(a+b\tan^2(e+fx))^{5/2}}{5b}-\frac{2}{3}(a+b\tan^2(e+}{2f}$$

input
```
Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-2*(a + b*Tan[e + f*x]^2)^(3/2))/3 + (2*(a + b*Tan[e + f*x]^2)^(5/2))/(5
*b) - (a - b)*(-2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a -
b]] + 2*Sqrt[a + b*Tan[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(100) = 200$.

Time = 0.65 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{(a+b\tan(fx+e)^2)^{\frac{5}{2}}}{5bf} - \frac{b\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3f} - \frac{4a\sqrt{a+b\tan(fx+e)^2}}{3f} - \frac{b^2\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right.}{f\sqrt{-a+b}}$
default	$\frac{(a+b\tan(fx+e)^2)^{\frac{5}{2}}}{5bf} - \frac{b\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3f} - \frac{4a\sqrt{a+b\tan(fx+e)^2}}{3f} - \frac{b^2\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right.}{f\sqrt{-a+b}}$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a+b*tan(f*x+e)^2)^(5/2)/b/f-1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(
1/2)-4/3/f*a*(a+b*tan(f*x+e)^2)^(1/2)-1/f*b^2/(-a+b)^(1/2)*arctan((a+b*tan
(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+b*(a+b*tan(f*x+e)^2)^(1/2)/f+2/f*a*b/(-a+b)
^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/f*a^2/(-a+b)^(1/2)*
arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.10

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{15\,(ab-b^2)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right.}{}\right.$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/60*(15*(a*b - b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3
*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)
^2 + 1)) - 4*(3*b^2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2 + 3*a^
2 - 20*a*b + 15*b^2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f), 1/30*(15*(a*b - b^
2)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) + 2*(3*b^
2*tan(f*x + e)^4 + (6*a*b - 5*b^2)*tan(f*x + e)^2 + 3*a^2 - 20*a*b + 15*b^
2)*sqrt(b*tan(f*x + e)^2 + a))/(b*f)]
```

Sympy [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\tan^3{(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^3\,dx$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.34

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}{5\,b\,f} - \left(\frac{a}{3\,b\,f}-\frac{a-b}{3\,b\,f}\right)\left(b\tan(e+f\,x)^2+a\right)^{3/2} - \left(\frac{a}{b\,f}-\frac{a-b}{b\,f}\right)\sqrt{b\tan\left(e+f\,x\right)^2+a}\,(a-b) - \frac{\mathrm{atan}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}\,(a-b)^{3/2}\,1\mathrm{i}}{a^2-2\,a\,b+b^2}\right)(a-b)^{3/2}\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(a + b*tan(e + f*x)^2)^(5/2)/(5*b*f) - (a/(3*b*f) - (a - b)/(3*b*f))*(a +
b*tan(e + f*x)^2)^(3/2) - (a/(b*f) - (a - b)/(b*f))*(a + b*tan(e + f*x)^2)
^(1/2)*(a - b) - (atan(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2)*1i)/(a^
2 - 2*a*b + b^2))*(a - b)^(3/2)*1i)/f
```

Reduce [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{3\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^4 b^2+6\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^2 ab-5\sqrt{\tan}}{}$$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
(3*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**4*b**2 + 6*sqrt(tan(e + f*x)*
*2*b + a)*tan(e + f*x)**2*a*b - 5*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)
**2*b**2 - 12*sqrt(tan(e + f*x)**2*b + a)*a**2 + 10*sqrt(tan(e + f*x)**2*b
 + a)*a*b + 15*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e +
f*x)**2*b + a),x)*a**2*b*f - 30*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f
*x)**3)/(tan(e + f*x)**2*b + a),x)*a*b**2*f + 15*int((sqrt(tan(e + f*x)**2
*b + a)*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*b**3*f)/(15*b*f)
```

3.308 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = -\frac{(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a-b)\sqrt{a+b\tan^2(e+fx)}}{f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3f}$$

output
```
-(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+(a-b)*(a+b*ta
n(f*x+e)^2)^(1/2)/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{-3(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + \sqrt{a+b\tan^2(e+fx)}\left(4a-3b+b\tan^2(e+fx)\right)}{3f}$$

input
```
Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a
 + b*Tan[e + f*x]^2]*(4*a - 3*b + b*Tan[e + f*x]^2))/(3*f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 353, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a-b)\int \frac{\sqrt{b\tan^2(e+fx)+a}}{\tan^2(e+fx)+1}d\tan^2(e+fx)+\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{(a-b)\left((a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)+2\sqrt{a+b\tan^2(e+fx)}\right)+\frac{2}{3}\left(a+b\tan^2(e+fx)\right)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a-b)\left(\frac{2(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}+2\sqrt{a+b\tan^2(e+fx)}\right)+\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

$$\Big\downarrow \; 221$$

$$\frac{(a-b)\left(2\sqrt{a+b\tan^2(e+fx)}-2\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)\right)+\frac{2}{3}\left(a+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

input
```
Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a + b*Tan[e + f*x]^2)^(3/2))/3 + (a - b)*(-2*Sqrt[a - b]*ArcTanh[Sqrt
[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + 2*Sqrt[a + b*Tan[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(78) = 156$.

Time = 0.61 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{b\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3f}+\frac{4a\sqrt{a+b\tan(fx+e)^2}}{3f}+\frac{b^2\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}-\frac{b\sqrt{a+b\tan(fx+e)^2}}{f}$
default	$\frac{b\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3f}+\frac{4a\sqrt{a+b\tan(fx+e)^2}}{3f}+\frac{b^2\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}-\frac{b\sqrt{a+b\tan(fx+e)^2}}{f}$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*b*tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)+4/3/f*a*(a+b*tan(f*x+e)^2)^(
1/2)+1/f*b^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-b*
(a+b*tan(f*x+e)^2)^(1/2)/f-2/f*a*b/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^
(1/2)/(-a+b)^(1/2))+1/f*a^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(
-a+b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.12

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\,(a-b)^{\frac{3}{2}}\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+\cdots}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right)\cdots}{12\,f}\right.$$

$$-\frac{3\,(a-b)\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right)-2\left(b\tan\left(fx+e\right)^2+4\,a-3\,b\right)\sqrt{b\cdots}}{6\,f}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f
*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) -
4*(b*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/f, -1/6*(3*(a
 - b)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) - 2*(b
*tan(f*x + e)^2 + 4*a - 3*b)*sqrt(b*tan(f*x + e)^2 + a))/f]
```

Sympy [F]

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Maxima [F]

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{3\,f}+\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(a-b\right)}{f}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}\,(a-b)^{3/2}}{a^2-2\,a\,b+b^2}\right)\,\left(a-b\right)^{3/2}}{f}$$

input
```
int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(a + b*tan(e + f*x)^2)^(3/2)/(3*f) + ((a + b*tan(e + f*x)^2)^(1/2)*(a - b)
)/f - (atanh(((a + b*tan(e + f*x)^2)^(1/2)*(a - b)^(3/2))/(a^2 - 2*a*b + b
^2))*(a - b)^(3/2))/f
```

Reduce [F]

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^2 b^2+3\sqrt{\tan(fx+e)^2 b+a}\,a^2-2\sqrt{\tan(fx+e)^2 b+a}$$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*b**2 + 3*sqrt(tan(e + f*x)**2
*b + a)*a**2 - 2*sqrt(tan(e + f*x)**2*b + a)*a*b - 3*int((sqrt(tan(e + f*x
)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*a**2*b*f + 6*int((
sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*a*
b**2*f - 3*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x)
**2*b + a),x)*b**3*f)/(3*b*f)
```

3.309 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{b\sqrt{a+b\tan^2(e+fx)}}{f}$$

output
```
-a^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f+(a-b)^(3/2)*arctanh((
a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+b*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{-a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right) + (a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) + b\sqrt{a+b\tan^2(e+f}}{f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]) + (a - b)^(3/2)*Ar
cTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + b*Sqrt[a + b*Tan[e + f*x]^
2])/f
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 354, 95, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{95}$$

$$\frac{\int \frac{\cot(e+fx)\left(a^2+(2a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)+2b\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{a^2\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-(a-b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)+2b\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a^2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{2(a-b)^2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}+2b\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+2(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+2b\sqrt{a+b\tan^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-2*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + 2*(a - b)^(3/2)*
ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + 2*b*Sqrt[a + b*Tan[e + f
*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(81) = 162$.

Time = 13.69 (sec) , antiderivative size = 817, normalized size of antiderivative = 8.60

method	result
default	$\frac{\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}\left(2a^{\frac{7}{2}}\ln\left(4\sqrt{a-b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\cos(fx+e)+4\sqrt{a-b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}+4a\cos(fx+e\right.\right.}{}$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^(3/2)/(a-b)^(1/2)*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(
f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)/(cos(f*x+e)+1)/(a*cos(f*x+e)^2+b*sin(f*x
+e)^2)*(2*a^(7/2)*ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f
*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*cos(f*x+e)^3-4*a^
(5/2)*ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^
(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e
)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*b*cos(f*x+e)^3+2*a^(3/2)*ln(4
*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(
f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1
/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*b^2*cos(f*x+e)^3-(a-b)^(1/2)*ln(2*(2*((
a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^
2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+
e)*b-a+2*b)/(cos(f*x+e)-1)^2)*a^3*cos(f*x+e)^3+(a-b)^(1/2)*ln(2/a^(1/2)*(a
^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)
+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*
x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^3*cos(f*x+e)^3+(2*cos(f*x+e)^3+2*co
s(f*x+e)^2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*b*(a-
b)^(1/2)*a^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 647, normalized size of antiderivative = 6.81

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((a - b)^(3/2)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x
+ e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
- b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 2*a
^(3/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)
/tan(f*x + e)^2) - 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/4*(4*sqrt(-a)*a*ar
ctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a)) - (a - b)^(3/2)*log(-(b^2*tan(f*
x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a -
b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x
+ e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*(
(a - b)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) + a^
(3/2)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/
tan(f*x + e)^2) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f, 1/2*((a - b)*sqrt(-a
+ b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) + 2*sqrt(-a)*a*arcta
n(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a)) + 2*sqrt(b*tan(f*x + e)^2 + a)*b)/f
]
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\left(a+b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\cot{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 546, normalized size of antiderivative = 5.75

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{b\sqrt{b\tan\left(e+f\,x\right)^2+a}}{f}+\frac{\mathrm{atanh}\left(\frac{6\,a^3\,b^3\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}-\frac{6\,a^2\,b^4\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7}+\frac{2\,a\,b^5\,\sqrt{b\tan(e+f\,x)^2+a}}{6\,a^5\,b^3-18\,a^4\,b^4+20\,a}\right.}{f}-\frac{\mathrm{atanh}\left(\frac{2\,b^6\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}-\frac{8\,a\,b^5\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}+\frac{12\,a^2\,b^4\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3}}{-6\,a^5\,b^3+12\,a^4\,b^4-8\,a^3\,b^5+2\,a^2\,b^6}-\frac{6\,a^3\,b^3\,\sqrt{b\,\mathrm{ta}}}{-6\,a^5\,b^3+12\,a}\right.}{f}$$

input
```
int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(b*(a + b*tan(e + f*x)^2)^(1/2))/f + (atanh((6*a^3*b^3*(a + b*tan(e + f*x)
^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 2
0*a^3*b^5 - 18*a^4*b^4 + 6*a^5*b^3) - (6*a^2*b^4*(a + b*tan(e + f*x)^2)^(1
/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*
b^5 - 18*a^4*b^4 + 6*a^5*b^3) + (2*a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a
*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(2*a*b^7 - 10*a^2*b^6 + 20*a^3*b^5 - 18
*a^4*b^4 + 6*a^5*b^3))*((a - b)^3)^(1/2))/f - (atanh((2*b^6*(a + b*tan(e +
 f*x)^2)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^
3) - (8*a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3
*b^5 + 12*a^4*b^4 - 6*a^5*b^3) + (12*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)*
(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4*b^4 - 6*a^5*b^3) - (6*a^3*b^3
*(a + b*tan(e + f*x)^2)^(1/2)*(a^3)^(1/2))/(2*a^2*b^6 - 8*a^3*b^5 + 12*a^4
*b^4 - 6*a^5*b^3))*(a^3)^(1/2))/f
```

Reduce [F]

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int\sqrt{\tan\left(fx+e\right)^2b+a}\,\cot\left(fx+e\right)\tan\left(fx+e\right)^2dx\right)b+\left(\int\sqrt{\tan\left(fx+e\right)^2b+a}\,\cot\left(fx+e\right)dx\right)a$$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x)*tan(e + f*x)**2,x)*b + int(sq
rt(tan(e + f*x)**2*b + a)*cot(e + f*x),x)*a
```

3.310 $\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} - \frac{a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*a^(1/2)*(2*a-3*b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/f-(a-b)^(3
/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f-1/2*a*cot(f*x+e)^2*(a+
b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

$$\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right) - 2(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right) - a\cot^2(e+f}{2f}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - 2*(a -
b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] - a*Cot[e + f*x]^
2*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4153, 354, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^3(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{109}$$

$$\frac{-\int \frac{\cot(e+fx)\left((a-2b)b\tan^2(e+fx)+a(2a-3b)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{2}\int \frac{\cot(e+fx)((a-2b)b\tan^2(e+fx)+a(2a-3b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx) - a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx) - a(2a-3b)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)\right) - a\cot}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{2}\left(\frac{4(a-b)^2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b} - \frac{2a(2a-3b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}\right) - a\cot(e+fx)\sqrt{a+b\tan}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{2}\left(2\sqrt{a}(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right) - 4(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)\right) - a\cot(e+fx)\sqrt{a+b\tan^2}}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] - 4*(a
 - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/2 - a*Cot[e +
 f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(98) = 196$.

Time = 6.67 (sec) , antiderivative size = 1080, normalized size of antiderivative = 9.31

method	result	size
default	Expression too large to display	1080

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(a-b)^(1/2)*((8*cos(f*x+e)-8)*ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*s
in(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b
)*a*b+(3*cos(f*x+e)-3)*a^(1/2)*b*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*
sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*
x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(
f*x+e)+1))*(a-b)^(1/2)+(-3*cos(f*x+e)+3)*a^(1/2)*ln(2*(2*((a*cos(f*x+e)^2+
b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^
2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(co
s(f*x+e)-1)^2)*b*(a-b)^(1/2)+(2-2*cos(f*x+e))*a^(3/2)*ln(2/a^(1/2)*(a^(1/2
)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*
cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+
cos(f*x+e)*b+b)/(cos(f*x+e)+1))*(a-b)^(1/2)+(-4*cos(f*x+e)+4)*a^2*ln(4*(a-
b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+
e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+
4*a*cos(f*x+e)-4*cos(f*x+e)*b)+(-4*cos(f*x+e)+4)*ln(4*(a-b)^(1/2)*((a*cos(
f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*
((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*
cos(f*x+e)*b)*b^2+(2*cos(f*x+e)-2)*a^(3/2)*ln(2*(2*((a*cos(f*x+e)^2+b*sin(
f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.85

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{2\,(a-b)^{\frac{3}{2}}\log\left(\frac{b\tan(fx+e)^2+2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+2\,a-b}{\tan(fx+e)^2+1}\right)\tan\left(fx+e\right)^2+(2\,a-3\,b)\sqrt{a}\log}{4\,f\tan\left(fx+\right.}\right.$$

$$-\frac{\sqrt{-a}(2\,a-3\,b)\arctan\left(\frac{\sqrt{-a}}{\sqrt{b\tan(fx+e)^2+a}}\right)\tan\left(fx+e\right)^2+(a-b)^{\frac{3}{2}}\log\left(\frac{b\tan(fx+e)^2+2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+2}{\tan(fx+e)^2+1}\right.}{2\,f\tan\left(fx+e\right)^2}$$

$$-\frac{\sqrt{-a}(2\,a-3\,b)\arctan\left(\frac{\sqrt{-a}}{\sqrt{b\tan(fx+e)^2+a}}\right)\tan\left(fx+e\right)^2-2\,(a-b)\sqrt{-a+b}\arctan\left(\frac{\sqrt{-a+b}}{\sqrt{b\tan(fx+e)^2+a}}\right)\tan}{2\,f\tan\left(fx+e\right)^2}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a - 3*b
)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2
*a)/tan(f*x + e)^2)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*ta
n(f*x + e)^2), 1/4*(4*(a - b)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(
f*x + e)^2 + a))*tan(f*x + e)^2 - (2*a - 3*b)*sqrt(a)*log((b*tan(f*x + e)^
2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x +
e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a)*
(2*a - 3*b)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 + (
a - b)^(3/2)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
 b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + sqrt(b*tan(f*x + e)^
2 + a)*a)/(f*tan(f*x + e)^2), -1/2*(sqrt(-a)*(2*a - 3*b)*arctan(sqrt(-a)/s
qrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 - 2*(a - b)*sqrt(-a + b)*arctan(
sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 + sqrt(b*tan(f*x +
 e)^2 + a)*a)/(f*tan(f*x + e)^2)]
```

Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\cot^3{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**3, x)
```

Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.85

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\operatorname{atanh}\left(\frac{3\,a^2\,b^4\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{2\left(-\frac{3\,a^4\,b^4}{2}+5\,a^3\,b^5-\frac{11\,a^2\,b^6}{2}+2\,a\,b^7\right)}-\frac{2\,a\,b^5\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{-\frac{3\,a^4\,b^4}{2}+5\,a^3\,b^5-\frac{11\,a^2\,b^6}{2}+2\,a\,b^7}\right)\sqrt{(a-}{f} + \frac{\sqrt{a}\operatorname{atanh}\left(\frac{3\,\sqrt{a}\,b^7\,\sqrt{b\tan(e+f\,x)^2+a}}{-\frac{3\,a^4\,b^4}{2}+\frac{23\,a^3\,b^5}{4}-\frac{29\,a^2\,b^6}{4}+3\,a\,b^7}-\frac{29\,a^{3/2}\,b^6\,\sqrt{b\tan(e+f\,x)^2+a}}{4\left(-\frac{3\,a^4\,b^4}{2}+\frac{23\,a^3\,b^5}{4}-\frac{29\,a^2\,b^6}{4}+3\,a\,b^7\right)}+\frac{23\,a^{5/2}\,b^5\,\sqrt{b\tan(e+f\,x)^2+a}}{4\left(-\frac{3\,a^4\,b^4}{2}+\frac{23\,a^3\,b^5}{4}-\frac{29\,a^2\,b^6}{4}+3\,a\,b^7\right)}-\right.}{2\,f} - \frac{a\,b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}}{2\,\left(f\,\left(b\tan\left(e+f\,x\right)^2+a\right)-a\,f\right)}$$

input
```
int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(atanh((3*a^2*b^4*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 -
b^3)^(1/2))/(2*(2*a*b^7 - (11*a^2*b^6)/2 + 5*a^3*b^5 - (3*a^4*b^4)/2)) - (
2*a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2)
)/(2*a*b^7 - (11*a^2*b^6)/2 + 5*a^3*b^5 - (3*a^4*b^4)/2))*((a - b)^3)^(1/2
))/f + (a^(1/2)*atanh((3*a^(1/2)*b^7*(a + b*tan(e + f*x)^2)^(1/2))/(3*a*b^
7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b^4)/2) - (29*a^(3/2)*b^6*(a
+ b*tan(e + f*x)^2)^(1/2))/(4*(3*a*b^7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4 -
 (3*a^4*b^4)/2)) + (23*a^(5/2)*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(3*a*b
^7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4 - (3*a^4*b^4)/2)) - (3*a^(7/2)*b^4*(a
 + b*tan(e + f*x)^2)^(1/2))/(2*(3*a*b^7 - (29*a^2*b^6)/4 + (23*a^3*b^5)/4
- (3*a^4*b^4)/2)))*(2*a - 3*b))/(2*f) - (a*b*(a + b*tan(e + f*x)^2)^(1/2))
/(2*(f*(a + b*tan(e + f*x)^2) - a*f))
```

Reduce [F]

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \cot(fx+e)^3\left(\tan(fx+e)^2b+a\right)^{\frac{3}{2}}dx$$

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.311 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = -\frac{(8a^2-12ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f} + \frac{(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(4a-5b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} - \frac{a\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

output
```
-1/8*(8*a^2-12*a*b+3*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2
)/f+(a-b)^(3/2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+1/8*(4*a-5
*b)*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/f-1/4*a*cot(f*x+e)^4*(a+b*tan(f*
x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{(-8a^2+12ab-3b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\right.}{8\sqrt{a}f}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-8*a^2 + 12*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + S
qrt[a]*(8*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] +
Cot[e + f*x]^2*(4*a - 5*b - 2*a*Cot[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2]
))/(8*Sqrt[a]*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4153, 354, 109, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^5(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^3(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{109}$$

$$\frac{-\frac{1}{2}\int \frac{\cot^2(e+fx)\left((3a-4b)b\tan^2(e+fx)+a(4a-5b)\right)}{2(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-\frac{1}{2}a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{4}\int \frac{\cot^2(e+fx)\left((3a-4b)b\tan^2(e+fx)+a(4a-5b)\right)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-\frac{1}{2}a\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{a\cot(e+fx)\left(8a^2-12ba+3b^2+(4a-5b)b\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a}+(4a-5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{2}a\cot^2(e}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{\cot(e+fx)\left(8a^2-12ba+3b^2+(4a-5b)b\tan^2(e+fx)\right)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)+(4a-5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{2}}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(8a^2-12ab+3b^2\right)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-8(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+f\right.\right.}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{2\left(8a^2-12ab+3b^2\right)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{16(a-b)^2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}\right)+(4a-5b)\cot(\right.}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(16(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)-\frac{2(8a^2-12ab+3b^2)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+(4a-5b)\cot(e+fx\right.}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*(a*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2]) + (((-2*(8*a^2 - 12*a*
b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + 16*(a -
b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/2 + (4*a - 5*b)*
Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/4)/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1505 vs. $2(139) = 278$.

Time = 6.60 (sec) , antiderivative size = 1506, normalized size of antiderivative = 9.35

method	result	size
default	Expression too large to display	1506

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(a-b)^(1/2)/a^(1/2)*((-16*cos(f*x+e)+16)*sin(f*x+e)^2*ln(4*(a-b)^(1
/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*
(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+4*a*c
os(f*x+e)-4*cos(f*x+e)*b)*a^(5/2)+(32*cos(f*x+e)-32)*sin(f*x+e)^2*ln(4*(a-
b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+
e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)+
4*a*cos(f*x+e)-4*cos(f*x+e)*b)*a^(3/2)*b+(-16*cos(f*x+e)+16)*sin(f*x+e)^2*
ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*
cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*a^(1/2)*b^2+(8*cos(f*x+e)-8)*sin(f*
x+e)^2*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^
(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*x+e)^2+2*a*cos(
f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*(a-b)^(1/2)*a^2+(-12*cos(f*
x+e)+12)*sin(f*x+e)^2*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)
+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^2+2*b*cos(f*
x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*(a-b)^(1/2)*
a*b+(3*cos(f*x+e)-3)*sin(f*x+e)^2*ln(2*(2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)
/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2-a*cos(f*x+e)^
2+2*b*cos(f*x+e)^2+2*a*cos(f*x+e)-4*cos(f*x+e)*b-a+2*b)/(cos(f*x+e)-1)^2)*
(a-b)^(1/2)*b^2+(-8*cos(f*x+e)+8)*sin(f*x+e)^2*ln(2/a^(1/2)*(a^(1/2)*((...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.52

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*(a^2 - a*b)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 -
 (8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*((4*a^2 - 5
*a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e
)^4), -1/16*(16*(a^2 - a*b)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*
x + e)^2 + a))*tan(f*x + e)^4 - (8*a^2 - 12*a*b + 3*b^2)*sqrt(a)*log((b*ta
n(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)
*tan(f*x + e)^4 - 2*((4*a^2 - 5*a*b)*tan(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*
x + e)^2 + a))/(a*f*tan(f*x + e)^4), 1/8*((8*a^2 - 12*a*b + 3*b^2)*sqrt(-a
)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^4 - 4*(a^2 - a*
b)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
 - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*t
an(f*x + e)^2 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4), 1
/8*((8*a^2 - 12*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^
2 + a))*tan(f*x + e)^4 - 8*(a^2 - a*b)*sqrt(-a + b)*arctan(sqrt(-a + b)/sq
rt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^4 + ((4*a^2 - 5*a*b)*tan(f*x + e)^2
 - 2*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f*tan(f*x + e)^4)]
```

Sympy [F]

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\cot^5{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**5, x)
```

Maxima [F]

$$\int \cot^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \int \left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}\cot(fx+e)^5 dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^5(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.59

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(\frac{3\,a\,b^2}{8}-\frac{a^2\,b}{2}\right)+\frac{b\,{\left(b\tan(e+f\,x)^2+a\right)}^{3/2}\,\left(4\,a-5\,b\right)}{8}}{f\,{\left(b\tan\left(e+f\,x\right)^2+a\right)}^2+a^2\,f-2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a\right)}-\frac{\mathrm{atanh}\left(\frac{9\,b^6\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{32\,\left(\frac{a^3\,b^5}{4}-\frac{25\,a^2\,b^6}{32}+\frac{13\,a\,b^7}{16}-\frac{9\,b^8}{32}\right)}-\frac{a\,b^5\,\sqrt{b\tan(e+f\,x)^2+a}\,\sqrt{a^3-3\,a^2\,b+3\,a\,b^2-b^3}}{4\,\left(\frac{a^3\,b^5}{4}-\frac{25\,a^2\,b^6}{32}+\frac{13\,a\,b^7}{16}-\frac{9\,b^8}{32}\right)}\right)\,\sqrt{{\left(a-b\right)}^3}}{f}-\frac{\mathrm{atanh}\left(\frac{75\,\sqrt{a}\,b^7\,\sqrt{b\tan(e+f\,x)^2+a}}{64\,\left(\frac{75\,a\,b^7}{64}-\frac{159\,b^8}{256}-\frac{29\,a^2\,b^6}{32}+\frac{a^3\,b^5}{4}+\frac{27\,b^9}{256\,a}\right)}-\frac{159\,b^8\,\sqrt{b\tan(e+f\,x)^2+a}}{256\,\sqrt{a}\,\left(\frac{75\,a\,b^7}{64}-\frac{159\,b^8}{256}-\frac{29\,a^2\,b^6}{32}+\frac{a^3\,b^5}{4}+\frac{27\,b^9}{256\,a}\right)}-\frac{29\,a^{3/2}\,b^6\,\sqrt{b\tan(e+f}{32\,\left(\frac{75\,a\,b^7}{64}-\frac{159\,b^8}{256}-\frac{29\,a^2\,b^6}{32}+\right.}}{8}$$

input
```
int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
((a + b*tan(e + f*x)^2)^(1/2)*((3*a*b^2)/8 - (a^2*b)/2) + (b*(a + b*tan(e
 + f*x)^2)^(3/2)*(4*a - 5*b))/8)/(f*(a + b*tan(e + f*x)^2)^2 + a^2*f - 2*a*
f*(a + b*tan(e + f*x)^2)) - (atanh((9*b^6*(a + b*tan(e + f*x)^2)^(1/2)*(3*
a*b^2 - 3*a^2*b + a^3 - b^3)^(1/2))/(32*((13*a*b^7)/16 - (9*b^8)/32 - (25*
a^2*b^6)/32 + (a^3*b^5)/4)) - (a*b^5*(a + b*tan(e + f*x)^2)^(1/2)*(3*a*b^2
 - 3*a^2*b + a^3 - b^3)^(1/2))/(4*((13*a*b^7)/16 - (9*b^8)/32 - (25*a^2*b^
6)/32 + (a^3*b^5)/4)))*((a - b)^3)^(1/2))/f - (atanh((75*a^(1/2)*b^7*(a +
b*tan(e + f*x)^2)^(1/2))/(64*((75*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)
/32 + (a^3*b^5)/4 + (27*b^9)/(256*a))) - (159*b^8*(a + b*tan(e + f*x)^2)^(
1/2))/(256*a^(1/2)*((75*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a^3
*b^5)/4 + (27*b^9)/(256*a))) - (29*a^(3/2)*b^6*(a + b*tan(e + f*x)^2)^(1/2
))/(32*((75*a*b^7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 + (2
7*b^9)/(256*a))) + (a^(5/2)*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*((75*a*b^
7)/64 - (159*b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*a)))
 + (27*b^9*(a + b*tan(e + f*x)^2)^(1/2))/(256*a^(3/2)*((75*a*b^7)/64 - (15
9*b^8)/256 - (29*a^2*b^6)/32 + (a^3*b^5)/4 + (27*b^9)/(256*a))))*(8*a^2 -
12*a*b + 3*b^2))/(8*a^(1/2)*f)
```

Reduce [F]

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \cot\left(fx+e\right)^5\left(\tan\left(fx+e\right)^2b+a\right)^{\frac{3}{2}}dx$$

input

```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output

```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.312 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\begin{aligned}\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx &= -\frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} \\ &\quad+\frac{(3a^4+8a^3b+48a^2b^2-192ab^3+128b^4)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} \\ &\quad-\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f} \\ &\quad+\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{192bf} \\ &\quad+\frac{(9a-8b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{48f} \\ &\quad+\frac{b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f}\end{aligned}$$

output
```
-(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/1
28*(3*a^4+8*a^3*b+48*a^2*b^2-192*a*b^3+128*b^4)*arctanh(b^(1/2)*tan(f*x+e)
/(a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+8*a^2*b-80*a*b^2+64*b^3)
*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/b^2/f+1/192*(3*a^2-56*a*b+48*b^2)*tan
(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/b/f+1/48*(9*a-8*b)*tan(f*x+e)^5*(a+b*ta
n(f*x+e)^2)^(1/2)/f+1/8*b*tan(f*x+e)^7*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.34 (sec) , antiderivative size = 908, normalized size of antiderivative = 3.09

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{-\frac{b(3a^4+8a^3b-16a^2b^2-64ab^3+64b^4)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{1+\cos(2(e+fx))}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(1+\cos(2(e+fx)))\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-}{}}}{a(a+b+(a-b)\cos(}}{} + \frac{\sqrt{\frac{a+b+a\cos(2(e+fx))-b\cos(2(e+fx))}{1+\cos(2(e+fx))}}\left(\frac{1}{48}\sec^5(e+fx)(9a\sin(e+fx)-26b\sin(e+fx))+\frac{\sec^3(e+fx)(3a^2\sin(e+fx)}{}\right.}{}$$

input
```
Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*(3*a^4 + 8*a^3*b - 16*a^2*b^2 - 64*a*b^3 + 64*b^4)*Sqrt[(a + b + (a
- b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b
)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a
- b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSi
n[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]
*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x)]))) - (4*b*(-64*a^2*b
^2 + 128*a*b^3 - 64*b^4)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*
Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*
Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b
)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[S
qrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Si
n[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(
e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x
)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f
*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b +
(a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/
(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)
]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(64*b^2*f) + (Sqrt[(a + b +
a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sec[e +
 f*x]^5*(9*a*Sin[e + f*x] - 26*b*Sin[e + f*x]))/48 + (Sec[e + f*x]^3*(3...
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4153, 379, 444, 27, 444, 27, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{379}$$

$$\frac{\frac{1}{8}\int \frac{\tan^6(e+fx)\left((9a-8b)b\tan^2(e+fx)+a(8a-7b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{8}b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}(9a-8b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{\int \frac{b\tan^4(e+fx)\left(5a(9a-8b)-\left(3a^2-56ba+48b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{6b}\right)+\frac{1}{8}b}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}(9a-8b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{1}{6}\int \frac{\tan^4(e+fx)\left(5a(9a-8b)-\left(3a^2-56ba+48b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx\right.}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int -\frac{3\tan^2(e+fx)\left(\left(3a^3+8ba^2-80b^2a+64b^3\right)\tan^2(e+fx)+a\left(3a^2-56ba+48b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{4b}+\frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e-}}{4b}\right.\right.}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-56ab+48b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\int \frac{\tan^2(e+fx)\left(\left(3a^3+8ba^2-80b^2a+64b^3\right)\tan^2(e+fx)+a\left(3a^2-56ba+48b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+}{4b}\right.\right.}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\int\frac{(3a^4+8ba^3+48b^2a^2-192\cdots}{(\tan\cdots}}{\cdots}\right)}{4b}\right.\right.$$

$\downarrow$ 398

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(3a^4+8a^3b+48a^2b^2-192ab\cdots}{\cdots}\right)}{\cdots}\right.\right.$$

$\downarrow$ 224

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(3a^4+8a^3b+48a^2b^2-192ab\cdots}{\cdots}\right)}{\cdots}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{(3a^4+8a^3b+48a^2b^2-192ab\cdots}{\cdots}}{\cdots}\right)}{\cdots}\right.\right.$$

$\downarrow$ 291

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(3a^4+8a^3b+48a^2b^2-192ab\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 216

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-56ab+48b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{3\left(\frac{(3a^3+8a^2b-80ab^2+64b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(3a^4+8a^3b+48a^2b^2-192ab\right.}{}\right.}{}\right.\right.$$

input
```
Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((b*Tan[e + f*x]^7*Sqrt[a + b*Tan[e + f*x]^2])/8 + (((9*a - 8*b)*Tan[e + f
*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/6 + (((3*a^2 - 56*a*b + 48*b^2)*Tan[e +
f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*b) - (3*(-1/2*(-128*(a - b)^(3/2)*b^
2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] + ((3*a^4
+ 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*ArcTanh[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[b])/b + ((3*a^3 + 8*a^2*b - 80*a*b^2
+ 64*b^3)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*b)))/(4*b))/6)/8)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(264) = 528$.

Time = 0.76 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.28

method	result
derivativedivides	$\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right.}{8f\sqrt{b}}$
default	$\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right.}{8f\sqrt{b}}$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/8/f*a*tan(f*x+e)*(a+b*tan(f*x+
e)^2)^(1/2)+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/
2))+1/8/f*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(5/2)/b-1/16/f*a/b^2*tan(f*x+e)*
(a+b*tan(f*x+e)^2)^(5/2)+1/64/f*a^2/b^2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2
)+3/128/f*a^3/b^2*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+3/128/f*a^4/b^(5/2)*
ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/6/f*tan(f*x+e)*(a+b*tan(
f*x+e)^2)^(5/2)/b+1/24/f*a/b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+1/16/f*a^
2/b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)+1/16/f*a^3/b^(3/2)*ln(b^(1/2)*tan(
f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/2*b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)
/f-3/2/f*b^(1/2)*a*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^
4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)
^(1/2)*tan(f*x+e))+1/f*b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1
/2))+2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arc
tan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 1043, normalized size of antiderivative = 3.55

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log
(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) +
a) - 384*(a*b^3 - b^4)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqr
t(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1
)) + 2*(48*b^4*tan(f*x + e)^7 + 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*
a^2*b^2 - 56*a*b^3 + 48*b^4)*tan(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*
a*b^3 + 64*b^4)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*
(3*(3*a^4 + 8*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(-b)*arctan(sq
rt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + 192*(a*b^3 - b^4)*sqrt(-
a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (48*b^4*tan(f*x + e)^7 +
 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 + 48*b^4)*ta
n(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/768*(768*(a*b^3 - b^4)*sqrt(a - b)*
arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - 3*(3*a^4 + 8
*a^3*b + 48*a^2*b^2 - 192*a*b^3 + 128*b^4)*sqrt(b)*log(2*b*tan(f*x + e)^2
+ 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(48*b^4*tan(f
*x + e)^7 + 8*(9*a*b^3 - 8*b^4)*tan(f*x + e)^5 + 2*(3*a^2*b^2 - 56*a*b^3 +
 48*b^4)*tan(f*x + e)^3 - 3*(3*a^3*b + 8*a^2*b^2 - 80*a*b^3 + 64*b^4)*tan(
f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^3*f), -1/384*(384*(a*b^3 - b^4...
```

Sympy [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^6\left(e+fx\right)dx$$

input
```
integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

Maxima [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^6\,dx$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \tan(e+f\,x)^6\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx = \left(\int \sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^8dx\right)b+\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^6dx\right)a$$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**8,x)*b + int(sqrt(tan(e + f*
x)**2*b + a)*tan(e + f*x)**6,x)*a
```

3.313 $\int \tan^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$\begin{aligned}\int \tan^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx &= \frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} \\ &- \frac{(a^3+6a^2b-24ab^2+16b^3)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} \\ &+ \frac{(a^2-10ab+8b^2)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{16bf} \\ &+ \frac{(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{24f} \\ &+ \frac{b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{6f}\end{aligned}$$

output
```
(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/16
*(a^3+6*a^2*b-24*a*b^2+16*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^
2)^(1/2))/b^(3/2)/f+1/16*(a^2-10*a*b+8*b^2)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^
(1/2)/b/f+1/24*(7*a-6*b)*tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f+1/6*b*tan
(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 4.85 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.97

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{\Big((30a^3-266a^2b+200ab^2+104b^3+(45a^3-433a^2b+296ab^2-84b^3)\cos(2(e+fx))+2(}{}$$

input
```
Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(((30*a^3 - 266*a^2*b + 200*a*b^2 + 104*b^3 + (45*a^3 - 433*a^2*b + 296*a*
b^2 - 84*b^3)*Cos[2*(e + f*x)] + 2*(9*a^3 - 107*a^2*b + 92*a*b^2 + 12*b^3)
*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)] - 47*a^2*b*Cos[6*(e + f*x)] + 8
8*a*b^2*Cos[6*(e + f*x)] - 44*b^3*Cos[6*(e + f*x)])*Csc[2*(e + f*x)]^4 - 3
*Sqrt[2]*a*(a^2 - 10*a*b + 8*b^2)*Cot[e + f*x]^2*Csc[e + f*x]^2*Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((
a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 48*Sqr
t[2]*a*b*(-a + b)*Cot[e + f*x]^2*Csc[e + f*x]^2*Sqrt[((a + b + (a - b)*Cos
[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a
 + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e +
f*x]^2*Tan[e + f*x]^3)/(48*Sqrt[2]*b*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*
x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4153, 379, 444, 27, 444, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4 \left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{379}$$

$$\frac{\frac{1}{6}\int \frac{\tan^4(e+fx)((7a-6b)b\tan^2(e+fx)+a(6a-5b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{6}b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{\int \frac{3b\tan^2(e+fx)\left(a(7a-6b)-\left(a^2-10ba+8b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{4b}\right)+\frac{1}{6}b\,t}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\int \frac{\tan^2(e+fx)(a(7a-6b)-(a^2-10ba+8b^2)\tan^2(e+fx))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\left(-\frac{\int -\frac{(a-2b)\left(a^2+8ba-8b^2\right)\tan^2(e+fx)+a\left(a^2-10ba+8b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}-(\right.\right.}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\left(\frac{\int \frac{(a-2b)\left(a^2+8ba-8b^2\right)\tan^2(e+fx)+a\left(a^2-10ba+8b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}-\frac{(a^2-}{}\right.\right.}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\left(\frac{(a-2b)\left(a^2+8ab-8b^2\right)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-16b(a-b)^2\int\frac{}{\left(\tan\right.}}{2b}\right.\right.$$

$\downarrow$ 224

$$\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\left(\frac{(a-2b)\left(a^2+8ab-8b^2\right)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-16b(a-b)^2}{2b}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\left(\frac{\frac{(a-2b)\left(a^2+8ab-8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-16b(a-b)^2\int\frac{}{\left(\tan^2(e\right.}}{2b}\right.\right.$$

$\downarrow$ 291

$$\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\left(\frac{\frac{(a-2b)\left(a^2+8ab-8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-16b(a-b)^2\int\frac{}{1-\frac{(b-a}{b\tan}}}{2b}\right.\right.$$
$$f$$

$\downarrow$ 216

$$\frac{1}{6}\left(\frac{1}{4}(7a-6b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{3}{4}\left(\frac{\frac{(a-2b)\left(a^2+8ab-8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-16b(a-b)^{3/2}\arctan(}{2b}\right.\right.$$
$$f$$

input
```
Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((b*Tan[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/6 + (((7*a - 6*b)*Tan[e + f
*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/4 - (3*((-16*(a - b)^(3/2)*b*ArcTan[(Sqr
t[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] + ((a - 2*b)*(a^2 + 8*a
*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sq
rt[b])/(2*b) - ((a^2 - 10*a*b + 8*b^2)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x
]^2])/(2*b)))/4)/6)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(198) = 396$.

Time = 0.69 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.28

method	result
derivativedivides	$\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}{6fb} - \frac{a\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{24fb} - \frac{a^2\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{16fb} - \frac{a^3 \ln}{}$
default	$\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}{6fb} - \frac{a\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{24fb} - \frac{a^2\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{16fb} - \frac{a^3 \ln}{}$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(5/2)/b-1/24/f*a/b*tan(f*x+e)*(a+b*tan
(f*x+e)^2)^(3/2)-1/16/f*a^2/b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)-1/16/f*a
^3/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/2*b*tan(f*x+e
)*(a+b*tan(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*ln(b^(1/2)*tan(f*x+e)+(a+b*ta
n(f*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*b^(3/2)*ln(b^(1/2)*tan(f*
x+e)+(a+b*tan(f*x+e)^2)^(1/2))-2/f*a/b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*
(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f*a^2*(b^4*
(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^
2)^(1/2)*tan(f*x+e))-1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)-3/8/f*a*tan
(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)-3/8/f*a^2/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(
a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 845, normalized size of antiderivative = 3.77

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^
2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 48*(a*b^2 - b
^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2
+ a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(8*b^3*tan(f
*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 + 8*b
^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b^2*f), 1/48*(3*(a^3 + 6*a^
2*b - 24*a*b^2 + 16*b^3)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(
f*x + e)^2 + a)) - 24*(a*b^2 - b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x +
 e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f
*x + e)^2 + 1)) + (8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)
^3 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)
)/(b^2*f), 1/96*(96*(a*b^2 - b^3)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x +
 e)/sqrt(b*tan(f*x + e)^2 + a)) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sq
rt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*
x + e) + a) + 2*(8*b^3*tan(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3
 + 3*(a^2*b - 10*a*b^2 + 8*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/
(b^2*f), 1/48*(48*(a*b^2 - b^3)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e
)/sqrt(b*tan(f*x + e)^2 + a)) + 3*(a^3 + 6*a^2*b - 24*a*b^2 + 16*b^3)*sqrt
(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (8*b^3*tan
(f*x + e)^5 + 2*(7*a*b^2 - 6*b^3)*tan(f*x + e)^3 + 3*(a^2*b - 10*a*b^2 ...
```

Sympy [F]

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\tan^4{(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \tan(e+f\,x)^4\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\tan(fx+e)^2b+a}\,\tan(fx+e)^6\,dx\right)b+\left(\int \sqrt{\tan(fx+e)^2b+a}\,\tan(fx+e)^4\,dx\right)a$$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**6,x)*b + int(sqrt(tan(e + f*
x)**2*b + a)*tan(e + f*x)**4,x)*a
```

3.314 $\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = -\frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-12ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4f}$$

output
```
-(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/8
*(3*a^2-12*a*b+8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))
/b^(1/2)/f+1/8*(5*a-4*b)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f+1/4*b*tan(f
*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.20 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.48

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \frac{b(a^2+4ab-4b^2)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{1+\cos(2(e+fx))}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(1+\cos(2(e+fx)))\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+}{b}}}{a(a+b+(a-b)\cos(2(e+fx)))} + \frac{\sqrt{\frac{a+b+a\cos(2(e+fx))-b\cos(2(e+fx))}{1+\cos(2(e+fx))}}\left(\frac{1}{8}\sec(e+fx)(5a\sin(e+fx)-6b\sin(e+fx))+\frac{1}{4}b\sec^2(e+fx)\tan(e+\right.}{f}$$

input
```
Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((b*(a^2 + 4*a*b - 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos
[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*
(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a
 - b)*Cos[2*(e + f*x)])) + (4*b*(4*a^2 - 8*a*b + 4*b^2)*Sqrt[1 + Cos[2*(e
+ f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*(
(Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[
2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc
[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)
]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)
]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a -
 b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(
a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[
a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]
])/(4*f) + (Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Co
s[2*(e + f*x)])]*((Sec[e + f*x]*(5*a*Sin[e + f*x] - 6*b*Sin[e + f*x]))/8 +
 (b*Sec[e + f*x]^2*Tan[e + f*x])/4))/f
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4153, 379, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{379}$$

$$\frac{\frac{1}{4}\int \frac{\tan^2(e+fx)\left((5a-4b)b\tan^2(e+fx)+a(4a-3b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{4}b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{\int \frac{b\left(a(5a-4b)-\left(3a^2-12ba+8b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}\right)+\frac{1}{4}b\tan^3(e+f}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(5a-4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}-\frac{1}{2}\int \frac{a(5a-4b)-\left(3a^2-12ba+8b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+\frac{1}{4}b\tan^3(}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2-12ab+8b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-8(a-b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right.\right.}{f}$$

$$\Big\downarrow 224$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2-12ab+8b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-8(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+\right.\right.}{f}$$

$$\Big\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{(3a^2-12ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+\frac{1}{2}(5a\right.}{f}$$

$$\Big\downarrow 291$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{(3a^2-12ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8(a-b)^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}\right)+\frac{1}{2}(5a-4b)\tan(\right.}{f}$$

$$\Big\downarrow 216$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{(3a^2-12ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}(5a-4b)\tan(e+fx)\right.}{f}$$

input
```
Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((b*Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/4 + ((-8*(a - b)^(3/2)*ArcT
an[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] + ((3*a^2 - 12*a
*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sq
rt[b])/2 + ((5*a - 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/2)/4)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(150) = 300$.

Time = 0.69 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.24

method	result
derivativedivides	$\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{8f\sqrt{b}}$
default	$\frac{\tan(fx+e)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}{4f}+\frac{3a\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{8f\sqrt{b}}$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2)+3/8/f*a*tan(f*x+e)*(a+b*tan(f*x+
e)^2)^(1/2)+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/
2))-1/2*b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-3/2/f*b^(1/2)*a*ln(b^(1/2)
*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b
^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/f*b^(3/2
)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+2/f*a/b*(b^4*(a-b))^(1/2
)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*
x+e))-1/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/
2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.02

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((3*a^2 - 12*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*
tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 8*(a*b - b^2)*sqrt(-a + b)
*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b
)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*(2*b^2*tan(f*x + e)^3 + (5*a
*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(b*f), -1/8*((3*a^2
- 12*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e
)^2 + a)) + 4*(a*b - b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*
sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2
+ 1)) - (2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f
*x + e)^2 + a))/(b*f), -1/16*(16*(a*b - b^2)*sqrt(a - b)*arctan(sqrt(a - b
)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (3*a^2 - 12*a*b + 8*b^2)*sqrt
(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x
+ e) + a) - 2*(2*b^2*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b
*tan(f*x + e)^2 + a))/(b*f), -1/8*(8*(a*b - b^2)*sqrt(a - b)*arctan(sqrt(a
 - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (3*a^2 - 12*a*b + 8*b^2)*
sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (2*b^2
*tan(f*x + e)^3 + (5*a*b - 4*b^2)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)
)/(b*f)]
```

Sympy [F]

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan^2\left(e+fx\right)dx$$

input
```
integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \tan(e+f\,x)^2\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \tan^2(e+fx)\,(a+b\tan^2(e+fx))^{3/2}\,dx=\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^4\,dx\right)b+\left(\int \sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^2\,dx\right)a$$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**4,x)*b + int(sqrt(tan(e + f*
x)**2*b + a)*tan(e + f*x)**2,x)*a
```

3.315 $\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-2b)\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2f}$$

output
```
(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/2*
(3*a-2*b)*b^(1/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1
/2*b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \frac{-2(a-b)^{3/2}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\tan^2(e+fx)-\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{b}(-3a+2b)\log\left(-\sqrt{b}\,\text{ta}\right.}{2f}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a - b)^(3/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[e + f*x]^2 - Tan[e + f*x]*
Sqrt[a + b*Tan[e + f*x]^2])/Sqrt[a - b]] + Sqrt[b]*(-3*a + 2*b)*Log[-(Sqrt
[b]*Tan[e + f*x]) + Sqrt[a + b*Tan[e + f*x]^2]] + b*Tan[e + f*x]*Sqrt[a +
b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(e+fx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{1}{2}\int \frac{(3a-2b)b\tan^2(e+fx)+a(2a-b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b(3a-2b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b(3a-2b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}\right)+\frac{1}{2}b\tan}{f}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}b\tan(e+}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{2}\left(2(a-b)^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a}}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{2}\left(2(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+\sqrt{b}(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x
]^2]] + (3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/2)/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(107) = 214.

Time = 0.68 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{b\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2f}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}}\right.}{f(a-b)}$
default	$\frac{b\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2f}+\frac{3\sqrt{b}\,a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2f}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-}{\sqrt{b^4(a-b)}}\right.}{f(a-b)}$

input
```
int((a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*ln(b^(1/2)*tan
(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+1/f*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(
a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/f*b^(3/2)*ln
(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))-2/f*a/b*(b^4*(a-b))^(1/2)/(a
-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)
)+1/f*a^2*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(
a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.17

$$\int (a+b\tan^2(e+fx))^{3/2}\,dx=\left[-\frac{(3a-2b)\sqrt{b}\log\left(2b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{b}\tan(fx+e)+a\right)+2}{}\right.$$

$$-\frac{(3a-2b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-(-a+b)^{\frac{3}{2}}\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+}{\tan(fx+e)^2+1}\right.}{2f}$$

input
```
integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*a - 2*b)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(a - b)*sqrt(-a + b)*log(-((a - 2*b)*
tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) -
a)/(tan(f*x + e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f,
 -1/2*((3*a - 2*b)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x +
e)^2 + a)) - (-a + b)^(3/2)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - sqr
t(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/4*(4*(a - b)^(3/2)*arctan(sqr
t(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (3*a - 2*b)*sqrt(b)*lo
g(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) +
 a) + 2*sqrt(b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f, 1/2*(2*(a - b)^(3/2)
*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (3*a - 2*b)
*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + sqrt(
b*tan(f*x + e)^2 + a)*b*tan(f*x + e))/f]
```

Sympy [F]

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left(\int \sqrt{\tan\left(fx+e\right)^2 b+a}dx\right)a+\left(\int \sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^2 dx\right)b$$

input
```
int((a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a),x)*a + int(sqrt(tan(e + f*x)**2*b + a)*tan
(e + f*x)**2,x)*b
```

3.316 $\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=-\frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f}-\frac{a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

output
```
-(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+b^(
3/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-a*cot(f*x+e)*(
a+b*tan(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 4.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.25

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=$$

$$-\frac{a\left(\left(a+b+(a-b)\cos(2(e+fx))\right)\csc^2(e+fx)+\sqrt{2}(a-2b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\text{EllipticF}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-((a*((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 + Sqrt[2]*(a - 2*b
)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[Ar
cSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]],
 1] + Sqrt[2]*(-a + b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*
x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[(a
+ b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4153, 376, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{376}$$

$$\frac{\int -\frac{a(a-2b)-b^2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \frac{a(a-2b)-b^2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$\Big\downarrow$ 398

$$\frac{b^2\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-b)^2\left(-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)-a\cot(e+fx)\sqrt{}}{f}$$

$\Big\downarrow$ 224

$$\frac{b^2\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+(a-b)^2\left(-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)-a\cot(e+fx)}{f}$$

$\Big\downarrow$ 219

$$\frac{(a-b)^2\left(-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-a\cot(e+fx)\sqrt{a+}}{f}$$

$\Big\downarrow$ 291

$$\frac{(a-b)^2\left(-\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-a\cot(e+fx)\sqrt{a+b\tan^2(e}}{f}$$

$\Big\downarrow$ 216

$$\frac{(a-b)^{3/2}\left(-\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)-a\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]
^2]]) + b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]
 - a*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(100) = 200.

Time = 23.59 (sec) , antiderivative size = 439, normalized size of antiderivative = 3.85

method	result
default	$-\frac{\left(b^{\frac{3}{2}}\sqrt{a-b}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{b}\,(\cos(fx+e)-1)}\right)\sin(fx+e)+\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a^2\sin}{}$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a-b)^(1/2)*(b^(3/2)*(a-b)^(1/2)*arctanh(1/b^(1/2)*((a*cos(f*x+e)^2+b
*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*sin(f*x+
e)+arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a^2*sin(f*x+e)-2*arctan(1/(a-b)^(1/2)*((
a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x
+e)-1))*a*b*sin(f*x+e)+arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^
2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*b^2*sin(f*x+e)+(cos(
f*x+e)+1)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a*(a-b)
^(1/2))*(a+b*tan(f*x+e)^2)^(3/2)/(cos(f*x+e)+1)/(a*cos(f*x+e)^2+b*sin(f*x+
e)^2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^
2*cot(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 710, normalized size of antiderivative = 6.23

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b^(3/2)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(b)*tan(f*x + e) + a)*tan(f*x + e) - (a - b)*sqrt(-a + b)*log(-((a^2 - 8*a
*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((
a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e) - 4*sqrt(b*
tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/4*(4*sqrt(-b)*b*arctan(sqrt(-b
)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e) + (a - b)*sqrt(-a
+ b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*
x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))*tan
(f*x + e) + 4*sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b
)^(3/2)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a
- 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e) - b^(3/2)*log(2*b*tan(f*x + e)^2
+ 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a)*tan(f*x + e) + 2*
sqrt(b*tan(f*x + e)^2 + a)*a)/(f*tan(f*x + e)), -1/2*((a - b)^(3/2)*arctan
(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x
 + e)^2 - a))*tan(f*x + e) + 2*sqrt(-b)*b*arctan(sqrt(-b)*tan(f*x + e)/sqr
t(b*tan(f*x + e)^2 + a))*tan(f*x + e) + 2*sqrt(b*tan(f*x + e)^2 + a)*a)/(f
*tan(f*x + e))]
```

Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^2\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \cot\left(fx+e\right)^2\left(\tan\left(fx+e\right)^2b+a\right)^{\frac{3}{2}}dx$$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.317 $\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} \, dx = \frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f} - \frac{a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

output

```
(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f+1/3*
(3*a-4*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/f-1/3*a*cot(f*x+e)^3*(a+b*ta
n(f*x+e)^2)^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} \, dx = -\frac{\cot(e+fx)\left(b+a\cot^2(e+fx)\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{(a-b)\tan^2(e+fx)}{a+b\tan^2(e+fx)}\right)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/3*(Cot[e + f*x]*(b + a*Cot[e + f*x]^2)*Hypergeometric2F1[-3/2, 1, -1/2,
 -(((a - b)*Tan[e + f*x]^2)/(a + b*Tan[e + f*x]^2))]*Sqrt[a + b*Tan[e + f*
x]^2])/f
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4153, 376, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{376}$$

$$\frac{\frac{1}{3}\int -\frac{\cot^2(e+fx)\left((2a-3b)b\tan^2(e+fx)+a(3a-4b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{3}a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{3}\int \frac{\cot^2(e+fx)\left((2a-3b)b\tan^2(e+fx)+a(3a-4b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{3}a\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{1}{3}\left(\frac{\int \frac{3a(a-b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}+(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{3}a\cot^3(e+fx)\sqrt{a+}}{f}$$

$\downarrow$ 27

$$\frac{\frac{1}{3}\left(3(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{3}a\cot}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{3}\left(3(a-b)^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{3}a\cot^3(e+fx}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{3}\left(3(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)+(3a-4b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{3}a\cot^3(e+fx)\sqrt{a+b}}{f}$$

input

```
Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output

```
(-1/3*(a*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2]) + (3*(a - b)^(3/2)*Arc
Tan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] + (3*a - 4*b)*C
ot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/3)/f
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(101) = 202.

Time = 23.00 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.79

method	result
default	$-\frac{\left((3\cos(fx+e)-3)\sin(fx+e)\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a^2+(-6\cos(fx+e)+6)\sin(fx+e)\arctan\left(\sqrt{\frac{a\cos}{}}\right.\right.}{}$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a-b)^(1/2)*((3*cos(f*x+e)-3)*sin(f*x+e)*arctan(1/(a-b)^(1/2)*((a*c
os(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)
-1))*a^2+(-6*cos(f*x+e)+6)*sin(f*x+e)*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^
2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*a*b+(
3*cos(f*x+e)-3)*sin(f*x+e)*arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x
+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*b^2+(4*cos(f*x+e
)^2-3)*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2
)*a+4*sin(f*x+e)^2*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e
)+1)^2)^(1/2)*b)*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2)
/(cos(f*x+e)+1)^2)^(1/2)/(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*cot(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.68

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\left[-\frac{3\,(a-b)\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2-4\left((a-2\,b)\tan(fx+e)^3\right.}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right.}{12}\right.$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a - b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4
- 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a
*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 +
2*tan(f*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a - 4*b)*tan(f*x + e)^2 - a)
*sqrt(b*tan(f*x + e)^2 + a))/(f*tan(f*x + e)^3), 1/6*(3*(a - b)^(3/2)*arct
an(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f
*x + e)^2 - a))*tan(f*x + e)^3 + 2*((3*a - 4*b)*tan(f*x + e)^2 - a)*sqrt(b
*tan(f*x + e)^2 + a))/(f*tan(f*x + e)^3)]
```

Sympy [F]

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(a+b\tan^2{(e+fx)}\right)^{\frac{3}{2}}\cot^4{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(3/2)*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^4\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \cot\left(fx+e\right)^4\left(\tan\left(fx+e\right)^2b+a\right)^{\frac{3}{2}}dx$$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.318 $\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = -\frac{(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15af} + \frac{(5a-6b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15f} - \frac{a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5f}$$

output
```
-(a-b)^(3/2)*arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/f-1/1
5*(15*a^2-20*a*b+3*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f+1/15*(5*a-
6*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/f-1/5*a*cot(f*x+e)^5*(a+b*tan(f
*x+e)^2)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.85

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{\cos(e+fx)\left(b+a\cot^2(e+fx)\right)^2\left(a(-2b+3a\cot^2(e+fx))\,\text{Hypergeometric2F1}\left(1,1,-\frac{1}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/15*(Cos[e + f*x]*(b + a*Cot[e + f*x]^2)^2*(a*(-2*b + 3*a*Cot[e + f*x]^2
)*Hypergeometric2F1[1, 1, -1/2, ((a - b)*Sin[e + f*x]^2)/a] + 2*(a - b)*(a
 + b + (a - b)*Cos[2*(e + f*x)])*Hypergeometric2F1[2, 2, 1/2, ((a - b)*Sin
[e + f*x]^2)/a])*Sin[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(a^3*f)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4153, 376, 25, 445, 27, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^{3/2}}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(b\tan^2(e+fx)+a\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 376$$

$$\frac{\frac{1}{5}\int -\frac{\cot^4(e+fx)\left((4a-5b)b\tan^2(e+fx)+a(5a-6b)\right)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{5}a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{1}{5}\int \frac{\cot^4(e+fx)\left((4a-5b)b\tan^2(e+fx)+a(5a-6b)\right)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{1}{5}a\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{1}{5}\left(\frac{\int \frac{a\cot^2(e+fx)\left(15a^2-20ba+3b^2+2(5a-6b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}+\frac{1}{3}(5a-6b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)-\frac{1}{5}a\,\mathrm{co}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\cot^2(e+fx)\left(15a^2-20ba+3b^2+2(5a-6b)b\tan^2(e+fx)\right)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{1}{3}(5a-6b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}\right)}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int \frac{15a(a-b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)+\frac{1}{3}(5a-6b)\cot^3(e+}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15(a-b)^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)+\frac{1}{3}(5}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15(a-b)^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}\right)+\frac{1}{3}(5a-6b)\,\mathrm{co}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-\frac{(15a^2-20ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}-15(a-b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)\right)+\frac{1}{3}(5a-6b)\cot^3(e+ }{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*(a*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2]) + (((5*a - 6*b)*Cot[e
+ f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/3 + (-15*(a - b)^(3/2)*ArcTan[(Sqrt[a
 - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]] - ((15*a^2 - 20*a*b + 3*b^
2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/3)/5)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(147) = 294$.

Time = 23.85 (sec) , antiderivative size = 534, normalized size of antiderivative = 3.24

method	result
default	$-\frac{\left(\sin(fx+e)^{3}(-15\cos(fx+e)+15)\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^{2}+b\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)a^{3}+\sin(fx+e)^{3}(30\cos(fx+e)-30)\arctan\left(\right.\right.}{}$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a/(a-b)^(1/2)*(sin(f*x+e)^3*(-15*cos(f*x+e)+15)*arctan(1/(a-b)^(1/
2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(co
s(f*x+e)-1))*a^3+sin(f*x+e)^3*(30*cos(f*x+e)-30)*arctan(1/(a-b)^(1/2)*((a*
cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e
)-1))*a^2*b+sin(f*x+e)^3*(-15*cos(f*x+e)+15)*arctan(1/(a-b)^(1/2)*((a*cos(
f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1)
)*a*b^2+(23*cos(f*x+e)^4-35*cos(f*x+e)^2+15)*(a-b)^(1/2)*((a*cos(f*x+e)^2+
b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^2+(26*cos(f*x+e)^2-20)*sin(f*x+e
)^2*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a
*b+3*sin(f*x+e)^4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)
+1)^2)^(1/2)*b^2)*(a+b*tan(f*x+e)^2)^(3/2)/((a*cos(f*x+e)^2+b*sin(f*x+e)^2
)/(cos(f*x+e)+1)^2)^(1/2)/(a*cos(f*x+e)^2+b*sin(f*x+e)^2)*cot(f*x+e)^3*csc
(f*x+e)^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.33

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^{3/2} dx = \left[-\frac{15\,(a^2-ab)\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2+4\,\left((a-2\,b)\tan(fx+\right.}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right.}{} \right.$$

$$-\frac{15\,(a^2-ab)\sqrt{a-b}\arctan\left(-\frac{2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan\left(fx+e\right)^5+2\,\left((15\,a^2-20\,ab+3\,b^2)\tan\right.}{30\,af\tan\left(fx+e\right)^5}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/60*(15*(a^2 - a*b)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x +
e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^
3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)
^4 + 2*tan(f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^2 - 20*a*b + 3*b^2)*
tan(f*x + e)^4 - (5*a^2 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x +
e)^2 + a))/(a*f*tan(f*x + e)^5), -1/30*(15*(a^2 - a*b)*sqrt(a - b)*arctan(
-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x
+ e)^2 - a))*tan(f*x + e)^5 + 2*((15*a^2 - 20*a*b + 3*b^2)*tan(f*x + e)^4
- (5*a^2 - 6*a*b)*tan(f*x + e)^2 + 3*a^2)*sqrt(b*tan(f*x + e)^2 + a))/(a*f
*tan(f*x + e)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^6\,\left(b\tan(e+f\,x)^2+a\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^{3/2}\,dx = \int \cot\left(fx+e\right)^6\left(\tan\left(fx+e\right)^2b+a\right)^{\frac{3}{2}}dx$$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.319 $\int \left(a+b\tan^2(c+dx)\right)^{5/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\begin{aligned}\int \left(a+b\tan^2(c+dx)\right)^{5/2} dx &= \frac{(a-b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{d} \\ &+ \frac{\sqrt{b}(15a^2-20ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)}{8d} \\ &+ \frac{(7a-4b)b\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{8d} \\ &+ \frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{4d}\end{aligned}$$

output
```
(a-b)^(5/2)*arctan((a-b)^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^2)^(1/2))/d+1/8*
b^(1/2)*(15*a^2-20*a*b+8*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b*tan(d*x+c)^2
)^(1/2))/d+1/8*(7*a-4*b)*b*tan(d*x+c)*(a+b*tan(d*x+c)^2)^(1/2)/d+1/4*b*tan
(d*x+c)*(a+b*tan(d*x+c)^2)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2} dx = \frac{-8(a-b)^{5/2}\arctan\left(\frac{\sqrt{b}+\sqrt{b}\tan^2(c+dx)-\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}}{\sqrt{a-b}}\right)-\sqrt{b}(15a^2-20ab+8b^2)\log\left(\right.$$

input
```
Integrate[(a + b*Tan[c + d*x]^2)^(5/2),x]
```

output
```
(-8*(a - b)^(5/2)*ArcTan[(Sqrt[b] + Sqrt[b]*Tan[c + d*x]^2 - Tan[c + d*x]*
Sqrt[a + b*Tan[c + d*x]^2])/Sqrt[a - b]] - Sqrt[b]*(15*a^2 - 20*a*b + 8*b^
2)*Log[-(Sqrt[b]*Tan[c + d*x]) + Sqrt[a + b*Tan[c + d*x]^2]] + b*Tan[c + d
*x]*Sqrt[a + b*Tan[c + d*x]^2]*(9*a - 4*b + 2*b*Tan[c + d*x]^2))/(8*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 4144, 318, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(c+dx)^2\right)^{5/2} dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a\right)^{5/2}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{318}$$

$$\frac{\frac{1}{4}\int \frac{\sqrt{b\tan^2(c+dx)+a}\left((7a-4b)b\tan^2(c+dx)+a(4a-b)\right)}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{1}{4}b\tan(c+dx)\left(a+b\tan^2(c+dx)\right)^{3/2}}{d}$$

$\big\downarrow$ 403

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{b(15a^2-20ba+8b^2)\tan^2(c+dx)+a(8a^2-9ba+4b^2)}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a}}d\tan(c+dx)+\frac{1}{2}b(7a-4b)\tan(c+dx)\sqrt{a+b\tan^2(c+dx)}\right)+}{d}$$

$\big\downarrow$ 398

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(b(15a^2-20ab+8b^2)\int \frac{1}{\sqrt{b\tan^2(c+dx)+a}}d\tan(c+dx)+8(a-b)^3\int \frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a}}d\tan(c+dx\right.\right.}{d}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(b(15a^2-20ab+8b^2)\int \frac{1}{1-\frac{b\tan^2(c+dx)}{b\tan^2(c+dx)+a}}d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)+a}}+8(a-b)^3\int \frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a}}d\tan(c+\right.\right.}{d}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^3\int \frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a}}d\tan(c+dx)+\sqrt{b}(15a^2-20ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)\right)\right.}{d}$$

$\big\downarrow$ 291

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8(a-b)^3\int \frac{1}{1-\frac{(b-a)\tan^2(c+dx)}{b\tan^2(c+dx)+a}}d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)+a}}+\sqrt{b}(15a^2-20ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)\right)+\frac{1}{2}b(7a\right.}{d}$$

$\big\downarrow$ 216

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{b}(15a^2-20ab+8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)+8(a-b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)}}\right)\right)+\frac{1}{2}b(7a-4b)\text{ta}\right.}{d}$$

input

```
Int[(a + b*Tan[c + d*x]^2)^(5/2),x]
```

output
```
((b*Tan[c + d*x]*(a + b*Tan[c + d*x]^2)^(3/2))/4 + ((8*(a - b)^(5/2)*ArcTa
n[(Sqrt[a - b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^2]] + Sqrt[b]*(15*a^2
 - 20*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]^
2]])/2 + ((7*a - 4*b)*b*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]^2])/2)/4)/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(148) = 296$.

Time = 1.06 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.71

method	result
derivativedivides	$\frac{b^{\frac{5}{2}} \ln\left(\sqrt{b}\, \tan(dx+c)+\sqrt{a+b\tan(dx+c)^{2}}\right)}{d}+\frac{b^{2} \tan(dx+c)^{3} \sqrt{a+b\tan(dx+c)^{2}}}{4d}+\frac{9ba\tan(dx+c)\sqrt{a+b\tan(dx+c}}{8d}$
default	$\frac{b^{\frac{5}{2}} \ln\left(\sqrt{b}\, \tan(dx+c)+\sqrt{a+b\tan(dx+c)^{2}}\right)}{d}+\frac{b^{2} \tan(dx+c)^{3} \sqrt{a+b\tan(dx+c)^{2}}}{4d}+\frac{9ba\tan(dx+c)\sqrt{a+b\tan(dx+c}}{8d}$

input
```
int((a+b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*b^(5/2)*ln(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)^2)^(1/2))+1/4/d*b^2*tan(
d*x+c)^3*(a+b*tan(d*x+c)^2)^(1/2)+9/8/d*b*a*tan(d*x+c)*(a+b*tan(d*x+c)^2)^
(1/2)+15/8/d*b^(1/2)*a^2*ln(b^(1/2)*tan(d*x+c)+(a+b*tan(d*x+c)^2)^(1/2))-1
/2/d*b^2*tan(d*x+c)*(a+b*tan(d*x+c)^2)^(1/2)-5/2/d*b^(3/2)*a*ln(b^(1/2)*ta
n(d*x+c)+(a+b*tan(d*x+c)^2)^(1/2))-1/d*b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^
2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))+3/d*a*(b^4*
(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(
1/2)*tan(d*x+c))-3/d*a^2/b*(b^4*(a-b))^(1/2)/(a-b)*arctan(b^2*(a-b)/(b^4*(
a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c))+1/d*a^3*(b^4*(a-b))^(1/2)
/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(d*x+c)^2)^(1/2)*tan
(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 687, normalized size of antiderivative = 4.04

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/16*((15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c)^2 + 2*sqrt(b
*tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 8*(a^2 - 2*a*b + b^2)*sqr
t(-a + b)*log(-((a - 2*b)*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sq
rt(-a + b)*tan(d*x + c) - a)/(tan(d*x + c)^2 + 1)) + 2*(2*b^2*tan(d*x + c)
^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/16*(16
*(a^2 - 2*a*b + b^2)*sqrt(a - b)*arctan(sqrt(a - b)*tan(d*x + c)/sqrt(b*ta
n(d*x + c)^2 + a)) + (15*a^2 - 20*a*b + 8*b^2)*sqrt(b)*log(2*b*tan(d*x + c
)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(b)*tan(d*x + c) + a) + 2*(2*b^2*ta
n(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d
, -1/8*((15*a^2 - 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(-b)*tan(d*x + c)/sq
rt(b*tan(d*x + c)^2 + a)) - 4*(a^2 - 2*a*b + b^2)*sqrt(-a + b)*log(-((a -
2*b)*tan(d*x + c)^2 + 2*sqrt(b*tan(d*x + c)^2 + a)*sqrt(-a + b)*tan(d*x +
c) - a)/(tan(d*x + c)^2 + 1)) - (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*ta
n(d*x + c))*sqrt(b*tan(d*x + c)^2 + a))/d, 1/8*(8*(a^2 - 2*a*b + b^2)*sqrt
(a - b)*arctan(sqrt(a - b)*tan(d*x + c)/sqrt(b*tan(d*x + c)^2 + a)) - (15*
a^2 - 20*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(-b)*tan(d*x + c)/sqrt(b*tan(d*x
 + c)^2 + a)) + (2*b^2*tan(d*x + c)^3 + (9*a*b - 4*b^2)*tan(d*x + c))*sqrt
(b*tan(d*x + c)^2 + a))/d]
```

Sympy [F]

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2} dx = \int \left(a+b\tan^2\left(c+dx\right)\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*tan(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx=\int \left(b\tan\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^2 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx=\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^2+a\right)}^{5/2}\,dx$$

input
```
int((a + b*tan(c + d*x)^2)^(5/2),x)
```

output
```
int((a + b*tan(c + d*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\tan^2(c+dx)\right)^{5/2}\,dx = \left(\int \sqrt{\tan(dx+c)^2 b+a}dx\right)a^2 + \left(\int \sqrt{\tan(dx+c)^2 b+a}\,\tan(dx+c)^4\,dx\right)b^2 + 2\left(\int \sqrt{\tan(dx+c)^2 b+a}\,\tan(dx+c)^2\,dx\right)ab$$

input
```
int((a+b*tan(d*x+c)^2)^(5/2),x)
```

output
```
int(sqrt(tan(c + d*x)**2*b + a),x)*a**2 + int(sqrt(tan(c + d*x)**2*b + a)*
tan(c + d*x)**4,x)*b**2 + 2*int(sqrt(tan(c + d*x)**2*b + a)*tan(c + d*x)**
2,x)*a*b
```

3.320 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f} - \frac{(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2f} + \frac{\left(a+b\tan^2(e+fx)\right)^{3/2}}{3b^2f}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/f-(a+b)*(a+b*ta
n(f*x+e)^2)^(1/2)/b^2/f+1/3*(a+b*tan(f*x+e)^2)^(3/2)/b^2/f
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{2(2a+3b-b\tan^2(e+fx))\sqrt{a+b\tan^2(e+fx)}}{3b^2}}{2f}$$

input
```
Integrate[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/2*((2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b] + (2
*(2*a + 3*b - b*Tan[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])/(3*b^2))/f
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^5(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{-a-b}{b\sqrt{b\tan^2(e+fx)+a}}+\frac{\sqrt{b\tan^2(e+fx)+a}}{b}+\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{2(a+b\tan^2(e+fx))^{3/2}}{3b^2}-\frac{2(a+b)\sqrt{a+b\tan^2(e+fx)}}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b] - (2*(a
+ b)*Sqrt[a + b*Tan[e + f*x]^2])/b^2 + (2*(a + b*Tan[e + f*x]^2)^(3/2))/(3
*b^2))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\frac{\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3b}-\frac{2a\sqrt{a+b\tan(fx+e)^2}}{3b^2}+\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{\sqrt{a+b\tan(fx+e)^2}}{b}}{f}$	103
default	$\frac{\frac{\tan(fx+e)^2\sqrt{a+b\tan(fx+e)^2}}{3b}-\frac{2a\sqrt{a+b\tan(fx+e)^2}}{3b^2}+\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{\sqrt{a+b\tan(fx+e)^2}}{b}}{f}$	103

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*tan(f*x+e)^2/b*(a+b*tan(f*x+e)^2)^(1/2)-2/3*a/b^2*(a+b*tan(f*x+e)
^2)^(1/2)+1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/b
*(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.58

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$=\left[\frac{3\sqrt{a-b}b^2\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)+}{12\,(ab^2-b^3)f}\right.$$
$$-\frac{3\sqrt{-a+b}b^2\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right)-2\left((ab-b^2)\tan\left(fx+e\right)^2-2\,a^2-\right.}{6\,(ab^2-b^3)f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*sqrt(a - b)*b^2*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(
f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqr
t(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) +
 4*((a*b - b^2)*tan(f*x + e)^2 - 2*a^2 - a*b + 3*b^2)*sqrt(b*tan(f*x + e)^
2 + a))/((a*b^2 - b^3)*f), -1/6*(3*sqrt(-a + b)*b^2*arctan(-1/2*(b*tan(f*x
 + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*ta
n(f*x + e)^2 + a^2 - a*b)) - 2*((a*b - b^2)*tan(f*x + e)^2 - 2*a^2 - a*b +
 3*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a*b^2 - b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\tan^5{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\tan{(fx+e)}^5}{\sqrt{b\tan{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^5/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\left(b\tan(e+f\,x)^2+a\right)^{3/2}}{3\,b^2\,f} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}} - \left(\frac{2\,a}{b^2\,f} - \frac{a-b}{b^2\,f}\right)\sqrt{b\tan\left(e+f\,x\right)^2+a}$$

input
```
int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
(a + b*tan(e + f*x)^2)^(3/2)/(3*b^2*f) - atanh((a + b*tan(e + f*x)^2)^(1/2
)/(a - b)^(1/2))/(f*(a - b)^(1/2)) - ((2*a)/(b^2*f) - (a - b)/(b^2*f))*(a
+ b*tan(e + f*x)^2)^(1/2)
```

Reduce [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^2 b - 2\sqrt{\tan\left(fx+e\right)^2 b+a}\,a - 3\left(\int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^3}{\tan(fx+e)^2 b+a}dx\right) b^2 f}{3b^2 f}$$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*b - 2*sqrt(tan(e + f*x)**2*b
+ a)*a - 3*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x)
**2*b + a),x)*b**2*f)/(3*b**2*f)
```

3.321 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}+\frac{\sqrt{a+b\tan^2(e+fx)}}{bf}$$

output
```
arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/f+(a+b*tan(f*x+e
)^2)^(1/2)/b/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a+b\tan^2(e+fx)}}{b}}{f}$$

input
```
Integrate[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b] + Sqrt[a + b*
Tan[e + f*x]^2]/b)/f
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4153, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^3(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{2\sqrt{a+b\tan^2(e+fx)}}{b}-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\sqrt{a+b\tan^2(e+fx)}}{b}-\frac{2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{2\sqrt{a+b\tan^2(e+fx)}}{b}}{2f}$$

input
```
Int[Tan[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b] + (2*Sqrt
[a + b*Tan[e + f*x]^2])/b)/(2*f)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{\sqrt{a+b\tan(fx+e)^2}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}}{f}$	56
default	$\frac{\frac{\sqrt{a+b\tan(fx+e)^2}}{b}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}}{f}$	56

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/b*(a+b*tan(f*x+e)^2)^(1/2)-1/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)
^(1/2)/(-a+b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.28

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{a-b}b\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4\,ab-3\,b^2\right)\tan(fx+e)^2+4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)+4}{4\,(ab-b^2)f}\right.$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a - b)*b*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x +
 e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a -
 b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*sq
rt(b*tan(f*x + e)^2 + a)*(a - b))/((a*b - b^2)*f), 1/2*(sqrt(-a + b)*b*arc
tan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
 b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) + 2*sqrt(b*tan(f*x + e)^2 +
a)*(a - b))/((a*b - b^2)*f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan^3\left(e+fx\right)}{\sqrt{a+b\tan^2\left(e+fx\right)}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^3}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}} + \frac{\sqrt{b\tan\left(e+f\,x\right)^2+a}}{b\,f}$$

input
```
int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))/(f*(a - b)^(1/2)) + (a +
 b*tan(e + f*x)^2)^(1/2)/(b*f)
```

Reduce [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^3}{\tan\left(fx+e\right)^2 b+a}dx$$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),
x)
```

3.322 $\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

input
```
Integrate[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4153, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\tan(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 353$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{bf}$$

$$\Big\downarrow \; 221$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

input
```
Int[Tan[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	35
default	$\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{f\sqrt{-a+b}}$	35

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 5.15

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \left[\frac{\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right)}{4\,\sqrt{a-b}f},\ -\frac{\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right)}{2\,(a-b)f}\right]$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*ta
n(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8
*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/(sqrt(a - b)*f), -1/2
*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)
^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b))/((a - b)*f)
]
```

Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan(e+f\,x)^2+a}}{\sqrt{a-b}}\right)}{f\,\sqrt{a-b}}$$

input
```
int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
-atanh((a + b*tan(e + f*x)^2)^(1/2)/(a - b)^(1/2))/(f*(a - b)^(1/2))
```

Reduce [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\;\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2 b+a}dx$$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x))/(tan(e + f*x)**2*b + a),x)
```

3.323 $\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(1/2)/f+arctanh((a+b*tan(f*x+
e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{f}$$

input
```
Integrate[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-(ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a +
 b*Tan[e + f*x]^2]/Sqrt[a - b]]/Sqrt[a - b])/f
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4153, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{97}$$

$$\frac{\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2f}$$

input
```
Int[Cot[e + f*x]/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*ArcTanh[Sqr
t[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/(2*f)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(62) = 124$.

Time = 6.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 6.00

method	result
default	$-\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\left(2\ln\left(4\sqrt{a-b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\cos(fx+e)+4\sqrt{a-b}\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}+4a\cos\right.\right.}{}$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a^(1/2)/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)*(2*ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1
)^2)^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(
f*x+e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*a^(1/2)-ln(2/(1-cos(f*x+
e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f
*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*(a-
b)^(1/2)+ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e
)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2
)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*(a-b)^(1/2))/
(a+b*tan(f*x+e)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 426, normalized size of antiderivative = 5.76

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \left[\frac{\sqrt{a-b}a\log\left(\frac{b\tan(fx+e)^2+2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}+2a-b}{\tan(fx+e)^2+1}\right)+(a-b)\sqrt{a}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a}+2a}{\tan(fx+e)^2}\right)}{2(a^2-ab)f}\right.$$
$$-\frac{2a\sqrt{-a+b}\arctan\left(\frac{\sqrt{-a+b}}{\sqrt{b\tan(fx+e)^2+a}}\right)-(a-b)\sqrt{a}\log\left(\frac{b\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{a}+2a}{\tan(fx+e)^2}\right)}{2(a^2-ab)f}, \frac{2\sqrt{-a}(a-\ldots}{\ldots}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a - b)*a*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + (a - b)*sqrt(a)*log((b*tan(f
*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/(
(a^2 - a*b)*f), -1/2*(2*a*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x
+ e)^2 + a)) - (a - b)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x +
e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2))/((a^2 - a*b)*f), 1/2*(2*sqrt(-a)
*(a - b)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a)) + sqrt(a - b)*a*log((
b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(ta
n(f*x + e)^2 + 1)))/((a^2 - a*b)*f), (sqrt(-a)*(a - b)*arctan(sqrt(-a)/sqr
t(b*tan(f*x + e)^2 + a)) - a*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f
*x + e)^2 + a)))/((a^2 - a*b)*f)]
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.14

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tan(e+fx)^2+a}}{\sqrt{a}}\right)}{\sqrt{a}\,f} - \frac{\operatorname{atanh}\left(\frac{4ab^2\sqrt{b\tan(e+fx)^2+a}}{\left(\frac{2b^4f^3}{af^3-bf^3}-\frac{2ab^3f^3}{af^3-bf^3}\right)\sqrt{a-b}} - \frac{2b^3\sqrt{b\tan(e+fx)^2+a}}{\left(\frac{2b^4f^3}{af^3-bf^3}-\frac{2ab^3f^3}{af^3-bf^3}\right)\sqrt{a-b}} + \frac{2\sqrt{b\tan(e+fx)^2+a}\,(af^3-bf^3)}{bf^3\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

input
```
int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
- atanh((a + b*tan(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f) - atanh((4*a*b^2
*(a + b*tan(e + f*x)^2)^(1/2))/(((2*b^4*f^3)/(a*f^3 - b*f^3) - (2*a*b^3*f^
3)/(a*f^3 - b*f^3))*(a - b)^(1/2)) - (2*b^3*(a + b*tan(e + f*x)^2)^(1/2))/
(((2*b^4*f^3)/(a*f^3 - b*f^3) - (2*a*b^3*f^3)/(a*f^3 - b*f^3))*(a - b)^(1/
2)) + (2*(a + b*tan(e + f*x)^2)^(1/2)*(a*f^3 - b*f^3))/(b*f^3*(a - b)^(1/2
)))/(f*(a - b)^(1/2))
```

Reduce [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\cot(fx+e)}{\tan(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x))/(tan(e + f*x)**2*b + a),x)
```

3.324 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f} - \frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2af}$$

output
```
1/2*(2*a+b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a
+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/f-1/2*cot(f*x+e)^2*(a+b*ta
n(f*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \frac{(2a^2-ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(-2a\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+(-a+b)\cot^2(e\right.}{2a^{3/2}(a-b)f}$$

input
```
Integrate[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((2*a^2 - a*b - b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]] + Sqrt[a]
*(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]] + (-a +
 b)*Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2]))/(2*a^(3/2)*(a - b)*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4153, 354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\tan(e+fx)^3\sqrt{a+b\tan(e+fx)^2}}dx$$
$$\Big\downarrow \text{ 4153}$$
$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$\Big\downarrow$ 114

$$\frac{-\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+2a+b\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{2f}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+2a+b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2a}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{2f}$$

$\Big\downarrow$ 174

$$\frac{-\frac{(2a+b)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-2a\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2a}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{2f}$$

$\Big\downarrow$ 73

$$\frac{-\frac{\frac{2(2a+b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{4a\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{2a}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{2f}$$

$\Big\downarrow$ 221

$$\frac{-\frac{\frac{4a\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{2f}$$

input
```
Int[Cot[e + f*x]^3/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/2*((-2*(2*a + b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a]
+ (4*a*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/a - (
Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(98) = 196$.

Time = 6.56 (sec) , antiderivative size = 875, normalized size of antiderivative = 7.54

method	result	size
default	Expression too large to display	875

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/a^(5/2)/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^
2)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)/(1-cos
(f*x+e))^2*(4*ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e
)+1)^2)^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(c
os(f*x+e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*a^(5/2)*(1-cos(f*x+e)
)^2-((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*(1-cos(f*x+e)
)^2*a^(3/2)*(a-b)^(1/2)-2*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-
cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)
*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*a^2*(1-cos(f*x+e))^2*(a-b)^(1/2)-ln
(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*
x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(
f*x+e)^2))*a*(1-cos(f*x+e))^2*(a-b)^(1/2)*b+2*ln(2/a^(1/2)*(a^(1/2)*((a*co
s(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+
e)*b+b)/(cos(f*x+e)+1))*a^2*(1-cos(f*x+e))^2*(a-b)^(1/2)+ln(2/a^(1/2)*(a^(
1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+(
(a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+
e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a*(1-cos(f*x+e))^2*(a-b)^(1/2)*b+((a*co
s(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(3/2)*(a-b)^(1/2)*sin
(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 676, normalized size of antiderivative = 5.83

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(a - b)*a^2*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^2 + (2*a^2 -
a*b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)
*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), 1/4*(4*a^2*sqrt(-a + b)*ar
ctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 + (2*a^2 - a*
b - b^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt
(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*(
a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), 1/2*(sqrt(a - b)*a^2*log((b*
tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(
f*x + e)^2 + 1))*tan(f*x + e)^2 - (2*a^2 - a*b - b^2)*sqrt(-a)*arctan(sqrt
(-a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 - sqrt(b*tan(f*x + e)^2 +
a)*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x + e)^2), 1/2*(2*a^2*sqrt(-a + b)*
arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^2 - (2*a^2 -
a*b - b^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x +
e)^2 - sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^3 - a^2*b)*f*tan(f*x +
e)^2)]
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**3/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot\left(fx+e\right)^3}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^3/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 830, normalized size of antiderivative = 7.16

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\text{Too large to display}$$

input
```
int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
(atan((((((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) - ((a + b*tan(e + f*x)
^2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a - b)^(1/2)))/(2
*f*(a - b)^(1/2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b^4 + 8*a^2*b
^2))/(4*a^2*f^2))*1i)/(f*(a - b)^(1/2)) - ((((2*a*b^4*f^2 + 2*a^2*b^3*f^2)
/(2*a^2*f^3) + ((a + b*tan(e + f*x)^2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*
f^2))/(8*a^2*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x
)^2)^(1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2))/(4*a^2*f^2))*1i)/(f*(a - b)^(1/2))
)/((((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^3) - ((a + b*tan(e + f*x)^2)^(
1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a^2*f^3*(a - b)^(1/2)))/(2*f*(a
 - b)^(1/2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b^3 + b^4 + 8*a^2*b^2))/
(4*a^2*f^2))/(f*(a - b)^(1/2)) + (((2*a*b^4*f^2 + 2*a^2*b^3*f^2)/(2*a^2*f^
3) + ((a + b*tan(e + f*x)^2)^(1/2)*(16*a^2*b^3*f^2 - 32*a^3*b^2*f^2))/(8*a
^2*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/2)
*(4*a*b^3 + b^4 + 8*a^2*b^2))/(4*a^2*f^2))/(f*(a - b)^(1/2)) - (a*b^3 + b^
4/2)/(a^2*f^3)))*1i)/(f*(a - b)^(1/2)) - (b*(a + b*tan(e + f*x)^2)^(1/2))/
(2*a*(f*(a + b*tan(e + f*x)^2) - a*f)) + (atanh((b^6*(a + b*tan(e + f*x)^2
)^(1/2))/(4*(a^3)^(1/2)*((3*a*b^4)/2 + (5*b^5)/4 + b^6/(4*a))) + (3*b^4*(a
 + b*tan(e + f*x)^2)^(1/2))/(2*(a^3)^(1/2)*((3*b^4)/(2*a) + (5*b^5)/(4*a^2
) + b^6/(4*a^3))) + (5*b^5*(a + b*tan(e + f*x)^2)^(1/2))/(4*(a^3)^(1/2)*((
3*b^4)/2 + (5*b^5)/(4*a) + b^6/(4*a^2))))*(2*a + b))/(2*f*(a^3)^(1/2))
```

Reduce [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^3}{\sqrt{\tan(fx+e)^2 b+a}}dx$$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x)
```

3.325 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{(8a^2+4ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f} + \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8a^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4af}$$

output
```
-1/8*(8*a^2+4*a*b+3*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)
/f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)/f+1/8*(4*a+3*
b)*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/4*cot(f*x+e)^4*(a+b*tan(f
*x+e)^2)^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \frac{(-8a^3+4a^2b+ab^2+3b^3)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a^2\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)+(-a+\right.}{8a^{5/2}(a-b)f}$$

input
```
Integrate[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((-8*a^3 + 4*a^2*b + a*b^2 + 3*b^3)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqr
t[a]] + Sqrt[a]*(8*a^2*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt
[a - b]] + (-a + b)*Cot[e + f*x]^2*(-4*a - 3*b + 2*a*Cot[e + f*x]^2)*Sqrt[
a + b*Tan[e + f*x]^2]))/(8*a^(5/2)*(a - b)*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4153, 354, 114, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^5\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\cot^5(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2f}$$

$\Big\downarrow$ 114

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(3b\tan^2(e+fx)+4a+3b\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2a}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2a}}{2f}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(3b\tan^2(e+fx)+4a+3b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{4a}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2a}}{2f}$$

$\Big\downarrow$ 168

$$\frac{-\frac{-\frac{\int \frac{\cot(e+fx)\left(8a^2+4ba+3b^2+b(4a+3b)\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a}-\frac{(4a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{4a}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2a}}{2f}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{\int \frac{\cot(e+fx)\left(8a^2+4ba+3b^2+b(4a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2a}-\frac{(4a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{4a}-\frac{\cot^2(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2a}}{2f}$$

$\Big\downarrow$ 174

$$\frac{-\frac{-\frac{\left(8a^2+4ab+3b^2\right)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-8a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{2a}-\frac{(4a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{4a}-\mathrm{co}}{2f}$$

$\Big\downarrow$ 73

$$\frac{-\frac{-\frac{\frac{2\left(8a^2+4ab+3b^2\right)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{16a^2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{2a}-\frac{(4a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{4a}-\cot^2}{2f}$$

$\Big\downarrow$ 221

$$-\frac{-\frac{\frac{16a^2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2\left(8a^2+4ab+3b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{(4a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{4a}-\frac{\cot^2(e+fx)\sqrt{a}}{2}}{2f}$$

input
```
Int[Cot[e + f*x]^5/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/2*(Cot[e + f*x]^2*Sqrt[a + b*Tan[e + f*x]^2])/a - (-1/2*((-2*(8*a^2 +
4*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2/Sqrt[a]])/Sqrt[a] + (16*
a^2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/a - ((4*
a + 3*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(4*a))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. $2(144) = 288$.

Time = 6.63 (sec) , antiderivative size = 1235, normalized size of antiderivative = 7.44

method	result	size
default	Expression too large to display	1235

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/a^(7/2)/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1
)^2)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)/(1-c
os(f*x+e))^4*(-16*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+
e))^2*b+2*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)
*sin(f*x+e)^2+a*sin(f*x+e)^2))*a^3*(1-cos(f*x+e))^4*(a-b)^(1/2)+16*ln(2/a^
(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*co
s(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-
a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1))*a^3*(1-cos(f*x+e))^4*(a-b)^(1
/2)+32*ln(4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)
^(1/2)*cos(f*x+e)+4*(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+
e)+1)^2)^(1/2)+4*a*cos(f*x+e)-4*cos(f*x+e)*b)*a^(7/2)*(1-cos(f*x+e))^4-8*l
n(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f
*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin
(f*x+e)^2))*a^2*(1-cos(f*x+e))^4*(a-b)^(1/2)*b-6*(a-b)^(1/2)*ln(2/(1-cos(f
*x+e))^2*(-a*(1-cos(f*x+e))^2+2*(1-cos(f*x+e))^2*b+2*((a*cos(f*x+e)^2+b*si
n(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2))*
a*b^2*(1-cos(f*x+e))^4+8*ln(2/a^(1/2)*(a^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+
e)^2)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/
(cos(f*x+e)+1)^2)^(1/2)*a^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b+b)/(cos(f*x+e)+1
))*a^2*(1-cos(f*x+e))^4*(a-b)^(1/2)*b+6*(a-b)^(1/2)*ln(2/a^(1/2)*(a^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 836, normalized size of antiderivative = 5.04

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))*tan(f*x + e)^4 + (8*a^3 -
 4*a^2*b - a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 - 2*(2*a^3 - 2*
a^2*b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a^4 - a^3*b)*f*tan(f*x + e)^4), -1/16*(16*a^3*sqrt(-a + b)*arctan(sq
rt(-a + b)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^4 - (8*a^3 - 4*a^2*b -
 a*b^2 - 3*b^3)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a) + 2*a)/tan(f*x + e)^2)*tan(f*x + e)^4 + 2*(2*a^3 - 2*a^2*b - (4
*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4
- a^3*b)*f*tan(f*x + e)^4), 1/8*(4*sqrt(a - b)*a^3*log((b*tan(f*x + e)^2 +
 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1))
*tan(f*x + e)^4 + (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*sqrt(-a)*arctan(sqrt(-
a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^4 - (2*a^3 - 2*a^2*b - (4*a^3
- a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3
*b)*f*tan(f*x + e)^4), -1/8*(8*a^3*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b
*tan(f*x + e)^2 + a))*tan(f*x + e)^4 - (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*s
qrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a))*tan(f*x + e)^4 + (2*a^
3 - 2*a^2*b - (4*a^3 - a^2*b - 3*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e
)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^4)]
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot^5{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**5/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 1215, normalized size of antiderivative = 7.32

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
- (((a + b*tan(e + f*x)^2)^(1/2)*(4*a*b + 5*b^2))/(8*a) - (b*(a + b*tan(e
+ f*x)^2)^(3/2)*(4*a + 3*b))/(8*a^2))/(f*(a + b*tan(e + f*x)^2)^2 + a^2*f
- 2*a*f*(a + b*tan(e + f*x)^2)) - (atan((((((((3*a^2*b^5*f^2)/2 + (a^3*b^4*
f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) - ((a + b*tan(e + f*x)^2)^(1/2)*(256*a
^4*b^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(
1/2)) - ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5 + 9*b^6 + 64*a^2*b^4 + 64*
a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2))*1i)/(f*(a - b)^(1/2)) - (((((3*a^2*b
^5*f^2)/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^3) + ((a + b*tan(e +
 f*x)^2)^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(128*a^4*f^3*(a - b)^(
1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*tan(e + f*x)^2)^(1/2)*(24*a*b^5 + 9*b
^6 + 64*a^2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f^2))*1i)/(f*(a - b)^
(1/2)))/(((((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^3*f^2)/(2*a^4*f^
3) - ((a + b*tan(e + f*x)^2)^(1/2)*(256*a^4*b^3*f^2 - 512*a^5*b^2*f^2))/(1
28*a^4*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) - ((a + b*tan(e + f*x)^2)^(
1/2)*(24*a*b^5 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^3 + 128*a^4*b^2))/(64*a^4*f
^2))/(f*(a - b)^(1/2)) + ((((3*a^2*b^5*f^2)/2 + (a^3*b^4*f^2)/2 + 2*a^4*b^
3*f^2)/(2*a^4*f^3) + ((a + b*tan(e + f*x)^2)^(1/2)*(256*a^4*b^3*f^2 - 512*
a^5*b^2*f^2))/(128*a^4*f^3*(a - b)^(1/2)))/(2*f*(a - b)^(1/2)) + ((a + b*t
an(e + f*x)^2)^(1/2)*(24*a*b^5 + 9*b^6 + 64*a^2*b^4 + 64*a^3*b^3 + 128*a^4
*b^2))/(64*a^4*f^2))/(f*(a - b)^(1/2)) - ((3*a*b^5)/4 + (9*b^6)/32 + (5...
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot\left(fx+e\right)^5}{\sqrt{\tan\left(fx+e\right)^2 b+a}}dx$$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x)
```

3.326 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} + \frac{(3a^2+4ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4bf}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f+1/8
*(3*a^2+4*a*b+8*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/
b^(5/2)/f-1/8*(3*a+4*b)*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/b^2/f+1/4*tan(
f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/b/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.24 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.34

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

$$= \frac{-\frac{b(3a^2+4ab+4b^2)\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{1+\cos(2(e+fx))}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(1+\cos(2(e+fx)))\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\csc(2(e+}{a(a+b+(a-b)\cos(2(e+fx)))}}{}$$

$$+\frac{\sqrt{\frac{a+b+a\cos(2(e+fx))-b\cos(2(e+fx))}{1+\cos(2(e+fx))}}\left(-\frac{3\sec(e+fx)(a\sin(e+fx)+2b\sin(e+fx))}{8b^2}+\frac{\sec^2(e+fx)\tan(e+fx)}{4b}\right)}{f}$$

input
```
Integrate[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-((b*(3*a^2 + 4*a*b + 4*b^2)*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 +
 Cos[2*(e + f*x)])]*Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e
+ f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[
e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Co
s[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b
+ (a - b)*Cos[2*(e + f*x)]))) + (16*b^3*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a
 + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e
 + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[(
(a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*Ell
ipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/
Sqrt[2]], 1]*Sin[e + f*x]^4)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b +
(a - b)*Cos[2*(e + f*x)]]) - (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 +
 Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f
*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[
Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*S
in[e + f*x]^4)/(2*(a - b)*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*
Cos[2*(e + f*x)]])))/Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])/(4*b^2*f) + (
Sqrt[(a + b + a*Cos[2*(e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x
)])]*((-3*Sec[e + f*x]*(a*Sin[e + f*x] + 2*b*Sin[e + f*x]))/(8*b^2) + (Sec
[e + f*x]^2*Tan[e + f*x])/(4*b)))/f
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4153, 381, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^6}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 381}$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{\int \frac{\tan^2(e+fx)\left((3a+4b)\tan^2(e+fx)+3a\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{4b}}{f}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\int \frac{\left(3a^2+4ba+8b^2\right)\tan^2(e+fx)+a(3a+4b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}}{4b}}{f}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\left(3a^2+4ab+8b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-8b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan\cdots}}\cdots}{2b}}{4b}}{f}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\left(3a^2+4ab+8b^2\right)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-8b^2\int\frac{}{\left(\tan^2(e+fx)\right.}}{2b}}{4b}}{f}$$

$\downarrow$ 219

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{\left(3a^2+4ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8b^2\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\,t}}}{2b}}{4b}}{f}$$

$\downarrow$ 291

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{\left(3a^2+4ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-8b^2\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}}{2b}}{4b}}{f}$$

$\downarrow$ 216

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{4b}-\frac{\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{\left(3a^2+4ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-\frac{8b^2\arctan\left(\frac{\sqrt{a-b}\tan(e+f}{\sqrt{a+b\tan^2(e+}\right.}{\sqrt{a-b}}}{2b}}{4b}}{f}$$

input
```
Int[Tan[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/(4*b) - (-1/2*((-8*b^2*ArcTan
[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b] + ((3
*a^2 + 4*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*
x]^2]])/Sqrt[b])/b + ((3*a + 4*b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])
/(2*b))/(4*b))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{\sqrt{b}}+\frac{\tan(fx+e)^3\sqrt{a+b\tan(fx+e)^2}}{4b}-\frac{3a\left(\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)\right.}{2}\right.}{4b}}{}$
default	$\frac{\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{\sqrt{b}}+\frac{\tan(fx+e)^3\sqrt{a+b\tan(fx+e)^2}}{4b}-\frac{3a\left(\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)\right.}{2}\right.}{4b}}{}$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)+1/4*tan(f*x+e
)^3/b*(a+b*tan(f*x+e)^2)^(1/2)-3/4*a/b*(1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2
)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)))-1/2
*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)+1/2*a/b^(3/2)*ln(b^(1/2)*tan(f*x+e)
+(a+b*tan(f*x+e)^2)^(1/2))-(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b
^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.53

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (3*a
^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(2*(a*b^2 - b^3)*tan(f*x +
 e)^3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)
)/((a*b^3 - b^4)*f), -1/8*(4*sqrt(-a + b)*b^3*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
+ e)^2 + 1)) + (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(-b)*
tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (2*(a*b^2 - b^3)*tan(f*x + e)^3
 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a
*b^3 - b^4)*f), -1/16*(16*sqrt(a - b)*b^3*arctan(sqrt(a - b)*tan(f*x + e)/
sqrt(b*tan(f*x + e)^2 + a)) - (3*a^3 + a^2*b + 4*a*b^2 - 8*b^3)*sqrt(b)*lo
g(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) +
 a) - 2*(2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 - 4*b^3)*tan(f*
x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f), -1/8*(8*sqrt(a - b)
*b^3*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (3*a^3
+ a^2*b + 4*a*b^2 - 8*b^3)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*ta
n(f*x + e)^2 + a)) - (2*(a*b^2 - b^3)*tan(f*x + e)^3 - (3*a^2*b + a*b^2 -
4*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a*b^3 - b^4)*f)]
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^6}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2b+a}\,\tan\left(fx+e\right)^6}{\tan\left(fx+e\right)^2b+a}dx$$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**6)/(tan(e + f*x)**2*b + a),
x)
```

3.327 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 125

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} - \frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2bf}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f-1/2*
(a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f+1/2
*tan(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/b/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 4.36 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.17

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx =$$
$$-\frac{\left(\sqrt{2}a(-a+b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right),1\right)\right.}{}$$

input
```
Integrate[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/2*((Sqrt[2]*a*(-a + b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
 f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[
e + f*x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2]*a*b*Sqrt[((a + b + (a - b)*Cos[2*(
e + f*x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b
 + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + (a - b)*(a
+ b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2)*Tan[e + f*x])/(Sqrt[2]*b*(
-a + b)*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4153, 381, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(e+fx)^4}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 381$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\int \frac{(a+2b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(a+2b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{(a+2b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-2b\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{2b}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}}{2b}}{f}$$

input
```
Int[Tan[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/2*((-2*b*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]
)/Sqrt[a - b] + ((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e
 + f*x]^2]])/Sqrt[b])/b + (Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/(2*b))
/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{b^2(a-b)}-\ln}{f}$
default	$\frac{\frac{\tan(fx+e)\sqrt{a+b\tan(fx+e)^2}}{2b}-\frac{a\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{b^2(a-b)}-\ln}{f}$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*tan(f*x+e)/b*(a+b*tan(f*x+e)^2)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*ta
n(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*
(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-ln(b^(1/2)*ta
n(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 631, normalized size of antiderivative = 5.05

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - (a^2
+ a*b - 2*b^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(b)*tan(f*x + e) + a) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*ta
n(f*x + e))/((a*b^2 - b^3)*f), -1/2*(sqrt(-a + b)*b^2*log(-((a - 2*b)*tan(
f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(
tan(f*x + e)^2 + 1)) - (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(-b)*tan(f*
x + e)/sqrt(b*tan(f*x + e)^2 + a)) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2
)*tan(f*x + e))/((a*b^2 - b^3)*f), 1/4*(4*sqrt(a - b)*b^2*arctan(sqrt(a -
b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (a^2 + a*b - 2*b^2)*sqrt(b)*
log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e)
 + a) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a*b^2 - b
^3)*f), 1/2*(2*sqrt(a - b)*b^2*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(
f*x + e)^2 + a)) + (a^2 + a*b - 2*b^2)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x +
e)/sqrt(b*tan(f*x + e)^2 + a)) + sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*ta
n(f*x + e))/((a*b^2 - b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^4}{\tan(fx+e)^2 b+a}dx$$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**4)/(tan(e + f*x)**2*b + a),
x)
```

3.328 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f+arc
tanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.73

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \frac{a\csc^2(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{a-b},\arcsin\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}{\sqrt{2}}\right),1\right)\sqrt{(a+b+(a-b)\cos(2(e+}{2(a-b)bf\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(a*Csc[e + f*x]^2*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*C
os[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a + b + (a - b)*Cos
[2*(e + f*x)])*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(2*(a - b)*b*f*Sqrt[((a +
 b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4153, 385, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{385}$$

$$\frac{\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}}{f}$$

input
```
Int[Tan[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/Sqrt[a -
b]) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/Sqrt[b])/
f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^{2}}\right)}{\sqrt{b}}-\frac{\sqrt{b^{4}(a-b)}\,\arctan\left(\frac{b^{2}(a-b)\tan(fx+e)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\tan(fx+e)^{2}}}\right)}{b^{2}(a-b)}}{f}$	100
default	$\frac{\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^{2}}\right)}{\sqrt{b}}-\frac{\sqrt{b^{4}(a-b)}\,\arctan\left(\frac{b^{2}(a-b)\tan(fx+e)}{\sqrt{b^{4}(a-b)}\,\sqrt{a+b\tan(fx+e)^{2}}}\right)}{b^{2}(a-b)}}{f}$	100

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))/b^(1/2)-(b^4*(a-b))^(
1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)
*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 463, normalized size of antiderivative = 5.38

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \left[\frac{(a-b)\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\left(fx+e\right)+a\right)-\sqrt{-a+b}b\log\left(-\frac{(a}{}\right.}{2\,(ab-b^2)f}\right.$$
$$-\frac{2\,(a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)+\sqrt{-a+b}b\log\left(-\frac{(a-2\,b)\tan(fx+e)^2+2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx}{\tan(fx+e)^2+1}\right.}{2\,(ab-b^2)f}$$
$$-\frac{2\,\sqrt{a-b}b\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-(a-b)\sqrt{b}\log\left(2\,b\tan\left(fx+e\right)^2+2\,\sqrt{b\tan\left(fx+e\right)^2+a}\sqrt{b}\tan\right.}{2\,(ab-b^2)f}$$
$$\left.-\frac{\sqrt{a-b}b\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)+(a-b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)}{(ab-b^2)f}\right]$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((a - b)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a
)*sqrt(b)*tan(f*x + e) + a) - sqrt(-a + b)*b*log(-((a - 2*b)*tan(f*x + e)^
2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x +
 e)^2 + 1)))/((a*b - b^2)*f), -1/2*(2*(a - b)*sqrt(-b)*arctan(sqrt(-b)*tan
(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + sqrt(-a + b)*b*log(-((a - 2*b)*tan
(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
(tan(f*x + e)^2 + 1)))/((a*b - b^2)*f), -1/2*(2*sqrt(a - b)*b*arctan(sqrt(
a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (a - b)*sqrt(b)*log(2*b*
tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a))/(
(a*b - b^2)*f), -(sqrt(a - b)*b*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan
(f*x + e)^2 + a)) + (a - b)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*t
an(f*x + e)^2 + a)))/((a*b - b^2)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\tan^2{(e+fx)}}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\tan{(fx+e)}^2}{\sqrt{b\tan{(fx+e)}^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\sqrt{b\tan(e+f\,x)^2+a}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2b+a}\,\tan(fx+e)^2}{\tan(fx+e)^2b+a}\,dx$$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2)/(tan(e + f*x)**2*b + a),
x)
```

3.329 $\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

input
```
Integrate[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]
*f)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4144, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

input
```
Int[1/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(Sqrt[a - b]
*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

method	result	size
derivativedivides	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{f\,b^2(a-b)}$	67
default	$\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{f\,b^2(a-b)}$	67

input
```
int(1/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^4*(a-b))^(1/2)/b^2/(a-b)*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \left[-\frac{\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan(fx+e)^2-2\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}\tan(fx+e)-a}{\tan(fx+e)^2+1}\right)}{2(a-b)f}, \frac{\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)}{\sqrt{a-b}f}\right]$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^
2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1))/((a - b)*f), a
rctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a))/(sqrt(a - b)*f)
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\tan^2{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)\,\sqrt{a-b}}{\sqrt{b\tan(e+f\,x)^2+a}}\right)}{f\,\sqrt{a-b}}$$

input
```
int(1/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
atan((tan(e + f*x)*(a - b)^(1/2))/(a + b*tan(e + f*x)^2)^(1/2))/(f*(a - b)
^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\sqrt{\tan(fx+e)^2 b+a}}{\tan(fx+e)^2 b+a}dx$$

input
```
int(1/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)/(tan(e + f*x)**2*b + a),x)
```

3.330 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{af}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f-cot
(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.78 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.29

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{2\cos^2(e+fx)\cot(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(2(a-b)\,\text{Hypergeometric2F1}\left(2,2,\frac{5}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)\text{si}\right.}{3a^2f\sqrt{a+b\tan^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-2*Cos[e + f*x]^2*Cot[e + f*x]*(1 + (b*Tan[e + f*x]^2)/a)*(2*(a - b)*Hype
rgeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b
*Tan[e + f*x]^2) + (3*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a + 2*b*
Tan[e + f*x]^2))/Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a + b*Tan[e + f*x]^2))/
a^2]))/(3*a^2*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4153, 382, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{a}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{a}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx) - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{-\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{f}$$

input
```
Int[Cot[e + f*x]^2/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/Sqrt[a -
b]) - (Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(70) = 140$.

Time = 23.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.13

method	result
default	$\frac{-a\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)^2+b\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)}{\sqrt{a-b}\,(\cos(fx+e)-1)}\right)(\sec(fx+e)+1)-\sqrt{a-b}\,a\cot(fx+e)-\sqrt{a-b}\,b\tan(}{fa\sqrt{a-b}\,\sqrt{a+b\tan(fx+e)^2}}$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*arctan(1/
(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f
*x+e)/(cos(f*x+e)-1))*(sec(f*x+e)+1)-(a-b)^(1/2)*a*cot(f*x+e)-(a-b)^(1/2)*
b*tan(f*x+e))/a/(a-b)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.71

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \left[-\frac{a\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2+4\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(fx+}}{\tan(fx+e)^4+2\tan(fx+e)^2+1}\right.}{4\,(a^2-ab)f\tan(fx+e)}\right.$$
$$\left. -\frac{\sqrt{a-b}a\arctan\left(-\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan(fx+e)+2\sqrt{b\tan(fx+e)^2+a}(a-b)}{2\,(a^2-ab)f\tan(fx+e)}\right]$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(a*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^
2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x
+ e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x
 + e)^2 + 1))*tan(f*x + e) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 -
 a*b)*f*tan(f*x + e)), -1/2*(sqrt(a - b)*a*arctan(-2*sqrt(b*tan(f*x + e)^2
 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e
) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2 - a*b)*f*tan(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^2}{\sqrt{b\tan(e+f\,x)^2+a}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\cot(fx+e)^2}{\tan(fx+e)^2 b+a}dx$$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x)**2)/(tan(e + f*x)**2*b + a),
x)
```

3.331 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2f} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3af}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f+1/3*
(3*a+2*b)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)^3*(a+b*
tan(f*x+e)^2)^(1/2)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.81 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.18

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx =$$

$$-\frac{\cos^2(e+fx)\cot^3(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(-24(a-b)b\,\text{Hypergeometric2F1}\left(2,2,\frac{5}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/9*(Cos[e + f*x]^2*Cot[e + f*x]^3*(1 + (b*Tan[e + f*x]^2)/a)*(-24*(a - b
)*b*Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^
2*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2) - 8*(a - b)*HypergeometricPFQ[{2,
2, 2}, {1, 5/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e +
 f*x]^2)^2 + (6*a*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a^2 - 4*a*b*Ta
n[e + f*x]^2 - 8*b^2*Tan[e + f*x]^4))/Sqrt[((a - b)*Sin[2*(e + f*x)]^2*(a
+ b*Tan[e + f*x]^2))/a^2]))/(a^3*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4153, 382, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^4\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 382$$

$$\frac{\frac{\int -\frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a+2b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a+2b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{\int \frac{3a^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-3a\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{-\frac{-3a\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{-\frac{-\frac{3a\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{f}$$

input
```
Int[Cot[e + f*x]^4/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/3*(Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/a - ((-3*a*ArcTan[(Sqrt[
a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b] - ((3*a + 2*
b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(3*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(106) = 212$.

Time = 25.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.88

method	result
default	$\frac{\left(-\frac{4\cot(fx+e)^3}{3}+\cot(fx+e)\csc(fx+e)^2\right)a^2\sqrt{a-b}+\frac{2\sqrt{a-b}\,b^2\tan(fx+e)}{3}+\left(\sec(fx+e)\csc(fx+e)-\frac{2\cot(fx+e)}{3}\right)ba\sqrt{a-b}-\frac{\sqrt{\frac{a\cos(f}{(c}}}{}}{f\,a^2\sqrt{a-b}\,\sqrt{a+b\tan(fx+e)^2}}$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((-4/3*cot(f*x+e)^3+cot(f*x+e)*csc(f*x+e)^2)*a^2*(a-b)^(1/2)+2/3*(a-b)
^(1/2)*b^2*tan(f*x+e)+(sec(f*x+e)*csc(f*x+e)-2/3*cot(f*x+e))*b*a*(a-b)^(1/
2)-1/3*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*arctan(1/(
a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*
x+e)/(cos(f*x+e)-1))*a^2*(-3-3*sec(f*x+e)))/a^2/(a-b)^(1/2)/(a+b*tan(f*x+e
)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.99

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$
$$= \left[-\frac{3\,a^2\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2-4\,\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan(f}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right.}{12\,(a^3-a^2b)f}\right.$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*a^2*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*
(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan
(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*ta
n(f*x + e)^2 + 1))*tan(f*x + e)^3 - 4*((3*a^2 - a*b - 2*b^2)*tan(f*x + e)^
2 - a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^3 - a^2*b)*f*tan(f*x + e)^3
), 1/6*(3*sqrt(a - b)*a^2*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)
*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^3 + 2*((3*a^2 -
 a*b - 2*b^2)*tan(f*x + e)^2 - a^2 + a*b)*sqrt(b*tan(f*x + e)^2 + a))/((a^
3 - a^2*b)*f*tan(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**4/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Giac [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\tan\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot(e+f\,x)^4}{\sqrt{b\tan\left(e+f\,x\right)^2+a}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot\left(fx+e\right)^4}{\sqrt{\tan\left(fx+e\right)^2b+a}}dx$$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(1/2),x)
```

3.332 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}f} - \frac{(15a^2+10ab+8b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3f} + \frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^2f} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5af}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(1/2)/f-1/1
5*(15*a^2+10*a*b+8*b^2)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/f+1/15*(5*
a+4*b)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^2/f-1/5*cot(f*x+e)^5*(a+b*t
an(f*x+e)^2)^(1/2)/a/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.65 (sec) , antiderivative size = 1214, normalized size of antiderivative = 7.14

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
-1/45*(Cos[e + f*x]^4*Cot[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*(9*a^4*Arc
Sin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]] + 9*a^4*ArcSin[Sqrt[((a - b)*Sin[e +
 f*x]^2)/a]]*Tan[e + f*x]^2 - 18*a^3*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2
)/a]]*Tan[e + f*x]^2 - 18*a^3*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*T
an[e + f*x]^4 + 72*a^2*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e
+ f*x]^4 + 72*a^2*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x
]^6 + 144*a*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^6 +
144*a*b^3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^8 - 4*a^4*
Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^2*Sq
rt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] + 4
*a^3*b*Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*
x]^2*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a
^2] - 12*a^3*b*Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x]^2)/a]*Ta
n[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*
x]^2))/a^2] + 12*a^2*b^2*Hypergeometric2F1[2, 2, 5/2, ((a - b)*Sin[e + f*x
]^2)/a]*Tan[e + f*x]^4*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x]^2*(a + b*
Tan[e + f*x]^2))/a^2] + 168*a^2*b^2*Hypergeometric2F1[2, 2, 5/2, ((a - b)*
Sin[e + f*x]^2)/a]*Tan[e + f*x]^6*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + f*x
]^2*(a + b*Tan[e + f*x]^2))/a^2] - 168*a*b^3*Hypergeometric2F1[2, 2, 5/2,
((a - b)*Sin[e + f*x]^2)/a]*Tan[e + f*x]^6*Sqrt[((a - b)*Cos[e + f*x]^2...
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4153, 382, 25, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\sqrt{a+b\tan(e+fx)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int -\frac{\cot^4(e+fx)\left(4b\tan^2(e+fx)+5a+4b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(4b\tan^2(e+fx)+5a+4b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{5a}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^2+10ba+8b^2+2b(5a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}}{f}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{-\frac{\int \frac{15a^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-\frac{\cot^5(\cdots}{\cdots}}{f}$$

$\downarrow$ 27

$$-\frac{-\frac{-15a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-\frac{\mathrm{co}\cdots}{\cdots}}{f}$$

$\downarrow$ 291

$$-\frac{-\frac{-15a^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-\frac{\cot^5(e\cdots}{\cdots}}{f}$$

$\downarrow$ 216

$$-\frac{-\frac{-\frac{15a^2\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{\left(15a^2+10ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(5a+4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-\frac{\cot^5(e+fx)\sqrt{a+b\,\mathrm{ta}\cdots}}{5a}}{f}$$

input
```
Int[Cot[e + f*x]^6/Sqrt[a + b*Tan[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/a - (-1/3*((5*a + 4*b)*C
ot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/a - ((-15*a^2*ArcTan[(Sqrt[a - b
]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b] - ((15*a^2 + 10*a
*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(3*a))/(5*a))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 13.85 (sec) , antiderivative size = 773, normalized size of antiderivative = 4.55

method	result
default	$-\frac{\sqrt{\frac{i\cos(fx+e)\sqrt{b}\,\sqrt{a-b}-i\sqrt{b}\,\sqrt{a-b}+a\cos(fx+e)-\cos(fx+e)b+b}{a(\cos(fx+e)+1)}}\,\sqrt{-\frac{i\cos(fx+e)\sqrt{b}\,\sqrt{a-b}-i\sqrt{b}\,\sqrt{a-b}-a\cos(fx+e)+\cos(fx+e)b-b}{a(\cos(fx+e)+1)}}\,\mathrm{EllipticPi}}{15}$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/15*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+a*
cos(f*x+e)-cos(f*x+e)*b+b)/(cos(f*x+e)+1))^(1/2)*(-1/a*(I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b-b)/(cos(f*x
+e)+1))^(1/2)*EllipticPi(((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(cot(f*
x+e)-csc(f*x+e)),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)
^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*a^3*(-60
-60*sec(f*x+e))-1/15*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b
)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/(cos(f*x+e)+1))^(1/2)*(-1/a*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b-b
)/(cos(f*x+e)+1))^(1/2)*EllipticF(((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2
)*(cot(f*x+e)-csc(f*x+e)),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2
)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*a^3*(30+30*sec(f*x+e))-1/15*(23*cos(f*x+e
)^4-35*cos(f*x+e)^2+15)*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^3*cot(
f*x+e)*csc(f*x+e)^4-1/15*(9*cos(f*x+e)^4-25*cos(f*x+e)^2+15)*((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b*sec(f*x+e)*csc(f*x+e)^3-1/15*((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a*b^2*(-6*cot(f*x+e)+10*sec(f*x+e)*csc(f*x
+e))-8/15*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*b^3*tan(f*x+e))/((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/a^3/(a+b*tan(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.57

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

$$= \left[-\frac{15\,a^3\sqrt{-a+b}\log\left(-\frac{(a^2-8\,ab+8\,b^2)\tan(fx+e)^4-2\,(3\,a^2-4\,ab)\tan(fx+e)^2+a^2+4\,\left((a-2\,b)\tan(fx+e)^3-a\tan(fx+e)\right)\sqrt{b\tan}}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}\right.}{}\right.$$

$$-\frac{15\,\sqrt{a-b}a^3\arctan\left(-\frac{2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2\,b)\tan(fx+e)^2-a}\right)\tan\left(fx+e\right)^5+2\,\left((15\,a^3-5\,a^2b-2\,ab^2-8\,b^3\right)}{30\,(a^4-a^3b)f\tan\left(f\right.}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/60*(15*a^3*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2
*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*ta
n(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*t
an(f*x + e)^2 + 1))*tan(f*x + e)^5 + 4*((15*a^3 - 5*a^2*b - 2*a*b^2 - 8*b^
3)*tan(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b - 4*a*b^2)*tan(f*x +
e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan(f*x + e)^5), -1/30*
(15*sqrt(a - b)*a^3*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f
*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))*tan(f*x + e)^5 + 2*((15*a^3 - 5*a^
2*b - 2*a*b^2 - 8*b^3)*tan(f*x + e)^4 + 3*a^3 - 3*a^2*b - (5*a^3 - a^2*b -
 4*a*b^2)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4 - a^3*b)*f*tan
(f*x + e)^5)]
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**6/sqrt(a + b*tan(e + f*x)**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot(fx+e)^6}{\sqrt{b\tan(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^6/sqrt(b*tan(f*x + e)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot(e+f\,x)^6}{\sqrt{b\tan(e+f\,x)^2+a}}\,dx$$

input
```
int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\,dx=\int \frac{\cot(fx+e)^6}{\sqrt{\tan(fx+e)^2\,b+a}}dx$$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

output
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(1/2),x)
```

3.333 $\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{a^2}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}} + \frac{\sqrt{a+b\tan^2(e+fx)}}{b^2f}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+a^2/(a-b)/b^2
/f/(a+b*tan(f*x+e)^2)^(1/2)+(a+b*tan(f*x+e)^2)^(1/2)/b^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(2a+b+b\tan^2(\right.}{(a-b)b^2f\sqrt{a+b\tan^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (a
- b)*(2*a + b + b*Tan[e + f*x]^2))/((a - b)*b^2*f*Sqrt[a + b*Tan[e + f*x]^
2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4153, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^5(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{98}$$

$$\frac{\int \left(-\frac{a^2}{(a-b)b\left(b\tan^2(e+fx)+a\right)^{3/2}}+\frac{1}{b\sqrt{b\tan^2(e+fx)+a}}+\frac{1}{(a-b)(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2a^2}{b^2(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}+\frac{2\sqrt{a+b\tan^2(e+fx)}}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + (2*a
^2)/((a - b)*b^2*Sqrt[a + b*Tan[e + f*x]^2]) + (2*Sqrt[a + b*Tan[e + f*x]^
2])/b^2)/(2*f)
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

method	result	si
derivativedivides	$\frac{\frac{\tan(fx+e)^2}{b\sqrt{a+b\tan(fx+e)^2}}+\frac{2a}{b^2\sqrt{a+b\tan(fx+e)^2}}+\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\tan(fx+e)^2}}+\frac{1}{b\sqrt{a+b\tan(fx+e)^2}}}{f}$	1
default	$\frac{\frac{\tan(fx+e)^2}{b\sqrt{a+b\tan(fx+e)^2}}+\frac{2a}{b^2\sqrt{a+b\tan(fx+e)^2}}+\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\tan(fx+e)^2}}+\frac{1}{b\sqrt{a+b\tan(fx+e)^2}}}{f}$	1

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(1/2)+2*a/b^2/(a+b*tan(f*x+e)^2)^(1
/2)+1/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/(
a-b)/(a+b*tan(f*x+e)^2)^(1/2)+1/b/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(88) = 176.

