Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.5-Secant/231-4.5.0

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**299**]. This is test number [231].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (299)	0.00 (0)
Mathematica	100.00 (299)	0.00 (0)
Maple	75.92 (227)	24.08 (72)
Fricas	72.91 (218)	27.09 (81)
Maxima	31.10 (93)	68.90 (206)
Giac	29.43 (88)	70.57 (211)
Reduce	27.76 (83)	72.24 (216)
Mupad	26.09 (78)	73.91 (221)
Sympy	9.70 (29)	90.30 (270)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

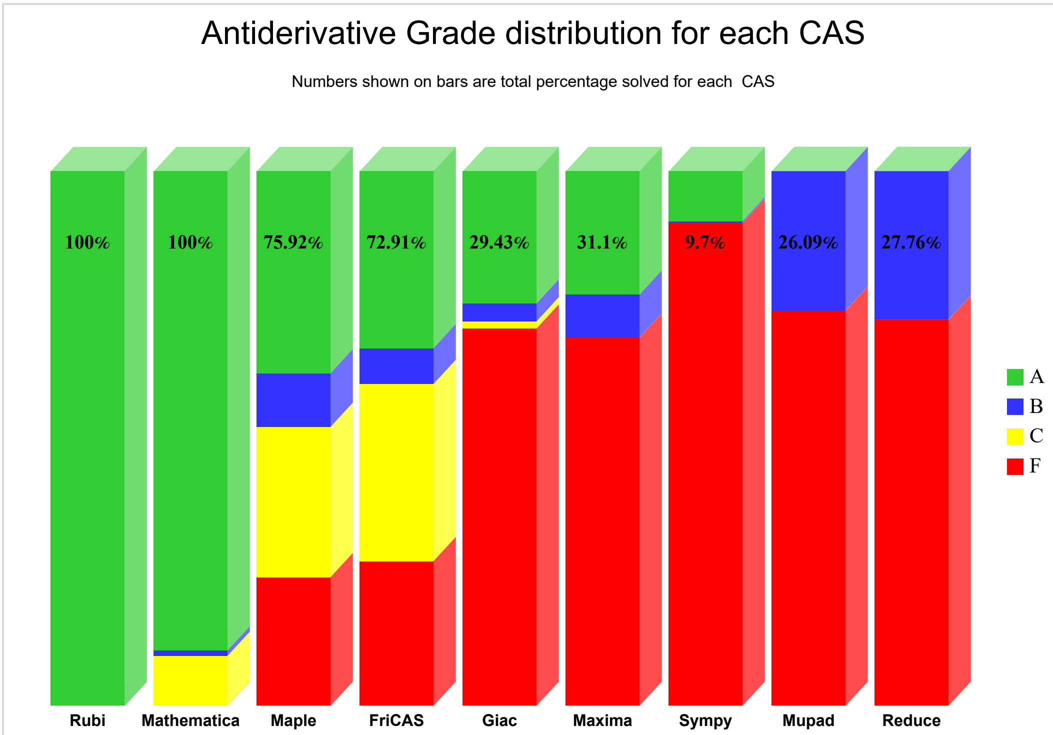
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

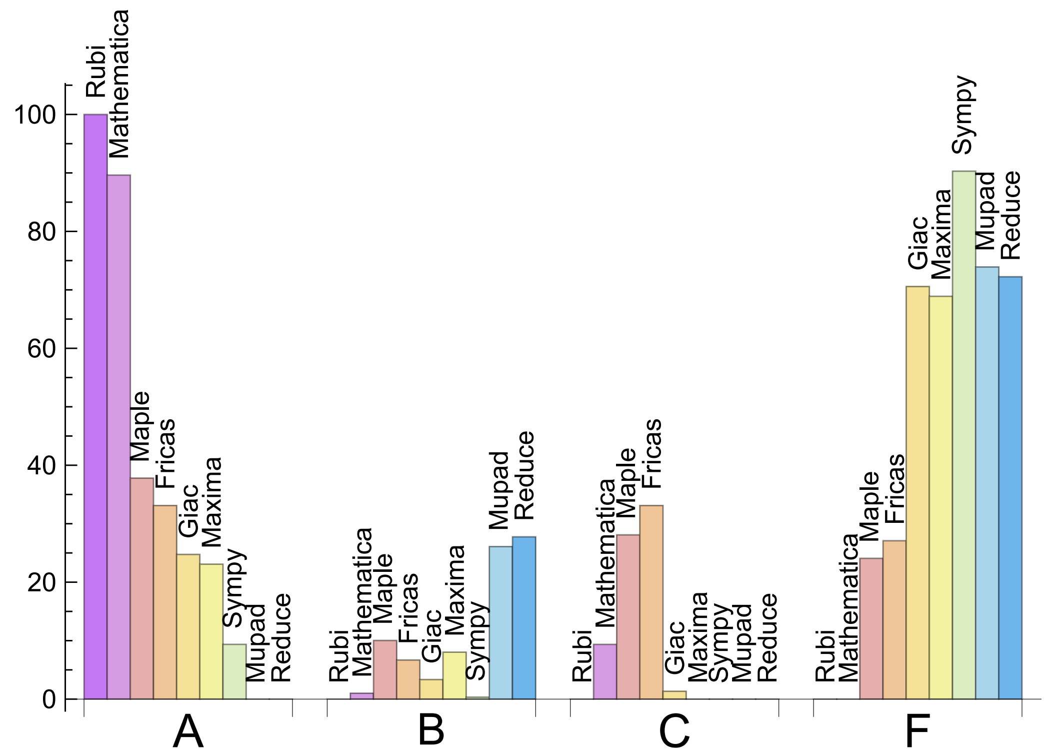
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	89.632	1.003	9.365	0.000
Maple	37.793	10.033	28.094	24.080
Fricas	33.110	6.689	33.110	27.090
Giac	24.749	3.344	1.338	70.569
Maxima	23.077	8.027	0.000	68.896
Sympy	9.365	0.334	0.000	90.301
Mupad	0.000	26.087	0.000	73.913
Reduce	0.000	27.759	0.000	72.241

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	72	100.00	0.00	0.00
Fricas	81	100.00	0.00	0.00
Maxima	206	100.00	0.00	0.00
Giac	211	99.05	0.00	0.95
Reduce	216	100.00	0.00	0.00
Mupad	221	0.00	100.00	0.00
Sympy	270	62.96	37.04	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.09
Maxima	0.16
Reduce	0.18
Giac	0.19
Rubi	0.34
Mathematica	0.54
Maple	3.64
Mupad	7.81
Sympy	18.05

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Reduce	46.70	0.94	28.00	0.60
Sympy	59.97	1.24	46.00	1.08
Giac	60.85	1.33	43.00	1.22
Mupad	61.22	1.19	46.00	1.05
Mathematica	62.39	0.90	56.00	0.86
Rubi	73.17	0.99	69.00	1.00
Fricas	114.23	1.48	87.00	1.27
Maple	126.79	1.60	90.00	1.25
Maxima	168.53	2.44	42.00	0.96

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

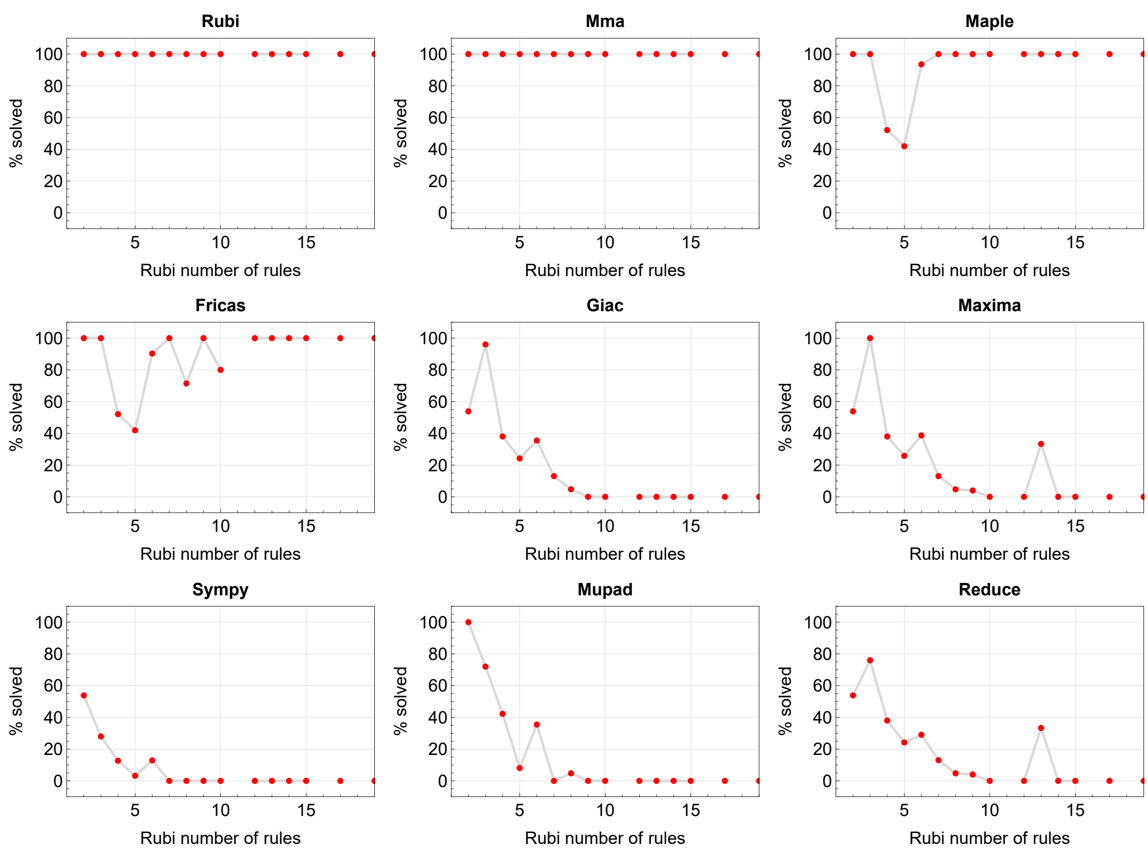


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

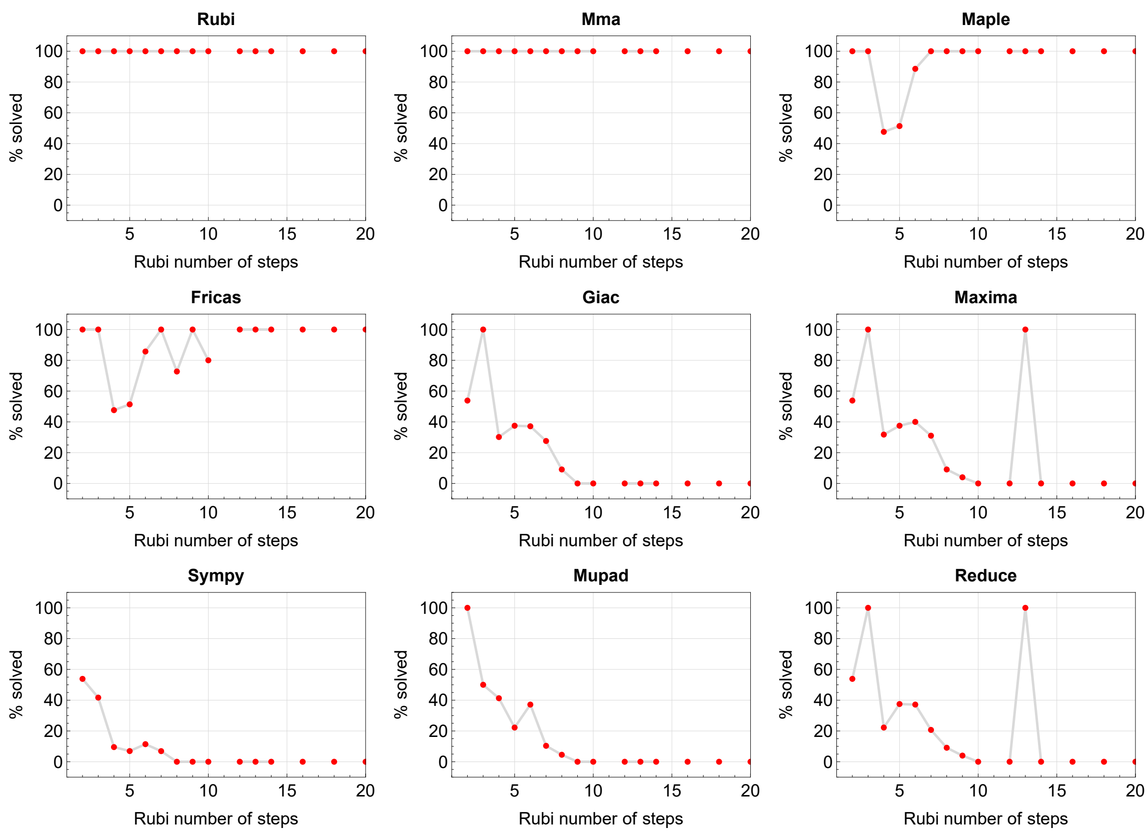


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

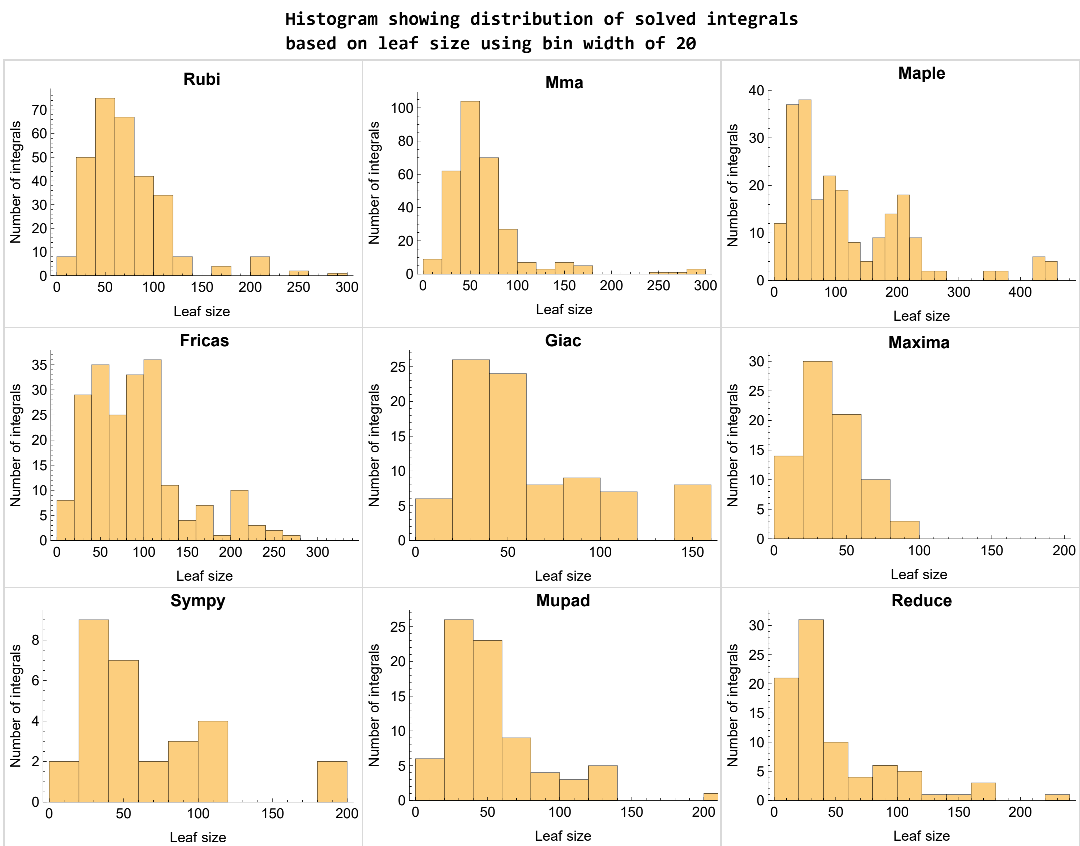


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

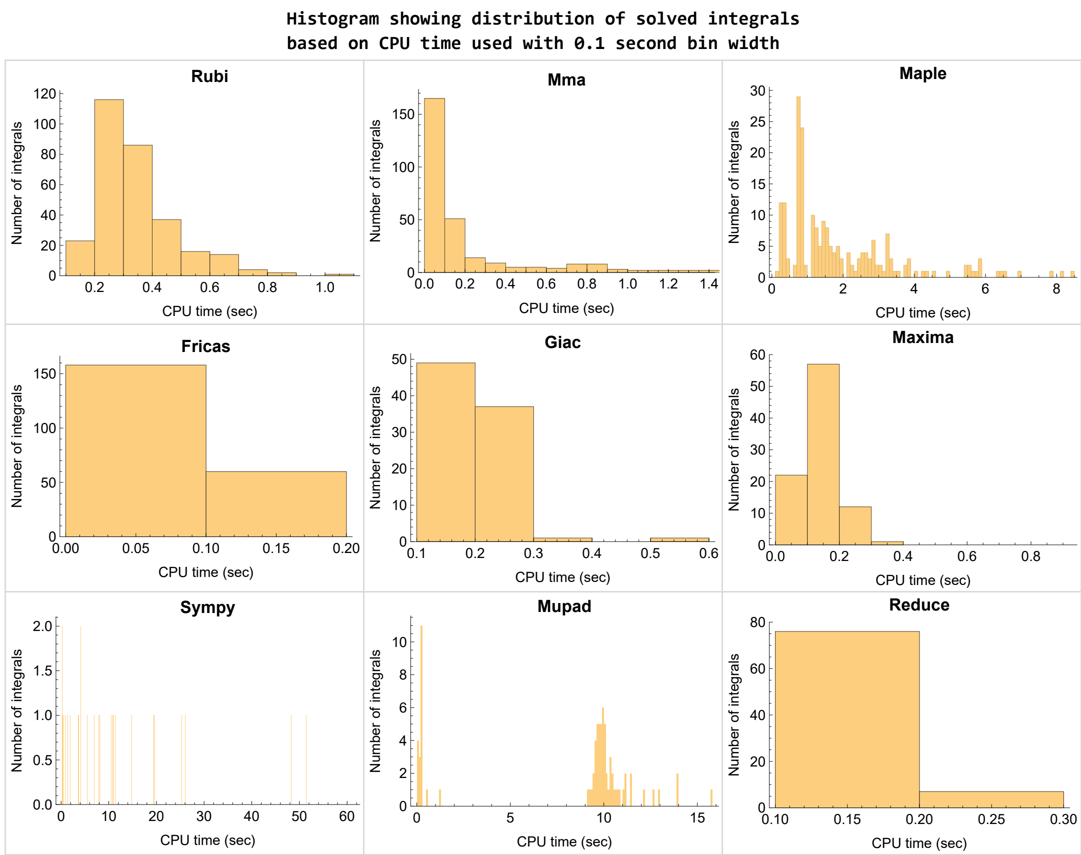


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

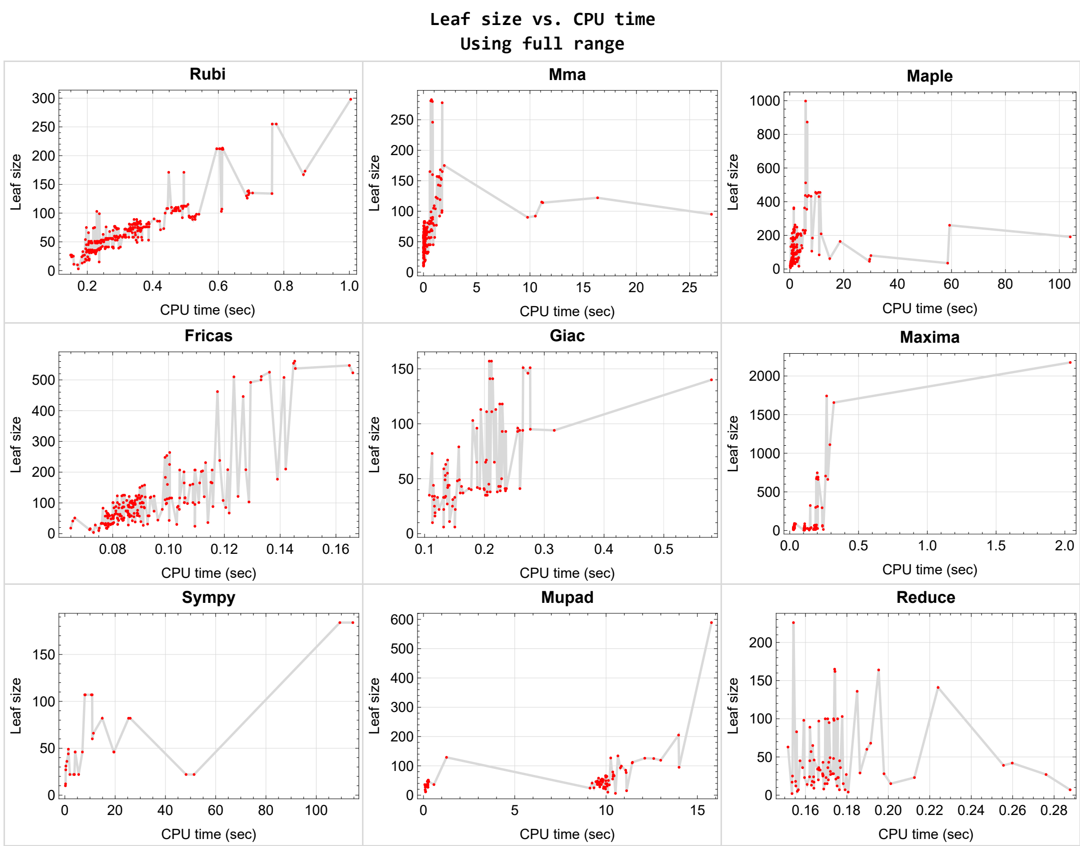


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {280, 281, 282, 283}

Maple {39, 40, 41, 43, 44, 45, 46, 237, 244, 253, 260, 266, 268, 269, 270, 275, 277}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

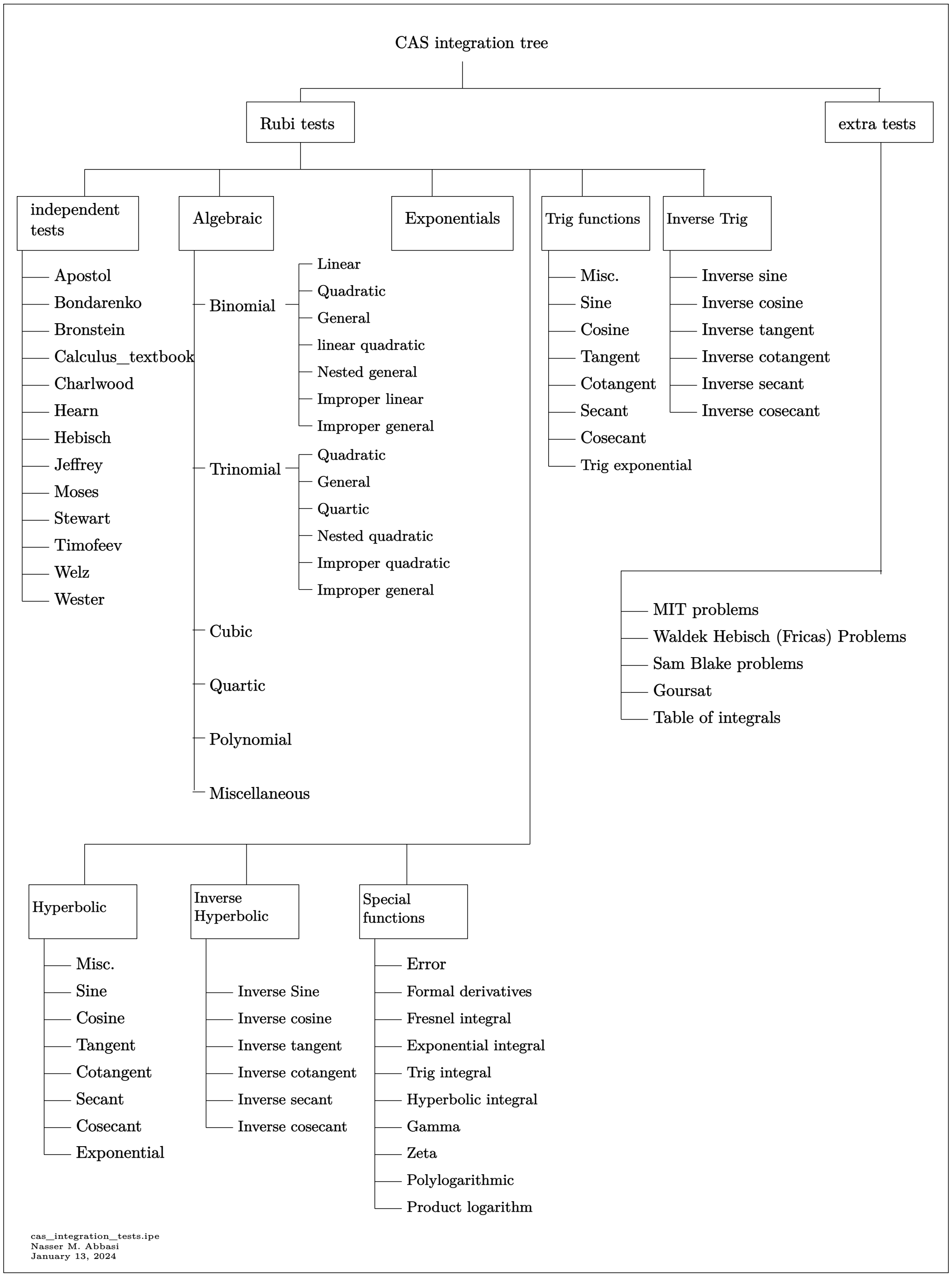
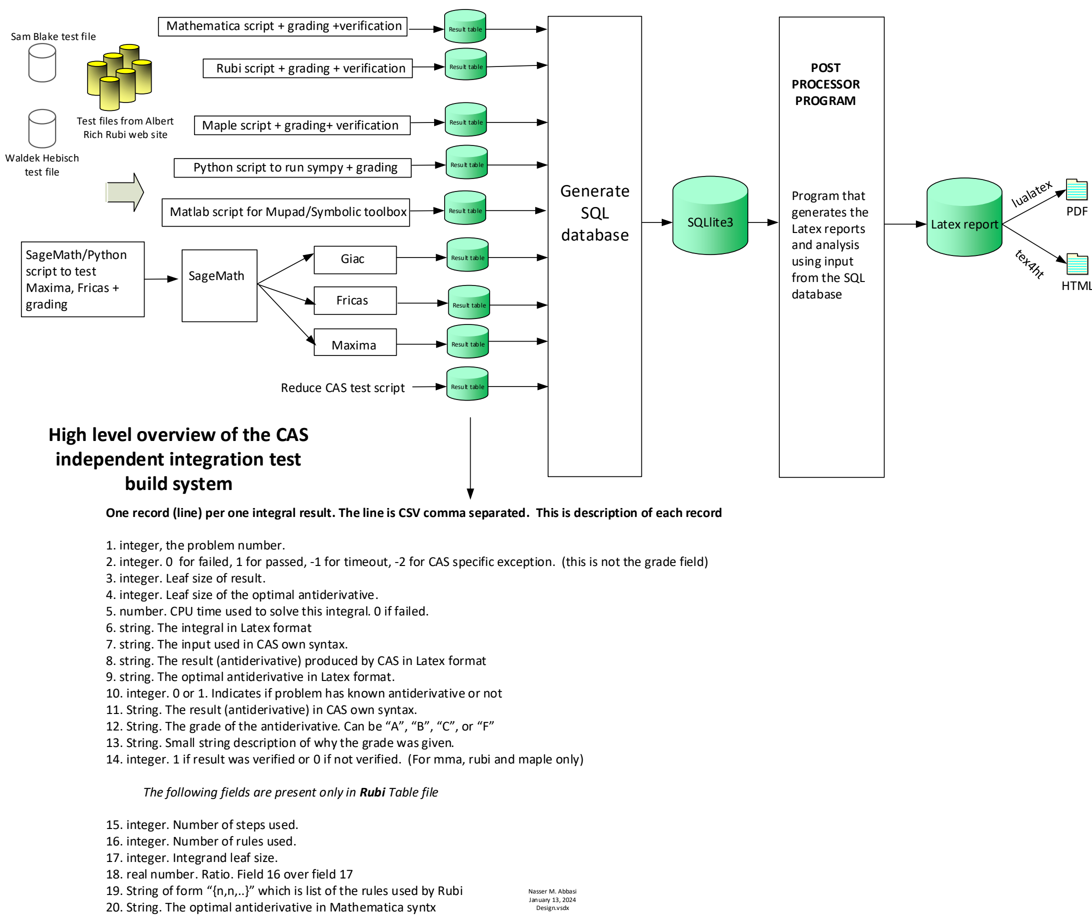


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 233, 235, 237, 238, 240, 242, 244, 247, 249, 251, 253, 254, 256, 258, 260, 262, 264, 266, 268, 270, 271, 273, 275, 277, 279, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299 }

B grade { 42, 294, 295 }

C grade { 230, 232, 234, 236, 239, 241, 243, 245, 246, 248, 250, 252, 255, 257, 259, 261, 263, 265, 267, 269, 272, 274, 276, 278, 280, 281, 282, 283 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 236, 238, 239, 240, 241, 242, 246, 247, 248, 249, 250, 251, 252, 254, 256, 262, 263, 264, 265, 267, 270, 271, 272, 277, 279, 287, 288, 289 }

B grade { 9, 10, 11, 12, 13, 14, 15, 16, 42, 228, 234, 235, 237, 243, 244, 245, 253, 255, 257, 258, 259, 260, 261, 266, 268, 273, 274, 275, 276, 278 }

C grade { 17, 18, 19, 20, 21, 22, 23, 24, 39, 40, 41, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 269 }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 68, 69, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 4, 5, 6, 7, 8, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 231, 233, 238, 240, 242, 247, 249, 254, 256, 262, 287, 288, 289 }

B grade { 1, 3, 41, 42, 235, 237, 244, 251, 253, 258, 260, 264, 266, 268, 270, 271, 273, 275, 277, 279 }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 230, 232, 234, 239, 241, 243, 246, 248, 250, 252, 255, 257, 263, 265, 272 }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 68, 69, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 236, 245, 259, 261, 267, 269, 274, 276, 278, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 135, 137, 138, 139, 140, 141, 145, 147, 148, 149, 150, 151, 155, 157, 158, 159, 160, 164, 165, 166, 167, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 287, 288, 289 }

B grade { 47, 48, 49, 132, 133, 134, 136, 142, 143, 144, 146, 152, 153, 154, 156, 161, 162, 163, 168, 169, 170, 176, 177, 178 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 2, 3, 4, 5, 6, 7, 8, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 180, 182, 225, 226, 228, 229 }

B grade { 1, 40, 41, 42, 139, 149, 166, 223, 224, 227 }

C grade { 175, 179, 181, 183 }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { 66, 67 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 12, 20, 43, 51, 61, 62, 63, 64, 74, 133, 135, 137, 138, 139, 140, 141, 143, 145, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 159, 160, 162, 164, 165, 166, 167, 169, 171, 172, 173, 174, 175, 177, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 231, 233, 238, 240, 242, 247, 249, 251, 254, 256, 262, 264, 271, 287, 288, 289 }

C grade { }

F normal fail { }

F(-1) timedout fail { 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 136, 142, 144, 146, 154, 156, 161, 163, 168, 170, 176, 178, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 232, 234, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 255, 257, 258, 259, 260, 261, 263, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-2) exception fail { }

Sympy

A grade { 43, 44, 45, 46, 51, 52, 53, 54, 137, 138, 139, 147, 148, 157, 164, 165, 166, 167, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183 }

B grade { 1 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 48, 49, 50, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 81, 82, 83, 84, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 145, 146, 163, 184, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 225, 226, 227, 234, 235, 236, 243, 244, 257, 258, 259, 260, 266, 267, 268, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 297, 298, 299 }

F(-1) timedout fail { 9, 17, 39, 47, 61, 77, 79, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 109, 120, 132, 133, 134, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 168, 169, 170, 176, 177, 178, 192, 193, 204, 216, 217, 223, 224, 228, 229, 230, 231, 232, 233, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 261, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 288, 289, 290, 296 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	18	28	36	44	31	11
N.S.	1	1.00	1.00	1.73	1.64	2.55	3.27	4.00	2.82	1.00
time (sec)	N/A	0.159	0.003	0.143	0.027	0.074	0.809	0.141	0.172	0.111

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	18	0	10	18	10
N.S.	1	1.00	1.00	1.10	1.00	1.80	0.00	1.00	1.80	1.00
time (sec)	N/A	0.170	0.004	0.237	0.029	0.065	0.000	0.113	0.163	10.098

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	46	61	0	48	95	36
N.S.	1	1.00	1.00	1.06	1.35	1.79	0.00	1.41	2.79	1.06
time (sec)	N/A	0.229	0.009	0.346	0.026	0.079	0.000	0.155	0.171	0.071

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	24	22	31	0	22	43	21
N.S.	1	1.00	0.88	0.92	0.85	1.19	0.00	0.85	1.65	0.81
time (sec)	N/A	0.187	0.034	0.312	0.025	0.079	0.000	0.124	0.171	9.972

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	55	49	71	74	0	63	162	58
N.S.	1	1.09	1.00	0.89	1.29	1.35	0.00	1.15	2.95	1.05
time (sec)	N/A	0.311	0.011	0.473	0.031	0.081	0.000	0.135	0.174	9.589

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	38	35	34	34	41	0	34	63	31
N.S.	1	0.93	0.85	0.83	0.83	1.00	0.00	0.83	1.54	0.76
time (sec)	N/A	0.195	0.085	0.385	0.026	0.066	0.000	0.135	0.152	10.056

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	86	76	59	91	84	0	73	226	79
N.S.	1	1.13	1.00	0.78	1.20	1.11	0.00	0.96	2.97	1.04
time (sec)	N/A	0.421	0.009	0.650	0.033	0.080	0.000	0.113	0.154	10.369

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	48	43	44	44	51	0	44	83	39
N.S.	1	0.91	0.81	0.83	0.83	0.96	0.00	0.83	1.57	0.74
time (sec)	N/A	0.204	0.132	0.480	0.033	0.066	0.000	0.139	0.156	10.054

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	86	59	358	0	110	0	0	18	0
N.S.	1	1.01	0.69	4.21	0.00	1.29	0.00	0.00	0.21	0.00
time (sec)	N/A	0.414	0.130	1.486	0.000	0.099	0.000	0.000	0.161	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	213	0	88	0	0	18	0
N.S.	1	1.00	0.74	3.44	0.00	1.42	0.00	0.00	0.29	0.00
time (sec)	N/A	0.316	0.057	0.868	0.000	0.081	0.000	0.000	0.167	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	45	182	0	73	0	0	16	0
N.S.	1	1.00	0.78	3.14	0.00	1.26	0.00	0.00	0.28	0.00
time (sec)	N/A	0.316	0.040	0.458	0.000	0.078	0.000	0.000	0.170	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	51	0	0	9	33
N.S.	1	1.00	1.00	3.69	0.00	1.42	0.00	0.00	0.25	0.92
time (sec)	N/A	0.230	0.026	0.366	0.000	0.082	0.000	0.000	0.172	0.094

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	57	0	0	18	0
N.S.	1	1.00	1.00	3.69	0.00	1.58	0.00	0.00	0.50	0.00
time (sec)	N/A	0.237	0.033	0.992	0.000	0.077	0.000	0.000	0.193	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	179	0	68	0	0	18	0
N.S.	1	1.00	0.79	2.89	0.00	1.10	0.00	0.00	0.29	0.00
time (sec)	N/A	0.315	0.047	1.435	0.000	0.085	0.000	0.000	0.168	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	202	0	74	0	0	18	0
N.S.	1	1.00	0.89	3.26	0.00	1.19	0.00	0.00	0.29	0.00
time (sec)	N/A	0.312	0.060	2.691	0.000	0.085	0.000	0.000	0.175	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	61	199	0	87	0	0	18	0
N.S.	1	1.06	0.72	2.34	0.00	1.02	0.00	0.00	0.21	0.00
time (sec)	N/A	0.404	0.073	4.207	0.000	0.090	0.000	0.000	0.239	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	100	62	207	0	125	0	0	24	0
N.S.	1	1.02	0.63	2.11	0.00	1.28	0.00	0.00	0.24	0.00
time (sec)	N/A	0.457	0.140	3.290	0.000	0.089	0.000	0.000	0.176	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	90	0	101	0	0	24	0
N.S.	1	1.00	0.73	1.29	0.00	1.44	0.00	0.00	0.34	0.00
time (sec)	N/A	0.329	0.059	2.468	0.000	0.086	0.000	0.000	0.202	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	182	0	84	0	0	20	0
N.S.	1	1.00	0.73	2.76	0.00	1.27	0.00	0.00	0.30	0.00
time (sec)	N/A	0.337	0.040	1.811	0.000	0.082	0.000	0.000	0.190	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	77	0	57	0	0	12	35
N.S.	1	1.00	1.00	2.03	0.00	1.50	0.00	0.00	0.32	0.92
time (sec)	N/A	0.241	0.026	1.260	0.000	0.090	0.000	0.000	0.192	0.156

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	175	0	66	0	0	24	0
N.S.	1	1.00	1.00	4.61	0.00	1.74	0.00	0.00	0.63	0.00
time (sec)	N/A	0.239	0.032	1.181	0.000	0.078	0.000	0.000	0.183	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	90	0	87	0	0	24	0
N.S.	1	1.00	0.82	1.25	0.00	1.21	0.00	0.00	0.33	0.00
time (sec)	N/A	0.339	0.057	1.487	0.000	0.087	0.000	0.000	0.184	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	191	0	95	0	0	24	0
N.S.	1	1.00	0.83	2.65	0.00	1.32	0.00	0.00	0.33	0.00
time (sec)	N/A	0.339	0.061	2.451	0.000	0.109	0.000	0.000	0.202	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	66	104	0	100	0	0	24	0
N.S.	1	1.08	0.66	1.04	0.00	1.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.443	0.081	3.066	0.000	0.091	0.000	0.000	0.198	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0	10	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.247	0.044	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0	10	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.250	0.029	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	10	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.245	0.028	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	10	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.244	0.057	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0	10	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.249	0.059	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0	10	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.242	0.045	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0	14	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.258	0.033	0.000	0.000	0.000	0.000	0.000	0.288	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0	14	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.263	0.029	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.257	0.028	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.262	0.039	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	14	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.258	0.038	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	14	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.267	0.047	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	0	0	0	0	0	10	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.279	0.036	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0	14	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.295	0.032	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	66	39	51	42	49	0	59	136	0
N.S.	1	1.32	0.78	1.02	0.84	0.98	0.00	1.18	2.72	0.00
time (sec)	N/A	0.204	0.040	2.869	0.105	0.082	0.000	0.132	0.185	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	36	45	33	43	30	43	0	53	98	0
N.S.	1	1.25	0.92	1.19	0.83	1.19	0.00	1.47	2.72	0.00
time (sec)	N/A	0.197	0.025	2.707	0.103	0.084	0.000	0.135	0.159	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	24	23	33	18	34	0	44	57	0
N.S.	1	1.09	1.05	1.50	0.82	1.55	0.00	2.00	2.59	0.00
time (sec)	N/A	0.183	0.013	0.316	0.105	0.080	0.000	0.115	0.163	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	14	19	3	17	0	35	17	0
N.S.	1	1.00	4.67	6.33	1.00	5.67	0.00	11.67	5.67	0.00
time (sec)	N/A	0.173	0.005	0.267	0.102	0.078	0.000	0.108	0.154	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	11	13	11	7	11	4	10	6	2	12
N.S.	1	1.18	1.00	0.64	1.00	0.36	0.91	0.55	0.18	1.09
time (sec)	N/A	0.184	0.018	0.257	0.025	0.073	0.222	0.151	0.154	0.121

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	29	33	28	19	25	10	27	16	12	0
N.S.	1	1.14	0.97	0.66	0.86	0.34	0.93	0.55	0.41	0.00
time (sec)	N/A	0.187	0.014	0.291	0.033	0.075	0.354	0.117	0.175	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	54	28	22	37	18	44	25	18	0
N.S.	1	1.26	0.65	0.51	0.86	0.42	1.02	0.58	0.42	0.00
time (sec)	N/A	0.194	0.018	0.336	0.028	0.078	1.370	0.116	0.155	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	57	75	34	28	49	24	60	34	24	0
N.S.	1	1.32	0.60	0.49	0.86	0.42	1.05	0.60	0.42	0.00
time (sec)	N/A	0.197	0.024	0.366	0.030	0.078	10.920	0.117	0.162	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	99	44	60	2175	65	0	79	141	0
N.S.	1	1.18	0.52	0.71	25.89	0.77	0.00	0.94	1.68	0.00
time (sec)	N/A	0.237	0.061	3.008	2.039	0.089	0.000	0.157	0.224	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	75	37	52	1111	56	0	55	103	0
N.S.	1	1.15	0.57	0.80	17.09	0.86	0.00	0.85	1.58	0.00
time (sec)	N/A	0.223	0.036	2.830	0.292	0.086	0.000	0.138	0.178	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	24	42	324	39	0	42	60	0
N.S.	1	1.11	0.52	0.91	7.04	0.85	0.00	0.91	1.30	0.00
time (sec)	N/A	0.205	0.025	0.318	0.149	0.090	0.000	0.139	0.190	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	27	16	21	38	55	0	31	20	0
N.S.	1	1.08	0.64	0.84	1.52	2.20	0.00	1.24	0.80	0.00
time (sec)	N/A	0.192	0.005	0.268	0.155	0.088	0.000	0.147	0.166	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	15	13	12	6	16	12	11	8	15
N.S.	1	1.15	1.00	0.92	0.46	1.23	0.92	0.85	0.62	1.15
time (sec)	N/A	0.192	0.016	0.259	0.141	0.072	0.278	0.142	0.169	11.123

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	45	25	24	14	24	31	19	17	0
N.S.	1	1.25	0.69	0.67	0.39	0.67	0.86	0.53	0.47	0.00
time (sec)	N/A	0.203	0.018	0.288	0.143	0.109	0.403	0.118	0.164	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	74	33	29	22	32	49	27	23	0
N.S.	1	1.35	0.60	0.53	0.40	0.58	0.89	0.49	0.42	0.00
time (sec)	N/A	0.219	0.017	0.310	0.145	0.080	1.401	0.138	0.160	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	103	39	35	28	38	66	33	29	0
N.S.	1	1.39	0.53	0.47	0.38	0.51	0.89	0.45	0.39	0.00
time (sec)	N/A	0.229	0.021	0.341	0.151	0.088	11.362	0.132	0.186	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	103	59	164	0	102	0	0	16	0
N.S.	1	0.88	0.50	1.40	0.00	0.87	0.00	0.00	0.14	0.00
time (sec)	N/A	0.609	0.084	18.677	0.000	0.091	0.000	0.000	0.157	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	71	43	68	0	74	0	0	14	0
N.S.	1	1.09	0.66	1.05	0.00	1.14	0.00	0.00	0.22	0.00
time (sec)	N/A	0.424	0.038	3.232	0.000	0.085	0.000	0.000	0.160	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	49	32	114	0	57	0	0	11	0
N.S.	1	1.17	0.76	2.71	0.00	1.36	0.00	0.00	0.26	0.00
time (sec)	N/A	0.352	0.023	1.269	0.000	0.083	0.000	0.000	0.158	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	53	31	57	0	58	0	0	16	0
N.S.	1	1.20	0.70	1.30	0.00	1.32	0.00	0.00	0.36	0.00
time (sec)	N/A	0.349	0.040	1.154	0.000	0.079	0.000	0.000	0.158	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	43	140	0	73	0	0	16	0
N.S.	1	1.00	0.59	1.92	0.00	1.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.434	0.079	4.363	0.000	0.085	0.000	0.000	0.161	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	107	59	84	0	79	0	0	16	0
N.S.	1	0.91	0.50	0.72	0.00	0.68	0.00	0.00	0.14	0.00
time (sec)	N/A	0.611	0.078	10.907	0.000	0.098	0.000	0.000	0.173	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	68	54	58	61	76	0	67	89	589
N.S.	1	0.42	0.33	0.36	0.37	0.47	0.00	0.41	0.55	3.61
time (sec)	N/A	0.268	0.155	29.658	0.107	0.089	0.000	0.138	0.162	15.774

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	56	42	46	43	58	0	34	65	119
N.S.	1	0.48	0.36	0.39	0.37	0.50	0.00	0.29	0.56	1.02
time (sec)	N/A	0.258	0.088	29.541	0.109	0.078	0.000	0.113	0.164	12.993

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	38	33	31	25	34	0	22	39	36
N.S.	1	0.62	0.54	0.51	0.41	0.56	0.00	0.36	0.64	0.59
time (sec)	N/A	0.254	0.027	0.304	0.107	0.076	0.000	0.151	0.256	0.572

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	6	13	0	6	9	6
N.S.	1	1.00	1.00	0.93	0.40	0.87	0.00	0.40	0.60	0.40
time (sec)	N/A	0.236	0.006	0.260	0.108	0.072	0.000	0.132	0.164	10.515

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	29	23	20	25	27	0	31	14	0
N.S.	1	0.81	0.64	0.56	0.69	0.75	0.00	0.86	0.39	0.00
time (sec)	N/A	0.226	0.030	0.277	0.111	0.091	0.000	0.115	0.161	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	62	38	39	58	43	0	0	33	0
N.S.	1	0.72	0.44	0.45	0.67	0.50	0.00	0.00	0.38	0.00
time (sec)	N/A	0.347	0.046	0.383	0.110	0.100	0.000	0.000	0.167	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	92	55	51	88	55	0	0	49	0
N.S.	1	0.70	0.42	0.39	0.67	0.42	0.00	0.00	0.37	0.00
time (sec)	N/A	0.490	0.082	0.657	0.108	0.086	0.000	0.000	0.172	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	69	0	0	0	0	0	18	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.381	0.082	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	0	0	0	0	0	21	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.359	0.056	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	108	69	103	0	114	0	0	21	0
N.S.	1	1.11	0.71	1.06	0.00	1.18	0.00	0.00	0.22	0.00
time (sec)	N/A	0.492	0.268	3.821	0.000	0.090	0.000	0.000	0.168	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	104	69	204	0	120	0	0	21	0
N.S.	1	1.09	0.73	2.15	0.00	1.26	0.00	0.00	0.22	0.00
time (sec)	N/A	0.480	0.196	2.875	0.000	0.113	0.000	0.000	0.174	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	51	87	0	98	0	0	21	0
N.S.	1	1.07	0.74	1.26	0.00	1.42	0.00	0.00	0.30	0.00
time (sec)	N/A	0.360	0.114	2.129	0.000	0.088	0.000	0.000	0.180	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	47	181	0	83	0	0	19	0
N.S.	1	1.11	0.75	2.87	0.00	1.32	0.00	0.00	0.30	0.00
time (sec)	N/A	0.361	0.048	1.801	0.000	0.077	0.000	0.000	0.182	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	77	0	57	0	0	12	35
N.S.	1	1.00	1.00	2.03	0.00	1.50	0.00	0.00	0.32	0.92
time (sec)	N/A	0.236	0.027	1.316	0.000	0.080	0.000	0.000	0.169	10.181

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	187	0	63	0	0	19	0
N.S.	1	1.00	1.00	4.79	0.00	1.62	0.00	0.00	0.49	0.00
time (sec)	N/A	0.296	0.033	1.271	0.000	0.082	0.000	0.000	0.203	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	76	51	93	0	84	0	0	21	0
N.S.	1	1.13	0.76	1.39	0.00	1.25	0.00	0.00	0.31	0.00
time (sec)	N/A	0.376	0.170	1.595	0.000	0.091	0.000	0.000	0.184	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	76	57	206	0	92	0	0	21	0
N.S.	1	1.09	0.81	2.94	0.00	1.31	0.00	0.00	0.30	0.00
time (sec)	N/A	0.388	0.130	2.704	0.000	0.085	0.000	0.000	0.180	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	112	63	107	0	97	0	0	21	0
N.S.	1	1.18	0.66	1.13	0.00	1.02	0.00	0.00	0.22	0.00
time (sec)	N/A	0.499	0.152	3.369	0.000	0.099	0.000	0.000	0.181	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	112	71	228	0	105	0	0	21	0
N.S.	1	1.14	0.72	2.33	0.00	1.07	0.00	0.00	0.21	0.00
time (sec)	N/A	0.491	0.265	5.662	0.000	0.086	0.000	0.000	0.244	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	108	64	104	0	117	0	0	22	0
N.S.	1	1.14	0.67	1.09	0.00	1.23	0.00	0.00	0.23	0.00
time (sec)	N/A	0.473	0.262	3.793	0.000	0.089	0.000	0.000	0.198	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	104	64	205	0	121	0	0	22	0
N.S.	1	1.06	0.65	2.09	0.00	1.23	0.00	0.00	0.22	0.00
time (sec)	N/A	0.471	0.184	2.894	0.000	0.086	0.000	0.000	0.180	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	74	49	88	0	99	0	0	22	0
N.S.	1	1.10	0.73	1.31	0.00	1.48	0.00	0.00	0.33	0.00
time (sec)	N/A	0.354	0.066	2.181	0.000	0.090	0.000	0.000	0.172	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	182	0	84	0	0	20	0
N.S.	1	1.00	0.73	2.76	0.00	1.27	0.00	0.00	0.30	0.00
time (sec)	N/A	0.346	0.008	1.833	0.000	0.083	0.000	0.000	0.173	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	78	0	57	0	0	26	0
N.S.	1	1.00	1.00	2.00	0.00	1.46	0.00	0.00	0.67	0.00
time (sec)	N/A	0.280	0.006	1.318	0.000	0.078	0.000	0.000	0.212	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	189	0	63	0	0	28	0
N.S.	1	1.00	1.00	4.61	0.00	1.54	0.00	0.00	0.68	0.00
time (sec)	N/A	0.279	0.019	1.196	0.000	0.086	0.000	0.000	0.200	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	76	52	95	0	85	0	0	28	0
N.S.	1	1.09	0.74	1.36	0.00	1.21	0.00	0.00	0.40	0.00
time (sec)	N/A	0.377	0.116	1.644	0.000	0.087	0.000	0.000	0.194	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	58	207	0	93	0	0	28	0
N.S.	1	1.06	0.81	2.88	0.00	1.29	0.00	0.00	0.39	0.00
time (sec)	N/A	0.383	0.109	2.742	0.000	0.086	0.000	0.000	0.193	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	112	64	108	0	99	0	0	28	0
N.S.	1	1.14	0.65	1.10	0.00	1.01	0.00	0.00	0.29	0.00
time (sec)	N/A	0.488	0.171	3.467	0.000	0.087	0.000	0.000	0.210	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	72	229	0	107	0	0	28	0
N.S.	1	1.12	0.72	2.29	0.00	1.07	0.00	0.00	0.28	0.00
time (sec)	N/A	0.495	0.311	5.826	0.000	0.087	0.000	0.000	0.209	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	108	61	106	0	121	0	0	24	0
N.S.	1	1.10	0.62	1.08	0.00	1.23	0.00	0.00	0.24	0.00
time (sec)	N/A	0.469	0.281	8.171	0.000	0.125	0.000	0.000	0.174	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	104	61	210	0	125	0	0	24	0
N.S.	1	1.07	0.63	2.16	0.00	1.29	0.00	0.00	0.25	0.00
time (sec)	N/A	0.483	0.148	4.025	0.000	0.084	0.000	0.000	0.175	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	90	0	101	0	0	24	0
N.S.	1	1.00	0.73	1.29	0.00	1.44	0.00	0.00	0.34	0.00
time (sec)	N/A	0.327	0.040	2.516	0.000	0.080	0.000	0.000	0.186	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	184	0	86	0	0	30	0
N.S.	1	1.00	0.74	2.71	0.00	1.26	0.00	0.00	0.44	0.00
time (sec)	N/A	0.362	0.013	8.410	0.000	0.103	0.000	0.000	0.199	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	80	0	57	0	0	32	0
N.S.	1	1.00	1.00	1.95	0.00	1.39	0.00	0.00	0.78	0.00
time (sec)	N/A	0.277	0.011	30.096	0.000	0.080	0.000	0.000	0.296	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	191	0	63	0	0	32	0
N.S.	1	1.00	0.93	4.66	0.00	1.54	0.00	0.00	0.78	0.00
time (sec)	N/A	0.301	0.123	104.040	0.000	0.081	0.000	0.000	0.195	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	54	96	0	87	0	0	32	0
N.S.	1	1.06	0.75	1.33	0.00	1.21	0.00	0.00	0.44	0.00
time (sec)	N/A	0.375	0.146	1.181	0.000	0.086	0.000	0.000	0.204	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	60	209	0	95	0	0	32	0
N.S.	1	1.06	0.83	2.90	0.00	1.32	0.00	0.00	0.44	0.00
time (sec)	N/A	0.379	0.125	11.629	0.000	0.088	0.000	0.000	0.194	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	66	110	0	103	0	0	32	0
N.S.	1	1.12	0.66	1.10	0.00	1.03	0.00	0.00	0.32	0.00
time (sec)	N/A	0.502	0.217	2.820	0.000	0.129	0.000	0.000	0.194	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	74	231	0	111	0	0	32	0
N.S.	1	1.12	0.74	2.31	0.00	1.11	0.00	0.00	0.32	0.00
time (sec)	N/A	0.487	0.360	4.918	0.000	0.089	0.000	0.000	0.201	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	100	62	210	0	125	0	0	24	0
N.S.	1	1.02	0.63	2.14	0.00	1.28	0.00	0.00	0.24	0.00
time (sec)	N/A	0.438	0.021	3.103	0.000	0.084	0.000	0.000	0.183	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	69	109	0	117	0	0	24	0
N.S.	1	1.08	0.69	1.09	0.00	1.17	0.00	0.00	0.24	0.00
time (sec)	N/A	0.469	0.225	3.813	0.000	0.093	0.000	0.000	0.181	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	104	61	200	0	123	0	0	24	0
N.S.	1	1.07	0.63	2.06	0.00	1.27	0.00	0.00	0.25	0.00
time (sec)	N/A	0.466	0.224	2.918	0.000	0.088	0.000	0.000	0.176	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	51	95	0	101	0	0	24	0
N.S.	1	1.03	0.71	1.32	0.00	1.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.359	0.119	2.206	0.000	0.082	0.000	0.000	0.180	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	48	177	0	86	0	0	22	0
N.S.	1	1.08	0.74	2.72	0.00	1.32	0.00	0.00	0.34	0.00
time (sec)	N/A	0.364	0.125	1.872	0.000	0.091	0.000	0.000	0.183	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	69	0	60	0	0	15	0
N.S.	1	1.00	1.00	1.68	0.00	1.46	0.00	0.00	0.37	0.00
time (sec)	N/A	0.255	0.007	1.249	0.000	0.078	0.000	0.000	0.178	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	175	0	66	0	0	24	0
N.S.	1	1.00	1.00	4.61	0.00	1.74	0.00	0.00	0.63	0.00
time (sec)	N/A	0.235	0.011	1.128	0.000	0.088	0.000	0.000	0.178	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	60	87	0	87	0	0	30	0
N.S.	1	1.07	0.87	1.26	0.00	1.26	0.00	0.00	0.43	0.00
time (sec)	N/A	0.368	0.058	1.516	0.000	0.084	0.000	0.000	0.187	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	76	60	188	0	95	0	0	32	0
N.S.	1	1.13	0.90	2.81	0.00	1.42	0.00	0.00	0.48	0.00
time (sec)	N/A	0.372	0.071	2.592	0.000	0.088	0.000	0.000	0.186	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	112	66	101	0	100	0	0	32	0
N.S.	1	1.15	0.68	1.04	0.00	1.03	0.00	0.00	0.33	0.00
time (sec)	N/A	0.496	0.165	3.270	0.000	0.088	0.000	0.000	0.273	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	112	70	212	0	108	0	0	32	0
N.S.	1	1.18	0.74	2.23	0.00	1.14	0.00	0.00	0.34	0.00
time (sec)	N/A	0.493	0.243	5.559	0.000	0.088	0.000	0.000	0.186	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	69	112	0	117	0	0	24	0
N.S.	1	1.08	0.69	1.12	0.00	1.17	0.00	0.00	0.24	0.00
time (sec)	N/A	0.467	0.275	3.819	0.000	0.106	0.000	0.000	0.210	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	64	201	0	123	0	0	24	0
N.S.	1	1.04	0.64	2.01	0.00	1.23	0.00	0.00	0.24	0.00
time (sec)	N/A	0.470	0.158	2.866	0.000	0.082	0.000	0.000	0.187	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	56	96	0	101	0	0	24	0
N.S.	1	1.03	0.78	1.33	0.00	1.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.364	0.158	2.183	0.000	0.084	0.000	0.000	0.188	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	51	178	0	86	0	0	22	0
N.S.	1	1.03	0.75	2.62	0.00	1.26	0.00	0.00	0.32	0.00
time (sec)	N/A	0.360	0.119	1.816	0.000	0.084	0.000	0.000	0.181	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	72	0	60	0	0	15	0
N.S.	1	1.00	1.00	1.76	0.00	1.46	0.00	0.00	0.37	0.00
time (sec)	N/A	0.260	0.008	1.245	0.000	0.080	0.000	0.000	0.189	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	178	0	66	0	0	24	0
N.S.	1	1.00	1.00	4.34	0.00	1.61	0.00	0.00	0.59	0.00
time (sec)	N/A	0.265	0.013	1.153	0.000	0.078	0.000	0.000	0.165	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	90	0	87	0	0	24	0
N.S.	1	1.00	0.82	1.25	0.00	1.21	0.00	0.00	0.33	0.00
time (sec)	N/A	0.331	0.031	1.545	0.000	0.084	0.000	0.000	0.175	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	60	195	0	95	0	0	30	0
N.S.	1	1.07	0.87	2.83	0.00	1.38	0.00	0.00	0.43	0.00
time (sec)	N/A	0.365	0.045	2.647	0.000	0.087	0.000	0.000	0.182	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	112	66	104	0	100	0	0	32	0
N.S.	1	1.14	0.67	1.06	0.00	1.02	0.00	0.00	0.33	0.00
time (sec)	N/A	0.493	0.082	3.263	0.000	0.087	0.000	0.000	0.169	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	112	73	215	0	108	0	0	32	0
N.S.	1	1.15	0.75	2.22	0.00	1.11	0.00	0.00	0.33	0.00
time (sec)	N/A	0.498	0.230	5.571	0.000	0.120	0.000	0.000	0.170	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	64	112	0	117	0	0	24	0
N.S.	1	1.08	0.64	1.12	0.00	1.17	0.00	0.00	0.24	0.00
time (sec)	N/A	0.477	0.301	3.787	0.000	0.090	0.000	0.000	0.169	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	64	203	0	123	0	0	24	0
N.S.	1	1.04	0.64	2.03	0.00	1.23	0.00	0.00	0.24	0.00
time (sec)	N/A	0.461	0.154	2.971	0.000	0.083	0.000	0.000	0.177	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	51	98	0	101	0	0	24	0
N.S.	1	1.03	0.71	1.36	0.00	1.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.357	0.142	2.228	0.000	0.080	0.000	0.000	0.181	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	51	180	0	86	0	0	22	0
N.S.	1	1.03	0.75	2.65	0.00	1.26	0.00	0.00	0.32	0.00
time (sec)	N/A	0.359	0.151	1.903	0.000	0.086	0.000	0.000	0.172	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	72	0	60	0	0	15	0
N.S.	1	1.00	1.00	1.76	0.00	1.46	0.00	0.00	0.37	0.00
time (sec)	N/A	0.258	0.007	1.259	0.000	0.086	0.000	0.000	0.270	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	178	0	66	0	0	24	0
N.S.	1	1.00	0.93	4.34	0.00	1.61	0.00	0.00	0.59	0.00
time (sec)	N/A	0.258	0.142	1.148	0.000	0.081	0.000	0.000	0.202	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	62	90	0	87	0	0	24	0
N.S.	1	1.06	0.86	1.25	0.00	1.21	0.00	0.00	0.33	0.00
time (sec)	N/A	0.354	0.021	1.529	0.000	0.087	0.000	0.000	0.165	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	191	0	95	0	0	24	0
N.S.	1	1.00	0.83	2.65	0.00	1.32	0.00	0.00	0.33	0.00
time (sec)	N/A	0.327	0.009	2.562	0.000	0.082	0.000	0.000	0.178	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	110	66	104	0	100	0	0	30	0
N.S.	1	1.13	0.68	1.07	0.00	1.03	0.00	0.00	0.31	0.00
time (sec)	N/A	0.477	0.024	3.234	0.000	0.091	0.000	0.000	0.180	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	112	73	215	0	108	0	0	32	0
N.S.	1	1.14	0.74	2.19	0.00	1.10	0.00	0.00	0.33	0.00
time (sec)	N/A	0.495	0.113	5.480	0.000	0.085	0.000	0.000	0.179	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	66	104	0	100	0	0	24	0
N.S.	1	1.08	0.66	1.04	0.00	1.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.446	0.013	3.227	0.000	0.088	0.000	0.000	0.176	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	83	64	109	1656	231	0	151	164	0
N.S.	1	0.78	0.60	1.02	15.48	2.16	0.00	1.41	1.53	0.00
time (sec)	N/A	0.387	0.111	0.893	0.320	0.113	0.000	0.265	0.195	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	48	45	48	294	43	0	94	45	126
N.S.	1	0.69	0.64	0.69	4.20	0.61	0.00	1.34	0.64	1.80
time (sec)	N/A	0.241	0.070	0.760	0.235	0.082	0.000	0.317	0.162	12.121

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	57	50	91	661	201	0	93	97	0
N.S.	1	0.79	0.69	1.26	9.18	2.79	0.00	1.29	1.35	0.00
time (sec)	N/A	0.295	0.041	0.808	0.277	0.106	0.000	0.230	0.166	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	35	54	30	0	39	20	46
N.S.	1	1.00	1.00	1.09	1.69	0.94	0.00	1.22	0.62	1.44
time (sec)	N/A	0.227	0.016	0.741	0.237	0.103	0.000	0.235	0.171	0.286

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	46	65	113	0	41	33	0
N.S.	1	1.00	1.00	1.39	1.97	3.42	0.00	1.24	1.00	0.00
time (sec)	N/A	0.211	0.012	0.747	0.238	0.099	0.000	0.237	0.167	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	26	98	22	35	4	24
N.S.	1	1.00	1.00	0.88	1.08	4.08	0.92	1.46	0.17	1.00
time (sec)	N/A	0.153	0.025	0.895	0.178	0.107	2.000	0.202	0.181	9.121

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	13	30	46	38	12	32
N.S.	1	1.00	1.00	0.91	0.41	0.94	1.44	1.19	0.38	1.00
time (sec)	N/A	0.205	0.153	0.740	0.244	0.082	4.100	0.205	0.155	0.216

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	48	45	45	25	158	107	140	24	41
N.S.	1	0.76	0.71	0.71	0.40	2.51	1.70	2.22	0.38	0.65
time (sec)	N/A	0.215	0.152	0.751	0.183	0.108	10.856	0.580	0.164	9.235

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	48	45	47	42	48	0	65	25	45
N.S.	1	0.69	0.64	0.67	0.60	0.69	0.00	0.93	0.36	0.64
time (sec)	N/A	0.229	0.079	0.786	0.183	0.079	0.000	0.217	0.154	9.494

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	74	55	64	49	202	0	141	42	52
N.S.	1	0.76	0.56	0.65	0.50	2.06	0.00	1.44	0.43	0.53
time (sec)	N/A	0.290	0.257	0.787	0.192	0.112	0.000	0.209	0.260	9.716

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	84	64	110	1742	238	0	151	165	0
N.S.	1	0.76	0.58	1.00	15.84	2.16	0.00	1.37	1.50	0.00
time (sec)	N/A	0.387	0.096	0.902	0.268	0.118	0.000	0.276	0.174	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	49	45	49	299	44	0	94	46	127
N.S.	1	0.68	0.62	0.68	4.15	0.61	0.00	1.31	0.64	1.76
time (sec)	N/A	0.238	0.068	0.822	0.188	0.084	0.000	0.259	0.164	10.255

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	58	50	92	691	204	0	93	98	0
N.S.	1	0.78	0.68	1.24	9.34	2.76	0.00	1.26	1.32	0.00
time (sec)	N/A	0.295	0.051	0.850	0.206	0.112	0.000	0.236	0.174	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	36	54	31	0	39	21	47
N.S.	1	1.00	0.97	1.09	1.64	0.94	0.00	1.18	0.64	1.42
time (sec)	N/A	0.223	0.017	0.780	0.187	0.079	0.000	0.226	0.175	0.209

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	47	68	114	0	41	34	0
N.S.	1	1.00	0.97	1.38	2.00	3.35	0.00	1.21	1.00	0.00
time (sec)	N/A	0.214	0.019	0.783	0.186	0.100	0.000	0.192	0.166	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	26	99	22	35	5	25
N.S.	1	1.00	1.00	0.84	1.04	3.96	0.88	1.40	0.20	1.00
time (sec)	N/A	0.158	0.036	0.882	0.129	0.099	3.799	0.205	0.156	9.336

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	13	31	46	38	13	33
N.S.	1	1.00	0.97	0.91	0.39	0.94	1.39	1.15	0.39	1.00
time (sec)	N/A	0.208	0.142	0.799	0.188	0.092	19.457	0.210	0.163	0.234

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	49	45	46	28	161	0	111	25	42
N.S.	1	0.75	0.69	0.71	0.43	2.48	0.00	1.71	0.38	0.65
time (sec)	N/A	0.213	0.163	0.805	0.180	0.109	0.000	0.204	0.168	9.514

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	49	45	48	45	51	0	65	26	46
N.S.	1	0.68	0.62	0.67	0.62	0.71	0.00	0.90	0.36	0.64
time (sec)	N/A	0.234	0.099	0.783	0.190	0.084	0.000	0.187	0.169	9.444

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	75	55	65	53	207	0	141	43	53
N.S.	1	0.74	0.54	0.64	0.52	2.05	0.00	1.40	0.43	0.52
time (sec)	N/A	0.285	0.298	0.843	0.187	0.116	0.000	0.214	0.168	9.846

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	63	57	62	705	63	0	146	68	205
N.S.	1	0.54	0.49	0.53	6.08	0.54	0.00	1.26	0.59	1.77
time (sec)	N/A	0.248	0.158	14.829	0.260	0.083	0.000	0.273	0.191	13.973

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	51	45	51	311	48	0	94	48	129
N.S.	1	0.67	0.59	0.67	4.09	0.63	0.00	1.24	0.63	1.70
time (sec)	N/A	0.249	0.075	0.800	0.202	0.083	0.000	0.264	0.174	1.261

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	60	50	95	747	210	0	93	100	0
N.S.	1	0.77	0.64	1.22	9.58	2.69	0.00	1.19	1.28	0.00
time (sec)	N/A	0.300	0.066	0.860	0.200	0.142	0.000	0.255	0.170	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	38	54	33	0	40	23	66
N.S.	1	1.00	0.91	1.09	1.54	0.94	0.00	1.14	0.66	1.89
time (sec)	N/A	0.225	0.025	0.793	0.189	0.076	0.000	0.200	0.176	10.440

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	49	72	116	0	43	36	0
N.S.	1	1.00	0.92	1.36	2.00	3.22	0.00	1.19	1.00	0.00
time (sec)	N/A	0.215	0.027	0.789	0.195	0.104	0.000	0.224	0.171	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	21	26	101	22	35	7	27
N.S.	1	1.00	0.89	0.78	0.96	3.74	0.81	1.30	0.26	1.00
time (sec)	N/A	0.153	0.040	0.884	0.146	0.109	48.213	0.204	0.288	0.099

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	13	33	0	38	15	35
N.S.	1	1.00	0.91	0.91	0.37	0.94	0.00	1.09	0.43	1.00
time (sec)	N/A	0.202	0.205	0.792	0.178	0.080	0.000	0.222	0.170	0.243

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	51	45	48	32	167	0	111	27	44
N.S.	1	0.74	0.65	0.70	0.46	2.42	0.00	1.61	0.39	0.64
time (sec)	N/A	0.223	0.226	0.793	0.195	0.115	0.000	0.212	0.172	9.802

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	51	45	50	49	55	0	65	28	48
N.S.	1	0.67	0.59	0.66	0.64	0.72	0.00	0.86	0.37	0.63
time (sec)	N/A	0.236	0.133	0.802	0.188	0.080	0.000	0.202	0.174	9.820

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	57	50	91	661	207	0	118	100	0
N.S.	1	0.79	0.69	1.26	9.18	2.88	0.00	1.64	1.39	0.00
time (sec)	N/A	0.294	0.045	0.812	0.206	0.104	0.000	0.230	0.174	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	35	59	33	0	42	23	51
N.S.	1	1.00	1.00	1.09	1.84	1.03	0.00	1.31	0.72	1.59
time (sec)	N/A	0.221	0.018	0.735	0.181	0.078	0.000	0.188	0.213	0.267

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	46	65	116	0	43	36	0
N.S.	1	1.00	1.00	1.39	1.97	3.52	0.00	1.30	1.09	0.00
time (sec)	N/A	0.206	0.011	0.742	0.184	0.106	0.000	0.162	0.166	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	26	101	22	37	7	27
N.S.	1	1.00	1.00	0.88	1.08	4.21	0.92	1.54	0.29	1.12
time (sec)	N/A	0.152	0.025	0.888	0.127	0.106	3.563	0.160	0.179	0.240

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	13	33	46	41	15	35
N.S.	1	1.00	1.00	0.91	0.41	1.03	1.44	1.28	0.47	1.09
time (sec)	N/A	0.197	0.104	0.724	0.172	0.080	6.978	0.174	0.178	9.629

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	48	45	45	25	165	107	113	27	44
N.S.	1	0.76	0.71	0.71	0.40	2.62	1.70	1.79	0.43	0.70
time (sec)	N/A	0.212	0.108	0.756	0.185	0.106	8.106	0.194	0.180	9.629

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	48	45	47	42	51	82	96	28	48
N.S.	1	0.69	0.64	0.67	0.60	0.73	1.17	1.37	0.40	0.69
time (sec)	N/A	0.230	0.065	0.755	0.183	0.084	26.002	0.256	0.198	9.526

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	60	50	94	670	207	0	118	100	0
N.S.	1	0.77	0.64	1.21	8.59	2.65	0.00	1.51	1.28	0.00
time (sec)	N/A	0.289	0.051	0.832	0.194	0.109	0.000	0.226	0.176	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	38	67	33	0	41	23	51
N.S.	1	1.00	0.91	1.09	1.91	0.94	0.00	1.17	0.66	1.46
time (sec)	N/A	0.222	0.027	0.783	0.185	0.077	0.000	0.259	0.176	0.258

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	49	65	116	0	45	36	0
N.S.	1	1.00	0.92	1.36	1.81	3.22	0.00	1.25	1.00	0.00
time (sec)	N/A	0.205	0.021	0.798	0.179	0.101	0.000	0.207	0.177	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	26	101	22	37	7	27
N.S.	1	1.00	1.00	0.78	0.96	3.74	0.81	1.37	0.26	1.00
time (sec)	N/A	0.149	0.029	0.887	0.129	0.106	5.486	0.164	0.176	0.242

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	13	33	46	41	15	39
N.S.	1	1.00	0.91	0.91	0.37	0.94	1.31	1.17	0.43	1.11
time (sec)	N/A	0.199	0.120	0.788	0.176	0.077	4.112	0.199	0.201	9.982

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	51	45	48	25	165	107	113	27	44
N.S.	1	0.74	0.65	0.70	0.36	2.39	1.55	1.64	0.39	0.64
time (sec)	N/A	0.211	0.124	0.804	0.178	0.116	7.911	0.219	0.276	9.606

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	51	45	50	42	51	82	67	28	48
N.S.	1	0.67	0.59	0.66	0.55	0.67	1.08	0.88	0.37	0.63
time (sec)	N/A	0.231	0.068	0.789	0.186	0.078	14.871	0.203	0.173	9.754

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	77	55	67	49	208	184	157	45	55
N.S.	1	0.72	0.51	0.63	0.46	1.94	1.72	1.47	0.42	0.51
time (sec)	N/A	0.283	0.188	0.858	0.188	0.128	114.570	0.208	0.176	10.360

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	60	53	95	688	207	0	95	100	0
N.S.	1	0.77	0.68	1.22	8.82	2.65	0.00	1.22	1.28	0.00
time (sec)	N/A	0.283	0.037	0.846	0.195	0.110	0.000	0.277	0.171	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	38	67	33	0	41	23	51
N.S.	1	1.00	0.91	1.09	1.91	0.94	0.00	1.17	0.66	1.46
time (sec)	N/A	0.211	0.025	0.796	0.180	0.078	0.000	0.235	0.170	0.255

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	49	65	116	0	43	36	0
N.S.	1	1.00	0.92	1.36	1.81	3.22	0.00	1.19	1.00	0.00
time (sec)	N/A	0.210	0.020	0.793	0.180	0.100	0.000	0.227	0.159	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	26	101	22	50	7	27
N.S.	1	1.00	1.00	0.78	0.96	3.74	0.81	1.85	0.26	1.00
time (sec)	N/A	0.151	0.029	0.888	0.127	0.111	51.402	0.229	0.157	9.961

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	13	33	46	40	15	39
N.S.	1	1.00	1.00	0.91	0.37	0.94	1.31	1.14	0.43	1.11
time (sec)	N/A	0.203	0.140	0.783	0.185	0.080	19.552	0.178	0.169	9.655

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	51	48	48	25	165	107	103	27	64
N.S.	1	0.74	0.70	0.70	0.36	2.39	1.55	1.49	0.39	0.93
time (sec)	N/A	0.212	0.111	0.818	0.202	0.111	10.556	0.181	0.168	10.043

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	51	48	50	42	51	82	96	28	48
N.S.	1	0.67	0.63	0.66	0.55	0.67	1.08	1.26	0.37	0.63
time (sec)	N/A	0.229	0.044	0.785	0.183	0.080	25.344	0.187	0.168	9.724

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	77	58	67	49	208	184	157	45	55
N.S.	1	0.72	0.54	0.63	0.46	1.94	1.72	1.47	0.42	0.51
time (sec)	N/A	0.285	0.138	0.856	0.184	0.121	109.395	0.212	0.157	9.777

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	14	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.295	0.052	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0	14	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.286	0.035	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.277	0.030	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.327	0.046	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	0	0	0	0	0	23	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.341	0.111	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	14	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.307	0.057	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	0	0	0	0	0	14	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.278	0.034	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0	14	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.256	0.001	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	0	0	21	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.318	0.002	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	23	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.297	0.006	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	14	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.288	0.042	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0	14	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.291	0.032	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.271	0.037	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.339	0.056	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	23	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.366	0.049	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	60	0	0	0	0	0	14	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.274	0.002	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	58	0	0	0	0	0	14	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.275	0.007	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	14	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.271	0.001	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.286	0.009	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	23	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.355	0.066	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	0	0	0	23	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.328	0.064	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	16	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.342	0.094	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	16	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.345	0.090	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	23	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.346	0.091	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	23	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.345	0.097	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	0	0	0	0	0	23	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.352	0.118	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	0	0	0	0	0	16	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.343	0.050	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	23	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.319	0.035	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	65	0	0	0	0	0	21	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.301	0.028	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0	14	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.290	0.031	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0	21	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.332	0.066	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.346	0.053	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	0	0	23	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.339	0.072	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	25	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.347	0.067	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	23	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.331	0.065	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	16	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.336	0.068	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	25	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.339	0.083	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	25	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.345	0.089	0.000	0.000	0.000	0.000	0.000	0.288	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	25	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.350	0.117	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	30	77
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.50	3.85
time (sec)	N/A	0.206	0.175	3.379	0.028	0.086	0.000	0.134	0.191	11.102

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	30	39
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.50	1.95
time (sec)	N/A	0.205	0.136	3.368	0.028	0.077	0.000	0.150	0.184	0.235

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	18	0	25	26	18
N.S.	1	1.00	1.00	0.94	1.28	1.00	0.00	1.39	1.44	1.00
time (sec)	N/A	0.198	0.125	0.282	0.027	0.075	0.000	0.147	0.180	0.092

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	23	0	22	19	23
N.S.	1	1.00	1.00	0.94	1.28	1.28	0.00	1.22	1.06	1.28
time (sec)	N/A	0.206	0.124	0.290	0.027	0.078	0.000	0.132	0.171	9.856

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	30	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.50	1.40
time (sec)	N/A	0.195	0.110	0.283	0.031	0.078	0.000	0.122	0.174	9.875

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	32	260	36	42	0	49	32	50
N.S.	1	0.98	0.78	6.34	0.88	1.02	0.00	1.20	0.78	1.22
time (sec)	N/A	0.241	0.349	59.284	0.035	0.087	0.000	0.158	0.191	10.129

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	42	35	38	44	0	49	32	95
N.S.	1	0.98	0.98	0.81	0.88	1.02	0.00	1.14	0.74	2.21
time (sec)	N/A	0.246	0.320	58.568	0.033	0.096	0.000	0.134	0.191	13.998

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	134	122	146	0	177	0	0	33	0
N.S.	1	1.05	0.95	1.14	0.00	1.38	0.00	0.00	0.26	0.00
time (sec)	N/A	0.764	16.354	1.576	0.000	0.139	0.000	0.000	0.181	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	53	0	67	0	0	33	85
N.S.	1	1.00	0.81	0.77	0.00	0.97	0.00	0.00	0.48	1.23
time (sec)	N/A	0.354	0.568	1.219	0.000	0.089	0.000	0.000	0.183	11.075

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	109	126	0	119	0	0	33	0
N.S.	1	1.00	1.17	1.35	0.00	1.28	0.00	0.00	0.35	0.00
time (sec)	N/A	0.527	1.009	1.380	0.000	0.094	0.000	0.000	0.194	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	33	0	36	0	0	29	36
N.S.	1	1.00	1.00	1.06	0.00	1.16	0.00	0.00	0.94	1.16
time (sec)	N/A	0.207	0.382	1.133	0.000	0.080	0.000	0.000	0.193	9.738

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	68	112	0	58	0	0	22	0
N.S.	1	1.00	1.28	2.11	0.00	1.09	0.00	0.00	0.42	0.00
time (sec)	N/A	0.387	0.806	1.336	0.000	0.093	0.000	0.000	0.181	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	171	121	363	0	446	0	0	33	0
N.S.	1	0.86	0.61	1.83	0.00	2.25	0.00	0.00	0.17	0.00
time (sec)	N/A	0.495	1.311	5.485	0.000	0.127	0.000	0.000	0.207	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	135	0	0	0	0	33	0
N.S.	1	1.00	0.86	1.45	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.511	0.881	2.486	0.000	0.000	0.000	0.000	0.266	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	249	212	157	437	0	500	0	0	33	0
N.S.	1	0.85	0.63	1.76	0.00	2.01	0.00	0.00	0.13	0.00
time (sec)	N/A	0.595	1.456	5.769	0.000	0.133	0.000	0.000	0.191	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	110	57	60	0	85	0	0	40	110
N.S.	1	1.06	0.55	0.58	0.00	0.82	0.00	0.00	0.38	1.06
time (sec)	N/A	0.457	0.716	1.550	0.000	0.121	0.000	0.000	0.207	11.431

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	167	114	231	0	264	0	0	40	0
N.S.	1	1.01	0.69	1.39	0.00	1.59	0.00	0.00	0.24	0.00
time (sec)	N/A	0.860	11.201	1.747	0.000	0.100	0.000	0.000	0.207	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	48	0	58	0	0	40	61
N.S.	1	1.00	0.65	0.70	0.00	0.84	0.00	0.00	0.58	0.88
time (sec)	N/A	0.329	0.462	1.457	0.000	0.087	0.000	0.000	0.226	9.991

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	126	99	217	0	158	0	0	36	0
N.S.	1	1.01	0.79	1.74	0.00	1.26	0.00	0.00	0.29	0.00
time (sec)	N/A	0.688	0.703	1.602	0.000	0.091	0.000	0.000	0.210	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	33	0	36	0	0	29	36
N.S.	1	1.00	1.00	1.06	0.00	1.16	0.00	0.00	0.94	1.16
time (sec)	N/A	0.213	0.320	1.408	0.000	0.114	0.000	0.000	0.193	9.522

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	222	0	155	0	0	40	0
N.S.	1	1.00	0.74	2.49	0.00	1.74	0.00	0.00	0.45	0.00
time (sec)	N/A	0.522	0.555	1.614	0.000	0.091	0.000	0.000	0.186	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	249	211	142	873	0	508	0	0	40	0
N.S.	1	0.85	0.57	3.51	0.00	2.04	0.00	0.00	0.16	0.00
time (sec)	N/A	0.612	1.573	6.460	0.000	0.141	0.000	0.000	0.195	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	69	236	0	0	0	0	40	0
N.S.	1	1.00	0.73	2.51	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.529	0.713	1.714	0.000	0.000	0.000	0.000	0.213	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	173	92	165	0	225	0	0	44	0
N.S.	1	1.04	0.55	0.99	0.00	1.36	0.00	0.00	0.27	0.00
time (sec)	N/A	0.865	1.514	1.138	0.000	0.100	0.000	0.000	0.214	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	110	57	68	0	89	0	0	44	112
N.S.	1	1.04	0.54	0.64	0.00	0.84	0.00	0.00	0.42	1.06
time (sec)	N/A	0.471	0.492	0.763	0.000	0.103	0.000	0.000	0.226	11.448

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	132	87	145	0	160	0	0	44	0
N.S.	1	1.01	0.66	1.11	0.00	1.22	0.00	0.00	0.34	0.00
time (sec)	N/A	0.692	0.855	0.848	0.000	0.100	0.000	0.000	0.211	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	46	0	58	0	0	40	64
N.S.	1	1.00	0.65	0.67	0.00	0.84	0.00	0.00	0.58	0.93
time (sec)	N/A	0.332	0.500	1.487	0.000	0.093	0.000	0.000	0.209	10.386

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	68	126	0	119	0	0	33	0
N.S.	1	1.00	0.73	1.35	0.00	1.28	0.00	0.00	0.35	0.00
time (sec)	N/A	0.516	0.701	1.717	0.000	0.092	0.000	0.000	0.224	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	0	58	0	0	44	66
N.S.	1	1.00	1.00	1.06	0.00	1.76	0.00	0.00	1.33	2.00
time (sec)	N/A	0.217	0.349	1.485	0.000	0.083	0.000	0.000	0.245	10.001

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	70	134	0	122	0	0	44	0
N.S.	1	1.00	0.71	1.37	0.00	1.24	0.00	0.00	0.45	0.00
time (sec)	N/A	0.541	0.740	1.638	0.000	0.090	0.000	0.000	0.183	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	250	213	143	512	0	554	0	0	44	0
N.S.	1	0.85	0.57	2.05	0.00	2.22	0.00	0.00	0.18	0.00
time (sec)	N/A	0.613	1.393	5.875	0.000	0.145	0.000	0.000	0.171	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	56	0	79	0	0	44	99
N.S.	1	1.00	0.65	0.81	0.00	1.14	0.00	0.00	0.64	1.43
time (sec)	N/A	0.324	0.645	1.238	0.000	0.104	0.000	0.000	0.173	10.838

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	130	104	222	0	248	0	0	44	0
N.S.	1	1.02	0.81	1.73	0.00	1.94	0.00	0.00	0.34	0.00
time (sec)	N/A	0.686	1.114	1.419	0.000	0.099	0.000	0.000	0.196	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	0	51	0	0	44	49
N.S.	1	1.00	1.00	1.06	0.00	1.55	0.00	0.00	1.33	1.48
time (sec)	N/A	0.218	0.446	1.168	0.000	0.080	0.000	0.000	0.175	9.988

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	80	364	0	150	0	0	40	0
N.S.	1	1.00	0.90	4.09	0.00	1.69	0.00	0.00	0.45	0.00
time (sec)	N/A	0.529	0.659	1.497	0.000	0.090	0.000	0.000	0.175	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	171	123	355	0	462	0	0	33	0
N.S.	1	0.87	0.62	1.80	0.00	2.35	0.00	0.00	0.17	0.00
time (sec)	N/A	0.449	1.173	6.305	0.000	0.118	0.000	0.000	0.170	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	236	0	0	0	0	44	0
N.S.	1	1.00	1.25	4.45	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.368	0.409	1.373	0.000	0.000	0.000	0.000	0.175	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	249	212	156	430	0	523	0	0	44	0
N.S.	1	0.85	0.63	1.73	0.00	2.10	0.00	0.00	0.18	0.00
time (sec)	N/A	0.603	1.443	6.513	0.000	0.166	0.000	0.000	0.199	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	84	249	0	0	0	0	44	0
N.S.	1	1.00	0.88	2.62	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.529	0.569	2.645	0.000	0.000	0.000	0.000	0.173	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	57	61	0	88	0	0	44	125
N.S.	1	1.05	0.52	0.55	0.00	0.80	0.00	0.00	0.40	1.14
time (sec)	N/A	0.507	1.052	2.338	0.000	0.116	0.000	0.000	0.181	12.610

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	139	119	145	0	183	0	0	44	0
N.S.	1	1.03	0.88	1.07	0.00	1.36	0.00	0.00	0.33	0.00
time (sec)	N/A	0.692	1.553	2.650	0.000	0.099	0.000	0.000	0.168	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	40	0	59	0	0	44	70
N.S.	1	1.00	1.36	1.21	0.00	1.79	0.00	0.00	1.33	2.12
time (sec)	N/A	0.219	0.591	2.168	0.000	0.089	0.000	0.000	0.167	10.402

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	105	129	0	122	0	0	44	0
N.S.	1	1.00	1.07	1.32	0.00	1.24	0.00	0.00	0.45	0.00
time (sec)	N/A	0.533	0.946	2.587	0.000	0.095	0.000	0.000	0.171	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	248	211	160	455	0	492	0	0	40	0
N.S.	1	0.85	0.65	1.83	0.00	1.98	0.00	0.00	0.16	0.00
time (sec)	N/A	0.614	0.850	10.618	0.000	0.129	0.000	0.000	0.192	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	124	0	0	0	0	33	0
N.S.	1	1.00	0.91	1.35	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.514	0.728	4.597	0.000	0.000	0.000	0.000	0.214	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	249	212	157	430	0	510	0	0	44	0
N.S.	1	0.85	0.63	1.73	0.00	2.05	0.00	0.00	0.18	0.00
time (sec)	N/A	0.605	1.275	10.816	0.000	0.123	0.000	0.000	0.193	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	89	998	0	0	0	0	44	0
N.S.	1	1.00	0.66	7.39	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.694	0.715	5.862	0.000	0.000	0.000	0.000	0.175	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	295	255	168	455	0	525	0	0	44	0
N.S.	1	0.86	0.57	1.54	0.00	1.78	0.00	0.00	0.15	0.00
time (sec)	N/A	0.777	1.586	11.237	0.000	0.136	0.000	0.000	0.178	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	40	0	67	0	0	44	93
N.S.	1	1.00	1.36	1.21	0.00	2.03	0.00	0.00	1.33	2.82
time (sec)	N/A	0.224	0.910	1.540	0.000	0.122	0.000	0.000	0.207	10.784

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	101	228	0	254	0	0	44	0
N.S.	1	1.00	0.75	1.69	0.00	1.88	0.00	0.00	0.33	0.00
time (sec)	N/A	0.705	1.738	1.666	0.000	0.099	0.000	0.000	0.171	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	213	154	437	0	561	0	0	44	0
N.S.	1	0.85	0.61	1.74	0.00	2.24	0.00	0.00	0.18	0.00
time (sec)	N/A	0.612	1.633	6.994	0.000	0.145	0.000	0.000	0.171	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	80	219	0	0	0	0	40	0
N.S.	1	1.00	0.85	2.33	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.522	0.824	1.556	0.000	0.000	0.000	0.000	0.176	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	249	212	152	432	0	511	0	0	33	0
N.S.	1	0.85	0.61	1.73	0.00	2.05	0.00	0.00	0.13	0.00
time (sec)	N/A	0.595	1.676	7.895	0.000	0.133	0.000	0.000	0.183	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	249	0	0	0	0	44	0
N.S.	1	1.00	0.83	2.62	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.511	0.541	1.778	0.000	0.000	0.000	0.000	0.185	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	295	255	165	455	0	537	0	0	44	0
N.S.	1	0.86	0.56	1.54	0.00	1.82	0.00	0.00	0.15	0.00
time (sec)	N/A	0.765	1.766	9.569	0.000	0.145	0.000	0.000	0.178	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	138	90	262	0	0	0	0	44	0
N.S.	1	1.02	0.67	1.94	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.690	0.858	1.944	0.000	0.000	0.000	0.000	0.201	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	298	175	449	0	547	0	0	44	0
N.S.	1	0.90	0.53	1.36	0.00	1.66	0.00	0.00	0.13	0.00
time (sec)	N/A	1.004	1.955	9.949	0.000	0.165	0.000	0.000	0.181	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	278	0	0	0	0	0	19	0
N.S.	1	1.00	3.43	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.354	1.756	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	280	0	0	0	0	0	23	0
N.S.	1	1.00	3.26	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.362	0.836	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	281	0	0	0	0	0	23	0
N.S.	1	1.00	3.35	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.358	0.685	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	283	0	0	0	0	0	26	0
N.S.	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.351	0.751	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	23	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.231	0.042	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	0	0	0	0	0	23	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.233	0.028	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	21	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.225	0.024	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	31	29	29	0	0	21	28
N.S.	1	1.00	0.96	1.29	1.21	1.21	0.00	0.00	0.88	1.17
time (sec)	N/A	0.201	0.018	1.934	0.035	0.079	0.000	0.000	0.190	9.658

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	45	67	58	49	0	0	23	66
N.S.	1	0.96	0.87	1.29	1.12	0.94	0.00	0.00	0.44	1.27
time (sec)	N/A	0.250	0.094	3.298	0.035	0.080	0.000	0.000	0.180	9.970

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	81	104	86	73	0	0	23	134
N.S.	1	0.97	1.04	1.33	1.10	0.94	0.00	0.00	0.29	1.72
time (sec)	N/A	0.258	0.379	3.521	0.042	0.095	0.000	0.000	0.182	10.649

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0	23	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.337	1.213	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0	23	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.327	0.991	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	98	0	0	0	0	0	23	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.316	1.728	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	0	0	0	0	0	14	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.291	0.094	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	165	0	0	0	0	0	23	0
N.S.	1	1.00	2.29	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.315	0.606	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	246	0	0	0	0	0	23	0
N.S.	1	1.00	3.42	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.315	0.858	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	0	0	0	0	0	31	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.383	10.509	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	90	0	0	0	0	0	24	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.363	9.774	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	95	0	0	0	0	0	35	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.363	27.049	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0	35	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.389	11.104	0.000	0.000	0.000	0.000	0.000	0.277	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [55] had the largest ratio of [1.39999999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	6	0.333
2	A	4	3	1.00	8	0.375
3	A	4	4	1.00	8	0.500
4	A	4	3	1.00	8	0.375
5	A	6	6	1.09	8	0.750
6	A	4	3	0.93	8	0.375
7	A	8	8	1.13	8	1.000
8	A	4	3	0.91	8	0.375
9	A	8	8	1.01	10	0.800
10	A	6	6	1.00	10	0.600
11	A	6	6	1.00	10	0.600
12	A	4	4	1.00	10	0.400
13	A	4	4	1.00	10	0.400
14	A	6	6	1.00	10	0.600
15	A	6	6	1.00	10	0.600
16	A	8	8	1.06	10	0.800
17	A	8	8	1.02	12	0.667
18	A	6	6	1.00	12	0.500
19	A	6	6	1.00	12	0.500
20	A	4	4	1.00	12	0.333
21	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	6	1.00	12	0.500
23	A	6	6	1.00	12	0.500
24	A	8	8	1.08	12	0.667
25	A	4	4	1.00	10	0.400
26	A	4	4	1.00	10	0.400
27	A	4	4	1.00	10	0.400
28	A	4	4	1.00	10	0.400
29	A	4	4	1.00	10	0.400
30	A	4	4	1.00	10	0.400
31	A	4	4	1.00	12	0.333
32	A	4	4	1.00	12	0.333
33	A	4	4	1.00	12	0.333
34	A	4	4	1.00	12	0.333
35	A	4	4	1.00	12	0.333
36	A	4	4	1.00	12	0.333
37	A	4	4	1.00	8	0.500
38	A	4	4	1.00	10	0.400
39	A	7	6	1.32	8	0.750
40	A	6	5	1.25	8	0.625
41	A	5	4	1.09	8	0.500
42	A	4	3	1.00	8	0.375
43	A	4	3	1.18	8	0.375
44	A	5	4	1.14	8	0.500
45	A	6	5	1.26	8	0.625
46	A	7	6	1.32	8	0.750
47	A	8	7	1.18	10	0.700
48	A	7	6	1.15	10	0.600
49	A	6	5	1.11	10	0.500
50	A	5	4	1.08	10	0.400
51	A	4	3	1.15	10	0.300
52	A	5	4	1.25	10	0.400
53	A	6	5	1.35	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	6	1.39	10	0.600
55	A	14	14	0.88	10	1.400
56	A	10	10	1.09	10	1.000
57	A	8	8	1.17	10	0.800
58	A	8	8	1.20	10	0.800
59	A	10	10	1.00	10	1.000
60	A	14	14	0.91	10	1.400
61	A	6	5	0.42	10	0.500
62	A	6	5	0.48	10	0.500
63	A	6	5	0.62	10	0.500
64	A	6	5	1.00	10	0.500
65	A	5	5	0.81	10	0.500
66	A	9	9	0.72	10	0.900
67	A	13	13	0.70	10	1.300
68	A	6	6	1.00	12	0.500
69	A	6	6	1.00	14	0.429
70	A	9	9	1.11	21	0.429
71	A	9	9	1.09	21	0.429
72	A	7	7	1.07	21	0.333
73	A	7	7	1.11	19	0.368
74	A	4	4	1.00	12	0.333
75	A	5	5	1.00	19	0.263
76	A	7	7	1.13	21	0.333
77	A	7	7	1.09	21	0.333
78	A	9	9	1.18	21	0.429
79	A	9	9	1.14	21	0.429
80	A	9	9	1.14	21	0.429
81	A	9	9	1.06	21	0.429
82	A	7	7	1.10	19	0.368
83	A	6	6	1.00	12	0.500
84	A	5	5	1.00	19	0.263
85	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	7	7	1.09	21	0.333
87	A	7	7	1.06	21	0.333
88	A	9	9	1.14	21	0.429
89	A	9	9	1.12	21	0.429
90	A	9	9	1.10	21	0.429
91	A	9	9	1.07	19	0.474
92	A	6	6	1.00	12	0.500
93	A	7	7	1.00	19	0.368
94	A	5	5	1.00	21	0.238
95	A	5	5	1.00	21	0.238
96	A	7	7	1.06	21	0.333
97	A	7	7	1.06	21	0.333
98	A	9	9	1.12	21	0.429
99	A	9	9	1.12	21	0.429
100	A	8	8	1.02	12	0.667
101	A	9	9	1.08	21	0.429
102	A	9	9	1.07	21	0.429
103	A	7	7	1.03	21	0.333
104	A	7	7	1.08	21	0.333
105	A	5	5	1.00	19	0.263
106	A	4	4	1.00	12	0.333
107	A	7	7	1.07	19	0.368
108	A	7	7	1.13	21	0.333
109	A	9	9	1.15	21	0.429
110	A	9	9	1.18	21	0.429
111	A	9	9	1.08	21	0.429
112	A	9	9	1.04	21	0.429
113	A	7	7	1.03	21	0.333
114	A	7	7	1.03	21	0.333
115	A	5	5	1.00	21	0.238
116	A	5	5	1.00	19	0.263
117	A	6	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	7	1.07	19	0.368
119	A	9	9	1.14	21	0.429
120	A	9	9	1.15	21	0.429
121	A	9	9	1.08	21	0.429
122	A	9	9	1.04	21	0.429
123	A	7	7	1.03	21	0.333
124	A	7	7	1.03	21	0.333
125	A	5	5	1.00	21	0.238
126	A	5	5	1.00	21	0.238
127	A	7	7	1.06	19	0.368
128	A	6	6	1.00	12	0.500
129	A	9	9	1.13	19	0.474
130	A	9	9	1.14	21	0.429
131	A	8	8	1.08	12	0.667
132	A	7	7	0.78	23	0.304
133	A	5	4	0.69	23	0.174
134	A	5	5	0.79	23	0.217
135	A	5	4	1.00	23	0.174
136	A	3	3	1.00	23	0.130
137	A	2	2	1.00	23	0.087
138	A	3	3	1.00	23	0.130
139	A	4	4	0.76	23	0.174
140	A	5	4	0.69	23	0.174
141	A	6	6	0.76	23	0.261
142	A	7	7	0.76	23	0.304
143	A	5	4	0.68	23	0.174
144	A	5	5	0.78	23	0.217
145	A	5	4	1.00	23	0.174
146	A	3	3	1.00	23	0.130
147	A	2	2	1.00	23	0.087
148	A	3	3	1.00	23	0.130
149	A	4	4	0.75	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	5	4	0.68	23	0.174
151	A	6	6	0.74	23	0.261
152	A	5	4	0.54	23	0.174
153	A	5	4	0.67	23	0.174
154	A	5	5	0.77	23	0.217
155	A	5	4	1.00	23	0.174
156	A	3	3	1.00	23	0.130
157	A	2	2	1.00	23	0.087
158	A	3	3	1.00	23	0.130
159	A	4	4	0.74	23	0.174
160	A	5	4	0.67	23	0.174
161	A	5	5	0.79	23	0.217
162	A	5	4	1.00	23	0.174
163	A	3	3	1.00	23	0.130
164	A	2	2	1.00	23	0.087
165	A	3	3	1.00	23	0.130
166	A	4	4	0.76	23	0.174
167	A	5	4	0.69	23	0.174
168	A	5	5	0.77	23	0.217
169	A	5	4	1.00	23	0.174
170	A	3	3	1.00	23	0.130
171	A	2	2	1.00	23	0.087
172	A	3	3	1.00	23	0.130
173	A	4	4	0.74	23	0.174
174	A	5	4	0.67	23	0.174
175	A	6	6	0.72	23	0.261
176	A	5	5	0.77	23	0.217
177	A	5	4	1.00	23	0.174
178	A	3	3	1.00	23	0.130
179	A	2	2	1.00	23	0.087
180	A	3	3	1.00	23	0.130
181	A	4	4	0.74	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	5	4	0.67	23	0.174
183	A	6	6	0.72	23	0.261
184	A	5	5	1.00	21	0.238
185	A	5	5	1.00	19	0.263
186	A	4	4	1.00	12	0.333
187	A	5	5	1.00	19	0.263
188	A	5	5	1.00	21	0.238
189	A	5	5	1.00	21	0.238
190	A	5	5	1.00	19	0.263
191	A	4	4	1.00	12	0.333
192	A	5	5	1.00	19	0.263
193	A	5	5	1.00	21	0.238
194	A	5	5	1.00	21	0.238
195	A	5	5	1.00	19	0.263
196	A	4	4	1.00	12	0.333
197	A	5	5	1.00	19	0.263
198	A	5	5	1.00	21	0.238
199	A	5	5	1.00	21	0.238
200	A	5	5	1.00	19	0.263
201	A	4	4	1.00	12	0.333
202	A	5	5	1.00	19	0.263
203	A	5	5	1.00	21	0.238
204	A	5	5	1.00	21	0.238
205	A	5	5	1.00	21	0.238
206	A	5	5	1.00	21	0.238
207	A	5	5	1.00	21	0.238
208	A	5	5	1.00	21	0.238
209	A	5	5	1.00	21	0.238
210	A	5	5	1.00	19	0.263
211	A	5	5	1.00	19	0.263
212	A	5	5	1.00	17	0.294
213	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	5	5	1.00	17	0.294
215	A	5	5	1.00	19	0.263
216	A	5	5	1.00	19	0.263
217	A	5	5	1.00	21	0.238
218	A	5	5	1.00	21	0.238
219	A	5	5	1.00	21	0.238
220	A	5	5	1.00	21	0.238
221	A	5	5	1.00	21	0.238
222	A	5	5	1.00	21	0.238
223	A	4	3	1.00	19	0.158
224	A	4	3	1.00	19	0.158
225	A	4	3	1.00	19	0.158
226	A	4	3	1.00	19	0.158
227	A	4	3	1.00	19	0.158
228	A	7	6	0.98	21	0.286
229	A	7	6	0.98	21	0.286
230	A	10	10	1.05	25	0.400
231	A	4	4	1.00	25	0.160
232	A	8	8	1.00	25	0.320
233	A	2	2	1.00	25	0.080
234	A	6	6	1.00	25	0.240
235	A	14	13	0.86	25	0.520
236	A	8	8	1.00	25	0.320
237	A	16	15	0.85	25	0.600
238	A	6	6	1.06	25	0.240
239	A	12	12	1.01	25	0.480
240	A	4	4	1.00	25	0.160
241	A	10	10	1.01	25	0.400
242	A	2	2	1.00	25	0.080
243	A	8	8	1.00	25	0.320
244	A	16	15	0.85	25	0.600
245	A	8	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	12	12	1.04	25	0.480
247	A	6	6	1.04	25	0.240
248	A	10	10	1.01	25	0.400
249	A	4	4	1.00	25	0.160
250	A	8	8	1.00	25	0.320
251	A	2	2	1.00	25	0.080
252	A	8	8	1.00	25	0.320
253	A	16	15	0.85	25	0.600
254	A	4	4	1.00	25	0.160
255	A	10	10	1.02	25	0.400
256	A	2	2	1.00	25	0.080
257	A	8	8	1.00	25	0.320
258	A	14	13	0.87	25	0.520
259	A	6	6	1.00	25	0.240
260	A	16	15	0.85	25	0.600
261	A	8	8	1.00	25	0.320
262	A	6	6	1.05	25	0.240
263	A	10	10	1.03	25	0.400
264	A	2	2	1.00	25	0.080
265	A	8	8	1.00	25	0.320
266	A	16	15	0.85	25	0.600
267	A	8	8	1.00	25	0.320
268	A	16	15	0.85	25	0.600
269	A	10	10	1.00	25	0.400
270	A	18	17	0.86	25	0.680
271	A	2	2	1.00	25	0.080
272	A	10	10	1.00	25	0.400
273	A	16	15	0.85	25	0.600
274	A	8	8	1.00	25	0.320
275	A	16	15	0.85	25	0.600
276	A	8	8	1.00	25	0.320
277	A	18	17	0.86	25	0.680

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	10	10	1.02	25	0.400
279	A	20	19	0.90	25	0.760
280	A	4	4	1.00	17	0.235
281	A	4	4	1.00	19	0.211
282	A	4	4	1.00	19	0.211
283	A	4	4	1.00	21	0.190
284	A	6	5	1.00	19	0.263
285	A	5	4	1.00	19	0.211
286	A	6	5	1.00	17	0.294
287	A	4	3	1.00	17	0.176
288	A	7	6	0.96	19	0.316
289	A	6	5	0.97	19	0.263
290	A	4	4	1.00	19	0.211
291	A	4	4	1.00	19	0.211
292	A	4	4	1.00	19	0.211
293	A	4	4	1.00	10	0.400
294	A	4	4	1.00	19	0.211
295	A	4	4	1.00	19	0.211
296	A	4	4	1.00	23	0.174
297	A	4	4	1.00	23	0.174
298	A	4	4	1.00	23	0.174
299	A	4	4	1.00	23	0.174

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sec(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \sec(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b}$$

output
```
arctanh(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\,dx = \frac{\coth^{-1}(\sin(a+bx))}{b}$$

input
```
Integrate[Sec[a + b*x],x]
```

output
```
ArcCoth[Sin[a + b*x]]/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(a+bx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{4257}$$
$$\frac{\text{arctanh}(\sin(a+bx))}{b}$$

input
```
Int[Sec[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

method	result	size
derivativedivides	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
default	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	32
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	35
risch	$-\frac{\ln(e^{i(bx+a)}-i)}{b}+\frac{\ln(e^{i(bx+a)}+i)}{b}$	37

input
```
int(sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \sec(a+bx)\,dx = \frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

input
```
integrate(sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(8) = 16$.

Time = 0.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.27

$$\int \sec(a+bx)\,dx = \begin{cases} \frac{\log(\tan(a+bx)+\sec(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x(\tan(a)\sec(a)+\sec^2(a))}{\tan(a)+\sec(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a),x)
```

output
```
Piecewise((log(tan(a + b*x) + sec(a + b*x))/b, Ne(b, 0)), (x*(tan(a)*sec(a
) + sec(a)**2)/(tan(a) + sec(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \sec(a+bx)\,dx = \frac{\log(\sec(bx+a)+\tan(bx+a))}{b}$$

input
```
integrate(sec(b*x+a),x, algorithm="maxima")
```

output
```
log(sec(b*x + a) + tan(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.00

$$\int \sec(a+bx)\,dx = \frac{\log\left(\left|\frac{1}{\sin(bx+a)}+\sin(bx+a)+2\right|\right)-\log\left(\left|\frac{1}{\sin(bx+a)}+\sin(bx+a)-2\right|\right)}{4b}$$

input
```
integrate(sec(b*x+a),x, algorithm="giac")
```

output
```
1/4*(log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) - log(abs(1/sin(b*x + a)
+ sin(b*x + a) - 2)))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\,dx = \frac{\operatorname{atanh}(\sin{(a+b\,x)})}{b}$$

input
```
int(1/cos(a + b*x),x)
```

output
```
atanh(sin(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \sec(a+bx)\,dx = \frac{-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$$

input
```
int(sec(b*x+a),x)
```

output
```
( - log(tan((a + b*x)/2) - 1) + log(tan((a + b*x)/2) + 1))/b
```

3.2 $\int \sec^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \sec^2(a+bx)\,dx = \frac{\tan(a+bx)}{b}$$

output
```
tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan(a+bx)}{b}$$

input
```
Integrate[Sec[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{ 4254}$$
$$-\frac{\int 1d(-\tan(a+bx))}{b}$$
$$\Big\downarrow \text{ 24}$$
$$\frac{\tan(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^2,x]
```

output
```
Tan[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\tan(bx+a)}{b}$	11
default	$\frac{\tan(bx+a)}{b}$	11
parallelrisch	$\frac{\sin(bx+a)}{\cos(bx+a)b}$	19
risch	$\frac{2i}{b(1+e^{2i(bx+a)})}$	20
norman	$-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}$	30

input
```
int(sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
tan(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \sec^2(a+bx)\,dx = \frac{\sin(bx+a)}{b\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F]

$$\int \sec^2(a+bx)\,dx = \int \sec^2{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**2,x)
```

output
```
Integral(sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan{(bx+a)}}{b}$$

input
```
integrate(sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
tan(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan{(bx+a)}}{b}$$

input
```
integrate(sec(b*x+a)^2,x, algorithm="giac")
```

output
```
tan(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\,dx = \frac{\tan(a+b\,x)}{b}$$

input
```
int(1/cos(a + b*x)^2,x)
```

output
```
tan(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \sec^2(a+bx)\,dx = \frac{\sin(bx+a)}{\cos(bx+a)\,b}$$

input
```
int(sec(b*x+a)^2,x)
```

output
```
sin(a + b*x)/(cos(a + b*x)*b)
```

3.3 $\int \sec^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \sec^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{2b} + \frac{\sec(a+bx)\tan(a+bx)}{2b}$$

output
```
1/2*arctanh(sin(b*x+a))/b+1/2*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{2b} + \frac{\sec(a+bx)\tan(a+bx)}{2b}$$

input
```
Integrate[Sec[a + b*x]^3,x]
```

output
```
ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[Sec[a + b*x]^3,x]
```

output
```
ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)\tan(bx+a)}{2}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	36
default	$\frac{\frac{\sec(bx+a)\tan(bx+a)}{2}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	36
risch	$-\frac{i\left(e^{3i(bx+a)}-e^{i(bx+a)}\right)}{b\left(1+e^{2i(bx+a)}\right)^{2}}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}-i\right)}{2b}$	78
parallelrisch	$\frac{(-1-\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(1+\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+2\sin(bx+a)}{2b(1+\cos(2bx+2a))}$	78
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}}{b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{2}}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2b}$	81

input
```
int(sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sec(b*x+a)*tan(b*x+a)+1/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \sec^3(a+bx)\,dx = \frac{\cos(bx+a)^2\log(\sin(bx+a)+1) - \cos(bx+a)^2\log(-\sin(bx+a)+1) + 2\sin(bx+a)}{4\,b\cos(bx+a)^2}$$

input
```
integrate(sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
a) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \sec^3(a+bx)\,dx = \int \sec^3(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**3,x)
```

output
```
Integral(sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \sec^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} - \log(\sin(bx+a)+1) + \log(\sin(bx+a)-1)}{4\,b}$$

input
```
integrate(sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(si
n(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \sec^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} - \log\left(|\sin\left(bx+a\right)+1|\right) + \log\left(|\sin\left(bx+a\right)-1|\right)}{4\,b}$$

input
```
integrate(sec(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(abs(sin(b*x + a) + 1)) + l
og(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \sec^3(a+bx)\,dx = \frac{\mathrm{atanh}(\sin\left(a+b\,x\right))}{2\,b} - \frac{\sin\left(a+b\,x\right)}{2\,b\,\left(\sin\left(a+b\,x\right)^2-1\right)}$$

input
```
int(1/cos(a + b*x)^3,x)
```

output
```
atanh(sin(a + b*x))/(2*b) - sin(a + b*x)/(2*b*(sin(a + b*x)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.79

$$\int \sec^3(a+bx)\,dx$$

$$= \frac{-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^2+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin\left(bx+a\right)^2-\mathrm{lo}}{2b\left(\sin\left(bx+a\right)^2-1\right)}$$

input
```
int(sec(b*x+a)^3,x)
```

output
```
( - log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 + log(tan((a + b*x)/2) - 1)
+ log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 - log(tan((a + b*x)/2) + 1) -
sin(a + b*x))/(2*b*(sin(a + b*x)**2 - 1))
```

3.4 $\int \sec^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \sec^4(a+bx)\,dx = \frac{\tan(a+bx)}{b} + \frac{\tan^3(a+bx)}{3b}$$

output
```
tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \sec^4(a+bx)\,dx = \frac{\tan(a+bx) + \frac{1}{3}\tan^3(a+bx)}{b}$$

input
```
Integrate[Sec[a + b*x]^4,x]
```

output
```
(Tan[a + b*x] + Tan[a + b*x]^3/3)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^2(a+bx)+1\right)d(-\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}\tan^3(a+bx)-\tan(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^4,x]
```

output
```
-((-Tan[a + b*x] - Tan[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\left(-\frac{2}{3}-\frac{\sec(bx+a)^2}{3}\right)\tan(bx+a)}{b}$	24
default	$-\frac{\left(-\frac{2}{3}-\frac{\sec(bx+a)^2}{3}\right)\tan(bx+a)}{b}$	24
risch	$\frac{4i\left(3\,\mathrm{e}^{2i(bx+a)}+1\right)}{3b\left(1+\mathrm{e}^{2i(bx+a)}\right)^3}$	33
norman	$\frac{-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3b}-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^3}$	64
parallelrisch	$\frac{-6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5+4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3-6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{3b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$	70

input
```
int(sec(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-2/3-1/3*sec(b*x+a)^2)*tan(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \sec^4(a+bx)\,dx=\frac{\left(2\cos(bx+a)^2+1\right)\sin(bx+a)}{3\,b\cos(bx+a)^3}$$

input
```
integrate(sec(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(2*cos(b*x + a)^2 + 1)*sin(b*x + a)/(b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \sec^4(a+bx)\,dx = \int \sec^4{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**4,x)
```

output
```
Integral(sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \sec^4(a+bx)\,dx = \frac{\tan{(bx+a)}^3 + 3\,\tan{(bx+a)}}{3\,b}$$

input
```
integrate(sec(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/3*(tan(b*x + a)^3 + 3*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \sec^4(a+bx)\,dx = \frac{\tan{(bx+a)}^3 + 3\,\tan{(bx+a)}}{3\,b}$$

input
```
integrate(sec(b*x+a)^4,x, algorithm="giac")
```

output
```
1/3*(tan(b*x + a)^3 + 3*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\,dx=\frac{\tan(a+b\,x)\,\left(\tan(a+b\,x)^2+3\right)}{3\,b}$$

input
```
int(1/cos(a + b*x)^4,x)
```

output
```
(tan(a + b*x)*(tan(a + b*x)^2 + 3))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \sec^4(a+bx)\,dx=\frac{\sin(bx+a)\left(2\sin(bx+a)^2-3\right)}{3\cos(bx+a)\,b\left(\sin(bx+a)^2-1\right)}$$

input
```
int(sec(b*x+a)^4,x)
```

output
```
(sin(a + b*x)*(2*sin(a + b*x)**2 - 3))/(3*cos(a + b*x)*b*(sin(a + b*x)**2
- 1))
```

3.5 $\int \sec^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \sec^5(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{8b} + \frac{3\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

output
```
3/8*arctanh(sin(b*x+a))/b+3/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)^3*tan
(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \sec^5(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{8b} + \frac{3\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

input
```
Integrate[Sec[a + b*x]^5,x]
```

output
```
(3*ArcTanh[Sin[a + b*x]])/(8*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (S
ec[a + b*x]^3*Tan[a + b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\int \sec^3(a+bx)dx+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{4}\left(\frac{\text{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

input
```
Int[Sec[a + b*x]^5,x]
```

output
```
(Sec[a + b*x]^3*Tan[a + b*x])/(4*b) + (3*(ArcTanh[Sin[a + b*x]]/(2*b) + (S
ec[a + b*x]*Tan[a + b*x])/(2*b)))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\left(-\frac{\sec(bx+a)^3}{4}-\frac{3\sec(bx+a)}{8}\right)\tan(bx+a)+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{-\left(-\frac{\sec(bx+a)^3}{4}-\frac{3\sec(bx+a)}{8}\right)\tan(bx+a)+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$-\frac{i\left(3\,e^{7i(bx+a)}+11\,e^{5i(bx+a)}-11\,e^{3i(bx+a)}-3\,e^{i(bx+a)}\right)}{4b\left(1+e^{2i(bx+a)}\right)^4}-\frac{3\ln\left(e^{i(bx+a)}-i\right)}{8b}+\frac{3\ln\left(e^{i(bx+a)}+i\right)}{8b}$
norman	$\frac{\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{4b}+\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^4}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{8b}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{8b}$
parallelrisch	$\frac{(-12\cos(2bx+2a)-3\cos(4bx+4a)-9)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(3\cos(4bx+4a)+12\cos(2bx+2a)+9)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right.}{8b(3+\cos(4bx+4a)+4\cos(2bx+2a))}$

input
```
int(sec(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-(-1/4*sec(b*x+a)^3-3/8*sec(b*x+a))*tan(b*x+a)+3/8*ln(sec(b*x+a)+tan(
b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \sec^5(a+bx)\,dx = \frac{3\cos(bx+a)^4\log(\sin(bx+a)+1) - 3\cos(bx+a)^4\log(-\sin(bx+a)+1) + 2\left(3\cos(bx+a)^2+2\right)}{16\,b\cos(bx+a)^4}$$

input
```
integrate(sec(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/16*(3*cos(b*x + a)^4*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^4*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^2 + 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \sec^5(a+bx)\,dx = \int \sec^5(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**5,x)
```

output
```
Integral(sec(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \sec^5(a+bx)\,dx$$
$$= -\frac{\frac{2\left(3\sin(bx+a)^3-5\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(\sin\left(bx+a\right)-1\right)}{16\,b}$$

input
```
integrate(sec(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*sin(b*x + a)^3 - 5*sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x +
 a)^2 + 1) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \sec^5(a+bx)\,dx$$
$$= -\frac{\frac{2\left(3\sin(bx+a)^3-5\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2} - 3\log\left(\left|\sin\left(bx+a\right)+1\right|\right) + 3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b}$$

input
```
integrate(sec(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/16*(2*(3*sin(b*x + a)^3 - 5*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 - 3*lo
g(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \sec^5(a+bx)\,dx = \frac{3\,\mathrm{atanh}(\sin{(a+b\,x)})}{8\,b}+\frac{\frac{5\,\sin(a+b\,x)}{8}-\frac{3\,\sin(a+b\,x)^3}{8}}{b\,\left(\sin{(a+b\,x)}^4-2\sin{(a+b\,x)}^2+1\right)}$$

input
```
int(1/cos(a + b*x)^5,x)
```

output
```
(3*atanh(sin(a + b*x)))/(8*b) + ((5*sin(a + b*x))/8 - (3*sin(a + b*x)^3)/8
)/(b*(sin(a + b*x)^4 - 2*sin(a + b*x)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.95

$$\int \sec^5(a+bx)\,dx$$
$$= \frac{-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin{(bx+a)}^4+6\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin{(bx+a)}^2-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{8b}$$

input
```
int(sec(b*x+a)^5,x)
```

output
```
( - 3*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 + 6*log(tan((a + b*x)/2) -
 1)*sin(a + b*x)**2 - 3*log(tan((a + b*x)/2) - 1) + 3*log(tan((a + b*x)/2)
 + 1)*sin(a + b*x)**4 - 6*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 + 3*lo
g(tan((a + b*x)/2) + 1) - 3*sin(a + b*x)**3 + 5*sin(a + b*x))/(8*b*(sin(a
+ b*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.6 $\int \sec^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \sec^6(a+bx)\,dx = \frac{\tan(a+bx)}{b} + \frac{2\tan^3(a+bx)}{3b} + \frac{\tan^5(a+bx)}{5b}$$

output
```
tan(b*x+a)/b+2/3*tan(b*x+a)^3/b+1/5*tan(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sec^6(a+bx)\,dx = \frac{\tan(a+bx) + \frac{2}{3}\tan^3(a+bx) + \frac{1}{5}\tan^5(a+bx)}{b}$$

input
```
Integrate[Sec[a + b*x]^6,x]
```

output
```
(Tan[a + b*x] + (2*Tan[a + b*x]^3)/3 + Tan[a + b*x]^5/5)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\tan^4(a+bx)+2\tan^2(a+bx)+1\right)d(-\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\tan^5(a+bx)-\frac{2}{3}\tan^3(a+bx)-\tan(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^6,x]
```

output
```
-((-Tan[a + b*x] - (2*Tan[a + b*x]^3)/3 - Tan[a + b*x]^5/5)/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\left(-\frac{8}{15}-\frac{\sec(bx+a)^4}{5}-\frac{4\sec(bx+a)^2}{15}\right)\tan(bx+a)}{b}$	34
default	$-\frac{\left(-\frac{8}{15}-\frac{\sec(bx+a)^4}{5}-\frac{4\sec(bx+a)^2}{15}\right)\tan(bx+a)}{b}$	34
risch	$\frac{16i\left(10\,\mathrm{e}^{4i(bx+a)}+5\,\mathrm{e}^{2i(bx+a)}+1\right)}{15b\left(1+\mathrm{e}^{2i(bx+a)}\right)^5}$	44
norman	$\frac{-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3b}-\frac{116\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{15b}+\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{3b}-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^5}$	96
parallelrisch	$\frac{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9+\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{3}-\frac{116\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{15}+\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3}-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5}$	96

input
```
int(sec(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-8/15-1/5*sec(b*x+a)^4-4/15*sec(b*x+a)^2)*tan(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sec^6(a+bx)\,dx = \frac{\left(8\cos(bx+a)^4+4\cos(bx+a)^2+3\right)\sin(bx+a)}{15\,b\cos(bx+a)^5}$$

input
```
integrate(sec(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/15*(8*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 3)*sin(b*x + a)/(b*cos(b*x + a
)^5)
```

Sympy [F]

$$\int \sec^6(a+bx)\,dx = \int \sec^6(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**6,x)
```

output
```
Integral(sec(a + b*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sec^6(a+bx)\,dx = \frac{3\tan(bx+a)^5 + 10\tan(bx+a)^3 + 15\tan(bx+a)}{15b}$$

input
```
integrate(sec(b*x+a)^6,x, algorithm="maxima")
```

output
```
1/15*(3*tan(b*x + a)^5 + 10*tan(b*x + a)^3 + 15*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sec^6(a+bx)\,dx = \frac{3\tan(bx+a)^5 + 10\tan(bx+a)^3 + 15\tan(bx+a)}{15b}$$

input
```
integrate(sec(b*x+a)^6,x, algorithm="giac")
```

output
```
1/15*(3*tan(b*x + a)^5 + 10*tan(b*x + a)^3 + 15*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\,dx=\frac{\frac{\tan(a+b\,x)^5}{5}+\frac{2\tan(a+b\,x)^3}{3}+\tan(a+b\,x)}{b}$$

input
```
int(1/cos(a + b*x)^6,x)
```

output
```
(tan(a + b*x) + (2*tan(a + b*x)^3)/3 + tan(a + b*x)^5/5)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \sec^6(a+bx)\,dx=\frac{\sin(bx+a)\left(8\sin(bx+a)^4-20\sin(bx+a)^2+15\right)}{15\cos(bx+a)\,b\left(\sin(bx+a)^4-2\sin(bx+a)^2+1\right)}$$

input
```
int(sec(b*x+a)^6,x)
```

output
```
(sin(a + b*x)*(8*sin(a + b*x)**4 - 20*sin(a + b*x)**2 + 15))/(15*cos(a + b
*x)*b*(sin(a + b*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.7 $\int \sec^7(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 76

$$\int \sec^7(a+bx)\,dx = \frac{5\text{arctanh}(\sin(a+bx))}{16b} + \frac{5\sec(a+bx)\tan(a+bx)}{16b} + \frac{5\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b}$$

output
```
5/16*arctanh(sin(b*x+a))/b+5/16*sec(b*x+a)*tan(b*x+a)/b+5/24*sec(b*x+a)^3*
tan(b*x+a)/b+1/6*sec(b*x+a)^5*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sec^7(a+bx)\,dx = \frac{5\text{arctanh}(\sin(a+bx))}{16b} + \frac{5\sec(a+bx)\tan(a+bx)}{16b} + \frac{5\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b}$$

input
```
Integrate[Sec[a + b*x]^7,x]
```

output
```
(5*ArcTanh[Sin[a + b*x]])/(16*b) + (5*Sec[a + b*x]*Tan[a + b*x])/(16*b) +
(5*Sec[a + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*
b)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^7dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\int \sec^5(a+bx)dx+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \csc\left(a+bx+\frac{\pi}{2}\right)^5dx+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sec^3(a+bx)dx+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

input
```
Int[Sec[a + b*x]^7,x]
```

output
```
(Sec[a + b*x]^5*Tan[a + b*x])/(6*b) + (5*((Sec[a + b*x]^3*Tan[a + b*x])/(4
*b) + (3*(ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))
)/4))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\left(-\frac{\sec(bx+a)^5}{6}-\frac{5\sec(bx+a)^3}{24}-\frac{5\sec(bx+a)}{16}\right)\tan(bx+a)+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
default	$\frac{-\left(-\frac{\sec(bx+a)^5}{6}-\frac{5\sec(bx+a)^3}{24}-\frac{5\sec(bx+a)}{16}\right)\tan(bx+a)+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
risch	$-\frac{i(15\,e^{11i(bx+a)}+85\,e^{9i(bx+a)}+198\,e^{7i(bx+a)}-198\,e^{5i(bx+a)}-85\,e^{3i(bx+a)}-15\,e^{i(bx+a)})}{24b(1+e^{2i(bx+a)})^6}+\frac{5\ln(e^{i(bx+a)}+i)}{16b}-5$
norman	$\frac{\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{24b}+\frac{15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}+\frac{15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{24b}+\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{8b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^6}-\frac{5\ln\left(\tan\left(\frac{bx}{2}\right.\right.}{16b}$
parallelrisch	$\frac{(-225\cos(2bx+2a)-90\cos(4bx+4a)-15\cos(6bx+6a)-150)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(15\cos(6bx+6a)+90\cos(4bx+4a)}{48b(\cos(6bx+6a)+6\cos(4bx+4a)+15}$

input
```
int(sec(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-(-1/6*sec(b*x+a)^5-5/24*sec(b*x+a)^3-5/16*sec(b*x+a))*tan(b*x+a)+5/1
6*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int \sec^7(a+bx)\,dx = \frac{15\cos(bx+a)^6\log(\sin(bx+a)+1)-15\cos(bx+a)^6\log(-\sin(bx+a)+1)+2\left(15\cos(bx+a)^4+\right.}{96\,b\cos(bx+a)^6}$$

input
```
integrate(sec(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/96*(15*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 15*cos(b*x + a)^6*log(-sin
(b*x + a) + 1) + 2*(15*cos(b*x + a)^4 + 10*cos(b*x + a)^2 + 8)*sin(b*x + a
))/(b*cos(b*x + a)^6)
```

Sympy [F]

$$\int \sec^7(a+bx)\,dx = \int \sec^7(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**7,x)
```

output
```
Integral(sec(a + b*x)**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \sec^7(a+bx)\,dx = -\frac{\frac{2\left(15\sin(bx+a)^5-40\sin(bx+a)^3+33\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1)}{96\,b}$$

input
```
integrate(sec(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/96*(2*(15*sin(b*x + a)^5 - 40*sin(b*x + a)^3 + 33*sin(b*x + a))/(sin(b*
x + a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 15*log(sin(b*x + a)
+ 1) + 15*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \sec^7(a+bx)\,dx = -\frac{\frac{2\left(15\sin(bx+a)^5-40\sin(bx+a)^3+33\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3} - 15\log(|\sin(bx+a)+1|) + 15\log(|\sin(bx+a)-1|)}{96\,b}$$

input
```
integrate(sec(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/96*(2*(15*sin(b*x + a)^5 - 40*sin(b*x + a)^3 + 33*sin(b*x + a))/(sin(b*
x + a)^2 - 1)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a)
- 1)))/b
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \sec^7(a+bx)\,dx = \frac{5\,\mathrm{atanh}(\sin(a+b\,x))}{16\,b} - \frac{\frac{5\sin(a+b\,x)^5}{16} - \frac{5\sin(a+b\,x)^3}{6} + \frac{11\,\sin(a+b\,x)}{16}}{b\,\left(\sin(a+b\,x)^6 - 3\sin(a+b\,x)^4 + 3\sin(a+b\,x)^2 - 1\right)}$$

input
```
int(1/cos(a + b*x)^7,x)
```

output
```
(5*atanh(sin(a + b*x)))/(16*b) - ((11*sin(a + b*x))/16 - (5*sin(a + b*x)^3
)/6 + (5*sin(a + b*x)^5)/16)/(b*(3*sin(a + b*x)^2 - 3*sin(a + b*x)^4 + sin
(a + b*x)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.97

$$\int \sec^7(a+bx)\,dx = \frac{-15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^6 + 45\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^4 - 45\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{}$$

input
```
int(sec(b*x+a)^7,x)
```

output
```
( - 15*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**6 + 45*log(tan((a + b*x)/2)
 - 1)*sin(a + b*x)**4 - 45*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 + 15*
log(tan((a + b*x)/2) - 1) + 15*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**6 -
 45*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4 + 45*log(tan((a + b*x)/2) +
1)*sin(a + b*x)**2 - 15*log(tan((a + b*x)/2) + 1) - 15*sin(a + b*x)**5 + 4
0*sin(a + b*x)**3 - 33*sin(a + b*x))/(48*b*(sin(a + b*x)**6 - 3*sin(a + b*
x)**4 + 3*sin(a + b*x)**2 - 1))
```

3.8 $\int \sec^8(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int \sec^8(a+bx)\,dx=\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}$$

output
```
tan(b*x+a)/b+tan(b*x+a)^3/b+3/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \sec^8(a+bx)\,dx=\frac{\tan(a+bx)+\tan^3(a+bx)+\frac{3}{5}\tan^5(a+bx)+\frac{1}{7}\tan^7(a+bx)}{b}$$

input
```
Integrate[Sec[a + b*x]^8,x]
```

output
```
(Tan[a + b*x] + Tan[a + b*x]^3 + (3*Tan[a + b*x]^5)/5 + Tan[a + b*x]^7/7)/
b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^8 dx$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\int \left(\tan^6(a+bx)+3\tan^4(a+bx)+3\tan^2(a+bx)+1\right)d(-\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{1}{7}\tan^7(a+bx)-\frac{3}{5}\tan^5(a+bx)-\tan^3(a+bx)-\tan(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^8,x]
```

output
```
-((-Tan[a + b*x] - Tan[a + b*x]^3 - (3*Tan[a + b*x]^5)/5 - Tan[a + b*x]^7/
7)/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\left(-\frac{16}{35}-\frac{\sec(bx+a)^{6}}{7}-\frac{6\sec(bx+a)^{4}}{35}-\frac{8\sec(bx+a)^{2}}{35}\right)\tan(bx+a)}{b}$
default	$-\frac{\left(-\frac{16}{35}-\frac{\sec(bx+a)^{6}}{7}-\frac{6\sec(bx+a)^{4}}{35}-\frac{8\sec(bx+a)^{2}}{35}\right)\tan(bx+a)}{b}$
risch	$\frac{32i\left(35\,\mathrm{e}^{6i(bx+a)}+21\,\mathrm{e}^{4i(bx+a)}+7\,\mathrm{e}^{2i(bx+a)}+1\right)}{35b\left(1+\mathrm{e}^{2i(bx+a)}\right)^{7}}$
parallelrisch	$-\frac{2\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}+\frac{43\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}}{5}-\frac{212\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}}{35}+\frac{43\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{5}-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\tan}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{7}}$
norman	$\frac{-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}}{b}-\frac{86\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}}{5b}+\frac{424\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}}{35b}-\frac{86\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}}{5b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{b}-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{7}}$

input
```
int(sec(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-16/35-1/7*sec(b*x+a)^6-6/35*sec(b*x+a)^4-8/35*sec(b*x+a)^2)*tan(b*x
+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \sec^{8}(a+bx)\,dx = \frac{\left(16\,\cos\left(bx+a\right)^{6}+8\,\cos\left(bx+a\right)^{4}+6\,\cos\left(bx+a\right)^{2}+5\right)\sin\left(bx+a\right)}{35\,b\cos\left(bx+a\right)^{7}}$$

input
```
integrate(sec(b*x+a)^8,x, algorithm="fricas")
```

output
```
1/35*(16*cos(b*x + a)^6 + 8*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 5)*sin(b*x
 + a)/(b*cos(b*x + a)^7)
```

Sympy [F]

$$\int \sec^8(a+bx)\,dx = \int \sec^8{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**8,x)
```

output
```
Integral(sec(a + b*x)**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \sec^8(a+bx)\,dx = \frac{5\tan{(bx+a)}^7 + 21\tan{(bx+a)}^5 + 35\tan{(bx+a)}^3 + 35\tan{(bx+a)}}{35\,b}$$

input
```
integrate(sec(b*x+a)^8,x, algorithm="maxima")
```

output
```
1/35*(5*tan(b*x + a)^7 + 21*tan(b*x + a)^5 + 35*tan(b*x + a)^3 + 35*tan(b*
x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \sec^8(a+bx)\,dx = \frac{5\,\tan\left(bx+a\right)^7+21\,\tan\left(bx+a\right)^5+35\,\tan\left(bx+a\right)^3+35\,\tan\left(bx+a\right)}{35\,b}$$

input
```
integrate(sec(b*x+a)^8,x, algorithm="giac")
```

output
```
1/35*(5*tan(b*x + a)^7 + 21*tan(b*x + a)^5 + 35*tan(b*x + a)^3 + 35*tan(b*
x + a))/b
```

Mupad [B] (verification not implemented)

Time = 10.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \sec^8(a+bx)\,dx = \frac{\frac{\tan(a+b\,x)^7}{7}+\frac{3\tan(a+b\,x)^5}{5}+\tan(a+b\,x)^3+\tan(a+b\,x)}{b}$$

input
```
int(1/cos(a + b*x)^8,x)
```

output
```
(tan(a + b*x) + tan(a + b*x)^3 + (3*tan(a + b*x)^5)/5 + tan(a + b*x)^7/7)/
b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \sec^8(a+bx)\,dx = \frac{\sin\left(bx+a\right)\left(16\sin\left(bx+a\right)^6-56\sin\left(bx+a\right)^4+70\sin\left(bx+a\right)^2-35\right)}{35\cos\left(bx+a\right)b\left(\sin\left(bx+a\right)^6-3\sin\left(bx+a\right)^4+3\sin\left(bx+a\right)^2-1\right)}$$

input
```
int(sec(b*x+a)^8,x)
```

output
```
(sin(a + b*x)*(16*sin(a + b*x)**6 - 56*sin(a + b*x)**4 + 70*sin(a + b*x)**
2 - 35))/(35*cos(a + b*x)*b*(sin(a + b*x)**6 - 3*sin(a + b*x)**4 + 3*sin(a
 + b*x)**2 - 1))
```

3.9 $\int \sec^{\frac{7}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx = -\frac{6\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{5b} + \frac{6\sqrt{\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b}$$

output
```
-6/5*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*sec(b*x+a)^(1/
2)/b+6/5*sec(b*x+a)^(1/2)*sin(b*x+a)/b+2/5*sec(b*x+a)^(5/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx = \frac{\sec^{\frac{5}{2}}(a+bx)\left(-12\cos^{\frac{5}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+7\sin(a+bx)+3\sin(3(a+bx))\right)}{10b}$$

input
```
Integrate[Sec[a + b*x]^(7/2),x]
```

output
```
(Sec[a + b*x]^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2] + 7*
Sin[a + b*x] + 3*Sin[3*(a + b*x)]))/(10*b)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}\int \sec^{\frac{3}{2}}(a+bx)dx+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int \frac{1}{\sqrt{\sec(a+bx)}}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\cos(a+bx)}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}$$

$\Big\downarrow$ 3042

$$\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}$$

$\Big\downarrow$ 3119

$$\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}+\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}\right)$$

input
```
Int[Sec[a + b*x]^(7/2),x]
```

output
```
(2*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(5*b) + (3*((-2*Sqrt[Cos[a + b*x]]*Ell
ipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b + (2*Sqrt[Sec[a + b*x]]*Sin[a
 + b*x])/b))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(73) = 146.

Time = 1.49 (sec) , antiderivative size = 358, normalized size of antiderivative = 4.21

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(24\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\right.}{}$

input
```
int(sec(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*b*x+1/2*a)^2+1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(8*sin(1/2*
b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x
+1/2*a)^3*(24*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*EllipticE(cos(1/2
*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^
2)^(1/2)*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)+1
2*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(
sin(1/2*b*x+1/2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^2+8*sin(1/2*b*x+1/2*a)^2*co
s(1/2*b*x+1/2*a)-3*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/
2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2))*(-2*sin(1/2*b*x+1/2*a)^4+sin
(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx$$
$$=\frac{-3i\sqrt{2}\cos{(bx+a)}^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(bx+a)}+i\,\sin{(bx+a)}))+}{}$$

input
```
integrate(sec(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*cos(b*x + a)^2*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*cos(b*x + a)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a
))) + 2*(3*cos(b*x + a)^2 + 1)*sin(b*x + a)/sqrt(cos(b*x + a)))/(b*cos(b*x
 + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(sec(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(sec(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(sec(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx = \int \left(\frac{1}{\cos(a+b\,x)}\right)^{7/2}\,dx$$

input
```
int((1/cos(a + b*x))^(7/2),x)
```

output
```
int((1/cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \sec^{\frac{7}{2}}(a+bx)\,dx=\int \sqrt{\sec (bx+a)}\,\sec (bx+a)^3\,dx$$

input
```
int(sec(b*x+a)^(7/2),x)
```

output
```
int(sqrt(sec(a + b*x))*sec(a + b*x)**3,x)
```

3.10 $\int \sec^{\frac{5}{2}}(a + bx)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{3b} + \frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b}$$

output
```
2/3*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*sec(b*x+a)^(1/
2)/b+2/3*sec(b*x+a)^(3/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \frac{2\sec^{\frac{3}{2}}(a+bx)\left(\cos^{\frac{3}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sin(a+bx)\right)}{3b}$$

input
```
Integrate[Sec[a + b*x]^(5/2),x]
```

output
```
(2*Sec[a + b*x]^(3/2)*(Cos[a + b*x]^(3/2)*EllipticF[(a + b*x)/2, 2] + Sin[
a + b*x]))/(3*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{3}\int \sqrt{\sec(a+bx)}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}$$

input
```
Int[Sec[a + b*x]^(5/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b)
+ (2*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(53) = 106$.

Time = 0.87 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx}{2}}\right.}{3\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\right.}$

input
```
int(sec(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ell
ipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-2*sin(1/2*b*x+1/2*
a)^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))*((2*cos(1/2*b*x+1/2*a)^
2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*
a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(3/2)/sin(1/2*b*x+1/2*a)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx$$
$$= \frac{-i\sqrt{2}\cos(bx+a)\ \text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\ \sin(bx+a))+i\sqrt{2}\cos(bx+a)\ \text{weierstra}}{3\,b\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*cos(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*s
in(b*x + a)) + I*sqrt(2)*cos(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x +
 a) - I*sin(b*x + a)) + 2*sin(b*x + a)/sqrt(cos(b*x + a)))/(b*cos(b*x + a)
)
```

Sympy [F]

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \int \sec^{\frac{5}{2}}(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**(5/2),x)
```

output
```
Integral(sec(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \int \left(\frac{1}{\cos(a+b\,x)}\right)^{5/2}\,dx$$

input
```
int((1/cos(a + b*x))^(5/2),x)
```

output
```
int((1/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \int \sqrt{\sec(bx+a)}\,\sec(bx+a)^2\,dx$$

input
```
int(sec(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sec(a + b*x))*sec(a + b*x)**2,x)
```

3.11 $\int \sec^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 58

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b} + \frac{2\sqrt{\sec(a+bx)}\sin(a+bx)}{b}$$

output
```
-2*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*sec(b*x+a)^(1/2)
/b+2*sec(b*x+a)^(1/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx = \frac{2\sqrt{\sec(a+bx)}\left(-\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\right)}{b}$$

input
```
Integrate[Sec[a + b*x]^(3/2),x]
```

output
```
(2*Sqrt[Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]) + S
in[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx$$
$$\Big\downarrow \text{4255}$$
$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int \frac{1}{\sqrt{\sec(a+bx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4258}$$
$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3119}$$
$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Int[Sec[a + b*x]^(3/2),x]
```

output
```
(-2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b + (
2*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(54) = 108$.