Time = 0.14 (sec) , antiderivative size = 458, normalized size of antiderivative = 4.67

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[-\frac{(b^3\tan(fx+e)^2+ab^2)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2+}{\tan(}\right.}{4}\right.$$

$$-\frac{(b^3\tan(fx+e)^2+ab^2)\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right)-2\,(2\,a^3-3\,a^2b+a}{2\left((a^2b^3-2\,ab^4+b^5)f\tan(fx+e)^2+(a^3b^2-2\,a^2b^3\right.}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 +
 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*
tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2
*tan(f*x + e)^2 + 1)) - 4*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^
3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*
f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), -1/2*((b^3*tan(f*x +
e)^2 + a*b^2)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)
) - 2*(2*a^3 - 3*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*s
qrt(b*tan(f*x + e)^2 + a))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (
a^3*b^2 - 2*a^2*b^3 + a*b^4)*f)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^5{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{b\tan(e+f\,x)^2+a}}{b^2\,f} + \frac{a^2}{b^2\,f\,\sqrt{b\tan(e+f\,x)^2+a}\,(a-b)} + \frac{\operatorname{atan}\left(\frac{a\,\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}-b\,\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}}{(a-b)^{3/2}}\right)\,1\mathrm{i}}{f\,(a-b)^{3/2}}$$

input
```
int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(a + b*tan(e + f*x)^2)^(1/2)/(b^2*f) + (atan((a*(a + b*tan(e + f*x)^2)^(1/
2)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(3/2))*1i)/(f*(a - b)^(
3/2)) + a^2/(b^2*f*(a + b*tan(e + f*x)^2)^(1/2)*(a - b))
```

Reduce [F]

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^2 b+2\sqrt{\tan\left(fx+e\right)^2 b+a}\,a+\sqrt{\tan(}{}$$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*b + 2*sqrt(tan(e + f*x)**2*b
+ a)*a + sqrt(tan(e + f*x)**2*b + a)*b + int((sqrt(tan(e + f*x)**2*b + a)*
tan(e + f*x))/(tan(e + f*x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2),x)*tan
(e + f*x)**2*b**3*f + int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x))/(tan(
e + f*x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2),x)*a*b**2*f)/(b**2*f*(tan
(e + f*x)**2*b + a))
```

3.334 $\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{a}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-a/(a-b)/b/f/(a
+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(-a+b)^2} + \frac{a}{b(-a+b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/(-a + b)^2
+ a/(b*(-a + b)*Sqrt[a + b*Tan[e + f*x]^2]))/f
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4153, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^3(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{-\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a-b}-\frac{2a}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{-\frac{2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b(a-b)}-\frac{2a}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}-\frac{2a}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) - (2*a)
/((a - b)*b*Sqrt[a + b*Tan[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{-\frac{1}{b\sqrt{a+b\tan(fx+e)^2}}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}-\frac{1}{(a-b)\sqrt{a+b\tan(fx+e)^2}}}{f}$	87
default	$\frac{-\frac{1}{b\sqrt{a+b\tan(fx+e)^2}}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}-\frac{1}{(a-b)\sqrt{a+b\tan(fx+e)^2}}}{f}$	87

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/b/(a+b*tan(f*x+e)^2)^(1/2)-1/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*
x+e)^2)^(1/2)/(-a+b)^(1/2))-1/(a-b)/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(65) = 130$.

Time = 0.14 (sec) , antiderivative size = 384, normalized size of antiderivative = 5.26

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[-\frac{\left(b^2\tan\left(fx+e\right)^2+ab\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\,(4\,ab-3\,b^2)\tan(fx+e)^2-4}{\tan(f}\right.}{4\,\left((a^2b^2-2\,ab^3+b^4)f\,t\right.}\right.$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b^2*tan(f*x + e)^2 + a*b)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2
*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*ta
n(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*t
an(f*x + e)^2 + 1)) + 4*sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^2*b^2
- 2*a*b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f), 1/2*((
b^2*tan(f*x + e)^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a
 - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2
+ a^2 - a*b)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a^2 - a*b))/((a^2*b^2 - 2*a*
b^3 + b^4)*f*tan(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^3\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{a}{b\,f\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,\left(a-b\right)} - \frac{\mathrm{atan}\left(\frac{a\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1{}\mathrm{i}-b\,\sqrt{b\tan\left(e+f\,x\right)^2+a}\,1{}\mathrm{i}}{{\left(a-b\right)}^{3/2}}\right)\,1{}\mathrm{i}}{f\,{\left(a-b\right)}^{3/2}}$$

input
```
int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
- (atan((a*(a + b*tan(e + f*x)^2)^(1/2)*1i - b*(a + b*tan(e + f*x)^2)^(1/2
)*1i)/(a - b)^(3/2))*1i)/(f*(a - b)^(3/2)) - a/(b*f*(a + b*tan(e + f*x)^2)
^(1/2)*(a - b))
```

Reduce [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{-\sqrt{\tan(fx+e)^2 b+a} - \left(\int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx\right)\tan(fx+e)}{bf\left(\tan(fx+e)^2 b+a\right)}$$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
( - (sqrt(tan(e + f*x)**2*b + a) + int((sqrt(tan(e + f*x)**2*b + a)*tan(e
+ f*x))/(tan(e + f*x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2),x)*tan(e + f
*x)**2*b**2*f + int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x))/(tan(e + f*
x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2),x)*a*b*f))/(b*f*(tan(e + f*x)**
2*b + a))
```

3.335 $\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f}+\frac{1}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/(a-b)/f/(a+
b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)}{(-a+b)f\sqrt{a+b\tan^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/((-a + b
)*f*Sqrt[a + b*Tan[e + f*x]^2]))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4153, 353, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)}{(a+b\tan(e+fx)^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2/((
a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\tan(fx+e)^2}}}{f}$	66
default	$\frac{\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}+\frac{1}{(a-b)\sqrt{a+b\tan(fx+e)^2}}}{f}$	66

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/
(a-b)/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(61) = 122$.

Time = 0.14 (sec) , antiderivative size = 358, normalized size of antiderivative = 5.19

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[-\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+2\left(4\,ab-3\,b^2\right)\tan(fx+e)^2+4\,\left(\right.}{\tan(fx\cdots}\right.}{4\left(\left(a^2b-2\,ab^2+b^3\right)f\tan\cdots\right.}\right.$$

$$-\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left(\left(ab-b^2\right)\tan(fx+e)^2+a^2-ab\right)}\right)-2\sqrt{b\tan\left(fx+e\right)^2+\cdots}}{2\left(\left(a^2b-2\,ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2\,a^2b+ab^2\right)f\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*tan(f*x + e)^2 + a)*sqrt(a - b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*
a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*
x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(tan(f*x + e)^4 + 2*tan(f
*x + e)^2 + 1)) - 4*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b^2
+ b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), -1/2*((b*tan(f*x + e
)^2 + a)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(
f*x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) - 2
*sqrt(b*tan(f*x + e)^2 + a)*(a - b))/((a^2*b - 2*a*b^2 + b^3)*f*tan(f*x +
e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(53) = 106$.

Time = 10.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{2f(a-b)\sqrt{a+b\tan^2(e+fx)}} + \frac{b\operatorname{atan}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{-a+b}}\right)}{2f\sqrt{-a+b}(a-b)}\right)}{b} & \text{for } b \neq 0 \\ \begin{cases} \tilde{\infty}\tan^2(e+fx) & \text{for } a^{\frac{3}{2}} = 0 \vee f = 0 \\ \frac{\log\left(2a^{\frac{3}{2}}f\tan^2(e+fx) + 2a^{\frac{3}{2}}f\right)}{2a^{\frac{3}{2}}f} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Piecewise((2*(b/(2*f*(a - b)*sqrt(a + b*tan(e + f*x)**2)) + b*atan(sqrt(a
+ b*tan(e + f*x)**2)/sqrt(-a + b))/(2*f*sqrt(-a + b)*(a - b)))/b, Ne(b, 0)
), (Piecewise((zoo*tan(e + f*x)**2, Eq(f, 0) | Eq(a**(3/2), 0)), (log(2*a*
*(3/2)*f*tan(e + f*x)**2 + 2*a**(3/2)*f)/(2*a**(3/2)*f), True)), True))
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{1}{f\sqrt{b\tan(e+fx)^2+a}\,(a-b)} + \frac{\operatorname{atan}\left(\frac{a\sqrt{b\tan(e+fx)^2+a}\,1\mathrm{i}-b\sqrt{b\tan(e+fx)^2+a}\,1\mathrm{i}}{(a-b)^{3/2}}\right)1\mathrm{i}}{f\,(a-b)^{3/2}}$$

input
```
int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
1/(f*(a + b*tan(e + f*x)^2)^(1/2)*(a - b)) + (atan((a*(a + b*tan(e + f*x)^
2)^(1/2)*1i - b*(a + b*tan(e + f*x)^2)^(1/2)*1i)/(a - b)^(3/2))*1i)/(f*(a
- b)^(3/2))
```

Reduce [F]

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x))/(tan(e + f*x)**4*b**2 + 2*t
an(e + f*x)**2*a*b + a**2),x)
```

3.336 $\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{b}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+arctanh((a+b*tan(f*x+
e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\,\text{Hypergeometri}}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (a
 - b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a])/(a*(a - b
)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 354, 96, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{96}$$

$$\frac{\frac{\int \frac{\cot(e+fx)\left(-b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}-\frac{2b}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{(a-b)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-a\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}-\frac{2b}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{2(a-b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{2a\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{a(a-b)}-\frac{2b}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{2a\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(a-b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a-b)}-\frac{2b}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(((-2*(a - b)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*
ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/(a*(a - b))
- (2*b)/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7771 vs. $2(92) = 184$.

Time = 7.26 (sec) , antiderivative size = 7772, normalized size of antiderivative = 73.32

method	result	size
default	Expression too large to display	7772

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(92) = 184.

Time = 0.15 (sec) , antiderivative size = 900, normalized size of antiderivative = 8.49

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((a^2*b*tan(f*x + e)^2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*
sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) -
(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(a)*l
og((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x
 + e)^2) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b
^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f), -1/2*(2*(a^
2*b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x
+ e)^2 + a)) - (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x +
e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a)
 + 2*a)/tan(f*x + e)^2) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((
a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*
f), 1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2
)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a)) - (a^2*b*tan(f*x +
e)^2 + a^3)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b
*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^2 + (a
^5 - 2*a^4*b + a^3*b^2)*f), ((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b
^3)*tan(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a)) -
 (a^2*b*tan(f*x + e)^2 + a^3)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(
f*x + e)^2 + a)) - (a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b ...
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{(b\tan(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 1922, normalized size of antiderivative = 18.13

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
b/(f*(a + b*tan(e + f*x)^2)^(1/2)*(a*b - a^2)) - atanh((2*a^2*b^8*f^2*(a +
 b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*
a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) - (12*a^3*
b^7*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b
^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)
) + (30*a^4*b^6*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^
2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*
a^6*b^3*f^2)) - (38*a^5*b^5*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^3)^(1/2)
*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f^2 + 24*a^5*
b^4*f^2 - 6*a^6*b^3*f^2)) + (24*a^6*b^4*f^2*(a + b*tan(e + f*x)^2)^(1/2))/
((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 38*a^4*b^5*f
^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)) - (6*a^7*b^3*f^2*(a + b*tan(e + f*x)
^2)^(1/2))/((a^3)^(1/2)*(2*a*b^8*f^2 - 12*a^2*b^7*f^2 + 30*a^3*b^6*f^2 - 3
8*a^4*b^5*f^2 + 24*a^5*b^4*f^2 - 6*a^6*b^3*f^2)))/(f*(a^3)^(1/2)) + (atan(
((((a + b*tan(e + f*x)^2)^(1/2)*(2*a^3*b^7*f^3 - 10*a^4*b^6*f^3 + 22*a^5*
b^5*f^3 - 26*a^6*b^4*f^3 + 16*a^7*b^3*f^3 - 4*a^8*b^2*f^3))/2 + (((a - b)^
3)^(1/2)*(12*a^5*b^7*f^4 - 2*a^4*b^8*f^4 - 28*a^6*b^6*f^4 + 32*a^7*b^5*f^4
 - 18*a^8*b^4*f^4 + 4*a^9*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)
^3)^(1/2)*(8*a^5*b^8*f^5 - 56*a^6*b^7*f^5 + 160*a^7*b^6*f^5 - 240*a^8*b^5*
f^5 + 200*a^9*b^4*f^5 - 88*a^10*b^3*f^5 + 16*a^11*b^2*f^5))/(4*f*(a - b...
```

Reduce [F]

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\cot(fx+e)}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x))/(tan(e + f*x)**4*b**2 + 2*t
an(e + f*x)**2*a*b + a**2),x)
```

3.337 $\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} - \frac{(a-3b)b}{2a^2(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^2(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)}}$$

output

```
1/2*(2*a+3*b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh(
(a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f-1/2*(a-3*b)*b/a^2/(a-b
)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/a/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.73

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{-2a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a\cot^2(e+f\right.}{2a^2(-a+b)f\sqrt{a+b\,t}}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-2*a^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] +
(a - b)*(a*Cot[e + f*x]^2 + (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ (b*Tan[e + f*x]^2)/a]))/(2*a^2*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4153, 354, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 114$$

$$\frac{-\frac{\int \frac{\cot(e+fx)\left(3b\tan^2(e+fx)+2a+3b\right)}{2\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\cot(e+fx)\left(3b\tan^2(e+fx)+2a+3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{2a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \; 169$$

$$\frac{-\frac{\frac{2b(a-3b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{2\int -\frac{\cot(e+fx)\left((a-3b)b\tan^2(e+fx)+(a-b)(2a+3b)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}}{2a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{\int \frac{\cot(e+fx)\left((a-3b)b\tan^2(e+fx)+(a-b)(2a+3b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b(a-3b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

$$\Big\downarrow \; 174$$

$$\frac{-\frac{\frac{(a-b)(2a+3b)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-2a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b(a-3b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+}}}{2f}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{\frac{\frac{2(a-b)(2a+3b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{4a^2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{a(a-b)}+\frac{2b(a-3b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx}}}{2f}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{\frac{4a^2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(a-b)(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a-b)}+\frac{2b(a-3b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{2a}-\frac{\cot(e+fx)}{a\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cot[e + f*x]/(a*Sqrt[a + b*Tan[e + f*x]^2])) - (((-2*(a - b)*(2*a + 3*b
)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (4*a^2*ArcTanh[Sq
rt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/(a*(a - b)) + (2*(a -
3*b)*b)/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(2*a))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 9633 vs. $2(135) = 270$.

Time = 7.73 (sec) , antiderivative size = 9634, normalized size of antiderivative = 61.36

method	result	size
default	Expression too large to display	9634

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(135) = 270$.

Time = 0.15 (sec) , antiderivative size = 1229, normalized size of antiderivative = 7.83

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(a^3*b*tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*ta
n(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*
x + e)^2 + 1)) - ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (
2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*
x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2
*(a^4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*
sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^
4 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), 1/4*(4*(a^3*b*tan(f*x + e
)^4 + a^4*tan(f*x + e)^2)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x
+ e)^2 + a)) + ((2*a^3*b - a^2*b^2 - 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*
a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x
+ e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(
a^4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + e)^2)*sq
rt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^4
+ (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x + e)^2), -1/2*(((2*a^3*b - a^2*b^2 -
 4*a*b^3 + 3*b^4)*tan(f*x + e)^4 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*t
an(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f*x + e)^2 + a)) + (a^3
*b*tan(f*x + e)^4 + a^4*tan(f*x + e)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2
+ 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)
) + (a^4 - 2*a^3*b + a^2*b^2 + (a^3*b - 4*a^2*b^2 + 3*a*b^3)*tan(f*x + ...
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\left(b\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 2483, normalized size of antiderivative = 15.82

$$\int \frac{\cot^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(b^2/(a*b - a^2) + (b*(a + b*tan(e + f*x)^2)*(a - 3*b))/(2*a*(a*b - a^2)))
/(f*(a + b*tan(e + f*x)^2)^(3/2) - a*f*(a + b*tan(e + f*x)^2)^(1/2)) - (at
an((((((a + b*tan(e + f*x)^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 5
44*a^8*b^7*f^3 + 160*a^9*b^6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 32
0*a^12*b^3*f^3 - 128*a^13*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(512*a^9*b^8*f^
4 - 96*a^8*b^9*f^4 - 1056*a^10*b^7*f^4 + 1024*a^11*b^6*f^4 - 416*a^12*b^5*
f^4 + 32*a^14*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(2
56*a^10*b^8*f^5 - 1792*a^11*b^7*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^
5 + 6400*a^14*b^4*f^5 - 2816*a^15*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b
)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3) + ((((a + b*ta
n(e + f*x)^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^3 + 544*a^8*b^7*f^3 +
 160*a^9*b^6*f^3 - 496*a^10*b^5*f^3 - 16*a^11*b^4*f^3 + 320*a^12*b^3*f^3 -
 128*a^13*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(96*a^8*b^9*f^4 - 512*a^9*b^8*f
^4 + 1056*a^10*b^7*f^4 - 1024*a^11*b^6*f^4 + 416*a^12*b^5*f^4 - 32*a^14*b^
3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(256*a^10*b^8*f^5
- 1792*a^11*b^7*f^5 + 5120*a^12*b^6*f^5 - 7680*a^13*b^5*f^5 + 6400*a^14*b^
4*f^5 - 2816*a^15*b^3*f^5 + 512*a^16*b^2*f^5))/(4*f*(a - b)^3)))/(2*f*(a -
 b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3))/(144*a^6*b^8*f^2 - 384*a^7*b^
7*f^2 + 256*a^8*b^6*f^2 + 96*a^9*b^5*f^2 - 144*a^10*b^4*f^2 + 32*a^11*b^3*
f^2 - ((((a + b*tan(e + f*x)^2)^(1/2)*(144*a^6*b^9*f^3 - 528*a^7*b^8*f^...
```

Reduce [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\int \frac{\cot(fx+e)^3}{(\tan(fx+e)^2 b+a)^{\frac{3}{2}}}dx$$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.338 $\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 215

$$\begin{aligned}\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx &= -\frac{(8a^2+12ab+15b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} \\ &+ \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{b(4a^2+3ab-15b^2)}{8a^3(a-b)f\sqrt{a+b\tan^2(e+fx)}} \\ &+ \frac{(4a+5b)\cot^2(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{\cot^4(e+fx)}{4af\sqrt{a+b\tan^2(e+fx)}}\end{aligned}$$

output
```
-1/8*(8*a^2+12*a*b+15*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/
2)/f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/8*b*(4*
a^2+3*a*b-15*b^2)/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/8*(4*a+5*b)*cot(f
*x+e)^2/a^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/a/f/(a+b*tan(f*x+e
)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.79 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{8a^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a\cot^2(e+fx\right.}{8a^3}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(8*a^3*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(a - b)] + (
a - b)*(a*Cot[e + f*x]^2*(-4*a - 5*b + 2*a*Cot[e + f*x]^2) - (8*a^2 + 12*a
*b + 15*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[e + f*x]^2)/a]))/(
8*a^3*(-a + b)*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

Failed to integrate

input
```
Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 11428 vs. 2(189) = 378.

Time = 8.27 (sec) , antiderivative size = 11429, normalized size of antiderivative = 53.16

method	result	size
default	Expression too large to display	11429

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1501, normalized size of antiderivative = 6.98

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(a - b)*log((b*t
an(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a - b)/(tan(f
*x + e)^2 + 1)) - ((8*a^4*b - 4*a^3*b^2 - a^2*b^3 - 18*a*b^4 + 15*b^5)*tan
(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x
+ e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(
a) + 2*a)/tan(f*x + e)^2) + 2*(2*a^5 - 4*a^4*b + 2*a^3*b^2 - (4*a^4*b - a^
3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b
^2 + 5*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^
5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x +
e)^4), -1/16*(16*(a^4*b*tan(f*x + e)^6 + a^5*tan(f*x + e)^4)*sqrt(-a + b)*
arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a)) - ((8*a^4*b - 4*a^3*b^2 -
a^2*b^3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 -
 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*
sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(2*a^5 - 4*a
^4*b + 2*a^3*b^2 - (4*a^4*b - a^3*b^2 - 18*a^2*b^3 + 15*a*b^4)*tan(f*x + e
)^4 - (4*a^5 - 3*a^4*b - 6*a^3*b^2 + 5*a^2*b^3)*tan(f*x + e)^2)*sqrt(b*tan
(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^6 + (a^7 -
 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^4), 1/8*(((8*a^4*b - 4*a^3*b^2 - a^2*b^
3 - 18*a*b^4 + 15*b^5)*tan(f*x + e)^6 + (8*a^5 - 4*a^4*b - a^3*b^2 - 18*a^
2*b^3 + 15*a*b^4)*tan(f*x + e)^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(f...
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^5{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 2118, normalized size of antiderivative = 9.85

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
(atan((((((a + b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*
b^10*f^3 + 854016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3
 - 680960*a^14*b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 32768
0*a^17*b^3*f^3 - 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(638976*a^13
*b^9*f^4 - 122880*a^12*b^10*f^4 - 1318912*a^14*b^8*f^4 + 1376256*a^15*b^7*
f^4 - 794624*a^16*b^6*f^4 + 311296*a^17*b^5*f^4 - 122880*a^18*b^4*f^4 + 32
768*a^19*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144
*a^15*b^8*f^5 - 1835008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18
*b^5*f^5 + 6553600*a^19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f
^5))/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3
) + ((((a + b*tan(e + f*x)^2)^(1/2)*(230400*a^9*b^11*f^3 - 783360*a^10*b^1
0*f^3 + 854016*a^11*b^9*f^3 - 387072*a^12*b^8*f^3 + 480256*a^13*b^7*f^3 -
680960*a^14*b^6*f^3 + 352256*a^15*b^5*f^3 - 262144*a^16*b^4*f^3 + 327680*a
^17*b^3*f^3 - 131072*a^18*b^2*f^3))/2 + (((a - b)^3)^(1/2)*(122880*a^12*b^
10*f^4 - 638976*a^13*b^9*f^4 + 1318912*a^14*b^8*f^4 - 1376256*a^15*b^7*f^4
 + 794624*a^16*b^6*f^4 - 311296*a^17*b^5*f^4 + 122880*a^18*b^4*f^4 - 32768
*a^19*b^3*f^4 + ((a + b*tan(e + f*x)^2)^(1/2)*((a - b)^3)^(1/2)*(262144*a^
15*b^8*f^5 - 1835008*a^16*b^7*f^5 + 5242880*a^17*b^6*f^5 - 7864320*a^18*b^
5*f^5 + 6553600*a^19*b^4*f^5 - 2883584*a^20*b^3*f^5 + 524288*a^21*b^2*f^5)
)/(4*f*(a - b)^3)))/(2*f*(a - b)^3))*((a - b)^3)^(1/2)*1i)/(f*(a - b)^3...
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^5}{\left(\tan(fx+e)^2 b+a\right)^{\frac{3}{2}}}dx$$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.339 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 182

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\tan^3(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2(a-b)b^2f}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-1/2
*(3*a+2*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-
a*tan(f*x+e)^3/(a-b)/b/f/(a+b*tan(f*x+e)^2)^(1/2)+1/2*(3*a-b)*tan(f*x+e)*(
a+b*tan(f*x+e)^2)^(1/2)/(a-b)/b^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 5.77 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.80

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \frac{\left(2(a-b)\left(3a^2-b^2+(3a^2-2ab+b^2)\cos(2(e+fx))\right)\csc(2(e+fx))-\sqrt{}\right.}{}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
((2*(a - b)*(3*a^2 - b^2 + (3*a^2 - 2*a*b + b^2)*Cos[2*(e + f*x)])*Csc[2*(
e + f*x)] - Sqrt[2]*a*(3*a^2 - 4*a*b + b^2)*Cot[e + f*x]*Sqrt[((a + b + (a
 - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b +
 (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + 2*Sqrt[2]*a*b
^2*Cot[e + f*x]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b
]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])
*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sec[e + f*x]^2*Sin[2*(e + f*x)]*Tan[e +
f*x])/(4*Sqrt[2]*(a - b)^2*b^2*f*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*S
ec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4153, 372, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(e+fx)^6}{(a+b\tan(e+fx)^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((3a-b)\tan^2(e+fx)+3a\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a\tan^3(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\int \frac{(a-b)(3a+2b)\tan^2(e+fx)+a(3a-b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}}{b(a-b)}-\frac{a\tan^3(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-b)(3a+2b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{2b}}{b(a-b)}-\frac{a\tan^3}{b(a-b)\sqrt{a+}}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-b)(3a+2b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{2b}}{b(a-b)}-\frac{}{b(a}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{(a-b)(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{2b}}{b(a-b)}-\frac{a\tan}{b(a-b)\sqrt{a}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{2b^2\int\frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\frac{(a-b)(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{2b}}{b(a-b)}-\frac{a\tan^3(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2b}-\frac{\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}+\frac{(a-b)(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{2b}}{b(a-b)}-\frac{a\tan^3(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-((a*Tan[e + f*x]^3)/((a - b)*b*Sqrt[a + b*Tan[e + f*x]^2])) + (-1/2*((2*
b^2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a
- b] + ((a - b)*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[
e + f*x]^2]])/Sqrt[b])/b + ((3*a - b)*Tan[e + f*x]*Sqrt[a + b*Tan[e + f*x]
^2])/(2*b))/((a - b)*b))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{a\sqrt{a+b\tan(fx+e)^2}}+\frac{\tan(fx+e)^3}{2b\sqrt{a+b\tan(fx+e)^2}}-\frac{3a\left(-\frac{\tan(fx+e)}{b\sqrt{a+b\tan(fx+e)^2}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{b^{\frac{3}{2}}}\right)}{2b}+\frac{\tan(fx+}{b\sqrt{a+b\tan(}}$
default	$\frac{\frac{\tan(fx+e)}{a\sqrt{a+b\tan(fx+e)^2}}+\frac{\tan(fx+e)^3}{2b\sqrt{a+b\tan(fx+e)^2}}-\frac{3a\left(-\frac{\tan(fx+e)}{b\sqrt{a+b\tan(fx+e)^2}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{b^{\frac{3}{2}}}\right)}{2b}+\frac{\tan(fx+}{b\sqrt{a+b\tan(}}$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+1/2*tan(f*x+e)^3/b/(a+b*tan(f*x
+e)^2)^(1/2)-3/2*a/b*(-tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)+1/b^(3/2)*ln(
b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2)))+tan(f*x+e)/b/(a+b*tan(f*x+e)
^2)^(1/2)-1/b^(3/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan(f*x+e)^2)^(1/2))+b/(a-b
)*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*ar
ctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 1191, normalized size of antiderivative = 6.54

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3
+ 2*b^4)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) + 2*(b^4*tan(f*x + e)^2 + a*b^3)*sq
rt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((a^2*b^2 - 2*a*b^
3 + b^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt
(b*tan(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (a^3
*b^3 - 2*a^2*b^4 + a*b^5)*f), 1/2*((3*a^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 +
(3*a^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*tan(f*x + e)^2)*sqrt(-b)*arctan(sqrt
(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (b^4*tan(f*x + e)^2 + a*b^
3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + ((a^2*b^2 - 2*a
*b^3 + b^4)*tan(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a))/((a^2*b^4 - 2*a*b^5 + b^6)*f*tan(f*x + e)^2 + (
a^3*b^3 - 2*a^2*b^4 + a*b^5)*f), -1/4*(4*(b^4*tan(f*x + e)^2 + a*b^3)*sqrt
(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) - (3*a
^4 - 4*a^3*b - a^2*b^2 + 2*a*b^3 + (3*a^3*b - 4*a^2*b^2 - a*b^3 + 2*b^4)*t
an(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(b)*tan(f*x + e) + a) - 2*((a^2*b^2 - 2*a*b^3 + b^4)*tan(f*x + e)^3
 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + ...
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^6{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^6}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^6}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**6)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.340 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{(a-b)bf\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f+arct
anh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x+e)/(a
-b)/b/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.03 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.03

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{a\left(-a+b+\frac{(a-b)\sqrt{\frac{(a+b+(a-b)\cos(2(e+fx)))\csc^2(e+fx)}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b+(a-b)\cos(2(e}{b}}}{\sqrt{2}}\right.\right.}{\sqrt{2}}\right.}{\sqrt{}}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(a*(-a + b + ((a - b)*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x
]^2)/b]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b]/Sqrt[2]], 1])/Sqrt[2] - (b*Sqrt[((a + b + (a - b)*Cos[2*(e + f*
x)])*Csc[e + f*x]^2)/b]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a
- b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])/Sqrt[2])*Sec[e + f
*x]^2*Sin[2*(e + f*x)])/(Sqrt[2]*(a - b)^2*b*f*Sqrt[(a + b + (a - b)*Cos[2
*(e + f*x)])*Sec[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4153, 372, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^4(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(a-b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{(a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+(a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{b(a-b)}-\frac{a\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)+\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{b(a-b)}-\frac{a\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{b\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}+\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{b(a-b)}-\frac{a\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{b\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}+\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}}{b(a-b)}-\frac{a\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(((b*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a
 - b] + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]
])/Sqrt[b])/((a - b)*b) - (a*Tan[e + f*x])/((a - b)*b*Sqrt[a + b*Tan[e + f
*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{-\frac{\tan(fx+e)}{b\sqrt{a+b\tan(fx+e)^2}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{b^{\frac{3}{2}}}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan}{\sqrt{b^4(a-b)}\,\sqrt{a+b}}\right.}{(a-b)^2b^2}}{f}$
default	$\frac{-\frac{\tan(fx+e)}{b\sqrt{a+b\tan(fx+e)^2}}+\frac{\ln\left(\sqrt{b}\,\tan(fx+e)+\sqrt{a+b\tan(fx+e)^2}\right)}{b^{\frac{3}{2}}}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan}{\sqrt{b^4(a-b)}\,\sqrt{a+b}}\right.}{(a-b)^2b^2}}{f}$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-tan(f*x+e)/b/(a+b*tan(f*x+e)^2)^(1/2)+1/b^(3/2)*ln(b^(1/2)*tan(f*x+e
)+(a+b*tan(f*x+e)^2)^(1/2))-b/(a-b)*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+
1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e))-tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 958, normalized size of antiderivative = 7.79

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqr
t(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x
 + e) + a) + (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan
(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/
(tan(f*x + e)^2 + 1)) - 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f
*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^
3 + a*b^4)*f), -1/2*(2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*ta
n(f*x + e)^2)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2
+ a)) - (b^3*tan(f*x + e)^2 + a*b^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x
+ e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(
f*x + e)^2 + 1)) + 2*(a^2*b - a*b^2)*sqrt(b*tan(f*x + e)^2 + a)*tan(f*x +
e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a
*b^4)*f), 1/2*(2*(b^3*tan(f*x + e)^2 + a*b^2)*sqrt(a - b)*arctan(sqrt(a -
b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + (a^3 - 2*a^2*b + a*b^2 + (a^
2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(b)*tan(f*x + e) + a) - 2*(a^2*b - a*b^2)*sqr
t(b*tan(f*x + e)^2 + a)*tan(f*x + e))/((a^2*b^3 - 2*a*b^4 + b^5)*f*tan(f*x
 + e)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*f), ((b^3*tan(f*x + e)^2 + a*b^2)*
sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) -
(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*tan(f*x + e)^2)*sqrt(-...
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^4{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^4}{(b\tan(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(b*tan(f*x + e)^2 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{(b\tan(e+f\,x)^2+a)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^4}{\tan\left(fx+e\right)^4 b^2+2\tan\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**4)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.341 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}+\frac{\tan(e+fx)}{(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f+tan
(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx=\frac{\tan(e+fx)\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(-a+b)\tan^2(e+fx)}{a}}}{\sqrt{1+\frac{b\tan^2(e+fx)}{a}}}\right)(b+a\cot^2(e+fx))\sqrt{\frac{(-a+b)\tan^2}{a}}\right.}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}\sqrt{1+\frac{b\tan^2(e-}{a}}}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]*(ArcTanh[Sqrt[((-a + b)*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e
 + f*x]^2)/a]]*(b + a*Cot[e + f*x]^2)*Sqrt[((-a + b)*Tan[e + f*x]^2)/a] +
(a - b)*Sqrt[1 + (b*Tan[e + f*x]^2)/a]))/((a - b)^2*f*Sqrt[a + b*Tan[e + f
*x]^2]*Sqrt[1 + (b*Tan[e + f*x]^2)/a])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4153, 373, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}} - \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(a - b)^(
3/2)) + Tan[e + f*x]/((a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

method	result	size
derivativedivides	$\frac{\frac{\tan(fx+e)}{a\sqrt{a+b\tan(fx+e)^2}}+\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^2b^2}}{f}$	126
default	$\frac{\frac{\tan(fx+e)}{a\sqrt{a+b\tan(fx+e)^2}}+\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^2b^2}}{f}$	126

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)+b/(a-b)*tan(f*x+e)/a/(a+b*tan(f
*x+e)^2)^(1/2)-1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b)
)^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.48

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2-2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{-a}}{\tan(fx+e)^2+1}\right.}{2\left(\left(a^2b-2\,ab^2+b^3\right)f\tan\left(fx+e\right)^2\right.}\right.$$

$$\left.-\frac{\left(b\tan\left(fx+e\right)^2+a\right)\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-\sqrt{b\tan\left(fx+e\right)^2+a}(a-b)\tan\left(fx+e\right)}{\left(a^2b-2\,ab^2+b^3\right)f\tan\left(fx+e\right)^2+\left(a^3-2\,a^2b+ab^2\right)f}\right]$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((b*tan(f*x + e)^2 + a)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 -
 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)
^2 + 1)) + 2*sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*
a*b^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f), -((b*tan(f*x +
 e)^2 + a)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)
^2 + a)) - sqrt(b*tan(f*x + e)^2 + a)*(a - b)*tan(f*x + e))/((a^2*b - 2*a*
b^2 + b^3)*f*tan(f*x + e)^2 + (a^3 - 2*a^2*b + a*b^2)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^2\left(e+fx\right)}{\left(a+b\tan^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)-\left(\int \frac{\sqrt{\tan(fx+e)^2b+a}}{\tan(fx+e)^4b^2+2\tan(fx+e)^2ab+a^2}dx\right)\mathrm{ta}}{af\left(\tan\left(fx+e\right)^2 b+\right.}$$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x) - int(sqrt(tan(e + f*x)**2*b + a
)/(tan(e + f*x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2),x)*tan(e + f*x)**2
*a*b*f - int(sqrt(tan(e + f*x)**2*b + a)/(tan(e + f*x)**4*b**2 + 2*tan(e +
 f*x)**2*a*b + a**2),x)*a**2*f)/(a*f*(tan(e + f*x)**2*b + a))
```

3.342 $\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f}-\frac{b\tan(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*ta
n(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.55 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx=\frac{\cos(e+fx)\sin(e+fx)\left(\frac{4(a-b)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+fx)}{a^2}\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

output
```
(Cos[e + f*x]*Sin[e + f*x]*((4*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a -
b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2 - (15*(2*
b + 3*a*Cot[e + f*x]^2)*(-(a*Sec[e + f*x]^2*Sqrt[((a - b)*(b + a*Cot[e + f
*x]^2)*Sin[e + f*x]^4)/a^2]) + ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*(a
 + b*Tan[e + f*x]^2)))/(a*(a - b)*Sqrt[((a - b)*(b + a*Cot[e + f*x]^2)*Sin
[e + f*x]^4)/a^2])))/(15*a*f*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4144, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{296}$$

$$\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}-\frac{b\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{b\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]]/(a - b)^(3/
2) - (b*Tan[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^2b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}}{f}$	102
default	$\frac{\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^2b^2}-\frac{b\tan(fx+e)}{(a-b)a\sqrt{a+b\tan(fx+e)^2}}}{f}$	102

input
```
int(1/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^2*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-b/(a-b)*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.61

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{\left(ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan(fx+e)^2+2\,\sqrt{b\tan(fx+e)^2+a}\sqrt{}}{\tan(fx+e)^2+1}\right.}{2\left(\left(a^3b-2\,a^2b^2+ab^3\right)f\tan\left(fx+e\right.\right.}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)
^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x
+ e)^2 + 1)) - 2*sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x + e))/((a^
3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^2*b^2)*f),
((a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sq
rt(b*tan(f*x + e)^2 + a)) - sqrt(b*tan(f*x + e)^2 + a)*(a*b - b^2)*tan(f*x
 + e))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*tan(f*x + e)^2 + (a^4 - 2*a^3*b + a^
2*b^2)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\tan^2{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}}{\tan\left(fx+e\right)^4 b^2+2\tan\left(fx+e\right)^2 ab+a^2}dx$$

input
```
int(1/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(sqrt(tan(e + f*x)**2*b + a)/(tan(e + f*x)**4*b**2 + 2*tan(e + f*x)**2*
a*b + a**2),x)
```

3.343 $\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a^2(a-b)f}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*c
ot(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-(a-2*b)*cot(f*x+e)*(a+b*tan(f
*x+e)^2)^(1/2)/a^2/(a-b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.65 (sec) , antiderivative size = 882, normalized size of antiderivative = 6.89

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-((Cos[e + f*x]^2*Cot[e + f*x]*((3*a*Csc[e + f*x]^2)/(a - b) + (12*b*Sec[e
 + f*x]^2)/(a - b) + (16*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin
[e + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*(a - b)*HypergeometricPFQ[{2,
2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(15*a) + (8*b
^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)) + (8*(a - b)*b*Hypergeometri
c2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)
/(3*a^2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*S
in[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(15*a^2) + (8*(a - b)*b^2
*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*T
an[e + f*x]^4)/(5*a^3) + (8*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7
/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(15*a^3) -
 (3*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/((((a - b)*Sin[e + f*x]^2)/a
)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (12*b*ArcSin[Sq
rt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*(((a - b)*Sin[e + f*x]^
2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) - (8*b^2*ArcS
in[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/(a^2*(((a - b)*Sin[e
+ f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a]) + (3*A
rcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]])/Sqrt[((a - b)*Cos[e + f*x]^2*Sin[
e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2] + (12*b*ArcSin[Sqrt[((a - b)*Sin[e
 + f*x]^2)/a]]*Tan[e + f*x]^2)/(a*Sqrt[((a - b)*Cos[e + f*x]^2*Sin[e + ...
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4153, 374, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \textbf{374}$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-2b\tan^2(e+fx)+a-2b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \textbf{445}$$

$$\frac{\frac{-\frac{\int \frac{a^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{-a\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \textbf{291}$$

$$\frac{\frac{-a\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{-\frac{a\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{(a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2])) + (-((a*ArcTan
[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b]) - ((
a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(118) = 236$.

Time = 25.71 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.90

method	result
default	$\frac{(-2\sec(fx+e)\csc(fx+e)+3\cot(fx+e))b\,a^2(a-b)^{\frac{3}{2}}+\left(-\tan(fx+e)\sec(fx+e)^2+4\tan(fx+e)\right)b^2a(a-b)^{\frac{3}{2}}-(a-b)^{\frac{3}{2}}a^3\cot(fx+e)}{}$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((-2*sec(f*x+e)*csc(f*x+e)+3*cot(f*x+e))*b*a^2*(a-b)^(3/2)+(-tan(f*x+e
)*sec(f*x+e)^2+4*tan(f*x+e))*b^2*a*(a-b)^(3/2)-(a-b)^(3/2)*a^3*cot(f*x+e)+
2*(a-b)^(3/2)*b^3*tan(f*x+e)^3-arctan(1/(a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin
(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*((a*cos(f*x+
e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^4*(sec(f*x+e)+1)-arctan(1/(
a-b)^(1/2)*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*
x+e)/(cos(f*x+e)-1))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1
/2)*a^3*b*(-2-2*sec(f*x+e)+sec(f*x+e)^2+sec(f*x+e)^3)-arctan(1/(a-b)^(1/2)
*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)/(cos(
f*x+e)-1))*((a*cos(f*x+e)^2+b*sin(f*x+e)^2)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b^
2*(-tan(f*x+e)^2-tan(f*x+e)^2*sec(f*x+e)))/a^2/(a-b)^(5/2)/(a+b*tan(f*x+e)
^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.68

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[\frac{(a^2b\tan(fx+e)^3 + a^3\tan(fx+e))\sqrt{-a+b}\log\left(-\frac{(a^2-8ab+8b^2)\tan(fx+\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{(a^2b\tan(fx+e)^3 + a^3\tan(fx+e))\sqrt{a-b}\arctan\left(-\frac{2\sqrt{b\tan(fx+e)^2+a}\sqrt{a-b}\tan(fx+e)}{(a-2b)\tan(fx+e)^2-a}\right) + 2(a^3-2a^2b+\cdots}{2((a^4b-2a^3b^2+a^2b^3)f\tan(fx+e)^3 + (a^5-2a^4b+a^3b^2\cdots}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqrt(-a + b)*log(-((a^2 -
8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^2 - 4
*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 - 2*a^2*b +
a*b^2 + (a^2*b - 3*a*b^2 + 2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 +
a))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3
*b^2)*f*tan(f*x + e)), -1/2*((a^2*b*tan(f*x + e)^3 + a^3*tan(f*x + e))*sqr
t(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a
 - 2*b)*tan(f*x + e)^2 - a)) + 2*(a^3 - 2*a^2*b + a*b^2 + (a^2*b - 3*a*b^2
 + 2*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b - 2*a^3*b^2
+ a^2*b^3)*f*tan(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*f*tan(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\cot(fx+e)^2}{\tan(fx+e)^4 b^2+2\tan(fx+e)^2 ab+a^2}dx$$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x)**2)/(tan(e + f*x)**4*b**2 +
2*tan(e + f*x)**2*a*b + a**2),x)
```

3.344 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} + \frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3(a-b)f} - \frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^2(a-b)f}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*co
t(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-4*b)*(a+2*b)*cot(f*
x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)/f-1/3*(a-4*b)*cot(f*x+e)^3*(a+b*ta
n(f*x+e)^2)^(1/2)/a^2/(a-b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.79 (sec) , antiderivative size = 1398, normalized size of antiderivative = 7.60

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output

```
-1/45*(Cos[e + f*x]^2*Cot[e + f*x]^3*((45*a*Csc[e + f*x]^2)/(a - b) - (270
*b*Sec[e + f*x]^2)/(a - b) + (4*(a - b)*Hypergeometric2F1[2, 2, 7/2, ((a -
 b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/a - (24*(a - b)*HypergeometricPFQ[{
2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/a - (16*(a
 - b)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2
)/a]*Sin[e + f*x]^2)/a - (1080*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a -
b)) - (132*(a - b)*b*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)
/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/a^2 - (144*(a - b)*b*HypergeometricPFQ[
{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*
x]^2)/a^2 - (48*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a
 - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/a^2 - (720*b^3*Sec
[e + f*x]^2*Tan[e + f*x]^4)/(a^2*(a - b)) - (312*(a - b)*b^2*Hypergeometri
c2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)
/a^3 - (216*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Si
n[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 - (48*(a - b)*b^2*Hype
rgeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e
 + f*x]^2*Tan[e + f*x]^4)/a^3 - (176*(a - b)*b^3*Hypergeometric2F1[2, 2, 7
/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/a^4 - (96*(
a - b)*b^3*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)
/a]*Sin[e + f*x]^2*Tan[e + f*x]^6)/a^4 - (16*(a - b)*b^3*Hypergeometric...
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4153, 374, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 374}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-4b\tan^2(e+fx)+a-4b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b\cot^3(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2(a-4b)b\tan^2(e+fx)+(3a-4b)(a+2b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}-\frac{b\cot^3(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\frac{-\frac{-\frac{\int \frac{3a^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}-\frac{b\cot^3}{a(a-b)\sqrt{a+}}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{-\frac{-3a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}-\frac{b\cot^3(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$\Big\downarrow$ 291

$$\frac{-\frac{-\frac{-3a^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}-\frac{b\cot^3(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$\Big\downarrow$ 216

$$\frac{-\frac{-\frac{-\frac{3a^2\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{(3a-4b)(a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{(a-4b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}-\frac{b\cot^3(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x]^3)/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2])) + (-1/3*((a
- 4*b)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/a - ((-3*a^2*ArcTan[(Sqr
t[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b] - ((3*a -
4*b)*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(3*a))/(a*(a -
b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 14.81 (sec) , antiderivative size = 1080, normalized size of antiderivative = 5.87

method	result	size
default	Expression too large to display	1080

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f/a^3/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(a-b)*(-((2*I*b^(1/2)
*(a-b)^(1/2)+a-2*b)/a)^(1/2)*a^2*b+((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/
2)*a^3+((1-cos(f*x+e))^8*csc(f*x+e)^8-16*(1-cos(f*x+e))^6*csc(f*x+e)^6+30*
(1-cos(f*x+e))^4*csc(f*x+e)^4-16*(1-cos(f*x+e))^2*csc(f*x+e)^2)*a^3*((2*I*
b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)+128*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4+(-(cos(f*x+e)-1)^8*csc(f*x+e)^8-
46*(1-cos(f*x+e))^4*csc(f*x+e)^4)*b*a^2*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a
)^(1/2)+(16*(1-cos(f*x+e))^6*csc(f*x+e)^6-64*(1-cos(f*x+e))^4*csc(f*x+e)^4
+16*(1-cos(f*x+e))^2*csc(f*x+e)^2)*b^2*a*((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/
a)^(1/2)+96*a^3*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/
2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/(cos(f*x+e)+1))^(1/2)*((-I*cos(f*x+e)*b^(1
/2)*(a-b)^(1/2)+I*b^(1/2)*(a-b)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/a/(cos(
f*x+e)+1))^(1/2)*EllipticF(((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(csc(
f*x+e)-cot(f*x+e)),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b^(1/2)*(a-b)^(1/2)*a+a^2
-8*a*b+8*b^2)/a^2)^(1/2))*(1-cos(f*x+e))^3*csc(f*x+e)^3-192*(1/a*(I*cos(f*
x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b
)/(cos(f*x+e)+1))^(1/2)*((-I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)+I*b^(1/2)*(a-b
)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/a/(cos(f*x+e)+1))^(1/2)*EllipticPi(((
2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(csc(f*x+e)-cot(f*x+e)),1/(-2*I*b^
(1/2)*(a-b)^(1/2)-a+2*b)*a,((-2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)/(...
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.15

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \left[\frac{3\left(a^3b\tan(fx+e)^5+a^4\tan(fx+e)^3\right)\sqrt{-a+b}\log\left(-\frac{(a^2-8ab+8b^2)\tan(f\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e)^3)*sqrt(-a + b)*log(-((a
^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a^
2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*((3*a^3*b -
a^2*b^2 - 10*a*b^3 + 8*b^4)*tan(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*
a^4 - 2*a^3*b - 5*a^2*b^2 + 4*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2
 + a))/((a^5*b - 2*a^4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b +
a^4*b^2)*f*tan(f*x + e)^3), 1/6*(3*(a^3*b*tan(f*x + e)^5 + a^4*tan(f*x + e
)^3)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x
+ e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((3*a^3*b - a^2*b^2 - 10*a*b^3 +
8*b^4)*tan(f*x + e)^4 - a^4 + 2*a^3*b - a^2*b^2 + (3*a^4 - 2*a^3*b - 5*a^2
*b^2 + 4*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^5*b - 2*a^
4*b^2 + a^3*b^3)*f*tan(f*x + e)^5 + (a^6 - 2*a^5*b + a^4*b^2)*f*tan(f*x +
e)^3)]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^4}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^4}{\left(\tan(fx+e)^2 b+a\right)^{\frac{3}{2}}}dx$$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.345 $\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a-b)f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15a^3+10a^2b+8ab^2-48b^3)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4(a-b)f} + \frac{(5a^2+4ab-24b^2)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^3(a-b)f} - \frac{(a-6b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^2(a-b)f}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*c
ot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+10*a^2*b+8*a*b
^2-48*b^3)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^4/(a-b)/f+1/15*(5*a^2+4*a
*b-24*b^2)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)/f-1/5*(a-6*b)*c
ot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2)/a^2/(a-b)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 15.66 (sec) , antiderivative size = 1994, normalized size of antiderivative = 7.91

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
-1/5*(Cos[e + f*x]^2*Cot[e + f*x]^5*((3*a*Csc[e + f*x]^2)/(a - b) - (8*b*S
ec[e + f*x]^2)/(a - b) - (16*(a - b)*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}
, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(9*a) + (32*(a - b)*Hypergeo
metricPFQ[{2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin
[e + f*x]^2)/(45*a) + (48*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(a*(a - b)) -
 (16*(a - b)*b*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Si
n[e + f*x]^2*Tan[e + f*x]^2)/(9*a^2) + (32*(a - b)*b*HypergeometricPFQ[{2,
 2, 2}, {1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^
2)/(9*a^2) + (64*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((
a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(9*a^2) + (128*(a
 - b)*b*HypergeometricPFQ[{2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, ((a - b)*Sin[e
+ f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(45*a^2) + (192*b^3*Sec[e + f*
x]^2*Tan[e + f*x]^4)/(a^2*(a - b)) + (80*(a - b)*b^2*Hypergeometric2F1[2,
2, 7/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(3*a^3)
 + (112*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 7/2}, ((a - b)*Sin[e
+ f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(3*a^3) + (64*(a - b)*b^2*Hype
rgeometricPFQ[{2, 2, 2, 2}, {1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e
 + f*x]^2*Tan[e + f*x]^4)/(3*a^3) + (64*(a - b)*b^2*HypergeometricPFQ[{2,
2, 2, 2, 2}, {1, 1, 1, 7/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Ta
n[e + f*x]^4)/(15*a^3) + (128*b^4*Sec[e + f*x]^2*Tan[e + f*x]^6)/(a^3*(...
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4153, 374, 445, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-6b\tan^2(e+fx)+a-6b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b\cot^5(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{\cot^4(e+fx)\left(5a^2+4ba-24b^2+4(a-6b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{5a}-\frac{(a-6b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}}{a(a-b)}-\frac{b\cot^5(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^3+10ba^2+8b^2a-48b^3+2b\left(5a^2+4ba-24b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{5a}-(a-6b)\cot^5(e+}{a(a-b)}}{f}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{15a^4}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(}{3a}}{5a}}{a(a-b)}}{f}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{-15a^3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\,t}}{3a}}{5a}}{a(a-b)}}{f}$$

$\Big\downarrow$ 291

$$-\frac{-\frac{-\frac{-15a^3\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e}}{3a}}{5a}}{a(a-b)}}{f}$$

$\Big\downarrow$ 216

$$-\frac{-\frac{-\frac{\left(5a^2+4ab-24b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}-\frac{-\frac{15a^3\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{\left(15a^3+10a^2b+8ab^2-48b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}}{5a}-\frac{(a-6b}{}}{a(a-b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x]^5)/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2])) + (-1/5*((a
- 6*b)*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/a - (-1/3*((5*a^2 + 4*a*
b - 24*b^2)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/a - ((-15*a^3*ArcTa
n[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b] - ((
15*a^3 + 10*a^2*b + 8*a*b^2 - 48*b^3)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]
^2])/a)/(3*a))/(5*a))/(a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 16.82 (sec) , antiderivative size = 1474, normalized size of antiderivative = 5.85

method	result	size
default	Expression too large to display	1474

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/15*sin(f*x+e)^5*cos(f*x+e)^2*(60*cos(f*x+e)+60)*EllipticPi(((2*I*b^
(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(cot(f*x+e)-csc(f*x+e)),-1/(2*I*b^(1/2)*
(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*b)/a)^(1/2)/((2*I*b^(1
/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*(1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*
b^(1/2)*(a-b)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/(cos(f*x+e)+1))^(1/2)*(-1
/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)-a*cos(f*x+e)+co
s(f*x+e)*b-b)/(cos(f*x+e)+1))^(1/2)*a^5+1/15*sin(f*x+e)^7*(60*cos(f*x+e)+6
0)*EllipticPi(((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2)*(cot(f*x+e)-csc(f*
x+e)),-1/(2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)*a,(-(2*I*b^(1/2)*(a-b)^(1/2)-a+2*
b)/a)^(1/2)/((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2))*(1/a*(I*cos(f*x+e)*
b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/(co
s(f*x+e)+1))^(1/2)*(-1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)
^(1/2)-a*cos(f*x+e)+cos(f*x+e)*b-b)/(cos(f*x+e)+1))^(1/2)*a^4*b+1/15*sin(f
*x+e)^5*cos(f*x+e)^2*(-30*cos(f*x+e)-30)*EllipticF(((2*I*b^(1/2)*(a-b)^(1/
2)+a-2*b)/a)^(1/2)*(cot(f*x+e)-csc(f*x+e)),((8*I*b^(3/2)*(a-b)^(1/2)-4*I*b
^(1/2)*(a-b)^(1/2)*a+a^2-8*a*b+8*b^2)/a^2)^(1/2))*(1/a*(I*cos(f*x+e)*b^(1/
2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2)+a*cos(f*x+e)-cos(f*x+e)*b+b)/(cos(f*x
+e)+1))^(1/2)*(-1/a*(I*cos(f*x+e)*b^(1/2)*(a-b)^(1/2)-I*b^(1/2)*(a-b)^(1/2
)-a*cos(f*x+e)+cos(f*x+e)*b-b)/(cos(f*x+e)+1))^(1/2)*a^5+1/15*sin(f*x+e)^7
*(-30*cos(f*x+e)-30)*EllipticF(((2*I*b^(1/2)*(a-b)^(1/2)+a-2*b)/a)^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.73

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/60*(15*(a^4*b*tan(f*x + e)^7 + a^5*tan(f*x + e)^5)*sqrt(-a + b)*log(-((
a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 - 4*a*b)*tan(f*x + e)^2 + a
^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 +
 a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*((15*a^4*b
- 5*a^3*b^2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a^
4*b + 3*a^3*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*ta
n(f*x + e)^4 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*s
qrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^7
 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5), -1/30*(15*(a^4*b*tan(f*x +
 e)^7 + a^5*tan(f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^4*b
 - 5*a^3*b^2 - 2*a^2*b^3 - 56*a*b^4 + 48*b^5)*tan(f*x + e)^6 + 3*a^5 - 6*a
^4*b + 3*a^3*b^2 + (15*a^5 - 10*a^4*b - a^3*b^2 - 28*a^2*b^3 + 24*a*b^4)*t
an(f*x + e)^4 - (5*a^5 - 4*a^4*b - 7*a^3*b^2 + 6*a^2*b^3)*tan(f*x + e)^2)*
sqrt(b*tan(f*x + e)^2 + a))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*f*tan(f*x + e)^
7 + (a^7 - 2*a^6*b + a^5*b^2)*f*tan(f*x + e)^5)]
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**6/(a + b*tan(e + f*x)**2)**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^6}{\left(\tan\left(fx+e\right)^2 b+a\right)^{\frac{3}{2}}}dx$$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

output
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(3/2),x)
```

3.346 $\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f}+\frac{a^2}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{a(a-2b)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/3*a^2/(a-b)
/b^2/f/(a+b*tan(f*x+e)^2)^(3/2)-a*(a-2*b)/(a-b)^2/b^2/f/(a+b*tan(f*x+e)^2)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\frac{b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)-(a-b)\left(2a-b+3b\,\text{ta}\right.}{3(a-b)b^2f\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)] - (a
 - b)*(2*a - b + 3*b*Tan[e + f*x]^2))/(3*(a - b)*b^2*f*(a + b*Tan[e + f*x]
^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4153, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^5(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 98}$$

$$\frac{\int \left(-\frac{a^2}{(a-b)b\left(b\tan^2(e+fx)+a\right)^{5/2}}+\frac{(a-2b)a}{(a-b)^2b\left(b\tan^2(e+fx)+a\right)^{3/2}}+\frac{1}{(a-b)^2(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a}}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2a^2}{3b^2(a-b)(a+b\tan^2(e+fx))^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}}-\frac{2a(a-2b)}{b^2(a-b)^2\sqrt{a+b\tan^2(e+fx)}}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(5/2) + (2*a
^2)/(3*(a - b)*b^2*(a + b*Tan[e + f*x]^2)^(3/2)) - (2*a*(a - 2*b))/((a - b
)^2*b^2*Sqrt[a + b*Tan[e + f*x]^2]))/(2*f)
```

Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{\tan(fx+e)^2}{b\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{3(a-b)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{(a-b)^2\sqrt{a+b\tan}}}{f}$
default	$\frac{-\frac{\tan(fx+e)^2}{b\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{3(a-b)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{(a-b)^2\sqrt{a+b\tan}}}{f}$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-tan(f*x+e)^2/b/(a+b*tan(f*x+e)^2)^(3/2)-2/3*a/b^2/(a+b*tan(f*x+e)^2)
^(3/2)+1/3/(a-b)/(a+b*tan(f*x+e)^2)^(3/2)+1/(a-b)^2/(-a+b)^(1/2)*arctan((a
+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+1/(a-b)^2/(a+b*tan(f*x+e)^2)^(1/2)+1/
3/b/(a+b*tan(f*x+e)^2)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(103) = 206.

Time = 0.16 (sec) , antiderivative size = 634, normalized size of antiderivative = 5.51

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(b^4\tan\left(fx+e\right)^4+2\,ab^3\tan\left(fx+e\right)^2+a^2b^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e}{}\right.}{12\left((a^3b^4-3\,a^2b^5+3\right.}\right.$$

$$-\frac{3\left(b^4\tan\left(fx+e\right)^4+2\,ab^3\tan\left(fx+e\right)^2+a^2b^2\right)\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right.}{6\left((a^3b^4-3\,a^2b^5+3\,ab^6-b^7)f\tan\left(fx+e\right)^4+2\left(a^4b^3-3\,a^3b^4+3\,a^2b^5\right.\right.}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*tan(f*x + e)^2 + a^2*b^2)*sqrt(a -
b)*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(
f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a
*b + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(2*a^4 - 7*a^3*b +
5*a^2*b^2 + 3*(a^3*b - 3*a^2*b^2 + 2*a*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x
 + e)^2 + a))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*
(a^4*b^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*
a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f), -1/6*(3*(b^4*tan(f*x + e)^4 + 2*a*b^3*t
an(f*x + e)^2 + a^2*b^2)*sqrt(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a
- b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 +
 a^2 - a*b)) + 2*(2*a^4 - 7*a^3*b + 5*a^2*b^2 + 3*(a^3*b - 3*a^2*b^2 + 2*a
*b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^4 - 3*a^2*b^5 +
3*a*b^6 - b^7)*f*tan(f*x + e)^4 + 2*(a^4*b^3 - 3*a^3*b^4 + 3*a^2*b^5 - a*b
^6)*f*tan(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f)]
```

Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 11.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\frac{a^2}{3(a-b)}+\frac{(b\tan(e+fx)^2+a)\,(2ab-a^2)}{(a-b)^2}}{b^2 f\,(b\tan(e+fx)^2+a)^{3/2}} + \frac{\operatorname{atan}\left(\frac{a^2\sqrt{b\tan(e+fx)^2+a}\,1\mathrm{i}+b^2\sqrt{b\tan(e+fx)^2+a}\,1\mathrm{i}-ab\sqrt{b\tan(e+fx)^2+a}\,2\mathrm{i}}{(a-b)^{5/2}}\right)1\mathrm{i}}{f\,(a-b)^{5/2}}$$

input
```
int(tan(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
(atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e + f*x)^2)^(1
/2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/(f*(a - b
)^(5/2)) + (a^2/(3*(a - b)) + ((a + b*tan(e + f*x)^2)*(2*a*b - a^2))/(a -
b)^2)/(b^2*f*(a + b*tan(e + f*x)^2)^(3/2))
```

Reduce [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{-3\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^2 b-2\sqrt{\tan(fx+e)^2 b+a}\,a+\sqrt{\mathrm{ta}}}{}$$

input
```
int(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
( - 3*sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2*b - 2*sqrt(tan(e + f*x)*
*2*b + a)*a + sqrt(tan(e + f*x)**2*b + a)*b + 3*int((sqrt(tan(e + f*x)**2*
b + a)*tan(e + f*x))/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x)**4*a*b**2 + 3*
tan(e + f*x)**2*a**2*b + a**3),x)*tan(e + f*x)**4*b**4*f + 6*int((sqrt(tan
(e + f*x)**2*b + a)*tan(e + f*x))/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x)**
4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)*tan(e + f*x)**2*a*b**3*f +
3*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x))/(tan(e + f*x)**6*b**3 + 3
*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)*a**2*b**2*f)
/(3*b**2*f*(tan(e + f*x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2))
```

3.347 $\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{a}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{1}{(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-1/3*a/(a-b)/b/
f/(a+b*tan(f*x+e)^2)^(3/2)-1/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{a(-a+b) - 3b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e\right.}{3(a-b)^2bf\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(a*(-a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[e + f*x]^2)/(
a - b)]*(a + b*Tan[e + f*x]^2))/(3*(a - b)^2*b*f*(a + b*Tan[e + f*x]^2)^(3
/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4153, 354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^3}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^3(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{a-b}-\frac{2a}{3b(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{-\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a-b}-\frac{2a}{3b(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{\frac{2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a-b}-\frac{2a}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{a-b}-\frac{2a}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
((-2*a)/(3*(a - b)*b*(a + b*Tan[e + f*x]^2)^(3/2)) - ((-2*ArcTanh[Sqrt[a +
 b*Tan[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2/((a - b)*Sqrt[a + b*Tan
[e + f*x]^2]))/(a - b))/(2*f)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{-\frac{1}{3b\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{1}{3(a-b)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}-\frac{1}{(a-b)^2\sqrt{a+b\tan(fx+e)^2}}}{f}$	110
default	$\frac{-\frac{1}{3b\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{1}{3(a-b)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}-\frac{1}{(a-b)^2\sqrt{a+b\tan(fx+e)^2}}}{f}$	110

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3/b/(a+b*tan(f*x+e)^2)^(3/2)-1/3/(a-b)/(a+b*tan(f*x+e)^2)^(3/2)-1/
(a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))-1/(a-b)
^2/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(91) = 182$.

Time = 0.16 (sec) , antiderivative size = 598, normalized size of antiderivative = 5.81

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(b^3\tan\left(fx+e\right)^4+2\,ab^2\tan\left(fx+e\right)^2+a^2b\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4}{\cdots}\right.}{12\left(\left(a^3b^3-3\,a^2b^4+3\,ab^5-b^6\cdots\right.\right.}\right.$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt(a - b)
*log(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 + 4*(b*tan(f*
x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b
 + b^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) - 4*(a^3 + a^2*b - 2*a*b^
2 + 3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4 + 2*(a^4*b^2 - 3*a^3*b^3 + 3
*a^2*b^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*
b^4)*f), 1/6*(3*(b^3*tan(f*x + e)^4 + 2*a*b^2*tan(f*x + e)^2 + a^2*b)*sqrt
(-a + b)*arctan(-1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 +
a)*sqrt(-a + b)/((a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) - 2*(a^3 + a^2*b
 - 2*a*b^2 + 3*(a*b^2 - b^3)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*f*tan(f*x + e)^4 + 2*(a^4*b^2 - 3*a^
3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b
^3 - a^2*b^4)*f)]
```

Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx=\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^3}{(b\tan(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 11.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \frac{\tan^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\frac{a}{3(a-b)}+\frac{b\left(b\tan(e+f\,x)^2+a\right)}{(a-b)^2}}{b\,f\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}} - \frac{\mathrm{atan}\left(\frac{a^2\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}+b^2\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}-a\,b\sqrt{b\tan(e+f\,x)^2+a}\,2\mathrm{i}}{(a-b)^{5/2}}\right)1\mathrm{i}}{f\,(a-b)^{5/2}}$$

input
```
int(tan(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
- (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e + f*x)^2)^
(1/2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1i)/(f*(a -
 b)^(5/2)) - (a/(3*(a - b)) + (b*(a + b*tan(e + f*x)^2))/(a - b)^2)/(b*f*(
a + b*tan(e + f*x)^2)^(3/2))
```

Reduce [F]

$$\int \frac{\tan^3(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{-\sqrt{\tan\left(fx+e\right)^2 b+a}-3\left(\int \frac{\sqrt{\tan(fx+e)^2b+a}\,\tan(fx+e)}{\tan(fx+e)^6b^3+3\tan(fx+e)^4a\,b^2+3\tan(fx+e)^2a^2b+a^3}d\right.}{}$$

input
```
int(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
( - sqrt(tan(e + f*x)**2*b + a) - 3*int((sqrt(tan(e + f*x)**2*b + a)*tan(e
 + f*x))/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)
**2*a**2*b + a**3),x)*tan(e + f*x)**4*b**3*f - 6*int((sqrt(tan(e + f*x)**2
*b + a)*tan(e + f*x))/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x)**4*a*b**2 + 3
*tan(e + f*x)**2*a**2*b + a**3),x)*tan(e + f*x)**2*a*b**2*f - 3*int((sqrt(
tan(e + f*x)**2*b + a)*tan(e + f*x))/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x
)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)*a**2*b*f)/(3*b*f*(tan(e
+ f*x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2))
```

3.348 $\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2 f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/3/(a-b)/f/(
a+b*tan(f*x+e)^2)^(3/2)+1/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]/(3*(a - b
)*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4153, 353, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{a-b}+\frac{2}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a-b}+\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b(a-b)}+\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a-b}+\frac{2}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{2}{(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}}{a-b}+\frac{2}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(2/(3*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Tan[
e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) + 2/((a - b)*Sqrt[a + b*Tan[e + f*
x]^2]))/(a - b))/(2*f)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{3(a-b)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{(a-b)^2\sqrt{a+b\tan(fx+e)^2}}}{f}$	89
default	$\frac{\frac{\arctan\left(\frac{\sqrt{a+b\tan(fx+e)^2}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{1}{3(a-b)\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{(a-b)^2\sqrt{a+b\tan(fx+e)^2}}}{f}$	89

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*tan(f*x+e)^2)^(1/2)/(-a+b)^(1/2))+
1/3/(a-b)/(a+b*tan(f*x+e)^2)^(3/2)+1/(a-b)^2/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(87) = 174$.

Time = 0.16 (sec) , antiderivative size = 570, normalized size of antiderivative = 5.76

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{a-b}\log\left(-\frac{b^2\tan(fx+e)^4+\cdots}{\cdots}\right.}{12\left((a^3b^2-3\,a^2b^3+3\,ab^4-b^5\cdots\right.}\right.$$

$$-\frac{3\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\arctan\left(-\frac{\left(b\tan(fx+e)^2+2\,a-b\right)\sqrt{b\tan(fx+e)^2+a}\sqrt{-a+b}}{2\left((ab-b^2)\tan(fx+e)^2+a^2-ab\right)}\right)}{6\left((a^3b^2-3\,a^2b^3+3\,ab^4-b^5)f\tan\left(fx+e\right)^4+2\left(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4\right)f\cdots\right.}$$

input

```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[1/12*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*log
(-(b^2*tan(f*x + e)^4 + 2*(4*a*b - 3*b^2)*tan(f*x + e)^2 - 4*(b*tan(f*x +
e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b
^2)/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1)) + 4*(3*(a*b - b^2)*tan(f*x +
e)^2 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 -
 a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f), -1/6*
(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*arctan(-
1/2*(b*tan(f*x + e)^2 + 2*a - b)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)/(
(a*b - b^2)*tan(f*x + e)^2 + a^2 - a*b)) - 2*(3*(a*b - b^2)*tan(f*x + e)^2
 + 4*a^2 - 5*a*b + b^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2 - 3*a^2*b^3
+ 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b
^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)]
```

Sympy [A] (verification not implemented)

Time = 11.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{6f(a-b)\left(a+b\tan^2(e+fx)\right)^{\frac{3}{2}}}+\frac{b}{2f(a-b)^2\sqrt{a+b\tan^2(e+fx)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{-a+b}}\right)}{2f\sqrt{-a+b}(a-b)^2}\right)}{b} & \text{for } b \\ \begin{cases} \tilde{\infty}\tan^2(e+fx) & \text{for } a^{\frac{5}{2}}=0 \vee f=0 \\ \frac{\log\left(2a^{\frac{5}{2}}f\tan^2(e+fx)+2a^{\frac{5}{2}}f\right)}{2a^{\frac{5}{2}}f} & \text{otherwise} \end{cases} & \text{othe} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Piecewise((2*(b/(6*f*(a - b)*(a + b*tan(e + f*x)**2)**(3/2)) + b/(2*f*(a -
 b)**2*sqrt(a + b*tan(e + f*x)**2)) + b*atan(sqrt(a + b*tan(e + f*x)**2)/s
qrt(-a + b))/(2*f*sqrt(-a + b)*(a - b)**2))/b, Ne(b, 0)), (Piecewise((zoo*
tan(e + f*x)**2, Eq(f, 0) | Eq(a**(5/2), 0)), (log(2*a**(5/2)*f*tan(e + f*
x)**2 + 2*a**(5/2)*f)/(2*a**(5/2)*f), True)), True))
```

Maxima [F]

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/(b*tan(f*x + e)^2 + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 11.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\frac{b\tan(e+f\,x)^2+a}{(a-b)^2}+\frac{1}{3\,(a-b)}}{f\,\left(b\tan\left(e+f\,x\right)^2+a\right)^{3/2}} + \frac{\mathrm{atan}\left(\frac{a^2\,\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}+b^2\,\sqrt{b\tan(e+f\,x)^2+a}\,1\mathrm{i}-a\,b\,\sqrt{b\tan(e+f\,x)^2+a}\,2\mathrm{i}}{(a-b)^{5/2}}\right)\,1\mathrm{i}}{f\,(a-b)^{5/2}}$$

input
```
int(tan(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
((a + b*tan(e + f*x)^2)/(a - b)^2 + 1/(3*(a - b)))/(f*(a + b*tan(e + f*x)^
2)^(3/2)) + (atan((a^2*(a + b*tan(e + f*x)^2)^(1/2)*1i + b^2*(a + b*tan(e
+ f*x)^2)^(1/2)*1i - a*b*(a + b*tan(e + f*x)^2)^(1/2)*2i)/(a - b)^(5/2))*1
i)/(f*(a - b)^(5/2))
```

Reduce [F]

$$\int \frac{\tan(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(tan(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x))/(tan(e + f*x)**6*b**3 + 3*t
an(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.349 $\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{b}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(2a-b)b}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+arctanh((a+b*tan(f*x+
e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-1/3*b/a/(a-b)/f/(a+b*tan(f*x+e)^2)^
(3/2)-(2*a-b)*b/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{-a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\,\text{Hypergeomet}}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x]^2)/(a - b)]) + (
a - b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])/(3*a*(a
 - b)*f*(a + b*Tan[e + f*x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4153, 354, 96, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan^2(e+fx)}{2f}$$

$\Big\downarrow$ 96

$$\frac{\frac{\int \frac{\cot(e+fx)\left(-b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{a(a-b)}-\frac{2b}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 169

$$\frac{\frac{-\frac{2\int -\frac{\cot(e+fx)\left((a-b)^2-(2a-b)b\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}-\frac{2b}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\cot(e+fx)\left((a-b)^2-(2a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}-\frac{2b}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 174

$$\frac{\frac{\frac{(a-b)^2\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}-\frac{2b}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{2(a-b)^2\int \frac{1}{\frac{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}{b}}d\sqrt{b\tan^2(e+fx)+a}-2a^2\int \frac{1}{\frac{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}{b}}d\sqrt{b\tan^2(e+fx)+a}}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}-\frac{2b}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(a-b)^2\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a-b)}-\frac{2b(2a-b)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}-\frac{2b}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
((-2*b)/(3*a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + (((-2*(a - b)^2*ArcTa
nh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*ArcTanh[Sqrt[a +
b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/(a*(a - b)) - (2*(2*a - b)*b)
/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(a*(a - b)))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 39147 vs. $2(129) = 258$.

Time = 15.18 (sec) , antiderivative size = 39148, normalized size of antiderivative = 266.31

method	result	size
default	Expression too large to display	39148

input
```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(129) = 258$.

Time = 0.14 (sec) , antiderivative size = 1627, normalized size of antiderivative = 11.07

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^3*b^2*tan(f*x + e)^4 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqrt(a - b
)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a - b) + 2*a -
 b)/(tan(f*x + e)^2 + 1)) + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*
b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3
*a^2*b^3 - a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b
*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(7*a^4*b - 11*a^3*
b^2 + 4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b
*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f
*x + e)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 +
 (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f), -1/6*(6*(a^3*b^2*tan(f*x + e)^4
 + 2*a^4*b*tan(f*x + e)^2 + a^5)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*t
an(f*x + e)^2 + a)) - 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 -
3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f
*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) + 2*(7*a^4*b - 11*a^3*b^2 +
4*a^2*b^3 + 3*(2*a^3*b^2 - 3*a^2*b^3 + a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f
*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e
)^4 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^2 + (a^8
- 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f), 1/6*(6*(a^5 - 3*a^4*b + 3*a^3*b^2 - a
^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*...
```

Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 2788, normalized size of antiderivative = 18.97

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
int(cot(e + f*x)/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output

```
(b/(3*(a*b - a^2)) - (b*(a + b*tan(e + f*x)^2)*(2*a - b))/(a*b - a^2)^2)/(
f*(a + b*tan(e + f*x)^2)^(3/2)) - atanh((2*a^5*b^13*f^2*(a + b*tan(e + f*x
)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*
f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f
^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f
^2)) - (22*a^6*b^12*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*
b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7
*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11
*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (110*a^7*b^11*f^2*(a + b*
tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 11
0*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 91
2*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10
*a^13*b^3*f^2)) - (330*a^8*b^10*f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(
1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f
^2 + 660*a^7*b^9*f^2 - 922*a^8*b^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^
2 + 290*a^11*b^5*f^2 - 80*a^12*b^4*f^2 + 10*a^13*b^3*f^2)) + (660*a^9*b^9*
f^2*(a + b*tan(e + f*x)^2)^(1/2))/((a^5)^(1/2)*(2*a^3*b^13*f^2 - 22*a^4*b^
12*f^2 + 110*a^5*b^11*f^2 - 330*a^6*b^10*f^2 + 660*a^7*b^9*f^2 - 922*a^8*b
^8*f^2 + 912*a^9*b^7*f^2 - 630*a^10*b^6*f^2 + 290*a^11*b^5*f^2 - 80*a^12*b
^4*f^2 + 10*a^13*b^3*f^2)) - (922*a^10*b^8*f^2*(a + b*tan(e + f*x)^2)^(...
```

Reduce [F]

$$\int \frac{\cot(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\cot(fx+e)}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input

```
int(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output

```
int((sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x))/(tan(e + f*x)**6*b**3 + 3*t
an(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.350 $\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} - \frac{(3a-5b)b}{6a^2(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\cot^2(e+fx)}{2af\,(a+b\tan^2(e+fx))^{3/2}} - \frac{b(a^2-8ab+5b^2)}{2a^3(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output

```
1/2*(2*a+5*b)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f-arctanh(
(a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f-1/6*(3*a-5*b)*b/a^2/(a
-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/a/f/(a+b*tan(f*x+e)^2)^(3/
2)-1/2*b*(a^2-8*a*b+5*b^2)/a^3/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\cot^2(e+fx)\left(-2a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-\right.}{6a^2(-a+b)f\,(b+a\,\text{co}}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Cot[e + f*x]^2*(-2*a^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*
x]^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + (b*Tan[e + f*x]^2)/a])))/(6*a^2*(-a + b)*f*(b + a*Co
t[e + f*x]^2)*Sqrt[a + b*Tan[e + f*x]^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4153, 354, 114, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\tan(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{114}$$

$$\frac{-\frac{\int \frac{\cot(e+fx)\left(5b\tan^2(e+fx)+2a+5b\right)}{2\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{a}-\frac{\cot(e+fx)}{a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\cot(e+fx)\left(5b\tan^2(e+fx)+2a+5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{2a}-\frac{\cot(e+fx)}{a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{169}$$

$$\frac{-\frac{\frac{2b(3a-5b)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{2\int -\frac{3\cot(e+fx)\left((3a-5b)b\tan^2(e+fx)+(a-b)(2a+5b)\right)}{2\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{3a(a-b)}}{2a}-\frac{\cot(e+fx)}{a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\frac{\int \frac{\cot(e+fx)\left((3a-5b)b\tan^2(e+fx)+(a-b)(2a+5b)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b(3a-5b)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2a}-\frac{\cot(e+fx)}{a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \text{169}$$

$$\frac{-\frac{\frac{\frac{2b\left(a^2-8ab+5b^2\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{2\int -\frac{\cot(e+fx)\left((2a+5b)(a-b)^2+b\left(a^2-8ba+5b^2\right)\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}}{a(a-b)}+\frac{2b(3a-5b)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2a}-\frac{\cot(e}{a(a+b\tan^2}}{2f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{\int \frac{\cot(e+fx)\left((2a+5b)(a-b)^2+b\left(a^2-8ba+5b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b\left(a^2-8ab+5b^2\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}+\frac{2b(3a-5b)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2a} - \frac{\cot(e+f}{a(a+b\tan^2(e+}}{2f}$$

$\big\downarrow$ 174

$$-\frac{\frac{\frac{(a-b)^2(2a+5b)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-2a^3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b\left(a^2-8ab+5b^2\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}+\frac{2b(3a-5b}{3a(a-b)\left(a+b\tan^2(\right.}}{2a}}{2f}$$

$\big\downarrow$ 73

$$-\frac{\frac{\frac{\frac{2(a-b)^2(2a+5b)\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\tan^2(e+fx)+a}}{b}-\frac{4a^3\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}}{a(a-b)}+\frac{2b\left(a^2-8ab+5b^2\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}+\frac{2b(3a-5b)}{3a(a-b)\left(a+b\tan^2(e+\right.}}{2a}}{2f}$$

$\big\downarrow$ 221

$$-\frac{\frac{\frac{\frac{4a^3\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(a-b)^2(2a+5b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a-b)}+\frac{2b\left(a^2-8ab+5b^2\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}+\frac{2b(3a-5b)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2a} - }{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-(Cot[e + f*x]/(a*(a + b*Tan[e + f*x]^2)^(3/2))) - ((2*(3*a - 5*b)*b)/(3*
a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + (((-2*(a - b)^2*(2*a + 5*b)*ArcT
anh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (4*a^3*ArcTanh[Sqrt[a +
 b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/(a*(a - b)) + (2*b*(a^2 - 8*
a*b + 5*b^2))/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(a*(a - b)))/(2*a))/
(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 56969 vs. $2(180) = 360$.

Time = 22.03 (sec) , antiderivative size = 56970, normalized size of antiderivative = 276.55

method	result	size
default	Expression too large to display	56970

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(180) = 360$.

Time = 0.22 (sec) , antiderivative size = 2061, normalized size of antiderivative = 10.00

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(6*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x +
e)^2)*sqrt(a - b)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqr
t(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((2*a^4*b^2 - a^3*b^3 - 9*a^
2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2*a^5*b - a^4*b^2 - 9*a^3*b^
3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a^6 - a^5*b - 9*a^4*b^2 + 13
*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*log((b*tan(f*x + e)^2 + 2*sq
rt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x + e)^2) - 2*(3*a^6 - 9*a^5
*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5
)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 + 26*a^3*b^3 - 10*a^2*b^4)*tan(
f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4
- a^4*b^5)*f*tan(f*x + e)^6 + 2*(a^8*b - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*
f*tan(f*x + e)^4 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^2)
, 1/12*(12*(a^4*b^2*tan(f*x + e)^6 + 2*a^5*b*tan(f*x + e)^4 + a^6*tan(f*x
+ e)^2)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a)) + 3*(
(2*a^4*b^2 - a^3*b^3 - 9*a^2*b^4 + 13*a*b^5 - 5*b^6)*tan(f*x + e)^6 + 2*(2
*a^5*b - a^4*b^2 - 9*a^3*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + (2*a
^6 - a^5*b - 9*a^4*b^2 + 13*a^3*b^3 - 5*a^2*b^4)*tan(f*x + e)^2)*sqrt(a)*l
og((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a) + 2*a)/tan(f*x
 + e)^2) - 2*(3*a^6 - 9*a^5*b + 9*a^4*b^2 - 3*a^3*b^3 + 3*(a^4*b^2 - 9*a^3
*b^3 + 13*a^2*b^4 - 5*a*b^5)*tan(f*x + e)^4 + 2*(3*a^5*b - 19*a^4*b^2 +...
```

Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 3429, normalized size of antiderivative = 16.65

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^3/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
(atan(((((a + b*tan(e + f*x)^2)^(1/2)*(400*a^9*b^14*f^3 - 3680*a^10*b^13*f
^3 + 14864*a^11*b^12*f^3 - 34240*a^12*b^11*f^3 + 48480*a^13*b^10*f^3 - 412
80*a^14*b^9*f^3 + 16864*a^15*b^8*f^3 + 2688*a^16*b^7*f^3 - 6000*a^17*b^6*f
^3 + 1440*a^18*b^5*f^3 + 1040*a^19*b^4*f^3 - 704*a^20*b^3*f^3 + 128*a^21*b
^2*f^3) - ((2*a + 5*b)*(320*a^12*b^14*f^4 - 3392*a^13*b^13*f^4 + 16192*a^1
4*b^12*f^4 - 45760*a^15*b^11*f^4 + 84608*a^16*b^10*f^4 - 106624*a^17*b^9*f
^4 + 92288*a^18*b^8*f^4 - 53632*a^19*b^7*f^4 + 19520*a^20*b^6*f^4 - 3648*a
^21*b^5*f^4 + 64*a^22*b^4*f^4 + 64*a^23*b^3*f^4 - ((a + b*tan(e + f*x)^2)^
(1/2)*(2*a + 5*b)*(256*a^15*b^13*f^5 - 3072*a^16*b^12*f^5 + 16640*a^17*b^1
1*f^5 - 53760*a^18*b^10*f^5 + 115200*a^19*b^9*f^5 - 172032*a^20*b^8*f^5 +
182784*a^21*b^7*f^5 - 138240*a^22*b^6*f^5 + 72960*a^23*b^5*f^5 - 25600*a^2
4*b^4*f^5 + 5376*a^25*b^3*f^5 - 512*a^26*b^2*f^5))/(4*f*(a^7)^(1/2))))/(4*
f*(a^7)^(1/2)))*(2*a + 5*b)*1i)/(4*f*(a^7)^(1/2)) + (((a + b*tan(e + f*x)^
2)^(1/2)*(400*a^9*b^14*f^3 - 3680*a^10*b^13*f^3 + 14864*a^11*b^12*f^3 - 34
240*a^12*b^11*f^3 + 48480*a^13*b^10*f^3 - 41280*a^14*b^9*f^3 + 16864*a^15*
b^8*f^3 + 2688*a^16*b^7*f^3 - 6000*a^17*b^6*f^3 + 1440*a^18*b^5*f^3 + 1040
*a^19*b^4*f^3 - 704*a^20*b^3*f^3 + 128*a^21*b^2*f^3) + ((2*a + 5*b)*(320*a
^12*b^14*f^4 - 3392*a^13*b^13*f^4 + 16192*a^14*b^12*f^4 - 45760*a^15*b^11*
f^4 + 84608*a^16*b^10*f^4 - 106624*a^17*b^9*f^4 + 92288*a^18*b^8*f^4 - 536
32*a^19*b^7*f^4 + 19520*a^20*b^6*f^4 - 3648*a^21*b^5*f^4 + 64*a^22*b^4*...
```

Reduce [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^3}{(\tan(fx+e)^2\,b+a)^{\frac{5}{2}}}dx$$

input
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x)
```

3.351 $\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 272

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{(8a^2+20ab+35b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{b(12a^2+15ab-35b^2)}{24a^3(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} + \frac{(4a+7b)\cot^2(e+fx)}{8a^2f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{\cot^4(e+fx)}{4af\,(a+b\tan^2(e+fx))^{3/2}} + \frac{b(4a^3+3a^2b-50ab^2+35b^3)}{8a^4(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-1/8*(8*a^2+20*a*b+35*b^2)*arctanh((a+b*tan(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/
2)/f+arctanh((a+b*tan(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/24*b*(1
2*a^2+15*a*b-35*b^2)/a^3/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)+1/8*(4*a+7*b)*co
t(f*x+e)^2/a^2/f/(a+b*tan(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/a/f/(a+b*tan(f*
x+e)^2)^(3/2)+1/8*b*(4*a^3+3*a^2*b-50*a*b^2+35*b^3)/a^4/(a-b)^2/f/(a+b*tan
(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.61

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\cot^2(e+fx)\left(8a^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b\right.}{24a}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Cot[e + f*x]^2*(8*a^3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[e + f*x
]^2)/(a - b)] + (a - b)*(3*a*Cot[e + f*x]^2*(-4*a - 7*b + 2*a*Cot[e + f*x]
^2) - (8*a^2 + 20*a*b + 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Ta
n[e + f*x]^2)/a])))/(24*a^3*(-a + b)*f*(b + a*Cot[e + f*x]^2)*Sqrt[a + b*T
an[e + f*x]^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4153, 354, 114, 27, 168, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\tan(e+fx)^2)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^5(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 354$$

$$\frac{\int \frac{\cot^3(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 114$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(7b\tan^2(e+fx)+4a+7b\right)}{2\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{2a}-\frac{\cot^2(e+fx)}{2a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(7b\tan^2(e+fx)+4a+7b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{4a}-\frac{\cot^2(e+fx)}{2a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 168$$

$$\frac{-\frac{-\frac{\int \frac{\cot(e+fx)\left(8a^2+20ba+35b^2+5b(4a+7b)\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{a}-\frac{(4a+7b)\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{4a}-\frac{\cot^2(e+fx)}{2a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{\int \frac{\cot(e+fx)\left(8a^2+20ba+35b^2+5b(4a+7b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{5/2}}d\tan^2(e+fx)}{2a}-\frac{(4a+7b)\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{4a}-\frac{\cot^2(e+fx)}{2a(a+b\tan^2(e+fx))^{3/2}}}{2f}$$

$$\Big\downarrow \; 169$$

$$\frac{-\frac{-\frac{\frac{2b\left(12a^2+15ab-35b^2\right)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}-\frac{2\int -\frac{3\cot(e+fx)\left(b\left(12a^2+15ba-35b^2\right)\tan^2(e+fx)+(a-b)\left(8a^2+20ba+35b^2\right)\right)}{2\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{3a(a-b)}}{2a}-\frac{(4a+7b)\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{4a}}{2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{\frac{\int \frac{\cot(e+fx)\left(b\left(12a^2+15ba-35b^2\right)\tan^2(e+fx)+(a-b)\left(8a^2+20ba+35b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b\left(12a^2+15ab-35b^2\right)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}}{2a}-\frac{(4a+7b)\cot(e+fx)}{a\left(a+b\tan^2(e+fx)\right)^{3/2}}}{4a}-\frac{\cdots}{2a\cdots}}{2f}$$

$\Big\downarrow$ 169

$$-\frac{-\frac{-\frac{\frac{\frac{2b\left(4a^3+3a^2b-50ab^2+35b^3\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}-\frac{2\int -\frac{\cot(e+fx)\left(\left(8a^2+20ba+35b^2\right)(a-b)^2+b\left(4a^3+3ba^2-50b^2a+35b^3\right)\tan^2(e+fx)\right)}{2\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}}{a(a-b)}+\frac{2b\left(12a^2+15ab-\cdots\right)}{3a(a-b)\left(a+b\tan^2(e\cdots\right)}}{2a}\cdots}{4a}\cdots}{2f}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{\frac{\frac{\int \frac{\cot(e+fx)\left(\left(8a^2+20ba+35b^2\right)(a-b)^2+b\left(4a^3+3ba^2-50b^2a+35b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b\left(4a^3+3a^2b-50ab^2+35b^3\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}+\frac{2b\left(12a^2+15ab-35b^{\cdots}\right)}{3a(a-b)\left(a+b\tan^2(e+f\cdots\right)}}{2a}\cdots}{4a}\cdots}{2f}$$

$\Big\downarrow$ 174

$$-\frac{-\frac{-\frac{\frac{\frac{(a-b)^2\left(8a^2+20ab+35b^2\right)\int \frac{\cot(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)-8a^4\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan^2(e+fx)}{a(a-b)}+\frac{2b\left(4a^3+3a^2b-50ab^2+35b^3\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}+\frac{\cdots}{3a\cdots}}{2a}\cdots}{4a}\cdots}{2f}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{-\frac{\frac{\frac{2(a-b)^2\left(8a^2+20ab+35b^2\right)\int \frac{1}{\frac{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}}{b}}d\sqrt{b\tan^2(e+fx)+a}}{a(a-b)}-\frac{16a^4\int \frac{1}{\frac{\tan^4(e+fx)}{b}-\frac{a}{b}+1}d\sqrt{b\tan^2(e+fx)+a}}{b}+\frac{2b\left(4a^3+3a^2b-50ab^2+35b^3\right)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}+\frac{2\cdots}{3a(a-\cdots}}{2a}\cdots}{4a}\cdots}{2f}$$

$\downarrow$ 221

$$-\frac{-\frac{-\frac{2b(12a^2+15ab-35b^2)}{3a(a-b)\left(a+b\tan^2(e+fx)\right)^{3/2}}+\frac{\frac{\frac{16a^4\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2(a-b)^2(8a^2+20ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a-b)}+\frac{2b\left(4a^3+3a^2b-5\cdots\right.}{a(a-b)\sqrt{a+b\,t\cdots}}}{a(a-b)}}{2a}}{4a}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/2*Cot[e + f*x]^2/(a*(a + b*Tan[e + f*x]^2)^(3/2)) - (-(((4*a + 7*b)*Co
t[e + f*x])/(a*(a + b*Tan[e + f*x]^2)^(3/2))) - ((2*b*(12*a^2 + 15*a*b - 3
5*b^2))/(3*a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + (((-2*(a - b)^2*(8*a^
2 + 20*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a]])/Sqrt[a]
+ (16*a^4*ArcTanh[Sqrt[a + b*Tan[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a - b])/(a
*(a - b)) + (2*b*(4*a^3 + 3*a^2*b - 50*a*b^2 + 35*b^3))/(a*(a - b)*Sqrt[a
+ b*Tan[e + f*x]^2]))/(a*(a - b)))/(2*a))/(4*a))/(2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 76696 vs. $2(242) = 484$.

Time = 38.76 (sec) , antiderivative size = 76697, normalized size of antiderivative = 281.97

method	result	size
default	Expression too large to display	76697

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(242) = 484$.

Time = 0.21 (sec) , antiderivative size = 2411, normalized size of antiderivative = 8.86

$$\int \frac{\cot^5(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(24*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x +
 e)^4)*sqrt(a - b)*log((b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sq
rt(a - b) + 2*a - b)/(tan(f*x + e)^2 + 1)) + 3*((8*a^5*b^2 - 4*a^4*b^3 - a
^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x + e)^8 + 2*(8*a^6*b - 4*a
^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^6 + (8
*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x +
 e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(a
) + 2*a)/tan(f*x + e)^2) - 2*(6*a^7 - 18*a^6*b + 18*a^5*b^2 - 6*a^4*b^3 -
3*(4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a*b^6)*tan(f*x + e)^
6 - 4*(6*a^6*b - 3*a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^4 - 35*a^2*b^5)*tan(f*x
 + e)^4 - 3*(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*tan(f*x
 + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b^3 + 3*a^6*b^4 - a
^5*b^5)*f*tan(f*x + e)^8 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*b^3 - a^6*b^4)*f*t
an(f*x + e)^6 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f*tan(f*x + e)^4),
-1/48*(48*(a^5*b^2*tan(f*x + e)^8 + 2*a^6*b*tan(f*x + e)^6 + a^7*tan(f*x +
 e)^4)*sqrt(-a + b)*arctan(sqrt(-a + b)/sqrt(b*tan(f*x + e)^2 + a)) - 3*((
8*a^5*b^2 - 4*a^4*b^3 - a^3*b^4 - 53*a^2*b^5 + 85*a*b^6 - 35*b^7)*tan(f*x
+ e)^8 + 2*(8*a^6*b - 4*a^5*b^2 - a^4*b^3 - 53*a^3*b^4 + 85*a^2*b^5 - 35*a
*b^6)*tan(f*x + e)^6 + (8*a^7 - 4*a^6*b - a^5*b^2 - 53*a^4*b^3 + 85*a^3*b^
4 - 35*a^2*b^5)*tan(f*x + e)^4)*sqrt(a)*log((b*tan(f*x + e)^2 - 2*sqrt(...
```

Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^5{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 4652, normalized size of antiderivative = 17.10

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^5/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
((b*(a + b*tan(e + f*x)^2)^2*(15*a^2*b - 250*a*b^2 + 12*a^3 + 175*b^3))/(2
4*(a^3*b - a^4)*(a - b)) - b^3/(3*a*(a - b)) + (b*(a + b*tan(e + f*x)^2)^3
*(3*a^2*b - 50*a*b^2 + 4*a^3 + 35*b^3))/(8*(a^3*b - a^4)*(a*b - a^2)) + (b
*(10*a*b^2 - 7*b^3)*(a + b*tan(e + f*x)^2))/(3*a*(a - b)*(a*b - a^2)))/(f*
(a + b*tan(e + f*x)^2)^(7/2) + a^2*f*(a + b*tan(e + f*x)^2)^(3/2) - 2*a*f*
(a + b*tan(e + f*x)^2)^(5/2)) - (atan((a^16*f^3*(a + b*tan(e + f*x)^2)^(1/
2)*128i - a^11*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4
*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*128i + b^11*f*(a + b
*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10
*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*1225i + a^8*b^8*f^3*(a + b*tan(e + f*x)^2)^
(1/2)*64i - a^9*b^7*f^3*(a + b*tan(e + f*x)^2)^(1/2)*576i + a^10*b^6*f^3*(
a + b*tan(e + f*x)^2)^(1/2)*2240i - a^11*b^5*f^3*(a + b*tan(e + f*x)^2)^(1
/2)*4928i + a^12*b^4*f^3*(a + b*tan(e + f*x)^2)^(1/2)*6720i - a^13*b^3*f^3
*(a + b*tan(e + f*x)^2)^(1/2)*5824i + a^14*b^2*f^3*(a + b*tan(e + f*x)^2)^
(1/2)*3136i - a^15*b*f^3*(a + b*tan(e + f*x)^2)^(1/2)*960i + a^2*b^9*f*(a
+ b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 -
 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)*16885i - a^3*b^8*f*(a + b*tan(e + f*x)^2
)^(1/2)*(a^5*f^2 - b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 +
10*a^3*b^2*f^2)*19875i + a^4*b^7*f*(a + b*tan(e + f*x)^2)^(1/2)*(a^5*f^2 -
 b^5*f^2 + 5*a*b^4*f^2 - 5*a^4*b*f^2 - 10*a^2*b^3*f^2 + 10*a^3*b^2*f^2)...
```

Reduce [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^5}{(\tan(fx+e)^2 b+a)^{\frac{5}{2}}}dx$$

input
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x)
```

3.352 $\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\tan^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\tan^3(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(a-2b)\tan(e+fx)}{(a-b)^2b^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+arc
tanh(b^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*a*tan(f*x+
e)^3/(a-b)/b/f/(a+b*tan(f*x+e)^2)^(3/2)-a*(a-2*b)*tan(f*x+e)/(a-b)^2/b^2/f
/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.08 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.73

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(a^2(a-b)(2ab+(3a-7b)(a+b+(a-b)\cos(2(e+f\right.}{}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-1/3*(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*(a^2*(a - b)
*(2*a*b + (3*a - 7*b)*(a + b + (a - b)*Cos[2*(e + f*x)]))*Sin[2*(e + f*x)]
 - (3*a^2*b*(((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*
((a^2 - 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e +
f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[-(b/(a - b)), ArcS
in[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1
])*Sin[e + f*x]^2*Sin[2*(e + f*x)])/Sqrt[2]))/(Sqrt[2]*a*(a - b)^3*b^2*f*(
a + b + (a - b)*Cos[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4153, 372, 27, 440, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^6}{(a+b\tan(e+fx)^2)^{5/2}}dx$$

$\Big\downarrow$ 4153

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 372

$$\frac{\frac{\int \frac{3\tan^2(e+fx)\left((a-b)\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((a-b)\tan^2(e+fx)+a\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 440

$$\frac{-\frac{\frac{\int -\frac{(a-b)^2\tan^2(e+fx)+a(a-2b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a(a-2b)\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{(a-b)^2\tan^2(e+fx)+a(a-2b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a(a-2b)\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 398

$$\frac{\frac{\frac{(a-b)^2\int \frac{1}{\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a(a-2b)\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/}}}{f}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{(a-b)^2\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a(a-2b)\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{(a-b)^2\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{b(a-b)}-\frac{a(a-2b)\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{(a-b)^2\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-b^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{b(a-b)}-\frac{a(a-2b)\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{(a-b)^2\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{b}}-\frac{b^2\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}}{b(a-b)}-\frac{a(a-2b)\tan(e+fx)}{b(a-b)\sqrt{a+b\tan^2(e+fx)}}}{b(a-b)}-\frac{a\tan^3(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(a*Tan[e + f*x]^3)/((a - b)*b*(a + b*Tan[e + f*x]^2)^(3/2)) + ((-((b
^2*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a -
 b]) + ((a - b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2
]])/Sqrt[b])/((a - b)*b) - (a*(a - 2*b)*Tan[e + f*x])/((a - b)*b*Sqrt[a +
b*Tan[e + f*x]^2]))/((a - b)*b))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(153) = 306$.

Time = 0.56 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.23

method	result
derivativedivides	$\frac{\tan(fx+e)}{3fa\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3f\,a^2\sqrt{a+b\tan(fx+e)^2}}-\frac{\tan(fx+e)^3}{3fb\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{\tan(fx+e)}{f\,b^2\sqrt{a+b\tan(fx+e)^2}}+$
default	$\frac{\tan(fx+e)}{3fa\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3f\,a^2\sqrt{a+b\tan(fx+e)^2}}-\frac{\tan(fx+e)^3}{3fb\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{\tan(fx+e)}{f\,b^2\sqrt{a+b\tan(fx+e)^2}}+$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/f/a^2*tan(f*x+e)/(a+b*tan(
f*x+e)^2)^(1/2)-1/3/f*tan(f*x+e)^3/b/(a+b*tan(f*x+e)^2)^(3/2)-1/f/b^2*tan(
f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)+1/f/b^(5/2)*ln(b^(1/2)*tan(f*x+e)+(a+b*tan
(f*x+e)^2)^(1/2))+1/3/f/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2)-1/3/f/a/b*ta
n(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)+1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+
e)^2)^(3/2)+2/3/f/(a-b)*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)-1/f/(a-b
)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e))+1/f/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(153) = 306$.

Time = 1.34 (sec) , antiderivative size = 1698, normalized size of antiderivative = 9.93

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*
b^4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(
f*x + e)^2)*sqrt(b)*log(2*b*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(b)*tan(f*x + e) + a) - 3*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2
 + a^2*b^3)*sqrt(-a + b)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x
 + e)^2 + a)*sqrt(-a + b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((4*
a^3*b^2 - 11*a^2*b^3 + 7*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*
a^2*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 +
 3*a*b^7 - b^8)*f*tan(f*x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*
b^7)*f*tan(f*x + e)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1
/6*(6*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^
4 - b^5)*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*tan(f*
x + e)^2)*sqrt(-b)*arctan(sqrt(-b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)
) + 3*(b^5*tan(f*x + e)^4 + 2*a*b^4*tan(f*x + e)^2 + a^2*b^3)*sqrt(-a + b)
*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b
)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((4*a^3*b^2 - 11*a^2*b^3 + 7
*a*b^4)*tan(f*x + e)^3 + 3*(a^4*b - 3*a^3*b^2 + 2*a^2*b^3)*tan(f*x + e))*s
qrt(b*tan(f*x + e)^2 + a))/((a^3*b^5 - 3*a^2*b^6 + 3*a*b^7 - b^8)*f*tan(f*
x + e)^4 + 2*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*f*tan(f*x + e)^2 +
(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*f), -1/6*(6*(b^5*tan(f*x + ...
```

Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^6{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^6}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^6}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(tan(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**6)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.353 $\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{a\tan(e+fx)}{3(a-b)bf\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-4b)\tan(e+fx)}{3(a-b)^2bf\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
a*tan(f*x+e)/(a-b)/b/f/(a+b*tan(f*x+e)^2)^(3/2)+1/3*(a-4*b)*tan(f*x+e)/(a-
b)^2/b/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 6.03 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.98

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\tan^5(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a}\right)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{-\tan^2(e+fx)+\frac{b\tan^2(e+fx)}{a}}}{\sqrt{1+\frac{b\tan^2(e+fx)}{a}}}\right)\sqrt{-\tan^2(e\cdots}}{\sqrt{1+\frac{b\tan^2(e+fx)}{a}}}\cdots\right.}{a^2f\sqrt{a+b\tan^2(e+fx)}\left(-\tan\cdots\right.}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]^5*(1 + (b*Tan[e + f*x]^2)/a)*((ArcTanh[Sqrt[-Tan[e + f*x]^2
+ (b*Tan[e + f*x]^2)/a]/Sqrt[1 + (b*Tan[e + f*x]^2)/a]]*Sqrt[-Tan[e + f*x]
^2 + (b*Tan[e + f*x]^2)/a])/Sqrt[1 + (b*Tan[e + f*x]^2)/a] - (-Tan[e + f*x
]^2 + (b*Tan[e + f*x]^2)/a)/(1 + (b*Tan[e + f*x]^2)/a) - (-Tan[e + f*x]^2
+ (b*Tan[e + f*x]^2)/a)^2/(3*(1 + (b*Tan[e + f*x]^2)/a)^2)))/(a^2*f*Sqrt[a
 + b*Tan[e + f*x]^2]*(-Tan[e + f*x]^2 + (b*Tan[e + f*x]^2)/a)^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4153, 372, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{\int \frac{(a-3b)\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3b(a-b)}-\frac{a\tan(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3ab}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}+\frac{(a-4b)\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3b(a-b)}-\frac{a\tan(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}+\frac{(a-4b)\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3b(a-b)}-\frac{a\tan(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{\frac{3b\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}+\frac{(a-4b)\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3b(a-b)}-\frac{a\tan(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{\frac{3b\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}+\frac{(a-4b)\tan(e+fx)}{(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3b(a-b)}-\frac{a\tan(e+fx)}{3b(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(a*Tan[e + f*x])/((a - b)*b*(a + b*Tan[e + f*x]^2)^(3/2)) + ((3*b*Ar
cTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(a - b)^(3/2)
 + ((a - 4*b)*Tan[e + f*x])/((a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(3*(a -
b)*b))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(117) = 234$.

Time = 0.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.22

method	result
derivativedivides	$-\frac{\tan(fx+e)}{3fb\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{\tan(fx+e)}{3fab\sqrt{a+b\tan(fx+e)^2}}-\frac{b\tan(fx+e)}{3a(a-b)f\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{2b\tan(fx+e)}{3f(a-b)a^2\sqrt{a+b\tan}}$
default	$-\frac{\tan(fx+e)}{3fb\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{\tan(fx+e)}{3fab\sqrt{a+b\tan(fx+e)^2}}-\frac{b\tan(fx+e)}{3a(a-b)f\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}-\frac{2b\tan(fx+e)}{3f(a-b)a^2\sqrt{a+b\tan}}$

input
```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/b*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2)+1/3/f/a/b*tan(f*x+e)/(a+b*tan
(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f
/(a-b)*b/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)+1/f/(a-b)^3*(b^4*(a-b))^(
1/2)/b^2*arctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f
*x+e))-1/f/(a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2)-1/3/f*tan(f*x+e
)/a/(a+b*tan(f*x+e)^2)^(3/2)-2/3/f/a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.78

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \left[-\frac{3\left(b^2\tan\left(fx+e\right)^4+2\,ab\tan\left(fx+e\right)^2+a^2\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)\tan}{}\right.}{6\left(\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)f\tan\right.}\right.$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(-a + b)*lo
g(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b)*t
an(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((a^2 - 5*a*b + 4*b^2)*tan(f*x
+ e)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/((a^3*b^2
 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 + 3*
a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*
f), 1/3*(3*(b^2*tan(f*x + e)^4 + 2*a*b*tan(f*x + e)^2 + a^2)*sqrt(a - b)*a
rctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 + a)) + ((a^2 - 5*a*b
 + 4*b^2)*tan(f*x + e)^3 - 3*(a^2 - a*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)
^2 + a))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*tan(f*x + e)^4 + 2*(a^4*
b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*tan(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a
^3*b^2 - a^2*b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^4{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e)^3 - 3\left(\int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\tan(fx+e}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a\,b^2+3\tan(}\right.}{}$$

input

```
int(tan(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output

```
(sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**3 - 3*int((sqrt(tan(e + f*x)**2
*b + a)*tan(e + f*x)**2)/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x)**4*a*b**2
+ 3*tan(e + f*x)**2*a**2*b + a**3),x)*tan(e + f*x)**4*a*b**2*f - 6*int((sq
rt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2)/(tan(e + f*x)**6*b**3 + 3*tan(e
 + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)*tan(e + f*x)**2*a*
*2*b*f - 3*int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2)/(tan(e + f*x)
**6*b**3 + 3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)*
a**3*f)/(3*a*f*(tan(e + f*x)**4*b**2 + 2*tan(e + f*x)**2*a*b + a**2))
```

3.354 $\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} + \frac{\tan(e+fx)}{3(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+b)\tan(e+fx)}{3a(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f+1/3
*tan(f*x+e)/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)+1/3*(2*a+b)*tan(f*x+e)/a/(a-b
)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.07 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.38

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \frac{\sin^2(e+fx)\tan(e+fx)\left(\frac{4(a-b)\,\text{Hypergeometric2F1}\left(2,2,\frac{9}{2},\frac{(a-b)\sin^2(e+fx)}{a}\right)\sin^2(e+f}{35a^2}\right.}{}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Sin[e + f*x]^2*Tan[e + f*x]*((4*(a - b)*Hypergeometric2F1[2, 2, 9/2, ((a
- b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/(35*a^2) +
(Cot[e + f*x]^4*(5*a + 2*b*Tan[e + f*x]^2)*(3*ArcSin[Sqrt[((a - b)*Sin[e +
 f*x]^2)/a]]*(a + b*Tan[e + f*x]^2)^2 + a*Sec[e + f*x]^2*Sqrt[((a - b)*Cos
[e + f*x]^2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2]*(-4*b*Tan[e + f*x]
^2 + a*(-3 + Tan[e + f*x]^2))))/(3*a*(a - b)^2*Sqrt[((a - b)*Cos[e + f*x]^
2*Sin[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a^2])))/(3*a^2*f*Sqrt[a + b*Tan[e
 + f*x]^2]*(1 + (b*Tan[e + f*x]^2)/a))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4153, 373, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 4153$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 373$$

$$\frac{\frac{\tan(e+fx)}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}-\frac{\int \frac{1-2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3(a-b)}}{f}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tan(e+fx)}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{\int \frac{3a}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{(2a+b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tan(e+fx)}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}-\frac{(2a+b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}}{f}$$

$\Big\downarrow$ 291

$$\frac{\frac{\tan(e+fx)}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{3\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{(2a+b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}}{f}$$

$\Big\downarrow$ 216

$$\frac{\frac{\tan(e+fx)}{3(a-b)(a+b\tan^2(e+fx))^{3/2}}-\frac{\frac{3\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{(2a+b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3(a-b)}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(3*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) - ((3*ArcTan[(Sqrt[
a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(a - b)^(3/2) - ((2*a +
b)*Tan[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(3*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.66

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}+\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a-b}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{}{\sqrt{b^4}}\right.}{(a-}}{f}$
default	$\frac{\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}+\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a-b}-\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{}{\sqrt{b^4}}\right.}{(a-}}{f}$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x+e)/(a+b*tan
(f*x+e)^2)^(1/2)+1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/
a^2*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))-1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*a
rctan(b^2*(a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))+1/(
a-b)^2*b*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(114) = 228.

Time = 0.12 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.11

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \Bigg[-\frac{3\left(ab^2\tan(fx+e)^4+2a^2b\tan(fx+e)^2+a^3\right)\sqrt{-a+b}\log\left(-\frac{(a-2b)\tan\cdots}{\cdots}\right)\cdots}{6\left((a^4b^2-3a^3b^3+3a^2b^4-ab^5)f\tan\cdots\right.}$$

$$-\frac{3\left(ab^2\tan(fx+e)^4+2a^2b\tan(fx+e)^2+a^3\right)\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a}}\right)-\left((2a^2b-ab^2-b^3)\tan\cdots\right.}{3\left((a^4b^2-3a^3b^3+3a^2b^4-ab^5)f\tan(fx+e)^4+2(a^5b-3a^4b^2+3a^3b^3-a^2b^4)f\tan\cdots\right.}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 + a^3)*sqrt(-a + b
)*log(-((a - 2*b)*tan(f*x + e)^2 + 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a +
b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) - 2*((2*a^2*b - a*b^2 - b^3)*ta
n(f*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2 + a))/(
(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*f*tan(f*x + e)^4 + 2*(a^5*b - 3*
a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x + e)^2 + (a^6 - 3*a^5*b + 3*a^4*b
^2 - a^3*b^3)*f), -1/3*(3*(a*b^2*tan(f*x + e)^4 + 2*a^2*b*tan(f*x + e)^2 +
 a^3)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x + e)/sqrt(b*tan(f*x + e)^2 +
a)) - ((2*a^2*b - a*b^2 - b^3)*tan(f*x + e)^3 + 3*(a^3 - a^2*b)*tan(f*x +
e))*sqrt(b*tan(f*x + e)^2 + a))/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)
*f*tan(f*x + e)^4 + 2*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*f*tan(f*x
+ e)^2 + (a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*f)]
```

Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}\,\tan\left(fx+e\right)^2}{\tan\left(fx+e\right)^6 b^3+3\tan\left(fx+e\right)^4 a\,b^2+3\tan\left(fx+e\right)^2 a^2b+a^3}dx$$

input
```
int(tan(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int((sqrt(tan(e + f*x)**2*b + a)*tan(e + f*x)**2)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.355 $\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\tan(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\tan(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
b*tan(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*(5*a-2*b)*b*tan(f*x+e)
/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.01 (sec) , antiderivative size = 1331, normalized size of antiderivative = 9.93

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

output
```
(Cos[e + f*x]*Sin[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]
- (3150*(a - b)*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2)/a
 + (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4
)/a^2 + (2100*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^2)/a
 - (4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2
*Tan[e + f*x]^2)/a^2 + (2100*(a - b)^2*b*ArcSin[Sqrt[((a - b)*Sin[e + f*x]
^2)/a]]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (840*b^2*ArcSin[Sqrt[((a - b)
*Sin[e + f*x]^2)/a]]*Tan[e + f*x]^4)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[(
(a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/a^3 + (840*(a -
 b)^2*b^2*ArcSin[Sqrt[((a - b)*Sin[e + f*x]^2)/a]]*Sin[e + f*x]^4*Tan[e +
f*x]^4)/a^4 + 2100*(((a - b)*Sin[e + f*x]^2)/a)^(3/2)*Sqrt[(Cos[e + f*x]^2
*(a + b*Tan[e + f*x]^2))/a] + 96*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin
[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*x]^2*(a
 + b*Tan[e + f*x]^2))/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a -
 b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin[e + f*x]^2)/a)^(7/2)*Sqrt[(Cos[e + f*
x]^2*(a + b*Tan[e + f*x]^2))/a] + (2800*b*(((a - b)*Sin[e + f*x]^2)/a)^(3/
2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f*x]^2))/a])/a + (16
8*b*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*(((a - b)*Sin
[e + f*x]^2)/a)^(7/2)*Tan[e + f*x]^2*Sqrt[(Cos[e + f*x]^2*(a + b*Tan[e + f
*x]^2))/a])/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*S...
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4144, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$
$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-2b\tan^2(e+fx)+3a-2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3a^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3a\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a-b}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3a\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}}{a-b}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{3a\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{3/2}}-\frac{b(5a-2b)\tan(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\tan(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^(-5/2),x]
```

output
```
(-1/3*(b*Tan[e + f*x])/(a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + ((3*a*Ar
cTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/(a - b)^(3/2)
 - ((5*a - 2*b)*b*Tan[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2]))/(3
*a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a-b}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^3b^2}-\frac{b\tan(fx+e}{(a-b)^2a\sqrt{a+b\tan(}}}{f}$
default	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{3a\left(a+b\tan(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\tan(fx+e)}{3a^2\sqrt{a+b\tan(fx+e)^2}}\right)}{a-b}+\frac{\sqrt{b^4(a-b)}\,\arctan\left(\frac{b^2(a-b)\tan(fx+e)}{\sqrt{b^4(a-b)}\,\sqrt{a+b\tan(fx+e)^2}}\right)}{(a-b)^3b^2}-\frac{b\tan(fx+e}{(a-b)^2a\sqrt{a+b\tan(}}}{f}$

input
```
int(1/(a+b*tan(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a-b)*b*(1/3*tan(f*x+e)/a/(a+b*tan(f*x+e)^2)^(3/2)+2/3/a^2*tan(f*x
+e)/(a+b*tan(f*x+e)^2)^(1/2))+1/(a-b)^3*(b^4*(a-b))^(1/2)/b^2*arctan(b^2*(
a-b)/(b^4*(a-b))^(1/2)/(a+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e))-1/(a-b)^2*b*ta
n(f*x+e)/a/(a+b*tan(f*x+e)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(120) = 240.

Time = 0.13 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.16

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \left[-\frac{3\left(a^2b^2\tan\left(fx+e\right)^4+2\,a^3b\tan\left(fx+e\right)^2+a^4\right)\sqrt{-a+b}\log\left(-\frac{(a-2\,b)}{}\right.}{6\left(\left(a^5b^2-3\,a^4b^3+3\,a^3b^4-\right.\right.}\right.$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(a^2*b^2*tan(f*x + e)^4 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(-a +
 b)*log(-((a - 2*b)*tan(f*x + e)^2 - 2*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a
+ b)*tan(f*x + e) - a)/(tan(f*x + e)^2 + 1)) + 2*((5*a^2*b^2 - 7*a*b^3 + 2
*b^4)*tan(f*x + e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(
b*tan(f*x + e)^2 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(
f*x + e)^4 + 2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2
+ (a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*f), 1/3*(3*(a^2*b^2*tan(f*x + e)^4
 + 2*a^3*b*tan(f*x + e)^2 + a^4)*sqrt(a - b)*arctan(sqrt(a - b)*tan(f*x +
e)/sqrt(b*tan(f*x + e)^2 + a)) - ((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*tan(f*x +
e)^3 + 3*(2*a^3*b - 3*a^2*b^2 + a*b^3)*tan(f*x + e))*sqrt(b*tan(f*x + e)^2
 + a))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*tan(f*x + e)^4 + 2*(
a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*f*tan(f*x + e)^2 + (a^7 - 3*a^6*b
 + 3*a^5*b^2 - a^4*b^3)*f)]
```

Sympy [F]

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**(-5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\tan\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\tan\left(fx+e\right)^2 b+a}}{\tan\left(fx+e\right)^6 b^3+3\tan\left(fx+e\right)^4 a\,b^2+3\tan\left(fx+e\right)^2 a^2b+a^3}dx$$

input

```
int(1/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output

```
int(sqrt(tan(e + f*x)**2*b + a)/(tan(e + f*x)**6*b**3 + 3*tan(e + f*x)**4*
a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.356 $\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(7a-4b)b\cot(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3(a-b)^2f}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3
*b*cot(f*x+e)/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*(7*a-4*b)*b*cot(f*x+e
)/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/3*(a-4*b)*(3*a-2*b)*cot(f*x+e)*
(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 12.03 (sec) , antiderivative size = 1890, normalized size of antiderivative = 10.16

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
-((Cos[e + f*x]^2*Cot[e + f*x]*((20*a*Csc[e + f*x]^2)/(3*(a - b)) - (5*a^2
*Csc[e + f*x]^4)/(a - b)^2 + (40*b*Sec[e + f*x]^2)/(a - b) - (30*a*b*Csc[e
 + f*x]^2*Sec[e + f*x]^2)/(a - b)^2 - (40*b^2*Sec[e + f*x]^4)/(a - b)^2 +
(92*(a - b)*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e
 + f*x]^2)/(105*a) + (24*(a - b)*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((
a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2)/(35*a) + (16*(a - b)*Hypergeomet
ricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]
^2)/(105*a) + (160*b^2*Sec[e + f*x]^2*Tan[e + f*x]^2)/(3*a*(a - b)) + (124
*(a - b)*b*Hypergeometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e
+ f*x]^2*Tan[e + f*x]^2)/(35*a^2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2,
 2}, {1, 9/2}, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/
(7*a^2) + (16*(a - b)*b*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a -
 b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(35*a^2) + (64*b^3*S
ec[e + f*x]^2*Tan[e + f*x]^4)/(3*a^2*(a - b)) + (152*(a - b)*b^2*Hypergeom
etric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x
]^4)/(35*a^3) + (88*(a - b)*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a
 - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) + (16*(a
- b)*b^2*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, ((a - b)*Sin[e + f*x
]^2)/a]*Sin[e + f*x]^2*Tan[e + f*x]^4)/(35*a^3) + (176*(a - b)*b^3*Hyperge
ometric2F1[2, 2, 9/2, ((a - b)*Sin[e + f*x]^2)/a]*Sin[e + f*x]^2*Tan[e ...
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4153, 374, 441, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 374}$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-4b\tan^2(e+fx)+3a-4b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a(a-b)}-\frac{b\cot(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{ 441}$$

$$\frac{\frac{\frac{\int \frac{\cot^2(e+fx)\left((a-4b)(3a-2b)-2(7a-4b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(7a-4b)\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\cot(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\frac{\frac{-\frac{\int \frac{3a^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b(7a-4b)\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\cot(e+fx)}{3a(a-b)(a+b\tan^2(e+fx}}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{-3a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b(7a-4b)\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\cot(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{-3a^2\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b(7a-4b)\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\cot(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{-\frac{3a^2\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{(a-4b)(3a-2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{a(a-b)}-\frac{b(7a-4b)\cot(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\cot(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

input

```
Int[Cot[e + f*x]^2/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output

```
(-1/3*(b*Cot[e + f*x])/(a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + (-(((7*a
 - 4*b)*b*Cot[e + f*x])/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2])) + ((-3*a^2
*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqrt[a - b
] - ((a - 4*b)*(3*a - 2*b)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(a*
(a - b)))/(3*a*(a - b)))/f
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\cot (fx+e)^2}{\left(a+b\tan (fx+e)^2\right)^{\frac{5}{2}}}dx$$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(168) = 336$.