Time = 0.46 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.14

method	result
default	$-\frac{2\left(-2\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\mathrm{si}}\right.}{\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$

input
```
int(sec(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*cos(1/2*b*x+1/
2*a)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*
a)^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*Ellipti
cE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a
)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{-i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+i\sqrt{2}\text{weierstras}}{b}$$

input
```
integrate(sec(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a)
 + I*sin(b*x + a))) + I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*sin(b*x + a)/sqrt(cos(b*x + a)
))/b
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx = \int \sec^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**(3/2),x)
```

output
```
Integral(sec(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx = \int \left(\frac{1}{\cos(a+b\,x)}\right)^{3/2}\,dx$$

input
```
int((1/cos(a + b*x))^(3/2),x)
```

output
```
int((1/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\sec(bx+a)}\,\sec(bx+a)\,dx$$

input
```
int(sec(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sec(a + b*x))*sec(a + b*x),x)
```

3.12 $\int \sqrt{\sec(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{\sec(a+bx)}\,dx = \frac{2\sqrt{\cos(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{b}$$

output
```
2*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*sec(b*x+a)^(1/2)
/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec(a+bx)}\,dx = \frac{2\sqrt{\cos(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{b}$$

input
```
Integrate[Sqrt[Sec[a + b*x]],x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

input
```
Int[Sqrt[Sec[a + b*x]],x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(33) = 66$.

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.69

method	result	size
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\operatorname{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$	133

input
```
int(sec(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/
2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+s
in(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))/sin(1/2*b
*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \sqrt{\sec(a+bx)}\,dx$$
$$= \frac{-i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx}{b}$$

input
```
integrate(sec(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*
sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int \sqrt{\sec(a+bx)}\,dx = \int \sqrt{\sec(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(sec(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\sec(a+bx)}\,dx = \int \sqrt{\sec(bx+a)}\,dx$$

input
```
integrate(sec(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\sec(a+bx)}\,dx = \int \sqrt{\sec\left(bx+a\right)}\,dx$$

input
```
integrate(sec(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \sqrt{\sec(a+bx)}\,dx = \frac{2\sqrt{\cos\left(a+b\,x\right)}\,\sqrt{\frac{1}{\cos(a+b\,x)}}\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int((1/cos(a + b*x))^(1/2),x)
```

output
```
(2*cos(a + b*x)^(1/2)*(1/cos(a + b*x))^(1/2)*ellipticF(a/2 + (b*x)/2, 2))/
b
```

Reduce [F]

$$\int \sqrt{\sec(a+bx)}\,dx = \int \sqrt{\sec\left(bx+a\right)}dx$$

input
```
int(sec(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sec(a + b*x)),x)
```

3.13 $\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx=\frac{2\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}$$

output
```
2*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*sec(b*x+a)^(1/2)/
b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx=\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}}$$

input
```
Integrate[1/Sqrt[Sec[a + b*x]],x]
```

output
```
(2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Int[1/Sqrt[Sec[a + b*x]],x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(34) = 68$.

Time = 0.99 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.69

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$
risch	$-\frac{i\sqrt{2}}{b\sqrt{\frac{e^{i(bx+a)}}{1+e^{2i(bx+a)}}}}-\frac{i\left(-\frac{2\left(1+e^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(1+e^{2i(bx+a)}\right)}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(bx+a)}-i\right)}\,\sqrt{ie^{i(bx+a)}}\,\left(-2i\operatorname{EllipticE}\left(\sqrt{-i\left(e^{i(bx+a)}\right.}\right.\right.}{\sqrt{e^{3i(bx+a)}+e^{i(bx+a)}}}}{b\sqrt{\frac{e^{i(bx+a)}}{1+e^{2i(bx+a)}}}\,\left(1+e^{2i(bx+a)}\right)}$

input
```
int(1/sec(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a
),2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*
x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx = \frac{i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\text{weierstrassZ}}{b}$$

input
```
integrate(1/sec(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a)
+ I*sin(b*x + a))) - I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sec(a+bx)}}\,dx$$

input
```
integrate(1/sec(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(sec(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sec(bx+a)}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sec\left(bx+a\right)}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(a+b\,x)}}}\,dx$$

input
```
int(1/(1/cos(a + b*x))^(1/2),x)
```

output
```
int(1/(1/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{\sqrt{\sec\left(bx+a\right)}}{\sec\left(bx+a\right)}dx$$

input
```
int(1/sec(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sec(a + b*x))/sec(a + b*x),x)
```

3.14 $\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{3b} + \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

output
```
2/3*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*sec(b*x+a)^(1/
2)/b+2/3*sin(b*x+a)/b/sec(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \frac{\sqrt{\sec(a+bx)}\left(2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sin(2(a+bx))\right)}{3b}$$

input
```
Integrate[Sec[a + b*x]^(-3/2),x]
```

output
```
(Sqrt[Sec[a + b*x]]*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[
2*(a + b*x)]))/(3*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{3}\int \sqrt{\sec(a+bx)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}$$

input
```
Int[Sec[a + b*x]^(-3/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b)
+ (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(53) = 106$.

Time = 1.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.89

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\left(4\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}\cdots\right.}\right.}{3\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$

input
```
int(1/sec(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b*
x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(s
in(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(
1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1
/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sqrt{\cos(bx+a)}\sin(bx+a) - i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) + i\sqrt{2}\text{weier}}{3b}$$

input
```
integrate(1/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(cos(b*x + a))*sin(b*x + a) - I*sqrt(2)*weierstrassPInverse(-4,
 0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0,
cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sec(b*x+a)**(3/2),x)
```

output
```
Integral(sec(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sec\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sec\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(1/cos(a + b*x))^(3/2),x)
```

output
```
int(1/(1/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sec(bx+a)}}{\sec(bx+a)^2}\,dx$$

input
```
int(1/sec(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sec(a + b*x))/sec(a + b*x)**2,x)
```

3.15 $\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \frac{6\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{5b} + \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}$$

output
```
6/5*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*sec(b*x+a)^(1/2
)/b+2/5*sin(b*x+a)/b/sec(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\sec(a+bx)}\left(12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)+\sin(3(a+bx))\right)}{10b}$$

input
```
Integrate[Sec[a + b*x]^(-5/2),x]
```

output
```
(Sqrt[Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + Sin
[a + b*x] + Sin[3*(a + b*x)]))/(10*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{3}{5}\int \frac{1}{\sqrt{\sec(a+bx)}}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{3}{5}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{5}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

input
```
Int[Sec[a + b*x]^(-5/2),x]
```

output
```
(6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b)
+ (2*Sin[a + b*x])/(5*b*Sec[a + b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(54) = 108$.

Time = 2.69 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.26

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(-8\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+8\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\cos\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{5\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right.}}$

input
```
int(1/sec(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(-8*sin(1/2*b
*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2
*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*EllipticE(cos(1/2*b*x+1/2*a),2^
(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2))/(-2*
sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos
(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx$$
$$= \frac{2\cos(bx+a)^{\frac{3}{2}}\sin(bx+a) + 3i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin}{5b}$$

input
```
integrate(1/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*cos(b*x + a)^(3/2)*sin(b*x + a) + 3*I*sqrt(2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - 3*I*sqrt(2)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*
x + a))))/b
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sec(b*x+a)**(5/2),x)
```

output
```
Integral(sec(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sec\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sec\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(1/cos(a + b*x))^(5/2),x)
```

output
```
int(1/(1/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sec(bx+a)}}{\sec(bx+a)^3}\,dx$$

input
```
int(1/sec(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sec(a + b*x))/sec(a + b*x)**3,x)
```

3.16 $\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \frac{10\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{21b} + \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}}$$

output
```
10/21*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*sec(b*x+a)^(
1/2)/b+2/7*sin(b*x+a)/b/sec(b*x+a)^(5/2)+10/21*sin(b*x+a)/b/sec(b*x+a)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \frac{\sqrt{\sec(a+bx)}\left(40\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+26\sin(2(a+bx))+3\sin(4(a+bx))\right)}{84b}$$

input
```
Integrate[Sec[a + b*x]^(-7/2),x]
```

output
```
(Sqrt[Sec[a + b*x]]*(40*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 26*
Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)]))/(84*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}dx+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(a+bx)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}+\frac{5}{7}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}\right)$$

input
```
Int[Sec[a + b*x]^(-7/2),x]
```

output
```
(2*Sin[a + b*x])/(7*b*Sec[a + b*x]^(5/2)) + (5*((2*Sqrt[Cos[a + b*x]]*Elli
pticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) + (2*Sin[a + b*x])/(3*b*Sq
rt[Sec[a + b*x]])))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(72) = 144$.

Time = 4.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\left(48\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^9-120\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^7+128\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^5-72\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^3+5\sqrt{\frac{1}{2}-\frac{\cos(b}{2}}\right.}{21\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cos\left(\frac{bx}{2}+\right.}}$

input
```
int(1/sec(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(48*cos(1/2*
b*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(1/2*
b*x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1
/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-2*sin(1
/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*
b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \frac{\frac{2\left(3\cos(bx+a)^3+5\cos(bx+a)\right)\sin(bx+a)}{\sqrt{\cos(bx+a)}} - 5i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) + 5i\sqrt{}}{21\,b}$$

input
```
integrate(1/sec(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(b*x + a)^3 + 5*cos(b*x + a))*sin(b*x + a)/sqrt(cos(b*x + a)
) - 5*I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))
+ 5*I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sec(b*x+a)**(7/2),x)
```

output
```
Integral(sec(a + b*x)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(a+b\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/(1/cos(a + b*x))^(7/2),x)
```

output
```
int(1/(1/cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sec(bx+a)}}{\sec(bx+a)^4}dx$$

input
```
int(1/sec(b*x+a)^(7/2),x)
```

output
```
int(sqrt(sec(a + b*x))/sec(a + b*x)**4,x)
```

3.17 $\int (c\sec(a+bx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int (c\sec(a+bx))^{7/2}\,dx = -\frac{6c^4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{6c^3\sqrt{c\sec(a+bx)}\sin(a+bx)}{5b} + \frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b}$$

output
```
-6/5*c^4*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/2)/(c*sec(b
*x+a))^(1/2)+6/5*c^3*(c*sec(b*x+a))^(1/2)*sin(b*x+a)/b+2/5*c*(c*sec(b*x+a)
)^(5/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int (c\sec(a+bx))^{7/2}\,dx = \frac{c(c\sec(a+bx))^{5/2}\left(-12\cos^{\frac{5}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+7\sin(a+bx)+3\sin(3(a+bx))\right)}{10b}$$

input
```
Integrate[(c*Sec[a + b*x])^(7/2),x]
```

output
```
(c*(c*Sec[a + b*x])^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2
] + 7*Sin[a + b*x] + 3*Sin[3*(a + b*x)]))/(10*b)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}c^2\int (c\sec(a+bx))^{3/2}dx+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}c^2\int \left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}c^2\left(\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-c^2\int \frac{1}{\sqrt{c\sec(a+bx)}}dx\right)+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}c^2\left(\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-c^2\int \frac{1}{\sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}}dx\right)+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3}{5}c^2\left(\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{c^2\int\sqrt{\cos(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\right)+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}c^2\left(\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{c^2\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\right)+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3}{5}c^2\left(\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\right)+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

input
```
Int[(c*Sec[a + b*x])^(7/2),x]
```

output
```
(2*c*(c*Sec[a + b*x])^(5/2)*Sin[a + b*x])/(5*b) + (3*c^2*((-2*c^2*Elliptic
E[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (2*c*Sqrt
[c*Sec[a + b*x]]*Sin[a + b*x])/b))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.11

method	result
default	$\frac{2\sqrt{c\sec(bx+a)}\, c^3 \left(3\sin(bx+a)+\tan(bx+a)+\sec(bx+a)\tan(bx+a)+i\left(3\cos(bx+a)^2+6\cos(bx+a)+3\right)\sqrt{\frac{1}{\cos(bx+a)+1}}\ \text{EllipticE}(i(c}{5}$

input
```
int((c*sec(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b*(c*sec(b*x+a))^(1/2)*c^3/(cos(b*x+a)+1)*(3*sin(b*x+a)+tan(b*x+a)+sec
(b*x+a)*tan(b*x+a)+I*(3*cos(b*x+a)^2+6*cos(b*x+a)+3)*(1/(cos(b*x+a)+1))^(1
/2)*EllipticE(I*(cot(b*x+a)-csc(b*x+a)),I)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/
2)+I*(-3*cos(b*x+a)^2-6*cos(b*x+a)-3)*(1/(cos(b*x+a)+1))^(1/2)*EllipticF(I
*(cot(b*x+a)-csc(b*x+a)),I)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int (c\sec(a+bx))^{7/2}\,dx = \frac{-3i\sqrt{2}c^{\frac{7}{2}}\cos(bx+a)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin}{}$$

input
```
integrate((c*sec(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*c^(7/2)*cos(b*x + a)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*c^(7/2)*cos
(b*x + a)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a)
 - I*sin(b*x + a))) + 2*(3*c^3*cos(b*x + a)^2 + c^3)*sqrt(c/cos(b*x + a))*
sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (c\sec(a+bx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((c*sec(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c\sec(a+bx))^{7/2}\,dx = \int (c\sec(bx+a))^{\frac{7}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(7/2), x)
```

Giac [F]

$$\int (c\sec(a+bx))^{7/2}\,dx = \int (c\sec(bx+a))^{\frac{7}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sec(a+bx))^{7/2}\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^{7/2}\,dx$$

input
```
int((c/cos(a + b*x))^(7/2),x)
```

output
```
int((c/cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int (c\sec(a+bx))^{7/2}\,dx = \sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sec(bx+a)^3\,dx\right)c^3$$

input
```
int((c*sec(b*x+a))^(7/2),x)
```

output
```
sqrt(c)*int(sqrt(sec(a + b*x))*sec(a + b*x)**3,x)*c**3
```

3.18 $\int (c\sec(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (c\sec(a+bx))^{5/2}\,dx = \frac{2c^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{3b} + \frac{2c(c\sec(a+bx))^{3/2}\sin(a+bx)}{3b}$$

output
```
2/3*c^2*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*(c*sec(b*x
+a))^(1/2)/b+2/3*c*(c*sec(b*x+a))^(3/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int (c\sec(a+bx))^{5/2}\,dx = \frac{2c^2\sqrt{c\sec(a+bx)}\left(\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\tan(a+bx)\right)}{3b}$$

input
```
Integrate[(c*Sec[a + b*x])^(5/2),x]
```

output
```
(2*c^2*Sqrt[c*Sec[a + b*x]]*(Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]
+ Tan[a + b*x]))/(3*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{3}c^2\int \sqrt{c\sec(a+bx)}dx+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}c^2\int \sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2c^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{3b}+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

input
```
Int[(c*Sec[a + b*x])^(5/2),x]
```

output
```
(2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/
(3*b) + (2*c*(c*Sec[a + b*x])^(3/2)*Sin[a + b*x])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{\left(-\frac{2 i \left(\cos \left(b x +a \right)+1\right) \sqrt{\frac{1}{\cos \left(b x +a \right)+1}}\, \operatorname{EllipticF}\left(i \left(-\cot \left(b x +a \right)+\csc \left(b x +a \right)\right), i\right) \sqrt{\frac{\cos \left(b x +a \right)}{\cos \left(b x +a \right)+1}}}{3}+\frac{2 \tan \left(b x +a \right)}{3}\right) c^{2} \sqrt{c \sec \left(b x +a \right)}}{b}$	90

input
```
int((c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/3*I*(cos(b*x+a)+1)*(1/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-cot(b*x+
a)+csc(b*x+a)),I)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)+2/3*tan(b*x+a))*c^2*(c
*sec(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int (c\sec(a+bx))^{5/2}\,dx = \frac{-i\sqrt{2}c^{\frac{5}{2}}\cos(bx+a)\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a))+i\sqrt{2}c^{\frac{5}{2}}\cos(}{3\,b\cos(}$$

input
```
integrate((c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*c^(5/2)*cos(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x +
a) + I*sin(b*x + a)) + I*sqrt(2)*c^(5/2)*cos(b*x + a)*weierstrassPInverse(
-4, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*c^2*sqrt(c/cos(b*x + a))*sin(b*x
 + a))/(b*cos(b*x + a))
```

Sympy [F]

$$\int (c\sec(a+bx))^{5/2}\,dx = \int (c\sec(a+bx))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(5/2),x)
```

output
```
Integral((c*sec(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int (c\sec(a+bx))^{5/2}\,dx=\int (c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (c\sec(a+bx))^{5/2}\,dx=\int (c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sec(a+bx))^{5/2}\,dx=\int \left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\,dx$$

input
```
int((c/cos(a + b*x))^(5/2),x)
```

output
```
int((c/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (c\sec(a+bx))^{5/2}\,dx=\sqrt{c}\left(\int\sqrt{\sec\left(bx+a\right)}\,\sec\left(bx+a\right)^2dx\right)c^2$$

input
```
int((c*sec(b*x+a))^(5/2),x)
```

output
```
sqrt(c)*int(sqrt(sec(a + b*x))*sec(a + b*x)**2,x)*c**2
```

3.19 $\int (c\sec(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (c\sec(a+bx))^{3/2}\,dx = -\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2c\sqrt{c\sec(a+bx)}\sin(a+bx)}{b}$$

output
```
-2*c^2*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/2)/(c*sec(b*x
+a))^(1/2)+2*c*(c*sec(b*x+a))^(1/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int (c\sec(a+bx))^{3/2}\,dx = \frac{2c\sqrt{c\sec(a+bx)}\left(-\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\right)}{b}$$

input
```
Integrate[(c*Sec[a + b*x])^(3/2),x]
```

output
```
(2*c*Sqrt[c*Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])
 + Sin[a + b*x]))/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-c^2\int \frac{1}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-c^2\int \frac{1}{\sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{c^2\int \sqrt{\cos(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{c^2\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c^2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*
x]]) + (2*c*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.76

method	result
default	$\frac{2\left(i\left(\cos(bx+a)^2+2\cos(bx+a)+1\right)\operatorname{EllipticF}(i(-\cot(bx+a)+\csc(bx+a)),i)\sqrt{\frac{1}{\cos(bx+a)+1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}+i\left(-\cos(bx+a)^2-2\cos(\right.\right.}{b(\cos(bx+a)+1)}$

input
```
int((c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(I*(cos(b*x+a)^2+2*cos(b*x+a)+1)*EllipticF(I*(-cot(b*x+a)+csc(b*x+a)),
I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)+I*(-cos(b*x+
a)^2-2*cos(b*x+a)-1)*EllipticE(I*(-cot(b*x+a)+csc(b*x+a)),I)*(1/(cos(b*x+a
)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)+sin(b*x+a))*(c*sec(b*x+a))^(
1/2)*c/(cos(b*x+a)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int (c\sec(a+bx))^{3/2}\,dx = \frac{-i\sqrt{2}c^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+}{}$$

input
```
integrate((c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*c^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*c^(3/2)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*c*sqrt(c/cos(b
*x + a))*sin(b*x + a))/b
```

Sympy [F]

$$\int (c\sec(a+bx))^{3/2}\,dx = \int (c\sec(a+bx))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(3/2),x)
```

output
```
Integral((c*sec(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (c\sec(a+bx))^{3/2}\,dx = \int (c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (c\sec(a+bx))^{3/2}\,dx = \int (c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sec(a+bx))^{3/2}\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\,dx$$

input
```
int((c/cos(a + b*x))^(3/2),x)
```

output
```
int((c/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (c\sec(a+bx))^{3/2}\,dx = \sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sec(bx+a)\,dx\right)c$$

input
```
int((c*sec(b*x+a))^(3/2),x)
```

output
```
sqrt(c)*int(sqrt(sec(a + b*x))*sec(a + b*x),x)*c
```

3.20 $\int \sqrt{c\sec(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \sqrt{c\sec(a+bx)}\,dx=\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{b}$$

output
```
2*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*(c*sec(b*x+a))^(
1/2)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sqrt{c\sec(a+bx)}\,dx=\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{b}$$

input
```
Integrate[Sqrt[c*Sec[a + b*x]],x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/b
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sec(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{b}$$

input
```
Int[Sqrt[c*Sec[a + b*x]],x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

method	result	size
default	$\frac{2i\sqrt{\frac{1}{\cos(bx+a)+1}}\,\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,(\cos(bx+a)+1)\sqrt{c\sec(bx+a)}\ \mathrm{EllipticF}(i(\cot(bx+a)-\csc(bx+a)),i)}{b}$	77

input
```
int((c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/b*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*(cos(b*x+
a)+1)*(c*sec(b*x+a))^(1/2)*EllipticF(I*(cot(b*x+a)-csc(b*x+a)),I)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \sqrt{c\sec(a+bx)}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{c}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\sqrt{c}\text{weierstrassPInverse}(-4,0,c}{b}$$

input
```
integrate((c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x +
a)) + I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*
x + a)))/b
```

Sympy [F]

$$\int \sqrt{c\sec(a+bx)}\,dx = \int \sqrt{c\sec(a+bx)}\,dx$$

input
```
integrate((c*sec(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sec(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{c\sec(a+bx)}\,dx = \int \sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{c\sec(a+bx)}\,dx = \int \sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sec(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sqrt{c\sec(a+bx)}\,dx = \frac{2\sqrt{\cos(a+b\,x)}\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int((c/cos(a + b*x))^(1/2),x)
```

output
```
(2*cos(a + b*x)^(1/2)*(c/cos(a + b*x))^(1/2)*ellipticF(a/2 + (b*x)/2, 2))/
b
```

Reduce [F]

$$\int \sqrt{c\sec(a+bx)}\,dx = \sqrt{c}\left(\int \sqrt{\sec(bx+a)}dx\right)$$

input
```
int((c*sec(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(sec(a + b*x)),x)
```

3.21 $\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

output
```
2*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/2)/(c*sec(b*x+a))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{\int \sqrt{\cos(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

input
```
Int[1/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.61

method	result
default	$\frac{2i\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,\mathrm{EllipticF}(i(-\cot(bx+a)+\csc(bx+a)),i)\sqrt{\frac{1}{\cos(bx+a)+1}}\,(-\cos(bx+a)-2-\sec(bx+a))+2i\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,\mathrm{EllipticE}\cdots}{b(\cos(bx+a)+1)\sqrt{c\sec(bx+a)}}$
risch	$-\frac{i\sqrt{2}}{b\sqrt{\frac{c\,\mathrm{e}^{i(bx+a)}}{1+\mathrm{e}^{2i(bx+a)}}}}-\frac{i\left(-\frac{2\left(\mathrm{e}^{2i(bx+a)}c+c\right)}{c\sqrt{\mathrm{e}^{i(bx+a)}\left(\mathrm{e}^{2i(bx+a)}c+c\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(bx+a)}-i\right)}\,\sqrt{i\mathrm{e}^{i(bx+a)}}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(\mathrm{e}^{i(bx+}\cdots\right.\right.\right.}{\sqrt{c\,\mathrm{e}^{3i(bx+a)}+c\,\mathrm{e}^{i(bx+a)}}}\cdots}{b\sqrt{\frac{c\,\mathrm{e}^{i(bx+a)}}{1+\mathrm{e}^{2i(bx+a)}}}\,\left(1+\mathrm{e}^{2i(bx+a)}\cdots\right.}$

input
```
int(1/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/(cos(b*x+a)+1)/(c*sec(b*x+a))^(1/2)*(I*(cos(b*x+a)/(cos(b*x+a)+1))^(1/
2)*EllipticF(I*(-cot(b*x+a)+csc(b*x+a)),I)*(1/(cos(b*x+a)+1))^(1/2)*(-cos(
b*x+a)-2-sec(b*x+a))+I*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticE(I*(-cot
(b*x+a)+csc(b*x+a)),I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)+2+sec(b*x+a))+
sin(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \frac{i\sqrt{2}\sqrt{c}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))) - i\sqrt{2}\sqrt{c}\text{weiers}}{bc}$$

input
```
integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b
*x + a) + I*sin(b*x + a))) - I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(c*sec(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{c}{\cos(a+b\,x)}}}\,dx$$

input
```
int(1/(c/cos(a + b*x))^(1/2),x)
```

output
```
int(1/(c/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}}{\sec(bx+a)}dx\right)}{c}$$

input
```
int(1/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(sec(a + b*x))/sec(a + b*x),x))/c
```

3.22 $\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{3bc^2} + \frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

output
```
2/3*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*(c*sec(b*x+a))
^(1/2)/b/c^2+2/3*sin(b*x+a)/b/c/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sec^2(a+bx)\left(2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sin(2(a+bx))\right)}{3b(c\sec(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sec[a + b*x])^(-3/2),x]
```

output
```
(Sec[a + b*x]^2*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[2*(a
 + b*x)]))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{\int \sqrt{c\sec(a+bx)}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{3bc^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(-3/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(3*b
*c^2) + (2*Sin[a + b*x])/(3*b*c*Sqrt[c*Sec[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{-\frac{2i\sqrt{\frac{1}{\cos(bx+a)+1}}\,\mathrm{EllipticF}(i(-\cot(bx+a)+\csc(bx+a)),i)\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,(1+\sec(bx+a))}{3}+\frac{2\sin(bx+a)}{3}}{b\sqrt{c\sec(bx+a)}\,c}$	90

input
```
int(1/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/3*I*(1/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-cot(b*x+a)+csc(b*x+a)),
I)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*(1+sec(b*x+a))+2/3*sin(b*x+a))/(c*sec
(b*x+a))^(1/2)/c
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \frac{2\sqrt{\frac{c}{\cos(bx+a)}}\cos(bx+a)\sin(bx+a) - i\sqrt{2}\sqrt{c}\,\mathrm{weierstrassPInverse}(-4,0,\cos(b$$

input
```
integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(c/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) - I*sqrt(2)*sqrt(c)*
weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*sqrt
(c)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/(b*c^2)
```

Sympy [F]

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sec{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))**(3/2),x)
```

output
```
Integral((c*sec(a + b*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(c/cos(a + b*x))^(3/2),x)
```

output
```
int(1/(c/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}}{\sec(bx+a)^2}dx\right)}{c^2}$$

input
```
int(1/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(c)*int(sqrt(sec(a + b*x))/sec(a + b*x)**2,x))/c**2
```

3.23 $\int \frac{1}{(c\sec(a+bx))^{5/2}} dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(c\sec(a+bx))^{5/2}} dx = \frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

output
```
6/5*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/c^2/cos(b*x+a)^(1/2)/(c*sec(b*
x+a))^(1/2)+2/5*sin(b*x+a)/b/c/(c*sec(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c\sec(a+bx))^{5/2}} dx = \frac{\sqrt{c\sec(a+bx)}\left(12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right) + \sin(a+bx) + \sin(3(a+bx)\right.}{10bc^3}$$

input
```
Integrate[(c*Sec[a + b*x])^(-5/2),x]
```

output
```
(Sqrt[c*Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + S
in[a + b*x] + Sin[3*(a + b*x)]))/(10*b*c^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{3\int \frac{1}{\sqrt{c\sec(a+bx)}}dx}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}}dx}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3\int \sqrt{\cos(a+bx)}dx}{5c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

input
```
Int[(c*Sec[a + b*x])^(-5/2),x]
```

output

```
(6*EllipticE[(a + b*x)/2, 2])/(5*b*c^2*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b
*x]]) + (2*Sin[a + b*x])/(5*b*c*(c*Sec[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.65

method	result
default	$\frac{\frac{2 \sin(bx+a)\left(\cos(bx+a)^{2}+\cos(bx+a)+3\right)}{5}-\frac{6i\sqrt{\frac{1}{\cos(bx+a)+1}}\,\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,(\cos(bx+a)+2+\sec(bx+a))\operatorname{EllipticF}(i(-\cot(bx+a)+\csc(bx+a)),i)}{5}+}{b(\cos(bx+a)+1)\sqrt{c\sec(bx+a)}\,c^{2}}$

input

```
int(1/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b/(cos(b*x+a)+1)/(c*sec(b*x+a))^(1/2)/c^2*(sin(b*x+a)*(cos(b*x+a)^2+co
s(b*x+a)+3)-3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)
*(cos(b*x+a)+2+sec(b*x+a))*EllipticF(I*(-cot(b*x+a)+csc(b*x+a)),I)+3*I*(1/
(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)+2+sec(
b*x+a))*EllipticE(I*(-cot(b*x+a)+csc(b*x+a)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx = \frac{2\sqrt{\frac{c}{\cos(bx+a)}}\cos(bx+a)^2\sin(bx+a) + 3i\sqrt{2}\sqrt{c}\text{weierstrassZeta}(-4,0,\text{weierstr}$$

input
```
integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(c/cos(b*x + a))*cos(b*x + a)^2*sin(b*x + a) + 3*I*sqrt(2)*sqrt
(c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin
(b*x + a))) - 3*I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/(b*c^3)
```

Sympy [F]

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sec{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))**(5/2),x)
```

output
```
Integral((c*sec(a + b*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(c/cos(a + b*x))^(5/2),x)
```

output
```
int(1/(c/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}}{\sec(bx+a)^3}dx\right)}{c^3}$$

input
```
int(1/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(c)*int(sqrt(sec(a + b*x))/sec(a + b*x)**3,x))/c**3
```

3.24 $\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 100

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \frac{10\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{21bc^4} + \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}}$$

output
```
10/21*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*(c*sec(b*x+a
))^(1/2)/b/c^4+2/7*sin(b*x+a)/b/c/(c*sec(b*x+a))^(5/2)+10/21*sin(b*x+a)/b/
c^3/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \frac{\sqrt{c\sec(a+bx)}\left(40\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+26\sin(2(a+bx))+\right.}{84bc^4}$$

input
```
Integrate[(c*Sec[a + b*x])^(-7/2),x]
```

output
```
(Sqrt[c*Sec[a + b*x]]*(40*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 2
6*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)]))/(84*b*c^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\int \frac{1}{(c\sec(a+bx))^{3/2}}dx}{7c^2}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{1}{\left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7c^2}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\left(\frac{\int \sqrt{c\sec(a+bx)}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}\right)}{7c^2}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \sqrt{c\csc\left(a+bx+\frac{\pi}{2}\right)}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}\right)}{7c^2}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5\left(\frac{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}\right)}{7c^2}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}\right)}{7c^2}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5\left(\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{c\sec(a+bx)}}{3bc^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}\right)}{7c^2}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

input
```
Int[(c*Sec[a + b*x])^(-7/2),x]
```

output
```
(2*Sin[a + b*x])/(7*b*c*(c*Sec[a + b*x])^(5/2)) + (5*((2*Sqrt[Cos[a + b*x]
]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(3*b*c^2) + (2*Sin[a + b
*x])/(3*b*c*Sqrt[c*Sec[a + b*x]])))/(7*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\frac{2\sin(bx+a)\left(3\cos(bx+a)^2+5\right)}{21}+\frac{2i\sqrt{\frac{1}{\cos(bx+a)+1}}\,\mathrm{EllipticF}(i(\cot(bx+a)-\csc(bx+a)),i)\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,(5+5\sec(bx+a))}{21}}{b\sqrt{c\sec(bx+a)}\,c^3}$	104

input
```
int(1/(c*sec(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/b/(c*sec(b*x+a))^(1/2)/c^3*(sin(b*x+a)*(3*cos(b*x+a)^2+5)+I*(1/(cos(b
*x+a)+1))^(1/2)*EllipticF(I*(cot(b*x+a)-csc(b*x+a)),I)*(cos(b*x+a)/(cos(b*
x+a)+1))^(1/2)*(5+5*sec(b*x+a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \frac{2\left(3\cos\left(bx+a\right)^3+5\cos\left(bx+a\right)\right)\sqrt{\frac{c}{\cos(bx+a)}}\sin\left(bx+a\right)-5i\sqrt{2}\sqrt{c}\,\mathrm{weierstras}}{}$$

input
```
integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(b*x + a)^3 + 5*cos(b*x + a))*sqrt(c/cos(b*x + a))*sin(b*x +
 a) - 5*I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(
b*x + a)) + 5*I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) -
I*sin(b*x + a)))/(b*c^4)
```

Sympy [F]

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sec{(a+bx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))**(7/2),x)
```

output
```
Integral((c*sec(a + b*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/(c/cos(a + b*x))^(7/2),x)
```

output
```
int(1/(c/cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}}{\sec(bx+a)^4}dx\right)}{c^4}$$

input
```
int(1/(c*sec(b*x+a))^(7/2),x)
```

output
```
(sqrt(c)*int(sqrt(sec(a + b*x))/sec(a + b*x)**4,x))/c**4
```

3.25 $\int \sec^{\frac{4}{3}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sqrt[3]{\sec(a+bx)}\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

output
```
3*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*sec(b*x+a)^(1/3)*sin(b*x+a)/b/
(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx = \frac{3\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(a+bx)\right)\sqrt[3]{\sec(a+bx)}\sqrt{-\tan^2(a+bx)}}{4b}$$

input
```
Integrate[Sec[a + b*x]^(4/3),x]
```

output
```
(3*Csc[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b*x]^2]*Sec[a + b
*x]^(1/3)*Sqrt[-Tan[a + b*x]^2])/(4*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^{4/3}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{4/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(a+bx)\sqrt[3]{\sec(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Sec[a + b*x]^(4/3),x]
```

output
```
(3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sec[a + b*x]^(1/3)*Si
n[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(bx+a)^{\frac{4}{3}}\,dx$$

input
```
int(sec(b*x+a)^(4/3),x)
```

output
```
int(sec(b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^(4/3), x)
```

Sympy [F]

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx=\int \sec^{\frac{4}{3}}\left(a+bx\right)dx$$

input
```
integrate(sec(b*x+a)**(4/3),x)
```

output
```
Integral(sec(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx=\int \sec\left(bx+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx=\int \sec\left(bx+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx=\int \left(\frac{1}{\cos\left(a+b\,x\right)}\right)^{4/3}\,dx$$

input
```
int((1/cos(a + b*x))^(4/3),x)
```

output
```
int((1/cos(a + b*x))^(4/3), x)
```

Reduce [F]

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx=\int \sec\left(bx+a\right)^{\frac{4}{3}}\,dx$$

input
```
int(sec(b*x+a)^(4/3),x)
```

output
```
int(sec(a + b*x)**(1/3)*sec(a + b*x),x)
```

3.26 $\int \sec^{\frac{2}{3}}(a + bx)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \sec^{\frac{2}{3}}(a+bx)\, dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\sec(a+bx)}\sqrt{\sin^2(a+bx)}}$$

output
```
-3*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(1/3)/
(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(a+bx)\, dx = \frac{3\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{2b\sqrt[3]{\sec(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^(2/3),x]
```

output
```
(3*Csc[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b*x]^2]*Sqrt[-Tan
[a + b*x]^2])/(2*b*Sec[a + b*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^{2/3}dx$$

$$\Big\downarrow \text{4259}$$

$$\cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\sec(a+bx)}}$$

input
```
Int[Sec[a + b*x]^(2/3),x]
```

output
```
(-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Sec[
a + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int \sec (bx+a)^{\frac{2}{3}} dx$$

input
```
int(sec(b*x+a)^(2/3),x)
```

output
```
int(sec(b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx = \int \sec (bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^(2/3), x)
```

Sympy [F]

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx = \int \sec^{\frac{2}{3}}(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**(2/3),x)
```

output
```
Integral(sec(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx = \int \sec(bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx = \int \left(\frac{1}{\cos\left(a+b\,x\right)}\right)^{2/3}\,dx$$

input
```
int((1/cos(a + b*x))^(2/3),x)
```

output
```
int((1/cos(a + b*x))^(2/3), x)
```

Reduce [F]

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx = \int \sec\left(bx+a\right)^{\frac{2}{3}}\,dx$$

input
```
int(sec(b*x+a)^(2/3),x)
```

output
```
int(sec(a + b*x)**(2/3),x)
```

3.27 $\int \sqrt[3]{\sec(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \sqrt[3]{\sec(a+bx)}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sin(a+bx)}{2b\sec^{\frac{2}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

output
```
-3/2*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(2/3
)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{\sec(a+bx)}\,dx = \frac{3\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{b\sec^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Sec[a + b*x]^(1/3),x]
```

output
```
(3*Csc[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b*x]^2]*Sqrt[-Tan
[a + b*x]^2])/(b*Sec[a + b*x]^(2/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sec(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\csc\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \frac{1}{\sqrt[3]{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \frac{1}{\sqrt[3]{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}\sec^{\frac{2}{3}}(a+bx)}$$

input
```
Int[Sec[a + b*x]^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*Se
c[a + b*x]^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec \left(bx+a\right)^{\frac{1}{3}}dx$$

input
```
int(sec(b*x+a)^(1/3),x)
```

output
```
int(sec(b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{\sec(a+bx)}\,dx = \int \sec \left(bx+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{\sec(a+bx)}\,dx = \int \sqrt[3]{\sec(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**(1/3),x)
```

output
```
Integral(sec(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{\sec(a+bx)}\,dx = \int \sec(bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{\sec(a+bx)}\,dx = \int \sec(bx+a)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec(a+bx)}\,dx=\int \left(\frac{1}{\cos\left(a+b\,x\right)}\right)^{1/3}\,dx$$

input
```
int((1/cos(a + b*x))^(1/3),x)
```

output
```
int((1/cos(a + b*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{\sec(a+bx)}\,dx=\int \sec\left(bx+a\right)^{\frac{1}{3}}dx$$

input
```
int(sec(b*x+a)^(1/3),x)
```

output
```
int(sec(a + b*x)**(1/3),x)
```

3.28 $\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sin(a+bx)}{4b\sec^{\frac{4}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

output
```
-3/4*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(4/3
)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx = -\frac{3\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{b\sec^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Sec[a + b*x]^(-1/3),x]
```

output
```
(-3*Csc[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b*x]^2]*Sqrt[-T
an[a + b*x]^2])/(b*Sec[a + b*x]^(4/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4259}$$

$$\cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \sqrt[3]{\cos(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \sqrt[3]{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}\sec^{\frac{4}{3}}(a+bx)}$$

input
```
Int[Sec[a + b*x]^(-1/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*Se
c[a + b*x]^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\sec \left(bx+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/sec(b*x+a)^(1/3),x)
```

output
```
int(1/sec(b*x+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx=\int \frac{1}{\sec \left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt[3]{\sec{(a+bx)}}}\,dx$$

input
```
integrate(1/sec(b*x+a)**(1/3),x)
```

output
```
Integral(sec(a + b*x)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx = \int \frac{1}{\sec{(bx+a)}^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx = \int \frac{1}{\sec{(bx+a)}^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx = \int \frac{1}{\left(\frac{1}{\cos(a+b\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(1/cos(a + b*x))^(1/3),x)
```

output
```
int(1/(1/cos(a + b*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx = \int \frac{1}{\sec\left(bx+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/sec(b*x+a)^(1/3),x)
```

output
```
int(1/sec(a + b*x)**(1/3),x)
```

3.29 $\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sin(a+bx)}{5b\sec^{\frac{5}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

output
```
-3/5*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(5/
3)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{2b\sec^{\frac{5}{3}}(a+bx)}$$

input
```
Integrate[Sec[a + b*x]^(-2/3),x]
```

output
```
(-3*Csc[a + b*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b*x]^2]*Sqrt[-T
an[a + b*x]^2])/(2*b*Sec[a + b*x]^(5/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \cos^{\frac{2}{3}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(a+bx)}\sqrt[3]{\sec(a+bx)}\int \sin\left(a+bx+\frac{\pi}{2}\right)^{2/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}\sec^{\frac{5}{3}}(a+bx)}$$

input
```
Int[Sec[a + b*x]^(-2/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b*S
ec[a + b*x]^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\sec (bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/sec(b*x+a)^(2/3),x)
```

output
```
int(1/sec(b*x+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{1}{\sec (bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^(-2/3), x)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(1/sec(b*x+a)**(2/3),x)
```

output
```
Integral(sec(a + b*x)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\cos(a+b\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/(1/cos(a + b*x))^(2/3),x)
```

output
```
int(1/(1/cos(a + b*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/sec(b*x+a)^(2/3),x)
```

output
```
int(1/sec(a + b*x)**(2/3),x)
```

3.30 $\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sin(a+bx)}{7b\sec^{\frac{7}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

output
```
-3/7*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(7/
3)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{4b\sec^{\frac{7}{3}}(a+bx)}$$

input
```
Integrate[Sec[a + b*x]^(-4/3),x]
```

output
```
(-3*Csc[a + b*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b*x]^2]*Sqrt[-T
an[a + b*x]^2])/(4*b*Sec[a + b*x]^(7/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 4259}$$

$$\cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \cos^{\frac{4}{3}}(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \sin\left(a+bx+\frac{\pi}{2}\right)^{4/3}dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}\sec^{\frac{7}{3}}(a+bx)}$$

input
```
Int[Sec[a + b*x]^(-4/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b*S
ec[a + b*x]^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\sec\left(bx+a\right)^{\frac{4}{3}}}dx$$

input
```
int(1/sec(b*x+a)^(4/3),x)
```

output
```
int(1/sec(b*x+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\sec\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^(-4/3), x)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(1/sec(b*x+a)**(4/3),x)
```

output
```
Integral(sec(a + b*x)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{{\left(\frac{1}{\cos(a+b\,x)}\right)}^{4/3}}\,dx$$

input
```
int(1/(1/cos(a + b*x))^(4/3),x)
```

output
```
int(1/(1/cos(a + b*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\sec(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(1/sec(b*x+a)^(4/3),x)
```

output
```
int(1/(sec(a + b*x)**(1/3)*sec(a + b*x)),x)
```

3.31 $\int (c\sec(a+bx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int (c\sec(a+bx))^{4/3}\,dx=\frac{3c\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

output
```
3*c*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*(c*sec(b*x+a))^(1/3)*sin(b*x
+a)/b/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (c\sec(a+bx))^{4/3}\,dx=\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(a+bx)\right)(c\sec(a+bx))^{4/3}\sqrt{-\tan^2(a+bx)}}{4b}$$

input
```
Integrate[(c*Sec[a + b*x])^(4/3),x]
```

output
```
(3*Cot[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b*x]^2]*(c*Sec[a
+ b*x])^(4/3)*Sqrt[-Tan[a + b*x]^2])/(4*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\left(\frac{\cos(a+bx)}{c}\right)^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\left(\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c}\right)^{4/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3c\sin(a+bx)\sqrt[3]{c\sec(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(4/3),x]
```

output
```
(3*c*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*(c*Sec[a + b*x])^(1
/3)*Sin[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int (c\sec (bx+a))^{\frac{4}{3}} dx$$

input
```
int((c*sec(b*x+a))^(4/3),x)
```

output
```
int((c*sec(b*x+a))^(4/3),x)
```

Fricas [F]

$$\int (c\sec(a+bx))^{4/3}\, dx = \int (c\sec (bx+a))^{\frac{4}{3}}\, dx$$

input
```
integrate((c*sec(b*x+a))^(4/3),x, algorithm="fricas")
```

output
```
integral((c*sec(b*x + a))^(1/3)*c*sec(b*x + a), x)
```

Sympy [F]

$$\int (c\sec(a+bx))^{4/3}\,dx = \int (c\sec{(a+bx)})^{\frac{4}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(4/3),x)
```

output
```
Integral((c*sec(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int (c\sec(a+bx))^{4/3}\,dx = \int (c\sec{(bx+a)})^{\frac{4}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(4/3), x)
```

Giac [F]

$$\int (c\sec(a+bx))^{4/3}\,dx = \int (c\sec{(bx+a)})^{\frac{4}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(4/3),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sec(a+bx))^{4/3}\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^{4/3}\,dx$$

input
```
int((c/cos(a + b*x))^(4/3),x)
```

output
```
int((c/cos(a + b*x))^(4/3), x)
```

Reduce [F]

$$\int (c\sec(a+bx))^{4/3}\,dx = c^{\frac{4}{3}}\left(\int \sec(bx+a)^{\frac{4}{3}}\,dx\right)$$

input
```
int((c*sec(b*x+a))^(4/3),x)
```

output
```
c**(1/3)*int(sec(a + b*x)**(1/3)*sec(a + b*x),x)*c
```

3.32 $\int (c\sec(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int (c\sec(a+bx))^{2/3}\,dx=-\frac{3c\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{c\sec(a+bx)}\sqrt{\sin^2(a+bx)}}$$

output
```
-3*c*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a))^
(1/3)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (c\sec(a+bx))^{2/3}\,dx=\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sqrt{-\tan^2(a+bx)}}{2b}$$

input
```
Integrate[(c*Sec[a + b*x])^(2/3),x]
```

output
```
(3*Cot[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b*x]^2]*(c*Sec[a
+ b*x])^(2/3)*Sqrt[-Tan[a + b*x]^2])/(2*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \frac{1}{\left(\frac{\cos(a+bx)}{c}\right)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c}\right)^{2/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3c\sin(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{c\sec(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(2/3),x]
```

output
```
(-3*c*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(c
*Sec[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (c\sec (bx+a))^{\frac{2}{3}}dx$$

input
```
int((c*sec(b*x+a))^(2/3),x)
```

output
```
int((c*sec(b*x+a))^(2/3),x)
```

Fricas [F]

$$\int (c\sec(a+bx))^{2/3}\,dx=\int (c\sec (bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(2/3),x, algorithm="fricas")
```

output
```
integral((c*sec(b*x + a))^(2/3), x)
```

Sympy [F]

$$\int (c\sec(a+bx))^{2/3}\,dx = \int (c\sec(a+bx))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(2/3),x)
```

output
```
Integral((c*sec(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int (c\sec(a+bx))^{2/3}\,dx = \int (c\sec(bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(2/3), x)
```

Giac [F]

$$\int (c\sec(a+bx))^{2/3}\,dx = \int (c\sec(bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sec(a+bx))^{2/3}\,dx=\int \left(\frac{c}{\cos(a+b\,x)}\right)^{2/3}\,dx$$

input
```
int((c/cos(a + b*x))^(2/3),x)
```

output
```
int((c/cos(a + b*x))^(2/3), x)
```

Reduce [F]

$$\int (c\sec(a+bx))^{2/3}\,dx=c^{\frac{2}{3}}\left(\int \sec(bx+a)^{\frac{2}{3}}\,dx\right)$$

input
```
int((c*sec(b*x+a))^(2/3),x)
```

output
```
c**(2/3)*int(sec(a + b*x)**(2/3),x)
```

3.33 $\int \sqrt[3]{c\sec(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \sqrt[3]{c\sec(a+bx)}\,dx=-\frac{3c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sin(a+bx)}{2b(c\sec(a+bx))^{2/3}\sqrt{\sin^2(a+bx)}}$$

output
```
-3/2*c*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a)
)^(2/3)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \sqrt[3]{c\sec(a+bx)}\,dx$$
$$=\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\sqrt{-\tan^2(a+bx)}}{b}$$

input
```
Integrate[(c*Sec[a + b*x])^(1/3),x]
```

output
```
(3*Cot[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b*x]^2]*(c*Sec[a
+ b*x])^(1/3)*Sqrt[-Tan[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c\sec(a+bx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt[3]{c\csc\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 4259$$

$$\sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\sqrt[3]{\frac{\cos(a+bx)}{c}}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\sqrt[3]{\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c}}}dx$$

$$\Big\downarrow \ 3122$$

$$-\frac{3c\sin(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{2/3}}$$

input
```
Int[(c*Sec[a + b*x])^(1/3),x]
```

output
```
(-3*c*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*
(c*Sec[a + b*x])^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int (c\sec(bx+a))^{\frac{1}{3}}\,dx$$

input
```
int((c*sec(b*x+a))^(1/3),x)
```

output
```
int((c*sec(b*x+a))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{c\sec(a+bx)}\,dx = \int (c\sec(bx+a))^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/3),x, algorithm="fricas")
```

output
```
integral((c*sec(b*x + a))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{c\sec(a+bx)}\,dx = \int \sqrt[3]{c\sec{(a+bx)}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(1/3),x)
```

output
```
Integral((c*sec(a + b*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{c\sec(a+bx)}\,dx = \int (c\sec{(bx+a)})^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{c\sec(a+bx)}\,dx = \int (c\sec{(bx+a)})^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\sec(a+bx)}\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^{1/3} dx$$

input
```
int((c/cos(a + b*x))^(1/3),x)
```

output
```
int((c/cos(a + b*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{c\sec(a+bx)}\,dx = c^{\frac{1}{3}}\left(\int \sec(bx+a)^{\frac{1}{3}}\,dx\right)$$

input
```
int((c*sec(b*x+a))^(1/3),x)
```

output
```
c**(1/3)*int(sec(a + b*x)**(1/3),x)
```

3.34 $\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = -\frac{3c\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sin(a+bx)}{4b(c\sec(a+bx))^{4/3}\sqrt{\sin^2(a+bx)}}$$

output
```
-3/4*c*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a)
)^(4/3)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = -\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{b\sqrt[3]{c\sec(a+bx)}}$$

input
```
Integrate[(c*Sec[a + b*x])^(-1/3),x]
```

output
```
(-3*Cot[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b*x]^2]*Sqrt[-T
an[a + b*x]^2])/(b*(c*Sec[a + b*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{c\csc\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \sqrt[3]{\frac{\cos(a+bx)}{c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \sqrt[3]{\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3c\sin(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{4/3}}$$

input
```
Int[(c*Sec[a + b*x])^(-1/3),x]
```

output
```
(-3*c*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*
(c*Sec[a + b*x])^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{(c\sec(bx+a))^{\frac{1}{3}}}dx$$

input
```
int(1/(c*sec(b*x+a))^(1/3),x)
```

output
```
int(1/(c*sec(b*x+a))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = \int \frac{1}{(c\sec(bx+a))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="fricas")
```

output
```
integral((c*sec(b*x + a))^(2/3)/(c*sec(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt[3]{c\sec{(a+bx)}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))**(1/3),x)
```

output
```
Integral((c*sec(a + b*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(c/cos(a + b*x))^(1/3),x)
```

output
```
int(1/(c/cos(a + b*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx = \frac{\int \frac{1}{\sec(bx+a)^{\frac{1}{3}}}dx}{c^{\frac{1}{3}}}$$

input
```
int(1/(c*sec(b*x+a))^(1/3),x)
```

output
```
int(1/sec(a + b*x)**(1/3),x)/c**(1/3)
```

3.35 $\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = -\frac{3c\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sin(a+bx)}{5b(c\sec(a+bx))^{5/3}\sqrt{\sin^2(a+bx)}}$$

output
```
-3/5*c*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a
))^(5/3)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = -\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{2b(c\sec(a+bx))^{2/3}}$$

input
```
Integrate[(c*Sec[a + b*x])^(-2/3),x]
```

output
```
(-3*Cot[a + b*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b*x]^2]*Sqrt[-T
an[a + b*x]^2])/(2*b*(c*Sec[a + b*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$
$$\Big\downarrow \text{4259}$$
$$\sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \left(\frac{\cos(a+bx)}{c}\right)^{2/3}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \left(\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c}\right)^{2/3}dx$$
$$\Big\downarrow \text{3122}$$
$$-\frac{3c\sin(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{5/3}}$$

input
```
Int[(c*Sec[a + b*x])^(-2/3),x]
```

output
```
(-3*c*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b
*(c*Sec[a + b*x])^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(c \sec \left(b x +a \right)\right)^{\frac{2}{3}}}d x$$

input
```
int(1/(c*sec(b*x+a))^(2/3),x)
```

output
```
int(1/(c*sec(b*x+a))^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = \int \frac{1}{\left(c \sec \left(b x + a\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="fricas")
```

output
```
integral((c*sec(b*x + a))^(1/3)/(c*sec(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sec{(a+bx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))**(2/3),x)
```

output
```
Integral((c*sec(a + b*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/(c/cos(a + b*x))^(2/3),x)
```

output
```
int(1/(c/cos(a + b*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx = \frac{\int \frac{1}{\sec(bx+a)^{\frac{2}{3}}}dx}{c^{\frac{2}{3}}}$$

input
```
int(1/(c*sec(b*x+a))^(2/3),x)
```

output
```
int(1/sec(a + b*x)**(2/3),x)/c**(2/3)
```

3.36 $\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = -\frac{3c\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sin(a+bx)}{7b(c\sec(a+bx))^{7/3}\sqrt{\sin^2(a+bx)}}$$

output
```
-3/7*c*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a
))^(7/3)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = -\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{4b(c\sec(a+bx))^{4/3}}$$

input
```
Integrate[(c*Sec[a + b*x])^(-4/3),x]
```

output
```
(-3*Cot[a + b*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b*x]^2]*Sqrt[-T
an[a + b*x]^2])/(4*b*(c*Sec[a + b*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \ 4259$$

$$\left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \left(\frac{\cos(a+bx)}{c}\right)^{4/3}dx$$

$$\Big\downarrow \ 3042$$

$$\left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \left(\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c}\right)^{4/3}dx$$

$$\Big\downarrow \ 3122$$

$$-\frac{3c\sin(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}(c\sec(a+bx))^{7/3}}$$

input
```
Int[(c*Sec[a + b*x])^(-4/3),x]
```

output
```
(-3*c*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b
*(c*Sec[a + b*x])^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{\left(c\sec\left(bx+a\right)\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(c*sec(b*x+a))^(4/3),x)
```

output
```
int(1/(c*sec(b*x+a))^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = \int \frac{1}{\left(c\sec\left(bx+a\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="fricas")
```

output
```
integral((c*sec(b*x + a))^(2/3)/(c^2*sec(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sec{(a+bx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))**(4/3),x)
```

output
```
Integral((c*sec(a + b*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sec{(bx+a)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/(c/cos(a + b*x))^(4/3),x)
```

output
```
int(1/(c/cos(a + b*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx = \frac{\int \frac{1}{\sec(bx+a)^{\frac{4}{3}}}dx}{c^{\frac{4}{3}}}$$

input
```
int(1/(c*sec(b*x+a))^(4/3),x)
```

output
```
int(1/(sec(a + b*x)**(1/3)*sec(a + b*x)),x)/(c**(1/3)*c)
```

3.37 $\int \sec^n(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 70

$$\int \sec^n(a+bx)\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)\sec^{-1+n}(a+bx)\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

output
```
-hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*sec(b*x+a)^(-1+n)*si
n(b*x+a)/b/(1-n)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \sec^n(a+bx)\,dx = \frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(a+bx)\right)\sec^{-1+n}(a+bx)\sqrt{-\tan^2(a+bx)}}{bn}$$

input
```
Integrate[Sec[a + b*x]^n,x]
```

output
```
(Csc[a + b*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b*x]^2]*Sec[a
 + b*x]^(-1 + n)*Sqrt[-Tan[a + b*x]^2])/(b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4259}$$

$$\cos^n(a+bx)\sec^n(a+bx)\int \cos^{-n}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^n(a+bx)\sec^n(a+bx)\int \sin\left(a+bx+\frac{\pi}{2}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(a+bx)\sec^{n-1}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Sec[a + b*x]^n,x]
```

output
```
-((Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*Sec[a + b*
x]^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(bx+a)^n\,dx$$

input
```
int(sec(b*x+a)^n,x)
```

output
```
int(sec(b*x+a)^n,x)
```

Fricas [F]

$$\int \sec^n(a+bx)\,dx = \int \sec(bx+a)^n\,dx$$

input
```
integrate(sec(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^n, x)
```

Sympy [F]

$$\int \sec^n(a+bx)\,dx = \int \sec^n{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**n,x)
```

output
```
Integral(sec(a + b*x)**n, x)
```

Maxima [F]

$$\int \sec^n(a+bx)\,dx = \int \sec{(bx+a)}^n\,dx$$

input
```
integrate(sec(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^n, x)
```

Giac [F]

$$\int \sec^n(a+bx)\,dx = \int \sec{(bx+a)}^n\,dx$$

input
```
integrate(sec(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(a+bx)\,dx=\int \left(\frac{1}{\cos\left(a+b\,x\right)}\right)^n\,dx$$

input
```
int((1/cos(a + b*x))^n,x)
```

output
```
int((1/cos(a + b*x))^n, x)
```

Reduce [F]

$$\int \sec^n(a+bx)\,dx=\int \sec\left(bx+a\right)^n\,dx$$

input
```
int(sec(b*x+a)^n,x)
```

output
```
int(sec(a + b*x)**n,x)
```

3.38 $\int (c\sec(a+bx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int (c\sec(a+bx))^n\,dx = -\frac{c\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(c\sec(a+bx))^{-1+n}\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

output
```
-c*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(c*sec(b*x+a))^(-1
+n)*sin(b*x+a)/b/(1-n)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (c\sec(a+bx))^n\,dx = \frac{\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(a+bx)\right)(c\sec(a+bx))^n\sqrt{-\tan^2(a+bx)}}{bn}$$

input
```
Integrate[(c*Sec[a + b*x])^n,x]
```

output
```
(Cot[a + b*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b*x]^2]*(c*Se
c[a + b*x])^n*Sqrt[-Tan[a + b*x]^2])/(b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\csc\left(a+bx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(a+bx)}{c}\right)^n (c\sec(a+bx))^n \int \left(\frac{\cos(a+bx)}{c}\right)^{-n} dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(a+bx)}{c}\right)^n (c\sec(a+bx))^n \int \left(\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{c\sin(a+bx)(c\sec(a+bx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^n,x]
```

output
```
-((c*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(c*Sec[a
 + b*x])^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (c\sec (bx+a))^n\,dx$$

input
```
int((c*sec(b*x+a))^n,x)
```

output
```
int((c*sec(b*x+a))^n,x)
```

Fricas [F]

$$\int (c\sec(a+bx))^n\,dx = \int (c\sec (bx+a))^n\,dx$$

input
```
integrate((c*sec(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((c*sec(b*x + a))^n, x)
```

Sympy [F]

$$\int (c\sec(a+bx))^n\,dx=\int (c\sec(a+bx))^n\,dx$$

input
```
integrate((c*sec(b*x+a))**n,x)
```

output
```
Integral((c*sec(a + b*x))**n, x)
```

Maxima [F]

$$\int (c\sec(a+bx))^n\,dx=\int (c\sec(bx+a))^n\,dx$$

input
```
integrate((c*sec(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^n, x)
```

Giac [F]

$$\int (c\sec(a+bx))^n\,dx=\int (c\sec(bx+a))^n\,dx$$

input
```
integrate((c*sec(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sec(a+bx))^n\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^n\,dx$$

input
```
int((c/cos(a + b*x))^n,x)
```

output
```
int((c/cos(a + b*x))^n, x)
```

Reduce [F]

$$\int (c\sec(a+bx))^n\,dx = c^n \left(\int \sec(bx+a)^n\,dx\right)$$

input
```
int((c*sec(b*x+a))^n,x)
```

output
```
c**n*int(sec(a + b*x)**n,x)
```

3.39 $\int \sec^2(x)^{7/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 50

$$\int \sec^2(x)^{7/2}\,dx = \frac{5}{16}\text{arcsinh}(\tan(x)) + \frac{5}{16}\sqrt{\sec^2(x)}\tan(x) + \frac{5}{24}\sec^2(x)^{3/2}\tan(x) + \frac{1}{6}\sec^2(x)^{5/2}\tan(x)$$

output
```
5/16*arcsinh(tan(x))+5/16*(sec(x)^2)^(1/2)*tan(x)+5/24*(sec(x)^2)^(3/2)*ta
n(x)+1/6*(sec(x)^2)^(5/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \sec^2(x)^{7/2}\,dx = \frac{\sec(x)\left(15\text{arctanh}(\sin(x)) + \sec(x)\left(15 + 10\sec^2(x) + 8\sec^4(x)\right)\tan(x)\right)}{48\sqrt{\sec^2(x)}}$$

input
```
Integrate[(Sec[x]^2)^(7/2),x]
```

output
```
(Sec[x]*(15*ArcTanh[Sin[x]] + Sec[x]*(15 + 10*Sec[x]^2 + 8*Sec[x]^4)*Tan[x
]))/(48*Sqrt[Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4610, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x)^{7/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\sec(x)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{ 4610}$$

$$\int \left(\tan^2(x)+1\right)^{5/2}d\tan(x)$$

$$\Big\downarrow \text{ 211}$$

$$\frac{5}{6}\int \left(\tan^2(x)+1\right)^{3/2}d\tan(x)+\frac{1}{6}\tan(x)\left(\tan^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sqrt{\tan^2(x)+1}d\tan(x)+\frac{1}{4}\tan(x)\left(\tan^2(x)+1\right)^{3/2}\right)+\frac{1}{6}\tan(x)\left(\tan^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\tan^2(x)+1}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+1}\tan(x)\right)+\frac{1}{4}\tan(x)\left(\tan^2(x)+1\right)^{3/2}\right)+\frac{1}{6}\tan(x)\left(\tan^2(x)+1\right)^{5/2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\text{arcsinh}(\tan(x))+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+1}\right)+\frac{1}{4}\tan(x)\left(\tan^2(x)+1\right)^{3/2}\right)+\frac{1}{6}\tan(x)\left(\tan^2(x)+1\right)^{5/2}$$

input
```
Int[(Sec[x]^2)^(7/2),x]
```

output
```
(Tan[x]*(1 + Tan[x]^2)^(5/2))/6 + (5*((Tan[x]*(1 + Tan[x]^2)^(3/2))/4 + (3
*(ArcSinh[Tan[x]]/2 + (Tan[x]*Sqrt[1 + Tan[x]^2])/2))/4))/6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

method	result
default	$\frac{\operatorname{csgn}(\sec(x))\left(15\ln(\csc(x)-\cot(x)+1)-15\ln(\csc(x)-\cot(x)-1)+15\tan(x)\sec(x)+10\tan(x)\sec(x)^3+8\tan(x)\sec(x)^5\right)}{48}$
risch	$-\frac{i\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(15\,e^{10ix}+85\,e^{8ix}+198\,e^{6ix}-198\,e^{4ix}-85\,e^{2ix}-15\right)}{24\left(e^{2ix}+1\right)^5}+\frac{5\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^2}}\ln\left(e^{ix}+i\right)\cos(x)}{8}-\frac{5\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^2}}\ln(e^{ix}}{8}$

input
```
int((sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*csgn(sec(x))*(15*ln(csc(x)-cot(x)+1)-15*ln(csc(x)-cot(x)-1)+15*tan(x)
*sec(x)+10*tan(x)*sec(x)^3+8*tan(x)*sec(x)^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \sec^2(x)^{7/2} dx = -\frac{15 \cos(x)^6 \log(\sin(x) + 1) - 15 \cos(x)^6 \log(-\sin(x) + 1) + 2 (15 \cos(x)^4 + 10 \cos(x)^2 + 8) \sin(x)}{96 \cos(x)^6}$$

input
```
integrate((sec(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/96*(15*cos(x)^6*log(sin(x) + 1) - 15*cos(x)^6*log(-sin(x) + 1) + 2*(15*
cos(x)^4 + 10*cos(x)^2 + 8)*sin(x))/cos(x)^6
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(x)^{7/2} dx = \text{Timed out}$$

input
```
integrate((sec(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \sec^2(x)^{7/2}\,dx = \frac{1}{6}\left(\tan(x)^2+1\right)^{\frac{5}{2}}\tan(x) + \frac{5}{24}\left(\tan(x)^2+1\right)^{\frac{3}{2}}\tan(x) + \frac{5}{16}\sqrt{\tan(x)^2+1}\tan(x) + \frac{5}{16}\operatorname{arsinh}(\tan(x))$$

input
```
integrate((sec(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/6*(tan(x)^2 + 1)^(5/2)*tan(x) + 5/24*(tan(x)^2 + 1)^(3/2)*tan(x) + 5/16*
sqrt(tan(x)^2 + 1)*tan(x) + 5/16*arcsinh(tan(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \sec^2(x)^{7/2}\,dx = \frac{5\log(\sin(x)+1)}{32\,\mathrm{sgn}(\cos(x))} - \frac{5\log(-\sin(x)+1)}{32\,\mathrm{sgn}(\cos(x))} - \frac{15\sin(x)^5 - 40\sin(x)^3 + 33\sin(x)}{48\left(\sin(x)^2-1\right)^3\mathrm{sgn}(\cos(x))}$$

input
```
integrate((sec(x)^2)^(7/2),x, algorithm="giac")
```

output
```
5/32*log(sin(x) + 1)/sgn(cos(x)) - 5/32*log(-sin(x) + 1)/sgn(cos(x)) - 1/4
8*(15*sin(x)^5 - 40*sin(x)^3 + 33*sin(x))/((sin(x)^2 - 1)^3*sgn(cos(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(x)^{7/2}\,dx = \int \left(\frac{1}{\cos(x)^2}\right)^{7/2}\,dx$$

input
```
int((1/cos(x)^2)^(7/2),x)
```

output
```
int((1/cos(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.72

$$\int \sec^2(x)^{7/2}\,dx = \frac{-15\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^6+45\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^4-45\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x$$

input
```
int((sec(x)^2)^(7/2),x)
```

output
```
( - 15*log(tan(x/2) - 1)*sin(x)**6 + 45*log(tan(x/2) - 1)*sin(x)**4 - 45*l
og(tan(x/2) - 1)*sin(x)**2 + 15*log(tan(x/2) - 1) + 15*log(tan(x/2) + 1)*s
in(x)**6 - 45*log(tan(x/2) + 1)*sin(x)**4 + 45*log(tan(x/2) + 1)*sin(x)**2
 - 15*log(tan(x/2) + 1) - 15*sin(x)**5 + 40*sin(x)**3 - 33*sin(x))/(48*(si
n(x)**6 - 3*sin(x)**4 + 3*sin(x)**2 - 1))
```

3.40 $\int \sec^2(x)^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 36

$$\int \sec^2(x)^{5/2}\,dx = \frac{3}{8}\text{arcsinh}(\tan(x)) + \frac{3}{8}\sqrt{\sec^2(x)}\tan(x) + \frac{1}{4}\sec^2(x)^{3/2}\tan(x)$$

output
```
3/8*arcsinh(tan(x))+3/8*(sec(x)^2)^(1/2)*tan(x)+1/4*(sec(x)^2)^(3/2)*tan(x
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \sec^2(x)^{5/2}\,dx = \frac{\sec(x)\left(3\text{arctanh}(\sin(x)) + \sec(x)\left(3 + 2\sec^2(x)\right)\tan(x)\right)}{8\sqrt{\sec^2(x)}}$$

input
```
Integrate[(Sec[x]^2)^(5/2),x]
```

output
```
(Sec[x]*(3*ArcTanh[Sin[x]] + Sec[x]*(3 + 2*Sec[x]^2)*Tan[x]))/(8*Sqrt[Sec[
x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 4610, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x)^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\sec(x)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{ 4610}$$

$$\int \left(\tan^2(x)+1\right)^{3/2}d\tan(x)$$

$$\Big\downarrow \text{ 211}$$

$$\frac{3}{4}\int \sqrt{\tan^2(x)+1}d\tan(x)+\frac{1}{4}\tan(x)\left(\tan^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\tan^2(x)+1}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+1}\tan(x)\right)+\frac{1}{4}\tan(x)\left(\tan^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{3}{4}\left(\frac{1}{2}\text{arcsinh}(\tan(x))+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+1}\right)+\frac{1}{4}\tan(x)\left(\tan^2(x)+1\right)^{3/2}$$

input
```
Int[(Sec[x]^2)^(5/2),x]
```

output
```
(Tan[x]*(1 + Tan[x]^2)^(3/2))/4 + (3*(ArcSinh[Tan[x]]/2 + (Tan[x]*Sqrt[1 +
 Tan[x]^2])/2))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.71 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
default	$-\frac{\operatorname{csgn}\left(\sec \left(x \right)\right) \left(3 \ln \left(\csc \left(x \right)-\cot \left(x \right)-1\right)-3 \ln \left(\csc \left(x \right)-\cot \left(x \right)+1\right)-3 \tan \left(x \right) \sec \left(x \right)-2 \tan \left(x \right) \sec \left(x \right)^{3}\right)}{8}$	43
risch	$-\frac{i \sqrt{\frac{{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \left(3 \,{\mathrm e}^{6 i x}+11 \,{\mathrm e}^{4 i x}-11 \,{\mathrm e}^{2 i x}-3\right)}{4 \left({\mathrm e}^{2 i x}+1\right)^{3}}-\frac{3 \sqrt{\frac{{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \ln \left({\mathrm e}^{i x}-i\right) \cos \left(x \right)}{4}+\frac{3 \sqrt{\frac{{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \ln \left({\mathrm e}^{i x}+i\right) \cos \left(x \right)}{4}$	114

input
```
int((sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*csgn(sec(x))*(3*ln(csc(x)-cot(x)-1)-3*ln(csc(x)-cot(x)+1)-3*tan(x)*se
c(x)-2*tan(x)*sec(x)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \sec^2(x)^{5/2}\,dx = -\frac{3\cos(x)^4\log(\sin(x)+1) - 3\cos(x)^4\log(-\sin(x)+1) + 2\left(3\cos(x)^2+2\right)\sin(x)}{16\cos(x)^4}$$

input
```
integrate((sec(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos
(x)^2 + 2)*sin(x))/cos(x)^4
```

Sympy [F]

$$\int \sec^2(x)^{5/2}\,dx = \int \left(\sec^2(x)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((sec(x)**2)**(5/2),x)
```

output
```
Integral((sec(x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \sec^2(x)^{5/2}\,dx = \frac{1}{4}\left(\tan(x)^2+1\right)^{\frac{3}{2}}\tan(x) + \frac{3}{8}\sqrt{\tan(x)^2+1}\tan(x) + \frac{3}{8}\operatorname{arsinh}(\tan(x))$$

input
```
integrate((sec(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(tan(x)^2 + 1)^(3/2)*tan(x) + 3/8*sqrt(tan(x)^2 + 1)*tan(x) + 3/8*arcs
inh(tan(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \sec^2(x)^{5/2}\,dx = \frac{3\,\log\left(\sin\left(x\right)+1\right)}{16\,\mathrm{sgn}\left(\cos\left(x\right)\right)} - \frac{3\,\log\left(-\sin\left(x\right)+1\right)}{16\,\mathrm{sgn}\left(\cos\left(x\right)\right)} - \frac{3\,\sin\left(x\right)^3 - 5\,\sin\left(x\right)}{8\,\left(\sin\left(x\right)^2-1\right)^2\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

input
```
integrate((sec(x)^2)^(5/2),x, algorithm="giac")
```

output
```
3/16*log(sin(x) + 1)/sgn(cos(x)) - 3/16*log(-sin(x) + 1)/sgn(cos(x)) - 1/8
*(3*sin(x)^3 - 5*sin(x))/((sin(x)^2 - 1)^2*sgn(cos(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(x)^{5/2}\,dx = \int \left(\frac{1}{\cos\left(x\right)^2}\right)^{5/2}\,dx$$

input
```
int((1/cos(x)^2)^(5/2),x)
```

output
```
int((1/cos(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.72

$$\int \sec^2(x)^{5/2}\,dx = \frac{-3\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin\left(x\right)^4+6\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin\left(x\right)^2-3\log\left(\tan\left(\frac{x}{2}\right)-1\right)+3\log(}{8\sin\left(x\right)^4}$$

input
```
int((sec(x)^2)^(5/2),x)
```

output
```
( - 3*log(tan(x/2) - 1)*sin(x)**4 + 6*log(tan(x/2) - 1)*sin(x)**2 - 3*log(
tan(x/2) - 1) + 3*log(tan(x/2) + 1)*sin(x)**4 - 6*log(tan(x/2) + 1)*sin(x)
**2 + 3*log(tan(x/2) + 1) - 3*sin(x)**3 + 5*sin(x))/(8*(sin(x)**4 - 2*sin(
x)**2 + 1))
```