Time = 0.20 (sec) , antiderivative size = 753, normalized size of antiderivative = 4.05

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e)^3 + a^5*tan(f*x +
 e))*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2 -
4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x + e)
)*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x + e
)^2 + 1)) + 4*(3*a^5 - 9*a^4*b + 9*a^3*b^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a
^2*b^3 + 22*a*b^4 - 8*b^5)*tan(f*x + e)^4 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^
2*b^3 - 4*a*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3
*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*f*tan(f*x + e)^5 + 2*(a^7*b - 3*a^6*b^2 +
3*a^5*b^3 - a^4*b^4)*f*tan(f*x + e)^3 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b
^3)*f*tan(f*x + e)), -1/6*(3*(a^3*b^2*tan(f*x + e)^5 + 2*a^4*b*tan(f*x + e
)^3 + a^5*tan(f*x + e))*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*s
qrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*(3*a^5 - 9*a^4
*b + 9*a^3*b^2 - 3*a^2*b^3 + (3*a^3*b^2 - 17*a^2*b^3 + 22*a*b^4 - 8*b^5)*t
an(f*x + e)^4 + 3*(2*a^4*b - 9*a^3*b^2 + 11*a^2*b^3 - 4*a*b^4)*tan(f*x + e
)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b
^5)*f*tan(f*x + e)^5 + 2*(a^7*b - 3*a^6*b^2 + 3*a^5*b^3 - a^4*b^4)*f*tan(f
*x + e)^3 + (a^8 - 3*a^7*b + 3*a^6*b^2 - a^5*b^3)*f*tan(f*x + e))]
```

Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(b\tan(e+f\,x)^2+a\right)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b*tan(e + f*x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cot^2(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx = \int \frac{\sqrt{\tan(fx+e)^2 b+a}\,\cot(fx+e)^2}{\tan(fx+e)^6 b^3+3\tan(fx+e)^4 a\,b^2+3\tan(fx+e)^2 a^2 b+a^3}dx$$

input
```
int(cot(f*x+e)^2/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output

```
int((sqrt(tan(e + f*x)**2*b + a)*cot(e + f*x)**2)/(tan(e + f*x)**6*b**3 +
3*tan(e + f*x)**4*a*b**2 + 3*tan(e + f*x)**2*a**2*b + a**3),x)
```

3.357 $\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}\,dx &= \frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} \\ &- \frac{b\cot^3(e+fx)}{3a(a-b)f\left(a+b\tan^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)b\cot^3(e+fx)}{a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} \\ &+ \frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^4(a-b)^2f} \\ &- \frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a^3(a-b)^2f}\end{aligned}$$

output
```
arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3*
b*cot(f*x+e)^3/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-(3*a-2*b)*b*cot(f*x+e)^3
/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)+1/3*(a-2*b)*(3*a^2+8*a*b-8*b^2)*co
t(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^4/(a-b)^2/f-1/3*(a^2-12*a*b+8*b^2)*cot
(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 17.38 (sec) , antiderivative size = 871, normalized size of antiderivative = 3.50

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{-\frac{b\sqrt{\frac{a+b+(a-b)\cos(2(e+fx))}{1+\cos(2(e+fx))}}\sqrt{-\frac{a\cot^2(e+fx)}{b}}\sqrt{-\frac{a(1+\cos(2(e+fx)))\csc^2(e+fx)}{b}}\sqrt{\frac{(a+b+(a-b)\cos(2(e+f}{b}}}{a(a+b+(a-}}{} + \frac{\sqrt{\frac{a+b+a\cos(2(e+fx))-b\cos(2(e+fx))}{1+\cos(2(e+fx))}}\left(\frac{4(a\cos(e+fx)+2b\cos(e+fx))\csc(e+fx)}{3a^4} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^3} + \frac{2b^4\sin(2}{3a^3(a-b)^2(a+b+a\cos(2(e}\right.}{f}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-((b*Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*Sqrt
[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)
/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e
+ f*x)]*EllipticF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(a*(a + b + (a - b)*Cos[2*(e + f*x
)]))) - (4*b*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[(a + b + (a - b)*Cos[2*(e + f
*x)])/(1 + Cos[2*(e + f*x)])]*((Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1
 + Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e +
 f*x)])*Csc[e + f*x]^2)/b]*Csc[2*(e + f*x)]*EllipticF[ArcSin[Sqrt[((a + b
+ (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4
)/(4*a*Sqrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])
- (Sqrt[-((a*Cot[e + f*x]^2)/b)]*Sqrt[-((a*(1 + Cos[2*(e + f*x)])*Csc[e +
f*x]^2)/b)]*Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]*Cs
c[2*(e + f*x)]*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos[
2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1]*Sin[e + f*x]^4)/(2*(a - b)*S
qrt[1 + Cos[2*(e + f*x)]]*Sqrt[a + b + (a - b)*Cos[2*(e + f*x)]])))/Sqrt[a
 + b + (a - b)*Cos[2*(e + f*x)]])/((a - b)^2*f) + (Sqrt[(a + b + a*Cos[2*(
e + f*x)] - b*Cos[2*(e + f*x)])/(1 + Cos[2*(e + f*x)])]*((4*(a*Cos[e + f*x
] + 2*b*Cos[e + f*x])*Csc[e + f*x])/(3*a^4) - (Cot[e + f*x]*Csc[e + f*x]^2
)/(3*a^3) + (2*b^4*Sin[2*(e + f*x)])/(3*a^3*(a - b)^2*(a + b + a*Cos[2*...
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4153, 374, 27, 441, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{\int \frac{3\cot^4(e+fx)\left(-2b\tan^2(e+fx)+a-2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a(a-b)}-\frac{b\cot^3(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-2b\tan^2(e+fx)+a-2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{a(a-b)}-\frac{b\cot^3(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{\frac{\int \frac{\cot^4(e+fx)\left(a^2-12ba+8b^2-4(3a-2b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(3a-2b)\cot^3(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{a(a-b)}-\frac{b\cot^3(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2b\left(a^2-12ba+8b^2\right)\tan^2(e+fx)+(a-2b)\left(3a^2+8ba-8b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}-\frac{b(3a-2b)\cot^3(e+\cdots}{a(a-b)\sqrt{a+b\tan^2(e\cdots}}}{a(a-b)}}{f}$$

$\downarrow$ 445

$$\frac{-\frac{-\frac{-\frac{\int \frac{3a^4}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}\cdots}{a(a-b)}}{f}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{-3a^3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+f\cdots}}{3a}}{a(a-b)}\cdots}{a(a-b)}}{f}$$

$\downarrow$ 291

$$\frac{-\frac{\frac{-3a^3\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}}{a(a-b)}-\cdots}{a(a-b)}}{f}$$

$\downarrow$ 216

$$\frac{-\frac{-\frac{\left(a^2-12ab+8b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}-\frac{-\frac{3a^3\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{(a-2b)\left(3a^2+8ab-8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}}{a(a-b)}-\frac{b(3a-2b)\cot\cdots}{a(a-b)\sqrt{a+b\,\mathrm{ta}\cdots}}}{a(a-b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x]^3)/(a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + (-(((3
*a - 2*b)*b*Cot[e + f*x]^3)/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2])) + (-1/
3*((a^2 - 12*a*b + 8*b^2)*Cot[e + f*x]^3*Sqrt[a + b*Tan[e + f*x]^2])/a - (
(-3*a^3*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]^2]])/Sqr
t[a - b] - ((a - 2*b)*(3*a^2 + 8*a*b - 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Tan[
e + f*x]^2])/a)/(3*a))/(a*(a - b)))/(a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\cot(fx+e)^4}{\left(a+b\tan(fx+e)^2\right)^{\frac{5}{2}}}dx$$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.53

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a^5*b*tan(f*x + e)^5 + a^6*tan(f*x +
 e)^3)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2
- 4*a*b)*tan(f*x + e)^2 + a^2 - 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x +
e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x +
 e)^2 + 1)) - 4*((3*a^4*b^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*ta
n(f*x + e)^6 - a^6 + 3*a^5*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2
- 13*a^3*b^3 + 20*a^2*b^4 - 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a
^4*b^2 + 5*a^3*b^3 - 2*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a)
)/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b
 - 3*a^7*b^2 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*
a^7*b^2 - a^6*b^3)*f*tan(f*x + e)^3), 1/6*(3*(a^4*b^2*tan(f*x + e)^7 + 2*a
^5*b*tan(f*x + e)^5 + a^6*tan(f*x + e)^3)*sqrt(a - b)*arctan(-2*sqrt(b*tan
(f*x + e)^2 + a)*sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a))
+ 2*((3*a^4*b^2 - a^3*b^3 - 26*a^2*b^4 + 40*a*b^5 - 16*b^6)*tan(f*x + e)^6
 - a^6 + 3*a^5*b - 3*a^4*b^2 + a^3*b^3 + 3*(2*a^5*b - a^4*b^2 - 13*a^3*b^3
 + 20*a^2*b^4 - 8*a*b^5)*tan(f*x + e)^4 + 3*(a^6 - a^5*b - 3*a^4*b^2 + 5*a
^3*b^3 - 2*a^2*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^7*b^2
- 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*f*tan(f*x + e)^7 + 2*(a^8*b - 3*a^7*b^2
 + 3*a^6*b^3 - a^5*b^4)*f*tan(f*x + e)^5 + (a^9 - 3*a^8*b + 3*a^7*b^2 - a^
6*b^3)*f*tan(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^4{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^4/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx=\int \frac{\cot\left(fx+e\right)^4}{\left(\tan\left(fx+e\right)^2b+a\right)^{\frac{5}{2}}}dx$$

input
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(cot(f*x+e)^4/(a+b*tan(f*x+e)^2)^(5/2),x)
```

3.358 $\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 327

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a-b)f\,(a+b\tan^2(e+fx))^{3/2}} - \frac{(11a-8b)b\cot^5(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\tan^2(e+fx)}} - \frac{(15a^4+10a^3b+8a^2b^2-176ab^3+128b^4)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^5(a-b)^2f} + \frac{(5a^3+4a^2b-88ab^2+64b^3)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{15a^4(a-b)^2f} - \frac{(a^2-22ab+16b^2)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a^3(a-b)^2f}$$

output
```
-arctan((a-b)^(1/2)*tan(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3
*b*cot(f*x+e)^5/a/(a-b)/f/(a+b*tan(f*x+e)^2)^(3/2)-1/3*(11*a-8*b)*b*cot(f*
x+e)^5/a^2/(a-b)^2/f/(a+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^4+10*a^3*b+8*a^2*
b^2-176*a*b^3+128*b^4)*cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2)/a^5/(a-b)^2/f+1
/15*(5*a^3+4*a^2*b-88*a*b^2+64*b^3)*cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2)/
a^4/(a-b)^2/f-1/5*(a^2-22*a*b+16*b^2)*cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2
)/a^3/(a-b)^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 15.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.35

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \frac{\sqrt{(a+b+(a-b)\cos(2(e+fx)))\sec^2(e+fx)}\left(-\frac{15a^5b\left(\frac{(a+b+(a-b)\cos(2(e+fx))}{b}\right.}{}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[(a + b + (a - b)*Cos[2*(e + f*x)])*Sec[e + f*x]^2]*((-15*a^5*b*(((a
+ b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b)^(3/2)*(2*(a - b)*Ellipt
icF[ArcSin[Sqrt[((a + b + (a - b)*Cos[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqr
t[2]], 1] - 2*a*EllipticPi[-(b/(a - b)), ArcSin[Sqrt[((a + b + (a - b)*Cos
[2*(e + f*x)])*Csc[e + f*x]^2)/b]/Sqrt[2]], 1])*Sin[e + f*x]^2*Sin[2*(e +
f*x)])/(2*Sqrt[2]) - (a - b)*((a - b)^2*(23*a^2 + 54*a*b + 73*b^2)*(a + b
+ (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x] - a*(a - b)^2*(11*a + 14*b)*(a
+ b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^2 + 3*a^2*(a -
 b)^2*(a + b + (a - b)*Cos[2*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^4 + 1
0*a*b^5*Sin[2*(e + f*x)] - 5*(15*a - 11*b)*b^4*(a + b + (a - b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])))/(15*Sqrt[2]*a^5*(a - b)^3*f*(a + b + (a - b)*C
os[2*(e + f*x)])^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4153, 374, 441, 27, 445, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\tan^2(e+fx)\right)^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\tan(e+fx)^6\left(a+b\tan(e+fx)^2\right)^{5/2}}dx$$

$\Big\downarrow$ 4153

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a)^{5/2}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 374

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-8b\tan^2(e+fx)+3a-8b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a\right)^{3/2}}d\tan(e+fx)}{3a(a-b)}-\frac{b\cot^5(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 441

$$\frac{\frac{\frac{\int \frac{3\cot^6(e+fx)\left(a^2-22ba+16b^2-2(11a-8b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(11a-8b)\cot^5(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\cot^5(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{\cot^6(e+fx)\left(a^2-22ba+16b^2-2(11a-8b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a(a-b)}-\frac{b(11a-8b)\cot^5(e+fx)}{a(a-b)\sqrt{a+b\tan^2(e+fx)}}}{3a(a-b)}-\frac{b\cot^5(e+fx)}{3a(a-b)(a+b\tan^2(e+fx))^{3/2}}}{f}$$

$\Big\downarrow$ 445

$$\frac{\frac{3\left(-\frac{\int \frac{\cot^4(e+fx)\left(5a^3+4ba^2-88b^2a+64b^3+4b\left(a^2-22ba+16b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{5a}-\frac{\left(a^2-22ab+16b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}\right)}{a(a-b)}-\frac{b(11a-8b)\,c}{a(a-b)\sqrt{a+b}}}{3a(a-b)}}{f}$$

$\Big\downarrow$ 445

$$\frac{3\left(-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^4+10ba^3+8b^2a^2-176b^3a+128b^4+2b\left(5a^3+4ba^2-88b^2a+64b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{3a}-\frac{\left(5a^3+4a^2b-88ab^2+64b^3\right)\cot^3(e+fx)\sqrt{a-}}{3a}}{5a}\right.}{a(a-b)}}{3a(a-b)} \over f$$

$$\Big\downarrow \text{445}$$

$$\frac{3\left(-\frac{-\frac{\frac{\int \frac{15a^5}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)}{a}-\frac{\left(15a^4+10a^3b+8a^2b^2-176ab^3+128b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(5a^3+4a^2b-88ab^2+64b^3\right)}{}}{5a}\right.}{a(a-b)}}{3a(a-b)} \over f$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(-\frac{-\frac{-15a^4\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a}}d\tan(e+fx)-\frac{\left(15a^4+10a^3b+8a^2b^2-176ab^3+128b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(5a^3+4a^2b-88ab^2+64\right.}{}}{5a}\right.}{a(a-b)}}{3a(a-b)} \over f$$

$$\Big\downarrow \text{291}$$

$$\frac{3\left(-\frac{-\frac{-15a^4\int \frac{1}{1-\frac{(b-a)\tan^2(e+fx)}{b\tan^2(e+fx)+a}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a}}-\frac{\left(15a^4+10a^3b+8a^2b^2-176ab^3+128b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{a}}{3a}-\frac{\left(5a^3+4a^2b-88ab^2+64b^3\right)\text{co}}{3}}{5a}\right.}{a(a-b)}}{3a(a-b)} \over f$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3\left(-\frac{\left(a^2-22ab+16b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)}}{5a}-\frac{-\frac{\left(5a^3+4a^2b-88ab^2+64b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3a}-\frac{-\frac{15a^4\arctan\left(\frac{\sqrt{a-b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)}}\right)}{\sqrt{a-b}}-\frac{\left(15a\cdots\right.}{\cdots}}{5a}\cdots\right.}{a(a-b)}\cdots}{3a(a-b)}\cdots}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Tan[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x]^5)/(a*(a - b)*(a + b*Tan[e + f*x]^2)^(3/2)) + (-(((1
1*a - 8*b)*b*Cot[e + f*x]^5)/(a*(a - b)*Sqrt[a + b*Tan[e + f*x]^2])) + (3*
(-1/5*((a^2 - 22*a*b + 16*b^2)*Cot[e + f*x]^5*Sqrt[a + b*Tan[e + f*x]^2])/
a - (-1/3*((5*a^3 + 4*a^2*b - 88*a*b^2 + 64*b^3)*Cot[e + f*x]^3*Sqrt[a + b
*Tan[e + f*x]^2])/a - ((-15*a^4*ArcTan[(Sqrt[a - b]*Tan[e + f*x])/Sqrt[a +
 b*Tan[e + f*x]^2]])/Sqrt[a - b] - ((15*a^4 + 10*a^3*b + 8*a^2*b^2 - 176*a
*b^3 + 128*b^4)*Cot[e + f*x]*Sqrt[a + b*Tan[e + f*x]^2])/a)/(3*a))/(5*a)))
/(a*(a - b)))/(3*a*(a - b)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\cot (fx+e)^{6}}{\left(a+b\tan (fx+e)^{2}\right)^{\frac{5}{2}}}dx$$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1023, normalized size of antiderivative = 3.13

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/60*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)^7 + a^7*tan(f*x
+ e)^5)*sqrt(-a + b)*log(-((a^2 - 8*a*b + 8*b^2)*tan(f*x + e)^4 - 2*(3*a^2
 - 4*a*b)*tan(f*x + e)^2 + a^2 + 4*((a - 2*b)*tan(f*x + e)^3 - a*tan(f*x +
 e))*sqrt(b*tan(f*x + e)^2 + a)*sqrt(-a + b))/(tan(f*x + e)^4 + 2*tan(f*x
+ e)^2 + 1)) + 4*((15*a^5*b^2 - 5*a^4*b^3 - 2*a^3*b^4 - 184*a^2*b^5 + 304*
a*b^6 - 128*b^7)*tan(f*x + e)^8 + 3*a^7 - 9*a^6*b + 9*a^5*b^2 - 3*a^4*b^3
+ 3*(10*a^6*b - 5*a^5*b^2 - a^4*b^3 - 92*a^3*b^4 + 152*a^2*b^5 - 64*a*b^6)
*tan(f*x + e)^6 + 3*(5*a^7 - 5*a^6*b + a^5*b^2 - 23*a^4*b^3 + 38*a^3*b^4 -
 16*a^2*b^5)*tan(f*x + e)^4 - (5*a^7 - 7*a^6*b - 9*a^5*b^2 + 19*a^4*b^3 -
8*a^3*b^4)*tan(f*x + e)^2)*sqrt(b*tan(f*x + e)^2 + a))/((a^8*b^2 - 3*a^7*b
^3 + 3*a^6*b^4 - a^5*b^5)*f*tan(f*x + e)^9 + 2*(a^9*b - 3*a^8*b^2 + 3*a^7*
b^3 - a^6*b^4)*f*tan(f*x + e)^7 + (a^10 - 3*a^9*b + 3*a^8*b^2 - a^7*b^3)*f
*tan(f*x + e)^5), -1/30*(15*(a^5*b^2*tan(f*x + e)^9 + 2*a^6*b*tan(f*x + e)
^7 + a^7*tan(f*x + e)^5)*sqrt(a - b)*arctan(-2*sqrt(b*tan(f*x + e)^2 + a)*
sqrt(a - b)*tan(f*x + e)/((a - 2*b)*tan(f*x + e)^2 - a)) + 2*((15*a^5*b^2
- 5*a^4*b^3 - 2*a^3*b^4 - 184*a^2*b^5 + 304*a*b^6 - 128*b^7)*tan(f*x + e)^
8 + 3*a^7 - 9*a^6*b + 9*a^5*b^2 - 3*a^4*b^3 + 3*(10*a^6*b - 5*a^5*b^2 - a^
4*b^3 - 92*a^3*b^4 + 152*a^2*b^5 - 64*a*b^6)*tan(f*x + e)^6 + 3*(5*a^7 - 5
*a^6*b + a^5*b^2 - 23*a^4*b^3 + 38*a^3*b^4 - 16*a^2*b^5)*tan(f*x + e)^4 -
(5*a^7 - 7*a^6*b - 9*a^5*b^2 + 19*a^4*b^3 - 8*a^3*b^4)*tan(f*x + e)^2)*...
```

Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^6{(e+fx)}}{(a+b\tan^2{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*tan(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**6/(a + b*tan(e + f*x)**2)**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^6/(a + b*tan(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\tan^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)^6}{\left(\tan(fx+e)^2 b+a\right)^{\frac{5}{2}}}dx$$

input
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

output
```
int(cot(f*x+e)^6/(a+b*tan(f*x+e)^2)^(5/2),x)
```

3.359 $\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+m+2p), \frac{1}{2}(3+m+2p), -\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m \left(b\right.}{f(1+m+2p)}$$

output
```
hypergeom([1, 1/2+1/2*m+p],[3/2+1/2*m+p],-tan(f*x+e)^2)*tan(f*x+e)*(d*tan(
f*x+e))^m*(b*tan(f*x+e)^2)^p/f/(1+m+2*p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}+p, \frac{3+m}{2}+p, -\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m \left(b\tan^2(e+fx\right.}{f(1+m+2p)}$$

input
```
Integrate[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + m)/2 + p, (3 + m)/2 + p, -Tan[e + f*x]^2]*Tan[e
 + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4061, 2034, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^2\right)^p (d\tan(e+fx))^m dx$$

$$\Big\downarrow \text{4061}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\int \tan^{2p}(e+fx)(d\tan(e+fx))^m dx$$

$$\Big\downarrow \text{2034}$$

$$\left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m\tan^{-m-2p}(e+fx)\int \tan^{m+2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m\tan^{-m-2p}(e+fx)\int \tan(e+fx)^{m+2p}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m\tan^{-m-2p}(e+fx)\int \frac{\tan^{m+2p}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+2p+1),\frac{1}{2}(m+2p+3),-\tan^2(e\right.}{f(m+2p+1)}$$

input
```
Int[(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -Tan[e + f*x]^2]*T
an[e + f*x]*(d*Tan[e + f*x])^m*(b*Tan[e + f*x]^2)^p)/(f*(1 + m + 2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4061
```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Tan[e + f*x
])^p)^FracPart[n]/(d*Tan[e + f*x])^(p*FracPart[n]))   Int[(a + b*Tan[e + f*
x])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\tan(fx+e))^m \left(b\tan(fx+e)^2\right)^p dx$$

input
```
int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \int \left(b\tan(fx+e)^2\right)^p (d\tan(fx+e))^m dx$$

input
```
integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p dx = \int \left(b\tan^2(e+fx)\right)^p (d\tan(e+fx))^m dx$$

input
```
integrate((d*tan(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((b*tan(e + f*x)**2)**p*(d*tan(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\tan\left(fx+e\right)\right)^m \,dx$$

input
```
integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Giac [F]

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan\left(fx+e\right)^2\right)^p \left(d\tan\left(fx+e\right)\right)^m \,dx$$

input
```
integrate((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int (d\tan(e+f\,x))^m \left(b\tan(e+f\,x)^2\right)^p \,dx$$

input
```
int((d*tan(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\tan(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = d^m b^p \left(\int \tan(fx+e)^{m+2p} \, dx\right)$$

input
```
int((d*tan(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*b**p*int(tan(e + f*x)**(m + 2*p),x)
```

3.360 $\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2},1,-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\tan(e+fx))^{1+m}\left(a+b\tan^2(e+fx)\right)^p \left(\frac{a+\cdots}{\cdots}\right.}{df(1+m)}$$

output
```
AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*tan(
f*x+e))^(1+m)*(a+b*tan(f*x+e)^2)^p/d/f/(1+m)/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m\left(a+b\tan^2(e+\cdots\right.}{f(1+m)}$$

input
```
Integrate[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f
*x]^2]*Tan[e + f*x]*(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + m
)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^m \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{(d\tan(e+fx))^m\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \frac{(d\tan(e+fx))^m\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{m+1}{2},1,-p,\frac{m+3}{2},-\tan^2(e+fx),-\frac{b\tan}{}\right.}{df(m+1)}$$

input
```
Int[(d*Tan[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)]*(d*Tan[e + f*x])^(1 + m)*(a + b*Tan[e + f*x]^2)^p)/(d*f*(1 + m)*(1
+ (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int (d\tan (fx+e))^m \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan (fx+e)^2+a\right)^p (d\tan (fx+e))^m dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \int (d\tan (e+fx))^m \left(a+b\tan^2 (e+fx)\right)^p dx$$

input
```
integrate((d*tan(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((d*tan(e + f*x))**m*(a + b*tan(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan{(fx+e)}^2+a\right)^p (d\tan{(fx+e)})^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Giac [F]

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan{(fx+e)}^2+a\right)^p (d\tan{(fx+e)})^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int (d\tan(e+f\,x))^m \left(b\tan(e+f\,x)^2+a\right)^p \, dx$$

input
```
int((d*tan(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\tan(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= d^m \left(\int \tan(fx+e)^m \left(\tan(fx+e)^2 b+a\right)^p dx\right)$$

input
```
int((d*tan(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*int(tan(e + f*x)**m*(tan(e + f*x)**2*b + a)**p,x)
```

3.361 $\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\begin{aligned}\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx &= -\frac{(a+b)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}\\ &\quad-\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}\\ &\quad+\frac{\left(a+b\tan^2(e+fx)\right)^{2+p}}{2b^2f(2+p)}\end{aligned}$$

output
```
-1/2*(a+b)*(a+b*tan(f*x+e)^2)^(p+1)/b^2/f/(p+1)-1/2*hypergeom([1, p+1],[2+
p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(p+1)/(a-b)/f/(p+1)+1/2*(a
+b*tan(f*x+e)^2)^(2+p)/b^2/f/(2+p)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$=\frac{\left(a+b\tan^2(e+fx)\right)^{1+p}\left(b^2(2+p)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a+b(2\right.\right.}{2b^2(-a+b)f(1+p)(2+p)}$$

input
```
Integrate[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
((a + b*Tan[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 + p, 2
 + p, (a + b*Tan[e + f*x]^2)/(a - b)] + (a - b)*(a + b*(2 + p) - b*(1 + p)
*Tan[e + f*x]^2)))/(2*b^2*(-a + b)*f*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5\left(a+b\tan(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^5(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{99}$$

$$\frac{\int \left(\frac{(-a-b)(b\tan^2(e+fx)+a)^p}{b}+\frac{(b\tan^2(e+fx)+a)^p}{\tan^2(e+fx)+1}+\frac{(b\tan^2(e+fx)+a)^{p+1}}{b}\right)d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{-\frac{(a+b)(a+b\tan^2(e+fx))^{p+1}}{b^2(p+1)}+\frac{(a+b\tan^2(e+fx))^{p+2}}{b^2(p+2)}-\frac{(a+b\tan^2(e+fx))^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)+a}{a-b}\right)}{(p+1)(a-b)}}{2f}$$

input
```
Int[Tan[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(-(((a + b)*(a + b*Tan[e + f*x]^2)^(1 + p))/(b^2*(1 + p))) - (Hypergeometr
ic2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]
^2)^(1 + p))/((a - b)*(1 + p)) + (a + b*Tan[e + f*x]^2)^(2 + p)/(b^2*(2 +
p)))/(2*f)
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \tan(fx+e)^5 \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan(fx+e)^2+a\right)^p \tan(fx+e)^5 dx$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Sympy [F]

$$\int \tan^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(a+b\tan^2\left(e+fx\right)\right)^p \tan^5\left(e+fx\right) dx$$

input
```
integrate(tan(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**5, x)
```

Maxima [F]

$$\int \tan^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^5 \, dx$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Giac [F]

$$\int \tan^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^5 \, dx$$

input
```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \tan(e+f\,x)^5\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$=\frac{\left(\tan\left(fx+e\right)^2b+a\right)^p\tan\left(fx+e\right)^4b^2p^2+\left(\tan\left(fx+e\right)^2b+a\right)^p\tan\left(fx+e\right)^4b^2p+\left(\tan\left(fx+e\right)^2b+\right.}{}$$

input
```
int(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**4*b**2*p**2 + (tan(e + f*x)**2*b
 + a)**p*tan(e + f*x)**4*b**2*p + (tan(e + f*x)**2*b + a)**p*tan(e + f*x)*
*2*a*b*p**2 - (tan(e + f*x)**2*b + a)**p*tan(e + f*x)**2*b**2*p**2 - 2*(ta
n(e + f*x)**2*b + a)**p*tan(e + f*x)**2*b**2*p - (tan(e + f*x)**2*b + a)**
p*a**2*p + (tan(e + f*x)**2*b + a)**p*a*b*p + 2*(tan(e + f*x)**2*b + a)**p
*a*b - 2*int(((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**3)/(tan(e + f*x)**2
*b + a),x)*a*b**2*f*p**3 - 6*int(((tan(e + f*x)**2*b + a)**p*tan(e + f*x)*
*3)/(tan(e + f*x)**2*b + a),x)*a*b**2*f*p**2 - 4*int(((tan(e + f*x)**2*b +
 a)**p*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*a*b**2*f*p + 2*int(((ta
n(e + f*x)**2*b + a)**p*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*b**3*f
*p**3 + 6*int(((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**3)/(tan(e + f*x)**
2*b + a),x)*b**3*f*p**2 + 4*int(((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**
3)/(tan(e + f*x)**2*b + a),x)*b**3*f*p)/(2*b**2*f*p*(p**2 + 3*p + 2))
```

3.362 $\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\left(a+b\tan^2(e+fx)\right)^{1+p}}{2bf(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

output
```
1/2*(a+b*tan(f*x+e)^2)^(p+1)/b/f/(p+1)+1/2*hypergeom([1, p+1],[2+p],(a+b*t
an(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(p+1)/(a-b)/f/(p+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\left(a-b+b\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2b(-a+b)f(1+p)}$$

input
```
Integrate[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-1/2*((a - b + b*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)
/(a - b)])*(a + b*Tan[e + f*x]^2)^(1 + p))/(b*(-a + b)*f*(1 + p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4153, 354, 90, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3 \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^3(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}}{b(p+1)}-\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)+a}{a-b}\right)}{(p+1)(a-b)}+\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}}{b(p+1)}}{2f}$$

input
```
Int[Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
((a + b*Tan[e + f*x]^2)^(1 + p)/(b*(1 + p)) + (Hypergeometric2F1[1, 1 + p,
 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^(1 + p))/((
a - b)*(1 + p)))/(2*f)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \tan(fx+e)^3 \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\, dx = \int \left(b\tan(fx+e)^2+a\right)^p \tan(fx+e)^3\, dx$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\, dx = \int \left(a+b\tan^2(e+fx)\right)^p \tan^3(e+fx)\, dx$$

input
```
integrate(tan(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3\,dx$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Giac [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3\,dx$$

input
```
integrate(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \tan(e+f\,x)^3\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{\left(\tan(fx+e)^2 b+a\right)^p \tan(fx+e)^2 bp - \left(\tan(fx+e)^2 b+a\right)^p a + 2\left(\int \frac{\left(\tan(fx+e)^2 b+a\right)^p \tan(fx+e)^3}{\tan(fx+e)^2 b+a}dx\right) ab}{}$$

input
```
int(tan(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**2*b*p - (tan(e + f*x)**2*b + a)*
*p*a + 2*int(((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**3)/(tan(e + f*x)**2
*b + a),x)*a*b*f*p**2 + 2*int(((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**3)
/(tan(e + f*x)**2*b + a),x)*a*b*f*p - 2*int(((tan(e + f*x)**2*b + a)**p*ta
n(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*b**2*f*p**2 - 2*int(((tan(e + f*
x)**2*b + a)**p*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*b**2*f*p)/(2*b
*f*p*(p + 1))
```

3.363 $\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)
^(p+1)/(a-b)/f/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

input
```
Integrate[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(
a + b*Tan[e + f*x]^2)^(1 + p))/((a - b)*f*(1 + p))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4153, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)\left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{ 78}$$

$$-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)+a}{a-b}\right)}{2f(p+1)(a-b)}$$

input
```
Int[Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output

```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(
a + b*Tan[e + f*x]^2)^(1 + p))/((a - b)*f*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \tan (fx+e) \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \tan(e + fx) \left(a + b\tan^2(e + fx)\right)^p \, dx = \int \left(b\tan\left(fx + e\right)^2 + a\right)^p \tan\left(fx + e\right) \, dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Sympy [F]

$$\int \tan(e + fx) \left(a + b\tan^2(e + fx)\right)^p \, dx = \int \left(a + b\tan^2\left(e + fx\right)\right)^p \tan\left(e + fx\right) dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x), x)
```

Maxima [F]

$$\int \tan(e + fx) \left(a + b\tan^2(e + fx)\right)^p \, dx = \int \left(b\tan\left(fx + e\right)^2 + a\right)^p \tan\left(fx + e\right) \, dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Giac [F]

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)\,dx$$

input
```
integrate(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \tan(e+f\,x)\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \tan(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$=\frac{\left(\tan\left(fx+e\right)^2b+a\right)^p a-2\left(\int \frac{\left(\tan(fx+e)^2b+a\right)^p\tan(fx+e)^3}{\tan(fx+e)^2b+a}dx\right)abfp+2\left(\int \frac{\left(\tan(fx+e)^2b+a\right)^p\tan(fx+e)^3}{\tan(fx+e)^2b+a}dx\right)b^{2}}{2bfp}$$

input
```
int(tan(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
((tan(e + f*x)**2*b + a)**p*a - 2*int(((tan(e + f*x)**2*b + a)**p*tan(e +
f*x)**3)/(tan(e + f*x)**2*b + a),x)*a*b*f*p + 2*int(((tan(e + f*x)**2*b +
a)**p*tan(e + f*x)**3)/(tan(e + f*x)**2*b + a),x)*b**2*f*p)/(2*b*f*p)
```

3.364 $\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],(a+b*tan(f*x+e)^2)/a)*(a+b*tan(f*x+e)^2)^(p+
1)/a/f/(p+1)+1/2*hypergeom([1, p+1],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*t
an(f*x+e)^2)^(p+1)/(a-b)/f/(p+1)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\left(a\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)+(-a+b)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\right.\right.}{2a(a-b)f(1+p)}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
((a*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)] + (
-a + b)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a])*(a +
 b*Tan[e + f*x]^2)^(1 + p))/(2*a*(a - b)*f*(1 + p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4153, 354, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{97}$$

$$\frac{\int \cot(e+fx)\left(b\tan^2(e+fx)+a\right)^p d\tan^2(e+fx)-\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \text{75}$$

$$\frac{-\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)}{a}+1\right)}{a(p+1)}}{2f}$$

$\Big\downarrow$ 78

$$\frac{\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)+a}{a-b}\right)}{(p+1)(a-b)}-\frac{\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+f}{a}\right.}{a(p+1)}}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
((Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a +
b*Tan[e + f*x]^2)^(1 + p))/((a - b)*(1 + p)) - (Hypergeometric2F1[1, 1 + p
, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(a*(1 +
 p)))/(2*f)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot \left(fx+e\right) \left(a+b\tan \left(fx+e\right)^{2}\right)^{p} dx$$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan \left(fx+e\right)^{2}+a\right)^{p} \cot \left(fx+e\right) \, dx$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Sympy [F]

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(a+b\tan^2\left(e+fx\right)\right)^p\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Giac [F]

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\cot\left(fx+e\right)dx$$

input
```
int(cot(f*x+e)*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*cot(e + f*x),x)
```

3.365 $\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=-\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2af}+\frac{(a-bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2a^2f(1+p)}-\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

output
```
-1/2*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(p+1)/a/f+1/2*(-b*p+a)*hypergeom([1,
p+1],[2+p],(a+b*tan(f*x+e)^2)/a)*(a+b*tan(f*x+e)^2)^(p+1)/a^2/f/(p+1)-1/2*
hypergeom([1, p+1],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)^(p+1
)/(a-b)/f/(p+1)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$= \frac{(b+a\cot^2(e+fx))\left(-a^2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)-(a-b)\left(a(1+p)\cot^2(e\right.\right.}{2a^2(a}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
((b + a*Cot[e + f*x]^2)*(-(a^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*T
an[e + f*x]^2)/(a - b)]) - (a - b)*(a*(1 + p)*Cot[e + f*x]^2 + (-a + b*p)*
Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x
]^2*(a + b*Tan[e + f*x]^2)^p)/(2*a^2*(a - b)*f*(1 + p))
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4153, 354, 114, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\tan(e+fx)^3}dx$$
$$\Big\downarrow \text{4153}$$
$$\frac{\int \frac{\cot^3(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\cot^2(e+fx)(b\tan^2(e+fx)+a)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$$\Big\downarrow \; 114$$

$$\frac{-\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^p\left(-bp\tan^2(e+fx)+a-bp\right)}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{a}-\frac{\cot(e+fx)(a+b\tan^2(e+fx))^{p+1}}{a}}{2f}$$

$$\Big\downarrow \; 174$$

$$\frac{-\frac{(a-bp)\int \cot(e+fx)(b\tan^2(e+fx)+a)^p d\tan^2(e+fx)-a\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{a}-\frac{\cot(e+fx)(a+b\tan^2(e+fx))^{p+1}}{a}}{2f}$$

$$\Big\downarrow \; 75$$

$$\frac{-\frac{-a\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)-\frac{(a-bp)\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)}{a}+1\right)}{a(p+1)}}{a}-\frac{\cot(e+fx)(a+b\tan^2(e+}{a}}{2f}$$

$$\Big\downarrow \; 78$$

$$\frac{-\frac{\frac{a\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)+a}{a-b}\right)}{(p+1)(a-b)}-\frac{(a-bp)\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)}{a}+\right.}{a(p+1)}}{a}}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(-((Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(1 + p))/a) - ((a*Hypergeometric2F
1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]^2)^
(1 + p))/((a - b)*(1 + p)) - ((a - b*p)*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]^2)^(1 + p))/(a*(1 + p)))/a)
/(2*f)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot (fx+e)^3 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan (fx+e)^2+a\right)^p \cot (fx+e)^3 dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Giac [F]

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^3\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^3*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot\left(fx+e\right)^3\left(\tan\left(fx+e\right)^2b+a\right)^p dx$$

input
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^p,x)
```

3.366 $\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 218

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{(2a+b-bp)\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^2f} - \frac{\cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4af} - \frac{(2a^2-2abp-b^2(1-p)p)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{4a^3f(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)\left(a+b\tan^2(e+fx)\right)^{1+p}}{2(a-b)f(1+p)}$$

output
```
1/4*(-b*p+2*a+b)*cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^(p+1)/a^2/f-1/4*cot(f*x+e
)^4*(a+b*tan(f*x+e)^2)^(p+1)/a/f-1/4*(2*a^2-2*a*b*p-b^2*(1-p)*p)*hypergeom
([1, p+1],[2+p],(a+b*tan(f*x+e)^2)/a)*(a+b*tan(f*x+e)^2)^(p+1)/a^3/f/(p+1)
+1/2*hypergeom([1, p+1],[2+p],(a+b*tan(f*x+e)^2)/(a-b))*(a+b*tan(f*x+e)^2)
^(p+1)/(a-b)/f/(p+1)
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.79

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx =$$

$$-\frac{(b+a\cot^2(e+fx))\left(-2a^3\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\tan^2(e+fx)}{a-b}\right)+(a-b)\left(a(1+p)\,\text{co}\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-1/4*((b + a*Cot[e + f*x]^2)*(-2*a^3*Hypergeometric2F1[1, 1 + p, 2 + p, (a
 + b*Tan[e + f*x]^2)/(a - b)] + (a - b)*(a*(1 + p)*Cot[e + f*x]^2*(-2*a +
b*(-1 + p) + a*Cot[e + f*x]^2) + (2*a^2 - 2*a*b*p + b^2*(-1 + p)*p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]))*Tan[e + f*x]^2*(a
 + b*Tan[e + f*x]^2)^p)/(a^3*(a - b)*f*(1 + p))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4153, 354, 114, 168, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^5(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\downarrow$ 354

$$\frac{\int \frac{\cot^3(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2f}$$

$\downarrow$ 114

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a\right)^p\left(b(1-p)\tan^2(e+fx)+2a+b-bp\right)}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{2a}-\frac{\cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{2a}}{2f}$$

$\downarrow$ 168

$$-\frac{-\frac{\int \frac{\cot(e+fx)\left(b\tan^2(e+fx)+a\right)^p\left(2a^2-2bpa-bp(2a+b-bp)\tan^2(e+fx)-b^2(1-p)p\right)}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{a}-\frac{(2a-bp+b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{p+1}}{a}}{2a}-\cot^2(\ldots}{2f}$$

$\downarrow$ 174

$$-\frac{-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\int \cot(e+fx)\left(b\tan^2(e+fx)+a\right)^p d\tan^2(e+fx)-2a^2\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)}{a}-\frac{(2a-bp+b)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^{\ldots}}{a}}{2a}}{2f}$$

$\downarrow$ 75

$$-\frac{-\frac{-2a^2\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan^2(e+fx)-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)}{a}+1\right)}{a(p+1)}}{a}-\frac{(2a-bp+b\ldots}{\ldots}}{2a}}{2f}$$

$\downarrow$ 78

$$-\frac{-\frac{\frac{2a^2\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\tan^2(e+fx)+a}{a-b}\right)}{(p+1)(a-b)}-\frac{\left(2a^2-2abp-b^2(1-p)p\right)\left(a+b\tan^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1\ldots\right)}{a(p+1)}}{a}}{2a}}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(-1/2*(Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(1 + p))/a - (-(((2*a + b - b
*p)*Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^(1 + p))/a) - ((2*a^2*Hypergeometr
ic2F1[1, 1 + p, 2 + p, (a + b*Tan[e + f*x]^2)/(a - b)]*(a + b*Tan[e + f*x]
^2)^(1 + p))/((a - b)*(1 + p)) - ((2*a^2 - 2*a*b*p - b^2*(1 - p)*p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/a]*(a + b*Tan[e + f*x]
^2)^(1 + p))/(a*(1 + p)))/a)/(2*a))/(2*f)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot (fx+e)^5 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \cot\left(fx+e\right)^5 \, dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p \cot\left(fx+e\right)^5 \, dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

Giac [F]

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^5\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^5*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot\left(fx+e\right)^5\left(\tan\left(fx+e\right)^2b+a\right)^p dx$$

input
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^p,x)
```

3.367 $\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$
$$=\frac{\operatorname{AppellF1}\left(\frac{7}{2},1,-p,\frac{9}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^7(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)}{7f}$$

output
```
1/7*AppellF1(7/2,1,-p,9/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^7*(a
+b*tan(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

input
```
Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
Integrate[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^6 \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^6(e+fx)(b\tan^2(e+fx)+a)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \int \frac{\tan^6(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{\tan^7(e+fx) \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{7}{2},1,-p,\frac{9}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{7f}$$

input
```
Int[Tan[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[7/2, 1, -p, 9/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
 + f*x]^7*(a + b*Tan[e + f*x]^2)^p)/(7*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \tan(fx+e)^6 \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^6\,dx$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(tan(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^6\,dx$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

Giac [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^6\,dx$$

input
```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \tan(e+f\,x)^6\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^6*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \tan^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\tan\left(fx+e\right)^6\,dx$$

input
```
int(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**6,x)
```

3.368 $\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)}{5f}$$

output
```
1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^5*(a
+b*tan(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1896 vs. $2(84) = 168$.

Time = 13.21 (sec) , antiderivative size = 1896, normalized size of antiderivative = 22.57

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(-2*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]*
(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p) + (Tan[e + f*x]
*(a + b*Tan[e + f*x]^2)^p*((-a + b*(3 + 2*p))*Hypergeometric2F1[1/2, -p, 3
/2, -((b*Tan[e + f*x]^2)/a)] + (a + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]
^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/
2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[
e + f*x]*(a + b*Tan[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((6*a*b*p*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Tan[e +
f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 + p))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((
b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3*a*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^
2*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*a*AppellF1[1/2, -p...
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\left(a+b\tan(e+fx)^2\right)^p\,dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 395

$$\frac{(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \frac{\tan^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 394

$$\frac{\tan^5(e+fx)\,(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

input
```
Int[Tan[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
 + f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \tan (fx+e)^4 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \tan^4(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan (fx+e)^2+a\right)^p \tan (fx+e)^4 dx$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Sympy [F]

$$\int \tan^4(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(a+b\tan^2{(e+fx)}\right)^p \tan^4{(e+fx)}\, dx$$

input
```
integrate(tan(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**4, x)
```

Maxima [F]

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4\,dx$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Giac [F]

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4\,dx$$

input
```
integrate(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \tan(e+f\,x)^4\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^4*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \tan^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(\tan\left(fx+e\right)^2b+a\right)^p\tan\left(fx+e\right)^4dx$$

input
```
int(tan(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**4,x)
```

3.369 $\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \frac{\text{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)}{3f}$$

output
```
1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)^3*(a
+b*tan(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1992 vs. 2(84) = 168.

Time = 13.44 (sec) , antiderivative size = 1992, normalized size of antiderivative = 23.71

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Too large to display}$$

input
```
Integrate[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(Tan[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(Hypergeometric2F1[1/2, -p, 3
/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)
/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
 + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
 f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*Tan[e + f*x]^2*(a + b*
Tan[e + f*x]^2)^(-1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x
]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*AppellF1[1/2, -p, 1, 3/2, -((
b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(-3*a*AppellF1[1/2,
 -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1
[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*Appell
F1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x
]^2)) + Sec[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p*(Hypergeometric2F1[1/2, -p
, 3/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p + (3*a*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]
^2)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]
^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2])*Tan[e + f*x]^2)) + Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((...
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^2 \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \frac{\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$\frac{\tan^3(e+fx)\,(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{3f}$$

input
```
Int[Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
 + f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \tan (fx+e)^2 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan (fx+e)^2+a\right)^p \tan (fx+e)^2 \, dx$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int \tan^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(a+b\tan^2(e+fx)\right)^p \tan^2(e+fx) \, dx$$

input
```
integrate(tan(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^2\,dx$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^2\,dx$$

input
```
integrate(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \tan(e+f\,x)^2\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \tan^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx = \int \left(\tan(fx+e)^2 b+a\right)^p \tan(fx+e)^2\,dx$$

input
```
int(tan(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*tan(e + f*x)**2,x)
```

3.370 $\int (a+b\tan^2(e+fx))^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int (a+b\tan^2(e+fx))^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\tan(e+fx)\,(a+b\tan^2(e+fx))^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)^{-}}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*tan(f*x+e)*(a+b*tan
(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 192 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.43

$$\int (a+b\tan^2(e+fx))^p \, dx$$
$$= \frac{3a\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx\right.}{6af\,\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+4f\left(bp\,\text{AppellF1}\left(\frac{3}{2},1-p,1,\frac{5}{2},-\frac{b\tan^2(e+fx)}{a}\right.\right.}$$

input
```
Integrate[(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*S
in[2*(e + f*x)]*(a + b*Tan[e + f*x]^2)^p)/(6*a*f*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 4*f*(b*p*AppellF1[3/2, 1 - p,
 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2
, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{334}$$

$$\frac{\left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \int \frac{\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx) \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

input
```
Int[(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Tan[e
 + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [F]

$$\int \left(a+b\tan \left(fx+e\right)^{2}\right)^{p}dx$$

input
```
int((a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \left(a+b\tan^{2}(e+fx)\right)^{p}\,dx=\int \left(b\tan \left(fx+e\right)^{2}+a\right)^{p}dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\tan^{2}(e+fx)\right)^{p}\,dx=\int \left(a+b\tan^{2}\left(e+fx\right)\right)^{p}\,dx$$

input
```
integrate((a+b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*tan(e + f*x)**2)**p, x)
```

Maxima [F]

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\,dx$$

input
```
integrate((a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int((a + b*tan(e + f*x)^2)^p,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \left(a + b\tan^2(e + fx)\right)^p \, dx = \int \left(\tan\left(fx + e\right)^2 b + a\right)^p dx$$

input
```
int((a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p,x)
```

3.371 $\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = -\frac{\text{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
-AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)*(a+b*t
an(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1989 vs. 2(80) = 160.

Time = 13.54 (sec) , antiderivative size = 1989, normalized size of antiderivative = 24.86

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^(2*p)*(-(Hypergeometric2F1[-1/2, -p
, 1/2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x
]^2)/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x
]^2] + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan
[e + f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[
e + f*x]^2])*Tan[e + f*x]^2)))/(f*(2*b*p*Sec[e + f*x]^2*(a + b*Tan[e + f*x
]^2)^(-1 + p)*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]
/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e
 + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(-3*a*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*(-(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]) + a*AppellF1[3/2,
 -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) -
 Csc[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/
2, -((b*Tan[e + f*x]^2)/a)]/(1 + (b*Tan[e + f*x]^2)/a)^p) + (3*a*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Sin[e + f*x]^2)
/(-3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]
 + 2*(-(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
 f*x]^2]) + a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -Tan[e +
f*x]^2])*Tan[e + f*x]^2)) + Cot[e + f*x]*(a + b*Tan[e + f*x]^2)^p*((2*b...
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^2(e+fx)(b\tan^2(e+fx)+a)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{(a+b\tan^2(e+fx))^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \frac{\cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Co
t[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p))
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot (fx+e)^2 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan (fx+e)^2+a\right)^p \cot (fx+e)^2 \, dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**2*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^2\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^2*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^2(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(\tan (fx+e)^2 b+a\right)^p \cot (fx+e)^2 \, dx$$

input
```
int(cot(f*x+e)^2*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((tan(e + f*x)**2*b + a)**p*cot(e + f*x)**2,x)
```

3.372 $\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx =$$

$$-\frac{\text{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e}{a}\right.}{3f}$$

output
```
-1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)^3
*(a+b*tan(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1887 vs. 2(84) = 168.

Time = 6.42 (sec) , antiderivative size = 1887, normalized size of antiderivative = 22.46

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(2*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*
(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p) + (Cot[e + f*x]
^3*(a + b*Tan[e + f*x]^2)^p*(-a - b*Tan[e + f*x]^2 - ((3*a + b*(-1 + 2*p))
*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/a)]*Tan[e + f*x]^2)
/(1 + (b*Tan[e + f*x]^2)/a)^p))/(3*a*f) + (3*a*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x]*(a + b*
Tan[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
+ f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2)*((6*a*b*p*AppellF1[1/2, -p,
1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Tan[e + f*x]^2*(a + b*Ta
n[e + f*x]^2)^(-1 + p))/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/a), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/a), -Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/a), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(a + b*Tan[e +
 f*x]^2)^p)/(3*a*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/a), -Tan[e
 + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/a),
-Tan[e + f*x]^2] - a*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/a), -T
an[e + f*x]^2])*Tan[e + f*x]^2) - (3*a*AppellF1[1/2, -p, 1, 3/2, -((b*T...
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \frac{\cot^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx}{a}\right.}{3f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*Cot[e + f*x]^3*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p
)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot(fx+e)^4 \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^4(e+fx) \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan(fx+e)^2+a\right)^p \cot(fx+e)^4 dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^4\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^4*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^4(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot\left(fx+e\right)^4\left(\tan\left(fx+e\right)^2b+a\right)^p dx$$

input
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^4*(a+b*tan(f*x+e)^2)^p,x)
```

3.373 $\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = -\frac{\text{AppellF1}\left(-\frac{5}{2},1,-p,-\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)\cot^5(e+fx)\left(a+b\tan^2(e+fx)\right)^p\left(\frac{a+b\tan^2(e+fx)}{a}\right)^{-p}}{5f}$$

output
```
-1/5*AppellF1(-5/2,1,-p,-3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*cot(f*x+e)^5
*(a+b*tan(f*x+e)^2)^p/f/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [F]

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
Integrate[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p, x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx) \left(a+b\tan^2(e+fx)\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(e+fx)^2\right)^p}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\cot^6(e+fx)\left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{395}$$

$$\frac{\left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \int \frac{\cot^6(e+fx)\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{394}$$

$$-\frac{\cot^5(e+fx) \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \text{AppellF1}\left(-\frac{5}{2},1,-p,-\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)}{5f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-1/5*(AppellF1[-5/2, 1, -p, -3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)
]*Cot[e + f*x]^5*(a + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/a)^p
)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot (fx+e)^6 \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**6*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

Giac [F]

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot(e+f\,x)^6\,\left(b\tan(e+f\,x)^2+a\right)^p\,dx$$

input
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^6*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int \cot^6(e+fx)\left(a+b\tan^2(e+fx)\right)^p\,dx=\int \cot\left(fx+e\right)^6\left(\tan\left(fx+e\right)^2b+a\right)^p\,dx$$

input
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^6*(a+b*tan(f*x+e)^2)^p,x)
```

3.374 $\int (a+b\tan^3(c+dx))^4\,dx$

Optimal result

Integrand size = 14, antiderivative size = 255

$$\int (a+b\tan^3(c+dx))^4\,dx = (a^4-6a^2b^2+b^4)\,x+\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d} +\frac{b^2(6a^2-b^2)\tan(c+dx)}{d}+\frac{2ab(a^2-b^2)\tan^2(c+dx)}{d} -\frac{b^2(6a^2-b^2)\tan^3(c+dx)}{3d}+\frac{ab^3\tan^4(c+dx)}{d} +\frac{b^2(6a^2-b^2)\tan^5(c+dx)}{5d}-\frac{2ab^3\tan^6(c+dx)}{3d} +\frac{b^4\tan^7(c+dx)}{7d}+\frac{ab^3\tan^8(c+dx)}{2d} -\frac{b^4\tan^9(c+dx)}{9d}+\frac{b^4\tan^{11}(c+dx)}{11d}$$

output
```
(a^4-6*a^2*b^2+b^4)*x+4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d+b^2*(6*a^2-b^2)*tan
(d*x+c)/d+2*a*b*(a^2-b^2)*tan(d*x+c)^2/d-1/3*b^2*(6*a^2-b^2)*tan(d*x+c)^3/
d+a*b^3*tan(d*x+c)^4/d+1/5*b^2*(6*a^2-b^2)*tan(d*x+c)^5/d-2/3*a*b^3*tan(d*
x+c)^6/d+1/7*b^4*tan(d*x+c)^7/d+1/2*a*b^3*tan(d*x+c)^8/d-1/9*b^4*tan(d*x+c
)^9/d+1/11*b^4*tan(d*x+c)^11/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.88

$$\int \left(a+b\tan^3(c+dx)\right)^4 \, dx$$
$$= \frac{-3465i((a-ib)^4\log(i-\tan(c+dx))-(a+ib)^4\log(i+\tan(c+dx)))-6930b^2(-6a^2+b^2)\tan(c+dx)}{}$$

input
```
Integrate[(a + b*Tan[c + d*x]^3)^4,x]
```

output
```
((-3465*I)*((a - I*b)^4*Log[I - Tan[c + d*x]] - (a + I*b)^4*Log[I + Tan[c
+ d*x]]) - 6930*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 13860*a*b*(a^2 - b^2)*Ta
n[c + d*x]^2 + 2310*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^3 + 6930*a*b^3*Tan[c +
 d*x]^4 - 1386*b^2*(-6*a^2 + b^2)*Tan[c + d*x]^5 - 4620*a*b^3*Tan[c + d*x]
^6 + 990*b^4*Tan[c + d*x]^7 + 3465*a*b^3*Tan[c + d*x]^8 - 770*b^4*Tan[c +
d*x]^9 + 630*b^4*Tan[c + d*x]^11)/(6930*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^3(c+dx)\right)^4 \, dx$$
$$\Big\downarrow \ 3042$$
$$\int \left(a+b\tan(c+dx)^3\right)^4 dx$$
$$\Big\downarrow \ 4144$$
$$\frac{\int \frac{\left(b\tan^3(c+dx)+a\right)^4}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$\downarrow$ 2341

$$\int \left(b^4 \tan^{10}(c+dx) - b^4 \tan^8(c+dx) + 4ab^3 \tan^7(c+dx) + b^4 \tan^6(c+dx) - 4ab^3 \tan^5(c+dx) + b^2\left(6a^2-b^2\right)$$

$\downarrow$ 2009

$$\frac{1}{5}b^2\left(6a^2-b^2\right)\tan^5(c+dx) - \frac{1}{3}b^2\left(6a^2-b^2\right)\tan^3(c+dx) + 2ab\left(a^2-b^2\right)\tan^2(c+dx) + b^2\left(6a^2-b^2\right)\tan(c+dx$$

input
```
Int[(a + b*Tan[c + d*x]^3)^4,x]
```

output
```
((a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]] - 2*a*b*(a^2 - b^2)*Log[1 +
Tan[c + d*x]^2] + b^2*(6*a^2 - b^2)*Tan[c + d*x] + 2*a*b*(a^2 - b^2)*Tan[c
 + d*x]^2 - (b^2*(6*a^2 - b^2)*Tan[c + d*x]^3)/3 + a*b^3*Tan[c + d*x]^4 +
(b^2*(6*a^2 - b^2)*Tan[c + d*x]^5)/5 - (2*a*b^3*Tan[c + d*x]^6)/3 + (b^4*T
an[c + d*x]^7)/7 + (a*b^3*Tan[c + d*x]^8)/2 - (b^4*Tan[c + d*x]^9)/9 + (b^
4*Tan[c + d*x]^11)/11)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.89

method	result
parts	$a^4x+\frac{b^4\left(\frac{\tan(dx+c)^{11}}{11}-\frac{\tan(dx+c)^9}{9}+\frac{\tan(dx+c)^7}{7}-\frac{\tan(dx+c)^5}{5}+\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+$
derivativedivides	$\frac{\frac{b^4\tan(dx+c)^{11}}{11}-\frac{b^4\tan(dx+c)^9}{9}+\frac{a\,b^3\tan(dx+c)^8}{2}+\frac{b^4\tan(dx+c)^7}{7}-\frac{2a\,b^3\tan(dx+c)^6}{3}+\frac{6a^2b^2\tan(dx+c)^5}{5}-\frac{b^4\tan(dx+c)^5}{5}+a}{}$
default	$\frac{\frac{b^4\tan(dx+c)^{11}}{11}-\frac{b^4\tan(dx+c)^9}{9}+\frac{a\,b^3\tan(dx+c)^8}{2}+\frac{b^4\tan(dx+c)^7}{7}-\frac{2a\,b^3\tan(dx+c)^6}{3}+\frac{6a^2b^2\tan(dx+c)^5}{5}-\frac{b^4\tan(dx+c)^5}{5}+a}{}$
norman	$(a^4-6a^2b^2+b^4)\,x+\frac{a\,b^3\tan(dx+c)^4}{d}+\frac{b^2(6a^2-b^2)\tan(dx+c)}{d}+\frac{b^4\tan(dx+c)^7}{7d}-\frac{b^4\tan(dx+c)^9}{9d}+$
parallelrisch	$-\frac{-630b^4\tan(dx+c)^{11}+770b^4\tan(dx+c)^9-3465a\,b^3\tan(dx+c)^8-990b^4\tan(dx+c)^7+4620a\,b^3\tan(dx+c)^6-8316a^2}{}$
risch	$-4ia^3bx+4ia\,b^3x+a^4x-6a^2b^2x+b^4x-\frac{8ia^3bc}{d}+\frac{8ia\,b^3c}{d}+\frac{4b(939015ia^2b\,\mathrm{e}^{16i(dx+c)}+627165}{}$

input
```
int((a+b*tan(d*x+c)^3)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+b^4/d*(1/11*tan(d*x+c)^11-1/9*tan(d*x+c)^9+1/7*tan(d*x+c)^7-1/5*tan(
d*x+c)^5+1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)))+4*a*b^3/d*(1/8*ta
n(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan
(d*x+c)^2))+6*a^2*b^2/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arct
an(tan(d*x+c)))+2*a^3*b/d*tan(d*x+c)^2-2*a^3*b/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx$$
$$=\frac{630\,b^4\tan\left(dx+c\right)^{11}-770\,b^4\tan\left(dx+c\right)^9+3465\,ab^3\tan\left(dx+c\right)^8+990\,b^4\tan\left(dx+c\right)^7-4620\,ab^3\tan}{}$$

input
```
integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="fricas")
```

output
```
1/6930*(630*b^4*tan(d*x + c)^11 - 770*b^4*tan(d*x + c)^9 + 3465*a*b^3*tan(
d*x + c)^8 + 990*b^4*tan(d*x + c)^7 - 4620*a*b^3*tan(d*x + c)^6 + 6930*a*b
^3*tan(d*x + c)^4 + 1386*(6*a^2*b^2 - b^4)*tan(d*x + c)^5 - 2310*(6*a^2*b^
2 - b^4)*tan(d*x + c)^3 + 6930*(a^4 - 6*a^2*b^2 + b^4)*d*x + 13860*(a^3*b
- a*b^3)*tan(d*x + c)^2 + 13860*(a^3*b - a*b^3)*log(1/(tan(d*x + c)^2 + 1)
) + 6930*(6*a^2*b^2 - b^4)*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.18

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx$$
$$=\begin{cases} a^4x-\frac{2a^3b\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{2a^3b\tan^2\left(c+dx\right)}{d}-6a^2b^2x+\frac{6a^2b^2\tan^5\left(c+dx\right)}{5d}-\frac{2a^2b^2\tan^3\left(c+dx\right)}{d}+\frac{6a^2b^2\tan\left(c+dx\right)}{d}- \\ x\left(a+b\tan^3\left(c\right)\right)^4 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**3)**4,x)
```

output
```
Piecewise((a**4*x - 2*a**3*b*log(tan(c + d*x)**2 + 1)/d + 2*a**3*b*tan(c +
 d*x)**2/d - 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**5/(5*d) - 2*a**2*b*
*2*tan(c + d*x)**3/d + 6*a**2*b**2*tan(c + d*x)/d + 2*a*b**3*log(tan(c + d
*x)**2 + 1)/d + a*b**3*tan(c + d*x)**8/(2*d) - 2*a*b**3*tan(c + d*x)**6/(3
*d) + a*b**3*tan(c + d*x)**4/d - 2*a*b**3*tan(c + d*x)**2/d + b**4*x + b**
4*tan(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d*x)
**7/(7*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b**4
*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.02

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx$$
$$= a^4x + \frac{2\left(3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx - 15\,c + 15\,\tan\left(dx+c\right)\right)a^2b^2}{5\,d}$$
$$+ \frac{\left(315\,\tan\left(dx+c\right)^{11} - 385\,\tan\left(dx+c\right)^9 + 495\,\tan\left(dx+c\right)^7 - 693\,\tan\left(dx+c\right)^5 + 1155\,\tan\left(dx+c\right)^{}\right.}{3465\,d}$$
$$+ \frac{ab^3\left(\frac{48\,\sin(dx+c)^6-108\,\sin(dx+c)^4+88\,\sin(dx+c)^2-25}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1} - 12\,\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{6\,d}$$
$$- \frac{2\,a^3b\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)^2-1\right)\right)}{d}$$

input
```
integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="maxima")
```

output
```
a^4*x + 2/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*a^2*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x + c)^9 + 4
95*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 3465*d*x +
3465*c - 3465*tan(d*x + c))*b^4/d + 1/6*a*b^3*((48*sin(d*x + c)^6 - 108*si
n(d*x + c)^4 + 88*sin(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6
+ 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d
 - 2*a^3*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.15

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx$$
$$= \frac{\left(a^4-6\,a^2b^2+b^4\right)\left(dx+c\right)}{d} - \frac{2\left(a^3b-ab^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{d}$$
$$+ \frac{630\,b^4d^{10}\tan\left(dx+c\right)^{11} - 770\,b^4d^{10}\tan\left(dx+c\right)^9 + 3465\,ab^3d^{10}\tan\left(dx+c\right)^8 + 990\,b^4d^{10}\tan\left(dx+c\right)^{7}}{}$$

input
```
integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="giac")
```

output
```
(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/d - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2
 + 1)/d + 1/6930*(630*b^4*d^10*tan(d*x + c)^11 - 770*b^4*d^10*tan(d*x + c)
^9 + 3465*a*b^3*d^10*tan(d*x + c)^8 + 990*b^4*d^10*tan(d*x + c)^7 - 4620*a
*b^3*d^10*tan(d*x + c)^6 + 8316*a^2*b^2*d^10*tan(d*x + c)^5 - 1386*b^4*d^1
0*tan(d*x + c)^5 + 6930*a*b^3*d^10*tan(d*x + c)^4 - 13860*a^2*b^2*d^10*tan
(d*x + c)^3 + 2310*b^4*d^10*tan(d*x + c)^3 + 13860*a^3*b*d^10*tan(d*x + c)
^2 - 13860*a*b^3*d^10*tan(d*x + c)^2 + 41580*a^2*b^2*d^10*tan(d*x + c) - 6
930*b^4*d^10*tan(d*x + c))/d^11
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.22

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx = \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(2\,a\,b^3-2\,a^3\,b\right)}{d}+\frac{\tan(c+d\,x)^3\,\left(\frac{b^4}{3}-2\,a^2\,b^2\right)}{d} - \frac{\tan(c+d\,x)^5\,\left(\frac{b^4}{5}-\frac{6\,a^2\,b^2}{5}\right)}{d}-\frac{\tan(c+d\,x)^2\,\left(2\,a\,b^3-2\,a^3\,b\right)}{d} - \frac{\tan(c+d\,x)\,\left(b^4-6\,a^2\,b^2\right)}{d}+\frac{b^4\tan(c+d\,x)^7}{7\,d}-\frac{b^4\tan(c+d\,x)^9}{9\,d}+\frac{b^4\tan(c+d\,x)^{11}}{11\,d} + \frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{a^4-6\,a^2\,b^2+b^4}\right)\,\left(-a^2+2\,a\,b+b^2\right)\,\left(a^2+2\,a\,b-b^2\right)}{d} + \frac{a\,b^3\tan(c+d\,x)^4}{d}-\frac{2\,a\,b^3\tan(c+d\,x)^6}{3\,d}+\frac{a\,b^3\tan(c+d\,x)^8}{2\,d}$$

input
```
int((a + b*tan(c + d*x)^3)^4,x)
```

output
```
(log(tan(c + d*x)^2 + 1)*(2*a*b^3 - 2*a^3*b))/d + (tan(c + d*x)^3*(b^4/3 -
 2*a^2*b^2))/d - (tan(c + d*x)^5*(b^4/5 - (6*a^2*b^2)/5))/d - (tan(c + d*x
)^2*(2*a*b^3 - 2*a^3*b))/d - (tan(c + d*x)*(b^4 - 6*a^2*b^2))/d + (b^4*tan
(c + d*x)^7)/(7*d) - (b^4*tan(c + d*x)^9)/(9*d) + (b^4*tan(c + d*x)^11)/(1
1*d) + (atan((tan(c + d*x)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^4 +
 b^4 - 6*a^2*b^2))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/d + (a*b^3*tan
(c + d*x)^4)/d - (2*a*b^3*tan(c + d*x)^6)/(3*d) + (a*b^3*tan(c + d*x)^8)/(
2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00

$$\int \left(a+b\tan^3(c+dx)\right)^4\,dx$$
$$=\frac{-13860\log\left(\tan\left(dx+c\right)^2+1\right)a^3b+13860\log\left(\tan\left(dx+c\right)^2+1\right)a\,b^3+630\tan\left(dx+c\right)^{11}b^4-770\tan}{}$$

input
```
int((a+b*tan(d*x+c)^3)^4,x)
```

output
```
( - 13860*log(tan(c + d*x)**2 + 1)*a**3*b + 13860*log(tan(c + d*x)**2 + 1)
*a*b**3 + 630*tan(c + d*x)**11*b**4 - 770*tan(c + d*x)**9*b**4 + 3465*tan(
c + d*x)**8*a*b**3 + 990*tan(c + d*x)**7*b**4 - 4620*tan(c + d*x)**6*a*b**
3 + 8316*tan(c + d*x)**5*a**2*b**2 - 1386*tan(c + d*x)**5*b**4 + 6930*tan(
c + d*x)**4*a*b**3 - 13860*tan(c + d*x)**3*a**2*b**2 + 2310*tan(c + d*x)**
3*b**4 + 13860*tan(c + d*x)**2*a**3*b - 13860*tan(c + d*x)**2*a*b**3 + 415
80*tan(c + d*x)*a**2*b**2 - 6930*tan(c + d*x)*b**4 + 6930*a**4*d*x - 41580
*a**2*b**2*d*x + 6930*b**4*d*x)/(6930*d)
```

3.375 $\int (a+b\tan^3(c+dx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 168

$$\int (a+b\tan^3(c+dx))^3\,dx = a(a^2-3b^2)\,x+\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b(3a^2-b^2)\tan^2(c+dx)}{2d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c+dx)}{4d}+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{b^3\tan^6(c+dx)}{6d}+\frac{b^3\tan^8(c+dx)}{8d}$$

output
```
a*(a^2-3*b^2)*x+b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b*
(3*a^2-b^2)*tan(d*x+c)^2/d-a*b^2*tan(d*x+c)^3/d+1/4*b^3*tan(d*x+c)^4/d+3/5
*a*b^2*tan(d*x+c)^5/d-1/6*b^3*tan(d*x+c)^6/d+1/8*b^3*tan(d*x+c)^8/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx$$
$$= \frac{60(-i(a-ib)^3\log(i-\tan(c+dx))+i(a+ib)^3\log(i+\tan(c+dx)))+360ab^2\tan(c+dx)-60b(-3a^2}{}$$

input
```
Integrate[(a + b*Tan[c + d*x]^3)^3,x]
```

output
```
(60*((-I)*(a - I*b)^3*Log[I - Tan[c + d*x]] + I*(a + I*b)^3*Log[I + Tan[c
+ d*x]]) + 360*a*b^2*Tan[c + d*x] - 60*b*(-3*a^2 + b^2)*Tan[c + d*x]^2 - 1
20*a*b^2*Tan[c + d*x]^3 + 30*b^3*Tan[c + d*x]^4 + 72*a*b^2*Tan[c + d*x]^5
- 20*b^3*Tan[c + d*x]^6 + 15*b^3*Tan[c + d*x]^8)/(120*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\tan(c+dx)^3\right)^3\,dx$$
$$\Big\downarrow \text{4144}$$
$$\frac{\int \frac{\left(b\tan^3(c+dx)+a\right)^3}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$
$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(b^3 \tan^7(c+dx) - b^3 \tan^5(c+dx) + 3ab^2 \tan^4(c+dx) + b^3 \tan^3(c+dx) - 3ab^2 \tan^2(c+dx) + b(3a^2-b^2) \text{ta}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(a^2-3b^2)\arctan(\tan(c+dx)) + \frac{1}{2}b(3a^2-b^2)\tan^2(c+dx) - \frac{1}{2}b(3a^2-b^2)\log\left(\tan^2(c+dx)+1\right) + \frac{3}{5}ab^2\tan^5}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^3)^3,x]
```

output
```
(a*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]] - (b*(3*a^2 - b^2)*Log[1 + Tan[c + d
*x]^2])/2 + 3*a*b^2*Tan[c + d*x] + (b*(3*a^2 - b^2)*Tan[c + d*x]^2)/2 - a*
b^2*Tan[c + d*x]^3 + (b^3*Tan[c + d*x]^4)/4 + (3*a*b^2*Tan[c + d*x]^5)/5 -
 (b^3*Tan[c + d*x]^6)/6 + (b^3*Tan[c + d*x]^8)/8)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

method	result
parts	$a^3x+\frac{b^3\left(\frac{\tan(dx+c)^8}{8}-\frac{\tan(dx+c)^6}{6}+\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{3a^2b\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan\right.}{2}\right.}{d}$
derivativedivides	$\frac{\frac{\tan(dx+c)^8b^3}{8}-\frac{\tan(dx+c)^6b^3}{6}+\frac{3\tan(dx+c)^5ab^2}{5}+\frac{\tan(dx+c)^4b^3}{4}-ab^2\tan(dx+c)^3+\frac{3\tan(dx+c)^2a^2b}{2}-\frac{\tan(dx+c)^2b^3}{2}+3\tan}{d}$
default	$\frac{\frac{\tan(dx+c)^8b^3}{8}-\frac{\tan(dx+c)^6b^3}{6}+\frac{3\tan(dx+c)^5ab^2}{5}+\frac{\tan(dx+c)^4b^3}{4}-ab^2\tan(dx+c)^3+\frac{3\tan(dx+c)^2a^2b}{2}-\frac{\tan(dx+c)^2b^3}{2}+3\tan}{d}$
parallelrisch	$-\frac{-15\tan(dx+c)^8b^3+20\tan(dx+c)^6b^3-72\tan(dx+c)^5ab^2-30\tan(dx+c)^4b^3+120ab^2\tan(dx+c)^3-120a^3dx+360a}{}$
norman	$(a^3-3ab^2)x+\frac{b^3\tan(dx+c)^4}{4d}-\frac{b^3\tan(dx+c)^6}{6d}+\frac{b^3\tan(dx+c)^8}{8d}+\frac{3ab^2\tan(dx+c)}{d}-\frac{ab^2\tan(dx+c)^3}{d}$
risch	$-3ia^2bx+ib^3x+a^3x-3ab^2x-\frac{6iba^2c}{d}+\frac{2ib^3c}{d}-\frac{2b\left(-1635iab\,e^{10i(dx+c)}-45a^2e^{14i(dx+c)}+60b^2e^{1}\right.}{}$

input
```
int((a+b*tan(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+b^3/d*(1/8*tan(d*x+c)^8-1/6*tan(d*x+c)^6+1/4*tan(d*x+c)^4-1/2*tan(d*
x+c)^2+1/2*ln(1+tan(d*x+c)^2))+3*a^2*b/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*
x+c)^2))+3*a*b^2/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(ta
n(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx$$
$$=\frac{15\,b^3\tan\left(dx+c\right)^8-20\,b^3\tan\left(dx+c\right)^6+72\,ab^2\tan\left(dx+c\right)^5+30\,b^3\tan\left(dx+c\right)^4-120\,ab^2\tan\left(dx+\right.}{}$$

input
```
integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/120*(15*b^3*tan(d*x + c)^8 - 20*b^3*tan(d*x + c)^6 + 72*a*b^2*tan(d*x +
c)^5 + 30*b^3*tan(d*x + c)^4 - 120*a*b^2*tan(d*x + c)^3 + 360*a*b^2*tan(d*
x + c) + 120*(a^3 - 3*a*b^2)*d*x + 60*(3*a^2*b - b^3)*tan(d*x + c)^2 + 60*
(3*a^2*b - b^3)*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.15

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx$$
$$=\begin{cases} a^3x-\frac{3a^2b\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{3a^2b\tan^2(c+dx)}{2d}-3ab^2x+\frac{3ab^2\tan^5(c+dx)}{5d}-\frac{ab^2\tan^3(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3 l \\ x\left(a+b\tan^3(c)\right)^3 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**3)**3,x)
```

output
```
Piecewise((a**3*x - 3*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*a**2*b*tan
(c + d*x)**2/(2*d) - 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**5/(5*d) - a*b**2*
tan(c + d*x)**3/d + 3*a*b**2*tan(c + d*x)/d + b**3*log(tan(c + d*x)**2 + 1
)/(2*d) + b**3*tan(c + d*x)**8/(8*d) - b**3*tan(c + d*x)**6/(6*d) + b**3*t
an(c + d*x)**4/(4*d) - b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*ta
n(c)**3)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx$$
$$=a^3x+\frac{\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)ab^2}{5\,d}$$
$$+\frac{b^3\left(\frac{48\,\sin(dx+c)^6-108\,\sin(dx+c)^4+88\,\sin(dx+c)^2-25}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-12\,\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{24\,d}$$
$$-\frac{3\,a^2b\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
a^3*x + 1/5*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(
d*x + c))*a*b^2/d + 1/24*b^3*((48*sin(d*x + c)^6 - 108*sin(d*x + c)^4 + 88
*sin(d*x + c)^2 - 25)/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^
4 - 4*sin(d*x + c)^2 + 1) - 12*log(sin(d*x + c)^2 - 1))/d - 3/2*a^2*b*(1/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx=\frac{(a^3-3\,ab^2)(dx+c)}{d}-\frac{(3\,a^2b-b^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}+\frac{15\,b^3d^7\tan\left(dx+c\right)^8-20\,b^3d^7\tan\left(dx+c\right)^6+72\,ab^2d^7\tan\left(dx+c\right)^5+30\,b^3d^7\tan\left(dx+c\right)^4-120\,ab^2}{120\,d^8}$$

input
```
integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
(a^3 - 3*a*b^2)*(d*x + c)/d - 1/2*(3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/
d + 1/120*(15*b^3*d^7*tan(d*x + c)^8 - 20*b^3*d^7*tan(d*x + c)^6 + 72*a*b^
2*d^7*tan(d*x + c)^5 + 30*b^3*d^7*tan(d*x + c)^4 - 120*a*b^2*d^7*tan(d*x +
 c)^3 + 180*a^2*b*d^7*tan(d*x + c)^2 - 60*b^3*d^7*tan(d*x + c)^2 + 360*a*b
^2*d^7*tan(d*x + c))/d^8
```

Mupad [B] (verification not implemented)

Time = 7.50 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx=\frac{\tan(c+d\,x)^2\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)+\frac{b^3\tan(c+d\,x)^4}{4}-\frac{b^3\tan(c+d\,x)^6}{6}+\frac{b^3\tan(c+d\,x)^8}{8}-\ln\left(\tan(c+d\,x)^2+1\right)\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right.}{d}$$

input
```
int((a + b*tan(c + d*x)^3)^3,x)
```

output
```
(tan(c + d*x)^2*((3*a^2*b)/2 - b^3/2) + (b^3*tan(c + d*x)^4)/4 - (b^3*tan(
c + d*x)^6)/6 + (b^3*tan(c + d*x)^8)/8 - log(tan(c + d*x)^2 + 1)*((3*a^2*b
)/2 - b^3/2) - a*atan((a*tan(c + d*x)*(a^2 - 3*b^2))/(3*a*b^2 - a^3))*(a^2
 - 3*b^2) - a*b^2*tan(c + d*x)^3 + (3*a*b^2*tan(c + d*x)^5)/5 + 3*a*b^2*ta
n(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \left(a+b\tan^3(c+dx)\right)^3\,dx$$
$$= \frac{-180\log\left(\tan\left(dx+c\right)^2+1\right)a^2b+60\log\left(\tan\left(dx+c\right)^2+1\right)b^3+15\tan\left(dx+c\right)^8b^3-20\tan\left(dx+c\right)^6b}{}$$

input
```
int((a+b*tan(d*x+c)^3)^3,x)
```

output
```
( - 180*log(tan(c + d*x)**2 + 1)*a**2*b + 60*log(tan(c + d*x)**2 + 1)*b**3
 + 15*tan(c + d*x)**8*b**3 - 20*tan(c + d*x)**6*b**3 + 72*tan(c + d*x)**5*
a*b**2 + 30*tan(c + d*x)**4*b**3 - 120*tan(c + d*x)**3*a*b**2 + 180*tan(c
+ d*x)**2*a**2*b - 60*tan(c + d*x)**2*b**3 + 360*tan(c + d*x)*a*b**2 + 120
*a**3*d*x - 360*a*b**2*d*x)/(120*d)
```

3.376 $\int \left(a+b\tan^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx=\left(a^2-b^2\right)x+\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}-\frac{b^2\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

output
```
(a^2-b^2)*x+2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d-1/3
*b^2*tan(d*x+c)^3/d+1/5*b^2*tan(d*x+c)^5/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx=\frac{-15i\left((a-ib)^2\log(i-\tan(c+dx))-(a+ib)^2\log(i+\tan(c+dx))\right)+30b^2\tan(c+dx)+30ab\tan^2(c}{30d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^3)^2,x]
```

output
```
((-15*I)*((a - I*b)^2*Log[I - Tan[c + d*x]] - (a + I*b)^2*Log[I + Tan[c +
d*x]]) + 30*b^2*Tan[c + d*x] + 30*a*b*Tan[c + d*x]^2 - 10*b^2*Tan[c + d*x]
^3 + 6*b^2*Tan[c + d*x]^5)/(30*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^3(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(c+dx)^3\right)^2 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^3(c+dx)+a\right)^2}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(b^2\tan^4(c+dx)-b^2\tan^2(c+dx)+2ab\tan(c+dx)+b^2+\frac{a^2-2b\tan(c+dx)a-b^2}{\tan^2(c+dx)+1}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2-b^2\right)\arctan(\tan(c+dx))+ab\tan^2(c+dx)-ab\log\left(\tan^2(c+dx)+1\right)+\frac{1}{5}b^2\tan^5(c+dx)-\frac{1}{3}b^2\tan^3(c+}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^3)^2,x]
```

output
```
((a^2 - b^2)*ArcTan[Tan[c + d*x]] - a*b*Log[1 + Tan[c + d*x]^2] + b^2*Tan[
c + d*x] + a*b*Tan[c + d*x]^2 - (b^2*Tan[c + d*x]^3)/3 + (b^2*Tan[c + d*x]
^5)/5)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

method	result
parts	$x\,a^2+\frac{b^2\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{ab\tan(dx+c)^2}{d}-\frac{ab\ln\left(1+\tan(dx+c)^2\right)}{d}$
derivativedivides	$\frac{\frac{\tan(dx+c)^5b^2}{5}-\frac{\tan(dx+c)^3b^2}{3}+\tan(dx+c)^2ab+\tan(dx+c)b^2-ab\ln\left(1+\tan(dx+c)^2\right)+(a^2-b^2)\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{\tan(dx+c)^5b^2}{5}-\frac{\tan(dx+c)^3b^2}{3}+\tan(dx+c)^2ab+\tan(dx+c)b^2-ab\ln\left(1+\tan(dx+c)^2\right)+(a^2-b^2)\arctan(\tan(dx+c))}{d}$
parallelrisch	$-\frac{-3\tan(dx+c)^5b^2+5\tan(dx+c)^3b^2-15a^2dx+15b^2dx-15\tan(dx+c)^2ab+15ab\ln\left(1+\tan(dx+c)^2\right)-15\tan(dx+c)b^2}{15d}$
norman	$(a^2-b^2)\,x+\frac{b^2\tan(dx+c)}{d}+\frac{ab\tan(dx+c)^2}{d}-\frac{b^2\tan(dx+c)^3}{3d}+\frac{b^2\tan(dx+c)^5}{5d}-\frac{ab\ln\left(1+\tan(dx+c)^2\right)}{d}$
risch	$-2iabx+x\,a^2-x\,b^2-\frac{4iabc}{d}+\frac{2b\left(45ib\,e^{8i(dx+c)}+30a\,e^{8i(dx+c)}+90ib\,e^{6i(dx+c)}+90a\,e^{6i(dx+c)}+140ib\,e^{4i(}\right.}{15d\left(e^{2i(dx+c)}+1\right)}$

input
```
int((a+b*tan(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+b^2/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(tan(d*x+c
)))+a*b*tan(d*x+c)^2/d-a*b/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx$$
$$=\frac{3\,b^2\tan\left(dx+c\right)^5-5\,b^2\tan\left(dx+c\right)^3+15\,ab\tan\left(dx+c\right)^2+15\left(a^2-b^2\right)dx+15\,ab\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+}{15\,d}$$

input
```
integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/15*(3*b^2*tan(d*x + c)^5 - 5*b^2*tan(d*x + c)^3 + 15*a*b*tan(d*x + c)^2
+ 15*(a^2 - b^2)*d*x + 15*a*b*log(1/(tan(d*x + c)^2 + 1)) + 15*b^2*tan(d*x
 + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x-\frac{ab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{ab\tan^2\left(c+dx\right)}{d}-b^2x+\frac{b^2\tan^5\left(c+dx\right)}{5d}-\frac{b^2\tan^3\left(c+dx\right)}{3d}+\frac{b^2\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\left(a+b\tan^3\left(c\right)\right)^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**3)**2,x)
```

output
```
Piecewise((a**2*x - a*b*log(tan(c + d*x)**2 + 1)/d + a*b*tan(c + d*x)**2/d
 - b**2*x + b**2*tan(c + d*x)**5/(5*d) - b**2*tan(c + d*x)**3/(3*d) + b**2
*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**3)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx$$
$$=a^2x+\frac{\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)b^2}{15\,d}$$
$$-\frac{ab\left(\frac{1}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{d}$$

input
```
integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan
(d*x + c))*b^2/d - a*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/
d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx=-\frac{ab\log\left(\tan\left(dx+c\right)^2+1\right)}{d}+\frac{(a^2-b^2)(dx+c)}{d}+\frac{3\,b^2d^4\tan\left(dx+c\right)^5-5\,b^2d^4\tan\left(dx+c\right)^3+15\,abd^4\tan\left(dx+c\right)^2+15\,b^2d^4\tan\left(dx+c\right)}{15\,d^5}$$

input
```
integrate((a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-a*b*log(tan(d*x + c)^2 + 1)/d + (a^2 - b^2)*(d*x + c)/d + 1/15*(3*b^2*d^4
*tan(d*x + c)^5 - 5*b^2*d^4*tan(d*x + c)^3 + 15*a*b*d^4*tan(d*x + c)^2 + 1
5*b^2*d^4*tan(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx=\frac{b^2\tan(c+d\,x)}{d}-\frac{b^2\tan(c+d\,x)^3}{3\,d}+\frac{b^2\tan(c+d\,x)^5}{5\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)\,(a-b)}{a^2-b^2}\right)\,(a+b)\,(a-b)}{d}-\frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}+\frac{a\,b\tan(c+d\,x)^2}{d}$$

input
```
int((a + b*tan(c + d*x)^3)^2,x)
```

output
```
(b^2*tan(c + d*x))/d - (b^2*tan(c + d*x)^3)/(3*d) + (b^2*tan(c + d*x)^5)/(
5*d) + (atan((tan(c + d*x)*(a + b)*(a - b))/(a^2 - b^2))*(a + b)*(a - b))/
d - (a*b*log(tan(c + d*x)^2 + 1))/d + (a*b*tan(c + d*x)^2)/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \left(a+b\tan^3(c+dx)\right)^2\,dx$$
$$=\frac{-15\log\left(\tan\left(dx+c\right)^2+1\right)ab+3\tan\left(dx+c\right)^5b^2-5\tan\left(dx+c\right)^3b^2+15\tan\left(dx+c\right)^2ab+15\tan\left(dx\right.}{15d}$$

input
```
int((a+b*tan(d*x+c)^3)^2,x)
```

output
```
( - 15*log(tan(c + d*x)**2 + 1)*a*b + 3*tan(c + d*x)**5*b**2 - 5*tan(c + d
*x)**3*b**2 + 15*tan(c + d*x)**2*a*b + 15*tan(c + d*x)*b**2 + 15*a**2*d*x
- 15*b**2*d*x)/(15*d)
```

3.377 $\int (a+b\tan^3(c+dx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int (a+b\tan^3(c+dx))\,dx = ax+\frac{b\log(\cos(c+dx))}{d}+\frac{b\tan^2(c+dx)}{2d}$$

output
```
a*x+b*ln(cos(d*x+c))/d+1/2*b*tan(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int (a+b\tan^3(c+dx))\,dx = ax+\frac{b(2\log(\cos(c+dx))+\sec^2(c+dx))}{2d}$$

input
```
Integrate[a + b*Tan[c + d*x]^3,x]
```

output
```
a*x + (b*(2*Log[Cos[c + d*x]] + Sec[c + d*x]^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^3(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b\tan^2(c+dx)}{2d}+\frac{b\log(\cos(c+dx))}{d}$$

input
```
Int[a + b*Tan[c + d*x]^3,x]
```

output
```
a*x + (b*Log[Cos[c + d*x]])/d + (b*Tan[c + d*x]^2)/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$-\frac{b\left(-\tan(dx+c)^{2}+\ln\left(1+\tan(dx+c)^{2}\right)\right)}{2d}+ax$	33
default	$ax+\frac{b\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}$	34
parts	$ax+\frac{b\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}$	34
norman	$ax+\frac{b\tan(dx+c)^{2}}{2d}-\frac{b\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$	36
derivativedivides	$\frac{\frac{b\tan(dx+c)^{2}}{2}-\frac{b\ln\left(1+\tan(dx+c)^{2}\right)}{2}+a\arctan(\tan(dx+c))}{d}$	40
risch	$ax-ibx-\frac{2ibc}{d}+\frac{2b\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	63

input
```
int(a+b*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/d+a*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \left(a+b\tan^{3}(c+dx)\right)\,dx=\frac{2\,adx+b\tan\left(dx+c\right)^{2}+b\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)}{2\,d}$$

input
```
integrate(a+b*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x + b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \left(a+b\tan^3(c+dx)\right)\,dx = ax+b\left(\begin{cases} -\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x\tan^3\left(c\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*tan(d*x+c)**3,x)
```

output
```
a*x + b*Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d)
, Ne(d, 0)), (x*tan(c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \left(a+b\tan^3(c+dx)\right)\,dx = ax-\frac{b\left(\frac{1}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)}{2\,d}$$

input
```
integrate(a+b*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
a*x - 1/2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \left(a+b\tan^3(c+dx)\right)\,dx = ax+\frac{1}{2}\,b\left(\frac{\tan\left(dx+c\right)^2}{d}-\frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{d}\right)$$

input
```
integrate(a+b*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
a*x + 1/2*b*(tan(d*x + c)^2/d - log(tan(d*x + c)^2 + 1)/d)
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(a+b\tan^3(c+dx)\right)\,dx=\frac{\frac{b\tan(c+d\,x)^2}{2}-\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}+a\,d\,x}{d}$$

input
```
int(a + b*tan(c + d*x)^3,x)
```

output
```
((b*tan(c + d*x)^2)/2 - (b*log(tan(c + d*x)^2 + 1))/2 + a*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \left(a+b\tan^3(c+dx)\right)\,dx=\frac{-\log\left(\tan\left(dx+c\right)^2+1\right)b+\tan\left(dx+c\right)^2b+2adx}{2d}$$

input
```
int(a+b*tan(d*x+c)^3,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*b + tan(c + d*x)**2*b + 2*a*d*x)/(2*d)
```

3.378 $\int \frac{1}{a+b\tan^3(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 256

$$\begin{aligned}\int \frac{1}{a+b\tan^3(c+dx)}\,dx &= \frac{ax}{a^2+b^2}+\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^2+b^2\right)d} \\ &\quad -\frac{b\log\left(a\cos^3(c+dx)+b\sin^3(c+dx)\right)}{3\left(a^2+b^2\right)d} \\ &\quad +\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3a^{2/3}\left(a^2+b^2\right)d} \\ &\quad -\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{6a^{2/3}\left(a^2+b^2\right)d}\end{aligned}$$

output
```
a*x/(a^2+b^2)+1/3*b^(1/3)*(a^(4/3)-b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*
tan(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/(a^2+b^2)/d-1/3*b*ln(a*cos(d*
x+c)^3+b*sin(d*x+c)^3)/(a^2+b^2)/d+1/3*b^(1/3)*(a^(4/3)+b^(4/3))*ln(a^(1/3
)+b^(1/3)*tan(d*x+c))/a^(2/3)/(a^2+b^2)/d-1/6*b^(1/3)*(a^(4/3)+b^(4/3))*ln
(a^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b^(2/3)*tan(d*x+c)^2)/a^(2/3)/(a^2+b^2
)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx = \frac{-2\sqrt{3}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-3ia^{5/3}\log(i-\tan(c+dx))+3a^{2/3}b\log(i-\tan(c+dx))+3ia}{}$$

input
```
Integrate[(a + b*Tan[c + d*x]^3)^(-1),x]
```

output
```
(-2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(
1/3))] - (3*I)*a^(5/3)*Log[I - Tan[c + d*x]] + 3*a^(2/3)*b*Log[I - Tan[c +
 d*x]] + (3*I)*a^(5/3)*Log[I + Tan[c + d*x]] + 3*a^(2/3)*b*Log[I + Tan[c +
 d*x]] + 2*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]] - b^(5/3)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2] - 2*a^(2/3)*b*
Log[a + b*Tan[c + d*x]^3] - 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((
b*Tan[c + d*x]^3)/a)]*Tan[c + d*x]^2)/(6*a^(2/3)*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+b\tan(c+dx)^3}dx$$

$$\Big\downarrow \; 4144$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^3(c+dx)+a)}d\tan(c+dx)}{d}$$

$\Big\downarrow$ 7276

$$\frac{\int \left(\frac{a+b\tan(c+dx)}{(a^2+b^2)(\tan^2(c+dx)+1)}-\frac{b(b\tan^2(c+dx)+a\tan(c+dx)-b)}{(a^2+b^2)(b\tan^3(c+dx)+a)}\right)d\tan(c+dx)}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{a\arctan(\tan(c+dx))}{a^2+b^2}-\frac{b\log(a+b\tan^3(c+dx))}{3(a^2+b^2)}+\frac{b\log(\tan^2(c+dx)+1)}{2(a^2+b^2)}+\frac{\sqrt[3]{b}(a^{4/3}-b^{4/3})\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a^2+b^2)}-\frac{\sqrt[3]{b}(a^{4/3}+ \cdots}{\cdots}}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^3)^(-1),x]
```

output
```
((a*ArcTan[Tan[c + d*x]])/(a^2 + b^2) + (b^(1/3)*(a^(4/3) - b^(4/3))*ArcTa
n[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*
(a^2 + b^2)) + (b^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(1/3) + b^(1/3)*Tan[c +
d*x]])/(3*a^(2/3)*(a^2 + b^2)) + (b*Log[1 + Tan[c + d*x]^2])/(2*(a^2 + b^2
)) - (b^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*
x] + b^(2/3)*Tan[c + d*x]^2])/(6*a^(2/3)*(a^2 + b^2)) - (b*Log[a + b*Tan[c
 + d*x]^3])/(3*(a^2 + b^2)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.04

method	result
risch	$\frac{x}{ib+a}+\frac{2iba^2d^3x}{a^4d^3+a^2b^2d^3}+\frac{2iba^2d^2c}{a^4d^3+a^2b^2d^3}+\left(\sum_{\_R=\mathrm{RootOf}\left((27a^4d^3+27a^2b^2d^3)\_Z^3+27\_Z^2a^2bd^2-b\right)}\_R\ln\left(\ldots\right.\right.$
derivativedivides	$\frac{\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}-\frac{\left(-b\left(\frac{\ln\left(\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,a\ldots}{\ldots}\right.\right.}{\ldots}$
default	$\frac{\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}-\frac{\left(-b\left(\frac{\ln\left(\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,a\ldots}{\ldots}\right.\right.}{\ldots}$

input
```
int(1/(a+b*tan(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
x/(a+I*b)+2*I*b*a^2*d^3/(a^4*d^3+a^2*b^2*d^3)*x+2*I*b*a^2*d^2/(a^4*d^3+a^2
*b^2*d^3)*c+sum(_R*ln(exp(2*I*(d*x+c))+(-18/(a^2-b^2)*d^2*a^4-18/(a^2-b^2)
*d^2*b^2*a^2)*_R^2+(6*I/(a^2-b^2)*d*a^3-6*I/(a^2-b^2)*d*b^2*a-6/(a^2-b^2)*
d*b*a^2)*_R+1/(a^2-b^2)*a^2+1/(a^2-b^2)*b^2),_R=RootOf((27*a^4*d^3+27*a^2*
b^2*d^3)*_Z^3+27*_Z^2*a^2*b*d^2-b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 4817, normalized size of antiderivative = 18.82

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx = \int \frac{1}{a+b\tan^3(c+dx)}\,dx$$

input
```
integrate(1/(a+b*tan(d*x+c)**3),x)
```

output
```
Integral(1/(a + b*tan(c + d*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx = -\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\tan(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{18\,(dx+c)a}{a^2+b^2}+\frac{3\left(b\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\tan(c\right.}{a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b}}{18\,d}$$

input
```
integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/18*(2*sqrt(3)*(a*(3*(a/b)^(2/3) - 2) - b*(3*(a/b)^(1/3) - 2*a/b))*arcta
n(-1/3*sqrt(3)*((a/b)^(1/3) - 2*tan(d*x + c))/(a/b)^(1/3))/((a^2*(a/b)^(2/
3) + b^2*(a/b)^(2/3))*(a/b)^(1/3)) - 18*(d*x + c)*a/(a^2 + b^2) + 3*(b*(2*
(a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log(tan(d*x + c)^2 - (a/b)^(1/3)*tan(d*x
 + c) + (a/b)^(2/3))/(a^2*(a/b)^(2/3) + b^2*(a/b)^(2/3)) - 9*b*log(tan(d*x
 + c)^2 + 1)/(a^2 + b^2) + 6*(b*((a/b)^(2/3) - 1) - a*(a/b)^(1/3))*log((a/
b)^(1/3) + tan(d*x + c))/(a^2*(a/b)^(2/3) + b^2*(a/b)^(2/3)))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{1}{a+b\tan^3(c+dx)}\,dx &= \frac{\left(a^3b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+ab^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3d-b^5d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\tan(dx+c)\right|\right)}{3\,(a^5bd^2+2\,a^3b^3d^2+ab^5d^2)} \\ &+\frac{(dx+c)a}{a^2d+b^2d}+\frac{b\log\left(\tan(dx+c)^2+1\right)}{2\,(a^2d+b^2d)}-\frac{b\log\left(\left|b\tan(dx+c)^3+a\right|\right)}{3\,(a^2d+b^2d)} \\ &+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{(\sqrt{3}a^3b+\sqrt{3}ab^3)d} \\ &+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\log\left(\tan(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(a^3b+ab^3)d}\end{aligned}$$

input
```
integrate(1/(a+b*tan(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*(a^3*b^2*d*(-a/b)^(1/3) + a*b^4*d*(-a/b)^(1/3) - a^2*b^3*d - b^5*d)*(-
a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + tan(d*x + c)))/(a^5*b*d^2 + 2*a^3*b^3*d
^2 + a*b^5*d^2) + (d*x + c)*a/(a^2*d + b^2*d) + 1/2*b*log(tan(d*x + c)^2 +
 1)/(a^2*d + b^2*d) - 1/3*b*log(abs(b*tan(d*x + c)^3 + a))/(a^2*d + b^2*d)
 + ((-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3)*((-a/b)^(1/3
) + 2*tan(d*x + c))/(-a/b)^(1/3))/((sqrt(3)*a^3*b + sqrt(3)*a*b^3)*d) + 1/
6*((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(tan(d*x + c)^2 + (-a/b)^(1/3
)*tan(d*x + c) + (-a/b)^(2/3))/((a^3*b + a*b^3)*d)
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.34

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx = \frac{\sum_{k=1}^{3} \ln\left(\mathrm{root}(27\,a^2\,b^2\,z^3+27\,a^4\,z^3+27\,a^2\,b\,z^2-b,z,k\right)\,\left(\mathrm{root}(27\,a^2\,b^2\,z^3+27\,a^4\,z^3+27\,a^2\,b\,z^2-b,z,k\right.}{} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int(1/(a + b*tan(c + d*x)^3),x)
```

output
```
symsum(log(root(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(roo
t(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(root(27*a^2*b^2*z
^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(tan(c + d*x)*(12*b^6 - 69*a^2*b
^4) + root(27*a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k)*(36*a*b^6
 - 180*a^3*b^4 + tan(c + d*x)*(162*a^2*b^5 - 54*a^4*b^3)) - 36*a*b^5 + 27*
a^3*b^3) + 13*a*b^4 - 16*b^5*tan(c + d*x)) + 5*b^4*tan(c + d*x)))*root(27*
a^2*b^2*z^3 + 27*a^4*z^3 + 27*a^2*b*z^2 - b, z, k), k, 1, 3)/d + log(tan(c
 + d*x) - 1i)/(2*d*(a*1i + b)) + (log(tan(c + d*x) + 1i)*1i)/(2*d*(a + b*1
i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.16

$$\int \frac{1}{a+b\tan^3(c+dx)}\,dx$$
$$= \frac{-2b^{\frac{7}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}\tan(dx+c)}{a^{\frac{1}{3}}\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}\tan(dx+c)}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2b-b^{\frac{7}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}\tan\left(dx+c\right.\right.}{}$$

input
```
int(1/(a+b*tan(d*x+c)^3),x)
```

output
```
( - 2*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*tan(c + d*x))/
(a**(1/3)*sqrt(3)))*b**2 + 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*tan(c + d
*x))/(a**(1/3)*sqrt(3)))*a**2*b - b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3
)*a**(1/3)*tan(c + d*x) + b**(2/3)*tan(c + d*x)**2)*b**2 + 2*b**(1/3)*a**(
2/3)*log(a**(1/3) + b**(1/3)*tan(c + d*x))*b**2 + 3*b**(2/3)*a**(1/3)*log(
tan(c + d*x)**2 + 1)*a*b - 2*b**(2/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**
(1/3)*tan(c + d*x) + b**(2/3)*tan(c + d*x)**2)*a*b - 2*b**(2/3)*a**(1/3)*l
og(a**(1/3) + b**(1/3)*tan(c + d*x))*a*b + 6*b**(2/3)*a**(1/3)*a**2*d*x -
log(a**(2/3) - b**(1/3)*a**(1/3)*tan(c + d*x) + b**(2/3)*tan(c + d*x)**2)*
a**2*b + 2*log(a**(1/3) + b**(1/3)*tan(c + d*x))*a**2*b)/(6*b**(2/3)*a**(1
/3)*a*d*(a**2 + b**2))
```

3.379 $\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 558

$$\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx$$
$$= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}-b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,(a^2+b^2)^2\,d}$$
$$+ \frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\,(a^2+b^2)\,d}$$
$$- \frac{2ab\log\left(a\cos^3(c+dx)+b\sin^3(c+dx)\right)}{3\,(a^2+b^2)^2\,d}$$
$$+ \frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}-b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{3\sqrt[3]{a}\,(a^2+b^2)^2\,d}$$
$$+ \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tan(c+dx)\right)}{9a^{5/3}\,(a^2+b^2)\,d}$$
$$- \frac{\sqrt[3]{b}\left(a^2+2a^{2/3}b^{4/3}-b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{6\sqrt[3]{a}\,(a^2+b^2)^2\,d}$$
$$- \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tan(c+dx)+b^{2/3}\tan^2(c+dx)\right)}{18a^{5/3}\,(a^2+b^2)\,d}$$
$$+ \frac{b(a+\tan(c+dx)(b-a\tan(c+dx)))}{3a\,(a^2+b^2)\,d\,\left(a+b\tan^3(c+dx)\right)}$$

output
```
(a^2-b^2)*x/(a^2+b^2)^2+1/3*b^(1/3)*(a^2-2*a^(2/3)*b^(4/3)-b^2)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*tan(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/(a^2+b^2)
^2/d+1/9*b^(1/3)*(a^(4/3)-2*b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*tan(d*x
+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/(a^2+b^2)/d-2/3*a*b*ln(a*cos(d*x+c)^
3+b*sin(d*x+c)^3)/(a^2+b^2)^2/d+1/3*b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)-b^2)*ln
(a^(1/3)+b^(1/3)*tan(d*x+c))/a^(1/3)/(a^2+b^2)^2/d+1/9*b^(1/3)*(a^(4/3)+2*
b^(4/3))*ln(a^(1/3)+b^(1/3)*tan(d*x+c))/a^(5/3)/(a^2+b^2)/d-1/6*b^(1/3)*(a
^2+2*a^(2/3)*b^(4/3)-b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b^(2/3)*ta
n(d*x+c)^2)/a^(1/3)/(a^2+b^2)^2/d-1/18*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(2
/3)-a^(1/3)*b^(1/3)*tan(d*x+c)+b^(2/3)*tan(d*x+c)^2)/a^(5/3)/(a^2+b^2)/d+1
/3*b*(a+tan(d*x+c)*(b-a*tan(d*x+c)))/a/(a^2+b^2)/d/(a+b*tan(d*x+c)^3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.83 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx$$
$$= \frac{-\frac{12\sqrt{3}\sqrt[3]{a}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{(a^2+b^2)^2}-\frac{4\sqrt{3}b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}(a^2+b^2)}-\frac{9i\log(i-\tan(c+dx))}{(a-ib)^2}+\frac{9i\log(i+\tan(c+}{(a+ib)^2}}{}$$

input
```
Integrate[(a + b*Tan[c + d*x]^3)^(-2),x]
```

output
```
((-12*Sqrt[3]*a^(1/3)*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(S
qrt[3]*a^(1/3))])/(a^2 + b^2)^2 - (4*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b
^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(a^(5/3)*(a^2 + b^2)) - ((9*I)*Lo
g[I - Tan[c + d*x]])/(a - I*b)^2 + ((9*I)*Log[I + Tan[c + d*x]])/(a + I*b)
^2 + (12*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(a^2 + b^2)^
2 + (4*b^(5/3)*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(a^(5/3)*(a^2 + b^2))
- (6*a^(1/3)*b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*
Tan[c + d*x]^2])/(a^2 + b^2)^2 - (2*b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/(a^(5/3)*(a^2 + b^2)) - (12*a*b*Lo
g[a + b*Tan[c + d*x]^3])/(a^2 + b^2)^2 + (9*b*(-a^2 + b^2)*Hypergeometric2
F1[2/3, 1, 5/3, -((b*Tan[c + d*x]^3)/a)]*Tan[c + d*x]^2)/(a*(a^2 + b^2)^2)
 - (9*b*Hypergeometric2F1[2/3, 2, 5/3, -((b*Tan[c + d*x]^3)/a)]*Tan[c + d*
x]^2)/(a*(a^2 + b^2)) + (6*b)/((a^2 + b^2)*(a + b*Tan[c + d*x]^3)) + (6*b^
2*Tan[c + d*x])/(a*(a^2 + b^2)*(a + b*Tan[c + d*x]^3)))/(18*d)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\left(a+b\tan(c+dx)^3\right)^2}dx$$

$\Big\downarrow$ 4144

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^3(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$\Big\downarrow$ 7276

$$\frac{\int \left(\frac{a^2+2b\tan(c+dx)a-b^2}{(a^2+b^2)^2(\tan^2(c+dx)+1)}+\frac{b(-2ab\tan^2(c+dx)-(a^2-b^2)\tan(c+dx)+2ab)}{(a^2+b^2)^2(b\tan^3(c+dx)+a)}-\frac{b(b\tan^2(c+dx)+a\tan(c+dx)-b)}{(a^2+b^2)(b\tan^3(c+dx)+a)^2}\right)d\tan(c+dx)}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(a^2-b^2)\arctan(\tan(c+dx))}{(a^2+b^2)^2}+\frac{b(\tan(c+dx)(b-a\tan(c+dx))+a)}{3a(a^2+b^2)(a+b\tan^3(c+dx))}-\frac{2ab\log(a+b\tan^3(c+dx))}{3(a^2+b^2)^2}+\frac{ab\log(\tan^2(c+dx)+1)}{(a^2+b^2)^2}+\frac{\sqrt[3]{b}(a^{4/3}-2b^{4/3}}{}}{}$$

input
```
Int[(a + b*Tan[c + d*x]^3)^(-2),x]
```

output
```
(((a^2 - b^2)*ArcTan[Tan[c + d*x]])/(a^2 + b^2)^2 + (b^(1/3)*(a^2 - 2*a^(2
/3)*b^(4/3) - b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(1/3)*(a^2 + b^2)^2) + (b^(1/3)*(a^(4/3) - 2*b^(4/3))*Ar
cTan[(a^(1/3) - 2*b^(1/3)*Tan[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
5/3)*(a^2 + b^2)) + (b^(1/3)*(a^2 + 2*a^(2/3)*b^(4/3) - b^2)*Log[a^(1/3) +
 b^(1/3)*Tan[c + d*x]])/(3*a^(1/3)*(a^2 + b^2)^2) + (b^(1/3)*(a^(4/3) + 2*
b^(4/3))*Log[a^(1/3) + b^(1/3)*Tan[c + d*x]])/(9*a^(5/3)*(a^2 + b^2)) + (a
*b*Log[1 + Tan[c + d*x]^2])/(a^2 + b^2)^2 - (b^(1/3)*(a^2 + 2*a^(2/3)*b^(4
/3) - b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*
x]^2])/(6*a^(1/3)*(a^2 + b^2)^2) - (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(2
/3) - a^(1/3)*b^(1/3)*Tan[c + d*x] + b^(2/3)*Tan[c + d*x]^2])/(18*a^(5/3)*
(a^2 + b^2)) - (2*a*b*Log[a + b*Tan[c + d*x]^3])/(3*(a^2 + b^2)^2) + (b*(a
 + Tan[c + d*x]*(b - a*Tan[c + d*x])))/(3*a*(a^2 + b^2)*(a + b*Tan[c + d*x
]^3)))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{ab\ln\left(1+\tan(dx+c)^2\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{a^4+2a^2b^2+b^4} - \frac{b\left(\frac{\left(\frac{a^2}{3}+\frac{b^2}{3}\right)\tan(dx+c)^2-\frac{b\left(a^2+b^2\right)\tan(dx+c)}{3a}-\frac{a^2}{3}-\frac{b^2}{3}}{a+b\tan(dx+c)^3}+\frac{2\left(-4a^2b-b^3\right.}{}\right.}{}}{}$
default	$\frac{\frac{ab\ln\left(1+\tan(dx+c)^2\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{a^4+2a^2b^2+b^4} - \frac{b\left(\frac{\left(\frac{a^2}{3}+\frac{b^2}{3}\right)\tan(dx+c)^2-\frac{b\left(a^2+b^2\right)\tan(dx+c)}{3a}-\frac{a^2}{3}-\frac{b^2}{3}}{a+b\tan(dx+c)^3}+\frac{2\left(-4a^2b-b^3\right.}{}\right.}{}}{}$
risch	Expression too large to display

input
```
int(1/(a+b*tan(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^4+2*a^2*b^2+b^4)*(a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*
x+c)))-b/(a^4+2*a^2*b^2+b^4)*(((1/3*a^2+1/3*b^2)*tan(d*x+c)^2-1/3*b*(a^2+b
^2)/a*tan(d*x+c)-1/3*a^2-1/3*b^2)/(a+b*tan(d*x+c)^3)+2/3/a*((-4*a^2*b-b^3)
*(1/3/b/(a/b)^(2/3)*ln(tan(d*x+c)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(tan(d*
x+c)^2-(a/b)^(1/3)*tan(d*x+c)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*tan(d*x+c)-1)))+(2*a^3-a*b^2)*(-1/3/b/(a/b)^(
1/3)*ln(tan(d*x+c)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(tan(d*x+c)^2-(a/b)^(1
/3)*tan(d*x+c)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*tan(d*x+c)-1)))+a^2*ln(a+b*tan(d*x+c)^3))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 11554, normalized size of antiderivative = 20.71

$$\int \frac{1}{(a+b\tan^3(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\tan^3(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*tan(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/9*(9*a*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*sqrt(3)*(2*
a^3*((a/b)^(2/3) - 1) - 2*a^2*b*(2*(a/b)^(1/3) - a/b) - a*b^2*(a/b)^(2/3)
- b^3*(a/b)^(1/3))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*tan(d*x + c))/(a/b
)^(1/3))/((a^5*(a/b)^(2/3) + 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3))*(a
/b)^(1/3)) + 9*(a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (2*a^2*b*(3
*(a/b)^(2/3) + 2) + 2*a^3*(a/b)^(1/3) - a*b^2*(a/b)^(1/3) + b^3)*log(tan(d
*x + c)^2 - (a/b)^(1/3)*tan(d*x + c) + (a/b)^(2/3))/(a^5*(a/b)^(2/3) + 2*a
^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3)) - 2*(a^2*b*(3*(a/b)^(2/3) - 4) - 2
*a^3*(a/b)^(1/3) + a*b^2*(a/b)^(1/3) - b^3)*log((a/b)^(1/3) + tan(d*x + c)
)/(a^5*(a/b)^(2/3) + 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2/3)) - 3*(a*b*t
an(d*x + c)^2 - b^2*tan(d*x + c) - a*b)/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*t
an(d*x + c)^3))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+b\tan^3(c+dx)\right)^2}\,dx$$
$$= \frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4d+2a^2b^2d+b^4d} - \frac{2ab\log\left(\left|b\tan(dx+c)^3+a\right|\right)}{3\left(a^4d+2a^2b^2d+b^4d\right)}$$
$$+ \frac{2\left(2a^8b^2d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3a^6b^4d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^8d\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4a^7b^3d-9a^5b^5d-6a^3b^7d-ab^9d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\right.\right.}{9\left(a^{11}bd^2+4a^9b^3d^2+6a^7b^5d^2+4a^5b^7d^2+a^3b^9d^2\right)}$$
$$+ \frac{(a^2-b^2)(dx+c)}{a^4d+2a^2b^2d+b^4d}$$
$$+ \frac{2\left(\left(2\sqrt{3}a^3-\sqrt{3}ab^2\right)\left(-ab^2\right)^{\frac{2}{3}}+\left(4\sqrt{3}a^2b^2+\sqrt{3}b^4\right)\left(-ab^2\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\tan(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(a^6b+2a^4b^3+a^2b^5\right)d}$$
$$- \frac{\left((2a^3-ab^2)(-ab^2)^{\frac{2}{3}}-(4a^2b^2+b^4)(-ab^2)^{\frac{1}{3}}\right)\log\left(\tan(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\tan(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(a^6b+2a^4b^3+a^2b^5\right)d}$$
$$+ \frac{a^3b+ab^3-(a^3b+ab^3)\tan(dx+c)^2+(a^2b^2+b^4)\tan(dx+c)}{3\left(b\tan(dx+c)^3+a\right)(a^2+b^2)^2ad}$$

input
```
integrate(1/(a+b*tan(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
a*b*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 2/3*a*b*log(ab
s(b*tan(d*x + c)^3 + a))/(a^4*d + 2*a^2*b^2*d + b^4*d) + 2/9*(2*a^8*b^2*d*
(-a/b)^(1/3) + 3*a^6*b^4*d*(-a/b)^(1/3) - a^2*b^8*d*(-a/b)^(1/3) - 4*a^7*b
^3*d - 9*a^5*b^5*d - 6*a^3*b^7*d - a*b^9*d)*(-a/b)^(1/3)*log(abs(-(-a/b)^(
1/3) + tan(d*x + c)))/(a^11*b*d^2 + 4*a^9*b^3*d^2 + 6*a^7*b^5*d^2 + 4*a^5*
b^7*d^2 + a^3*b^9*d^2) + (a^2 - b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*
d) + 2/9*((2*sqrt(3)*a^3 - sqrt(3)*a*b^2)*(-a*b^2)^(2/3) + (4*sqrt(3)*a^2*
b^2 + sqrt(3)*b^4)*(-a*b^2)^(1/3))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*ta
n(d*x + c))/(-a/b)^(1/3))/((a^6*b + 2*a^4*b^3 + a^2*b^5)*d) - 1/9*((2*a^3
- a*b^2)*(-a*b^2)^(2/3) - (4*a^2*b^2 + b^4)*(-a*b^2)^(1/3))*log(tan(d*x +
c)^2 + (-a/b)^(1/3)*tan(d*x + c) + (-a/b)^(2/3))/((a^6*b + 2*a^4*b^3 + a^2
*b^5)*d) + 1/3*(a^3*b + a*b^3 - (a^3*b + a*b^3)*tan(d*x + c)^2 + (a^2*b^2
+ b^4)*tan(d*x + c))/((b*tan(d*x + c)^3 + a)*(a^2 + b^2)^2*a*d)
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.77

$$\int \frac{1}{(a+b\tan^3(c+dx))^2} dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(c + d*x)^3)^2,x)
```

output
```
symsum(log(root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^
6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k)*(((32*a*b^7)/27 - (128*a
^3*b^5)/27)/(a^7 + a^3*b^4 + 2*a^5*b^2) - root(1458*a^7*b^2*z^3 + 729*a^5*
b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3,
 z, k)*(root(1458*a^7*b^2*z^3 + 729*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b
*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3, z, k)*((16*a^3*b^9 + 77*a^5*b^7 +
 34*a^7*b^5 - 27*a^9*b^3)/(a^7 + a^3*b^4 + 2*a^5*b^2) + root(1458*a^7*b^2*
z^3 + 729*a^5*b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*
a^2*b - 8*b^3, z, k)*((108*a^6*b^8 - 36*a^4*b^10 + 324*a^8*b^6 + 180*a^10*
b^4)/(a^7 + a^3*b^4 + 2*a^5*b^2) - (tan(c + d*x)*(4374*a^5*b^9 + 7290*a^7*
b^7 + 1458*a^9*b^5 - 1458*a^11*b^3))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) - (
tan(c + d*x)*(216*a^2*b^10 + 864*a^4*b^8 - 1836*a^6*b^6 - 2484*a^8*b^4))/(
27*(a^7 + a^3*b^4 + 2*a^5*b^2))) - ((64*a^2*b^8)/9 + (353*a^4*b^6)/9 + (38
8*a^6*b^4)/9)/(a^7 + a^3*b^4 + 2*a^5*b^2) + (tan(c + d*x)*(96*a*b^9 + 408*
a^3*b^7 + 447*a^5*b^5))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) + (tan(c + d*x)*
(134*a^2*b^6 - 16*b^8 + 236*a^4*b^4))/(27*(a^7 + a^3*b^4 + 2*a^5*b^2))) -
((8*b^6)/27 + (16*a^2*b^4)/27)/(a^7 + a^3*b^4 + 2*a^5*b^2) - (8*a*b^5*tan(
c + d*x))/(9*(a^7 + a^3*b^4 + 2*a^5*b^2)))*root(1458*a^7*b^2*z^3 + 729*a^5
*b^4*z^3 + 729*a^9*z^3 + 1458*a^6*b*z^2 + 108*a^3*b^2*z - 64*a^2*b - 8*b^3
, z, k), k, 1, 3)/d + (log(tan(c + d*x) - 1i)*1i)/(2*d*(a*b*2i - a^2 + ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1262, normalized size of antiderivative = 2.26

$$\int \frac{1}{(a+b\tan^3(c+dx))^2} dx = \text{Too large to display}$$

input
```
int(1/(a+b*tan(d*x+c)^3)^2,x)
```

output
```
( - 8*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*tan(c + d*x))/
(a**(1/3)*sqrt(3)))*tan(c + d*x)**3*a**2*b**3 - 2*b**(1/3)*a**(2/3)*sqrt(3
)*atan((a**(1/3) - 2*b**(1/3)*tan(c + d*x))/(a**(1/3)*sqrt(3)))*tan(c + d*
x)**3*b**5 - 8*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*tan(c
 + d*x))/(a**(1/3)*sqrt(3)))*a**3*b**2 - 2*b**(1/3)*a**(2/3)*sqrt(3)*atan(
(a**(1/3) - 2*b**(1/3)*tan(c + d*x))/(a**(1/3)*sqrt(3)))*a*b**4 + 4*sqrt(3
)*atan((a**(1/3) - 2*b**(1/3)*tan(c + d*x))/(a**(1/3)*sqrt(3)))*tan(c + d*
x)**3*a**4*b**2 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*tan(c + d*x))/(a**
(1/3)*sqrt(3)))*tan(c + d*x)**3*a**2*b**4 + 4*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*tan(c + d*x))/(a**(1/3)*sqrt(3)))*a**5*b - 2*sqrt(3)*atan((a**(1/3
) - 2*b**(1/3)*tan(c + d*x))/(a**(1/3)*sqrt(3)))*a**3*b**3 - 4*b**(1/3)*a*
*(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*tan(c + d*x) + b**(2/3)*tan(c + d*
x)**2)*tan(c + d*x)**3*a**2*b**3 - b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/
3)*a**(1/3)*tan(c + d*x) + b**(2/3)*tan(c + d*x)**2)*tan(c + d*x)**3*b**5
- 4*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*tan(c + d*x) + b**(
2/3)*tan(c + d*x)**2)*a**3*b**2 - b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3
)*a**(1/3)*tan(c + d*x) + b**(2/3)*tan(c + d*x)**2)*a*b**4 + 8*b**(1/3)*a*
*(2/3)*log(a**(1/3) + b**(1/3)*tan(c + d*x))*tan(c + d*x)**3*a**2*b**3 + 2
*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*tan(c + d*x))*tan(c + d*x)**3*b
**5 + 8*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*tan(c + d*x))*a**3*b*...
```

3.380 $\int \frac{1}{1+\tan^3(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{1}{1+\tan^3(x)}\,dx = \frac{x}{2}-\frac{1}{2}\log(\cos(x))+\frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)$$

output
```
1/2*x-1/2*ln(cos(x))+1/6*ln(1+tan(x))-1/3*ln(1-tan(x)+tan(x)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{1}{1+\tan^3(x)}\,dx = \left(\frac{1}{4}-\frac{i}{4}\right)\log(i-\tan(x))+\left(\frac{1}{4}+\frac{i}{4}\right)\log(i+\tan(x)) + \frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)$$

input
```
Integrate[(1 + Tan[x]^3)^(-1),x]
```

output
```
(1/4 - I/4)*Log[I - Tan[x]] + (1/4 + I/4)*Log[I + Tan[x]] + Log[1 + Tan[x]
]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan^3(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(x)^3+1}dx$$
$$\Big\downarrow \text{4144}$$
$$\int \frac{1}{\left(\tan^2(x)+1\right)\left(\tan^3(x)+1\right)}d\tan(x)$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{1-2\tan(x)}{3\left(\tan^2(x)-\tan(x)+1\right)}+\frac{\tan(x)+1}{2\left(\tan^2(x)+1\right)}+\frac{1}{6(\tan(x)+1)}\right)d\tan(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\arctan(\tan(x))+\frac{1}{4}\log\left(\tan^2(x)+1\right)-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)$$

input
```
Int[(1 + Tan[x]^3)^(-1),x]
```

output
```
ArcTan[Tan[x]]/2 + Log[1 + Tan[x]]/6 + Log[1 + Tan[x]^2]/4 - Log[1 - Tan[x
] + Tan[x]^2]/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
norman	$\frac{x}{2}+\frac{\ln\left(1+\tan\left(x\right)\right)}{6}+\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{4}-\frac{\ln\left(1-\tan\left(x\right)+\tan\left(x\right)^{2}\right)}{3}$	34
parallelrisch	$\frac{x}{2}+\frac{\ln\left(1+\tan\left(x\right)\right)}{6}+\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{4}-\frac{\ln\left(1-\tan\left(x\right)+\tan\left(x\right)^{2}\right)}{3}$	34
derivativedivides	$\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{4}+\frac{\arctan\left(\tan\left(x\right)\right)}{2}+\frac{\ln\left(1+\tan\left(x\right)\right)}{6}-\frac{\ln\left(1-\tan\left(x\right)+\tan\left(x\right)^{2}\right)}{3}$	36
default	$\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{4}+\frac{\arctan\left(\tan\left(x\right)\right)}{2}+\frac{\ln\left(1+\tan\left(x\right)\right)}{6}-\frac{\ln\left(1-\tan\left(x\right)+\tan\left(x\right)^{2}\right)}{3}$	36
risch	$\frac{x}{2}+\frac{ix}{2}+\frac{\ln\left({\mathrm e}^{2 i x}+i\right)}{6}-\frac{\ln\left({\mathrm e}^{4 i x}-4 i {\mathrm e}^{2 i x}-1\right)}{3}$	38

input
```
int(1/(1+tan(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/6*ln(1+tan(x))+1/4*ln(1+tan(x)^2)-1/3*ln(1-tan(x)+tan(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{1}{1+\tan^3(x)}\,dx = \frac{1}{2}\,x + \frac{1}{12}\,\log\left(\frac{\tan\left(x\right)^2 + 2\,\tan\left(x\right) + 1}{\tan\left(x\right)^2 + 1}\right) - \frac{1}{3}\,\log\left(\frac{\tan\left(x\right)^2 - \tan\left(x\right) + 1}{\tan\left(x\right)^2 + 1}\right)$$

input
```
integrate(1/(1+tan(x)^3),x, algorithm="fricas")
```

output
```
1/2*x + 1/12*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/3*log((tan(
x)^2 - tan(x) + 1)/(tan(x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+\tan^3(x)}\,dx = \frac{x}{2} + \frac{\log\left(\tan\left(x\right) + 1\right)}{6} + \frac{\log\left(\tan^2\left(x\right) + 1\right)}{4} - \frac{\log\left(\tan^2\left(x\right) - \tan\left(x\right) + 1\right)}{3}$$

input
```
integrate(1/(1+tan(x)**3),x)
```

output
```
x/2 + log(tan(x) + 1)/6 + log(tan(x)**2 + 1)/4 - log(tan(x)**2 - tan(x) +
1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+\tan^3(x)}\,dx = \frac{1}{2}\,x - \frac{1}{3}\,\log\left(\tan\left(x\right)^2 - \tan\left(x\right) + 1\right) + \frac{1}{4}\,\log\left(\tan\left(x\right)^2 + 1\right) + \frac{1}{6}\,\log\left(\tan\left(x\right) + 1\right)$$

input
```
integrate(1/(1+tan(x)^3),x, algorithm="maxima")
```

output
```
1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(t
an(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+\tan^3(x)}\,dx = \frac{1}{2}\,x - \frac{1}{3}\,\log\left(\tan\left(x\right)^2 - \tan\left(x\right) + 1\right) + \frac{1}{4}\,\log\left(\tan\left(x\right)^2 + 1\right) + \frac{1}{6}\,\log\left(\left|\tan\left(x\right) + 1\right|\right)$$

input
```
integrate(1/(1+tan(x)^3),x, algorithm="giac")
```

output
```
1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(a
bs(tan(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{1}{1+\tan^3(x)}\,dx = \frac{\ln\left(\tan(x)+1\right)}{6} - \frac{\ln\left(\tan(x)^2-\tan(x)+1\right)}{3} + \ln\left(\tan(x)-\mathrm{i}\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln\left(\tan(x)+1\mathrm{i}\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(tan(x)^3 + 1),x)
```

output
```
log(tan(x) + 1)/6 - log(tan(x)^2 - tan(x) + 1)/3 + log(tan(x) - 1i)*(1/4 -
 1i/4) + log(tan(x) + 1i)*(1/4 + 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+\tan^3(x)}\,dx = -\frac{\log\left(\tan(x)^2-\tan(x)+1\right)}{3} + \frac{\log\left(\tan(x)^2+1\right)}{4} + \frac{\log\left(\tan(x)+1\right)}{6} + \frac{x}{2}$$

input
```
int(1/(1+tan(x)^3),x)
```

output
```
( - 4*log(tan(x)**2 - tan(x) + 1) + 3*log(tan(x)**2 + 1) + 2*log(tan(x) +
1) + 6*x)/12
```

3.381 $\int \left(a+b\tan^4(c+dx)\right)^4 dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int \left(a+b\tan^4(c+dx)\right)^4 dx = (a+b)^4x - \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d} + \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx)}{3d} - \frac{b^2(6a^2+4ab+b^2)\tan^5(c+dx)}{5d} + \frac{b^2(6a^2+4ab+b^2)\tan^7(c+dx)}{7d} - \frac{b^3(4a+b)\tan^9(c+dx)}{9d} + \frac{b^3(4a+b)\tan^{11}(c+dx)}{11d} - \frac{b^4\tan^{13}(c+dx)}{13d} + \frac{b^4\tan^{15}(c+dx)}{15d}$$

output
```
(a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b*(2*a+b)*(2*a^2+2*
a*b+b^2)*tan(d*x+c)^3/d-1/5*b^2*(6*a^2+4*a*b+b^2)*tan(d*x+c)^5/d+1/7*b^2*(
6*a^2+4*a*b+b^2)*tan(d*x+c)^7/d-1/9*b^3*(4*a+b)*tan(d*x+c)^9/d+1/11*b^3*(4
*a+b)*tan(d*x+c)^11/d-1/13*b^4*tan(d*x+c)^13/d+1/15*b^4*tan(d*x+c)^15/d
```

Mathematica [A] (verified)

Time = 3.67 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.91

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx=\frac{(a+b)^4\arctan(\tan(c+dx))}{d}$$
$$+\frac{b\tan(c+dx)\left(-45045(4a^3+6a^2b+4ab^2+b^3)+15015(4a^3+6a^2b+4ab^2+b^3)\tan^2(c+dx)-9009\right.}{}$$

input
```
Integrate[(a + b*Tan[c + d*x]^4)^4,x]
```

output
```
((a + b)^4*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-45045*(4*a^3 + 6*a^
2*b + 4*a*b^2 + b^3) + 15015*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*Tan[c + d*x
]^2 - 9009*b*(6*a^2 + 4*a*b + b^2)*Tan[c + d*x]^4 + 6435*b*(6*a^2 + 4*a*b
+ b^2)*Tan[c + d*x]^6 - 5005*b^2*(4*a + b)*Tan[c + d*x]^8 + 4095*b^2*(4*a
+ b)*Tan[c + d*x]^10 - 3465*b^3*Tan[c + d*x]^12 + 3003*b^3*Tan[c + d*x]^14
))/(45045*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\tan(c+dx)^4\right)^4dx$$
$$\Big\downarrow \text{4144}$$
$$\frac{\int \frac{\left(b\tan^4(c+dx)+a\right)^4}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1468}$$

$$\int \Big(b^4\tan^{14}(c+dx)-b^4\tan^{12}(c+dx)+b^3(4a+b)\tan^{10}(c+dx)-b^3(4a+b)\tan^8(c+dx)+b^2(6a^2+4ba+b^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}b^2\left(6a^2+4ab+b^2\right)\tan^7(c+dx)-\frac{1}{5}b^2\left(6a^2+4ab+b^2\right)\tan^5(c+dx)+\frac{1}{3}b(2a+b)\left(2a^2+2ab+b^2\right)\tan^3(c+dx$$

input
```
Int[(a + b*Tan[c + d*x]^4)^4,x]
```

output
```
((a + b)^4*ArcTan[Tan[c + d*x]] - b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c
+ d*x] + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x]^3)/3 - (b^2*(6*a^
2 + 4*a*b + b^2)*Tan[c + d*x]^5)/5 + (b^2*(6*a^2 + 4*a*b + b^2)*Tan[c + d*
x]^7)/7 - (b^3*(4*a + b)*Tan[c + d*x]^9)/9 + (b^3*(4*a + b)*Tan[c + d*x]^1
1)/11 - (b^4*Tan[c + d*x]^13)/13 + (b^4*Tan[c + d*x]^15)/15)/d
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

method	result
norman	$(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)\,x-\frac{b^4\tan(dx+c)^{13}}{13d}+\frac{b^4\tan(dx+c)^{15}}{15d}-\frac{b(4a^3+6a^2b+4a\,b^2+b^3)\ \mathrm{ta}}{d}$
parts	$a^4x+\frac{b^4\left(\frac{\tan(dx+c)^{15}}{15}-\frac{\tan(dx+c)^{13}}{13}+\frac{\tan(dx+c)^{11}}{11}-\frac{\tan(dx+c)^{9}}{9}+\frac{\tan(dx+c)^{7}}{7}-\frac{\tan(dx+c)^{5}}{5}+\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)\right.}{d}$
derivativedivides	$\frac{b^4\tan(dx+c)^{15}}{15}-\frac{b^4\tan(dx+c)^{13}}{13}+\frac{4a\,b^3\tan(dx+c)^{11}}{11}+\frac{b^4\tan(dx+c)^{11}}{11}-\frac{4a\,b^3\tan(dx+c)^{9}}{9}-\frac{b^4\tan(dx+c)^{9}}{9}+\frac{6a^2b^2\tan(dx+c)^{7}}{7}$
default	$\frac{b^4\tan(dx+c)^{15}}{15}-\frac{b^4\tan(dx+c)^{13}}{13}+\frac{4a\,b^3\tan(dx+c)^{11}}{11}+\frac{b^4\tan(dx+c)^{11}}{11}-\frac{4a\,b^3\tan(dx+c)^{9}}{9}-\frac{b^4\tan(dx+c)^{9}}{9}+\frac{6a^2b^2\tan(dx+c)^{7}}{7}$
parallelrisch	$4095b^4\tan(dx+c)^{11}-5005b^4\tan(dx+c)^{9}+15015b^4\tan(dx+c)^{3}-45045\tan(dx+c)b^4+6435b^4\tan(dx+c)^{7}-9009b^4\tan$
risch	Expression too large to display

input
```
int((a+b*tan(d*x+c)^4)^4,x,method=_RETURNVERBOSE)
```

output
```
(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*x-1/13*b^4*tan(d*x+c)^13/d+1/15*b^4*ta
n(d*x+c)^15/d-b*(4*a^3+6*a^2*b+4*a*b^2+b^3)/d*tan(d*x+c)+1/3*b*(4*a^3+6*a^
2*b+4*a*b^2+b^3)/d*tan(d*x+c)^3-1/5*b^2*(6*a^2+4*a*b+b^2)*tan(d*x+c)^5/d+1
/7*b^2*(6*a^2+4*a*b+b^2)*tan(d*x+c)^7/d-1/9*b^3*(4*a+b)*tan(d*x+c)^9/d+1/1
1*b^3*(4*a+b)*tan(d*x+c)^11/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx$$
$$=\frac{3003\,b^4\tan\left(dx+c\right)^{15}-3465\,b^4\tan\left(dx+c\right)^{13}+4095\left(4\,ab^3+b^4\right)\tan\left(dx+c\right)^{11}-5005\left(4\,ab^3+b^4\right)\tan}{}$$

input
```
integrate((a+b*tan(d*x+c)^4)^4,x, algorithm="fricas")
```

output
```
1/45045*(3003*b^4*tan(d*x + c)^15 - 3465*b^4*tan(d*x + c)^13 + 4095*(4*a*b
^3 + b^4)*tan(d*x + c)^11 - 5005*(4*a*b^3 + b^4)*tan(d*x + c)^9 + 6435*(6*
a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^7 - 9009*(6*a^2*b^2 + 4*a*b^3 + b^4)
*tan(d*x + c)^5 + 15015*(4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)
^3 + 45045*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x - 45045*(4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.79

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx$$
$$= \begin{cases} a^4x+4a^3bx+\frac{4a^3b\tan^3(c+dx)}{3d}-\frac{4a^3b\tan(c+dx)}{d}+6a^2b^2x+\frac{6a^2b^2\tan^7(c+dx)}{7d}-\frac{6a^2b^2\tan^5(c+dx)}{5d}+\frac{2a^2b^2\tan^3(c+dx)}{d} \\ x\left(a+b\tan^4(c)\right)^4 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**4)**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*x + 4*a**3*b*tan(c + d*x)**3/(3*d) - 4*a**3*b
*tan(c + d*x)/d + 6*a**2*b**2*x + 6*a**2*b**2*tan(c + d*x)**7/(7*d) - 6*a*
*2*b**2*tan(c + d*x)**5/(5*d) + 2*a**2*b**2*tan(c + d*x)**3/d - 6*a**2*b**
2*tan(c + d*x)/d + 4*a*b**3*x + 4*a*b**3*tan(c + d*x)**11/(11*d) - 4*a*b**
3*tan(c + d*x)**9/(9*d) + 4*a*b**3*tan(c + d*x)**7/(7*d) - 4*a*b**3*tan(c
+ d*x)**5/(5*d) + 4*a*b**3*tan(c + d*x)**3/(3*d) - 4*a*b**3*tan(c + d*x)/d
 + b**4*x + b**4*tan(c + d*x)**15/(15*d) - b**4*tan(c + d*x)**13/(13*d) +
b**4*tan(c + d*x)**11/(11*d) - b**4*tan(c + d*x)**9/(9*d) + b**4*tan(c + d
*x)**7/(7*d) - b**4*tan(c + d*x)**5/(5*d) + b**4*tan(c + d*x)**3/(3*d) - b
**4*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.23

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx = a^4x + \frac{4\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^3b}{3\,d}$$
$$+\frac{2\left(15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+105\,dx+105\,c-105\,\tan\left(dx+c\right)\right)a^2b^2}{35\,d}$$
$$+\frac{4\left(315\,\tan\left(dx+c\right)^{11}-385\,\tan\left(dx+c\right)^9+495\,\tan\left(dx+c\right)^7-693\,\tan\left(dx+c\right)^5+1155\,\tan\left(dx+c\right)\right.}{3465\,d}$$
$$+\frac{\left(3003\,\tan\left(dx+c\right)^{15}-3465\,\tan\left(dx+c\right)^{13}+4095\,\tan\left(dx+c\right)^{11}-5005\,\tan\left(dx+c\right)^9+6435\,\tan\left(d\right.\right.}{45045}$$

input
```
integrate((a+b*tan(d*x+c)^4)^4,x, algorithm="maxima")
```

output
```
a^4*x + 4/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3*b/d + 2/35
*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 10
5*c - 105*tan(d*x + c))*a^2*b^2/d + 4/3465*(315*tan(d*x + c)^11 - 385*tan(
d*x + c)^9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3
 + 3465*d*x + 3465*c - 3465*tan(d*x + c))*a*b^3/d + 1/45045*(3003*tan(d*x
+ c)^15 - 3465*tan(d*x + c)^13 + 4095*tan(d*x + c)^11 - 5005*tan(d*x + c)^
9 + 6435*tan(d*x + c)^7 - 9009*tan(d*x + c)^5 + 15015*tan(d*x + c)^3 + 450
45*d*x + 45045*c - 45045*tan(d*x + c))*b^4/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.74