3.41 $\int \sec^2(x)^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 22

$$\int \sec^2(x)^{3/2}\,dx = \frac{1}{2}\text{arcsinh}(\tan(x)) + \frac{1}{2}\sqrt{\sec^2(x)}\tan(x)$$

output
```
1/2*arcsinh(tan(x))+1/2*(sec(x)^2)^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \sec^2(x)^{3/2}\,dx = \frac{\sec(x)(\text{arctanh}(\sin(x)) + \sec(x)\tan(x))}{2\sqrt{\sec^2(x)}}$$

input
```
Integrate[(Sec[x]^2)^(3/2),x]
```

output
```
(Sec[x]*(ArcTanh[Sin[x]] + Sec[x]*Tan[x]))/(2*Sqrt[Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4610, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sec(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \sqrt{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\tan^2(x)+1}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+1}\tan(x)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\text{arcsinh}(\tan(x))+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+1}$$

input
```
Int[(Sec[x]^2)^(3/2),x]
```

output
```
ArcSinh[Tan[x]]/2 + (Tan[x]*Sqrt[1 + Tan[x]^2])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

method	result	size
default	$-\frac{\operatorname{csgn}(\sec(x))(\ln(\csc(x)-\cot(x)-1)-\ln(\csc(x)-\cot(x)+1)-\tan(x)\sec(x))}{2}$	33
risch	$-\frac{i\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\left(e^{2ix}-1\right)}{e^{2ix}+1}+\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}+i\right)\cos\left(x\right)-\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}-i\right)\cos\left(x\right)$	97

input
```
int((sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(sec(x))*(ln(csc(x)-cot(x)-1)-ln(csc(x)-cot(x)+1)-tan(x)*sec(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \sec^2(x)^{3/2}\,dx = -\frac{\cos(x)^2\log(\sin(x)+1) - \cos(x)^2\log(-\sin(x)+1) + 2\sin(x)}{4\cos(x)^2}$$

input
```
integrate((sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/cos
(x)^2
```

Sympy [F]

$$\int \sec^2(x)^{3/2}\,dx = \int \left(\sec^2(x)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((sec(x)**2)**(3/2),x)
```

output
```
Integral((sec(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(x)^{3/2}\,dx = \frac{1}{2}\sqrt{\tan(x)^2+1}\tan(x) + \frac{1}{2}\operatorname{arsinh}(\tan(x))$$

input
```
integrate((sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*arcsinh(tan(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \sec^2(x)^{3/2}\,dx = \frac{\log(\sin(x)+1)}{4\,\mathrm{sgn}(\cos(x))} - \frac{\log(-\sin(x)+1)}{4\,\mathrm{sgn}(\cos(x))} - \frac{\sin(x)}{2\,(\sin(x)^2-1)\,\mathrm{sgn}(\cos(x))}$$

input
```
integrate((sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*log(sin(x) + 1)/sgn(cos(x)) - 1/4*log(-sin(x) + 1)/sgn(cos(x)) - 1/2*s
in(x)/((sin(x)^2 - 1)*sgn(cos(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(x)^{3/2}\,dx = \int \left(\frac{1}{\cos(x)^2}\right)^{3/2} dx$$

input
```
int((1/cos(x)^2)^(3/2),x)
```

output
```
int((1/cos(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \sec^2(x)^{3/2}\,dx = \frac{-\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^2 + \log\left(\tan\left(\frac{x}{2}\right)-1\right) + \log\left(\tan\left(\frac{x}{2}\right)+1\right)\sin(x)^2 - \log\left(\tan\left(\frac{x}{2}\right.\right.}{2\sin(x)^2-2}$$

input
```
int((sec(x)^2)^(3/2),x)
```

output
```
( - log(tan(x/2) - 1)*sin(x)**2 + log(tan(x/2) - 1) + log(tan(x/2) + 1)*si
n(x)**2 - log(tan(x/2) + 1) - sin(x))/(2*(sin(x)**2 - 1))
```

3.42 $\int \sqrt{\sec^2(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \sqrt{\sec^2(x)}\,dx = \operatorname{arcsinh}(\tan(x))$$

output
```
arcsinh(tan(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 14 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 4.67

$$\int \sqrt{\sec^2(x)}\,dx = \coth^{-1}(\sin(x))\cos(x)\sqrt{\sec^2(x)}$$

input
```
Integrate[Sqrt[Sec[x]^2],x]
```

output
```
ArcCoth[Sin[x]]*Cos[x]*Sqrt[Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4610, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\sec(x)^2}dx$$

$$\Big\downarrow \; 4610$$

$$\int \frac{1}{\sqrt{\tan^2(x)+1}}d\tan(x)$$

$$\Big\downarrow \; 222$$

$$\text{arcsinh}(\tan(x))$$

input
```
Int[Sqrt[Sec[x]^2],x]
```

output
```
ArcSinh[Tan[x]]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(3) = 6$.

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 6.33

method	result	size
default	$-2\sqrt{\sec\left(x\right)^{2}}\,\operatorname{arctanh}\left(-\csc\left(x\right)+\cot\left(x\right)\right)\cos\left(x\right)$	19
risch	$2\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}+i\right)\cos\left(x\right)-2\sqrt{\frac{e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,\ln\left(e^{ix}-i\right)\cos\left(x\right)$	62

input
```
int((sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(sec(x)^2)^(1/2)*arctanh(-csc(x)+cot(x))*cos(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sqrt{\sec^2(x)}\,dx = -\frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\,\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate((sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1)
```

Sympy [F]

$$\int \sqrt{\sec^2(x)}\,dx = \int \sqrt{\sec^2(x)}\,dx$$

input
```
integrate((sec(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(sec(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec^2(x)}\,dx = \operatorname{arsinh}(\tan(x))$$

input
```
integrate((sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(tan(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(3) = 6.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 11.67

$$\int \sqrt{\sec^2(x)}\,dx = \frac{\log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right)}{4\,\mathrm{sgn}(\cos(x))} - \frac{\log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right)}{4\,\mathrm{sgn}(\cos(x))}$$

input
```
integrate((sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*log(abs(1/sin(x) + sin(x) + 2))/sgn(cos(x)) - 1/4*log(abs(1/sin(x) + s
in(x) - 2))/sgn(cos(x))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec^2(x)}\,dx = \int \sqrt{\frac{1}{\cos(x)^2}}\,dx$$

input
```
int((1/cos(x)^2)^(1/2),x)
```

output
```
int((1/cos(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sqrt{\sec^2(x)}\,dx = -\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
int((sec(x)^2)^(1/2),x)
```

output
```
- log(tan(x/2) - 1) + log(tan(x/2) + 1)
```

3.43 $\int \frac{1}{\sqrt{\sec^2(x)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{\sqrt{\sec^2(x)}}\,dx=\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

output
```
tan(x)/(sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\sec^2(x)}}\,dx=\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

input
```
Integrate[1/Sqrt[Sec[x]^2],x]
```

output
```
Tan[x]/Sqrt[Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec^2(x)}} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\sec(x)^2}} dx$$
$$\Big\downarrow \text{4610}$$
$$\int \frac{1}{\left(\tan^2(x)+1\right)^{3/2}} d\tan(x)$$
$$\Big\downarrow \text{208}$$
$$\frac{\tan(x)}{\sqrt{\tan^2(x)+1}}$$

input
```
Int[1/Sqrt[Sec[x]^2],x]
```

output
```
Tan[x]/Sqrt[1 + Tan[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
default	$\sin(x)\operatorname{csgn}(\sec(x))$	7
risch	$-\frac{i e^{2ix}}{2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}+\frac{i}{2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}$	65

input
```
int(1/(sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sin(x)*csgn(sec(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{\sec^2(x)}}\,dx = -\sin(x)$$

input
```
integrate(1/(sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{\sec^2(x)}} \, dx = \frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

input
```
integrate(1/(sec(x)**2)**(1/2),x)
```

output
```
tan(x)/sqrt(sec(x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\sec^2(x)}} \, dx = \frac{\tan(x)}{\sqrt{\tan(x)^2 + 1}}$$

input
```
integrate(1/(sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
tan(x)/sqrt(tan(x)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{\sec^2(x)}} \, dx = \operatorname{sgn}(\cos(x)) \sin(x)$$

input
```
integrate(1/(sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sgn(cos(x))*sin(x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\sec^2(x)}}\,dx = \frac{\sqrt{2}\,\sin(2\,x)}{2\,\sqrt{2\cos(x)^2}}$$

input
```
int(1/(1/cos(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*sin(2*x))/(2*(2*cos(x)^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{\sec^2(x)}}\,dx = \sin(x)$$

input
```
int(1/(sec(x)^2)^(1/2),x)
```

output
```
sin(x)
```

3.44 $\int \frac{1}{\sec^2(x)^{3/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx=\frac{\tan(x)}{3\sec^2(x)^{3/2}}+\frac{2\tan(x)}{3\sqrt{\sec^2(x)}}$$

output
```
1/3*tan(x)/(sec(x)^2)^(3/2)+2/3*tan(x)/(sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx=\frac{\tan(x)}{\sqrt{\sec^2(x)}}-\frac{\tan^3(x)}{3\sec^2(x)^{3/2}}$$

input
```
Integrate[(Sec[x]^2)^(-3/2),x]
```

output
```
Tan[x]/Sqrt[Sec[x]^2] - Tan[x]^3/(3*(Sec[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^2(x)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)^{5/2}}d\tan(x)$$

$$\Big\downarrow \text{209}$$

$$\frac{2}{3}\int \frac{1}{\left(\tan^2(x)+1\right)^{3/2}}d\tan(x)+\frac{\tan(x)}{3\left(\tan^2(x)+1\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2\tan(x)}{3\sqrt{\tan^2(x)+1}}+\frac{\tan(x)}{3\left(\tan^2(x)+1\right)^{3/2}}$$

input
```
Int[(Sec[x]^2)^(-3/2),x]
```

output
```
Tan[x]/(3*(1 + Tan[x]^2)^(3/2)) + (2*Tan[x])/(3*Sqrt[1 + Tan[x]^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{2\,\mathrm{csgn}(\sec(x))\left(-\sin(x)-\frac{\sin(x)\cos(x)^{2}}{2}\right)}{3}$	19
risch	$-\frac{i e^{4 i x}}{24\left(e^{2 i x}+1\right)\sqrt{\frac{e^{2 i x}}{\left(e^{2 i x}+1\right)^{2}}}}-\frac{3 i e^{2 i x}}{8\sqrt{\frac{e^{2 i x}}{\left(e^{2 i x}+1\right)^{2}}}\left(e^{2 i x}+1\right)}+\frac{3 i}{8\sqrt{\frac{e^{2 i x}}{\left(e^{2 i x}+1\right)^{2}}}\left(e^{2 i x}+1\right)}+\frac{i e^{-2 i x}}{24\left(e^{2 i x}+1\right)\sqrt{\frac{e^{2 i x}}{\left(e^{2 i x}+1\right)^{2}}}}$	133

input
```
int(1/(sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*csgn(sec(x))*(-sin(x)-1/2*sin(x)*cos(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx = -\frac{1}{3}\left(\cos\left(x\right)^2+2\right)\sin\left(x\right)$$

input
```
integrate(1/(sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx = \frac{2\tan^3\left(x\right)}{3\left(\sec^2\left(x\right)\right)^{\frac{3}{2}}}+\frac{\tan\left(x\right)}{\left(\sec^2\left(x\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(sec(x)**2)**(3/2),x)
```

output
```
2*tan(x)**3/(3*(sec(x)**2)**(3/2)) + tan(x)/(sec(x)**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx = \frac{2\,\tan\left(x\right)}{3\,\sqrt{\tan\left(x\right)^2+1}}+\frac{\tan\left(x\right)}{3\left(\tan\left(x\right)^2+1\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
2/3*tan(x)/sqrt(tan(x)^2 + 1) + 1/3*tan(x)/(tan(x)^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx = -\frac{1}{3}\,\mathrm{sgn}(\cos(x))\sin(x)^3 + \mathrm{sgn}(\cos(x))\sin(x)$$

input
```
integrate(1/(sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*sgn(cos(x))*sin(x)^3 + sgn(cos(x))*sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cos(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(1/cos(x)^2)^(3/2),x)
```

output
```
int(1/(1/cos(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sec^2(x)^{3/2}}\,dx = \frac{\sin(x)\left(-\sin(x)^2+3\right)}{3}$$

input
```
int(1/(sec(x)^2)^(3/2),x)
```

output
```
(sin(x)*( - sin(x)**2 + 3))/3
```

3.45 $\int \frac{1}{\sec^2(x)^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx=\frac{\tan(x)}{5\sec^2(x)^{5/2}}+\frac{4\tan(x)}{15\sec^2(x)^{3/2}}+\frac{8\tan(x)}{15\sqrt{\sec^2(x)}}$$

output
```
1/5*tan(x)/(sec(x)^2)^(5/2)+4/15*tan(x)/(sec(x)^2)^(3/2)+8/15*tan(x)/(sec(
x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx=\frac{(15-10\sin^2(x)+3\sin^4(x))\tan(x)}{15\sqrt{\sec^2(x)}}$$

input
```
Integrate[(Sec[x]^2)^(-5/2),x]
```

output
```
((15 - 10*Sin[x]^2 + 3*Sin[x]^4)*Tan[x])/(15*Sqrt[Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 4610, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^2(x)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec(x)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)^{7/2}}d\tan(x)$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{5}\int \frac{1}{\left(\tan^2(x)+1\right)^{5/2}}d\tan(x)+\frac{\tan(x)}{5\left(\tan^2(x)+1\right)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(\tan^2(x)+1\right)^{3/2}}d\tan(x)+\frac{\tan(x)}{3\left(\tan^2(x)+1\right)^{3/2}}\right)+\frac{\tan(x)}{5\left(\tan^2(x)+1\right)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\tan(x)}{5\left(\tan^2(x)+1\right)^{5/2}}+\frac{4}{5}\left(\frac{2\tan(x)}{3\sqrt{\tan^2(x)+1}}+\frac{\tan(x)}{3\left(\tan^2(x)+1\right)^{3/2}}\right)$$

input
```
Int[(Sec[x]^2)^(-5/2),x]
```

output
```
Tan[x]/(5*(1 + Tan[x]^2)^(5/2)) + (4*(Tan[x]/(3*(1 + Tan[x]^2)^(3/2)) + (2
*Tan[x])/(3*Sqrt[1 + Tan[x]^2])))/5
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.51

method	result
default	$-\frac{2\sin(x)\operatorname{csgn}(\sec(x))\left(-4-\frac{3\cos(x)^4}{2}-2\cos(x)^2\right)}{15}$
risch	$-\frac{i e^{6ix}}{160(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{5i e^{2ix}}{16\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}(e^{2ix}+1)}+\frac{5i}{16\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}(e^{2ix}+1)}+\frac{5i e^{-2ix}}{96(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{11}{240\sqrt{(e^2}}$

input
```
int(1/(sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*sin(x)*csgn(sec(x))*(-4-3/2*cos(x)^4-2*cos(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx = -\frac{1}{15}\left(3\,\cos\left(x\right)^{4}+4\,\cos\left(x\right)^{2}+8\right)\sin\left(x\right)$$

input
```
integrate(1/(sec(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx = \frac{8\tan^{5}\left(x\right)}{15\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{4\tan^{3}\left(x\right)}{3\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{\tan\left(x\right)}{\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(sec(x)**2)**(5/2),x)
```

output
```
8*tan(x)**5/(15*(sec(x)**2)**(5/2)) + 4*tan(x)**3/(3*(sec(x)**2)**(5/2)) +
 tan(x)/(sec(x)**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx = \frac{8\,\tan\left(x\right)}{15\,\sqrt{\tan\left(x\right)^{2}+1}}+\frac{4\,\tan\left(x\right)}{15\,\left(\tan\left(x\right)^{2}+1\right)^{\frac{3}{2}}}+\frac{\tan\left(x\right)}{5\,\left(\tan\left(x\right)^{2}+1\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(sec(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
8/15*tan(x)/sqrt(tan(x)^2 + 1) + 4/15*tan(x)/(tan(x)^2 + 1)^(3/2) + 1/5*ta
n(x)/(tan(x)^2 + 1)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx = \frac{1}{5}\operatorname{sgn}(\cos(x))\sin(x)^5 - \frac{2}{3}\operatorname{sgn}(\cos(x))\sin(x)^3 + \operatorname{sgn}(\cos(x))\sin(x)$$

input
```
integrate(1/(sec(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/5*sgn(cos(x))*sin(x)^5 - 2/3*sgn(cos(x))*sin(x)^3 + sgn(cos(x))*sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cos(x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(1/cos(x)^2)^(5/2),x)
```

output
```
int(1/(1/cos(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^2(x)^{5/2}}\,dx = \frac{\sin(x)\left(3\sin(x)^4 - 10\sin(x)^2 + 15\right)}{15}$$

input
```
int(1/(sec(x)^2)^(5/2),x)
```

output
```
(sin(x)*(3*sin(x)**4 - 10*sin(x)**2 + 15))/15
```

3.46 $\int \frac{1}{\sec^2(x)^{7/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6\tan(x)}{35\sec^2(x)^{5/2}} + \frac{8\tan(x)}{35\sec^2(x)^{3/2}} + \frac{16\tan(x)}{35\sqrt{\sec^2(x)}}$$

output
```
1/7*tan(x)/(sec(x)^2)^(7/2)+6/35*tan(x)/(sec(x)^2)^(5/2)+8/35*tan(x)/(sec(
x)^2)^(3/2)+16/35*tan(x)/(sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = \frac{(35 - 35\sin^2(x) + 21\sin^4(x) - 5\sin^6(x))\tan(x)}{35\sqrt{\sec^2(x)}}$$

input
```
Integrate[(Sec[x]^2)^(-7/2),x]
```

output
```
((35 - 35*Sin[x]^2 + 21*Sin[x]^4 - 5*Sin[x]^6)*Tan[x])/(35*Sqrt[Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4610, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^2(x)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sec(x)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)^{9/2}}d\tan(x)$$

$$\Big\downarrow \text{209}$$

$$\frac{6}{7}\int \frac{1}{\left(\tan^2(x)+1\right)^{7/2}}d\tan(x)+\frac{\tan(x)}{7\left(\tan^2(x)+1\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{1}{\left(\tan^2(x)+1\right)^{5/2}}d\tan(x)+\frac{\tan(x)}{5\left(\tan^2(x)+1\right)^{5/2}}\right)+\frac{\tan(x)}{7\left(\tan^2(x)+1\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\left(\tan^2(x)+1\right)^{3/2}}d\tan(x)+\frac{\tan(x)}{3\left(\tan^2(x)+1\right)^{3/2}}\right)+\frac{\tan(x)}{5\left(\tan^2(x)+1\right)^{5/2}}\right)+\frac{\tan(x)}{7\left(\tan^2(x)+1\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\tan(x)}{7\left(\tan^2(x)+1\right)^{7/2}}+\frac{6}{7}\left(\frac{\tan(x)}{5\left(\tan^2(x)+1\right)^{5/2}}+\frac{4}{5}\left(\frac{2\tan(x)}{3\sqrt{\tan^2(x)+1}}+\frac{\tan(x)}{3\left(\tan^2(x)+1\right)^{3/2}}\right)\right)$$

input
```
Int[(Sec[x]^2)^(-7/2),x]
```

output
```
Tan[x]/(7*(1 + Tan[x]^2)^(7/2)) + (6*(Tan[x]/(5*(1 + Tan[x]^2)^(5/2)) + (4
*(Tan[x]/(3*(1 + Tan[x]^2)^(3/2)) + (2*Tan[x])/(3*Sqrt[1 + Tan[x]^2])))/5)
)/7
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.49

method	result
default	$-\frac{2\sin(x)\operatorname{csgn}(\sec(x))\left(-8-\frac{5\cos(x)^6}{2}-3\cos(x)^4-4\cos(x)^2\right)}{35}$
risch	$-\frac{i e^{8ix}}{896(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{35 i e^{2ix}}{128\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}(e^{2ix}+1)}+\frac{35 i}{128\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}(e^{2ix}+1)}+\frac{7 i e^{-2ix}}{128(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{}{1120}$

input
```
int(1/(sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*sin(x)*csgn(sec(x))*(-8-5/2*cos(x)^6-3*cos(x)^4-4*cos(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = -\frac{1}{35}\left(5\,\cos\left(x\right)^{6} + 6\,\cos\left(x\right)^{4} + 8\,\cos\left(x\right)^{2} + 16\right)\sin\left(x\right)$$

input
```
integrate(1/(sec(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/35*(5*cos(x)^6 + 6*cos(x)^4 + 8*cos(x)^2 + 16)*sin(x)
```

Sympy [A] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = \frac{16\tan^{7}\left(x\right)}{35\left(\sec^{2}\left(x\right)\right)^{\frac{7}{2}}} + \frac{8\tan^{5}\left(x\right)}{5\left(\sec^{2}\left(x\right)\right)^{\frac{7}{2}}} + \frac{2\tan^{3}\left(x\right)}{\left(\sec^{2}\left(x\right)\right)^{\frac{7}{2}}} + \frac{\tan\left(x\right)}{\left(\sec^{2}\left(x\right)\right)^{\frac{7}{2}}}$$

input
```
integrate(1/(sec(x)**2)**(7/2),x)
```

output
```
16*tan(x)**7/(35*(sec(x)**2)**(7/2)) + 8*tan(x)**5/(5*(sec(x)**2)**(7/2))
+ 2*tan(x)**3/(sec(x)**2)**(7/2) + tan(x)/(sec(x)**2)**(7/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = \frac{16\,\tan(x)}{35\,\sqrt{\tan(x)^2+1}} + \frac{8\,\tan(x)}{35\,\left(\tan(x)^2+1\right)^{\frac{3}{2}}} + \frac{6\,\tan(x)}{35\,\left(\tan(x)^2+1\right)^{\frac{5}{2}}} + \frac{\tan(x)}{7\,\left(\tan(x)^2+1\right)^{\frac{7}{2}}}$$

input
```
integrate(1/(sec(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
16/35*tan(x)/sqrt(tan(x)^2 + 1) + 8/35*tan(x)/(tan(x)^2 + 1)^(3/2) + 6/35*
tan(x)/(tan(x)^2 + 1)^(5/2) + 1/7*tan(x)/(tan(x)^2 + 1)^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = -\frac{1}{7}\operatorname{sgn}(\cos(x))\sin(x)^7 + \frac{3}{5}\operatorname{sgn}(\cos(x))\sin(x)^5 - \operatorname{sgn}(\cos(x))\sin(x)^3 + \operatorname{sgn}(\cos(x))\sin(x)$$

input
```
integrate(1/(sec(x)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/7*sgn(cos(x))*sin(x)^7 + 3/5*sgn(cos(x))*sin(x)^5 - sgn(cos(x))*sin(x)^
3 + sgn(cos(x))*sin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cos(x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(1/cos(x)^2)^(7/2),x)
```

output
```
int(1/(1/cos(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^2(x)^{7/2}}\,dx = \frac{\sin(x)\left(-5\sin(x)^6+21\sin(x)^4-35\sin(x)^2+35\right)}{35}$$

input
```
int(1/(sec(x)^2)^(7/2),x)
```

output
```
(sin(x)*( - 5*sin(x)**6 + 21*sin(x)**4 - 35*sin(x)**2 + 35))/35
```

3.47 $\int (a\sec^2(x))^{7/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\int (a\sec^2(x))^{7/2}\,dx = \frac{5}{16}a^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{5}{16}a^3\sqrt{a\sec^2(x)}\tan(x) + \frac{5}{24}a^2(a\sec^2(x))^{3/2}\tan(x)+\frac{1}{6}a(a\sec^2(x))^{5/2}\tan(x)$$

output
```
5/16*a^(7/2)*arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))+5/16*a^3*(a*sec(x)
^2)^(1/2)*tan(x)+5/24*a^2*(a*sec(x)^2)^(3/2)*tan(x)+1/6*a*(a*sec(x)^2)^(5/
2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int (a\sec^2(x))^{7/2}\,dx = \frac{1}{48}a^3\cos(x)\sqrt{a\sec^2(x)}(15\text{arctanh}(\sin(x)) + \sec(x)\left(15+10\sec^2(x)+8\sec^4(x)\right)\tan(x))$$

input
```
Integrate[(a*Sec[x]^2)^(7/2),x]
```

output
```
(a^3*Cos[x]*Sqrt[a*Sec[x]^2]*(15*ArcTanh[Sin[x]] + Sec[x]*(15 + 10*Sec[x]^
2 + 8*Sec[x]^4)*Tan[x]))/48
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 4610, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sec^2(x)\right)^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(x)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \left(a\tan^2(x)+a\right)^{5/2}d\tan(x)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{5}{6}a\int \left(a\tan^2(x)+a\right)^{3/2}d\tan(x)+\frac{1}{6}\tan(x)\left(a\tan^2(x)+a\right)^{5/2}\right)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{a\tan^2(x)+a}d\tan(x)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\right)+\frac{1}{6}\tan(x)\left(a\tan^2(x)+a\right)^{5/2}\right)$$

$$\Big\downarrow \text{211}$$

$$a\Bigg(\frac{5}{6}a\Bigg(\frac{3}{4}a\Bigg(\frac{1}{2}a\int \frac{1}{\sqrt{a\tan^2(x)+a}}d\tan(x)+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\Bigg)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\Bigg)+\frac{1}{6}\tan$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{a\tan^2(x)}{a\tan^2(x)+a}}d\frac{\tan(x)}{\sqrt{a\tan^2(x)+a}}+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\tan^2(x)+a}}\right)+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\right)+\frac{1}{6}\right.$$

input
```
Int[(a*Sec[x]^2)^(7/2),x]
```

output
```
a*((Tan[x]*(a + a*Tan[x]^2)^(5/2))/6 + (5*a*((Tan[x]*(a + a*Tan[x]^2)^(3/2
))/4 + (3*a*((Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a + a*Tan[x]^2]])/2 +
(Tan[x]*Sqrt[a + a*Tan[x]^2])/2))/4))/6)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

method	result
default	$\left(\frac{5\ln(\csc(x)-\cot(x)+1)\cos(x)}{16}-\frac{5\ln(\csc(x)-\cot(x)-1)\cos(x)}{16}+\frac{5\tan(x)}{16}+\frac{5\tan(x)\sec(x)^2}{24}+\frac{\tan(x)\sec(x)^4}{6}\right)\sqrt{a\sec}$
risch	$-\frac{ia^3\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(15\,e^{10ix}+85\,e^{8ix}+198\,e^{6ix}-198\,e^{4ix}-85\,e^{2ix}-15\right)}{24\left(e^{2ix}+1\right)^5}-\frac{5a^3\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\ln\left(e^{ix}-i\right)\cos(x)}{8}+\frac{5a^3\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}{}$

input
```
int((a*sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(5/16*ln(csc(x)-cot(x)+1)*cos(x)-5/16*ln(csc(x)-cot(x)-1)*cos(x)+5/16*tan(
x)+5/24*tan(x)*sec(x)^2+1/6*tan(x)*sec(x)^4)*(a*sec(x)^2)^(1/2)*a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \left(a\sec^2(x)\right)^{7/2}\,dx = -\frac{\left(15\,a^3\cos\left(x\right)^6\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(15\,a^3\cos\left(x\right)^4+10\,a^3\cos\left(x\right)^2+8\,a^3\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^2}}}{96\,\cos\left(x\right)^5}$$

input
```
integrate((a*sec(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/96*(15*a^3*cos(x)^6*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(15*a^3*cos(x)^
4 + 10*a^3*cos(x)^2 + 8*a^3)*sin(x))*sqrt(a/cos(x)^2)/cos(x)^5
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\sec^2(x)\right)^{7/2} \, dx = \text{Timed out}$$

input
```
integrate((a*sec(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2175 vs. $2(64) = 128$.

Time $= 2.04$ (sec) , antiderivative size $= 2175$, normalized size of antiderivative $= 25.89$

$$\int \left(a\sec^2(x)\right)^{7/2} \, dx = \text{Too large to display}$$

input
```
integrate((a*sec(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/96*(2040*a^3*cos(3*x)*sin(2*x) + 360*a^3*cos(x)*sin(2*x) - 360*a^3*cos(2
*x)*sin(x) - 60*a^3*sin(x) + 4*(15*a^3*sin(11*x) + 85*a^3*sin(9*x) + 198*a
^3*sin(7*x) - 198*a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(12*x
) - 60*(6*a^3*sin(10*x) + 15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4
*x) + 6*a^3*sin(2*x))*cos(11*x) + 24*(85*a^3*sin(9*x) + 198*a^3*sin(7*x) -
 198*a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(10*x) - 340*(15*a
^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*cos(9*x)
 + 60*(198*a^3*sin(7*x) - 198*a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(
x))*cos(8*x) - 792*(20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*co
s(7*x) - 80*(198*a^3*sin(5*x) + 85*a^3*sin(3*x) + 15*a^3*sin(x))*cos(6*x)
+ 2376*(5*a^3*sin(4*x) + 2*a^3*sin(2*x))*cos(5*x) - 300*(17*a^3*sin(3*x) +
 3*a^3*sin(x))*cos(4*x) + 15*(a^3*cos(12*x)^2 + 36*a^3*cos(10*x)^2 + 225*a
^3*cos(8*x)^2 + 400*a^3*cos(6*x)^2 + 225*a^3*cos(4*x)^2 + 36*a^3*cos(2*x)^
2 + a^3*sin(12*x)^2 + 36*a^3*sin(10*x)^2 + 225*a^3*sin(8*x)^2 + 400*a^3*si
n(6*x)^2 + 225*a^3*sin(4*x)^2 + 180*a^3*sin(4*x)*sin(2*x) + 36*a^3*sin(2*x
)^2 + 12*a^3*cos(2*x) + a^3 + 2*(6*a^3*cos(10*x) + 15*a^3*cos(8*x) + 20*a^
3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(12*x) + 12*(15*a^
3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos
(10*x) + 30*(20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos
(8*x) + 40*(15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(6*x) + 30*(6*a^...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \left(a\sec^2(x)\right)^{7/2}\,dx = \frac{1}{96}\left(15\,a^3\log\left(\sin\left(x\right)+1\right)\text{sgn}(\cos\left(x\right)) - 15\,a^3\log\left(-\sin\left(x\right)+1\right)\text{sgn}(\cos\left(x\right)) - \frac{2\left(\right.}{}\right.$$

input
```
integrate((a*sec(x)^2)^(7/2),x, algorithm="giac")
```

output
```
1/96*(15*a^3*log(sin(x) + 1)*sgn(cos(x)) - 15*a^3*log(-sin(x) + 1)*sgn(cos
(x)) - 2*(15*a^3*sgn(cos(x))*sin(x)^5 - 40*a^3*sgn(cos(x))*sin(x)^3 + 33*a
^3*sgn(cos(x))*sin(x))/(sin(x)^2 - 1)^3)*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sec^2(x)\right)^{7/2} dx = \int \left(\frac{a}{\cos(x)^2}\right)^{7/2} dx$$

input
```
int((a/cos(x)^2)^(7/2),x)
```

output
```
int((a/cos(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.68

$$\int \left(a\sec^2(x)\right)^{7/2} dx = \frac{\sqrt{a}\,a^3\left(-15\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^6+45\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^4-45\log\left(\tan\left(\frac{x}{2}\right)\right.\right.}{}$$

input
```
int((a*sec(x)^2)^(7/2),x)
```

output
```
(sqrt(a)*a**3*( - 15*log(tan(x/2) - 1)*sin(x)**6 + 45*log(tan(x/2) - 1)*si
n(x)**4 - 45*log(tan(x/2) - 1)*sin(x)**2 + 15*log(tan(x/2) - 1) + 15*log(t
an(x/2) + 1)*sin(x)**6 - 45*log(tan(x/2) + 1)*sin(x)**4 + 45*log(tan(x/2)
+ 1)*sin(x)**2 - 15*log(tan(x/2) + 1) - 15*sin(x)**5 + 40*sin(x)**3 - 33*s
in(x)))/(48*(sin(x)**6 - 3*sin(x)**4 + 3*sin(x)**2 - 1))
```

3.48 $\int \left(a\sec^2(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \left(a\sec^2(x)\right)^{5/2} dx = \frac{3}{8}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right) + \frac{3}{8}a^2\sqrt{a\sec^2(x)}\tan(x) + \frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x)$$

output
```
3/8*a^(5/2)*arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))+3/8*a^2*(a*sec(x)^2
)^(1/2)*tan(x)+1/4*a*(a*sec(x)^2)^(3/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \left(a\sec^2(x)\right)^{5/2} dx = \frac{1}{8}\cos(x)\left(a\sec^2(x)\right)^{5/2}\left(3\text{arctanh}(\sin(x))\cos^4(x)+\left(2+3\cos^2(x)\right)\sin(x)\right)$$

input
```
Integrate[(a*Sec[x]^2)^(5/2),x]
```

output
```
(Cos[x]*(a*Sec[x]^2)^(5/2)*(3*ArcTanh[Sin[x]]*Cos[x]^4 + (2 + 3*Cos[x]^2)*
Sin[x]))/8
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4610, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sec^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(x)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \left(a\tan^2(x)+a\right)^{3/2}d\tan(x)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{3}{4}a\int \sqrt{a\tan^2(x)+a}d\tan(x)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a\tan^2(x)+a}}d\tan(x)+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{a\tan^2(x)}{a\tan^2(x)+a}}d\frac{\tan(x)}{\sqrt{a\tan^2(x)+a}}+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{3}{4}a\left(\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\tan^2(x)+a}}\right)+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)+\frac{1}{4}\tan(x)\left(a\tan^2(x)+a\right)^{3/2}\right)$$

input
```
Int[(a*Sec[x]^2)^(5/2),x]
```

output
```
a*((Tan[x]*(a + a*Tan[x]^2)^(3/2))/4 + (3*a*((Sqrt[a]*ArcTanh[(Sqrt[a]*Tan
[x])/Sqrt[a + a*Tan[x]^2]])/2 + (Tan[x]*Sqrt[a + a*Tan[x]^2])/2))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	si
default	$\left(-\frac{3\ln\left(\csc\left(x\right)-\cot\left(x\right)-1\right)\cos\left(x\right)}{8}+\frac{3\ln\left(\csc\left(x\right)-\cot\left(x\right)+1\right)\cos\left(x\right)}{8}+\frac{3\tan\left(x\right)}{8}+\frac{\tan\left(x\right)\sec\left(x\right)^{2}}{4}\right)\sqrt{a\sec\left(x\right)^{2}}\,a^{2}$	5
risch	$-\frac{i a^{2}\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\left(3\,{\mathrm e}^{6 i x}+11\,{\mathrm e}^{4 i x}-11\,{\mathrm e}^{2 i x}-3\right)}{4\left({\mathrm e}^{2 i x}+1\right)^{3}}-\frac{3 a^{2}\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\ln\left({\mathrm e}^{i x}-i\right)\cos\left(x\right)}{4}+\frac{3 a^{2}\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\,\ln\left({\mathrm e}^{i x}+i\right)\cos\left(x\right)}{4}$	1

input
```
int((a*sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-3/8*ln(csc(x)-cot(x)-1)*cos(x)+3/8*ln(csc(x)-cot(x)+1)*cos(x)+3/8*tan(x)
+1/4*tan(x)*sec(x)^2)*(a*sec(x)^2)^(1/2)*a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \left(a\sec^2(x)\right)^{5/2}\,dx = -\frac{\left(3\,a^2\cos\left(x\right)^4\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(3\,a^2\cos\left(x\right)^2+2\,a^2\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^2}}}{16\,\cos\left(x\right)^3}$$

input
```
integrate((a*sec(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(3*a^2*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*a^2*cos(x)^2
+ 2*a^2)*sin(x))*sqrt(a/cos(x)^2)/cos(x)^3
```

Sympy [F]

$$\int \left(a\sec^2(x)\right)^{5/2}\,dx = \int \left(a\sec^2\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sec(x)**2)**(5/2),x)
```

output
```
Integral((a*sec(x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 1111, normalized size of antiderivative = 17.09

$$\int \left(a\sec^2(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*sec(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*(176*a^2*cos(3*x)*sin(2*x) + 48*a^2*cos(x)*sin(2*x) - 48*a^2*cos(2*x)
*sin(x) - 12*a^2*sin(x) + 4*(3*a^2*sin(7*x) + 11*a^2*sin(5*x) - 11*a^2*sin
(3*x) - 3*a^2*sin(x))*cos(8*x) - 24*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a
^2*sin(2*x))*cos(7*x) + 16*(11*a^2*sin(5*x) - 11*a^2*sin(3*x) - 3*a^2*sin(
x))*cos(6*x) - 88*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*cos(5*x) - 24*(11*a^2*
sin(3*x) + 3*a^2*sin(x))*cos(4*x) + 3*(a^2*cos(8*x)^2 + 16*a^2*cos(6*x)^2
+ 36*a^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 16*a^2*sin(6*x)
^2 + 36*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*sin(2*x)^2 + 8*
a^2*cos(2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(2*x) +
 a^2)*cos(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(6*x) + 12*(
4*a^2*cos(2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a^
2*sin(2*x))*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6*x))*log(
cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 3*(a^2*cos(8*x)^2 + 16*a^2*cos(6*x)^
2 + 36*a^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 16*a^2*sin(6*
x)^2 + 36*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*sin(2*x)^2 +
8*a^2*cos(2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(2*x)
 + a^2)*cos(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(6*x) + 12
*(4*a^2*cos(2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*
a^2*sin(2*x))*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6*x))*lo
g(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(3*a^2*cos(7*x) + 11*a^2*cos(...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \left(a\sec^2(x)\right)^{5/2} dx = \frac{1}{16}\left(3\log\left(\sin\left(x\right)+1\right)\mathrm{sgn}(\cos\left(x\right))-3\log\left(-\sin\left(x\right)+1\right)\mathrm{sgn}(\cos\left(x\right))-\frac{2\left(3\,\mathrm{sgn}(\mathrm{c}\right.}{}\right.$$

input
```
integrate((a*sec(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*(3*log(sin(x) + 1)*sgn(cos(x)) - 3*log(-sin(x) + 1)*sgn(cos(x)) - 2*(
3*sgn(cos(x))*sin(x)^3 - 5*sgn(cos(x))*sin(x))/(sin(x)^2 - 1)^2)*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sec^2(x)\right)^{5/2} dx = \int \left(\frac{a}{\cos\left(x\right)^2}\right)^{5/2} dx$$

input
```
int((a/cos(x)^2)^(5/2),x)
```

output
```
int((a/cos(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \left(a\sec^2(x)\right)^{5/2} dx = \frac{\sqrt{a}\,a^2\left(-3\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin\left(x\right)^4+6\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin\left(x\right)^2-3\log\left(\tan\left(\frac{x}{2}\right)-\right.\right.}{8\!}$$

input
```
int((a*sec(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - 3*log(tan(x/2) - 1)*sin(x)**4 + 6*log(tan(x/2) - 1)*sin(
x)**2 - 3*log(tan(x/2) - 1) + 3*log(tan(x/2) + 1)*sin(x)**4 - 6*log(tan(x/
2) + 1)*sin(x)**2 + 3*log(tan(x/2) + 1) - 3*sin(x)**3 + 5*sin(x)))/(8*(sin
(x)**4 - 2*sin(x)**2 + 1))
```

3.49 $\int \left(a\sec^2(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \left(a\sec^2(x)\right)^{3/2} \, dx = \frac{1}{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right) + \frac{1}{2}a\sqrt{a\sec^2(x)}\tan(x)$$

output
```
1/2*a^(3/2)*arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))+1/2*a*(a*sec(x)^2)^
(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \left(a\sec^2(x)\right)^{3/2} \, dx = \frac{1}{2}a\sqrt{a\sec^2(x)}(\text{arctanh}(\sin(x))\cos(x) + \tan(x))$$

input
```
Integrate[(a*Sec[x]^2)^(3/2),x]
```

output
```
(a*Sqrt[a*Sec[x]^2]*(ArcTanh[Sin[x]]*Cos[x] + Tan[x]))/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4610, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sec^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(x)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \sqrt{a\tan^2(x)+a}d\tan(x)$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a\tan^2(x)+a}}d\tan(x)+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{2}a\int \frac{1}{1-\frac{a\tan^2(x)}{a\tan^2(x)+a}}d\frac{\tan(x)}{\sqrt{a\tan^2(x)+a}}+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\tan^2(x)+a}}\right)+\frac{1}{2}\tan(x)\sqrt{a\tan^2(x)+a}\right)$$

input
```
Int[(a*Sec[x]^2)^(3/2),x]
```

output
```
a*((Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a + a*Tan[x]^2]])/2 + (Tan[x]*Sq
rt[a + a*Tan[x]^2])/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
default	$\left(-\frac{\ln(\csc(x)-\cot(x)-1)\cos(x)}{2}+\frac{\ln(\csc(x)-\cot(x)+1)\cos(x)}{2}+\frac{\tan(x)}{2}\right) a\sqrt{a\sec\left(x\right)^{2}}$	42
risch	$-\frac{ia\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\,\left(\mathrm{e}^{2ix}-1\right)}{\mathrm{e}^{2ix}+1}-a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\,\ln\left(\mathrm{e}^{ix}-i\right)\cos\left(x\right)+a\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\,\ln\left(\mathrm{e}^{ix}+i\right)\cos\left(x\right)$	103

input
```
int((a*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*ln(csc(x)-cot(x)-1)*cos(x)+1/2*ln(csc(x)-cot(x)+1)*cos(x)+1/2*tan(x)
)*a*(a*sec(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \left(a\sec^2(x)\right)^{3/2}\,dx = -\frac{\left(a\cos\left(x\right)^2\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right) - 2\,a\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^2}}}{4\,\cos\left(x\right)}$$

input
```
integrate((a*sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(a*cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*a*sin(x))*sqrt(a/cos(
x)^2)/cos(x)
```

Sympy [F]

$$\int \left(a\sec^2(x)\right)^{3/2}\,dx = \int \left(a\sec^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sec(x)**2)**(3/2),x)
```

output
```
Integral((a*sec(x)**2)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(34) = 68.

Time = 0.15 (sec) , antiderivative size = 324, normalized size of antiderivative = 7.04

$$\int \left(a\sec^2(x)\right)^{3/2}\,dx = -\frac{(8\,a\cos(3\,x)\sin(2\,x) - 8\,a\cos(x)\sin(2\,x) + 8\,a\cos(2\,x)\sin(x) - 4\,(a\sin(3\,x) - a\sin(x))\cos(4\,x) - (}{}$$

input
```
integrate((a*sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(8*a*cos(3*x)*sin(2*x) - 8*a*cos(x)*sin(2*x) + 8*a*cos(2*x)*sin(x) -
4*(a*sin(3*x) - a*sin(x))*cos(4*x) - (a*cos(4*x)^2 + 4*a*cos(2*x)^2 + a*si
n(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(2*a*cos(2*x) + a)*c
os(4*x) + 4*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*c
os(4*x)^2 + 4*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*si
n(2*x)^2 + 2*(2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*log(cos(x)^2
+ sin(x)^2 - 2*sin(x) + 1) + 4*(a*cos(3*x) - a*cos(x))*sin(4*x) - 4*(2*a*c
os(2*x) + a)*sin(3*x) + 4*a*sin(x))*sqrt(a)/(2*(2*cos(2*x) + 1)*cos(4*x) +
 cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)
^2 + 4*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a\sec^2(x)\right)^{3/2}\,dx = \frac{1}{4}\left(\log(\sin(x)+1)\,\mathrm{sgn}(\cos(x)) - \log(-\sin(x)+1)\,\mathrm{sgn}(\cos(x)) - \frac{2\,\mathrm{sgn}(\cos(x))\,\mathrm{s}}{\sin(x)^2 -}\right.$$

input
```
integrate((a*sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(log(sin(x) + 1)*sgn(cos(x)) - log(-sin(x) + 1)*sgn(cos(x)) - 2*sgn(co
s(x))*sin(x)/(sin(x)^2 - 1))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sec^2(x)\right)^{3/2} dx = \int \left(\frac{a}{\cos(x)^2}\right)^{3/2} dx$$

input
```
int((a/cos(x)^2)^(3/2),x)
```

output
```
int((a/cos(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \left(a\sec^2(x)\right)^{3/2} dx = \frac{\sqrt{a}\,a\left(-\log\left(\tan\left(\frac{x}{2}\right)-1\right)\sin(x)^2+\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)\sin(x)^2-\mathrm{l}\right.}{2\sin(x)^2-2}$$

input
```
int((a*sec(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*a*( - log(tan(x/2) - 1)*sin(x)**2 + log(tan(x/2) - 1) + log(tan(x
/2) + 1)*sin(x)**2 - log(tan(x/2) + 1) - sin(x)))/(2*(sin(x)**2 - 1))
```

3.50 $\int \sqrt{a \sec^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \sqrt{a \sec^2(x)}\,dx = \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a \sec^2(x)}}\right)$$

output
```
a^(1/2)*arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \sqrt{a \sec^2(x)}\,dx = \coth^{-1}(\sin(x))\cos(x)\sqrt{a \sec^2(x)}$$

input
```
Integrate[Sqrt[a*Sec[x]^2],x]
```

output
```
ArcCoth[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4610, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sec^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a \sec(x)^2}dx$$

$$\Big\downarrow \text{ 4610}$$

$$a\int \frac{1}{\sqrt{a \tan^2(x)+a}}d\tan(x)$$

$$\Big\downarrow \text{ 224}$$

$$a\int \frac{1}{1-\frac{a \tan^2(x)}{a \tan^2(x)+a}}d\frac{\tan(x)}{\sqrt{a \tan^2(x)+a}}$$

$$\Big\downarrow \text{ 219}$$

$$\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a \tan^2(x)+a}}\right)$$

input
```
Int[Sqrt[a*Sec[x]^2],x]
```

output
```
Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a + a*Tan[x]^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$-2\sqrt{a \sec \left(x \right)^{2}}\, \operatorname{arctanh}\left(-\csc \left(x \right)+\cot \left(x \right)\right) \cos \left(x \right)$	21
risch	$-2\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \ln \left({\mathrm e}^{i x}-i\right) \cos \left(x \right)+2\sqrt{\frac{a\,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \ln \left({\mathrm e}^{i x}+i\right) \cos \left(x \right)$	64

input
```
int((a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a*sec(x)^2)^(1/2)*arctanh(-csc(x)+cot(x))*cos(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \sqrt{a\sec^2(x)}\,dx = \left[-\frac{1}{2}\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right), -\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\sin(x)}{a}\right)\right]$$

input
```
integrate((a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(a/cos(x)^2)*cos(x)*log(-(sin(x) - 1)/(sin(x) + 1)), -sqrt(-a)*a
rctan(sqrt(-a)*sqrt(a/cos(x)^2)*cos(x)*sin(x)/a)]
```

Sympy [F]

$$\int \sqrt{a\sec^2(x)}\,dx = \int \sqrt{a\sec^2(x)}\,dx$$

input
```
integrate((a*sec(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*sec(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \sqrt{a\sec^2(x)}\,dx = \frac{1}{2}\sqrt{a}\left(\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\right)$$

input
```
integrate((a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(
x)^2 - 2*sin(x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \sqrt{a\sec^2(x)}\,dx$$
$$= \frac{1}{4}\sqrt{a}\left(\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)\right)\operatorname{sgn}(\cos(x))$$

input
```
integrate((a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a)*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) -
 2)))*sgn(cos(x))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sec^2(x)}\,dx = \int \sqrt{\frac{a}{\cos(x)^2}}\,dx$$

input
```
int((a/cos(x)^2)^(1/2),x)
```

output
```
int((a/cos(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \sqrt{a \sec^2(x)}\, dx = \sqrt{a}\left(-\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)\right)$$

input
```
int((a*sec(x)^2)^(1/2),x)
```

output
```
sqrt(a)*( - log(tan(x/2) - 1) + log(tan(x/2) + 1))
```

3.51 $\int \frac{1}{\sqrt{a \sec^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{a \sec^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

output
```
tan(x)/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a \sec^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Sec[x]^2],x]
```

output
```
Tan[x]/Sqrt[a*Sec[x]^2]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sec^2(x)}} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a \sec(x)^2}} dx$$
$$\Big\downarrow \text{4610}$$
$$a \int \frac{1}{\left(a \tan^2(x)+a\right)^{3/2}} d\tan(x)$$
$$\Big\downarrow \text{208}$$
$$\frac{\tan(x)}{\sqrt{a \tan^2(x)+a}}$$

input
```
Int[1/Sqrt[a*Sec[x]^2],x]
```

output
```
Tan[x]/Sqrt[a + a*Tan[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\tan(x)}{\sqrt{a\sec(x)^{2}}}$	12
risch	$-\frac{i\,e^{2ix}}{2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\left(e^{2ix}+1\right)}+\frac{i}{2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\left(e^{2ix}+1\right)}$	67

input
```
int(1/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
tan(x)/(a*sec(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx=\frac{\sqrt{\frac{a}{\cos(x)^{2}}}\cos(x)\sin(x)}{a}$$

input
```
integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a/cos(x)^2)*cos(x)*sin(x)/a
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx = \frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

input
```
integrate(1/(a*sec(x)**2)**(1/2),x)
```

output
```
tan(x)/sqrt(a*sec(x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx = \frac{\sin(x)}{\sqrt{a}}$$

input
```
integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sin(x)/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx = \frac{\sin(x)}{\sqrt{a}\,\mathrm{sgn}(\cos(x))}$$

input
```
integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sin(x)/(sqrt(a)*sgn(cos(x)))
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx = \frac{\sqrt{2}\,\sin(2\,x)}{2\,\sqrt{a}\,\sqrt{2\cos(x)^2}}$$

input
```
int(1/(a/cos(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*sin(2*x))/(2*a^(1/2)*(2*cos(x)^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{a\sec^2(x)}}\,dx = \frac{\sqrt{a}\,\sin(x)}{a}$$

input
```
int(1/(a*sec(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*sin(x))/a
```

3.52 $\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx=\frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{2\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

output
```
1/3*tan(x)/(a*sec(x)^2)^(3/2)+2/3*tan(x)/a/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx=\frac{(5+\cos(2x))\tan(x)}{6a\sqrt{a\sec^2(x)}}$$

input
```
Integrate[(a*Sec[x]^2)^(-3/2),x]
```

output
```
((5 + Cos[2*x])*Tan[x])/(6*a*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4610, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(x)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \frac{1}{\left(a\tan^2(x)+a\right)^{5/2}}d\tan(x)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{2\int \frac{1}{\left(a\tan^2(x)+a\right)^{3/2}}d\tan(x)}{3a}+\frac{\tan(x)}{3a\left(a\tan^2(x)+a\right)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$a\left(\frac{2\tan(x)}{3a^2\sqrt{a\tan^2(x)+a}}+\frac{\tan(x)}{3a\left(a\tan^2(x)+a\right)^{3/2}}\right)$$

input
```
Int[(a*Sec[x]^2)^(-3/2),x]
```

output
```
a*(Tan[x]/(3*a*(a + a*Tan[x]^2)^(3/2)) + (2*Tan[x])/(3*a^2*Sqrt[a + a*Tan[
x]^2]))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\sin(x)\cos(x)+2\tan(x)}{3\sqrt{a\sec(x)^{2}}\,a}$	24
risch	$-\frac{i\,\mathrm{e}^{4ix}}{24a(\mathrm{e}^{2ix}+1)\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}}-\frac{3i\,\mathrm{e}^{2ix}}{8a(\mathrm{e}^{2ix}+1)\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}}+\frac{3i}{8a(\mathrm{e}^{2ix}+1)\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}}+\frac{i\,\mathrm{e}^{-2ix}}{24a(\mathrm{e}^{2ix}+1)\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^{2}}}}$	149

input
```
int(1/(a*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*sec(x)^2)^(1/2)/a*(sin(x)*cos(x)+2*tan(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx = \frac{\left(\cos\left(x\right)^3 + 2\,\cos\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^2}}\,\sin\left(x\right)}{3\,a^2}$$

input
```
integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(cos(x)^3 + 2*cos(x))*sqrt(a/cos(x)^2)*sin(x)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx = \frac{2\tan^3\left(x\right)}{3\left(a\sec^2\left(x\right)\right)^{\frac{3}{2}}} + \frac{\tan\left(x\right)}{\left(a\sec^2\left(x\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sec(x)**2)**(3/2),x)
```

output
```
2*tan(x)**3/(3*(a*sec(x)**2)**(3/2)) + tan(x)/(a*sec(x)**2)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx = \frac{\sin\left(3\,x\right) + 9\,\sin\left(x\right)}{12\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*x) + 9*sin(x))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a\sec^2(x))^{3/2}}\,dx = -\frac{\sin(x)^3 - 3\sin(x)}{3\,a^{\frac{3}{2}}\mathrm{sgn}(\cos(x))}$$

input
```
integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(sin(x)^3 - 3*sin(x))/(a^(3/2)*sgn(cos(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sec^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cos(x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a/cos(x)^2)^(3/2),x)
```

output
```
int(1/(a/cos(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a\sec^2(x))^{3/2}}\,dx = \frac{\sqrt{a}\,\sin(x)\left(-\sin(x)^2 + 3\right)}{3a^2}$$

input
```
int(1/(a*sec(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*sin(x)*( - sin(x)**2 + 3))/(3*a**2)
```

3.53 $\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx=\frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}}+\frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}}+\frac{8\tan(x)}{15a^2\sqrt{a\sec^2(x)}}$$

output
```
1/5*tan(x)/(a*sec(x)^2)^(5/2)+4/15*tan(x)/a/(a*sec(x)^2)^(3/2)+8/15*tan(x)
/a^2/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx=\frac{(15-10\sin^2(x)+3\sin^4(x))\tan(x)}{15a^2\sqrt{a\sec^2(x)}}$$

input
```
Integrate[(a*Sec[x]^2)^(-5/2),x]
```

output
```
((15 - 10*Sin[x]^2 + 3*Sin[x]^4)*Tan[x])/(15*a^2*Sqrt[a*Sec[x]^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4610, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \sec^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a \sec(x)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$a \int \frac{1}{\left(a \tan^2(x)+a\right)^{7/2}}d\tan(x)$$

$$\Big\downarrow \text{209}$$

$$a \left(\frac{4 \int \frac{1}{\left(a \tan^2(x)+a\right)^{5/2}}d\tan(x)}{5 a}+\frac{\tan(x)}{5 a \left(a \tan^2(x)+a\right)^{5/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$a \left(\frac{4 \left(\frac{2 \int \frac{1}{\left(a \tan^2(x)+a\right)^{3/2}}d\tan(x)}{3 a}+\frac{\tan(x)}{3 a \left(a \tan^2(x)+a\right)^{3/2}}\right)}{5 a}+\frac{\tan(x)}{5 a \left(a \tan^2(x)+a\right)^{5/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$a \left(\frac{4 \left(\frac{2 \tan(x)}{3 a^2 \sqrt{a \tan^2(x)+a}}+\frac{\tan(x)}{3 a \left(a \tan^2(x)+a\right)^{3/2}}\right)}{5 a}+\frac{\tan(x)}{5 a \left(a \tan^2(x)+a\right)^{5/2}}\right)$$

input
```
Int[(a*Sec[x]^2)^(-5/2),x]
```

output
```
a*(Tan[x]/(5*a*(a + a*Tan[x]^2)^(5/2)) + (4*(Tan[x]/(3*a*(a + a*Tan[x]^2)^
(3/2)) + (2*Tan[x])/(3*a^2*Sqrt[a + a*Tan[x]^2])))/(5*a))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

method	result
default	$\frac{\left(\frac{\cos(x)^4}{5}+\frac{4\cos(x)^2}{15}+\frac{8}{15}\right)\tan(x)}{\sqrt{a\sec(x)^2}\,a^2}$
risch	$-\frac{i e^{6ix}}{160a^2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{5i e^{2ix}}{16a^2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{5i}{16a^2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{5i e^{-2ix}}{96a^2(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-$

input
```
int(1/(a*sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/5*cos(x)^4+4/15*cos(x)^2+8/15)/(a*sec(x)^2)^(1/2)/a^2*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx = \frac{\left(3\,\cos\left(x\right)^{5}+4\,\cos\left(x\right)^{3}+8\,\cos\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\,\sin\left(x\right)}{15\,a^{3}}$$

input
```
integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*cos(x)^5 + 4*cos(x)^3 + 8*cos(x))*sqrt(a/cos(x)^2)*sin(x)/a^3
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx = \frac{8\tan^{5}\left(x\right)}{15\left(a\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{4\tan^{3}\left(x\right)}{3\left(a\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{\tan\left(x\right)}{\left(a\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sec(x)**2)**(5/2),x)
```

output
```
8*tan(x)**5/(15*(a*sec(x)**2)**(5/2)) + 4*tan(x)**3/(3*(a*sec(x)**2)**(5/2
)) + tan(x)/(a*sec(x)**2)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.40

$$\int \frac{1}{(a\sec^2(x))^{5/2}}\,dx = \frac{3\,\sin(5\,x) + 25\,\sin(3\,x) + 150\,\sin(x)}{240\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/240*(3*sin(5*x) + 25*sin(3*x) + 150*sin(x))/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a\sec^2(x))^{5/2}}\,dx = \frac{3\,\sin(x)^5 - 10\,\sin(x)^3 + 15\,\sin(x)}{15\,a^{\frac{5}{2}}\mathrm{sgn}(\cos(x))}$$

input
```
integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/(a^(5/2)*sgn(cos(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sec^2(x))^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cos(x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a/cos(x)^2)^(5/2),x)
```

output
```
int(1/(a/cos(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,\sin(x)\left(3\sin(x)^4 - 10\sin(x)^2 + 15\right)}{15a^3}$$

input
```
int(1/(a*sec(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*sin(x)*(3*sin(x)**4 - 10*sin(x)**2 + 15))/(15*a**3)
```

3.54 $\int \frac{1}{\left(a\sec^2(x)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \frac{1}{\left(a\sec^2(x)\right)^{7/2}}\,dx = \frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}} + \frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}} + \frac{8\tan(x)}{35a^2\left(a\sec^2(x)\right)^{3/2}} + \frac{16\tan(x)}{35a^3\sqrt{a\sec^2(x)}}$$

output
```
1/7*tan(x)/(a*sec(x)^2)^(7/2)+6/35*tan(x)/a/(a*sec(x)^2)^(5/2)+8/35*tan(x)
/a^2/(a*sec(x)^2)^(3/2)+16/35*tan(x)/a^3/(a*sec(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a\sec^2(x)\right)^{7/2}}\,dx = \frac{\left(35-35\sin^2(x)+21\sin^4(x)-5\sin^6(x)\right)\tan(x)}{35a^3\sqrt{a\sec^2(x)}}$$

input
```
Integrate[(a*Sec[x]^2)^(-7/2),x]
```

output
```
((35 - 35*Sin[x]^2 + 21*Sin[x]^4 - 5*Sin[x]^6)*Tan[x])/(35*a^3*Sqrt[a*Sec[
x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4610, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\sec^2(x)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sec(x)^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{4610}$$

$$a\int \frac{1}{\left(a\tan^2(x)+a\right)^{9/2}}d\tan(x)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{6\int \frac{1}{\left(a\tan^2(x)+a\right)^{7/2}}d\tan(x)}{7a}+\frac{\tan(x)}{7a\left(a\tan^2(x)+a\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{6\left(\frac{4\int \frac{1}{\left(a\tan^2(x)+a\right)^{5/2}}d\tan(x)}{5a}+\frac{\tan(x)}{5a\left(a\tan^2(x)+a\right)^{5/2}}\right)}{7a}+\frac{\tan(x)}{7a\left(a\tan^2(x)+a\right)^{7/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$a\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(a\tan^2(x)+a\right)^{3/2}}d\tan(x)}{3a}+\frac{\tan(x)}{3a\left(a\tan^2(x)+a\right)^{3/2}}\right)}{5a}+\frac{\tan(x)}{5a(a\tan^2(x)+a)^{5/2}}\right)}{7a}+\frac{\tan(x)}{7a\left(a\tan^2(x)+a\right)^{7/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$a\left(\frac{6\left(\frac{4\left(\frac{2\tan(x)}{3a^2\sqrt{a\tan^2(x)+a}}+\frac{\tan(x)}{3a\left(a\tan^2(x)+a\right)^{3/2}}\right)}{5a}+\frac{\tan(x)}{5a(a\tan^2(x)+a)^{5/2}}\right)}{7a}+\frac{\tan(x)}{7a\left(a\tan^2(x)+a\right)^{7/2}}\right)$$

input
```
Int[(a*Sec[x]^2)^(-7/2),x]
```

output
```
a*(Tan[x]/(7*a*(a + a*Tan[x]^2)^(7/2)) + (6*(Tan[x]/(5*a*(a + a*Tan[x]^2)^
(5/2)) + (4*(Tan[x]/(3*a*(a + a*Tan[x]^2)^(3/2)) + (2*Tan[x])/(3*a^2*Sqrt[
a + a*Tan[x]^2])))/(5*a)))/(7*a))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.47

method	result
default	$\frac{\left(\frac{\cos(x)^6}{7}+\frac{6\cos(x)^4}{35}+\frac{8\cos(x)^2}{35}+\frac{16}{35}\right)\tan(x)}{\sqrt{a\sec(x)^2}\,a^3}$
risch	$-\frac{i e^{8ix}}{896a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{35i e^{2ix}}{128a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{35i}{128a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}+\frac{7i e^{-2ix}}{128a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}$

input
```
int(1/(a*sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(1/7*cos(x)^6+6/35*cos(x)^4+8/35*cos(x)^2+16/35)/(a*sec(x)^2)^(1/2)/a^3*ta
n(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a\sec^2(x))^{7/2}}\,dx=\frac{\left(5\cos(x)^7+6\cos(x)^5+8\cos(x)^3+16\cos(x)\right)\sqrt{\frac{a}{\cos(x)^2}}\sin(x)}{35\,a^4}$$

input
```
integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/35*(5*cos(x)^7 + 6*cos(x)^5 + 8*cos(x)^3 + 16*cos(x))*sqrt(a/cos(x)^2)*s
in(x)/a^4
```

Sympy [A] (verification not implemented)

Time = 11.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a\sec^2(x))^{7/2}}\,dx = \frac{16\tan^7{(x)}}{35\,(a\sec^2{(x)})^{\frac{7}{2}}} + \frac{8\tan^5{(x)}}{5\,(a\sec^2{(x)})^{\frac{7}{2}}} + \frac{2\tan^3{(x)}}{(a\sec^2{(x)})^{\frac{7}{2}}} + \frac{\tan{(x)}}{(a\sec^2{(x)})^{\frac{7}{2}}}$$

input
```
integrate(1/(a*sec(x)**2)**(7/2),x)
```

output
```
16*tan(x)**7/(35*(a*sec(x)**2)**(7/2)) + 8*tan(x)**5/(5*(a*sec(x)**2)**(7/
2)) + 2*tan(x)**3/(a*sec(x)**2)**(7/2) + tan(x)/(a*sec(x)**2)**(7/2)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a\sec^2(x))^{7/2}}\,dx = \frac{5\,\sin{(7\,x)} + 49\,\sin{(5\,x)} + 245\,\sin{(3\,x)} + 1225\,\sin{(x)}}{2240\,a^{\frac{7}{2}}}$$

input
```
integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2240*(5*sin(7*x) + 49*sin(5*x) + 245*sin(3*x) + 1225*sin(x))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a\sec^2(x))^{7/2}}\,dx = -\frac{5\,\sin(x)^7 - 21\,\sin(x)^5 + 35\,\sin(x)^3 - 35\,\sin(x)}{35\,a^{\frac{7}{2}}\mathrm{sgn}(\cos(x))}$$

input
```
integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/(a^(7/2)*sgn(co
s(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sec^2(x))^{7/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cos(x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(a/cos(x)^2)^(7/2),x)
```

output
```
int(1/(a/cos(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int \frac{1}{(a\sec^2(x))^{7/2}}\,dx = \frac{\sqrt{a}\,\sin(x)\left(-5\sin(x)^6 + 21\sin(x)^4 - 35\sin(x)^2 + 35\right)}{35a^4}$$

input
```
int(1/(a*sec(x)^2)^(7/2),x)
```

output
```
(sqrt(a)*sin(x)*( - 5*sin(x)**6 + 21*sin(x)**4 - 35*sin(x)**2 + 35))/(35*a
**4)
```

3.55 $\int (a\sec^3(x))^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \left(a\sec^3(x)\right)^{5/2}\,dx = -\frac{154}{195}a^2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)} + \frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\sin(x) + \frac{154}{585}a^2\sqrt{a\sec^3(x)}\tan(x) + \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(x)}\tan(x) + \frac{2}{13}a^2\sec^4(x)\sqrt{a\sec^3(x)}\tan(x)$$

output

```
-154/195*a^2*cos(x)^(3/2)*EllipticE(sin(1/2*x),2^(1/2))*(a*sec(x)^3)^(1/2)
+154/195*a^2*cos(x)*(a*sec(x)^3)^(1/2)*sin(x)+154/585*a^2*(a*sec(x)^3)^(1/
2)*tan(x)+22/117*a^2*sec(x)^2*(a*sec(x)^3)^(1/2)*tan(x)+2/13*a^2*sec(x)^4*
(a*sec(x)^3)^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \left(a\sec^3(x)\right)^{5/2}\,dx = -\frac{2}{585}a\sec(x)\left(a\sec^3(x)\right)^{3/2}\left(231\cos^{\frac{11}{2}}(x)E\left(\frac{x}{2}\middle|2\right) - 55\cos(x)\sin(x) - 77\cos^3(x)\sin(x) - 231\cos^5(x)\sin(x$$

input
```
Integrate[(a*Sec[x]^3)^(5/2),x]
```

output
```
(-2*a*Sec[x]*(a*Sec[x]^3)^(3/2)*(231*Cos[x]^(11/2)*EllipticE[x/2, 2] - 55*
Cos[x]*Sin[x] - 77*Cos[x]^3*Sin[x] - 231*Cos[x]^5*Sin[x] - 45*Tan[x]))/585
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {3042, 4611, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \sec ^3(x)\right)^{5/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sec (x)^3\right)^{5/2}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{a^2 \sqrt{a \sec ^3(x)} \int \sec ^{\frac{15}{2}}(x)dx}{\sec ^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2 \sqrt{a \sec ^3(x)} \int \csc \left(x+\frac{\pi }{2}\right)^{15/2}dx}{\sec ^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a^2 \sqrt{a \sec ^3(x)} \left(\frac{11}{13} \int \sec ^{\frac{11}{2}}(x)dx+\frac{2}{13} \sin (x) \sec ^{\frac{13}{2}}(x)\right)}{\sec ^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2 \sqrt{a \sec ^3(x)} \left(\frac{11}{13} \int \csc \left(x+\frac{\pi }{2}\right)^{11/2}dx+\frac{2}{13} \sin (x) \sec ^{\frac{13}{2}}(x)\right)}{\sec ^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\int \sec^{\frac{7}{2}}(x)dx+\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(x)\right)+\frac{2}{13}\sin(x)\sec^{\frac{13}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\int \csc\left(x+\frac{\pi}{2}\right)^{7/2}dx+\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(x)\right)+\frac{2}{13}\sin(x)\sec^{\frac{13}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(x)dx+\frac{2}{5}\sin(x)\sec^{\frac{5}{2}}(x)\right)+\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(x)\right)+\frac{2}{13}\sin(x)\sec^{\frac{13}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int \csc\left(x+\frac{\pi}{2}\right)^{3/2}dx+\frac{2}{5}\sin(x)\sec^{\frac{5}{2}}(x)\right)+\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(x)\right)+\frac{2}{13}\sin(x)\sec^{\frac{13}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\Big(\frac{11}{13}\Big(\frac{7}{9}\Big(\frac{3}{5}\Big(2\sin(x)\sqrt{\sec(x)}-\int\frac{1}{\sqrt{\sec(x)}}dx\Big)+\frac{2}{5}\sin(x)\sec^{\frac{5}{2}}(x)\Big)+\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(x)\Big)+\frac{2}{13}\sin(x)\sec}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\Bigg(\frac{11}{13}\Bigg(\frac{7}{9}\Bigg(\frac{3}{5}\Bigg(2\sin(x)\sqrt{\sec(x)}-\int\frac{1}{\sqrt{\csc\left(x+\frac{\pi}{2}\right)}}dx\Bigg)+\frac{2}{5}\sin(x)\sec^{\frac{5}{2}}(x)\Bigg)+\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(x)\Bigg)+\frac{2}{13}\sin}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\Big(\frac{11}{13}\Big(\frac{7}{9}\Big(\frac{3}{5}\Big(2\sin(x)\sqrt{\sec(x)}-\sqrt{\cos(x)}\sqrt{\sec(x)}\int\sqrt{\cos(x)}dx\Big)+\frac{2}{5}\sin(x)\sec^{\frac{5}{2}}(x)\Big)+\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(2\sin(x)\sqrt{\sec(x)}-\sqrt{\cos(x)}\sqrt{\sec(x)}\int\sqrt{\sin\left(x+\frac{\pi}{2}\right)}dx\right)+\frac{2}{5}\sin(x)\sec^{\frac{5}{2}}(x)\right)+\frac{2}{9}\sin(x)\right.\right.}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a^2\sqrt{a\sec^3(x)}\left(\frac{2}{13}\sin(x)\sec^{\frac{13}{2}}(x)+\frac{11}{13}\left(\frac{2}{9}\sin(x)\sec^{\frac{9}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sin(x)\sec^{\frac{5}{2}}(x)+\frac{3}{5}\left(2\sin(x)\sqrt{\sec(x)}-2\sqrt{\cos(x)}\right.\right.\right.\right.}{\sec^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sec[x]^3)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Sec[x]^3]*((2*Sec[x]^(13/2)*Sin[x])/13 + (11*((2*Sec[x]^(9/2)*
Sin[x])/9 + (7*((2*Sec[x]^(5/2)*Sin[x])/5 + (3*(-2*Sqrt[Cos[x]]*EllipticE[
x/2, 2]*Sqrt[Sec[x]] + 2*Sqrt[Sec[x]]*Sin[x]))/5))/9))/13))/Sec[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 18.68 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.40

method	result
default	$\frac{2a^2\left(\left(231\cos(x)^6+77\cos(x)^5+77\cos(x)^4+55\cos(x)^3+55\cos(x)^2+45\cos(x)+45\right)\tan(x)\sec(x)^4+231i\,\mathrm{EllipticE}(i(-\csc(x)+\cot(\ldots}{}$

input
```
int((a*sec(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/585*a^2*((231*cos(x)^6+77*cos(x)^5+77*cos(x)^4+55*cos(x)^3+55*cos(x)^2+4
5*cos(x)+45)*tan(x)*sec(x)^4+231*I*EllipticE(I*(-csc(x)+cot(x)),I)*(cos(x)
/(cos(x)+1))^(1/2)*(1/(cos(x)+1))^(1/2)*(cos(x)^3+2*cos(x)^2+cos(x))+231*I
*EllipticF(I*(-csc(x)+cot(x)),I)*(cos(x)/(cos(x)+1))^(1/2)*(1/(cos(x)+1))^
(1/2)*(-cos(x)^3-2*cos(x)^2-cos(x)))*(a*sec(x)^3)^(1/2)/(cos(x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

$$\int \left(a\sec^3(x)\right)^{5/2}\,dx = \frac{231i\sqrt{2}a^{\frac{5}{2}}\cos(x)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)+i\sin\ldots}{}$$

input
```
integrate((a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/585*(231*I*sqrt(2)*a^(5/2)*cos(x)^5*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(x) + I*sin(x))) - 231*I*sqrt(2)*a^(5/2)*cos(x)^5*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) - I*sin(x))) + 2*(231*a^
2*cos(x)^6 + 77*a^2*cos(x)^4 + 55*a^2*cos(x)^2 + 45*a^2)*sqrt(a/cos(x)^3)*
sin(x))/cos(x)^5
```

Sympy [F]

$$\int \left(a\sec^3(x)\right)^{5/2}\,dx=\int \left(a\sec^3\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sec(x)**3)**(5/2),x)
```

output
```
Integral((a*sec(x)**3)**(5/2), x)
```

Maxima [F]

$$\int \left(a\sec^3(x)\right)^{5/2}\,dx=\int \left(a\sec\left(x\right)^{3}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sec(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a \sec^3(x)\right)^{5/2} dx = \int \left(a \sec(x)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*sec(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a \sec^3(x)\right)^{5/2} dx = \int \left(\frac{a}{\cos(x)^3}\right)^{5/2} dx$$

input
```
int((a/cos(x)^3)^(5/2),x)
```

output
```
int((a/cos(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \left(a \sec^3(x)\right)^{5/2} dx = \sqrt{a} \left(\int \sqrt{\sec(x)} \sec(x)^7 dx\right) a^2$$

input
```
int((a*sec(x)^3)^(5/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(x))*sec(x)**7,x)*a**2
```

3.56 $\int \left(a\sec^3(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \left(a\sec^3(x)\right)^{3/2}\,dx=\frac{10}{21}a\cos^{\frac{3}{2}}(x)\,\text{EllipticF}\left(\frac{x}{2},2\right)\sqrt{a\sec^3(x)}+\frac{10}{21}a\sqrt{a\sec^3(x)}\sin(x)+\frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\tan(x)$$

output
```
10/21*a*cos(x)^(3/2)*InverseJacobiAM(1/2*x,2^(1/2))*(a*sec(x)^3)^(1/2)+10/
21*a*(a*sec(x)^3)^(1/2)*sin(x)+2/7*a*sec(x)*(a*sec(x)^3)^(1/2)*tan(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(a\sec^3(x)\right)^{3/2}\,dx=\frac{2}{21}a\sec(x)\sqrt{a\sec^3(x)}\left(5\cos^{\frac{5}{2}}(x)\,\text{EllipticF}\left(\frac{x}{2},2\right)+5\cos(x)\sin(x)+3\tan(x)\right)$$

input
```
Integrate[(a*Sec[x]^3)^(3/2),x]
```

output
```
(2*a*Sec[x]*Sqrt[a*Sec[x]^3]*(5*Cos[x]^(5/2)*EllipticF[x/2, 2] + 5*Cos[x]*
Sin[x] + 3*Tan[x]))/21
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4611, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sec^3(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(x)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{a\sqrt{a\sec^3(x)}\int \sec^{\frac{9}{2}}(x)dx}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sec^3(x)}\int \csc\left(x+\frac{\pi}{2}\right)^{9/2}dx}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\sqrt{a\sec^3(x)}\left(\frac{5}{7}\int \sec^{\frac{5}{2}}(x)dx+\frac{2}{7}\sin(x)\sec^{\frac{7}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sec^3(x)}\left(\frac{5}{7}\int \csc\left(x+\frac{\pi}{2}\right)^{5/2}dx+\frac{2}{7}\sin(x)\sec^{\frac{7}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\sqrt{a\sec^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\sec(x)}dx+\frac{2}{3}\sin(x)\sec^{\frac{3}{2}}(x)\right)+\frac{2}{7}\sin(x)\sec^{\frac{7}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sec^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc\left(x+\frac{\pi}{2}\right)}dx+\frac{2}{3}\sin(x)\sec^{\frac{3}{2}}(x)\right)+\frac{2}{7}\sin(x)\sec^{\frac{7}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sqrt{a\sec^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\int\frac{1}{\sqrt{\cos(x)}}dx+\frac{2}{3}\sin(x)\sec^{\frac{3}{2}}(x)\right)+\frac{2}{7}\sin(x)\sec^{\frac{7}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sec^3(x)}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2}{3}\sin(x)\sec^{\frac{3}{2}}(x)\right)+\frac{2}{7}\sin(x)\sec^{\frac{7}{2}}(x)\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a\sqrt{a\sec^3(x)}\left(\frac{2}{7}\sin(x)\sec^{\frac{7}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sin(x)\sec^{\frac{3}{2}}(x)+\frac{2}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)\right)\right)}{\sec^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Sec[x]^3)^(3/2),x]
```

output
```
(a*Sqrt[a*Sec[x]^3]*((2*Sec[x]^(7/2)*Sin[x])/7 + (5*((2*Sqrt[Cos[x]]*Ellip
ticF[x/2, 2]*Sqrt[Sec[x]])/3 + (2*Sec[x]^(3/2)*Sin[x])/3))/7))/Sec[x]^(3/2
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

method	result
default	$\left(\frac{10\sin(x)}{21}+\frac{2\tan(x)\sec(x)}{7}+\frac{2i\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\operatorname{EllipticF}(i(\csc(x)-\cot(x)),i)\left(-5\cos(x)^{2}-5\cos(x)\right)}{21}\right)a\sqrt{a\sec($

input
```
int((a*sec(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(10/21*sin(x)+2/7*tan(x)*sec(x)+2/21*I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x
)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)*(-5*cos(x)^2-5*cos(x)))*a*(a*se
c(x)^3)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \left(a\sec^3(x)\right)^{3/2}\,dx = \frac{5i\,\sqrt{2}a^{\frac{3}{2}}\cos\left(x\right)^{2}\,\text{weierstrassPInverse}(-4,0,\cos\left(x\right)+i\,\sin\left(x\right))-5i\,\sqrt{2}a^{\frac{3}{2}}\cos\left(x\right)^{2}\,\text{w}}{21\,\text{co}}$$

input
```
integrate((a*sec(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/21*(5*I*sqrt(2)*a^(3/2)*cos(x)^2*weierstrassPInverse(-4, 0, cos(x) + I*s
in(x)) - 5*I*sqrt(2)*a^(3/2)*cos(x)^2*weierstrassPInverse(-4, 0, cos(x) -
I*sin(x)) + 2*(5*a*cos(x)^2 + 3*a)*sqrt(a/cos(x)^3)*sin(x))/cos(x)^2
```

Sympy [F]

$$\int \left(a\sec^3(x)\right)^{3/2}\,dx = \int \left(a\sec^{3}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sec(x)**3)**(3/2),x)
```

output
```
Integral((a*sec(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\sec^3(x)\right)^{3/2}\,dx = \int \left(a\sec\left(x\right)^{3}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sec(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\sec^3(x)\right)^{3/2} dx = \int \left(a\sec{(x)}^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*sec(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sec^3(x)\right)^{3/2} dx = \int \left(\frac{a}{\cos{(x)}^3}\right)^{3/2} dx$$

input
```
int((a/cos(x)^3)^(3/2),x)
```

output
```
int((a/cos(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(a\sec^3(x)\right)^{3/2} dx = \sqrt{a}\left(\int \sqrt{\sec{(x)}}\,\sec{(x)}^4\,dx\right)a$$

input
```
int((a*sec(x)^3)^(3/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(x))*sec(x)**4,x)*a
```

3.57 $\int \sqrt{a \sec^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \sqrt{a \sec^3(x)}\,dx = -2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)} + 2\cos(x)\sqrt{a\sec^3(x)}\sin(x)$$

output
```
-2*cos(x)^(3/2)*EllipticE(sin(1/2*x),2^(1/2))*(a*sec(x)^3)^(1/2)+2*cos(x)*
(a*sec(x)^3)^(1/2)*sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \sqrt{a \sec^3(x)}\,dx = 2\cos(x)\sqrt{a\sec^3(x)}\left(-\sqrt{\cos(x)}E\left(\frac{x}{2}\middle|2\right) + \sin(x)\right)$$

input
```
Integrate[Sqrt[a*Sec[x]^3],x]
```

output
```
2*Cos[x]*Sqrt[a*Sec[x]^3]*(-(Sqrt[Cos[x]]*EllipticE[x/2, 2]) + Sin[x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4611, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sec^3(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a \sec(x)^3}dx$$

$$\Big\downarrow \text{ 4611}$$

$$\frac{\sqrt{a \sec^3(x)} \int \sec^{\frac{3}{2}}(x)dx}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a \sec^3(x)} \int \csc\left(x+\frac{\pi}{2}\right)^{3/2}dx}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{a \sec^3(x)}\left(2\sin(x)\sqrt{\sec(x)}-\int \frac{1}{\sqrt{\sec(x)}}dx\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a \sec^3(x)}\left(2\sin(x)\sqrt{\sec(x)}-\int \frac{1}{\sqrt{\csc\left(x+\frac{\pi}{2}\right)}}dx\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\sqrt{a \sec^3(x)}\left(2\sin(x)\sqrt{\sec(x)}-\sqrt{\cos(x)}\sqrt{\sec(x)}\int \sqrt{\cos(x)}dx\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a\sec^3(x)}\left(2\sin(x)\sqrt{\sec(x)}-\sqrt{\cos(x)}\sqrt{\sec(x)}\int\sqrt{\sin\left(x+\frac{\pi}{2}\right)}dx\right)}{\sec^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sqrt{a\sec^3(x)}\left(2\sin(x)\sqrt{\sec(x)}-2\sqrt{\cos(x)}\sqrt{\sec(x)}E\left(\frac{x}{2}\middle|2\right)\right)}{\sec^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[a*Sec[x]^3],x]
```

output
```
(Sqrt[a*Sec[x]^3]*(-2*Sqrt[Cos[x]]*EllipticE[x/2, 2]*Sqrt[Sec[x]] + 2*Sqrt
[Sec[x]]*Sin[x]))/Sec[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.71

method	result
default	$\frac{2\left(i\left(\cos(x)^2+2\cos(x)+1\right)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\operatorname{EllipticF}(i(\csc(x)-\cot(x)),i)+i\left(-\cos(x)^2-2\cos(x)-1\right)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}}{\cos(x)+1}$

input
```
int((a*sec(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(I*(cos(x)^2+2*cos(x)+1)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*
EllipticF(I*(csc(x)-cot(x)),I)+I*(-cos(x)^2-2*cos(x)-1)*(1/(cos(x)+1))^(1/
2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(csc(x)-cot(x)),I)+sin(x))*cos(x)
*(a*sec(x)^3)^(1/2)/(cos(x)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \sqrt{a\sec^3(x)}\,dx = 2\sqrt{\frac{a}{\cos(x)^3}}\cos(x)\sin(x) + i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x) + i\sin(x))) - i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x) - i\sin(x)))$$

input
```
integrate((a*sec(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a/cos(x)^3)*cos(x)*sin(x) + I*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(x) + I*sin(x))) - I*sqrt(2)*sqrt(a)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) - I*sin(x)))
```

Sympy [F]

$$\int \sqrt{a\sec^3(x)}\,dx = \int \sqrt{a\sec^3{(x)}}\,dx$$

input
```
integrate((a*sec(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*sec(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\sec^3(x)}\,dx = \int \sqrt{a\sec{(x)}^3}\,dx$$

input
```
integrate((a*sec(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\sec^3(x)}\,dx = \int \sqrt{a\sec{(x)}^3}\,dx$$

input
```
integrate((a*sec(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a \sec^3(x)}\, dx = \int \sqrt{\frac{a}{\cos(x)^3}}\, dx$$

input
```
int((a/cos(x)^3)^(1/2),x)
```

output
```
int((a/cos(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a \sec^3(x)}\, dx = \sqrt{a}\left(\int \sqrt{\sec(x)}\, \sec(x)\, dx\right)$$

input
```
int((a*sec(x)^3)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(x))*sec(x),x)
```

3.58 $\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 44

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx=\frac{2\,\text{EllipticF}\left(\frac{x}{2},2\right)}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}$$

output
```
2/3*InverseJacobiAM(1/2*x,2^(1/2))/cos(x)^(3/2)/(a*sec(x)^3)^(1/2)+2/3*tan
(x)/(a*sec(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx=\frac{2\left(\frac{\text{EllipticF}\left(\frac{x}{2},2\right)}{\cos^{\frac{3}{2}}(x)}+\tan(x)\right)}{3\sqrt{a\sec^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Sec[x]^3],x]
```

output
```
(2*(EllipticF[x/2, 2]/Cos[x]^(3/2) + Tan[x]))/(3*Sqrt[a*Sec[x]^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4611, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sec(x)^3}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{3}{2}}(x)}dx}{\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\csc\left(x+\frac{\pi}{2}\right)^{3/2}}dx}{\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{1}{3}\int \sqrt{\sec(x)}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)}{\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{1}{3}\int \sqrt{\csc\left(x+\frac{\pi}{2}\right)}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)}{\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{1}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\int \frac{1}{\sqrt{\cos(x)}}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)}{\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{1}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)}{\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \ 3120$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{3\sqrt{\sec(x)}}+\frac{2}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\operatorname{EllipticF}\left(\frac{x}{2},2\right)\right)}{\sqrt{a\sec^3(x)}}$$

input
```
Int[1/Sqrt[a*Sec[x]^3],x]
```

output
```
(Sec[x]^(3/2)*((2*Sqrt[Cos[x]]*EllipticF[x/2, 2]*Sqrt[Sec[x]])/3 + (2*Sin[
x])/(3*Sqrt[Sec[x]])))/Sqrt[a*Sec[x]^3]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{-\frac{2i\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\operatorname{EllipticF}(i(\csc(x)-\cot(x)),i)\left(\sec(x)+\sec(x)^2\right)}{3}+\frac{2\tan(x)}{3}}{\sqrt{a\sec(x)^3}}$	57

input
```
int(1/(a*sec(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)
-cot(x)),I)*(sec(x)+sec(x)^2)+2/3*tan(x))/(a*sec(x)^3)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx = \frac{2\sqrt{\frac{a}{\cos(x)^3}}\cos(x)^2\sin(x) + i\sqrt{2}\sqrt{a}\text{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)) - i\sqrt{2}\sqrt{a}\text{weierstrassI}}{3a}$$

input
```
integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(a/cos(x)^3)*cos(x)^2*sin(x) + I*sqrt(2)*sqrt(a)*weierstrassPIn
verse(-4, 0, cos(x) + I*sin(x)) - I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4
, 0, cos(x) - I*sin(x)))/a
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sec^3{(x)}}}\,dx$$

input
```
integrate(1/(a*sec(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sec(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sec{(x)}^3}}\,dx$$

input
```
integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*sec(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sec{(x)}^3}}\,dx$$

input
```
integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*sec(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\cos(x)^3}}}\,dx$$

input
```
int(1/(a/cos(x)^3)^(1/2),x)
```

output
```
int(1/(a/cos(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(x)}}{\sec(x)^2}dx\right)}{a}$$

input
```
int(1/(a*sec(x)^3)^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(sec(x))/sec(x)**2,x))/a
```

3.59 $\int \frac{1}{\left(a\sec^3(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int \frac{1}{\left(a\sec^3(x)\right)^{3/2}}\,dx=\frac{14E\left(\frac{x}{2}\middle|2\right)}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}}+\frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}}$$

output
```
14/15*EllipticE(sin(1/2*x),2^(1/2))/a/cos(x)^(3/2)/(a*sec(x)^3)^(1/2)+14/4
5*sin(x)/a/(a*sec(x)^3)^(1/2)+2/9*cos(x)^2*sin(x)/a/(a*sec(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a\sec^3(x)\right)^{3/2}}\,dx=\frac{\frac{84E\left(\frac{x}{2}\middle|2\right)}{\cos^{\frac{3}{2}}(x)}+33\sin(x)+5\sin(3x)}{90a\sqrt{a\sec^3(x)}}$$

input
```
Integrate[(a*Sec[x]^3)^(-3/2),x]
```

output
```
((84*EllipticE[x/2, 2])/Cos[x]^(3/2) + 33*Sin[x] + 5*Sin[3*x])/(90*a*Sqrt[
a*Sec[x]^3])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4611, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec^3(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sec(x)^3)^{3/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{9}{2}}(x)}dx}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\csc\left(x+\frac{\pi}{2}\right)^{9/2}}dx}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{7}{9}\int \frac{1}{\sec^{\frac{5}{2}}(x)}dx+\frac{2\sin(x)}{9\sec^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{7}{9}\int \frac{1}{\csc\left(x+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(x)}{9\sec^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{7}{9}\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(x)}}dx+\frac{2\sin(x)}{5\sec^{\frac{3}{2}}(x)}\right)+\frac{2\sin(x)}{9\sec^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{7}{9}\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc(x+\frac{\pi}{2})}}dx+\frac{2\sin(x)}{5\sec^{\frac{3}{2}}(x)}\right)+\frac{2\sin(x)}{9\sec^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{7}{9}\left(\frac{3}{5}\sqrt{\cos(x)}\sqrt{\sec(x)}\int \sqrt{\cos(x)}dx+\frac{2\sin(x)}{5\sec^{\frac{3}{2}}(x)}\right)+\frac{2\sin(x)}{9\sec^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{7}{9}\left(\frac{3}{5}\sqrt{\cos(x)}\sqrt{\sec(x)}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2\sin(x)}{5\sec^{\frac{3}{2}}(x)}\right)+\frac{2\sin(x)}{9\sec^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{9\sec^{\frac{7}{2}}(x)}+\frac{7}{9}\left(\frac{2\sin(x)}{5\sec^{\frac{3}{2}}(x)}+\frac{6}{5}\sqrt{\cos(x)}\sqrt{\sec(x)}E\left(\frac{x}{2}\middle|2\right)\right)\right)}{a\sqrt{a\sec^3(x)}}$$

input
```
Int[(a*Sec[x]^3)^(-3/2),x]
```

output
```
(Sec[x]^(3/2)*((2*Sin[x])/(9*Sec[x]^(7/2)) + (7*((6*Sqrt[Cos[x]]*EllipticE
[x/2, 2]*Sqrt[Sec[x]])/5 + (2*Sin[x])/(5*Sec[x]^(3/2))))/9))/(a*Sqrt[a*Sec
[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.92

method	result
default	$\frac{\frac{2\left(5\cos(x)^4+5\cos(x)^3+7\cos(x)^2+7\cos(x)+21\right)\tan(x)}{45}+\frac{14i\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\operatorname{EllipticF}(i(\csc(x)-\cot(x)),i)\left(-\sec(x)^2-2\sec(x)-1\right)}{15}+\frac{14i\sqrt{}}{}}{a(\cos(x)+1)\sqrt{a\sec(x)^3}}$

input
```
int(1/(a*sec(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/a*((5*cos(x)^4+5*cos(x)^3+7*cos(x)^2+7*cos(x)+21)*tan(x)+21*I*(1/(cos
(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)*(-s
ec(x)^2-2*sec(x)-1)+21*I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*El
lipticE(I*(csc(x)-cot(x)),I)*(sec(x)^2+2*sec(x)+1))/(cos(x)+1)/(a*sec(x)^3
)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx = \frac{2\left(5\cos(x)^5+7\cos(x)^3\right)\sqrt{\frac{a}{\cos(x)^3}}\sin(x)-21i\sqrt{2}\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weiers}$$

input
```
integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(x)^5 + 7*cos(x)^3)*sqrt(a/cos(x)^3)*sin(x) - 21*I*sqrt(2)*s
qrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) + I*sin(x)
)) + 21*I*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(x) - I*sin(x))))/a^2
```

Sympy [F]

$$\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx = \int \frac{1}{(a\sec^3(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)**3)**(3/2),x)
```

output
```
Integral((a*sec(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx = \int \frac{1}{\left(a\sec(x)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx = \int \frac{1}{\left(a\sec\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cos(x)^3}\right)^{3/2}}\,dx$$

input
```
int(1/(a/cos(x)^3)^(3/2),x)
```

output
```
int(1/(a/cos(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(x)}}{\sec(x)^5}dx\right)}{a^2}$$

input
```
int(1/(a*sec(x)^3)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(sec(x))/sec(x)**5,x))/a**2
```

3.60 $\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx=\frac{26\,\text{EllipticF}\left(\frac{x}{2},2\right)}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}}+\frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}}+\frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}}+\frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}}$$

output
```
26/77*InverseJacobiAM(1/2*x,2^(1/2))/a^2/cos(x)^(3/2)/(a*sec(x)^3)^(1/2)+7
8/385*cos(x)*sin(x)/a^2/(a*sec(x)^3)^(1/2)+26/165*cos(x)^3*sin(x)/a^2/(a*s
ec(x)^3)^(1/2)+2/15*cos(x)^5*sin(x)/a^2/(a*sec(x)^3)^(1/2)+26/77*tan(x)/a^
2/(a*sec(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx=\frac{\cos(x)\sqrt{a\sec^3(x)}\left(24960\sqrt{\cos(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)+19122\sin(2x)+4406\sin(4x)+\right.}{73920a^3}$$

input
```
Integrate[(a*Sec[x]^3)^(-5/2),x]
```

output
```
(Cos[x]*Sqrt[a*Sec[x]^3]*(24960*Sqrt[Cos[x]]*EllipticF[x/2, 2] + 19122*Sin
[2*x] + 4406*Sin[4*x] + 826*Sin[6*x] + 77*Sin[8*x]))/(73920*a^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {3042, 4611, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec^3(x))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a\sec(x)^3)^{5/2}}dx$$

$$\Big\downarrow \ 4611$$

$$\frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\csc\left(x+\frac{\pi}{2}\right)^{15/2}}dx}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \ 4256$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\int \frac{1}{\sec^{\frac{11}{2}}(x)}dx+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\int \frac{1}{\csc\left(x+\frac{\pi}{2}\right)^{11/2}}dx+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\int\frac{1}{\sec^{\frac{7}{2}}(x)}dx+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\int\frac{1}{\csc\left(x+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int\frac{1}{\sec^{\frac{3}{2}}(x)}dx+\frac{2\sin(x)}{7\sec^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int\frac{1}{\csc\left(x+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(x)}{7\sec^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 4256$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\sec(x)}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)+\frac{2\sin(x)}{7\sec^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc\left(x+\frac{\pi}{2}\right)}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)+\frac{2\sin(x)}{7\sec^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 4258$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\int\frac{1}{\sqrt{\cos(x)}}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)+\frac{2\sin(x)}{7\sec^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2\sin(x)}{3\sqrt{\sec(x)}}\right)+\frac{2\sin(x)}{7\sec^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\sec^3(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sec^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{15\sec^{\frac{13}{2}}(x)}+\frac{13}{15}\left(\frac{2\sin(x)}{11\sec^{\frac{9}{2}}(x)}+\frac{9}{11}\left(\frac{2\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{5}{7}\left(\frac{2\sin(x)}{3\sqrt{\sec(x)}}+\frac{2}{3}\sqrt{\cos(x)}\sqrt{\sec(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)\right)\right)\right)\right)}{a^2\sqrt{a\sec^3(x)}}$$

input
```
Int[(a*Sec[x]^3)^(-5/2),x]
```

output
```
(Sec[x]^(3/2)*((2*Sin[x])/(15*Sec[x]^(13/2)) + (13*((2*Sin[x])/(11*Sec[x]^
(9/2)) + (9*((2*Sin[x])/(7*Sec[x]^(5/2)) + (5*((2*Sqrt[Cos[x]]*EllipticF[x
/2, 2]*Sqrt[Sec[x]])/3 + (2*Sin[x])/(3*Sqrt[Sec[x]])))/7))/11))/15))/(a^2*
Sqrt[a*Sec[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.91 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\frac{2\left(77\cos(x)^6+91\cos(x)^4+117\cos(x)^2+195\right)\tan(x)}{1155}+\frac{2i\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\operatorname{EllipticF}(i(\csc(x)-\cot(x)),i)\left(-195\sec(x)-195\sec(x)^2\right)}{1155}}{a^2\sqrt{a\sec(x)^3}}$	84

input
```
int(1/(a*sec(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/1155*(77*cos(x)^6+91*cos(x)^4+117*cos(x)^2+195)*tan(x)+2/1155*I*(1/(cos
(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)*(-1
95*sec(x)-195*sec(x)^2))/a^2/(a*sec(x)^3)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx = \frac{2\left(77\cos(x)^8+91\cos(x)^6+117\cos(x)^4+195\cos(x)^2\right)\sqrt{\frac{a}{\cos(x)^3}}\sin(x)+195i\sqrt{2}$$

input
```
integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/1155*(2*(77*cos(x)^8 + 91*cos(x)^6 + 117*cos(x)^4 + 195*cos(x)^2)*sqrt(a
/cos(x)^3)*sin(x) + 195*I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4, 0, cos(x
) + I*sin(x)) - 195*I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4, 0, cos(x) -
I*sin(x)))/a^3
```

Sympy [F]

$$\int \frac{1}{(a\sec^3(x))^{5/2}}\,dx = \int \frac{1}{(a\sec^3{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)**3)**(5/2),x)
```

output
```
Integral((a*sec(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\sec^3(x))^{5/2}}\,dx = \int \frac{1}{\left(a\sec{(x)}^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sec\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cos(x)^3}\right)^{5/2}}\,dx$$

input
```
int(1/(a/cos(x)^3)^(5/2),x)
```

output
```
int(1/(a/cos(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(x)}}{\sec(x)^8}dx\right)}{a^3}$$

input
```
int(1/(a*sec(x)^3)^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(sec(x))/sec(x)**8,x))/a**3
```

3.61 $\int \left(a\sec^4(x)\right)^{7/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 163

$$\int \left(a\sec^4(x)\right)^{7/2}\,dx = a^3\cos(x)\sqrt{a\sec^4(x)}\sin(x) + 2a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan(x) + 3a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan^3(x) + \frac{20}{7}a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan^5(x) + \frac{5}{3}a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan^7(x) + \frac{6}{11}a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan^9(x) + \frac{1}{13}a^3\sqrt{a\sec^4(x)}\sin^2(x)\tan^{11}(x)$$

output
```
a^3*cos(x)*(a*sec(x)^4)^(1/2)*sin(x)+2*a^3*(a*sec(x)^4)^(1/2)*sin(x)^2*tan
(x)+3*a^3*(a*sec(x)^4)^(1/2)*sin(x)^2*tan(x)^3+20/7*a^3*(a*sec(x)^4)^(1/2)
*sin(x)^2*tan(x)^5+5/3*a^3*(a*sec(x)^4)^(1/2)*sin(x)^2*tan(x)^7+6/11*a^3*(
a*sec(x)^4)^(1/2)*sin(x)^2*tan(x)^9+1/13*a^3*(a*sec(x)^4)^(1/2)*sin(x)^2*t
an(x)^11
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \left(a\sec^4(x)\right)^{7/2}\,dx = \frac{\cos(x)(2048 + 2380\cos(2x) + 1093\cos(4x) + 378\cos(6x) + 92\cos(8x) + 14\cos(10x}{6006}$$

input
```
Integrate[(a*Sec[x]^4)^(7/2),x]
```

output
```
(Cos[x]*(2048 + 2380*Cos[2*x] + 1093*Cos[4*x] + 378*Cos[6*x] + 92*Cos[8*x]
 + 14*Cos[10*x] + Cos[12*x])*(a*Sec[x]^4)^(7/2)*Sin[x])/6006
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.42, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4611, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sec^4(x)\right)^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(x)^4\right)^{7/2}dx$$

$$\Big\downarrow \text{4611}$$

$$a^3\cos^2(x)\sqrt{a\sec^4(x)}\int \sec^{14}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^3\cos^2(x)\sqrt{a\sec^4(x)}\int \csc\left(x+\frac{\pi}{2}\right)^{14}dx$$

$$\Big\downarrow \text{4254}$$

$$-a^3\cos^2(x)\sqrt{a\sec^4(x)}\int\left(\tan^{12}(x)+6\tan^{10}(x)+15\tan^8(x)+20\tan^6(x)+15\tan^4(x)+6\tan^2(x)+1\right)d(-\mathrm{ta}$$

$$\Big\downarrow \text{2009}$$

$$-a^3\cos^2(x)\left(-\frac{1}{13}\tan^{13}(x)-\frac{6\tan^{11}(x)}{11}-\frac{5\tan^9(x)}{3}-\frac{20\tan^7(x)}{7}-3\tan^5(x)-2\tan^3(x)-\tan(x)\right)\sqrt{a\sec^4(}$$

input
```
Int[(a*Sec[x]^4)^(7/2),x]
```

output
```
-(a^3*Cos[x]^2*Sqrt[a*Sec[x]^4]*(-Tan[x] - 2*Tan[x]^3 - 3*Tan[x]^5 - (20*T
an[x]^7)/7 - (5*Tan[x]^9)/3 - (6*Tan[x]^11)/11 - Tan[x]^13/13))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 29.66 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{\left(1024\cos(x)^{12}+512\cos(x)^{10}+384\cos(x)^{8}+320\cos(x)^{6}+280\cos(x)^{4}+252\cos(x)^{2}+231\right)\sqrt{a\sec(x)^{4}}\,a^{3}\tan(x)\sec(x)^{10}}{3003}$	58
risch	$\frac{2048ia^{3}\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^{4}}}\left(1716\,e^{10ix}+1287\,e^{8ix}+715\,e^{6ix}+286\,e^{4ix}+13+79\cos(2x)+77i\sin(2x)\right)}{3003\left(e^{2ix}+1\right)^{11}}$	77

input
```
int((a*sec(x)^4)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/3003*(1024*cos(x)^12+512*cos(x)^10+384*cos(x)^8+320*cos(x)^6+280*cos(x)^
4+252*cos(x)^2+231)*(a*sec(x)^4)^(1/2)*a^3*tan(x)*sec(x)^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \left(a\sec^4(x)\right)^{7/2}\,dx = \frac{\left(1024\,a^3\cos(x)^{12} + 512\,a^3\cos(x)^{10} + 384\,a^3\cos(x)^{8} + 320\,a^3\cos(x)^{6} + 280\,a^3\cos(\right.}{3003\,\cos(x)^{11}}$$

input
```
integrate((a*sec(x)^4)^(7/2),x, algorithm="fricas")
```

output
```
1/3003*(1024*a^3*cos(x)^12 + 512*a^3*cos(x)^10 + 384*a^3*cos(x)^8 + 320*a^
3*cos(x)^6 + 280*a^3*cos(x)^4 + 252*a^3*cos(x)^2 + 231*a^3)*sqrt(a/cos(x)^
4)*sin(x)/cos(x)^11
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\sec^4(x)\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a*sec(x)**4)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \left(a\sec^4(x)\right)^{7/2} dx = \frac{1}{13}a^{\frac{7}{2}}\tan(x)^{13} + \frac{6}{11}a^{\frac{7}{2}}\tan(x)^{11} + \frac{5}{3}a^{\frac{7}{2}}\tan(x)^{9} + \frac{20}{7}a^{\frac{7}{2}}\tan(x)^{7} + 3a^{\frac{7}{2}}\tan(x)^{5} + 2a^{\frac{7}{2}}\tan(x)^{3} + a^{\frac{7}{2}}\tan(x)$$

input
```
integrate((a*sec(x)^4)^(7/2),x, algorithm="maxima")
```

output
```
1/13*a^(7/2)*tan(x)^13 + 6/11*a^(7/2)*tan(x)^11 + 5/3*a^(7/2)*tan(x)^9 + 2
0/7*a^(7/2)*tan(x)^7 + 3*a^(7/2)*tan(x)^5 + 2*a^(7/2)*tan(x)^3 + a^(7/2)*t
an(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \left(a\sec^4(x)\right)^{7/2} dx = \frac{1}{3003}\left(231a^3\tan(x)^{13} + 1638a^3\tan(x)^{11} + 5005a^3\tan(x)^{9} + 8580a^3\tan(x)^{7} + 90\right.$$

input
```
integrate((a*sec(x)^4)^(7/2),x, algorithm="giac")
```

output
```
1/3003*(231*a^3*tan(x)^13 + 1638*a^3*tan(x)^11 + 5005*a^3*tan(x)^9 + 8580*
a^3*tan(x)^7 + 9009*a^3*tan(x)^5 + 6006*a^3*tan(x)^3 + 3003*a^3*tan(x))*sq
rt(a)
```

Mupad [B] (verification not implemented)

Time = 15.77 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.61

$$\int \left(a\sec^4(x)\right)^{7/2}\,dx = \text{Too large to display}$$

input
```
int((a/cos(x)^4)^(7/2),x)
```

output
```
(a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) +
 4*exp(x*6i) + exp(x*8i) + 1)*2048i)/(7*(exp(x*2i) + 1)^7*(exp(x*2i) + 2*e
xp(x*4i) + exp(x*6i))) - (a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*
exp(x*2i) + 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8i) + 1)*1536i)/((exp(x*2i)
+ 1)^8*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) + (a^3*(a/(exp(-x*1i)/2 + ex
p(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8i) +
 1)*10240i)/(3*(exp(x*2i) + 1)^9*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) -
(a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) +
 4*exp(x*6i) + exp(x*8i) + 1)*4096i)/((exp(x*2i) + 1)^10*(exp(x*2i) + 2*ex
p(x*4i) + exp(x*6i))) + (a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*e
xp(x*2i) + 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8i) + 1)*30720i)/(11*(exp(x*2
i) + 1)^11*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) - (a^3*(a/(exp(-x*1i)/2
+ exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8
i) + 1)*1024i)/((exp(x*2i) + 1)^12*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i)))
+ (a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i)
 + 4*exp(x*6i) + exp(x*8i) + 1)*2048i)/(13*(exp(x*2i) + 1)^13*(exp(x*2i) +
 2*exp(x*4i) + exp(x*6i)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int \left(a\sec^4(x)\right)^{7/2}\,dx = \frac{\sqrt{a}\,\sin(x)\,a^3\left(1024\sin(x)^{12}-6656\sin(x)^{10}+18304\sin(x)^{8}-27456\sin(x)^{6}+240\right.}{3003\cos(x)\left(\sin(x)^{12}-6\sin(x)^{10}+15\sin(x)^{8}-20\sin(x)^{6}+15\sin\right.}$$

input
```
int((a*sec(x)^4)^(7/2),x)
```

output
```
(sqrt(a)*sin(x)*a**3*(1024*sin(x)**12 - 6656*sin(x)**10 + 18304*sin(x)**8
- 27456*sin(x)**6 + 24024*sin(x)**4 - 12012*sin(x)**2 + 3003))/(3003*cos(x
)*(sin(x)**12 - 6*sin(x)**10 + 15*sin(x)**8 - 20*sin(x)**6 + 15*sin(x)**4
- 6*sin(x)**2 + 1))
```

3.62 $\int \left(a\sec^4(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx = a^2\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{4}{3}a^2\sqrt{a\sec^4(x)}\sin^2(x)\tan(x) + \frac{6}{5}a^2\sqrt{a\sec^4(x)}\sin^2(x)\tan^3(x) + \frac{4}{7}a^2\sqrt{a\sec^4(x)}\sin^2(x)\tan^5(x) + \frac{1}{9}a^2\sqrt{a\sec^4(x)}\sin^2(x)\tan^7(x)$$

output
```
a^2*cos(x)*(a*sec(x)^4)^(1/2)*sin(x)+4/3*a^2*(a*sec(x)^4)^(1/2)*sin(x)^2*t
an(x)+6/5*a^2*(a*sec(x)^4)^(1/2)*sin(x)^2*tan(x)^3+4/7*a^2*(a*sec(x)^4)^(1
/2)*sin(x)^2*tan(x)^5+1/9*a^2*(a*sec(x)^4)^(1/2)*sin(x)^2*tan(x)^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.36

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx = \frac{1}{315}\cos(x)(128+130\cos(2x) + 46\cos(4x)+10\cos(6x)+\cos(8x))\left(a\sec^4(x)\right)^{5/2}\sin(x)$$

input
```
Integrate[(a*Sec[x]^4)^(5/2),x]
```

output
```
(Cos[x]*(128 + 130*Cos[2*x] + 46*Cos[4*x] + 10*Cos[6*x] + Cos[8*x])*(a*Sec
[x]^4)^(5/2)*Sin[x])/315
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4611, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(x)^4\right)^{5/2}dx$$

$$\Big\downarrow \text{4611}$$

$$a^2\cos^2(x)\sqrt{a\sec^4(x)}\int \sec^{10}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\cos^2(x)\sqrt{a\sec^4(x)}\int \csc\left(x+\frac{\pi}{2}\right)^{10}dx$$

$$\Big\downarrow \text{4254}$$

$$-a^2\cos^2(x)\sqrt{a\sec^4(x)}\int \left(\tan^8(x)+4\tan^6(x)+6\tan^4(x)+4\tan^2(x)+1\right)d(-\tan(x))$$

$$\Big\downarrow \text{2009}$$

$$-a^2\cos^2(x)\left(-\frac{1}{9}\tan^9(x)-\frac{4\tan^7(x)}{7}-\frac{6\tan^5(x)}{5}-\frac{4\tan^3(x)}{3}-\tan(x)\right)\sqrt{a\sec^4(x)}$$

input
```
Int[(a*Sec[x]^4)^(5/2),x]
```

output
```
-(a^2*Cos[x]^2*Sqrt[a*Sec[x]^4]*(-Tan[x] - (4*Tan[x]^3)/3 - (6*Tan[x]^5)/5
 - (4*Tan[x]^7)/7 - Tan[x]^9/9))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 29.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.39

method	result	size
default	$\frac{\left(128 \cos \left(x \right)^{8}+64 \cos \left(x \right)^{6}+48 \cos \left(x \right)^{4}+40 \cos \left(x \right)^{2}+35\right) \sqrt{a \sec \left(x \right)^{4}}\, a^{2} \tan \left(x \right) \sec \left(x \right)^{6}}{315}$	46
risch	$\frac{256 i a^{2} \sqrt{\frac{a \,{\mathrm e}^{4 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{4}}}\, \left(126 \,{\mathrm e}^{6 i x}+84 \,{\mathrm e}^{4 i x}+9+37 \cos \left(2 x \right)+35 i \sin \left(2 x \right)\right)}{315 \left({\mathrm e}^{2 i x}+1\right)^{7}}$	63

input
```
int((a*sec(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(128*cos(x)^8+64*cos(x)^6+48*cos(x)^4+40*cos(x)^2+35)*(a*sec(x)^4)^(
1/2)*a^2*tan(x)*sec(x)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx = \frac{\left(128\,a^2\cos\left(x\right)^8 + 64\,a^2\cos\left(x\right)^6 + 48\,a^2\cos\left(x\right)^4 + 40\,a^2\cos\left(x\right)^2 + 35\,a^2\right)\sqrt{\frac{a}{\cos(x)^4}}\,\sin}{315\,\cos\left(x\right)^7}$$

input
```
integrate((a*sec(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(128*a^2*cos(x)^8 + 64*a^2*cos(x)^6 + 48*a^2*cos(x)^4 + 40*a^2*cos(x
)^2 + 35*a^2)*sqrt(a/cos(x)^4)*sin(x)/cos(x)^7
```

Sympy [F]

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx = \int \left(a\sec^4\left(x\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*sec(x)**4)**(5/2),x)
```

output
```
Integral((a*sec(x)**4)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.37

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx = \frac{1}{9}\,a^{\frac{5}{2}}\tan\left(x\right)^9+\frac{4}{7}\,a^{\frac{5}{2}}\tan\left(x\right)^7+\frac{6}{5}\,a^{\frac{5}{2}}\tan\left(x\right)^5+\frac{4}{3}\,a^{\frac{5}{2}}\tan\left(x\right)^3+a^{\frac{5}{2}}\tan\left(x\right)$$

input
```
integrate((a*sec(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
1/9*a^(5/2)*tan(x)^9 + 4/7*a^(5/2)*tan(x)^7 + 6/5*a^(5/2)*tan(x)^5 + 4/3*a
^(5/2)*tan(x)^3 + a^(5/2)*tan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx = \frac{1}{315}\left(35\,\tan\left(x\right)^9+180\,\tan\left(x\right)^7+378\,\tan\left(x\right)^5+420\,\tan\left(x\right)^3+315\,\tan\left(x\right)\right)a^{\frac{5}{2}}$$

input
```
integrate((a*sec(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(
x))*a^(5/2)
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \left(a\sec^4(x)\right)^{5/2}\,dx = \frac{128\,a^{5/2}\left(\mathrm{e}^{x\,46\mathrm{i}}\,1\mathrm{i}+\mathrm{e}^{x\,48\mathrm{i}}\,9\mathrm{i}+\mathrm{e}^{x\,50\mathrm{i}}\,36\mathrm{i}+\mathrm{e}^{x\,52\mathrm{i}}\,84\mathrm{i}+\mathrm{e}^{x\,54\mathrm{i}}\,126\mathrm{i}\right)}{315\,\left(\frac{\mathrm{e}^{-x\,2\mathrm{i}}}{2}+\frac{\mathrm{e}^{x\,2\mathrm{i}}}{2}+1\right)\left(\mathrm{e}^{x\,48\mathrm{i}}+7\,\mathrm{e}^{x\,50\mathrm{i}}+21\,\mathrm{e}^{x\,52\mathrm{i}}+35\,\mathrm{e}^{x\,54\mathrm{i}}+35\,\mathrm{e}^{x\,56\mathrm{i}}+21\,\mathrm{e}^{x\,58\mathrm{i}}+7\,\mathrm{e}\right.}$$

input
```
int((a/cos(x)^4)^(5/2),x)
```

output
```
(128*a^(5/2)*(exp(x*46i)*1i + exp(x*48i)*9i + exp(x*50i)*36i + exp(x*52i)*
84i + exp(x*54i)*126i))/(315*(exp(-x*2i)/2 + exp(x*2i)/2 + 1)*(exp(x*48i)
+ 7*exp(x*50i) + 21*exp(x*52i) + 35*exp(x*54i) + 35*exp(x*56i) + 21*exp(x*
58i) + 7*exp(x*60i) + exp(x*62i)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \left(a\sec^4(x)\right)^{5/2} dx = \frac{\sqrt{a}\,\sin(x)\,a^2\left(128\sin(x)^8-576\sin(x)^6+1008\sin(x)^4-840\sin(x)^2+315\right)}{315\cos(x)\left(\sin(x)^8-4\sin(x)^6+6\sin(x)^4-4\sin(x)^2+1\right)}$$

input
```
int((a*sec(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*sin(x)*a**2*(128*sin(x)**8 - 576*sin(x)**6 + 1008*sin(x)**4 - 840
*sin(x)**2 + 315))/(315*cos(x)*(sin(x)**8 - 4*sin(x)**6 + 6*sin(x)**4 - 4*
sin(x)**2 + 1))
```

3.63 $\int \left(a\sec^4(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx = a\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{2}{3}a\sqrt{a\sec^4(x)}\sin^2(x)\tan(x) + \frac{1}{5}a\sqrt{a\sec^4(x)}\sin^2(x)\tan^3(x)$$

output
```
a*cos(x)*(a*sec(x)^4)^(1/2)*sin(x)+2/3*a*(a*sec(x)^4)^(1/2)*sin(x)^2*tan(x
)+1/5*a*(a*sec(x)^4)^(1/2)*sin(x)^2*tan(x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx = \frac{1}{15}a\cos(x)\sqrt{a\sec^4(x)}\sin(x)\left(15+10\tan^2(x)+3\tan^4(x)\right)$$

input
```
Integrate[(a*Sec[x]^4)^(3/2),x]
```

output
```
(a*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]*(15 + 10*Tan[x]^2 + 3*Tan[x]^4))/15
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4611, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\sec^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sec(x)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{4611}$$

$$a\cos^2(x)\sqrt{a\sec^4(x)}\int \sec^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\cos^2(x)\sqrt{a\sec^4(x)}\int \csc\left(x+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{4254}$$

$$-a\cos^2(x)\sqrt{a\sec^4(x)}\int \left(\tan^4(x)+2\tan^2(x)+1\right)d(-\tan(x))$$

$$\Big\downarrow \text{2009}$$

$$-a\cos^2(x)\left(-\frac{1}{5}\tan^5(x)-\frac{2\tan^3(x)}{3}-\tan(x)\right)\sqrt{a\sec^4(x)}$$

input
```
Int[(a*Sec[x]^4)^(3/2),x]
```

output
```
-(a*Cos[x]^2*Sqrt[a*Sec[x]^4]*(-Tan[x] - (2*Tan[x]^3)/3 - Tan[x]^5/5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{a\sqrt{a\sec\left(x\right)^{4}}\,\left(8\sin\left(x\right)\cos\left(x\right)+4\tan\left(x\right)+3\tan\left(x\right)\sec\left(x\right)^{2}\right)}{15}$	31
risch	$\frac{16ia\sqrt{\frac{a\,{\mathrm e}^{4ix}}{\left({\mathrm e}^{2ix}+1\right)^{4}}}\,\left(5+11\cos\left(2x\right)+9i\sin\left(2x\right)\right)}{15\left({\mathrm e}^{2ix}+1\right)^{3}}$	47

input
```
int((a*sec(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*a*(a*sec(x)^4)^(1/2)*(8*sin(x)*cos(x)+4*tan(x)+3*tan(x)*sec(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx = \frac{\left(8\,a\cos\left(x\right)^4 + 4\,a\cos\left(x\right)^2 + 3\,a\right)\sqrt{\frac{a}{\cos\left(x\right)^4}}\sin\left(x\right)}{15\,\cos\left(x\right)^3}$$

input
```
integrate((a*sec(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(8*a*cos(x)^4 + 4*a*cos(x)^2 + 3*a)*sqrt(a/cos(x)^4)*sin(x)/cos(x)^3
```

Sympy [F]

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx = \int \left(a\sec^4\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sec(x)**4)**(3/2),x)
```

output
```
Integral((a*sec(x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.41

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx = \frac{1}{5}\,a^{\frac{3}{2}}\tan\left(x\right)^5 + \frac{2}{3}\,a^{\frac{3}{2}}\tan\left(x\right)^3 + a^{\frac{3}{2}}\tan\left(x\right)$$

input
```
integrate((a*sec(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
1/5*a^(3/2)*tan(x)^5 + 2/3*a^(3/2)*tan(x)^3 + a^(3/2)*tan(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx=\frac{1}{15}\left(3\tan\left(x\right)^5+10\tan\left(x\right)^3+15\tan\left(x\right)\right)a^{\frac{3}{2}}$$

input
```
integrate((a*sec(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))*a^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx=\frac{4\,a^{3/2}\,\sin\left(x\right)}{5\cos\left(x\right)^3}+\frac{a^{3/2}\,\sin\left(x\right)}{5\cos\left(x\right)^5}-\frac{8\,a^{3/2}\sin\left(x\right)^3}{15\cos\left(x\right)^3}$$

input
```
int((a/cos(x)^4)^(3/2),x)
```

output
```
(4*a^(3/2)*sin(x))/(5*cos(x)^3) + (a^(3/2)*sin(x))/(5*cos(x)^5) - (8*a^(3/
2)*sin(x)^3)/(15*cos(x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \left(a\sec^4(x)\right)^{3/2}\,dx=\frac{\sqrt{a}\,\sin\left(x\right)a\left(8\sin\left(x\right)^4-20\sin\left(x\right)^2+15\right)}{15\cos\left(x\right)\left(\sin\left(x\right)^4-2\sin\left(x\right)^2+1\right)}$$

input
```
int((a*sec(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*sin(x)*a*(8*sin(x)**4 - 20*sin(x)**2 + 15))/(15*cos(x)*(sin(x)**4
 - 2*sin(x)**2 + 1))
```

3.64 $\int \sqrt{a \sec^4(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \sqrt{a \sec^4(x)}\, dx = \cos(x)\sqrt{a \sec^4(x)} \sin(x)$$

output
```
cos(x)*(a*sec(x)^4)^(1/2)*sin(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sqrt{a \sec^4(x)}\, dx = \cos(x)\sqrt{a \sec^4(x)} \sin(x)$$

input
```
Integrate[Sqrt[a*Sec[x]^4],x]
```

output
```
Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4611, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sec^4(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a \sec(x)^4}dx$$
$$\Big\downarrow \text{4611}$$
$$\cos^2(x)\sqrt{a \sec^4(x)}\int \sec^2(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\cos^2(x)\sqrt{a \sec^4(x)}\int \csc\left(x+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{4254}$$
$$-\cos^2(x)\sqrt{a \sec^4(x)}\int 1d(-\tan(x))$$
$$\Big\downarrow \text{24}$$
$$\sin(x)\cos(x)\sqrt{a \sec^4(x)}$$

input
```
Int[Sqrt[a*Sec[x]^4],x]
```

output
```
Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\cos(x)\sqrt{a\sec(x)^4}\,\sin(x)$	14
risch	$2i\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(1+e^{-2ix})$	29

input
```
int((a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(x)*(a*sec(x)^4)^(1/2)*sin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sqrt{a \sec^4(x)}\, dx = \sqrt{\frac{a}{\cos(x)^4}} \cos(x) \sin(x)$$

input
```
integrate((a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a/cos(x)^4)*cos(x)*sin(x)
```

Sympy [F]

$$\int \sqrt{a \sec^4(x)}\, dx = \int \sqrt{a \sec^4(x)}\, dx$$

input
```
integrate((a*sec(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a*sec(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sqrt{a \sec^4(x)}\, dx = \sqrt{a} \tan(x)$$

input
```
integrate((a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*tan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sqrt{a\sec^4(x)}\,dx = \sqrt{a}\tan(x)$$

input
```
integrate((a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
sqrt(a)*tan(x)
```

Mupad [B] (verification not implemented)

Time = 10.51 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sqrt{a\sec^4(x)}\,dx = \sqrt{a}\tan(x)$$

input
```
int((a/cos(x)^4)^(1/2),x)
```

output
```
a^(1/2)*tan(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \sqrt{a\sec^4(x)}\,dx = \frac{\sqrt{a}\,\sin(x)}{\cos(x)}$$

input
```
int((a*sec(x)^4)^(1/2),x)
```

output
```
(sqrt(a)*sin(x))/cos(x)
```

3.65 $\int \frac{1}{\sqrt{a \sec^4(x)}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\sqrt{a \sec^4(x)}} \, dx = \frac{x \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a \sec^4(x)}}$$

output
```
1/2*x*sec(x)^2/(a*sec(x)^4)^(1/2)+1/2*tan(x)/(a*sec(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{a \sec^4(x)}} \, dx = \frac{x \sec^2(x) + \tan(x)}{2\sqrt{a \sec^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Sec[x]^4],x]
```

output
```
(x*Sec[x]^2 + Tan[x])/(2*Sqrt[a*Sec[x]^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4611, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sec(x)^4}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\sec^2(x)\int \cos^2(x)dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^2(x)\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sec^2(x)\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sec^2(x)\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{\sqrt{a\sec^4(x)}}$$

input
```
Int[1/Sqrt[a*Sec[x]^4],x]
```

output
```
(Sec[x]^2*(x/2 + (Cos[x]*Sin[x])/2))/Sqrt[a*Sec[x]^4]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\tan(x)+x\sec(x)^{2}}{2\sqrt{a\sec(x)^{4}}}$	20
risch	$\frac{e^{2ix}x}{2\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^{4}}}\,\left(e^{2ix}+1\right)^{2}}-\frac{i e^{4ix}}{8\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^{4}}}\,\left(e^{2ix}+1\right)^{2}}+\frac{i}{8\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^{4}}}\,\left(e^{2ix}+1\right)^{2}}$	102

input
```
int(1/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*sec(x)^4)^(1/2)*(tan(x)+x*sec(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx = \frac{\left(\cos(x)^3\sin(x) + x\cos(x)^2\right)\sqrt{\frac{a}{\cos(x)^4}}}{2\,a}$$

input
```
integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(cos(x)^3*sin(x) + x*cos(x)^2)*sqrt(a/cos(x)^4)/a
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx = \int \frac{1}{\sqrt{a\sec^4(x)}}\,dx$$

input
```
integrate(1/(a*sec(x)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a*sec(x)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx = \frac{x}{2\sqrt{a}} + \frac{\tan(x)}{2\left(\sqrt{a}\tan(x)^2 + \sqrt{a}\right)}$$

input
```
integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x/sqrt(a) + 1/2*tan(x)/(sqrt(a)*tan(x)^2 + sqrt(a))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx = -\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor - x - \frac{\tan(x)}{\tan(x)^2+1}}{2\sqrt{a}}$$

input
```
integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(pi*floor(x/pi + 1/2) - x - tan(x)/(tan(x)^2 + 1))/sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\cos(x)^4}}}\,dx$$

input
```
int(1/(a/cos(x)^4)^(1/2),x)
```

output
```
int(1/(a/cos(x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx = \frac{\sqrt{a}\,(\cos(x)\sin(x)+x)}{2a}$$

input
```
int(1/(a*sec(x)^4)^(1/2),x)
```

output
```
(sqrt(a)*(cos(x)*sin(x) + x))/(2*a)
```

3.66 $\int \frac{1}{\left(a\sec^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{\left(a\sec^4(x)\right)^{3/2}}\,dx=\frac{5x\sec^2(x)}{16a\sqrt{a\sec^4(x)}}+\frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}}+\frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}}+\frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}}$$

output
```
5/16*x*sec(x)^2/a/(a*sec(x)^4)^(1/2)+5/24*cos(x)*sin(x)/a/(a*sec(x)^4)^(1/
2)+1/6*cos(x)^3*sin(x)/a/(a*sec(x)^4)^(1/2)+5/16*tan(x)/a/(a*sec(x)^4)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a\sec^4(x)\right)^{3/2}}\,dx=\frac{\sec^6(x)(60x+45\sin(2x)+9\sin(4x)+\sin(6x))}{192\left(a\sec^4(x)\right)^{3/2}}$$

input
```
Integrate[(a*Sec[x]^4)^(-3/2),x]
```

output
```
(Sec[x]^6*(60*x + 45*Sin[2*x] + 9*Sin[4*x] + Sin[6*x]))/(192*(a*Sec[x]^4)^
(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 4611, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec^4(x))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sec(x)^4)^{3/2}}dx$$

$$\Big\downarrow \text{4611}$$

$$\frac{\sec^2(x)\int \cos^6(x)dx}{a\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^2(x)\int \sin\left(x+\frac{\pi}{2}\right)^6dx}{a\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sec^2(x)\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)}{a\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^2(x)\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)}{a\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sec^2(x)\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)}{a\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^2(x)\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)}{a\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sec^2(x)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)}{a\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sec^2(x)\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)}{a\sqrt{a\sec^4(x)}}$$

input
```
Int[(a*Sec[x]^4)^(-3/2),x]
```

output
```
(Sec[x]^2*((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[
x]*Sin[x])/2))/4))/6))/(a*Sqrt[a*Sec[x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

method	result
default	$\frac{\left(8\cos(x)^4+10\cos(x)^2+15\right)\tan(x)+15x\sec(x)^2}{48a\sqrt{a\sec(x)^4}}$
risch	$\frac{5\,\mathrm{e}^{2ix}x}{16a(\mathrm{e}^{2ix}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}}-\frac{i\mathrm{e}^{8ix}}{384a(\mathrm{e}^{2ix}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}}-\frac{3i\mathrm{e}^{6ix}}{128a(\mathrm{e}^{2ix}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}}-\frac{15i\mathrm{e}^{4ix}}{128a(\mathrm{e}^{2ix}+1)^2\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}}+$

input
```
int(1/(a*sec(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/a/(a*sec(x)^4)^(1/2)*((8*cos(x)^4+10*cos(x)^2+15)*tan(x)+15*x*sec(x)^
2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\sec^4(x)\right)^{3/2}}\,dx=\frac{\left(15\,x\cos\left(x\right)^2+\left(8\,\cos\left(x\right)^7+10\,\cos\left(x\right)^5+15\,\cos\left(x\right)^3\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos(x)^4}}}{48\,a^2}$$

input
```
integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/48*(15*x*cos(x)^2 + (8*cos(x)^7 + 10*cos(x)^5 + 15*cos(x)^3)*sin(x))*sqr
t(a/cos(x)^4)/a^2
```