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx = \frac{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(dx+c)}{d}$$
$$+\frac{3003\,b^4d^{14}\tan\left(dx+c\right)^{15}-3465\,b^4d^{14}\tan\left(dx+c\right)^{13}+16380\,ab^3d^{14}\tan\left(dx+c\right)^{11}+4095\,b^4d^{14}\tan\left(d\right.}{}$$

input
```
integrate((a+b*tan(d*x+c)^4)^4,x, algorithm="giac")
```

output
```
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(d*x + c)/d + 1/45045*(3003*b^
4*d^14*tan(d*x + c)^15 - 3465*b^4*d^14*tan(d*x + c)^13 + 16380*a*b^3*d^14*
tan(d*x + c)^11 + 4095*b^4*d^14*tan(d*x + c)^11 - 20020*a*b^3*d^14*tan(d*x
 + c)^9 - 5005*b^4*d^14*tan(d*x + c)^9 + 38610*a^2*b^2*d^14*tan(d*x + c)^7
 + 25740*a*b^3*d^14*tan(d*x + c)^7 + 6435*b^4*d^14*tan(d*x + c)^7 - 54054*
a^2*b^2*d^14*tan(d*x + c)^5 - 36036*a*b^3*d^14*tan(d*x + c)^5 - 9009*b^4*d
^14*tan(d*x + c)^5 + 60060*a^3*b*d^14*tan(d*x + c)^3 + 90090*a^2*b^2*d^14*
tan(d*x + c)^3 + 60060*a*b^3*d^14*tan(d*x + c)^3 + 15015*b^4*d^14*tan(d*x
+ c)^3 - 180180*a^3*b*d^14*tan(d*x + c) - 270270*a^2*b^2*d^14*tan(d*x + c)
 - 180180*a*b^3*d^14*tan(d*x + c) - 45045*b^4*d^14*tan(d*x + c))/d^15
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.25

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx=\frac{\tan(c+d\,x)^3\,\left(\frac{4\,a^3\,b}{3}+2\,a^2\,b^2+\frac{4\,a\,b^3}{3}+\frac{b^4}{3}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)^4}{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}\right)\,(a+b)^4}{d}-\frac{\tan(c+d\,x)\,\left(4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)}{d}-\frac{b^4\tan(c+d\,x)^{13}}{13\,d}+\frac{b^4\tan(c+d\,x)^{15}}{15\,d}-\frac{\tan(c+d\,x)^5\,\left(\frac{6\,a^2\,b^2}{5}+\frac{4\,a\,b^3}{5}+\frac{b^4}{5}\right)}{d}+\frac{\tan(c+d\,x)^7\,\left(\frac{6\,a^2\,b^2}{7}+\frac{4\,a\,b^3}{7}+\frac{b^4}{7}\right)}{d}-\frac{\tan(c+d\,x)^9\,\left(\frac{b^4}{9}+\frac{4\,a\,b^3}{9}\right)}{d}+\frac{\tan(c+d\,x)^{11}\,\left(\frac{b^4}{11}+\frac{4\,a\,b^3}{11}\right)}{d}$$

input
```
int((a + b*tan(c + d*x)^4)^4,x)
```

output
```
(tan(c + d*x)^3*((4*a*b^3)/3 + (4*a^3*b)/3 + b^4/3 + 2*a^2*b^2))/d + (atan
((tan(c + d*x)*(a + b)^4)/(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))*(a
+ b)^4)/d - (tan(c + d*x)*(4*a*b^3 + 4*a^3*b + b^4 + 6*a^2*b^2))/d - (b^4*
tan(c + d*x)^13)/(13*d) + (b^4*tan(c + d*x)^15)/(15*d) - (tan(c + d*x)^5*(
(4*a*b^3)/5 + b^4/5 + (6*a^2*b^2)/5))/d + (tan(c + d*x)^7*((4*a*b^3)/7 + b
^4/7 + (6*a^2*b^2)/7))/d - (tan(c + d*x)^9*((4*a*b^3)/9 + b^4/9))/d + (tan
(c + d*x)^11*((4*a*b^3)/11 + b^4/11))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.47

$$\int \left(a+b\tan^4(c+dx)\right)^4\,dx$$
$$=\frac{-36036\tan{(dx+c)}^5\,a\,b^3+90090\tan{(dx+c)}^3\,a^2b^2+60060\tan{(dx+c)}^3\,a\,b^3-180180\tan{(dx+c)}\,a^3}{}$$

input
```
int((a+b*tan(d*x+c)^4)^4,x)
```

output
```
(3003*tan(c + d*x)**15*b**4 - 3465*tan(c + d*x)**13*b**4 + 16380*tan(c + d
*x)**11*a*b**3 + 4095*tan(c + d*x)**11*b**4 - 20020*tan(c + d*x)**9*a*b**3
 - 5005*tan(c + d*x)**9*b**4 + 38610*tan(c + d*x)**7*a**2*b**2 + 25740*tan
(c + d*x)**7*a*b**3 + 6435*tan(c + d*x)**7*b**4 - 54054*tan(c + d*x)**5*a*
*2*b**2 - 36036*tan(c + d*x)**5*a*b**3 - 9009*tan(c + d*x)**5*b**4 + 60060
*tan(c + d*x)**3*a**3*b + 90090*tan(c + d*x)**3*a**2*b**2 + 60060*tan(c +
d*x)**3*a*b**3 + 15015*tan(c + d*x)**3*b**4 - 180180*tan(c + d*x)*a**3*b -
 270270*tan(c + d*x)*a**2*b**2 - 180180*tan(c + d*x)*a*b**3 - 45045*tan(c
+ d*x)*b**4 + 45045*a**4*d*x + 180180*a**3*b*d*x + 270270*a**2*b**2*d*x +
180180*a*b**3*d*x + 45045*b**4*d*x)/(45045*d)
```

3.382 $\int (a+b\tan^4(c+dx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 144

$$\begin{aligned}\int (a+b\tan^4(c+dx))^3\,dx &= (a+b)^3x-\frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d}\\ &\quad+\frac{b(3a^2+3ab+b^2)\tan^3(c+dx)}{3d}\\ &\quad-\frac{b^2(3a+b)\tan^5(c+dx)}{5d}+\frac{b^2(3a+b)\tan^7(c+dx)}{7d}\\ &\quad-\frac{b^3\tan^9(c+dx)}{9d}+\frac{b^3\tan^{11}(c+dx)}{11d}\end{aligned}$$

output
```
(a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b*(3*a^2+3*a*b+b^2)*tan(d*x
+c)^3/d-1/5*b^2*(3*a+b)*tan(d*x+c)^5/d+1/7*b^2*(3*a+b)*tan(d*x+c)^7/d-1/9*
b^3*tan(d*x+c)^9/d+1/11*b^3*tan(d*x+c)^11/d
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx=\frac{(a+b)^3\arctan(\tan(c+dx))}{d}$$
$$+\frac{b\tan(c+dx)\left(-3465(3a^2+3ab+b^2)+1155(3a^2+3ab+b^2)\tan^2(c+dx)-693b(3a+b)\tan^4(c+d\right.}{3465d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^4)^3,x]
```

output
```
((a + b)^3*ArcTan[Tan[c + d*x]])/d + (b*Tan[c + d*x]*(-3465*(3*a^2 + 3*a*b
 + b^2) + 1155*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x]^2 - 693*b*(3*a + b)*Tan[
c + d*x]^4 + 495*b*(3*a + b)*Tan[c + d*x]^6 - 385*b^2*Tan[c + d*x]^8 + 315
*b^2*Tan[c + d*x]^10))/(3465*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \left(a+b\tan(c+dx)^4\right)^3dx$$
$$\Big\downarrow \ 4144$$
$$\frac{\int \frac{\left(b\tan^4(c+dx)+a\right)^3}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$
$$\Big\downarrow \ 1468$$

$$\frac{\int \left(b^3 \tan^{10}(c+dx) - b^3 \tan^8(c+dx) + b^2(3a+b)\tan^6(c+dx) - b^2(3a+b)\tan^4(c+dx) + b(3a^2+3ba+b^2)\text{ t}}{d}$$

$\downarrow$ 2009

$$\frac{\frac{1}{3}b(3a^2+3ab+b^2)\tan^3(c+dx) - b(3a^2+3ab+b^2)\tan(c+dx) + (a+b)^3\arctan(\tan(c+dx)) + \frac{1}{7}b^2(3a+b)\text{ t}}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^4)^3,x]
```

output
```
((a + b)^3*ArcTan[Tan[c + d*x]] - b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x] + (
b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x]^3)/3 - (b^2*(3*a + b)*Tan[c + d*x]^5)
/5 + (b^2*(3*a + b)*Tan[c + d*x]^7)/7 - (b^3*Tan[c + d*x]^9)/9 + (b^3*Tan[
c + d*x]^11)/11)/d
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

method	result
norman	$\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)x-\frac{b^{3}\tan(dx+c)^{9}}{9d}+\frac{b^{3}\tan(dx+c)^{11}}{11d}-\frac{b(3a^{2}+3ab+b^{2})\tan(dx+c)}{d}+\frac{b(3a^{2}+$
parts	$a^{3}x+\frac{b^{3}\left(\frac{\tan(dx+c)^{11}}{11}-\frac{\tan(dx+c)^{9}}{9}+\frac{\tan(dx+c)^{7}}{7}-\frac{\tan(dx+c)^{5}}{5}+\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+$
derivativedivides	$\frac{b^{3}\tan(dx+c)^{11}}{11}-\frac{\tan(dx+c)^{9}b^{3}}{9}+\frac{3a\,b^{2}\tan(dx+c)^{7}}{7}+\frac{b^{3}\tan(dx+c)^{7}}{7}-\frac{3\tan(dx+c)^{5}a\,b^{2}}{5}-\frac{b^{3}\tan(dx+c)^{5}}{5}+\tan(dx+c)^{3}a^{2}b+a\,b$
default	$\frac{b^{3}\tan(dx+c)^{11}}{11}-\frac{\tan(dx+c)^{9}b^{3}}{9}+\frac{3a\,b^{2}\tan(dx+c)^{7}}{7}+\frac{b^{3}\tan(dx+c)^{7}}{7}-\frac{3\tan(dx+c)^{5}a\,b^{2}}{5}-\frac{b^{3}\tan(dx+c)^{5}}{5}+\tan(dx+c)^{3}a^{2}b+a\,b$
parallelrisch	$315b^{3}\tan(dx+c)^{11}-385\tan(dx+c)^{9}b^{3}+1485a\,b^{2}\tan(dx+c)^{7}+495b^{3}\tan(dx+c)^{7}-2079\tan(dx+c)^{5}a\,b^{2}-693b^{3}\tan(d$
risch	$a^{3}x+3a^{2}bx+3a\,b^{2}x+b^{3}x-\frac{4ib(6930a^{2}+3254b^{2}+8712ab+928620a^{2}e^{14i(dx+c)}+438900b^{2}e^{14i(dx+c)}+15$

input
```
int((a+b*tan(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
(a^3+3*a^2*b+3*a*b^2+b^3)*x-1/9*b^3*tan(d*x+c)^9/d+1/11*b^3*tan(d*x+c)^11/
d-b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b*(3*a^2+3*a*b+b^2)*tan(d*x+c)^3/d-
1/5*b^2*(3*a+b)*tan(d*x+c)^5/d+1/7*b^2*(3*a+b)*tan(d*x+c)^7/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \left(a+b\tan^{4}(c+dx)\right)^{3}\,dx$$
$$=\frac{315\,b^{3}\tan\left(dx+c\right)^{11}-385\,b^{3}\tan\left(dx+c\right)^{9}+495\left(3\,ab^{2}+b^{3}\right)\tan\left(dx+c\right)^{7}-693\left(3\,ab^{2}+b^{3}\right)\tan\left(dx+\right.}{}$$

input
```
integrate((a+b*tan(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
1/3465*(315*b^3*tan(d*x + c)^11 - 385*b^3*tan(d*x + c)^9 + 495*(3*a*b^2 +
b^3)*tan(d*x + c)^7 - 693*(3*a*b^2 + b^3)*tan(d*x + c)^5 + 1155*(3*a^2*b +
 3*a*b^2 + b^3)*tan(d*x + c)^3 + 3465*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x
- 3465*(3*a^2*b + 3*a*b^2 + b^3)*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.56

$$\int \left(a + b\tan^4(c + dx)\right)^3\, dx$$
$$= \begin{cases} a^3x + 3a^2bx + \frac{a^2b\tan^3(c+dx)}{d} - \frac{3a^2b\tan(c+dx)}{d} + 3ab^2x + \frac{3ab^2\tan^7(c+dx)}{7d} - \frac{3ab^2\tan^5(c+dx)}{5d} + \frac{ab^2\tan^3(c+dx)}{d} - \frac{3 \\ x(a + b\tan^4(c))^3 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**4)**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*x + a**2*b*tan(c + d*x)**3/d - 3*a**2*b*tan(c
 + d*x)/d + 3*a*b**2*x + 3*a*b**2*tan(c + d*x)**7/(7*d) - 3*a*b**2*tan(c +
 d*x)**5/(5*d) + a*b**2*tan(c + d*x)**3/d - 3*a*b**2*tan(c + d*x)/d + b**3
*x + b**3*tan(c + d*x)**11/(11*d) - b**3*tan(c + d*x)**9/(9*d) + b**3*tan(
c + d*x)**7/(7*d) - b**3*tan(c + d*x)**5/(5*d) + b**3*tan(c + d*x)**3/(3*d
) - b**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)**4)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.16

$$\int \left(a + b\tan^4(c + dx)\right)^3\, dx = a^3x + \frac{\left(\tan(dx + c)^3 + 3\,dx + 3\,c - 3\,\tan(dx + c)\right)a^2b}{d}$$
$$+ \frac{\left(15\,\tan(dx + c)^7 - 21\,\tan(dx + c)^5 + 35\,\tan(dx + c)^3 + 105\,dx + 105\,c - 105\,\tan(dx + c)\right)ab^2}{35\,d}$$
$$+ \frac{\left(315\,\tan(dx + c)^{11} - 385\,\tan(dx + c)^9 + 495\,\tan(dx + c)^7 - 693\,\tan(dx + c)^5 + 1155\,\tan(dx + c)^{}\right.}{3465\,d}$$

input
```
integrate((a+b*tan(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
a^3*x + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2*b/d + 1/35*(15
*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105*c
- 105*tan(d*x + c))*a*b^2/d + 1/3465*(315*tan(d*x + c)^11 - 385*tan(d*x +
c)^9 + 495*tan(d*x + c)^7 - 693*tan(d*x + c)^5 + 1155*tan(d*x + c)^3 + 346
5*d*x + 3465*c - 3465*tan(d*x + c))*b^3/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.58

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx=\frac{(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)}{d}$$
$$+\frac{315\,b^3d^{10}\tan{(dx+c)}^{11}-385\,b^3d^{10}\tan{(dx+c)}^{9}+1485\,ab^2d^{10}\tan{(dx+c)}^{7}+495\,b^3d^{10}\tan{(dx+c)}^{7}}{}$$

input
```
integrate((a+b*tan(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c)/d + 1/3465*(315*b^3*d^10*tan(d*x
 + c)^11 - 385*b^3*d^10*tan(d*x + c)^9 + 1485*a*b^2*d^10*tan(d*x + c)^7 +
495*b^3*d^10*tan(d*x + c)^7 - 2079*a*b^2*d^10*tan(d*x + c)^5 - 693*b^3*d^1
0*tan(d*x + c)^5 + 3465*a^2*b*d^10*tan(d*x + c)^3 + 3465*a*b^2*d^10*tan(d*
x + c)^3 + 1155*b^3*d^10*tan(d*x + c)^3 - 10395*a^2*b*d^10*tan(d*x + c) -
10395*a*b^2*d^10*tan(d*x + c) - 3465*b^3*d^10*tan(d*x + c))/d^11
```

Mupad [B] (verification not implemented)

Time = 7.65 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx=\frac{\tan(c+d\,x)^3\,\left(a^2\,b+a\,b^2+\frac{b^3}{3}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,{(a+b)}^3}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}\right)\,{(a+b)}^3}{d}-\frac{b^3\,\tan(c+d\,x)^9}{9\,d}+\frac{b^3\,\tan(c+d\,x)^{11}}{11\,d}-\frac{\tan(c+d\,x)\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{d}-\frac{\tan(c+d\,x)^5\,\left(\frac{b^3}{5}+\frac{3\,a\,b^2}{5}\right)}{d}+\frac{\tan(c+d\,x)^7\,\left(\frac{b^3}{7}+\frac{3\,a\,b^2}{7}\right)}{d}$$

input
```
int((a + b*tan(c + d*x)^4)^3,x)
```

output
```
(tan(c + d*x)^3*(a*b^2 + a^2*b + b^3/3))/d + (atan((tan(c + d*x)*(a + b)^3
)/(3*a*b^2 + 3*a^2*b + a^3 + b^3))*(a + b)^3)/d - (b^3*tan(c + d*x)^9)/(9*
d) + (b^3*tan(c + d*x)^11)/(11*d) - (tan(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3
))/d - (tan(c + d*x)^5*((3*a*b^2)/5 + b^3/5))/d + (tan(c + d*x)^7*((3*a*b^
2)/7 + b^3/7))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.33

$$\int \left(a+b\tan^4(c+dx)\right)^3\,dx = \frac{315\tan(dx+c)^{11}b^3-385\tan(dx+c)^9b^3+1485\tan(dx+c)^7ab^2+495\tan(dx+c)^7b^3-2079\tan(\ldots}{}$$

input
```
int((a+b*tan(d*x+c)^4)^3,x)
```

output
```
(315*tan(c + d*x)**11*b**3 - 385*tan(c + d*x)**9*b**3 + 1485*tan(c + d*x)*
*7*a*b**2 + 495*tan(c + d*x)**7*b**3 - 2079*tan(c + d*x)**5*a*b**2 - 693*t
an(c + d*x)**5*b**3 + 3465*tan(c + d*x)**3*a**2*b + 3465*tan(c + d*x)**3*a
*b**2 + 1155*tan(c + d*x)**3*b**3 - 10395*tan(c + d*x)*a**2*b - 10395*tan(
c + d*x)*a*b**2 - 3465*tan(c + d*x)*b**3 + 3465*a**3*d*x + 10395*a**2*b*d*
x + 10395*a*b**2*d*x + 3465*b**3*d*x)/(3465*d)
```

3.383 $\int (a+b\tan^4(c+dx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int (a+b\tan^4(c+dx))^2\,dx = (a+b)^2x - \frac{b(2a+b)\tan(c+dx)}{d} + \frac{b(2a+b)\tan^3(c+dx)}{3d} - \frac{b^2\tan^5(c+dx)}{5d} + \frac{b^2\tan^7(c+dx)}{7d}$$

output
```
(a+b)^2*x-b*(2*a+b)*tan(d*x+c)/d+1/3*b*(2*a+b)*tan(d*x+c)^3/d-1/5*b^2*tan(
d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int (a+b\tan^4(c+dx))^2\,dx = \frac{105(a+b)^2\arctan(\tan(c+dx)) + b\tan(c+dx)\left(-105(2a+b)+35(2a+b)\tan^2(c+dx)-21b\tan^4(c\right.}{105d}$$

input
```
Integrate[(a + b*Tan[c + d*x]^4)^2,x]
```

output
```
(105*(a + b)^2*ArcTan[Tan[c + d*x]] + b*Tan[c + d*x]*(-105*(2*a + b) + 35*
(2*a + b)*Tan[c + d*x]^2 - 21*b*Tan[c + d*x]^4 + 15*b*Tan[c + d*x]^6))/(10
5*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^4(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\tan(c+dx)^4\right)^2 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tan^4(c+dx)+a\right)^2}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1468}$$

$$\frac{\int \left(b^2\tan^6(c+dx)-b^2\tan^4(c+dx)+b(2a+b)\tan^2(c+dx)-b(2a+b)+\frac{(a+b)^2}{\tan^2(c+dx)+1}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b)^2\arctan(\tan(c+dx))+\frac{1}{3}b(2a+b)\tan^3(c+dx)-b(2a+b)\tan(c+dx)+\frac{1}{7}b^2\tan^7(c+dx)-\frac{1}{5}b^2\tan^5(c}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^4)^2,x]
```

output
```
((a + b)^2*ArcTan[Tan[c + d*x]] - b*(2*a + b)*Tan[c + d*x] + (b*(2*a + b)*
Tan[c + d*x]^3)/3 - (b^2*Tan[c + d*x]^5)/5 + (b^2*Tan[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

method	result
norman	$\left(a^2+2ab+b^2\right)x-\frac{b^2\tan(dx+c)^5}{5d}+\frac{b^2\tan(dx+c)^7}{7d}-\frac{b(2a+b)\tan(dx+c)}{d}+\frac{b(2a+b)\tan(dx+c)^3}{3d}$
parts	$x\,a^2+\frac{b^2\left(\frac{\tan(dx+c)^7}{7}-\frac{\tan(dx+c)^5}{5}+\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{2ab\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c\right.}{d}$
derivativedivides	$\frac{\frac{b^2\tan(dx+c)^7}{7}-\frac{\tan(dx+c)^5b^2}{5}+\frac{2ab\tan(dx+c)^3}{3}+\frac{\tan(dx+c)^3b^2}{3}-2ab\tan(dx+c)-\tan(dx+c)b^2+(a^2+2ab+b^2)\arctan(\tan}{d}$
default	$\frac{\frac{b^2\tan(dx+c)^7}{7}-\frac{\tan(dx+c)^5b^2}{5}+\frac{2ab\tan(dx+c)^3}{3}+\frac{\tan(dx+c)^3b^2}{3}-2ab\tan(dx+c)-\tan(dx+c)b^2+(a^2+2ab+b^2)\arctan(\tan}{d}$
parallelrisch	$\frac{15b^2\tan(dx+c)^7-21\tan(dx+c)^5b^2+70ab\tan(dx+c)^3+35\tan(dx+c)^3b^2+105a^2dx+210abdx+105b^2dx-210ab\tan(d}{105d}$
risch	$x\,a^2+2xab+x\,b^2-\frac{8ib\left(105a\,e^{12i(dx+c)}+105b\,e^{12i(dx+c)}+525a\,e^{10i(dx+c)}+315b\,e^{10i(dx+c)}+1120a\,e^{8i(dx+c)}\right.}{}$

input
```
int((a+b*tan(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
(a^2+2*a*b+b^2)*x-1/5*b^2*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d-b*(2*a+b)*
tan(d*x+c)/d+1/3*b*(2*a+b)*tan(d*x+c)^3/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \left(a+b\tan^4(c+dx)\right)^2\,dx$$
$$=\frac{15\,b^2\tan\left(dx+c\right)^7-21\,b^2\tan\left(dx+c\right)^5+35\left(2\,ab+b^2\right)\tan\left(dx+c\right)^3+105\left(a^2+2\,ab+b^2\right)dx-105\,(2}{105\,d}$$

input
```
integrate((a+b*tan(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 - 21*b^2*tan(d*x + c)^5 + 35*(2*a*b + b^2)*ta
n(d*x + c)^3 + 105*(a^2 + 2*a*b + b^2)*d*x - 105*(2*a*b + b^2)*tan(d*x + c
))/d
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int \left(a+b\tan^4(c+dx)\right)^2\,dx$$
$$=\begin{cases} a^2x+2abx+\frac{2ab\tan^3(c+dx)}{3d}-\frac{2ab\tan(c+dx)}{d}+b^2x+\frac{b^2\tan^7(c+dx)}{7d}-\frac{b^2\tan^5(c+dx)}{5d}+\frac{b^2\tan^3(c+dx)}{3d}-\frac{b^2\tan(c+dx)}{d} \\ x\left(a+b\tan^4(c)\right)^2 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c)**4)**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x + 2*a*b*tan(c + d*x)**3/(3*d) - 2*a*b*tan(c +
d*x)/d + b**2*x + b**2*tan(c + d*x)**7/(7*d) - b**2*tan(c + d*x)**5/(5*d)
+ b**2*tan(c + d*x)**3/(3*d) - b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*t
an(c)**4)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+b\tan^4(c+dx)\right)^2\,dx=a^2x+\frac{2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)ab}{3\,d}$$
$$+\frac{\left(15\,\tan(dx+c)^7-21\,\tan(dx+c)^5+35\,\tan(dx+c)^3+105\,dx+105\,c-105\,\tan(dx+c)\right)b^2}{105\,d}$$

input
```
integrate((a+b*tan(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
a^2*x + 2/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b/d + 1/105*
(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x + 105
*c - 105*tan(d*x + c))*b^2/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.43

$$\int \left(a+b\tan^4(c+dx)\right)^2\,dx=\frac{(a^2+2\,ab+b^2)(dx+c)}{d}+\frac{15\,b^2d^6\tan\left(dx+c\right)^7-21\,b^2d^6\tan\left(dx+c\right)^5+70\,abd^6\tan\left(dx+c\right)^3+35\,b^2d^6\tan\left(dx+c\right)^3-210\,abd^6}{105\,d^7}$$

input
```
integrate((a+b*tan(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
(a^2 + 2*a*b + b^2)*(d*x + c)/d + 1/105*(15*b^2*d^6*tan(d*x + c)^7 - 21*b^
2*d^6*tan(d*x + c)^5 + 70*a*b*d^6*tan(d*x + c)^3 + 35*b^2*d^6*tan(d*x + c)
^3 - 210*a*b*d^6*tan(d*x + c) - 105*b^2*d^6*tan(d*x + c))/d^7
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

$$\int \left(a+b\tan^4(c+dx)\right)^2\,dx=\frac{\tan(c+d\,x)^3\,\left(\frac{b^2}{3}+\frac{2\,a\,b}{3}\right)}{d}-\frac{b^2\tan(c+d\,x)^5}{5\,d}+\frac{b^2\tan(c+d\,x)^7}{7\,d}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)^2}{a^2+2\,a\,b+b^2}\right)\,(a+b)^2}{d}-\frac{\tan(c+d\,x)\,\left(b^2+2\,a\,b\right)}{d}$$

input
```
int((a + b*tan(c + d*x)^4)^2,x)
```

output
```
(tan(c + d*x)^3*((2*a*b)/3 + b^2/3))/d - (b^2*tan(c + d*x)^5)/(5*d) + (b^2
*tan(c + d*x)^7)/(7*d) + (atan((tan(c + d*x)*(a + b)^2)/(2*a*b + a^2 + b^2
))*(a + b)^2)/d - (tan(c + d*x)*(2*a*b + b^2))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \left(a+b\tan^4(c+dx)\right)^2\,dx$$
$$=\frac{15\tan(dx+c)^7 b^2-21\tan(dx+c)^5 b^2+70\tan(dx+c)^3 ab+35\tan(dx+c)^3 b^2-210\tan(dx+c)\,ab}{105d}$$

input
```
int((a+b*tan(d*x+c)^4)^2,x)
```

output
```
(15*tan(c + d*x)**7*b**2 - 21*tan(c + d*x)**5*b**2 + 70*tan(c + d*x)**3*a*
b + 35*tan(c + d*x)**3*b**2 - 210*tan(c + d*x)*a*b - 105*tan(c + d*x)*b**2
 + 105*a**2*d*x + 210*a*b*d*x + 105*b**2*d*x)/(105*d)
```

3.384 $\int \left(a+b\tan^4(c+dx)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \left(a+b\tan^4(c+dx)\right)\,dx=ax+bx-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

output
```
a*x+b*x-b*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \left(a+b\tan^4(c+dx)\right)\,dx=ax+\frac{b\arctan(\tan(c+dx))}{d}-\frac{b\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

input
```
Integrate[a + b*Tan[c + d*x]^4,x]
```

output
```
a*x + (b*ArcTan[Tan[c + d*x]])/d - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)
/(3*d)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tan^4(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b\tan^3(c+dx)}{3d}-\frac{b\tan(c+dx)}{d}+bx$$

input
```
Int[a + b*Tan[c + d*x]^4,x]
```

output
```
a*x + b*x - (b*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
parallelrisch	$\frac{b\left(\tan(dx+c)^{3}+3dx-3\tan(dx+c)\right)}{3d}+ax$	32
norman	$(a+b)\,x-\frac{b\tan(dx+c)}{d}+\frac{b\tan(dx+c)^{3}}{3d}$	33
default	$ax+\frac{b\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}$	36
parts	$ax+\frac{b\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}$	36
derivativedivides	$\frac{\frac{b\tan(dx+c)^{3}}{3}-b\tan(dx+c)+(a+b)\arctan(\tan(dx+c))}{d}$	37
risch	$ax+bx-\frac{4ib\left(3\,e^{4i(dx+c)}+3\,e^{2i(dx+c)}+2\right)}{3d\left(e^{2i(dx+c)}+1\right)^{3}}$	52

input
```
int(a+b*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*b*(tan(d*x+c)^3+3*d*x-3*tan(d*x+c))/d+a*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a+b\tan^{4}(c+dx)\right)\,dx=\frac{b\tan\left(dx+c\right)^{3}+3\,(a+b)dx-3\,b\tan\left(dx+c\right)}{3\,d}$$

input
```
integrate(a+b*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(b*tan(d*x + c)^3 + 3*(a + b)*d*x - 3*b*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a+b\tan^4(c+dx)\right)\,dx=ax+b\left(\begin{cases} x+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\tan^4(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*tan(d*x+c)**4,x)
```

output
```
a*x + b*Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)),
(x*tan(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \left(a+b\tan^4(c+dx)\right)\,dx=ax+\frac{\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)b}{3\,d}$$

input
```
integrate(a+b*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
a*x + 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*b/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \left(a+b\tan^4(c+dx)\right)\,dx=ax+\frac{1}{3}\,b\left(\frac{3\,(dx+c)}{d}+\frac{d^2\tan(dx+c)^3-3\,d^2\tan(dx+c)}{d^3}\right)$$

input
```
integrate(a+b*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
a*x + 1/3*b*(3*(d*x + c)/d + (d^2*tan(d*x + c)^3 - 3*d^2*tan(d*x + c))/d^3
)
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+b\tan^4(c+dx)\right)\,dx=\frac{\frac{b\tan(c+d\,x)^3}{3}-b\tan(c+d\,x)+d\,x\,(a+b)}{d}$$

input
```
int(a + b*tan(c + d*x)^4,x)
```

output
```
((b*tan(c + d*x)^3)/3 - b*tan(c + d*x) + d*x*(a + b))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a+b\tan^4(c+dx)\right)\,dx=\frac{\tan\left(dx+c\right)^3b-3\tan\left(dx+c\right)b+3adx+3bdx}{3d}$$

input
```
int(a+b*tan(d*x+c)^4,x)
```

output
```
(tan(c + d*x)**3*b - 3*tan(c + d*x)*b + 3*a*d*x + 3*b*d*x)/(3*d)
```

3.385 $\int \frac{1}{a+b\tan^4(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 222

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx = \frac{x}{a+b} + \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)}{\sqrt{a}+\sqrt{b}\tan^2(c+dx)}\right)}{2\sqrt{2}a^{3/4}(a+b)d}$$

output
```
x/(a+b)+1/4*(a^(1/2)-b^(1/2))*b^(1/4)*arctan(1-2^(1/2)*b^(1/4)*tan(d*x+c)/
a^(1/4))*2^(1/2)/a^(3/4)/(a+b)/d-1/4*(a^(1/2)-b^(1/2))*b^(1/4)*arctan(1+2^
(1/2)*b^(1/4)*tan(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/(a+b)/d+1/4*(a^(1/2)+b^(
1/2))*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*tan(d*x+c)/(a^(1/2)+b^(1/2)*
tan(d*x+c)^2))*2^(1/2)/a^(3/4)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx$$
$$= \frac{8a^{3/4}\arctan(\tan(c+dx))+\sqrt{2}\sqrt[4]{b}\left(2\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)-2\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\right.\right.}{}$$

input
```
Integrate[(a + b*Tan[c + d*x]^4)^(-1),x]
```

output
```
(8*a^(3/4)*ArcTan[Tan[c + d*x]] + Sqrt[2]*b^(1/4)*(2*(Sqrt[a] - Sqrt[b])*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - 2*(Sqrt[a] - Sqrt[b])*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)] - (Sqrt[a] + Sqrt[b])*(
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^
2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c +
d*x]^2])))/(8*a^(3/4)*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 1485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{a+b\tan(c+dx)^4}dx$$
$$\Big\downarrow \ 4144$$
$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^4(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1485}$$

$$\frac{\int \left(\frac{b-b\tan^2(c+dx)}{(a+b)(b\tan^4(c+dx)+a)}+\frac{1}{(a+b)(\tan^2(c+dx)+1)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(a+b)}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx\right.}{4\sqrt{2}a^{3/4}(a+b)}}{d}$$

input
```
Int[(a + b*Tan[c + d*x]^4)^(-1),x]
```

output
```
(ArcTan[Tan[c + d*x]]/(a + b) + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)) - ((Sqr
t[a] - Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*(a + b)) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2
]*a^(3/4)*(a + b)) + ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/4)*(
a + b)))/d
```

Defintions of rubi rules used

rule 1485
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.71

method	result
risch	$\frac{x}{a+b}+\left(\sum_{\_R=\text{RootOf}\left(\left(256a^5d^4+512a^4b\,d^4+256a^3b^2d^4\right)\_Z^4-64\_Z^2a^2b\,d^2+b\right)}\_R\ln\left(\mathrm{e}^{2i(dx+c)}+\left(-\frac{32a^{\cdots}}{a-\cdots}\cdots\right)\right)\right)$
derivativedivides	$\frac{\arctan(\tan(dx+c))}{a+b}-\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}{\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\tan(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{\cdots}}{\cdots}\right)\cdots\right)}{8a}\cdots\right)}{\cdots}$
default	$\frac{\arctan(\tan(dx+c))}{a+b}-\frac{b\left(-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}{\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\tan(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{\cdots}}{\cdots}\right)\cdots\right)}{8a}\cdots\right)}{\cdots}$

input
```
int(1/(a+b*tan(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
x/(a+b)+sum(_R*ln(exp(2*I*(d*x+c))+(-32/(a-b)*a^3*d^2-32/(a-b)*a^2*b*d^2)*
_R^2+(8*I/(a-b)*a^2*d-8*I/(a-b)*a*b*d)*_R+a/(a-b)+1/(a-b)*b),_R=RootOf((25
6*a^5*d^4+512*a^4*b*d^4+256*a^3*b^2*d^4)*_Z^4-64*_Z^2*a^2*b*d^2+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1541 vs. $2(164) = 328$.

Time = 0.13 (sec) , antiderivative size = 1541, normalized size of antiderivative = 6.94

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*((a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^
3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 +
 2*a^2*b + a*b^2)*d^2))*log((2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b + a*b^
2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c) + (
a*b - b^2)*tan(d*x + c)^2 + a^2 - a*b + ((a^4 + 2*a^3*b + a^2*b^2)*d^2*tan
(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 + b^3)
/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x + c)^2
 + 1)) - (a + b)*sqrt(((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2
+ b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a
^3 + 2*a^2*b + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(((a^3 + 2*a^2*b +
 a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*
a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*tan(d*x + c
) - (a*b - b^2)*tan(d*x + c)^2 - a^2 + a*b - ((a^4 + 2*a^3*b + a^2*b^2)*d^
2*tan(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d^2)*sqrt(-(a^2*b - 2*a*b^2 +
 b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)))/(tan(d*x +
 c)^2 + 1)) + (a + b)*sqrt(-((a^3 + 2*a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*
a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*
b)/((a^3 + 2*a^2*b + a*b^2)*d^2))*log(-(2*(a^3 - a*b^2)*d*sqrt(-((a^3 + 2*
a^2*b + a*b^2)*d^2*sqrt(-(a^2*b - 2*a*b^2 + b^3)/((a^7 + 4*a^6*b + 6*a^...
```

Sympy [F]

$$\int \frac{1}{a + b\tan^4(c + dx)}\,dx = \int \frac{1}{a + b\tan^4(c + dx)}\,dx$$

input
```
integrate(1/(a+b*tan(d*x+c)**4),x)
```

output
```
Integral(1/(a + b*tan(c + d*x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.18

$$\int \frac{1}{a + b\tan^4(c + dx)}\,dx = -\frac{\frac{b\left(\frac{2\sqrt{2}(\sqrt{a}-\sqrt{b})\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}(\sqrt{a}-\sqrt{b})\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}(\sqrt{a}+\sqrt{b})\log\left(\sqrt{b}\tan(dx\cdots\right.}{a\cdots}\right.}{a+b}\cdots}{8d}$$

input
```
integrate(1/(a+b*tan(d*x+c)^4),x, algorithm="maxima")
```

output
```
-1/8*(b*(2*sqrt(2)*(sqrt(a) - sqrt(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*tan(d
*x + c) + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*(sqrt(a) - sqrt(b))*arctan(1/2*sqrt(2)
*(2*sqrt(b)*tan(d*x + c) - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*(sqrt(a) + sqrt(b))*log
(sqrt(b)*tan(d*x + c)^2 + sqrt(2)*a^(1/4)*b^(1/4)*tan(d*x + c) + sqrt(a))/
(a^(3/4)*b^(3/4)) + sqrt(2)*(sqrt(a) + sqrt(b))*log(sqrt(b)*tan(d*x + c)^2
 - sqrt(2)*a^(1/4)*b^(1/4)*tan(d*x + c) + sqrt(a))/(a^(3/4)*b^(3/4)))/(a +
 b) - 8*(d*x + c)/(a + b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(164) = 328$.

Time = 0.25 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.51

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx$$
$$= \frac{dx+c}{ad+bd} + \frac{\left((ab^3)^{\frac{1}{4}}b^2 - (ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2+\sqrt{2}ab^3\right)d}$$
$$+ \frac{\left((ab^3)^{\frac{1}{4}}b^2 - (ab^3)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2+\sqrt{2}ab^3\right)d}$$
$$+ \frac{\left((ab^3)^{\frac{1}{4}}b^2 + (ab^3)^{\frac{3}{4}}\right)\log\left(\tan(dx+c)^2 + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c) + \sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2+\sqrt{2}ab^3\right)d}$$
$$- \frac{\left((ab^3)^{\frac{1}{4}}b^2 + (ab^3)^{\frac{3}{4}}\right)\log\left(\tan(dx+c)^2 - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c) + \sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2+\sqrt{2}ab^3\right)d}$$

input
```
integrate(1/(a+b*tan(d*x+c)^4),x, algorithm="giac")
```

output
```
(d*x + c)/(a*d + b*d) + 1/2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*tan(d*x + c))/(a/b)^(1/4))/((sqrt(2)*a^2
*b^2 + sqrt(2)*a*b^3)*d) + 1/2*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*tan(d*x + c))/(a/b)^(1/4))/((sqrt(2)
*a^2*b^2 + sqrt(2)*a*b^3)*d) + 1/4*((a*b^3)^(1/4)*b^2 + (a*b^3)^(3/4))*log
(tan(d*x + c)^2 + sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/b))/((sqrt(2)*
a^2*b^2 + sqrt(2)*a*b^3)*d) - 1/4*((a*b^3)^(1/4)*b^2 + (a*b^3)^(3/4))*log(
tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/b))/((sqrt(2)*a
^2*b^2 + sqrt(2)*a*b^3)*d)
```

Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 4038, normalized size of antiderivative = 18.19

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(c + d*x)^4),x)
```

output
```
(2*atan(((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 +
256*a^4*b^4 - (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5 - 512
*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + tan(c + d*x)*(32*a*b^6 + 16*b
^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - 6*b^5*tan(c + d*x))/
(2*a + 2*b) - (((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448*a^3*b
^5 + 256*a^4*b^4 + (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^4*b^5
- 512*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - tan(c + d*x)*(32*a*b^6 +
 16*b^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + 6*b^5*tan(c + d
*x))/(2*a + 2*b))/(((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a*b^7 + 448
*a^3*b^5 + 256*a^4*b^4 - (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^6 - 512*a^
4*b^5 - 512*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) + tan(c + d*x)*(32*a
*b^6 + 16*b^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - 6*b^5*tan
(c + d*x))*1i)/(2*a + 2*b) + ((((20*a*b^5 + 4*b^6 - ((((128*a^2*b^6 - 64*a
*b^7 + 448*a^3*b^5 + 256*a^4*b^4 + (tan(c + d*x)*(512*a^2*b^7 + 512*a^3*b^
6 - 512*a^4*b^5 - 512*a^5*b^4)*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) - tan(c +
d*x)*(32*a*b^6 + 16*b^7 - 240*a^2*b^5))*1i)/(2*a + 2*b))*1i)/(2*a + 2*b) +
 6*b^5*tan(c + d*x))*1i)/(2*a + 2*b))))/(d*(2*a + 2*b)) - (atan((((20*a*b^
5 - (((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*b + a^5 +
 a^3*b^2)))^(1/2)*(128*a^2*b^6 - 64*a*b^7 + 448*a^3*b^5 + 256*a^4*b^4 + ta
n(c + d*x)*((2*a^2*b + a*(-a^3*b)^(1/2) - b*(-a^3*b)^(1/2))/(16*(2*a^4*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.56

$$\int \frac{1}{a+b\tan^4(c+dx)}\,dx = \frac{2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\tan(dx+c)}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) - 2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\tan(dx+c)}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) - 2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$$

input
```
int(1/(a+b*tan(d*x+c)^4),x)
```

output
```
(2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*t
an(c + d*x))/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*tan(c + d*x))/(b**(1/4)*a**(1/4)
*sqrt(2))) - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
 2*sqrt(b)*tan(c + d*x))/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*b**(3/4)*a**(1/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*tan(c + d*x))/(b**(1
/4)*a**(1/4)*sqrt(2))) - b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4
)*sqrt(2)*tan(c + d*x) + sqrt(a) + sqrt(b)*tan(c + d*x)**2) + b**(1/4)*a**
(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*tan(c + d*x) + sqrt(a) + sqrt(
b)*tan(c + d*x)**2) - b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*s
qrt(2)*tan(c + d*x) + sqrt(a) + sqrt(b)*tan(c + d*x)**2) + b**(3/4)*a**(1/
4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*tan(c + d*x) + sqrt(a) + sqrt(b)*
tan(c + d*x)**2) + 8*a*d*x)/(8*a*d*(a + b))
```

3.386 $\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 304

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2} + \frac{\sqrt[4]{b}\left(5a+b-\frac{\sqrt{b}(7a+3b)}{\sqrt{a}}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(a+b)^2d} - \frac{\sqrt[4]{b}\left(5a+b-\frac{\sqrt{b}(7a+3b)}{\sqrt{a}}\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(a+b)^2d} + \frac{\sqrt[4]{b}\left(5a+b+\frac{\sqrt{b}(7a+3b)}{\sqrt{a}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)}{\sqrt{a}+\sqrt{b}\tan^2(c+dx)}\right)}{8\sqrt{2}a^{5/4}(a+b)^2d} + \frac{b\tan(c+dx)\left(1-\tan^2(c+dx)\right)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)}$$

output
```
x/(a+b)^2+1/16*b^(1/4)*(5*a+b-b^(1/2)*(7*a+3*b)/a^(1/2))*arctan(1-2^(1/2)*
b^(1/4)*tan(d*x+c)/a^(1/4))*2^(1/2)/a^(5/4)/(a+b)^2/d-1/16*b^(1/4)*(5*a+b-
b^(1/2)*(7*a+3*b)/a^(1/2))*arctan(1+2^(1/2)*b^(1/4)*tan(d*x+c)/a^(1/4))*2^
(1/2)/a^(5/4)/(a+b)^2/d+1/16*b^(1/4)*(5*a+b+b^(1/2)*(7*a+3*b)/a^(1/2))*arc
tanh(2^(1/2)*a^(1/4)*b^(1/4)*tan(d*x+c)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2))*2^
(1/2)/a^(5/4)/(a+b)^2/d+1/4*b*tan(d*x+c)*(1-tan(d*x+c)^2)/a/(a+b)/d/(a+tan
(d*x+c)^4*b)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.19 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.00

$$
\begin{aligned}
\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx &= \frac{\arctan(\tan(c+dx))}{(a+b)^2d} \\
&- \frac{3b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(a+b)d} + \frac{3b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(a+b)d} \\
&+ \frac{\left(\sqrt{a}-\sqrt{b}\right)\left(\frac{\sqrt{2}\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}-\frac{\sqrt{2}\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}\right)}{4\sqrt{a}(a+b)^2d} \\
&- \frac{3b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{16\sqrt{2}a^{7/4}(a+b)d} \\
&+ \frac{3b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{16\sqrt{2}a^{7/4}(a+b)d} \\
&- \frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+dx)\right)}{\sqrt[4]{a}}-\frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\tan(c+dx)+\sqrt{b}\tan^2(c+\right.}{\sqrt[4]{a}}\right.}{8\sqrt{a}(a+b)^2d} \\
&- \frac{b\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-\frac{b\tan^4(c+dx)}{a}\right)\tan^3(c+dx)}{3a^2(a+b)d} \\
&+ \frac{b\tan(c+dx)}{4a(a+b)d\left(a+b\tan^4(c+dx)\right)}
\end{aligned}
$$

input
```
Integrate[(a + b*Tan[c + d*x]^4)^(-2),x]
```

output
```
ArcTan[Tan[c + d*x]]/((a + b)^2*d) - (3*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(a + b)*d) + (3*b^(3/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(a + b)*
d) + ((Sqrt[a] - Sqrt[b])*((Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Ta
n[c + d*x])/a^(1/4)])/a^(1/4) - (Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Tan[c + d*x])/a^(1/4)])/a^(1/4)))/(4*Sqrt[a]*(a + b)^2*d) - (3*b^(3/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^
2])/(16*Sqrt[2]*a^(7/4)*(a + b)*d) + (3*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a
 + b)*d) - ((Sqrt[a] + Sqrt[b])*((Sqrt[2]*b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/a^(1/4) - (Sqrt[2]*b
^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c
+ d*x]^2])/a^(1/4)))/(8*Sqrt[a]*(a + b)^2*d) - (b*Hypergeometric2F1[3/4, 2
, 7/4, -((b*Tan[c + d*x]^4)/a)]*Tan[c + d*x]^3)/(3*a^2*(a + b)*d) + (b*Tan
[c + d*x])/(4*a*(a + b)*d*(a + b*Tan[c + d*x]^4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 631 vs. $2(304) = 608$.

Time = 0.70 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 1568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\tan(c+dx)^4\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^4(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1568}$$

$$\frac{\int \left(\frac{b-b\tan^2(c+dx)}{(a+b)^2(b\tan^4(c+dx)+a)}+\frac{b-b\tan^2(c+dx)}{(a+b)(b\tan^4(c+dx)+a)^2}+\frac{1}{(a+b)^2(\tan^2(c+dx)+1)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(a+b)}+\frac{\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+b)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}-3\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right.}{8\sqrt{2}a^{7/4}(a+b)}$$

input

```
Int[(a + b*Tan[c + d*x]^4)^(-2),x]
```

output

```
(ArcTan[Tan[c + d*x]]/(a + b)^2 + ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2) + (
(Sqrt[a] - 3*Sqrt[b])*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^
(1/4)])/(8*Sqrt[2]*a^(7/4)*(a + b)) - ((Sqrt[a] - Sqrt[b])*b^(1/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b)^2)
 - ((Sqrt[a] - 3*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Tan[c + d*x]
)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(a + b)) - ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])
/(4*Sqrt[2]*a^(3/4)*(a + b)^2) - ((Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sq
rt[2]*a^(7/4)*(a + b)) + ((Sqrt[a] + Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(4*Sqrt[2]*a^(3/
4)*(a + b)^2) + ((Sqrt[a] + 3*Sqrt[b])*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Tan[c + d*x] + Sqrt[b]*Tan[c + d*x]^2])/(16*Sqrt[2]*a^(7/4)*(a
+ b)) + (b*Tan[c + d*x]*(1 - Tan[c + d*x]^2))/(4*a*(a + b)*(a + b*Tan[c +
d*x]^4)))/d
```

Defintions of rubi rules used

rule 1568
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\arctan(\tan(dx+c))}{(a+b)^2}-\frac{b\left(\frac{\frac{(a+b)\tan(dx+c)^3}{4a}-\frac{(a+b)\tan(dx+c)}{4a}}{a+b\tan(dx+c)^4}+\frac{(-7a-3b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}{\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{}\right.}{}$
default	$\frac{\arctan(\tan(dx+c))}{(a+b)^2}-\frac{b\left(\frac{\frac{(a+b)\tan(dx+c)^3}{4a}-\frac{(a+b)\tan(dx+c)}{4a}}{a+b\tan(dx+c)^4}+\frac{(-7a-3b)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}{\tan(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan(dx+c)\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{}\right.}{}$
risch	Expression too large to display

input

```
int(1/(a+b*tan(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/(a+b)^2*arctan(tan(d*x+c))-1/(a+b)^2*b*((1/4*(a+b)/a*tan(d*x+c)^3-1
/4*(a+b)/a*tan(d*x+c))/(a+b*tan(d*x+c)^4)+1/4/a*(1/8*(-7*a-3*b)*(a/b)^(1/4
)/a*2^(1/2)*(ln((tan(d*x+c)^2+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2))/
(tan(d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*tan(d*x+c)+1)-2*arctan(-2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1))+1
/8*(5*a+b)/b/(a/b)^(1/4)*2^(1/2)*(ln((tan(d*x+c)^2-(a/b)^(1/4)*tan(d*x+c)*
2^(1/2)+(a/b)^(1/2))/(tan(d*x+c)^2+(a/b)^(1/4)*tan(d*x+c)*2^(1/2)+(a/b)^(1
/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*tan(d*x+c)+1)-2*arctan(-2^(1/2)/(a/b)^(
1/4)*tan(d*x+c)+1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4291 vs. $2(242) = 484$.

Time = 0.36 (sec) , antiderivative size = 4291, normalized size of antiderivative = 14.12

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tan(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(a+b\tan^4(c+dx))^2}\,dx=\int \frac{1}{(a+b\tan^4(c+dx))^2}\,dx$$

input
```
integrate(1/(a+b*tan(d*x+c)**4)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**4)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+b\tan^4(c+dx))^2}\,dx=$$
$$-\frac{b\left(\frac{2\sqrt{2}\left(b\left(\sqrt{a}-3\sqrt{b}\right)+5a^{\frac{3}{2}}-7a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\left(b\left(\sqrt{a}-3\sqrt{b}\right)+5a^{\frac{3}{2}}-7a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\tan(dx+c)-\sqrt{2}a\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{}$$

input
```
integrate(1/(a+b*tan(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/32*(b*(2*sqrt(2)*(b*(sqrt(a) - 3*sqrt(b)) + 5*a^(3/2) - 7*a*sqrt(b))*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*tan(d*x + c) + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*(b*(s
qrt(a) - 3*sqrt(b)) + 5*a^(3/2) - 7*a*sqrt(b))*arctan(1/2*sqrt(2)*(2*sqrt(
b)*tan(d*x + c) - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*(b*(sqrt(a) + 3*sqrt(b)) + 5*a^(
3/2) + 7*a*sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 + sqrt(2)*a^(1/4)*b^(1/4)*t
an(d*x + c) + sqrt(a))/(a^(3/4)*b^(3/4)) + sqrt(2)*(b*(sqrt(a) + 3*sqrt(b)
) + 5*a^(3/2) + 7*a*sqrt(b))*log(sqrt(b)*tan(d*x + c)^2 - sqrt(2)*a^(1/4)*
b^(1/4)*tan(d*x + c) + sqrt(a))/(a^(3/4)*b^(3/4)))/(a^3 + 2*a^2*b + a*b^2)
 + 8*(b*tan(d*x + c)^3 - b*tan(d*x + c))/((a^2*b + a*b^2)*tan(d*x + c)^4 +
 a^3 + a^2*b) - 32*(d*x + c)/(a^2 + 2*a*b + b^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(242) = 484$.

Time = 0.28 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \frac{dx+c}{a^2d+2\,abd+b^2d} - \frac{\left(\left(ab^3\right)^{\frac{3}{4}}(5\,a+b)-\left(ab^3\right)^{\frac{1}{4}}\left(7\,ab^2+3\,b^3\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^4b^2+2\,\sqrt{2}a^3b^3+\sqrt{2}a^2b^4\right)d} - \frac{\left(\left(ab^3\right)^{\frac{3}{4}}(5\,a+b)-\left(ab^3\right)^{\frac{1}{4}}\left(7\,ab^2+3\,b^3\right)\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\tan(dx+c)\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^4b^2+2\,\sqrt{2}a^3b^3+\sqrt{2}a^2b^4\right)d} + \frac{\left(\left(ab^3\right)^{\frac{3}{4}}(5\,a+b)+\left(ab^3\right)^{\frac{1}{4}}\left(7\,ab^2+3\,b^3\right)\right)\log\left(\tan\left(dx+c\right)^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan\left(dx+c\right)+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^4b^2+2\,\sqrt{2}a^3b^3+\sqrt{2}a^2b^4\right)d} - \frac{\left(\left(ab^3\right)^{\frac{3}{4}}(5\,a+b)+\left(ab^3\right)^{\frac{1}{4}}\left(7\,ab^2+3\,b^3\right)\right)\log\left(\tan\left(dx+c\right)^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\tan\left(dx+c\right)+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^4b^2+2\,\sqrt{2}a^3b^3+\sqrt{2}a^2b^4\right)d} - \frac{b\tan\left(dx+c\right)^3-b\tan\left(dx+c\right)}{4\left(b\tan\left(dx+c\right)^4+a\right)\left(a^2d+abd\right)}$$

input
```
integrate(1/(a+b*tan(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
(d*x + c)/(a^2*d + 2*a*b*d + b^2*d) - 1/8*((a*b^3)^(3/4)*(5*a + b) - (a*b^
3)^(1/4)*(7*a*b^2 + 3*b^3))*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*ta
n(d*x + c))/(a/b)^(1/4))/((sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a
^2*b^4)*d) - 1/8*((a*b^3)^(3/4)*(5*a + b) - (a*b^3)^(1/4)*(7*a*b^2 + 3*b^3
))*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*tan(d*x + c))/(a/b)^(1/4))
/((sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4)*d) + 1/16*((a*b^
3)^(3/4)*(5*a + b) + (a*b^3)^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d*x + c)^2 +
 sqrt(2)*(a/b)^(1/4)*tan(d*x + c) + sqrt(a/b))/((sqrt(2)*a^4*b^2 + 2*sqrt(
2)*a^3*b^3 + sqrt(2)*a^2*b^4)*d) - 1/16*((a*b^3)^(3/4)*(5*a + b) + (a*b^3)
^(1/4)*(7*a*b^2 + 3*b^3))*log(tan(d*x + c)^2 - sqrt(2)*(a/b)^(1/4)*tan(d*x
 + c) + sqrt(a/b))/((sqrt(2)*a^4*b^2 + 2*sqrt(2)*a^3*b^3 + sqrt(2)*a^2*b^4
)*d) - 1/4*(b*tan(d*x + c)^3 - b*tan(d*x + c))/((b*tan(d*x + c)^4 + a)*(a^
2*d + a*b*d))
```

Mupad [B] (verification not implemented)

Time = 11.86 (sec) , antiderivative size = 11516, normalized size of antiderivative = 37.88

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tan(c + d*x)^4)^2,x)
```

output
```
((b*tan(c + d*x))/(4*a*(a + b)) - (b*tan(c + d*x)^3)/(4*a*(a + b)))/(d*(a
+ b*tan(c + d*x)^4)) - (2*atan(((((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a
^6*b^9 - 48*a^4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a
^11*b^4)*1i)/(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) - (tan(c +
d*x)*(65536*a^6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 -
 327680*a^10*b^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(1
28*(4*a*b + 2*a^2 + 2*b^2)*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^
2)))*1i)/(4*a*b + 2*a^2 + 2*b^2) - (tan(c + d*x)*(1152*a^2*b^11 + 7936*a^3
*b^10 + 20352*a^4*b^9 + 8704*a^5*b^8 - 66688*a^6*b^7 - 110848*a^7*b^6 - 49
024*a^8*b^5)*1i)/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*
1i)/(4*a*b + 2*a^2 + 2*b^2) - (((45*a*b^10)/16 + (305*a^2*b^9)/16 + (385*a
^3*b^8)/8 + (657*a^4*b^7)/8 + (2081*a^5*b^6)/16 + (1277*a^6*b^5)/16)*1i)/(
4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2))/(4*a*b + 2*a^2 + 2*b^2)
- (tan(c + d*x)*(612*a*b^8 + 81*b^9 + 1894*a^2*b^7 + 2532*a^3*b^6 + 1425*a
^4*b^5))/(128*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))/(4*a*b +
 2*a^2 + 2*b^2) - ((((((((960*a^7*b^8 - 224*a^5*b^10 - 144*a^6*b^9 - 48*a^
4*b^11 + 2480*a^8*b^7 + 2592*a^9*b^6 + 1296*a^10*b^5 + 256*a^11*b^4)*1i)/(
4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2) + (tan(c + d*x)*(65536*a^
6*b^11 + 327680*a^7*b^10 + 589824*a^8*b^9 + 327680*a^9*b^8 - 327680*a^10*b
^7 - 589824*a^11*b^6 - 327680*a^12*b^5 - 65536*a^13*b^4))/(128*(4*a*b +...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1586, normalized size of antiderivative = 5.22

$$\int \frac{1}{\left(a+b\tan^4(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*tan(d*x+c)^4)^2,x)
```

output
```
(10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
tan(c + d*x))/(b**(1/4)*a**(1/4)*sqrt(2)))*tan(c + d*x)**4*a*b + 2*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*tan(c + d*x
))/(b**(1/4)*a**(1/4)*sqrt(2)))*tan(c + d*x)**4*b**2 + 10*b**(1/4)*a**(3/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*tan(c + d*x))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a**2 + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(b)*tan(c + d*x))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b
 - 14*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*tan(c + d*x))/(b**(1/4)*a**(1/4)*sqrt(2)))*tan(c + d*x)**4*a*b - 6*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*tan(c + d
*x))/(b**(1/4)*a**(1/4)*sqrt(2)))*tan(c + d*x)**4*b**2 - 14*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*tan(c + d*x))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a**2 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(b)*tan(c + d*x))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
*b - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(b)*tan(c + d*x))/(b**(1/4)*a**(1/4)*sqrt(2)))*tan(c + d*x)**4*a*b - 2*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*tan(c +
 d*x))/(b**(1/4)*a**(1/4)*sqrt(2)))*tan(c + d*x)**4*b**2 - 10*b**(1/4)*a**
(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*tan(c + d*x))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**...
```

3.387 $\int \sqrt{a+b\tan^4(c+dx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 516

$$\begin{aligned}
&\int \sqrt{a+b\tan^4(c+dx)}\,dx \\
&= \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2d} + \frac{\sqrt{b}\tan(c+dx)\sqrt{a+b\tan^4(c+dx)}}{d\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)} \\
&\quad - \frac{\sqrt[4]{a}\sqrt[4]{b}E\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}}{d\sqrt{a+b\tan^4(c+dx)}} \\
&\quad - \frac{\sqrt[4]{a}b^{3/4}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}}{\left(\sqrt{a}-\sqrt{b}\right)d\sqrt{a+b\tan^4(c+dx)}} \\
&\quad + \frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\cdots}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}d\sqrt{a+b\tan^4(c+dx)}}
\end{aligned}$$

output
```
1/2*(a+b)^(1/2)*arctan((a+b)^(1/2)*tan(d*x+c)/(a+tan(d*x+c)^4*b)^(1/2))/d+
b^(1/2)*tan(d*x+c)*(a+tan(d*x+c)^4*b)^(1/2)/d/(a^(1/2)+b^(1/2)*tan(d*x+c)^
2)-a^(1/4)*b^(1/4)*EllipticE(sin(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1
/2)*tan(d*x+c)^2)^2)^(1/2)/d/(a+tan(d*x+c)^4*b)^(1/2)-a^(1/4)*b^(3/4)*Inve
rseJacobiAM(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)),1/2*2^(1/2))*(a^(1/2)+b^(
1/2)*tan(d*x+c)^2)*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(
1/2)/(a^(1/2)-b^(1/2))/d/(a+tan(d*x+c)^4*b)^(1/2)+1/4*(a^(1/2)+b^(1/2))*(a
+b)*EllipticPi(sin(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4))),-1/4*(a^(1/2)-b^(
1/2))^2/a^(1/2)/b^(1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)*((a+ta
n(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(1/2)/a^(1/4)/(a^(1/2)-b^(
1/2))/b^(1/4)/d/(a+tan(d*x+c)^4*b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.67 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.42

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx = \frac{\left(\sqrt{a}\sqrt{b}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)\middle|-1\right)+\left(\sqrt{a}-i\sqrt{b}\right)\left(-\sqrt{b}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+\cdots\right.\right.\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}d\sqrt{a+b\cdots}}$$

input
```
Integrate[Sqrt[a + b*Tan[c + d*x]^4],x]
```

output
```
((Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*
x]], -1] + (Sqrt[a] - I*Sqrt[b])*(-(Sqrt[b]*EllipticF[I*ArcSinh[Sqrt[(I*Sq
rt[b])/Sqrt[a]]*Tan[c + d*x]], -1]) + ((-I)*Sqrt[a] + Sqrt[b])*EllipticPi[
((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]],
 -1]))*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a
 + b*Tan[c + d*x]^4])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 4144, 1524, 27, 1512, 27, 761, 1510, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\sqrt{b\tan^4(c+dx)+a}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1524}$$

$$\frac{\frac{(a+b)\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\sqrt{a}\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{1-\frac{\sqrt{b}}{\sqrt{a}}}-\frac{\int \frac{\sqrt{b}\left(-\left(\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\sqrt{b}\tan^2(c+dx)\right)+\sqrt{a}+\sqrt{b}\right)}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{1-\frac{\sqrt{b}}{\sqrt{a}}}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(a+b)\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)}-\frac{\sqrt{b}\int \frac{-\left(\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\sqrt{b}\tan^2(c+dx)\right)+\sqrt{a}+\sqrt{b}}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{1-\frac{\sqrt{b}}{\sqrt{a}}}}{d}$$

$$\Big\downarrow \text{1512}$$

$$\frac{\frac{(a+b)\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)}-\frac{\sqrt{b}\left(2\sqrt{b}\int \frac{1}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)+\left(\sqrt{a}-\sqrt{b}\right)\int \frac{\sqrt{a}-\sqrt{b}\tan^2(c+dx)}{\sqrt{a}\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx\right.}{1-\frac{\sqrt{b}}{\sqrt{a}}}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(a+b)\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)}-\frac{\sqrt{b}\left(2\sqrt{b}\int \frac{1}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)+\frac{\left(\sqrt{a}-\sqrt{b}\right)\int \frac{\sqrt{a}-\sqrt{b}\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}}\right)}{1-\frac{\sqrt{b}}{\sqrt{a}}}}{d}$$

$\downarrow$ 761

$$\frac{\frac{(a+b)\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)}-\frac{\sqrt{b}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\int \frac{\sqrt{a}-\sqrt{b}\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}}+\frac{\sqrt[4]{b}\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\left(\sqrt{a}+\sqrt{b}\tan\right.}{\sqrt[4]{a}\sqrt{}}\right.}{1-\frac{\sqrt{b}}{\sqrt{a}}}}{d}$$

$\downarrow$ 1510

$$\frac{\frac{(a+b)\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)}-\frac{\sqrt{b}\left(\frac{\sqrt[4]{b}\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{a}\sqrt{a+b\tan^4(c+dx)}}\right.}{}}{d}$$

$\downarrow$ 2221

$$\frac{(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\arctan\left(\frac{\sqrt[4]{b}\tan(c+d}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(c+dx)}}\right.}{\sqrt{a}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)}$$

input
```
Int[Sqrt[a + b*Tan[c + d*x]^4],x]
```

output
```
(((a + b)*(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a +
 b*Tan[c + d*x]^4]])/(2*Sqrt[a + b]) + ((Sqrt[a] + Sqrt[b])*EllipticPi[-1/
4*(Sqrt[a] - Sqrt[b])^2/(Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x])
/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x
]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*
Tan[c + d*x]^4])))/(Sqrt[a]*(1 - Sqrt[b]/Sqrt[a])) - (Sqrt[b]*((b^(1/4)*El
lipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*
Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x
]^2)^2])/(a^(1/4)*Sqrt[a + b*Tan[c + d*x]^4]) + ((Sqrt[a] - Sqrt[b])*(-((T
an[c + d*x]*Sqrt[a + b*Tan[c + d*x]^4])/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)
) + (a^(1/4)*EllipticE[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(Sqr
t[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt
[b]*Tan[c + d*x]^2)^2])/(b^(1/4)*Sqrt[a + b*Tan[c + d*x]^4])))/Sqrt[a]))/(
1 - Sqrt[b]/Sqrt[a]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1524
```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(c*d^2 + a*e^2)/(e*(e - d*q))   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x] - Simp[1/(e*(e - d*q))   Int[(c*d + a*e*q - (
c*e - a*d*q^3)*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && N
eQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{b\sqrt{1-\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(dx+c)^4}}+\frac{i\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(dx+c}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(dx}}$
default	$-\frac{b\sqrt{1-\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(dx+c)^4}}+\frac{i\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(dx+c}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(dx}}$

input
```
int((a+b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*Ellipti
cF(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticF(tan(d*x+c)*(I/a^(1
/2)*b^(1/2))^(1/2),I)-I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/
(a+b*tan(d*x+c)^4)^(1/2)*EllipticE(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I)
+a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(t
an(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))+b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b
*tan(d*x+c)^4)^(1/2)*EllipticPi(tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(
1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a + b\tan^4(c + dx)}\, dx = \int \sqrt{a + b\tan^4{(c + dx)}}\, dx$$

input
```
integrate((a+b*tan(d*x+c)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(c + d*x)**4), x)
```

Maxima [F]

$$\int \sqrt{a + b\tan^4(c + dx)}\, dx = \int \sqrt{b\tan{(dx + c)}^4 + a}\, dx$$

input
```
integrate((a+b*tan(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

Giac [F]

$$\int \sqrt{a + b\tan^4(c + dx)}\, dx = \int \sqrt{b\tan{(dx + c)}^4 + a}\, dx$$

input
```
integrate((a+b*tan(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(d*x + c)^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx=\int \sqrt{b\,\tan\left(c+d\,x\right)^4+a}\,dx$$

input
```
int((a + b*tan(c + d*x)^4)^(1/2),x)
```

output
```
int((a + b*tan(c + d*x)^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\tan^4(c+dx)}\,dx=\int \sqrt{\tan\left(dx+c\right)^4 b+a}dx$$

input
```
int((a+b*tan(d*x+c)^4)^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)**4*b + a),x)
```

3.388 $\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 348

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2\sqrt{a+b}d} - \frac{\sqrt[4]{b}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)d\sqrt{a+b\tan^4(c+dx)}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\,\mathrm{ta}\ldots}{\left(\sqrt{a}+\sqrt{b}\,\mathrm{t}\ldots\right.}}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}d\sqrt{a+b\tan^4(c+dx)}}$$

output

```
1/2*arctan((a+b)^(1/2)*tan(d*x+c)/(a+tan(d*x+c)^4*b)^(1/2))/(a+b)^(1/2)/d-
1/2*b^(1/4)*InverseJacobiAM(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)),1/2*2^(1/
2))*(a^(1/2)+b^(1/2)*tan(d*x+c)^2)*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*ta
n(d*x+c)^2)^2)^(1/2)/a^(1/4)/(a^(1/2)-b^(1/2))/d/(a+tan(d*x+c)^4*b)^(1/2)+
1/4*(a^(1/2)+b^(1/2))*EllipticPi(sin(2*arctan(b^(1/4)*tan(d*x+c)/a^(1/4)))
,-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b^(1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2)*ta
n(d*x+c)^2)*((a+tan(d*x+c)^4*b)/(a^(1/2)+b^(1/2)*tan(d*x+c)^2)^2)^(1/2)/a^
(1/4)/(a^(1/2)-b^(1/2))/b^(1/4)/d/(a+tan(d*x+c)^4*b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = -\frac{i\,\text{EllipticPi}\left(-\frac{i\sqrt{a}}{\sqrt{b}},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right),-1\right)\sqrt{1+\frac{b\tan^4(c+dx)}{a}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}d\sqrt{a+b\tan^4(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```

output
```
((-I)*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a
]]*Tan[c + d*x]], -1]*Sqrt[1 + (b*Tan[c + d*x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sq
rt[a]]*d*Sqrt[a + b*Tan[c + d*x]^4])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4144, 1541, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\tan(c+dx)^4}}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{\sqrt{a}\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\sqrt{a}\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt{b}\int \frac{1}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}-\sqrt{b}}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt{b}\int \frac{1}{\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}-\sqrt{b}}}{d}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\int \frac{\sqrt{b}\tan^2(c+dx)+\sqrt{a}}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^4(c+dx)+a}}d\tan(c+dx)}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(c+dx)}}}{d}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a+b\tan^4(c+dx)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)\sqrt{\frac{a+b\tan^4(c+dx)}{\left(\sqrt{a}+\sqrt{b}\tan^2(c+dx)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\arctan\left(\frac{\sqrt[4]{b}\tan(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(c+dx)}}}{\sqrt{a}-\sqrt{b}}}{d}$$

input
```
Int[1/Sqrt[a + b*Tan[c + d*x]^4],x]
```

output
```
(-1/2*(b^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[c + d*x])/a^(1/4)], 1/2]*(S
qrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*x]^4)/(Sqrt[a] + Sq
rt[b]*Tan[c + d*x]^2)^2])/(a^(1/4)*(Sqrt[a] - Sqrt[b])*Sqrt[a + b*Tan[c +
d*x]^4]) + (((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a
+ b*Tan[c + d*x]^4]])/(2*Sqrt[a + b]) + ((Sqrt[a] + Sqrt[b])*EllipticPi[-1
/4*(Sqrt[a] - Sqrt[b])^2/(Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[c + d*x]
)/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)*Sqrt[(a + b*Tan[c + d*
x]^4)/(Sqrt[a] + Sqrt[b]*Tan[c + d*x]^2)^2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b
*Tan[c + d*x]^4]))/(Sqrt[a] - Sqrt[b]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{1-\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}}{\sqrt{b}},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(dx+c)^4}}$	123
default	$\frac{\sqrt{1-\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(dx+c)^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(\tan(dx+c)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\frac{i\sqrt{a}}{\sqrt{b}},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(dx+c)^4}}$	123

input
```
int(1/(a+b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*tan(d*x+c)^2)^(1/2)/(a+b*tan(d*x+c)^4)^(1/2)*EllipticPi(
tan(d*x+c)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2)
)^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(1/(a+b*tan(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\tan^4{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*tan(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*tan(c + d*x)**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(1/(a+b*tan(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*tan(d*x + c)^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(dx+c\right)^4+a}}\,dx$$

input
```
integrate(1/(a+b*tan(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^4+a}}\,dx$$

input
```
int(1/(a + b*tan(c + d*x)^4)^(1/2),x)
```

output
```
int(1/(a + b*tan(c + d*x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\tan^4(c+dx)}}\,dx=\int \frac{\sqrt{\tan\left(dx+c\right)^4 b+a}}{\tan\left(dx+c\right)^4 b+a}dx$$

input
```
int(1/(a+b*tan(d*x+c)^4)^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)**4*b + a)/(tan(c + d*x)**4*b + a),x)
```

3.389 $\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx = \frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{4\sqrt{b}} + \frac{1}{2}\sqrt{a+b}\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tan^4(x)}+\frac{1}{4}\tan^2(x)\sqrt{a+b\tan^4(x)}$$

output
```
1/4*(a+2*b)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2*(a+
b)^(1/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))-1/2*(a+b
*tan(x)^4)^(1/2)+1/4*tan(x)^2*(a+b*tan(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx$$
$$=\frac{1}{4}\left(2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+2\sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\frac{(-2+\tan^2(x))\,(a+b\tan^4(x))+\frac{a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan^4(x)}{a}}}{\sqrt{b}}}{\sqrt{a+b\tan^4(x)}}\right)$$

input
```
Integrate[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```

output
```
(2*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*Sqrt[a + b
]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + ((-2 + Ta
n[x]^2)*(a + b*Tan[x]^4) + (a^(3/2)*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sq
rt[1 + (b*Tan[x]^4)/a])/Sqrt[b])/Sqrt[a + b*Tan[x]^4])/4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 4153, 1579, 591, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \tan(x)^3\sqrt{a+b\tan(x)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan^3(x)\sqrt{a+b\tan^4(x)}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{\tan^2(x)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{591}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{a-(a+2b)\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\frac{1}{2}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{a-(a+2b)\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\frac{1}{2}\left(2-\tan^2(x)\right)\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\left(\frac{1}{2}\left((a+2b)\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)-2(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)-\frac{1}{2}(2-\tan$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{2}\left((a+2b)\int \frac{1}{1-b\tan^4(x)}d\frac{\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-2(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)-\frac{1}{2}(2$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}-2(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)-\frac{1}{2}\left(2-\tan^2(x)\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}+\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\left(2-\tan^2(x)\right)\sqrt{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}+2\sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)\right)-\frac{1}{2}(2-\tan^2(x))\sqrt{a+b\tan^4(x)}\right)$$

input
```
Int[Tan[x]^3*Sqrt[a + b*Tan[x]^4],x]
```

output
```
((((a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/Sqrt[b] + 2
*Sqrt[a + b]*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])
/2 - ((2 - Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{\tan(x)^2\sqrt{a+b\tan(x)^4}}{4}+\frac{a\ln\left(\sqrt{b}\,\tan(x)^2+\sqrt{a+b\tan(x)^4}\right)}{4\sqrt{b}}-\frac{\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{2}+\sqrt{b}\,\mathrm{l}$
default	$\frac{\tan(x)^2\sqrt{a+b\tan(x)^4}}{4}+\frac{a\ln\left(\sqrt{b}\,\tan(x)^2+\sqrt{a+b\tan(x)^4}\right)}{4\sqrt{b}}-\frac{\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{2}+\sqrt{b}\,\mathrm{l}$

input
```
int(tan(x)^3*(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)^2*(a+b*tan(x)^4)^(1/2)+1/4*a/b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*t
an(x)^4)^(1/2))-1/2*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2)+1/2*b^(1
/2)*ln((b*(1+tan(x)^2)-b)/b^(1/2)+(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^
(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+ta
n(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.83

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(x)^3*(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 - a) + 2*sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x
)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b
, -1/4*((a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^
2)) - sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(
b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*
tan(x)^2 + 1)) - sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b))/b, 1/8*(4*sqrt(-
a - b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b +
 b^2)*tan(x)^4 + a^2 + a*b)) + (a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqr
t(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)
^2 - 2*b))/b, 1/4*(2*sqrt(-a - b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^
2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - (a + 2*b)*sqrt(-
b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + sqrt(b*tan(x)^4 +
a)*(b*tan(x)^2 - 2*b))/b]
```

Sympy [F]

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx = \int \sqrt{a+b\tan^4{(x)}}\tan^3{(x)}\,dx$$

input
```
integrate(tan(x)**3*(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(x)**4)*tan(x)**3, x)
```

Maxima [F]

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx = \int \sqrt{b\tan{(x)}^4+a}\tan{(x)}^3\,dx$$

input
```
integrate(tan(x)^3*(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(x)^4 + a)*tan(x)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.16

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx = \frac{1}{4}\sqrt{b\tan{(x)}^4+a}\left(\tan{(x)}^2-2\right)$$

input
```
integrate(tan(x)^3*(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*tan(x)^4 + a)*(tan(x)^2 - 2)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx=\int \tan(x)^3\,\sqrt{b\tan(x)^4+a}\,dx$$

input
```
int(tan(x)^3*(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(tan(x)^3*(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \tan^3(x)\sqrt{a+b\tan^4(x)}\,dx=\frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^2}{4}-\frac{\sqrt{\tan(x)^4 b+a}}{2}+\frac{\left(\int \frac{\sqrt{\tan(x)^4b+a}\,\tan(x)^3}{\tan(x)^4b+a}dx\right)a}{2}+\left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^3}{\tan(x)^4 b+a}dx\right)b-\frac{\left(\int \frac{\sqrt{\tan(x)^4b+a}\,\tan(x)}{\tan(x)^4b+a}dx\right)a}{2}$$

input
```
int(tan(x)^3*(a+b*tan(x)^4)^(1/2),x)
```

output
```
(sqrt(tan(x)**4*b + a)*tan(x)**2 - 2*sqrt(tan(x)**4*b + a) + 2*int((sqrt(t
an(x)**4*b + a)*tan(x)**3)/(tan(x)**4*b + a),x)*a + 4*int((sqrt(tan(x)**4*
b + a)*tan(x)**3)/(tan(x)**4*b + a),x)*b - 2*int((sqrt(tan(x)**4*b + a)*ta
n(x))/(tan(x)**4*b + a),x)*a)/4
```

3.390 $\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx = -\frac{1}{2}\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}\sqrt{a+b\tan^4(x)}$$

output
```
-1/2*b^(1/2)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))-1/2*(a+b)^(1/2
)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))+1/2*(a+b*tan(x)
^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx = \frac{1}{2}\left(-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) + \sqrt{a+b\tan^4(x)}\right)$$

input
```
Integrate[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

output
```
(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]) - Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + Sqrt[a + b
*Tan[x]^4])/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4153, 1577, 493, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\sqrt{a+b\tan(x)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)\sqrt{a+b\tan^4(x)}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{493}$$

$$\frac{1}{2}\left(\int \frac{a-b\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)+\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\left(-b\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)+(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)+\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(-b\int \frac{1}{1-b\tan^4(x)}d\frac{\tan^2(x)}{\sqrt{b\tan^4(x)+a}}+(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)+\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(-(a+b)\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-\sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\sqrt{a+b\tan^4(x)}\right)$$

input
```
Int[Tan[x]*Sqrt[a + b*Tan[x]^4],x]
```

output
```
(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]) - Sqrt[a + b]
*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + Sqrt[a + b
*Tan[x]^4])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b\left(1+\tan(x)^2\right)-b}{\sqrt{b}}+\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}\right)}{2}-$
default	$\frac{\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b\left(1+\tan(x)^2\right)-b}{\sqrt{b}}+\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}\right)}{2}-$

input
```
int(tan(x)*(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(1+tan
(x)^2)-b)/b^(1/2)+(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))-1/2*(a+b)
^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1
+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 475, normalized size of antiderivative = 5.28

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx$$

$$= \left[\frac{1}{4}\sqrt{b}\log\left(-2\,b\tan(x)^4+2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2-a\right)\right.$$

$$+\frac{1}{4}\sqrt{a+b}\log\left(\frac{(ab+2\,b^2)\tan(x)^4-2\,ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2\,a^2+}{\tan(x)^4+2\,\tan(x)^2+1}\right.$$

$$+\frac{1}{2}\sqrt{b\tan(x)^4+a},\ \frac{1}{2}\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\sqrt{-b}}{b\tan(x)^2}\right)$$

$$+\frac{1}{4}\sqrt{a+b}\log\left(\frac{(ab+2\,b^2)\tan(x)^4-2\,ab\tan(x)^2+2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2\,a^2+}{\tan(x)^4+2\,\tan(x)^2+1}\right.$$

$$+\frac{1}{2}\sqrt{b\tan(x)^4+a},$$

$$-\frac{1}{2}\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)$$

$$+\frac{1}{4}\sqrt{b}\log\left(-2\,b\tan(x)^4+2\sqrt{b\tan(x)^4+a}\sqrt{b}\tan(x)^2-a\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a},$$

$$-\frac{1}{2}\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)$$

$$\left.+\frac{1}{2}\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\sqrt{-b}}{b\tan(x)^2}\right)+\frac{1}{2}\sqrt{b\tan(x)^4+a}\right]$$

input
```
integrate(tan(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 -
 a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqr
t(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 +
2*tan(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a), 1/2*sqrt(-b)*arctan(sqrt(b*ta
n(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*ta
n(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a +
 b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*sqrt(b*tan(x)^4 + a)
, -1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a -
 b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*sqrt(b)*log(-2*b*tan(x)^4 +
2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 1/2*sqrt(b*tan(x)^4 + a), -
1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)
/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/2*sqrt(-b)*arctan(sqrt(b*tan(x)^4
 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/2*sqrt(b*tan(x)^4 + a)]
```

Sympy [F]

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx=\int \sqrt{a+b\tan^4{(x)}}\tan{(x)}\,dx$$

input
```
integrate(tan(x)*(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(x)**4)*tan(x), x)
```

Maxima [F]

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx=\int \sqrt{b\tan{(x)}^4+a}\tan{(x)}\,dx$$

input
```
integrate(tan(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(x)^4 + a)*tan(x), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx = \frac{(a+b)\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{1}{2}\sqrt{b}\log\left(\left|-\sqrt{b}\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right) + \frac{1}{2}\sqrt{b\tan(x)^4+a}$$

input
```
integrate(tan(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
(a + b)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-
a - b))/sqrt(-a - b) + 1/2*sqrt(b)*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(
x)^4 + a))) + 1/2*sqrt(b*tan(x)^4 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx = \int \tan(x)\,\sqrt{b\tan(x)^4+a}\,dx$$

input
```
int(tan(x)*(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(tan(x)*(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \tan(x)\sqrt{a+b\tan^4(x)}\,dx = \int \sqrt{\tan(x)^4 b+a}\,\tan(x)\,dx$$

input
```
int(tan(x)*(a+b*tan(x)^4)^(1/2),x)
```

output
```
int(sqrt(tan(x)**4*b + a)*tan(x),x)
```

3.391 $\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \frac{1}{2}\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)$$

output

```
1/2*b^(1/2)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))+1/2*(a+b)^(1/2)
*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))-1/2*a^(1/2)*arct
anh((a+b*tan(x)^4)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \frac{1}{2}\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + Sqrt[a + b]*Ar
cTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - Sqrt[a]*ArcTa
nh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4153, 1579, 606, 243, 73, 221, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(x)^4}}{\tan(x)}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\cot(x)\sqrt{a+b\tan^4(x)}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{\cot(x)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{ 606}$$

$$\frac{1}{2}\left(a\int \frac{\cot(x)}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\int \frac{a-b\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \frac{\cot(x)}{\sqrt{b\tan^4(x)+a}}d\tan^4(x)-\int \frac{a-b\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\frac{\sqrt{b\tan^4(x)+a}}{b}-\frac{a}{b}}d\sqrt{b\tan^4(x)+a}}{b}-\int \frac{a-b\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(-\int \frac{a-b\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)$$

$$\Big\downarrow \text{ 719}$$

$$\frac{1}{2}\left(b\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)-(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan}}{\sqrt{a}}\right.\right.$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}\left(b\int \frac{1}{1-b\tan^4(x)}d\frac{\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+}}{}\right.\right.$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(-(a+b)\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{ta}}{\sqrt{a+b}}\right.\right.$$

$$\Big\downarrow \text{ 488}$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\sqrt{a+b}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right.\right.$$

input
```
Int[Cot[x]*Sqrt[a + b*Tan[x]^4],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + Sqrt[a + b]*Ar
cTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - Sqrt[a]*ArcTa
nh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]])/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot(x)\sqrt{a+b\tan(x)^4}dx$$

input
```
int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

output
```
int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1073, normalized size of antiderivative = 10.52

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 +
a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt
(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2
*tan(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqr
t(a) + 2*a)/tan(x)^4), -1/2*sqrt(-b)*arctan(sqrt(-b)*tan(x)^2/sqrt(b*tan(x
)^4 + a)) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 -
 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x
)^4 + 2*tan(x)^2 + 1)) + 1/4*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 +
 a)*sqrt(a) + 2*a)/tan(x)^4), 1/2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(x)^4
 + a)) + 1/4*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan
(x)^2 + a) + 1/4*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2
- 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(
x)^4 + 2*tan(x)^2 + 1)), -1/2*sqrt(-b)*arctan(sqrt(-b)*tan(x)^2/sqrt(b*tan
(x)^4 + a)) + 1/2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*tan(x)^4 + a)) + 1/4*sqr
t(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4
+ a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 +
1)), 1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a
 - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + 1/4*sqrt(b)*log(2*b*tan(x)^4 +
 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1/4*sqrt(a)*log((b*tan(x)^
4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4), -1/2*sqrt(-b)*arct...
```

Sympy [F]

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \int \sqrt{a+b\tan^4{(x)}}\cot{(x)}\,dx$$

input
```
integrate(cot(x)*(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(x)**4)*cot(x), x)
```

Maxima [F]

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \int \sqrt{b\tan{(x)}^4+a}\cot{(x)}\,dx$$

input
```
integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(x)^4 + a)*cot(x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)*(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionError: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx=\int \cot(x)\ \sqrt{b\tan{(x)}^4+a}\,dx$$

input
```
int(cot(x)*(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(cot(x)*(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \cot(x)\sqrt{a+b\tan^4(x)}\,dx=\int \sqrt{\tan{(x)}^4b+a}\ \cot{(x)}\,dx$$

input
```
int(cot(x)*(a+b*tan(x)^4)^(1/2),x)
```

output
```
int(sqrt(tan(x)**4*b + a)*cot(x),x)
```

3.392 $\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 458

$$
\begin{aligned}
\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx =& -\frac{1}{2}\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right) \\
&+\frac{1}{3}\tan(x)\sqrt{a+b\tan^4(x)}-\frac{\sqrt{b}\tan(x)\sqrt{a+b\tan^4(x)}}{\sqrt{a}+\sqrt{b}\tan^2(x)} \\
&+\frac{\sqrt[4]{a}\sqrt[4]{b}E\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}}{\sqrt{a+b\tan^4(x)}} \\
&+\frac{\sqrt[4]{a}\left(a-\sqrt{a}\sqrt{b}+3b\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}}{3\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\sqrt{a+b\tan^4(x)}} \\
&-\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan}{\left(\sqrt{a}+\sqrt{b}\tan\right.}}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}
\end{aligned}
$$

output
```
-1/2*(a+b)^(1/2)*arctan((a+b)^(1/2)*tan(x)/(a+b*tan(x)^4)^(1/2))+1/3*tan(x
)*(a+b*tan(x)^4)^(1/2)-b^(1/2)*tan(x)*(a+b*tan(x)^4)^(1/2)/(a^(1/2)+b^(1/2
)*tan(x)^2)+a^(1/4)*b^(1/4)*EllipticE(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+b^(1/2)*tan(x)^2)*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)
*tan(x)^2)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)+1/3*a^(1/4)*(a-a^(1/2)*b^(1/2)+3*
b)*InverseJacobiAM(2*arctan(b^(1/4)*tan(x)/a^(1/4)),1/2*2^(1/2))*(a^(1/2)+
b^(1/2)*tan(x)^2)*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)/(a^(
1/2)-b^(1/2))/b^(1/4)/(a+b*tan(x)^4)^(1/2)-1/4*(a^(1/2)+b^(1/2))*(a+b)*Ell
ipticPi(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(
1/2)/b^(1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2)*tan(x)^2)*((a+b*tan(x)^4)/(a^(1
/2)+b^(1/2)*tan(x)^2)^2)^(1/2)/a^(1/4)/(a^(1/2)-b^(1/2))/b^(1/4)/(a+b*tan(
x)^4)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.16 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.88

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx$$
$$= \sqrt{\frac{3a+3b+4a\cos(2x)-4b\cos(2x)+a\cos(4x)+b\cos(4x)}{3+4\cos(2x)+\cos(4x)}}\left(-\frac{1}{2}\sin(2x)+\frac{\tan(x)}{3}\right)$$
$$+ \frac{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\cos(x)\sin(x)+3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}b\sin^2(x)\tan^3(x)-3\sqrt{a}\sqrt{b}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right)\middle|-1\right)\sqrt{1+\frac{b\tan}{}}}{}$$

input
```
Integrate[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```

output
```
Sqrt[(3*a + 3*b + 4*a*Cos[2*x] - 4*b*Cos[2*x] + a*Cos[4*x] + b*Cos[4*x])/(
3 + 4*Cos[2*x] + Cos[4*x])]*(-1/2*Sin[2*x] + Tan[x]/3) + (3*a*Sqrt[(I*Sqrt
[b])/Sqrt[a]]*Cos[x]*Sin[x] + 3*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b*Sin[x]^2*Tan[x
]^3 - 3*Sqrt[a]*Sqrt[b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[
x]], -1]*Sqrt[1 + (b*Tan[x]^4)/a] + ((-2*I)*a + 3*Sqrt[a]*Sqrt[b] - (3*I)*
b)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Sqrt[1 + (b*
Tan[x]^4)/a] + (3*I)*a*EllipticPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(
I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1]*Sqrt[1 + (b*Tan[x]^4)/a] + (3*I)*b*Ellipt
icPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]],
-1]*Sqrt[1 + (b*Tan[x]^4)/a])/(3*Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*Tan[
x]^4])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 4153, 1631, 25, 27, 2221, 2427, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)^2\sqrt{a+b\tan(x)^4}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\int \frac{\tan^2(x)\sqrt{a+b\tan^4(x)}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 1631}$$

$$-\frac{(a+b)\int \frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{b}\tan^2(x)+\sqrt{a}\right)}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan(x)}{a-b}-\frac{\int -\frac{(a-b)b\tan^4(x)-(a-b)b\tan^2(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)(a+b)}{\sqrt{b\tan^4(x)+a}}d\tan(x)}{a-b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(a-b)b\tan^4(x)-(a-b)b\tan^2(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)(a+b)}{\sqrt{b\tan^4(x)+a}}d\tan(x)}{a-b}-\frac{(a+b)\int \frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{b}\tan^2(x)+\sqrt{a}\right)}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan(x)}{a-b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a-b)b\tan^4(x)-(a-b)b\tan^2(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)(a+b)}{\sqrt{b\tan^4(x)+a}}d\tan(x)}{a-b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\int \frac{\sqrt{b}\tan^2(x)+\sqrt{a}}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan(x)}{a-b}$$

$$\Big\downarrow \; 2221$$

$$\frac{\int \frac{(a-b)b\tan^4(x)-(a-b)b\tan^2(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)(a+b)}{\sqrt{b\tan^4(x)+a}}d\tan(x)}{a-b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

$$\Big\downarrow \; 2427$$

$$\frac{\frac{\int \frac{b\left(\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)\left(2a+\sqrt{b}\sqrt{a}+3b\right)-3(a-b)b\tan^2(x)\right)}{\sqrt{b\tan^4(x)+a}}d\tan(x)}{3b}+\frac{1}{3}(a-b)\tan(x)\sqrt{a+b\tan^4(x)}}{a-b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}\int \frac{\sqrt{a}\left(2a^{3/2}+3\sqrt{b}a+4b\sqrt{a}+3b^{3/2}\right)-3(a-b)b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}d\tan(x)+\frac{1}{3}(a-b)\tan(x)\sqrt{a+b\tan^4(x)}}{a-b}-\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

$$\Big\downarrow \text{1512}$$

$$\frac{\frac{1}{3}\left(2\sqrt{a}\left(a^{3/2}+2\sqrt{a}b+3b^{3/2}\right)\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan(x)+3\sqrt{a}\sqrt{b}(a-b)\int \frac{\sqrt{a}-\sqrt{b}\tan^2(x)}{\sqrt{a}\sqrt{b\tan^4(x)+a}}d\tan(x)\right)+\frac{1}{3}(a-b)\tan(}{a-b}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\left(2\sqrt{a}\left(a^{3/2}+2\sqrt{a}b+3b^{3/2}\right)\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan(x)+3\sqrt{b}(a-b)\int \frac{\sqrt{a}-\sqrt{b}\tan^2(x)}{\sqrt{b\tan^4(x)+a}}d\tan(x)\right)+\frac{1}{3}(a-b)\tan(x)\sqrt{}}{a-b}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{1}{3}\left(3\sqrt{b}(a-b)\int \frac{\sqrt{a}-\sqrt{b}\tan^2(x)}{\sqrt{b\tan^4(x)+a}}d\tan(x)+\frac{\sqrt[4]{a}\left(a^{3/2}+2\sqrt{a}b+3b^{3/2}\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{}\right.\right.}{\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{1}{3}\left(\frac{\sqrt[4]{a}(a^{3/2}+2\sqrt{a}b+3b^{3/2})\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}+3\sqrt{b}(a-b)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right.}{}\right.\right.}{a-b}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)(a+b)\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}\right.}{a-b}$$

input
```
Int[Tan[x]^2*Sqrt[a + b*Tan[x]^4],x]
```

output
```
-(((Sqrt[a] + Sqrt[b])*(a + b)*(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a + b]*T
an[x])/Sqrt[a + b*Tan[x]^4]])/(2*Sqrt[a + b]) + ((Sqrt[a] + Sqrt[b])*Ellip
ticPi[-1/4*(Sqrt[a] - Sqrt[b])^2/(Sqrt[a]*Sqrt[b]), 2*ArcTan[(b^(1/4)*Tan[
x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqr
t[a] + Sqrt[b]*Tan[x]^2)^2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*Tan[x]^4])))/(a
 - b)) + (((a - b)*Tan[x]*Sqrt[a + b*Tan[x]^4])/3 + ((a^(1/4)*(a^(3/2) + 2
*Sqrt[a]*b + 3*b^(3/2))*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]
*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan
[x]^2)^2])/(b^(1/4)*Sqrt[a + b*Tan[x]^4]) + 3*(a - b)*Sqrt[b]*(-((Tan[x]*S
qrt[a + b*Tan[x]^4])/(Sqrt[a] + Sqrt[b]*Tan[x]^2)) + (a^(1/4)*EllipticE[2*
ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(
a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(b^(1/4)*Sqrt[a + b*Tan[x
]^4])))/3)/(a - b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1631
```
Int[((x_)^(m_)*((a_) + (c_.)*(x_)^4)^(p_))/((d_) + (e_.)*(x_)^2), x_Symbol]
 :> Simp[(-(-d/e)^(m/2))*((c*d^2 + a*e^2)^(p + 1/2)/(e^(2*p)*(c*d^2 - a*e^2
)))   Int[(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2)/((d + e*x^2)*
Sqrt[a + c*x^4]), x], x] + Simp[1/(e^(2*p)*(c*d^2 - a*e^2))   Int[(1/Sqrt[a
 + c*x^4])*ExpandToSum[(e^(2*p)*(c*d^2 - a*e^2)*x^m*(a + c*x^4)^(p + 1/2) +
 (-d/e)^(m/2)*(c*d^2 + a*e^2)^(p + 1/2)*(a*d*Rt[c/a, 2] + a*e + (c*d + a*e*
Rt[c/a, 2])*x^2))/(d + e*x^2), x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ
[p + 1/2, 0] && IGtQ[m/2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\tan(x)\sqrt{a+b\tan(x)^4}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan(x)^4}}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\sqrt{1\cdots}}{\sqrt{\cdots}}$
default	$\frac{\tan(x)\sqrt{a+b\tan(x)^4}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan(x)^4}}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\sqrt{1\cdots}}{\sqrt{\cdots}}$

input
```
int(tan(x)^2*(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(x)*(a+b*tan(x)^4)^(1/2)+2/3*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x
)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+b/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x
)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2)
,I)-I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x
)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*Ellip
ticF(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)+I*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x
)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticE(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2)
,I)-a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticPi(tan(x)*(I/
a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(
1/2)*b^(1/2))^(1/2))-b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(
x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*Elli
pticPi(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1
/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(tan(x)^2*(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx = \int \sqrt{a+b\tan^4{(x)}}\tan^2{(x)}\,dx$$

input
```
integrate(tan(x)**2*(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(x)**4)*tan(x)**2, x)
```

Maxima [F]

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx=\int \sqrt{b\tan\left(x\right)^4+a}\tan\left(x\right)^2\,dx$$

input
```
integrate(tan(x)^2*(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

Giac [F]

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx=\int \sqrt{b\tan\left(x\right)^4+a}\tan\left(x\right)^2\,dx$$

input
```
integrate(tan(x)^2*(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(x)^4 + a)*tan(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx=\int \tan(x)^2\,\sqrt{b\tan\left(x\right)^4+a}\,dx$$

input
```
int(tan(x)^2*(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(tan(x)^2*(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \tan^2(x)\sqrt{a+b\tan^4(x)}\,dx = \int \sqrt{\tan(x)^4 b + a}\,\tan(x)^2\,dx$$

input
```
int(tan(x)^2*(a+b*tan(x)^4)^(1/2),x)
```

output
```
int(sqrt(tan(x)**4*b + a)*tan(x)**2,x)
```

3.393 $\int \tan^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 171

$$\int \tan^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{16\sqrt{b}} + \frac{1}{2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}(a+b)\sqrt{a+b\tan^4(x)} + \frac{1}{16}(3a+4b)\tan^2(x)\sqrt{a+b\tan^4(x)} - \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2} + \frac{1}{8}\tan^2(x)\left(a+b\tan^4(x)\right)^{3/2}$$

output
```
1/16*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b
^(1/2)+1/2*(a+b)^(3/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(
1/2))-1/2*(a+b)*(a+b*tan(x)^4)^(1/2)+1/16*(3*a+4*b)*tan(x)^2*(a+b*tan(x)^4
)^(1/2)-1/6*(a+b*tan(x)^4)^(3/2)+1/8*tan(x)^2*(a+b*tan(x)^4)^(3/2)
```

Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

$$\int \tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx=\frac{1}{48}\left(24\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right.$$
$$+24(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\frac{3\sqrt{a}(3a+4b)\text{arcsinh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)\sqrt{a+b\tan^4(x)}}{\sqrt{b}\sqrt{1+\frac{b\tan^4(x)}{a}}}+\sqrt{a+$$

input
```
Integrate[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(24*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 24*
(a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]
 + (3*Sqrt[a]*(3*a + 4*b)*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*T
an[x]^4])/(Sqrt[b]*Sqrt[1 + (b*Tan[x]^4)/a]) + Sqrt[a + b*Tan[x]^4]*(-8*(4
*a + 3*b) + 3*(5*a + 4*b)*Tan[x]^2 - 8*b*Tan[x]^4 + 6*b*Tan[x]^6))/48
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {3042, 4153, 1579, 591, 25, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \tan(x)^3\left(a+b\tan(x)^4\right)^{3/2}dx$$
$$\Big\downarrow \; 4153$$
$$\int \frac{\tan^3(x)\left(a+b\tan^4(x)\right)^{3/2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{\tan^2(x)\left(b\tan^4(x)+a\right)^{3/2}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{591}$$

$$\frac{1}{2}\left(\frac{1}{4}\int -\frac{\left(a-(3a+4b)\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)-\frac{1}{12}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{\left(a-(3a+4b)\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)-\frac{1}{12}\left(4-3\tan^2(x)\right)\left(a+b\tan^4(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{\int \frac{b\left(a(5a+4b)-\left(3a^2+12ba+8b^2\right)\tan^2(x)\right)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{2b}-\frac{1}{2}\sqrt{a+b\tan^4(x)}\left(8(a+b)-(3a+4b)\tan^2(x)\right)\right)-\frac{1}{12}\left(\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{a(5a+4b)-\left(3a^2+12ba+8b^2\right)\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\frac{1}{2}\sqrt{a+b\tan^4(x)}\left(8(a+b)-(3a+4b)\tan^2(x\right.\right.\right.$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2+12ab+8b^2\right)\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)-8(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x\right.\right.\right.\right.$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2+12ab+8b^2\right)\int \frac{1}{1-b\tan^4(x)}d\frac{\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-8(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\,\mathrm{ta}\right.\right.\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}-8(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)-\right.\right.$$

$$\Big\downarrow \ 488$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}+\frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(\right.\right.$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}+8(a+b)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)\right)-\frac{1}{2}(8(\right.\right.$$

input
```
Int[Tan[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(-1/12*((4 - 3*Tan[x]^2)*(a + b*Tan[x]^4)^(3/2)) + ((((3*a^2 + 12*a*b + 8*
b^2)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/Sqrt[b] + 8*(a + b)
^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/2 - (
(8*(a + b) - (3*a + 4*b)*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/2)/4)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(139) = 278$.

Time = 0.20 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{3a^2 \ln\left(\sqrt{b}\,\tan(x)^2+\sqrt{a+b\tan(x)^4}\right)}{16\sqrt{b}} + \frac{b\tan(x)^6\sqrt{a+b\tan(x)^4}}{8} + \frac{5a\tan(x)^2\sqrt{a+b\tan(x)^4}}{16} - \frac{b\sqrt{a+b\tan(x)^4}}{2}$
default	$\frac{3a^2 \ln\left(\sqrt{b}\,\tan(x)^2+\sqrt{a+b\tan(x)^4}\right)}{16\sqrt{b}} + \frac{b\tan(x)^6\sqrt{a+b\tan(x)^4}}{8} + \frac{5a\tan(x)^2\sqrt{a+b\tan(x)^4}}{16} - \frac{b\sqrt{a+b\tan(x)^4}}{2}$

input
```
int(tan(x)^3*(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/16*a^2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/8*b*tan(x)^6*
(a+b*tan(x)^4)^(1/2)+5/16*a*tan(x)^2*(a+b*tan(x)^4)^(1/2)-1/2*b*(a+b*tan(x
)^4)^(1/2)-1/2*b^2*(1/3*tan(x)^4/b*(a+b*tan(x)^4)^(1/2)-2/3*a/b^2*(a+b*tan
(x)^4)^(1/2))+1/2*(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)
+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2)
)+1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))+a*b^(1/2)*ln(b^(1/
2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))+1/2*b^2*(1/2*tan(x)^2/b*(a+b*tan(x)^4)^(
1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2)))-a*(a+b*tan(x
)^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 758, normalized size of antiderivative = 4.43

$$\int \tan^3(x) \left(a + b \tan^4(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan
(x)^4 + a)*sqrt(b)*tan(x)^2 - a) + 24*(a*b + b^2)*sqrt(a + b)*log(((a*b +
2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)
*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*(6*b^2*tan(x)
^6 - 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b
*tan(x)^4 + a))/b, -1/48*(3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(-b)*arctan(sqrt(
b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - 12*(a*b + b^2)*sqrt(a + b)*log(((
a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^
2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - (6*b^2*ta
n(x)^6 - 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sq
rt(b*tan(x)^4 + a))/b, 1/96*(48*(a*b + b^2)*sqrt(-a - b)*arctan(sqrt(b*tan
(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b
)) + 3*(3*a^2 + 12*a*b + 8*b^2)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x
)^4 + a)*sqrt(b)*tan(x)^2 - a) + 2*(6*b^2*tan(x)^6 - 8*b^2*tan(x)^4 + 3*(5
*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqrt(b*tan(x)^4 + a))/b, 1/48*(2
4*(a*b + b^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sq
rt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) - 3*(3*a^2 + 12*a*b + 8*b^2
)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) + (6*b^2*tan
(x)^6 - 8*b^2*tan(x)^4 + 3*(5*a*b + 4*b^2)*tan(x)^2 - 32*a*b - 24*b^2)*sqr
t(b*tan(x)^4 + a))/b]
```

Sympy [F]

$$\int \tan^3(x) \left(a + b \tan^4(x)\right)^{3/2} dx = \int \left(a + b \tan^4(x)\right)^{\frac{3}{2}} \tan^3(x)\, dx$$

input
```
integrate(tan(x)**3*(a+b*tan(x)**4)**(3/2),x)
```

output
```
Integral((a + b*tan(x)**4)**(3/2)*tan(x)**3, x)
```

Maxima [F]

$$\int \tan^3(x) \left(a + b\tan^4(x)\right)^{3/2} dx = \int \left(b\tan(x)^4 + a\right)^{\frac{3}{2}} \tan(x)^3 \, dx$$

input
```
integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(x)^4 + a)^(3/2)*tan(x)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.41

$$\int \tan^3(x) (a + b\tan^4(x))^{3/2} dx = \frac{1}{48}\sqrt{b\tan(x)^4 + a}\left(\left(2\left(3b\tan(x)^2 - 4b\right)\tan(x)^2 + \frac{3\left(5ab^2 + 4b^3\right)}{b^2}\right)\tan(x)^2 - \frac{8(4}{}\right.$$

input
```
integrate(tan(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/48*sqrt(b*tan(x)^4 + a)*((2*(3*b*tan(x)^2 - 4*b)*tan(x)^2 + 3*(5*a*b^2 +
 4*b^3)/b^2)*tan(x)^2 - 8*(4*a*b^2 + 3*b^3)/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \int \tan(x)^3 \left(b\tan(x)^4+a\right)^{3/2} \, dx$$

input
```
int(tan(x)^3*(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(tan(x)^3*(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \tan^3(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^6 b}{8} - \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^4 b}{6} + \frac{5\sqrt{\tan(x)^4 b+a}\,\tan(x)^2 a}{16} + \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^2 b}{4} - \frac{2\sqrt{\tan(x)^4 b+a}\,a}{3} - \frac{\sqrt{\tan(x)^4 b+a}\,b}{2} + \frac{3\left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^3}{\tan(x)^4 b+a}dx\right)a^2}{8} + \frac{3\left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^3}{\tan(x)^4 b+a}dx\right)ab}{2} + \left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^3}{\tan(x)^4 b+a}dx\right)b^2 - \frac{5\left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)}{\tan(x)^4 b+a}dx\right)a^2}{8} - \frac{\left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)}{\tan(x)^4 b+a}dx\right)ab}{2}$$

input
```
int(tan(x)^3*(a+b*tan(x)^4)^(3/2),x)
```

output
```
(6*sqrt(tan(x)**4*b + a)*tan(x)**6*b - 8*sqrt(tan(x)**4*b + a)*tan(x)**4*b
 + 15*sqrt(tan(x)**4*b + a)*tan(x)**2*a + 12*sqrt(tan(x)**4*b + a)*tan(x)*
*2*b - 32*sqrt(tan(x)**4*b + a)*a - 24*sqrt(tan(x)**4*b + a)*b + 18*int((s
qrt(tan(x)**4*b + a)*tan(x)**3)/(tan(x)**4*b + a),x)*a**2 + 72*int((sqrt(t
an(x)**4*b + a)*tan(x)**3)/(tan(x)**4*b + a),x)*a*b + 48*int((sqrt(tan(x)*
*4*b + a)*tan(x)**3)/(tan(x)**4*b + a),x)*b**2 - 30*int((sqrt(tan(x)**4*b
+ a)*tan(x))/(tan(x)**4*b + a),x)*a**2 - 24*int((sqrt(tan(x)**4*b + a)*tan
(x))/(tan(x)**4*b + a),x)*a*b)/48
```

3.394 $\int \tan(x) \left(a+b\tan^4(x)\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int \tan(x) \left(a+b\tan^4(x)\right)^{3/2} dx = -\frac{1}{4}\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}(a+b)\sqrt{a+b\tan^4(x)} - \frac{1}{4}b\tan^2(x)\sqrt{a+b\tan^4(x)} + \frac{1}{6}\left(a+b\tan^4(x)\right)^{3/2}$$

output
```
-1/4*b^(1/2)*(3*a+2*b)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))-1/2*
(a+b)^(3/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))+1/2*(
a+b)*(a+b*tan(x)^4)^(1/2)-1/4*b*tan(x)^2*(a+b*tan(x)^4)^(1/2)+1/6*(a+b*tan
(x)^4)^(3/2)
```

Mathematica [A] (verified)

Time = 3.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

$$\int \tan(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \frac{1}{12}\left(-6\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right.$$
$$-6(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\sqrt{a+b\tan^4(x)}\left(8a+6b-3b\tan^2(x)+2b\tan^4(x)\right)-\frac{3\sqrt{a}\sqrt{}}{}$$

input
```
Integrate[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] - 6*(
a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]
+ Sqrt[a + b*Tan[x]^4]*(8*a + 6*b - 3*b*Tan[x]^2 + 2*b*Tan[x]^4) - (3*Sqrt
[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt
[1 + (b*Tan[x]^4)/a])/12
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 4153, 1577, 493, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x) \left(a+b\tan^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x) \left(a+b\tan(x)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4153}$$

$$\int \frac{\tan(x) \left(a+b\tan^4(x)\right)^{3/2}}{\tan^2(x)+1} d\tan(x)$$

$$\Big\downarrow \; 1577$$

$$\frac{1}{2}\int \frac{\left(b\tan^4(x)+a\right)^{3/2}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \; 493$$

$$\frac{1}{2}\left(\int \frac{\left(a-b\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)+\frac{1}{3}\left(a+b\tan^4(x)\right)^{3/2}\right)$$

$$\Big\downarrow \; 682$$

$$\frac{1}{2}\left(\frac{\int \frac{b\left(a(2a+b)-b(3a+2b)\tan^2(x)\right)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{2b}+\frac{1}{3}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{2}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{a(2a+b)-b(3a+2b)\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)+\frac{1}{3}\left(a+b\tan^4(x)\right)^{3/2}+\frac{1}{2}\left(2(a+b)-b\tan^2(x)\right)\sqrt{a+b\tan^4(x)}\right.$$

$$\Big\downarrow \; 719$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-b(3a+2b)\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)+\frac{1}{3}(a+\right.$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-b(3a+2b)\int \frac{1}{1-b\tan^4(x)}d\frac{\tan^2(x)}{\sqrt{b\tan^4(x)+a}}\right)+\frac{1}{3}\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)+\frac{1}{3}(a+\right.$$

$$\Big\downarrow \; 488$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2(a+b)^2\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)+\frac{1}{3}(a+b\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)+\frac{1}{3}(a+b\tan$$

input
```
Int[Tan[x]*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
((-(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])
- 2*(a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^
4])])/2 + ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/2 + (a + b*Tan[x
]^4)^(3/2)/3)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(109) = 218.