Sympy [F]

$$\int \frac{1}{(a\sec^4(x))^{3/2}}\,dx = \int \frac{1}{(a\sec^4(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)**4)**(3/2),x)
```

output
```
Integral((a*sec(x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a\sec^4(x))^{3/2}}\,dx = \frac{15\,\tan(x)^5 + 40\,\tan(x)^3 + 33\,\tan(x)}{48\left(a^{\frac{3}{2}}\tan(x)^6 + 3\,a^{\frac{3}{2}}\tan(x)^4 + 3\,a^{\frac{3}{2}}\tan(x)^2 + a^{\frac{3}{2}}\right)} + \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
1/48*(15*tan(x)^5 + 40*tan(x)^3 + 33*tan(x))/(a^(3/2)*tan(x)^6 + 3*a^(3/2)
*tan(x)^4 + 3*a^(3/2)*tan(x)^2 + a^(3/2)) + 5/16*x/a^(3/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a\sec^4(x))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sec^4(x))^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cos(x)^4}\right)^{3/2}}\,dx$$

input
```
int(1/(a/cos(x)^4)^(3/2),x)
```

output
```
int(1/(a/cos(x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a\sec^4(x))^{3/2}}\,dx = \frac{\sqrt{a}\,(8\cos(x)\sin(x)^5 - 26\cos(x)\sin(x)^3 + 33\cos(x)\sin(x) + 15x)}{48a^2}$$

input
```
int(1/(a*sec(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*(8*cos(x)*sin(x)**5 - 26*cos(x)*sin(x)**3 + 33*cos(x)*sin(x) + 15
*x))/(48*a**2)
```

3.67 $\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx=\frac{63x\sec^2(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}}+\frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}}+\frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}}+\frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}}+\frac{63\tan(x)}{256a^2\sqrt{a\sec^4(x)}}$$

output
```
63/256*x*sec(x)^2/a^2/(a*sec(x)^4)^(1/2)+21/128*cos(x)*sin(x)/a^2/(a*sec(x
)^4)^(1/2)+21/160*cos(x)^3*sin(x)/a^2/(a*sec(x)^4)^(1/2)+9/80*cos(x)^5*sin
(x)/a^2/(a*sec(x)^4)^(1/2)+1/10*cos(x)^7*sin(x)/a^2/(a*sec(x)^4)^(1/2)+63/
256*tan(x)/a^2/(a*sec(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx=\frac{\cos^2(x)\sqrt{a\sec^4(x)}(2520x+2100\sin(2x)+600\sin(4x)+150\sin(6x)+25\sin(8x)+}{10240a^3}$$

input
```
Integrate[(a*Sec[x]^4)^(-5/2),x]
```

output
```
(Cos[x]^2*Sqrt[a*Sec[x]^4]*(2520*x + 2100*Sin[2*x] + 600*Sin[4*x] + 150*Si
n[6*x] + 25*Sin[8*x] + 2*Sin[10*x]))/(10240*a^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 4611, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \sec^4(x)\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a \sec(x)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4611}$$

$$\frac{\sec^2(x) \int \cos^{10}(x)dx}{a^2 \sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sec^2(x) \int \sin\left(x+\frac{\pi}{2}\right)^{10}dx}{a^2 \sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sec^2(x) \left(\frac{9}{10} \int \cos^8(x)dx+\frac{1}{10} \sin(x) \cos^9(x)\right)}{a^2 \sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sec^2(x) \left(\frac{9}{10} \int \sin\left(x+\frac{\pi}{2}\right)^8dx+\frac{1}{10} \sin(x) \cos^9(x)\right)}{a^2 \sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sec^2(x) \left(\frac{9}{10} \left(\frac{7}{8} \int \cos^6(x)dx+\frac{1}{8} \sin(x) \cos^7(x)\right)+\frac{1}{10} \sin(x) \cos^9(x)\right)}{a^2 \sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6dx+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)}{a^2\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{\sec^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)}{a^2\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)}{a^2\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{\sec^2(x)\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)}{a^2\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sec^2(x)\Big(\frac{9}{10}\Big(\frac{7}{8}\Big(\frac{5}{6}\Big(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\Big)+\frac{1}{6}\sin(x)\cos^5(x)\Big)+\frac{1}{8}\sin(x)\cos^7(x)\Big)+\frac{1}{10}\sin(x)\cos}{a^2\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{\sec^2(x)\Big(\frac{9}{10}\Big(\frac{7}{8}\Big(\frac{5}{6}\Big(\frac{3}{4}\Big(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\Big)+\frac{1}{4}\sin(x)\cos^3(x)\Big)+\frac{1}{6}\sin(x)\cos^5(x)\Big)+\frac{1}{8}\sin(x)\cos^7(x)\Big)+\frac{1}{10}\mathrm{si}}{a^2\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \; 24$$

$$\frac{\sec^2(x)\Big(\frac{1}{10}\sin(x)\cos^9(x)+\frac{9}{10}\Big(\frac{1}{8}\sin(x)\cos^7(x)+\frac{7}{8}\Big(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\Big(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\Big(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(}{a^2\sqrt{a\sec^4(x)}}$$

input
```
Int[(a*Sec[x]^4)^(-5/2),x]
```

output
```
(Sec[x]^2*((Cos[x]^9*Sin[x])/10 + (9*((Cos[x]^7*Sin[x])/8 + (7*((Cos[x]^5*
Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4))/6)
)/8))/10))/(a^2*Sqrt[a*Sec[x]^4])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.39

method	result
default	$\frac{\frac{\left(128 \cos(x)^{8}+144 \cos(x)^{6}+168 \cos(x)^{4}+210 \cos(x)^{2}+315\right) \tan(x)}{1280}+\frac{63 x \sec(x)^{2}}{256}}{\sqrt{a \sec(x)^{4}}\, a^{2}}$
risch	$\frac{63\, e^{2 i x} x}{256 a^{2} \left(e^{2 i x}+1\right)^{2} \sqrt{\frac{a\, e^{4 i x}}{\left(e^{2 i x}+1\right)^{4}}}}-\frac{i e^{12 i x}}{10240 a^{2} \left(e^{2 i x}+1\right)^{2} \sqrt{\frac{a\, e^{4 i x}}{\left(e^{2 i x}+1\right)^{4}}}}-\frac{5 i e^{10 i x}}{4096 a^{2} \left(e^{2 i x}+1\right)^{2} \sqrt{\frac{a\, e^{4 i x}}{\left(e^{2 i x}+1\right)^{4}}}}-\frac{105 i e^{4 i x}}{1024 a^{2} \left(e^{2 i x}+1\right)^{2} \sqrt{\frac{a\, e^{4 i x}}{\left(e^{2 i x}+1\right)^{4}}}}$

input
```
int(1/(a*sec(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/1280*(128*cos(x)^8+144*cos(x)^6+168*cos(x)^4+210*cos(x)^2+315)*tan(x)+6
3/256*x*sec(x)^2)/(a*sec(x)^4)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx = \frac{\left(315\,x\cos\left(x\right)^{2} + \left(128\,\cos\left(x\right)^{11} + 144\,\cos\left(x\right)^{9} + 168\,\cos\left(x\right)^{7} + 210\,\cos\left(x\right)^{5} + 315\right.}{1280\,a^{3}}$$

input
```
integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/1280*(315*x*cos(x)^2 + (128*cos(x)^11 + 144*cos(x)^9 + 168*cos(x)^7 + 21
0*cos(x)^5 + 315*cos(x)^3)*sin(x))*sqrt(a/cos(x)^4)/a^3
```

Sympy [F]

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sec^{4}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*sec(x)**4)**(5/2),x)
```

output
```
Integral((a*sec(x)**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx = \frac{315\tan(x)^9 + 1470\tan(x)^7 + 2688\tan(x)^5 + 2370\tan(x)^3 + 965\tan(x)}{1280\left(a^{\frac{5}{2}}\tan(x)^{10} + 5\,a^{\frac{5}{2}}\tan(x)^8 + 10\,a^{\frac{5}{2}}\tan(x)^6 + 10\,a^{\frac{5}{2}}\tan(x)^4 + 5\,a^{\frac{5}{2}}\tan(x)^2\right.} + \frac{63\,x}{256\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
1/1280*(315*tan(x)^9 + 1470*tan(x)^7 + 2688*tan(x)^5 + 2370*tan(x)^3 + 965
*tan(x))/(a^(5/2)*tan(x)^10 + 5*a^(5/2)*tan(x)^8 + 10*a^(5/2)*tan(x)^6 + 1
0*a^(5/2)*tan(x)^4 + 5*a^(5/2)*tan(x)^2 + a^(5/2)) + 63/256*x/a^(5/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cos(x)^4}\right)^{5/2}}\,dx$$

input
```
int(1/(a/cos(x)^4)^(5/2),x)
```

output
```
int(1/(a/cos(x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,\left(128\cos(x)\sin(x)^9 - 656\cos(x)\sin(x)^7 + 1368\cos(x)\sin(x)^5 - 1490\cos(x)\,\mathrm{s}\right.}{1280a^3}$$

input
```
int(1/(a*sec(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*(128*cos(x)*sin(x)**9 - 656*cos(x)*sin(x)**7 + 1368*cos(x)*sin(x)
**5 - 1490*cos(x)*sin(x)**3 + 965*cos(x)*sin(x) + 315*x))/(1280*a**3)
```

3.68 $\int ((b\sec(c+dx))^p)^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int ((b\sec(c+dx))^p)^n \, dx = -\frac{\cos(c+dx) \, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1-np), \frac{1}{2}(3-np), \cos^2(c+dx)\right) ((b\sec(c+dx))^p)^n \sin(c+dx}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

output
```
-cos(d*x+c)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(d*x+c)^2)*((b
*sec(d*x+c))^p)^n*sin(d*x+c)/d/(-n*p+1)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int ((b\sec(c+dx))^p)^n \, dx = \frac{\cot(c+dx) \, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{np}{2}, \frac{1}{2}(2+np), \sec^2(c+dx)\right) ((b\sec(c+dx))^p)^n \sqrt{-\tan^2(c+dx)}}{dnp}$$

input
```
Integrate[((b*Sec[c + d*x])^p)^n,x]
```

output
```
(Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]
*((b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4611, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int ((b\sec(c+dx))^p)^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int ((b\sec(c+dx))^p)^n dx$$

$$\Big\downarrow \text{4611}$$

$$(b\sec(c+dx))^{-np}\left((b\sec(c+dx))^p\right)^n\int (b\sec(c+dx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(b\sec(c+dx))^{-np}\left((b\sec(c+dx))^p\right)^n\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{np}dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^{np}\left((b\sec(c+dx))^p\right)^n\int \left(\frac{\cos(c+dx)}{b}\right)^{-np}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^{np}\left((b\sec(c+dx))^p\right)^n\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{-np}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(c+dx)\cos(c+dx)\left((b\sec(c+dx))^p\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(c+dx)\right)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[((b*Sec[c + d*x])^p)^n,x]
```

output
```
-((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d
*x]^2]*((b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*x]
^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&& !IntegerQ[p]
```

Maple [F]

$$\int ((b\sec(dx+c))^p)^n\,dx$$

input
```
int(((b*sec(d*x+c))^p)^n,x)
```

output
```
int(((b*sec(d*x+c))^p)^n,x)
```

Fricas [F]

$$\int ((b\sec(c+dx))^p)^n\,dx=\int ((b\sec(dx+c))^p)^n\,dx$$

input
```
integrate(((b*sec(d*x+c))^p)^n,x, algorithm="fricas")
```

output
```
integral(((b*sec(d*x + c))^p)^n, x)
```

Sympy [F]

$$\int ((b\sec(c+dx))^p)^n\,dx=\int ((b\sec(c+dx))^p)^n\,dx$$

input
```
integrate(((b*sec(d*x+c))**p)**n,x)
```

output
```
Integral(((b*sec(c + d*x))**p)**n, x)
```

Maxima [F]

$$\int ((b\sec(c+dx))^p)^n\,dx=\int ((b\sec(dx+c))^p)^n\,dx$$

input
```
integrate(((b*sec(d*x+c))^p)^n,x, algorithm="maxima")
```

output
```
integrate(((b*sec(d*x + c))^p)^n, x)
```

Giac [F]

$$\int ((b\sec(c+dx))^p)^n\,dx = \int ((b\sec(dx+c))^p)^n\,dx$$

input
```
integrate(((b*sec(d*x+c))^p)^n,x, algorithm="giac")
```

output
```
integrate(((b*sec(d*x + c))^p)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int ((b\sec(c+dx))^p)^n\,dx = \int \left(\left(\frac{b}{\cos(c+d\,x)}\right)^p\right)^n\,dx$$

input
```
int(((b/cos(c + d*x))^p)^n,x)
```

output
```
int(((b/cos(c + d*x))^p)^n, x)
```

Reduce [F]

$$\int ((b\sec(c+dx))^p)^n\,dx = b^{np}\left(\int \sec(dx+c)^{np}\,dx\right)$$

input
```
int(((b*sec(d*x+c))^p)^n,x)
```

output
```
b**(n*p)*int(sec(c + d*x)**(n*p),x)
```

3.69 $\int (a(b\sec(c+dx))^p)^n \, dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (a(b\sec(c+dx))^p)^n \, dx = -\frac{\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(c+dx)\right)(a(b\sec(c+dx))^p)^n\sin(c+d}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

output
```
-cos(d*x+c)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(d*x+c)^2)*(a*
(b*sec(d*x+c))^p)^n*sin(d*x+c)/d/(-n*p+1)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int (a(b\sec(c+dx))^p)^n \, dx = \frac{\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},\frac{1}{2}(2+np),\sec^2(c+dx)\right)(a(b\sec(c+dx))^p)^n\sqrt{-\tan^2(c+dx)}}{dnp}$$

input
```
Integrate[(a*(b*Sec[c + d*x])^p)^n,x]
```

output
```
(Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]
*(a*(b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4611, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(b\sec(c+dx))^p)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a(b\sec(c+dx))^p)^n dx$$

$$\Big\downarrow \text{4611}$$

$$(b\sec(c+dx))^{-np}(a(b\sec(c+dx))^p)^n\int (b\sec(c+dx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(b\sec(c+dx))^{-np}(a(b\sec(c+dx))^p)^n\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{np}dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^{np}(a(b\sec(c+dx))^p)^n\int \left(\frac{\cos(c+dx)}{b}\right)^{-np}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^{np}(a(b\sec(c+dx))^p)^n\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{-np}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(c+dx)\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(c+dx)\right)(a(b\sec(c+dx))^p)^n}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(a*(b*Sec[c + d*x])^p)^n,x]
```

output
```
-((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d
*x]^2]*(a*(b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*
x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4611
```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Simp[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p]))   Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x]
&& !IntegerQ[p]
```

Maple [F]

$$\int (a(b\sec(dx+c))^p)^n\,dx$$

input
```
int((a*(b*sec(d*x+c))^p)^n,x)
```

output
```
int((a*(b*sec(d*x+c))^p)^n,x)
```

Fricas [F]

$$\int (a(b\sec(c+dx))^p)^n \, dx = \int \left(\left(b \sec\left(dx+c\right)\right)^p a\right)^n \, dx$$

input
```
integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="fricas")
```

output
```
integral(((b*sec(d*x + c))^p*a)^n, x)
```

Sympy [F]

$$\int (a(b\sec(c+dx))^p)^n \, dx = \int \left(a(b\sec\left(c+dx\right))^p\right)^n \, dx$$

input
```
integrate((a*(b*sec(d*x+c))**p)**n,x)
```

output
```
Integral((a*(b*sec(c + d*x))**p)**n, x)
```

Maxima [F]

$$\int (a(b\sec(c+dx))^p)^n \, dx = \int \left(\left(b \sec\left(dx+c\right)\right)^p a\right)^n \, dx$$

input
```
integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="maxima")
```

output
```
integrate(((b*sec(d*x + c))^p*a)^n, x)
```

Giac [F]

$$\int (a(b\sec(c+dx))^p)^n \, dx = \int ((b\sec(dx+c))^p a)^n \, dx$$

input
```
integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="giac")
```

output
```
integrate(((b*sec(d*x + c))^p*a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a(b\sec(c+dx))^p)^n \, dx = \int \left(a\left(\frac{b}{\cos(c+d\,x)}\right)^p\right)^n dx$$

input
```
int((a*(b/cos(c + d*x))^p)^n,x)
```

output
```
int((a*(b/cos(c + d*x))^p)^n, x)
```

Reduce [F]

$$\int (a(b\sec(c+dx))^p)^n \, dx = b^{np}a^n\left(\int \sec(dx+c)^{np}\,dx\right)$$

input
```
int((a*(b*sec(d*x+c))^p)^n,x)
```

output
```
b**(n*p)*a**n*int(sec(c + d*x)**(n*p),x)
```

3.70 $\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\begin{aligned}\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx &= \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}\\&\quad+\frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21bd}+\frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^3d}\end{aligned}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/d+10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*(b*sec(d*x+c))^(7
/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\begin{aligned}&\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx\\&=\frac{\sec^2(c+dx)\sqrt{b\sec(c+dx)}\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(10*Cos[c + d*x]^(5/2)*EllipticF[(c +
 d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{9/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\int (b\sec(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^4}$$

input
```
Int[Sec[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

output
```
((2*b*(b*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b*(b
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7)/b^4
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.82 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

method	result
default	$\frac{\left(\frac{10\tan(dx+c)}{21}+\frac{2\tan(dx+c)\sec(dx+c)^2}{7}-\frac{10i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)}{21}\right)\sqrt{b\sec(}}{d}$

input
```
int(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(10/21*tan(d*x+c)+2/7*tan(d*x+c)*sec(d*x+c)^2-10/21*I*(cos(d*x+c)+1)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-co
t(d*x+c)+csc(d*x+c)),I))*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^3 \,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}\sqrt{b}\cos(dx+c}{21\,d\cos}$$

input
```
integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*
sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(c+dx)}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Maxima [F]

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

output
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)$$

input
```
int(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)
```

3.71 $\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d}$$

output
```
-6/5*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x
+c))^(1/2)+6/5*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*(b*sec(d*x+c))^(5/2)*
sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sec^2(c+dx)\sqrt{b\sec(c+dx)}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[Sec[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c
+ d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\sec(c+dx))^{7/2}dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^3}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{5}b^2\int (b\sec(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{5}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^3}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int\frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^3}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^3}$$

input
```
Int[Sec[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

output
```
((2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*b^2*((-2*b^2*Ellipti
cE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqr
t[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5)/b^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.88 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.15

method	result
default	$\frac{2\sqrt{b\sec(dx+c)}\left(3\sin(dx+c)+\tan(dx+c)+\sec(dx+c)\tan(dx+c)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \text{EllipticF}(i(-c$

input
```
int(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*(b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(3*sin(d*x+c)+tan(d*x+c)+sec(d*x
+c)*tan(d*x+c)+I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
I*(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-
cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.26

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^2\ \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{}$$

input
```
integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(c+dx)}}\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}}\sec{(dx+c)}^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)$$

input
```
int(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)
```

3.72 $\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/d+2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{2(b\sec(c+dx))^{3/2}\left(\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{3bd}$$

input
```
Integrate[Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*(b*Sec[c + d*x])^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
Sin[c + d*x]))/(3*b*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{5/2}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^2}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^2}$$

input
```
Int[Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\left(-\frac{2i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)}{3}+\frac{2\tan(dx+c)}{3}\right)\sqrt{b\sec(dx+c)}}{d}$	87

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/3*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+
c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2/3*tan(d*x+c))*(b*sec
(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$= \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{we}}{3\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(c+dx)}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}}\sec{(dx+c)}^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}}\sec{(dx+c)}^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^2 dx\right)$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)
```

3.73 $\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c
))^(1/2)+2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{2\sqrt{b\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
 Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{3/2}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b}$$

input
```
Int[Sec[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

output
```
((-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d)/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

method	result
default	$\frac{2\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\left(-\cos(dx+c)^2-2\cos(\right.}{d(\cos(dx+c)+1)}$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),
I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-cos(d*x+
c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)+sin(d*x+c))*(b*sec(d*x+c))^(
1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\text{weie}}{d}$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(c+dx)}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}}\sec{(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}}\sec{(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
int((b/cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)dx\right)$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x),x)
```

3.74 $\int \sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \sqrt{b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

output
```
2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))^(
1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sqrt{b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

input
```
Integrate[Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

input
```
Int[Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

method	result	size
default	$\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}\ \mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)}{d}$	77

input
```
int((b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1)*(b*sec(d*x+c))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,c}{d}$$

input
```
integrate((b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/d
```

Sympy [F]

$$\int \sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(c+dx)}}\,dx$$

input
```
integrate((b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\,dx$$

input
```
integrate((b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sqrt{b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+d\,x)}\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input
```
int((b/cos(c + d*x))^(1/2),x)
```

output
```
(2*cos(c + d*x)^(1/2)*(b/cos(c + d*x))^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/
d
```

Reduce [F]

$$\int \sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}dx\right)$$

input
```
int((b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x)),x)
```

3.75 $\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
2*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \frac{1}{\sqrt{b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{b\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Int[Cos[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.79

method	result
default	$-\frac{2\sqrt{b\sec(dx+c)}\left(2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)-2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{Ellip}\right.}{d\left((1-\cos(dx+c))^2\csc(dx+c)^2+1\right)}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{d}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}b+b\right)}{b\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}b+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+}}}{}\right.}{}$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(b*sec(d*x+c))^(1/2)*(2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)-2*I*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(
d*x+c)),I)+(1-cos(d*x+c))^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))/((1-cos(d*
x+c))^2*csc(d*x+c)^2+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))-i\sqrt{2}\sqrt{b}\text{weiers}}{d}$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(c+dx)}}\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \cos(c+d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)*(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx=\sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x),x)
```

3.76 $\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/d+2/3*b*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Si
n[2*(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^2\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

input
```
Int[Cos[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*b^2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.39

method	result	si
default	$\frac{2\sqrt{b\sec(dx+c)}\left(i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \text{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\sin(dx+c)\cos(dx+c)\right)}{3d}$	9

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(b*sec(d*x+c))^(1/2)*(I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d
*x+c)*cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.25

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))}{3\,d}$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt
(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx=\int \sqrt{b\sec{(c+dx)}}\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx=\int \sqrt{b\sec{(dx+c)}}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx=\int \sqrt{b\sec{(dx+c)}}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^2 dx\right)$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**2,x)
```

3.77 $\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx=\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

output
```
6/5*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+
c))^(1/2)+2/5*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + S
in[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^3\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^3\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^3 \left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{5b^2 d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}} \right)$$

input
```
Int[Cos[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^3*((6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[
c + d*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.94

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)+i\left(-3\cos(dx+c)^2-6\cos(dx+c)-3\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticE}(i(\cot(dx+c)-}{5}$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*(sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)^2+cos(d*x+c)+3)+I*(-3*cos(d*x+c)^
2-6*cos(d*x+c)-3)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c
)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cot(d*x+c)-csc(d*x+c)),I))*(b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c) + 3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^3dx\right)$$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**3,x)
```

3.78 $\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/d+2/7*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*b*sin(d*x+c)/d/
(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84d}$$

input
```
Integrate[Cos[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2
6*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^4\left(\frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{5\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^4\left(\frac{5\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{5\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^4\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^4\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

input

```
Int[Cos[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

output

```
b^4*((2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^2*d) + (2*Sin[
c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(3\cos(dx+c)^2+5\right)+i(-5\cos(dx+c)-5)\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c}}\right.}{21d}$

input
```
int(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*(sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2+5)+I*(-5*cos(d*x+c)-5)*Ellip
ticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2))*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2\left(3\cos\left(dx+c\right)^3+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx\right.}{21\,d}$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
 c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)))/d
```

Sympy [F]

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(c+dx\right)}\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*cos(c + d*x)**4, x)
```

Maxima [F]

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \cos(c+d\,x)^4\sqrt{\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^4\,dx\right)$$

input
```
int(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**4,x)
```

3.79 $\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

output
```
14/15*b*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*
x+c))^(1/2)+2/9*b^4*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b^2*sin(d*x+c)
/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(84\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))\right)}{90d}$$

input
```
Integrate[Cos[c + d*x]^5*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C
os[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^5\left(\frac{7\int \frac{1}{(b\sec(c+dx))^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{7\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^5\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 4258

$$b^5\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^5\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3119

$$b^5\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

input
```
Int[Cos[c + d*x]^5*Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^5*((2*Sin[c + d*x])/(9*b*d*(b*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c
 + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sin[
c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))))/(9*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.66 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.33

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(5\cos(dx+c)^{4}+5\cos(dx+c)^{3}+7\cos(dx+c)^{2}+7\cos(dx+c)+21\right)+21i\left(\cos(dx+c)^{2}+2\cos(dx+c)+1\right)\sqrt{\frac{\cos}{\cos(}}}{}$

input
```
int(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d*(sin(d*x+c)*cos(d*x+c)*(5*cos(d*x+c)^4+5*cos(d*x+c)^3+7*cos(d*x+c)^
2+7*cos(d*x+c)+21)+21*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c))
,I)+21*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I))*(b*sec(d*
x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2\left(5\cos\left(dx+c\right)^4+7\cos\left(dx+c\right)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstra}}{}$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)
```

Giac [F]

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \cos(c+d\,x)^5\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^5\,dx\right)$$

input
```
int(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**5,x)
```

3.80 $\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{10b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^2d}$$

output
```
10/21*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x
+c))^(1/2)/d+10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(b*sec(d*x+c))^(7
/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+d}{21bd}$$

input
```
Integrate[Sec[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{9/2}dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\int (b\sec(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^3}$$

input
```
Int[Sec[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b*(b
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7)/b^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(\frac{10\tan(dx+c)}{21}+\frac{2\tan(dx+c)\sec(dx+c)^2}{7}-\frac{10i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)}{21}\right)b\sqrt{b\sec}}{d}$

input
```
int(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(10/21*tan(d*x+c)+2/7*tan(d*x+c)*sec(d*x+c)^2-10/21*I*(cos(d*x+c)+1)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-co
t(d*x+c)+csc(d*x+c)),I))*b*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{3}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b^{\frac{3}{2}}\cos\cdots}{}$$

input
```
integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^3*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b*cos(d*x + c)^2 + 3
*b)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(c+dx))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx=\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output
```
int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)b$$

input
```
int(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)*b
```

3.81 $\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = -\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6b\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd}$$

output
```
-6/5*b^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d
*x+c))^(1/2)+6/5*b*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*(b*sec(d*x+c))^(5
/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{(b\sec(c+dx))^{5/2}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10bd}$$

input
```
Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]
+ 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*b*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{7/2}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\int (b\sec(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^2}$$

input
```
Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*b^2*((-2*b^2*Ellipti
cE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqr
t[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5)/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.09

method	result
default	$\frac{2\sqrt{b\sec(dx+c)}\, b\left(3\sin(dx+c)+\tan(dx+c)+\sec(dx+c)\tan(dx+c)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \text{EllipticF}(i(-\right.}{}$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*(b*sec(d*x+c))^(1/2)*b/(cos(d*x+c)+1)*(3*sin(d*x+c)+tan(d*x+c)+sec(d
*x+c)*tan(d*x+c)+I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+I*(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*
(-cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{-3i\sqrt{2}b^{\frac{3}{2}}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\ \text{si}}{}$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(3/2)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*b*cos(d*x + c)^2 + b)*sqrt(b/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec{(c+dx)})^{\frac{3}{2}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec{(dx+c)})^{\frac{3}{2}}\sec{(dx+c)}^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)b$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*b
```

3.82 $\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}$$

output
```
2/3*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/d+2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d}$$

input
```
Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
Tan[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\sec(c+dx))^{5/2}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b}$$

input
```
Int[Sec[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{\left(-\frac{2i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)}{3}+\frac{2\tan(dx+c)}{3}\right)b\sqrt{b\sec(dx+c)}}{d}$	88

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/3*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+
c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2/3*tan(d*x+c))*b*(b*s
ec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}b^{\frac{3}{2}}\cos(}{3\,d\cos(}$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + I*sqrt(2)*b^(3/2)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*b*sqrt(b/cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((b/cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
int((b/cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^2 dx\right) b$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*b
```

3.83 $\int (b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (b\sec(c+dx))^{3/2}\,dx = -\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*b^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x
+c))^(1/2)+2*b*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int (b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
 + Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.76

method	result
default	$\frac{2\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\left(-\cos(dx+c)^2-2\cos(\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int((b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),
I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-cos(d*x+
c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)+sin(d*x+c))*(b*sec(d*x+c))^(
1/2)*b/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int (b\sec(c+dx))^{3/2}\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\ldots}{}$$

input
```
integrate((b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*b*sqrt(b/cos(d
*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int (b\sec(c+dx))^{3/2}\,dx = \int (b\sec(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{3/2}\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((b/cos(c + d*x))^(3/2),x)
```

output
```
int((b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)b$$

input
```
int((b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x),x)*b
```

3.84 $\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

output
```
2*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

input
```
Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \sqrt{b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\Big\downarrow \text{ 4258}$$

$$b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.00

method	result	size
default	$-\frac{2 i b (\cos(dx+c)+1) \sqrt{b \sec(dx+c)}\, \sqrt{\frac{1}{\cos(dx+c)+1}}\, \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, \operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)), i)}{d}$	78

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*b/d*(cos(d*x+c)+1)*(b*sec(d*x+c))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInve}}{d}$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/d
```

Sympy [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(c+dx))^{\frac{3}{2}}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**(3/2)*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx=\int \cos(c+d\,x)\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx=\sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)b$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*b
```

3.85 $\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
2*b^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+
c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 2030, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\sqrt{b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{b^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.61

method	result
default	$\frac{2\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)+i\left(-\cos(dx+c)^2-2\cos(dx\right.\right.}{d(\cos(dx+c)+1}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{d}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}b+b\right)}{b\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}b+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx-}}}{}\right.}{}$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+I*(-cos(d*x+c
)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)+sin(d*x+c)*cos(d*x+c))*(b*sec(
d*x+c))^(1/2)*b/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i}{d}$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*b
```

3.86 $\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

output
```
2/3*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/d+2/3*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{b\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
Sin[2*(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^3\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^3\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

input
```
Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*b^2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

method	result
default	$-\frac{2\left(i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)-\sin(dx+c)\cos(dx+c)\right)b\sqrt{b\sec(dx+c)}}{3d}$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)-sin(d*x+c)*cos(d*x+c))*b
*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{2\,b\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}b^{\frac{3}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\text{s}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*b^(3/2
)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*b^
(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^3\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)dx\right)b$$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*b
```

3.87 $\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

output
```
6/5*b^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*
x+c))^(1/2)+2/5*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[Cos[c + d*x]^4*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^4\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^4\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4 \left(\frac{3 \int \sqrt{\sin \left(c + dx + \frac{\pi}{2}\right)} dx}{5b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2 \sin(c + dx)}{5bd(b \sec(c + dx))^{3/2}} \right)$$

$$\Big\downarrow \text{3119}$$

$$b^4 \left(\frac{6E\left(\frac{1}{2}(c + dx)\middle| 2\right)}{5b^2 d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2 \sin(c + dx)}{5bd(b \sec(c + dx))^{3/2}} \right)$$

input
```
Int[Cos[c + d*x]^4*(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^4*((6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[
c + d*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.88

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)+i\left(-3\cos(dx+c)^2-6\cos(dx+c)-3\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticE}(i(\cot(dx+c)-\right.}{}$

input
```
int(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*(sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)^2+cos(d*x+c)+3)+I*(-3*cos(d*x+c)^
2-6*cos(d*x+c)-3)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c
)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cot(d*x+c)-csc(d*x+c)),I))*b*(b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.29

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{2\,b\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c)+3i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}}{}$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(2*b*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*b^
(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) - 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^4\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)dx\right)b$$

input
```
int(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x)*b
```

3.88 $\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{10b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

output
```
10/21*b*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x
+c))^(1/2)/d+2/7*b^4*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*b^2*sin(d*x+c
)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{b\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+\right.}{84d}$$

input
```
Integrate[Cos[c + d*x]^5*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^5\left(\frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{5\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^5\left(\frac{5\left(\frac{\int\sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{5\left(\frac{\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^5\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^5\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

input
```
Int[Cos[c + d*x]^5*(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^5*((2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^2*d) + (2*Sin[
c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

method	result
default	$\frac{2\left(\sin(dx+c)\cos(dx+c)\left(3\cos(dx+c)^2+5\right)+i(5\cos(dx+c)+5)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)\right.}{21d}$

input
```
int(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*(sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2+5)+I*(5*cos(d*x+c)+5)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+
c)-csc(d*x+c)),I))*b*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPIn}$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 2*(3*b*cos(d*x + c)^3 + 5*b*cos(d*x + c))*sqrt(b/cos(d*
x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^5, x)
```

Giac [F]

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^5\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^5\sec(dx+c)\,dx\right)b$$

input
```
int(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**5*sec(c + d*x),x)*b
```

3.89 $\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

output
```
14/15*b^2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(
d*x+c))^(1/2)+2/9*b^5*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b^3*sin(d*x+
c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{b\sec(c+dx)}\left(84\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(33\sin(c+dx)+5\sin(3(c\right.}{90d}$$

input
```
Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{2030}$$

$$b^6\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^6\left(\frac{7\int \frac{1}{(b\sec(c+dx))^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{7\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^6\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^6\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^6\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

input
```
Int[Cos[c + d*x]^6*(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^6*((2*Sin[c + d*x])/(9*b*d*(b*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c
 + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sin[
c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))))/(9*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.83 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.29

method	result
default	$\frac{2b\left(\sin(dx+c)\cos(dx+c)\left(5\cos(dx+c)^4+5\cos(dx+c)^3+7\cos(dx+c)^2+7\cos(dx+c)+21\right)+21i\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)\sqrt{\cdots}}{\cdots}$

input
```
int(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d*b*(sin(d*x+c)*cos(d*x+c)*(5*cos(d*x+c)^4+5*cos(d*x+c)^3+7*cos(d*x+c
)^2+7*cos(d*x+c)+21)+21*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x
+c)),I)+21*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I))*(b*se
c(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{21i\,\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-$$

input
```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/45*(21*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(3/2)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b
*cos(d*x + c)^4 + 7*b*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)
```

Giac [F]

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c))^{\frac{3}{2}}\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^6\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx=\sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^6\sec(dx+c)\,dx\right)b$$

input
```
int(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**6*sec(c + d*x),x)*b
```

3.90 $\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{10b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{10b(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7bd}$$

output
```
10/21*b^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d
*x+c))^(1/2)/d+10/21*b*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(b*sec(d*x+c)
)^(7/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+d\right.}{21d}$$

input
```
Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{9/2}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\int (b\sec(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^2}$$

input
```
Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b*(b
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7)/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

method	result
default	$\frac{\left(\frac{10\tan(dx+c)}{21}+\frac{2\tan(dx+c)\sec(dx+c)^{2}}{7}-\frac{10i(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)}{21}\right)b^{2}\sqrt{b\,\mathrm{se}}}{d}$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(10/21*tan(d*x+c)+2/7*tan(d*x+c)*sec(d*x+c)^2-10/21*I*(cos(d*x+c)+1)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-co
t(d*x+c)+csc(d*x+c)),I))*b^2*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{5}{2}}\cos{(dx+c)}^3\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5i\sqrt{2}b^{\frac{5}{2}}\,\mathrm{c}}{}$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^3*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b^2*cos(d*x + c)^2 +
 3*b^2)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec{(c+dx)})^{\frac{5}{2}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((b*sec(c + d*x))**(5/2)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\;dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\;dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx=\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((b/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
int((b/cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)b^2$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)*b**2
```

3.91 $\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = -\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

output
```
-6/5*b^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d
*x+c))^(1/2)+6/5*b^2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*(b*sec(d*x+c))^
(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{(b\sec(c+dx))^{5/2}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]
+ 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{7/2}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\int (b\sec(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b}$$

input
```
Int[Sec[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*b^2*((-2*b^2*Ellipti
cE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqr
t[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5)/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.02 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.16

method	result
default	$-\frac{2\sqrt{b\sec(dx+c)}\, b^2 \left(-3\sin(dx+c)-\tan(dx+c)-\sec(dx+c)\tan(dx+c)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\text{EllipticE}(i(-\cot(dx+c$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d*(b*sec(d*x+c))^(1/2)*b^2/(cos(d*x+c)+1)*(-3*sin(d*x+c)-tan(d*x+c)-s
ec(d*x+c)*tan(d*x+c)+I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*EllipticE(I*(-cot(d
*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)+I*(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.29

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{-3i\sqrt{2}b^{\frac{5}{2}}\cos(dx+c)^2 \,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\text{si}$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*b^(5/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(5/2)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*b^2*cos(d*x + c)^2 + b^2)*sqrt(b/cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx=\int \left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((b*sec(c + d*x))**(5/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx=\int \left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((b/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
int((b/cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)b^2$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*b**2
```

3.92 $\int (b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}$$

output
```
2/3*b^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x
+c))^(1/2)/d+2/3*b*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int (b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d}$$

input
```
Integrate[(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ Tan[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

input
```
Int[(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{\left(-\frac{2 i \left(\cos \left(d x+c\right)+1\right) \sqrt{\frac{1}{\cos \left(d x+c\right)+1}}\, \sqrt{\frac{\cos \left(d x+c\right)}{\cos \left(d x+c\right)+1}}\, \operatorname{EllipticF}\left(i \left(-\cot \left(d x+c\right)+\csc \left(d x+c\right)\right), i\right)}{3}+\frac{2 \tan \left(d x+c\right)}{3}\right) b^{2} \sqrt{b \sec \left(d x+c\right)}}{d}$	90

input
```
int((b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/3*I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+
c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2/3*tan(d*x+c))*b^2*(b
*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int (b\sec(c+dx))^{5/2}\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}b^{\frac{5}{2}}\cos(}{3\,d\cos}$$

input
```
integrate((b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + I*sqrt(2)*b^(5/2)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*b^2*sqrt(b/cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (b\sec(c+dx))^{5/2}\,dx = \int (b\sec(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{5/2}\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int((b/cos(c + d*x))^(5/2),x)
```

output
```
int((b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^2\,dx\right)b^2$$

input
```
int((b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*b**2
```

3.93 $\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = -\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*b^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x
+c))^(1/2)+2*b^2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\sqrt{b\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]) + Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 2030, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4255}$$

$$b\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{ 4258}$$

$$b\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)$$

input
```
Int[Cos[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

output
```
b*((-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
 d*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.71

method	result
default	$\frac{2b^2\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\left(-\cos(dx+c)^2-2\cos\right.}{d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^2/d*(I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*EllipticF(I*(-cot(d*x+c)+csc(d*x+
c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*(-cos(
d*x+c)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I)+sin(d*x+c))*(b*sec(d*x+c
))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+}{}$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*b^2*sqrt(b/cos
(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)\,\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)b^2$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*b**2
```

3.94 $\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx=\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

output
```
2*b^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx=\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 2030, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \sqrt{b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
d
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 30.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2 i b^{2} \sqrt{\frac{1}{\cos(dx+c)+1}}\, \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, (\cos(dx+c)+1) \sqrt{b \sec(dx+c)}\, \operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)}{d}$	80

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I*b^2/d*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1)*(b*sec(d*x+c))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInve}}{d}$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b^2$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*b**2
```

3.95 $\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
2*b^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+
c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{2\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)(b\sec(c+dx))^{5/2}}{d}$$

input
```
Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]*(b*Sec[c + d*x])^(5/2))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 2030, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\sqrt{b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{b^3\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 104.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 4.66

method	result
default	$\frac{2\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)+i\left(-\cos(dx+c)^2-2\cos(dx\right.\right.}{d(\cos(dx+c)+}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,b^2e^{-i(dx+c)}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{d}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}b+b\right)}{b\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}b+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx\cdots}}}{\cdots}\right.}{\cdots}$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)+I*(-cos(d*x+c
)^2-2*cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)+sin(d*x+c)*cos(d*x+c))*(b*sec(
d*x+c))^(1/2)*b^2/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{}}{d}$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^3\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b^2$$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*b**2
```

3.96 $\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx=\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

output
```
2/3*b^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x
+c))^(1/2)/d+2/3*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx=\frac{b^2\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^4*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ Sin[2*(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^4\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^4\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4 \left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)} \int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2} + \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} \right)$$

$\Big\downarrow$ 3120

$$b^4 \left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} \right)$$

input
```
Int[Cos[c + d*x]^4*(b*Sec[c + d*x])^(5/2),x]
```

output
```
b^4*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*b^2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\frac{2\sqrt{b\sec\left(dx+c\right)}\,b^{2}\left(i(\cos\left(dx+c\right)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(i(\cot\left(dx+c\right)-\csc\left(dx+c\right)),i\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{3d}$$

input
```
int(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x)
```

output
```
2/3/d*(b*sec(d*x+c))^(1/2)*b^2*(I*(cos(d*x+c)+1)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+s
in(d*x+c)*cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{2\,b^{2}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right)-i\,\sqrt{2}b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,}{3\,d}$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*b^(5
/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*
b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^4\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2 dx\right)b^2$$

input
```
int(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x)*b**2
```

3.97 $\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

output
```
6/5*b^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*
x+c))^(1/2)+2/5*b^4*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[Cos[c + d*x]^5*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
 + Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^5\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^5\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$b^5\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

input
```
Int[Cos[c + d*x]^5*(b*Sec[c + d*x])^(5/2),x]
```

output
```
b^5*((6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[
c + d*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.63 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.90

$$\frac{2\Big(\sin(dx+c)\cos(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)+i\left(-3\cos(dx+c)^2-6\cos(dx+c)-3\right)\sqrt{}}{}$$

input
```
int(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x)
```

output
```
2/5/d*(sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)^2+cos(d*x+c)+3)+I*(-3*cos(d*x+c)^
2-6*cos(d*x+c)-3)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c
)-csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cot(d*x+c)-csc(d*x+c)),I))*b^2*(b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{2\,b^2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c)+3i\sqrt{2}b^{\frac{5}{2}}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInvers}}{}$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*
b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) - 3*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

Giac [F]

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^5\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^5\sec(dx+c)^2\,dx\right)b^2$$

input
```
int(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**5*sec(c + d*x)**2,x)*b**2
```

3.98 $\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{10b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

output
```
10/21*b^2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d
*x+c))^(1/2)/d+2/7*b^5*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*b^3*sin(d*x
+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+\right.}{84d}$$

input
```
Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + 26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{2030}$$

$$b^6\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^6\left(\frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{5\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^6\left(\frac{5\left(\frac{\int\sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{5\left(\frac{\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^6\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^6\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

input
```
Int[Cos[c + d*x]^6*(b*Sec[c + d*x])^(5/2),x]
```

output
```
b^6*((2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^2*d) + (2*Sin[
c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\frac{2\left(\sin\left(dx+c\right)\cos\left(dx+c\right)\left(3\cos\left(dx+c\right)^{2}+5\right)+i(5\cos\left(dx+c\right)+5)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticI}}{21d}$$

input
```
int(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x)
```

output
```
2/21/d*(sin(d*x+c)*cos(d*x+c)*(3*cos(d*x+c)^2+5)+I*(5*cos(d*x+c)+5)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+
c)-csc(d*x+c)),I))*b^2*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPIn}}{}$$

input
```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 2*(3*b^2*cos(d*x + c)^3 + 5*b^2*cos(d*x + c))*sqrt(b/co
s(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^6, x)
```

Giac [F]

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^6\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^6\sec(dx+c)^2\,dx\right)b^2$$

input
```
int(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**6*sec(c + d*x)**2,x)*b**2
```

3.99 $\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{14b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

output
```
14/15*b^3*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(
d*x+c))^(1/2)+2/9*b^6*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b^4*sin(d*x+
c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{b\sec(c+dx)}\left(84\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(33\sin(c+dx)+5\sin(3(\ldots}{90d}$$

input
```
Integrate[Cos[c + d*x]^7*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
 + Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{2030}$$

$$b^7\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^7\left(\frac{7\int \frac{1}{(b\sec(c+dx))^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{7\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^7\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \textbf{4258}$$

$$b^7\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^7\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \textbf{3119}$$

$$b^7\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

input
```
Int[Cos[c + d*x]^7*(b*Sec[c + d*x])^(5/2),x]
```

output
```
b^7*((2*Sin[c + d*x])/(9*b*d*(b*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c
 + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sin[
c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))))/(9*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.92 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.31

$$\frac{2b^2\left(\sin\left(dx+c\right)\cos\left(dx+c\right)\left(5\cos\left(dx+c\right)^4+5\cos\left(dx+c\right)^3+7\cos\left(dx+c\right)^2+7\cos\left(dx+c\right)+21\right)+2}{}$$

input
```
int(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x)
```

output
```
2/45/d*b^2*(sin(d*x+c)*cos(d*x+c)*(5*cos(d*x+c)^4+5*cos(d*x+c)^3+7*cos(d*x
+c)^2+7*cos(d*x+c)+21)+21*I*(cos(d*x+c)^2+2*cos(d*x+c)+1)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x
+c)),I)+21*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I))*(b*se
c(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{21i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - }{}$$

input
```
integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(21*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(5/2)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b
^2*cos(d*x + c)^4 + 7*b^2*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c
))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^7\,dx$$

input
```
integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)
```

Giac [F]

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c))^{\frac{5}{2}}\cos(dx+c)^7\,dx$$

input
```
integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^7\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\cos(dx+c)^7\sec(dx+c)^2\,dx\right)b^2$$

input
```
int(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*cos(c + d*x)**7*sec(c + d*x)**2,x)*b**2
```

3.100 $\int (b\sec(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int (b\sec(c+dx))^{7/2}\,dx = -\frac{6b^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6b^3\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

output
```
-6/5*b^4*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d
*x+c))^(1/2)+6/5*b^3*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/5*b*(b*sec(d*x+c)
)^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int (b\sec(c+dx))^{7/2}\,dx = \frac{b(b\sec(c+dx))^{5/2}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[(b*Sec[c + d*x])^(7/2),x]
```

output
```
(b*(b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2
] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}b^2\int (b\sec(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \ 4258$$

$$\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \ 3119$$

$$\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

input
```
Int[(b*Sec[c + d*x])^(7/2),x]
```

output
```
(2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*b^2*((-2*b^2*Elliptic
E[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqrt
[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.14

method	result
default	$-\frac{2\sqrt{b\sec(dx+c)}\,b^3\left(-3\sin(dx+c)-\tan(dx+c)-\sec(dx+c)\tan(dx+c)+i\left(3\cos(dx+c)^2+6\cos(dx+c)+3\right)\text{EllipticE}(i(-\cot(dx+c}{}$

input
```
int((b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d*(b*sec(d*x+c))^(1/2)*b^3/(cos(d*x+c)+1)*(-3*sin(d*x+c)-tan(d*x+c)-s
ec(d*x+c)*tan(d*x+c)+I*(3*cos(d*x+c)^2+6*cos(d*x+c)+3)*EllipticE(I*(-cot(d
*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)+I*(-3*cos(d*x+c)^2-6*cos(d*x+c)-3)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int (b\sec(c+dx))^{7/2}\,dx = \frac{-3i\sqrt{2}b^{\frac{7}{2}}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\text{si}}{}$$

input
```
integrate((b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(7/2)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*b^3*cos(d*x + c)^2 + b^3)*sqrt(b/cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(c+dx))^{7/2}\,dx = \int (b\sec(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (b\sec(c+dx))^{7/2}\,dx = \int (b\sec(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{7/2}\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^{7/2}\,dx$$

input
```
int((b/cos(c + d*x))^(7/2),x)
```

output
```
int((b/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int (b\sec(c+dx))^{7/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)b^3$$

input
```
int((b*sec(d*x+c))^(7/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*b**3
```

3.101 $\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21bd} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^2d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^4d}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/b/d+10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*(b*sec(d*x+c)
)^(7/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sec^3(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^3*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2
*(c + d*x)] + 6*Tan[c + d*x]))/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{9/2}dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^5}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\int (b\sec(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^5}$$

$$\Big\downarrow \ 4258$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^5}$$

$$\Big\downarrow \ 3120$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^5}$$

input
```
Int[Sec[c + d*x]^5/Sqrt[b*Sec[c + d*x]],x]
```

output
```
((2*b*(b*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b*(b
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7)/b^5
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

method	result	siz
default	$\frac{\frac{2\left(5\cos(dx+c)^2+3\right)\tan(dx+c)\sec(dx+c)^3}{21}+\frac{2i\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(-5-5\sec(dx+c))}{21}}{d\sqrt{b\sec(dx+c)}}$	109

input
```
int(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/21*(5*cos(d*x+c)^2+3)*tan(d*x+c)*sec(d*x+c)^3+2/21*I*EllipticF(I*(-
cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*(-5-5*sec(d*x+c)))/(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\sqrt{b}\cos(dx+c)}{21\,bd\cos}$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*
sqrt(b/cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**5/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^4 dx\right)}{b}$$

input
```
int(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x))/b
```

3.102 $\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5bd} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d}$$

output
```
-6/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c
))^(1/2)+6/5*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/b/d+2/5*(b*sec(d*x+c))^(5/2)*
sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2(3+\sec^2(c+dx))\tan(c+dx)}{5d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

output
```
((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2
)*Tan[c + d*x])/(5*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{7/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\int (b\sec(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^4}$$

input

```
Int[Sec[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

output

```
((2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*b^2*((-2*b^2*Ellipti
cE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqr
t[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5)/b^4
```

Defintions of rubi rules used

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\left(\left(-3\cos(dx+c)^2-\cos(dx+c)-1\right)\tan(dx+c)\sec(dx+c)^2-3i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,(2+\cos(dx+c)+\sec(dx+c))\,\text{Elliptic}}{5d(\cos(dx+}$

input
```
int(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*((-3*cos(d*x+c)^2-cos(d*x+c)-1)
*tan(d*x+c)*sec(d*x+c)^2-3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticF(I*(-cot(d*x+c)+csc(d*x+c
)),I)+3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(2+co
s(d*x+c)+sec(d*x+c))*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.27

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))}{}$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*
x + c))/(b*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)}{b}$$

input
```
int(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x))/b
```

3.103 $\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3bd} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^2d}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/b/d+2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3bd}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Ta
n[c + d*x]))/(3*b*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{5/2}dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^3}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^3}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d))/b^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\frac{2 i \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, \operatorname{EllipticF}\left(i(-\cot(dx+c)+\csc(dx+c)), i\right) \sqrt{\frac{1}{\cos(dx+c)+1}}\,(-1-\sec(dx+c))}{3}+\frac{2\sec(dx+c)\tan(dx+c)}{3}}{d\sqrt{b\sec(dx+c)}}$	95

input
```
int(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-1-sec(d*x+c))+2/3*sec(d*x+c)*tan(d*x
+c))/(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{we}}{3\,bd\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c
))/(b*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec^3{(c+dx)}}{\sqrt{b\sec{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^3}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^3}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^2dx\right)}{b}$$

input
```
int(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x))/b
```

3.104 $\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd}$$

output
```
-2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))
^(1/2)+2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}+2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

output
```
((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(d*Sq
rt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{3/2}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^2}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

output
```
((-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d)/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.72

method	result
default	$-\frac{2\left(i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(-\cos(dx+c)-2-\sec(dx+c))+i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \sqrt{\cos(\cdots}\right.}{d(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}}$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(I*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-cos
(d*x+c)-2-sec(d*x+c))+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-co
t(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))
-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\text{weie}\cdots}{b\cdots}$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x
 + c))*sin(d*x + c))/(b*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec^2{(c+dx)}}{\sqrt{b\sec{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)dx\right)}{b}$$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x),x))/b
```

3.105 $\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd}$$

output
```
2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))^(
1/2)/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b*d
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \sqrt{b\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{bd}$$

input
```
Int[Sec[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b*d
)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.68

method	result	size
default	$-\frac{2 i \sqrt{\frac{1}{\cos \left(d x +c \right)+1}}\, \operatorname{EllipticF}\left(i \left(-\cot \left(d x +c \right)+\csc \left(d x +c \right)\right), i\right)}{d \sqrt{b \sec \left(d x +c \right)}\, \sqrt{\frac{\cos \left(d x +c \right)}{\cos \left(d x +c \right)+1}}}$	69

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)/(b
*sec(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,c}{bd}$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/(b*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec{(c+dx)}}{\sqrt{b\sec{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}dx\right)}{b}$$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x)),x))/b
```

3.106 $\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.61

method	result
default	$\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(2+\cos(dx+c)+\sec(dx+c))+2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cdots}{d(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}}$
risch	$-\frac{i\sqrt{2}}{d\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}b+b\right)}{b\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}b+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\left(-2i\operatorname{EllipticE}\left(\sqrt{-i\left(e^{i(dx+\cdots}\right.}\right.\right.}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}}\cdots\right)}{d\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\cdots\right.}$

input
```
int(1/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(2+cos(
d*x+c)+sec(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)-2-sec(d*x+c))+si
n(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - i\sqrt{2}\sqrt{b}\text{weiers}}{bd}$$

input
```
integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(b/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)}{b}$$

input
```
int(1/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))/sec(c + d*x),x))/b
```

3.107 $\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3bd} + \frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/b/d+2/3*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{b\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(b*Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*
(c + d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$b\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$b\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)$$

input
```
Int[Cos[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

output
```
b*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(
3*b^2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)    Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(1+\sec(dx+c))}{3}+\frac{2\sin(dx+c)}{3}}{d\sqrt{b\sec(dx+c)}}$	87

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1+sec(d*x+c))+2/3*sin(d*x+c))/(b*sec(d
*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{3\,bd}$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt
(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/(b*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)/(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)/(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)}dx\right)}{b}$$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x))/sec(c + d*x),x))/b
```

3.108 $\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

output
```
6/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c)
)^(1/2)+2/5*b*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10bd}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + S
in[c + d*x] + Sin[3*(c + d*x)]))/(10*b*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^2\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{3\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3119$$

$$b^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

input
```
Int[Cos[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^2*((6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[
c + d*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.81

method	result
default	$\frac{\frac{2\sin(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)}{5}+\frac{6i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(2+\cos(dx+c)+\sec(dx+c))}{5}-\frac{6}{}}{d(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}}$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(sin(d*x+c)*(cos(d*x+c)^2+cos(d*
x+c)+3)+3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(2+
cos(d*x+c)+sec(d*x+c))*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)-3*I*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c
))*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c)+3i\sqrt{2}\sqrt{b}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos^2{(c+dx)}}{\sqrt{b\sec{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)}dx\right)}{b}$$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*x),x))/b
```

3.109 $\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21bd} + \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/b/d+2/7*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/d/
(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84bd}$$

input
```
Integrate[Cos[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2
6*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$b^3\left(\frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{5\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 4256}$$

$$b^3\left(\frac{5\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$b^3\left(\frac{5\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 4258

$$b^3\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$b^3\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3120

$$b^3\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

input
```
Int[Cos[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^3*((2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^2*d) + (2*Sin[
c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\frac{2 \sin \left(d x +c \right) \left(3 \cos \left(d x +c \right)^{2}+5\right)}{21}+\frac{2 i \sqrt{\frac{1}{\cos \left(d x +c \right)+1}}\, \sqrt{\frac{\cos \left(d x +c \right)}{\cos \left(d x +c \right)+1}}\, \operatorname{EllipticF}\left(i \left(\cot \left(d x +c \right)-\csc \left(d x +c \right)\right), i\right) \left(5+5 \sec \left(d x +c \right)\right)}{21}}{d \sqrt{b \sec \left(d x +c \right)}}$	101

input
```
int(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/(b*sec(d*x+c))^(1/2)*(sin(d*x+c)*(3*cos(d*x+c)^2+5)+I*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(
d*x+c)),I)*(5+5*sec(d*x+c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\left(3\cos(dx+c)^3+5\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)-5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx}{21\,bd}$$

input
```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
 c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^3/(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3/(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^3}{\sec(dx+c)}dx\right)}{b}$$

input
```
int(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x)**3)/sec(c + d*x),x))/b
```

3.110 $\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

output
```
14/15*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+
c))^(1/2)+2/9*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b*sin(d*x+c)/d/(
b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{\frac{336E\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}}+4\cos(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))}{360d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

output
```
((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*S
in[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^4\left(\frac{7\int \frac{1}{(b\sec(c+dx))^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{7\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^4\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^4\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^4\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

input
```
Int[Cos[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^4*((2*Sin[c + d*x])/(9*b*d*(b*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c
 + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sin[
c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))))/(9*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.56 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.23

method	result
default	$\frac{\frac{2\sin(dx+c)\left(5\cos(dx+c)^4+5\cos(dx+c)^3+7\cos(dx+c)^2+7\cos(dx+c)+21\right)}{45}+\frac{14i\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c}{15}}{d(\cos(dx+c)+1)\sqrt{b s}}$

input
```
int(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(sin(d*x+c)*(5*cos(d*x+c)^4+5*c
os(d*x+c)^3+7*cos(d*x+c)^2+7*cos(d*x+c)+21)+21*I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticF(I*(c
ot(d*x+c)-csc(d*x+c)),I)-21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticE(I*(cot(d*x+c)-csc(d*x+c
)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\left(5\cos\left(dx+c\right)^4+7\cos\left(dx+c\right)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstra}$$

input
```
integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos^4\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**4/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\sqrt{b\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^4/(b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4/(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^4}{\sec(dx+c)}dx\right)}{b}$$

input
```
int(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x)**4)/sec(c + d*x),x))/b
```

3.111 $\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^3d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/b^2/d+10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*(b*sec(d*x+
c))^(7/2)*sin(d*x+c)/b^5/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^4(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan\right.}{21d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^4*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2
*(c + d*x)] + 6*Tan[c + d*x]))/(21*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{9/2}dx}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^6}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\int (b\sec(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^6}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^6}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^6}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^6}$$

input
```
Int[Sec[c + d*x]^6/(b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b*(b
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7)/b^6
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

method	result	si
default	$\frac{-\frac{2\left(-5\cos(dx+c)^2-3\right)\tan(dx+c)\sec(dx+c)^3}{21}-\frac{2i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\ (5+5\sec(dx+c))}{21}}{d\sqrt{b\sec(dx+c)}\,b}$	1

input
```
int(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/21*(-5*cos(d*x+c)^2-3)*tan(d*x+c)*sec(d*x+c)^3-2/21*I*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+
c)+1))^(1/2)*(5+5*sec(d*x+c)))/(b*sec(d*x+c))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{-5i\sqrt{2}\sqrt{b}\cos{(dx+c)}^3\ \text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})}{}$$

input
```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*
sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**6/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^6}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^6}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^6\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)}{b^2}$$

input
```
int(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x))/b**2
```

3.112 $\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^2d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d}$$

output
```
-6/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x
+c))^(1/2)+6/5*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/b^2/d+2/5*(b*sec(d*x+c))^(5
/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2(3+\sec^2(c+dx))\tan(c+dx)}{5bd\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/(b*Sec[c + d*x])^(3/2),x]
```

output
```
((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2
)*Tan[c + d*x])/(5*b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\sec(c+dx))^{7/2}dx}{b^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^5}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{5}b^2\int (b\sec(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{5}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^5}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^5}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^5}$$

$$\Big\downarrow \ 4258$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^5}$$

$$\Big\downarrow \ 3119$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^5}$$

input
```
Int[Sec[c + d*x]^5/(b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*b^2*((-2*b^2*Ellipti
cE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqr
t[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5)/b^5
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.01

method	result
default	$\frac{\frac{2\left(3\cos(dx+c)^2+\cos(dx+c)+1\right)\tan(dx+c)\sec(dx+c)^2}{5}+\frac{6i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,(2+\cos(dx+c)+\sec(dx+c))\,\mathrm{EllipticF}(i(-\cot(dx+c)+\mathrm{cs}}{5}}{d(\cos(dx+c)+1)\sqrt{b\sec(dx+c}}$

input
```
int(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b*((3*cos(d*x+c)^2+cos(d*x+c)+1)
*tan(d*x+c)*sec(d*x+c)^2+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticF(I*(-cot(d*x+c)+csc(d*x+c
)),I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2+co
s(d*x+c)+sec(d*x+c))*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*
x + c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^5{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^3 dx\right)}{b^2}$$

input
```
int(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x))/b**2
```

3.113 $\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^3d}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/b^2/d+2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sec^3(c+dx)\left(\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{3d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^4/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^3*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin[c +
d*x]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{5/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^4}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^4}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^4}$$

input
```
Int[Sec[c + d*x]^4/(b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d))/b^4
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{-\frac{2i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(1+\sec(dx+c))}{3}+\frac{2\sec(dx+c)\tan(dx+c)}{3}}{d\sqrt{b\sec(dx+c)}\,b}$	96

input
```
int(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(1+sec(d*x+c))+2/3*sec(d*x+c)*tan(d*x
+c))/(b*sec(d*x+c))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+}{}$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c
))/(b^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^4{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^4}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^4}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^2 dx\right)}{b^2}$$

input
```
int(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x))/b**2
```

3.114 $\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^2d}$$

output
```
-2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c
))^(1/2)+2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}}+2\tan(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b*d*
Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\sec(c+dx))^{3/2}dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b^3}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^3}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^3}$$

input
```
Int[Sec[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d)/b^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.62

method	result
default	$\frac{2i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,(2+\cos(dx+c)+\sec(dx+c))\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)+2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\mathrm{EllipticE}(i($

$d(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}\,b$

input
```
int(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b*(I*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2+c
os(d*x+c)+sec(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cot(d*x+c)+
csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-2-sec(d*x+c)
)+tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(}}{}$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x
 + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)\,dx\right)}{b^2}$$

input
```
int(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x),x))/b**2
```

3.115 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

output
```
2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))^(
1/2)/b^2/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^2
*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \sqrt{b\sec(c+dx)}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

input
```
Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^2
*d)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\ \text{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)}{db\sqrt{b\sec(dx+c)}\ \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$	72

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)/b/(b
*sec(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\sqrt{2}\sqrt{b}\text{weie}}{b^2d}$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/(b^2*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^2{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^2}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec(dx+c)}dx\right)}{b^2}$$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x)),x))/b**2
```

3.116 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{ 2030}$$
$$\frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{b}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$
$$\Big\downarrow \text{ 4258}$$
$$\frac{\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$\Big\downarrow \text{ 3119}$$
$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.34

method	result
default	$\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(2+\cos(dx+c)+\sec(dx+c))+2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{bd(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}}$
risch	$-\frac{i\sqrt{2}}{db\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}b+b\right)}{b\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}b+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\left(-2i\operatorname{EllipticE}\left(\sqrt{-i\left(e^{i(dx}\right.}\right.\right.}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}}}{db\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c}}{e^{2i(dx+c)}}}}$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(2+co
s(d*x+c)+sec(d*x+c))+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)*(-cos(d*x+c)-2-sec(d*x+c))+
sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(d}$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)}{b^2}$$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))/sec(c + d*x),x))/b**2
```

3.117 $\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/b^2/d+2/3*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[(b*Sec[c + d*x])^(-3/2),x]
```

output
```
(Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c
 + d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(-3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b
^2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{\frac{2i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(1+\sec(dx+c))}{3}+\frac{2\sin(dx+c)}{3}}{d\sqrt{b\sec(dx+c)}\,b}$	90

input
```
int(1/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(1+sec(d*x+c))+2/3*sin(d*x+c))/(b*sec
(d*x+c))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(d$$

input
```
integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt
(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/(b^2*d)
```

Sympy [F]

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(b/cos(c + d*x))^(3/2),x)
```

output
```
int(1/(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)}{b^2}$$

input
```
int(1/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x))/b**2
```

3.118 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

output
```
6/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+
c))^(1/2)+2/5*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + \sin(c+dx) + \sin(3(c+dx)\right.}{10b^2d}$$

input
```
Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + S
in[c + d*x] + Sin[3*(c + d*x)]))/(10*b^2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)$$

input
```
Int[Cos[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

output
```
b*((6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c
+ d*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.65 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

method	result
default	$-\frac{2\left(\sin(dx+c)\left(-\cos(dx+c)^{2}-\cos(dx+c)-3\right)+3i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(2+\cos(dx+c)+\sec(dx+c))\,\mathrm{EllipticE}(i(\cot(dx+c}{5d(\cos(dx+c)+1)\sqrt{b\,s}}$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b*(sin(d*x+c)*(-cos(d*x+c)^2-co
s(d*x+c)-3)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(2+cos(d*x+c)+sec(d*x+c))*EllipticE(I*(cot(d*x+c)-csc(d*x+c)),I)-3*I*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d
*x+c))*EllipticF(I*(cot(d*x+c)-csc(d*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{2}\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstr}}{}$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)^2}dx\right)}{b^2}$$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x))/sec(c + d*x)**2,x))/b**2
```

3.119 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d} + \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/b^2/d+2/7*b*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/b/
d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\sec(c+dx)}\Big(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 26\sin(2(c+dx)) +}{84b^2d}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2
6*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^2*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^2\left(\frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{5\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^2\left(\frac{5\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{5\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

input
```
Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^2*((2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^2*d) + (2*Sin[
c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{\frac{2\sin(dx+c)\left(3\cos(dx+c)^2+5\right)}{21}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(5+5\sec(dx+c))}{21}}{d\sqrt{b\sec(dx+c)}\,b}$	104

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/(b*sec(d*x+c))^(1/2)/b*(sin(d*x+c)*(3*cos(d*x+c)^2+5)+I*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-cs
c(d*x+c)),I)*(5+5*sec(d*x+c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(3\cos\left(dx+c\right)^3+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\text{weierstras}}{}$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
 c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)))/(b^2*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)^2}dx\right)}{b^2}$$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*x)**2,x))/b**2
```

3.120 $\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

output
```
14/15*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*
x+c))^(1/2)+2/9*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*sin(d*x+c)/d/(
b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{84E\left(\frac{1}{2}(c+dx)\middle|2\right) + \cos^{\frac{3}{2}}(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))}{90d\cos^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(84*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^(3/2)*(33*Sin[c + d*x] + 5*Si
n[3*(c + d*x)]))/(90*d*Cos[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 4256}$$

$$b^3\left(\frac{7\int \frac{1}{(b\sec(c+dx))^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{7\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 4256}$$

$$b^3\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$\big\downarrow$ 4258

$$b^3\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$\big\downarrow$ 3042

$$b^3\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$\big\downarrow$ 3119

$$b^3\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

input
```
Int[Cos[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^3*((2*Sin[c + d*x])/(9*b*d*(b*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c
 + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sin[
c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))))/(9*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.22

method	result
default	$-\frac{2\left(\sin(dx+c)\left(-5\cos(dx+c)^4-5\cos(dx+c)^3-7\cos(dx+c)^2-7\cos(dx+c)-21\right)+21i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(2+\cos(dx+c)}{45d(}$

input
```
int(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b*(sin(d*x+c)*(-5*cos(d*x+c)^4
-5*cos(d*x+c)^3-7*cos(d*x+c)^2-7*cos(d*x+c)-21)+21*I*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticE(
I*(cot(d*x+c)-csc(d*x+c)),I)-21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticF(I*(cot(d*x+c)-csc(d
*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(5\cos\left(dx+c\right)^4+7\cos\left(dx+c\right)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\text{weierstr}}{}$$

input
```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*
d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3/(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^3}{\sec(dx+c)^2}dx\right)}{b^2}$$

input
```
int(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x)**3)/sec(c + d*x)**2,x))/b**2
```

3.121 $\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^3d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^4d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/b^3/d+10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*(b*sec(d*x+
c))^(7/2)*sin(d*x+c)/b^6/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+\right.}{21b^5d}$$

input
```
Integrate[Sec[c + d*x]^7/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b^5*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{9/2}dx}{b^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^7}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\int (b\sec(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^7}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^7}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^7}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^7}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{7/2}}{7d}}{b^7}$$

input
```
Int[Sec[c + d*x]^7/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b*(b
*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7)/b^7
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

method	result	siz
default	$\frac{\frac{2\left(5\cos(dx+c)^2+3\right)\tan(dx+c)\sec(dx+c)^3}{21}+\frac{2i\,\mathrm{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(-5-5\sec(dx+c))}{21}}{d\sqrt{b\sec(dx+c)}\,b^2}$	11

input
```
int(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/21*(5*cos(d*x+c)^2+3)*tan(d*x+c)*sec(d*x+c)^3+2/21*I*EllipticF(I*(-
cot(d*x+c)+csc(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*(-5-5*sec(d*x+c)))/(b*sec(d*x+c))^(1/2)/b^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-5i\,\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^3 \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)}{}$$

input
```
integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*
sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^7{(c+dx)}}{(b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**7/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^7}{(b\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^7}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^7\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^4\,dx\right)}{b^3}$$

input
```
int(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x))/b**3
```

3.122 $\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^3d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d}$$

output
```
-6/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d
*x+c))^(1/2)+6/5*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/b^3/d+2/5*(b*sec(d*x+c))^
(5/2)*sin(d*x+c)/b^5/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2(3+\sec^2(c+dx))\tan(c+dx)}{5b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2
)*Tan[c + d*x])/(5*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{7/2}dx}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^6}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\int (b\sec(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^6}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^6}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^6}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^6}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{3}{5}b^2\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}}{b^6}$$

input
```
Int[Sec[c + d*x]^6/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*b^2*((-2*b^2*Ellipti
cE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqr
t[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5)/b^6
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.97 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\left(\left(-3\cos(dx+c)^2-\cos(dx+c)-1\right)\tan(dx+c)\sec(dx+c)^2-3i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,(2+\cos(dx+c)+\sec(dx+c))\,\text{Elliptic}}{5d(\cos(dx+c)}$

input
```
int(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^2*((-3*cos(d*x+c)^2-cos(d*x+c
)-1)*tan(d*x+c)*sec(d*x+c)^2-3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticF(I*(-cot(d*x+c)+csc(d
*x+c)),I)+3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(
2+cos(d*x+c)+sec(d*x+c))*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{b}\cos{(dx+c)}^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(}}{}$$

input
```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*
x + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^6\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**6/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^6}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec(dx+c)^6}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos(c+d\,x)^6\,\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\,\left(\int \sqrt{\sec(dx+c)}\,\sec(dx+c)^3\,dx\right)}{b^3}$$

input
```
int(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x))/b**3
```

3.123 $\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^4d}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/b^3/d+2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3b^3d}$$

input
```
Integrate[Sec[c + d*x]^5/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Ta
n[c + d*x]))/(3*b^3*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{5/2}dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^5}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^5}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^5}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}}{b^5}$$

input
```
Int[Sec[c + d*x]^5/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])
/(3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d))/b^5
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\frac{2i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(-1-\sec(dx+c))}{3}+\frac{2\sec(dx+c)\tan(dx+c)}{3}}{d\sqrt{b\sec(dx+c)}\,b^{2}}$	98

input
```
int(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-1-sec(d*x+c))+2/3*sec(d*x+c)*tan(d*x
+c))/(b*sec(d*x+c))^(1/2)/b^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+}{}$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c
))/(b^3*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^5{(c+dx)}}{(b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(b\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(b\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^2 dx\right)}{b^3}$$

input
```
int(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x))/b**3
```

3.124 $\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{b^3d}$$

output
```
-2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x
+c))^(1/2)+2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}+2\tan(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^4/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b^2*
d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{3/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^4}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^4}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}}{b^4}$$

input
```
Int[Sec[c + d*x]^4/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d)/b^4
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.65

method	result
default	$-\frac{2\left(i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(-\cos(dx+c)-2-\sec(dx+c))+i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{}{\cos(}}}{d(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}\,b^2}$

input
```
int(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^2*(I*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(
-cos(d*x+c)-2-sec(d*x+c))+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*
(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x
+c))-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(}{}$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x
 + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**4/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)dx\right)}{b^3}$$

input
```
int(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))*sec(c + d*x),x))/b**3
```