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{b\sqrt{a+b\tan(x)^4}}{2}+\frac{b^2\left(\frac{\tan(x)^4\sqrt{a+b\tan(x)^4}}{3b}-\frac{2a\sqrt{a+b\tan(x)^4}}{3b^2}\right)}{2}-\frac{(a^2+2ab+b^2)\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{}}{1+}\right.}{2\sqrt{a+b}}$
default	$\frac{b\sqrt{a+b\tan(x)^4}}{2}+\frac{b^2\left(\frac{\tan(x)^4\sqrt{a+b\tan(x)^4}}{3b}-\frac{2a\sqrt{a+b\tan(x)^4}}{3b^2}\right)}{2}-\frac{(a^2+2ab+b^2)\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{}}{1+}\right.}{2\sqrt{a+b}}$

input
```
int(tan(x)*(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*(a+b*tan(x)^4)^(1/2)+1/2*b^2*(1/3*tan(x)^4/b*(a+b*tan(x)^4)^(1/2)-2/
3*a/b^2*(a+b*tan(x)^4)^(1/2))-1/2*(a^2+2*a*b+b^2)/(a+b)^(1/2)*ln((2*a+2*b-
2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/
2))/(1+tan(x)^2))-1/2*b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))-a*
b^(1/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))-1/2*b^2*(1/2*tan(x)^2/b*
(a+b*tan(x)^4)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2
)))+a*(a+b*tan(x)^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.33

$$\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b
)*tan(x)^2 - a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*ta
n(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b
)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a +
 6*b)*sqrt(b*tan(x)^4 + a), 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^
4 + a)*sqrt(-b)/(b*tan(x)^2)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x
)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b)
 + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/12*(2*b*tan(x)^4 - 3*b*ta
n(x)^2 + 8*a + 6*b)*sqrt(b*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan
(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4
+ a^2 + a*b)) + 1/8*(3*a + 2*b)*sqrt(b)*log(-2*b*tan(x)^4 + 2*sqrt(b*tan(x
)^4 + a)*sqrt(b)*tan(x)^2 - a) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a +
 6*b)*sqrt(b*tan(x)^4 + a), -1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x)
^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b))
+ 1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^
2)) + 1/12*(2*b*tan(x)^4 - 3*b*tan(x)^2 + 8*a + 6*b)*sqrt(b*tan(x)^4 + a)]
```

Sympy [F]

$$\int \tan(x) \left(a + b\tan^4(x)\right)^{3/2} \, dx = \int \left(a + b\tan^4{(x)}\right)^{\frac{3}{2}} \tan{(x)}\, dx$$

input
```
integrate(tan(x)*(a+b*tan(x)**4)**(3/2),x)
```

output
```
Integral((a + b*tan(x)**4)**(3/2)*tan(x), x)
```

Maxima [F]

$$\int \tan(x) \left(a + b\tan^4(x)\right)^{3/2} \, dx = \int \left(b\tan{(x)}^4 + a\right)^{\frac{3}{2}} \tan{(x)}\, dx$$

input
```
integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(x)^4 + a)^(3/2)*tan(x), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \tan(x) \left(a + b\tan^4(x)\right)^{3/2} dx = \frac{1}{4}\left(3\,a\sqrt{b} + 2\,b^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}\tan\left(x\right)^2 + \sqrt{b\tan\left(x\right)^4 + a}\right|\right) + \frac{1}{12}\sqrt{b\tan\left(x\right)^4 + a}\left(\left(2\,b\tan\left(x\right)^2 - 3\,b\right)\tan\left(x\right)^2 + \frac{2\left(4\,ab + 3\,b^2\right)}{b}\right) + \frac{\left(a^2 + 2\,ab + b^2\right)\arctan\left(-\frac{\sqrt{b}\tan(x)^2 - \sqrt{b\tan(x)^4 + a} + \sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

input
```
integrate(tan(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/4*(3*a*sqrt(b) + 2*b^(3/2))*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4
+ a))) + 1/12*sqrt(b*tan(x)^4 + a)*((2*b*tan(x)^2 - 3*b)*tan(x)^2 + 2*(4*a
*b + 3*b^2)/b) + (a^2 + 2*a*b + b^2)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*ta
n(x)^4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-a - b)
```

Mupad [F(-1)]

Timed out.

$$\int \tan(x) \left(a + b\tan^4(x)\right)^{3/2} dx = \int \tan(x)\,\left(b\tan(x)^4 + a\right)^{3/2} dx$$

input
```
int(tan(x)*(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(tan(x)*(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \tan(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx=\frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^4 b}{6}-\frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^2 b}{4}+\frac{2\sqrt{\tan(x)^4 b+a}\,a}{3}+\frac{\sqrt{\tan(x)^4 b+a}\,b}{2}-\frac{3\left(\int\frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^3}{\tan(x)^4 b+a}dx\right)ab}{2}-\left(\int\frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^3}{\tan(x)^4 b+a}dx\right)b^2+\left(\int\frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)}{\tan(x)^4 b+a}dx\right)a^2+\frac{\left(\int\frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)}{\tan(x)^4 b+a}dx\right)ab}{2}$$

input
```
int(tan(x)*(a+b*tan(x)^4)^(3/2),x)
```

output
```
(2*sqrt(tan(x)**4*b + a)*tan(x)**4*b - 3*sqrt(tan(x)**4*b + a)*tan(x)**2*b
 + 8*sqrt(tan(x)**4*b + a)*a + 6*sqrt(tan(x)**4*b + a)*b - 18*int((sqrt(ta
n(x)**4*b + a)*tan(x)**3)/(tan(x)**4*b + a),x)*a*b - 12*int((sqrt(tan(x)**
4*b + a)*tan(x)**3)/(tan(x)**4*b + a),x)*b**2 + 12*int((sqrt(tan(x)**4*b +
 a)*tan(x))/(tan(x)**4*b + a),x)*a**2 + 6*int((sqrt(tan(x)**4*b + a)*tan(x
))/(tan(x)**4*b + a),x)*a*b)/12
```

3.395 $\int \cot(x) \left(a+b\tan^4(x)\right)^{3/2} dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \cot(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \frac{1}{4}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right) - \frac{1}{2}b\sqrt{a+b\tan^4(x)} + \frac{1}{4}b\tan^2(x)\sqrt{a+b\tan^4(x)}$$

output
```
1/4*b^(1/2)*(3*a+2*b)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))+1/2*(
a+b)^(3/2)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))-1/2*a^
(3/2)*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))-1/2*b*(a+b*tan(x)^4)^(1/2)+1/4
*b*tan(x)^2*(a+b*tan(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.29

$$\int \cot(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx=\frac{1}{4}\left(2\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right.$$

$$+2(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)-2b\sqrt{a+b\tan^4(x)}+b\,\text{ta}$$

input
```
Integrate[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(2*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*(a
 + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] -
 2*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] - 2*b*Sqrt[a + b*Tan[x]^4
] + b*Tan[x]^2*Sqrt[a + b*Tan[x]^4] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Ta
n[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/Sqrt[1 + (b*Tan[x]^4)/a])/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4153, 1579, 606, 243, 60, 73, 221, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(x)^4\right)^{3/2}}{\tan(x)}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\cot(x)\left(a+b\tan^4(x)\right)^{3/2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{\cot(x)\left(b\tan^4(x)+a\right)^{3/2}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{ 606}$$

$$\frac{1}{2}\left(a\int \cot(x)\sqrt{b\tan^4(x)+a}d\tan^2(x)-\int \frac{\left(a-b\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \cot(x)\sqrt{b\tan^4(x)+a}d\tan^4(x)-\int \frac{\left(a-b\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{1}{2}a\left(a\int \frac{\cot(x)}{\sqrt{b\tan^4(x)+a}}d\tan^4(x)+2\sqrt{a+b\tan^4(x)}\right)-\int \frac{\left(a-b\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{1}{2}a\left(\frac{2a\int \frac{1}{\frac{\sqrt{b\tan^4(x)+a}}{b}-\frac{a}{b}}d\sqrt{b\tan^4(x)+a}}{b}+2\sqrt{a+b\tan^4(x)}\right)-\int \frac{\left(a-b\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x\right.$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{1}{2}a\left(2\sqrt{a+b\tan^4(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)-\int \frac{\left(a-b\tan^2(x)\right)\sqrt{b\tan^4(x)+a}}{\tan^2(x)+1}d\tan^2(x)\right)$$

$$\Big\downarrow \text{ 682}$$

$$\frac{1}{2}\left(-\frac{\int \frac{b\left(a(2a+b)-b(3a+2b)\tan^2(x)\right)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{2b}+\frac{1}{2}a\left(2\sqrt{a+b\tan^4(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)-\frac{1}{2}\sqrt{a\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{a(2a+b)-b(3a+2b)\tan^2(x)}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)+\frac{1}{2}a\left(2\sqrt{a+b\tan^4(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)\right)$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(b(3a+2b)\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)-2(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)+\frac{1}{2}a\left(2\right.\right.$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(b(3a+2b)\int \frac{1}{1-b\tan^4(x)}d\frac{\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-2(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)+\frac{1}{2}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)-2(a+b)^2\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)+\frac{1}{2}a\left(2\right.\right.$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}+\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right)+\frac{1}{2}a\left(2\sqrt{a}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}a\left(2\sqrt{a+b\tan^4(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)\right)+\frac{1}{2}\left(2(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right.\right.\right.$$

input
```
Int[Cot[x]*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
((Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2
*(a + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])
])/2 - ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*Tan[x]^4])/2 + (a*(-2*Sqrt[a]*
ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] + 2*Sqrt[a + b*Tan[x]^4]))/2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot (x) \left(a+b\tan (x)^{4}\right)^{\frac{3}{2}}dx$$

input
```
int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 21.43 (sec) , antiderivative size = 1321, normalized size of antiderivative = 8.99

$$\int \cot(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a + 2*b)*sqrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)
*tan(x)^2 + a) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan
(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)
/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan
(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - 2*b), -1/4*(3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*tan(x)^2/sqrt(b*tan(x)^
4 + a)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 -
 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x
)^4 + 2*tan(x)^2 + 1)) + 1/4*a^(3/2)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 +
 a)*sqrt(a) + 2*a)/tan(x)^4) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b)
, 1/2*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*tan(x)^4 + a)) + 1/8*(3*a + 2*b)*s
qrt(b)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a) + 1
/4*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*t
an(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan
(x)^2 + 1)) + 1/4*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - 2*b), -1/4*(3*a + 2*b
)*sqrt(-b)*arctan(sqrt(-b)*tan(x)^2/sqrt(b*tan(x)^4 + a)) + 1/2*sqrt(-a)*a
*arctan(sqrt(-a)/sqrt(b*tan(x)^4 + a)) + 1/4*(a + b)^(3/2)*log(((a*b + 2*b
^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sq
rt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/4*sqrt(b*tan(x)^
4 + a)*(b*tan(x)^2 - 2*b), 1/2*(a + b)*sqrt(-a - b)*arctan(sqrt(b*tan(x...
```

Sympy [F]

$$\int \cot(x) \left(a + b\tan^4(x)\right)^{3/2} \, dx = \int \left(a + b\tan^4\left(x\right)\right)^{\frac{3}{2}} \cot\left(x\right) dx$$

input
```
integrate(cot(x)*(a+b*tan(x)**4)**(3/2),x)
```

output
```
Integral((a + b*tan(x)**4)**(3/2)*cot(x), x)
```

Maxima [F]

$$\int \cot(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \int \left(b\tan\left(x\right)^4+a\right)^{\frac{3}{2}} \cot\left(x\right) dx$$

input
```
integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(x)^4 + a)^(3/2)*cot(x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionError: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \cot(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \int \cot(x) \left(b\tan(x)^4+a\right)^{3/2} dx$$

input
```
int(cot(x)*(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)*(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \cot(x) \left(a + b\tan^4(x)\right)^{3/2} dx = \left(\int \sqrt{\tan(x)^4 b + a}\, \cot(x) \tan(x)^4 dx\right) b + \left(\int \sqrt{\tan(x)^4 b + a}\, \cot(x)\, dx\right) a$$

input

```
int(cot(x)*(a+b*tan(x)^4)^(3/2),x)
```

output

```
int(sqrt(tan(x)**4*b + a)*cot(x)*tan(x)**4,x)*b + int(sqrt(tan(x)**4*b + a
)*cot(x),x)*a
```

3.396 $\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = -\frac{1}{2}b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)$$
$$-\frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)+\frac{1}{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}b\sqrt{a+b\tan^4(x)}-\frac{1}{2}$$

output
```
-1/2*b^(3/2)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))-1/2*(a+b)^(3/2
)*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))+1/2*a^(3/2)*arc
tanh((a+b*tan(x)^4)^(1/2)/a^(1/2))+1/2*b*(a+b*tan(x)^4)^(1/2)-1/2*a*cot(x)
^2*(a+b*tan(x)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.77

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \frac{-\sqrt{a}\sqrt{b}\text{arcsinh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)\sqrt{a+b\tan^4(x)}-2a\cot^2(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2}\right.}{}$$

input
```
Integrate[Cot[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(-(Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4
]) - 2*a*Cot[x]^2*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*Tan[x]^4)/a)]*Sq
rt[a + b*Tan[x]^4] + Sqrt[1 + (b*Tan[x]^4)/a]*(-2*Sqrt[b]*(a + b)*ArcTanh[
(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] - 2*(a + b)^(3/2)*ArcTanh[(a - b*
Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + 2*a^(3/2)*ArcTanh[Sqrt[a +
 b*Tan[x]^4]/Sqrt[a]] + 2*b*Sqrt[a + b*Tan[x]^4] - b*Tan[x]^2*Sqrt[a + b*T
an[x]^4]))/(4*Sqrt[1 + (b*Tan[x]^4)/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4153, 1579, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(x)^4\right)^{3/2}}{\tan(x)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\cot^3(x) \left(a+b\tan^4(x)\right)^{3/2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{\cot^2(x) \left(b\tan^4(x)+a\right)^{3/2}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{617}$$

$$\frac{1}{2}\int \left(\left(b\tan^4(x)+a\right)^{3/2}\cot^2(x)-\left(b\tan^4(x)+a\right)^{3/2}\cot(x)+\frac{\left(b\tan^4(x)+a\right)^{3/2}}{\tan^2(x)+1}\right)d\tan^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)+\frac{3}{2}a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right.\right.$$

input
```
Int[Cot[x]^3*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
((3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 - (Sqrt[
b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 - (a +
b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] + a^
(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] - a*Sqrt[a + b*Tan[x]^4] + (3*
b*Tan[x]^2*Sqrt[a + b*Tan[x]^4])/2 + ((2*(a + b) - b*Tan[x]^2)*Sqrt[a + b*
Tan[x]^4])/2 - Cot[x]*(a + b*Tan[x]^4)^(3/2))/2
```

Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot (x)^{3} \left(a+b\tan (x)^{4}\right)^{\frac{3}{2}}dx$$

input
```
int(cot(x)^3*(a+b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)^3*(a+b*tan(x)^4)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 20.40 (sec) , antiderivative size = 1404, normalized size of antiderivative = 10.03

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(cot(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(b^(3/2)*log(2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 +
 a)*tan(x)^2 + (a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2
+ 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(
x)^4 + 2*tan(x)^2 + 1))*tan(x)^2 + a^(3/2)*log((b*tan(x)^4 + 2*sqrt(b*tan(
x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4)*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*ta
n(x)^2 - a))/tan(x)^2, 1/4*(2*sqrt(-b)*b*arctan(sqrt(-b)*tan(x)^2/sqrt(b*t
an(x)^4 + a))*tan(x)^2 + (a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b
*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
a*b)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^2 + a^(3/2)*log((b*tan(x)^4 + 2*s
qrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4)*tan(x)^2 + 2*sqrt(b*tan(x)^4
+ a)*(b*tan(x)^2 - a))/tan(x)^2, -1/4*(2*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b
*tan(x)^4 + a))*tan(x)^2 - b^(3/2)*log(2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 +
a)*sqrt(b)*tan(x)^2 + a)*tan(x)^2 - (a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x
)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b)
 + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^2 - 2*sqrt(b*tan(x)^4
+ a)*(b*tan(x)^2 - a))/tan(x)^2, 1/4*(2*sqrt(-b)*b*arctan(sqrt(-b)*tan(x)^
2/sqrt(b*tan(x)^4 + a))*tan(x)^2 - 2*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*tan
(x)^4 + a))*tan(x)^2 + (a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*t
an(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*
b)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*ta...
```

Sympy [F]

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \int \left(a + b\tan^4{\left(x \right)}\right)^{\frac{3}{2}} \cot^3{\left(x \right)}\, dx$$

input
```
integrate(cot(x)**3*(a+b*tan(x)**4)**(3/2),x)
```

output
```
Integral((a + b*tan(x)**4)**(3/2)*cot(x)**3, x)
```

Maxima [F]

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \int \left(b\tan(x)^4+a\right)^{\frac{3}{2}} \cot(x)^3 \, dx$$

input
```
integrate(cot(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(x)^4 + a)^(3/2)*cot(x)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx = -\frac{a^2 \arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{2} b^{\frac{3}{2}} \log\left(\left|-\sqrt{b}\tan(x)^2+\sqrt{b\tan(x)^4+a}\right|\right) + \frac{a^2\sqrt{b}}{\left(\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}\right)^2-a} + \frac{1}{2}\sqrt{b\tan(x)^4+ab} + \frac{(a^2+2ab+b^2)\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

input
```
integrate(cot(x)^3*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-a^2*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a))/sqrt(-a))/sqrt(-a)
+ 1/2*b^(3/2)*log(abs(-sqrt(b)*tan(x)^2 + sqrt(b*tan(x)^4 + a))) + a^2*sqr
t(b)/((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a))^2 - a) + 1/2*sqrt(b*tan(x)
^4 + a)*b + (a^2 + 2*a*b + b^2)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^
4 + a) + sqrt(b))/sqrt(-a - b))/sqrt(-a - b)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \int \cot(x)^3 \left(b\tan(x)^4+a\right)^{3/2} dx$$

input
```
int(cot(x)^3*(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)^3*(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \cot^3(x) \left(a+b\tan^4(x)\right)^{3/2} dx = \int \cot(x)^3 \left(\tan(x)^4 b+a\right)^{\frac{3}{2}} dx$$

input
```
int(cot(x)^3*(a+b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)^3*(a+b*tan(x)^4)^(3/2),x)
```

3.397 $\int \cot^5(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 151

$$\int \cot^5(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \frac{1}{2}b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right) - \frac{1}{4}\sqrt{a}(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right) + \frac{1}{2}a\cot^2(x)\sqrt{a$$

output
```
1/2*b^(3/2)*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))+1/2*(a+b)^(3/2)
*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))-1/4*a^(1/2)*(2*a
+3*b)*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))+1/2*a*cot(x)^2*(a+b*tan(x)^4)^
(1/2)-1/4*a*cot(x)^4*(a+b*tan(x)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.64 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.10

$$\int \cot^5(x)\left(a+b\tan^4(x)\right)^{3/2}\,dx = \frac{b(4(a-b)\cos(2x)+(a+b)(3+\cos(4x)))^2\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1+\frac{b\tan^4(x)}{a}\right)}{640a^2}$$
$$+\frac{\sqrt{a}\sqrt{b}\,\text{arcsinh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a}}\right)\sqrt{a+b\tan^4(x)}}{4\sqrt{1+\frac{b\tan^4(x)}{a}}}$$
$$+\frac{a\cot^2(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-\frac{b\tan^4(x)}{a}\right)\sqrt{a+b\tan^4(x)}}{2\sqrt{1+\frac{b\tan^4(x)}{a}}}$$
$$+\frac{1}{4}\left(2\sqrt{b}(a+b)\,\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)\right.$$
$$+2(a+b)^{3/2}\,\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)-2a^{3/2}\,\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)-2b\sqrt{a+b\tan^4(x)}+b\,\text{ta}$$

input
```
Integrate[Cot[x]^5*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(b*(4*(a - b)*Cos[2*x] + (a + b)*(3 + Cos[4*x]))^2*Hypergeometric2F1[2, 5/
2, 7/2, 1 + (b*Tan[x]^4)/a]*Sec[x]^8*Sqrt[a + b*Tan[x]^4])/(640*a^2) + (Sq
rt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tan[x]^2)/Sqrt[a]]*Sqrt[a + b*Tan[x]^4])/(4
*Sqrt[1 + (b*Tan[x]^4)/a]) + (a*Cot[x]^2*Hypergeometric2F1[-3/2, -1/2, 1/2
, -((b*Tan[x]^4)/a)]*Sqrt[a + b*Tan[x]^4])/(2*Sqrt[1 + (b*Tan[x]^4)/a]) +
(2*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]] + 2*(a
 + b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] -
 2*a^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] - 2*b*Sqrt[a + b*Tan[x]^4
] + b*Tan[x]^2*Sqrt[a + b*Tan[x]^4])/4
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4153, 1579, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(x) \left(a+b\tan^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\tan(x)^4\right)^{3/2}}{\tan(x)^5}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\int \frac{\cot^5(x)\left(a+b\tan^4(x)\right)^{3/2}}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{\cot^3(x)\left(b\tan^4(x)+a\right)^{3/2}}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{ 617}$$

$$\frac{1}{2}\int \left(\left(b\tan^4(x)+a\right)^{3/2}\cot^3(x)-\left(b\tan^4(x)+a\right)^{3/2}\cot^2(x)+\left(b\tan^4(x)+a\right)^{3/2}\cot(x)+\frac{\left(b\tan^4(x)+a\right)^{3/2}}{-\tan^2(x)-1}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)-\frac{3}{2}\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x}{\sqrt{a+b\tan^4}}\right.\right.$$

input
```
Int[Cot[x]^5*(a + b*Tan[x]^4)^(3/2),x]
```

output
```
((-3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 + (Sqrt
[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]])/2 + (a +
 b)^(3/2)*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])] - a
^(3/2)*ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]] - (3*Sqrt[a]*b*ArcTanh[Sqrt[a
 + b*Tan[x]^4]/Sqrt[a]])/2 + a*Sqrt[a + b*Tan[x]^4] + (3*b*Sqrt[a + b*Tan[
x]^4])/2 - (3*b*Tan[x]^2*Sqrt[a + b*Tan[x]^4])/2 - ((2*(a + b) - b*Tan[x]^
2)*Sqrt[a + b*Tan[x]^4])/2 + Cot[x]*(a + b*Tan[x]^4)^(3/2) - (Cot[x]^2*(a
+ b*Tan[x]^4)^(3/2))/2)/2
```

Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \cot(x)^5 \left(a + b\tan(x)^4\right)^{\frac{3}{2}} dx$$

input
```
int(cot(x)^5*(a+b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)^5*(a+b*tan(x)^4)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 21.45 (sec) , antiderivative size = 1464, normalized size of antiderivative = 9.70

$$\int \cot^5(x) \left(a + b\tan^4(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(cot(x)^5*(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(2*b^(3/2)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2
 + a)*tan(x)^4 + 2*(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x
)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(
tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^4 + (2*a + 3*b)*sqrt(a)*log((b*tan(x)^4
 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4)*tan(x)^4 + 2*sqrt(b*tan
(x)^4 + a)*(2*a*tan(x)^2 - a))/tan(x)^4, -1/8*(4*sqrt(-b)*b*arctan(sqrt(-b
)*tan(x)^2/sqrt(b*tan(x)^4 + a))*tan(x)^4 - 2*(a + b)^(3/2)*log(((a*b + 2*
b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*s
qrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^4 - (2*a + 3
*b)*sqrt(a)*log((b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)
^4)*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*(2*a*tan(x)^2 - a))/tan(x)^4, 1/4*(s
qrt(-a)*(2*a + 3*b)*arctan(sqrt(-a)/sqrt(b*tan(x)^4 + a))*tan(x)^4 + b^(3/
2)*log(2*b*tan(x)^4 + 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 + a)*tan(x)^
4 + (a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*
tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*ta
n(x)^2 + 1))*tan(x)^4 + sqrt(b*tan(x)^4 + a)*(2*a*tan(x)^2 - a))/tan(x)^4,
 -1/4*(2*sqrt(-b)*b*arctan(sqrt(-b)*tan(x)^2/sqrt(b*tan(x)^4 + a))*tan(x)^
4 - sqrt(-a)*(2*a + 3*b)*arctan(sqrt(-a)/sqrt(b*tan(x)^4 + a))*tan(x)^4 -
(a + b)^(3/2)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(
x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan...
```

Sympy [F]

$$\int \cot^5(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \int \left(a+b\tan^4{(x)}\right)^{\frac{3}{2}} \cot^5{(x)}\, dx$$

input
```
integrate(cot(x)**5*(a+b*tan(x)**4)**(3/2),x)
```

output
```
Integral((a + b*tan(x)**4)**(3/2)*cot(x)**5, x)
```

Maxima [F]

$$\int \cot^5(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \int \left(b\tan\left(x\right)^4+a\right)^{\frac{3}{2}} \cot\left(x\right)^5 \, dx$$

input
```
integrate(cot(x)^5*(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(x)^4 + a)^(3/2)*cot(x)^5, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13

$$\int \cot^5(x) \left(a+b\tan^4(x)\right)^{3/2} \, dx = \frac{\left(2\,a^2+3\,ab\right)\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} + \frac{\left(\sqrt{b}\tan\left(x\right)^2-\sqrt{b\tan\left(x\right)^4+a}\right)^3 ab - 2\left(\sqrt{b}\tan\left(x\right)^2-\sqrt{b\tan\left(x\right)^4+a}\right)^2 a^2\sqrt{b} + \left(\sqrt{b}\tan\left(x\right)^2-\sqrt{b}\right.}{2\left(\left(\sqrt{b}\tan\left(x\right)^2-\sqrt{b\tan\left(x\right)^4+a}\right)^2-a\right)^2}$$

input
```
integrate(cot(x)^5*(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*a^2 + 3*a*b)*arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a))/sqrt
(-a))/sqrt(-a) + 1/2*((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a))^3*a*b - 2*
(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a))^2*a^2*sqrt(b) + (sqrt(b)*tan(x)^
2 - sqrt(b*tan(x)^4 + a))*a^2*b + 2*a^3*sqrt(b))/((sqrt(b)*tan(x)^2 - sqrt
(b*tan(x)^4 + a))^2 - a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \cot^5(x) \left(a + b\tan^4(x)\right)^{3/2} dx = \int \cot(x)^5 \left(b\tan(x)^4 + a\right)^{3/2} dx$$

input
```
int(cot(x)^5*(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)^5*(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \cot^5(x) \left(a + b\tan^4(x)\right)^{3/2} dx = \int \cot(x)^5 \left(\tan(x)^4 b + a\right)^{\frac{3}{2}} dx$$

input
```
int(cot(x)^5*(a+b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)^5*(a+b*tan(x)^4)^(3/2),x)
```

3.398 $\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

output
```
1/2*arctanh(b^(1/2)*tan(x)^2/(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2*arctanh((a-
b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Integrate[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

output
```
ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/(2*Sqrt[b]) + ArcTanh[(a
- b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4153, 1579, 605, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^3}{\sqrt{a+b\tan(x)^4}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan^3(x)}{(\tan^2(x)+1)\sqrt{a+b\tan^4(x)}}d\tan(x)$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{\tan^2(x)}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan^2(x)$$

$$\Big\downarrow \text{605}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan^2(x)-\int \frac{1}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\int \frac{1}{1-b\tan^4(x)}d\frac{\tan^2(x)}{\sqrt{b\tan^4(x)+a}}-\int \frac{1}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}-\int \frac{1}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan^2(x)\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{b}}+\frac{\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{a+b}}\right)$$

input
```
Int[Tan[x]^3/Sqrt[a + b*Tan[x]^4],x]
```

output
```
(ArcTanh[(Sqrt[b]*Tan[x]^2)/Sqrt[a + b*Tan[x]^4]]/Sqrt[b] + ArcTanh[(a - b
*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/Sqrt[a + b])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

method	result	siz
derivativedivides	$\frac{\ln\left(\sqrt{b}\,\tan(x)^2+\sqrt{a+b\tan(x)^4}\right)}{2\sqrt{b}}+\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\sqrt{a+b}}$	91
default	$\frac{\ln\left(\sqrt{b}\,\tan(x)^2+\sqrt{a+b\tan(x)^4}\right)}{2\sqrt{b}}+\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\sqrt{a+b}}$	91

input
```
int(tan(x)^3/(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(b^(1/2)*tan(x)^2+(a+b*tan(x)^4)^(1/2))/b^(1/2)+1/2/(a+b)^(1/2)*ln((
2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+
a+b)^(1/2))/(1+tan(x)^2))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 483, normalized size of antiderivative = 6.53

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$
$$= \left[\frac{(a+b)\sqrt{b}\log\left(-2\,b\tan{(x)}^4-2\,\sqrt{b\tan{(x)}^4+a}\sqrt{b}\tan{(x)}^2-a\right)+\sqrt{a+b}b\log\left(\frac{(ab+2\,b^2)\tan(x)^4-2\,ab\tan\right.}{4\,(ab+b^2)}\right.$$
$$-\frac{2\,(a+b)\sqrt{-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\sqrt{-b}}{b\tan(x)^2}\right)-\sqrt{a+b}b\log\left(\frac{(ab+2\,b^2)\tan(x)^4-2\,ab\tan(x)^2-2\,\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-\right.}{\tan(x)^4+2\,\tan(x)^2+1}\right.}{4\,(ab+b^2)}$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(b)*t
an(x)^2 - a) + sqrt(a + b)*b*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2
- 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(
x)^4 + 2*tan(x)^2 + 1)))/(a*b + b^2), -1/4*(2*(a + b)*sqrt(-b)*arctan(sqrt
(b*tan(x)^4 + a)*sqrt(-b)/(b*tan(x)^2)) - sqrt(a + b)*b*log(((a*b + 2*b^2)
*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(
a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a*b + b^2), 1/4*(2*sq
rt(-a - b)*b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a
*b + b^2)*tan(x)^4 + a^2 + a*b)) + (a + b)*sqrt(b)*log(-2*b*tan(x)^4 - 2*s
qrt(b*tan(x)^4 + a)*sqrt(b)*tan(x)^2 - a))/(a*b + b^2), 1/2*(sqrt(-a - b)*
b*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*t
an(x)^4 + a^2 + a*b)) - (a + b)*sqrt(-b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(
-b)/(b*tan(x)^2)))/(a*b + b^2)]
```

Sympy [F]

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

input
```
integrate(tan(x)**3/(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(tan(x)**3/sqrt(a + b*tan(x)**4), x)
```

Maxima [F]

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)^3}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)^3/sqrt(b*tan(x)^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionDegree mismatch inside factorisation over extensionError:
Bad Argum
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)^3}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
int(tan(x)^3/(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(tan(x)^3/(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^3(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^3}{\tan(x)^4 b+a}dx$$

input
```
int(tan(x)^3/(a+b*tan(x)^4)^(1/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*tan(x)**3)/(tan(x)**4*b + a),x)
```

3.399 $\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

output
```
-1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Integrate[Tan[x]/Sqrt[a + b*Tan[x]^4],x]
```

output
```
-1/2*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/Sqrt[a +
 b]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4153, 1577, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\sqrt{a+b\tan(x)^4}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{(\tan^2(x)+1)\sqrt{a+b\tan^4(x)}}d\tan(x)$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{1}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan^2(x)$$

$$\Big\downarrow \text{488}$$

$$-\frac{1}{2}\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Int[Tan[x]/Sqrt[a + b*Tan[x]^4],x]
```

output
```
-1/2*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/Sqrt[a +
 b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\sqrt{a+b}}$	65
default	$-\frac{\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\sqrt{a+b}}$	65

input
```
int(tan(x)/(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2
)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.66

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \left[\frac{\log\left(\frac{(ab+2\,b^2)\tan(x)^4-2\,ab\tan(x)^2+2\,\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2\,a^2+ab}{\tan(x)^4+2\,\tan(x)^2+1}\right)}{4\,\sqrt{a+b}}, -\frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right)}{2\,(a+b)}\right]$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)
*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1))/
sqrt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b))/(a + b)]
```

Sympy [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

input
```
integrate(tan(x)/(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(tan(x)/sqrt(a + b*tan(x)**4), x)
```

Maxima [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/sqrt(b*tan(x)^4 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \frac{\arctan\left(-\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
arctan(-(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/
sqrt(-a - b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)}{\sqrt{b\tan\left(x\right)^4+a}}\,dx$$

input
```
int(tan(x)/(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(tan(x)/(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\sqrt{\tan\left(x\right)^4 b+a}\,\tan\left(x\right)}{\tan\left(x\right)^4 b+a}dx$$

input
```
int(tan(x)/(a+b*tan(x)^4)^(1/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*tan(x))/(tan(x)**4*b + a),x)
```

3.400 $\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)-1
/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Integrate[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

output
```
ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(2*Sqrt[a + b
]) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4153, 1579, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(x)\sqrt{a+b\tan(x)^4}}dx$$

$$\Big\downarrow \; 4153$$

$$\int \frac{\cot(x)}{(\tan^2(x)+1)\sqrt{a+b\tan^4(x)}}d\tan(x)$$

$$\Big\downarrow \; 1579$$

$$\frac{1}{2}\int \frac{\cot(x)}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan^2(x)$$

$$\Big\downarrow \; 617$$

$$\frac{1}{2}\int \left(\frac{\cot(x)}{\sqrt{b\tan^4(x)+a}}+\frac{1}{(-\tan^2(x)-1)\sqrt{b\tan^4(x)+a}}\right)d\tan^2(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{\sqrt{a+b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Int[Cot[x]/Sqrt[a + b*Tan[x]^4],x]
```

output
```
(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/Sqrt[a + b]
- ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/Sqrt[a])/2
```

Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\tan(x)^4}}dx$$

input
```
int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

output
```
int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 475, normalized size of antiderivative = 6.79

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

$$= \left[\frac{\sqrt{a+b}a\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a+b}+2a^2+ab}{\tan(x)^4+2\tan(x)^2+1}\right)+(a+b)\sqrt{a}\log\left(-\frac{b\tan(x)}{}\right.}{4(a^2+ab)}\right.$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(
b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*
tan(x)^2 + 1)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)
*sqrt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/4*(2*sqrt(-a)*(a + b)*arctan(sqr
t(b*tan(x)^4 + a)*sqrt(-a)/a) + sqrt(a + b)*a*log(((a*b + 2*b^2)*tan(x)^4
- 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2
*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(a^2 + a*b), 1/4*(2*a*sqrt(-a -
b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*
tan(x)^4 + a^2 + a*b)) + (a + b)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x
)^4 + a)*sqrt(a) + 2*a)/tan(x)^4))/(a^2 + a*b), 1/2*(a*sqrt(-a - b)*arctan
(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4
+ a^2 + a*b)) + sqrt(-a)*(a + b)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a))/
(a^2 + a*b)]
```

Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

input
```
integrate(cot(x)/(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(cot(x)/sqrt(a + b*tan(x)**4), x)
```

Maxima [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)/sqrt(b*tan(x)^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\cot(x)}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
int(cot(x)/(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(cot(x)/(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cot(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\sqrt{\tan(x)^4 b + a}\,\cot(x)}{\tan(x)^4 b + a}dx$$

input
```
int(cot(x)/(a+b*tan(x)^4)^(1/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*cot(x))/(tan(x)**4*b + a),x)
```

3.401 $\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 291

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}} + \frac{\sqrt[4]{a}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\sqrt{a+b\tan^4(x)}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}}{4\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}$$

output
```
-1/2*arctan((a+b)^(1/2)*tan(x)/(a+b*tan(x)^4)^(1/2))/(a+b)^(1/2)+1/2*a^(1/
4)*InverseJacobiAM(2*arctan(b^(1/4)*tan(x)/a^(1/4)),1/2*2^(1/2))*(a^(1/2)+
b^(1/2)*tan(x)^2)*((a+b*tan(x)^4)/(a^(1/2)+b^(1/2)*tan(x)^2)^2)^(1/2)/(a^(
1/2)-b^(1/2))/b^(1/4)/(a+b*tan(x)^4)^(1/2)-1/4*(a^(1/2)+b^(1/2))*EllipticP
i(sin(2*arctan(b^(1/4)*tan(x)/a^(1/4))),-1/4*(a^(1/2)-b^(1/2))^2/a^(1/2)/b
^(1/2),1/2*2^(1/2))*(a^(1/2)+b^(1/2)*tan(x)^2)*((a+b*tan(x)^4)/(a^(1/2)+b^
(1/2)*tan(x)^2)^2)^(1/2)/a^(1/4)/(a^(1/2)-b^(1/2))/b^(1/4)/(a+b*tan(x)^4)^
(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.42

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx =$$
$$-\frac{i\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right),-1\right)-\text{EllipticPi}\left(-\frac{i\sqrt{a}}{\sqrt{b}},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\tan(x)\right),-1\right)\right)\sqrt{1+}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\tan^4(x)}}$$

input
```
Integrate[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

output
```
((-I)*(EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]], -1] - Ellipt
icPi[((-I)*Sqrt[a])/Sqrt[b], I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*Tan[x]],
-1])*Sqrt[1 + (b*Tan[x]^4)/a])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*Tan[x
]^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4153, 1657, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(x)^2}{\sqrt{a+b\tan(x)^4}}dx$$

$$\Big\downarrow \; 4153$$

$$\int \frac{\tan^2(x)}{(\tan^2(x)+1)\sqrt{a+b\tan^4(x)}}d\tan(x)$$

$\Big\downarrow$ 1657

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan(x)}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{b}\tan^2(x)+\sqrt{a}}{\sqrt{a}(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan(x)}{\sqrt{a}-\sqrt{b}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{b\tan^4(x)+a}}d\tan(x)}{\sqrt{a}-\sqrt{b}}-\frac{\int \frac{\sqrt{b}\tan^2(x)+\sqrt{a}}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan(x)}{\sqrt{a}-\sqrt{b}}$$

$\Big\downarrow$ 761

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\int \frac{\sqrt{b}\tan^2(x)+\sqrt{a}}{(\tan^2(x)+1)\sqrt{b\tan^4(x)+a}}d\tan(x)}{\sqrt{a}-\sqrt{b}}$$

$\Big\downarrow$ 2221

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{a+b\tan^4(x)}}-\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a+b\tan^4(x)}}\right)}{2\sqrt{a+b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)\sqrt{\frac{a+b\tan^4(x)}{\left(\sqrt{a}+\sqrt{b}\tan^2(x)\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{4\sqrt{a}\sqrt{b}},2\arctan\left(\frac{\sqrt[4]{b}\tan(x)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b\tan^4(x)}}}{\sqrt{a}-\sqrt{b}}$$

input
```
Int[Tan[x]^2/Sqrt[a + b*Tan[x]^4],x]
```

output
```
(a^(1/4)*EllipticF[2*ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqr
t[b]*Tan[x]^2)*Sqrt[(a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(2*(
Sqrt[a] - Sqrt[b])*b^(1/4)*Sqrt[a + b*Tan[x]^4]) - (((Sqrt[a] - Sqrt[b])*A
rcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a + b*Tan[x]^4]])/(2*Sqrt[a + b]) + ((Sqrt
[a] + Sqrt[b])*EllipticPi[-1/4*(Sqrt[a] - Sqrt[b])^2/(Sqrt[a]*Sqrt[b]), 2*
ArcTan[(b^(1/4)*Tan[x])/a^(1/4)], 1/2]*(Sqrt[a] + Sqrt[b]*Tan[x]^2)*Sqrt[(
a + b*Tan[x]^4)/(Sqrt[a] + Sqrt[b]*Tan[x]^2)^2])/(4*a^(1/4)*b^(1/4)*Sqrt[a
 + b*Tan[x]^4]))/(Sqrt[a] - Sqrt[b])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1657
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2]}, Simp[(-a)*((e + d*q)/(c*d^2 - a*e^2))   Int[1/Sqrt[a
 + c*x^4], x], x] + Simp[a*d*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/
((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PosQ[c/
a] && NeQ[c*d^2 - a*e^2, 0]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(x)^4}}-\frac{\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(\tan\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(x)^4}}$
default	$\frac{\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(\tan(x)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(x)^4}}-\frac{\sqrt{1-\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,\tan(x)^2}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(\tan\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\tan(x)^4}}$

input
```
int(tan(x)^2/(a+b*tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*EllipticF(tan(x)*(I/a^(1/
2)*b^(1/2))^(1/2),I)-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tan(
x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tan(x)^2)^(1/2)/(a+b*tan(x)^4)^(1/2)*Elli
pticPi(tan(x)*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1
/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Sympy [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx$$

input
```
integrate(tan(x)**2/(a+b*tan(x)**4)**(1/2),x)
```

output
```
Integral(tan(x)**2/sqrt(a + b*tan(x)**4), x)
```

Maxima [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Giac [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
integrate(tan(x)^2/(a+b*tan(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(x)^2/sqrt(b*tan(x)^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\tan(x)^2}{\sqrt{b\tan(x)^4+a}}\,dx$$

input
```
int(tan(x)^2/(a + b*tan(x)^4)^(1/2),x)
```

output
```
int(tan(x)^2/(a + b*tan(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^2(x)}{\sqrt{a+b\tan^4(x)}}\,dx = \int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^2}{\tan(x)^4 b+a}\,dx$$

input
```
int(tan(x)^2/(a+b*tan(x)^4)^(1/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*tan(x)**2)/(tan(x)**4*b + a),x)
```

3.402 $\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{1-\tan^2(x)}{2(a+b)\sqrt{a+b\tan^4(x)}}$$

output
```
1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)-1
/2*(1-tan(x)^2)/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}} + \frac{-1+\tan^2(x)}{(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

input
```
Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2
) + (-1 + Tan[x]^2)/((a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4153, 1579, 593, 25, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^3}{\left(a+b\tan(x)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan^3(x)}{\left(\tan^2(x)+1\right)\left(a+b\tan^4(x)\right)^{3/2}}d\tan(x)$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{\tan^2(x)}{\left(\tan^2(x)+1\right)\left(b\tan^4(x)+a\right)^{3/2}}d\tan^2(x)$$

$$\Big\downarrow \text{593}$$

$$\frac{1}{2}\left(\frac{\int -\frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{a+b}-\frac{1-\tan^2(x)}{(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{a+b}-\frac{1-\tan^2(x)}{(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}}{a+b}-\frac{1-\tan^2(x)}{(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}-\frac{1-\tan^2(x)}{(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

input
```
Int[Tan[x]^3/(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2
) - (1 - Tan[x]^2)/((a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(59) = 118$.

Time = 29.78 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.76

method	result
derivativedivides	$\frac{\tan(x)^2}{2\sqrt{a+b\tan(x)^4}\,a}-\frac{b\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\left(b+\sqrt{-ab}\right)\left(-b+\sqrt{-ab}\right)\sqrt{a+b}}-\frac{\sqrt{\left(\tan(x)^2-\frac{\sqrt{-}}{b}\right.}}{4a(b+\sqrt{}}$
default	$\frac{\tan(x)^2}{2\sqrt{a+b\tan(x)^4}\,a}-\frac{b\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\left(b+\sqrt{-ab}\right)\left(-b+\sqrt{-ab}\right)\sqrt{a+b}}-\frac{\sqrt{\left(\tan(x)^2-\frac{\sqrt{-}}{b}\right.}}{4a(b+\sqrt{}}$

input
```
int(tan(x)^3/(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b*tan(x)^4)^(1/2)/a*tan(x)^2-1/2*b/(b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2
))/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^
2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+tan(x)^2))-1/4/a/(b+(-a*b)^(1/2))/(tan(x
)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^
2-(-a*b)^(1/2)/b))^(1/2)+1/4/a/(-b+(-a*b)^(1/2))/(tan(x)^2+(-a*b)^(1/2)/b)
*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(59) = 118$.

Time = 0.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.11

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \left[\frac{(b\tan(x)^4+a)\sqrt{a+b}\log\left(\frac{(ab+2b^2)\tan(x)^4-2ab\tan(x)^2-2\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{a}}{\tan(x)^4+2\tan(x)^2+1}\right.}{4\left((a^2b+2ab^2+b^3)\tan(x)^4+a^3\right.}\right.$$

input

```
integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output

```
[1/4*((b*tan(x)^4 + a)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan
(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)
/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a + b)*tan(x)^2 -
 a - b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2), 1/2*(
(b*tan(x)^4 + a)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)
*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + sqrt(b*tan(x)^4 + a)*(
(a + b)*tan(x)^2 - a - b))/((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2
*b + a*b^2)]
```

Sympy [F]

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\tan^3(x)}{(a+b\tan^4(x))^{\frac{3}{2}}}\,dx$$

input

```
integrate(tan(x)**3/(a+b*tan(x)**4)**(3/2),x)
```

output

```
Integral(tan(x)**3/(a + b*tan(x)**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\tan(x)^3}{\left(b\tan(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)^3/(b*tan(x)^4 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.45

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \frac{\frac{(a+b)\tan(x)^2}{a^2+2\,ab+b^2} - \frac{a+b}{a^2+2\,ab+b^2}}{2\sqrt{b\tan(x)^4+a}} + \frac{\arctan\left(\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/2*((a + b)*tan(x)^2/(a^2 + 2*a*b + b^2) - (a + b)/(a^2 + 2*a*b + b^2))/s
qrt(b*tan(x)^4 + a) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sq
rt(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\tan(x)^3}{\left(b\tan(x)^4+a\right)^{3/2}}\,dx$$

input
```
int(tan(x)^3/(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(tan(x)^3/(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx = \frac{-\sqrt{\tan\left(x\right)^4 b+a}-2\left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^5}{\tan(x)^8 b^2+2\tan(x)^4 ab+a^2}dx\right)\tan\left(x\right)^4 b^2-2\left(\int \frac{\sqrt{\tan(x)^4 b\cdots}}{\tan(x)^8 b^2+2\,\mathrm{t}\cdots}\right.}{2b\left(\tan\left(x\right)^4 b+a\right)}$$

input
```
int(tan(x)^3/(a+b*tan(x)^4)^(3/2),x)
```

output
```
( - sqrt(tan(x)**4*b + a) - 2*int((sqrt(tan(x)**4*b + a)*tan(x)**5)/(tan(x
)**8*b**2 + 2*tan(x)**4*a*b + a**2),x)*tan(x)**4*b**2 - 2*int((sqrt(tan(x)
**4*b + a)*tan(x)**5)/(tan(x)**8*b**2 + 2*tan(x)**4*a*b + a**2),x)*a*b)/(2
*b*(tan(x)**4*b + a))
```

3.403 $\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}}+\frac{a+b\tan^2(x)}{2a(a+b)\sqrt{a+b\tan^4(x)}}$$

output
```
-1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)+
1/2*(a+b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}+\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

input
```
Integrate[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(-(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3
/2)) + (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 4153, 1577, 496, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(a+b\tan(x)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{\left(\tan^2(x)+1\right)\left(a+b\tan^4(x)\right)^{3/2}}d\tan(x)$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{1}{\left(\tan^2(x)+1\right)\left(b\tan^4(x)+a\right)^{3/2}}d\tan^2(x)$$

$$\Big\downarrow \text{496}$$

$$\frac{1}{2}\left(\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\int -\frac{a}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{a(a+b)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{a}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{a(a+b)}+\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{a+b}+\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}}{a+b}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}\right)$$

input
```
Int[Tan[x]/(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(-(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3
/2)) + (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(62) = 124$.

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.35

method	result
derivativedivides	$\frac{b\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\left(b+\sqrt{-ab}\right)\left(-b+\sqrt{-ab}\right)\sqrt{a+b}} + \frac{\sqrt{\left(\tan(x)^2-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(\tan(x\right.}}{4a\left(b+\sqrt{-ab}\right)\left(\tan(x)^2-\frac{\sqrt{-ab}}{b}\right.}$
default	$\frac{b\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\left(b+\sqrt{-ab}\right)\left(-b+\sqrt{-ab}\right)\sqrt{a+b}} + \frac{\sqrt{\left(\tan(x)^2-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\,\left(\tan(x\right.}}{4a\left(b+\sqrt{-ab}\right)\left(\tan(x)^2-\frac{\sqrt{-ab}}{b}\right.}$

input
```
int(tan(x)/(a+b*tan(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/(b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2))/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+ta
n(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2))/(1+ta
n(x)^2))+1/4/a/(b+(-a*b)^(1/2))/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b
)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/4/a/(-b+(
-a*b)^(1/2))/(tan(x)^2+(-a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-
a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(64) = 128$.

Time = 0.30 (sec) , antiderivative size = 319, normalized size of antiderivative = 4.31

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx = \left[\frac{\left(ab\tan\left(x\right)^4+a^2\right)\sqrt{a+b}\log\left(\frac{\left(ab+2\,b^2\right)\tan\left(x\right)^4-2\,ab\tan\left(x\right)^2+2\,\sqrt{b\tan\left(x\right)^4+a}\left(b\tan\left(x\right)^2-a\right)}{\tan\left(x\right)^4+2\,\tan\left(x\right)^2+1}\right.}{4\left(\left(a^3b+2\,a^2b^2+ab^3\right)\tan\left(x\right)^4+\right.}\right.$$

$$-\frac{\left(ab\tan\left(x\right)^4+a^2\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan\left(x\right)^4+a}\left(b\tan\left(x\right)^2-a\right)\sqrt{-a-b}}{\left(ab+b^2\right)\tan\left(x\right)^4+a^2+ab}\right)-\sqrt{b\tan\left(x\right)^4+a}\left(\left(ab+b^2\right)\tan\left(x\right)^2\right.}{2\left(\left(a^3b+2\,a^2b^2+ab^3\right)\tan\left(x\right)^4+a^4+2\,a^3b+a^2b^2\right)}$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*b*tan(x)^4 + a^2)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b
*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 +
a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*((a*b + b^2)*ta
n(x)^2 + a^2 + a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(x)^4 + a^4 + 2*a^3*b
 + a^2*b^2), -1/2*((a*b*tan(x)^4 + a^2)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^
4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) -
 sqrt(b*tan(x)^4 + a)*((a*b + b^2)*tan(x)^2 + a^2 + a*b))/((a^3*b + 2*a^2*
b^2 + a*b^3)*tan(x)^4 + a^4 + 2*a^3*b + a^2*b^2)]
```

Sympy [F]

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\tan{(x)}}{(a+b\tan^4{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*tan(x)**4)**(3/2),x)
```

output
```
Integral(tan(x)/(a + b*tan(x)**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\tan{(x)}}{(b\tan{(x)}^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(b*tan(x)^4 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \frac{\frac{(ab+b^2)\tan(x)^2}{a^3+2a^2b+ab^2}+\frac{a^2+ab}{a^3+2a^2b+ab^2}}{2\sqrt{b\tan{(x)}^4+a}} - \frac{\arctan\left(\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/2*((a*b + b^2)*tan(x)^2/(a^3 + 2*a^2*b + a*b^2) + (a^2 + a*b)/(a^3 + 2*a
^2*b + a*b^2))/sqrt(b*tan(x)^4 + a) - arctan((sqrt(b)*tan(x)^2 - sqrt(b*ta
n(x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a + b)*sqrt(-a - b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\tan(x)}{\left(b\tan\left(x\right)^4+a\right)^{3/2}}\,dx$$

input
```
int(tan(x)/(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(tan(x)/(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\sqrt{\tan\left(x\right)^4 b+a}\,\tan\left(x\right)}{\tan\left(x\right)^8 b^2+2\tan\left(x\right)^4 ab+a^2}dx$$

input
```
int(tan(x)/(a+b*tan(x)^4)^(3/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*tan(x))/(tan(x)**8*b**2 + 2*tan(x)**4*a*b + a**
2),x)
```

3.404 $\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{b(1-\tan^2(x))}{2a(a+b)\sqrt{a+b\tan^4(x)}}$$

output
```
1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(3/2)-1
/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(3/2)+1/2*b*(1-tan(x)^2)/a/(a+b
)/(a+b*tan(x)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}} + \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\tan^4(x)}{a}\right)}{a\sqrt{a+b\tan^4(x)}} - \frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

input
```
Integrate[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2
) + Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tan[x]^4)/a]/(a*Sqrt[a + b*Tan[
x]^4]) - (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4153, 1579, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)\left(a+b\tan(x)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\cot(x)}{(\tan^2(x)+1)\left(a+b\tan^4(x)\right)^{3/2}}d\tan(x)$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{\cot(x)}{(\tan^2(x)+1)\left(b\tan^4(x)+a\right)^{3/2}}d\tan^2(x)$$

$\Big\downarrow$ 617

$$\frac{1}{2}\int \left(\frac{\cot(x)}{\left(b\tan^4(x)+a\right)^{3/2}}+\frac{1}{(-\tan^2(x)-1)\left(b\tan^4(x)+a\right)^{3/2}}\right)d\tan^2(x)$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}+\frac{1}{a\sqrt{a+b\tan^4(x)}}-\frac{a+b\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}\right)$$

input
```
Int[Cot[x]/(a + b*Tan[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(3/2
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/a^(3/2) + 1/(a*Sqrt[a + b*Tan[x]
^4]) - (a + b*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/2
```

Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\cot(x)}{\left(a+b\tan(x)^{4}\right)^{\frac{3}{2}}}dx$$

input
```
int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(83) = 166$.

Time = 0.33 (sec) , antiderivative size = 954, normalized size of antiderivative = 9.26

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a
*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2
+ a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 +
a^3 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^4 - 2*sqrt(b*tan(x)^4 + a)*s
qrt(a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b +
 a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3
)*tan(x)^4), 1/4*(2*((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*
b^2)*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(-a)/a) + (a^2*b*tan(x)^4 +
a^3)*sqrt(a + b)*log(((a*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*t
an(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan
(x)^2 + 1)) + 2*sqrt(b*tan(x)^4 + a)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*tan(
x)^2))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(x)^4),
 1/4*(2*(a^2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b
*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((a^2*b
+ 2*a*b^2 + b^3)*tan(x)^4 + a^3 + 2*a^2*b + a*b^2)*sqrt(a)*log(-(b*tan(x)^
4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) + 2*sqrt(b*tan(x)^4 +
a)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*tan(x)^2))/(a^5 + 2*a^4*b + a^3*b^2 +
(a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(x)^4), 1/2*((a^2*b*tan(x)^4 + a^3)*sqrt(
-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b +
b^2)*tan(x)^4 + a^2 + a*b)) + ((a^2*b + 2*a*b^2 + b^3)*tan(x)^4 + a^3 +...
```

Sympy [F]

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\cot{(x)}}{(a+b\tan^4{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)/(a+b*tan(x)**4)**(3/2),x)
```

output
```
Integral(cot(x)/(a + b*tan(x)**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{3/2}}\,dx = \int \frac{\cot(x)}{\left(b\tan(x)^4+a\right)^{3/2}}\,dx$$

input
```
int(cot(x)/(a + b*tan(x)^4)^(3/2),x)
```

output
```
int(cot(x)/(a + b*tan(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{3/2}}\,dx = \int \frac{\sqrt{\tan(x)^4 b+a}\,\cot(x)}{\tan(x)^8 b^2+2\tan(x)^4 ab+a^2}dx$$

input
```
int(cot(x)/(a+b*tan(x)^4)^(3/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*cot(x))/(tan(x)**8*b**2 + 2*tan(x)**4*a*b + a**
2),x)
```

3.405 $\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 112

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{1-\tan^2(x)}{6(a+b)\left(a+b\tan^4(x)\right)^{3/2}} - \frac{3a-(2a-b)\tan^2(x)}{6a(a+b)^2\sqrt{a+b\tan^4(x)}}$$

output
```
1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)-1
/6*(1-tan(x)^2)/(a+b)/(a+b*tan(x)^4)^(3/2)-1/6*(3*a-(2*a-b)*tan(x)^2)/a/(a
+b)^2/(a+b*tan(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \frac{1}{6}\left(\frac{3\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}} + \frac{-a(4a+b)+3a^2\tan^2(x)-3ab\tan^4(x)+(2a-b)b\tan^6(x)}{a(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

input
```
Integrate[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

output
```
((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^
(5/2) + (-(a*(4*a + b)) + 3*a^2*Tan[x]^2 - 3*a*b*Tan[x]^4 + (2*a - b)*b*Ta
n[x]^6)/(a*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4153, 1579, 593, 25, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^3}{\left(a+b\tan(x)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan^3(x)}{\left(\tan^2(x)+1\right)\left(a+b\tan^4(x)\right)^{5/2}}d\tan(x)$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2}\int \frac{\tan^2(x)}{\left(\tan^2(x)+1\right)\left(b\tan^4(x)+a\right)^{5/2}}d\tan^2(x)$$

$$\Big\downarrow \text{593}$$

$$\frac{1}{2}\left(\frac{\int -\frac{1-2\tan^2(x)}{\left(\tan^2(x)+1\right)\left(b\tan^4(x)+a\right)^{3/2}}d\tan^2(x)}{3(a+b)}-\frac{1-\tan^2(x)}{3(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1-2\tan^2(x)}{(\tan^2(x)+1)(b\tan^4(x)+a)^{3/2}}d\tan^2(x)}{3(a+b)}-\frac{1-\tan^2(x)}{3(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$\Big\downarrow$ 686

$$\frac{1}{2}\left(-\frac{\frac{3a-(2a-b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\int -\frac{3ab}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{ab(a+b)}}{3(a+b)}-\frac{1-\tan^2(x)}{3(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{\frac{3\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{a+b}+\frac{3a-(2a-b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}}{3(a+b)}-\frac{1-\tan^2(x)}{3(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$\Big\downarrow$ 488

$$\frac{1}{2}\left(-\frac{\frac{3a-(2a-b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{3\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}}{a+b}}{3(a+b)}-\frac{1-\tan^2(x)}{3(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(-\frac{\frac{3a-(2a-b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{3\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}}{3(a+b)}-\frac{1-\tan^2(x)}{3(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

input
```
Int[Tan[x]^3/(a + b*Tan[x]^4)^(5/2),x]
```

output
```
(-1/3*(1 - Tan[x]^2)/((a + b)*(a + b*Tan[x]^4)^(3/2)) - ((-3*ArcTanh[(a -
b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^(3/2) + (3*a - (2
*a - b)*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/(3*(a + b)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(96) = 192$.

Time = 0.43 (sec) , antiderivative size = 638, normalized size of antiderivative = 5.70

method	result
derivativedivides	$\frac{\sqrt{a+b\tan(x)^4}\,\tan(x)^2\left(2b\tan(x)^4+3a\right)}{6a^2\left(b^2\tan(x)^8+2ab\tan(x)^4+a^2\right)}+\frac{b^2\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right.}{2\left(b+\sqrt{-ab}\right)^2\left(-b+\sqrt{-ab}\right)^2\sqrt{a+b}}$
default	$\frac{\sqrt{a+b\tan(x)^4}\,\tan(x)^2\left(2b\tan(x)^4+3a\right)}{6a^2\left(b^2\tan(x)^8+2ab\tan(x)^4+a^2\right)}+\frac{b^2\ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right.}{2\left(b+\sqrt{-ab}\right)^2\left(-b+\sqrt{-ab}\right)^2\sqrt{a+b}}$

input
```
int(tan(x)^3/(a+b*tan(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a+b*tan(x)^4)^(1/2)*tan(x)^2*(2*b*tan(x)^4+3*a)/a^2/(b^2*tan(x)^8+2*a
*b*tan(x)^4+a^2)+1/2*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^(1/2
)*ln((2*a+2*b-2*b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(
x)^2)+a+b)^(1/2))/(1+tan(x)^2))+1/8/(b+(-a*b)^(1/2))/a*(-1/3/(-a*b)^(1/2)/
(tan(x)^2-(-a*b)^(1/2)/b)^2*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*
(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)-1/3/a/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^
2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2))-1/8
/(-b+(-a*b)^(1/2))/a*(1/3/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/2)/b)^2*((tan(x
)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/
3/a/(tan(x)^2+(-a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2
)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2))+1/8*(2*(-a*b)^(1/2)-b)/(-b+(-a*b)^(1/2
))^2/a^2/(tan(x)^2+(-a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)
^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/8*(2*(-a*b)^(1/2)+b)/(b+(-a*b)^(
1/2))^2/a^2/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a
*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(95) = 190$.

Time = 0.29 (sec) , antiderivative size = 556, normalized size of antiderivative = 4.96

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \left[\frac{3\left(ab^2\tan\left(x\right)^8+2\,a^2b\tan\left(x\right)^4+a^3\right)\sqrt{a+b}\log\left(\frac{\left(ab+2\,b^2\right)\tan(x)^4-2\,ab\tan(x)^2-2\,\sqrt{b\cdots}}{\tan(x)^4+\cdots}\right.}{12\left(\left(a^4b^2+3\,a^3b^3+3\cdots\right.\right.}\right.$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*b^2*tan(x)^8 + 2*a^2*b*tan(x)^4 + a^3)*sqrt(a + b)*log(((a*b +
 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a
)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((2*a^2*b +
a*b^2 - b^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b
^2 + 3*(a^3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3
 + 3*a^2*b^4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(
a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4), 1/6*(3*(a*b^2*tan(x)^8
 + 2*a^2*b*tan(x)^4 + a^3)*sqrt(-a - b)*arctan(sqrt(b*tan(x)^4 + a)*(b*tan
(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(x)^4 + a^2 + a*b)) + ((2*a^2*b +
a*b^2 - b^3)*tan(x)^6 - 3*(a^2*b + a*b^2)*tan(x)^4 - 4*a^3 - 5*a^2*b - a*b
^2 + 3*(a^3 + a^2*b)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^4*b^2 + 3*a^3*b^3
 + 3*a^2*b^4 + a*b^5)*tan(x)^8 + a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + 2*(
a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(x)^4)]
```

Sympy [F]

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx=\int \frac{\tan^3{(x)}}{\left(a+b\tan^4{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)**3/(a+b*tan(x)**4)**(5/2),x)
```

output
```
Integral(tan(x)**3/(a + b*tan(x)**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tan^3(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx=\int \frac{\tan{(x)}^3}{\left(b\tan{(x)}^4+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)^3/(b*tan(x)^4 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(95) = 190$.

Time = 0.21 (sec) , antiderivative size = 597, normalized size of antiderivative = 5.33

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \frac{\left(\left(\frac{(2a^7b^2+11a^6b^3+24a^5b^4+25a^4b^5+10a^3b^6-3a^2b^7-4ab^8-b^9)\tan(x)^2}{a^9b+8a^8b^2+28a^7b^3+56a^6b^4+70a^5b^5+56a^4b^6+28a^3b^7+8a^2b^8+ab^9} - \frac{3(a^7b^2+6a^6b^3+15a^5}{a^9b+8a^8b^2+28a^7b^3+56a^6b}\right.\right.}{}$$

$$+ \frac{\arctan\left(\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a^2+2ab+b^2)\sqrt{-a-b}}$$

input
```
integrate(tan(x)^3/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/6*((((2*a^7*b^2 + 11*a^6*b^3 + 24*a^5*b^4 + 25*a^4*b^5 + 10*a^3*b^6 - 3*
a^2*b^7 - 4*a*b^8 - b^9)*tan(x)^2/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6
*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9) - 3*(a^7*
b^2 + 6*a^6*b^3 + 15*a^5*b^4 + 20*a^4*b^5 + 15*a^3*b^6 + 6*a^2*b^7 + a*b^8
)/(a^9*b + 8*a^8*b^2 + 28*a^7*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 +
 28*a^3*b^7 + 8*a^2*b^8 + a*b^9))*tan(x)^2 + 3*(a^8*b + 6*a^7*b^2 + 15*a^6
*b^3 + 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)/(a^9*b + 8*a^8*b^2 +
 28*a^7*b^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^
8 + a*b^9))*tan(x)^2 - (4*a^8*b + 25*a^7*b^2 + 66*a^6*b^3 + 95*a^5*b^4 + 8
0*a^4*b^5 + 39*a^3*b^6 + 10*a^2*b^7 + a*b^8)/(a^9*b + 8*a^8*b^2 + 28*a^7*b
^3 + 56*a^6*b^4 + 70*a^5*b^5 + 56*a^4*b^6 + 28*a^3*b^7 + 8*a^2*b^8 + a*b^9
))/(b*tan(x)^4 + a)^(3/2) + arctan((sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a
) + sqrt(b))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a - b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \int \frac{\tan(x)^3}{\left(b\tan(x)^4+a\right)^{5/2}}\,dx$$

input
```
int(tan(x)^3/(a + b*tan(x)^4)^(5/2),x)
```

output
```
int(tan(x)^3/(a + b*tan(x)^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^3(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \frac{-\sqrt{\tan(x)^4 b+a}-6\left(\int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)^5}{\tan(x)^{12} b^3+3\tan(x)^8 a\,b^2+3\tan(x)^4 a^2 b+a^3}dx\right)\tan(x)^8 b^3-12}{6b\,(\tan}$$

input
```
int(tan(x)^3/(a+b*tan(x)^4)^(5/2),x)
```

output
```
( - sqrt(tan(x)**4*b + a) - 6*int((sqrt(tan(x)**4*b + a)*tan(x)**5)/(tan(x
)**12*b**3 + 3*tan(x)**8*a*b**2 + 3*tan(x)**4*a**2*b + a**3),x)*tan(x)**8*
b**3 - 12*int((sqrt(tan(x)**4*b + a)*tan(x)**5)/(tan(x)**12*b**3 + 3*tan(x
)**8*a*b**2 + 3*tan(x)**4*a**2*b + a**3),x)*tan(x)**4*a*b**2 - 6*int((sqrt
(tan(x)**4*b + a)*tan(x)**5)/(tan(x)**12*b**3 + 3*tan(x)**8*a*b**2 + 3*tan
(x)**4*a**2*b + a**3),x)*a**2*b)/(6*b*(tan(x)**8*b**2 + 2*tan(x)**4*a*b +
a**2))
```

3.406 $\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} + \frac{a+b\tan^2(x)}{6a(a+b)\left(a+b\tan^4(x)\right)^{3/2}} + \frac{3a^2+b(5a+2b)\tan^2(x)}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}$$

output
```
-1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)+
1/6*(a+b*tan(x)^2)/a/(a+b)/(a+b*tan(x)^4)^(3/2)+1/6*(3*a^2+b*(5*a+2*b)*tan
(x)^2)/a^2/(a+b)^2/(a+b*tan(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}} \, dx = \frac{1}{6}\left(-\frac{3\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}} + \frac{a^2(4a+b)+3ab(2a+b)\tan^2(x)+3a^2b\tan^4(x)+b^2(5a+2b)\tan^6(x)}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

input
```
Integrate[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

output
```
((-3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)
^(5/2) + (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]^4 + b^
2*(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4153, 1577, 496, 25, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\left(a+b\tan^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\left(a+b\tan(x)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tan(x)}{\left(\tan^2(x)+1\right)\left(a+b\tan^4(x)\right)^{5/2}}d\tan(x)$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{1}{\left(\tan^2(x)+1\right)\left(b\tan^4(x)+a\right)^{5/2}}d\tan^2(x)$$

$$\Big\downarrow \text{496}$$

$$\frac{1}{2}\left(\frac{a+b\tan^2(x)}{3a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}-\frac{\int -\frac{2b\tan^2(x)+3a+2b}{\left(\tan^2(x)+1\right)\left(b\tan^4(x)+a\right)^{3/2}}d\tan^2(x)}{3a(a+b)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b\tan^2(x)+3a+2b}{(\tan^2(x)+1)(b\tan^4(x)+a)^{3/2}}d\tan^2(x)}{3a(a+b)}+\frac{a+b\tan^2(x)}{3a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{686}$$

$$\frac{1}{2}\left(\frac{\frac{3a^2+b(5a+2b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{\int -\frac{3a^2b}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{ab(a+b)}}{3a(a+b)}+\frac{a+b\tan^2(x)}{3a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{3a\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{b\tan^4(x)+a}}d\tan^2(x)}{a+b}+\frac{3a^2+b(5a+2b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}}{3a(a+b)}+\frac{a+b\tan^2(x)}{3a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(\frac{\frac{3a^2+b(5a+2b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{3a\int \frac{1}{-\tan^4(x)+a+b}d\frac{a-b\tan^2(x)}{\sqrt{b\tan^4(x)+a}}}{a+b}}{3a(a+b)}+\frac{a+b\tan^2(x)}{3a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{3a^2+b(5a+2b)\tan^2(x)}{a(a+b)\sqrt{a+b\tan^4(x)}}-\frac{3a\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{3/2}}}{3a(a+b)}+\frac{a+b\tan^2(x)}{3a(a+b)\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

input
```
Int[Tan[x]/(a + b*Tan[x]^4)^(5/2),x]
```

output
```
((a + b*Tan[x]^2)/(3*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + ((-3*a*ArcTanh[(a
 - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]])/(a + b)^(3/2) + (3*a^2
 + b*(5*a + 2*b)*Tan[x]^2)/(a*(a + b)*Sqrt[a + b*Tan[x]^4]))/(3*a*(a + b))
)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(101) = 202$.

Time = 0.14 (sec) , antiderivative size = 586, normalized size of antiderivative = 5.01

method	result
derivativedivides	$-\frac{b^2 \ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\left(b+\sqrt{-ab}\right)^2\left(-b+\sqrt{-ab}\right)^2\sqrt{a+b}} - \frac{-\frac{\sqrt{\left(\tan(x)^2-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(\tan \cdots\right.}}{3\sqrt{-ab}\left(\tan(x)^2-\frac{\sqrt{-ab}}{b} \cdots\right.}}{\cdots}$
default	$-\frac{b^2 \ln\left(\frac{2a+2b-2b\left(1+\tan(x)^2\right)+2\sqrt{a+b}\,\sqrt{b\left(1+\tan(x)^2\right)^2-2b\left(1+\tan(x)^2\right)+a+b}}{1+\tan(x)^2}\right)}{2\left(b+\sqrt{-ab}\right)^2\left(-b+\sqrt{-ab}\right)^2\sqrt{a+b}} - \frac{-\frac{\sqrt{\left(\tan(x)^2-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(\tan \cdots\right.}}{3\sqrt{-ab}\left(\tan(x)^2-\frac{\sqrt{-ab}}{b} \cdots\right.}}{\cdots}$

input
```
int(tan(x)/(a+b*tan(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^(1/2)*ln((2*a+2*b-2*
b*(1+tan(x)^2)+2*(a+b)^(1/2)*(b*(1+tan(x)^2)^2-2*b*(1+tan(x)^2)+a+b)^(1/2)
)/(1+tan(x)^2))-1/8/(b+(-a*b)^(1/2))/a*(-1/3/(-a*b)^(1/2)/(tan(x)^2-(-a*b)
^(1/2)/b)^2*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)
^(1/2)/b))^(1/2)-1/3/a/(tan(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/b
)^2*b+2*(-a*b)^(1/2)*(tan(x)^2-(-a*b)^(1/2)/b))^(1/2))+1/8/(-b+(-a*b)^(1/2
))/a*(1/3/(-a*b)^(1/2)/(tan(x)^2+(-a*b)^(1/2)/b)^2*((tan(x)^2+(-a*b)^(1/2)
/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*b)^(1/2)/b))^(1/2)-1/3/a/(tan(x)^2+(-
a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2+(-a*
b)^(1/2)/b))^(1/2))-1/8*(2*(-a*b)^(1/2)-b)/(-b+(-a*b)^(1/2))^2/a^2/(tan(x)
^2+(-a*b)^(1/2)/b)*((tan(x)^2+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(tan(x)^2
+(-a*b)^(1/2)/b))^(1/2)+1/8*(2*(-a*b)^(1/2)+b)/(b+(-a*b)^(1/2))^2/a^2/(tan
(x)^2-(-a*b)^(1/2)/b)*((tan(x)^2-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(tan(x
)^2-(-a*b)^(1/2)/b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 599, normalized size of antiderivative = 5.12

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \left[\frac{3\left(a^2b^2\tan(x)^8 + 2a^3b\tan(x)^4 + a^4\right)\sqrt{a+b}\log\left(\frac{(ab+2b^2)\tan(x)^4 - 2ab\tan(x)^2 + 2\sqrt{}}{\tan(x)^4}\right.}{12\left((a^5b^2 + 3\right.}\right.$$

$$-\frac{3\left(a^2b^2\tan(x)^8 + 2a^3b\tan(x)^4 + a^4\right)\sqrt{-a-b}\arctan\left(\frac{\sqrt{b\tan(x)^4+a}\left(b\tan(x)^2-a\right)\sqrt{-a-b}}{(ab+b^2)\tan(x)^4+a^2+ab}\right) - \left((5a^2b^2 + 7ab^3\right.}{6\left((a^5b^2 + 3a^4b^3 + 3a^3b^4 + a^2b^5)\tan(x)^8 + a^7 + 3a\right.}$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(a + b)*log(((a*b
 + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 + 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 -
 a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 2*((5*a^2*b^
2 + 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b + a^2*b^2)*tan(x)^4 + 4*a^4 + 5*a
^3*b + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*tan(x)^2)*sqrt(b*tan(x)^4
 + a))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*tan(x)^8 + a^7 + 3*a^6
*b + 3*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*tan
(x)^4), -1/6*(3*(a^2*b^2*tan(x)^8 + 2*a^3*b*tan(x)^4 + a^4)*sqrt(-a - b)*a
rctan(sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 - a)*sqrt(-a - b)/((a*b + b^2)*tan(
x)^4 + a^2 + a*b)) - ((5*a^2*b^2 + 7*a*b^3 + 2*b^4)*tan(x)^6 + 3*(a^3*b +
a^2*b^2)*tan(x)^4 + 4*a^4 + 5*a^3*b + a^2*b^2 + 3*(2*a^3*b + 3*a^2*b^2 + a
*b^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 +
a^2*b^5)*tan(x)^8 + a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + 2*(a^6*b + 3*a^5
*b^2 + 3*a^4*b^3 + a^3*b^4)*tan(x)^4)]
```

Sympy [F]

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \int \frac{\tan(x)}{(a+b\tan^4(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*tan(x)**4)**(5/2),x)
```

output
```
Integral(tan(x)/(a + b*tan(x)**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \int \frac{\tan(x)}{(b\tan(x)^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(x)/(b*tan(x)^4 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(103) = 206$.

Time = 0.21 (sec) , antiderivative size = 618, normalized size of antiderivative = 5.28

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \frac{\left(\left(\frac{(5\,a^7b^3+32\,a^6b^4+87\,a^5b^5+130\,a^4b^6+115\,a^3b^7+60\,a^2b^8+17\,ab^9+2\,b^{10})\tan(x)^2}{a^{10}b+8\,a^9b^2+28\,a^8b^3+56\,a^7b^4+70\,a^6b^5+56\,a^5b^6+28\,a^4b^7+8\,a^3b^8+a^2b^9} + \frac{3\,(a^8b^2+6\,a^7b}{a^{10}b+8\,a^9b^2+28\,a^8b^3}\right.\right.}{}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}\tan(x)^2-\sqrt{b\tan(x)^4+a}+\sqrt{b}}{\sqrt{-a-b}}\right)}{(a^2+2\,ab+b^2)\sqrt{-a-b}}$$

input
```
integrate(tan(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/6*((((5*a^7*b^3 + 32*a^6*b^4 + 87*a^5*b^5 + 130*a^4*b^6 + 115*a^3*b^7 +
60*a^2*b^8 + 17*a*b^9 + 2*b^10)*tan(x)^2/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3
+ 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9)
 + 3*(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b
^7 + a^2*b^8)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 +
 56*a^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + 3*(2*a^8*b^2 +
 13*a^7*b^3 + 36*a^6*b^4 + 55*a^5*b^5 + 50*a^4*b^6 + 27*a^3*b^7 + 8*a^2*b^
8 + a*b^9)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56
*a^5*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))*tan(x)^2 + (4*a^9*b + 25*a^8
*b^2 + 66*a^7*b^3 + 95*a^6*b^4 + 80*a^5*b^5 + 39*a^4*b^6 + 10*a^3*b^7 + a^
2*b^8)/(a^10*b + 8*a^9*b^2 + 28*a^8*b^3 + 56*a^7*b^4 + 70*a^6*b^5 + 56*a^5
*b^6 + 28*a^4*b^7 + 8*a^3*b^8 + a^2*b^9))/(b*tan(x)^4 + a)^(3/2) - arctan(
(sqrt(b)*tan(x)^2 - sqrt(b*tan(x)^4 + a) + sqrt(b))/sqrt(-a - b))/((a^2 +
2*a*b + b^2)*sqrt(-a - b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \int \frac{\tan(x)}{\left(b\tan(x)^4+a\right)^{5/2}}\,dx$$

input
```
int(tan(x)/(a + b*tan(x)^4)^(5/2),x)
```

output
```
int(tan(x)/(a + b*tan(x)^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \int \frac{\sqrt{\tan(x)^4 b+a}\,\tan(x)}{\tan(x)^{12} b^3+3\tan(x)^8 a\,b^2+3\tan(x)^4 a^2 b+a^3}dx$$

input
```
int(tan(x)/(a+b*tan(x)^4)^(5/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*tan(x))/(tan(x)**12*b**3 + 3*tan(x)**8*a*b**2 +
 3*tan(x)**4*a**2*b + a**3),x)
```

3.407 $\int \frac{\cot(x)}{(a+b\tan^4(x))^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{2a^{5/2}} + \frac{b(1-\tan^2(x))}{6a(a+b)\,(a+b\tan^4(x))^{3/2}} + \frac{b(3(2a+b)-(5a+2b)\tan^2(x))}{6a^2(a+b)^2\sqrt{a+b\tan^4(x)}}$$

output
```
1/2*arctanh((a-b*tan(x)^2)/(a+b)^(1/2)/(a+b*tan(x)^4)^(1/2))/(a+b)^(5/2)-1
/2*arctanh((a+b*tan(x)^4)^(1/2)/a^(1/2))/a^(5/2)+1/6*b*(1-tan(x)^2)/a/(a+b
)/(a+b*tan(x)^4)^(3/2)+1/6*b*(6*a+3*b-(5*a+2*b)*tan(x)^2)/a^2/(a+b)^2/(a+b
*tan(x)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \frac{1}{6}\left(\frac{3\operatorname{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}} + \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1+\frac{b\tan^4(x)}{a}\right)}{a\left(a+b\tan^4(x)\right)^{3/2}} - \frac{a^2(4a+b)+3ab(2a+b)\tan^2(x)+3a^2b\tan^4(x)+b^2(5a+2b)\tan^6(x)}{a^2(a+b)^2\left(a+b\tan^4(x)\right)^{3/2}}\right)$$

input
```
Integrate[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

output
```
((3*ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])])/(a + b)^
(5/2) + Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tan[x]^4)/a]/(a*(a + b*Tan
[x]^4)^(3/2)) - (a^2*(4*a + b) + 3*a*b*(2*a + b)*Tan[x]^2 + 3*a^2*b*Tan[x]
^4 + b^2*(5*a + 2*b)*Tan[x]^6)/(a^2*(a + b)^2*(a + b*Tan[x]^4)^(3/2)))/6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4153, 1579, 617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(x)\left(a+b\tan(x)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\cot(x)}{(\tan^2(x)+1)\left(a+b\tan^4(x)\right)^{5/2}}d\tan(x)$$

$$\Big\downarrow \ 1579$$

$$\frac{1}{2}\int \frac{\cot(x)}{(\tan^2(x)+1)\left(b\tan^4(x)+a\right)^{5/2}}d\tan^2(x)$$

$$\Big\downarrow \ 617$$

$$\frac{1}{2}\int \left(\frac{\cot(x)}{\left(b\tan^4(x)+a\right)^{5/2}}+\frac{1}{(-\tan^2(x)-1)\left(b\tan^4(x)+a\right)^{5/2}}\right)d\tan^2(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tan^4(x)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{1}{a^2\sqrt{a+b\tan^4(x)}}-\frac{3a^2+b(5a+2b)\tan^2(x)}{3a^2(a+b)^2\sqrt{a+b\tan^4(x)}}+\frac{\text{arctanh}\left(\frac{a-b\tan^2(x)}{\sqrt{a+b}\sqrt{a+b\tan^4(x)}}\right)}{(a+b)^{5/2}}+\right.$$

input
```
Int[Cot[x]/(a + b*Tan[x]^4)^(5/2),x]
```

output
```
(ArcTanh[(a - b*Tan[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan[x]^4])]/(a + b)^(5/2
) - ArcTanh[Sqrt[a + b*Tan[x]^4]/Sqrt[a]]/a^(5/2) + 1/(3*a*(a + b*Tan[x]^4
)^(3/2)) - (a + b*Tan[x]^2)/(3*a*(a + b)*(a + b*Tan[x]^4)^(3/2)) + 1/(a^2*
Sqrt[a + b*Tan[x]^4]) - (3*a^2 + b*(5*a + 2*b)*Tan[x]^2)/(3*a^2*(a + b)^2*
Sqrt[a + b*Tan[x]^4]))/2
```

Defintions of rubi rules used

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\cot(x)}{(a+b\tan(x)^4)^{\frac{5}{2}}}dx$$

input
```
int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
```

output
```
int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(123) = 246$.

Time = 0.49 (sec) , antiderivative size = 1749, normalized size of antiderivative = 11.74

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a*b
 + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2 -
 a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 3*((a^3*b^2
+ 3*a^2*b^3 + 3*a*b^4 + b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^
3 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(a)*log(-(b*ta
n(x)^4 - 2*sqrt(b*tan(x)^4 + a)*sqrt(a) + 2*a)/tan(x)^4) - 2*((5*a^3*b^2 +
 7*a^2*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3*(2*a
^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b^3)*t
an(x)^2)*sqrt(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5
)*tan(x)^8 + a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^6*b^2 +
3*a^5*b^3 + a^4*b^4)*tan(x)^4), 1/12*(6*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 +
b^5)*tan(x)^8 + a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + 2*(a^4*b + 3*a^3*b^2
 + 3*a^2*b^3 + a*b^4)*tan(x)^4)*sqrt(-a)*arctan(sqrt(b*tan(x)^4 + a)*sqrt(
-a)/a) + 3*(a^3*b^2*tan(x)^8 + 2*a^4*b*tan(x)^4 + a^5)*sqrt(a + b)*log(((a
*b + 2*b^2)*tan(x)^4 - 2*a*b*tan(x)^2 - 2*sqrt(b*tan(x)^4 + a)*(b*tan(x)^2
 - a)*sqrt(a + b) + 2*a^2 + a*b)/(tan(x)^4 + 2*tan(x)^2 + 1)) - 2*((5*a^3*
b^2 + 7*a^2*b^3 + 2*a*b^4)*tan(x)^6 - 7*a^4*b - 11*a^3*b^2 - 4*a^2*b^3 - 3
*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(x)^4 + 3*(2*a^4*b + 3*a^3*b^2 + a^2*b
^3)*tan(x)^2)*sqrt(b*tan(x)^4 + a))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^
3*b^5)*tan(x)^8 + a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3 + 2*(a^7*b + 3*a^...
```

Sympy [F]

$$\int \frac{\cot(x)}{(a+b\tan^4(x))^{5/2}}\,dx = \int \frac{\cot{(x)}}{(a+b\tan^4{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)/(a+b*tan(x)**4)**(5/2),x)
```

output
```
Integral(cot(x)/(a + b*tan(x)**4)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(x)/(a+b*tan(x)^4)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(x)/(a + b*tan(x)^4)^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot(x)}{\left(a+b\tan^4(x)\right)^{5/2}}\,dx = \int \frac{\sqrt{\tan\left(x\right)^4 b+a}\;\cot\left(x\right)}{\tan\left(x\right)^{12} b^3+3\tan\left(x\right)^8 a\,b^2+3\tan\left(x\right)^4 a^2b+a^3}dx$$

input
```
int(cot(x)/(a+b*tan(x)^4)^(5/2),x)
```

output
```
int((sqrt(tan(x)**4*b + a)*cot(x))/(tan(x)**12*b**3 + 3*tan(x)**8*a*b**2 +
 3*tan(x)**4*a**2*b + a**3),x)
```

3.408 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$

Optimal result

Integrand size = 29, antiderivative size = 212

$$\begin{aligned}\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx &= \frac{\left(a^2-b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2f(1+m)}\\ &+\frac{\left(a^2+b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2f(1+m)}\\ &+\frac{4ab\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),-\tan^2(e+fx)\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m}{cf(3+2m)}\end{aligned}$$

output
```
1/2*(a^2-b^2*(-c^2)^(1/2))*hypergeom([1, 1+m],[2+m],-c*tan(f*x+e)/(-c^2)^(
1/2))*tan(f*x+e)*(d*tan(f*x+e))^m/f/(1+m)+1/2*(a^2+b^2*(-c^2)^(1/2))*hyper
geom([1, 1+m],[2+m],c*tan(f*x+e)/(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m
/f/(1+m)+4*a*b*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(f*x+e)^2)*(c*tan(
f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/f/(3+2*m)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$$
$$= \frac{\tan(e+fx)(d\tan(e+fx))^m \left(\frac{a^2 \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(e+fx)\right)}{1+m}+b\left(\frac{bc\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\right.}{2+m}\right.\right.}{f}$$

input
```
Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

output
```
(Tan[e + f*x]*(d*Tan[e + f*x])^m*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -Tan[e + f*x]^2])/(1 + m) + b*((b*c*Hypergeometric2F1[1, (2 + m)/2
, (4 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + m) + (4*a*Hypergeometric2
F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*Sqrt[c*Tan[e + f*x]])/(3
+ 2*m))))/f
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3042, 4153, 7267, 30, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$$
$$\Big\downarrow \text{3042}$$
$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$$
$$\Big\downarrow \text{4153}$$
$$\frac{c\int \frac{(d\tan(e+fx))^m\left(a+b\sqrt{c\tan(e+fx)}\right)^2}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{7267}$$

$$\frac{2c\int \frac{\sqrt{c\tan(e+fx)}\left(cd\tan^2(e+fx)\right)^m(a+bc\tan(e+fx))^2}{c^4\tan^4(e+fx)+c^2}d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \text{30}$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int \frac{(c\tan(e+fx))^{\frac{1}{2}(2m+1)}\left(a+b\sqrt{c\tan(e+fx)}\right)^2}{c^4\tan^4(e+fx)+c^2}d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \text{2370}$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int \left(\frac{(a^2+b^2c^2\tan^2(e+fx))(c\tan(e+fx))^{\frac{1}{2}(2m+1)}}{c^4\tan^4(e+fx)+c^2}+\frac{2ab(c\tan(e+fx))^{\frac{1}{2}(2m+2)}}{c^4\tan^4(e+fx)+c^2}\right)d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \text{2009}$$

$$2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\left(\frac{\left(a^2-b^2\sqrt{-c^2}\right)(c\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{c^2\tan^2(e+fx)}{\sqrt{-c^2}}\right)}{4c^2(m+1)}\right.$$

input

```
Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

output

```
(2*c*(c*d*Tan[e + f*x]^2)^m*(((a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1,
1 + m, 2 + m, -((c^2*Tan[e + f*x]^2)/Sqrt[-c^2])]*(c*Tan[e + f*x])^(1 + m)
)/(4*c^2*(1 + m)) + ((a^2 + b^2*Sqrt[-c^2])*Hypergeometric2F1[1, 1 + m, 2
+ m, (c^2*Tan[e + f*x]^2)/Sqrt[-c^2]]*(c*Tan[e + f*x])^(1 + m))/(4*c^2*(1
+ m)) + (2*a*b*Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -(c^2*Tan[e
+ f*x]^4)]*(c*Tan[e + f*x])^((3 + 2*m)/2))/(c^2*(3 + 2*m))))/(f*(c*Tan[e +
 f*x])^m)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [F]

$$\int (d\tan (fx+e))^m \left(a+b\sqrt{c\tan (fx+e)}\right)^2 dx$$

input
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

output
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

Fricas [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$$
$$= \int \left(\sqrt{c\tan (fx+e)}b+a\right)^2 (d\tan (fx+e))^m dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="fric
as")
```

output
```
integral(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b + (b^2*c*tan(f*x +
e) + a^2)*(d*tan(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$$
$$= \int (d\tan (e+fx))^m \left(a+b\sqrt{c\tan (e+fx)}\right)^2 dx$$

input
```
integrate((d*tan(f*x+e))**m*(a+b*(c*tan(f*x+e))**(1/2))**2,x)
```

output
```
Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

Maxima [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx = \int \left(\sqrt{c\tan(fx+e)}b+a\right)^2 (d\tan(fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="maxi
ma")
```

output
```
integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

Giac [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx = \int \left(\sqrt{c\tan(fx+e)}b+a\right)^2 (d\tan(fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="giac
")
```

output
```
integrate((sqrt(c*tan(f*x + e))*b + a)^2*(d*tan(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx = \int \left(a+b\sqrt{c\tan(e+f\,x)}\right)^2 (d\tan(e+f\,x))^m \, dx$$

input
```
int((a + b*(c*tan(e + f*x))^(1/2))^2*(d*tan(e + f*x))^m,x)
```

output
```
int((a + b*(c*tan(e + f*x))^(1/2))^2*(d*tan(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)^2 dx$$
$$= \frac{d^m \left(\tan(fx+e)^m b^2 c + 2\sqrt{c}\left(\int \tan(fx+e)^{m+\frac{1}{2}} dx\right) abfm + \left(\int \tan(fx+e)^m dx\right) a^2 fm - \left(\int \frac{\tan(fx+e}{\tan(fx+}\right.\right.}{fm}$$

input
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

output
```
(d**m*(tan(e + f*x)**m*b**2*c + 2*sqrt(c)*int(tan(e + f*x)**((2*m + 1)/2),
x)*a*b*f*m + int(tan(e + f*x)**m,x)*a**2*f*m - int(tan(e + f*x)**m/tan(e +
 f*x),x)*b**2*c*f*m))/(f*m)
```

3.409 $\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m}{f(1+m)} + \frac{2b\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),-\tan^2(e+fx)\right)(c\tan(e+fx))^{3/2}(d\tan(e+fx))^m}{cf(3+2m)}$$

output
```
a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(f*x+e)^2)*tan(f*x+e)*(d*tan(f*
x+e))^m/f/(1+m)+2*b*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(f*x+e)^2)*(c
*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/f/(3+2*m)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.51

$$\int (d\tan(e+fx))^m\left(a+b\sqrt{c\tan(e+fx)}\right)\,dx$$
$$=\frac{\left(\left(a-b\sqrt[4]{-c^2}\right)\text{Hypergeometric2F1}\left(1,2(1+m),3+2m,-\frac{\sqrt{c\tan(e+fx)}}{\sqrt[4]{-c^2}}\right)+\left(a+ib\sqrt[4]{-c^2}\right)\text{Hypergeom}\right.}{}$$

input
```
Integrate[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

output
```
(((a - b*(-c^2)^(1/4))*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -(Sqrt[c*T
an[e + f*x]]/(-c^2)^(1/4))] + (a + I*b*(-c^2)^(1/4))*Hypergeometric2F1[1,
2*(1 + m), 3 + 2*m, ((-I)*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] + a*Hypergeo
metric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e + f*x]])/(-c^2)^(1/4)] -
I*b*(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, (I*Sqrt[c*Tan[e
+ f*x]])/(-c^2)^(1/4)] + a*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, Sqrt[c
*Tan[e + f*x]]/(-c^2)^(1/4)] + b*(-c^2)^(1/4)*Hypergeometric2F1[1, 2*(1 +
m), 3 + 2*m, Sqrt[c*Tan[e + f*x]]/(-c^2)^(1/4)])*Tan[e + f*x]*(d*Tan[e + f
*x])^m)/(4*f*(1 + m))
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4153, 7267, 30, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m\left(a+b\sqrt{c\tan(e+fx)}\right)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (d\tan(e+fx))^m\left(a+b\sqrt{c\tan(e+fx)}\right)dx$$
$$\Big\downarrow \; 4153$$

$$\frac{c\int \frac{(d\tan(e+fx))^m\left(a+b\sqrt{c\tan(e+fx)}\right)}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{7267}$$

$$\frac{2c\int \frac{\sqrt{c\tan(e+fx)}\left(cd\tan^2(e+fx)\right)^m(a+bc\tan(e+fx))}{c^4\tan^4(e+fx)+c^2}d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \text{30}$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int \frac{(c\tan(e+fx))^{\frac{1}{2}(2m+1)}\left(a+b\sqrt{c\tan(e+fx)}\right)}{c^4\tan^4(e+fx)+c^2}d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \text{2370}$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int \left(\frac{a(c\tan(e+fx))^{\frac{1}{2}(2m+1)}}{c^4\tan^4(e+fx)+c^2}+\frac{b(c\tan(e+fx))^{\frac{1}{2}(2m+2)}}{c^4\tan^4(e+fx)+c^2}\right)d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\left(\frac{a(c\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-c^2\tan^4(e+fx)\right)}{2c^2(m+1)}+\frac{b(c\tan(e+f}{}\right.}{f}$$

input
```
Int[(d*Tan[e + f*x])^m*(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

output
```
(2*c*(c*d*Tan[e + f*x]^2)^m*((a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
 -(c^2*Tan[e + f*x]^4)]*(c*Tan[e + f*x])^(1 + m))/(2*c^2*(1 + m)) + (b*Hyp
ergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -(c^2*Tan[e + f*x]^4)]*(c*Tan[
e + f*x])^((3 + 2*m)/2))/(c^2*(3 + 2*m))))/(f*(c*Tan[e + f*x])^m)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [F]

$$\int (d\tan(fx+e))^m \left(a+b\sqrt{c\tan(fx+e)}\right) dx$$

input
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2)),x)
```

output
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2)),x)
```

Fricas [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \int \left(\sqrt{c\tan(fx+e)}b+a\right)(d\tan(fx+e))^m dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b + (d*tan(f*x + e))^m*a,
 x)
```

Sympy [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx$$

input
```
integrate((d*tan(f*x+e))**m*(a+b*(c*tan(f*x+e))**(1/2)),x)
```

output
```
Integral((d*tan(e + f*x))**m*(a + b*sqrt(c*tan(e + f*x))), x)
```

Maxima [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \int \left(\sqrt{c\tan(fx+e)}b+a\right)(d\tan(fx+e))^m dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="maxima
")
```

output
```
integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

Giac [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \int \left(\sqrt{c\tan(fx+e)}b+a\right)(d\tan(fx+e))^m dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="giac")
```

output
```
integrate((sqrt(c*tan(f*x + e))*b + a)*(d*tan(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right) dx = \int \left(a+b\sqrt{c\tan(e+fx)}\right) (d\tan(e+fx))^m dx$$

input
```
int((a + b*(c*tan(e + f*x))^(1/2))*(d*tan(e + f*x))^m,x)
```

output
```
int((a + b*(c*tan(e + f*x))^(1/2))*(d*tan(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\tan(e+fx))^m \left(a+b\sqrt{c\tan(e+fx)}\right)\,dx = d^m\left(\sqrt{c}\left(\int \tan\left(fx+e\right)^{m+\frac{1}{2}}dx\right)b+\left(\int \tan\left(fx+e\right)^{m}dx\right)a\right)$$

input
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^(1/2)),x)
```

output
```
d**m*(sqrt(c)*int(tan(e + f*x)**((2*m + 1)/2),x)*b + int(tan(e + f*x)**m,x
)*a)
```

3.410 $\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 460

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx$$
$$= \frac{a\left(a^2-b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2\left(a^4+b^4c^2\right)f(1+m)}$$
$$+\frac{a\left(a^2+b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+fx)(d\tan(e+fx))^m}{2\left(a^4+b^4c^2\right)f(1+m)}$$
$$+\frac{b^4c^2\,\text{Hypergeometric2F1}\left(1,2(1+m),3+2m,-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a\left(a^4+b^4c^2\right)f(1+m)}$$
$$-\frac{b\left(a^2-b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3+2m),\frac{1}{2}(5+2m),-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)(c\tan(e+fx))^{3/2}(d\tan}{c\left(a^4+b^4c^2\right)f(3+2m)}$$
$$-\frac{b\left(a^2+b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3+2m),\frac{1}{2}(5+2m),\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)(c\tan(e+fx))^{3/2}(d\tan(e}{c\left(a^4+b^4c^2\right)f(3+2m)}$$

output
```
1/2*a*(a^2-b^2*(-c^2)^(1/2))*hypergeom([1, 1+m],[2+m],-c*tan(f*x+e)/(-c^2)
^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)/f/(1+m)+1/2*a*(a^2+b^2*(
-c^2)^(1/2))*hypergeom([1, 1+m],[2+m],c*tan(f*x+e)/(-c^2)^(1/2))*tan(f*x+e
)*(d*tan(f*x+e))^m/(b^4*c^2+a^4)/f/(1+m)+b^4*c^2*hypergeom([1, 2+2*m],[3+2
*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f*x+e)*(d*tan(f*x+e))^m/a/(b^4*c^2+a^4)
/f/(1+m)-b*(a^2-b^2*(-c^2)^(1/2))*hypergeom([1, 3/2+m],[5/2+m],-c*tan(f*x+
e)/(-c^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)/f/(
3+2*m)-b*(a^2+b^2*(-c^2)^(1/2))*hypergeom([1, 3/2+m],[5/2+m],c*tan(f*x+e)/
(-c^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)/f/(3+2
*m)
```

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.57

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \frac{\tan(e+fx)(d\tan(e+fx))^m\left(\frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(e+fx)\right)}{1+m}+b\left(\frac{b^3c^2\,\text{Hypergeometric2F1}\left(1,2(1+m),3\right.}{a+am}\right.\right.}{}$$

input
```
Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

output
```
(Tan[e + f*x]*(d*Tan[e + f*x])^m*((a^3*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -Tan[e + f*x]^2])/(1 + m) + b*((b^3*c^2*Hypergeometric2F1[1, 2*(1
+ m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)])/(a + a*m) + (a*b*c*Hypergeo
metric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(2 + m)
+ 2*Sqrt[c*Tan[e + f*x]]*(-((a^2*Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*
m)/4, -Tan[e + f*x]^2])/(3 + 2*m)) - (b^2*c*Hypergeometric2F1[1, (5 + 2*m)
/4, (9 + 2*m)/4, -Tan[e + f*x]^2]*Tan[e + f*x])/(5 + 2*m)))))/((a^4 + b^4*
c^2)*f)
```

Rubi [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3042, 4153, 7267, 30, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}dx$$

$$\Big\downarrow \ 4153$$

$$\frac{c\int \frac{(d\tan(e+fx))^m}{(\tan^2(e+fx)c^2+c^2)\left(a+b\sqrt{c\tan(e+fx)}\right)}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \ 7267$$

$$\frac{2c\int \frac{\sqrt{c\tan(e+fx)}(cd\tan^2(e+fx))^m}{(a+bc\tan(e+fx))(c^4\tan^4(e+fx)+c^2)}d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \ 30$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int \frac{(c\tan(e+fx))^{\frac{1}{2}(2m+1)}}{(c^4\tan^4(e+fx)+c^2)\left(a+b\sqrt{c\tan(e+fx)}\right)}d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \ 7276$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int \left(\frac{\left(a^3-b\sqrt{c\tan(e+fx)}a^2+b^2c^2\tan^2(e+fx)a-b^3c^3\tan^3(e+fx)\right)(c\tan(e+fx))^{\frac{1}{2}(2m+1)}}{(a^4+b^4c^2)(c^4\tan^4(e+fx)+c^2)}\right.}{f}$$

$$\Big\downarrow \ 2009$$

$$2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\left(\frac{b^4(c\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,2(m+1),2m+3,-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{2a(m+1)(a^4+b^4c^2)}+\frac{a\left(a^2-\right.}{}\right.$$

input
```
Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]]),x]
```

output
```
(2*c*(c*d*Tan[e + f*x]^2)^m*((a*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1[1
, 1 + m, 2 + m, -((c^2*Tan[e + f*x]^2)/Sqrt[-c^2])]*(c*Tan[e + f*x])^(1 +
m))/(4*c^2*(a^4 + b^4*c^2)*(1 + m)) + (a*(a^2 + b^2*Sqrt[-c^2])*Hypergeome
tric2F1[1, 1 + m, 2 + m, (c^2*Tan[e + f*x]^2)/Sqrt[-c^2]]*(c*Tan[e + f*x])
^(1 + m))/(4*c^2*(a^4 + b^4*c^2)*(1 + m)) + (b^4*Hypergeometric2F1[1, 2*(1
 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*(c*Tan[e + f*x])^(1 + m))/(
2*a*(a^4 + b^4*c^2)*(1 + m)) - (b*(a^2 - b^2*Sqrt[-c^2])*Hypergeometric2F1
[1, (3 + 2*m)/2, (5 + 2*m)/2, -((c^2*Tan[e + f*x]^2)/Sqrt[-c^2])]*(c*Tan[e
 + f*x])^((3 + 2*m)/2))/(2*c^2*(a^4 + b^4*c^2)*(3 + 2*m)) - (b*(a^2 + b^2*
Sqrt[-c^2])*Hypergeometric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, (c^2*Tan[e + f*
x]^2)/Sqrt[-c^2]]*(c*Tan[e + f*x])^((3 + 2*m)/2))/(2*c^2*(a^4 + b^4*c^2)*(
3 + 2*m))))/(f*(c*Tan[e + f*x])^m)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{(d\tan(fx+e))^m}{a+b\sqrt{c\tan(fx+e)}}dx$$

input
```
int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

output
```
int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

Fricas [F]

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^m}{\sqrt{c\tan(fx+e)}b+a}\,dx$$

input
```
integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="fricas
")
```

output
```
integral((sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*b - (d*tan(f*x + e))^m*a
)/(b^2*c*tan(f*x + e) - a^2), x)
```

Sympy [F]

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \int \frac{(d\tan{(e+fx)})^m}{a+b\sqrt{c\tan{(e+fx)}}}\,dx$$

input
```
integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2)),x)
```

output
```
Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \int \frac{(d\tan{(fx+e)})^m}{\sqrt{c\tan{(fx+e)}}b+a}\,dx$$

input
```
integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="maxima
")
```

output
```
integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

Giac [F]

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \int \frac{(d\tan{(fx+e)})^m}{\sqrt{c\tan{(fx+e)}}b+a}\,dx$$

input
```
integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \int \frac{(d\tan(e+f\,x))^m}{a+b\,\sqrt{c\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2)),x)
```

output
```
int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(d\tan(e+fx))^m}{a+b\sqrt{c\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2)),x)
```

output
```
(d**m*(2*sqrt(c)*tan(e + f*x)**((2*m + 1)/2)*b*m - 2*tan(e + f*x)**m*a*m -
 tan(e + f*x)**m*a + 2*sqrt(c)*int(tan(e + f*x)**((2*m + 1)/2)/(tan(e + f*
x)**2*b**2*c - tan(e + f*x)*a**2),x)*a**2*b*f*m**2 + sqrt(c)*int(tan(e + f
*x)**((2*m + 1)/2)/(tan(e + f*x)**2*b**2*c - tan(e + f*x)*a**2),x)*a**2*b*
f*m - 2*sqrt(c)*int((tan(e + f*x)**((2*m + 1)/2)*tan(e + f*x)**2)/(tan(e +
 f*x)*b**2*c - a**2),x)*b**3*c*f*m**2 - sqrt(c)*int((tan(e + f*x)**((2*m +
 1)/2)*tan(e + f*x)**2)/(tan(e + f*x)*b**2*c - a**2),x)*b**3*c*f*m + 2*sqr
t(c)*int((tan(e + f*x)**((2*m + 1)/2)*tan(e + f*x))/(tan(e + f*x)*b**2*c -
 a**2),x)*a**2*b*f*m**2 + sqrt(c)*int((tan(e + f*x)**((2*m + 1)/2)*tan(e +
 f*x))/(tan(e + f*x)*b**2*c - a**2),x)*a**2*b*f*m - 2*int(tan(e + f*x)**m/
(tan(e + f*x)**2*b**2*c - tan(e + f*x)*a**2),x)*a**3*f*m**2 - int(tan(e +
f*x)**m/(tan(e + f*x)**2*b**2*c - tan(e + f*x)*a**2),x)*a**3*f*m + 2*int((
tan(e + f*x)**m*tan(e + f*x)**2)/(tan(e + f*x)*b**2*c - a**2),x)*a*b**2*c*
f*m**2 + int((tan(e + f*x)**m*tan(e + f*x)**2)/(tan(e + f*x)*b**2*c - a**2
),x)*a*b**2*c*f*m - 2*int((tan(e + f*x)**m*tan(e + f*x))/(tan(e + f*x)*b**
2*c - a**2),x)*a**3*f*m**2 - int((tan(e + f*x)**m*tan(e + f*x))/(tan(e + f
*x)*b**2*c - a**2),x)*a**3*f*m))/(b**2*c*f*m*(2*m + 1))
```

3.411 $\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 617

$$\begin{aligned}
&\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx \\
&= \frac{\left(a^6-3a^2b^4c^2-3a^4b^2\sqrt{-c^2}-b^6(-c^2)^{3/2}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+\ldots}{2\left(a^4+b^4c^2\right)^2 f(1+m)} \\
&+\frac{\left(a^6-3a^2b^4c^2+3a^4b^2\sqrt{-c^2}+b^6(-c^2)^{3/2}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)\tan(e+\ldots}{2\left(a^4+b^4c^2\right)^2 f(1+m)} \\
&+\frac{4a^2b^4c^2\,\text{Hypergeometric2F1}\left(1,2(1+m),3+2m,-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{\left(a^4+b^4c^2\right)^2 f(1+m)} \\
&+\frac{b^4c^2\,\text{Hypergeometric2F1}\left(2,2(1+m),3+2m,-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)\tan(e+fx)(d\tan(e+fx))^m}{a^2\left(a^4+b^4c^2\right) f(1+m)} \\
&-\frac{2ab\left(a^4-b^4c^2-2a^2b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3+2m),\frac{1}{2}(5+2m),-\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)(c\tan(e+\ldots}{c\left(a^4+b^4c^2\right)^2 f(3+2m)} \\
&-\frac{2ab\left(a^4-b^4c^2+2a^2b^2\sqrt{-c^2}\right)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3+2m),\frac{1}{2}(5+2m),\frac{c\tan(e+fx)}{\sqrt{-c^2}}\right)(c\tan(e+fx\ldots}{c\left(a^4+b^4c^2\right)^2 f(3+2m)}
\end{aligned}$$

output
```
1/2*(a^6-3*a^2*b^4*c^2-3*a^4*b^2*(-c^2)^(1/2)-b^6*(-c^2)^(3/2))*hypergeom(
[1, 1+m],[2+m],-c*tan(f*x+e)/(-c^2)^(1/2))*tan(f*x+e)*(d*tan(f*x+e))^m/(b^
4*c^2+a^4)^2/f/(1+m)+1/2*(a^6-3*a^2*b^4*c^2+3*a^4*b^2*(-c^2)^(1/2)+b^6*(-c
^2)^(3/2))*hypergeom([1, 1+m],[2+m],c*tan(f*x+e)/(-c^2)^(1/2))*tan(f*x+e)*
(d*tan(f*x+e))^m/(b^4*c^2+a^4)^2/f/(1+m)+4*a^2*b^4*c^2*hypergeom([1, 2+2*m
],[3+2*m],-b*(c*tan(f*x+e))^(1/2)/a)*tan(f*x+e)*(d*tan(f*x+e))^m/(b^4*c^2+
a^4)^2/f/(1+m)+b^4*c^2*hypergeom([2, 2+2*m],[3+2*m],-b*(c*tan(f*x+e))^(1/2
)/a)*tan(f*x+e)*(d*tan(f*x+e))^m/a^2/(b^4*c^2+a^4)/f/(1+m)-2*a*b*(a^4-b^4*
c^2-2*a^2*b^2*(-c^2)^(1/2))*hypergeom([1, 3/2+m],[5/2+m],-c*tan(f*x+e)/(-c
^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c^2+a^4)^2/f/(3+2*
m)-2*a*b*(a^4-b^4*c^2+2*a^2*b^2*(-c^2)^(1/2))*hypergeom([1, 3/2+m],[5/2+m]
,c*tan(f*x+e)/(-c^2)^(1/2))*(c*tan(f*x+e))^(3/2)*(d*tan(f*x+e))^m/c/(b^4*c
^2+a^4)^2/f/(3+2*m)
```

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.62

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \frac{c(d\tan(e+fx))^m\left(\frac{a^2(a^4-3b^4c^2)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(e+fx)\right)\tan(e+fx)}{c(1+m)}+\frac{4a^2b^4c\,\text{Hypergeometric2F1}\left(1,2(1\right.}{}\right.}{}$$

input
```
Integrate[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

output
```
(c*(d*Tan[e + f*x])^m*((a^2*(a^4 - 3*b^4*c^2)*Hypergeometric2F1[1, (1 + m)
/2, (3 + m)/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(c*(1 + m)) + (4*a^2*b^4*c*H
ypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan
[e + f*x])/(1 + m) + (b^4*c*(a^4 + b^4*c^2)*Hypergeometric2F1[2, 2*(1 + m)
, 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*Tan[e + f*x])/(a^2*(1 + m)) + (b
^2*(3*a^4 - b^4*c^2)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[e + f
*x]^2]*Tan[e + f*x]^2)/(2 + m) + (4*a*b*(-a^4 + b^4*c^2)*Hypergeometric2F1
[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*(c*Tan[e + f*x])^(3/2))/(c^
2*(3 + 2*m)) - (8*a^3*b^3*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4, -
Tan[e + f*x]^2]*(c*Tan[e + f*x])^(5/2))/(c^2*(5 + 2*m))))/((a^4 + b^4*c^2)
^2*f)
```

Rubi [A] (warning: unable to verify)

Time = 1.52 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3042, 4153, 7267, 30, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{c\int \frac{(d\tan(e+fx))^m}{(\tan^2(e+fx)c^2+c^2)\left(a+b\sqrt{c\tan(e+fx)}\right)^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{7267}$$

$$\frac{2c\int \frac{\sqrt{c\tan(e+fx)}\left(cd\tan^2(e+fx)\right)^m}{(a+bc\tan(e+fx))^2(c^4\tan^4(e+fx)+c^2)}d\sqrt{c\tan(e+fx)}}{f}$$

$$\Big\downarrow \text{30}$$

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int\frac{(c\tan(e+fx))^{\frac{1}{2}(2m+1)}}{(c^4\tan^4(e+fx)+c^2)\left(a+b\sqrt{c\tan(e+fx)}\right)^2}d\sqrt{c\tan(e+fx)}}{f}$$

$\Big\downarrow$ 7276

$$\frac{2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\int\left(\frac{\left(-4a^3b^3c^3\tan^3(e+fx)+b^2c^2\left(3a^4-b^4c^2\right)\tan^2(e+fx)+a^2\left(a^4-3b^4c^2\right)-2ab\left(a^4-b^4c^2\right)\right.}{(a^4+b^4c^2)^2(c^4\tan^4(e+fx)+c^2)}\right.}{f}$$

$\Big\downarrow$ 2009

$$2c(c\tan(e+fx))^{-m}\left(cd\tan^2(e+fx)\right)^m\left(\frac{2a^2b^4(c\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,2(m+1),2m+3,-\frac{b\sqrt{c\tan(e+fx)}}{a}\right)}{(m+1)(a^4+b^4c^2)^2}+\frac{b^4}{}\right.$$

input
```
Int[(d*Tan[e + f*x])^m/(a + b*Sqrt[c*Tan[e + f*x]])^2,x]
```

output
```
(2*c*(c*d*Tan[e + f*x]^2)^m*(((a^6 - 3*a^2*b^4*c^2 - 3*a^4*b^2*Sqrt[-c^2]
- b^6*(-c^2)^(3/2))*Hypergeometric2F1[1, 1 + m, 2 + m, -((c^2*Tan[e + f*x]
^2)/Sqrt[-c^2])]*(c*Tan[e + f*x])^(1 + m))/(4*c^2*(a^4 + b^4*c^2)^2*(1 + m
)) + ((a^6 - 3*a^2*b^4*c^2 + 3*a^4*b^2*Sqrt[-c^2] + b^6*(-c^2)^(3/2))*Hype
rgeometric2F1[1, 1 + m, 2 + m, (c^2*Tan[e + f*x]^2)/Sqrt[-c^2]]*(c*Tan[e +
 f*x])^(1 + m))/(4*c^2*(a^4 + b^4*c^2)^2*(1 + m)) + (2*a^2*b^4*Hypergeomet
ric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*(c*Tan[e + f*
x])^(1 + m))/((a^4 + b^4*c^2)^2*(1 + m)) + (b^4*Hypergeometric2F1[2, 2*(1
+ m), 3 + 2*m, -((b*Sqrt[c*Tan[e + f*x]])/a)]*(c*Tan[e + f*x])^(1 + m))/(2
*a^2*(a^4 + b^4*c^2)*(1 + m)) - (a*b*(a^4 - b^4*c^2 - 2*a^2*b^2*Sqrt[-c^2]
)*Hypergeometric2F1[1, (3 + 2*m)/2, (5 + 2*m)/2, -((c^2*Tan[e + f*x]^2)/Sq
rt[-c^2])]*(c*Tan[e + f*x])^((3 + 2*m)/2))/(c^2*(a^4 + b^4*c^2)^2*(3 + 2*m
)) - (a*b*(a^4 - b^4*c^2 + 2*a^2*b^2*Sqrt[-c^2])*Hypergeometric2F1[1, (3 +
 2*m)/2, (5 + 2*m)/2, (c^2*Tan[e + f*x]^2)/Sqrt[-c^2]]*(c*Tan[e + f*x])^((
3 + 2*m)/2))/(c^2*(a^4 + b^4*c^2)^2*(3 + 2*m))))/(f*(c*Tan[e + f*x])^m)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{(d\tan (fx+e))^m}{\left(a+b\sqrt{c\tan (fx+e)}\right)^2}dx$$

input
```
int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

output
```
int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

Fricas [F]

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \int \frac{(d\tan (fx+e))^m}{\left(\sqrt{c\tan (fx+e)}b+a\right)^2}\,dx$$

input
```
integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="fric
as")
```

output
```
integral(-(2*sqrt(c*tan(f*x + e))*(d*tan(f*x + e))^m*a*b - (b^2*c*tan(f*x
+ e) + a^2)*(d*tan(f*x + e))^m)/(b^4*c^2*tan(f*x + e)^2 - 2*a^2*b^2*c*tan(
f*x + e) + a^4), x)
```

Sympy [F]

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \int \frac{(d\tan (e+fx))^m}{\left(a+b\sqrt{c\tan (e+fx)}\right)^2}\,dx$$

input
```
integrate((d*tan(f*x+e))**m/(a+b*(c*tan(f*x+e))**(1/2))**2,x)
```

output
```
Integral((d*tan(e + f*x))**m/(a + b*sqrt(c*tan(e + f*x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \int \frac{(d\tan(fx+e))^m}{\left(\sqrt{c\tan(fx+e)}b+a\right)^2}\,dx$$

input
```
integrate((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x, algorithm="giac
")
```

output
```
integrate((d*tan(f*x + e))^m/(sqrt(c*tan(f*x + e))*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \int \frac{(d\tan(e+f\,x))^m}{\left(a+b\,\sqrt{c\tan(e+f\,x)}\right)^2}\,dx$$

input
```
int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2))^2,x)
```

output
```
int((d*tan(e + f*x))^m/(a + b*(c*tan(e + f*x))^(1/2))^2, x)
```

Reduce [F]

$$\int \frac{(d\tan(e+fx))^m}{\left(a+b\sqrt{c\tan(e+fx)}\right)^2}\,dx = \text{too large to display}$$

input
```
int((d*tan(f*x+e))^m/(a+b*(c*tan(f*x+e))^(1/2))^2,x)
```

output
```
(d**m*(tan(e + f*x)**m*a**4*m + 8*sqrt(c)*int(tan(e + f*x)**((2*m + 1)/2)/
(4*tan(e + f*x)**3*b**4*c**2*m**2 - tan(e + f*x)**3*b**4*c**2 - 8*tan(e +
f*x)**2*a**2*b**2*c*m**2 + 2*tan(e + f*x)**2*a**2*b**2*c + 4*tan(e + f*x)*
a**4*m**2 - tan(e + f*x)*a**4),x)*tan(e + f*x)*a**5*b**3*c*f*m**4 - 6*sqrt
(c)*int(tan(e + f*x)**((2*m + 1)/2)/(4*tan(e + f*x)**3*b**4*c**2*m**2 - ta
n(e + f*x)**3*b**4*c**2 - 8*tan(e + f*x)**2*a**2*b**2*c*m**2 + 2*tan(e + f
*x)**2*a**2*b**2*c + 4*tan(e + f*x)*a**4*m**2 - tan(e + f*x)*a**4),x)*tan(
e + f*x)*a**5*b**3*c*f*m**2 - 2*sqrt(c)*int(tan(e + f*x)**((2*m + 1)/2)/(4
*tan(e + f*x)**3*b**4*c**2*m**2 - tan(e + f*x)**3*b**4*c**2 - 8*tan(e + f*
x)**2*a**2*b**2*c*m**2 + 2*tan(e + f*x)**2*a**2*b**2*c + 4*tan(e + f*x)*a*
*4*m**2 - tan(e + f*x)*a**4),x)*tan(e + f*x)*a**5*b**3*c*f*m - 8*sqrt(c)*i
nt(tan(e + f*x)**((2*m + 1)/2)/(4*tan(e + f*x)**3*b**4*c**2*m**2 - tan(e +
 f*x)**3*b**4*c**2 - 8*tan(e + f*x)**2*a**2*b**2*c*m**2 + 2*tan(e + f*x)**
2*a**2*b**2*c + 4*tan(e + f*x)*a**4*m**2 - tan(e + f*x)*a**4),x)*a**7*b*f*
m**4 + 6*sqrt(c)*int(tan(e + f*x)**((2*m + 1)/2)/(4*tan(e + f*x)**3*b**4*c
**2*m**2 - tan(e + f*x)**3*b**4*c**2 - 8*tan(e + f*x)**2*a**2*b**2*c*m**2
+ 2*tan(e + f*x)**2*a**2*b**2*c + 4*tan(e + f*x)*a**4*m**2 - tan(e + f*x)*
a**4),x)*a**7*b*f*m**2 + 2*sqrt(c)*int(tan(e + f*x)**((2*m + 1)/2)/(4*tan(
e + f*x)**3*b**4*c**2*m**2 - tan(e + f*x)**3*b**4*c**2 - 8*tan(e + f*x)**2
*a**2*b**2*c*m**2 + 2*tan(e + f*x)**2*a**2*b**2*c + 4*tan(e + f*x)*a**4...
```

3.412 $\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+m+np), \frac{1}{2}(3+m+np), -\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m (b}{f(1+m+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2*m+1/2],[1/2*n*p+1/2*m+3/2],-tan(f*x+e)^2)*tan(f*
x+e)*(d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p/f/(n*p+m+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+m+np), \frac{1}{2}(3+m+np), -\tan^2(e+fx)\right)\tan(e+fx)(d\tan(e+fx))^m (b}{f(1+m+np)}$$

input
```
Integrate[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + m + n*p)/2, (3 + m + n*p)/2, -Tan[e + f*x]^2]*T
an[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4061, 2034, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{ 4061}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (c\tan(e+fx))^{np}(d\tan(e+fx))^m dx$$

$$\Big\downarrow \text{ 2034}$$

$$(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p (c\tan(e+fx))^{-m-np} \int (c\tan(e+fx))^{m+np} dx$$

$$\Big\downarrow \text{ 3042}$$

$$(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p (c\tan(e+fx))^{-m-np} \int (c\tan(e+fx))^{m+np} dx$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{c(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p (c\tan(e+fx))^{-m-np} \int \frac{(c\tan(e+fx))^{m+np}}{\tan^2(e+fx)c^2+c^2} d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\tan(e+fx)(d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(1, \frac{1}{2}(m+np+1), \frac{1}{2}(m+np+3), -\tan^2\right.}{f(m+np+1)}$$

input
```
Int[(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + m + n*p)/2, (3 + m + n*p)/2, -Tan[e + f*x]^2]*T
an[e + f*x]*(d*Tan[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + m + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4061
```
Int[((c_.)*((d_.)*tan[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Tan[e + f*x
])^p)^FracPart[n]/(d*Tan[e + f*x])^(p*FracPart[n]))   Int[(a + b*Tan[e + f*
x])^m*(d*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\tan(fx+e))^m \left(b(c\tan(fx+e))^n\right)^p dx$$

input
```
int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (d\tan(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\tan(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \int (b(c\tan(e+fx))^n)^p (d\tan(e+fx))^m \, dx$$

input
```
integrate((d*tan(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*(d*tan(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \int ((c\tan(fx+e))^n b)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Giac [F]

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \int ((c\tan(fx+e))^n b)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*tan(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \int (d\tan(e+f\,x))^m \left(b\,(c\tan(e+f\,x))^n\right)^p \, dx$$

input
```
int((d*tan(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (d\tan(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = d^m c^{np} b^p \left(\int \tan\left(fx+e\right)^{np+m} dx\right)$$

input
```
int((d*tan(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*c**(n*p)*b**p*int(tan(e + f*x)**(m + n*p),x)
```

3.413 $\int \tan^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \tan^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(3+np), \frac{1}{2}(5+np), -\tan^2(e+fx)\right) \tan^3(e+fx) (b(c\tan(e+fx))^n)^p}{f(3+np)}$$

output
```
hypergeom([1, 1/2*n*p+3/2],[1/2*n*p+5/2],-tan(f*x+e)^2)*tan(f*x+e)^3*(b*(c
*tan(f*x+e))^n)^p/f/(n*p+3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \tan^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(3+np), \frac{1}{2}(5+np), -\tan^2(e+fx)\right) \tan^3(e+fx) (b(c\tan(e+fx))^n)^p}{f(3+np)}$$

input
```
Integrate[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 2030, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \tan^2(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int(c\tan(e+fx))^{np+2}dx}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int(c\tan(e+fx))^{np+2}dx}{c^2}$$

$$\Big\downarrow \text{3957}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int\frac{(c\tan(e+fx))^{np+2}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{cf}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan^3(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+3),\frac{1}{2}(np+5),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+3)}$$

input
```
Int[Tan[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (3 + n*p)/2, (5 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^3*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \tan (fx+e)^2 (b(c\tan (fx+e))^n)^p \, dx$$

input
```
int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \tan^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^n b)^p \tan (fx+e)^2 \, dx$$

input
```
integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Sympy [F]

$$\int \tan^2(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \int (b(c\tan (e+fx))^n)^p \tan^2 (e+fx) \, dx$$

input
```
integrate(tan(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

Maxima [F]

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\tan\left(fx+e\right)^2\,dx$$

input
```
integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Giac [F]

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\tan\left(fx+e\right)^2\,dx$$

input
```
integrate(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \tan(e+f\,x)^2\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int(tan(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \tan^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\tan\left(fx+e\right)^2dx\right)$$

input

```
int(tan(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output

```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*tan(e + f*x)**2,x)
```

3.414 $\int (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*t
an(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(e+fx)\right)\tan(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(1+np)}$$

input
```
Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b(c\tan(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b(c\tan(e+fx))^n)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p\int (c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p\int (c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c(c\tan(e+fx))^{-np}(b(c\tan(e+fx))^n)^p\int \frac{(c\tan(e+fx))^{np}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

input
```
Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (b(c\tan (fx+e))^n)^p\,dx$$

input
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int (b(c\tan(e+fx))^n)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Maxima [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Giac [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int (b\,(c\tan(e+f\,x))^n)^p \, dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = c^{np} b^p \left(\int \tan(fx+e)^{np} \, dx\right)$$

input
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p),x)
```

3.415 $\int \cot^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \cot^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = -\frac{\cot(e+fx) \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+np),\frac{1}{2}(1+np),-\tan^2(e+fx)\right) \left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

output
```
-cot(f*x+e)*hypergeom([1, 1/2*n*p-1/2],[1/2*n*p+1/2],-tan(f*x+e)^2)*(b*(c*
tan(f*x+e))^n)^p/f/(-n*p+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \cot^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\cot(e+fx) \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+np),\frac{1}{2}(1+np),-\tan^2(e+fx)\right) \left(b(c\tan(e+fx))^n\right)^p}{f(-1+np)}$$

input
```
Integrate[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f*x
]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-1 + n*p))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4142, 3042, 2030, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \cot^2(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$c^2(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np-2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c^3(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np-2}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-1),\frac{1}{2}(np+1),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(1-np)}$$

input
```
Int[Cot[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[1, (-1 + n*p)/2, (1 + n*p)/2, -Tan[e + f
*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - n*p)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cot(fx+e)^2 (b(c\tan(fx+e))^n)^p\, dx$$

input
```
int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cot^2(e+fx) (b(c\tan(e+fx))^n)^p\ dx = \int ((c\tan(fx+e))^n b)^p \cot(fx+e)^2\ dx$$

input
```
integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Sympy [F]

$$\int \cot^2(e+fx) (b(c\tan(e+fx))^n)^p\ dx = \int (b(c\tan(e+fx))^n)^p \cot^2(e+fx)\, dx$$

input
```
integrate(cot(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

Maxima [F]

$$\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int\left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Giac [F]

$$\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int\left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \cot(e+f\,x)^2\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int(cot(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cot^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\cot\left(fx+e\right)^2dx\right)$$

input
```
int(cot(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cot(e + f*x)**2,x)
```

3.416 $\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = -\frac{\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+np),\frac{1}{2}(-1+np),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

output
```
-cot(f*x+e)^3*hypergeom([1, 1/2*n*p-3/2],[1/2*n*p-1/2],-tan(f*x+e)^2)*(b*(
c*tan(f*x+e))^n)^p/f/(-n*p+3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+np),\frac{1}{2}(-1+np),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(-3+np)}$$

input
```
Integrate[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, (-1 + n*p)/2, -Tan[e +
f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-3 + n*p))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 3042, 2030, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \cot^4(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \frac{(c\tan(e+fx))^{np}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$c^4(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (c\tan(e+fx))^{np-4}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c^5(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \frac{(c\tan(e+fx))^{np-4}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\cot^3(e+fx) \text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-3),\frac{1}{2}(np-1),-\tan^2(e+fx)\right) \left(b(c\tan(e+fx))^n\right)^p}{f(3-np)}$$

input
```
Int[Cot[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
-((Cot[e + f*x]^3*Hypergeometric2F1[1, (-3 + n*p)/2, (-1 + n*p)/2, -Tan[e
+ f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(3 - n*p)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cot(fx+e)^4 \left(b(c\tan(fx+e))^n\right)^p dx$$

input
```
int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\, dx = \int \left((c\tan(fx+e))^n b\right)^p \cot(fx+e)^4\, dx$$

input
```
integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

Sympy [F]

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\, dx = \int \left(b(c\tan(e+fx))^n\right)^p \cot^4(e+fx)\, dx$$

input
```
integrate(cot(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**4, x)
```

Maxima [F]

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int\left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

Giac [F]

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int\left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \cot(e+f\,x)^4\left(b\left(c\tan(e+f\,x)\right)^n\right)^p\,dx$$

input
```
int(cot(e + f*x)^4*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^4*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cot^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = c^{np} b^p \left(\int \tan(fx+e)^{np} \cot(fx+e)^4 \, dx\right)$$

input
```
int(cot(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cot(e + f*x)**4,x)
```

3.417 $\int \cot^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \cot^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = -\frac{\cot^5(e+fx)\, \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-5+np), \frac{1}{2}(-3+np), -\tan^2(e+fx)\right) \left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

output
```
-cot(f*x+e)^5*hypergeom([1, 1/2*n*p-5/2],[1/2*n*p-3/2],-tan(f*x+e)^2)*(b*(
c*tan(f*x+e))^n)^p/f/(-n*p+5)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \cot^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\cot^5(e+fx)\, \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-5+np), \frac{1}{2}(-3+np), -\tan^2(e+fx)\right) \left(b(c\tan(e+fx))^n\right)^p}{f(-5+np)}$$

input
```
Integrate[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, (-3 + n*p)/2, -Tan[e +
f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-5 + n*p))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 3042, 2030, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \cot^6(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \frac{(c\tan(e+fx))^{np}}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{2030}$$

$$c^6(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (c\tan(e+fx))^{np-6}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c^7(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \frac{(c\tan(e+fx))^{np-6}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-5),\frac{1}{2}(np-3),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(5-np)}$$

input
```
Int[Cot[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
-((Cot[e + f*x]^5*Hypergeometric2F1[1, (-5 + n*p)/2, (-3 + n*p)/2, -Tan[e
+ f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(5 - n*p)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cot (fx+e)^6 (b(c\tan (fx+e))^n)^p \, dx$$

input
```
int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cot^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan (fx+e))^n b\right)^p \cot (fx+e)^6 \, dx$$

input
```
integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

Sympy [F]

$$\int \cot^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(b(c\tan (e+fx))^n\right)^p \cot^6 (e+fx) \, dx$$

input
```
integrate(cot(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**6, x)
```

Maxima [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left((c\tan\left(fx+e\right))^n b\right)^p \cot\left(fx+e\right)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \cot(e+f\,x)^6\,\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int(cot(e + f*x)^6*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^6*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cot^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\cot\left(fx+e\right)^6dx\right)$$

input
```
int(cot(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cot(e + f*x)**6,x)
```

3.418 $\int \tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(4+np),\frac{1}{2}(6+np),-\tan^2(e+fx)\right)\tan^4(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(4+np)}$$

output
```
hypergeom([1, 1/2*n*p+2],[1/2*n*p+3],-tan(f*x+e)^2)*tan(f*x+e)^4*(b*(c*tan
(f*x+e))^n)^p/f/(n*p+4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \tan^3(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,2+\frac{np}{2},3+\frac{np}{2},-\tan^2(e+fx)\right)\tan^4(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(4+np)}$$

input
```
Integrate[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, 2 + (n*p)/2, 3 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 2030, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^3\left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{ 4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \tan^3(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np+3}dx}{c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np+3}dx}{c^3}$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np+3}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{c^2f}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\tan^4(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+4),\frac{1}{2}(np+6),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+4)}$$

input
```
Int[Tan[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (4 + n*p)/2, (6 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^4*(b*(c*Tan[e + f*x])^n)^p)/(f*(4 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \tan (fx+e)^3 (b(c\tan (fx+e))^n)^p \, dx$$

input
```
int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \tan^3(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^n b)^p \tan (fx+e)^3 \, dx$$

input
```
integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Sympy [F]

$$\int \tan^3(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \int (b(c\tan (e+fx))^n)^p \tan^3 (e+fx) \, dx$$

input
```
integrate(tan(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

Maxima [F]

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p \tan\left(fx+e\right)^3\,dx$$

input
```
integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Giac [F]

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p \tan\left(fx+e\right)^3\,dx$$

input
```
integrate(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int {\mathrm{tan}\left(e+f\,x\right)}^3\,{\left(b\,{\left(c\,\mathrm{tan}\left(e+f\,x\right)\right)}^n\right)}^p \,d x$$

input
```
int(tan(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$
$$= \frac{c^{np}b^p\left(\tan(fx+e)^{np}\tan(fx+e)^2 np - \tan(fx+e)^{np} np - 2\tan(fx+e)^{np} + \left(\int \frac{\tan(fx+e)^{np}}{\tan(fx+e)}dx\right) f\,n^2p\right.}{fnp\,(np+2)}$$

input
```
int(tan(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
(c**(n*p)*b**p*(tan(e + f*x)**(n*p)*tan(e + f*x)**2*n*p - tan(e + f*x)**(n
*p)*n*p - 2*tan(e + f*x)**(n*p) + int(tan(e + f*x)**(n*p)/tan(e + f*x),x)*
f*n**2*p**2 + 2*int(tan(e + f*x)**(n*p)/tan(e + f*x),x)*f*n*p))/(f*n*p*(n*
p + 2))
```

3.419 $\int \tan(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \tan(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(2+np),\frac{1}{2}(4+np),-\tan^2(e+fx)\right)\tan^2(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(2+np)}$$

output
```
hypergeom([1, 1/2*n*p+1],[1/2*n*p+2],-tan(f*x+e)^2)*tan(f*x+e)^2*(b*(c*tan
(f*x+e))^n)^p/f/(n*p+2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \tan(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{\text{Hypergeometric2F1}\left(1,1+\frac{np}{2},2+\frac{np}{2},-\tan^2(e+fx)\right)\tan^2(e+fx)\,(b(c\tan(e+fx))^n)^p}{f(2+np)}$$

input
```
Integrate[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, 1 + (n*p)/2, 2 + (n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4142, 2030, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \tan(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np+1}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np+1}dx}{c}$$

$$\Big\downarrow \text{3957}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np+1}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan^2(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+2),\frac{1}{2}(np+4),-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(np+2)}$$

input
```
Int[Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (2 + n*p)/2, (4 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]^2*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \tan(fx+e)\left(b(c\tan(fx+e))^n\right)^p dx$$

input
```
int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left((c\tan(fx+e))^n b\right)^p \tan(fx+e)\,dx$$

input
```
integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Sympy [F]

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left(b(c\tan(e+fx))^n\right)^p \tan(e+fx)\,dx$$

input
```
integrate(tan(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*tan(e + f*x), x)
```

Maxima [F]

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b)^p\tan(fx+e)\,dx$$

input
```
integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Giac [F]

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b)^p\tan(fx+e)\,dx$$

input
```
integrate(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*tan(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \tan(e+f\,x)\,\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int(tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \tan(e+fx)\,(b(c\tan(e+fx))^n)^p\,dx = \frac{c^{np}b^p\left(\tan\left(fx+e\right)^{np}-\left(\int\frac{\tan(fx+e)^{np}}{\tan(fx+e)}dx\right)fnp\right)}{fnp}$$

input
```
int(tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
(c**(n*p)*b**p*(tan(e + f*x)**(n*p) - int(tan(e + f*x)**(n*p)/tan(e + f*x)
,x)*f*n*p))/(f*n*p)
```

3.420 $\int \cot(e+fx) (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cot(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{np}{2}, 1+\frac{np}{2}, -\tan^2(e+fx)\right) (b(c\tan(e+fx))^n)^p}{fnp}$$

output
```
hypergeom([1, 1/2*n*p],[1/2*n*p+1],-tan(f*x+e)^2)*(b*(c*tan(f*x+e))^n)^p/f
/n/p
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \cot(e+fx) (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{np}{2}, 1+\frac{np}{2}, -\tan^2(e+fx)\right) (b(c\tan(e+fx))^n)^p}{fnp}$$

input
```
Integrate[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e +
f*x])^n)^p)/(f*n*p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4142, 3042, 2030, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \cot(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{2030}$$

$$c(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np-1}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c^2(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np-1}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\text{Hypergeometric2F1}\left(1,\frac{np}{2},\frac{np}{2}+1,-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{fnp}$$

input
```
Int[Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (n*p)/2, 1 + (n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e +
 f*x])^n)^p)/(f*n*p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cot (fx+e) \left(b(c\tan (fx+e))^n\right)^p \, dx$$

input
```
int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cot(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan (fx+e))^n b\right)^p \cot (fx+e) \, dx$$

input
```
integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Sympy [F]

$$\int \cot(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(b(c\tan (e+fx))^n\right)^p \cot (e+fx) \, dx$$

input
```
integrate(cot(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x), x)
```

Maxima [F]

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Giac [F]

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \cot(e+f\,x)\,\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)^p\,dx$$

input
```
int(cot(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan(fx+e)^{np}\cot(fx+e)\,dx\right)$$

input
```
int(cot(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cot(e + f*x),x)
```

3.421 $\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = -\frac{\cot^2(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-2+np),\frac{np}{2},-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

output
```
-cot(f*x+e)^2*hypergeom([1, 1/2*n*p-1],[1/2*n*p],-tan(f*x+e)^2)*(b*(c*tan(
f*x+e))^n)^p/f/(-n*p+2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\cot^2(e+fx)\,\text{Hypergeometric2F1}\left(1,-1+\frac{np}{2},\frac{np}{2},-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(-2+np)}$$

input
```
Integrate[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]^2*Hypergeometric2F1[1, -1 + (n*p)/2, (n*p)/2, -Tan[e + f*x]^
2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(-2 + n*p))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4142, 3042, 2030, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \cot^3(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$c^3(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np-3}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c^4(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np-3}}{\tan^2(e+fx)c^2+c^2}d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\cot^2(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np-2),\frac{np}{2},-\tan^2(e+fx)\right)\left(b(c\tan(e+fx))^n\right)^p}{f(2-np)}$$

input
```
Int[Cot[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
-((Cot[e + f*x]^2*Hypergeometric2F1[1, (-2 + n*p)/2, (n*p)/2, -Tan[e + f*x
]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(2 - n*p)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cot(fx+e)^3 \left(b(c\tan(fx+e))^n\right)^p dx$$

input
```
int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\, dx = \int \left((c\tan(fx+e))^n b\right)^p \cot(fx+e)^3\, dx$$

input
```
integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Sympy [F]

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\, dx = \int \left(b(c\tan(e+fx))^n\right)^p \cot^3(e+fx)\, dx$$

input
```
integrate(cot(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*cot(e + f*x)**3, x)
```

Maxima [F]

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Giac [F]

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b\right)^p\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cot(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \cot(e+f\,x)^3\left(b\left(c\tan(e+f\,x)\right)^n\right)^p\,dx$$

input
```
int(cot(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cot^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan(fx+e)^{np}\cot(fx+e)^3\,dx\right)$$

input
```
int(cot(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cot(e + f*x)**3,x)
```

3.422 $\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \,dx = \mathrm{Int}((d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p, x)$$

output
```
Defer(Int)((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \,dx = \int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \,dx$$

input
```
Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \; 4155$$

$$\int (d\tan(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx$$

input
```
Int[(d*Tan[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4155
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Tan[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (d\tan (fx+e))^m (a+b(c\tan (fx+e))^n)^p \, dx$$

input
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^n b+a)^p (d\tan (fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [N/A]

Not integrable

Time = 36.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int (d\tan (e+fx))^m (a+b(c\tan (e+fx))^n)^p \, dx$$

input
```
integrate((d*tan(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((d*tan(e + f*x))**m*(a + b*(c*tan(e + f*x))**n)**p, x)
```

Maxima [N/A]

Not integrable

Time = 6.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b+a)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b+a)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 9.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$
$$= \int (d\tan(e+f\,x))^m \,(a+b\,(c\tan(e+f\,x))^n)^p \, dx$$

input
```
int((d*tan(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int (d\tan(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$
$$= d^m \left(\int \tan(fx+e)^m (c^n \tan(fx+e)^n b + a)^p \, dx \right)$$

input
```
int((d*tan(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*int(tan(e + f*x)**m*(c**n*tan(e + f*x)**n*b + a)**p,x)
```

3.423 $\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \frac{(d\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+2p),\frac{1}{2}(3-m+2p),-\tan^2(e+fx)\right)\tan(e+fx)\,(b}{f(1-m+2p)}$$

output
```
(d*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+p],[3/2-1/2*m+p],-tan(f*x+e)^2)*t
an(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1-m+2*p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = -\frac{d(d\cot(e+fx))^{-1+m}\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{m}{2}+p,\frac{3}{2}-\frac{m}{2}+p,-\tan^2(e+fx)\right)\left(b\tan^2(e+fx)\right)}{f(-1+m-2p)}$$

input
```
Integrate[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
-((d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, 1/2 - m/2 + p, 3/2 - m
/2 + p, -Tan[e + f*x]^2]*(b*Tan[e + f*x]^2)^p)/(f*(-1 + m - 2*p)))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4141, 3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^2\right)^p (d\cot(e+fx))^m dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\int (d\cot(e+fx))^m\tan^{2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\int (d\cot(e+fx))^m\tan(e+fx)^{2p}dx$$

$$\Big\downarrow \text{3084}$$

$$\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m\tan^{m-2p}(e+fx)\int \tan^{2p-m}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m\tan^{m-2p}(e+fx)\int \tan(e+fx)^{2p-m}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m\tan^{m-2p}(e+fx)\int \frac{\tan^{2p-m}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\cot(e+fx))^m\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+2p+1),\frac{1}{2}(-m+2p+3),-\tan\right.}{f(-m+2p+1)}$$

input
```
Int[(d*Cot[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + 2*p)/2, (3 - m + 2*p)/2,
 -Tan[e + f*x]^2]*Tan[e + f*x]*(b*Tan[e + f*x]^2)^p)/(f*(1 - m + 2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\cot(fx+e))^m \left(b\tan(fx+e)^2\right)^p dx$$

input
```
int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan(fx+e)^2\right)^p (d\cot(fx+e))^m \, dx$$

input
```
integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan^2(e+fx)\right)^p (d\cot(e+fx))^m \, dx$$

input
```
integrate((d*cot(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((b*tan(e + f*x)**2)**p*(d*cot(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\cot(e+fx))^m\left(b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2\right)^p\left(d\cot\left(fx+e\right)\right)^m\,dx$$

input
```
integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cot(e+fx))^m\left(b\tan^2(e+fx)\right)^p\,dx=\int \left(b\tan\left(fx+e\right)^2\right)^p\left(d\cot\left(fx+e\right)\right)^m\,dx$$

input
```
integrate((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*cot(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cot(e+fx))^m\left(b\tan^2(e+fx)\right)^p\,dx=\int (d\cot(e+f\,x))^m\left(b\tan(e+f\,x)^2\right)^p\,dx$$

input
```
int((d*cot(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*cot(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\cot(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = d^m b^p \left(\int \tan(fx+e)^{2p} \cot(fx+e)^m \,dx\right)$$

input
```
int((d*cot(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*b**p*int(tan(e + f*x)**(2*p)*cot(e + f*x)**m,x)
```

3.424 $\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \frac{\text{AppellF1}\left(\frac{1-m}{2},1,-p,\frac{3-m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\cot(e+fx))^m\tan(e+fx)\left(a+b\tan^2(e+\right.}{f(1-m)}$$

output
```
AppellF1(1/2-1/2*m,1,-p,3/2-1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*cot(
f*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1-m)/(((a+b*tan(f*x+e)^2)/a)^
p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 265 vs. 2(108) = 216.

Time = 1.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.45

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx =$$

$$-\frac{a(-3+m)\,\text{AppellF1}\left(\frac{1-m}{2},-p,1,\frac{3-m}{2},-\frac{b\tan^2(e+fx)}{a},\right.}{f(-1+m)\left(-2bp\,\text{AppellF1}\left(\frac{3-m}{2},1-p,1,\frac{5-m}{2},-\frac{b\tan^2(e+fx)}{a},-\tan^2(e+fx)\right)+2a\,\text{AppellF1}\left(\frac{3-m}{2},\right.\right.}$$

input
```
Integrate[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((a*(-3 + m)*AppellF1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a
), -Tan[e + f*x]^2]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Cot[e + f*x])^m*(a + b*
Tan[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[(3 - m)/2, 1 - p, 1, (5 -
m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + 2*a*AppellF1[(3 - m)/2,
-p, 2, (5 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x]^2] + a*(-3 + m)*A
ppellF1[(1 - m)/2, -p, 1, (3 - m)/2, -((b*Tan[e + f*x]^2)/a), -Tan[e + f*x
]^2]*Cot[e + f*x]^2)))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4157, 3042, 4153, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cot(e+fx))^m \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4157}$$

$$\left(\frac{\tan(e+fx)}{d}\right)^m (d\cot(e+fx))^m \int \left(\frac{\tan(e+fx)}{d}\right)^{-m} \left(b\tan^2(e+fx)+a\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\tan(e+fx)}{d}\right)^m (d\cot(e+fx))^m \int \left(\frac{\tan(e+fx)}{d}\right)^{-m} \left(b\tan(e+fx)^2+a\right)^p dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\left(\frac{\tan(e+fx)}{d}\right)^m (d\cot(e+fx))^m \int \frac{\left(\frac{\tan(e+fx)}{d}\right)^{-m} \left(b\tan^2(e+fx)+a\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$\downarrow$ 395

$$\frac{\left(\frac{\tan(e+fx)}{d}\right)^m (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \int \frac{\left(\frac{\tan(e+fx)}{d}\right)^{-m}\left(\frac{b\tan^2(e+fx)}{a}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e}{f}$$

$\downarrow$ 394

$$\frac{\tan(e+fx)(d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1-m}{2},1,-p,\frac{3-m}{2},-\tan^2(e+}{f(1-m)}$$

input
```
Int[(d*Cot[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[(1 - m)/2, 1, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)]*(d*Cot[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(1 - m
)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

rule 4157
```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[(d*Cot[e + f*x])^FracPart[m]*(Tan
[e + f*x]/d)^FracPart[m]   Int[(a + b*(c*Tan[e + f*x])^n)^p/(Tan[e + f*x]/d
)^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\cot (fx+e))^m \left(a+b\tan (fx+e)^2\right)^p dx$$

input
```
int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx = \int \left(b\tan (fx+e)^2+a\right)^p (d\cot (fx+e))^m dx$$

input
```
integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*cot(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\cot\left(fx+e\right))^m \, dx$$

input
```
integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\cot\left(fx+e\right))^m \, dx$$

input
```
integrate((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*cot(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \int (d\cot(e+f\,x))^m \left(b\tan(e+f\,x)^2+a\right)^p dx$$

input
```
int((d*cot(e + f*x))^m*(a + b*tan(e + f*x)^2)^p,x)
```

output
```
int((d*cot(e + f*x))^m*(a + b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\cot(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= d^m \left(\int \left(\tan(fx+e)^2 b + a\right)^p \cot(fx+e)^m dx \right)$$

input
```
int((d*cot(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*int((tan(e + f*x)**2*b + a)**p*cot(e + f*x)**m,x)
```

3.425 $\int (d\cot(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 80

$$\int (d\cot(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \frac{(d\cot(e+fx))^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+np), \frac{1}{2}(3-m+np), -\tan^2(e+fx)\right)\tan(e+fx)\,(b}{f(1-m+np)}$$

output
```
(d*cot(f*x+e))^m*hypergeom([1, 1/2*n*p-1/2*m+1/2],[1/2*n*p-1/2*m+3/2],-tan
(f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p-m+1)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int (d\cot(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \frac{d(d\cot(e+fx))^{-1+m} \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-m+np), \frac{1}{2}(3-m+np), -\tan^2(e+fx)\right)(b(c\tan(e}{f(1-m+np)}$$

input
```
Integrate[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(d*(d*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m
+ n*p)/2, -Tan[e + f*x]^2]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4142, 3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (d\cot(e+fx))^m (c\tan(e+fx))^{np} dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (d\cot(e+fx))^m (c\tan(e+fx))^{np} dx$$

$$\Big\downarrow \text{3084}$$

$$(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p (c\tan(e+fx))^{m-np} \int (c\tan(e+fx))^{np-m} dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p (c\tan(e+fx))^{m-np} \int (c\tan(e+fx))^{np-m} dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c(d\cot(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p (c\tan(e+fx))^{m-np} \int \frac{(c\tan(e+fx))^{np-m}}{\tan^2(e+fx)c^2+c^2} d(c\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(e+fx)(d\cot(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+np+1),\frac{1}{2}(-m+np+3),-\right.}{f(-m+np+1)}$$

input
```
Int[(d*Cot[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((d*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n*p)/2, (3 - m + n*p)/2,
 -Tan[e + f*x]^2]*Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\cot (fx+e))^m (b(c\tan (fx+e))^n)^p\,dx$$

input
```
int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (d\cot(e+fx))^m (b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan (fx+e))^n b)^p (d\cot (fx+e))^m\,dx$$

input
```
integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cot(e+fx))^m (b(c\tan(e+fx))^n)^p\,dx = \int (b(c\tan (e+fx))^n)^p (d\cot (e+fx))^m\,dx$$

input
```
integrate((d*cot(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*(d*cot(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\cot(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,dx=\int\left((c\tan\left(fx+e\right))^nb\right)^p(d\cot\left(fx+e\right))^m\,dx$$

input
```
integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cot(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,dx=\int\left((c\tan\left(fx+e\right))^nb\right)^p(d\cot\left(fx+e\right))^m\,dx$$

input
```
integrate((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*cot(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cot(e+fx))^m\left(b(c\tan(e+fx))^n\right)^p\,dx=\int (d\cot(e+f\,x))^m\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int((d*cot(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d*cot(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (d\cot(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = d^m c^{np} b^p \left(\int \tan(fx+e)^{np} \cot(fx+e)^m \, dx\right)$$

input
```
int((d*cot(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cot(e + f*x)**m,x)
```

3.426 $\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx = (d\cot(e+fx))^m \left(\frac{\tan(e+fx)}{d}\right)^m \operatorname{Int}\left(\left(\frac{\tan(e+fx)}{d}\right)^{-m} (a+b(c\tan(e+fx))^n)^p, x\right)$$

output
```
(d*cot(f*x+e))^m*(tan(f*x+e)/d)^m*Defer(Int)((a+b*(c*tan(f*x+e))^n)^p/((ta
n(f*x+e)/d)^m),x)
```

Mathematica [N/A]

Not integrable

Time = 7.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx = \int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p\,dx$$

input
```
Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4157, 3042, 4155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cot(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cot(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4157}$$

$$\left(\frac{\tan(e+fx)}{d}\right)^m(d\cot(e+fx))^m\int\left(\frac{\tan(e+fx)}{d}\right)^{-m}\left(b(c\tan(e+fx))^n+a\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\tan(e+fx)}{d}\right)^m(d\cot(e+fx))^m\int\left(\frac{\tan(e+fx)}{d}\right)^{-m}\left(b(c\tan(e+fx))^n+a\right)^p dx$$

$$\Big\downarrow \text{4155}$$

$$\left(\frac{\tan(e+fx)}{d}\right)^m(d\cot(e+fx))^m\int\left(\frac{\tan(e+fx)}{d}\right)^{-m}\left(b(c\tan(e+fx))^n+a\right)^p dx$$

input
```
Int[(d*Cot[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4155
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Tan[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 4157
```
Int[(cot[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[(d*Cot[e + f*x])^FracPart[m]*(Tan
[e + f*x]/d)^FracPart[m]   Int[(a + b*(c*Tan[e + f*x])^n)^p/(Tan[e + f*x]/d
)^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Maple [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (d\cot (fx+e))^{m} (a+b(c\tan (fx+e))^{n})^{p} dx$$

input
```
int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^{n} b+a)^{p}(d\cot (fx+e))^{m} \, dx$$

input
```
integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cot(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx = \text{Timed out}$$

input
```
integrate((d*cot(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 6.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cot(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n\,b+a)^p(d\cot(fx+e))^m\,dx$$

input
```
integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b+a)^p (d\cot(fx+e))^m \, dx$$

input
```
integrate((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*cot(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 8.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int (d\cot(e+f\,x))^m\,(a+b\,(c\tan(e+f\,x))^n)^p \, dx$$

input
```
int((d*cot(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d*cot(e + f*x))^m*(a + b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int (d\cot(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$
$$= d^m \left(\int (c^n \tan(fx+e)^n b + a)^p \cot(fx+e)^m \, dx \right)$$

input
```
int((d*cot(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*int((c**n*tan(e + f*x)**n*b + a)**p*cot(e + f*x)**m,x)
```

3.427 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{(4a-b)\text{arctanh}(\sin(c+dx))}{8d}+\frac{(4a-b)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
1/8*(4*a-b)*arctanh(sin(d*x+c))/d+1/8*(4*a-b)*sec(d*x+c)*tan(d*x+c)/d+1/4*
b*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{2d}-\frac{b\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}-\frac{b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) - (b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Sec
[c + d*x]*Tan[c + d*x])/(2*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*S
ec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4159, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3\left(a+b\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{a-(a-b)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}(4a-b)\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)+\frac{b\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}(4a-b)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{b\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}(4a-b)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{b\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

output
```
((b*Sin[c + d*x])/(4*(1 - Sin[c + d*x]^2)^2) + ((4*a - b)*(ArcTanh[Sin[c +
 d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{b\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
default	$\frac{b\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
risch	$-\frac{ie^{i(dx+c)}\left(4a\,e^{6i(dx+c)}-b\,e^{6i(dx+c)}+4a\,e^{4i(dx+c)}+7b\,e^{4i(dx+c)}-4a\,e^{2i(dx+c)}-7b\,e^{2i(dx+c)}-4a+b\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{i(dx+c)}\right.}{2d}$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*si
n(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*l
n(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{(4\,a-b)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(4\,a-b)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left((4\,a-\right.}{16\,d\cos\left(dx+c\right)^4}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/16*((4*a - b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*a - b)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*((4*a - b)*cos(d*x + c)^2 + 2*b)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\int\left(a+b\tan^2{(c+dx)}\right)\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{(4a-b)\log\left(\sin\left(dx+c\right)+1\right)-(4a-b)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((4a-b)\sin(dx+c)^3-(4a+b)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/16*((4*a - b)*log(sin(d*x + c) + 1) - (4*a - b)*log(sin(d*x + c) - 1) -
2*((4*a - b)*sin(d*x + c)^3 - (4*a + b)*sin(d*x + c))/(sin(d*x + c)^4 - 2*
sin(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{(4a-b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(4a-b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(4a\sin(dx+c)^3-b\sin(dx+c)^3-4a\sin(dx+c)-\right.}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*((4*a - b)*log(abs(sin(d*x + c) + 1)) - (4*a - b)*log(abs(sin(d*x + c
) - 1)) - 2*(4*a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 4*a*sin(d*x + c) - b*
sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.83 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.10

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\left(a+\frac{b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{7\,b}{4}-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{7\,b}{4}-a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(a+\frac{b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a-\frac{b}{4}\right)}{d}$$

input
```
int((a + b*tan(c + d*x)^2)/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^7*(a + b/4) - tan(c/2 + (d*x)/2)^3*(a - (7*b)/4) - tan
(c/2 + (d*x)/2)^5*(a - (7*b)/4) + tan(c/2 + (d*x)/2)*(a + b/4))/(d*(6*tan(
c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c
/2 + (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(a - b/4))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.40

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4 a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4 b+8\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{}$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2),x)
```

output
```
( - 4*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a + log(tan((c + d*x)/2) -
 1)*sin(c + d*x)**4*b + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*
log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b - 4*log(tan((c + d*x)/2) - 1)*
a + log(tan((c + d*x)/2) - 1)*b + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**4*a - log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*b - 8*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)**2*a + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b
+ 4*log(tan((c + d*x)/2) + 1)*a - log(tan((c + d*x)/2) + 1)*b - 4*sin(c +
d*x)**3*a + sin(c + d*x)**3*b + 4*sin(c + d*x)*a + sin(c + d*x)*b)/(8*d*(s
in(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.428 $\int \sec(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \sec(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{(2a-b)\text{arctanh}(\sin(c+dx))}{2d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*(2*a-b)*arctanh(sin(d*x+c))/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \sec(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{a\coth^{-1}(\sin(c+dx))}{d} - \frac{b\text{arctanh}(\sin(c+dx))}{2d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*ArcCoth[Sin[c + d*x]])/d - (b*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4159, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)\left(a+b\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{a-(a-b)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{2}(2a-b)\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{b\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}(2a-b)\text{arctanh}(\sin(c+dx))+\frac{b\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

output
```
(((2*a - b)*ArcTanh[Sin[c + d*x]])/2 + (b*Sin[c + d*x])/(2*(1 - Sin[c + d*
x]^2)))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

method	result	size
derivativedivides	$\frac{b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	67
default	$\frac{b\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	67
risch	$-\frac{ib\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)a}{d}-\frac{\ln\left(e^{i(dx+c)}+i\right)b}{2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)a}{d}+\frac{\ln\left(e^{i(dx+c)}-i\right)b}{2d}$	118

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan
(d*x+c)))+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.81

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{(2\,a-b)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-(2\,a-b)\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)}+2\,b\sin{(dx}}{4\,d\cos{(dx+c)}^2}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*((2*a - b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - b)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \int \left(a+b\tan^2{(c+dx)}\right)\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{(2a-b)\log\left(\sin\left(dx+c\right)+1\right)-(2a-b)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2b\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*((2*a - b)*log(sin(d*x + c) + 1) - (2*a - b)*log(sin(d*x + c) - 1) - 2
*b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{(2a-b)\log\left(|\sin\left(dx+c\right)+1|\right)-(2a-b)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2b\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((2*a - b)*log(abs(sin(d*x + c) + 1)) - (2*a - b)*log(abs(sin(d*x + c)
 - 1)) - 2*b*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(2\,a-b)}{d} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + b*tan(c + d*x)^2)/cos(c + d*x),x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(2*a - b))/d + (b*tan(c/2 + (d*x)/2) + b*tan(c/
2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.21

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$= \frac{-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 b+2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{}$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c)^2),x)
```

output
```
( - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + log(tan((c + d*x)/2) -
 1)*sin(c + d*x)**2*b + 2*log(tan((c + d*x)/2) - 1)*a - log(tan((c + d*x)/
2) - 1)*b + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - log(tan((c + d
*x)/2) + 1)*sin(c + d*x)**2*b - 2*log(tan((c + d*x)/2) + 1)*a + log(tan((c
 + d*x)/2) + 1)*b - sin(c + d*x)*b)/(2*d*(sin(c + d*x)**2 - 1))
```

3.429 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right) dx = \frac{b\,\text{arctanh}(\sin(c+dx))}{d} + \frac{(a-b)\sin(c+dx)}{d}$$

output
```
b*arctanh(sin(d*x+c))/d+(a-b)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right) dx = \frac{b\,\text{arctanh}(\sin(c+dx))}{d} + \frac{a\cos(dx)\sin(c)}{d} + \frac{a\cos(c)\sin(dx)}{d} - \frac{b\sin(c+dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/
d - (b*Sin[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4159, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\tan(c+dx)^2}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{a-(a-b)\sin^2(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{b\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+(a-b)\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(a-b)\sin(c+dx)+b\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]] + (a - b)*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+\sin(dx+c)a}{d}$	39
default	$\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+\sin(dx+c)a}{d}$	39
risch	$-\frac{i e^{i(dx+c)}a}{2d}+\frac{i e^{i(dx+c)}b}{2d}+\frac{i e^{-i(dx+c)}a}{2d}-\frac{i e^{-i(dx+c)}b}{2d}+\frac{\ln(e^{i(dx+c)}+i)b}{d}-\frac{\ln(e^{i(dx+c)}-i)b}{d}$	103

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+sin(d*x+c)*a)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a-b\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) + 2*(a - b)*sin(d*
x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \int \left(a+b\tan^2\left(c+dx\right)\right)\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))+2\,a\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) +
2*a*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \cos(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{b(\log(|\sin(dx+c)+1|) - \log(|\sin(dx+c)-1|) - 2\sin(dx+c)) + 2a\sin(dx+c)}{2d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(b*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)) - 2*sin(d*
x + c)) + 2*a*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \cos(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\sin(c+dx)\ (a-b)}{d} + \frac{2b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x)^2),x)
```

output
```
(sin(c + d*x)*(a - b))/d + (2*b*atanh(tan(c/2 + (d*x)/2)))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b+\sin\left(dx+c\right)a-\sin\left(dx+c\right)b}{d}$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c)^2),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*b + log(tan((c + d*x)/2) + 1)*b + sin(c + d*
x)*a - sin(c + d*x)*b)/d
```

3.430 $\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{a\sin(c+dx)}{d} - \frac{(a-b)\sin^3(c+dx)}{3d}$$

output
```
a*sin(d*x+c)/d-1/3*(a-b)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} + \frac{b\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{a+b\tan(c+dx)^2}{\sec(c+dx)^3}dx$$
$$\Big\downarrow \text{ 4159}$$
$$\frac{\int \left(a-(a-b)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a\sin(c+dx)-\frac{1}{3}(a-b)\sin^3(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Sin[c + d*x] - ((a - b)*Sin[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+\frac{b\sin(dx+c)^3}{3}}{d}$	36
default	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+\frac{b\sin(dx+c)^3}{3}}{d}$	36
risch	$\frac{3a\sin(dx+c)}{4d}+\frac{\sin(dx+c)b}{4d}+\frac{\sin(3dx+3c)a}{12d}-\frac{\sin(3dx+3c)b}{12d}$	56

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*b*sin(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{\left((a-b)\cos\left(dx+c\right)^2+2\,a+b\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/3*((a - b)*cos(d*x + c)^2 + 2*a + b)*sin(d*x + c)/d
```

Sympy [F]

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\int \left(a+b\tan^2(c+dx)\right)\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=-\frac{(a-b)\sin(dx+c)^3-3\,a\sin(dx+c)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/3*((a - b)*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=-\frac{a\sin(dx+c)^3-b\sin(dx+c)^3-3\,a\sin(dx+c)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/3*(a*sin(d*x + c)^3 - b*sin(d*x + c)^3 - 3*a*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 7.99 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{9\,a\,\sin\left(c+d\,x\right)+3\,b\,\sin\left(c+d\,x\right)+a\,\sin\left(3\,c+3\,d\,x\right)-b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x)^2),x)
```

output
```
(9*a*sin(c + d*x) + 3*b*sin(c + d*x) + a*sin(3*c + 3*d*x) - b*sin(3*c + 3*
d*x))/(12*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^2 a+\sin\left(dx+c\right)^2 b+3a\right)}{3d}$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*( - sin(c + d*x)**2*a + sin(c + d*x)**2*b + 3*a))/(3*d)
```

3.431 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{a\sin(c+dx)}{d}-\frac{(2a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)\sin^5(c+dx)}{5d}$$

output
```
a*sin(d*x+c)/d-1/3*(2*a-b)*sin(d*x+c)^3/d+1/5*(a-b)*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{(89a+11b+4(7a-2b)\cos(2(c+dx))+3(a-b)\cos(4(c+dx)))\sin(c+dx)}{120d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
```

output
```
((89*a + 11*b + 4*(7*a - 2*b)*Cos[2*(c + d*x)] + 3*(a - b)*Cos[4*(c + d*x)
])*Sin[c + d*x])/(120*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\tan(c+dx)^2}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 290}$$

$$\frac{\int \left((a-b)\sin^4(c+dx)-(2a-b)\sin^2(c+dx)+a\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{5}(a-b)\sin^5(c+dx)-\frac{1}{3}(2a-b)\sin^3(c+dx)+a\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Sin[c + d*x] - ((2*a - b)*Sin[c + d*x]^3)/3 + ((a - b)*Sin[c + d*x]^5)/
5)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 14.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+b\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)}{d}$	72
default	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+b\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)}{d}$	72
risch	$\frac{5a\sin(dx+c)}{8d}+\frac{\sin(dx+c)b}{8d}+\frac{\sin(5dx+5c)a}{80d}-\frac{\sin(5dx+5c)b}{80d}+\frac{5\sin(3dx+3c)a}{48d}-\frac{\sin(3dx+3c)b}{48d}$	86

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b*(-1/5*cos(d*x+
c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\left(3\,(a-b)\cos\left(dx+c\right)^4+(4\,a+b)\cos\left(dx+c\right)^2+8\,a+2\,b\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/15*(3*(a - b)*cos(d*x + c)^4 + (4*a + b)*cos(d*x + c)^2 + 8*a + 2*b)*sin
(d*x + c)/d
```

Sympy [F]

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \int \left(a+b\tan^2\left(c+dx\right)\right)\cos^5\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{3\,(a-b)\sin\left(dx+c\right)^5-5\,(2\,a-b)\sin\left(dx+c\right)^3+15\,a\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/15*(3*(a - b)*sin(d*x + c)^5 - 5*(2*a - b)*sin(d*x + c)^3 + 15*a*sin(d*x
 + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(50) = 100$.

Time = 14.35 (sec) , antiderivative size = 2147, normalized size of antiderivative = 39.76

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/15*(15*a*tan(1/2*d*x)^10*tan(1/2*c)^9 + 15*a*tan(1/2*d*x)^9*tan(1/2*c)^
10 + 20*a*tan(1/2*d*x)^10*tan(1/2*c)^7 + 20*b*tan(1/2*d*x)^10*tan(1/2*c)^7
 - 75*a*tan(1/2*d*x)^9*tan(1/2*c)^8 + 60*b*tan(1/2*d*x)^9*tan(1/2*c)^8 - 7
5*a*tan(1/2*d*x)^8*tan(1/2*c)^9 + 60*b*tan(1/2*d*x)^8*tan(1/2*c)^9 + 20*a*
tan(1/2*d*x)^7*tan(1/2*c)^10 + 20*b*tan(1/2*d*x)^7*tan(1/2*c)^10 + 58*a*ta
n(1/2*d*x)^10*tan(1/2*c)^5 - 8*b*tan(1/2*d*x)^10*tan(1/2*c)^5 + 150*a*tan(
1/2*d*x)^9*tan(1/2*c)^6 - 180*b*tan(1/2*d*x)^9*tan(1/2*c)^6 + 700*a*tan(1/
2*d*x)^8*tan(1/2*c)^7 - 500*b*tan(1/2*d*x)^8*tan(1/2*c)^7 + 700*a*tan(1/2*
d*x)^7*tan(1/2*c)^8 - 500*b*tan(1/2*d*x)^7*tan(1/2*c)^8 + 150*a*tan(1/2*d*
x)^6*tan(1/2*c)^9 - 180*b*tan(1/2*d*x)^6*tan(1/2*c)^9 + 58*a*tan(1/2*d*x)^
5*tan(1/2*c)^10 - 8*b*tan(1/2*d*x)^5*tan(1/2*c)^10 + 20*a*tan(1/2*d*x)^10*
tan(1/2*c)^3 + 20*b*tan(1/2*d*x)^10*tan(1/2*c)^3 - 150*a*tan(1/2*d*x)^9*ta
n(1/2*c)^4 + 180*b*tan(1/2*d*x)^9*tan(1/2*c)^4 - 610*a*tan(1/2*d*x)^8*tan(
1/2*c)^5 + 1040*b*tan(1/2*d*x)^8*tan(1/2*c)^5 - 2200*a*tan(1/2*d*x)^7*tan(
1/2*c)^6 + 2360*b*tan(1/2*d*x)^7*tan(1/2*c)^6 - 2200*a*tan(1/2*d*x)^6*tan(
1/2*c)^7 + 2360*b*tan(1/2*d*x)^6*tan(1/2*c)^7 - 610*a*tan(1/2*d*x)^5*tan(1
/2*c)^8 + 1040*b*tan(1/2*d*x)^5*tan(1/2*c)^8 - 150*a*tan(1/2*d*x)^4*tan(1/
2*c)^9 + 180*b*tan(1/2*d*x)^4*tan(1/2*c)^9 + 20*a*tan(1/2*d*x)^3*tan(1/2*c
)^10 + 20*b*tan(1/2*d*x)^3*tan(1/2*c)^10 + 15*a*tan(1/2*d*x)^10*tan(1/2*c)
 + 75*a*tan(1/2*d*x)^9*tan(1/2*c)^2 - 60*b*tan(1/2*d*x)^9*tan(1/2*c)^2 ...
```

Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\frac{5\,a\,\sin(c+d\,x)}{8}+\frac{b\,\sin(c+d\,x)}{8}+\frac{5\,a\,\sin(3\,c+3\,d\,x)}{48}+\frac{a\,\sin(5\,c+5\,d\,x)}{80}-\frac{b\,\sin(3\,c+3\,d\,x)}{48}-\frac{b\,\sin(5\,c+5\,d\,x)}{80}}{d}$$

input
```
int(cos(c + d*x)^5*(a + b*tan(c + d*x)^2),x)
```

output
```
((5*a*sin(c + d*x))/8 + (b*sin(c + d*x))/8 + (5*a*sin(3*c + 3*d*x))/48 + (
a*sin(5*c + 5*d*x))/80 - (b*sin(3*c + 3*d*x))/48 - (b*sin(5*c + 5*d*x))/80
)/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\sin(dx+c)\left(3\sin(dx+c)^4 a-3\sin(dx+c)^4 b-10\sin(dx+c)^2 a+5\sin(dx+c)^2 b+15a\right)}{15d}$$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*(3*sin(c + d*x)**4*a - 3*sin(c + d*x)**4*b - 10*sin(c + d*x)
**2*a + 5*sin(c + d*x)**2*b + 15*a))/(15*d)
```

3.432 $\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{a\sin(c+dx)}{d}-\frac{(3a-b)\sin^3(c+dx)}{3d}+\frac{(3a-2b)\sin^5(c+dx)}{5d}-\frac{(a-b)\sin^7(c+dx)}{7d}$$

output
```
a*sin(d*x+c)/d-1/3*(3*a-b)*sin(d*x+c)^3/d+1/5*(3*a-2*b)*sin(d*x+c)^5/d-1/7
*(a-b)*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{(2286a+206b+(897a-113b)\cos(2(c+dx))+6(27a-13b)\cos(4(c+dx))+15a\cos(6(c+dx))-15}{3360d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
```

output
```
((2286*a + 206*b + (897*a - 113*b)*Cos[2*(c + d*x)] + 6*(27*a - 13*b)*Cos[
4*(c + d*x)] + 15*a*Cos[6*(c + d*x)] - 15*b*Cos[6*(c + d*x)])*Sin[c + d*x]
)/(3360*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\tan(c+dx)^2}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)^2\left(a-(a-b)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 290}$$

$$\frac{\int \left(-\left((a-b)\sin^6(c+dx)\right)+(3a-2b)\sin^4(c+dx)-(3a-b)\sin^2(c+dx)+a\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{7}(a-b)\sin^7(c+dx)+\frac{1}{5}(3a-2b)\sin^5(c+dx)-\frac{1}{3}(3a-b)\sin^3(c+dx)+a\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Sin[c + d*x] - ((3*a - b)*Sin[c + d*x]^3)/3 + ((3*a - 2*b)*Sin[c + d*x]
^5)/5 - ((a - b)*Sin[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 47.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{a\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}+b\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\mathrm{si}}{35}\right.}{d}$
default	$\frac{\frac{a\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}+b\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\mathrm{si}}{35}\right.}{d}$
risch	$\frac{35a\sin(dx+c)}{64d}+\frac{5\sin(dx+c)b}{64d}+\frac{\sin(7dx+7c)a}{448d}-\frac{\sin(7dx+7c)b}{448d}+\frac{7\sin(5dx+5c)a}{320d}-\frac{3\sin(5dx+5c)b}{320d}+\frac{7\sin}{}$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c
)+b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\left(15\,(a-b)\cos\left(dx+c\right)^6+3\,(6\,a+b)\cos\left(dx+c\right)^4+4\,(6\,a+b)\cos\left(dx+c\right)^2+48\,a+8\,b\right)\sin\left(dx+c\right)}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/105*(15*(a - b)*cos(d*x + c)^6 + 3*(6*a + b)*cos(d*x + c)^4 + 4*(6*a + b
)*cos(d*x + c)^2 + 48*a + 8*b)*sin(d*x + c)/d
```

Sympy [F]

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \int \left(a+b\tan^2\left(c+dx\right)\right)\cos^7\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = -\frac{15\,(a-b)\sin\left(dx+c\right)^7-21\,(3\,a-2\,b)\sin\left(dx+c\right)^5+35\,(3\,a-b)\sin\left(dx+c\right)^3-105\,a\sin\left(dx+c\right)}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/105*(15*(a - b)*sin(d*x + c)^7 - 21*(3*a - 2*b)*sin(d*x + c)^5 + 35*(3*
a - b)*sin(d*x + c)^3 - 105*a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29589 vs. 2(70) = 140.

Time = 21.67 (sec) , antiderivative size = 29589, normalized size of antiderivative = 389.33

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/6720*(12705*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^13
 + 147*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)*tan(1/2*c)^14 + 12705*a
*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^14 + 147*a*tan(5/2
*d*x)*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^14 + 34230*a*tan(5/2*d*x)^2*
tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^11 + 17920*b*tan(5/2*d*x)^2*tan(1/
2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^11 + 1029*a*tan(5/2*d*x)^2*tan(1/2*d*x)^
14*tan(5/2*c)*tan(1/2*c)^12 - 62475*a*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5
/2*c)^2*tan(1/2*c)^12 + 53760*b*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(5/2*c)^
2*tan(1/2*c)^12 + 1029*a*tan(5/2*d*x)*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2
*c)^12 + 12705*a*tan(5/2*d*x)^2*tan(1/2*d*x)^14*tan(1/2*c)^13 - 62475*a*ta
n(5/2*d*x)^2*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^13 + 53760*b*tan(5/2*
d*x)^2*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*c)^13 + 12705*a*tan(1/2*d*x)^1
4*tan(5/2*c)^2*tan(1/2*c)^13 + 12705*a*tan(5/2*d*x)^2*tan(1/2*d*x)^13*tan(
1/2*c)^14 - 147*a*tan(5/2*d*x)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 1029*a*tan(
5/2*d*x)^2*tan(1/2*d*x)^12*tan(5/2*c)*tan(1/2*c)^14 - 147*a*tan(1/2*d*x)^1
4*tan(5/2*c)*tan(1/2*c)^14 + 34230*a*tan(5/2*d*x)^2*tan(1/2*d*x)^11*tan(5/
2*c)^2*tan(1/2*c)^14 + 17920*b*tan(5/2*d*x)^2*tan(1/2*d*x)^11*tan(5/2*c)^2
*tan(1/2*c)^14 + 1029*a*tan(5/2*d*x)*tan(1/2*d*x)^12*tan(5/2*c)^2*tan(1/2*
c)^14 + 12705*a*tan(1/2*d*x)^13*tan(5/2*c)^2*tan(1/2*c)^14 + 113967*a*tan(
5/2*d*x)^2*tan(1/2*d*x)^14*tan(5/2*c)^2*tan(1/2*c)^9 - 14336*b*tan(5/2*...
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\frac{35\,a\,\sin(c+d\,x)}{64}+\frac{5\,b\,\sin(c+d\,x)}{64}+\frac{7\,a\,\sin(3\,c+3\,d\,x)}{64}+\frac{7\,a\,\sin(5\,c+5\,d\,x)}{320}+\frac{a\,\sin(7\,c+7\,d\,x)}{448}-\frac{b\,\sin(3\,c+3\,d\,x)}{192}-\frac{3\,b\,\sin(5\,c+5\,d\,x)}{320}-}{d}$$

input
```
int(cos(c + d*x)^7*(a + b*tan(c + d*x)^2),x)
```

output
```
((35*a*sin(c + d*x))/64 + (5*b*sin(c + d*x))/64 + (7*a*sin(3*c + 3*d*x))/6
4 + (7*a*sin(5*c + 5*d*x))/320 + (a*sin(7*c + 7*d*x))/448 - (b*sin(3*c + 3
*d*x))/192 - (3*b*sin(5*c + 5*d*x))/320 - (b*sin(7*c + 7*d*x))/448)/d
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right) dx$$
$$= \frac{\sin(dx+c)\left(-15\sin(dx+c)^6 a+15\sin(dx+c)^6 b+63\sin(dx+c)^4 a-42\sin(dx+c)^4 b-105\sin(d\right.}{105d}$$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*( - 15*sin(c + d*x)**6*a + 15*sin(c + d*x)**6*b + 63*sin(c +
 d*x)**4*a - 42*sin(c + d*x)**4*b - 105*sin(c + d*x)**2*a + 35*sin(c + d*x
)**2*b + 105*a))/(105*d)
```

3.433 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{a\tan(c+dx)}{d}+\frac{(2a+b)\tan^3(c+dx)}{3d}+\frac{(a+2b)\tan^5(c+dx)}{5d}+\frac{b\tan^7(c+dx)}{7d}$$

output
```
a*tan(d*x+c)/d+1/3*(2*a+b)*tan(d*x+c)^3/d+1/5*(a+2*b)*tan(d*x+c)^5/d+1/7*b
*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{\tan(c+dx)\left(105a-8b-4b\sec^2(c+dx)-3b\sec^4(c+dx)+15b\sec^6(c+dx)+70a\tan^2(c+dx)+21a\right.}{105d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

output
```
(Tan[c + d*x]*(105*a - 8*b - 4*b*Sec[c + d*x]^2 - 3*b*Sec[c + d*x]^4 + 15*
b*Sec[c + d*x]^6 + 70*a*Tan[c + d*x]^2 + 21*a*Tan[c + d*x]^4))/(105*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^6\left(a+b\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \left(\tan^2(c+dx)+1\right)^2\left(b\tan^2(c+dx)+a\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 290}$$

$$\frac{\int \left(b\tan^6(c+dx)+(a+2b)\tan^4(c+dx)+(2a+b)\tan^2(c+dx)+a\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{5}(a+2b)\tan^5(c+dx)+\frac{1}{3}(2a+b)\tan^3(c+dx)+a\tan(c+dx)+\frac{1}{7}b\tan^7(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Tan[c + d*x] + ((2*a + b)*Tan[c + d*x]^3)/3 + ((a + 2*b)*Tan[c + d*x]^5
)/5 + (b*Tan[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 11.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{b\tan(dx+c)^7}{7}+\frac{(a+2b)\tan(dx+c)^5}{5}+\frac{(2a+b)\tan(dx+c)^3}{3}+a\tan(dx+c)}{d}$
default	$\frac{\frac{b\tan(dx+c)^7}{7}+\frac{(a+2b)\tan(dx+c)^5}{5}+\frac{(2a+b)\tan(dx+c)^3}{3}+a\tan(dx+c)}{d}$
risch	$\frac{16i(70a\,e^{8i(dx+c)}-70b\,e^{8i(dx+c)}+175a\,e^{6i(dx+c)}+35b\,e^{6i(dx+c)}+147a\,e^{4i(dx+c)}-21b\,e^{4i(dx+c)}+49a\,e^{2i(dx+c)}-7b\,e^{2i}}{105d\left(e^{2i(dx+c)}+1\right)^7}$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*b*tan(d*x+c)^7+1/5*(a+2*b)*tan(d*x+c)^5+1/3*(2*a+b)*tan(d*x+c)^3+
a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\left(8\,(7\,a-b)\cos\left(dx+c\right)^6+4\,(7\,a-b)\cos\left(dx+c\right)^4+3\,(7\,a-b)\cos\left(dx+c\right)^2+15\,b\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/105*(8*(7*a - b)*cos(d*x + c)^6 + 4*(7*a - b)*cos(d*x + c)^4 + 3*(7*a -
b)*cos(d*x + c)^2 + 15*b)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

Sympy [F]

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \int \left(a+b\tan^2\left(c+dx\right)\right)\sec^6\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{15\,b\tan\left(dx+c\right)^7+21\,(a+2\,b)\tan\left(dx+c\right)^5+35\,(2\,a+b)\tan\left(dx+c\right)^3+105\,a\tan\left(dx+c\right)}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/105*(15*b*tan(d*x + c)^7 + 21*(a + 2*b)*tan(d*x + c)^5 + 35*(2*a + b)*ta
n(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{15\,b\tan\left(dx+c\right)^7+21\,a\tan\left(dx+c\right)^5+42\,b\tan\left(dx+c\right)^5+70\,a\tan\left(dx+c\right)^3+35\,b\tan\left(dx+c\right)^3+10}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/105*(15*b*tan(d*x + c)^7 + 21*a*tan(d*x + c)^5 + 42*b*tan(d*x + c)^5 + 7
0*a*tan(d*x + c)^3 + 35*b*tan(d*x + c)^3 + 105*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 7.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{\frac{b\tan(c+d\,x)^7}{7}+\left(\frac{a}{5}+\frac{2\,b}{5}\right)\,\tan(c+d\,x)^5+\left(\frac{2\,a}{3}+\frac{b}{3}\right)\,\tan(c+d\,x)^3+a\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x)^2)/cos(c + d*x)^6,x)
```

output
```
(tan(c + d*x)^3*((2*a)/3 + b/3) + tan(c + d*x)^5*(a/5 + (2*b)/5) + a*tan(c
 + d*x) + (b*tan(c + d*x)^7)/7)/d
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.78

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$= \frac{\sin(dx+c)\,(56\sin(dx+c)^6 a - 8\sin(dx+c)^6 b - 196\sin(dx+c)^4 a + 28\sin(dx+c)^4 b + 245\sin(dx}{105\cos(dx+c)\,d\,(\sin(dx+c)^6 - 3\sin(dx+c)^4 + 3\sin(dx+c)^2 - 1}$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*(56*sin(c + d*x)**6*a - 8*sin(c + d*x)**6*b - 196*sin(c + d*
x)**4*a + 28*sin(c + d*x)**4*b + 245*sin(c + d*x)**2*a - 35*sin(c + d*x)**
2*b - 105*a))/(105*cos(c + d*x)*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3
*sin(c + d*x)**2 - 1))
```

3.434 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{a\tan(c+dx)}{d}+\frac{(a+b)\tan^3(c+dx)}{3d}+\frac{b\tan^5(c+dx)}{5d}$$

output
```
a*tan(d*x+c)/d+1/3*(a+b)*tan(d*x+c)^3/d+1/5*b*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{\tan(c+dx)\left(15a-2b-b\sec^2(c+dx)+3b\sec^4(c+dx)+5a\tan^2(c+dx)\right)}{15d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

output
```
(Tan[c + d*x]*(15*a - 2*b - b*Sec[c + d*x]^2 + 3*b*Sec[c + d*x]^4 + 5*a*Ta
n[c + d*x]^2))/(15*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4\left(a+b\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(b\tan^4(c+dx)+(a+b)\tan^2(c+dx)+a\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}(a+b)\tan^3(c+dx)+a\tan(c+dx)+\frac{1}{5}b\tan^5(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Tan[c + d*x] + ((a + b)*Tan[c + d*x]^3)/3 + (b*Tan[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{b\tan(dx+c)^5}{5}+\frac{(a+b)\tan(dx+c)^3}{3}+a\tan(dx+c)}{d}$	38
default	$\frac{\frac{b\tan(dx+c)^5}{5}+\frac{(a+b)\tan(dx+c)^3}{3}+a\tan(dx+c)}{d}$	38
risch	$\frac{4i\left(15a\,\mathrm{e}^{6i(dx+c)}-15b\,\mathrm{e}^{6i(dx+c)}+35a\,\mathrm{e}^{4i(dx+c)}+5b\,\mathrm{e}^{4i(dx+c)}+25a\,\mathrm{e}^{2i(dx+c)}-5b\,\mathrm{e}^{2i(dx+c)}+5a-b\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$	99

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b*tan(d*x+c)^5+1/3*(a+b)*tan(d*x+c)^3+a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \sec^4(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\left(2\left(5a-b\right)\cos\left(dx+c\right)^4+\left(5a-b\right)\cos\left(dx+c\right)^2+3b\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*a - b)*cos(d*x + c)^4 + (5*a - b)*cos(d*x + c)^2 + 3*b)*sin(d*x
 + c)/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^4(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \int \left(a+b\tan^2\left(c+dx\right)\right)\sec^4\left(c+dx\right) dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \sec^4(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{3\,b\tan\left(dx+c\right)^5+5\left(a+b\right)\tan\left(dx+c\right)^3+15\,a\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/15*(3*b*tan(d*x + c)^5 + 5*(a + b)*tan(d*x + c)^3 + 15*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{3\,b\tan\left(dx+c\right)^5+5\,a\tan\left(dx+c\right)^3+5\,b\tan\left(dx+c\right)^3+15\,a\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/15*(3*b*tan(d*x + c)^5 + 5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 15*a*
tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 7.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{\frac{b\tan(c+d\,x)^5}{5}+\left(\frac{a}{3}+\frac{b}{3}\right)\,\tan(c+d\,x)^3+a\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x)^2)/cos(c + d*x)^4,x)
```

output
```
(tan(c + d*x)^3*(a/3 + b/3) + a*tan(c + d*x) + (b*tan(c + d*x)^5)/5)/d
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{\sin(dx+c)\left(10\sin(dx+c)^4a-2\sin(dx+c)^4b-25\sin(dx+c)^2a+5\sin(dx+c)^2b+15a\right)}{15\cos(dx+c)\,d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*(10*sin(c + d*x)**4*a - 2*sin(c + d*x)**4*b - 25*sin(c + d*x
)**2*a + 5*sin(c + d*x)**2*b + 15*a))/(15*cos(c + d*x)*d*(sin(c + d*x)**4
- 2*sin(c + d*x)**2 + 1))
```

3.435 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

output
```
a*tan(d*x+c)/d+1/3*b*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{a\tan(c+dx)}{d}+\frac{b\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Tan[c + d*x])/d + (b*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2\left(a+b\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(b\tan^2(c+dx)+a\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\tan(c+dx)+\frac{1}{3}b\tan^3(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

output
```
(a*Tan[c + d*x] + (b*Tan[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\tan(dx+c)^3}{3}+a\tan(dx+c)}{d}$	25
default	$\frac{\frac{b\tan(dx+c)^3}{3}+a\tan(dx+c)}{d}$	25
risch	$-\frac{2i\left(-3a\,\mathrm{e}^{4i(dx+c)}+3b\,\mathrm{e}^{4i(dx+c)}-6a\,\mathrm{e}^{2i(dx+c)}-3a+b\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	61

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b*tan(d*x+c)^3+a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{\left((3\,a-b)\cos\left(dx+c\right)^2+b\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/3*((3*a - b)*cos(d*x + c)^2 + b)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\begin{cases}\frac{a\tan(c+dx)+\frac{b\tan^3(c+dx)}{3}}{d} & \text{for } d\neq 0\\ x(a+b\tan^2(c))\sec^2(c) & \text{otherwise}\end{cases}$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
```

output
```
Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**3/3)/d, Ne(d, 0)), (x*(a + b*
tan(c)**2)*sec(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{b\tan(dx+c)^3+3\,a\tan(dx+c)}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{b\tan(dx+c)^3+3\,a\tan(dx+c)}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{\tan(c+d\,x)\,\left(b\tan(c+d\,x)^2+3\,a\right)}{3\,d}$$

input
```
int((a + b*tan(c + d*x)^2)/cos(c + d*x)^2,x)
```

output
```
(tan(c + d*x)*(3*a + b*tan(c + d*x)^2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.04

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{\sin(dx+c)\left(3\sin(dx+c)^2a-\sin(dx+c)^2b-3a\right)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*(3*sin(c + d*x)**2*a - sin(c + d*x)**2*b - 3*a))/(3*cos(c +
d*x)*d*(sin(c + d*x)**2 - 1))
```

3.436 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{1}{2}(a+b)x+\frac{(a-b)\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*(a+b)*x+1/2*(a-b)*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{2(a+b)(c+dx)+(a-b)\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

output
```
(2*(a + b)*(c + d*x) + (a - b)*Sin[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4158, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)^2}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{b\tan^2(c+dx)+a}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{(a-b)\tan(c+dx)}{2(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}(a+b)\arctan(\tan(c+dx))+\frac{(a-b)\tan(c+dx)}{2(\tan^2(c+dx)+1)}}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2),x]
```

output
```
(((a + b)*ArcTan[Tan[c + d*x]])/2 + ((a - b)*Tan[c + d*x])/(2*(1 + Tan[c +
 d*x]^2)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{ax}{2}+\frac{bx}{2}+\frac{\sin(2dx+2c)a}{4d}-\frac{\sin(2dx+2c)b}{4d}$	40
derivativedivides	$\frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
default	$\frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+1/2*b*x+1/4/d*sin(2*d*x+2*c)*a-1/4/d*sin(2*d*x+2*c)*b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{(a+b)dx+(a-b)\cos(dx+c)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a + b)*d*x + (a - b)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\int \left(a+b\tan^2(c+dx)\right)\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{(dx+c)(a+b)+\frac{(a-b)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)*(a + b) + (a - b)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{(dx+c)(a+b)}{2\,d} + \frac{a\tan\left(dx+c\right) - b\tan\left(dx+c\right)}{2\left(\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(d*x + c)*(a + b)/d + 1/2*(a*tan(d*x + c) - b*tan(d*x + c))/((tan(d*x
+ c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\sin\left(2\,c+2\,d\,x\right)\,\left(\frac{a}{4}-\frac{b}{4}\right)+d\,x\,\left(\frac{a}{2}+\frac{b}{2}\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x)^2),x)
```

output
```
(sin(2*c + 2*d*x)*(a/4 - b/4) + d*x*(a/2 + b/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{\cos\left(dx+c\right)\sin\left(dx+c\right)a - \cos\left(dx+c\right)\sin\left(dx+c\right)b + adx + bdx}{2d}$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2),x)
```

output
```
(cos(c + d*x)*sin(c + d*x)*a - cos(c + d*x)*sin(c + d*x)*b + a*d*x + b*d*x
)/(2*d)
```

3.437 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{1}{8}(3a+b)x+\frac{(3a+b)\cos(c+dx)\sin(c+dx)}{8d}+\frac{(a-b)\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
1/8*(3*a+b)*x+1/8*(3*a+b)*cos(d*x+c)*sin(d*x+c)/d+1/4*(a-b)*cos(d*x+c)^3*s
in(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{4bdx+12a(c+dx)+8a\sin(2(c+dx))+(a-b)\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

output
```
(4*b*d*x + 12*a*(c + d*x) + 8*a*Sin[2*(c + d*x)] + (a - b)*Sin[4*(c + d*x)
])/(32*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4158, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)^2}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{b\tan^2(c+dx)+a}{\left(\tan^2(c+dx)+1\right)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}(3a+b)\int \frac{1}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)+\frac{(a-b)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}(3a+b)\left(\frac{1}{2}\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)+\frac{(a-b)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}(3a+b)\left(\frac{1}{2}\arctan(\tan(c+dx))+\frac{\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)+\frac{(a-b)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2),x]
```

output
```
(((a - b)*Tan[c + d*x])/(4*(1 + Tan[c + d*x]^2)^2) + ((3*a + b)*(ArcTan[Ta
n[c + d*x]]/2 + Tan[c + d*x]/(2*(1 + Tan[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 7.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{3ax}{8}+\frac{bx}{8}+\frac{\sin(4dx+4c)a}{32d}-\frac{\sin(4dx+4c)b}{32d}+\frac{\sin(2dx+2c)a}{4d}$	55
derivativedivides	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\cos(dx+c)^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$	81
default	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\cos(dx+c)^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$	81

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
3/8*a*x+1/8*b*x+1/32/d*sin(4*d*x+4*c)*a-1/32/d*sin(4*d*x+4*c)*b+1/4/d*sin(
2*d*x+2*c)*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{(3\,a+b)dx+\left(2\,(a-b)\cos\left(dx+c\right)^3+(3\,a+b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*((3*a + b)*d*x + (2*(a - b)*cos(d*x + c)^3 + (3*a + b)*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\int \left(a+b\tan^2{(c+dx)}\right)\cos^4{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{(dx+c)(3\,a+b)+\frac{(3\,a+b)\tan(dx+c)^3+(5\,a-b)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*((d*x + c)*(3*a + b) + ((3*a + b)*tan(d*x + c)^3 + (5*a - b)*tan(d*x +
 c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{(dx+c)(3\,a+b)}{8\,d}+\frac{3\,a\tan{(dx+c)}^3+b\tan{(dx+c)}^3+5\,a\tan{(dx+c)}-b\tan{(dx+c)}}{8\left(\tan{(dx+c)}^2+1\right)^2d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(d*x + c)*(3*a + b)/d + 1/8*(3*a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + 5
*a*tan(d*x + c) - b*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \cos^4(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = x\left(\frac{3a}{8}+\frac{b}{8}\right)+\frac{\left(\frac{3a}{8}+\frac{b}{8}\right)\tan(c+d\,x)^3+\left(\frac{5a}{8}-\frac{b}{8}\right)\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x)^2),x)
```

output
```
x*((3*a)/8 + b/8) + (tan(c + d*x)^3*((3*a)/8 + b/8) + tan(c + d*x)*((5*a)/
8 - b/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int \cos^4(c+dx) \left(a+b\tan^2(c+dx)\right) \, dx = \frac{-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3 a+2\cos\left(dx+c\right)\sin\left(dx+c\right)^3 b+5\cos\left(dx+c\right)\sin\left(dx+c\right)a-\cos\left(dx+\right.}{8d}$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*a + 2*cos(c + d*x)*sin(c + d*x)**3*b +
5*cos(c + d*x)*sin(c + d*x)*a - cos(c + d*x)*sin(c + d*x)*b + 3*a*d*x + b*
d*x)/(8*d)
```

3.438 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{1}{16}(5a+b)x+\frac{(5a+b)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(5a+b)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{(a-b)\cos^5(c+dx)\sin(c+dx)}{6d}$$

output
```
1/16*(5*a+b)*x+1/16*(5*a+b)*cos(d*x+c)*sin(d*x+c)/d+1/24*(5*a+b)*cos(d*x+c
)^3*sin(d*x+c)/d+1/6*(a-b)*cos(d*x+c)^5*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{60ac+60adx+12bdx+3(15a+b)\sin(2(c+dx))+(9a-3b)\sin(4(c+dx))+a\sin(6(c+dx))-b\sin}{192d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

output
```
(60*a*c + 60*a*d*x + 12*b*d*x + 3*(15*a + b)*Sin[2*(c + d*x)] + (9*a - 3*b
)*Sin[4*(c + d*x)] + a*Sin[6*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4158, 298, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)^2}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{b\tan^2(c+dx)+a}{(\tan^2(c+dx)+1)^4}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}(5a+b)\int \frac{1}{(\tan^2(c+dx)+1)^3}d\tan(c+dx)+\frac{(a-b)\tan(c+dx)}{6(\tan^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(5a+b)\left(\frac{3}{4}\int \frac{1}{(\tan^2(c+dx)+1)^2}d\tan(c+dx)+\frac{\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}\right)+\frac{(a-b)\tan(c+dx)}{6(\tan^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}(5a+b)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{\tan(c+dx)}{2(\tan^2(c+dx)+1)}\right)+\frac{\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}\right)+\frac{(a-b)\tan(c+dx)}{6(\tan^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{6}(5a+b)\left(\frac{3}{4}\left(\frac{1}{2}\arctan(\tan(c+dx))+\frac{\tan(c+dx)}{2(\tan^2(c+dx)+1)}\right)+\frac{\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}\right)+\frac{(a-b)\tan(c+dx)}{6(\tan^2(c+dx)+1)^3}}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2),x]
```

output
```
(((a - b)*Tan[c + d*x])/(6*(1 + Tan[c + d*x]^2)^3) + ((5*a + b)*(Tan[c + d
*x]/(4*(1 + Tan[c + d*x]^2)^2) + (3*(ArcTan[Tan[c + d*x]]/2 + Tan[c + d*x]
/(2*(1 + Tan[c + d*x]^2))))/4))/6)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 26.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

method	result
risch	$\frac{5ax}{16}+\frac{bx}{16}+\frac{\sin(6dx+6c)a}{192d}-\frac{\sin(6dx+6c)b}{192d}+\frac{3\sin(4dx+4c)a}{64d}-\frac{\sin(4dx+4c)b}{64d}+\frac{15\sin(2dx+2c)a}{64d}+\frac{\sin(2c}{6}$
derivativedivides	$\frac{b\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c}{8}\right.}{6}\right.}{d}$
default	$\frac{b\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c}{8}\right.}{6}\right.}{d}$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
5/16*a*x+1/16*b*x+1/192/d*sin(6*d*x+6*c)*a-1/192/d*sin(6*d*x+6*c)*b+3/64/d
*sin(4*d*x+4*c)*a-1/64/d*sin(4*d*x+4*c)*b+15/64/d*sin(2*d*x+2*c)*a+1/64/d*
sin(2*d*x+2*c)*b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$=\frac{3\,(5\,a+b)dx+\left(8\,(a-b)\cos\left(dx+c\right)^5+2\,(5\,a+b)\cos\left(dx+c\right)^3+3\,(5\,a+b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/48*(3*(5*a + b)*d*x + (8*(a - b)*cos(d*x + c)^5 + 2*(5*a + b)*cos(d*x +
c)^3 + 3*(5*a + b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\int \left(a+b\tan^2\left(c+dx\right)\right)\cos^6\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2),x)
```

output
```
Integral((a + b*tan(c + d*x)**2)*cos(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx = \frac{3\,(dx+c)(5\,a+b)+\frac{3\,(5\,a+b)\tan(dx+c)^5+8\,(5\,a+b)\tan(dx+c)^3+3\,(11\,a-b)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/48*(3*(d*x + c)*(5*a + b) + (3*(5*a + b)*tan(d*x + c)^5 + 8*(5*a + b)*ta
n(d*x + c)^3 + 3*(11*a - b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)
^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx=\frac{(dx+c)(5\,a+b)}{16\,d}+\frac{15\,a\tan\left(dx+c\right)^5+3\,b\tan\left(dx+c\right)^5+40\,a\tan\left(dx+c\right)^3+8\,b\tan\left(dx+c\right)^3+33\,a\tan\left(dx+c\right)-3\,b}{48\left(\tan\left(dx+c\right)^2+1\right)^3d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*(d*x + c)*(5*a + b)/d + 1/48*(15*a*tan(d*x + c)^5 + 3*b*tan(d*x + c)^
5 + 40*a*tan(d*x + c)^3 + 8*b*tan(d*x + c)^3 + 33*a*tan(d*x + c) - 3*b*tan
(d*x + c))/((tan(d*x + c)^2 + 1)^3*d)
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$= x\left(\frac{5a}{16}+\frac{b}{16}\right) + \frac{\left(\frac{5a}{16}+\frac{b}{16}\right)\tan(c+dx)^5+\left(\frac{5a}{6}+\frac{b}{6}\right)\tan(c+dx)^3+\left(\frac{11a}{16}-\frac{b}{16}\right)\tan(c+dx)}{d\left(\tan(c+dx)^6+3\tan(c+dx)^4+3\tan(c+dx)^2+1\right)}$$

input
```
int(cos(c + d*x)^6*(a + b*tan(c + d*x)^2),x)
```

output
```
x*((5*a)/16 + b/16) + (tan(c + d*x)^3*((5*a)/6 + b/6) + tan(c + d*x)^5*((5
*a)/16 + b/16) + tan(c + d*x)*((11*a)/16 - b/16))/(d*(3*tan(c + d*x)^2 + 3
*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)\,dx$$
$$= \frac{8\cos(dx+c)\sin(dx+c)^5a-8\cos(dx+c)\sin(dx+c)^5b-26\cos(dx+c)\sin(dx+c)^3a+14\cos(dx}{48d}$$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x)
```

output
```
(8*cos(c + d*x)*sin(c + d*x)**5*a - 8*cos(c + d*x)*sin(c + d*x)**5*b - 26*
cos(c + d*x)*sin(c + d*x)**3*a + 14*cos(c + d*x)*sin(c + d*x)**3*b + 33*co
s(c + d*x)*sin(c + d*x)*a - 3*cos(c + d*x)*sin(c + d*x)*b + 15*a*d*x + 3*b
*d*x)/(48*d)
```

3.439 $\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{(8a^2-4ab+b^2)\,\text{arctanh}(\sin(c+dx))}{16d}+\frac{(8a^2-4ab+b^2)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(12a-7b)b\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}$$

output
```
1/16*(8*a^2-4*a*b+b^2)*arctanh(sin(d*x+c))/d+1/16*(8*a^2-4*a*b+b^2)*sec(d*
x+c)*tan(d*x+c)/d+1/24*(12*a-7*b)*b*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b^2*sec(
d*x+c)^5*tan(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.19 (sec) , antiderivative size = 875, normalized size of antiderivative = 7.74

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx = \text{Too large to display}$$

input

```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

output

```
(Sin[c + d*x]*(65625*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]] - 36855*a^2*ArcTanh
[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^2 - 91875*a*(a - b)*ArcTanh[Sqrt[Sin[c
 + d*x]^2]]*Sin[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c
+ d*x]^4 + 54180*a*(a - b)*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 +
32970*(a - b)^2*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^4 - 1365*a*(a -
 b)*ArcTanh[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^6 - 19845*(a - b)^2*ArcTanh
[Sqrt[Sin[c + d*x]^2]]*Sin[c + d*x]^6 + 525*(a - b)^2*ArcTanh[Sqrt[Sin[c +
 d*x]^2]]*Sin[c + d*x]^8 - 65625*a^2*Sqrt[Sin[c + d*x]^2] - 23555*a*(a - b
)*Sin[c + d*x]^4*Sqrt[Sin[c + d*x]^2] - 32970*(a - b)^2*Sin[c + d*x]^4*Sqr
t[Sin[c + d*x]^2] + 8855*(a - b)^2*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 6
20*a^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[
c + d*x]^6*Sqrt[Sin[c + d*x]^2] + 160*a^2*HypergeometricPFQ[{3/2, 2, 2, 2,
 2}, {1, 1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] +
 16*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c +
 d*x]^2]*Sin[c + d*x]^6*Sqrt[Sin[c + d*x]^2] - 968*a*(a - b)*Hypergeometri
cPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqrt[Sin[
c + d*x]^2] - 288*a*(a - b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1,
 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] - 32*a*(a - b)*
HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, Sin[c + d*x]^2]
*Sin[c + d*x]^8*Sqrt[Sin[c + d*x]^2] + 380*(a - b)^2*HypergeometricPFQ[...
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4159, 315, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^3\left(a+b\tan(c+dx)^2\right)^2dx$$

$$\Big\downarrow \ 4159$$

$$\frac{\int \frac{\left(a-(a-b)\sin^2(c+dx)\right)^2}{\left(1-\sin^2(c+dx)\right)^4}d\sin(c+dx)}{d}$$

$$\Big\downarrow \ 315$$

$$\frac{\frac{b\sin(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)}{6\left(1-\sin^2(c+dx)\right)^3}-\frac{1}{6}\int -\frac{a(6a-b)-3(a-b)(2a-b)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{6}\int \frac{a(6a-b)-3(a-b)(2a-b)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)+\frac{b\sin(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)}{6\left(1-\sin^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2-4ab+b^2\right)\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)+\frac{b(8a-3b)\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)+\frac{b\sin(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)}{6\left(1-\sin^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2-4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{b(8a-3b)\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)+\frac{b\sin(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)}{6\left(1-\sin^2(c+dx)\right)^3}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}(8a^2-4ab+b^2)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{b(8a-3b)\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b\sin(c+dx)(a-(a-b)\sin^2(c+dx))}{6(1-\sin^2(c+dx))^3}}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((b*Sin[c + d*x]*(a - (a - b)*Sin[c + d*x]^2))/(6*(1 - Sin[c + d*x]^2)^3)
+ (((8*a - 3*b)*b*Sin[c + d*x])/(4*(1 - Sin[c + d*x]^2)^2) + (3*(8*a^2 - 4
*a*b + b^2)*(ArcTanh[Sin[c + d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c + d*x]^2
))))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+2ab\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+2ab\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}\right.}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(24a^2 e^{10i(dx+c)}-12ab\, e^{10i(dx+c)}+3b^2 e^{10i(dx+c)}+72a^2 e^{8i(dx+c)}+60ab\, e^{8i(dx+c)}-47b^2 e^{8i(dx+c)}+48a^2 e^{6i}\right.}{}$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/6*sin(d*x+c)^5/cos(d*x+c)^6+1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/4
8*sin(d*x+c)^5/cos(d*x+c)^2-1/48*sin(d*x+c)^3-1/16*sin(d*x+c)+1/16*ln(sec(
d*x+c)+tan(d*x+c)))+2*a*b*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/
cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(1/2*sec(d*
x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \frac{3\left(8a^2-4ab+b^2\right)\cos(dx+c)^6\log(\sin(dx+c)+1)-3\left(8a^2-4ab+b^2\right)\cos(dx+c)^6\log(-\sin(dx+c)$$
$$96\,d\cos(dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/96*(3*(8*a^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(8*
a^2 - 4*a*b + b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(8*a^2 - 4
*a*b + b^2)*cos(d*x + c)^4 + 2*(12*a*b - 7*b^2)*cos(d*x + c)^2 + 8*b^2)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \sec^3(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \int \left(a+b\tan^2(c+dx)\right)^2 \sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \frac{3\left(8a^2-4ab+b^2\right)\log(\sin(dx+c)+1)-3\left(8a^2-4ab+b^2\right)\log(\sin(dx+c)-1)-\frac{2\left(3\left(8a^2-4ab+b^2\right)\sin\right.}{\mathrm{si}}}{96\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/96*(3*(8*a^2 - 4*a*b + b^2)*log(sin(d*x + c) + 1) - 3*(8*a^2 - 4*a*b + b
^2)*log(sin(d*x + c) - 1) - 2*(3*(8*a^2 - 4*a*b + b^2)*sin(d*x + c)^5 - 8*
(6*a^2 - b^2)*sin(d*x + c)^3 + 3*(8*a^2 + 4*a*b - b^2)*sin(d*x + c))/(sin(
d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{3\left(8a^2-4ab+b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(8a^2-4ab+b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(24a^2\sin(dx+c)^5\right.}{}}{96\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/96*(3*(8*a^2 - 4*a*b + b^2)*log(abs(sin(d*x + c) + 1)) - 3*(8*a^2 - 4*a*
b + b^2)*log(abs(sin(d*x + c) - 1)) - 2*(24*a^2*sin(d*x + c)^5 - 12*a*b*si
n(d*x + c)^5 + 3*b^2*sin(d*x + c)^5 - 48*a^2*sin(d*x + c)^3 + 8*b^2*sin(d*
x + c)^3 + 24*a^2*sin(d*x + c) + 12*a*b*sin(d*x + c) - 3*b^2*sin(d*x + c))
/(sin(d*x + c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.38

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\left(a^2+\frac{a\,b}{2}-\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(-3\,a^2+\frac{5\,a\,b}{2}+\frac{17\,b^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(2\,a^2-3\,a\,b+\frac{19\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.} + \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{a\,b}{2}+\frac{b^2}{8}\right)}{d}$$

input
```
int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(2*a^2 - 3*a*b + (19*b^2)/4) + tan(c/2 + (d*x)/2)^7*
(2*a^2 - 3*a*b + (19*b^2)/4) + tan(c/2 + (d*x)/2)^3*((5*a*b)/2 - 3*a^2 + (
17*b^2)/24) + tan(c/2 + (d*x)/2)^9*((5*a*b)/2 - 3*a^2 + (17*b^2)/24) + tan
(c/2 + (d*x)/2)*((a*b)/2 + a^2 - b^2/8) + tan(c/2 + (d*x)/2)^11*((a*b)/2 +
 a^2 - b^2/8))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*t
an(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 +
tan(c/2 + (d*x)/2)^12 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(a^2 - (a*b)/2 +
b^2/8))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 678, normalized size of antiderivative = 6.00

$$\int \sec^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x)
```

output

```
( - 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a**2 + 12*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)**6*a*b - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*
*6*b**2 + 72*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2 - 36*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**4*a*b + 9*log(tan((c + d*x)/2) - 1)*sin(c +
 d*x)**4*b**2 - 72*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2 + 36*log
(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b - 9*log(tan((c + d*x)/2) - 1)*s
in(c + d*x)**2*b**2 + 24*log(tan((c + d*x)/2) - 1)*a**2 - 12*log(tan((c +
d*x)/2) - 1)*a*b + 3*log(tan((c + d*x)/2) - 1)*b**2 + 24*log(tan((c + d*x)
/2) + 1)*sin(c + d*x)**6*a**2 - 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*
*6*a*b + 3*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**6*b**2 - 72*log(tan((c
+ d*x)/2) + 1)*sin(c + d*x)**4*a**2 + 36*log(tan((c + d*x)/2) + 1)*sin(c +
 d*x)**4*a*b - 9*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*b**2 + 72*log(t
an((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2 - 36*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*a*b + 9*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 - 24
*log(tan((c + d*x)/2) + 1)*a**2 + 12*log(tan((c + d*x)/2) + 1)*a*b - 3*log
(tan((c + d*x)/2) + 1)*b**2 - 24*sin(c + d*x)**5*a**2 + 12*sin(c + d*x)**5
*a*b - 3*sin(c + d*x)**5*b**2 + 48*sin(c + d*x)**3*a**2 - 8*sin(c + d*x)**
3*b**2 - 24*sin(c + d*x)*a**2 - 12*sin(c + d*x)*a*b + 3*sin(c + d*x)*b**2)
/(48*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.440 $\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{(8a^2-8ab+3b^2)\,\text{arctanh}(\sin(c+dx))}{8d}+\frac{(8a-5b)b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
1/8*(8*a^2-8*a*b+3*b^2)*arctanh(sin(d*x+c))/d+1/8*(8*a-5*b)*b*sec(d*x+c)*t
an(d*x+c)/d+1/4*b^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.63 (sec) , antiderivative size = 347, normalized size of antiderivative = 4.28

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$

$$=\frac{\csc^3(c+dx)\left(128\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{9}{2};\sin^2(c+dx)\right)\sin^6(c+dx)\left(a+(-a+b)\sin^2(c+dx)\right)^2+12\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(Csc[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2},
Sin[c + d*x]^2]*Sin[c + d*x]^6*(a + (-a + b)*Sin[c + d*x]^2)^2 + 128*Hyper
geometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, Sin[c + d*x]^2]*Sin[c + d*x]^6*(
(a^2*Cos[c + d*x]^2*(9 + 5*Cos[2*(c + d*x)]))/2 + b*Sin[c + d*x]^2*(7*a +
5*a*Cos[2*(c + d*x)] + 5*b*Sin[c + d*x]^2)) + 35*(-3375*a^2 + 3*a*(1969*a
- 1750*b)*Sin[c + d*x]^2 + (-3161*a^2 + 5108*a*b - 1947*b^2)*Sin[c + d*x]^
4 + 485*(a - b)^2*Sin[c + d*x]^6 + (3*ArcTanh[Sqrt[Sin[c + d*x]^2]]*(1125*
a^2 - 2*a*(1172*a - 875*b)*Sin[c + d*x]^2 + (1674*a^2 - 2286*a*b + 649*b^2
)*Sin[c + d*x]^4 + (-400*a^2 + 778*a*b - 378*b^2)*Sin[c + d*x]^6 + 9*(a -
b)^2*Sin[c + d*x]^8))/Sqrt[Sin[c + d*x]^2]))/(6720*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4159, 315, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)\left(a+b\tan(c+dx)^2\right)^2dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(a-(a-b)\sin^2(c+dx)\right)^2}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{b\sin(c+dx)\left(a-(a-b)\sin^2(c+dx)\right)}{4\left(1-\sin^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{a(4a-b)-(4a-3b)(a-b)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\int \frac{a(4a-b)-(4a-3b)(a-b)\sin^2(c+dx)}{(1-\sin^2(c+dx))^2}d\sin(c+dx)+\frac{b\sin(c+dx)(a-(a-b)\sin^2(c+dx))}{4(1-\sin^2(c+dx))^2}}{d}$$

$\Big\downarrow$ 298

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8a^2-8ab+3b^2)\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{3b(2a-b)\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{b\sin(c+dx)(a-(a-b)\sin^2(c+dx))}{4(1-\sin^2(c+dx))^2}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8a^2-8ab+3b^2)\operatorname{arctanh}(\sin(c+dx))+\frac{3b(2a-b)\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{b\sin(c+dx)(a-(a-b)\sin^2(c+dx))}{4(1-\sin^2(c+dx))^2}}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((b*Sin[c + d*x]*(a - (a - b)*Sin[c + d*x]^2))/(4*(1 - Sin[c + d*x]^2)^2)
+ (((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sin[c + d*x]])/2 + (3*(2*a - b)*b*Sin[
c + d*x])/(2*(1 - Sin[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\right.}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\right.}{d}$
risch	$\frac{ib\,e^{i(dx+c)}\left(-8a\,e^{6i(dx+c)}+5b\,e^{6i(dx+c)}-8a\,e^{4i(dx+c)}-3b\,e^{4i(dx+c)}+8a\,e^{2i(dx+c)}+3b\,e^{2i(dx+c)}+8a-5b\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}+\frac{\ln(e^{i(dx+}}{}$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*
sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(1/2*sin(
d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*ln
(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \sec(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx$$
$$= \frac{(8a^2-8ab+3b^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-(8a^2-8ab+3b^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left((8ab-5b^2)\cos(dx+c)^2+2b^2\right)\sin(dx+c)}{16\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/16*((8*a^2 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^
2 - 8*a*b + 3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*((8*a*b - 5*b
^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \int \left(a+b\tan^2(c+dx)\right)^2 \sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int \sec(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx$$
$$= \frac{(8a^2-8ab+3b^2)\log(\sin(dx+c)+1)-(8a^2-8ab+3b^2)\log(\sin(dx+c)-1)-\frac{2\left((8ab-5b^2)\sin(dx+c)^3 \ldots\right)}{\sin(dx+c)^4-\ldots}}{16\,d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/16*((8*a^2 - 8*a*b + 3*b^2)*log(sin(d*x + c) + 1) - (8*a^2 - 8*a*b + 3*b
^2)*log(sin(d*x + c) - 1) - 2*((8*a*b - 5*b^2)*sin(d*x + c)^3 - (8*a*b - 3
*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{(8a^2-8ab+3b^2)\log(|\sin(dx+c)+1|)-(8a^2-8ab+3b^2)\log(|\sin(dx+c)-1|)-\frac{2\left(8ab\sin(dx+c)^3-\right.}{}}{16d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/16*((8*a^2 - 8*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1)) - (8*a^2 - 8*a*b
+ 3*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(8*a*b*sin(d*x + c)^3 - 5*b^2*sin(
d*x + c)^3 - 8*a*b*sin(d*x + c) + 3*b^2*sin(d*x + c))/(sin(d*x + c)^2 - 1)
^2)/d
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.19

$$\int \sec(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^2-2ab+\frac{3b^2}{4}\right)}{d} + \frac{\left(2ab-\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{11b^2}{4}-2ab\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{11b^2}{4}-2ab\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2ab-\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

input
```
int((a + b*tan(c + d*x)^2)^2/cos(c + d*x),x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(2*a^2 - 2*a*b + (3*b^2)/4))/d + (tan(c/2 + (d*
x)/2)^7*(2*a*b - (3*b^2)/4) - tan(c/2 + (d*x)/2)^3*(2*a*b - (11*b^2)/4) -
tan(c/2 + (d*x)/2)^5*(2*a*b - (11*b^2)/4) + tan(c/2 + (d*x)/2)*(2*a*b - (3
*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 470, normalized size of antiderivative = 5.80

$$\int \sec(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a**2 + 8*log(tan((c + d*x)
/2) - 1)*sin(c + d*x)**4*a*b - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4
*b**2 + 16*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2 - 16*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**2*a*b + 6*log(tan((c + d*x)/2) - 1)*sin(c + d
*x)**2*b**2 - 8*log(tan((c + d*x)/2) - 1)*a**2 + 8*log(tan((c + d*x)/2) -
1)*a*b - 3*log(tan((c + d*x)/2) - 1)*b**2 + 8*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**4*a**2 - 8*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a*b + 3*l
og(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*b**2 - 16*log(tan((c + d*x)/2) +
1)*sin(c + d*x)**2*a**2 + 16*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b
 - 6*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 + 8*log(tan((c + d*x)/
2) + 1)*a**2 - 8*log(tan((c + d*x)/2) + 1)*a*b + 3*log(tan((c + d*x)/2) +
1)*b**2 - 8*sin(c + d*x)**3*a*b + 5*sin(c + d*x)**3*b**2 + 8*sin(c + d*x)*
a*b - 3*sin(c + d*x)*b**2)/(8*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.441 $\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{(4a-3b)b\text{arctanh}(\sin(c+dx))}{2d}+\frac{(a-b)^2\sin(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*(4*a-3*b)*b*arctanh(sin(d*x+c))/d+(a-b)^2*sin(d*x+c)/d+1/2*b^2*sec(d*x
+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{(4a-3b)b\text{arctanh}(\sin(c+dx))+(a^2-2ab+2b^2+(a-b)^2\cos(2(c+dx)))\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]] + (a^2 - 2*a*b + 2*b^2 + (a - b)^2*Co
s[2*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(a-(a-b)\sin^2(c+dx)\right)^2}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left((a-b)^2+\frac{(2a-b)b-2(a-b)b\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b(4a-3b)\text{arctanh}(\sin(c+dx))+(a-b)^2\sin(c+dx)+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((4*a - 3*b)*b*ArcTanh[Sin[c + d*x]])/2 + (a - b)^2*Sin[c + d*x] + (b^2*S
in[c + d*x])/(2*(1 - Sin[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$-\frac{i e^{i(dx+c)}a^2}{2d}+\frac{i e^{i(dx+c)}ab}{d}-\frac{i e^{i(dx+c)}b^2}{2d}+\frac{i e^{-i(dx+c)}a^2}{2d}-\frac{i e^{-i(dx+c)}ab}{d}+\frac{i e^{-i(dx+c)}b^2}{2d}-\frac{ib^2(e^{3i(dx+c)}}{d(e^{2i(dx+}}$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/
2*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))
+sin(d*x+c)*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.71

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{(4\,ab-3\,b^2)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-(4\,ab-3\,b^2)\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)}+2}{4\,d\cos{(dx+c)}^2}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/4*((4*a*b - 3*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (4*a*b - 3*b^2
)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(a^2 - 2*a*b + b^2)*cos(d*x
 + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \int \left(a+b\tan^2{(c+dx)}\right)^2\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = -\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log{(\sin{(dx+c)}+1)}-3\log{(\sin{(dx+c)}-1)}-4\sin{(dx+c)}\right)-4\,ab(\log{(\sin{(}}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 4*a*b*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 4*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$= \frac{4\,a^2\sin\left(dx+c\right)-8\,ab\sin\left(dx+c\right)+4\,b^2\sin\left(dx+c\right)+\left(4\,ab-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(4\,ab-3\,b^2\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/4*(4*a^2*sin(d*x + c) - 8*a*b*sin(d*x + c) + 4*b^2*sin(d*x + c) + (4*a*b
 - 3*b^2)*log(abs(sin(d*x + c) + 1)) - (4*a*b - 3*b^2)*log(abs(sin(d*x + c
) - 1)) - 2*b^2*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.39

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a-3\,b\right)}{d}$$
$$-\frac{\left(2\,a^2-4\,a\,b+3\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,a^2+8\,a\,b-2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2-4\,a\,b+3\,b^2\right)\,\tan\left(\frac{c}{2}\right.}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
(b*atanh(tan(c/2 + (d*x)/2))*(4*a - 3*b))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2
 - 4*a*b + 3*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^2 - 8*a*b + 2*b^2) + tan(c/2
 + (d*x)/2)*(2*a^2 - 4*a*b + 3*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/2 +
(d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.06

$$\int \cos(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$= \frac{-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2ab+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2b^2+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 4*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b + 3*log(tan((c + d*x)/
2) - 1)*sin(c + d*x)**2*b**2 + 4*log(tan((c + d*x)/2) - 1)*a*b - 3*log(tan
((c + d*x)/2) - 1)*b**2 + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b
- 3*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b**2 - 4*log(tan((c + d*x)/2
) + 1)*a*b + 3*log(tan((c + d*x)/2) + 1)*b**2 + 2*sin(c + d*x)**3*a**2 - 4
*sin(c + d*x)**3*a*b + 2*sin(c + d*x)**3*b**2 - 2*sin(c + d*x)*a**2 + 4*si
n(c + d*x)*a*b - 3*sin(c + d*x)*b**2)/(2*d*(sin(c + d*x)**2 - 1))
```

3.442 $\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{b^2\text{arctanh}(\sin(c+dx))}{d}+\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{(a-b)^2\sin^3(c+dx)}{3d}$$

output
```
b^2*arctanh(sin(d*x+c))/d+(a^2-b^2)*sin(d*x+c)/d-1/3*(a-b)^2*sin(d*x+c)^3/
d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{\sin(c+dx)\left(\frac{3b^2\text{arctanh}\left(\sqrt{\sin^2(c+dx)}\right)}{\sqrt{\sin^2(c+dx)}}+(a-b)\left(3(a+b)+(-a+b)\sin^2(c+dx)\right)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(Sin[c + d*x]*((3*b^2*ArcTanh[Sqrt[Sin[c + d*x]^2]])/Sqrt[Sin[c + d*x]^2]
+ (a - b)*(3*(a + b) + (-a + b)*Sin[c + d*x]^2)))/(3*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(a-(a-b)\sin^2(c+dx)\right)^2}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(a^2-b^2-(a-b)^2\sin^2(c+dx)+\frac{b^2}{1-\sin^2(c+dx)}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)-\frac{1}{3}(a-b)^2\sin^3(c+dx)+b^2\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTanh[Sin[c + d*x]] + (a^2 - b^2)*Sin[c + d*x] - ((a - b)^2*Sin[c +
 d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 8.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{2ab\sin(dx+c)^3}{3}+\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{2ab\sin(dx+c)^3}{3}+\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$-\frac{3ie^{i(dx+c)}a^2}{8d}-\frac{ie^{i(dx+c)}ab}{4d}+\frac{5ie^{i(dx+c)}b^2}{8d}+\frac{3ie^{-i(dx+c)}a^2}{8d}+\frac{ie^{-i(dx+c)}ab}{4d}-\frac{5ie^{-i(dx+c)}b^2}{8d}+\frac{\ln(e^{i(dx+}}{d}$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b*
sin(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{3\,b^2\log\left(\sin\left(dx+c\right)+1\right)-3\,b^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left((a^2-2\,ab+b^2)\cos\left(dx+c\right)^2+2\,a^2+2\,ab}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1) + 2*((a^2
- 2*a*b + b^2)*cos(d*x + c)^2 + 2*a^2 + 2*a*b - 4*b^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \int \left(a+b\tan^2\left(c+dx\right)\right)^2\cos^3\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \cos^3(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = -\frac{2\,(a^2-2\,ab+b^2)\sin\left(dx+c\right)^3-3\,b^2\log\left(\sin\left(dx+c\right)+1\right)+3\,b^2\log\left(\sin\left(dx+c\right)-1\right)-6\,(a^2-b^2)\,\mathrm{si}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(a^2 - 2*a*b + b^2)*sin(d*x + c)^3 - 3*b^2*log(sin(d*x + c) + 1) +
 3*b^2*log(sin(d*x + c) - 1) - 6*(a^2 - b^2)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = -\frac{2\,a^2\sin\left(dx+c\right)^3-4\,ab\sin\left(dx+c\right)^3+2\,b^2\sin\left(dx+c\right)^3-3\,b^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,b^2\log\left(\left|\sin\left(\right.\right.\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(2*a^2*sin(d*x + c)^3 - 4*a*b*sin(d*x + c)^3 + 2*b^2*sin(d*x + c)^3 -
 3*b^2*log(abs(sin(d*x + c) + 1)) + 3*b^2*log(abs(sin(d*x + c) - 1)) - 6*a
^2*sin(d*x + c) + 6*b^2*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.43

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{\left(2\,a^2-2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{4\,a^2}{3}+\frac{16\,a\,b}{3}-\frac{20\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2-2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
(2*b^2*atanh(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*b^2
) + tan(c/2 + (d*x)/2)^3*((16*a*b)/3 + (4*a^2)/3 - (20*b^2)/3) + tan(c/2 +
 (d*x)/2)*(2*a^2 - 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/
2)^4 + tan(c/2 + (d*x)/2)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.79

$$\int \cos^3(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$=\frac{-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^2-\sin\left(dx+c\right)^3a^2+2\sin\left(dx+c\right)^3ab-\sin\left(dx\right.}{3d}$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 3*log(tan((c + d*x)/2) - 1)*b**2 + 3*log(tan((c + d*x)/2) + 1)*b**2 -
sin(c + d*x)**3*a**2 + 2*sin(c + d*x)**3*a*b - sin(c + d*x)**3*b**2 + 3*si
n(c + d*x)*a**2 - 3*sin(c + d*x)*b**2)/(3*d)
```

3.443 $\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{a^2\sin(c+dx)}{d}-\frac{2a(a-b)\sin^3(c+dx)}{3d}+\frac{(a-b)^2\sin^5(c+dx)}{5d}$$

output
```
a^2*sin(d*x+c)/d-2/3*a*(a-b)*sin(d*x+c)^3/d+1/5*(a-b)^2*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{15a^2\sin(c+dx)-10a(a-b)\sin^3(c+dx)+3(a-b)^2\sin^5(c+dx)}{15d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(15*a^2*Sin[c + d*x] - 10*a*(a - b)*Sin[c + d*x]^3 + 3*(a - b)^2*Sin[c + d
*x]^5)/(15*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4159, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \left(a-(a-b)\sin^2(c+dx)\right)^2 d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\int \left((a-b)^2\sin^4(c+dx)-2a(a-b)\sin^2(c+dx)+a^2\right) d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\sin(c+dx)+\frac{1}{5}(a-b)^2\sin^5(c+dx)-\frac{2}{3}a(a-b)\sin^3(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*Sin[c + d*x] - (2*a*(a - b)*Sin[c + d*x]^3)/3 + ((a - b)^2*Sin[c + d*
x]^5)/5)/d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 32.62 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{\frac{\sin(dx+c)^5 b^2}{5}+2ab\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)+\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{\frac{\sin(dx+c)^5 b^2}{5}+2ab\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)+\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$\frac{5a^2\sin(dx+c)}{8d}+\frac{\sin(dx+c)ab}{4d}+\frac{\sin(dx+c)b^2}{8d}+\frac{\sin(5dx+5c)a^2}{80d}-\frac{\sin(5dx+5c)ab}{40d}+\frac{\sin(5dx+5c)b^2}{80d}+\frac{5\sin(3}{}$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*sin(d*x+c)^5*b^2+2*a*b*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(
d*x+c)^2)*sin(d*x+c))+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{\left(3\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^4+2\left(2\,a^2+ab-3\,b^2\right)\cos\left(dx+c\right)^2+8\,a^2+4\,ab+3\,b^2\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/15*(3*(a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(2*a^2 + a*b - 3*b^2)*cos(d
*x + c)^2 + 8*a^2 + 4*a*b + 3*b^2)*sin(d*x + c)/d
```

Sympy [F]

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \int \left(a+b\tan^2\left(c+dx\right)\right)^2\cos^5\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{3\left(a^2-2\,ab+b^2\right)\sin\left(dx+c\right)^5-10\left(a^2-ab\right)\sin\left(dx+c\right)^3+15\,a^2\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*(a^2 - 2*a*b + b^2)*sin(d*x + c)^5 - 10*(a^2 - a*b)*sin(d*x + c)^3
 + 15*a^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2946 vs. $2(53) = 106$.

Time = 136.94 (sec) , antiderivative size = 2946, normalized size of antiderivative = 51.68

$$\int \cos^5(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/15*(15*a^2*tan(1/2*d*x)^10*tan(1/2*c)^9 + 15*a^2*tan(1/2*d*x)^9*tan(1/2
*c)^10 + 20*a^2*tan(1/2*d*x)^10*tan(1/2*c)^7 + 40*a*b*tan(1/2*d*x)^10*tan(
1/2*c)^7 - 75*a^2*tan(1/2*d*x)^9*tan(1/2*c)^8 + 120*a*b*tan(1/2*d*x)^9*tan
(1/2*c)^8 - 75*a^2*tan(1/2*d*x)^8*tan(1/2*c)^9 + 120*a*b*tan(1/2*d*x)^8*ta
n(1/2*c)^9 + 20*a^2*tan(1/2*d*x)^7*tan(1/2*c)^10 + 40*a*b*tan(1/2*d*x)^7*t
an(1/2*c)^10 + 58*a^2*tan(1/2*d*x)^10*tan(1/2*c)^5 - 16*a*b*tan(1/2*d*x)^1
0*tan(1/2*c)^5 + 48*b^2*tan(1/2*d*x)^10*tan(1/2*c)^5 + 150*a^2*tan(1/2*d*x
)^9*tan(1/2*c)^6 - 360*a*b*tan(1/2*d*x)^9*tan(1/2*c)^6 + 240*b^2*tan(1/2*d
*x)^9*tan(1/2*c)^6 + 700*a^2*tan(1/2*d*x)^8*tan(1/2*c)^7 - 1000*a*b*tan(1/
2*d*x)^8*tan(1/2*c)^7 + 480*b^2*tan(1/2*d*x)^8*tan(1/2*c)^7 + 700*a^2*tan(
1/2*d*x)^7*tan(1/2*c)^8 - 1000*a*b*tan(1/2*d*x)^7*tan(1/2*c)^8 + 480*b^2*t
an(1/2*d*x)^7*tan(1/2*c)^8 + 150*a^2*tan(1/2*d*x)^6*tan(1/2*c)^9 - 360*a*b
*tan(1/2*d*x)^6*tan(1/2*c)^9 + 240*b^2*tan(1/2*d*x)^6*tan(1/2*c)^9 + 58*a^
2*tan(1/2*d*x)^5*tan(1/2*c)^10 - 16*a*b*tan(1/2*d*x)^5*tan(1/2*c)^10 + 48*
b^2*tan(1/2*d*x)^5*tan(1/2*c)^10 + 20*a^2*tan(1/2*d*x)^10*tan(1/2*c)^3 + 4
0*a*b*tan(1/2*d*x)^10*tan(1/2*c)^3 - 150*a^2*tan(1/2*d*x)^9*tan(1/2*c)^4 +
 360*a*b*tan(1/2*d*x)^9*tan(1/2*c)^4 - 240*b^2*tan(1/2*d*x)^9*tan(1/2*c)^4
 - 610*a^2*tan(1/2*d*x)^8*tan(1/2*c)^5 + 2080*a*b*tan(1/2*d*x)^8*tan(1/2*c
)^5 - 1200*b^2*tan(1/2*d*x)^8*tan(1/2*c)^5 - 2200*a^2*tan(1/2*d*x)^7*tan(1
/2*c)^6 + 4720*a*b*tan(1/2*d*x)^7*tan(1/2*c)^6 - 2400*b^2*tan(1/2*d*x)^...
```

Mupad [B] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.09

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{\frac{5 a^2 \sin(c+d x)}{8} + \frac{b^2 \sin(c+d x)}{8} + \frac{5 a^2 \sin(3 c+3 d x)}{48} + \frac{a^2 \sin(5 c+5 d x)}{80} - \frac{b^2 \sin(3 c+3 d x)}{16} + \frac{b^2 \sin(5 c+5 d x)}{80} + \frac{a b \sin(c+d x)}{4} -}{d}$$

input
```
int(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
((5*a^2*sin(c + d*x))/8 + (b^2*sin(c + d*x))/8 + (5*a^2*sin(3*c + 3*d*x))/
48 + (a^2*sin(5*c + 5*d*x))/80 - (b^2*sin(3*c + 3*d*x))/16 + (b^2*sin(5*c
+ 5*d*x))/80 + (a*b*sin(c + d*x))/4 - (a*b*sin(3*c + 3*d*x))/24 - (a*b*sin
(5*c + 5*d*x))/40)/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \cos^5(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{\sin(dx+c)\left(3\sin(dx+c)^4 a^2 - 6\sin(dx+c)^4 ab + 3\sin(dx+c)^4 b^2 - 10\sin(dx+c)^2 a^2 + 10\sin(dx+c)}{15d}$$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(sin(c + d*x)*(3*sin(c + d*x)**4*a**2 - 6*sin(c + d*x)**4*a*b + 3*sin(c +
d*x)**4*b**2 - 10*sin(c + d*x)**2*a**2 + 10*sin(c + d*x)**2*a*b + 15*a**2)
)/(15*d)
```

3.444 $\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{a^2\sin(c+dx)}{d} - \frac{a(3a-2b)\sin^3(c+dx)}{3d} + \frac{(a-b)(3a-b)\sin^5(c+dx)}{5d} - \frac{(a-b)^2\sin^7(c+dx)}{7d}$$

output
```
a^2*sin(d*x+c)/d-1/3*a*(3*a-2*b)*sin(d*x+c)^3/d+1/5*(a-b)*(3*a-b)*sin(d*x+
c)^5/d-1/7*(a-b)^2*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{105a^2\sin(c+dx) - 35a(3a-2b)\sin^3(c+dx) + 21(3a^2-4ab+b^2)\sin^5(c+dx) - 15(a-b)^2\sin^7(c+d}{105d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(105*a^2*Sin[c + d*x] - 35*a*(3*a - 2*b)*Sin[c + d*x]^3 + 21*(3*a^2 - 4*a*
b + b^2)*Sin[c + d*x]^5 - 15*(a - b)^2*Sin[c + d*x]^7)/(105*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)^2 d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(-(a-b)^2\sin^6(c+dx)+\left(3a^2-4ba+b^2\right)\sin^4(c+dx)-a(3a-2b)\sin^2(c+dx)+a^2\right) d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin(c+dx)-\frac{1}{7}(a-b)^2\sin^7(c+dx)+\frac{1}{5}(a-b)(3a-b)\sin^5(c+dx)-\frac{1}{3}a(3a-2b)\sin^3(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*Sin[c + d*x] - (a*(3*a - 2*b)*Sin[c + d*x]^3)/3 + ((a - b)*(3*a - b)*
Sin[c + d*x]^5)/5 - ((a - b)^2*Sin[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 111.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}+2ab\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right.}{35}\right.}{d}$
default	$\frac{\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}+2ab\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right.}{35}\right.}{d}$
risch	$\frac{35a^2\sin(dx+c)}{64d}+\frac{5\sin(dx+c)ab}{32d}+\frac{3\sin(dx+c)b^2}{64d}+\frac{\sin(7dx+7c)a^2}{448d}-\frac{\sin(7dx+7c)ab}{224d}+\frac{\sin(7dx+7c)b^2}{448d}+7$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x
+c)+2*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))+b^2*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*cos(d*x+c)^4*s
in(d*x+c)+1/35*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$= \frac{\left(15\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^6+6\left(3\,a^2+ab-4\,b^2\right)\cos\left(dx+c\right)^4+\left(24\,a^2+8\,ab+3\,b^2\right)\cos\left(dx+c\right)^2}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/105*(15*(a^2 - 2*a*b + b^2)*cos(d*x + c)^6 + 6*(3*a^2 + a*b - 4*b^2)*cos
(d*x + c)^4 + (24*a^2 + 8*a*b + 3*b^2)*cos(d*x + c)^2 + 48*a^2 + 16*a*b +
6*b^2)*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = -\frac{15\,(a^2-2\,ab+b^2)\sin{(dx+c)}^7-21\,(3\,a^2-4\,ab+b^2)\sin{(dx+c)}^5+35\,(3\,a^2-2\,ab)\sin{(dx+c)}^3-}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/105*(15*(a^2 - 2*a*b + b^2)*sin(d*x + c)^7 - 21*(3*a^2 - 4*a*b + b^2)*s
in(d*x + c)^5 + 35*(3*a^2 - 2*a*b)*sin(d*x + c)^3 - 105*a^2*sin(d*x + c))/
d
```

Giac [F(-1)]

Timed out.

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.86

$$\int \cos^7(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \frac{\frac{35\,a^2\,\sin(c+d\,x)}{64}+\frac{3\,b^2\,\sin(c+d\,x)}{64}+\frac{7\,a^2\,\sin(3\,c+3\,d\,x)}{64}+\frac{7\,a^2\,\sin(5\,c+5\,d\,x)}{320}+\frac{a^2\,\sin(7\,c+7\,d\,x)}{448}-\frac{b^2\,\sin(3\,c+3\,d\,x)}{64}-\frac{b^2\,\sin(5\,c+5}{320}}{d}$$

input
```
int(cos(c + d*x)^7*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
((35*a^2*sin(c + d*x))/64 + (3*b^2*sin(c + d*x))/64 + (7*a^2*sin(3*c + 3*d
*x))/64 + (7*a^2*sin(5*c + 5*d*x))/320 + (a^2*sin(7*c + 7*d*x))/448 - (b^2
*sin(3*c + 3*d*x))/64 - (b^2*sin(5*c + 5*d*x))/320 + (b^2*sin(7*c + 7*d*x)
)/448 + (5*a*b*sin(c + d*x))/32 - (a*b*sin(3*c + 3*d*x))/96 - (3*a*b*sin(5
*c + 5*d*x))/160 - (a*b*sin(7*c + 7*d*x))/224)/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \cos^7(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$= \frac{\sin(dx+c)\left(-15\sin(dx+c)^6a^2+30\sin(dx+c)^6ab-15\sin(dx+c)^6b^2+63\sin(dx+c)^4a^2-84\sin\right.}{105d}$$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(sin(c + d*x)*( - 15*sin(c + d*x)**6*a**2 + 30*sin(c + d*x)**6*a*b - 15*si
n(c + d*x)**6*b**2 + 63*sin(c + d*x)**4*a**2 - 84*sin(c + d*x)**4*a*b + 21
*sin(c + d*x)**4*b**2 - 105*sin(c + d*x)**2*a**2 + 70*sin(c + d*x)**2*a*b
 + 105*a**2))/(105*d)
```

3.445 $\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{a^2\sin(c+dx)}{d} - \frac{2a(2a-b)\sin^3(c+dx)}{3d} + \frac{(6a^2-6ab+b^2)\sin^5(c+dx)}{5d} - \frac{2(a-b)(2a-b)\sin^7(c+dx)}{7d} + \frac{(a-b)^2\sin^9(c+dx)}{9d}$$

output
```
a^2*sin(d*x+c)/d-2/3*a*(2*a-b)*sin(d*x+c)^3/d+1/5*(6*a^2-6*a*b+b^2)*sin(d*
x+c)^5/d-2/7*(a-b)*(2*a-b)*sin(d*x+c)^7/d+1/9*(a-b)^2*sin(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{630(63a^2+14ab+3b^2)\sin(c+dx)+420(21a^2-b^2)\sin(3(c+dx))+252(9a^2-4ab-b^2)\sin(5(c+dx)}{80640d}$$

input
```
Integrate[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(630*(63*a^2 + 14*a*b + 3*b^2)*Sin[c + d*x] + 420*(21*a^2 - b^2)*Sin[3*(c
+ d*x)] + 252*(9*a^2 - 4*a*b - b^2)*Sin[5*(c + d*x)] + 45*(a - b)*(9*a - b
)*Sin[7*(c + d*x)] + 35*(a - b)^2*Sin[9*(c + d*x)])/(80640*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)^2\left(a-(a-b)\sin^2(c+dx)\right)^2 d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left((a-b)^2\sin^8(c+dx)-2\left(2a^2-3ba+b^2\right)\sin^6(c+dx)+\left(6a^2-6ba+b^2\right)\sin^4(c+dx)-2a(2a-b)\sin^2(c+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\left(6a^2-6ab+b^2\right)\sin^5(c+dx)+a^2\sin(c+dx)+\frac{1}{9}(a-b)^2\sin^9(c+dx)-\frac{2}{7}(a-b)(2a-b)\sin^7(c+dx)-\frac{2}{3}a(2}{d}$$

input
```
Int[Cos[c + d*x]^9*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*Sin[c + d*x] - (2*a*(2*a - b)*Sin[c + d*x]^3)/3 + ((6*a^2 - 6*a*b + b
^2)*Sin[c + d*x]^5)/5 - (2*(a - b)*(2*a - b)*Sin[c + d*x]^7)/7 + ((a - b)^
2*Sin[c + d*x]^9)/9)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 315.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.61

method	result
derivativedivides	$b^2\left(-\frac{\sin(dx+c)^3\cos(dx+c)^6}{9}-\frac{\sin(dx+c)\cos(dx+c)^6}{21}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{105}\right)+2ab\left(\right.$
default	$b^2\left(-\frac{\sin(dx+c)^3\cos(dx+c)^6}{9}-\frac{\sin(dx+c)\cos(dx+c)^6}{21}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{105}\right)+2ab\left(\right.$
risch	$\frac{63a^2\sin(dx+c)}{128d}+\frac{7\sin(dx+c)ab}{64d}+\frac{3\sin(dx+c)b^2}{128d}+\frac{\sin(9dx+9c)a^2}{2304d}-\frac{\sin(9dx+9c)ab}{1152d}+\frac{\sin(9dx+9c)b^2}{2304d}+9$

input
```
int(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/10
5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/9*sin(d*x+c)*c
os(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c))+1/9*a^2*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+
64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

$$\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\left(35\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)^8+10\left(4\,a^2+ab-5\,b^2\right)\cos\left(dx+c\right)^6+3\left(16\,a^2+4\,ab+b^2\right)\cos\left(dx+c\right)}{315\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/315*(35*(a^2 - 2*a*b + b^2)*cos(d*x + c)^8 + 10*(4*a^2 + a*b - 5*b^2)*co
s(d*x + c)^6 + 3*(16*a^2 + 4*a*b + b^2)*cos(d*x + c)^4 + 4*(16*a^2 + 4*a*b
 + b^2)*cos(d*x + c)^2 + 128*a^2 + 32*a*b + 8*b^2)*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^9(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**9*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \cos^9(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{35\,(a^2-2\,ab+b^2)\sin{(dx+c)}^9 - 90\,(2\,a^2-3\,ab+b^2)\sin{(dx+c)}^7 + 63\,(6\,a^2-6\,ab+b^2)\sin{(dx+c)}^5}{315\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/315*(35*(a^2 - 2*a*b + b^2)*sin(d*x + c)^9 - 90*(2*a^2 - 3*a*b + b^2)*si
n(d*x + c)^7 + 63*(6*a^2 - 6*a*b + b^2)*sin(d*x + c)^5 - 210*(2*a^2 - a*b)
*sin(d*x + c)^3 + 315*a^2*sin(d*x + c))/d
```

Giac [F(-1)]

Timed out.

$$\int \cos^9(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.65

$$\int \cos^9(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{\frac{63\,a^2\,\sin(c+d\,x)}{128} + \frac{3\,b^2\,\sin(c+d\,x)}{128} + \frac{7\,a^2\,\sin(3\,c+3\,d\,x)}{64} + \frac{9\,a^2\,\sin(5\,c+5\,d\,x)}{320} + \frac{9\,a^2\,\sin(7\,c+7\,d\,x)}{1792} + \frac{a^2\,\sin(9\,c+9\,d\,x)}{2304} - \frac{b^2\,\sin(3\,c+}{192}}{}$$

input
```
int(cos(c + d*x)^9*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
((63*a^2*sin(c + d*x))/128 + (3*b^2*sin(c + d*x))/128 + (7*a^2*sin(3*c + 3
*d*x))/64 + (9*a^2*sin(5*c + 5*d*x))/320 + (9*a^2*sin(7*c + 7*d*x))/1792 +
 (a^2*sin(9*c + 9*d*x))/2304 - (b^2*sin(3*c + 3*d*x))/192 - (b^2*sin(5*c +
 5*d*x))/320 + (b^2*sin(7*c + 7*d*x))/1792 + (b^2*sin(9*c + 9*d*x))/2304 +
 (7*a*b*sin(c + d*x))/64 - (a*b*sin(5*c + 5*d*x))/80 - (5*a*b*sin(7*c + 7*
d*x))/896 - (a*b*sin(9*c + 9*d*x))/1152)/d
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.37

$$\int \cos^9(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx$$
$$= \frac{\sin\left(dx+c\right)\left(35\sin\left(dx+c\right)^8 a^2 - 70\sin\left(dx+c\right)^8 ab + 35\sin\left(dx+c\right)^8 b^2 - 180\sin\left(dx+c\right)^6 a^2 + 270\sin\right.}{}$$

input
```
int(cos(d*x+c)^9*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(sin(c + d*x)*(35*sin(c + d*x)**8*a**2 - 70*sin(c + d*x)**8*a*b + 35*sin(c
 + d*x)**8*b**2 - 180*sin(c + d*x)**6*a**2 + 270*sin(c + d*x)**6*a*b - 90*
sin(c + d*x)**6*b**2 + 378*sin(c + d*x)**4*a**2 - 378*sin(c + d*x)**4*a*b
+ 63*sin(c + d*x)**4*b**2 - 420*sin(c + d*x)**2*a**2 + 210*sin(c + d*x)**2
*a*b + 315*a**2))/(315*d)
```

3.446 $\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{a^2\tan(c+dx)}{d}+\frac{2a(a+b)\tan^3(c+dx)}{3d}+\frac{(a^2+4ab+b^2)\tan^5(c+dx)}{5d}+\frac{2b(a+b)\tan^7(c+dx)}{7d}+\frac{b^2\tan^9(c+dx)}{9d}$$

output
```
a^2*tan(d*x+c)/d+2/3*a*(a+b)*tan(d*x+c)^3/d+1/5*(a^2+4*a*b+b^2)*tan(d*x+c)
^5/d+2/7*b*(a+b)*tan(d*x+c)^7/d+1/9*b^2*tan(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{(8(21a^2-6ab+b^2)+4(21a^2-6ab+b^2)\sec^2(c+dx)+3(21a^2-6ab+b^2)\sec^4(c+dx)+10(9a-5b)}{315d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((8*(21*a^2 - 6*a*b + b^2) + 4*(21*a^2 - 6*a*b + b^2)*Sec[c + d*x]^2 + 3*(
21*a^2 - 6*a*b + b^2)*Sec[c + d*x]^4 + 10*(9*a - 5*b)*b*Sec[c + d*x]^6 + 3
5*b^2*Sec[c + d*x]^8)*Tan[c + d*x])/(315*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6\left(a+b\tan(c+dx)^2\right)^2dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(\tan^2(c+dx)+1\right)^2\left(b\tan^2(c+dx)+a\right)^2d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(b^2\tan^8(c+dx)+2b(a+b)\tan^6(c+dx)+\left(a^2+4ba+b^2\right)\tan^4(c+dx)+2a(a+b)\tan^2(c+dx)+a^2\right)d\tan}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\left(a^2+4ab+b^2\right)\tan^5(c+dx)+a^2\tan(c+dx)+\frac{2}{7}b(a+b)\tan^7(c+dx)+\frac{2}{3}a(a+b)\tan^3(c+dx)+\frac{1}{9}b^2\tan^9(c}{d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*Tan[c + d*x] + (2*a*(a + b)*Tan[c + d*x]^3)/3 + ((a^2 + 4*a*b + b^2)*
Tan[c + d*x]^5)/5 + (2*b*(a + b)*Tan[c + d*x]^7)/7 + (b^2*Tan[c + d*x]^9)/
9)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 28.79 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^5}{9\cos(dx+c)^9}+\frac{4\sin(dx+c)^5}{63\cos(dx+c)^7}+\frac{8\sin(dx+c)^5}{315\cos(dx+c)^5}\right)+2ab\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)-a^2\left(-\frac{8}{15}-\frac{\sec(}{}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^5}{9\cos(dx+c)^9}+\frac{4\sin(dx+c)^5}{63\cos(dx+c)^7}+\frac{8\sin(dx+c)^5}{315\cos(dx+c)^5}\right)+2ab\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)-a^2\left(-\frac{8}{15}-\frac{\sec(}{}\right.}{d}$
risch	$\frac{16i\left(210a^2e^{12i(dx+c)}-420ab\,e^{12i(dx+c)}+210b^2e^{12i(dx+c)}+945a^2e^{10i(dx+c)}-630ab\,e^{10i(dx+c)}-315b^2e^{10i(dx+c)}+1701\right.}{}$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/3
15*sin(d*x+c)^5/cos(d*x+c)^5)+2*a*b*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*si
n(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*(-8/15-1/5*se
c(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \,dx = \frac{(8\,(21\,a^2-6\,ab+b^2)\cos{(dx+c)}^8+4\,(21\,a^2-6\,ab+b^2)\cos{(dx+c)}^6+3\,(21\,a^2-6\,ab+b^2)\cos{(dx+}}{315\,d\cos{(dx+c)}^9}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/315*(8*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^8 + 4*(21*a^2 - 6*a*b + b^2)*
cos(d*x + c)^6 + 3*(21*a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 10*(9*a*b - 5*b
^2)*cos(d*x + c)^2 + 35*b^2)*sin(d*x + c)/(d*cos(d*x + c)^9)
```

Sympy [F]

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \,dx = \int \left(a+b\tan^2{(c+dx)}\right)^2 \sec^6{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{35\,b^2\tan(dx+c)^9 + 90\,(ab+b^2)\tan(dx+c)^7 + 63\,(a^2+4\,ab+b^2)\tan(dx+c)^5 + 210\,(a^2+ab)\tan(d$$

$$315\,d$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/315*(35*b^2*tan(d*x + c)^9 + 90*(a*b + b^2)*tan(d*x + c)^7 + 63*(a^2 + 4
*a*b + b^2)*tan(d*x + c)^5 + 210*(a^2 + a*b)*tan(d*x + c)^3 + 315*a^2*tan(
d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{35\,b^2\tan(dx+c)^9 + 90\,ab\tan(dx+c)^7 + 90\,b^2\tan(dx+c)^7 + 63\,a^2\tan(dx+c)^5 + 252\,ab\tan(dx+c)$$

$$315\,d$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/315*(35*b^2*tan(d*x + c)^9 + 90*a*b*tan(d*x + c)^7 + 90*b^2*tan(d*x + c)
^7 + 63*a^2*tan(d*x + c)^5 + 252*a*b*tan(d*x + c)^5 + 63*b^2*tan(d*x + c)^
5 + 210*a^2*tan(d*x + c)^3 + 210*a*b*tan(d*x + c)^3 + 315*a^2*tan(d*x + c)
)/d
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{a^2\tan(c+d\,x)+\frac{b^2\tan(c+d\,x)^9}{9}+\tan(c+d\,x)^5\left(\frac{a^2}{5}+\frac{4\,a\,b}{5}+\frac{b^2}{5}\right)+\frac{2\,a\tan(c+d\,x)^3\,(a+b)}{3}+\frac{2\,b\tan(c+d\,x)^7\,(a+b)}{7}}{d}$$

input
```
int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^6,x)
```

output
```
(a^2*tan(c + d*x) + (b^2*tan(c + d*x)^9)/9 + tan(c + d*x)^5*((4*a*b)/5 + a
^2/5 + b^2/5) + (2*a*tan(c + d*x)^3*(a + b))/3 + (2*b*tan(c + d*x)^7*(a +
b))/7)/d
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.15

$$\int \sec^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{\sin(dx+c)\left(168\sin(dx+c)^8a^2-48\sin(dx+c)^8ab+8\sin(dx+c)^8b^2-756\sin(dx+c)^6a^2+216\sin\cdots\right)}{315\cos(dx+c)\,d\,\cdots}$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(sin(c + d*x)*(168*sin(c + d*x)**8*a**2 - 48*sin(c + d*x)**8*a*b + 8*sin(c
 + d*x)**8*b**2 - 756*sin(c + d*x)**6*a**2 + 216*sin(c + d*x)**6*a*b - 36*
sin(c + d*x)**6*b**2 + 1323*sin(c + d*x)**4*a**2 - 378*sin(c + d*x)**4*a*b
 + 63*sin(c + d*x)**4*b**2 - 1050*sin(c + d*x)**2*a**2 + 210*sin(c + d*x)*
*2*a*b + 315*a**2))/(315*cos(c + d*x)*d*(sin(c + d*x)**8 - 4*sin(c + d*x)*
*6 + 6*sin(c + d*x)**4 - 4*sin(c + d*x)**2 + 1))
```

3.447 $\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{a^2\tan(c+dx)}{d}+\frac{a(a+2b)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}$$

output
```
a^2*tan(d*x+c)/d+1/3*a*(a+2*b)*tan(d*x+c)^3/d+1/5*b*(2*a+b)*tan(d*x+c)^5/d
+1/7*b^2*tan(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$=\frac{(70a^2-28ab+6b^2+(35a^2-14ab+3b^2)\sec^2(c+dx)+6(7a-4b)b\sec^4(c+dx)+15b^2\sec^6(c+dx))\,t}{105d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((70*a^2 - 28*a*b + 6*b^2 + (35*a^2 - 14*a*b + 3*b^2)*Sec[c + d*x]^2 + 6*(
7*a - 4*b)*b*Sec[c + d*x]^4 + 15*b^2*Sec[c + d*x]^6)*Tan[c + d*x])/(105*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4\left(a+b\tan(c+dx)^2\right)^2dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)^2d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(b^2\tan^6(c+dx)+b(2a+b)\tan^4(c+dx)+a(a+2b)\tan^2(c+dx)+a^2\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\tan(c+dx)+\frac{1}{5}b(2a+b)\tan^5(c+dx)+\frac{1}{3}a(a+2b)\tan^3(c+dx)+\frac{1}{7}b^2\tan^7(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*Tan[c + d*x] + (a*(a + 2*b)*Tan[c + d*x]^3)/3 + (b*(2*a + b)*Tan[c +
d*x]^5)/5 + (b^2*Tan[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 9.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{b^{2}\left(\frac{\sin(dx+c)^{5}}{7\cos(dx+c)^{7}}+\frac{2\sin(dx+c)^{5}}{35\cos(dx+c)^{5}}\right)+2ab\left(\frac{\sin(dx+c)^{3}}{5\cos(dx+c)^{5}}+\frac{2\sin(dx+c)^{3}}{15\cos(dx+c)^{3}}\right)-a^{2}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}$
default	$\frac{b^{2}\left(\frac{\sin(dx+c)^{5}}{7\cos(dx+c)^{7}}+\frac{2\sin(dx+c)^{5}}{35\cos(dx+c)^{5}}\right)+2ab\left(\frac{\sin(dx+c)^{3}}{5\cos(dx+c)^{5}}+\frac{2\sin(dx+c)^{3}}{15\cos(dx+c)^{3}}\right)-a^{2}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}$
risch	$\frac{4i(105a^{2}e^{10i(dx+c)}-210ab\,e^{10i(dx+c)}+105b^{2}e^{10i(dx+c)}+455a^{2}e^{8i(dx+c)}-350ab\,e^{8i(dx+c)}-105b^{2}e^{8i(dx+c)}+770a^{2}e^{6i}}{}$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)+2*
a*b*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)-a^2*(-2
/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$=\frac{(2\,(35\,a^2-14\,ab+3\,b^2)\cos{(dx+c)}^6+(35\,a^2-14\,ab+3\,b^2)\cos{(dx+c)}^4+6\,(7\,ab-4\,b^2)\cos{(dx+c)}}{105\,d\cos{(dx+c)}^7}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/105*(2*(35*a^2 - 14*a*b + 3*b^2)*cos(d*x + c)^6 + (35*a^2 - 14*a*b + 3*b
^2)*cos(d*x + c)^4 + 6*(7*a*b - 4*b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x +
c)/(d*cos(d*x + c)^7)
```

Sympy [F]

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\int \left(a+b\tan^2{(c+dx)}\right)^2\sec^4{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{15\,b^2\tan\left(dx+c\right)^7+21\left(2\,ab+b^2\right)\tan\left(dx+c\right)^5+35\left(a^2+2\,ab\right)\tan\left(dx+c\right)^3+105\,a^2\tan\left(dx+c\right)}{105\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 + 21*(2*a*b + b^2)*tan(d*x + c)^5 + 35*(a^2 +
 2*a*b)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \sec^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{15\,b^2\tan\left(dx+c\right)^7+42\,ab\tan\left(dx+c\right)^5+21\,b^2\tan\left(dx+c\right)^5+35\,a^2\tan\left(dx+c\right)^3+70\,ab\tan\left(dx+c\right)^3}{105\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 + 42*a*b*tan(d*x + c)^5 + 21*b^2*tan(d*x + c)
^5 + 35*a^2*tan(d*x + c)^3 + 70*a*b*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))
/d
```

Mupad [B] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \sec^4(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{a^2\tan(c+d\,x)+\frac{b^2\tan(c+d\,x)^7}{7}+\frac{a\tan(c+d\,x)^3\,(a+2\,b)}{3}+\frac{b\tan(c+d\,x)^5\,(2\,a+b)}{5}}{d}$$

input
```
int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^4,x)
```

output
```
(a^2*tan(c + d*x) + (b^2*tan(c + d*x)^7)/7 + (a*tan(c + d*x)^3*(a + 2*b))/
3 + (b*tan(c + d*x)^5*(2*a + b))/5)/d
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.14

$$\int \sec^4(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{\sin(dx+c)\left(70\sin(dx+c)^6a^2-28\sin(dx+c)^6ab+6\sin(dx+c)^6b^2-245\sin(dx+c)^4a^2+98\sin(\right.}{105\cos(dx+c)\,d\left(\sin(dx+c)^6-3\sin(dx+\right.}$$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(sin(c + d*x)*(70*sin(c + d*x)**6*a**2 - 28*sin(c + d*x)**6*a*b + 6*sin(c
+ d*x)**6*b**2 - 245*sin(c + d*x)**4*a**2 + 98*sin(c + d*x)**4*a*b - 21*si
n(c + d*x)**4*b**2 + 280*sin(c + d*x)**2*a**2 - 70*sin(c + d*x)**2*a*b - 1
05*a**2))/(105*cos(c + d*x)*d*(sin(c + d*x)**6 - 3*sin(c + d*x)**4 + 3*sin
(c + d*x)**2 - 1))
```

3.448 $\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

output
```
a^2*tan(d*x+c)/d+2/3*a*b*tan(d*x+c)^3/d+1/5*b^2*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{b^2\tan^5(c+dx)}{5d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*Tan[c + d*x])/d + (2*a*b*Tan[c + d*x]^3)/(3*d) + (b^2*Tan[c + d*x]^5)
/(5*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2\left(a+b\tan(c+dx)^2\right)^2 dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(b\tan^2(c+dx)+a\right)^2 d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(b^2\tan^4(c+dx)+2ab\tan^2(c+dx)+a^2\right) d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\tan(c+dx)+\frac{2}{3}ab\tan^3(c+dx)+\frac{1}{5}b^2\tan^5(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(a^2*Tan[c + d*x] + (2*a*b*Tan[c + d*x]^3)/3 + (b^2*Tan[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{b^2 \sin(dx+c)^5}{5 \cos(dx+c)^5}+\frac{2ab \sin(dx+c)^3}{3 \cos(dx+c)^3}+a^2 \tan(dx+c)}{d}$
default	$\frac{\frac{b^2 \sin(dx+c)^5}{5 \cos(dx+c)^5}+\frac{2ab \sin(dx+c)^3}{3 \cos(dx+c)^3}+a^2 \tan(dx+c)}{d}$
risch	$\frac{2i\left(15a^2 e^{8i(dx+c)}-30ab\, e^{8i(dx+c)}+15b^2 e^{8i(dx+c)}+60a^2 e^{6i(dx+c)}-60ab\, e^{6i(dx+c)}+90a^2 e^{4i(dx+c)}-40\, e^{4i(dx+c)} ab+30\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b^2*sin(d*x+c)^5/cos(d*x+c)^5+2/3*a*b*sin(d*x+c)^3/cos(d*x+c)^3+a
^2*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{\left((15\,a^2-10\,ab+3\,b^2)\cos\left(dx+c\right)^4+2\,(5\,ab-3\,b^2)\cos\left(dx+c\right)^2+3\,b^2\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/15*((15*a^2 - 10*a*b + 3*b^2)*cos(d*x + c)^4 + 2*(5*a*b - 3*b^2)*cos(d*x
 + c)^2 + 3*b^2)*sin(d*x + c)/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \int \left(a+b\tan^2\left(c+dx\right)\right)^2\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{3\,b^2\tan\left(dx+c\right)^5+10\,ab\tan\left(dx+c\right)^3+15\,a^2\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/
d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{3\, b^2 \tan\left(dx+c\right)^5 + 10\, ab \tan\left(dx+c\right)^3 + 15\, a^2 \tan\left(dx+c\right)}{15\, d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*b^2*tan(d*x + c)^5 + 10*a*b*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/
d
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \sec^2(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{a^2 \tan(c+d\,x) + \frac{2\,a\,b\,\tan(c+d\,x)^3}{3} + \frac{b^2\,\tan(c+d\,x)^5}{5}}{d}$$

input
```
int((a + b*tan(c + d*x)^2)^2/cos(c + d*x)^2,x)
```

output
```
(a^2*tan(c + d*x) + (b^2*tan(c + d*x)^5)/5 + (2*a*b*tan(c + d*x)^3)/3)/d
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.24

$$\int \sec^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$=\frac{\sin(dx+c)\left(15\sin(dx+c)^4a^2-10\sin(dx+c)^4ab+3\sin(dx+c)^4b^2-30\sin(dx+c)^2a^2+10\sin(d\right.}{15\cos(dx+c)\,d\left(\sin(dx+c)^4-2\sin(dx+c)^2+1\right)}$$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(sin(c + d*x)*(15*sin(c + d*x)**4*a**2 - 10*sin(c + d*x)**4*a*b + 3*sin(c
+ d*x)**4*b**2 - 30*sin(c + d*x)**2*a**2 + 10*sin(c + d*x)**2*a*b + 15*a**
2))/(15*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.449 $\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{1}{2}(a-b)(a+3b)x+\frac{(a-b)^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}$$

output
```
1/2*(a-b)*(a+3*b)*x+1/2*(a-b)^2*cos(d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{2(a^2+2ab-3b^2)(c+dx)+(a-b)^2\sin(2(c+dx))+4b^2\tan(c+dx)}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(2*(a^2 + 2*a*b - 3*b^2)*(c + d*x) + (a - b)^2*Sin[2*(c + d*x)] + 4*b^2*Ta
n[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a\right)^2}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(b^2+\frac{a^2-b^2+2(a-b)b\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)^2}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}(a+3b)(a-b)\arctan(\tan(c+dx))+\frac{(a-b)^2\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}+b^2\tan(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((a - b)*(a + 3*b)*ArcTan[Tan[c + d*x]])/2 + b^2*Tan[c + d*x] + ((a - b)^
2*Tan[c + d*x])/(2*(1 + Tan[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 5.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ab\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\left(\frac{\cos(dx+c}{}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ab\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2\left(\frac{\cos(dx+c}{}\right.}{d}$
risch	$\frac{x\,a^2}{2}+xab-\frac{3x\,b^2}{2}-\frac{i e^{2i(dx+c)}a^2}{8d}+\frac{i e^{2i(dx+c)}ab}{4d}-\frac{i e^{2i(dx+c)}b^2}{8d}+\frac{i e^{-2i(dx+c)}a^2}{8d}-\frac{i e^{-2i(dx+c)}ab}{4d}+i$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)
-3/2*d*x-3/2*c)+2*a*b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*(1/2*
cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{(a^2+2\,ab-3\,b^2)dx\cos(dx+c) + \left((a^2-2\,ab+b^2)\cos(dx+c)^2+2\,b^2\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 + 2*a*b - 3*b^2)*d*x*cos(d*x + c) + ((a^2 - 2*a*b + b^2)*cos(d*x
 + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \int \left(a+b\tan^2(c+dx)\right)^2\cos^2(c+dx)\, dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{2\,b^2\tan\left(dx+c\right)+(a^2+2\,ab-3\,b^2)(dx+c)+\frac{(a^2-2\,ab+b^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(2*b^2*tan(d*x + c) + (a^2 + 2*a*b - 3*b^2)*(d*x + c) + (a^2 - 2*a*b +
 b^2)*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \cos^2(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{b^2\tan\left(dx+c\right)}{d}+\frac{(a^2+2\,ab-3\,b^2)(dx+c)}{2\,d} + \frac{a^2\tan\left(dx+c\right)-2\,ab\tan\left(dx+c\right)+b^2\tan\left(dx+c\right)}{2\left(\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
b^2*tan(d*x + c)/d + 1/2*(a^2 + 2*a*b - 3*b^2)*(d*x + c)/d + 1/2*(a^2*tan(
d*x + c) - 2*a*b*tan(d*x + c) + b^2*tan(d*x + c))/((tan(d*x + c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{b^2 \tan(c+d\,x)}{d} + \frac{\sin\left(2\,c+2\,d\,x\right)\,\left(\frac{a^2}{2}-a\,b+\frac{b^2}{2}\right)}{2\,d} + \frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a-b)\,(a+3\,b)}{2\,\left(\frac{a^2}{2}+a\,b-\frac{3\,b^2}{2}\right)}\right)\,(a-b)\,(a+3\,b)}{2\,d}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
(b^2*tan(c + d*x))/d + (sin(2*c + 2*d*x)*(a^2/2 - a*b + b^2/2))/(2*d) + (a
tan((tan(c + d*x)*(a - b)*(a + 3*b))/(2*(a*b + a^2/2 - (3*b^2)/2)))*(a - b
)*(a + 3*b))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.80

$$\int \cos^2(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{\cos\left(dx+c\right)a^2c+\cos\left(dx+c\right)a^2dx+2\cos\left(dx+c\right)abc+2\cos\left(dx+c\right)abdx-3\cos\left(dx+c\right)b^2c-3\,\mathrm{co}}{}$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(cos(c + d*x)*a**2*c + cos(c + d*x)*a**2*d*x + 2*cos(c + d*x)*a*b*c + 2*co
s(c + d*x)*a*b*d*x - 3*cos(c + d*x)*b**2*c - 3*cos(c + d*x)*b**2*d*x - sin
(c + d*x)**3*a**2 + 2*sin(c + d*x)**3*a*b - sin(c + d*x)**3*b**2 + sin(c +
 d*x)*a**2 - 2*sin(c + d*x)*a*b + 3*sin(c + d*x)*b**2)/(2*cos(c + d*x)*d)
```

3.450 $\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{1}{8}\left(3a^2+2ab+3b^2\right)x+\frac{(a-b)(3a+5b)\cos(c+dx)\sin(c+dx)}{8d}+\frac{(a-b)^2\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
1/8*(3*a^2+2*a*b+3*b^2)*x+1/8*(a-b)*(3*a+5*b)*cos(d*x+c)*sin(d*x+c)/d+1/4*
(a-b)^2*cos(d*x+c)^3*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\frac{4\left(3a^2+2ab+3b^2\right)(c+dx)+8\left(a^2-b^2\right)\sin(2(c+dx))+(a-b)^2\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(4*(3*a^2 + 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sin[2*(c + d*x)] + (a
 - b)^2*Sin[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4158, 315, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a\right)^2}{\left(\tan^2(c+dx)+1\right)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{1}{4}\int \frac{b(a+3b)\tan^2(c+dx)+a(3a+b)}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)+\frac{(a-b)\tan(c+dx)\left(a+b\tan^2(c+dx)\right)}{4\left(\tan^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+2ab+3b^2\right)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{3\left(a^2-b^2\right)\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)+\frac{(a-b)\tan(c+dx)\left(a+b\tan^2(c+dx)\right)}{4\left(\tan^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+2ab+3b^2\right)\arctan(\tan(c+dx))+\frac{3\left(a^2-b^2\right)\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)+\frac{(a-b)\tan(c+dx)\left(a+b\tan^2(c+dx)\right)}{4\left(\tan^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((a - b)*Tan[c + d*x]*(a + b*Tan[c + d*x]^2))/(4*(1 + Tan[c + d*x]^2)^2)
+ (((3*a^2 + 2*a*b + 3*b^2)*ArcTan[Tan[c + d*x]])/2 + (3*(a^2 - b^2)*Tan[c
 + d*x])/(2*(1 + Tan[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 16.84 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

method	result
risch	$\frac{3x\,a^2}{8}+\frac{xab}{4}+\frac{3x\,b^2}{8}+\frac{\sin(4dx+4c)a^2}{32d}-\frac{\sin(4dx+4c)ab}{16d}+\frac{\sin(4dx+4c)b^2}{32d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c}{4d}$
derivativedivides	$\frac{b^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\cos(dx+c)^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a^2}{d}$
default	$\frac{b^2\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\cos(dx+c)^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a^2}{d}$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8*x*a^2+1/4*x*a*b+3/8*x*b^2+1/32/d*sin(4*d*x+4*c)*a^2-1/16/d*sin(4*d*x+4
*c)*a*b+1/32/d*sin(4*d*x+4*c)*b^2+1/4/d*sin(2*d*x+2*c)*a^2-1/4/d*sin(2*d*x
+2*c)*b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$=\frac{(3a^2+2ab+3b^2)dx+\left(2\left(a^2-2ab+b^2\right)\cos(dx+c)^3+\left(3a^2+2ab-5b^2\right)\cos(dx+c)\right)\sin(dx+c)}{8d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((3*a^2 + 2*a*b + 3*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3 +
 (3*a^2 + 2*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx=\int \left(a+b\tan^2{(c+dx)}\right)^2\cos^4{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tan(c + d*x)**2)**2*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{(3\,a^2+2\,ab+3\,b^2)(dx+c)+\frac{(3\,a^2+2\,ab-5\,b^2)\tan(dx+c)^3+(5\,a^2-2\,ab-3\,b^2)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 2*a*b + 3*b^2)*(d*x + c) + ((3*a^2 + 2*a*b - 5*b^2)*tan(d*x
+ c)^3 + (5*a^2 - 2*a*b - 3*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x
 + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \cos^4(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{(3\,a^2+2\,ab+3\,b^2)(dx+c)}{8\,d} + \frac{3\,a^2\tan{(dx+c)}^3+2\,ab\tan{(dx+c)}^3-5\,b^2\tan{(dx+c)}^3+5\,a^2\tan{(dx+c)}-2\,ab\tan{(dx+c)}-3\,b}{8\left(\tan{(dx+c)}^2+1\right)^2d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/8*(3*a^2 + 2*a*b + 3*b^2)*(d*x + c)/d + 1/8*(3*a^2*tan(d*x + c)^3 + 2*a*
b*tan(d*x + c)^3 - 5*b^2*tan(d*x + c)^3 + 5*a^2*tan(d*x + c) - 2*a*b*tan(d
*x + c) - 3*b^2*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*d)
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \cos^4(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = x\left(\frac{3a^2}{8}+\frac{ab}{4}+\frac{3b^2}{8}\right) - \frac{\tan(c+dx)\left(-\frac{5a^2}{8}+\frac{ab}{4}+\frac{3b^2}{8}\right)-\tan(c+dx)^3\left(\frac{3a^2}{8}+\frac{ab}{4}-\frac{5b^2}{8}\right)}{d\left(\tan(c+dx)^4+2\tan(c+dx)^2+1\right)}$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
x*((a*b)/4 + (3*a^2)/8 + (3*b^2)/8) - (tan(c + d*x)*((a*b)/4 - (5*a^2)/8 +
 (3*b^2)/8) - tan(c + d*x)^3*((a*b)/4 + (3*a^2)/8 - (5*b^2)/8))/(d*(2*tan(
c + d*x)^2 + tan(c + d*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.65

$$\int \cos^4(c+dx) \left(a+b\tan^2(c+dx)\right)^2 dx = \frac{-2\cos(dx+c)\sin(dx+c)^3 a^2+4\cos(dx+c)\sin(dx+c)^3 ab-2\cos(dx+c)\sin(dx+c)^3 b^2+5\cos(}{}$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*a**2 + 4*cos(c + d*x)*sin(c + d*x)**3*a
*b - 2*cos(c + d*x)*sin(c + d*x)**3*b**2 + 5*cos(c + d*x)*sin(c + d*x)*a**
2 - 2*cos(c + d*x)*sin(c + d*x)*a*b - 3*cos(c + d*x)*sin(c + d*x)*b**2 + 3
*a**2*d*x + 2*a*b*d*x + 3*b**2*d*x)/(8*d)
```

3.451 $\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{1}{16}\left(5a^2+2ab+b^2\right)x + \frac{(5a^2+2ab+b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(a-b)(5a+7b)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{(a-b)^2\cos^5(c+dx)\sin(c+dx)}{6d}$$

output
```
1/16*(5*a^2+2*a*b+b^2)*x+1/16*(5*a^2+2*a*b+b^2)*cos(d*x+c)*sin(d*x+c)/d+1/
24*(a-b)*(5*a+7*b)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*(a-b)^2*cos(d*x+c)^5*sin(
d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx$$
$$= \frac{12((1-2i)a+b)((1+2i)a+b)(c+dx)+3(5a-b)(3a+b)\sin(2(c+dx))+3(a-b)(3a+b)\sin(4(c-}{192d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(12*((1 - 2*I)*a + b)*((1 + 2*I)*a + b)*(c + d*x) + 3*(5*a - b)*(3*a + b)*
Sin[2*(c + d*x)] + 3*(a - b)*(3*a + b)*Sin[4*(c + d*x)] + (a - b)^2*Sin[6*
(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4158, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\tan(c+dx)^2\right)^2}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a\right)^2}{\left(\tan^2(c+dx)+1\right)^4}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{1}{6}\int \frac{3b(a+b)\tan^2(c+dx)+a(5a+b)}{\left(\tan^2(c+dx)+1\right)^3}d\tan(c+dx)+\frac{(a-b)\tan(c+dx)\left(a+b\tan^2(c+dx)\right)}{6\left(\tan^2(c+dx)+1\right)^3}}{d}$$

$\big\downarrow$ 298

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+2ab+b^2\right)\int \frac{1}{\left(\tan^2(c+dx)+1\right)^2}d\tan(c+dx)+\frac{(a-b)(5a+3b)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\tan(c+dx)\left(a+b\tan^2(c+dx)\right)}{6\left(\tan^2(c+dx)+1\right)^3}}{d}$$

$\big\downarrow$ 215

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+2ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)+\frac{(a-b)(5a+3b)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\tan(c+dx)(}{6(\tan^2(c+}}{d}$$

$\big\downarrow$ 216

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+2ab+b^2\right)\left(\frac{1}{2}\arctan(\tan(c+dx))+\frac{\tan(c+dx)}{2\left(\tan^2(c+dx)+1\right)}\right)+\frac{(a-b)(5a+3b)\tan(c+dx)}{4\left(\tan^2(c+dx)+1\right)^2}\right)+\frac{(a-b)\tan(c+dx)(a+b\tan^2}{6\left(\tan^2(c+dx)+1\right)^3}}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((a - b)*Tan[c + d*x]*(a + b*Tan[c + d*x]^2))/(6*(1 + Tan[c + d*x]^2)^3)
+ (((a - b)*(5*a + 3*b)*Tan[c + d*x])/(4*(1 + Tan[c + d*x]^2)^2) + (3*(5*a
^2 + 2*a*b + b^2)*(ArcTan[Tan[c + d*x]]/2 + Tan[c + d*x]/(2*(1 + Tan[c + d
*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 58.92 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)^3\cos(dx+c)^3}{6}-\frac{\cos(dx+c)^3\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\frac{\left(\cos(\right.}{}\right.}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)^3\cos(dx+c)^3}{6}-\frac{\cos(dx+c)^3\sin(dx+c)}{8}+\frac{\cos(dx+c)\sin(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\frac{\left(\cos(\right.}{}\right.}{d}$
risch	$\frac{5x\,a^2}{16}+\frac{xab}{8}+\frac{x\,b^2}{16}+\frac{\sin(6dx+6c)a^2}{192d}-\frac{\sin(6dx+6c)ab}{96d}+\frac{\sin(6dx+6c)b^2}{192d}+\frac{3\sin(4dx+4c)a^2}{64d}-\frac{\sin(4dx+4}{32d}$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*
cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(-1/6*cos(d*x+c)^5*sin(d*x+c)
+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*(1/6*(
cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx$$
$$= \frac{3\left(5a^2+2ab+b^2\right)dx+\left(8\left(a^2-2ab+b^2\right)\cos\left(dx+c\right)^5+2\left(5a^2+2ab-7b^2\right)\cos\left(dx+c\right)^3+3\left(5a^2+\right.\right.}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/48*(3*(5*a^2 + 2*a*b + b^2)*d*x + (8*(a^2 - 2*a*b + b^2)*cos(d*x + c)^5
+ 2*(5*a^2 + 2*a*b - 7*b^2)*cos(d*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cos(d
*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.17

$$\int \cos^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{3\,(5\,a^2+2\,ab+b^2)(dx+c)+\frac{3\,(5\,a^2+2\,ab+b^2)\tan(dx+c)^5+8\,(5\,a^2+2\,ab-b^2)\tan(dx+c)^3+3\,(11\,a^2-2\,ab-b^2)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/48*(3*(5*a^2 + 2*a*b + b^2)*(d*x + c) + (3*(5*a^2 + 2*a*b + b^2)*tan(d*x
 + c)^5 + 8*(5*a^2 + 2*a*b - b^2)*tan(d*x + c)^3 + 3*(11*a^2 - 2*a*b - b^2
)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1)
)/d
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.34

$$\int \cos^6(c+dx) \left(a+b\tan^2(c+dx)\right)^2 \, dx = \frac{(5\,a^2+2\,ab+b^2)(dx+c)}{16\,d} + \frac{15\,a^2\tan\left(dx+c\right)^5+6\,ab\tan\left(dx+c\right)^5+3\,b^2\tan\left(dx+c\right)^5+40\,a^2\tan\left(dx+c\right)^3+16\,ab\tan\left(dx+c\right)^3}{48\,\left(\tan\left(dx+c\right)^2+1\right)^3}$$

input
```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/16*(5*a^2 + 2*a*b + b^2)*(d*x + c)/d + 1/48*(15*a^2*tan(d*x + c)^5 + 6*a
*b*tan(d*x + c)^5 + 3*b^2*tan(d*x + c)^5 + 40*a^2*tan(d*x + c)^3 + 16*a*b*
tan(d*x + c)^3 - 8*b^2*tan(d*x + c)^3 + 33*a^2*tan(d*x + c) - 6*a*b*tan(d*
x + c) - 3*b^2*tan(d*x + c))/((tan(d*x + c)^2 + 1)^3*d)
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = x\left(\frac{5\,a^2}{16}+\frac{a\,b}{8}+\frac{b^2}{16}\right) + \frac{\left(\frac{5\,a^2}{16}+\frac{a\,b}{8}+\frac{b^2}{16}\right)\tan(c+d\,x)^5+\left(\frac{5\,a^2}{6}+\frac{a\,b}{3}-\frac{b^2}{6}\right)\tan(c+d\,x)^3+\left(\frac{11\,a^2}{16}-\frac{a\,b}{8}-\frac{b^2}{16}\right)\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^6+3\tan\left(c+d\,x\right)^4+3\tan\left(c+d\,x\right)^2+1\right)}$$

input
```
int(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)^2,x)
```

output
```
x*((a*b)/8 + (5*a^2)/16 + b^2/16) + (tan(c + d*x)^3*((a*b)/3 + (5*a^2)/6 -
 b^2/6) - tan(c + d*x)*((a*b)/8 - (11*a^2)/16 + b^2/16) + tan(c + d*x)^5*(
(a*b)/8 + (5*a^2)/16 + b^2/16))/(d*(3*tan(c + d*x)^2 + 3*tan(c + d*x)^4 +
tan(c + d*x)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.68

$$\int \cos^6(c+dx)\left(a+b\tan^2(c+dx)\right)^2\,dx = \frac{8\cos\left(dx+c\right)\sin\left(dx+c\right)^5a^2-16\cos\left(dx+c\right)\sin\left(dx+c\right)^5ab+8\cos\left(dx+c\right)\sin\left(dx+c\right)^5b^2-26\cos}{}$$

input
```
int(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(8*cos(c + d*x)*sin(c + d*x)**5*a**2 - 16*cos(c + d*x)*sin(c + d*x)**5*a*b
 + 8*cos(c + d*x)*sin(c + d*x)**5*b**2 - 26*cos(c + d*x)*sin(c + d*x)**3*a
**2 + 28*cos(c + d*x)*sin(c + d*x)**3*a*b - 2*cos(c + d*x)*sin(c + d*x)**3
*b**2 + 33*cos(c + d*x)*sin(c + d*x)*a**2 - 6*cos(c + d*x)*sin(c + d*x)*a*
b - 3*cos(c + d*x)*sin(c + d*x)*b**2 + 15*a**2*d*x + 6*a*b*d*x + 3*b**2*d*
x)/(48*d)
```

3.452 $\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{(2a-3b)\operatorname{arctanh}(\sin(c+dx))}{2b^2d} + \frac{(a-b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

output
```
-1/2*(2*a-3*b)*arctanh(sin(d*x+c))/b^2/d+(a-b)^(3/2)*arctanh((a-b)^(1/2)*s
in(d*x+c)/a^(1/2))/a^(1/2)/b^2/d+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(90) = 180.