3.125 $\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

output
```
2*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))^(
1/2)/b^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^3/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^3
*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2030, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \sqrt{b\sec(c+dx)}dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

input
```
Int[Sec[c + d*x]^3/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^3
*d)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

method	result	size
default	$-\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(i(-\cot(dx+c)+\csc(dx+c)),i\right)}{d\,b^{2}\sqrt{b\sec(dx+c)}\ \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$	72

input
```
int(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)/b^
2/(b*sec(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weie}}{b^3d}$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/(b^3*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**3/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\left(\int \sqrt{\sec(dx+c)}dx\right)}{b^3}$$

input
```
int(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x)),x))/b**3
```

3.126 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
2*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+
c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\cos^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*EllipticE[(c + d*x)/2, 2])/(d*Cos[c + d*x]^(5/2)*(b*Sec[c + d*x])^(5/2)
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.34

method	result
default	$\frac{2i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\ (-\cos(dx+c)-2-\sec(dx+c))+2i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \sqrt{\frac{1}{\cos(dx+}}}{b^2 d(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}}$
risch	$-\frac{i\sqrt{2}}{d\,b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}b+b\right)}{b\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}b+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\ \sqrt{2}\ \sqrt{i\left(e^{i(dx+c)}-i\right)}\ \sqrt{ie^{i(dx+c)}}\ \left(-2i\ \text{EllipticE}\left(\sqrt{-i\left(e^{i(d}\right.\right.}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c}}}}{d\,b^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx}}{e^{2i(dx+}}}}$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)*(I*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-
cos(d*x+c)-2-sec(d*x+c))+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
-cot(d*x+c)+csc(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+
c))+sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(d$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)}{b^3}$$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))/sec(c + d*x),x))/b**3
```

3.127 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^3d} + \frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

output
```
2/3*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c))
^(1/2)/b^3/d+2/3*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)}{3bd(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c
 + d*x)]))/(3*b*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2030, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}$$
$$\Big\downarrow \text{4256}$$
$$\frac{\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}}{b}$$
$$\Big\downarrow \text{4258}$$
$$\frac{\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}}{b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}}{b}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}}{b}$$

input
```
Int[Sec[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*
b^2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(1+\sec(dx+c))}{3}+\frac{2\sin(dx+c)}{3}}{d\sqrt{b\sec(dx+c)}\,b^2}$	90

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(cot(d*x+c)-csc(d*x+c)),I)*(1+sec(d*x+c))+2/3*sin(d*x+c))/(b*sec(d
*x+c))^(1/2)/b^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos(d$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt
(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/(b^3*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)}{b^3}$$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x))/b**3
```

3.128 $\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

output
```
6/5*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*
x+c))^(1/2)+2/5*sin(d*x+c)/b/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx)\right.}{10b^3d}$$

input
```
Integrate[(b*Sec[c + d*x])^(-5/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + S
in[c + d*x] + Sin[3*(c + d*x)]))/(10*b^3*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

input
```
Int[(b*Sec[c + d*x])^(-5/2),x]
```

output
```
(6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.65

method	result
default	$\frac{\frac{2\sin(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)}{5}-\frac{6i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(2+\cos(dx+c)+\sec(dx+c))\,\text{EllipticF}(i(-\cot(dx+c)+\csc(dx+c)),i)}{5}+}{d(\cos(dx+c)+1)\sqrt{b\sec(dx+c)}\,b^2}$

input
```
int(1/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^2*(sin(d*x+c)*(cos(d*x+c)^2+co
s(d*x+c)+3)-3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*(2+cos(d*x+c)+sec(d*x+c))*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I)+3*I*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(
d*x+c))*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c) + 3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstr}$$

input
```
integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F]

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((b*sec(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(b/cos(c + d*x))^(5/2),x)
```

output
```
int(1/(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)}{b^3}$$

input
```
int(1/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x))/b**3
```

3.129 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^3d} + \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/b^3/d+2/7*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/b^2/
d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\sec(c+dx)}\Big(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 26\sin(2(c+dx)) +}{84b^3d}$$

input
```
Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2
6*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^3*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b\left(\frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{5\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b\left(\frac{5\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{5\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}\right)$$

input
```
Int[Cos[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

output
```
b*((2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^2*d) + (2*Sin[c
+ d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\frac{2\sin(dx+c)\left(3\cos(dx+c)^2+5\right)}{21}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(5+5\sec(dx+c))}{21}}{d\sqrt{b\sec(dx+c)}\,b^2}$	104

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/(b*sec(d*x+c))^(1/2)/b^2*(sin(d*x+c)*(3*cos(d*x+c)^2+5)+I*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-
csc(d*x+c)),I)*(5+5*sec(d*x+c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\left(3\,\cos\left(dx+c\right)^3+5\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin\left(dx+c\right)-5i\,\sqrt{2}\sqrt{b}\mathrm{weierstras}}{}$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
 c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)))/(b^3*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)}{\sec(dx+c)^3}dx\right)}{b^3}$$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x))/sec(c + d*x)**3,x))/b**3
```

3.130 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}$$

output
```
14/15*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(
d*x+c))^(1/2)+2/9*b*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*sin(d*x+c)/b/d
/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\frac{336E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 4\cos(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))}{360b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*S
in[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$b^2\left(\frac{7\int \frac{1}{(b\sec(c+dx))^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{7\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\sec(c+dx))^{7/2}}\right)$$

input
```
Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

output
```
b^2*((2*Sin[c + d*x])/(9*b*d*(b*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c
 + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sin[
c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))))/(9*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.48 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.19

method	result
default	$\frac{\frac{2\sin(dx+c)\left(5\cos(dx+c)^4+5\cos(dx+c)^3+7\cos(dx+c)^2+7\cos(dx+c)+21\right)}{45}+\frac{14i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \operatorname{EllipticF}(i(\cot(dx+c)-\csc(dx+c}{15}}{d(\cos(dx+c)+1)\sqrt{b\,\mathrm{se}}}$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d/(cos(d*x+c)+1)/(b*sec(d*x+c))^(1/2)/b^2*(sin(d*x+c)*(5*cos(d*x+c)^4
+5*cos(d*x+c)^3+7*cos(d*x+c)^2+7*cos(d*x+c)+21)+21*I*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticF(
I*(cot(d*x+c)-csc(d*x+c)),I)-21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*EllipticE(I*(cot(d*x+c)-csc(d
*x+c)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\left(5\cos\left(dx+c\right)^4+7\cos\left(dx+c\right)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\text{weierstr}}{}$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*
d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^2}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)^3}dx\right)}{b^3}$$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(c + d*x))*cos(c + d*x)**2)/sec(c + d*x)**3,x))/b**3
```

3.131 $\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 100

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^4d} + \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}$$

output
```
10/21*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec(d*x+c
))^(1/2)/b^4/d+2/7*sin(d*x+c)/b/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/b^
3/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+\right.}{84b^4d}$$

input
```
Integrate[(b*Sec[c + d*x])^(-7/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2
6*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^4*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3120$$

$$\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

input
```
Int[(b*Sec[c + d*x])^(-7/2),x]
```

output
```
(2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^2*d) + (2*Sin[c + d
*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))/(7*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\frac{2\sin(dx+c)\left(3\cos(dx+c)^2+5\right)}{21}+\frac{2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}(i(\cot(dx+c)-\csc(dx+c)),i)(5+5\sec(dx+c))}{21}}{d\sqrt{b\sec(dx+c)}\,b^3}$	104

input
```
int(1/(b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/(b*sec(d*x+c))^(1/2)/b^3*(sin(d*x+c)*(3*cos(d*x+c)^2+5)+I*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot(d*x+c)-
csc(d*x+c)),I)*(5+5*sec(d*x+c)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \frac{2\left(3\cos\left(dx+c\right)^3+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\text{weierstras}}{}$$

input
```
integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
 c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)))/(b^4*d)
```

Sympy [F]

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec{(c+dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))**(7/2),x)
```

output
```
Integral((b*sec(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/(b/cos(c + d*x))^(7/2),x)
```

output
```
int(1/(b/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^4}dx\right)}{b^4}$$

input
```
int(1/(b*sec(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(c + d*x))/sec(c + d*x)**4,x))/b**4
```

3.132 $\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d} + \frac{\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
3/8*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+3/8*sec(d*
x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*sec(d*x+c)^(7/2)*(b*sec(d
*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}(3\text{arctanh}(\sin(c+dx)) + \sec(c+dx)\,(3+2\sec^2(c+dx))\tan(c+dx))}{8d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(9/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec[c
 + d*x]^2)*Tan[c + d*x]))/(8*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2031, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sec^5(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \ 4257$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(9/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[S
in[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/Sqrt[Sec[c +
d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

method	result
default	$\frac{\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{9}{2}}\left(3\cos(dx+c)^5\ln(-\cot(dx+c)+\csc(dx+c)+1)-3\cos(dx+c)^5\ln(-\cot(dx+c)+\csc(dx+c)-1)+3\cos(dx\right.}{8d}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3\right)}{4\left(e^{2i(dx+c)}+1\right)^3 d}-\frac{3\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}-i\right)\,c}{4d}$

input
```
int(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(b*sec(d*x+c))^(1/2)*sec(d*x+c)^(9/2)*(3*cos(d*x+c)^5*ln(-cot(d*x+c)
+csc(d*x+c)+1)-3*cos(d*x+c)^5*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*cos(d*x+c)^3*
sin(d*x+c)+2*sin(d*x+c)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.16

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$=\left[\frac{3\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\left(3\cos(dx+c)^2+2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx}{\sqrt{\cos(dx+c)}}}{16\,d\cos(dx+c)^3}\right.$$

$$\left.-\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos(dx+c)^3-\frac{\left(3\cos(dx+c)^2+2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{8\,d\cos(dx+c)^3}\right]$$

input
```
integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*(
3*cos(d*x + c)^2 + 2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(d*cos(d*x + c)^3), -1/8*(3*sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c)
)*sqrt(cos(d*x + c))/(b*sin(d*x + c)))*cos(d*x + c)^3 - (3*cos(d*x + c)^2
+ 2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)
^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(89) = 178$.

Time $= 0.32$ (sec) , antiderivative size $= 1656$, normalized size of antiderivative $= 15.48$

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
4*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8
*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8
*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*
x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) +
 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8
*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16
*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x +...
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{b}\left(\frac{2\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-3\log\right.}{8d}$$

input
```
integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b)*(2*(5*tan(1/2*d*x + 1/2*c)^7 + 3*tan(1/2*d*x + 1/2*c)^5 + 3*ta
n(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^8 - 4
*tan(1/2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x + 1/2*c)^4 - 4*tan(1/2*d*x + 1/2*c
)^2 + 1) + 3*log(tan(1/2*d*x + 1/2*c) + 1) - 3*log(tan(1/2*d*x + 1/2*c) -
1))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}dx$$

input
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(9/2),x)
```

output
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b}\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4+6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

input
```
int(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 6*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**2 - 3*log(tan((c + d*x)/2) - 1) + 3*log(tan((c
+ d*x)/2) + 1)*sin(c + d*x)**4 - 6*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*
*2 + 3*log(tan((c + d*x)/2) + 1) - 3*sin(c + d*x)**3 + 5*sin(c + d*x)))/(8
*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.133 $\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d}$$

output
```
sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/3*sec(d*x+c)^(5/2)*(b
*sec(d*x+c))^(1/2)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[Sec[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sec^4(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{b\sec(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
-((Sqrt[b*Sec[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(d*Sqrt[Sec[c
+ d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\sin(dx+c)\left(2\cos(dx+c)^2+1\right)\sec(dx+c)^{\frac{7}{2}}\sqrt{b\sec(dx+c)}\,\cos(dx+c)}{3d}$	48
risch	$\frac{4i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(4\cos(dx+c)+2i\sin(dx+c))}{3\left(e^{2i(dx+c)}+1\right)^2 d}$	89

input
```
int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)*sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2)*
cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\left(2\,\cos\left(dx+c\right)^{2}+1\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c)^(5/2))
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(60) = 120.

Time = 0.23 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.20

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$= \frac{4\left(\left(3\,\cos\left(2\right.\right.\right.}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\left(3\,\cos\left(2\,dx+2\,c\right.\right.\right.}$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1
)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4
*c)*sin(2*d*x + 2*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c)
 + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d
*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*si
n(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*
c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = -\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{b}\mathrm{sgn}(\cos\left(dx+c\right))}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)d}$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2/3*(3*tan(1/2*d*x + 1/2*c)^5 - 2*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x
+ 1/2*c))*sqrt(b)*sgn(cos(d*x + c))/((tan(1/2*d*x + 1/2*c)^6 - 3*tan(1/2*d
*x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.80

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{2\cos\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(4\sin\left(c+d\,x\right)+5\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+\cos\left(c+d\right.\right.}{3\,d\,\left(10\cos\left(c+d\,x\right)+5\cos\left(3\,c+3\,d\,x\right)+\cos\left(5\,c+5\,d\,x\right.\right.}$$

input
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/2),x)
```

output
```
(2*cos(c + d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)*(cos(c + d*x
)*10i + 4*sin(c + d*x) + cos(3*c + 3*d*x)*5i + cos(5*c + 5*d*x)*1i + 5*sin
(3*c + 3*d*x) + sin(5*c + 5*d*x)))/(3*d*(10*cos(c + d*x) + 5*cos(3*c + 3*d
*x) + cos(5*c + 5*d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\left(2\sin(dx+c)^2-3\right)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*(2*sin(c + d*x)**2 - 3))/(3*cos(c + d*x)*d*(sin(c +
d*x)**2 - 1))
```

3.134 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/2*sec(d*
x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}(\operatorname{arctanh}(\sin(c+dx))+\sec(c+dx)\tan(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))
/(2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c +
 d*x])/(2*d)))/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

method	result
default	$\frac{\sec(dx+c)^{\frac{5}{2}}\sqrt{b\sec(dx+c)}\left(\cos(dx+c)^3\ln(-\cot(dx+c)+\csc(dx+c)+1)-\cos(dx+c)^3\ln(-\cot(dx+c)+\csc(dx+c)-1)+\sin(dx+c)\,c\right.}{2d}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)d}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}+i\right)\cos(dx+c)}{d}-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{}$

input
```
int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2)*(cos(d*x+c)^3*ln(-cot(d*x+c)+c
sc(d*x+c)+1)-cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+sin(d*x+c)*cos(d*x+
c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.79

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4d\cos\left(dx+c\right)},\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos\left(dx+c\right)-\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2d\cos\left(dx+c\right)}\right]$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sq
rt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*x
+ c)))*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 661, normalized size of antiderivative = 9.18

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.29

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{b}\left(\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)\right)\mathrm{sgn}(\cos\left(dx\right.}{2\,d}$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b)*(2*(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1) + log(tan(1/2*d*x + 1/2*c) +
1) - log(tan(1/2*d*x + 1/2*c) - 1))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx=\int \sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

input
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$=\frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^{2}+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sin\left(dx+c\right)\right.}{2d\left(\sin\left(dx+c\right)^{2}-1\right)}$$

input
```
int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c + d*x)
/2) - 1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d*x)/2
) + 1) - sin(c + d*x)))/(2*d*(sin(c + d*x)**2 - 1))
```

3.135 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx=\frac{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx=\frac{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{b\sec(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

input
```
Int[Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sin(dx+c)\sec(dx+c)^{\frac{3}{2}}\sqrt{b\sec(dx+c)}\,\cos(dx+c)}{d}$	35
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{d}$	67

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*sin(d*x+c)*sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx=\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx=\int\sqrt{b\sec\left(c+dx\right)}\sec^{\frac{3}{2}}\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$$
$$=\frac{2\sqrt{b}\sin\left(2\,dx+2\,c\right)}{\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = -\frac{2\sqrt{b}\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)d}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(b)*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{(\cos(d\,x)-\sin(d\,x)\ 1\mathrm{i})\ (\sin(c)+\cos(c)\ 1\mathrm{i})\ \sqrt{\frac{b}{\cos(c+d\,x)}}\ \sqrt{\frac{1}{\cos(c+d\,x)}}}{d}$$

input
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
((cos(d*x) - sin(d*x)*1i)*(cos(c)*1i + sin(c))*(b/cos(c + d*x))^(1/2)*(1/c
os(c + d*x))^(1/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b}\,\sin(dx+c)}{\cos(dx+c)\,d}$$

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/(cos(c + d*x)*d)
```

3.136 $\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx=\frac{\coth^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(ArcCoth[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sec(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\text{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

method	result	size
default	$-\frac{2\sqrt{\sec(dx+c)}\,\sqrt{b\sec(dx+c)}\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\cos(dx+c)}{d}$	46
risch	$-\frac{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}-i\right)\cos(dx+c)}{d}+\frac{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}+i\right)\cos(dx+c)}{d}$	152

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*arctanh(cot(d*x+c)-csc(d*x+c))*
cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.42

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx = \left[\frac{\sqrt{b}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^2}\right)}{2\,d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(b)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.97

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(\right.\right.}{2d}$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b}\left(\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)\right)\mathrm{sgn}(\cos\left(dx+c\right))}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(b)*(log(tan(1/2*d*x + 1/2*c) + 1) - log(tan(1/2*d*x + 1/2*c) - 1))*sg
n(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx = \int \sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}$$

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)))/d
```

3.137 $\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

output
```
x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int 1dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
orering	$\frac{x\sqrt{b\sec(dx+c)}}{\sqrt{\sec(dx+c)}}$	21
default	$\frac{(dx+c)\sqrt{b\sec(dx+c)}}{d\sqrt{\sec(dx+c)}}$	28
risch	$\frac{\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$	54

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \left[\frac{\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{2\,d}, \frac{\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\,s}{\sqrt{b}\sqrt{\cos(}}\right.}{d}\right.$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-b)*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + 2*b*cos(d*x + c)^2 - b)/d, sqrt(b)*arctan(sqrt(b/cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{x\sqrt{b\sec{(c+dx)}}}{\sqrt{\sec{(c+dx)}}}$$

input
```
integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
x*sqrt(b*sec(c + d*x))/sqrt(sec(c + d*x))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\left(dx-2\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+c\right)\sqrt{b}\mathrm{sgn}(\cos{(dx+c)})}{d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
(d*x - 2*pi*floor(1/2*(d*x + c)/pi + 1/2) + c)*sqrt(b)*sgn(cos(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{x\sqrt{\frac{b}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
(x*(b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \sqrt{b}\,x$$

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

output
```
sqrt(b)*x
```

3.138 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

output
```
(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \cos(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sqrt{b\sec(dx+c)}\,\tan(dx+c)}{d\sec(dx+c)^{\frac{3}{2}}}$	29
risch	$-\frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{i(dx+c)}}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	134

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos{(dx+c)}}\sin{(dx+c)}}{d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \begin{cases} \frac{\sqrt{b\sec(c+dx)}\tan(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)} & \text{for } d \neq 0 \\ \frac{x\sqrt{b\sec(c)}}{\sec^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Piecewise((sqrt(b*sec(c + d*x))*tan(c + d*x)/(d*sec(c + d*x)**(3/2)), Ne(d
, 0)), (x*sqrt(b*sec(c))/sec(c)**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\sin(dx+c)}{d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(b)*sin(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{b}\,\mathrm{sgn}(\cos(dx+c))}{d\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} + \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(b)*sgn(cos(d*x + c))/(d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2
*c)))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{\sin\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
(sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,\sin\left(dx+c\right)}{d}$$

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/d
```

3.139 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*(b*sec(d*x+c))^(1/2)*sin(d
*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[Sec[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[Sec[
c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{\sqrt{b\sec(dx+c)}\left(\tan(dx+c)+(dx+c)\sec(dx+c)^{2}\right)}{2d\sec(dx+c)^{\frac{5}{2}}}$	45
risch	$\frac{\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,x}{2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{2i(dx+c)}}{8\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{i\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-2i(dx+c)}}{8\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	188

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)*(tan(d*x+c)+(d*x+c)*sec(d*x+c)
^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.51

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\,c\right.}{4\,d}\right.$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)*lo
g(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*c
os(d*x + c)^2 - b))/d, 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*
x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(co
s(d*x + c)))))/d]
```

Sympy [A] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \begin{cases} \frac{x\sqrt{b\sec(c+dx)}\tan^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}+\frac{x\sqrt{b\sec(c+dx)}}{2\sec^{\frac{5}{2}}(c+dx)}+\frac{\sqrt{b\sec(c+dx)}\tan(c+dx)}{2d\sec^{\frac{5}{2}}(c+dx)} & \text{for } d\neq 0 \\ \frac{x\sqrt{b\sec(c)}}{\sec^{\frac{5}{2}}(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

output
```
Piecewise((x*sqrt(b*sec(c + d*x))*tan(c + d*x)**2/(2*sec(c + d*x)**(5/2))
+ x*sqrt(b*sec(c + d*x))/(2*sec(c + d*x)**(5/2)) + sqrt(b*sec(c + d*x))*ta
n(c + d*x)/(2*d*sec(c + d*x)**(5/2)), Ne(d, 0)), (x*sqrt(b*sec(c))/sec(c)*
*(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})\sqrt{b}}{4\,d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*sqrt(b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(51) = 102.

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b}dx\text{sgn}(\cos{(dx+c)})\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^4+2\,\sqrt{b}dx\text{sgn}(\cos{(dx+c)})\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^2-2\,\sqrt{b}\text{sgn}(\cos{(dx}}{2\left(d\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}^4+2\,d\tan{}\right.}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(b)*d*x*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^4 + 2*sqrt(b)*d*x*
sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 2*sqrt(b)*sgn(cos(d*x + c))*tan
(1/2*d*x + 1/2*c)^3 + sqrt(b)*d*x*sgn(cos(d*x + c)) + 2*sqrt(b)*sgn(cos(d*
x + c))*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(1/2*d*x
+ 1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{4\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
((sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*d*(1/cos(c + d*x))^
(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx)}{2d}$$

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x) + d*x))/(2*d)
```

3.140 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)-1/3*(b*sec(d*x+c))^(1/2
)*sin(d*x+c)^3/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\frac{(5+\cos(2(c+dx)))\sqrt{b\sec(c+dx)}\sin(c+dx)}{6d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
((5 + Cos[2*(c + d*x)])*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(6*d*Sqrt[Sec[c
 + d*x]])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b \sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b \sec(c+dx)} \int \cos^3(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b \sec(c+dx)} \int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{\sqrt{b \sec(c+dx)} \int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{b \sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
-((Sqrt[b*Sec[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[Sec[c
+ d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\sqrt{b\sec(dx+c)}\left(\tan(dx+c)+2\tan(dx+c)\sec(dx+c)^2\right)}{3d\sec(dx+c)^{\frac{7}{2}}}$	47
risch	$-\frac{3i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{3i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sin(3dx+3c)}{12\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	199

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2)*(tan(d*x+c)+2*tan(d*x+c)*sec(d
*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d
*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)}{12\,d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(3\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2-4\right)\sqrt{b}\mathrm{sgn}(\cos\left(dx+c\right))}{3\,d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^3}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
2/3*(3*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4)*sqrt(b)*sgn(
cos(d*x + c))/(d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^3)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{12\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

input
```
int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*d*(1/cos(
c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\left(-\sin(dx+c)^2+3\right)}{3d}$$

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2 + 3))/(3*d)
```

3.141 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{3x\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
3/8*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/4*(b*sec(d*x+c))^(1/2)*sin(d
*x+c)/d/sec(d*x+c)^(7/2)+3/8*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^
(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(9/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(32*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2031, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \cos^4(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{b\sec(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(9/2),x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Co
s[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\sqrt{b\sec(dx+c)}\left(2\tan(dx+c)+3\tan(dx+c)\sec(dx+c)^2+3(dx+c)\sec(dx+c)^4\right)}{8d\sec(dx+c)^{\frac{9}{2}}}$	64
risch	$\frac{3\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sin(4dx+4c)}{32\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	253

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2)*(2*tan(d*x+c)+3*tan(d*x+c)*sec
(d*x+c)^2+3*(d*x+c)*sec(d*x+c)^4)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \left[\frac{\frac{2\left(2\cos(dx+c)^4+3\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+3\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+\right.}{16\,d}\right.$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)) + 3*sqrt(-b)*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c)
)*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/d, 1/8*((2*co
s(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
sqrt(cos(d*x + c)))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)\sqrt{b}}{32\,d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(3\,\pi\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(5\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{3}+\frac{12}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}+4\right)^{2}}+6\,\arctan\left(\frac{\mathrm{ta}}{2}\right.\right.}{16\,d}$$

input
```
integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
1/16*(3*pi*sgn(tan(1/2*d*x + 1/2*c)) + 4*(5*(1/tan(1/2*d*x + 1/2*c) - tan(
1/2*d*x + 1/2*c))^3 + 12/tan(1/2*d*x + 1/2*c) - 12*tan(1/2*d*x + 1/2*c))/(
(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))^2 + 4)^2 + 6*arctan(1/2*(t
an(1/2*d*x + 1/2*c)^2 - 1)/tan(1/2*d*x + 1/2*c)))*sqrt(b)*sgn(cos(d*x + c)
)/d
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}\,(8\,\sin(2\,c+2\,d\,x)+\sin(4\,c+4\,d\,x)+12\,d\,x)}{32\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
((b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x))/
(32*d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\left(-2\cos(dx+c)\sin(dx+c)^3+5\cos(dx+c)\sin(dx+c)+3dx\right)}{8d}$$

input
```
int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)**3 + 5*cos(c + d*x)*sin(c + d*x)
+ 3*d*x))/(8*d)
```

3.142 $\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{3b\mathrm{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}} + \frac{3b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d} + \frac{b\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
3/8*b*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+3/8*b*se
c(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*b*sec(d*x+c)^(7/2)*(b
*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{(b\sec(c+dx))^{3/2}\left(3\mathrm{arctanh}(\sin(c+dx)) + \sec(c+dx)\left(3+2\sec^2(c+dx)\right)\tan(c+dx)\right)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((b*Sec[c + d*x])^(3/2)*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec
[c + d*x]^2)*Tan[c + d*x]))/(8*d*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2031, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sec^5(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

input

```
Int[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(3/2),x]
```

output

```
(b*Sqrt[b*Sec[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh
[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/Sqrt[Sec[c
+ d*x]]
```

Defintions of rubi rules used

rule 2031

```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

method	result
default	$\frac{b\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{9}{2}}\left(3\cos(dx+c)^{5}\ln(-\cot(dx+c)+\csc(dx+c)+1)-3\cos(dx+c)^{5}\ln(-\cot(dx+c)+\csc(dx+c)-1)+3\cos(d\right.}{8d}$
risch	$-\frac{ib\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3\right)}{4\left(e^{2i(dx+c)}+1\right)^{3}d}+\frac{3b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}+i\right.}{4d}$

input
```
int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*b/d*(b*sec(d*x+c))^(1/2)*sec(d*x+c)^(9/2)*(3*cos(d*x+c)^5*ln(-cot(d*x+
c)+csc(d*x+c)+1)-3*cos(d*x+c)^5*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*cos(d*x+c)^
3*sin(d*x+c)+2*sin(d*x+c)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.16

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx=\left[\frac{3\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{2}-2\sqrt{b}b\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^{2}}\right)+\frac{2\left(3\,b\cos(dx+c)^{2}+2\,b\right)}{\sqrt{\cos(}}}{16\,d\cos\left(dx+c\right)^{3}}\right.$$

$$\left.-\frac{3\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos\left(dx+c\right)^{3}-\frac{\left(3\,b\cos(dx+c)^{2}+2\,b\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{8\,d\cos\left(dx+c\right)^{3}}\right]$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*(
3*b*cos(d*x + c)^2 + 2*b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(d*cos(d*x + c)^3), -1/8*(3*sqrt(-b)*b*arctan(sqrt(-b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))/(b*sin(d*x + c)))*cos(d*x + c)^3 - (3*b*cos(d*x
 + c)^2 + 2*b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*co
s(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(92) = 184$.

Time = 0.27 (sec) , antiderivative size = 1742, normalized size of antiderivative = 15.84

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x
+ 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x
 + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4
*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c)
 + 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) +
 b)*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*
b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(
4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36...
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{b^{\frac{3}{2}}\left(\frac{2\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\ldots\right.}{8d}$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*b^(3/2)*(2*(5*tan(1/2*d*x + 1/2*c)^7 + 3*tan(1/2*d*x + 1/2*c)^5 + 3*ta
n(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^8 - 4
*tan(1/2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x + 1/2*c)^4 - 4*tan(1/2*d*x + 1/2*c
)^2 + 1) + 3*log(tan(1/2*d*x + 1/2*c) + 1) - 3*log(tan(1/2*d*x + 1/2*c) -
1))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}dx$$

input
```
int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/2),x)
```

output
```
int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.50

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{b}\,b\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4+6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2-3\log\left(\right.\right.}{}$$

input
```
int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*b*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4 + 6*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**2 - 3*log(tan((c + d*x)/2) - 1) + 3*log(tan((
c + d*x)/2) + 1)*sin(c + d*x)**4 - 6*log(tan((c + d*x)/2) + 1)*sin(c + d*x
)**2 + 3*log(tan((c + d*x)/2) + 1) - 3*sin(c + d*x)**3 + 5*sin(c + d*x)))/
(8*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.143 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{b\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d}$$

output
```
b*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/3*b*sec(d*x+c)^(5/2
)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{(b\sec(c+dx))^{3/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((b*Sec[c + d*x])^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sec[c + d*x]
^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sec^4(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{b\sqrt{b\sec(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2),x]
```

output
```
-((b*Sqrt[b*Sec[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(d*Sqrt[Sec[
c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{b\sin(dx+c)\left(2\cos(dx+c)^{2}+1\right)\sec(dx+c)^{\frac{7}{2}}\sqrt{b\sec(dx+c)}\,\cos(dx+c)}{3d}$	49
risch	$\frac{4ib\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(4\cos(dx+c)+2i\sin(dx+c))}{3\left(e^{2i(dx+c)}+1\right)^{2}d}$	90

input
```
int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)*sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2
)*cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\left(2\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*cos(d*x + c)^2 + b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x
+ c)^(5/2))
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(62) = 124.

Time = 0.19 (sec) , antiderivative size = 299, normalized size of antiderivative = 4.15

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx =$$

$$-\frac{4\,(3\,b\cos\,(6\,d}{3\,\left(2\,\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\,\left(3\,\cos\left(2\,dx+2\,c\right)\right.}$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-4/3*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin(2*d
*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*d*x
 + 2*c) + b)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6
*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)
^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2
*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = -\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)b^{\frac{3}{2}}\mathrm{sgn}(\cos\left(dx+c\right))}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)d}$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-2/3*(3*tan(1/2*d*x + 1/2*c)^5 - 2*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x
+ 1/2*c))*b^(3/2)*sgn(cos(d*x + c))/((tan(1/2*d*x + 1/2*c)^6 - 3*tan(1/2*d
*x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.76

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{2\,b\,\cos\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(4\,\sin\left(c+d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)\right.}{3\,d\,\left(10\,\cos\left(c+d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+\mathrm{co}\right.}$$

input
```
int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
(2*b*cos(c + d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)*(cos(c + d
*x)*10i + 4*sin(c + d*x) + cos(3*c + 3*d*x)*5i + cos(5*c + 5*d*x)*1i + 5*s
in(3*c + 3*d*x) + sin(5*c + 5*d*x)))/(3*d*(10*cos(c + d*x) + 5*cos(3*c + 3
*d*x) + cos(5*c + 5*d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b\big(2\sin(dx+c)^2-3\big)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b*(2*sin(c + d*x)**2 - 3))/(3*cos(c + d*x)*d*(sin(c
+ d*x)**2 - 1))
```

3.144 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*b*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/2*b*se
c(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{(b\sec(c+dx))^{3/2}(\operatorname{arctanh}(\sin(c+dx))+\sec(c+dx)\tan(c+dx))}{2d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((b*Sec[c + d*x])^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]
))/(2*d*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
 + d*x])/(2*d)))/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

method	result
default	$\frac{b\sec(dx+c)^{\frac{5}{2}}\sqrt{b\sec(dx+c)}\left(\cos(dx+c)^3\ln(-\cot(dx+c)+\csc(dx+c)+1)-\cos(dx+c)^3\ln(-\cot(dx+c)+\csc(dx+c)-1)+\sin(dx+c)\right.}{2d}$
risch	$-\frac{ib\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}-1)}{(e^{2i(dx+c)}+1)d}+\frac{b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln(e^{i(dx+c)}+i)\cos(dx+c)}{d}-\frac{b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+}}}{}$

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/d*sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2)*(cos(d*x+c)^3*ln(-cot(d*x+c)
+csc(d*x+c)+1)-cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+sin(d*x+c)*cos(d*
x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.76

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \left[\frac{b^{\frac{3}{2}}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4d\cos(dx+c)}, -\frac{\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos(dx+c)-\frac{b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2d\cos(dx+c)}\right]$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*b*sqrt
(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(
sqrt(-b)*b*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(
d*x + c)))*cos(d*x + c) - b*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(62) = 124$.

Time = 0.21 (sec) , antiderivative size = 691, normalized size of antiderivative = 9.34

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2
*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x +
 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) +
 b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)
^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2
*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) +
 b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x
+ 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos
(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d
)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{b^{\frac{3}{2}}\left(\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/2*b^(3/2)*(2*(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1) + log(tan(1/2*d*x + 1/2*c) +
1) - log(tan(1/2*d*x + 1/2*c) - 1))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx=\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx=\frac{\sqrt{b}\,b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^{2}+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right.}{2d\left(\sin\left(dx+c\right)^{2}-1\right)}$$

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*b*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c + d*
x)/2) - 1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d*x)
/2) + 1) - sin(c + d*x)))/(2*d*(sin(c + d*x)**2 - 1))
```

3.145 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
b*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = \frac{(b\sec(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2),x]
```

output
```
((b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{b\sqrt{b\sec(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{b\sin(dx+c)\sec(dx+c)^{\frac{3}{2}}\sqrt{b\sec(dx+c)}\,\cos(dx+c)}{d}$	36
risch	$\frac{2ib\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-i(dx+c)}\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}{d}$	68

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b/d*sin(d*x+c)*sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
b*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = \int (b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = \frac{2\,b^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*b^(3/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = -\frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)d}$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-2*b^(3/2)*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = \frac{b\,(\cos(d\,x)-\sin(d\,x)\;1\mathrm{i})\;(\sin(c)+\cos(c)\;1\mathrm{i})\;\sqrt{\frac{b}{\cos(c+d\,x)}}\;\sqrt{\frac{1}{\cos(c+d\,x)}}}{d}$$

input
```
int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
(b*(cos(d*x) - sin(d*x)*1i)*(cos(c)*1i + sin(c))*(b/cos(c + d*x))^(1/2)*(1
/cos(c + d*x))^(1/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b}{\cos(dx+c)\,d}$$

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b)/(cos(c + d*x)*d)
```

3.146 $\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

output
```
b*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\coth^{-1}(\sin(c+dx))(b\sec(c+dx))^{3/2}}{d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(ArcCoth[Sin[c + d*x]]*(b*Sec[c + d*x])^(3/2))/(d*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sec(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b\text{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{2b\sqrt{\sec(dx+c)}\,\sqrt{b\sec(dx+c)}\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\cos(dx+c)}{d}$	47
risch	$-\frac{b\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{b\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	141

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b/d*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*arctanh(cot(d*x+c)-csc(d*x+c)
)*cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.35

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \left[\frac{b^{\frac{3}{2}}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2d},\ -\frac{\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*b^(3/2)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*b*arctan(sq
rt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(30) = 60.

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\left(b\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-b\log\left(\cos\left(dx+c\right)^{2}\right.\right.}{2\,d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b^{\frac{3}{2}}\left(\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)\right)\operatorname{sgn}(\cos(dx+c))}{d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
b^(3/2)*(log(tan(1/2*d*x + 1/2*c) + 1) - log(tan(1/2*d*x + 1/2*c) - 1))*sg
n(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{b}\,b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}$$

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

output
```
(sqrt(b)*b*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)))/d
```

3.147 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

output
```
b*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(b*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{b\sqrt{b\sec(c+dx)}\int 1dx}{\sqrt{\sec(c+dx)}}$$
$$\Big\downarrow \text{24}$$
$$\frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(b*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
orering	$\frac{x(b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}$	21
default	$\frac{b(dx+c)\sqrt{b\sec(dx+c)}}{d\sqrt{\sec(dx+c)}}$	29
risch	$\frac{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	55

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{\sqrt{-b}b\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right.}{2\,d}\right.$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-b)*b*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*si
n(d*x + c) + 2*b*cos(d*x + c)^2 - b)/d, b^(3/2)*arctan(sqrt(b/cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [A] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{x(b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(c+dx)}$$

input
```
integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
x*(b*sec(c + d*x))**(3/2)/sec(c + d*x)**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
2*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(dx-2\,\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+c\right)b^{\frac{3}{2}}\mathrm{sgn}(\cos(dx+c))}{d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
(d*x - 2*pi*floor(1/2*(d*x + c)/pi + 1/2) + c)*b^(3/2)*sgn(cos(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{b\,x\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
(b*x*(b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.20

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \sqrt{b}\,bx$$

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

output
```
sqrt(b)*b*x
```

3.148 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

output
```
b*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(b\sec(c+dx))^{3/2}\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
((b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \cos(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b\sqrt{b\sec\left(dx+c\right)}\,\tan\left(dx+c\right)}{d\sec\left(dx+c\right)^{\frac{3}{2}}}$	30
risch	$-\frac{ib\sqrt{\frac{b\,{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,{\mathrm e}^{i(dx+c)}}{2\sqrt{\frac{{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,d}+\frac{ib\sqrt{\frac{b\,{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,{\mathrm e}^{-i(dx+c)}}{2\sqrt{\frac{{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,d}$	136

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
b/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\frac{b\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
b*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [A] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \begin{cases} \frac{(b\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)} & \text{for } d\neq 0 \\ \frac{x(b\sec(c))^{\frac{3}{2}}}{\sec^{\frac{5}{2}}(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

output
```
Piecewise(((b*sec(c + d*x))**(3/2)*tan(c + d*x)/(d*sec(c + d*x)**(5/2)), N
e(d, 0)), (x*(b*sec(c))**(3/2)/sec(c)**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.39

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^{\frac{3}{2}}\sin(dx+c)}{d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
b^(3/2)*sin(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(\cos(dx+c))}{d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
2*b^(3/2)*sgn(cos(d*x + c))/(d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2
*c)))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\frac{b\,\sin\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(b*sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.39

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,\sin\left(dx+c\right)b}{d}$$

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b)/d
```

3.149 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{bx\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*b*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*b*(b*sec(d*x+c))^(1/2)*s
in(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(b\sec(c+dx))^{3/2}(2(c+dx)+\sin(2(c+dx)))}{4d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
((b*Sec[c + d*x])^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sec[c + d*x
]^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[Se
c[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{b\sqrt{b\sec\left(dx+c\right)}\left(\tan\left(dx+c\right)+\left(dx+c\right)\sec\left(dx+c\right)^{2}\right)}{2d\sec\left(dx+c\right)^{\frac{5}{2}}}$	46
risch	$\frac{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	191

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)*(tan(d*x+c)+(d*x+c)*sec(d*x+
c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.48

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \left[\frac{2\,b\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+\sqrt{-b}b\log\left(-2\,\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx\right.\right.}{4\,d}\right.$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)*
b*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2
*b*cos(d*x + c)^2 - b))/d, 1/2*(b*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*
sin(d*x + c) + b^(3/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*s
qrt(cos(d*x + c))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.43

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(2\,(dx+c)b+b\sin\left(2\,dx+2\,c\right))\sqrt{b}}{4\,d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/4*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*sqrt(b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(53) = 106$.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(\pi\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}+4}+2\,\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\cdots\right.}{2\,\tan\left(\frac{1}{2}\,dx\cdots\right.}\right.\right.\cdots}{4\,d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
1/4*(pi*sgn(tan(1/2*d*x + 1/2*c)) + 4*(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*
x + 1/2*c))/((1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))^2 + 4) + 2*ar
ctan(1/2*(tan(1/2*d*x + 1/2*c)^2 - 1)/tan(1/2*d*x + 1/2*c)))*b^(3/2)*sgn(c
os(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{b\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{4\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(b*(sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*d*(1/cos(c + d*x)
)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.38

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx)}{2d}$$

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

output
```
(sqrt(b)*b*(cos(c + d*x)*sin(c + d*x) + d*x))/(2*d)
```

3.150 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}} - \frac{b\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
b*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)-1/3*b*(b*sec(d*x+c))^
(1/2)*sin(d*x+c)^3/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{(5+\cos(2(c+dx)))(b\sec(c+dx))^{3/2}\sin(c+dx)}{6d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

output
```
((5 + Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d*Sec[c +
d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \cos^3(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{b\sqrt{b\sec(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

output
```
-((b*Sqrt[b*Sec[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[Sec[
c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{b\sqrt{b\sec(dx+c)}\left(\tan(dx+c)+2\tan(dx+c)\sec(dx+c)^{2}\right)}{3d\sec(dx+c)^{\frac{7}{2}}}$	48
risch	$-\frac{3ib\sqrt{\frac{b\,{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,{\mathrm e}^{i(dx+c)}}{8\sqrt{\frac{{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,d}+\frac{3ib\sqrt{\frac{b\,{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,{\mathrm e}^{-i(dx+c)}}{8\sqrt{\frac{{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,d}+\frac{b\sqrt{\frac{b\,{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,\sin(3dx+3c)}{12\sqrt{\frac{{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,d}$	202

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2)*(tan(d*x+c)+2*tan(d*x+c)*sec
(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b\cos\left(dx+c\right)^{3}+2\,b\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/3*(b*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c
)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/12*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))))*sqrt(b)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(3\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}-4\right)b^{\frac{3}{2}}\mathrm{sgn}(\cos\left(dx+c\right))}{3\,d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{3}}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
2/3*(3*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4)*b^(3/2)*sgn(
cos(d*x + c))/(d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^3)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{b\,\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{12\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
(b*(9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*d*(1/co
s(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)b\left(-\sin\left(dx+c\right)^{2}+3\right)}{3d}$$

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b*( - sin(c + d*x)**2 + 3))/(3*d)
```

3.151 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{3bx\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{b\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3b\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
3/8*b*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/4*b*(b*sec(d*x+c))^(1/2)*s
in(d*x+c)/d/sec(d*x+c)^(7/2)+3/8*b*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d
*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.54

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{(b\sec(c+dx))^{3/2}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(11/2),x]
```

output
```
((b*Sec[c + d*x])^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*
x)]))/(32*d*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = $\{2031, 3042, 3115, 3042, 3115, 24\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 2031$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \cos^4(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 24$$

$$\frac{b\sqrt{b\sec(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(11/2),x]
```

output
```
(b*Sqrt[b*Sec[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (
Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{b\sqrt{b\sec(dx+c)}\left(2\tan(dx+c)+3\tan(dx+c)\sec(dx+c)^2+3(dx+c)\sec(dx+c)^4\right)}{8d\sec(dx+c)^{\frac{9}{2}}}$	65
risch	$\frac{3b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sin(4dx+4c)}{32\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	257

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*b/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2)*(2*tan(d*x+c)+3*tan(d*x+c)*s
ec(d*x+c)^2+3*(d*x+c)*sec(d*x+c)^4)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.05

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{3\sqrt{-b}b\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-\right.}{16\,d}\right.$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(-b)*b*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2
)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b) + 2*(2*b*cos(d*x + c)^4 + 3*b*cos
(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d, 1/8*
(3*b^(3/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x
+ c)))) + (2*b*cos(d*x + c)^4 + 3*b*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{(12\,(dx+c)b + b\sin\left(4\,dx+4\,c\right) + 8\,b\sin\left(\frac{1}{2}\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+\right.\right.\right.}{32\,d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
1/32*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.40

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(3\,\pi\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right) + \frac{4\left(5\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{3} + \frac{12}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - 12\,\tan\left(\frac{1}{2}\right.\right.}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}+4\right)^{2}}\right.}{16\,d}$$

input
```
integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
1/16*(3*pi*sgn(tan(1/2*d*x + 1/2*c)) + 4*(5*(1/tan(1/2*d*x + 1/2*c) - tan(
1/2*d*x + 1/2*c))^3 + 12/tan(1/2*d*x + 1/2*c) - 12*tan(1/2*d*x + 1/2*c))/(
(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))^2 + 4)^2 + 6*arctan(1/2*(t
an(1/2*d*x + 1/2*c)^2 - 1)/tan(1/2*d*x + 1/2*c)))*b^(3/2)*sgn(cos(d*x + c)
)/d
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\frac{b\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\left(8\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)+12\,d\,x\right)}{32\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(11/2),x)
```

output
```
(b*(b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x)
)/(32*d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.43

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,b\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^{3}+5\cos\left(dx+c\right)\sin\left(dx+c\right)+3dx\right)}{8d}$$

input
```
int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x)
```

output
```
(sqrt(b)*b*( - 2*cos(c + d*x)*sin(c + d*x)**3 + 5*cos(c + d*x)*sin(c + d*x
) + 3*d*x))/(8*d)
```

3.152 $\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d} + \frac{b^2\sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^5(c+dx)}{5d}$$

output
```
b^2*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+2/3*b^2*sec(d*x+c)^
(5/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)^3/d+1/5*b^2*sec(d*x+c)^(9/2)*(b*sec(
d*x+c))^(1/2)*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.49

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{(b\sec(c+dx))^{5/2}\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(5/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x
]^5/5))/(d*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \sec^6(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{b^2\sqrt{b\sec(c+dx)}\int \left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(5/2),x]
```

output
```
-((b^2*Sqrt[b*Sec[c + d*x]]*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Tan[c
+ d*x]^5/5))/(d*Sqrt[Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 14.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2 b^{2} \sqrt{b \sec \left(d x +c \right)}\, \sec \left(d x +c \right)^{\frac{11}{2}} \sin \left(d x +c \right) \left(-4 \cos \left(d x +c \right)^{5}-2 \cos \left(d x +c \right)^{3}-\frac{3 \cos \left(d x +c \right)}{2}\right)}{15 d}$	62
risch	$\frac{16 i b^{2} \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, \sqrt{\frac{b\,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, \left(10 \,{\mathrm e}^{3 i \left(d x +c \right)}+6 \cos \left(d x +c \right)+4 i \sin \left(d x +c \right)\right)}{15 \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{4} d}$	103

input
```
int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*b^2*(b*sec(d*x+c))^(1/2)*sec(d*x+c)^(11/2)*sin(d*x+c)*(-4*cos(d*x+
c)^5-2*cos(d*x+c)^3-3/2*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{\left(8\,b^2\cos\left(dx+c\right)^4+4\,b^2\cos\left(dx+c\right)^2+3\,b^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{\frac{9}{2}}}$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/15*(8*b^2*cos(d*x + c)^4 + 4*b^2*cos(d*x + c)^2 + 3*b^2)*sqrt(b/cos(d*x
+ c))*sin(d*x + c)/(d*cos(d*x + c)^(9/2))
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. 2(100) = 200.

Time = 0.26 (sec) , antiderivative size = 705, normalized size of antiderivative = 6.08

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output

```
-16/15*(5*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(10*d*x + 10*
c) + 25*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) +
 50*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - (10
*b^2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(10*d*x + 10*c) -
 5*(10*b^2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(8*d*x + 8*
c) - 10*(10*b^2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x
 + 6*c))*sqrt(b)/((2*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4*
d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*
c)^2 + 10*(10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c)
+ 1)*cos(8*d*x + 8*c) + 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) +
5*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2
*d*x + 2*c)^2 + 10*(sin(8*d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x +
4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2
*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c
) + 25*sin(8*d*x + 8*c)^2 + 100*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 100*sin(6*d*x + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*
c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = -\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-20\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+58\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-20\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\tan\left(\frac{1}{2}\right.\right.}{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+5\,\mathrm{t}\right.}$$

input

```
integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output

```
-2/15*(15*tan(1/2*d*x + 1/2*c)^9 - 20*tan(1/2*d*x + 1/2*c)^7 + 58*tan(1/2*
d*x + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x + 1/2*c))*b^(5
/2)*sgn(cos(d*x + c))/((tan(1/2*d*x + 1/2*c)^10 - 5*tan(1/2*d*x + 1/2*c)^8
 + 10*tan(1/2*d*x + 1/2*c)^6 - 10*tan(1/2*d*x + 1/2*c)^4 + 5*tan(1/2*d*x +
 1/2*c)^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.77

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{\sqrt{-\frac{b}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\left(\frac{b^2\sqrt{-\frac{1}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\,8\mathrm{i}}{15\,d}+\frac{b^2\sqrt{-\frac{1}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\left(-2\sin(2\,c+2\,d\,x)^2+\sin(4\,c+4\,d\,x)\,1\mathrm{i}+1\right)16\mathrm{i}}{3\,d}+\frac{b^2\sqrt{-\frac{}{2\sin}}}{}\right.}{16\left(\sin\left(c+d\,x\right)^2-\right.}$$

input
```
int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/2),x)
```

output
```
-((-b/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*((b^2*(-1/(2*sin(c/2 + (d*x)/2)^
2 - 1))^(1/2)*8i)/(15*d) + (b^2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(s
in(4*c + 4*d*x)*1i - 2*sin(2*c + 2*d*x)^2 + 1)*16i)/(3*d) + (b^2*(-1/(2*si
n(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin(2*c + 2*d*x)*1i - 2*sin(c + d*x)^2 + 1
)*8i)/(3*d))*(sin(5*c + 5*d*x)*1i + 2*sin((5*c)/2 + (5*d*x)/2)^2 - 1))/(16
*(sin(c + d*x)^2 - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)b^2\left(8\sin\left(dx+c\right)^4-20\sin\left(dx+c\right)^2+15\right)}{15\cos\left(dx+c\right)d\left(\sin\left(dx+c\right)^4-2\sin\left(dx+c\right)^2+1\right)}$$

input
```
int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2*(8*sin(c + d*x)**4 - 20*sin(c + d*x)**2 + 15))/
(15*cos(c + d*x)*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.153 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d}$$

output
```
b^2*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d+1/3*b^2*sec(d*x+c)^
(5/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{(b\sec(c+dx))^{5/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sec[c + d*x]
^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \sec^4(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{b^2\sqrt{b\sec(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2),x]
```

output
```
-((b^2*Sqrt[b*Sec[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(d*Sqrt[Se
c[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{b^{2} \sin \left(d x +c \right) \left(2 \cos \left(d x +c \right)^{2}+1\right) \sqrt{b \sec \left(d x +c \right)}\, \sec \left(d x +c \right)^{\frac{7}{2}} \cos \left(d x +c \right)}{3 d}$	51
risch	$\frac{4 i b^{2} \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, \sqrt{\frac{b \,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, \left(4 \cos \left(d x +c \right)+2 i \sin \left(d x +c \right)\right)}{3 \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{2} d}$	92

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)*(b*sec(d*x+c))^(1/2)*sec(d*x+c)^(7
/2)*cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{\left(2\,b^2\cos\left(dx+c\right)^2+b^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b^2*cos(d*x + c)^2 + b^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(
d*x + c)^(5/2))
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(66) = 132.

Time = 0.20 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.09

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{4\left(3\,b^2\cos\left(6\,dx\right.\right.}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^2+6\left(3\,\cos\left(2\,dx+2\,c\right)\right.\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-4/3*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2
*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c)
 + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*co
s(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
+ 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = -\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)b^{\frac{5}{2}}\mathrm{sgn}(\cos\left(dx+c\right))}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)d}$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*tan(1/2*d*x + 1/2*c)^5 - 2*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x
+ 1/2*c))*b^(5/2)*sgn(cos(d*x + c))/((tan(1/2*d*x + 1/2*c)^6 - 3*tan(1/2*d
*x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.70

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{2\,b^2\,\cos\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(4\,\sin\left(c+d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right.\right.}{3\,d\,\left(10\,\cos\left(c+d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+c\right.}$$

input
```
int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
(2*b^2*cos(c + d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)*(cos(c +
 d*x)*10i + 4*sin(c + d*x) + cos(3*c + 3*d*x)*5i + cos(5*c + 5*d*x)*1i + 5
*sin(3*c + 3*d*x) + sin(5*c + 5*d*x)))/(3*d*(10*cos(c + d*x) + 5*cos(3*c +
 3*d*x) + cos(5*c + 5*d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b^2\left(2\sin(dx+c)^2-3\right)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2*(2*sin(c + d*x)**2 - 3))/(3*cos(c + d*x)*d*(sin
(c + d*x)**2 - 1))
```

3.154 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx = \frac{b^2\text{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*b^2*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/2*b^
2*sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx = \frac{(b\sec(c+dx))^{5/2}(\text{arctanh}(\sin(c+dx))+\sec(c+dx)\tan(c+dx))}{2d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]
))/(2*d*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan
[c + d*x])/(2*d)))/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

method	result
default	$-\frac{b^{2}\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{5}{2}}\left(\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)-1)-\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)+1)-\sin(dx\right.}{2d}$
risch	$-\frac{ib^{2}\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)d}+\frac{b^{2}\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}+i\right)\cos(dx+c)}{d}-\frac{b^{2}\sqrt{\frac{e^{i(dx-}}{e^{2i(dx+}}}}{}$

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/d*(b*sec(d*x+c))^(1/2)*sec(d*x+c)^(5/2)*(cos(d*x+c)^3*ln(-cot(d*x
+c)+csc(d*x+c)-1)-cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-sin(d*x+c)*cos
(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.69

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx = \left[\frac{b^{\frac{5}{2}}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2b^2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4d\cos(dx+c)}, -\frac{\sqrt{-b}b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos(dx+c)-\frac{b^2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2d\cos(dx+c)}\right]$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*b^2*sq
rt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2
*(sqrt(-b)*b^2*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*
sin(d*x + c)))*cos(d*x + c) - b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(66) = 132$.

Time = 0.20 (sec) , antiderivative size = 747, normalized size of antiderivative = 9.58

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2
*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*c
os(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*
b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*c
os(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x
 + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*co
s(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*
(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d
*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqr
t(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx = \frac{b^{\frac{5}{2}}\left(\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/2*b^(5/2)*(2*(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1) + log(tan(1/2*d*x + 1/2*c) +
1) - log(tan(1/2*d*x + 1/2*c) - 1))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx=\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}\,dx=\frac{\sqrt{b}\,b^2\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.\right.}{2d\left(\sin\left(dx+c\right)^2-1\right)}$$

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*b**2*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c +
 d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d
*x)/2) + 1) - sin(c + d*x)))/(2*d*(sin(c + d*x)**2 - 1))
```

3.155 $\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
b^2*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{(b\sec(c+dx))^{5/2}\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
((b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{b^2\sqrt{b\sec(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}{d}$$

input
```
Int[(b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{b^{2} \sin\left(d x +c \right) \sec\left(d x +c \right)^{\frac{3}{2}} \sqrt{b \sec\left(d x +c \right)}\, \cos\left(d x +c \right)}{d}$	38
risch	$\frac{2 i b^{2} \sqrt{\frac{b\,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}}{\sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}$	74

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b^2/d*sin(d*x+c)*sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2)*cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b^2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\,b^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2*b^(5/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*d)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\,b^{\frac{5}{2}}\operatorname{sgn}(\cos(dx+c))}{d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2*b^(5/2)*sgn(cos(d*x + c))/(d*(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2
*c)))
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b^2\sqrt{\frac{b}{\cos(c+d\,x)}}\,(\cos(2\,c+2\,d\,x)\,1\mathrm{i}+\sin(2\,c+2\,d\,x)+1\mathrm{i})}{d\,(\cos(2\,c+2\,d\,x)+1)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
(b^2*(b/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))
/(d*(cos(2*c + 2*d*x) + 1)*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b^2}{\cos(dx+c)\,d}$$

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2)/(cos(c + d*x)*d)
```

3.156 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{b^2\operatorname{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

output
```
b^2*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{\coth^{-1}(\sin(c+dx))(b\sec(c+dx))^{5/2}}{d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(ArcCoth[Sin[c + d*x]]*(b*Sec[c + d*x])^(5/2))/(d*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \sec(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b^2\text{arctanh}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{2b^2\sqrt{\sec(dx+c)}\,\sqrt{b\sec(dx+c)}\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))\cos(dx+c)}{d}$	49
risch	$-\frac{b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	145

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b^2/d*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)*arctanh(cot(d*x+c)-csc(d*x+
c))*cos(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{b^{\frac{5}{2}}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2d},\right.$$
$$\left.-\frac{\sqrt{-b}b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*b^(5/2)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*b^2*arctan(
sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*x + c)))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(32) = 64$.

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.00

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(b^2\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-b^2\log\left(\cos\left(dx+c\right)\right.\right.}{2\,d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^{\frac{5}{2}}\left(\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)\right)\mathrm{sgn}(\cos\left(dx+c\right))}{d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
b^(5/2)*(log(abs(tan(1/2*d*x + 1/2*c) + 1)) - log(abs(tan(1/2*d*x + 1/2*c)
 - 1)))*sgn(cos(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}$$

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

output
```
(sqrt(b)*b**2*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)))/
d
```

3.157 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

output
```
b^2*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{x(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(x*(b*Sec[c + d*x])^(5/2))/Sec[c + d*x]^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{ 2031}$$
$$\frac{b^2\sqrt{b\sec(c+dx)}\int 1dx}{\sqrt{\sec(c+dx)}}$$
$$\Big\downarrow \text{ 24}$$
$$\frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(b^2*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
orering	$\frac{x(b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}$	21
default	$\frac{b^{2}(dx+c)\sqrt{b\sec(dx+c)}}{d\sqrt{\sec(dx+c)}}$	31
risch	$\frac{b^{2}\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$	57

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{\sqrt{-b}b^{2}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^{2}-\right.}{2\,d}\right.$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-b)*b^2*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*
sin(d*x + c) + 2*b*cos(d*x + c)^2 - b)/d, b^(5/2)*arctan(sqrt(b/cos(d*x +
c))*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [A] (verification not implemented)

Time = 48.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{x(b\sec{(c+dx)})^{\frac{5}{2}}}{\sec^{\frac{5}{2}}{(c+dx)}}$$

input
```
integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
x*(b*sec(c + d*x))**(5/2)/sec(c + d*x)**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
2*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(dx-2\,\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+c\right)b^{\frac{5}{2}}\mathrm{sgn}(\cos{(dx+c)})}{d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
(d*x - 2*pi*floor(1/2*(d*x + c)/pi + 1/2) + c)*b^(5/2)*sgn(cos(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2\,x\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(b^2*x*(b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \sqrt{b}\,b^2x$$

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

output
```
sqrt(b)*b**2*x
```

3.158 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)} \, dx = \frac{b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

output
```
b^2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)} \, dx = \frac{(b\sec(c+dx))^{5/2}\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int\cos(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b^{2} \sqrt{b \sec\left(d x +c \right)}\, \tan\left(d x +c \right)}{d \sec\left(d x +c \right)^{\frac{3}{2}}}$	32
risch	$-\frac{i b^{2} \sqrt{\frac{b\, {\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, {\mathrm e}^{i \left(d x +c \right)}}{2 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}+\frac{i b^{2} \sqrt{\frac{b\, {\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, {\mathrm e}^{-i \left(d x +c \right)}}{2 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}$	140

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
b^2/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{b^2\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
b^2*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{b^{\frac{5}{2}}\sin(dx+c)}{d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
b^(5/2)*sin(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{5}{2}}\mathrm{sgn}(\cos(dx+c))}{d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
2*b^(5/2)*sgn(cos(d*x + c))/(d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2
*c)))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{b^2\,\sin(c+d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(b^2*sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b^2}{d}$$

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2)/d
```

3.159 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*b^2*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*b^2*(b*sec(d*x+c))^(1/
2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\sec(c+dx))^{5/2}(2(c+dx)+\sin(2(c+dx)))}{4d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
((b*Sec[c + d*x])^(5/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sec[c + d*x
]^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(b^2*Sqrt[b*Sec[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[
Sec[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{b^{2} \sqrt{b \sec \left(d x +c \right)}\, \left(\tan \left(d x +c \right)+\left(d x +c \right) \sec \left(d x +c \right)^{2}\right)}{2 d \sec \left(d x +c \right)^{\frac{5}{2}}}$	48
risch	$\frac{b^{2} \sqrt{\frac{b\,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, x}{2 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}}-\frac{i b^{2} \sqrt{\frac{b\,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, {\mathrm e}^{2 i \left(d x +c \right)}}{8 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}+\frac{i b^{2} \sqrt{\frac{b\,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, {\mathrm e}^{-2 i \left(d x +c \right)}}{8 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}$	197

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)*(tan(d*x+c)+(d*x+c)*sec(d*
x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.42

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \left[\frac{2\,b^2\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+\sqrt{-b}b^2\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(\right.\right.}{4\,d}\right.$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b
)*b^2*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c)
 + 2*b*cos(d*x + c)^2 - b))/d, 1/2*(b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^
(3/2)*sin(d*x + c) + b^(5/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqr
t(b)*sqrt(cos(d*x + c)))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(2\,(dx+c)b^2 + b^2\sin\left(2\,dx+2\,c\right)\right)\sqrt{b}}{4\,d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/4*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*sqrt(b)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(\pi \mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right) + \frac{4\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2+4} + 2\,\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)\right)b^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{4\,d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
1/4*(pi*sgn(tan(1/2*d*x + 1/2*c)) + 4*(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*
x + 1/2*c))/((1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))^2 + 4) + 2*ar
ctan(1/2*(tan(1/2*d*x + 1/2*c)^2 - 1)/tan(1/2*d*x + 1/2*c)))*b^(5/2)*sgn(c
os(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2\,(\sin{(2\,c+2\,d\,x)}+2\,d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{4\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
(b^2*(sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*d*(1/cos(c + d*
x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2(\cos{(dx+c)}\sin{(dx+c)}+dx)}{2d}$$

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)
```

output
```
(sqrt(b)*b**2*(cos(c + d*x)*sin(c + d*x) + d*x))/(2*d)
```

3.160 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}} - \frac{b^2\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
b^2*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)-1/3*b^2*(b*sec(d*x+
c))^(1/2)*sin(d*x+c)^3/d/sec(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{(5+\cos(2(c+dx)))(b\sec(c+dx))^{5/2}\sin(c+dx)}{6d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

output
```
((5 + Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d*Sec[c +
d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int\cos^3(c+dx)dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sec(c+dx)}\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{b^2\sqrt{b\sec(c+dx)}\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

output
```
-((b^2*Sqrt[b*Sec[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[Se
c[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{b^{2} \sqrt{b \sec \left(d x +c \right)}\, \left(\tan \left(d x +c \right)+2 \tan \left(d x +c \right) \sec \left(d x +c \right)^{2}\right)}{3 d \sec \left(d x +c \right)^{\frac{7}{2}}}$	50
risch	$-\frac{3 i b^{2} \sqrt{\frac{b \,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, {\mathrm e}^{i \left(d x +c \right)}}{8 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}+\frac{3 i b^{2} \sqrt{\frac{b \,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, {\mathrm e}^{-i \left(d x +c \right)}}{8 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}+\frac{b^{2} \sqrt{\frac{b \,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, \sin \left(3 d x +3 c \right)}{12 \sqrt{\frac{{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}$	208

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2/d*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2)*(tan(d*x+c)+2*tan(d*x+c)*s
ec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b^2\cos\left(dx+c\right)^3+2\,b^2\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
1/3*(b^2*cos(d*x + c)^3 + 2*b^2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x
 + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
1/12*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*sqrt(b)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\left(3\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2-4\right)b^{\frac{5}{2}}\mathrm{sgn}(\cos\left(dx+c\right))}{3\,d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^3}$$

input
```
integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
2/3*(3*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4)*b^(5/2)*sgn(
cos(d*x + c))/(d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^3)
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{b^2\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{12\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(11/2),x)
```

output
```
(b^2*(9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*d*(1/
cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)b^2\left(-\sin\left(dx+c\right)^2+3\right)}{3d}$$

input
```
int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2*( - sin(c + d*x)**2 + 3))/(3*d)
```

3.161 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2d\sqrt{b\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\sec(c+dx)}}$$

output
```
1/2*arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/d/(b*sec(d*x+c))^(1/2)+1/2*sec(d*
x+c)^(5/2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}(\operatorname{arctanh}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx))}{2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(
2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(7/2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/Sqrt[b*Sec[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

method	result
default	$\frac{\sec(dx+c)^{\frac{7}{2}}\left(\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)+1)-\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)-1)+\sin(dx+c)\cos(dx+c)\right)}{2d\sqrt{b\sec(dx+c)}}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$

input
```
int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2)*(cos(d*x+c)^3*ln(-cot(d*x+c)+c
sc(d*x+c)+1)-cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+sin(d*x+c)*cos(d*x+
c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.88

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \left[\frac{\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4bd\cos(dx+c)}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos(dx+c)-\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2bd\cos(dx+c)}\right]$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b*d*cos(d*x + c)), -1/2*(
sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*
x + c)))*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(60) = 120$.

Time = 0.21 (sec) , antiderivative size = 661, normalized size of antiderivative = 9.18

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((2*(2*cos(2*d*x + 2*c) + 1)*
cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*c
os(2*d*x + 2*c) + 1)*sqrt(b)*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{\frac{4\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2-4}+\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right|\right)-\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\tan\left(\frac{1}{2}\right.\right.\right.}{4\sqrt{b}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(4*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))/((1/tan(1/2*d*x + 1
/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4) + log(abs(1/tan(1/2*d*x + 1/2*c) + ta
n(1/2*d*x + 1/2*c) + 2)) - log(abs(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x +
1/2*c) - 2)))/(sqrt(b)*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^{2}+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\sin\left(dx+c\right)^{2}}{2bd\left(\sin\left(dx+c\right)^{2}-1\right)}$$

input

```
int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x)
```

output

```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c + d*x)
/2) - 1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d*x)/2
) + 1) - sin(c + d*x)))/(2*b*d*(sin(c + d*x)**2 - 1))
```

3.162 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{\sec(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sin(dx+c)\sec(dx+c)^{\frac{5}{2}}\cos(dx+c)}{d\sqrt{b\sec(dx+c)}}$	35
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)}$	71

input
```
int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*sin(d*x+c)*sec(d*x+c)^(5/2)*cos(d*x+c)/(b*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{bd\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\sqrt{b}\sin\left(2\,dx+2\,c\right)}{\left(b\cos\left(2\,dx+2\,c\right)^{2}+b\sin\left(2\,dx+2\,c\right)^{2}+2\,b\cos\left(2\,dx+2\,c\right)+b\right)d}$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 +
 2*b*cos(2*d*x + 2*c) + b)*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2}{\sqrt{b}d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2/(sqrt(b)*d*(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))*sgn(cos(d*x +
 c)))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,1\mathrm{i}}{b\,d}$$

input
```
int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(1/2),x)
```

output
```
((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*(b/cos(c + d*x))^(1/2)*(1/c
os(c + d*x))^(1/2)*1i)/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\,\sin(dx+c)}{\cos(dx+c)\,bd}$$

input
```
int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/(cos(c + d*x)*b*d)
```

3.163 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\coth^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(ArcCoth[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\sec(c+dx)}\text{arctanh}(\sin(c+dx))}{d\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

method	result	size
default	$-\frac{2 \sec(dx+c)^{\frac{3}{2}} \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c)) \cos(dx+c)}{d\sqrt{b\sec(dx+c)}}$	46
risch	$-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, \ln\left(e^{i(dx+c)}-i\right)}{\sqrt{\frac{b\, e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, \ln\left(e^{i(dx+c)}+i\right)}{\sqrt{\frac{b\, e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}$	139

input
```
int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*sec(d*x+c)^(3/2)*arctanh(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)/(b*sec(d*x
+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.52

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^2}\right)}{2\sqrt{b}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)}{bd}\right]$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(sqrt(b)*d), -sqrt(-b)*arctan(sq
rt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*x + c)))/(b*d)]
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(b*sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.97

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx-\right.}{2\sqrt{b}d}$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(sqrt(b)*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{\sqrt{b}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
(log(tan(1/2*d*x + 1/2*c) + 1) - log(tan(1/2*d*x + 1/2*c) - 1))/(sqrt(b)*d
*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{bd}$$

input
```
int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)))/(b*d)
```

3.164 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

output
```
x*sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(x*Sqrt[Sec[c + d*x]])/Sqrt[b*Sec[c + d*x]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int 1dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[b*Sec[c + d*x]],x]
```

output
```
(x*Sqrt[Sec[c + d*x]])/Sqrt[b*Sec[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
orering	$\frac{x\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)}}$	21
default	$\frac{(dx+c)\sqrt{\sec(dx+c)}}{d\sqrt{b\sec(dx+c)}}$	28
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	54

input
```
int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.21

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{2\,bd}, \frac{\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(d}{\sqrt{b}\sqrt{\cos(dx+c}}\right.}{\sqrt{b}d}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + 2*b*cos(d*x + c)^2 - b)/(b*d), arctan(sqrt(b/cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(sqrt(b)*d)]
```

Sympy [A] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \frac{x\sqrt{\sec{(c+dx)}}}{\sqrt{b\sec{(c+dx)}}}$$

input
```
integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
x*sqrt(sec(c + d*x))/sqrt(b*sec(c + d*x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(b)*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \frac{dx-2\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+c}{\sqrt{b}d\mathrm{sgn}\left(\cos{(dx+c)}\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
(d*x - 2*pi*floor(1/2*(d*x + c)/pi + 1/2) + c)/(sqrt(b)*d*sgn(cos(d*x + c)
))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \frac{x\sqrt{\frac{b}{\cos(c+d\,x)}}}{b\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((1/cos(c + d*x))^(1/2)/(b/cos(c + d*x))^(1/2),x)
```

output
```
(x*(b/cos(c + d*x))^(1/2))/(b*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\,x}{b}$$

input
```
int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*x)/b
```

3.165 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(1/2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2032, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\tan(dx+c)}{d\sqrt{\sec(dx+c)}\,\sqrt{b\sec(dx+c)}}$	29
risch	$-\frac{i\,e^{2i(dx+c)}}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+1)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{i}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+1)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	151

input
```
int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{bd}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d)
```

Sympy [A] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \begin{cases} \frac{\tan(c+dx)}{d\sqrt{b\sec(c+dx)}\sqrt{\sec(c+dx)}} & \text{for } d\neq 0 \\ \frac{x}{\sqrt{b\sec(c)}\sqrt{\sec(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Piecewise((tan(c + d*x)/(d*sqrt(b*sec(c + d*x))*sqrt(sec(c + d*x))), Ne(d,
 0)), (x/(sqrt(b*sec(c))*sqrt(sec(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \frac{\sin(dx+c)}{\sqrt{b}d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(sqrt(b)*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)\sqrt{b}d\operatorname{sgn}(\cos(dx+c))}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*sqrt(b)*d*sgn(cos(d*x
 + c)))
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \frac{\sin\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{b\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
(sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(b*d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)}{bd}$$

input
```
int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/(b*d)
```

3.166 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
1/2*x*sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+1/2*sin(d*x+c)/d/sec(d*x+c)^(1
/2)/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Sec[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2032, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[b*Sec[
c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

method	result
default	$\frac{\tan(dx+c)+(dx+c)\sec(dx+c)^2}{2d\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{3}{2}}}$
risch	$\frac{e^{i(dx+c)}x}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)}-\frac{i e^{3i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)d}+\frac{i e^{-i(dx+c}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+}}{e^{2i(dx+c)}}}}$

input
```
int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)*(tan(d*x+c)+(d*x+c)*sec(d*x+c)
^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$$
$$= \left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c) - \sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c) + 2b\cos\right.}{4bd}\right.$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*lo
g(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*co
s(d*x + c)^2 - b))/(b*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin
(d*x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt
(cos(d*x + c)))))/(b*d)]
```

Sympy [A] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$$
$$= \begin{cases} \frac{x\tan^2(c+dx)}{2\sqrt{b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{x}{2\sqrt{b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\tan(c+dx)}{2d\sqrt{b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{\sqrt{b\sec(c)}\sec^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Piecewise((x*tan(c + d*x)**2/(2*sqrt(b*sec(c + d*x))*sec(c + d*x)**(3/2))
+ x/(2*sqrt(b*sec(c + d*x))*sec(c + d*x)**(3/2)) + tan(c + d*x)/(2*d*sqrt(
b*sec(c + d*x))*sec(c + d*x)**(3/2)), Ne(d, 0)), (x/(sqrt(b*sec(c))*sec(c)
**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,\sqrt{b}d}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(sqrt(b)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(51) = 102.