Time = 0.91 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.30

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{2(2a-3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(-2a+3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

output
```
(2*(2*a - 3*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-2*a + 3*b)*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*(a - b)^(3/2)*Log[Sqrt[a] - S
qrt[a - b]*Sin[c + d*x]])/Sqrt[a] + (2*(a - b)^(3/2)*Log[Sqrt[a] + Sqrt[a
- b]*Sin[c + d*x]])/Sqrt[a] + b/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 -
b/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*b^2*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4159, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int -\frac{(a-b)\sin^2(c+dx)+a-2b}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{2b}+\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)}-\frac{\int \frac{(a-b)\sin^2(c+dx)+a-2b}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{2b}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\sin(c+dx)}{2b(1-\sin^2(c+dx))}-\frac{\frac{(2a-3b)\int\frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)}{b}-\frac{2(a-b)^2\int\frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\sin(c+dx)}{2b(1-\sin^2(c+dx))}-\frac{\frac{(2a-3b)\text{arctanh}(\sin(c+dx))}{b}-\frac{2(a-b)^2\int\frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{2b}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\sin(c+dx)}{2b(1-\sin^2(c+dx))}-\frac{\frac{(2a-3b)\text{arctanh}(\sin(c+dx))}{b}-\frac{2(a-b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{2b}}{d}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

output
```
(-1/2*(((2*a - 3*b)*ArcTanh[Sin[c + d*x]])/b - (2*(a - b)^(3/2)*ArcTanh[(S
qrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/b + Sin[c + d*x]/(2*b*(1 -
 Sin[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 14.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{-\frac{\left(-a^2+2ab-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b^2\sqrt{a(a-b)}}-\frac{1}{4b(1+\sin(dx+c))}+\frac{(-2a+3b)\ln(1+\sin(dx+c))}{4b^2}-\frac{1}{4b(\sin(dx+c)-1)}+\frac{(2a-3b)\ln(\mathrm{si}}{4b}}{d}$
default	$\frac{-\frac{\left(-a^2+2ab-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b^2\sqrt{a(a-b)}}-\frac{1}{4b(1+\sin(dx+c))}+\frac{(-2a+3b)\ln(1+\sin(dx+c))}{4b^2}-\frac{1}{4b(\sin(dx+c)-1)}+\frac{(2a-3b)\ln(\mathrm{si}}{4b}}{d}$
risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{db\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+i\right)a}{d\,b^2}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{2db}+\frac{\ln\left(e^{i(dx+c)}-i\right)a}{d\,b^2}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{2db}+\frac{\sqrt{a}}{}$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-a^2+2*a*b-b^2)/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(
a-b))^(1/2))-1/4/b/(1+sin(d*x+c))+1/4/b^2*(-2*a+3*b)*ln(1+sin(d*x+c))-1/4/
b/(sin(d*x+c)-1)+1/4*(2*a-3*b)/b^2*ln(sin(d*x+c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.24

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \left[-\frac{2\,(a-b)\sqrt{\frac{a-b}{a}}\cos{(dx+c)}^2\log\left(-\frac{(a-b)\cos(dx+c)^2+2\,a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+(2\,a-3\,b)\cos{(dx+c)}^2\log\ldots}{4\,b^2d\cos{(dx+\ldots}} \right.$$

$$-\frac{4\,(a-b)\sqrt{-\frac{a-b}{a}}\arctan\left(\sqrt{-\frac{a-b}{a}}\sin{(dx+c)}\right)\cos{(dx+c)}^2+(2\,a-3\,b)\cos{(dx+c)}^2\log{(\sin{(dx+\ldots}}}{4\,b^2d\cos{(dx+c)}^2}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(a - b)*sqrt((a - b)/a)*cos(d*x + c)^2*log(-((a - b)*cos(d*x + c)
^2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 +
 b)) + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a - 3*b)*cos(
d*x + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*cos(d*x + c)^
2), -1/4*(4*(a - b)*sqrt(-(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c))
*cos(d*x + c)^2 + (2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a
- 3*b)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*b*sin(d*x + c))/(b^2*d*co
s(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{(2a-3b)\log(|\sin(dx+c)+1|)}{b^2} - \frac{(2a-3b)\log(|\sin(dx+c)-1|)}{b^2} - \frac{4\left(a^2-2ab+b^2\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2} + \frac{2\sin(dx+c)}{\left(\sin(dx+c)^2-1\right)}}{4d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/4*((2*a - 3*b)*log(abs(sin(d*x + c) + 1))/b^2 - (2*a - 3*b)*log(abs(sin
(d*x + c) - 1))/b^2 - 4*(a^2 - 2*a*b + b^2)*arctan(-(a*sin(d*x + c) - b*si
n(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*b^2) + 2*sin(d*x + c)/((si
n(d*x + c)^2 - 1)*b))/d
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.98

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\left(\frac{\operatorname{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,{(a-b)}^{3/2}\,1\mathrm{i}}{2}-a^{3/2}\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-a^{3/2}\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(2\,c+2\,d\,x\right)+\cdots}{\sqrt{a}\,b^2\,d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)} - \frac{\left(\frac{\sqrt{a}\,\sin(c+d\,x)\,1\mathrm{i}}{2}+\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(2\,c+2\,d\,x)}{2}\right)\,1\mathrm{i}}{\sqrt{a}\,b\,d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)),x)
```

output
```
- (((atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(3/2)*1i)/2 - a^(
3/2)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)) - a^(3/2)*atan((sin(
c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (cos(2*c + 2*d*x
)*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(3/2)*1i)/2)*1i)/(a^
(1/2)*b^2*d*(cos(2*c + 2*d*x)/2 + 1/2)) - (((a^(1/2)*sin(c + d*x)*1i)/2 +
(3*a^(1/2)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)))/2 + (3*a^(1/2
)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2)*1i
)/(a^(1/2)*b*d*(cos(2*c + 2*d*x)/2 + 1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 594, normalized size of antiderivative = 6.60

$$\int \frac{\sec^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2),x)
```

output
```
( - sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*ta
n((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a + sqrt(a)*sqrt(a - b)*log(
- 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*
sin(c + d*x)**2*b + sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)
/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a - sqrt(a)*sqrt(a - b)*log(
- 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*
b + sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((
c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a - sqrt(a)*sqrt(a - b)*log(2*sq
rt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c
+ d*x)**2*b - sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqr
t(a)*tan((c + d*x)/2)**2 + sqrt(a))*a + sqrt(a)*sqrt(a - b)*log(2*sqrt(a -
 b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*b + 2*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2 - 3*log(tan((c + d*x)/2) - 1)*sin
(c + d*x)**2*a*b - 2*log(tan((c + d*x)/2) - 1)*a**2 + 3*log(tan((c + d*x)/
2) - 1)*a*b - 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2 + 3*log(tan
((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b + 2*log(tan((c + d*x)/2) + 1)*a**2
- 3*log(tan((c + d*x)/2) + 1)*a*b - sin(c + d*x)*a*b)/(2*a*b**2*d*(sin(c +
 d*x)**2 - 1))
```

3.453 $\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

output
```
arctanh(sin(d*x+c))/b/d-(a-b)^(1/2)*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2)
)/a^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))-\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}}{bd}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

output
```
(ArcTanh[Sin[c + d*x]] - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/S
qrt[a]])/Sqrt[a])/(b*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4159, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^3}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{1}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 303}$$

$$\frac{\frac{\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)}{b}-\frac{(a-b)\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}(\sin(c+dx))}{b}-\frac{(a-b)\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\text{arctanh}(\sin(c+dx))}{b}-\frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{d}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])
/Sqrt[a]])/(Sqrt[a]*b))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{-\frac{(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b\sqrt{a(a-b)}}+\frac{\ln(1+\sin(dx+c))}{2b}-\frac{\ln(\sin(dx+c)-1)}{2b}}{d}$
default	$\frac{-\frac{(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{b\sqrt{a(a-b)}}+\frac{\ln(1+\sin(dx+c))}{2b}-\frac{\ln(\sin(dx+c)-1)}{2b}}{d}$
risch	$\frac{\ln(\mathrm{e}^{i(dx+c)}+i)}{db}-\frac{\ln(\mathrm{e}^{i(dx+c)}-i)}{db}+\frac{\sqrt{a(a-b)}\,\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{2i\sqrt{a(a-b)}\,\mathrm{e}^{i(dx+c)}}{a-b}-1\right)}{2adb}-\frac{\sqrt{a(a-b)}\,\ln\left(\mathrm{e}^{2i(dx+c)}\right.}{2a}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a-b)/b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))+1/
2/b*ln(1+sin(d*x+c))-1/2/b*ln(sin(d*x+c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \left[\frac{\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2\,a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(-\sin\left(dx+c\right)+1\right)}{2\,bd}, \right.$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*s
in(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + log(sin(d*x + c) +
1) - log(-sin(d*x + c) + 1))/(b*d), 1/2*(2*sqrt(-(a - b)/a)*arctan(sqrt(-(
a - b)/a)*sin(d*x + c)) + log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/
(b*d)]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec^3\left(c+dx\right)}{a+b\tan^2\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{2\,(a-b)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b} - \frac{\log(|\sin(dx+c)+1|)}{b} + \frac{\log(|\sin(dx+c)-1|)}{b}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(2*(a - b)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b)
)/(sqrt(-a^2 + a*b)*b) - log(abs(sin(d*x + c) + 1))/b + log(abs(sin(d*x +
c) - 1))/b)/d
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} - \frac{\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,\sqrt{a-b}}{\sqrt{a}\,b\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)),x)
```

output
```
(2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) - (atanh((sin(c + d
*x)*(a - b)^(1/2))/a^(1/2))*(a - b)^(1/2))/(a^(1/2)*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.22

$$\int \frac{\sec^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\sqrt{a}\,\sqrt{a-b}\log\left(-2\sqrt{a-b}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{a}\right)-\sqrt{a}\,\sqrt{a-b}\log\left(2\sqrt{a-b}\,\tan\left(\frac{dx}{2}\right.\right.}{2abd}$$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2),x)
```

output
```
(sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((
c + d*x)/2)**2 + sqrt(a)) - sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c +
 d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a)) - 2*log(tan((c + d*x)/2)
 - 1)*a + 2*log(tan((c + d*x)/2) + 1)*a)/(2*a*b*d)
```

3.454 $\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}d}$$

output
```
arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

output
```
ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4159, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

input
```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

output
```
ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{d\sqrt{a(a-b)}}$	36
default	$\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{d\sqrt{a(a-b)}}$	36
risch	$\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,d}-\frac{\ln\left(e^{2i(dx+c)}-\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,d}$	102

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.05

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx = \left[\frac{\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)}{2\sqrt{a^2-ab}d},\right.$$
$$\left.-\frac{\sqrt{-a^2+ab}\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)}{(a^2-ab)d}\right]$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a +
 b)/((a - b)*cos(d*x + c)^2 + b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*a
rctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a)/((a^2 - a*b)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a
*b)*d)
```

Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\operatorname{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)}{\sqrt{a}\,d\,\sqrt{a-b}}$$

input
```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x)^2)),x)
```

output
```
atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))/(a^(1/2)*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.40

$$\int \frac{\sec(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\sqrt{a}\,\sqrt{a-b}\,\left(-\log\left(-2\sqrt{a-b}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{a}\right)+\log\left(2\sqrt{a-b}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.\right.}{2ad\,(a-b)}$$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c)^2),x)
```

output
```
(sqrt(a)*sqrt(a - b)*( - log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*t
an((c + d*x)/2)**2 + sqrt(a)) + log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(
a)*tan((c + d*x)/2)**2 + sqrt(a))))/(2*a*d*(a - b))
```

3.455 $\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}d}+\frac{\sin(c+dx)}{(a-b)d}$$

output
```
-b*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(3/2)/d+sin(d*x+c
)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}d}+\frac{\sin(c+dx)}{(a-b)d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

output
```
-((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Sin[c + d*x]/((a - b)*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sec(c+dx)\left(a+b\tan(c+dx)^2\right)}dx$$

$$\Big\downarrow \ 4159$$

$$\frac{\int \frac{1-\sin^2(c+dx)}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\sin(c+dx)}{a-b}-\frac{b\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{a-b}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\sin(c+dx)}{a-b}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}}}{d}$$

input
```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2))
) + Sin[c + d*x]/(a - b))/d
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{\frac{\sin(dx+c)}{a-b}-\frac{b\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)\sqrt{a(a-b)}}}{d}$	61
default	$\frac{\frac{\sin(dx+c)}{a-b}-\frac{b\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)\sqrt{a(a-b)}}}{d}$	61
risch	$-\frac{i e^{i(dx+c)}}{2(a-b)d}+\frac{i e^{-i(dx+c)}}{2(a-b)d}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,(a-b)d}-\frac{b\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,(a-b)d}$	162

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(sin(d*x+c)/(a-b)-1/(a-b)*b/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(
a*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.03

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = \left[-\frac{\sqrt{a^2-ab}\,b\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\,(a^2-ab)\sin(dx+c)}{2\,(a^3-2\,a^2b+ab^2)d}, \frac{\sqrt{-a^2+ab}\,b\arctan\right.$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a^2 - a*b)*b*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*
sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(a^2 - a*b)*sin(
d*x + c))/((a^3 - 2*a^2*b + a*b^2)*d), (sqrt(-a^2 + a*b)*b*arctan(sqrt(-a^
2 + a*b)*sin(d*x + c)/a) + (a^2 - a*b)*sin(d*x + c))/((a^3 - 2*a^2*b + a*b
^2)*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{b\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}(a-b)} - \frac{\sin(dx+c)}{a-b}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-(b*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2
 + a*b)*(a - b)) - sin(d*x + c)/(a - b))/d
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\sin(c+d\,x)}{d\,(a-b)} + \frac{b\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,(a-b)^{3/2}}{\sqrt{a}\,b-a^{3/2}}\right)}{\sqrt{a}\,d\,(a-b)^{3/2}}$$

input
```
int(cos(c + d*x)/(a + b*tan(c + d*x)^2),x)
```

output
```
sin(c + d*x)/(d*(a - b)) + (b*atanh((sin(c + d*x)*(a - b)^(3/2))/(a^(1/2)*
b - a^(3/2))))/(a^(1/2)*d*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.23

$$\int \frac{\cos(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$= \frac{\sqrt{a}\,\sqrt{a-b}\log\Big(-2\sqrt{a-b}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{a}\Big)\,b-\sqrt{a}\,\sqrt{a-b}\log\Big(2\sqrt{a-b}\,\tan\left(\frac{dx}{2}\right.}{2ad\,(a^2-2ab+b^2)}$$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c)^2),x)
```

output
```
(sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((
c + d*x)/2)**2 + sqrt(a))*b - sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c
 + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*b + 2*sin(c + d*x)*a**
2 - 2*sin(c + d*x)*a*b)/(2*a*d*(a**2 - 2*a*b + b**2))
```

3.456 $\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}d}+\frac{(a-2b)\sin(c+dx)}{(a-b)^2d}-\frac{\sin^3(c+dx)}{3(a-b)d}$$

output
```
b^2*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(5/2)/d+(a-2*b)*
sin(d*x+c)/(a-b)^2/d-1/3*sin(d*x+c)^3/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\frac{6b^2\left(-\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)+\log\left(\sqrt{a}+\sqrt{a-b}\sin(c+dx)\right)\right)}{\sqrt{a}(a-b)^{5/2}}+\frac{3(3a-7b)\sin(c+dx)}{(a-b)^2}+\frac{\sin(3(c+dx))}{a-b}}{12d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

output
```
((6*b^2*(-Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] + Log[Sqrt[a] + Sqrt[a -
 b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(5/2)) + (3*(3*a - 7*b)*Sin[c + d*x])
/(a - b)^2 + Sin[3*(c + d*x)]/(a - b))/(12*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)^3\left(a+b\tan(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{\left(1-\sin^2(c+dx)\right)^2}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(\frac{b^2}{(a-b)^2\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\sin^2(c+dx)}{a-b}+\frac{a-2b}{(a-b)^2}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^2\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}}-\frac{\sin^3(c+dx)}{3(a-b)}+\frac{(a-2b)\sin(c+dx)}{(a-b)^2}}{d}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2),x]
```

output
```
((b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2))
 + ((a - 2*b)*Sin[c + d*x])/(a - b)^2 - Sin[c + d*x]^3/(3*(a - b)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(dx+c)^3}{3}-\frac{b\sin(dx+c)^3}{3}-\sin(dx+c)a+2b\sin(dx+c)}{(a-b)^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)^2\sqrt{a(a-b)}}}{d}$
default	$\frac{-\frac{\frac{a\sin(dx+c)^3}{3}-\frac{b\sin(dx+c)^3}{3}-\sin(dx+c)a+2b\sin(dx+c)}{(a-b)^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{(a-b)^2\sqrt{a(a-b)}}}{d}$
risch	$-\frac{3i\mathrm{e}^{i(dx+c)}a}{8(a-b)^2d}+\frac{7i\mathrm{e}^{i(dx+c)}b}{8(a-b)^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a}{8(a-b)^2d}-\frac{7i\mathrm{e}^{-i(dx+c)}b}{8(a-b)^2d}+\frac{b^2\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\,\mathrm{e}^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{2\sqrt{a^2-ab}\,(a-b)^2d}-\frac{b^2\ln\left(\mathrm{e}^{2}\right.}{}$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a-b)^2*(1/3*a*sin(d*x+c)^3-1/3*b*sin(d*x+c)^3-sin(d*x+c)*a+2*b*si
n(d*x+c))+b^2/(a-b)^2/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.14

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

$$= \left[\frac{3\sqrt{a^2-ab}b^2\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(2a^3-7a^2b+5ab^2+(a^3-2a^2b+ab^2)\right.}{6\left(a^4-3a^3b+3a^2b^2-ab^3\right)d}\right.$$

$$-\frac{3\sqrt{-a^2+ab}b^2\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)-\left(2a^3-7a^2b+5ab^2+(a^3-2a^2b+ab^2)\cos(dx+c)^2\right)\mathrm{si}}{3\left(a^4-3a^3b+3a^2b^2-ab^3\right)d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a^2 - a*b)*b^2*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*
b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(2*a^3 - 7*a^
2*b + 5*a*b^2 + (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^
4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d), -1/3*(3*sqrt(-a^2 + a*b)*b^2*arctan(s
qrt(-a^2 + a*b)*sin(d*x + c)/a) - (2*a^3 - 7*a^2*b + 5*a*b^2 + (a^3 - 2*a^
2*b + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a
*b^3)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(78) = 156.

Time = 0.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.83

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\frac{3\,b^2\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{-a^2+ab}} - \frac{a^2\sin(dx+c)^3-2\,ab\sin(dx+c)^3+b^2\sin(dx+c)^3-3\,a^2\sin(dx+c)+9\,ab\sin(dx+c)-6\,b^2\sin(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*b^2*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a
^2 - 2*a*b + b^2)*sqrt(-a^2 + a*b)) - (a^2*sin(d*x + c)^3 - 2*a*b*sin(d*x
+ c)^3 + b^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c) + 9*a*b*sin(d*x + c) - 6*
b^2*sin(d*x + c))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3))/d
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.85

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a-2b)}{a^2-2ab+b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-4b)}{3(a^2-2ab+b^2)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-2b)}{a^2-2ab+b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} - \frac{b^2\,\mathrm{atan}\left(\frac{2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}{\sqrt{a}\,(a-b)^{5/2}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}\right)1\mathrm{i}}{\sqrt{a}\,d\,(a-b)^{5/2}}$$

input
```
int(cos(c + d*x)^3/(a + b*tan(c + d*x)^2),x)
```

output
```
((2*tan(c/2 + (d*x)/2)*(a - 2*b))/(a^2 - 2*a*b + b^2) + (4*tan(c/2 + (d*x)
/2)^3*(a - 4*b))/(3*(a^2 - 2*a*b + b^2)) + (2*tan(c/2 + (d*x)/2)^5*(a - 2*
b))/(a^2 - 2*a*b + b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)
^4 + tan(c/2 + (d*x)/2)^6 + 1)) - (b^2*atan((a^3*tan(c/2 + (d*x)/2)*2i - b
^3*tan(c/2 + (d*x)/2)*2i + a*b^2*tan(c/2 + (d*x)/2)*6i - a^2*b*tan(c/2 + (
d*x)/2)*6i)/(a^(1/2)*(a - b)^(5/2)*(tan(c/2 + (d*x)/2)^2 + 1)))*1i)/(a^(1/
2)*d*(a - b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.32

$$\int \frac{\cos^3(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{-3\sqrt{a}\,\sqrt{a-b}\log\left(-2\sqrt{a-b}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{a}\right)b^2+3\sqrt{a}\,\sqrt{a-b}\log\left(2\sqrt{a-b}\,\mathrm{ta}\right.}{}$$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2),x)
```

output
```
( - 3*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*
tan((c + d*x)/2)**2 + sqrt(a))*b**2 + 3*sqrt(a)*sqrt(a - b)*log(2*sqrt(a -
 b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*b**2 - 2*sin
(c + d*x)**3*a**3 + 4*sin(c + d*x)**3*a**2*b - 2*sin(c + d*x)**3*a*b**2 +
6*sin(c + d*x)*a**3 - 18*sin(c + d*x)*a**2*b + 12*sin(c + d*x)*a*b**2)/(6*
a*d*(a**3 - 3*a**2*b + 3*a*b**2 - b**3))
```

3.457 $\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{b^3\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\sin(c+dx)}{(a-b)^3d} - \frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2d} + \frac{\sin^5(c+dx)}{5(a-b)d}$$

output
```
-b^3*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(1/2)/(a-b)^(7/2)/d+(a^2-3*
a*b+3*b^2)*sin(d*x+c)/(a-b)^3/d-1/3*(2*a-3*b)*sin(d*x+c)^3/(a-b)^2/d+1/5*s
in(d*x+c)^5/(a-b)/d
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\frac{120b^3\left(\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{a-b}\sin(c+dx)\right)\right)}{\sqrt{a}(a-b)^{7/2}} + \frac{30(5a^2-16ab+19b^2)\sin(c+dx)}{(a-b)^3} + \frac{5(5a-9b)\sin(3(c+dx))}{(a-b)^2} + \frac{3\sin(5(c+}{a-b}}{240d}$$

input
```
Integrate[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

output
```
((120*b^3*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a] + Sqrt[a
- b]*Sin[c + d*x]]))/(Sqrt[a]*(a - b)^(7/2)) + (30*(5*a^2 - 16*a*b + 19*b^
2)*Sin[c + d*x])/(a - b)^3 + (5*(5*a - 9*b)*Sin[3*(c + d*x)])/(a - b)^2 +
(3*Sin[5*(c + d*x)])/(a - b))/(240*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^5\left(a+b\tan(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(1-\sin^2(c+dx)\right)^3}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\sin^4(c+dx)}{a-b}-\frac{(2a-3b)\sin^2(c+dx)}{(a-b)^2}+\frac{a^2-3ba+3b^2}{(a-b)^3}-\frac{b^3}{(a-b)^3\left(a-(a-b)\sin^2(c+dx)\right)}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a^2-3ab+3b^2)\sin(c+dx)}{(a-b)^3}-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}}+\frac{\sin^5(c+dx)}{5(a-b)}-\frac{(2a-3b)\sin^3(c+dx)}{3(a-b)^2}}{d}$$

input
```
Int[Cos[c + d*x]^5/(a + b*Tan[c + d*x]^2),x]
```

output
```
(-((b^3*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2
))) + ((a^2 - 3*a*b + 3*b^2)*Sin[c + d*x])/(a - b)^3 - ((2*a - 3*b)*Sin[c
+ d*x]^3)/(3*(a - b)^2) + Sin[c + d*x]^5/(5*(a - b)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 23.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{\frac{a^2\sin(dx+c)^5}{5}-\frac{2ab\sin(dx+c)^5}{5}+\frac{\sin(dx+c)^5b^2}{5}-\frac{2\sin(dx+c)^3a^2}{3}+\frac{5ab\sin(dx+c)^3}{3}-\sin(dx+c)^3b^2+\sin(dx+c)a^2-3ab\sin(dx+c)+}{(a-b)^3}}{d}$
default	$\frac{\frac{\frac{a^2\sin(dx+c)^5}{5}-\frac{2ab\sin(dx+c)^5}{5}+\frac{\sin(dx+c)^5b^2}{5}-\frac{2\sin(dx+c)^3a^2}{3}+\frac{5ab\sin(dx+c)^3}{3}-\sin(dx+c)^3b^2+\sin(dx+c)a^2-3ab\sin(dx+c)+}{(a-b)^3}}{d}$
risch	$-\frac{5i e^{i(dx+c)}a^2}{16(a-b)^3d}+\frac{i e^{i(dx+c)}ab}{(a-b)^3d}-\frac{19i e^{i(dx+c)}b^2}{16(a-b)^3d}+\frac{5i e^{-i(dx+c)}a^2}{16(a-b)(a^2-2ab+b^2)d}-\frac{i e^{-i(dx+c)}ab}{(a-b)(a^2-2ab+b^2)d}+\frac{19i e^{-}}{16(a-b)(}$

input
```
int(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)^3*(1/5*a^2*sin(d*x+c)^5-2/5*a*b*sin(d*x+c)^5+1/5*sin(d*x+c)^5
*b^2-2/3*sin(d*x+c)^3*a^2+5/3*a*b*sin(d*x+c)^3-sin(d*x+c)^3*b^2+sin(d*x+c)
*a^2-3*a*b*sin(d*x+c)+3*sin(d*x+c)*b^2)-b^3/(a-b)^3/(a*(a-b))^(1/2)*arctan
h((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 395, normalized size of antiderivative = 3.13

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \left[-\frac{15\sqrt{a^2-ab}b^3\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2\left(3\left(a^4-3a^3b+3a^2b^2-ab^3\right)\cos\left(dx+\right.\right.}{30\left(a^5-4a^4b+6a\right.}\right.$$

input
```
integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/30*(15*sqrt(a^2 - a*b)*b^3*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 -
 a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(3*(a^4 -
3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^
2 - 33*a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d), 1/15*(15
*sqrt(-a^2 + a*b)*b^3*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (3*(a^4 -
3*a^3*b + 3*a^2*b^2 - a*b^3)*cos(d*x + c)^4 + 8*a^4 - 34*a^3*b + 59*a^2*b^
2 - 33*a*b^3 + (4*a^4 - 17*a^3*b + 22*a^2*b^2 - 9*a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a+b*tan(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(114) = 228$.

Time = 0.47 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.53

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{15\,b^3\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{-a^2+ab}} - \frac{3\,a^4\sin(dx+c)^5-12\,a^3b\sin(dx+c)^5+18\,a^2b^2\sin(dx+c)^5-12\,ab^3\sin(dx+c)^5+3\,b^4\sin(dx+c)^5-}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/15*(15*b^3*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/
((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a^2 + a*b)) - (3*a^4*sin(d*x + c)^5
 - 12*a^3*b*sin(d*x + c)^5 + 18*a^2*b^2*sin(d*x + c)^5 - 12*a*b^3*sin(d*x
+ c)^5 + 3*b^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^3 + 45*a^3*b*sin(d*x +
 c)^3 - 75*a^2*b^2*sin(d*x + c)^3 + 55*a*b^3*sin(d*x + c)^3 - 15*b^4*sin(d
*x + c)^3 + 15*a^4*sin(d*x + c) - 75*a^3*b*sin(d*x + c) + 150*a^2*b^2*sin(
d*x + c) - 135*a*b^3*sin(d*x + c) + 45*b^4*sin(d*x + c))/(a^5 - 5*a^4*b +
10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5))/d
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 1493, normalized size of antiderivative = 11.85

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^5/(a + b*tan(c + d*x)^2),x)
```

output
```
((2*tan(c/2 + (d*x)/2)*(a^2 - 3*a*b + 3*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b
^3) + (tan(c/2 + (d*x)/2)^9*(2*a^2 - 6*a*b + 6*b^2))/(3*a*b^2 - 3*a^2*b +
a^3 - b^3) + (tan(c/2 + (d*x)/2)^3*((8*a^2)/3 - (32*a*b)/3 + 16*b^2))/(3*a
*b^2 - 3*a^2*b + a^3 - b^3) + (tan(c/2 + (d*x)/2)^7*((8*a^2)/3 - (32*a*b)/
3 + 16*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (tan(c/2 + (d*x)/2)^5*((116
*a^2)/15 - (332*a*b)/15 + (132*b^2)/5))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(
d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2
)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (b^3*atan(((b
^3*((tan(c/2 + (d*x)/2)*(16*a*b^10 - 96*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^
7 + 240*a^5*b^6 - 96*a^6*b^5 + 16*a^7*b^4))/2 + (b^3*(tan(c/2 + (d*x)/2)^2
*(4*a^12 - 44*a^11*b + 8*a^2*b^10 - 76*a^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7
 + 1344*a^6*b^6 - 1512*a^7*b^5 + 1176*a^8*b^4 - 624*a^9*b^3 + 216*a^10*b^2
) + 36*a^11*b - 4*a^12 + 4*a^3*b^9 - 36*a^4*b^8 + 144*a^5*b^7 - 336*a^6*b^
6 + 504*a^7*b^5 - 504*a^8*b^4 + 336*a^9*b^3 - 144*a^10*b^2))/(a^(1/2)*(a -
 b)^(7/2)))*1i)/(a^(1/2)*(a - b)^(7/2)) + (b^3*((tan(c/2 + (d*x)/2)*(16*a*
b^10 - 96*a^2*b^9 + 240*a^3*b^8 - 320*a^4*b^7 + 240*a^5*b^6 - 96*a^6*b^5 +
 16*a^7*b^4))/2 - (b^3*(tan(c/2 + (d*x)/2)^2*(4*a^12 - 44*a^11*b + 8*a^2*b
^10 - 76*a^3*b^9 + 324*a^4*b^8 - 816*a^5*b^7 + 1344*a^6*b^6 - 1512*a^7*b^5
 + 1176*a^8*b^4 - 624*a^9*b^3 + 216*a^10*b^2) + 36*a^11*b - 4*a^12 + 4*a^3
*b^9 - 36*a^4*b^8 + 144*a^5*b^7 - 336*a^6*b^6 + 504*a^7*b^5 - 504*a^8*b...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.36

$$\int \frac{\cos^5(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$= \frac{15\sqrt{a}\,\sqrt{a-b}\log\left(-2\sqrt{a-b}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{a}\right)b^3-15\sqrt{a}\,\sqrt{a-b}\log\left(2\sqrt{a-b}\,\text{t}\right.}{}$$

input
```
int(cos(d*x+c)^5/(a+b*tan(d*x+c)^2),x)
```

output
```
(15*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*ta
n((c + d*x)/2)**2 + sqrt(a))*b**3 - 15*sqrt(a)*sqrt(a - b)*log(2*sqrt(a -
b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*b**3 + 6*sin(
c + d*x)**5*a**4 - 18*sin(c + d*x)**5*a**3*b + 18*sin(c + d*x)**5*a**2*b**
2 - 6*sin(c + d*x)**5*a*b**3 - 20*sin(c + d*x)**3*a**4 + 70*sin(c + d*x)**
3*a**3*b - 80*sin(c + d*x)**3*a**2*b**2 + 30*sin(c + d*x)**3*a*b**3 + 30*s
in(c + d*x)*a**4 - 120*sin(c + d*x)*a**3*b + 180*sin(c + d*x)*a**2*b**2 -
90*sin(c + d*x)*a*b**3)/(30*a*d*(a**4 - 4*a**3*b + 6*a**2*b**2 - 4*a*b**3
+ b**4))
```

3.458 $\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{(a-b)^3\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d} + \frac{(a^2-3ab+3b^2)\tan(c+dx)}{b^3d} - \frac{(a-3b)\tan^3(c+dx)}{3b^2d} + \frac{\tan^5(c+dx)}{5bd}$$

output
```
-(a-b)^3*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(1/2)/b^(7/2)/d+(a^2-3*a*b+3
*b^2)*tan(d*x+c)/b^3/d-1/3*(a-3*b)*tan(d*x+c)^3/b^2/d+1/5*tan(d*x+c)^5/b/d
```

Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{-\frac{15(a-b)^3\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + \sqrt{b}(15a^2-40ab+33b^2-(5a-9b)b\sec^2(c+dx)+3b^2\sec^4(c+dx))\tan(c}{15b^{7/2}d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

output
```
((-15*(a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*
(15*a^2 - 40*a*b + 33*b^2 - (5*a - 9*b)*b*Sec[c + d*x]^2 + 3*b^2*Sec[c + d
*x]^4)*Tan[c + d*x])/(15*b^(7/2)*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^3}{b\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\tan^4(c+dx)}{b}-\frac{(a-3b)\tan^2(c+dx)}{b^2}+\frac{a^2-3ba+3b^2}{b^3}+\frac{-a^3+3ba^2-3b^2a+b^3}{b^3\left(b\tan^2(c+dx)+a\right)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(a^2-3ab+3b^2\right)\tan(c+dx)}{b^3}-\frac{(a-b)^3\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{(a-3b)\tan^3(c+dx)}{3b^2}+\frac{\tan^5(c+dx)}{5b}}{d}$$

input
```
Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2),x]
```

output
```
(-(((a - b)^3*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2))) +
 ((a^2 - 3*a*b + 3*b^2)*Tan[c + d*x])/b^3 - ((a - 3*b)*Tan[c + d*x]^3)/(3*
b^2) + Tan[c + d*x]^5/(5*b))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 87.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{\tan(dx+c)^5 b^2}{5}-\frac{ab\tan(dx+c)^3}{3}+\tan(dx+c)^3 b^2+a^2\tan(dx+c)-3ab\tan(dx+c)+3\tan(dx+c)b^2}{b^3}+\frac{\left(-a^3+3a^2b-3a\,b^2+b^3\right)\arctan(\cdots}{b^3\sqrt{ab}}}{d}$
default	$\frac{\frac{\frac{\tan(dx+c)^5 b^2}{5}-\frac{ab\tan(dx+c)^3}{3}+\tan(dx+c)^3 b^2+a^2\tan(dx+c)-3ab\tan(dx+c)+3\tan(dx+c)b^2}{b^3}+\frac{\left(-a^3+3a^2b-3a\,b^2+b^3\right)\arctan(\cdots}{b^3\sqrt{ab}}}{d}$
risch	$\frac{2i(15a^2e^{8i(dx+c)}-30ab\,e^{8i(dx+c)}+15b^2e^{8i(dx+c)}+60a^2e^{6i(dx+c)}-150ab\,e^{6i(dx+c)}+90b^2e^{6i(dx+c)}+90a^2e^{4i(dx+c)}-25\cdots}{15d\,b^3(e^{2i(dx+c)}+1\cdots}$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*(1/5*tan(d*x+c)^5*b^2-1/3*a*b*tan(d*x+c)^3+tan(d*x+c)^3*b^2+a^2
*tan(d*x+c)-3*a*b*tan(d*x+c)+3*tan(d*x+c)*b^2)+(-a^3+3*a^2*b-3*a*b^2+b^3)/
b^3/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.94

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$= \left[\frac{15\,(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{-ab}\cos(dx+c)^5\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\,\left((a+b)\cos(dx\right.}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/60*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a*b)*cos(d*x + c)^5*log(((
a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a
 + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^
2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4
*((15*a^3*b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2
 - 9*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5), 1/30*(
15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x +
 c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*(
(15*a^3*b - 40*a^2*b^2 + 33*a*b^3)*cos(d*x + c)^4 + 3*a*b^3 - (5*a^2*b^2 -
 9*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^5)]
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**8/(a + b*tan(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{15\,(a^3-3\,a^2b+3\,ab^2-b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}-\frac{3\,b^2\tan(dx+c)^5-5\,(ab-3\,b^2)\tan(dx+c)^3+15\,(a^2-3\,ab+3\,b^2)\tan(dx+c)}{b^3}}{15\,d}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))
/(sqrt(a*b)*b^3) - (3*b^2*tan(d*x + c)^5 - 5*(a*b - 3*b^2)*tan(d*x + c)^3
+ 15*(a^2 - 3*a*b + 3*b^2)*tan(d*x + c))/b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{(a^3-3\,a^2b+3\,ab^2-b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3d} + \frac{3\,b^4d^4\tan(dx+c)^5-5\,ab^3d^4\tan(dx+c)^3+15\,b^4d^4\tan(dx+c)^3+15\,a^2b^2d^4\tan(dx+c)-45\,ab^3d^4}{15\,b^5d^5}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*
b)*b^3*d) + 1/15*(3*b^4*d^4*tan(d*x + c)^5 - 5*a*b^3*d^4*tan(d*x + c)^3 +
15*b^4*d^4*tan(d*x + c)^3 + 15*a^2*b^2*d^4*tan(d*x + c) - 45*a*b^3*d^4*tan
(d*x + c) + 45*b^4*d^4*tan(d*x + c))/(b^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\tan(c+d\,x)\,\left(\frac{3}{b}+\frac{a\,\left(\frac{a}{b^2}-\frac{3}{b}\right)}{b}\right)}{d} + \frac{{\tan(c+d\,x)}^5}{5\,b\,d} - \frac{{\tan(c+d\,x)}^3\,\left(\frac{a}{3\,b^2}-\frac{1}{b}\right)}{d} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)\,{(a-b)}^3}{\sqrt{a}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}\right)\,{(a-b)}^3}{\sqrt{a}\,b^{7/2}\,d}$$

input
```
int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x)^2)),x)
```

output
```
(tan(c + d*x)*(3/b + (a*(a/b^2 - 3/b))/b))/d + tan(c + d*x)^5/(5*b*d) - (t
an(c + d*x)^3*(a/(3*b^2) - 1/b))/d - (atan((b^(1/2)*tan(c + d*x)*(a - b)^3
)/(a^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))*(a - b)^3)/(a^(1/2)*b^(7/2)*d
)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1280, normalized size of antiderivative = 11.85

$$\int \frac{\sec^8(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*cos(c + d*x)*sin(c + d*x)**4*a**3 - 45*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
- sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**4*a**2*b +
 45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*cos(c + d*x)*sin(c + d*x)**4*a*b**2 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b
) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**4*b**3 -
 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*cos(c + d*x)*sin(c + d*x)**2*a**3 + 90*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
- sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a**2*b -
 90*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*cos(c + d*x)*sin(c + d*x)**2*a*b**2 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a - b
) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*b**3 +
 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*cos(c + d*x)*a**3 - 45*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c
 + d*x)/2))/sqrt(b))*cos(c + d*x)*a**2*b + 45*sqrt(b)*sqrt(a)*atan((sqrt(a
 - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a*b**2 - 15*sqrt(b
)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d
*x)*b**3 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2)
)/sqrt(b))*cos(c + d*x)*sin(c + d*x)**4*a**3 + 45*sqrt(b)*sqrt(a)*atan((sq
rt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x...
```

3.459 $\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{(a-b)^2\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{(a-2b)\tan(c+dx)}{b^2d} + \frac{\tan^3(c+dx)}{3bd}$$

output
```
(a-b)^2*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(1/2)/b^(5/2)/d-(a-2*b)*tan(d
*x+c)/b^2/d+1/3*tan(d*x+c)^3/b/d
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\frac{3(a-b)^2\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + \sqrt{b}(-3a+5b+b\sec^2(c+dx))\tan(c+dx)}{3b^{5/2}d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
```

output
```
((3*(a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a] + Sqrt[b]*(-
3*a + 5*b + b*Sec[c + d*x]^2)*Tan[c + d*x])/(3*b^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)^6}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \ 4158$$

$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^2}{b\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left(\frac{\tan^2(c+dx)}{b}+\frac{a^2-2ba+b^2}{b^2\left(b\tan^2(c+dx)+a\right)}-\frac{a-2b}{b^2}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{(a-b)^2\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a-2b)\tan(c+dx)}{b^2}+\frac{\tan^3(c+dx)}{3b}}{d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2),x]
```

output
```
(((a - b)^2*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((
a - 2*b)*Tan[c + d*x])/b^2 + Tan[c + d*x]^3/(3*b))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 26.82 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(dx+c)^3}{3}+a\tan(dx+c)-2b\tan(dx+c)}{b^2}+\frac{\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}}{d}$
default	$\frac{-\frac{-\frac{b\tan(dx+c)^3}{3}+a\tan(dx+c)-2b\tan(dx+c)}{b^2}+\frac{\left(a^2-2ab+b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}}{d}$
risch	$-\frac{2i\left(3a\,e^{4i(dx+c)}-3b\,e^{4i(dx+c)}+6a\,e^{2i(dx+c)}-12b\,e^{2i(dx+c)}+3a-5b\right)}{3d\,b^2\left(e^{2i(dx+c)}+1\right)^3}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)a^2}{2\sqrt{-ab}\,d\,b^2}+$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/3*b*tan(d*x+c)^3+a*tan(d*x+c)-2*b*tan(d*x+c))+(a^2-2*a*b+b
^2)/b^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.40

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

$$= \left[-\frac{3\,(a^2-2\,ab+b^2)\sqrt{-ab}\cos{(dx+c)}^3\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\,\left((a+b)\cos(dx+c)^3-b\cos\right.}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+}\right.}{12\,ab^3d\cos{(dx+c)}^3}\right.$$

$$-\frac{3\,(a^2-2\,ab+b^2)\sqrt{ab}\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)\cos{(dx+c)}^3-2\,\left(ab^2-(3\,a^2b-5\,ab^2)\cos{(dx}\right.}{6\,ab^3d\cos{(dx+c)}^3}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^2 - 2*a*b + b^2)*sqrt(-a*b)*cos(d*x + c)^3*log(((a^2 + 6*a*b
+ b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*
x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b +
b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*(a*b^2 - (3
*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3),
-1/6*(3*(a^2 - 2*a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 -
 b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^3 - 2*(a*b^2 -
 (3*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^3*d*cos(d*x + c)^3
)]
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec^6{(c+dx)}}{a+b\tan^2{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\frac{3\,(a^2-2\,ab+b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{b\tan(dx+c)^3-3\,(a-2\,b)\tan(dx+c)}{b^2}}{3\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*(3*(a^2 - 2*a*b + b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b^2
) + (b*tan(d*x + c)^3 - 3*(a - 2*b)*tan(d*x + c))/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{(a^2-2\,ab+b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2d} + \frac{b^2d^2\tan\left(dx+c\right)^3-3\,abd^2\tan\left(dx+c\right)+6\,b^2d^2\tan\left(dx+c\right)}{3\,b^3d^3}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
(a^2 - 2*a*b + b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b^2*d) + 1
/3*(b^2*d^2*tan(d*x + c)^3 - 3*a*b*d^2*tan(d*x + c) + 6*b^2*d^2*tan(d*x +
c))/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\tan(c+d\,x)^3}{3\,b\,d} - \frac{\tan(c+d\,x)\,\left(\frac{a}{b^2}-\frac{2}{b}\right)}{d} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(c+d\,x)\,(a-b)^2}{\sqrt{a}\,(a^2-2\,a\,b+b^2)}\right)\,(a-b)^2}{\sqrt{a}\,b^{5/2}\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)),x)
```

output
```
tan(c + d*x)^3/(3*b*d) - (tan(c + d*x)*(a/b^2 - 2/b))/d + (atan((b^(1/2)*t
an(c + d*x)*(a - b)^2)/(a^(1/2)*(a^2 - 2*a*b + b^2)))*(a - b)^2)/(a^(1/2)*
b^(5/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 614, normalized size of antiderivative = 7.97

$$\int \frac{\sec^6(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b
))*cos(c + d*x)*sin(c + d*x)**2*a**2 + 6*sqrt(b)*sqrt(a)*atan((sqrt(a - b)
 - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a*b - 3
*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*co
s(c + d*x)*sin(c + d*x)**2*b**2 + 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sq
rt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a**2 - 6*sqrt(b)*sqrt(a)*ata
n((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a*b + 3*s
qrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(
c + d*x)*b**2 + 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x
)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a**2 - 6*sqrt(b)*sqrt(a)*atan(
(sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x
)**2*a*b + 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))
/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*b**2 - 3*sqrt(b)*sqrt(a)*atan((sqrt
(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a**2 + 6*sqrt(b)
*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*
x)*a*b - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/s
qrt(b))*cos(c + d*x)*b**2 - 3*sin(c + d*x)**3*a**2*b + 5*sin(c + d*x)**3*a
*b**2 + 3*sin(c + d*x)*a**2*b - 6*sin(c + d*x)*a*b**2)/(3*cos(c + d*x)*a*b
**3*d*(sin(c + d*x)**2 - 1))
```

3.460 $\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx=-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}+\frac{\tan(c+dx)}{bd}$$

output
```
-(a-b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(1/2)/b^(3/2)/d+tan(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx=-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}+\frac{\tan(c+dx)}{bd}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

output
```
-(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) +
Tan[c + d*x]/(b*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4158, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^4}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \frac{\tan^2(c+dx)+1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\tan(c+dx)}{b}-\frac{(a-b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\tan(c+dx)}{b}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}}{d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

output
```
(-(((a - b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + T
an[c + d*x]/b)/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 6.93 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b}+\frac{(-a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b\sqrt{ab}}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b}+\frac{(-a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{b\sqrt{ab}}}{d}$
risch	$\frac{2i}{db\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{-2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)a}{2\sqrt{-ab}\,db}+\frac{\ln\left(e^{2i(dx+c)}+\frac{-2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2i(d\right.}}{}$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)/b+(-a+b)/b/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(44) = 88$.

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 5.13

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \left[\frac{\sqrt{-ab}(a-b)\cos(dx+c)\log\left(\frac{(a^2+6ab+b^2)\cos(dx+c)^4-2(3ab+b^2)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\sin}{(a^2-2ab+b^2)\cos(dx+c)^4+2(ab-b^2)\cos(dx+c)^2+b^2}\right.}{4ab^2d\cos(dx+c)}\right.$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-a*b)*(a - b)*cos(d*x + c)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c
)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d
*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^
4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*a*b*sin(d*x + c))/(a*b^2*d*co
s(d*x + c)), 1/2*(sqrt(a*b)*(a - b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b
)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*a*b*sin(d*x
+ c))/(a*b^2*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{(a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{\tan(dx+c)}{b}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
-((a - b)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b) - tan(d*x + c)/b)
/d
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{(a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}bd} + \frac{\tan(dx+c)}{bd}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-(a - b)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*b*d) + tan(d*x + c)/(
b*d)
```

Mupad [B] (verification not implemented)

Time = 7.83 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\tan(c+d\,x)}{b\,d} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(c+d\,x)}{\sqrt{a}}\right)(a-b)}{\sqrt{a}\,b^{3/2}\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)),x)
```

output
```
tan(c + d*x)/(b*d) - (atan((b^(1/2)*tan(c + d*x))/a^(1/2))*(a - b))/(a^(1/
2)*b^(3/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.46

$$\int \frac{\sec^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)\cos(dx+c)\,a-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)\cos(dx+c)\,b-\sqrt{b}}{\cos(}$$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*co
s(c + d*x)*a - sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2
))/sqrt(b))*cos(c + d*x)*b - sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*t
an((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a + sqrt(b)*sqrt(a)*atan((sqrt(a -
b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*b + sin(c + d*x)*a*b)
/(cos(c + d*x)*a*b**2*d)
```

3.461 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

output
```
arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(1/2)/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{a+b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input

```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

output

```
ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
default	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
risch	$-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2i(dx+c)}-\frac{2iab-\sqrt{-ab}\,a-\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}$	121

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 6.41

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab}\log\left(\frac{(a^2+6ab+b^2)\cos(dx+c)^4-2\,(3ab+b^2)\cos(dx+c)^2+4\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\sin(dx+c)+b^2}{(a^2-2ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)}{4\,abd}, -\frac{\sqrt{ab}\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)}{2\,abd}\right]$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)
*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*s
in(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos
(d*x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x +
c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)
```

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(c+d\,x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
```

output
```
atan((b^(1/2)*tan(c + d*x))/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$=\frac{\sqrt{b}\,\sqrt{a}\left(-atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)+atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)\right)}{abd}$$

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*( - atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b)
) + atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))))/(a*b*d)
```

3.462 $\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{(a-3b)x}{2(a-b)^2}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d}$$

output
```
1/2*(a-3*b)*x/(a-b)^2+b^(3/2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(1/2)/(
a-b)^2/d+1/2*cos(d*x+c)*sin(d*x+c)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{4b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a}(2(a-3b)(c+dx)+(a-b)\sin(2(c+dx)))}{4\sqrt{a}(a-b)^2d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

output
```
(4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(2*(a - 3*b)*(
c + d*x) + (a - b)*Sin[2*(c + d*x)]))/(4*Sqrt[a]*(a - b)^2*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4158, 316, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2\,(a+b\tan(c+dx)^2)}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)^2(b\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)}-\frac{\int -\frac{b\tan^2(c+dx)+a-2b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2(a-b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\tan^2(c+dx)+a-2b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}+\frac{(a-3b)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a-b}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}+\frac{(a-3b)\arctan(\tan(c+dx))}{a-b}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}+\frac{(a-3b)\arctan(\tan(c+dx))}{a-b}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)}}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

output
```
((((a - 3*b)*ArcTan[Tan[c + d*x]])/(a - b) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Ta
n[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*(a - b)) + Tan[c + d*x]/(2*(a
- b)*(1 + Tan[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan(dx+c)^2}+\frac{(a-3b)\arctan(\tan(dx+c))}{2}}{(a-b)^2}}{d}$
default	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^2\sqrt{ab}}+\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan(dx+c)^2}+\frac{(a-3b)\arctan(\tan(dx+c))}{2}}{(a-b)^2}}{d}$
risch	$\frac{xa}{2(a-b)^2}-\frac{3xb}{2(a-b)^2}-\frac{i e^{2i(dx+c)}}{8(a-b)d}+\frac{i e^{-2i(dx+c)}}{8(a-b)d}+\frac{\sqrt{-ab}\,b\ln\left(e^{2i(dx+c)}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)^2 d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2i(dx+\cdots}\right.}{2a(a-\cdots}$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2/(a-b)^2/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/(a-b)^2*((
1/2*a-1/2*b)*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2*(a-3*b)*arctan(tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.49

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$= \left[\frac{2\,(a-3\,b)dx + 2\,(a-b)\cos\left(dx+c\right)\sin\left(dx+c\right) + b\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2 \cdots}{(a^2-2\,ab+b^2)\cos(dx+ \cdots}\right.}{4\,(a^2-2\,ab+b^2)d}\right.$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a - 3*b)*d*x + 2*(a - b)*cos(d*x + c)*sin(d*x + c) + b*sqrt(-b/a)
*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2
- 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x + c
) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^
2 + b^2)))/((a^2 - 2*a*b + b^2)*d), 1/2*((a - 3*b)*d*x + (a - b)*cos(d*x +
 c)*sin(d*x + c) - b*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqr
t(b/a)/(b*cos(d*x + c)*sin(d*x + c))))/((a^2 - 2*a*b + b^2)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\frac{2\,b^2\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^2-2\,ab+b^2)\sqrt{ab}} + \frac{(dx+c)(a-3\,b)}{a^2-2\,ab+b^2} + \frac{\tan(dx+c)}{(a-b)\tan(dx+c)^2+a-b}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(2*b^2*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^2 - 2*a*b + b^2)*sqrt(a*b)
) + (d*x + c)*(a - 3*b)/(a^2 - 2*a*b + b^2) + tan(d*x + c)/((a - b)*tan(d*
x + c)^2 + a - b))/d
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{b^2\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^2d-2\,abd+b^2d)\sqrt{ab}} + \frac{(dx+c)(a-3\,b)}{2\,(a^2d-2\,abd+b^2d)} + \frac{\tan(dx+c)}{2\,(ad-bd)\left(\tan(dx+c)^2+1\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
b^2*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^2*d - 2*a*b*d + b^2*d)*sqrt(a*b))
 + 1/2*(d*x + c)*(a - 3*b)/(a^2*d - 2*a*b*d + b^2*d) + 1/2*tan(d*x + c)/((
a*d - b*d)*(tan(d*x + c)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.06

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx =$$
$$-\frac{6\,a\,b\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)-a^2\,\sin\left(2\,c+2\,d\,x\right)-2\,a^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)+a\,b\,\sin\left(2\,c+2\,d\,x\right)+\mathrm{atan}\left(\frac{a^2\,b^3\,\sin}{\cdots}\right.}{4\,d\,a^3-8\,d\,a^2\,b+4\,c}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x)^2),x)
```

output
```
-(atan((a^2*b^3*sin(c + d*x)*(-a*b^3)^(1/2)*9i - a^3*b^2*sin(c + d*x)*(-a*
b^3)^(1/2)*6i - a*b^4*sin(c + d*x)*(-a*b^3)^(1/2)*4i + a^4*b*sin(c + d*x)*
(-a*b^3)^(1/2)*1i)/(4*a^2*b^5*cos(c + d*x) - 9*a^3*b^4*cos(c + d*x) + 6*a^
4*b^3*cos(c + d*x) - a^5*b^2*cos(c + d*x)))*(-a*b^3)^(1/2)*4i - 2*a^2*atan
(sin(c + d*x)/cos(c + d*x)) - a^2*sin(2*c + 2*d*x) + 6*a*b*atan(sin(c + d*
x)/cos(c + d*x)) + a*b*sin(2*c + 2*d*x))/(4*a^3*d + 4*a*b^2*d - 8*a^2*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{\cos^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$=\frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)b+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)b+\cos\left(dx+c\right)\sin\left(dx+c\right)}{2ad\left(a^2-2ab+b^2\right)}$$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b
))*b + 2*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqr
t(b))*b + cos(c + d*x)*sin(c + d*x)*a**2 - cos(c + d*x)*sin(c + d*x)*a*b +
 a**2*c + a**2*d*x - 3*a*b*c - 3*a*b*d*x)/(2*a*d*(a**2 - 2*a*b + b**2))
```

3.463 $\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{(3a^2-10ab+15b^2)\,x}{8(a-b)^3} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^3 d} + \frac{(3a-7b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2 d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d}$$

output
```
1/8*(3*a^2-10*a*b+15*b^2)*x/(a-b)^3-b^(5/2)*arctan(b^(1/2)*tan(d*x+c)/a^(1
/2))/a^(1/2)/(a-b)^3/d+1/8*(3*a-7*b)*cos(d*x+c)*sin(d*x+c)/(a-b)^2/d+1/4*c
os(d*x+c)^3*sin(d*x+c)/(a-b)/d
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{-32b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a}(4(3a^2-10ab+15b^2)(c+dx) + 8(a^2-3ab+2b^2)\sin(2(c+dx)) +}{32\sqrt{a}(a-b)^3 d}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

output
```
(-32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*(4*(3*a^2 -
10*a*b + 15*b^2)*(c + d*x) + 8*(a^2 - 3*a*b + 2*b^2)*Sin[2*(c + d*x)] + (a
 - b)^2*Sin[4*(c + d*x)]))/(32*Sqrt[a]*(a - b)^3*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4158, 316, 25, 402, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4\left(a+b\tan(c+dx)^2\right)}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)^3(b\tan^2(c+dx)+a)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2}-\frac{\int -\frac{3b\tan^2(c+dx)+3a-4b}{\left(\tan^2(c+dx)+1\right)^2\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{4(a-b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3b\tan^2(c+dx)+3a-4b}{\left(\tan^2(c+dx)+1\right)^2\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{(3a-7b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)}-\frac{\int -\frac{3a^2-7ba+8b^2+(3a-7b)b\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2(a-b)}}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{3a^2-7ba+8b^2+(3a-7b)b\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2(a-b)}+\frac{(3a-7b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)}}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{\left(3a^2-10ab+15b^2\right)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a-b}-\frac{8b^3\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{2(a-b)}+\frac{(3a-7b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)}}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2}}{d}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{\frac{\left(3a^2-10ab+15b^2\right)\arctan(\tan(c+dx))}{a-b}-\frac{8b^3\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{2(a-b)}+\frac{(3a-7b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)}}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{\left(3a^2-10ab+15b^2\right)\arctan(\tan(c+dx))}{a-b}-\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{2(a-b)}+\frac{(3a-7b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)}}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2}}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2),x]
```

output
```
(Tan[c + d*x]/(4*(a - b)*(1 + Tan[c + d*x]^2)^2) + ((((3*a^2 - 10*a*b + 15
*b^2)*ArcTan[Tan[c + d*x]])/(a - b) - (8*b^(5/2)*ArcTan[(Sqrt[b]*Tan[c + d
*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(2*(a - b)) + ((3*a - 7*b)*Tan[c + d*x])
/(2*(a - b)*(1 + Tan[c + d*x]^2)))/(4*(a - b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 11.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{b^3 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}+\frac{\frac{\left(\frac{3}{8}a^2-\frac{5}{4}ab+\frac{7}{8}b^2\right)\tan(dx+c)^3+\left(-\frac{7}{4}ab+\frac{9}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan(dx+c)^2\right)^2}+\frac{\left(3a^2-10ab+15b^2\right)\arctan(\tan(dx-}{8}}{(a-b)^3}}{d}$
default	$\frac{-\frac{b^3 \arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a-b)^3\sqrt{ab}}+\frac{\frac{\left(\frac{3}{8}a^2-\frac{5}{4}ab+\frac{7}{8}b^2\right)\tan(dx+c)^3+\left(-\frac{7}{4}ab+\frac{9}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan(dx+c)^2\right)^2}+\frac{\left(3a^2-10ab+15b^2\right)\arctan(\tan(dx-}{8}}{(a-b)^3}}{d}$
risch	$\frac{3x\,a^2}{8(a-b)^3}-\frac{5xab}{4(a-b)^3}+\frac{15x\,b^2}{8(a-b)^3}-\frac{i e^{2i(dx+c)}a}{8(a-b)^2 d}+\frac{i e^{2i(dx+c)}b}{4(a-b)^2 d}+\frac{i e^{-2i(dx+c)}a}{8(a^2-2ab+b^2)d}-\frac{i e^{-2i(dx+c)}b}{4(a^2-2ab+b^2)d}+\frac{\sqrt{-a}}{}$

input

```
int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/(a-b)^3*b^3/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))+1/(a-b)^3
*(((3/8*a^2-5/4*a*b+7/8*b^2)*tan(d*x+c)^3+(-7/4*a*b+9/8*b^2+5/8*a^2)*tan(d
*x+c))/(1+tan(d*x+c)^2)^2+1/8*(3*a^2-10*a*b+15*b^2)*arctan(tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.11

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

$$= \left[-\frac{2\,b^2\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2-4\,\left((a^2+ab)\cos(dx+c)^3-ab\cos(dx+c)\right)\sqrt{-\frac{b}{a}}\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)}{8\,(a^3-3}\right.$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/8*(2*b^2*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b
 + b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))
*sqrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a
*b - b^2)*cos(d*x + c)^2 + b^2)) - (3*a^2 - 10*a*b + 15*b^2)*d*x - (2*(a^2
 - 2*a*b + b^2)*cos(d*x + c)^3 + (3*a^2 - 10*a*b + 7*b^2)*cos(d*x + c))*si
n(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d), 1/8*(4*b^2*sqrt(b/a)*arct
an(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c)
)) + (3*a^2 - 10*a*b + 15*b^2)*d*x + (2*(a^2 - 2*a*b + b^2)*cos(d*x + c)^3
 + (3*a^2 - 10*a*b + 7*b^2)*cos(d*x + c))*sin(d*x + c))/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.43

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{\frac{8\,b^3\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3-3\,a^2b+3\,ab^2-b^3)\sqrt{ab}} - \frac{(3\,a^2-10\,ab+15\,b^2)(dx+c)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{(3\,a-7\,b)\tan(dx+c)^3+(5\,a-9\,b)\tan(dx+c)}{(a^2-2\,ab+b^2)\tan(dx+c)^4+2\,(a^2-2\,ab+b^2)\tan(dx+c)^2+a^2-2\,ab+b^2}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(8*b^3*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*sqrt(a*b)) - (3*a^2 - 10*a*b + 15*b^2)*(d*x + c)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) - ((3*a - 7*b)*tan(d*x + c)^3 + (5*a - 9*b)*tan(d*x + c))/((a^2
 - 2*a*b + b^2)*tan(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*tan(d*x + c)^2 + a^
2 - 2*a*b + b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.36

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{b^3\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^3d-3\,a^2bd+3\,ab^2d-b^3d)\sqrt{ab}} + \frac{(3\,a^2-10\,ab+15\,b^2)(dx+c)}{8\,(a^3d-3\,a^2bd+3\,ab^2d-b^3d)} + \frac{3\,a\tan\left(dx+c\right)^3-7\,b\tan\left(dx+c\right)^3+5\,a\tan\left(dx+c\right)-9\,b\tan\left(dx+c\right)}{8\,(a^2d-2\,abd+b^2d)\left(\tan\left(dx+c\right)^2+1\right)^2}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
-b^3*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^
3*d)*sqrt(a*b)) + 1/8*(3*a^2 - 10*a*b + 15*b^2)*(d*x + c)/(a^3*d - 3*a^2*b
*d + 3*a*b^2*d - b^3*d) + 1/8*(3*a*tan(d*x + c)^3 - 7*b*tan(d*x + c)^3 + 5
*a*tan(d*x + c) - 9*b*tan(d*x + c))/((a^2*d - 2*a*b*d + b^2*d)*(tan(d*x +
c)^2 + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 3681, normalized size of antiderivative = 28.53

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x)^2),x)
```

output
```
((tan(c + d*x)*(5*a - 9*b))/(8*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^3*(3*a
 - 7*b))/(8*(a^2 - 2*a*b + b^2)))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 +
1)) - (atan(((((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b
^4 + 9*a^4*b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (((256
*b^10 - 1760*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5
*b^5 + 3040*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b
^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) - (tan(c + d*x)*(3*a^2 -
 10*a*b + 15*b^2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 12
80*a^4*b^5 - 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i -
a^2*b*3i + a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*
(3*a^2 - 10*a*b + 15*b^2))/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)))*(
3*a^2 - 10*a*b + 15*b^2)*1i)/(16*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i))
+ (((tan(c + d*x)*(289*b^7 - 300*a*b^6 + 190*a^2*b^5 - 60*a^3*b^4 + 9*a^4*
b^3))/(32*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (((256*b^10 - 176
0*a*b^9 + 5280*a^2*b^8 - 9056*a^3*b^7 + 9760*a^4*b^6 - 6816*a^5*b^5 + 3040
*a^6*b^4 - 800*a^7*b^3 + 96*a^8*b^2)/(64*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 +
15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (tan(c + d*x)*(3*a^2 - 10*a*b + 1
5*b^2)*(1280*a*b^8 - 256*b^9 - 2304*a^2*b^7 + 1280*a^3*b^6 + 1280*a^4*b^5
- 2304*a^5*b^4 + 1280*a^6*b^3 - 256*a^7*b^2))/(512*(a*b^2*3i - a^2*b*3i +
a^3*1i - b^3*1i)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(3*a^2 -...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.98

$$\int \frac{\cos^4(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)b^2 - 8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}+\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)b^2 - 2\cos(dx+c)\sin(dx+}{}$$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2),x)
```

output
```
(8*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*
b**2 - 8*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqr
t(b))*b**2 - 2*cos(c + d*x)*sin(c + d*x)**3*a**3 + 4*cos(c + d*x)*sin(c +
d*x)**3*a**2*b - 2*cos(c + d*x)*sin(c + d*x)**3*a*b**2 + 5*cos(c + d*x)*si
n(c + d*x)*a**3 - 14*cos(c + d*x)*sin(c + d*x)*a**2*b + 9*cos(c + d*x)*sin
(c + d*x)*a*b**2 + 3*a**3*c + 3*a**3*d*x - 10*a**2*b*c - 10*a**2*b*d*x + 1
5*a*b**2*c + 15*a*b**2*d*x)/(8*a*d*(a**3 - 3*a**2*b + 3*a*b**2 - b**3))
```

3.464 $\int \frac{\sec^7(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{\sec^7(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = -\frac{(4a-5b)\text{arctanh}(\sin(c+dx))}{2b^3d} + \frac{(a-b)^{3/2}(4a+b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(a-b)(2a-b)\sin(c+dx)}{2ab^2d\,(a-(a-b)\sin^2(c+dx))} + \frac{\sec(c+dx)\tan(c+dx)}{2bd\,(a-(a-b)\sin^2(c+dx))}$$

output
```
-1/2*(4*a-5*b)*arctanh(sin(d*x+c))/b^3/d+1/2*(a-b)^(3/2)*(4*a+b)*arctanh((
a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a-b)*(2*a-b)*sin(d*x+c)/
a/b^2/d/(a-(a-b)*sin(d*x+c)^2)+1/2*sec(d*x+c)*tan(d*x+c)/b/d/(a-(a-b)*sin(
d*x+c)^2)
```

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.52

$$\int \frac{\sec^7(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \frac{2(4a-5b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(-4a+5b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input
```
Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(2*(4*a - 5*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-4*a + 5*b)*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - ((a - b)^(3/2)*(4*a + b)*Log[Sqr
t[a] - Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + ((a - b)^(3/2)*(4*a + b)*Log[S
qrt[a] + Sqrt[a - b]*Sin[c + d*x]])/a^(3/2) + b/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 - b/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(a - b)^2*b
*Sin[c + d*x])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))/(4*b^3*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4159, 316, 25, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^7}{(a+b\tan(c+dx)^2)^2}dx$$
$$\Big\downarrow \text{4159}$$
$$\frac{\int \frac{1}{(1-\sin^2(c+dx))^2(a-(a-b)\sin^2(c+dx))^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$\frac{\frac{\int -\frac{3(a-b)\sin^2(c+dx)+a-2b}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)^2}d\sin(c+dx)}{2b}+\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\int \frac{3(a-b)\sin^2(c+dx)+a-2b}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)^2}d\sin(c+dx)}{2b}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{-\frac{\int -\frac{2\left(2a^2-2ba-b^2+(a-b)(2a-b)\sin^2(c+dx)\right)}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{2ab}-\frac{(a-b)(2a-b)\sin(c+dx)}{ab\left(a-(a-b)\sin^2(c+dx)\right)}}{2b}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\frac{\int \frac{2a^2-2ba-b^2+(a-b)(2a-b)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{ab}-\frac{(a-b)(2a-b)\sin(c+dx)}{ab\left(a-(a-b)\sin^2(c+dx)\right)}}{2b}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\frac{\frac{a(4a-5b)\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)}{b}-\frac{(a-b)^2(4a+b)\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{ab}-\frac{(a-b)(2a-b)\sin}{ab\left(a-(a-b)\sin^2\right.}}{2b}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\frac{\frac{a(4a-5b)\text{arctanh}(\sin(c+dx))}{b}-\frac{(a-b)^2(4a+b)\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{ab}-\frac{(a-b)(2a-b)\sin(c+d}{ab\left(a-(a-b)\sin^2(c+d\right.}}{2b}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\sin(c+dx)}{2b\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\frac{\frac{a(4a-5b)\text{arctanh}(\sin(c+dx))}{b}-\frac{(a-b)^{3/2}(4a+b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{ab}-\frac{(a-b)(2a-b)\sin(c+dx}{ab\left(a-(a-b)\sin^2(c+dx\right.}}{2b}}{d}$$

input
```
Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(Sin[c + d*x]/(2*b*(1 - Sin[c + d*x]^2)*(a - (a - b)*Sin[c + d*x]^2)) - ((
(a*(4*a - 5*b)*ArcTanh[Sin[c + d*x]])/b - ((a - b)^(3/2)*(4*a + b)*ArcTanh
[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b) - ((a - b)*(2*a -
 b)*Sin[c + d*x])/(a*b*(a - (a - b)*Sin[c + d*x]^2)))/(2*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 117.79 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{1}{4b^2(1+\sin(dx+c))}+\frac{(-4a+5b)\ln(1+\sin(dx+c))}{4b^3}-\frac{(a^2-2ab+b^2)\left(\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(4a+b)\operatorname{arctanh}\left(\frac{(a-b)\,s\cdots}{\sqrt{a\cdots}}\right)}{2a\sqrt{a(a-b)}\cdots}\cdots\right)}{b^3}\cdots}{d}$
default	$\frac{-\frac{1}{4b^2(1+\sin(dx+c))}+\frac{(-4a+5b)\ln(1+\sin(dx+c))}{4b^3}-\frac{(a^2-2ab+b^2)\left(\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(4a+b)\operatorname{arctanh}\left(\frac{(a-b)\,s\cdots}{\sqrt{a\cdots}}\right)}{2a\sqrt{a(a-b)}\cdots}\cdots\right)}{b^3}\cdots}{d}$
risch	$\frac{i\left(2a^2e^{7i(dx+c)}-3ab\,e^{7i(dx+c)}+b^2e^{7i(dx+c)}+2a^2e^{5i(dx+c)}+ab\,e^{5i(dx+c)}+b^2e^{5i(dx+c)}-2a^2e^{3i(dx+c)}-ab\,e^{3i(dx+c)}-b^2\cdots\right)}{d\,b^2\left(e^{2i(dx+c)}+1\right)^2a\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}\cdots\right)}$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/b^2/(1+sin(d*x+c))+1/4/b^3*(-4*a+5*b)*ln(1+sin(d*x+c))-(a^2-2*a*
b+b^2)/b^3*(1/2/a*b*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)-1/2*(4*a+
b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))-1/4/b^2/(s
in(d*x+c)-1)+1/4*(4*a-5*b)/b^3*ln(sin(d*x+c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 635, normalized size of antiderivative = 3.80

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(((4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*
b^2 - b^3)*cos(d*x + c)^2)*sqrt((a - b)/a)*log(-((a - b)*cos(d*x + c)^2 +
2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2 + b))
+ ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*
x + c)^2)*log(sin(d*x + c) + 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c
)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^
2 + (2*a^2*b - 3*a*b^2 + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d
*x + c)^2 + (a^2*b^3 - a*b^4)*d*cos(d*x + c)^4), -1/4*(2*((4*a^3 - 7*a^2*b
 + 2*a*b^2 + b^3)*cos(d*x + c)^4 + (4*a^2*b - 3*a*b^2 - b^3)*cos(d*x + c)^
2)*sqrt(-(a - b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((4*a^3 - 9*a^
2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b - 5*a*b^2)*cos(d*x + c)^2)*log(si
n(d*x + c) + 1) - ((4*a^3 - 9*a^2*b + 5*a*b^2)*cos(d*x + c)^4 + (4*a^2*b -
 5*a*b^2)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a*b^2 + (2*a^2*b - 3
*a*b^2 + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^2 + (a^2
*b^3 - a*b^4)*d*cos(d*x + c)^4)]
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \int \frac{\sec^7\left(c+dx\right)}{\left(a+b\tan^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(sec(c + d*x)**7/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.47

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = -\frac{\frac{(4a-5b)\log(|\sin(dx+c)+1|)}{b^3} - \frac{(4a-5b)\log(|\sin(dx+c)-1|)}{b^3} - \frac{2\left(4a^3-7a^2b+2ab^2+b^3\right)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ab^3} + \frac{2\left(2a^2 \cdots\right.}{\cdots}}{4d}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/4*((4*a - 5*b)*log(abs(sin(d*x + c) + 1))/b^3 - (4*a - 5*b)*log(abs(sin
(d*x + c) - 1))/b^3 - 2*(4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*arctan(-(a*sin(d
*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^3) + 2*(
2*a^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^3 + b^2*sin(d*x + c)^3 - 2*a^2*s
in(d*x + c) + 2*a*b*sin(d*x + c) - b^2*sin(d*x + c))/((a*sin(d*x + c)^4 -
b*sin(d*x + c)^4 - 2*a*sin(d*x + c)^2 + b*sin(d*x + c)^2 + a)*a*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 4304, normalized size of antiderivative = 25.77

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
((tan(c/2 + (d*x)/2)*(2*a^2 - 2*a*b + b^2))/(a*b^2) - (tan(c/2 + (d*x)/2)^
3*(2*a^2 - 6*a*b + b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^7*(2*a^2 - 2*a*b +
b^2))/(a*b^2) - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 6*a*b + b^2))/(a*b^2))/(d*(
a - tan(c/2 + (d*x)/2)^2*(4*a - 4*b) - tan(c/2 + (d*x)/2)^6*(4*a - 4*b) +
tan(c/2 + (d*x)/2)^4*(6*a - 8*b) + a*tan(c/2 + (d*x)/2)^8)) - (atan((((4*a
 - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 - 2236*a^4*b^12 + 76
8*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*b^8 - 13552*a^9*b^
7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + (((((256*(256*a^4*b^16 + 192
*a^5*b^15 - 1088*a^6*b^14 - 192*a^7*b^13 + 1600*a^8*b^12 - 768*a^9*b^11))/
(a^3*b^10) - (256*tan(c/2 + (d*x)/2)*(4*a - 5*b)*(1024*a^5*b^15 - 2304*a^6
*b^14 + 1664*a^7*b^13 - 384*a^8*b^12))/(a^3*b^11))*(4*a - 5*b))/(2*b^3) -
(512*tan(c/2 + (d*x)/2)*(64*a^2*b^14 + 160*a^3*b^13 - 984*a^4*b^12 - 6560*
a^5*b^11 + 28720*a^6*b^10 - 42400*a^7*b^9 + 29512*a^8*b^8 - 9664*a^9*b^7 +
 1152*a^10*b^6))/(a^3*b^8))*(4*a - 5*b))/(2*b^3))*(4*a - 5*b))/(2*b^3) + (
512*tan(c/2 + (d*x)/2)*(8*a*b^11 - 8960*a^11*b + 768*a^12 + b^12 + 396*a^2
*b^10 + 440*a^3*b^9 - 7144*a^4*b^8 + 6656*a^5*b^7 + 34712*a^6*b^6 - 106784
*a^7*b^5 + 138675*a^8*b^4 - 100016*a^9*b^3 + 41248*a^10*b^2))/(a^3*b^8))*1
i)/(2*b^3) - ((4*a - 5*b)*((((256*(16*a*b^15 + 92*a^2*b^14 - 8*a^3*b^13 -
2236*a^4*b^12 + 768*a^5*b^11 + 18228*a^6*b^10 - 41560*a^7*b^9 + 37420*a^8*
b^8 - 13552*a^9*b^7 + 64*a^10*b^6 + 768*a^11*b^5))/(a^3*b^10) + ((((25...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1718, normalized size of antiderivative = 10.29

$$\int \frac{\sec^7(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 4*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*
tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**4*a**3 + 7*sqrt(a)*sqrt(a - b
)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sq
rt(a))*sin(c + d*x)**4*a**2*b - 2*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)
*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**4
*a*b**2 - sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt
(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**4*b**3 + 8*sqrt(a)*sqrt(a
 - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2
+ sqrt(a))*sin(c + d*x)**2*a**3 - 10*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a -
 b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)
**2*a**2*b + sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + s
qrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a*b**2 + sqrt(a)*sqr
t(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)*
*2 + sqrt(a))*sin(c + d*x)**2*b**3 - 4*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a
 - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a**3 + 3*s
qrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c
+ d*x)/2)**2 + sqrt(a))*a**2*b + sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*
tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a*b**2 + 4*sqrt(
a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/
2)**2 + sqrt(a))*sin(c + d*x)**4*a**3 - 7*sqrt(a)*sqrt(a - b)*log(2*sqr...
```

3.465 $\int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a-b}(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b)\sin(c+dx)}{2abd\,(a-(a-b)\sin^2(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^2/d-1/2*(a-b)^(1/2)*(2*a+b)*arctanh((a-b)^(1/2)*sin(
d*x+c)/a^(1/2))/a^(3/2)/b^2/d-1/2*(a-b)*sin(d*x+c)/a/b/d/(a-(a-b)*sin(d*x+
c)^2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.75

$$\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$= \frac{-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sqrt{a-b}(2a+b)\log(\sqrt{}}{a}}{4b^2d}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(-4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (Sqrt[a - b]*(2*a + b)*Log[Sqrt[a] - Sqrt[a - b]*Sin[c +
 d*x]])/a^(3/2) + ((-2*a^2 + a*b + b^2)*Log[Sqrt[a] + Sqrt[a - b]*Sin[c +
d*x]])/(a^(3/2)*Sqrt[a - b]) + (4*b*(-a + b)*Sin[c + d*x])/(a*(a + b + (a
- b)*Cos[2*(c + d*x)])))/(4*b^2*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4159, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sec(c+dx)^5}{\left(a+b\tan(c+dx)^2\right)^2}dx$$
$$\Big\downarrow \; 4159$$
$$\frac{\int \frac{1}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$\frac{-\frac{\int -\frac{(a-b)\sin^2(c+dx)+a+b}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{2ab}-\frac{(a-b)\sin(c+dx)}{2ab(a-(a-b)\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(a-b)\sin^2(c+dx)+a+b}{\left(1-\sin^2(c+dx)\right)\left(a-(a-b)\sin^2(c+dx)\right)}d\sin(c+dx)}{2ab}-\frac{(a-b)\sin(c+dx)}{2ab(a-(a-b)\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{2a\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)}{b}-\frac{(a-b)(2a+b)\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{2ab}-\frac{(a-b)\sin(c+dx)}{2ab(a-(a-b)\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{2a\text{arctanh}(\sin(c+dx))}{b}-\frac{(a-b)(2a+b)\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{b}}{2ab}-\frac{(a-b)\sin(c+dx)}{2ab(a-(a-b)\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2a\text{arctanh}(\sin(c+dx))}{b}-\frac{\sqrt{a-b}(2a+b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}}{2ab}-\frac{(a-b)\sin(c+dx)}{2ab(a-(a-b)\sin^2(c+dx))}}{d}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((2*a*ArcTanh[Sin[c + d*x]])/b - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a -
 b]*Sin[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(2*a*b) - ((a - b)*Sin[c + d*x])/
(2*a*b*(a - (a - b)*Sin[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 38.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{\ln(1+\sin(dx+c))}{2b^2}+\frac{(a-b)\left(\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(2a+b)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{b^2}-\frac{\ln(\sin(dx+c)-1)}{2b^2}}{d}$
default	$\frac{\frac{\ln(1+\sin(dx+c))}{2b^2}+\frac{(a-b)\left(\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(2a+b)\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{b^2}-\frac{\ln(\sin(dx+c)-1)}{2b^2}}{d}$
risch	$\frac{i(a-b)\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{abd\left(a\,\mathrm{e}^{4i(dx+c)}-b\,\mathrm{e}^{4i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}+a-b\right)}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b^2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b^2d}+\frac{\sqrt{a(a-b)}}{}$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b^2*ln(1+sin(d*x+c))+(a-b)/b^2*(1/2/a*b*sin(d*x+c)/(a*sin(d*x+c)^
2-b*sin(d*x+c)^2-a)-1/2*(2*a+b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)
/(a*(a-b))^(1/2)))-1/2/b^2*ln(sin(d*x+c)-1))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.73

$$\int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \left[\frac{\left((2\,a^2-ab-b^2)\cos{(dx+c)}^2+2\,ab+b^2\right)\sqrt{\frac{a-b}{a}}\log\left(-\frac{(a-b)\cos(dx+c)^2+2\,a\sqrt{\frac{a-b}{a}}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left((a^2\right.}{4\,(ab}\right.$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt((a - b)/a)*l
og(-((a - b)*cos(d*x + c)^2 + 2*a*sqrt((a - b)/a)*sin(d*x + c) - 2*a + b)/
((a - b)*cos(d*x + c)^2 + b)) + 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(s
in(d*x + c) + 1) - 2*((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-sin(d*x + c)
+ 1) - 2*(a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*d*cos(d*x
+ c)^2), 1/2*(((2*a^2 - a*b - b^2)*cos(d*x + c)^2 + 2*a*b + b^2)*sqrt(-(a
- b)/a)*arctan(sqrt(-(a - b)/a)*sin(d*x + c)) + ((a^2 - a*b)*cos(d*x + c)^
2 + a*b)*log(sin(d*x + c) + 1) - ((a^2 - a*b)*cos(d*x + c)^2 + a*b)*log(-s
in(d*x + c) + 1) - (a*b - b^2)*sin(d*x + c))/(a*b^3*d + (a^2*b^2 - a*b^3)*
d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \frac{\frac{\log(|\sin(dx+c)+1|)}{b^2} - \frac{\log(|\sin(dx+c)-1|)}{b^2} - \frac{(2\,a^2-ab-b^2)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ab^2} + \frac{a\sin(dx+c)-b\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)ab}}{2\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(log(abs(sin(d*x + c) + 1))/b^2 - log(abs(sin(d*x + c) - 1))/b^2 - (2*
a^2 - a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b
))/(sqrt(-a^2 + a*b)*a*b^2) + (a*sin(d*x + c) - b*sin(d*x + c))/((a*sin(d*
x + c)^2 - b*sin(d*x + c)^2 - a)*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 10.29 (sec) , antiderivative size = 946, normalized size of antiderivative = 8.68

$$\int \frac{\sec^5(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
((a^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d
*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*1i - a
^(5/2)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i + (b^2*atan((sin(c/
2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*x)))/(2*a^(1/2)*c
os(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*1i)/2 - a^(3/2)*b*atanh(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*2i - a^(1/2)*b^2*sin(c + d*x)*1i +
a^2*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2))
)*(b - a)^(1/2)*1i + (b^2*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (
d*x)/2)*(b - a)^(1/2)))*(b - a)^(1/2)*1i)/2 - a^(5/2)*atanh(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*2i + a^(3/2)*b*sin(c + d*x)*1i +
 a^2*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)
))*cos(2*c + 2*d*x)*(b - a)^(1/2)*1i - (b^2*atan((a^(1/2)*sin(c/2 + (d*x)/
2))/(2*cos(c/2 + (d*x)/2)*(b - a)^(1/2)))*cos(2*c + 2*d*x)*(b - a)^(1/2)*1
i)/2 + (a*b*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos
(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b - a)^(1/2)))*(b - a)^(1/2)*
3i)/2 + (a*b*atan((a^(1/2)*sin(c/2 + (d*x)/2))/(2*cos(c/2 + (d*x)/2)*(b -
a)^(1/2)))*(b - a)^(1/2)*3i)/2 + a^2*atan((sin(c/2 + (d*x)/2)*(a - 2*b + 2
*a*cos(c + d*x) - 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 + (d*x)/2)^3*(b -
a)^(1/2)))*cos(2*c + 2*d*x)*(b - a)^(1/2)*1i - (b^2*atan((sin(c/2 + (d*x)/
2)*(a - 2*b + 2*a*cos(c + d*x) - 2*b*cos(c + d*x)))/(2*a^(1/2)*cos(c/2 ...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 720, normalized size of antiderivative = 6.61

$$\int \frac{\sec^5(c + dx)}{(a + b \tan^2(c + dx))^2} \, dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c)^2)^2,x)
```

output

```
(2*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan
((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a**2 - sqrt(a)*sqrt(a - b)*log
( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a)
)*sin(c + d*x)**2*a*b - sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c +
d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*b**2 - 2*
sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c
 + d*x)/2)**2 + sqrt(a))*a**2 - sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*t
an((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a*b - 2*sqrt(a)*s
qrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**
2 + sqrt(a))*sin(c + d*x)**2*a**2 + sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*
tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*
a*b + sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan
((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*b**2 + 2*sqrt(a)*sqrt(a - b)*l
og(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))
*a**2 + sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*t
an((c + d*x)/2)**2 + sqrt(a))*a*b - 4*log(tan((c + d*x)/2) - 1)*sin(c + d*
x)**2*a**3 + 4*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**2*b + 4*log(ta
n((c + d*x)/2) - 1)*a**3 + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**
3 - 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b - 4*log(tan((c + d*
x)/2) + 1)*a**3 + 2*sin(c + d*x)*a**2*b - 2*sin(c + d*x)*a*b**2)/(4*a**...
```

3.466 $\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}d}+\frac{\sin(c+dx)}{2ad\left(a-(a-b)\sin^2(c+dx)\right)}$$

output
```
1/2*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(1/2)/d+1/2*sin(
d*x+c)/a/d/(a-(a-b)*sin(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx=\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}+\frac{\sqrt{a}\sin(c+dx)}{a+(-a+b)\sin^2(c+dx)}}{2a^{3/2}d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sin[c
+ d*x])/(a + (-a + b)*Sin[c + d*x]^2))/(2*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{\left(a+b\tan(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{\left(a-(a-b)\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{2a}+\frac{\sin(c+dx)}{2a\left(a-(a-b)\sin^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}}+\frac{\sin(c+dx)}{2a\left(a-(a-b)\sin^2(c+dx)\right)}}{d}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]) + Sin
[c + d*x]/(2*a*(a - (a - b)*Sin[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 10.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}+\frac{\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}}{d}$
risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{ad\left(a\,e^{4i(dx+c)}-b\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+a-b\right)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^2-ab}}-1\right)}{4\sqrt{a^2-ab}\,da}-\frac{\ln\left(e^{2i(dx+c}\right.}{4\sqrt{}}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*sin(d*x+c)/a/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2/a/(a*(a-b))^(
1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.37

$$\int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \left[\frac{\left((a-b)\cos(dx+c)^2+b\right)\sqrt{a^2-ab}\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)+2\left(a^2-ab\right)\sin(dx+}{4\left((a^4-2a^3b+a^2b^2)d\cos(dx+c)^2+(a^3b-a^2b^2)d\right)}\right.$$
$$\left.-\frac{\left((a-b)\cos(dx+c)^2+b\right)\sqrt{-a^2+ab}\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)-\left(a^2-ab\right)\sin(dx+c)}{2\left((a^4-2a^3b+a^2b^2)d\cos(dx+c)^2+(a^3b-a^2b^2)d\right)}\right]$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((a - b)*cos(d*x + c)^2 + b)*sqrt(a^2 - a*b)*log(-((a - b)*cos(d*x +
 c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/((a - b)*cos(d*x + c)^2
+ b)) + 2*(a^2 - a*b)*sin(d*x + c))/((a^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x +
 c)^2 + (a^3*b - a^2*b^2)*d), -1/2*(((a - b)*cos(d*x + c)^2 + b)*sqrt(-a^2
 + a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (a^2 - a*b)*sin(d*x + c)
)/((a^4 - 2*a^3*b + a^2*b^2)*d*cos(d*x + c)^2 + (a^3*b - a^2*b^2)*d)]
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\frac{\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a} - \frac{\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)a}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(arctan(-(a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))/(sqrt(-a^
2 + a*b)*a) - sin(d*x + c)/((a*sin(d*x + c)^2 - b*sin(d*x + c)^2 - a)*a))/
d
```

Mupad [B] (verification not implemented)

Time = 10.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.37

$$\int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(4b-2a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}$$
$$-\frac{\operatorname{atanh}\left(\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3/2}\sqrt{a-b}\left(\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a-b}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a}+\frac{2}{a}-\frac{2}{a-b}+\frac{4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{ab-a^2}\right)}\right)}{2a^{3/2}d\sqrt{a-b}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
(tan(c/2 + (d*x)/2)^3/a + tan(c/2 + (d*x)/2)/a)/(d*(a - tan(c/2 + (d*x)/2)
^2*(2*a - 4*b) + a*tan(c/2 + (d*x)/2)^4)) - atanh((4*b*tan(c/2 + (d*x)/2))
/(a^(3/2)*(a - b)^(1/2)*((2*tan(c/2 + (d*x)/2)^2)/(a - b) - (2*tan(c/2 + (
d*x)/2)^2)/a + 2/a - 2/(a - b) + (4*b*tan(c/2 + (d*x)/2)^2)/(a*b - a^2)))
/(2*a^(3/2)*d*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.84

$$\int \frac{\sec^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \frac{-\sqrt{a}\sqrt{a-b}\log\left(-2\sqrt{a-b}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{a}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{a}\right)\sin(dx+c)^2a+\sqrt{a}\sqrt{a-b}\log\Big(-}{}$$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*ta
n((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a + sqrt(a)*sqrt(a - b)*log(
- 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*
sin(c + d*x)**2*b + sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)
/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a + sqrt(a)*sqrt(a - b)*log(2
*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin
(c + d*x)**2*a - sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) +
sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*b - sqrt(a)*sqrt(a
- b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sq
rt(a))*a - 2*sin(c + d*x)*a**2 + 2*sin(c + d*x)*a*b)/(4*a**2*d*(sin(c + d*
x)**2*a**2 - 2*sin(c + d*x)**2*a*b + sin(c + d*x)**2*b**2 - a**2 + a*b))
```

3.467 $\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\sin(c+dx)}{2a(a-b)d\left(a-(a-b)\sin^2(c+dx)\right)}$$

output
```
1/2*(2*a-b)*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(3/2)/d-
1/2*b*sin(d*x+c)/a/(a-b)/d/(a-(a-b)*sin(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \frac{\sec(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{-\frac{1}{2}(2a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)(a+b+(a-b)\cos(2(c+dx)))+\sqrt{a}\sqrt{a-b}b\sin(c+dx)}{2a^{3/2}(a-b)^{3/2}d\left(-a+(a-b)\sin^2(c+dx)\right)}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(-1/2*((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]*(a + b + (a -
 b)*Cos[2*(c + d*x)])) + Sqrt[a]*Sqrt[a - b]*b*Sin[c + d*x])/(2*a^(3/2)*(a
 - b)^(3/2)*d*(-a + (a - b)*Sin[c + d*x]^2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx)^2)^2}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{1-\sin^2(c+dx)}{(a-(a-b)\sin^2(c+dx))^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{(2a-b)\int \frac{1}{a-(a-b)\sin^2(c+dx)}d\sin(c+dx)}{2a(a-b)}-\frac{b\sin(c+dx)}{2a(a-b)(a-(a-b)\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}}-\frac{b\sin(c+dx)}{2a(a-b)(a-(a-b)\sin^2(c+dx))}}{d}$$

input
```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((2*a - b)*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b
)^(3/2)) - (b*Sin[c + d*x])/(2*a*(a - b)*(a - (a - b)*Sin[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{b\sin(dx+c)}{2a(a-b)\left(a\sin(dx+c)^{2}-b\sin(dx+c)^{2}-a\right)}+\frac{(2a-b)\,\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a(a-b)\sqrt{a(a-b)}}}{d}$
default	$\frac{\frac{b\sin(dx+c)}{2a(a-b)\left(a\sin(dx+c)^{2}-b\sin(dx+c)^{2}-a\right)}+\frac{(2a-b)\,\operatorname{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a(a-b)\sqrt{a(a-b)}}}{d}$
risch	$\frac{ib\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{a(-a+b)d\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-a+b\right)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{\sqrt{a^{2}-ab}}-1\right)}{2\sqrt{a^{2}-ab}\,(a-b)d}-\frac{\ln\left(e\right.}{}$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b/a/(a-b)*sin(d*x+c)/(a*sin(d*x+c)^2-b*sin(d*x+c)^2-a)+1/2*(2*a-b
)/a/(a-b)/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.59

$$\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \left[\frac{((2a^2-3ab+b^2)\cos(dx+c)^2+2ab-b^2)\sqrt{a^2-ab}\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right)-2}{4\left((a^5-3a^4b+3a^3b^2-a^2b^3)d\cos(dx+c)^2+(a^4b-2a^3b^2+a^2b^3)d\right)}\right.$$

$$-\frac{((2a^2-3ab+b^2)\cos(dx+c)^2+2ab-b^2)\sqrt{-a^2+ab}\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)+(a^2b-ab^2)\sin(c}{2\left((a^5-3a^4b+3a^3b^2-a^2b^3)d\cos(dx+c)^2+(a^4b-2a^3b^2+a^2b^3)d\right)}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(a^2 - a*b)
*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*a + b)/
((a - b)*cos(d*x + c)^2 + b)) - 2*(a^2*b - a*b^2)*sin(d*x + c))/((a^5 - 3*
a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b
^3)*d), -1/2*(((2*a^2 - 3*a*b + b^2)*cos(d*x + c)^2 + 2*a*b - b^2)*sqrt(-a
^2 + a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (a^2*b - a*b^2)*sin(d*
x + c))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cos(d*x + c)^2 + (a^4*b -
 2*a^3*b^2 + a^2*b^3)*d)]
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = -\frac{\frac{(2a-b)\arctan\left(\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^2-ab)\sqrt{-a^2+ab}} - \frac{b\sin(dx+c)}{\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)(a^2-ab)}}{2d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((2*a - b)*arctan((a*sin(d*x + c) - b*sin(d*x + c))/sqrt(-a^2 + a*b))
/((a^2 - a*b)*sqrt(-a^2 + a*b)) - b*sin(d*x + c)/((a*sin(d*x + c)^2 - b*si
n(d*x + c)^2 - a)*(a^2 - a*b)))/d
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.54

$$\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx =$$

$$-\frac{\left(a^2\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}}{2}+a^2\,\cos\left(2\,c+2\,d\,x\right)\,\mathrm{atanh}\left(\frac{\sin(c+d\,x)\,\sqrt{a-b}}{\sqrt{a}}\right)\,1\mathrm{i}+\right.}{2\,a^{3/2}\,d\,(a-b)}$$

input
```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
-((a^2*atanh((sin(c + d*x)*(a - b)^(1/2))/a^(1/2))*1i - (b^2*atanh((sin(c
+ d*x)*(a - b)^(1/2))/a^(1/2))*1i)/2 + a^2*cos(2*c + 2*d*x)*atanh((sin(c +
 d*x)*(a - b)^(1/2))/a^(1/2))*1i + (b^2*cos(2*c + 2*d*x)*atanh((sin(c + d*
x)*(a - b)^(1/2))/a^(1/2))*1i)/2 + (a*b*atanh((sin(c + d*x)*(a - b)^(1/2))
/a^(1/2))*1i)/2 - (a*b*cos(2*c + 2*d*x)*atanh((sin(c + d*x)*(a - b)^(1/2))
/a^(1/2))*3i)/2 - a^(1/2)*b*sin(c + d*x)*(a - b)^(1/2)*1i)*1i)/(2*a^(3/2)*
d*(a - b)^(3/2)*(a/2 + b/2 + (a*cos(2*c + 2*d*x))/2 - (b*cos(2*c + 2*d*x))
/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 628, normalized size of antiderivative = 6.68

$$\int \frac{\sec(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 2*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*
tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a**2 + 3*sqrt(a)*sqrt(a - b
)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sq
rt(a))*sin(c + d*x)**2*a*b - sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan(
(c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*b**2
 + 2*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*t
an((c + d*x)/2)**2 + sqrt(a))*a**2 - sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a -
 b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a*b + 2*sqrt
(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)
/2)**2 + sqrt(a))*sin(c + d*x)**2*a**2 - 3*sqrt(a)*sqrt(a - b)*log(2*sqrt(
a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d
*x)**2*a*b + sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt
(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*b**2 - 2*sqrt(a)*sqrt(a
 - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + s
qrt(a))*a**2 + sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sq
rt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a*b + 2*sin(c + d*x)*a**2*b - 2*sin(c
 + d*x)*a*b**2)/(4*a**2*d*(sin(c + d*x)**2*a**3 - 3*sin(c + d*x)**2*a**2*b
 + 3*sin(c + d*x)**2*a*b**2 - sin(c + d*x)**2*b**3 - a**3 + 2*a**2*b - a*b
**2))
```

3.468 $\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = -\frac{(4a-b)b\,\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d}+\frac{\sin(c+dx)}{(a-b)^2d}+\frac{b^2\sin(c+dx)}{2a(a-b)^2d\left(a-(a-b)\sin^2(c+dx)\right)}$$

output
```
-1/2*(4*a-b)*b*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(5/2)
/d+sin(d*x+c)/(a-b)^2/d+1/2*b^2*sin(d*x+c)/a/(a-b)^2/d/(a-(a-b)*sin(d*x+c)
^2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

$$\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\frac{1}{2}(4a-b)b\,\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)(a+b+(a-b)\cos(2(c+dx)))-\sqrt{a}\sqrt{a-b}(a^2+ab+b^2+a(a-b)}{2a^{3/2}(a-b)^{5/2}d\left(-a+(a-b)\sin^2(c+dx)\right)}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]]*(a + b + (a - b)
*Cos[2*(c + d*x)]))/2 - Sqrt[a]*Sqrt[a - b]*(a^2 + a*b + b^2 + a*(a - b)*C
os[2*(c + d*x)])*Sin[c + d*x])/(2*a^(3/2)*(a - b)^(5/2)*d*(-a + (a - b)*Si
n[c + d*x]^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)\,(a+b\tan(c+dx)^2)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{(1-\sin^2(c+dx))^2}{(a-(a-b)\sin^2(c+dx))^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{(a-b)^2}-\frac{(2a-b)b-2(a-b)b\sin^2(c+dx)}{(a-b)^2((b-a)\sin^2(c+dx)+a)^2}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(4a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}}+\frac{b^2\sin(c+dx)}{2a(a-b)^2(a-(a-b)\sin^2(c+dx))}+\frac{\sin(c+dx)}{(a-b)^2}}{d}$$

input
```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(-1/2*((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(a^(3/2)*(
a - b)^(5/2)) + Sin[c + d*x]/(a - b)^2 + (b^2*Sin[c + d*x])/(2*a*(a - b)^2
*(a - (a - b)*Sin[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\sin(dx+c)}{a^2-2ab+b^2}+\frac{b\left(-\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(4a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{(a-b)^2}}{d}$
default	$\frac{\frac{\sin(dx+c)}{a^2-2ab+b^2}+\frac{b\left(-\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(4a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin(dx+c)}{\sqrt{a(a-b)}}\right)}{2a\sqrt{a(a-b)}}\right)}{(a-b)^2}}{d}$
risch	$-\frac{ie^{i(dx+c)}}{2d(a^2-2ab+b^2)}+\frac{ie^{-i(dx+c)}}{2d(a^2-2ab+b^2)}+\frac{ib^2\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{da(-a+b)^2\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-a+b\right)}$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(sin(d*x+c)/(a^2-2*a*b+b^2)+b/(a-b)^2*(-1/2/a*b*sin(d*x+c)/(a*sin(d*x+
c)^2-b*sin(d*x+c)^2-a)-1/2*(4*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x
+c)/(a*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(103) = 206.

Time = 0.15 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.96

$$\int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \left[-\frac{\left(4\,ab^2-b^3+(4\,a^2b-5\,ab^2+b^3)\cos(dx+c)^2\right)\sqrt{a^2-ab}\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2\,a+b}{(a-b)\cos(dx+c)^2+b}\right)}{4\left((a^6-4\,a^5b+6\,a^4b^2-4\,a^3b^3+a^2b^4)d\cos(dx+c)^2+(\right.}\right.$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(a^2
 - a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c) - 2*
a + b)/((a - b)*cos(d*x + c)^2 + b)) - 2*(2*a^3*b - a^2*b^2 - a*b^3 + 2*(a
^4 - 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 4*a^5*b + 6*
a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^2 + (a^5*b - 3*a^4*b^2 + 3*a
^3*b^3 - a^2*b^4)*d), 1/2*((4*a*b^2 - b^3 + (4*a^2*b - 5*a*b^2 + b^3)*cos(
d*x + c)^2)*sqrt(-a^2 + a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) + (2*
a^3*b - a^2*b^2 - a*b^3 + 2*(a^4 - 2*a^3*b + a^2*b^2)*cos(d*x + c)^2)*sin(
d*x + c))/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d*cos(d*x + c
)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d)]
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.33

$$\int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = -\frac{\frac{b^2\sin(dx+c)}{(a^3-2a^2b+ab^2)\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}+\frac{(4ab-b^2)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^3-2a^2b+ab^2)\sqrt{-a^2+ab}}-\frac{2\sin(dx+c)}{a^2-2ab+b^2}}{2d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*sin(d*x + c)/((a^3 - 2*a^2*b + a*b^2)*(a*sin(d*x + c)^2 - b*sin(
d*x + c)^2 - a)) + (4*a*b - b^2)*arctan(-(a*sin(d*x + c) - b*sin(d*x + c))
/sqrt(-a^2 + a*b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) - 2*sin(d*x
+ c)/(a^2 - 2*a*b + b^2))/d
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.36

$$\int \frac{\cos(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^2+b^2)}{a(a-b)^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^2+b^2)}{a(a-b)^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(-2a^2+4ab+b^2)}{a(a-b)^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+(4b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(4b-a)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)} + \frac{b\,\mathrm{atan}\left(\frac{2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^3-6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b+6\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}{\sqrt{a}(a-b)^{5/2}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}\right)(4a-b)\,1\mathrm{i}}{2a^{3/2}d(a-b)^{5/2}}$$

input
```
int(cos(c + d*x)/(a + b*tan(c + d*x)^2)^2,x)
```

output
```
((tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(a*(a - b)^2) + (tan(c/2 + (d*x)/2)^5*
(2*a^2 + b^2))/(a*(a - b)^2) + (2*tan(c/2 + (d*x)/2)^3*(4*a*b - 2*a^2 + b^
2))/(a*(a - b)^2))/(d*(a - tan(c/2 + (d*x)/2)^2*(a - 4*b) - tan(c/2 + (d*x
)/2)^4*(a - 4*b) + a*tan(c/2 + (d*x)/2)^6)) + (b*atan((a^3*tan(c/2 + (d*x)
/2)*2i - b^3*tan(c/2 + (d*x)/2)*2i + a*b^2*tan(c/2 + (d*x)/2)*6i - a^2*b*t
an(c/2 + (d*x)/2)*6i)/(a^(1/2)*(a - b)^(5/2)*(tan(c/2 + (d*x)/2)^2 + 1)))*
(4*a - b)*1i)/(2*a^(3/2)*d*(a - b)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 730, normalized size of antiderivative = 6.40

$$\int \frac{\cos(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(4*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan
((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a**2*b - 5*sqrt(a)*sqrt(a - b)
*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqr
t(a))*sin(c + d*x)**2*a*b**2 + sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*ta
n((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*b*
*3 - 4*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)
*tan((c + d*x)/2)**2 + sqrt(a))*a**2*b + sqrt(a)*sqrt(a - b)*log( - 2*sqrt
(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a*b**2 -
 4*sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c
 + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a**2*b + 5*sqrt(a)*sqrt(a - b)*lo
g(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*
sin(c + d*x)**2*a*b**2 - sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*
x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*b**3 + 4*sq
rt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*
x)/2)**2 + sqrt(a))*a**2*b - sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c
+ d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a*b**2 + 4*sin(c + d*x)
**3*a**4 - 8*sin(c + d*x)**3*a**3*b + 4*sin(c + d*x)**3*a**2*b**2 - 4*sin(
c + d*x)*a**4 + 4*sin(c + d*x)*a**3*b - 2*sin(c + d*x)*a**2*b**2 + 2*sin(c
 + d*x)*a*b**3)/(4*a**2*d*(sin(c + d*x)**2*a**4 - 4*sin(c + d*x)**2*a**3*b
 + 6*sin(c + d*x)**2*a**2*b**2 - 4*sin(c + d*x)**2*a*b**3 + sin(c + d*x...
```

3.469 $\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{(6a-b)b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d}+\frac{(a-3b)\sin(c+dx)}{(a-b)^3d}-\frac{\sin^3(c+dx)}{3(a-b)^2d}-\frac{b^3\sin(c+dx)}{2a(a-b)^3d\left(a-(a-b)\sin^2(c+dx)\right)}$$

output
```
1/2*(6*a-b)*b^2*arctanh((a-b)^(1/2)*sin(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(7/2
)/d+(a-3*b)*sin(d*x+c)/(a-b)^3/d-1/3*sin(d*x+c)^3/(a-b)^2/d-1/2*b^3*sin(d*
x+c)/a/(a-b)^3/d/(a-(a-b)*sin(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\frac{3b^2(-6a+b)\left(\log\left(\sqrt{a}-\sqrt{a-b}\sin(c+dx)\right)-\log\left(\sqrt{a}+\sqrt{a-b}\sin(c+dx)\right)\right)}{a^{3/2}(a-b)^{7/2}}+\frac{3\left(3a-11b-\frac{4b^3}{a(a+b+(a-b)\cos(2(c+dx)))}\right)\sin(c+dx)}{(a-b)^3}+\frac{\sin(3(c+dx))}{(a-b)^2}}{12d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((3*b^2*(-6*a + b)*(Log[Sqrt[a] - Sqrt[a - b]*Sin[c + d*x]] - Log[Sqrt[a]
+ Sqrt[a - b]*Sin[c + d*x]]))/(a^(3/2)*(a - b)^(7/2)) + (3*(3*a - 11*b - (
4*b^3)/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))*Sin[c + d*x])/(a - b)^3 + S
in[3*(c + d*x)]/(a - b)^2)/(12*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3\left(a+b\tan(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(1-\sin^2(c+dx)\right)^3}{\left(a-(a-b)\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-\frac{\sin^2(c+dx)}{(a-b)^2}+\frac{(3a-b)b^2-3(a-b)b^2\sin^2(c+dx)}{(a-b)^3\left((b-a)\sin^2(c+dx)+a\right)^2}+\frac{a-3b}{(a-b)^3}\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(6a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\sin(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}}-\frac{b^3\sin(c+dx)}{2a(a-b)^3\left(a-(a-b)\sin^2(c+dx)\right)}-\frac{\sin^3(c+dx)}{3(a-b)^2}+\frac{(a-3b)\sin(c+dx)}{(a-b)^3}}{d}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Sin[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a
 - b)^(7/2)) + ((a - 3*b)*Sin[c + d*x])/(a - b)^3 - Sin[c + d*x]^3/(3*(a -
 b)^2) - (b^3*Sin[c + d*x])/(2*a*(a - b)^3*(a - (a - b)*Sin[c + d*x]^2)))/
d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 16.76 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(dx+c)^3}{3}-\frac{b\sin(dx+c)^3}{3}-\sin(dx+c)a+3b\sin(dx+c)}{(a^2-2ab+b^2)(a-b)}-\frac{b^2\left(-\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(6a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin}{\sqrt{a(a}}\right.}{2a\sqrt{a(a-b)}}\right.}{(a-b)^3}}{d}$
default	$\frac{-\frac{\frac{a\sin(dx+c)^3}{3}-\frac{b\sin(dx+c)^3}{3}-\sin(dx+c)a+3b\sin(dx+c)}{(a^2-2ab+b^2)(a-b)}-\frac{b^2\left(-\frac{b\sin(dx+c)}{2a\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)}-\frac{(6a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\sin}{\sqrt{a(a}}\right.}{2a\sqrt{a(a-b)}}\right.}{(a-b)^3}}{d}$
risch	$-\frac{ie^{3i(dx+c)}}{24d(a^2-2ab+b^2)}-\frac{3ie^{i(dx+c)}a}{8(a^2-2ab+b^2)(a-b)d}+\frac{11ie^{i(dx+c)}b}{8(a^2-2ab+b^2)(a-b)d}+\frac{3ie^{-i(dx+c)}a}{8d(a^3-3a^2b+3a\,b^2-b^3)}-\frac{11ie^{-}}{8d(a^3-3a^2}$

input

```
int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*sin(d*x+c)^3-1/3*b*sin(d*x+c)^3-sin(d
*x+c)*a+3*b*sin(d*x+c))-b^2/(a-b)^3*(-1/2/a*b*sin(d*x+c)/(a*sin(d*x+c)^2-b
*sin(d*x+c)^2-a)-1/2*(6*a-b)/a/(a*(a-b))^(1/2)*arctanh((a-b)*sin(d*x+c)/(a
*(a-b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(130) = 260.

Time = 0.16 (sec) , antiderivative size = 600, normalized size of antiderivative = 4.20

$$\int \frac{\cos^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

$$= \left[\frac{3\left(6\,ab^3-b^4+(6\,a^2b^2-7\,ab^3+b^4)\cos{(dx+c)}^2\right)\sqrt{a^2-ab}\log\left(-\frac{(a-b)\cos(dx+c)^2-2\sqrt{a^2-ab}\sin(dx+c)-2a+b}{(a-b)\cos(dx+c)^2+b}\right.}{12\left((a^7-5\,a^6b+10\,a^5b^2-10\,a\right.}\right.$$

$$-\frac{3\left(6\,ab^3-b^4+(6\,a^2b^2-7\,ab^3+b^4)\cos{(dx+c)}^2\right)\sqrt{-a^2+ab}\arctan\left(\frac{\sqrt{-a^2+ab}\sin(dx+c)}{a}\right)-\left(4\,a^4b-20\right.}{6\left((a^7-5\,a^6b+10\,a^5b^2-10\,a^4b^3+5\,a\right.}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(3*(6*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt
(a^2 - a*b)*log(-((a - b)*cos(d*x + c)^2 - 2*sqrt(a^2 - a*b)*sin(d*x + c)
- 2*a + b)/((a - b)*cos(d*x + c)^2 + b)) + 2*(4*a^4*b - 20*a^3*b^2 + 13*a^
2*b^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^4 +
 2*(2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c
))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*d*cos(
d*x + c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d), -1/
6*(3*(6*a*b^3 - b^4 + (6*a^2*b^2 - 7*a*b^3 + b^4)*cos(d*x + c)^2)*sqrt(-a^
2 + a*b)*arctan(sqrt(-a^2 + a*b)*sin(d*x + c)/a) - (4*a^4*b - 20*a^3*b^2 +
 13*a^2*b^3 + 3*a*b^4 + 2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x +
c)^4 + 2*(2*a^5 - 11*a^4*b + 16*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d
*x + c))/((a^7 - 5*a^6*b + 10*a^5*b^2 - 10*a^4*b^3 + 5*a^3*b^4 - a^2*b^5)*
d*cos(d*x + c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(130) = 260.

Time = 0.77 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.30

$$\int \frac{\cos^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \frac{\frac{3\,b^3\sin(dx+c)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\left(a\sin(dx+c)^2-b\sin(dx+c)^2-a\right)} + \frac{3\,(6\,ab^2-b^3)\arctan\left(-\frac{a\sin(dx+c)-b\sin(dx+c)}{\sqrt{-a^2+ab}}\right)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\sqrt{-a^2+ab}} - \frac{2\left(a^4\sin(dx+c)^3-4\,a^3b\sin(\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*b^3*sin(d*x + c)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a*sin(d*x +
c)^2 - b*sin(d*x + c)^2 - a)) + 3*(6*a*b^2 - b^3)*arctan(-(a*sin(d*x + c)
- b*sin(d*x + c))/sqrt(-a^2 + a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*s
qrt(-a^2 + a*b)) - 2*(a^4*sin(d*x + c)^3 - 4*a^3*b*sin(d*x + c)^3 + 6*a^2*
b^2*sin(d*x + c)^3 - 4*a*b^3*sin(d*x + c)^3 + b^4*sin(d*x + c)^3 - 3*a^4*s
in(d*x + c) + 18*a^3*b*sin(d*x + c) - 36*a^2*b^2*sin(d*x + c) + 30*a*b^3*s
in(d*x + c) - 9*b^4*sin(d*x + c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3
 + 15*a^2*b^4 - 6*a*b^5 + b^6))/d
```

Mupad [B] (verification not implemented)

Time = 12.72 (sec) , antiderivative size = 1690, normalized size of antiderivative = 11.82

$$\int \frac{\cos^3(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a + b*tan(c + d*x)^2)^2,x)
```

output
```
- ((tan(c/2 + (d*x)/2)*(6*a^2*b - 2*a^3 + b^3))/(a*(3*a*b^2 - 3*a^2*b + a^
3 - b^3)) + (tan(c/2 + (d*x)/2)^9*(6*a^2*b - 2*a^3 + b^3))/(a*(3*a*b^2 - 3
*a^2*b + a^3 - b^3)) + (4*tan(c/2 + (d*x)/2)^3*(18*a*b^2 - 8*a^2*b + 2*a^3
 + 3*b^3))/(3*a*(a - b)*(a^2 - 2*a*b + b^2)) + (4*tan(c/2 + (d*x)/2)^7*(18
*a*b^2 - 8*a^2*b + 2*a^3 + 3*b^3))/(3*a*(a - b)*(a^2 - 2*a*b + b^2)) + (2*
tan(c/2 + (d*x)/2)^5*(56*a*b^2 - 18*a^2*b - 2*a^3 + 9*b^3))/(3*a*(a - b)*(
a^2 - 2*a*b + b^2)))/(d*(a + tan(c/2 + (d*x)/2)^2*(a + 4*b) + tan(c/2 + (d
*x)/2)^8*(a + 4*b) - tan(c/2 + (d*x)/2)^4*(2*a - 12*b) - tan(c/2 + (d*x)/2
)^6*(2*a - 12*b) + a*tan(c/2 + (d*x)/2)^10)) - (b^2*atan(((b^2*(tan(c/2 +
(d*x)/2)*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b^8 - 880*a^6*b^7 + 1080*a^7*b
^6 - 768*a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3) - (b^2*(6*a - b)*(tan(c/2 +
(d*x)/2)^2*(16*a^15 - 176*a^14*b + 32*a^5*b^10 - 304*a^6*b^9 + 1296*a^7*b^
8 - 3264*a^8*b^7 + 5376*a^9*b^6 - 6048*a^10*b^5 + 4704*a^11*b^4 - 2496*a^1
2*b^3 + 864*a^13*b^2) + 144*a^14*b - 16*a^15 + 16*a^6*b^9 - 144*a^7*b^8 +
576*a^8*b^7 - 1344*a^9*b^6 + 2016*a^10*b^5 - 2016*a^11*b^4 + 1344*a^12*b^3
 - 576*a^13*b^2))/(4*a^(3/2)*(a - b)^(7/2)))*(6*a - b)*1i)/(4*a^(3/2)*(a -
 b)^(7/2)) + (b^2*(tan(c/2 + (d*x)/2)*(8*a^3*b^10 - 96*a^4*b^9 + 408*a^5*b
^8 - 880*a^6*b^7 + 1080*a^7*b^6 - 768*a^8*b^5 + 296*a^9*b^4 - 48*a^10*b^3)
 + (b^2*(6*a - b)*(tan(c/2 + (d*x)/2)^2*(16*a^15 - 176*a^14*b + 32*a^5*b^1
0 - 304*a^6*b^9 + 1296*a^7*b^8 - 3264*a^8*b^7 + 5376*a^9*b^6 - 6048*a^1...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 855, normalized size of antiderivative = 5.98

$$\int \frac{\cos^3(c+dx)}{(a+b\tan^2(c+dx))^2} \, dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 18*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)
*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a**2*b**2 + 21*sqrt(a)*sqr
t(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)*
*2 + sqrt(a))*sin(c + d*x)**2*a*b**3 - 3*sqrt(a)*sqrt(a - b)*log( - 2*sqrt
(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c +
d*x)**2*b**4 + 18*sqrt(a)*sqrt(a - b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2
) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a**2*b**2 - 3*sqrt(a)*sqrt(a -
b)*log( - 2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + s
qrt(a))*a*b**3 + 18*sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2)
 + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a**2*b**2 - 21*s
qrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d
*x)/2)**2 + sqrt(a))*sin(c + d*x)**2*a*b**3 + 3*sqrt(a)*sqrt(a - b)*log(2*
sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*sin(
c + d*x)**2*b**4 - 18*sqrt(a)*sqrt(a - b)*log(2*sqrt(a - b)*tan((c + d*x)/
2) + sqrt(a)*tan((c + d*x)/2)**2 + sqrt(a))*a**2*b**2 + 3*sqrt(a)*sqrt(a -
 b)*log(2*sqrt(a - b)*tan((c + d*x)/2) + sqrt(a)*tan((c + d*x)/2)**2 + sqr
t(a))*a*b**3 - 4*sin(c + d*x)**5*a**5 + 12*sin(c + d*x)**5*a**4*b - 12*sin
(c + d*x)**5*a**3*b**2 + 4*sin(c + d*x)**5*a**2*b**3 + 16*sin(c + d*x)**3*
a**5 - 68*sin(c + d*x)**3*a**4*b + 88*sin(c + d*x)**3*a**3*b**2 - 36*sin(c
 + d*x)**3*a**2*b**3 - 12*sin(c + d*x)*a**5 + 48*sin(c + d*x)*a**4*b - ...
```

3.470 $\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{(a-b)^2(5a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}d} - \frac{(2a-3b)\tan(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d} - \frac{(a-b)^3\tan(c+dx)}{2ab^3d\left(a+b\tan^2(c+dx)\right)}$$

output
```
1/2*(a-b)^2*(5*a+b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(7/2)/d-(
2*a-3*b)*tan(d*x+c)/b^3/d+1/3*tan(d*x+c)^3/b^2/d-1/2*(a-b)^3*tan(d*x+c)/a/
b^3/d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 4.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\frac{3(a-b)^2(5a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{3\sqrt{b}(-a+b)^3\sin(2(c+dx))}{a(a+b+(a-b)\cos(2(c+dx)))}+4\sqrt{b}(-3a+4b)\tan(c+dx)+2b^{3/2}\sec^2(c+dx)}{6b^{7/2}d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((3*(a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) +
(3*Sqrt[b]*(-a + b)^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x
)])) + 4*Sqrt[b]*(-3*a + 4*b)*Tan[c + d*x] + 2*b^(3/2)*Sec[c + d*x]^2*Tan[
c + d*x])/(6*b^(7/2)*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^8}{\left(a+b\tan(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)+1\right)^3}{\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(\frac{\tan^2(c+dx)}{b^2}+\frac{3b\tan^2(c+dx)(a-b)^2+(2a+b)(a-b)^2}{b^3\left(b\tan^2(c+dx)+a\right)^2}-\frac{2a-3b}{b^3}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(5a+b)(a-b)^2\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}-\frac{(a-b)^3\tan(c+dx)}{2ab^3(a+b\tan^2(c+dx))}-\frac{(2a-3b)\tan(c+dx)}{b^3}+\frac{\tan^3(c+dx)}{3b^2}}{d}$$

input
```
Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((a - b)^2*(5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b
^(7/2)) - ((2*a - 3*b)*Tan[c + d*x])/b^3 + Tan[c + d*x]^3/(3*b^2) - ((a -
b)^3*Tan[c + d*x])/(2*a*b^3*(a + b*Tan[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 201.97 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(dx+c)^3}{3}+2a\tan(dx+c)-3b\tan(dx+c)}{b^3}+\frac{-\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{\left(5a^3-9a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}}{d}$
default	$\frac{-\frac{-\frac{b\tan(dx+c)^3}{3}+2a\tan(dx+c)-3b\tan(dx+c)}{b^3}+\frac{-\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{\left(5a^3-9a^2b+3a\,b^2+b^3\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}}{d}$
risch	$\frac{i\left(15a^3\mathrm{e}^{8i(dx+c)}-27a^2b\,\mathrm{e}^{8i(dx+c)}+9a\,b^2\mathrm{e}^{8i(dx+c)}+3b^3\mathrm{e}^{8i(dx+c)}+60a^3\mathrm{e}^{6i(dx+c)}-78a^2b\,\mathrm{e}^{6i(dx+c)}+12a\,b^2\mathrm{e}^{6i(dx+c)}+6\right.}{3d\,b^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^3a}$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^3*(-1/3*b*tan(d*x+c)^3+2*a*tan(d*x+c)-3*b*tan(d*x+c))+1/b^3*(-1/
2*(a^3-3*a^2*b+3*a*b^2-b^3)/a*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(5*a^3-9*a
^2*b+3*a*b^2+b^3)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(113) = 226.

Time = 0.15 (sec) , antiderivative size = 597, normalized size of antiderivative = 4.70

$$\int \frac{\sec^8(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

$$= \left[-\frac{3\left(\left(5\,a^4-14\,a^3b+12\,a^2b^2-2\,ab^3-b^4\right)\cos\left(dx+c\right)^5+\left(5\,a^3b-9\,a^2b^2+3\,ab^3+b^4\right)\cos\left(dx+c\right)^3\right)\sqrt{}}{}\right.$$

$$-\frac{3\left(\left(5\,a^4-14\,a^3b+12\,a^2b^2-2\,ab^3-b^4\right)\cos\left(dx+c\right)^5+\left(5\,a^3b-9\,a^2b^2+3\,ab^3+b^4\right)\cos\left(dx+c\right)^3\right)\sqrt{a}}{12\left(a^2\right.}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/24*(3*((5*a^4 - 14*a^3*b + 12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5
+ (5*a^3*b - 9*a^2*b^2 + 3*a*b^3 + b^4)*cos(d*x + c)^3)*sqrt(-a*b)*log(((a
^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a
+ b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2
 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) - 4*
(2*a^2*b^3 - (15*a^4*b - 37*a^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4
 - 2*(5*a^3*b^2 - 7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(
d*x + c)^3 + (a^3*b^4 - a^2*b^5)*d*cos(d*x + c)^5), -1/12*(3*((5*a^4 - 14*
a^3*b + 12*a^2*b^2 - 2*a*b^3 - b^4)*cos(d*x + c)^5 + (5*a^3*b - 9*a^2*b^2
+ 3*a*b^3 + b^4)*cos(d*x + c)^3)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c
)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) - 2*(2*a^2*b^3 - (15*a
^4*b - 37*a^3*b^2 + 25*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4 - 2*(5*a^3*b^2 -
7*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^5*d*cos(d*x + c)^3 + (a^3*
b^4 - a^2*b^5)*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^8(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{\sec^8(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = -\frac{\frac{3\,(a^3-3\,a^2b+3\,ab^2-b^3)\tan(dx+c)}{ab^4\tan(dx+c)^2+a^2b^3} - \frac{2\left(b\tan(dx+c)^3-3\,(2\,a-3\,b)\tan(dx+c)\right)}{b^3} - \frac{3\,(5\,a^3-9\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}ab^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*tan(d*x + c)/(a*b^4*tan(d*x + c)^2
 + a^2*b^3) - 2*(b*tan(d*x + c)^3 - 3*(2*a - 3*b)*tan(d*x + c))/b^3 - 3*(5
*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b
)*a*b^3))/d
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int \frac{\sec^8(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{(5a^3-9a^2b+3ab^2+b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^3d} - \frac{a^3\tan(dx+c)-3a^2b\tan(dx+c)+3ab^2\tan(dx+c)-b^3\tan(dx+c)}{2\left(b\tan(dx+c)^2+a\right)ab^3d} + \frac{b^4d^2\tan(dx+c)^3-6ab^3d^2\tan(dx+c)+9b^4d^2\tan(dx+c)}{3b^6d^3}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sq
rt(a*b)*a*b^3*d) - 1/2*(a^3*tan(d*x + c) - 3*a^2*b*tan(d*x + c) + 3*a*b^2*
tan(d*x + c) - b^3*tan(d*x + c))/((b*tan(d*x + c)^2 + a)*a*b^3*d) + 1/3*(b
^4*d^2*tan(d*x + c)^3 - 6*a*b^3*d^2*tan(d*x + c) + 9*b^4*d^2*tan(d*x + c))
/(b^6*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.31

$$\int \frac{\sec^8(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\tan(c+dx)^3}{3b^2d} - \frac{\tan(c+dx)\left(\frac{2a}{b^3}-\frac{3}{b^2}\right)}{d} - \frac{\tan(c+dx)\left(a^3-3a^2b+3ab^2-b^3\right)}{2ad\left(b^4\tan(c+dx)^2+ab^3\right)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(c+dx)(a-b)^2(5a+b)}{\sqrt{a}(5a^3-9a^2b+3ab^2+b^3)}\right)(a-b)^2(5a+b)}{2a^{3/2}b^{7/2}d}$$

input
```
int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
tan(c + d*x)^3/(3*b^2*d) - (tan(c + d*x)*((2*a)/b^3 - 3/b^2))/d - (tan(c +
 d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*a*d*(a*b^3 + b^4*tan(c + d*x)^2)
) + (atan((b^(1/2)*tan(c + d*x)*(a - b)^2*(5*a + b))/(a^(1/2)*(3*a*b^2 - 9
*a^2*b + 5*a^3 + b^3)))*(a - b)^2*(5*a + b))/(2*a^(3/2)*b^(7/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1562, normalized size of antiderivative = 12.30

$$\int \frac{\sec^8(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(
b))*cos(c + d*x)*sin(c + d*x)**4*a**4 + 42*sqrt(b)*sqrt(a)*atan((sqrt(a -
b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**4*a**3*
b - 36*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(
b))*cos(c + d*x)*sin(c + d*x)**4*a**2*b**2 + 6*sqrt(b)*sqrt(a)*atan((sqrt(
a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**4*a
*b**3 + 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sq
rt(b))*cos(c + d*x)*sin(c + d*x)**4*b**4 + 30*sqrt(b)*sqrt(a)*atan((sqrt(a
 - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a*
*4 - 69*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt
(b))*cos(c + d*x)*sin(c + d*x)**2*a**3*b + 45*sqrt(b)*sqrt(a)*atan((sqrt(a
 - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a*
*2*b**2 - 3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/
sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a*b**3 - 3*sqrt(b)*sqrt(a)*atan((sqr
t(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2
*b**4 - 15*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/s
qrt(b))*cos(c + d*x)*a**4 + 27*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)
*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a**3*b - 9*sqrt(b)*sqrt(a)*atan((
sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a**2*b**2 -
3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b)...
```

3.471 $\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = -\frac{(3a^2-2ab-b^2)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\tan(c+dx)}{b^2d} + \frac{(a-b)^2\tan(c+dx)}{2ab^2d\left(a+b\tan^2(c+dx)\right)}$$

output
```
-1/2*(3*a^2-2*a*b-b^2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)/
d+tan(d*x+c)/b^2/d+1/2*(a-b)^2*tan(d*x+c)/a/b^2/d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{\sec^6(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{-\frac{(a-b)(3a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(a-b)^2\sqrt{b}\sin(2(c+dx))}{a(a+b+(a-b)\cos(2(c+dx)))} + 2\sqrt{b}\tan(c+dx)}{2b^{5/2}d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(-(((a - b)*(3*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2)) + (
(a - b)^2*Sqrt[b]*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)]))
 + 2*Sqrt[b]*Tan[c + d*x])/(2*b^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+b\tan(c+dx)^2)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{(\tan^2(c+dx)+1)^2}{(b\tan^2(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2(a-b)b\tan^2(c+dx)}{b^2(b\tan^2(c+dx)+a)^2}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(3a^2-2ab-b^2)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a-b)^2\tan(c+dx)}{2ab^2(a+b\tan^2(c+dx))}+\frac{\tan(c+dx)}{b^2}}{d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(-1/2*((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(a^(3
/2)*b^(5/2)) + Tan[c + d*x]/b^2 + ((a - b)^2*Tan[c + d*x])/(2*a*b^2*(a + b
*Tan[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 67.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{-\frac{\left(a^2-2ab+b^2\right)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{\left(3a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{-\frac{\left(a^2-2ab+b^2\right)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{\left(3a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}}{d}$
risch	$\frac{i\left(-3a^2e^{4i(dx+c)}+2\,e^{4i(dx+c)}ab+b^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-2ab\,e^{2i(dx+c)}-3a^2+4ab-b^2\right)}{a\,b^2d\left(-a\,e^{4i(dx+c)}+b\,e^{4i(dx+c)}-2a\,e^{2i(dx+c)}-2b\,e^{2i(dx+c)}-a+b\right)\left(e^{2i(dx+c)}+1\right)}-\frac{3a\ln\left(e^{2i(dx+c)}-\frac{2iab-}{}\right.}{4\sqrt{-ab}}$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)/b^2-1/b^2*(-1/2*(a^2-2*a*b+b^2)/a*tan(d*x+c)/(a+b*tan(d*x+
c)^2)+1/2*(3*a^2-2*a*b-b^2)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

Time = 0.18 (sec) , antiderivative size = 479, normalized size of antiderivative = 4.61

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \left[\frac{((3a^3-5a^2b+ab^2+b^3)\cos(dx+c)^3+(3a^2b-2ab^2-b^3)\cos(dx+c))\sqrt{-ab}\log\left(\frac{(a^2+6ab+b^2)\cos(dx\cdots}{\cdots}\right.}{8\left(a^2b^4d\cos(\cdots\right.}\right.$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*x + c)^3 + (3*a^2*b - 2*a*b^2
 - b^3)*cos(d*x + c))*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 -
 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x +
c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2
*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + 4*(2*a^2*b^2 + (3*a^3*b - 4*a^2*b^2
+ a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^2*b^4*d*cos(d*x + c) + (a^3*b^3
- a^2*b^4)*d*cos(d*x + c)^3), 1/4*(((3*a^3 - 5*a^2*b + a*b^2 + b^3)*cos(d*
x + c)^3 + (3*a^2*b - 2*a*b^2 - b^3)*cos(d*x + c))*sqrt(a*b)*arctan(1/2*((
a + b)*cos(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))) + 2*
(2*a^2*b^2 + (3*a^3*b - 4*a^2*b^2 + a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(
a^2*b^4*d*cos(d*x + c) + (a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\frac{(a^2-2\,ab+b^2)\tan(dx+c)}{ab^3\tan(dx+c)^2+a^2b^2} + \frac{2\,\tan(dx+c)}{b^2} - \frac{(3\,a^2-2\,ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a^2 - 2*a*b + b^2)*tan(d*x + c)/(a*b^3*tan(d*x + c)^2 + a^2*b^2) + 2
*tan(d*x + c)/b^2 - (3*a^2 - 2*a*b - b^2)*arctan(b*tan(d*x + c)/sqrt(a*b))
/(sqrt(a*b)*a*b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\tan(dx+c)}{b^2d} - \frac{(3\,a^2-2\,ab-b^2)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2d} + \frac{a^2\tan(dx+c) - 2\,ab\tan(dx+c) + b^2\tan(dx+c)}{2\,(b\tan(dx+c)^2+a)ab^2d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
tan(d*x + c)/(b^2*d) - 1/2*(3*a^2 - 2*a*b - b^2)*arctan(b*tan(d*x + c)/sqr
t(a*b))/(sqrt(a*b)*a*b^2*d) + 1/2*(a^2*tan(d*x + c) - 2*a*b*tan(d*x + c) +
 b^2*tan(d*x + c))/((b*tan(d*x + c)^2 + a)*a*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\tan(c+d\,x)}{b^2\,d}+\frac{\tan(c+d\,x)\,\left(a^2-2\,a\,b+b^2\right)}{2\,a\,d\,\left(b^3\tan\left(c+d\,x\right)^2+a\,b^2\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(c+d\,x)\,(a-b)\,(3\,a+b)}{\sqrt{a}\,(-3\,a^2+2\,a\,b+b^2)}\right)\,(a-b)\,(3\,a+b)}{2\,a^{3/2}\,b^{5/2}\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
tan(c + d*x)/(b^2*d) + (tan(c + d*x)*(a^2 - 2*a*b + b^2))/(2*a*d*(a*b^2 +
b^3*tan(c + d*x)^2)) + (atan((b^(1/2)*tan(c + d*x)*(a - b)*(3*a + b))/(a^(
1/2)*(2*a*b - 3*a^2 + b^2)))*(a - b)*(3*a + b))/(2*a^(3/2)*b^(5/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 764, normalized size of antiderivative = 7.35

$$\int \frac{\sec^6(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*
cos(c + d*x)*sin(c + d*x)**2*a**3 - 5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) -
sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a**2*b + s
qrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(
c + d*x)*sin(c + d*x)**2*a*b**2 + sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt
(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*b**3 - 3*sqrt(
b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c +
d*x)*a**3 + 2*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2)
)/sqrt(b))*cos(c + d*x)*a**2*b + sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(
a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a*b**2 - 3*sqrt(b)*sqrt(a)*atan
((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*
x)**2*a**3 + 5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2
))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**2*a**2*b - sqrt(b)*sqrt(a)*atan((sq
rt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*sin(c + d*x)**
2*a*b**2 - sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/s
qrt(b))*cos(c + d*x)*sin(c + d*x)**2*b**3 + 3*sqrt(b)*sqrt(a)*atan((sqrt(a
 - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)*a**3 - 2*sqrt(b)*s
qrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*cos(c + d*x)
*a**2*b - sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sq
rt(b))*cos(c + d*x)*a*b**2 + 3*sin(c + d*x)**3*a**3*b - 4*sin(c + d*x)*...
```

3.472 $\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx=\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\tan(c+dx)}{2abd\,(a+b\tan^2(c+dx))}$$

output
```
1/2*(a+b)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d-1/2*(a-b)*t
an(d*x+c)/a/b/d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx=\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)+\frac{\sqrt{a}\sqrt{b}(-a+b)\sin(2(c+dx))}{a+b+(a-b)\cos(2(c+dx))}}{2a^{3/2}b^{3/2}d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(-a + b
)*Sin[2*(c + d*x)])/(a + b + (a - b)*Cos[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2)
*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{\left(a+b\tan(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\tan^2(c+dx)+1}{\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(a+b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{2ab}-\frac{(a-b)\tan(c+dx)}{2ab(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{(a-b)\tan(c+dx)}{2ab(a+b\tan^2(c+dx))}}{d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(((a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) - ((
a - b)*Tan[c + d*x])/(2*a*b*(a + b*Tan[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 19.96 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{(a-b)\tan(dx+c)}{2ab\left(a+b\tan(dx+c)^2\right)}+\frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}}{d}$
default	$\frac{-\frac{(a-b)\tan(dx+c)}{2ab\left(a+b\tan(dx+c)^2\right)}+\frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}}{d}$
risch	$-\frac{i\left(a\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+a-b\right)}{abd\left(a\,e^{4i(dx+c)}-b\,e^{4i(dx+c)}+2a\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+a-b\right)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,db}-\frac{\ln\left(e^{2\cdots\right.}{}$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*(a-b)/a/b*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(a+b)/a/b/(a*b)^(1/2
)*arctan(b*tan(d*x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(65) = 130$.

Time = 0.14 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.77

$$\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$
$$= \left[-\frac{4\left(a^2b-ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+ab+b^2\right)\sqrt{-ab}\log\left(\frac{\left(a^2+6\,ab+b^2\right)\cos(\ldots}{\ldots}\right.}{8\left(a^2b^3d+(a^3b^2-a^2b^3)d\cos\left(dx+\ldots\right.\right.}\right.$$
$$-\frac{2\left(a^2b-ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+ab+b^2\right)\sqrt{ab}\arctan\left(\frac{\left((a+b)\cos(dx+c\ldots\right.}{2\,ab\cos(dx+c)\sin\ldots}\right.}{4\left(a^2b^3d+(a^3b^2-a^2b^3)d\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b^2)*cos(d*x
+ c)^2 + a*b + b^2)*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2
*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c)
)*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(
a*b - b^2)*cos(d*x + c)^2 + b^2)))/(a^2*b^3*d + (a^3*b^2 - a^2*b^3)*d*cos(
d*x + c)^2), -1/4*(2*(a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + ((a^2 - b
^2)*cos(d*x + c)^2 + a*b + b^2)*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)
^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/(a^2*b^3*d + (a^3*b^2
- a^2*b^3)*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = -\frac{\frac{(a-b)\tan(dx+c)}{ab^2\tan(dx+c)^2+a^2b} - \frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}ab}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((a - b)*tan(d*x + c)/(a*b^2*tan(d*x + c)^2 + a^2*b) - (a + b)*arctan
(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*a*b))/d
```

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{(a+b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}abd} - \frac{a\tan(dx+c) - b\tan(dx+c)}{2\left(b\tan(dx+c)^2+a\right)abd}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a + b)*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*a*b*d) - 1/2*(a*ta
n(d*x + c) - b*tan(d*x + c))/((b*tan(d*x + c)^2 + a)*a*b*d)
```

Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(c+d\,x)}{\sqrt{a}}\right)\,(a+b)}{2\,a^{3/2}\,b^{3/2}\,d} - \frac{\tan(c+d\,x)\,(a-b)}{2\,a\,b\,d\,\left(b\tan\left(c+d\,x\right)^2+a\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
(atan((b^(1/2)*tan(c + d*x))/a^(1/2))*(a + b))/(2*a^(3/2)*b^(3/2)*d) - (ta
n(c + d*x)*(a - b))/(2*a*b*d*(a + b*tan(c + d*x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.88

$$\int \frac{\sec^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)\sin\left(dx+c\right)^2 a^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)\sin\left(dx+c\right)^2 b^2 +}{}$$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*sin(c + d*x)**2*a**2 + sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c
 + d*x)/2))/sqrt(b))*sin(c + d*x)**2*b**2 + sqrt(b)*sqrt(a)*atan((sqrt(a -
 b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*a**2 + sqrt(b)*sqrt(a)*atan((sqrt
(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*a*b + sqrt(b)*sqrt(a)*atan((s
qrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*a**2 - sqr
t(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c
+ d*x)**2*b**2 - sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)
/2))/sqrt(b))*a**2 - sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c +
d*x)/2))/sqrt(b))*a*b + cos(c + d*x)*sin(c + d*x)*a**2*b - cos(c + d*x)*si
n(c + d*x)*a*b**2)/(2*a**2*b**2*d*(sin(c + d*x)**2*a - sin(c + d*x)**2*b -
 a))
```

3.473 $\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tan(c+dx)}{2ad\,(a+b\tan^2(c+dx))}$$

output
```
1/2*arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/b^(1/2)/d+1/2*tan(d*x+c)/a/
d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}\tan(c+dx)}{a+b\tan^2(c+dx)}}{2a^{3/2}d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tan[c + d*x])/(
a + b*Tan[c + d*x]^2))/(2*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4158, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+b\tan(c+dx)^2)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{(b\tan^2(c+dx)+a)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{2a}+\frac{\tan(c+dx)}{2a(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\tan(c+dx)}{2a(a+b\tan^2(c+dx))}}{d}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Tan[c + d*x]
/(2*a*(a + b*Tan[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$
default	$\frac{\frac{\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{d}$
risch	$\frac{i\left(a\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+a-b\right)}{a(a-b)d\left(a\,\mathrm{e}^{4i(dx+c)}-b\,\mathrm{e}^{4i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}+a-b\right)}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2iab+\sqrt{-ab}\,a+\sqrt{-ab}\,b}{(a-b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,da}+\frac{\ln\left(\right.}{}$

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(d*x+c)/a/(a+b*tan(d*x+c)^2)+1/2/a/(a*b)^(1/2)*arctan(b*tan(d*
x+c)/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(54) = 108$.

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.95

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \left[\frac{4\,ab\cos(dx+c)\sin(dx+c) - \left((a-b)\cos(dx+c)^2+b\right)\sqrt{-ab}\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(\cdots}{(a^2-2\,ab+b^2)\,\mathrm{co}\cdots}\right.}{8\left(a^2b^2d+(a^3b-a^2b^2)d\cos(dx+c)^2\right)}\right.$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*a*b*cos(d*x + c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(
-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x +
c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c)
 + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2
 + b^2)))/(a^2*b^2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2), 1/4*(2*a*b*cos
(d*x + c)*sin(d*x + c) - ((a - b)*cos(d*x + c)^2 + b)*sqrt(a*b)*arctan(1/2
*((a + b)*cos(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c))))/
(a^2*b^2*d + (a^3*b - a^2*b^2)*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\frac{\tan(dx+c)}{ab\tan(dx+c)^2+a^2} + \frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}a}}{2\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(tan(d*x + c)/(a*b*tan(d*x + c)^2 + a^2) + arctan(b*tan(d*x + c)/sqrt(
a*b))/(sqrt(a*b)*a))/d
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}ad} + \frac{\tan(dx+c)}{2\left(b\tan(dx+c)^2+a\right)ad}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*tan(d*x + c)/((
b*tan(d*x + c)^2 + a)*a*d)
```

Mupad [B] (verification not implemented)

Time = 8.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{\sec^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\tan(c+d\,x)}{2\,a\,d\,\left(b\tan(c+d\,x)^2+a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\tan(c+d\,x)}{\sqrt{a}}\right)}{2\,a^{3/2}\sqrt{b}\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)^2),x)
```

output
```
tan(c + d*x)/(2*a*d*(a + b*tan(c + d*x)^2)) + atan((b^(1/2)*tan(c + d*x))/
a^(1/2))/(2*a^(3/2)*b^(1/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.23

$$\int \frac{\sec^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$=\frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)\sin\left(dx+c\right)^2 a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{a-b}-\sqrt{a}\,\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{b}}\right)\sin\left(dx+c\right)^2 b+}{}$$

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*sin(c + d*x)**2*a + sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c +
d*x)/2))/sqrt(b))*sin(c + d*x)**2*b + sqrt(b)*sqrt(a)*atan((sqrt(a - b) -
sqrt(a)*tan((c + d*x)/2))/sqrt(b))*a + sqrt(b)*sqrt(a)*atan((sqrt(a - b) +
 sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*a - sqrt(b)*sqrt(a)*at
an((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*b - s
qrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*a -
cos(c + d*x)*sin(c + d*x)*a*b)/(2*a**2*b*d*(sin(c + d*x)**2*a - sin(c + d*
x)**2*b - a))
```

3.474 $\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\cos^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{(a-5b)x}{2(a-b)^3}+\frac{(5a-b)b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2(a-b)d\,(a+b\tan^2(c+dx))}+\frac{b(a+b)\tan(c+dx)}{2a(a-b)^2d\,(a+b\tan^2(c+dx))}$$

output
```
1/2*(a-5*b)*x/(a-b)^3+1/2*(5*a-b)*b^(3/2)*arctan(b^(1/2)*tan(d*x+c)/a^(1/2
))/a^(3/2)/(a-b)^3/d+1/2*cos(d*x+c)*sin(d*x+c)/(a-b)/d/(a+b*tan(d*x+c)^2)+
1/2*b*(a+b)*tan(d*x+c)/a/(a-b)^2/d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{2(a-5b)(c+dx)-\frac{2b^{3/2}(-5a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+(a-b)\sin(2(c+dx))+\frac{2(a-b)b^2\sin(2(c+dx))}{a(a+b+(a-b)\cos(2(c+dx)))}}{4(a-b)^3d}$$

input

```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

output

```
(2*(a - 5*b)*(c + d*x) - (2*b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[b]*Tan[c + d*x
])/Sqrt[a]])/a^(3/2) + (a - b)*Sin[2*(c + d*x)] + (2*(a - b)*b^2*Sin[2*(c
+ d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)])))/(4*(a - b)^3*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4158, 316, 25, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)^2\left(a+b\tan(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \frac{1}{\left(\tan^2(c+dx)+1\right)^2\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)(a+b\tan^2(c+dx))}-\frac{\int -\frac{3b\tan^2(c+dx)+a-2b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{2(a-b)}}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{3b\tan^2(c+dx)+a-2b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\frac{\int \frac{2\left(a^2-4ba+b^2+b(a+b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2a(a-b)}+\frac{b(a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{\int \frac{a^2-4ba+b^2+b(a+b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{a(a-b)}+\frac{b(a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{\frac{b^2(5a-b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}+\frac{a(a-5b)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a-b}}{a(a-b)}+\frac{b(a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{\frac{\frac{b^2(5a-b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}+\frac{a(a-5b)\arctan(\tan(c+dx))}{a-b}}{a(a-b)}+\frac{b(a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)(a+b\tan^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{\frac{b^{3/2}(5a-b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}+\frac{a(a-5b)\arctan(\tan(c+dx))}{a-b}}{a(a-b)}+\frac{b(a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{\tan(c+dx)}{2(a-b)(\tan^2(c+dx)+1)(a+b\tan^2(c+dx))}}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(Tan[c + d*x]/(2*(a - b)*(1 + Tan[c + d*x]^2)*(a + b*Tan[c + d*x]^2)) + ((
(a*(a - 5*b)*ArcTan[Tan[c + d*x]])/(a - b) + ((5*a - b)*b^(3/2)*ArcTan[(Sq
rt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)))/(a*(a - b)) + (b*(a + b)*
Tan[c + d*x])/(a*(a - b)*(a + b*Tan[c + d*x]^2)))/(2*(a - b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 7.88 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan(dx+c)^2}+\frac{(a-5b)\arctan(\tan(dx+c))}{2}}{(a-b)^3}+\frac{b^2\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{(5a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^3}}{d}$
default	$\frac{\frac{\frac{\left(\frac{a}{2}-\frac{b}{2}\right)\tan(dx+c)}{1+\tan(dx+c)^2}+\frac{(a-5b)\arctan(\tan(dx+c))}{2}}{(a-b)^3}+\frac{b^2\left(\frac{(a-b)\tan(dx+c)}{2a\left(a+b\tan(dx+c)^2\right)}+\frac{(5a-b)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a-b)^3}}{d}$
risch	$\frac{xa}{2(a^2-2ab+b^2)(a-b)}-\frac{5xb}{2(a^2-2ab+b^2)(a-b)}-\frac{i e^{2i(dx+c)}}{8d(a^2-2ab+b^2)}+\frac{i e^{-2i(dx+c)}}{8d(a^2-2ab+b^2)}+\frac{ib^2(a e}{d(-a+b)^3 a(-a e^{4i(dx+c)}}$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)^3*((1/2*a-1/2*b)*tan(d*x+c)/(1+tan(d*x+c)^2)+1/2*(a-5*b)*arct
an(tan(d*x+c)))+1/(a-b)^3*b^2*(1/2/a*(a-b)*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1
/2*(5*a-b)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.15

$$\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^3 - 6*a^2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 4*(a^2*b - 5*a*b^2)
*d*x + (5*a*b^2 - b^3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-b/
a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^
2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos(d*x + c))*sqrt(-b/a)*sin(d*x +
 c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c
)^2 + b^2)) + 4*((a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*co
s(d*x + c))*sin(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)
*d*cos(d*x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d), 1/4*(2*(a^
3 - 6*a^2*b + 5*a*b^2)*d*x*cos(d*x + c)^2 + 2*(a^2*b - 5*a*b^2)*d*x - (5*a
*b^2 - b^3 + (5*a^2*b - 6*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(b/a)*arctan(1/
2*((a + b)*cos(d*x + c)^2 - b)*sqrt(b/a)/(b*cos(d*x + c)*sin(d*x + c))) +
2*((a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^3 + (a^2*b - b^3)*cos(d*x + c))*si
n(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cos(d*x + c
)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.41

$$\int \frac{\cos^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \frac{\frac{(dx+c)(a-5\,b)}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{(5\,ab^2-b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^4-3\,a^3b+3\,a^2b^2-ab^3)\sqrt{ab}} + \frac{(ab+b^2)\tan(dx+c)^3+(a^2+b^2)\tan(dx+c)}{(a^3b-2\,a^2b^2+ab^3)\tan(dx+c)^4+a^4-2\,a^3b+a^2b^2+(a^4-a^3b-a^2b^2+ab^3)\tan(dx+c)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((d*x + c)*(a - 5*b)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (5*a*b^2 - b^3)
*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqr
t(a*b)) + ((a*b + b^2)*tan(d*x + c)^3 + (a^2 + b^2)*tan(d*x + c))/((a^3*b
- 2*a^2*b^2 + a*b^3)*tan(d*x + c)^4 + a^4 - 2*a^3*b + a^2*b^2 + (a^4 - a^3
*b - a^2*b^2 + a*b^3)*tan(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.39

$$\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$=\frac{(dx+c)(a-5\,b)}{2\,(a^3d-3\,a^2bd+3\,ab^2d-b^3d)}+\frac{(5\,ab^2-b^3)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2\,(a^4d-3\,a^3bd+3\,a^2b^2d-ab^3d)\sqrt{ab}}$$
$$+\frac{ab\tan{(dx+c)}^3+b^2\tan{(dx+c)}^3+a^2\tan{(dx+c)}+b^2\tan{(dx+c)}}{2\left(b\tan{(dx+c)}^4+a\tan{(dx+c)}^2+b\tan{(dx+c)}^2+a\right)(a^3d-2\,a^2bd+ab^2d)}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(d*x + c)*(a - 5*b)/(a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d) + 1/2*(5*a
*b^2 - b^3)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^4*d - 3*a^3*b*d + 3*a^2*b
^2*d - a*b^3*d)*sqrt(a*b)) + 1/2*(a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3
+ a^2*tan(d*x + c) + b^2*tan(d*x + c))/((b*tan(d*x + c)^4 + a*tan(d*x + c)
^2 + b*tan(d*x + c)^2 + a)*(a^3*d - 2*a^2*b*d + a*b^2*d))
```

Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 3843, normalized size of antiderivative = 25.97

$$\int \frac{\cos^2(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x)^2)^2,x)
```

output
```
((tan(c + d*x)*(a^2 + b^2))/(2*a*(a^2 - 2*a*b + b^2)) + (b*tan(c + d*x)^3*
(a + b))/(2*a*(a^2 - 2*a*b + b^2)))/(d*(a + tan(c + d*x)^2*(a + b) + b*tan
(c + d*x)^4)) - (atan((((((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b
^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8
 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) -
 (tan(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b
^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a
^2*b*3i + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^
2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) - (tan(c + d*x
)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b
+ a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b)*1i)/(4*(a*b^2*3i - a^2*b*3i
 + a^3*1i - b^3*1i)) - (((((2*a*b^10 - 20*a^2*b^9 + 80*a^3*b^8 - 172*a^4*b
^7 + 220*a^5*b^6 - 172*a^6*b^5 + 80*a^7*b^4 - 20*a^8*b^3 + 2*a^9*b^2)/(a^8
 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2) +
 (tan(c + d*x)*(a - 5*b)*(16*a^2*b^9 - 80*a^3*b^8 + 144*a^4*b^7 - 80*a^5*b
^6 - 80*a^6*b^5 + 144*a^7*b^4 - 80*a^8*b^3 + 16*a^9*b^2))/(8*(a*b^2*3i - a
^2*b*3i + a^3*1i - b^3*1i)*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^
2)))*(a - 5*b))/(4*(a*b^2*3i - a^2*b*3i + a^3*1i - b^3*1i)) + (tan(c + d*x
)*(b^7 - 10*a*b^6 + 50*a^2*b^5 - 10*a^3*b^4 + a^4*b^3))/(2*(a^6 - 4*a^5*b
+ a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*(a - 5*b)*1i)/(4*(a*b^2*3i - a^2*b...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 754, normalized size of antiderivative = 5.09

$$\int \frac{\cos^2(c+dx)}{(a+b\tan^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c)^2)^2,x)
```

output

```
( - 5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b
))*sin(c + d*x)**2*a**2*b + 6*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*
tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*a*b**2 - sqrt(b)*sqrt(a)*atan((
sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*b**3 + 5*
sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*a**
2*b - sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b
))*a*b**2 + 5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2)
)/sqrt(b))*sin(c + d*x)**2*a**2*b - 6*sqrt(b)*sqrt(a)*atan((sqrt(a - b) +
sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*a*b**2 + sqrt(b)*sqrt(a
)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*b
**3 - 5*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt
(b))*a**2*b + sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2)
)/sqrt(b))*a*b**2 + cos(c + d*x)*sin(c + d*x)**3*a**4 - 2*cos(c + d*x)*sin
(c + d*x)**3*a**3*b + cos(c + d*x)*sin(c + d*x)**3*a**2*b**2 - cos(c + d*x
)*sin(c + d*x)*a**4 + cos(c + d*x)*sin(c + d*x)*a**3*b - cos(c + d*x)*sin(
c + d*x)*a**2*b**2 + cos(c + d*x)*sin(c + d*x)*a*b**3 + sin(c + d*x)**2*a*
*4*c + sin(c + d*x)**2*a**4*d*x - 6*sin(c + d*x)**2*a**3*b*c - 6*sin(c + d
*x)**2*a**3*b*d*x + 5*sin(c + d*x)**2*a**2*b**2*c + 5*sin(c + d*x)**2*a**2
*b**2*d*x - a**4*c - a**4*d*x + 5*a**3*b*c + 5*a**3*b*d*x)/(2*a**2*d*(sin(
c + d*x)**2*a**4 - 4*sin(c + d*x)**2*a**3*b + 6*sin(c + d*x)**2*a**2*b*...
```

3.475 $\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 212

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \frac{(3a^2-14ab+35b^2)\,x}{8(a-b)^4} - \frac{(7a-b)b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^4d} + \frac{3(a-3b)\cos(c+dx)\sin(c+dx)}{8(a-b)^2d\left(a+b\tan^2(c+dx)\right)} + \frac{\cos^3(c+dx)\sin(c+dx)}{4(a-b)d\left(a+b\tan^2(c+dx)\right)} + \frac{(a-4b)b(3a+b)\tan(c+dx)}{8a(a-b)^3d\left(a+b\tan^2(c+dx)\right)}$$

output
```
1/8*(3*a^2-14*a*b+35*b^2)*x/(a-b)^4-1/2*(7*a-b)*b^(5/2)*arctan(b^(1/2)*tan
(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^4/d+3/8*(a-3*b)*cos(d*x+c)*sin(d*x+c)/(a-b)
^2/d/(a+b*tan(d*x+c)^2)+1/4*cos(d*x+c)^3*sin(d*x+c)/(a-b)/d/(a+b*tan(d*x+c
)^2)+1/8*(a-4*b)*b*(3*a+b)*tan(d*x+c)/a/(a-b)^3/d/(a+b*tan(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$= \frac{4(3a^2-14ab+35b^2)(c+dx)+\frac{16b^{5/2}(-7a+b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+8(a-3b)(a-b)\sin(2(c+dx))-\frac{16(a-}{a(a+b+}}{32(a-b)^4d}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(4*(3*a^2 - 14*a*b + 35*b^2)*(c + d*x) + (16*b^(5/2)*(-7*a + b)*ArcTan[(Sq
rt[b]*Tan[c + d*x])/Sqrt[a]])/a^(3/2) + 8*(a - 3*b)*(a - b)*Sin[2*(c + d*x
)] - (16*(a - b)*b^3*Sin[2*(c + d*x)])/(a*(a + b + (a - b)*Cos[2*(c + d*x)
])) + (a - b)^2*Sin[4*(c + d*x)])/(32*(a - b)^4*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4158, 316, 25, 402, 25, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4\left(a+b\tan(c+dx)^2\right)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{\left(\tan^2(c+dx)+1\right)^3\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2(a+b\tan^2(c+dx))}-\frac{\int -\frac{5b\tan^2(c+dx)+3a-4b}{\left(\tan^2(c+dx)+1\right)^2\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{4(a-b)}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{5b\tan^2(c+dx)+3a-4b}{\left(\tan^2(c+dx)+1\right)^2\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2(a+b\tan^2(c+dx))}}{d}$$

$\downarrow$ 402

$$\frac{\frac{\frac{3(a-3b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)\left(a+b\tan^2(c+dx)\right)}-\frac{\int -\frac{3a^2-5ba+8b^2+9(a-3b)b\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{2(a-b)}}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2(a+b\tan^2(c+dx))}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{3a^2-5ba+8b^2+9(a-3b)b\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)^2}d\tan(c+dx)}{2(a-b)}+\frac{3(a-3b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)\left(a+b\tan^2(c+dx)\right)}}{4(a-b)}+\frac{\tan(c+dx)}{4(a-b)(\tan^2(c+dx)+1)^2(a+b\tan^2(c+dx))}}{d}$$

$\downarrow$ 402

$$\frac{\frac{\frac{\frac{\int \frac{2\left(3a^3-11ba^2+24b^2a-4b^3+(a-4b)b(3a+b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{2a(a-b)}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{3(a-3b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)\left(a+b\tan^2(c+dx)\right)}}{4(a-b)}+\frac{\cdots}{4(a-b)(\mathrm{t}\cdots}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{3a^3-11ba^2+24b^2a-4b^3+(a-4b)b(3a+b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a\right)}d\tan(c+dx)}{a(a-b)}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{3(a-3b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)\left(a+b\tan^2(c+dx)\right)}}{4(a-b)}+\frac{\cdots}{4(a-b)(\tan^2\cdots}}{d}$$

$\downarrow$ 397

$$\frac{\frac{\frac{\frac{\frac{a\left(3a^2-14ab+35b^2\right)\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a-b}-\frac{4b^3(7a-b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{a(a-b)}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{3(a-3b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)\left(a+b\tan^2 \cdots\right.}}{4(a-b)}\cdots}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{\frac{\frac{a\left(3a^2-14ab+35b^2\right)\arctan(\tan(c+dx))}{a-b}-\frac{4b^3(7a-b)\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{a-b}}{a(a-b)}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{3(a-3b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)\left(a+b\tan^2(c+dx)\right)}}{4(a-b)}\cdots}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{\frac{a\left(3a^2-14ab+35b^2\right)\arctan(\tan(c+dx))}{a-b}-\frac{4b^{5/2}(7a-b)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}}{a(a-b)}+\frac{b(a-4b)(3a+b)\tan(c+dx)}{a(a-b)\left(a+b\tan^2(c+dx)\right)}}{2(a-b)}+\frac{3(a-3b)\tan(c+dx)}{2(a-b)\left(\tan^2(c+dx)+1\right)\left(a+b\tan^2(c+dx)\right)}}{4(a-b)}+\frac{\cdots}{4\cdots}}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]^2)^2,x]
```

output
```
(Tan[c + d*x]/(4*(a - b)*(1 + Tan[c + d*x]^2)^2*(a + b*Tan[c + d*x]^2)) +
((3*(a - 3*b)*Tan[c + d*x])/(2*(a - b)*(1 + Tan[c + d*x]^2)*(a + b*Tan[c +
 d*x]^2)) + (((a*(3*a^2 - 14*a*b + 35*b^2)*ArcTan[Tan[c + d*x]])/(a - b) -
 (4*(7*a - b)*b^(5/2)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a
- b)))/(a*(a - b)) + ((a - 4*b)*b*(3*a + b)*Tan[c + d*x])/(a*(a - b)*(a +
b*Tan[c + d*x]^2)))/(2*(a - b)))/(4*(a - b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 33.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-\frac{7}{4}ab+\frac{11}{8}b^2\right)\tan(dx+c)^3+\left(-\frac{9}{4}ab+\frac{13}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan(dx+c)^2\right)^2}+\frac{\left(3a^2-14ab+35b^2\right)\arctan(\tan(dx+c))}{8}}{(a-b)^4}-\frac{b^3\left(\frac{(a-b)\tan(dx-}{2a\left(a+b\tan(dx+\right.}\right.}{}}{d}$
default	$\frac{\frac{\frac{\left(\frac{3}{8}a^2-\frac{7}{4}ab+\frac{11}{8}b^2\right)\tan(dx+c)^3+\left(-\frac{9}{4}ab+\frac{13}{8}b^2+\frac{5}{8}a^2\right)\tan(dx+c)}{\left(1+\tan(dx+c)^2\right)^2}+\frac{\left(3a^2-14ab+35b^2\right)\arctan(\tan(dx+c))}{8}}{(a-b)^4}-\frac{b^3\left(\frac{(a-b)\tan(dx-}{2a\left(a+b\tan(dx+\right.}\right.}{}}{d}$
risch	$\frac{3x\,a^2}{8(a^2-2ab+b^2)(a-b)^2}-\frac{7xab}{4(a^2-2ab+b^2)(a-b)^2}+\frac{35x\,b^2}{8(a^2-2ab+b^2)(a-b)^2}-\frac{i\mathrm{e}^{4i(dx+c)}}{64(a-b)^2d}-\frac{i\mathrm{e}^{2i(dx+c)}a}{8(a-b)^3d}+\frac{3i\mathrm{e}^{2i(}}{8(a-}$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a-b)^4*(((3/8*a^2-7/4*a*b+11/8*b^2)*tan(d*x+c)^3+(-9/4*a*b+13/8*b^
2+5/8*a^2)*tan(d*x+c))/(1+tan(d*x+c)^2)^2+1/8*(3*a^2-14*a*b+35*b^2)*arctan
(tan(d*x+c)))-b^3/(a-b)^4*(1/2/a*(a-b)*tan(d*x+c)/(a+b*tan(d*x+c)^2)+1/2*(
7*a-b)/a/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 801, normalized size of antiderivative = 3.78

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*((3*a^4 - 17*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a
^3*b - 14*a^2*b^2 + 35*a*b^3)*d*x - (7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3
+ b^4)*cos(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4
- 2*(3*a*b + b^2)*cos(d*x + c)^2 - 4*((a^2 + a*b)*cos(d*x + c)^3 - a*b*cos
(d*x + c))*sqrt(-b/a)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c
)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2)) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2
 - a*b^3)*cos(d*x + c)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x
 + c)^3 + (3*a^3*b - 14*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x +
 c))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(
d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d), 1/8*(
(3*a^4 - 17*a^3*b + 49*a^2*b^2 - 35*a*b^3)*d*x*cos(d*x + c)^2 + (3*a^3*b -
 14*a^2*b^2 + 35*a*b^3)*d*x + 2*(7*a*b^3 - b^4 + (7*a^2*b^2 - 8*a*b^3 + b^
4)*cos(d*x + c)^2)*sqrt(b/a)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(
b/a)/(b*cos(d*x + c)*sin(d*x + c))) + (2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^
3)*cos(d*x + c)^5 + 3*(a^4 - 5*a^3*b + 7*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^3
 + (3*a^3*b - 14*a^2*b^2 + 7*a*b^3 + 4*b^4)*cos(d*x + c))*sin(d*x + c))/((
a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d*cos(d*x + c
)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.67

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{(3a^2-14ab+35b^2)(dx+c)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{4(7ab^3-b^4)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{(a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4)\sqrt{ab}} + \frac{(3a^2b-11ab^2-4b^3)\tan(dx+c)^5+(3a^3-6}{(a^4b-3a^3b^2+3a^2b^3-ab^4)\tan(dx+c)^6+a^5-3a^4b+3a^3b^2-a^2b^3+(a^5}}{8d}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 - 14*a*b + 35*b^2)*(d*x + c)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*
b^3 + b^4) - 4*(7*a*b^3 - b^4)*arctan(b*tan(d*x + c)/sqrt(a*b))/((a^5 - 4*
a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*sqrt(a*b)) + ((3*a^2*b - 11*a*b^2 -
 4*b^3)*tan(d*x + c)^5 + (3*a^3 - 6*a^2*b - 13*a*b^2 - 8*b^3)*tan(d*x + c)
^3 + (5*a^3 - 13*a^2*b - 4*b^3)*tan(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*tan(d*x + c)^6 + a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3 + (a^5 -
 a^4*b - 3*a^3*b^2 + 5*a^2*b^3 - 2*a*b^4)*tan(d*x + c)^4 + (2*a^5 - 5*a^4*
b + 3*a^3*b^2 + a^2*b^3 - a*b^4)*tan(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.27

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx$$
$$= -\frac{b^3\tan(dx+c)}{2\,(a^4d-3\,a^3bd+3\,a^2b^2d-ab^3d)\left(b\tan(dx+c)^2+a\right)}$$
$$+\frac{(3\,a^2-14\,ab+35\,b^2)(dx+c)}{8\,(a^4d-4\,a^3bd+6\,a^2b^2d-4\,ab^3d+b^4d)}$$
$$-\frac{(7\,ab^3-b^4)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{2\,(a^5d-4\,a^4bd+6\,a^3b^2d-4\,a^2b^3d+ab^4d)\sqrt{ab}}$$
$$+\frac{3\,a\tan(dx+c)^3-11\,b\tan(dx+c)^3+5\,a\tan(dx+c)-13\,b\tan(dx+c)}{8\,(a^3d-3\,a^2bd+3\,ab^2d-b^3d)\left(\tan(dx+c)^2+1\right)^2}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*b^3*tan(d*x + c)/((a^4*d - 3*a^3*b*d + 3*a^2*b^2*d - a*b^3*d)*(b*tan(
d*x + c)^2 + a)) + 1/8*(3*a^2 - 14*a*b + 35*b^2)*(d*x + c)/(a^4*d - 4*a^3*
b*d + 6*a^2*b^2*d - 4*a*b^3*d + b^4*d) - 1/2*(7*a*b^3 - b^4)*arctan(b*tan(
d*x + c)/sqrt(a*b))/((a^5*d - 4*a^4*b*d + 6*a^3*b^2*d - 4*a^2*b^3*d + a*b^
4*d)*sqrt(a*b)) + 1/8*(3*a*tan(d*x + c)^3 - 11*b*tan(d*x + c)^3 + 5*a*tan(
d*x + c) - 13*b*tan(d*x + c))/((a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d)*(ta
n(d*x + c)^2 + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 12.71 (sec) , antiderivative size = 5272, normalized size of antiderivative = 24.87

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x)^2)^2,x)
```

output
```
- ((tan(c + d*x)^5*(11*a*b^2 - 3*a^2*b + 4*b^3))/(8*a*(3*a*b^2 - 3*a^2*b +
 a^3 - b^3)) + (tan(c + d*x)^3*(13*a*b^2 + 6*a^2*b - 3*a^3 + 8*b^3))/(8*a*
(a - b)*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)*(13*a^2*b - 5*a^3 + 4*b^3))/(
8*a*(a - b)*(a^2 - 2*a*b + b^2)))/(d*(a + b*tan(c + d*x)^6 + tan(c + d*x)^
2*(2*a + b) + tan(c + d*x)^4*(a + 2*b))) - (atan((((((((2*a*b^13 - 28*a^2*b
^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^5*b^9 - 1302*a^6*b^8 + 1
197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^10*b^4 + (35*a^11*b^3)/2 -
(3*a^12*b^2)/2)/(9*a^10*b - a^11 + a^2*b^9 - 9*a^3*b^8 + 36*a^4*b^7 - 84*a
^5*b^6 + 126*a^6*b^5 - 126*a^7*b^4 + 84*a^8*b^3 - 36*a^9*b^2) - (tan(c + d
*x)*(a^2*3i - a*b*14i + b^2*35i)*(256*a^2*b^11 - 1792*a^3*b^10 + 5120*a^4*
b^9 - 7168*a^5*b^8 + 3584*a^6*b^7 + 3584*a^7*b^6 - 7168*a^8*b^5 + 5120*a^9
*b^4 - 1792*a^10*b^3 + 256*a^11*b^2))/(512*(a^4 - 4*a^3*b - 4*a*b^3 + b^4
+ 6*a^2*b^2)*(a^8 - 6*a^7*b + a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^
3 + 15*a^6*b^2)))*(a^2*3i - a*b*14i + b^2*35i))/(16*(a^4 - 4*a^3*b - 4*a*b
^3 + b^4 + 6*a^2*b^2)) - (tan(c + d*x)*(16*b^9 - 224*a*b^8 + 2009*a^2*b^7
- 980*a^3*b^6 + 406*a^4*b^5 - 84*a^5*b^4 + 9*a^6*b^3))/(32*(a^8 - 6*a^7*b
+ a^2*b^6 - 6*a^3*b^5 + 15*a^4*b^4 - 20*a^5*b^3 + 15*a^6*b^2)))*(a^2*3i -
a*b*14i + b^2*35i)*1i)/(16*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) -
(((((2*a*b^13 - 28*a^2*b^12 + (315*a^3*b^11)/2 - (987*a^4*b^10)/2 + 978*a^
5*b^9 - 1302*a^6*b^8 + 1197*a^7*b^7 - 765*a^8*b^6 + 336*a^9*b^5 - 98*a^...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 975, normalized size of antiderivative = 4.60

$$\int \frac{\cos^4(c+dx)}{\left(a+b\tan^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c)^2)^2,x)
```

output
```
(28*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))
*sin(c + d*x)**2*a**2*b**2 - 32*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a
)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*a*b**3 + 4*sqrt(b)*sqrt(a)*at
an((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*b**4
- 28*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/2))/sqrt(b)
)*a**2*b**2 + 4*sqrt(b)*sqrt(a)*atan((sqrt(a - b) - sqrt(a)*tan((c + d*x)/
2))/sqrt(b))*a*b**3 - 28*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((
c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*a**2*b**2 + 32*sqrt(b)*sqrt(a)*atan(
(sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*sin(c + d*x)**2*a*b**3 -
 4*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan((c + d*x)/2))/sqrt(b))*
sin(c + d*x)**2*b**4 + 28*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + sqrt(a)*tan(
(c + d*x)/2))/sqrt(b))*a**2*b**2 - 4*sqrt(b)*sqrt(a)*atan((sqrt(a - b) + s
qrt(a)*tan((c + d*x)/2))/sqrt(b))*a*b**3 - 2*cos(c + d*x)*sin(c + d*x)**5*
a**5 + 6*cos(c + d*x)*sin(c + d*x)**5*a**4*b - 6*cos(c + d*x)*sin(c + d*x)
**5*a**3*b**2 + 2*cos(c + d*x)*sin(c + d*x)**5*a**2*b**3 + 7*cos(c + d*x)*
sin(c + d*x)**3*a**5 - 27*cos(c + d*x)*sin(c + d*x)**3*a**4*b + 33*cos(c +
 d*x)*sin(c + d*x)**3*a**3*b**2 - 13*cos(c + d*x)*sin(c + d*x)**3*a**2*b**
3 - 5*cos(c + d*x)*sin(c + d*x)*a**5 + 18*cos(c + d*x)*sin(c + d*x)*a**4*b
 - 13*cos(c + d*x)*sin(c + d*x)*a**3*b**2 + 4*cos(c + d*x)*sin(c + d*x)*a*
*2*b**3 - 4*cos(c + d*x)*sin(c + d*x)*a*b**4 + 3*sin(c + d*x)**2*a**5*c...
```

3.476 $\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+2p)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2p),\frac{1}{2}(1+m+2p),\frac{1}{2}(3+2p),\sin^2(e+fx)\right)(d\sec(e}{f(1+2p)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*m+p)*hypergeom([1/2+p, 1/2+1/2*m+p],[3/2+p],sin(f*
x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{m}{2},\frac{1}{2}-p,\frac{2+m}{2},\sec^2(e+fx)\right)(d\sec(e+fx))^m\left(-\tan^2(e+fx)\right)^{\frac{1}{2}-p}(b}{fm}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[m/2, 1/2 - p, (2 + m)/2, Sec[e + f*x]^2]*(
d*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^(1/2 - p)*(b*Tan[e + f*x]^2)^p)/(f*m)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4141, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(e+fx)\right)^p (d\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^2\right)^p (d\sec(e+fx))^m dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\int (d\sec(e+fx))^m\tan^{2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\int (d\sec(e+fx))^m\tan(e+fx)^{2p}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan(e+fx)\left(b\tan^2(e+fx)\right)^p(d\sec(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(m+2p+1)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(m+2\right.}{f(2p+1)}$$

input
```
Int[(d*Sec[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
((Cos[e + f*x]^2)^((1 + m + 2*p)/2)*Hypergeometric2F1[(1 + 2*p)/2, (1 + m
+ 2*p)/2, (3 + 2*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*
Tan[e + f*x]^2)^p)/(f*(1 + 2*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\sec (fx+e))^m \left(b\tan (fx+e)^2\right)^p dx$$

input
```
int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan (fx+e)^2\right)^p (d\sec (fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan^2{(e+fx)}\right)^p (d\sec{(e+fx)})^m \, dx$$

input
```
integrate((d*sec(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((b*tan(e + f*x)**2)**p*(d*sec(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan{(fx+e)}^2\right)^p (d\sec{(fx+e)})^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan{(fx+e)}^2\right)^p (d\sec{(fx+e)})^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^m \left(b\tan(e+f\,x)^2\right)^p \, dx$$

input
```
int((d/cos(e + f*x))^m*(b*tan(e + f*x)^2)^p,x)
```

output
```
int((d/cos(e + f*x))^m*(b*tan(e + f*x)^2)^p, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = d^m b^p \left(\int \tan(fx+e)^{2p} \sec(fx+e)^m \, dx\right)$$

input
```
int((d*sec(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*b**p*int(tan(e + f*x)**(2*p)*sec(e + f*x)**m,x)
```

3.477 $\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},1-\frac{m}{2},-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\tan(e+fx}{f}$$

output
```
AppellF1(1/2,1-1/2*m,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(d*sec(f*x+e)
)^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/((sec(f*x+e)^2)^(1/2*m))/(((a+b*tan(
f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2033 vs. 2(109) = 218.

Time = 15.24 (sec) , antiderivative size = 2033, normalized size of antiderivative = 18.65

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/a)]*(d*Sec[e + f*x])^m*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]*(a + b*Ta
n[e + f*x]^2)^(2*p))/(f*(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/
2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2)*((6
*a*b*p*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/a)]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2*(a + b*Tan[e + f*x]^2)^(-1 +
p))/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*x]^2) + (3*a*AppellF1[1/2, 1
- m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)
^(m/2)*(a + b*Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] + a*(-2 + m)*AppellF1[3/
2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)])*Tan[e + f*
x]^2) + (6*a*(-1 + m/2)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/a)]*(Sec[e + f*x]^2)^(-1 + m/2)*Tan[e + f*x]^2*(a + b*
Tan[e + f*x]^2)^p)/(3*a*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2...
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4162, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (d\sec(e+fx))^m \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{ 4162}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int \left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}\left(b\tan^2(e+fx)+a\right)^p d\tan(e+fx)}{f}$$

$\Big\downarrow$ 334

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int \left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}\left(\frac{b\tan^2(e+\cdots}{a}\right.}{f}$$

$\Big\downarrow$ 333

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},\frac{2-m}{2},-p\cdots\right.}{f}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, (2 - m)/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a
)]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(a + b*Tan[e + f*x]^2)^p)/(f*(Sec[e + f
*x]^2)^(m/2)*(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4162
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff
*((d*Sec[e + f*x])^m/(f*(Sec[e + f*x]^2)^(m/2)))   Subst[Int[(1 + ff^2*x^2)
^(m/2 - 1)*(a + b*ff^2*x^2)^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\sec (fx+e))^{m} \left(a+b\tan (fx+e)^{2}\right)^{p}dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan (fx+e)^{2}+a\right)^{p}(d\sec (fx+e))^{m} \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int (b\tan(fx+e)^2+a)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int (b\tan(fx+e)^2+a)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int (b\tan(e+f\,x)^2+a)^p \left(\frac{d}{\cos(e+f\,x)}\right)^m \, dx$$

input
```
int((a + b*tan(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$
$$= d^m\left(\int \sec(fx+e)^m \left(\tan(fx+e)^2 b+a\right)^p dx\right)$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*int(sec(e + f*x)**m*(tan(e + f*x)**2*b + a)**p,x)
```

3.478 $\int (d\sec(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int (d\sec(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+np)} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+np), \frac{1}{2}(1+m+np), \frac{1}{2}(3+np), \sin^2(e+fx)\right) (d\sec(e}{f(1+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p+1/2*m+1/2)*hypergeom([1/2*n*p+1/2, 1/2*n*p+1/2*m+1
/2],[1/2*n*p+3/2],sin(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)*(b*(c*tan(f*x+
e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx$$
$$= \frac{\cot(e+fx) \text{Hypergeometric2F1}\left(\frac{m}{2}, \frac{1}{2}(1-np), \frac{2+m}{2}, \sec^2(e+fx)\right) (d\sec(e+fx))^m \left(-\tan^2(e+fx)\right)^{\frac{1}{2}(}}{fm}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[m/2, (1 - n*p)/2, (2 + m)/2, Sec[e + f*x]^
2]*(d*Sec[e + f*x])^m*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^
n)^p)/(f*m)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4142, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p \int (d\sec(e+fx))^m(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p \int (d\sec(e+fx))^m(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan(e+fx)(d\sec(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(m+np+1)}\left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(np+1), \frac{1}{2}(m+\right.}{f(np+1)}$$

input
```
Int[(d*Sec[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((1 + m + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (1 + m
+ n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]*(b*
(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\sec (fx+e))^m (b(c\tan (fx+e))^n)^p \, dx$$

input
```
int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^n b)^p (d\sec (fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left(b(c\tan{(e+fx)})^n\right)^p (d\sec{(e+fx)})^m\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*(d*sec(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left((c\tan{(fx+e)})^n b\right)^p (d\sec{(fx+e)})^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left((c\tan{(fx+e)})^n b\right)^p (d\sec{(fx+e)})^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^m \left(b\,(c\tan(e+f\,x))^n\right)^p \, dx$$

input
```
int((d/cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d/cos(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (d\sec(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = d^m c^{np} b^p \left(\int \tan(fx+e)^{np} \sec(fx+e)^m \, dx\right)$$

input
```
int((d*sec(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sec(e + f*x)**m,x)
```

3.479 $\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}+\frac{2\tan^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(3+np)}+\frac{\tan^5(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(5+np)}$$

output
```
tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)+2*tan(f*x+e)^3*(b*(c*tan(f*x+e
))^n)^p/f/(n*p+3)+tan(f*x+e)^5*(b*(c*tan(f*x+e))^n)^p/f/(n*p+5)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.23

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$
$$=\frac{\cot(e+fx)\left(b(c\tan(e+fx))^n\right)^p\left((8+6np+n^2p^2+2(3+np)\cos(2(e+fx))+\cos(4(e+fx)))\sec^4(e\right.}{f(1+np)(3+np)(5+np)}$$

input
```
Integrate[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((8 + 6*n*p + n^2*p^2 + 2*(3 + n*p)
*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Sec[e + f*x]^4*Tan[e + f*x]^2 + 8*(-
Tan[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p)*(5 + n*p))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4142, 3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6 \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p \int \sec^6(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p \int \sec(e+fx)^6(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p \int (c\tan(e+fx))^{np}\left(\tan^2(e+fx)+1\right)^2 d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p \int \left((c\tan(e+fx))^{np}+\frac{2(c\tan(e+fx))^{np+2}}{c^2}+\frac{(c\tan(e+fx))^{np+4}}{c^4}\right) d\tan(e+fx}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c\tan(e+fx))^{-np}\left(\frac{(c\tan(e+fx))^{np+5}}{c^5(np+5)}+\frac{2(c\tan(e+fx))^{np+3}}{c^3(np+3)}+\frac{(c\tan(e+fx))^{np+1}}{c(np+1)}\right)(b(c\tan(e+fx))^n)^p}{f}$$

input
```
Int[Sec[e + f*x]^6*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((b*(c*Tan[e + f*x])^n)^p*((c*Tan[e + f*x])^(1 + n*p)/(c*(1 + n*p)) + (2*(
c*Tan[e + f*x])^(3 + n*p))/(c^3*(3 + n*p)) + (c*Tan[e + f*x])^(5 + n*p)/(c
^5*(5 + n*p))))/(f*(c*Tan[e + f*x])^(n*p))
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.10 (sec) , antiderivative size = 60672, normalized size of antiderivative = 612.85

output too large to display

input
```
int(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\left(n^2p^2+8\,\cos\left(fx+e\right)^4+4\,(np+1)\cos\left(fx+e\right)^2+4\,np+3\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}\sin\left(fx+e\right)}{\left(fn^3p^3+9\,fn^2p^2+23\,fnp+15\,f\right)\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
(n^2*p^2 + 8*cos(f*x + e)^4 + 4*(n*p + 1)*cos(f*x + e)^2 + 4*n*p + 3)*e^(n
*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n^3*p^3 +
 9*f*n^2*p^2 + 23*f*n*p + 15*f)*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left(b(c\tan{(e+fx)})^n\right)^p\sec^6{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**6*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\frac{b^p c^{np}(\tan(fx+e)^n)^p\tan(fx+e)^5}{np+5}+\frac{2\,b^p c^{np}(\tan(fx+e)^n)^p\tan(fx+e)^3}{np+3}+\frac{b^p c^{np}(\tan(fx+e)^n)^p\tan(fx+e)}{np+1}}{f}$$

input
```
integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
(b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)^5/(n*p + 5) + 2*b^p*c^(n*p)*(
tan(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*c^(n*p)*(tan(f*x + e)^n)^
p*tan(f*x + e)/(n*p + 1))/f
```

Giac [A] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.31

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\frac{c^5e^{(np\log(c\tan(fx+e))+p\log(b))}\tan(fx+e)^5}{c^4np+5\,c^4}+\frac{2\,c^3e^{(np\log(c\tan(fx+e))+p\log(b))}\tan(fx+e)^3}{c^2np+3\,c^2}+\frac{ce^{(np\log(c\tan(fx+e))+p\log(b))}\tan(fx+e)}{np+1}}{cf}$$

input
```
integrate(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
(c^5*e^(n*p*log(c*tan(f*x + e)) + p*log(b))*tan(f*x + e)^5/(c^4*n*p + 5*c^
4) + 2*c^3*e^(n*p*log(c*tan(f*x + e)) + p*log(b))*tan(f*x + e)^3/(c^2*n*p
+ 3*c^2) + c*e^(n*p*log(c*tan(f*x + e)) + p*log(b))*tan(f*x + e)/(n*p + 1)
)/(c*f)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(b\,(c\tan(e+f\,x))^n\right)^p}{\cos\left(e+f\,x\right)^6}\,dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6, x)
```

Reduce [F]

$$\int \sec^6(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\sec\left(fx+e\right)^6dx\right)$$

input
```
int(sec(f*x+e)^6*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sec(e + f*x)**6,x)
```

3.480 $\int \sec^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \sec^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\tan(e+fx) \left(b(c\tan(e+fx))^n\right)^p}{f(1+np)} + \frac{\tan^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p}{f(3+np)}$$

output
```
tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)+tan(f*x+e)^3*(b*(c*tan(f*x+e))
^n)^p/f/(n*p+3)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \sec^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\cot(e+fx) \left(b(c\tan(e+fx))^n\right)^p \left(\left(2+(1+np)\sec^2(e+fx)\right)\tan^2(e+fx) + 2\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\right.}{f(1+np)(3+np)}$$

input
```
Integrate[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Cot[e + f*x]*(b*(c*Tan[e + f*x])^n)^p*((2 + (1 + n*p)*Sec[e + f*x]^2)*Tan
[e + f*x]^2 + 2*(-Tan[e + f*x]^2)^((1 - n*p)/2)))/(f*(1 + n*p)*(3 + n*p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4142, 3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4\left(b(c\tan(e+fx))^n\right)^pdx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sec^4(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sec(e+fx)^4(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int (c\tan(e+fx))^{np}\left(\tan^2(e+fx)+1\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \left((c\tan(e+fx))^{np}+\frac{(c\tan(e+fx))^{np+2}}{c^2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c\tan(e+fx))^{-np}\left(\frac{(c\tan(e+fx))^{np+3}}{c^3(np+3)}+\frac{(c\tan(e+fx))^{np+1}}{c(np+1)}\right)\left(b(c\tan(e+fx))^n\right)^p}{f}$$

input
```
Int[Sec[e + f*x]^4*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((b*(c*Tan[e + f*x])^n)^p*((c*Tan[e + f*x])^(1 + n*p)/(c*(1 + n*p)) + (c*T
an[e + f*x])^(3 + n*p)/(c^3*(3 + n*p))))/(f*(c*Tan[e + f*x])^(n*p))
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 29779, normalized size of antiderivative = 458.14

output too large to display

input
```
int(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{\left(np+2\cos\left(fx+e\right)^2+1\right)e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}\sin\left(fx+e\right)}{\left(fn^2p^2+4\,fnp+3\,f\right)\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
(n*p + 2*cos(f*x + e)^2 + 1)*e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*l
og(b))*sin(f*x + e)/((f*n^2*p^2 + 4*f*n*p + 3*f)*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec^4(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left(b(c\tan\left(e+fx\right))^n\right)^p\sec^4\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**4*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\frac{b^p c^{np} (\tan(fx+e)^n)^p \tan(fx+e)^3}{np+3} + \frac{b^p c^{np} (\tan(fx+e)^n)^p \tan(fx+e)}{np+1}}{f}$$

input
```
integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
(b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)^3/(n*p + 3) + b^p*c^(n*p)*(ta
n(f*x + e)^n)^p*tan(f*x + e)/(n*p + 1))/f
```

Giac [A] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int \sec^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\frac{c^3 e^{(np\log(c\tan(fx+e))+p\log(b))} \tan(fx+e)^3}{c^2np+3\,c^2} + \frac{ce^{(np\log(c\tan(fx+e))+p\log(b))} \tan(fx+e)}{np+1}}{cf}$$

input
```
integrate(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
(c^3*e^(n*p*log(c*tan(f*x + e)) + p*log(b))*tan(f*x + e)^3/(c^2*n*p + 3*c^
2) + c*e^(n*p*log(c*tan(f*x + e)) + p*log(b))*tan(f*x + e)/(n*p + 1))/(c*f
)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \frac{\left(b \left(c \tan(e+f\,x)\right)^n\right)^p}{\cos\left(e+f\,x\right)^4} \, dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = c^{np} b^p \left(\int \tan\left(fx+e\right)^{np} \sec\left(fx+e\right)^4 dx \right)$$

input
```
int(sec(f*x+e)^4*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sec(e + f*x)**4,x)
```

3.481 $\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

output
```
tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

input
```
Integrate[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4142, 3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)^2 \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{ 4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \sec^2(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \sec(e+fx)^2(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (c\tan(e+fx))^{np}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{\tan(e+fx) \left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

input
```
Int[Sec[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Tan[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 16.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{\tan(fx+e)\,e^{p\ln\left(b\,e^{n\ln(c\tan(fx+e))}\right)}}{f(np+1)}$	36
default	$\frac{\tan(fx+e)\,e^{p\ln\left(b\,e^{n\ln(c\tan(fx+e))}\right)}}{f(np+1)}$	36
risch	Expression too large to display	9188

input
```
int(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x,method=_RETURNVERBOSE)
```

output
```
1/f/(n*p+1)*tan(f*x+e)*exp(p*ln(b*exp(n*ln(c*tan(f*x+e)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{e^{\left(np\log\left(\frac{c\sin(fx+e)}{\cos(fx+e)}\right)+p\log(b)\right)}\sin\left(fx+e\right)}{\left(fnp+f\right)\cos\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
e^(n*p*log(c*sin(f*x + e)/cos(f*x + e)) + p*log(b))*sin(f*x + e)/((f*n*p +
 f)*cos(f*x + e))
```

Sympy [F]

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \int \left(b(c\tan\left(e+fx\right))^n\right)^p\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \frac{b^pc^{np}\left(\tan\left(fx+e\right)^n\right)^p\tan\left(fx+e\right)}{(np+1)f}$$

input
```
integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
b^p*c^(n*p)*(tan(f*x + e)^n)^p*tan(f*x + e)/((n*p + 1)*f)
```

Giac [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{e^{(np\log(c\tan(fx+e))+p\log(b))}\tan(fx+e)}{(np+1)f}$$

input
```
integrate(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
e^(n*p*log(c*tan(f*x + e)) + p*log(b))*tan(f*x + e)/((n*p + 1)*f)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\frac{\tan(e+f\,x)\,\left(b\,(c\tan(e+f\,x))^n\right)^p}{f\,(n\,p+1)}$$

input
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^2,x)
```

output
```
(tan(e + f*x)*(b*(c*tan(e + f*x))^n)^p)/(f*(n*p + 1))
```

Reduce [F]

$$\int \sec^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=c^{np}b^p\left(\int \tan(fx+e)^{np}\sec(fx+e)^2\,dx\right)$$

input
```
int(sec(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sec(e + f*x)**2,x)
```

3.482 $\int (b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*t
an(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int (b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

input
```
Integrate[(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b(c\tan (e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b(c\tan (e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan (e+fx))^{-np} \left(b(c\tan (e+fx))^n\right)^p \int (c\tan (e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan (e+fx))^{-np} \left(b(c\tan (e+fx))^n\right)^p \int (c\tan (e+fx))^{np}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{c(c\tan (e+fx))^{-np} \left(b(c\tan (e+fx))^n\right)^p \int \frac{(c\tan (e+fx))^{np}}{\tan ^2(e+fx)c^2+c^2}d(c\tan (e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan (e+fx) \text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan ^2(e+fx)\right) \left(b(c\tan (e+fx))^n\right)^p}{f(np+1)}$$

input
```
Int[(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(b (c \tan \left(f x +e \right))^{n}\right)^{p}d x$$

input
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p, x)
```

Sympy [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int (b(c\tan(e+fx))^n)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p, x)
```

Maxima [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Giac [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int \left((c\tan\left(fx+e\right))^n b\right)^p \, dx$$

input
```
integrate((b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b(c\tan(e+fx))^n)^p \, dx = \int \left(b\left(c\tan(e+f\,x)\right)^n\right)^p \, dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (b(c\tan(e+fx))^n)^p \, dx = c^{np} b^p \left(\int \tan\left(fx+e\right)^{np} dx\right)$$

input
```
int((b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p),x)
```

3.483 $\int \cos^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \cos^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

output
```
hypergeom([2, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(f*x+e)^2)*tan(f*x+e)*(b*(c*t
an(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \cos^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(e+fx)\right)\tan(e+fx)\left(b(c\tan(e+fx))^n\right)^p}{f(1+np)}$$

input
```
Integrate[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[2, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4142, 3042, 3087, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \cos^2(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \frac{(c\tan(e+fx))^{np}}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \frac{(c\tan(e+fx))^{np}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\tan(e+fx) \text{Hypergeometric2F1}\left(2,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(e+fx)\right) \left(b(c\tan(e+fx))^n\right)^p}{f(np+1)}$$

input
```
Int[Cos[e + f*x]^2*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[2, (1 + n*p)/2, (3 + n*p)/2, -Tan[e + f*x]^2]*Tan[e + f
*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cos(fx+e)^2 \left(b(c\tan(fx+e))^n\right)^p dx$$

input
```
int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cos^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan\left(fx+e\right))^n b\right)^p \cos\left(fx+e\right)^2 \, dx$$

input
```
integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Sympy [F]

$$\int \cos^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(b(c\tan\left(e+fx\right))^n\right)^p \cos^2\left(e+fx\right) dx$$

input
```
integrate(cos(f*x+e)**2*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*cos(e + f*x)**2, x)
```

Maxima [F]

$$\int \cos^2(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan\left(fx+e\right))^n b\right)^p \cos\left(fx+e\right)^2 \, dx$$

input
```
integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b)^p\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \cos(e+f\,x)^2\,\left(b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int(cos(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cos(e + f*x)^2*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cos^2(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=c^{np}b^p\left(\int \tan(fx+e)^{np}\cos(fx+e)^2\,dx\right)$$

input
```
int(cos(f*x+e)^2*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cos(e + f*x)**2,x)
```

3.484 $\int \sec^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \sec^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(4+np)} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+np), \frac{1}{2}(4+np), \frac{1}{2}(3+np), \sin^2(e+fx)\right) \sec^3(e+fx) \text{ta}}{f(1+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p+2)*hypergeom([1/2*n*p+2, 1/2*n*p+1/2],[1/2*n*p+3/2
],sin(f*x+e)^2)*sec(f*x+e)^3*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int \sec^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\csc(e+fx) \text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1}{2}(1-np), \frac{5}{2}, \sec^2(e+fx)\right) \sec^2(e+fx) \left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)} (b(}{3f}$$

input
```
Integrate[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n*p)/2, 5/2, Sec[e + f*x]^2]*Sec
[e + f*x]^2*(-Tan[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/(3*f
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4142, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sec^3(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sec(e+fx)^3(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan(e+fx)\sec^3(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+4)}\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+4),\frac{1}{2}(np+3),\sin^2(e+fx\right.}{f(np+1)}$$

input
```
Int[Sec[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((4 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (4 + n*p)/2
, (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*Tan[e + f*x]*(b*(c*Tan[e + f
*x])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sec (fx+e)^3 (b(c\tan (fx+e))^n)^p \, dx$$

input
```
int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sec^3(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int ((c\tan (fx+e))^n b)^p \sec (fx+e)^3 \, dx$$

input
```
integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Sympy [F]

$$\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left(b(c\tan{(e+fx)})^n\right)^p\sec^3{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan{(fx+e)})^n b\right)^p\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Giac [F]

$$\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan{(fx+e)})^n b\right)^p\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(b\,(c\tan(e+f\,x))^n\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3,x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3, x)
```

Reduce [F]

$$\int \sec^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\sec\left(fx+e\right)^3dx\right)$$

input
```
int(sec(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sec(e + f*x)**3,x)
```

3.485 $\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$
$$=\frac{\cos^2(e+fx)^{\frac{1}{2}(2+np)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+np),\frac{1}{2}(2+np),\frac{1}{2}(3+np),\sin^2(e+fx)\right)\sec(e+fx)\tan}{f(1+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p+1)*hypergeom([1/2*n*p+1, 1/2*n*p+1/2],[1/2*n*p+3/2
],sin(f*x+e)^2)*sec(f*x+e)*tan(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$
$$=\frac{\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{3}{2},\sec^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{\frac{1}{2}(1-np)}\left(b(c\tan(e+fx\right.}{f}$$

input
```
Integrate[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Csc[e + f*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, 3/2, Sec[e + f*x]^2]*(-T
an[e + f*x]^2)^((1 - n*p)/2)*(b*(c*Tan[e + f*x])^n)^p)/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4142, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \ 4142$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sec(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \ 3042$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \sec(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \ 3097$$

$$\frac{\tan(e+fx)\sec(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np+2)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(np+1),\frac{1}{2}(np+2),\frac{1}{2}(np+3),\sin^2(e+fx)\right)}{f(np+1)}$$

input
```
Int[Sec[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((2 + n*p)/2)*Hypergeometric2F1[(1 + n*p)/2, (2 + n*p)/2
, (3 + n*p)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x]*(b*(c*Tan[e + f*x
])^n)^p)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sec \left(f x +e \right) \left(b \left(c \tan \left(f x +e \right)\right)^{n}\right)^{p}d x$$

input
```
int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sec(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(\left(c \tan \left(f x + e\right)\right)^{n} b\right)^{p} \sec \left(f x + e\right) \,d x$$

input
```
integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Sympy [F]

$$\int \sec(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(b(c\tan{(e+fx)})^n\right)^p \sec{(e+fx)}\, dx$$

input
```
integrate(sec(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

Maxima [F]

$$\int \sec(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan{(fx+e)})^n b\right)^p \sec{(fx+e)} \, dx$$

input
```
integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan{(fx+e)})^n b\right)^p \sec{(fx+e)} \, dx$$

input
```
integrate(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x),x)
```

output
```
int((b*(c*tan(e + f*x))^n)^p/cos(e + f*x), x)
```

Reduce [F]

$$\int \sec(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\sec\left(fx+e\right)dx\right)$$

input
```
int(sec(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*sec(e + f*x),x)
```

3.486 $\int \cos(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \cos(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx = \frac{\cos^2(e+fx)^{\frac{np}{2}} \text{Hypergeometric2F1}\left(\frac{np}{2}, \frac{1}{2}(1+np), \frac{1}{2}(3+np), \sin^2(e+fx)\right) \sin(e+fx) \left(b(c\tan(e+fx\right.$$
$$f(1+np)$$

output
```
(cos(f*x+e)^2)^(1/2*n*p)*hypergeom([1/2*n*p, 1/2*n*p+1/2],[1/2*n*p+3/2],si
n(f*x+e)^2)*sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.72 (sec) , antiderivative size = 482, normalized size of antiderivative = 6.10

$$\int \cos(e+fx) \left(b(c\tan(e+fx))^n\right)^p \, dx$$
$$= \frac{}{2f(1+np)\left((3+np)\text{AppellF1}\left(\frac{1}{2}(1+np), np, 1, \frac{1}{2}(3+np), \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) - 2\right.}$$

input
```
Integrate[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((3 + n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] - 2*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[2*(e + f*x)]*(b*(c*Tan[e + f*x])^
n)^p)/(2*f*(1 + n*p)*((3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((3 + n*p)*AppellF1[(1 + n*p
)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (Appe
llF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] - 4*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] - n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*p*AppellF1[(3 + n*p)/2, 1 +
n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4142, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \cos(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \frac{(c\tan(e+fx))^{np}}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\,\text{Hypergeometric2F1}\left(\frac{np}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(e+fx)\right)(b(c\tan(e+fx))^n)^p}{f(np+1)}$$

input
```
Int[Cos[e + f*x]*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((n*p)/2)*Hypergeometric2F1[(n*p)/2, (1 + n*p)/2, (3 + n
*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p)/(f*(1 + n*p)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cos(fx+e)\left(b(c\tan(fx+e))^n\right)^p\,dx$$

input
```
int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan(fx+e))^n\,b\right)^p\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Sympy [F]

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left(b(c\tan(e+fx))^n\right)^p\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*cos(e + f*x), x)
```

Maxima [F]

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Giac [F]

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left(\left(c\tan\left(fx+e\right)\right)^n b\right)^p\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \cos\left(e+f\,x\right)\,{\left(b\,{\left(c\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

input
```
int(cos(e + f*x)*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cos(e + f*x)*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cos(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\cos\left(fx+e\right)dx\right)$$

input
```
int(cos(f*x+e)*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cos(e + f*x),x)
```

3.487 $\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$
$$= \frac{\cos^2(e+fx)^{\frac{np}{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+np),\frac{1}{2}(1+np),\frac{1}{2}(3+np),\sin^2(e+fx)\right)\sin(e+fx)\,(b(c\,\mathrm{ta}}{f(1+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p)*hypergeom([1/2*n*p-1, 1/2*n*p+1/2],[1/2*n*p+3/2],
sin(f*x+e)^2)*sin(f*x+e)*(b*(c*tan(f*x+e))^n)^p/f/(n*p+1)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 4.79 (sec) , antiderivative size = 1552, normalized size of antiderivative = 18.93

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((6 + 2*n*p)*(AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - 6*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 12*AppellF1[(1 + n*p)/2, n*p, 3, (
3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*AppellF1[(1 + n*p
)/2, n*p, 4, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e
 + f*x)/2]^3*Cos[e + f*x]^3*Sin[(e + f*x)/2]*(b*(c*Tan[e + f*x])^n)^p)/(f*
(1 + n*p)*(-AppellF1[(3 + n*p)/2, n*p, 2, (5 + n*p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + 12*AppellF1[(3 + n*p)/2, n*p, 3, (5 + n*p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 36*AppellF1[(3 + n*p)/2, n*p, 4, (5
 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 32*AppellF1[(3 + n*p
)/2, n*p, 5, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*A
ppellF1[(3 + n*p)/2, 1 + n*p, 1, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] - 6*n*p*AppellF1[(3 + n*p)/2, 1 + n*p, 2, (5 + n*p)/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 12*n*p*AppellF1[(3 + n*p)/2, 1 + n*p,
 3, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 8*n*p*AppellF1
[(3 + n*p)/2, 1 + n*p, 4, (5 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (3 + n*p)*AppellF1[(1 + n*p)/2, n*p, 1, (3 + n*p)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 18*AppellF1[(1 + n*p)/2,
n*p, 2, (3 + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x
)/2]^2 - 6*n*p*AppellF1[(1 + n*p)/2, n*p, 2, (3 + n*p)/2, Tan[(e + f*x)...
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4142, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b(c\tan(e+fx))^n\right)^p}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int \cos^3(e+fx)(c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(c\tan(e+fx))^{-np}\left(b(c\tan(e+fx))^n\right)^p\int\frac{(c\tan(e+fx))^{np}}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-2)+1}\text{Hypergeometric2F1}\left(\frac{1}{2}(np-2),\frac{1}{2}(np+1),\frac{1}{2}(np+3),\sin^2(e+fx)\right)(b(c\tan(e}{f(np+1)}$$

input
```
Int[Cos[e + f*x]^3*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^(1 + (-2 + n*p)/2)*Hypergeometric2F1[(-2 + n*p)/2, (1 +
n*p)/2, (3 + n*p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*(c*Tan[e + f*x])^n)^p
)/(f*(1 + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \cos(fx+e)^3 (b(c\tan(fx+e))^n)^p\,dx$$

input
```
int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \cos^3(e+fx)\,(b(c\tan(e+fx))^n)^p\;dx = \int ((c\tan(fx+e))^n\,b)^p \cos(fx+e)^3\;dx$$

input
```
integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\,(b(c\tan(e+fx))^n)^p\;dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^nb\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Giac [F]

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^nb\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*cos(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx=\int \cos\left(e+f\,x\right)^3\,{\left(b\,{\left(c\,\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

input
```
int(cos(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cos(e + f*x)^3*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int \cos^3(e+fx)\left(b(c\tan(e+fx))^n\right)^p\,dx = c^{np}b^p\left(\int \tan\left(fx+e\right)^{np}\cos\left(fx+e\right)^3dx\right)$$

input
```
int(cos(f*x+e)^3*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*cos(e + f*x)**3,x)
```

3.488 $\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \text{Int}((d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p, x)$$

output
```
Defer(Int)((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4163}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p dx$$

$$\Big\downarrow \text{ 4163}$$

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p dx$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4163
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (d\sec (fx+e))^m (a+b(c\tan (fx+e))^n)^p \, dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^n b+a)^p (d\sec (fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [N/A]

Not integrable

Time = 103.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int (d\sec (e+fx))^m (a+b(c\tan (e+fx))^n)^p \, dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*(c*tan(e + f*x))**n)**p, x)
```

Maxima [N/A]

Not integrable

Time = 6.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b+a)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\sec(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b+a)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*sec(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 9.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int (d\sec(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\;dx = \int (a+b\,(c\tan(e+f\,x))^n)^p\,\left(\frac{d}{\cos\,(e+f\,x)}\right)^m\;dx$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p*(d/cos(e + f*x))^m,x)
```

output
```
int((a + b*(c*tan(e + f*x))^n)^p*(d/cos(e + f*x))^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int (d\sec(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\;dx = d^m\left(\int \sec\,(fx+e)^m\,(c^n\tan\,(fx+e)^n\,b+a)^p\,dx\right)$$

input
```
int((d*sec(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*int(sec(e + f*x)**m*(c**n*tan(e + f*x)**n*b + a)**p,x)
```

3.489 $\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\operatorname{Int}\left(\sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

output
```
Defer(Int)(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 3.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4163}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{4163}$$

$$\int \sec^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

input
```
Int[Sec[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4163
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sec (fx+e)^3 (a+b(c\tan (fx+e))^n)^p \, dx$$

input
```
int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan (fx+e))^n b + a)^p \sec (fx+e)^3 \, dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 10.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^3(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n b + a)^p \sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Giac [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^3(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n b + a)^p \sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^3, x)
```

Mupad [N/A]

Not integrable

Time = 9.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^3(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int \frac{(a+b\,(c\tan(e+f\,x))^n)^p}{\cos(e+f\,x)^3}\,dx$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3,x)
```

output
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \sec^3(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int (c^n\tan(fx+e)^n\,b+a)^p\sec(fx+e)^3\,dx$$

input
```
int(sec(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*sec(e + f*x)**3,x)
```

3.490 $\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\text{Int}\left(\sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p,x\right)$$

output
```
Defer(Int)(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

input
```
Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4163}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \ 4163$$

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p dx$$

input
```
Int[Sec[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4163
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sec \left(fx+e\right) \left(a+b(c\tan \left(fx+e\right))^{n}\right)^{p} dx$$

input
```
int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int \left(\left(c\tan\left(fx+e\right)\right)^{n} b+a\right)^{p} \sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 35.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int \left(a+b(c\tan\left(e+fx\right))^{n}\right)^{p} \sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*tan(e + f*x))**n)**p*sec(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 5.71 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b+a)^p\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b+a)^p\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 8.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(a+b\,(c\tan(e+f\,x))^n\right)^p}{\cos(e+f\,x)}\,dx$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x),x)
```

output
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \sec(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left(c^n\tan\left(fx+e\right)^n b+a\right)^p\sec\left(fx+e\right)dx$$

input
```
int(sec(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*sec(e + f*x),x)
```

3.491 $\int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx = \text{Int}\left(\cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p, x\right)$$

output
```
Defer(Int)(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx = \int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

input
```
Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4163}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b(c\tan(e+fx))^n\right)^p}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{4163}$$

$$\int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p dx$$

input
```
Int[Cos[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4163
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos \left(fx+e\right) \left(a+b \left(c\tan \left(fx+e\right)\right)^{n}\right)^{p}dx$$

input
```
int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx = \int \left(\left(c\tan \left(fx+e\right)\right)^{n}b+a\right)^{p}\cos \left(fx+e\right) \, dx$$

input
```
integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 8.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b+a)^p\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 69.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b+a)^p\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 8.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \cos(e+f\,x)\,\left(a+b\,(c\tan(e+f\,x))^n\right)^p\,dx$$

input
```
int(cos(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cos(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \cos(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int\left(c^n\tan\left(fx+e\right)^n b+a\right)^p\cos\left(fx+e\right)dx$$

input
```
int(cos(f*x+e)*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*cos(e + f*x),x)
```

3.492 $\int \cos^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cos^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \text{Int}\left(\cos^3(e+fx) (a+b(c\tan(e+fx))^n)^p ,x\right)$$

output
```
Defer(Int)(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 4.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \int \cos^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4163}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b(c\tan(e+fx))^n\right)^p}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \; 4163$$

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

input
```
Int[Cos[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4163
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 3.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos{(fx+e)}^{3} (a+b(c\tan{(fx+e)})^{n})^{p} \,dx$$

input
```
int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan{(fx+e)})^{n} b+a)^{p} \cos{(fx+e)}^{3} \,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 10.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^nb+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Giac [N/A]

Not integrable

Time = 65.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^nb+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^3, x)
```

Mupad [N/A]

Not integrable

Time = 10.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \cos\left(e+f\,x\right)^3\,{\left(a+b\,{\left(c\tan(e+f\,x)\right)}^n\right)}^p\,dx$$

input
```
int(cos(e + f*x)^3*(a + b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cos(e + f*x)^3*(a + b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \cos^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left(c^n\tan\left(fx+e\right)^n b+a\right)^p\cos\left(fx+e\right)^3dx$$

input
```
int(cos(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*cos(e + f*x)**3,x)
```

3.493 $\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 247

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{a+b(c\tan(e+fx))}{a}\right.}{f}$$
$$+\frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{a+b(c\tan(e+}{a}\right.}{3f}$$
$$+\frac{\text{Hypergeometric2F1}\left(\frac{5}{n},-p,\frac{5+n}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)\tan^5(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{a+b(c\tan(e+fx}{a}\right.}{5f}$$

output
```
hypergeom([-p, 1/n],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(
f*x+e))^n)^p/f/(((a+b*(c*tan(f*x+e))^n)/a)^p)+2/3*hypergeom([-p, 3/n],[(3+
n)/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p/f/(((a+
b*(c*tan(f*x+e))^n)/a)^p)+1/5*hypergeom([-p, 5/n],[(5+n)/n],-b*(c*tan(f*x+
e))^n/a)*tan(f*x+e)^5*(a+b*(c*tan(f*x+e))^n)^p/f/(((a+b*(c*tan(f*x+e))^n)/
a)^p)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.67

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$=\frac{\tan(e+fx)\left(15\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)+10\,\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3}{}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Tan[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e
+ f*x])^n)/a)] + 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e +
f*x])^n)/a)]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b
*(c*Tan[e + f*x])^n)/a)]*Tan[e + f*x]^4)*(a + b*(c*Tan[e + f*x])^n)^p)/(15
*f*(1 + (b*(c*Tan[e + f*x])^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4158, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6\left(a+b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(\tan^2(e+fx)c^2+c^2\right)^2\left(b(c\tan(e+fx))^n+a\right)^p d(c\tan(e+fx))}{c^5 f}$$

$$\Big\downarrow \text{2432}$$

$$\frac{\int \left(c^4(b(c\tan(e+fx))^n+a)^p+c^4\tan^4(e+fx)\left(b(c\tan(e+fx))^n+a\right)^p+2c^4\tan^2(e+fx)\left(b(c\tan(e+fx))^n\right.\right.}{c^5f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}c^5\tan^5(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{n},-p,\frac{n+5}{n},-\frac{b(c\tan(e+fx)}{a}\right.}{}$$

input
```
Int[Sec[e + f*x]^6*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((c^5*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a
)]*Tan[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(1 + (b*(c*Tan[e + f*x])^n)/
a)^p + (2*c^5*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^
n)/a)]*Tan[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*(1 + (b*(c*Tan[e +
f*x])^n)/a)^p) + (c^5*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*(c*Tan[e
+ f*x])^n)/a)]*Tan[e + f*x]^5*(a + b*(c*Tan[e + f*x])^n)^p)/(5*(1 + (b*(c*
Tan[e + f*x])^n)/a)^p))/(c^5*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [F]

$$\int \sec(fx+e)^6 (a+b(c\tan(fx+e))^n)^p \, dx$$

input
```
int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sec^6(e+fx)\,(a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b + a)^p \sec(fx+e)^6 \, dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\,(a+b(c\tan(e+fx))^n)^p \, dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**6*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b+a)^p \sec(fx+e)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

Giac [F]

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int ((c\tan(fx+e))^n b+a)^p \sec(fx+e)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(a+b\,(c\tan(e+f\,x))^n\right)^p}{\cos(e+f\,x)^6}\,dx$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6,x)
```

output
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^6, x)
```

Reduce [F]

$$\int \sec^6(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left(c^n\tan\left(fx+e\right)^n b+a\right)^p\sec\left(fx+e\right)^6dx$$

input
```
int(sec(f*x+e)^6*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*sec(e + f*x)**6,x)
```

3.494 $\int \sec^4(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \sec^4(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)(a+b(c\tan(e+fx))^n)^p\left(\frac{a+b(c\tan(e+fx))}{a}\right.}{f} + \frac{\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)\tan^3(e+fx)(a+b(c\tan(e+fx))^n)^p\left(\frac{a+b(c\tan(e+fx}{a}\right.}{3f}$$

output
```
hypergeom([-p, 1/n],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(
f*x+e))^n)^p/f/(((a+b*(c*tan(f*x+e))^n)/a)^p)+1/3*hypergeom([-p, 3/n],[(3+
n)/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)^3*(a+b*(c*tan(f*x+e))^n)^p/f/(((a+
b*(c*tan(f*x+e))^n)/a)^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$=\frac{\tan(e+fx)\left(3\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)+\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},\cdot\right.}{3f}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Tan[e + f*x]*(3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e +
 f*x])^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x]
)^n)/a)]*Tan[e + f*x]^2)*(a + b*(c*Tan[e + f*x])^n)^p)/(3*f*(1 + (b*(c*Tan
[e + f*x])^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4158, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)^4\left(a+b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \left(\tan^2(e+fx)c^2+c^2\right)\left(b(c\tan(e+fx))^n+a\right)^p d(c\tan(e+fx))}{c^3 f}$$

$$\Big\downarrow \text{ 2432}$$

$$\frac{\int \left(c^2(b(c\tan(e+fx))^n+a)^p+c^2\tan^2(e+fx)\left(b(c\tan(e+fx))^n+a\right)^p\right)d(c\tan(e+fx))}{c^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}c^3\tan^3(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{n+3}{n},-\frac{b(c\tan(e+fx)}{a}\right.}{}$$

input
```
Int[Sec[e + f*x]^4*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((c^3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a
)]*Tan[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(1 + (b*(c*Tan[e + f*x])^n)/
a)^p + (c^3*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*Tan[e + f*x])^n)
/a)]*Tan[e + f*x]^3*(a + b*(c*Tan[e + f*x])^n)^p)/(3*(1 + (b*(c*Tan[e + f*
x])^n)/a)^p))/(c^3*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [F]

$$\int \sec(fx+e)^4 \left(a+b(c\tan(fx+e))^n\right)^p dx$$

input
```
int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p \, dx = \int \left((c\tan(fx+e))^n b+a\right)^p \sec(fx+e)^4 \, dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p \, dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^nb+a\right)^p\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^nb+a\right)^p\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \frac{\left(a+b\left(c\tan(e+f\,x)\right)^n\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4,x)
```

output
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx)\,(a+b(c\tan(e+fx))^n)^p \,dx = \int (c^n\tan(fx+e)^n\,b+a)^p\sec(fx+e)^4\,dx$$

input
```
int(sec(f*x+e)^4*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*sec(e + f*x)**4,x)
```

3.495 $\int \sec^2(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \sec^2(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)(a+b(c\tan(e+fx))^n)^p\left(\frac{a+b(c\tan(e+fx))^n}{a}\right)}{f}$$

output
```
hypergeom([-p, 1/n],[1+1/n],-b*(c*tan(f*x+e))^n/a)*tan(f*x+e)*(a+b*(c*tan(
f*x+e))^n)^p/f/(((a+b*(c*tan(f*x+e))^n)/a)^p)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \sec^2(e+fx) (a+b(c\tan(e+fx))^n)^p \, dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)\tan(e+fx)(a+b(c\tan(e+fx))^n)^p\left(1+\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Ta
n[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)
^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4158, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)^2 \left(a+b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int (b(c\tan(e+fx))^n+a)^p d(c\tan(e+fx))}{cf}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{(a+b(c\tan(e+fx))^n)^p \left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p} \int \left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^p d(c\tan(e+fx))}{cf}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{\tan(e+fx) \left(a+b(c\tan(e+fx))^n\right)^p \left(\frac{b(c\tan(e+fx))^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{b(c\tan(e+fx))^n}{a}\right)}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
(Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*Tan[e + f*x])^n)/a)]*Ta
n[e + f*x]*(a + b*(c*Tan[e + f*x])^n)^p)/(f*(1 + (b*(c*Tan[e + f*x])^n)/a)
^p)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [F]

$$\int \sec (fx+e)^2 (a+b(c\tan (fx+e))^n)^p \, dx$$

input
```
int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left(a+b(c\tan\left(e+fx\right))^n\right)^p\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*tan(e + f*x))**n)**p*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int \sec^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx=\int \left((c\tan\left(fx+e\right))^n b+a\right)^p\sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx=\int ((c\tan(fx+e))^n b+a)^p \sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*sec(f*x + e)^2, x)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \frac{\tan(e+f\,x)\,\left(a+b\,(c\tan(e+f\,x))^n\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,(c\tan(e+f\,x))^n}{a}\right)}{f\,\left(\frac{b\,(c\tan(e+f\,x))^n}{a}+1\right)^p}$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p/cos(e + f*x)^2,x)
```

output
```
(tan(e + f*x)*(a + b*(c*tan(e + f*x))^n)^p*hypergeom([1/n, -p], 1/n + 1, -
(b*(c*tan(e + f*x))^n)/a))/(f*((b*(c*tan(e + f*x))^n)/a + 1)^p)
```

Reduce [F]

$$\int \sec^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx=\int (c^n\tan(fx+e)^n b+a)^p \sec(fx+e)^2\,dx$$

input
```
int(sec(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*sec(e + f*x)**2,x)
```

3.496 $\int (a+b(c\tan(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (a+b(c\tan(e+fx))^n)^p\,dx = \text{Int}\left((a+b(c\tan(e+fx))^n)^p,x\right)$$

output
```
Defer(Int)((a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\tan(e+fx))^n)^p\,dx = \int (a+b(c\tan(e+fx))^n)^p\,dx$$

input
```
Integrate[(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4145}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b(c\tan(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b(c\tan(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{4145}$$

$$\int (a+b(c\tan(e+fx))^n)^p\,dx$$

input
```
Int[(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4145
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :>
 Unintegrable[(a + b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n,
p}, x]
```

Maple [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+b(c\tan(fx+e))^n)^p\,dx$$

input
```
int((a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\tan(e+fx))^n)^p\ dx = \int ((c\tan(fx+e))^n\,b+a)^p\,dx$$

input
```
integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (a+b(c\tan(e+fx))^n)^p\ dx = \int (a+b(c\tan(e+fx))^n)^p\ dx$$

input
```
integrate((a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*tan(e + f*x))**n)**p, x)
```

Maxima [N/A]

Not integrable

Time = 5.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b+a)^p \, dx$$

input
```
integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b+a)^p \, dx$$

input
```
integrate((a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 9.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\tan(e+fx))^n)^p \, dx = \int (a+b\,(c\tan(e+f\,x))^n)^p \, dx$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((a + b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int (a+b(c\tan(e+fx))^n)^p \, dx = \int (c^n \tan (fx+e)^n b+a)^p \, dx$$

input
```
int((a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p,x)
```

3.497 $\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \text{Int}\left(\cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,,x\right)$$

output
```
Defer(Int)(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Mathematica [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4163}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b(c\tan(e+fx))^n\right)^p}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{4163}$$

$$\int \cos^2(e+fx)\left(a+b(c\tan(e+fx))^n\right)^p\,dx$$

input
```
Int[Cos[e + f*x]^2*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4163
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos(fx+e)^2 (a+b(c\tan(fx+e))^n)^p \, dx$$

input
```
int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^2(e+fx)(a+b(c\tan(e+fx))^n)^p \, dx = \int ((c\tan(fx+e))^n b + a)^p \cos(fx+e)^2 \, dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(a+b(c\tan(e+fx))^n)^p \, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 7.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n b + a)^p \cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int ((c\tan(fx+e))^n b + a)^p \cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*cos(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 8.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int \cos(e+f\,x)^2\,(a+b\,(c\tan(e+f\,x))^n)^p\,dx$$

input
```
int(cos(e + f*x)^2*(a + b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int(cos(e + f*x)^2*(a + b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \cos^2(e+fx)\,(a+b(c\tan(e+fx))^n)^p\,dx = \int (c^n\tan(fx+e)^n b+a)^p\cos(fx+e)^2\,dx$$

input
```
int(cos(f*x+e)^2*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((c**n*tan(e + f*x)**n*b + a)**p*cos(e + f*x)**2,x)
```

3.498 $\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \frac{\cos^2(e+fx)^{\frac{1}{2}+p}(d\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(1+2p), \frac{1}{2}(1-m+2p), \frac{1}{2}(3-m+2p), \sin^2(e\right.}{f(1-m+2p)}$$

output
```
(cos(f*x+e)^2)^(1/2+p)*(d*csc(f*x+e))^m*hypergeom([1/2+p, 1/2-1/2*m+p],[3/
2-1/2*m+p],sin(f*x+e)^2)*tan(f*x+e)*(b*tan(f*x+e)^2)^p/f/(1-m+2*p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.84 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.05

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = -\frac{d(-}{f(-1+m-2p)\left((-3+m-2p)\text{AppellF1}\left(\frac{1}{2}-\frac{m}{2}+p, 2p, 1-m, \frac{3}{2}-\frac{m}{2}+p, \tan^2\left(\frac{1}{2}(e+fx)\right), -\text{ta}\right.\right.}$$

input
```
Integrate[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
-((d*(-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p, 1 - m, 3/2 - m/2 + p, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*Tan[e +
 f*x]^2)^p)/(f*(-1 + m - 2*p)*((-3 + m - 2*p)*AppellF1[1/2 - m/2 + p, 2*p,
 1 - m, 3/2 - m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*(-((-1
 + m)*AppellF1[3/2 - m/2 + p, 2*p, 2 - m, 5/2 - m/2 + p, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]) - 2*p*AppellF1[3/2 - m/2 + p, 1 + 2*p, 1 - m, 5/2
 - m/2 + p, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4141, 3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(e+fx)\right)^p (d\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(e+fx)^2\right)^p (d\csc(e+fx))^m dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p \int (d\csc(e+fx))^m \tan^{2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p \int (d\csc(e+fx))^m \tan(e+fx)^{2p}dx$$

$$\Big\downarrow \text{3098}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p \left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} \tan^{2p}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2p}(e+fx)\left(b\tan^2(e+fx)\right)^p\left(\frac{\sin(e+fx)}{d}\right)^m(d\csc(e+fx))^m\int\left(\frac{\sin(e+fx)}{d}\right)^{-m}\tan(e+fx)^{2p}dx$$

$\Big\downarrow$ 3082

$$\frac{\sin(e+fx)\cos^{2p}(e+fx)\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\left(\frac{\sin(e+fx)}{d}\right)^{m-2p-1}\int\cos^{-2p}(e+fx)\left(\frac{\sin(e+fx)}{d}\right)^{2p-}}{d}$$

$\Big\downarrow$ 3042

$$\frac{\sin(e+fx)\cos^{2p}(e+fx)\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\left(\frac{\sin(e+fx)}{d}\right)^{m-2p-1}\int\cos(e+fx)^{-2p}\left(\frac{\sin(e+fx)}{d}\right)^{2p-}}{d}$$

$\Big\downarrow$ 3057

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}\left(b\tan^2(e+fx)\right)^p(d\csc(e+fx))^m\text{Hypergeometric2F1}\left(\frac{1}{2}(2p+1),\frac{1}{2}(-m+2p+1\right.}{f(-m+2p+1)}$$

input
```
Int[(d*Csc[e + f*x])^m*(b*Tan[e + f*x]^2)^p,x]
```

output
```
((Cos[e + f*x]^2)^(1/2 + p)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 + 2*p)
/2, (1 - m + 2*p)/2, (3 - m + 2*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*Tan[
e + f*x]^2)^p)/(f*(1 - m + 2*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\csc(fx+e))^m \left(b\tan(fx+e)^2\right)^p dx$$

input
```
int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p\, dx = \int \left(b\tan(fx+e)^2\right)^p (d\csc(fx+e))^m\, dx$$

input
```
integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan^2{(e+fx)}\right)^p (d\csc{(e+fx)})^m \,dx$$

input
```
integrate((d*csc(f*x+e))**m*(b*tan(f*x+e)**2)**p,x)
```

output
```
Integral((b*tan(e + f*x)**2)**p*(d*csc(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan{(fx+e)}^2\right)^p (d\csc{(fx+e)})^m \,dx$$

input
```
integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Giac [F]

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \,dx = \int \left(b\tan{(fx+e)}^2\right)^p (d\csc{(fx+e)})^m \,dx$$

input
```
integrate((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2)^p*(d*csc(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = \int \left(b\tan(e+f\,x)^2\right)^p \left(\frac{d}{\sin(e+f\,x)}\right)^m dx$$

input
```
int((b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m,x)
```

output
```
int((b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\csc(e+fx))^m \left(b\tan^2(e+fx)\right)^p \, dx = d^m b^p \left(\int \tan(fx+e)^{2p} \csc(fx+e)^m \, dx\right)$$

input
```
int((d*csc(f*x+e))^m*(b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*b**p*int(tan(e + f*x)**(2*p)*csc(e + f*x)**m,x)
```

3.499 $\int (d\csc(e+fx))^m (a+b\tan^2(e+fx))^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{1-m}{2},1-\frac{m}{2},-p,\frac{3-m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)(d\csc(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e}{f(1-m)}$$

output
```
AppellF1(1/2-1/2*m,1-1/2*m,-p,3/2-1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/a)*(
d*csc(f*x+e))^m*tan(f*x+e)*(a+b*tan(f*x+e)^2)^p/f/(1-m)/((sec(f*x+e)^2)^(1
/2*m))/(((a+b*tan(f*x+e)^2)/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(128) = 256.

Time = 3.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.28

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\,dx =$$

$$-\frac{a(-3+m)\,\text{AppellF1}\left(\frac{1}{2}-\frac{m}{2},1-\frac{m}{2},-\right.}{f(-1+m)\left(-2bp\,\text{AppellF1}\left(\frac{3}{2}-\frac{m}{2},1-\frac{m}{2},1-p,\frac{5}{2}-\frac{m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a}\right)-a(-2+m\right.}$$

input
```
Integrate[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
-((a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/a)]*Cos[e + f*x]^2*Cot[e + f*x]*(d*Csc[e + f*x])^m*(
a + b*Tan[e + f*x]^2)^p)/(f*(-1 + m)*(-2*b*p*AppellF1[3/2 - m/2, 1 - m/2,
1 - p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/a)] - a*(-2 + m)*A
ppellF1[3/2 - m/2, 2 - m/2, -p, 5/2 - m/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/a)] + a*(-3 + m)*AppellF1[1/2 - m/2, 1 - m/2, -p, 3/2 - m/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/a)]*Cot[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4164, 3042, 4150, 393, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\csc(e+fx))^m \left(a+b\tan(e+fx)^2\right)^p dx$$

$$\Big\downarrow \text{4164}$$

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} \left(b\tan^2(e+fx)+a\right)^p dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} \left(b\tan(e+fx)^2+a\right)^p dx$$

$$\Big\downarrow \text{4150}$$

$$\frac{\tan^m(e+fx)\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m \int \tan^{-m}(e+fx)\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}\left(b\tan^2(e+fx)+a\right)^p}{f}$$

$\downarrow$ 393

$$\frac{\cot(e+fx)\tan^2(e+fx)^{\frac{m+1}{2}}\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m\int\tan^2(e+fx)^{\frac{1}{2}(-m-1)}\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}(}{2f}$$

$\downarrow$ 152

$$\frac{\cot(e+fx)\tan^2(e+fx)^{\frac{m+1}{2}}\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}\int\tan}{2f}$$

$\downarrow$ 150

$$\frac{\cot(e+fx)\tan^2(e+fx)^{\frac{1-m}{2}+\frac{m+1}{2}}\sec^2(e+fx)^{-m/2}(d\csc(e+fx))^m\left(a+b\tan^2(e+fx)\right)^p\left(\frac{b\tan^2(e+fx)}{a}+1\right)^{-p}}{f(1-m)}$$

input
```
Int[(d*Csc[e + f*x])^m*(a + b*Tan[e + f*x]^2)^p,x]
```

output
```
(AppellF1[(1 - m)/2, (2 - m)/2, -p, (3 - m)/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/a)]*Cot[e + f*x]*(d*Csc[e + f*x])^m*(Tan[e + f*x]^2)^((1 - m)/2
 + (1 + m)/2)*(a + b*Tan[e + f*x]^2)^p)/(f*(1 - m)*(Sec[e + f*x]^2)^(m/2)*
(1 + (b*Tan[e + f*x]^2)/a)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4150
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff
*(d*Sin[e + f*x])^m*((Sec[e + f*x]^2)^(m/2)/(f*Tan[e + f*x]^m))   Subst[Int
[(ff*x)^m*((a + b*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x
]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4164
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[(d*Csc[e + f*x])^FracPart[m]*(Sin
[e + f*x]/d)^FracPart[m]   Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sin[e + f*x]/d
)^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Maple [F]

$$\int (d\csc(fx+e))^m \left(a+b\tan(fx+e)^2\right)^p dx$$

input
```
int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

Fricas [F]

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\csc\left(fx+e\right))^m\, dx$$

input
```
integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\, dx = \text{Timed out}$$

input
```
integrate((d*csc(f*x+e))**m*(a+b*tan(f*x+e)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p\, dx = \int \left(b\tan\left(fx+e\right)^2+a\right)^p (d\csc\left(fx+e\right))^m\, dx$$

input
```
integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Giac [F]

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int (b\tan(fx+e)^2+a)^p (d\csc(fx+e))^m \, dx$$

input
```
integrate((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e)^2 + a)^p*(d*csc(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = \int (b\tan(e+fx)^2+a)^p \left(\frac{d}{\sin(e+fx)}\right)^m \, dx$$

input
```
int((a + b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m,x)
```

output
```
int((a + b*tan(e + f*x)^2)^p*(d/sin(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\csc(e+fx))^m \left(a+b\tan^2(e+fx)\right)^p \, dx = d^m \left(\int (\tan(fx+e)^2 b+a)^p \csc(fx+e)^m \, dx\right)$$

input
```
int((d*csc(f*x+e))^m*(a+b*tan(f*x+e)^2)^p,x)
```

output
```
d**m*int((tan(e + f*x)**2*b + a)**p*csc(e + f*x)**m,x)
```

3.500 $\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+np)}(d\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(1+np), \frac{1}{2}(1-m+np), \frac{1}{2}(3-m+np), \sin\right.}{f(1-m+np)}$$

output
```
(cos(f*x+e)^2)^(1/2*n*p+1/2)*(d*csc(f*x+e))^m*hypergeom([1/2*n*p+1/2, 1/2*
n*p-1/2*m+1/2],[1/2*n*p-1/2*m+3/2],sin(f*x+e)^2)*tan(f*x+e)*(b*(c*tan(f*x+
e))^n)^p/f/(n*p-m+1)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.28 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.07

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx =$$

$$-\frac{}{f(-1+m-np)\left((-3+m-np)\text{AppellF1}\left(\frac{1}{2}(1-m+np), np, 1-m, \frac{1}{2}(3-m+np), \tan^2\left(\frac{1}{2}(e+f\right.\right.\right.}$$

input
```
Integrate[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
-((d*(-3 + m - n*p)*AppellF1[(1 - m + n*p)/2, n*p, 1 - m, (3 - m + n*p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(d*Csc[e + f*x])^(-1 + m)*(b*(c*
Tan[e + f*x])^n)^p)/(f*(-1 + m - n*p)*((-3 + m - n*p)*AppellF1[(1 - m + n*
p)/2, n*p, 1 - m, (3 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] - 2*((-1 + m)*AppellF1[(3 - m + n*p)/2, n*p, 2 - m, (5 - m + n*p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*p*AppellF1[(3 - m + n*p)/2, 1 +
n*p, 1 - m, (5 - m + n*p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4142, 3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p dx$$

$$\Big\downarrow \text{4142}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (d\csc(e+fx))^m (c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int (d\csc(e+fx))^m (c\tan(e+fx))^{np}dx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m (c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} (c\tan(e+fx))^{np} dx$$

$\Big\downarrow$ 3042

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m (c\tan(e+fx))^{-np} \left(b(c\tan(e+fx))^n\right)^p \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} (c\tan(e+fx))^{np} dx$$

$\Big\downarrow$ 3082

$$\frac{\sin(e+fx)(d\csc(e+fx))^m \cos^{np}(e+fx) \left(b(c\tan(e+fx))^n\right)^p \left(\frac{\sin(e+fx)}{d}\right)^{m-np-1} \int \cos^{-np}(e+fx) \left(\frac{\sin(e+fx)}{d}\right)}{d}$$

$\Big\downarrow$ 3042

$$\frac{\sin(e+fx)(d\csc(e+fx))^m \cos^{np}(e+fx) \left(b(c\tan(e+fx))^n\right)^p \left(\frac{\sin(e+fx)}{d}\right)^{m-np-1} \int \cos(e+fx)^{-np} \left(\frac{\sin(e+fx)}{d}\right)}{d}$$

$\Big\downarrow$ 3057

$$\frac{\tan(e+fx)(d\csc(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(np+1)} \left(b(c\tan(e+fx))^n\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2}(np+1), \frac{1}{2}(-m+\right.}{f(-m+np+1)}$$

input
```
Int[(d*Csc[e + f*x])^m*(b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n*p)/2)*(d*Csc[e + f*x])^m*Hypergeometric2F1[(1 +
n*p)/2, (1 - m + n*p)/2, (3 - m + n*p)/2, Sin[e + f*x]^2]*Tan[e + f*x]*(b*
(c*Tan[e + f*x])^n)^p)/(f*(1 - m + n*p))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int (d\csc(fx+e))^m (b(c\tan(fx+e))^n)^p\,dx$$

input
```
int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [F]

$$\int (d\csc(e+fx))^m (b(c\tan(e+fx))^n)^p\ dx = \int ((c\tan(fx+e))^n b)^p (d\csc(fx+e))^m\ dx$$

input
```
integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\csc(e+fx))^m (b(c\tan(e+fx))^n)^p\ dx = \int (b(c\tan(e+fx))^n)^p (d\csc(e+fx))^m\ dx$$

input
```
integrate((d*csc(f*x+e))**m*(b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Integral((b*(c*tan(e + f*x))**n)**p*(d*csc(e + f*x))**m, x)
```

Maxima [F]

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \int ((c\tan(fx+e))^n b)^p (d\csc(fx+e))^m \, dx$$

input
```
integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Giac [F]

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \int ((c\tan(fx+e))^n b)^p (d\csc(fx+e))^m \, dx$$

input
```
integrate((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b)^p*(d*csc(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^m \left(b(c\tan(e+fx))^n\right)^p \, dx = \int \left(\frac{d}{\sin(e+fx)}\right)^m \left(b(c\tan(e+fx))^n\right)^p \, dx$$

input
```
int((d/sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p,x)
```

output
```
int((d/sin(e + f*x))^m*(b*(c*tan(e + f*x))^n)^p, x)
```

Reduce [F]

$$\int (d\csc(e+fx))^m\,(b(c\tan(e+fx))^n)^p\,dx = d^m c^{np} b^p \left(\int \tan(fx+e)^{np}\csc(fx+e)^m\,dx\right)$$

input
```
int((d*csc(f*x+e))^m*(b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*c**(n*p)*b**p*int(tan(e + f*x)**(n*p)*csc(e + f*x)**m,x)
```

3.501 $\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = (d\csc(e+fx))^m \left(\frac{\sin(e+fx)}{d}\right)^m \text{Int}\left(\left(\frac{\sin(e+fx)}{d}\right)^{-m} (a + b(c\tan(e+fx))^n)^p, x\right)$$

output
```
(d*csc(f*x+e))^m*(sin(f*x+e)/d)^m*Defer(Int)((a+b*(c*tan(f*x+e))^n)^p/((si
n(f*x+e)/d)^m),x)
```

Mathematica [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx = \int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \, dx$$

input
```
Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
Integrate[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4164, 3042, 4151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\csc(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\csc(e+fx))^m\,(a+b(c\tan(e+fx))^n)^p\,dx$$

$$\Big\downarrow \text{4164}$$

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} (b(c\tan(e+fx))^n+a)^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} (b(c\tan(e+fx))^n+a)^p\,dx$$

$$\Big\downarrow \text{4151}$$

$$\left(\frac{\sin(e+fx)}{d}\right)^m (d\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{d}\right)^{-m} (b(c\tan(e+fx))^n+a)^p\,dx$$

input
```
Int[(d*Csc[e + f*x])^m*(a + b*(c*Tan[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4151
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sin[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

rule 4164
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (
f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[(d*Csc[e + f*x])^FracPart[m]*(Sin
[e + f*x]/d)^FracPart[m]   Int[(a + b*(c*Tan[e + f*x])^n)^p/(Sin[e + f*x]/d
)^m, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !IntegerQ[m]
```

Maple [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(d \csc \left(f x +e \right)\right)^{m} \left(a +b \left(c \tan \left(f x +e \right)\right)^{n}\right)^{p}d x$$

input
```
int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\csc(e+fx))^m\left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int \left(\left(c \tan \left(f x + e\right)\right)^{n} b + a\right)^{p} \left(d \csc \left(f x + e\right)\right)^{m} \,d x$$

input
```
integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx = \text{Timed out}$$

input
```
integrate((d*csc(f*x+e))**m*(a+b*(c*tan(f*x+e))**n)**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 7.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p\,dx = \int ((c\tan(fx+e))^n b + a)^p (d\csc(fx+e))^m\,dx$$

input
```
integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Giac [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx = \int ((c\tan(fx+e))^n b+a)^p (d\csc(fx+e))^m dx$$

input
```
integrate((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*tan(f*x + e))^n*b + a)^p*(d*csc(f*x + e))^m, x)
```

Mupad [N/A]

Not integrable

Time = 9.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int (d\csc(e+fx))^m \left(a+b(c\tan(e+fx))^n\right)^p dx = \int \left(a+b\,(c\tan(e+f\,x))^n\right)^p \left(\frac{d}{\sin(e+f\,x)}\right)^m dx$$

input
```
int((a + b*(c*tan(e + f*x))^n)^p*(d/sin(e + f*x))^m,x)
```

output
```
int((a + b*(c*tan(e + f*x))^n)^p*(d/sin(e + f*x))^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int (d\csc(e+fx))^m (a+b(c\tan(e+fx))^n)^p \,dx$$
$$= d^m \left(\int (c^n \tan(fx+e)^n b + a)^p \csc(fx+e)^m \,dx \right)$$

input
```
int((d*csc(f*x+e))^m*(a+b*(c*tan(f*x+e))^n)^p,x)
```

output
```
d**m*int((c**n*tan(e + f*x)**n*b + a)**p*csc(e + f*x)**m,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
          'sin','cos','tan','cot','sec','csc',
          'arcsin','arccos','arctan','arccot','arcsec','arccsc',
          'sinh','cosh','tanh','coth','sech','csch',
          'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file