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{\pi\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}+4}+2\,\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{4\,\sqrt{b}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(pi*sgn(tan(1/2*d*x + 1/2*c)) + 4*(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*
x + 1/2*c))/((1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))^2 + 4) + 2*ar
ctan(1/2*(tan(1/2*d*x + 1/2*c)^2 - 1)/tan(1/2*d*x + 1/2*c)))/(sqrt(b)*d*sg
n(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{(\sin(2\,c+2\,d\,x)+2\,d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{4\,b\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
((sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*b*d*(1/cos(c + d*x)
)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{b}\,(\cos(dx+c)\sin(dx+c)+dx)}{2bd}$$

input
```
int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x) + d*x))/(2*b*d)
```

3.167 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{b\sec(c+dx)}} - \frac{\sqrt{\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(1/2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)-1/3*sec(d*x+c)^(1/2)*si
n(d*x+c)^3/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{(5+\cos(2(c+dx)))\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]),x]
```

output
```
((5 + Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*Sqrt[b*Sec[c
 + d*x]])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2032, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos^3(c+dx)dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{\sqrt{\sec(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]),x]
```

output
```
-((Sqrt[Sec[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[b*Sec[c
+ d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

method	result
default	$\frac{\tan(dx+c)+2\tan(dx+c)\sec(dx+c)^{2}}{3d\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\,\mathrm{e}^{4i(dx+c)}}{24\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}-\frac{3i\,\mathrm{e}^{2i(dx+c)}}{8\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{}{8\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx\right.}}$

input
```
int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)*(tan(d*x+c)+2*tan(d*x+c)*sec(d
*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{3\,bd\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b
*d*sqrt(cos(d*x + c)))
```

Sympy [A] (verification not implemented)

Time = 26.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \begin{cases} \frac{2\tan^{3}(c+dx)}{3d\sqrt{b\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}+\frac{\tan(c+dx)}{d\sqrt{b\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{\sqrt{b\sec(c)}\sec^{\frac{5}{2}}(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Piecewise((2*tan(c + d*x)**3/(3*d*sqrt(b*sec(c + d*x))*sec(c + d*x)**(5/2)
) + tan(c + d*x)/(d*sqrt(b*sec(c + d*x))*sec(c + d*x)**(5/2)), Ne(d, 0)),
(x/(sqrt(b*sec(c))*sec(c)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)}{12\,\sqrt{b}d}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))))/(sqrt(b)*d)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx = \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)\sqrt{b}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^5 + 2*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^6 + 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d
*x + 1/2*c)^2 + 1)*sqrt(b)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx=\frac{(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right))\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{12\,b\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*b*d*(1/co
s(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx=\frac{\sqrt{b}\,\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^{2}+3\right)}{3bd}$$

input
```
int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2 + 3))/(3*b*d)
```

3.168 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2bd\sqrt{b\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\sec(c+dx)}}$$

output
```
1/2*arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/b/d/(b*sec(d*x+c))^(1/2)+1/2*sec(
d*x+c)^(5/2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)(\operatorname{arctanh}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx))}{2d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(
2*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/(b*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sec(dx+c)^{\frac{7}{2}}\left(\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)+1)-\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)-1)+\sin(dx+c)\cos(dx+c)\right)}{2bd\sqrt{b\sec(dx+c)}}$
risch	$-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{4i(dx+c)}\ln(e^{i(dx+c)}-i)-e^{4i(dx+c)}\ln(e^{i(dx+c)}+i)+2ie^{3i(dx+c)}-2ie^{i(dx+c)}+2\,e^{2i(dx+c)}\ln(e^{i(dx+c)}-i)-2\,e^{2i(}}{2b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)^{2}}$

input
```
int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/d*sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2)*(cos(d*x+c)^3*ln(-cot(d*x+c)
+csc(d*x+c)+1)-cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+sin(d*x+c)*cos(d*
x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.65

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx}}}{\sqrt{\mathrm{co}}}}{4b^2d\cos(dx+c)}\right.$$

$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos(dx+c)-\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2b^2d\cos(dx+c)}\right]$$

input
```
integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^2*d*cos(d*x + c)), -1/2
*(sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(
d*x + c)))*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(66) = 132.

Time = 0.19 (sec) , antiderivative size = 670, normalized size of antiderivative = 8.59

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((b*cos(4*d*x + 4*c)^2 + 4*b*
cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\frac{4\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2-4}+\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)-\mathrm{log}}{4\,b^{\frac{3}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*(4*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))/((1/tan(1/2*d*x + 1
/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4) + log(abs(1/tan(1/2*d*x + 1/2*c) + ta
n(1/2*d*x + 1/2*c) + 2)) - log(abs(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x +
1/2*c) - 2)))/(b^(3/2)*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(b/cos(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{d}{2}\right.\right.\right.}{2b^2d\left(\sin\left(dx+c\right)^2\right.}$$

input
```
int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c + d*x)
/2) - 1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d*x)/2
) + 1) - sin(c + d*x)))/(2*b**2*d*(sin(c + d*x)**2 - 1))
```

3.169 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{\sec(c+dx)}\int 1d(-\tan(c+dx))}{bd\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sin(dx+c)\sec(dx+c)^{\frac{5}{2}}\cos(dx+c)}{bd\sqrt{b\sec(dx+c)}}$	38
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)}$	74

input
```
int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*sin(d*x+c)*sec(d*x+c)^(5/2)*cos(d*x+c)/(b*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{b^2d\sqrt{\cos(dx+c)}}$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(31) = 62.

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2\right)d}$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)
^2 + 2*b^2*cos(2*d*x + 2*c) + b^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = -\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^{\frac{3}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^(3/2)*d*sgn(cos(d*
x + c)))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,1\mathrm{i}}{b^2\,d}$$

input
```
int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(3/2),x)
```

output
```
((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*(b/cos(c + d*x))^(1/2)*(1/c
os(c + d*x))^(1/2)*1i)/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)b^2d}$$

input
```
int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/(cos(c + d*x)*b**2*d)
```

3.170 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/b/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\coth^{-1}(\sin(c+dx))\sec^{\frac{3}{2}}(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(ArcCoth[Sin[c + d*x]]*Sec[c + d*x]^(3/2))/(d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\sec(c+dx)}\text{arctanh}(\sin(c+dx))}{bd\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{2 \sec(dx+c)^{\frac{3}{2}} \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c)) \cos(dx+c)}{bd\sqrt{b\sec(dx+c)}}$	49
risch	$-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, \ln\left(e^{i(dx+c)}-i\right)}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, \ln\left(e^{i(dx+c)}+i\right)}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}$	145

input
```
int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/d*sec(d*x+c)^(3/2)*arctanh(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)/(b*sec(d
*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^2}\right)}{2\,b^{\frac{3}{2}}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)}{b^2d}\right]$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(b^(3/2)*d), -sqrt(-b)*arctan(sq
rt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*x + c)))/(b^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\mathrm{si}\right.}{2\,b^{\frac{3}{2}}d}$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^{\frac{3}{2}}d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
(log(abs(tan(1/2*d*x + 1/2*c) + 1)) - log(abs(tan(1/2*d*x + 1/2*c) - 1)))/
(b^(3/2)*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{b^2d}$$

input
```
int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)))/(b**2
*d)
```

3.171 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}$$

output
```
x*sec(d*x+c)^(1/2)/b/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{x\sqrt{b\sec(c+dx)}}{b^2\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(x*Sqrt[b*Sec[c + d*x]])/(b^2*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int 1dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(x*Sqrt[Sec[c + d*x]])/(b*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
orering	$\frac{x \sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c))^{\frac{3}{2}}}$	21
default	$\frac{(dx+c)\sqrt{\sec(dx+c)}}{bd\sqrt{b\sec(dx+c)}}$	31
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, x}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	57

input
```
int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{2\,b^2d}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + 2*b*cos(d*x + c)^2 - b)/(b^2*d), arctan(sqrt(b/cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(b^(3/2)*d)]
```

Sympy [A] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{x\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}$$

input
```
integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
x*sec(c + d*x)**(3/2)/(b*sec(c + d*x))**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}d}$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{dx-2\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+c}{b^{\frac{3}{2}}d\,\mathrm{sgn}(\cos(dx+c))}$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
(d*x - 2*pi*floor(1/2*(d*x + c)/pi + 1/2) + c)/(b^(3/2)*d*sgn(cos(d*x + c)
))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{x\sqrt{\frac{b}{\cos(c+d\,x)}}}{b^2\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(3/2),x)
```

output
```
(x*(b/cos(c + d*x))^(1/2))/(b^2*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,x}{b^2}$$

input
```
int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*x)/b**2
```

3.172 $\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(1/2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\tan(dx+c)}{bd\sqrt{\sec(dx+c)}\,\sqrt{b\sec(dx+c)}}$	32
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, e^{i(dx+c)}}{2b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}+\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, e^{-i(dx+c)}}{2b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}$	140

input
```
int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/d/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx=\frac{\sqrt{\frac{b}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\sin(dx+c)}{b^2d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d)
```

Sympy [A] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx = \begin{cases} \frac{\tan(c+dx)\sqrt{\sec(c+dx)}}{d(b\sec(c+dx))^{\frac{3}{2}}} & \text{for } d\neq 0 \\ \frac{x\sqrt{\sec(c)}}{(b\sec(c))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Piecewise((tan(c + d*x)*sqrt(sec(c + d*x))/(d*(b*sec(c + d*x))**(3/2)), Ne
(d, 0)), (x*sqrt(sec(c))/(b*sec(c))**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sin(dx+c)}{b^{\frac{3}{2}}d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(b^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b^{\frac{3}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^(3/2)*d*sgn(cos(d*x
 + c)))
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx=\frac{\sin\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{2\,b^2\,d}$$

input
```
int((1/cos(c + d*x))^(1/2)/(b/cos(c + d*x))^(3/2),x)
```

output
```
(sin(2*c + 2*d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2))/(2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx=\frac{\sqrt{b}\,\sin\left(dx+c\right)}{b^2d}$$

input
```
int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/(b**2*d)
```

3.173 $\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
1/2*x*sec(d*x+c)^(1/2)/b/(b*sec(d*x+c))^(1/2)+1/2*sin(d*x+c)/b/d/sec(d*x+c
)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)(2(c+dx)+\sin(2(c+dx)))}{4d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sec[c + d*x]^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*(b*Sec[c + d*x]
)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2032, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{ 2032}$$
$$\frac{\sqrt{\sec(c+dx)}\int \cos^2(c+dx)dx}{b\sqrt{b\sec(c+dx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{b\sqrt{b\sec(c+dx)}}$$
$$\Big\downarrow \text{ 3115}$$
$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{b\sqrt{b\sec(c+dx)}}$$
$$\Big\downarrow \text{ 24}$$
$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{b\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(b*Sqrt[b*S
ec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

method	result
default	$\frac{\tan(dx+c)+(dx+c)\sec(dx+c)^2}{2bd\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{3}{2}}}$
risch	$\frac{e^{i(dx+c)}x}{2b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{ie^{3i(dx+c)}}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{ie^{-i(dx}}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+\right.}}$

input
```
int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/d/(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)*(tan(d*x+c)+(d*x+c)*sec(d*x+
c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.39

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c) - \sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx}}\right.}{4\,b^2d}\right.$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*lo
g(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*co
s(d*x + c)^2 - b))/(b^2*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*s
in(d*x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sq
rt(cos(d*x + c)))))/(b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \begin{cases} \frac{x\tan^2(c+dx)}{2(b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}} + \frac{x}{2(b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}} + \frac{\tan(c+d}{2d(b\sec(c+dx))^{\frac{3}{2}}} \\ \frac{x}{(b\sec(c))^{\frac{3}{2}}\sqrt{\sec(c)}} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Piecewise((x*tan(c + d*x)**2/(2*(b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))
) + x/(2*(b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))) + tan(c + d*x)/(2*d*(
b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), Ne(d, 0)), (x/((b*sec(c))**(3/
2)*sqrt(sec(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,b^{\frac{3}{2}}d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \frac{\pi\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}+4}+2\,\mathrm{arctan}}{4\,b^{\frac{3}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*(pi*sgn(tan(1/2*d*x + 1/2*c)) + 4*(1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*
x + 1/2*c))/((1/tan(1/2*d*x + 1/2*c) - tan(1/2*d*x + 1/2*c))^2 + 4) + 2*ar
ctan(1/2*(tan(1/2*d*x + 1/2*c)^2 - 1)/tan(1/2*d*x + 1/2*c)))/(b^(3/2)*d*sg
n(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \frac{(\sin(2\,c+2\,d\,x)+2\,d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{4\,b^2\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
((sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*b^2*d*(1/cos(c + d*
x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,(\cos(dx+c)\sin(dx+c)+dx)}{2b^2d}$$

input
```
int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x) + d*x))/(2*b**2*d)
```

3.174 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}} - \frac{\sqrt{\sec(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(1/2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)-1/3*sec(d*x+c)^(1/2)*
sin(d*x+c)^3/b/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{(5+\cos(2(c+dx)))\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2)),x]
```

output
```
((5 + Cos[2*(c + d*x)])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(b*Sec[c + d
*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2032, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos^3(c+dx)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sqrt{\sec(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{bd\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2)),x]
```

output
```
-((Sqrt[Sec[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(b*d*Sqrt[b*Sec[
c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

method	result
default	$\frac{\tan(dx+c)+2\tan(dx+c)\sec(dx+c)^2}{3bd\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\mathrm{e}^{4i(dx+c)}}{24b\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}-\frac{3i\mathrm{e}^{2i(dx+c)}}{8b\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{}{8b\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{2}}$

input
```
int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d/(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)*(tan(d*x+c)+2*tan(d*x+c)*sec
(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\,b^{2}d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b
^2*d*sqrt(cos(d*x + c)))
```

Sympy [A] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \begin{cases} \frac{2\tan^{3}(c+dx)}{3d(b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)} + \frac{\tan(c+dx)}{d(b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(b\sec(c))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Piecewise((2*tan(c + d*x)**3/(3*d*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3
/2)) + tan(c + d*x)/(d*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/2)), Ne(d,
 0)), (x/((b*sec(c))**(3/2)*sec(c)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right.}{12\,b^{\frac{3}{2}}d}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))))/(b^(3/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{2\left(3\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}-4\right)}{3\,b^{\frac{3}{2}}d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
2/3*(3*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^2 - 4)/(b^(3/2)*d*(
1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))^3*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{12\,b^2\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*b^2*d*(1/
cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^{2}+3\right)}{3b^{2}d}$$

input
```
int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2 + 3))/(3*b**2*d)
```

3.175 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
3/8*x*sec(d*x+c)^(1/2)/b/(b*sec(d*x+c))^(1/2)+1/4*sin(d*x+c)/b/d/sec(d*x+c
)^(5/2)/(b*sec(d*x+c))^(1/2)+3/8*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)/(b*sec(d*
x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d(b\sec(c+dx))^{3/2}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sec[c + d*x]^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])
)/(32*d*(b*Sec[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2032, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos^4(c+dx)dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{b\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Sec[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[
c + d*x]*Sin[c + d*x])/(2*d)))/4))/(b*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result
default	$\frac{2\tan(dx+c)+3\tan(dx+c)\sec(dx+c)^2+3(dx+c)\sec(dx+c)^4}{8bd\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{7}{2}}}$
risch	$\frac{3\,e^{i(dx+c)}x}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+1)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{ie^{5i(dx+c)}}{64b(e^{2i(dx+c)}+1)\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{ie^{-i(d}}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx-}}$

input
```
int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b/d/(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2)*(2*tan(d*x+c)+3*tan(d*x+c)*s
ec(d*x+c)^2+3*(d*x+c)*sec(d*x+c)^4)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \left[\frac{\frac{2\left(2\cos(dx+c)^4+3\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}} - 3\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{}{c}}\right.}{16\,b^2d}\right.$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)) - 3*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))
*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/(b^2*d), 1/8*(
(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sq
rt(cos(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqr
t(b)*sqrt(cos(d*x + c)))))/(b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 114.57 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \begin{cases} \frac{3x\tan^4(c+dx)}{8(b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{5}{2}}(c+dx)} + \frac{3x\tan^2(c+dx)}{4(b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{5}{2}}(c+dx)} + \frac{3x}{8(b\sec(c+dx))^{\frac{3}{2}}\sec \\ \frac{x}{(b\sec(c))^{\frac{3}{2}}\sec^{\frac{5}{2}}(c)} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Piecewise((3*x*tan(c + d*x)**4/(8*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(5
/2)) + 3*x*tan(c + d*x)**2/(4*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(5/2))
 + 3*x/(8*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(5/2)) + 3*tan(c + d*x)**3
/(8*d*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(5/2)) + 5*tan(c + d*x)/(8*d*(
b*sec(c + d*x))**(3/2)*sec(c + d*x)**(5/2)), Ne(d, 0)), (x/((b*sec(c))**(3
/2)*sec(c)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{12\,dx + 12\,c + \sin(4\,dx + 4\,c) + 8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin(4\,dx + 4\,c), \mathrm{co}\right.\right.}{32\,b^{\frac{3}{2}}d}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))/(b^(3/2)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{2\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1} - 3i\log\left(i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)+3i\log\left(-\right.}{8\,b^{\frac{3}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/8*(2*(5*tan(1/2*d*x + 1/2*c)^7 - 3*tan(1/2*d*x + 1/2*c)^5 + 3*tan(1/2*d
*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^8 + 4*tan(1/
2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x + 1/2*c)^4 + 4*tan(1/2*d*x + 1/2*c)^2 + 1
) - 3*I*log(I*tan(1/2*d*x + 1/2*c) - 1) + 3*I*log(-I*tan(1/2*d*x + 1/2*c)
- 1))/(b^(3/2)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}\,\left(8\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)+12\,d\,x\right)}{32\,b^2\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
((b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x))/
(32*b^2*d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-2\cos(dx+c)\sin(dx+c)^{3}+5\cos(dx+c)\sin(dx+c)+3dx\right)}{8b^{2}d}$$

input
```
int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)**3 + 5*cos(c + d*x)*sin(c + d*x)
+ 3*d*x))/(8*b**2*d)
```

3.176 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2b^2d\sqrt{b\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}$$

output
```
1/2*arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/b^2/d/(b*sec(d*x+c))^(1/2)+1/2*se
c(d*x+c)^(5/2)*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{\sec(c+dx)}(\operatorname{arctanh}(\sin(c+dx))+\sec(c+dx)\tan(c+dx))}{2b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(11/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(
2*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(11/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/(b^2*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sec(dx+c)^{\frac{7}{2}}\left(\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)-1)-\cos(dx+c)^{3}\ln(-\cot(dx+c)+\csc(dx+c)+1)-\sin(dx+c)\cos(dx+c)\right)}{2b^{2}d\sqrt{b\sec(dx+c)}}$
risch	$-\frac{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{4i(dx+c)}\ln(\mathrm{e}^{i(dx+c)}-i)-\mathrm{e}^{4i(dx+c)}\ln(\mathrm{e}^{i(dx+c)}+i)+2i\mathrm{e}^{3i(dx+c)}-2i\mathrm{e}^{i(dx+c)}+2\,\mathrm{e}^{2i(dx+c)}\ln(\mathrm{e}^{i(dx+c)}-i)-2\cdots}{2b^{2}\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}$

input
```
int(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2/d*sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2)*(cos(d*x+c)^3*ln(-cot(d*x
+c)+csc(d*x+c)-1)-cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)-sin(d*x+c)*cos
(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.65

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \left[\frac{\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx}}\ldots}{\sqrt{co}\ldots}}{4b^3d\cos(dx+c)}\right.$$

$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)\cos(dx+c)-\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2b^3d\cos(dx+c)}\right]$$

input
```
integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^3*d*cos(d*x + c)), -1/2
*(sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(
d*x + c)))*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(11/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(66) = 132$.

Time = 0.20 (sec) , antiderivative size = 688, normalized size of antiderivative = 8.82

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((b^2*cos(4*d*x + 4*c)^2 + 4*
b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2
+ 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)*d)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{2\,b^{\frac{5}{2}}d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/2*(2*(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1) + log(tan(1/2*d*x + 1/2*c) + 1) - log
(tan(1/2*d*x + 1/2*c) - 1))/(b^(5/2)*d*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(11/2)/(b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(11/2)/(b/cos(c + d*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{d}{2}\right.\right.\right.}{2b^3d\left(\sin(dx+c)^2\right.}$$

input
```
int(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2 + log(tan((c + d*x)
/2) - 1) + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2 - log(tan((c + d*x)/2
) + 1) - sin(c + d*x)))/(2*b**3*d*(sin(c + d*x)**2 - 1))
```

3.177 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(b\sec(c+dx))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\sqrt{\sec(c+dx)}\int 1d(-\tan(c+dx))}{b^2d\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sin(dx+c)\sec(dx+c)^{\frac{5}{2}}\cos(dx+c)}{b^{2}d\sqrt{b\sec(dx+c)}}$	38
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{b^{2}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)}$	74

input
```
int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/d*sin(d*x+c)*sec(d*x+c)^(5/2)*cos(d*x+c)/(b*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{b^3d\sqrt{\cos(dx+c)}}$$

input
```
integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^3*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(31) = 62.

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3\right)d}$$

input
```
integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)
^2 + 2*b^3*cos(2*d*x + 2*c) + b^3)*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = -\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^{\frac{5}{2}}d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^(5/2)*d*sgn(cos(d*
x + c)))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1\mathrm{i}\right)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,1\mathrm{i}}{b^3\,d}$$

input
```
int((1/cos(c + d*x))^(9/2)/(b/cos(c + d*x))^(5/2),x)
```

output
```
((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*(b/cos(c + d*x))^(1/2)*(1/c
os(c + d*x))^(1/2)*1i)/(b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)b^3d}$$

input
```
int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/(cos(c + d*x)*b**3*d)
```

3.178 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\coth^{-1}(\sin(c+dx))\sec^{\frac{5}{2}}(c+dx)}{d(b\sec(c+dx))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(ArcCoth[Sin[c + d*x]]*Sec[c + d*x]^(5/2))/(d*(b*Sec[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\sec(c+dx)}\text{arctanh}(\sin(c+dx))}{b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{2 \sec(dx+c)^{\frac{3}{2}} \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c)) \cos(dx+c)}{b^{2} d \sqrt{b \sec(dx+c)}}$	49
risch	$-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, \ln(e^{i(dx+c)}-i)}{b^{2} \sqrt{\frac{b\, e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, \ln(e^{i(dx+c)}+i)}{b^{2} \sqrt{\frac{b\, e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}$	145

input
```
int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^2/d*sec(d*x+c)^(3/2)*arctanh(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)/(b*sec
(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \left[\frac{\log\left(-\frac{b\cos(dx+c)^{2}-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b}{\cos(dx+c)^{2}}\right)}{2\,b^{\frac{5}{2}}d},\ -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{b\sin(dx+c)}\right)}{b^{3}d}\right]$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(b^(5/2)*d), -sqrt(-b)*arctan(sq
rt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))/(b*sin(d*x + c)))/(b^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\mathrm{si}\right.}{2\,b^{\frac{5}{2}}d}$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(5/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{b^{\frac{5}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
(log(tan(1/2*d*x + 1/2*c) + 1) - log(tan(1/2*d*x + 1/2*c) - 1))/(b^(5/2)*d
*sgn(cos(d*x + c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{b^3d}$$

input
```
int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1) + log(tan((c + d*x)/2) + 1)))/(b**3
*d)
```

3.179 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}$$

output
```
x*sec(d*x+c)^(1/2)/b^2/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{x\sqrt{b\sec(c+dx)}}{b^3\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(x*Sqrt[b*Sec[c + d*x]])/(b^3*Sqrt[Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int 1dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(x*Sqrt[Sec[c + d*x]])/(b^2*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
orering	$\frac{x\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c))^{\frac{5}{2}}}$	21
default	$\frac{(dx+c)\sqrt{\sec(dx+c)}}{b^2 d\sqrt{b\sec(dx+c)}}$	31
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, x}{b^2\sqrt{\frac{b\, e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	57

input
```
int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{2\,b^3 d}\right.$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + 2*b*cos(d*x + c)^2 - b)/(b^3*d), arctan(sqrt(b/cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(b^(5/2)*d)]
```

Sympy [A] (verification not implemented)

Time = 51.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{x\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}$$

input
```
integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
x*sec(c + d*x)**(5/2)/(b*sec(c + d*x))**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}d}$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(5/2)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = -\frac{-i\log\left(i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)+i\log\left(-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{b^{\frac{5}{2}}d\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-(-I*log(I*tan(1/2*d*x + 1/2*c) - 1) + I*log(-I*tan(1/2*d*x + 1/2*c) - 1))
/(b^(5/2)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{x\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{b^3\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(5/2),x)
```

output
```
(x*(b/cos(c + d*x))^(1/2))/(b^3*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,x}{b^3}$$

input
```
int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*x)/b**3
```

3.180 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\tan(dx+c)}{b^2 d\sqrt{\sec(dx+c)}\,\sqrt{b\sec(dx+c)}}$	32
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{i(dx+c)}}{2b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{2b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	140

input
```
int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/d/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)*tan(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b^3d}$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d)
```

Sympy [A] (verification not implemented)

Time = 19.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \begin{cases} \frac{\tan(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(b\sec(c+dx))^{\frac{5}{2}}} & \text{for } d\neq 0 \\ \frac{x\sec^{\frac{3}{2}}(c)}{(b\sec(c))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Piecewise((tan(c + d*x)*sec(c + d*x)**(3/2)/(d*(b*sec(c + d*x))**(5/2)), N
e(d, 0)), (x*sec(c)**(3/2)/(b*sec(c))**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sin(dx+c)}{b^{\frac{5}{2}}d}$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(b^(5/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2}{b^{\frac{5}{2}}d\left(\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\operatorname{sgn}(\cos(dx+c))}$$

input
```
integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
2/(b^(5/2)*d*(1/tan(1/2*d*x + 1/2*c) + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x +
 c)))
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sin(2\,c+2\,d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{2\,b^3\,d}$$

input
```
int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(5/2),x)
```

output
```
(sin(2*c + 2*d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2))/(2*b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sin(dx+c)}{b^3d}$$

input
```
int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x))/(b**3*d)
```

3.181 $\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
1/2*x*sec(d*x+c)^(1/2)/b^2/(b*sec(d*x+c))^(1/2)+1/2*sin(d*x+c)/b^2/d/sec(d
*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}(2(c+dx)+\sin(2(c+dx)))}{4b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Sec[
c + d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos^2(c+dx)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(b^2*Sqrt[b
*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\tan(dx+c)+(dx+c)\sec(dx+c)^2}{2b^2d\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{3}{2}}}$	48
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{2b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	197

input
```
int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2/d/(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2)*(tan(d*x+c)+(d*x+c)*sec(d*
x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c) - \sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+\right.}{4\,b^3d}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*lo
g(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*co
s(d*x + c)^2 - b))/(b^3*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*s
in(d*x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sq
rt(cos(d*x + c)))))/(b^3*d)]
```

Sympy [A] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \begin{cases} \frac{x\tan^2(c+dx)\sqrt{\sec(c+dx)}}{2(b\sec(c+dx))^{\frac{5}{2}}} + \frac{x\sqrt{\sec(c+dx)}}{2(b\sec(c+dx))^{\frac{5}{2}}} + \frac{\tan(c+dx)\sqrt{\sec(c+dx)}}{2d(b\sec(c+dx))^{\frac{5}{2}}} & \text{for } d \neq 0 \\ \frac{x\sqrt{\sec(c)}}{(b\sec(c))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Piecewise((x*tan(c + d*x)**2*sqrt(sec(c + d*x))/(2*(b*sec(c + d*x))**(5/2)
) + x*sqrt(sec(c + d*x))/(2*(b*sec(c + d*x))**(5/2)) + tan(c + d*x)*sqrt(s
ec(c + d*x))/(2*d*(b*sec(c + d*x))**(5/2)), Ne(d, 0)), (x*sqrt(sec(c))/(b*
sec(c))**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,b^{\frac{5}{2}}d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(5/2)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = -\frac{\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}-i\,\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+i\,\log\left(-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{2\,b^{\frac{5}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/2*(2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^4 + 2*tan(1/2*d*x + 1/2*c)^2 + 1) - I*log(I*tan(1/2*d*x + 1/2*c) - 1)
+ I*log(-I*tan(1/2*d*x + 1/2*c) - 1))/(b^(5/2)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}\,(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+4\,d\,x\,\cos(c+d\,x))}{8\,b^3\,d\,\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int((1/cos(c + d*x))^(1/2)/(b/cos(c + d*x))^(5/2),x)
```

output
```
((b/cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x) + 4*d*x*cos(c + d
*x)))/(8*b^3*d*cos(c + d*x)*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,(\cos(dx+c)\sin(dx+c)+dx)}{2b^3d}$$

input
```
int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x) + d*x))/(2*b**3*d)
```

3.182 $\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}} - \frac{\sqrt{\sec(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

output
```
sec(d*x+c)^(1/2)*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)-1/3*sec(d*x+c)^(1/2
)*sin(d*x+c)^3/b^2/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \frac{(5+\cos(2(c+dx)))\sqrt{\sec(c+dx)}\sin(c+dx)}{6b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2)),x]
```

output
```
((5 + Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*b^2*d*Sqrt[b*S
ec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2032, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos^3(c+dx)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sqrt{\sec(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{b^2d\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2)),x]
```

output
```
-((Sqrt[Sec[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(b^2*d*Sqrt[b*Se
c[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

method	result
default	$\frac{\tan(dx+c)+2\tan(dx+c)\sec(dx+c)^2}{3b^2 d\sqrt{b\sec(dx+c)}\,\sec(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i e^{4i(dx+c)}}{24b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)d}-\frac{3i e^{2i(dx+c)}}{8b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)d}+\frac{}{8b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$

input
```
int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b^2/d/(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2)*(tan(d*x+c)+2*tan(d*x+c)*s
ec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \frac{\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{3\,b^{3}d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b
^3*d*sqrt(cos(d*x + c)))
```

Sympy [A] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \begin{cases} \frac{2\tan^{3}(c+dx)}{3d(b\sec(c+dx))^{\frac{5}{2}}\sqrt{\sec(c+dx)}} + \frac{\tan(c+dx)}{d(b\sec(c+dx))^{\frac{5}{2}}\sqrt{\sec(c+dx)}} & \text{for } d \neq 0 \\ \frac{x}{(b\sec(c))^{\frac{5}{2}}\sqrt{\sec(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Piecewise((2*tan(c + d*x)**3/(3*d*(b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x
))) + tan(c + d*x)/(d*(b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x))), Ne(d, 0
)), (x/((b*sec(c))**(5/2)*sqrt(sec(c))), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right.\right.}{12\,b^{\frac{5}{2}}d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))))/(b^(5/2)*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\tan\left(\right.\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\right.}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^5 + 2*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^6 + 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d
*x + 1/2*c)^2 + 1)*b^(5/2)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \frac{(9\sin(c+d\,x)+\sin(3\,c+3\,d\,x))\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{12\,b^3\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*b^3*d*(1/
cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^{2}+3\right)}{3b^{3}d}$$

input
```
int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2 + 3))/(3*b**3*d)
```

3.183 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

output
```
3/8*x*sec(d*x+c)^(1/2)/b^2/(b*sec(d*x+c))^(1/2)+1/4*sin(d*x+c)/b^2/d/sec(d
*x+c)^(5/2)/(b*sec(d*x+c))^(1/2)+3/8*sin(d*x+c)/b^2/d/sec(d*x+c)^(1/2)/(b*
sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])
)/(32*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2032, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\sec(c+dx)}\int \cos^4(c+dx)dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\sec(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{b^2\sqrt{b\sec(c+dx)}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Sec[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[
c + d*x]*Sin[c + d*x])/(2*d)))/4))/(b^2*Sqrt[b*Sec[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

method	result
default	$\frac{2\tan(dx+c)+3\tan(dx+c)\sec(dx+c)^2+3(dx+c)\sec(dx+c)^4}{8b^2d\sec(dx+c)^{\frac{7}{2}}\sqrt{b\sec(dx+c)}}$
risch	$\frac{3\,\mathrm{e}^{i(dx+c)}x}{8b^2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\mathrm{e}^{5i(dx+c)}}{64b^2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{i\mathrm{e}^{-}}{8b^2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i}\right.}$

input
```
int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b^2/d/sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2)*(2*tan(d*x+c)+3*tan(d*x+c)
*sec(d*x+c)^2+3*(d*x+c)*sec(d*x+c)^4)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx=\left[\frac{\frac{2\left(2\cos(dx+c)^4+3\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}-3\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{}{\mathrm{c}}}\right.}{16\,b^3d}\right.$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)) - 3*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))
*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/(b^3*d), 1/8*(
(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sq
rt(cos(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqr
t(b)*sqrt(cos(d*x + c)))))/(b^3*d)]
```

Sympy [A] (verification not implemented)

Time = 109.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \begin{cases} \frac{3x\tan^4(c+dx)}{8(b\sec(c+dx))^{\frac{5}{2}}\sec^{\frac{3}{2}}(c+dx)} + \frac{3x\tan^2(c+dx)}{4(b\sec(c+dx))^{\frac{5}{2}}\sec^{\frac{3}{2}}(c+dx)} + \frac{3x}{8(b\sec(c+dx))^{\frac{5}{2}}\sec \\ \frac{x}{(b\sec(c))^{\frac{5}{2}}\sec^{\frac{3}{2}}(c)} \end{cases}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Piecewise((3*x*tan(c + d*x)**4/(8*(b*sec(c + d*x))**(5/2)*sec(c + d*x)**(3
/2)) + 3*x*tan(c + d*x)**2/(4*(b*sec(c + d*x))**(5/2)*sec(c + d*x)**(3/2))
 + 3*x/(8*(b*sec(c + d*x))**(5/2)*sec(c + d*x)**(3/2)) + 3*tan(c + d*x)**3
/(8*d*(b*sec(c + d*x))**(5/2)*sec(c + d*x)**(3/2)) + 5*tan(c + d*x)/(8*d*(
b*sec(c + d*x))**(5/2)*sec(c + d*x)**(3/2)), Ne(d, 0)), (x/((b*sec(c))**(5
/2)*sec(c)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \frac{12\,dx + 12\,c + \sin(4\,dx + 4\,c) + 8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin(4\,dx + 4\,c), \mathrm{co}\right.\right.}{32\,b^{\frac{5}{2}}d}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))/(b^(5/2)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{\frac{2\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}-3i\,\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+3i\,\log\left(-\right.}{8\,b^{\frac{5}{2}}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/8*(2*(5*tan(1/2*d*x + 1/2*c)^7 - 3*tan(1/2*d*x + 1/2*c)^5 + 3*tan(1/2*d
*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^8 + 4*tan(1/
2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x + 1/2*c)^4 + 4*tan(1/2*d*x + 1/2*c)^2 + 1
) - 3*I*log(I*tan(1/2*d*x + 1/2*c) - 1) + 3*I*log(-I*tan(1/2*d*x + 1/2*c)
- 1))/(b^(5/2)*d*sgn(cos(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 9.78 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}\,\left(8\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)+12\,d\,x\right)}{32\,b^3\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

input
```
int(1/((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
((b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x))/
(32*b^3*d*(1/cos(c + d*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^{3}+5\cos\left(dx+c\right)\sin\left(dx+c\right)+3dx\right)}{8b^{3}d}$$

input
```
int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)**3 + 5*cos(c + d*x)*sin(c + d*x)
+ 3*d*x))/(8*b**3*d)
```

3.184 $\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{3\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sec^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{7bd}$$

input
```
Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Csc[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2]*(b*Sec[c
 + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(7*b*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{7/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{b^2}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{7/3}}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3
)*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(dx+c)^2 (b\sec(dx+c))^{\frac{1}{3}} dx$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F]

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int \sqrt[3]{b\sec(c+dx)}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((b/cos(c + d*x))^(1/3)/cos(c + d*x)^2,x)
```

output
```
int((b/cos(c + d*x))^(1/3)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \sec(dx+c)^{\frac{7}{3}}\,dx\right)$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*sec(c + d*x)**2,x)
```

3.185 $\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

output
```
3*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+c
)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{4bd}$$

input
```
Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{4/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{4/3}}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3
)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec (dx + c) (b \sec (dx + c))^{\frac{1}{3}} dx$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int \sqrt[3]{b\sec(c+dx)}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((b/cos(c + d*x))^(1/3)/cos(c + d*x),x)
```

output
```
int((b/cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=b^{\frac{1}{3}}\left(\int \sec(dx+c)^{\frac{4}{3}}\,dx\right)$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*sec(c + d*x),x)
```

3.186 $\int \sqrt[3]{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \sqrt[3]{b\sec(c+dx)}\,dx = -\frac{3b\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/2*b*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c)
)^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \sqrt[3]{b\sec(c+dx)}\,dx = \frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sqrt{-\tan^2(c+dx)}}{d}$$

input
```
Integrate[(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\sqrt[3]{\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

input
```
Int[(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*
(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int (b\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
int((b*sec(d*x+c))^(1/3),x)
```

output
```
int((b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{b\sec(c+dx)}\,dx = \int \sqrt[3]{b\sec{(c+dx)}}\,dx$$

input
```
integrate((b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec{(dx+c)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec{(dx+c)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sec(c+dx)}\,dx=\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{1/3}\,dx$$

input
```
int((b/cos(c + d*x))^(1/3),x)
```

output
```
int((b/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{b\sec(c+dx)}\,dx=b^{\frac{1}{3}}\left(\int \sec\left(dx+c\right)^{\frac{1}{3}}dx\right)$$

input
```
int((b*sec(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3),x)
```

3.187 $\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = -\frac{3b^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\sec(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/5*b^2*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x
+c))^(5/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = -\frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{2d(b\sec(c+dx))^{2/3}}$$

input
```
Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d*x]^2]*Sqrt[
-Tan[c + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt[3]{b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 2030$$

$$b\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \ 4259$$

$$b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}dx$$

$$\Big\downarrow \ 3042$$

$$b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{2/3}dx$$

$$\Big\downarrow \ 3122$$

$$-\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

input
```
Int[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5
*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Sympy [F]

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int \sqrt[3]{b\sec(c+dx)}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((b*sec(c + d*x))**(1/3)*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int \cos(c+d\,x)\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}\,dx$$

input
```
int(cos(c + d*x)*(b/cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)*(b/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \sec(dx+c)^{\frac{1}{3}}\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*cos(c + d*x),x)
```

3.188 $\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = -\frac{3b^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\sec(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/8*b^3*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(8/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(2(c+dx))}{10d\sqrt{-\tan^2(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[c + d*x]^2]*(b*Sec[c + d*x])^(1/3
)*Sin[2*(c + d*x)])/(10*d*Sqrt[-Tan[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{4259}$$

$$b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \left(\frac{\cos(c+dx)}{b}\right)^{5/3}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{5/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{8/3}}$$

input
```
Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b^3*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*
d*(b*Sec[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \cos(dx+c)^2 (b\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Sympy [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int \sqrt[3]{b\sec(c+dx)}\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((b*sec(c + d*x))**(1/3)*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx=\int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int \cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}\,dx$$

input
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \sec(dx+c)^{\frac{1}{3}}\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*cos(c + d*x)**2,x)
```

3.189 $\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)(b\sec(c+dx))^{7/3}\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(7/3)*sin(d*
x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \frac{3\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\sec^2(c+dx)\right)(b\sec(c+dx))^{7/3}\sqrt{-\tan^2(c+dx)}}{10bd}$$

input
```
Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*Csc[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(10*b*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{10/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}dx}{b^2}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{10/3}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{10/3}}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/
3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec (dx+c)^2 (b\sec (dx+c))^{\frac{4}{3}} dx$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^3, x)
```

Sympy [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(c+dx))^{\frac{4}{3}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((b*sec(c + d*x))**(4/3)*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((b/cos(c + d*x))^(4/3)/cos(c + d*x)^2,x)
```

output
```
int((b/cos(c + d*x))^(4/3)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = b^{\frac{4}{3}}\left(\int \sec(dx+c)^{\frac{10}{3}}\,dx\right)$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*sec(c + d*x)**3,x)*b
```

3.190 $\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx=\frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*x
+c)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx=\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sec^2(c+dx)\right)(b\sec(c+dx))^{7/3}\sqrt{-\tan^2(c+dx)}}{7bd}$$

input
```
Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2]*(b*Sec[c
 + d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(7*b*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{7/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{b}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{7/3}}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3
)*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^2, x)
```

Sympy [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(c+dx))^{\frac{4}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((b*sec(c + d*x))**(4/3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((b/cos(c + d*x))^(4/3)/cos(c + d*x),x)
```

output
```
int((b/cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx = b^{\frac{4}{3}}\left(\int \sec(dx+c)^{\frac{7}{3}}\,dx\right)$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*sec(c + d*x)**2,x)*b
```

3.191 $\int (b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int (b\sec(c+dx))^{4/3}\,dx=\frac{3b\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

output
```
3*b*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x
+c)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (b\sec(c+dx))^{4/3}\,dx=\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{4d}$$

input
```
Integrate[(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{4/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1
/3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (b\sec (dx+c))^{\frac{4}{3}}dx$$

input
```
int((b*sec(d*x+c))^(4/3),x)
```

output
```
int((b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int (b\sec(c+dx))^{4/3}\,dx=\int (b\sec (dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c), x)
```

Sympy [F]

$$\int (b\sec(c+dx))^{4/3}\,dx = \int (b\sec{(c+dx)})^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((b*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int (b\sec(c+dx))^{4/3}\,dx = \int (b\sec{(dx+c)})^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int (b\sec(c+dx))^{4/3}\,dx = \int (b\sec{(dx+c)})^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^{4/3}\,dx = \int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{4/3}\,dx$$

input
```
int((b/cos(c + d*x))^(4/3),x)
```

output
```
int((b/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int (b\sec(c+dx))^{4/3}\,dx = b^{\frac{4}{3}}\left(\int \sec\left(dx+c\right)^{\frac{4}{3}}dx\right)$$

input
```
int((b*sec(d*x+c))^(4/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*sec(c + d*x),x)*b
```

3.192 $\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = -\frac{3b^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/2*b^2*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = \frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sqrt{-\tan^2(c+dx)}}{d}$$

input
```
Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*b*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2]*(b*Sec[
c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \sqrt[3]{b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\Big\downarrow \text{4259}$$

$$b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}dx$$

$$\Big\downarrow \text{3042}$$

$$b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\sqrt[3]{\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b^2\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

input
```
Int[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*b^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*
d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \cos(dx + c) (b \sec(dx + c))^{\frac{4}{3}} dx$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int \cos(c+d\,x)\,\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

input
```
int(cos(c + d*x)*(b/cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)*(b/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = b^{\frac{4}{3}}\left(\int \sec(dx+c)^{\frac{4}{3}}\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x),x)*b
```

3.193 $\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = -\frac{3b^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\sec(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/5*b^3*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x
+c))^(5/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = -\frac{3b^2\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{2d(b\sec(c+dx))^{2/3}}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d*x]^2]*Sqr
t[-Tan[c + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 4259}$$

$$b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{2/3}dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3b^3\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{5/3}}$$

input
```
Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*b^3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5
*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \cos(dx+c)^2 (b\sec(dx+c))^{\frac{4}{3}}\, dx$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*b*cos(d*x + c)^2*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int \cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

input
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = b^{\frac{4}{3}}\left(\int \sec(dx+c)^{\frac{4}{3}}\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**(1/3)*cos(c + d*x)**2*sec(c + d*x),x)*b
```

3.194 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(2/3)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{3\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{5bd}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Csc[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2]*(b*Sec[c
 + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(5*b*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{5/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b^2}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{5/3}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{5/3}}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3
)*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sec (dx+c)^2}{(b\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)/b, x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec^2\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int \sec(dx+c)^{\frac{5}{3}}\,dx}{b^{\frac{1}{3}}}$$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**2/sec(c + d*x)**(1/3),x)/b**(1/3)
```

3.195 $\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1
/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{2bd}$$

input
```
Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*b*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int (b\sec(c+dx))^{2/3}dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}$$
$$\Big\downarrow \text{4259}$$
$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{2/3}}dx}{b}$$
$$\Big\downarrow \text{3122}$$
$$-\frac{3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*S
ec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)/b, x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int \sec\left(dx+c\right)^{\frac{2}{3}}dx}{b^{\frac{1}{3}}}$$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)/sec(c + d*x)**(1/3),x)/b**(1/3)
```

3.196 $\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/4*b*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c)
)^(4/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{d\sqrt[3]{b\sec(c+dx)}}$$

input
```
Integrate[(b*Sec[c + d*x])^(-1/3),x]
```

output
```
(-3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2]*Sqrt[-T
an[c + d*x]^2])/(d*(b*Sec[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \sqrt[3]{\frac{\cos(c+dx)}{b}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \sqrt[3]{\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

input
```
Int[(b*Sec[c + d*x])^(-1/3),x]
```

output
```
(-3*b*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*
(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{(b\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(1/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(1/(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)/(b*sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((b*sec(c + d*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{1}{\left(b\sec{(dx+c)}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{1}{\left(b\sec{(dx+c)}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(b/cos(c + d*x))^(1/3),x)
```

output
```
int(1/(b/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(1/sec(c + d*x)**(1/3),x)/b**(1/3)
```

3.197 $\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3b^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\sec(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/7*b^2*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x
+c))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{4d(b\sec(c+dx))^{4/3}}$$

input
```
Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2]*Sqrt[
-Tan[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{4259}$$

$$b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\cos(c+dx)}{b}\right)^{4/3}dx$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{4/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

input
```
Int[Cos[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7
*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\cos (dx + c)}{(b \sec (dx + c))^{\frac{1}{3}}} dx$$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)/(b*sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(cos(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{1/3}}\,dx$$

input
```
int(cos(c + d*x)/(b/cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)/(b/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx=\frac{\int \frac{\cos(dx+c)}{\sec(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(c + d*x)/sec(c + d*x)**(1/3),x)/b**(1/3)
```

3.198 $\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3b^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\sec(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/10*b^3*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x
+c))^(10/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3b^2\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{7d(b\sec(c+dx))^{7/3}}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d*x]^2]*Sq
rt[-Tan[c + d*x]^2])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt[3]{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{4259}$$

$$b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\cos(c+dx)}{b}\right)^{7/3}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{7/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{10/3}}$$

input
```
Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*b^3*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10
*d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\cos (dx+c)^2}{(b\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)^2/(b*sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos^2\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int \frac{\cos(dx+c)^2}{\sec(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(c + d*x)**2/sec(c + d*x)**(1/3),x)/b**(1/3)
```

3.199 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^
(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sec^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sqrt{-\mathrm{ta}}}{2b^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*b^2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{2/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^2}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{2/3}}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b
*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx=\int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx=\frac{\int \sec\left(dx+c\right)^{\frac{2}{3}}dx}{b^{\frac{4}{3}}}$$

input
```
int(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(c + d*x)/sec(c + d*x)**(1/3),x)/(b**(1/3)*b)
```

3.200 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/4*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))^
(4/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{bd\sqrt[3]{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2]*Sqrt[-T
an[c + d*x]^2])/(b*d*(b*Sec[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt[3]{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \sqrt[3]{\frac{\cos(c+dx)}{b}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \sqrt[3]{\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{4/3}}$$

input
```
Int[Sec[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b
*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sec (dx+c)}{(b\sec (dx+c))^{\frac{4}{3}}}dx$$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)), x)
```

Reduce [F]

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(1/sec(c + d*x)**(1/3),x)/(b**(1/3)*b)
```

3.201 $\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\sec(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/7*b*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c
))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{4d(b\sec(c+dx))^{4/3}}$$

input
```
Integrate[(b*Sec[c + d*x])^(-4/3),x]
```

output
```
(-3*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2]*Sqrt[-T
an[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$
$$\Big\downarrow \text{4259}$$
$$\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\cos(c+dx)}{b}\right)^{4/3}dx$$
$$\Big\downarrow \text{3042}$$
$$\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{4/3}dx$$
$$\Big\downarrow \text{3122}$$
$$-\frac{3b\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{7/3}}$$

input
```
Int[(b*Sec[c + d*x])^(-4/3),x]
```

output
```
(-3*b*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d
*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{1}{(b\sec(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(1/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(1/(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sec{(c+dx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((b*sec(c + d*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sec{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sec{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/(b/cos(c + d*x))^(4/3),x)
```

output
```
int(1/(b/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\sec(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(1/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(1/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)/(b**(1/3)*b)
```

3.202 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3b^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\sec(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/10*b^2*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x
+c))^(10/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{7d(b\sec(c+dx))^{7/3}}$$

input
```
Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*b*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d*x]^2]*Sqrt
[-Tan[c + d*x]^2])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{4259}$$

$$b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\cos(c+dx)}{b}\right)^{7/3}dx$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{7/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{10/3}}$$

input
```
Int[Cos[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*b^2*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10
*d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\cos (dx + c)}{(b \sec (dx + c))^{\frac{4}{3}}} dx$$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral(cos(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(cos(c + d*x)/(b/cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)/(b/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{\cos(dx+c)}{\sec(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(cos(c + d*x)/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)/(b**(1/3)*b)
```

3.203 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3b^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13d(b\sec(c+dx))^{13/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3/13*b^3*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d
*x+c))^(13/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3b^2\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{10d(b\sec(c+dx))^{10/3}}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-5/3, 1/2, -2/3, Sec[c + d*x]^2]*Sq
rt[-Tan[c + d*x]^2])/(10*d*(b*Sec[c + d*x])^(10/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{4259}$$

$$b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\cos(c+dx)}{b}\right)^{10/3}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{10/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b^3\sin(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)}{13d\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{13/3}}$$

input
```
Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*b^3*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
13*d*(b*Sec[c + d*x])^(13/3)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)^2/(b^2*sec(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\cos(c+d\,x)^2}{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^2/(b/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{\cos(dx+c)^2}{\sec(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(cos(c + d*x)**2/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)/(b**(1/3)*b)
```

3.204 $\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx=\frac{3b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1-3m),\frac{1}{6}(5-3m),\cos^2(c+dx)\right)\sec^m(c+dx)\sqrt[3]{b\sec(c+d}{d(1+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
3*b*hypergeom([1/2, -1/6-1/2*m],[5/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*(b*
sec(d*x+c))^(1/3)*sin(d*x+c)/d/(1+3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx=\frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{4}{3}+m\right),\frac{1}{2}\left(\frac{10}{3}+m\right),\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)(b}{d\left(\frac{4}{3}+m\right)}$$

input
```
Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (4/3 + m)/2, (10/3 + m)/2, Sec[c + d*
x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/
(d*(4/3 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{4/3}\sec^m(c+dx)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{b\sqrt[3]{b\sec(c+dx)}\int \sec^{m+\frac{4}{3}}(c+dx)dx}{\sqrt[3]{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx}{\sqrt[3]{\sec(c+dx)}}$$

$$\Big\downarrow \text{4259}$$

$$b\sqrt[3]{b\sec(c+dx)}\cos^{m+\frac{1}{3}}(c+dx)\sec^m(c+dx)\int \cos^{-m-\frac{4}{3}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\sqrt[3]{b\sec(c+dx)}\cos^{m+\frac{1}{3}}(c+dx)\sec^m(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-m-\frac{4}{3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-3m-1),\frac{1}{6}(5-3m),\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3),x]
```

output
```
(3*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c +
 d*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec (dx+c)^m (b\sec (dx+c))^{\frac{4}{3}} dx$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^m*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Giac [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c))^{\frac{4}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}\left(\frac{1}{\cos(c+d\,x)}\right)^m\,dx$$

input
```
int((b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m,x)
```

output
```
int((b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m, x)
```

Reduce [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = b^{\frac{4}{3}}\left(\int \sec(dx+c)^{m+\frac{1}{3}}\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**((3*m + 1)/3)*sec(c + d*x),x)*b
```

3.205 $\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1-3m),\frac{1}{6}(7-3m),\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c+}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*hypergeom([1/2, 1/6-1/2*m],[7/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*
(b*sec(d*x+c))^(2/3)*sin(d*x+c)/d/(1-3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{3}+m\right),\frac{1}{2}\left(\frac{8}{3}+m\right),\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\,s}{d\left(\frac{2}{3}+m\right)}$$

input
```
Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (2/3 + m)/2, (8/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(
d*(2/3 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{2/3}\sec^m(c+dx)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{(b\sec(c+dx))^{2/3}\int \sec^{m+\frac{2}{3}}(c+dx)dx}{\sec^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(b\sec(c+dx))^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx}{\sec^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{4259}$$

$$(b\sec(c+dx))^{2/3}\cos^{m+\frac{2}{3}}(c+dx)\sec^m(c+dx)\int \cos^{-m-\frac{2}{3}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$(b\sec(c+dx))^{2/3}\cos^{m+\frac{2}{3}}(c+dx)\sec^m(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-m-\frac{2}{3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1-3m),\frac{1}{6}(7-3m),\cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[Si
n[c + d*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec \left(dx+c\right)^{m} \left(b\sec \left(dx+c\right)\right)^{\frac{2}{3}}dx$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Sympy [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \int (b\sec(c+dx))^{\frac{2}{3}}\sec^m(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((b*sec(c + d*x))**(2/3)*sec(c + d*x)**m, x)
```

Maxima [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Giac [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c))^{\frac{2}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^{2/3}\left(\frac{1}{\cos(c+d\,x)}\right)^m\,dx$$

input
```
int((b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m,x)
```

output
```
int((b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m, x)
```

Reduce [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = b^{\frac{2}{3}}\left(\int \sec(dx+c)^{m+\frac{2}{3}}\,dx\right)$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)
```

output
```
b**(2/3)*int(sec(c + d*x)**((3*m + 2)/3),x)
```

3.206 $\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2-3m),\frac{1}{6}(8-3m),\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sin(c}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*hypergeom([1/2, 1/3-1/2*m],[4/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*
(b*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(2-3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{3}+m\right),\frac{1}{2}\left(\frac{7}{3}+m\right),\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt[3]{b\sec(c+dx}}{d\left(\frac{1}{3}+m\right)}$$

input
```
Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (1/3 + m)/2, (7/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(
d*(1/3 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{m+\frac{1}{3}}(c+dx)dx}{\sqrt[3]{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{b\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx}{\sqrt[3]{\sec(c+dx)}}$$

$$\Big\downarrow \text{4259}$$

$$\sqrt[3]{b\sec(c+dx)}\cos^{m+\frac{1}{3}}(c+dx)\sec^m(c+dx)\int \cos^{-m-\frac{1}{3}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{b\sec(c+dx)}\cos^{m+\frac{1}{3}}(c+dx)\sec^m(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-m-\frac{1}{3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\sec^{m-1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2-3m),\frac{1}{6}(8-3m),\cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(2 - 3*m)*Sqrt[Si
n[c + d*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(dx+c)^m (b\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Sympy [F]

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int \sqrt[3]{b\sec(c+dx)}\sec^m(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x)**m, x)
```

Maxima [F]

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Giac [F]

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int (b\sec(dx+c))^{\frac{1}{3}}\sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}\left(\frac{1}{\cos(c+d\,x)}\right)^m\,dx$$

input
```
int((b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m,x)
```

output
```
int((b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m, x)
```

Reduce [F]

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \sec(dx+c)^{m+\frac{1}{3}}\,dx\right)$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(sec(c + d*x)**((3*m + 1)/3),x)
```

3.207 $\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4-3m),\frac{1}{6}(10-3m),\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(4-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*hypergeom([1/2, 2/3-1/2*m],[5/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*
sin(d*x+c)/d/(4-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{3}+m\right),\frac{1}{2}\left(\frac{5}{3}+m\right),\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt{-\tan^2(c+}{d\left(-\frac{1}{3}+m\right)\sqrt[3]{b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/3 + m)/2, (5/3 + m)/2, Sec[c + d*
x]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1/3 + m)*(b*Sec[c
+ d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \ 2034$$

$$\frac{\sqrt[3]{\sec(c+dx)}\int \sec^{m-\frac{1}{3}}(c+dx)dx}{\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt[3]{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}dx}{\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \ 4259$$

$$\frac{\cos^{m+\frac{2}{3}}(c+dx)\sec^{m+1}(c+dx)\int \cos^{\frac{1}{3}-m}(c+dx)dx}{\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cos^{m+\frac{2}{3}}(c+dx)\sec^{m+1}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{\frac{1}{3}-m}dx}{\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \ 3122$$

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4-3m),\frac{1}{6}(10-3m),\cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec^m\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^m}{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(1/3),x)
```

output
```
int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int \frac{\sec(dx+c)^m}{\sec(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(c + d*x)**m/sec(c + d*x)**(1/3),x)/b**(1/3)
```

3.208 $\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5-3m),\frac{1}{6}(11-3m),\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(5-3m)(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*hypergeom([1/2, 5/6-1/2*m],[11/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)
*sin(d*x+c)/d/(5-3*m)/(b*sec(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{2}{3}+m\right),\frac{1}{2}\left(\frac{4}{3}+m\right),\sec^2(c+dx)\right)\sec^{-1\cdots}}{d\left(-\frac{2}{3}+m\right)(b\sec(c+dx))^{2/3}}$$

input
```
Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(2/3),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (-2/3 + m)/2, (4/3 + m)/2, Sec[c + d*
x]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-2/3 + m)*(b*Sec[c
+ d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{m-\frac{2}{3}}(c+dx)dx}{(b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^{\frac{2}{3}}(c+dx)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}dx}{(b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\cos^{m+\frac{1}{3}}(c+dx)\sec^{m+1}(c+dx)\int \cos^{\frac{2}{3}-m}(c+dx)dx}{(b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{m+\frac{1}{3}}(c+dx)\sec^{m+1}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{\frac{2}{3}-m}dx}{(b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5-3m),\frac{1}{6}(11-3m),\cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}(b\sec(c+dx))^{2/3}}$$

input
```
Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(2/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[S
in[c + d*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sec (dx+c)^m}{(b\sec (dx+c))^{\frac{2}{3}}}dx$$

input
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx=\int \frac{\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx=\int \frac{\sec^m\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(2/3),x)
```

output
```
Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx=\int \frac{\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^m}{\left(\frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(2/3),x)
```

output
```
int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int \frac{\sec(dx+c)^m}{\sec(dx+c)^{\frac{2}{3}}}dx}{b^{\frac{2}{3}}}$$

input
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(c + d*x)**m/sec(c + d*x)**(2/3),x)/b**(2/3)
```

3.209 $\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7-3m),\frac{1}{6}(13-3m),\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{bd(7-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*hypergeom([1/2, 7/6-1/2*m],[13/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-2+m)
*sin(d*x+c)/b/d/(7-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{4}{3}+m\right),\frac{1}{2}\left(\frac{2}{3}+m\right),\sec^2(c+dx)\right)\sec^{-1}}{d\left(-\frac{4}{3}+m\right)(b\sec(c+dx))^{4/3}}$$

input
```
Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (-4/3 + m)/2, (2/3 + m)/2, Sec[c + d*
x]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-4/3 + m)*(b*Sec[c
+ d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\sec(c+dx)}\int \sec^{m-\frac{4}{3}}(c+dx)dx}{b\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}dx}{b\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\cos^{m+\frac{2}{3}}(c+dx)\sec^{m+1}(c+dx)\int \cos^{\frac{4}{3}-m}(c+dx)dx}{b\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{m+\frac{2}{3}}(c+dx)\sec^{m+1}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{\frac{4}{3}-m}dx}{b\sqrt[3]{b\sec(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\sec^{m-2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7-3m),\frac{1}{6}(13-3m),\cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(4/3),x]
```

output
```
(-3*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(7 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt
[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{\sec (dx + c)^m}{(b \sec (dx + c))^{\frac{4}{3}}} dx$$

input
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*sec(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec^m\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec\left(dx+c\right)^m}{\left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)^m}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^m}{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(4/3),x)
```

output
```
int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{\sec(dx+c)^m}{\sec(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(c + d*x)**m/(sec(c + d*x)**(1/3)*sec(c + d*x)),x)/(b**(1/3)*b)
```

3.210 $\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-m-n),\frac{1}{2}(3-m-n),\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^n\,\mathrm{s}}{d(1-m-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-hypergeom([1/2, 1/2-1/2*m-1/2*n],[3/2-1/2*m-1/2*n],cos(d*x+c)^2)*sec(d*x+
c)^(-1+m)*(b*sec(d*x+c))^n*sin(d*x+c)/d/(1-m-n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n}{2},\frac{1}{2}(2+m+n),\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^n\sqrt{}}{d(m+n)}$$

input
```
Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (m + n)/2, (2 + m + n)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(m
 + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \sec^{m+n}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \csc\left(c+dx+\frac{\pi}{2}\right)^{m+n}dx$$

$$\Big\downarrow \text{ 4259}$$

$$\sec^m(c+dx)(b\sec(c+dx))^n\cos^{m+n}(c+dx)\int \cos^{-m-n}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sec^m(c+dx)(b\sec(c+dx))^n\cos^{m+n}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-m-n}dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-n+1),\frac{1}{2}(-m-n+3),\cos^2(c+d\right.}{d(-m-n+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
```

output
```
-((Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*Se
c[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[S
in[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(dx+c)^m (b\sec(dx+c))^n \, dx$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Sympy [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(c+dx))^n \sec^m(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n*sec(c + d*x)**m, x)
```

Maxima [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Giac [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^m\,dx$$

input
```
integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^n \left(\frac{1}{\cos(c+d\,x)}\right)^m\,dx$$

input
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^m,x)
```

output
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^m, x)
```

Reduce [F]

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^{m+n}\,dx\right)$$

input
```
int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**(m + n),x)
```

3.211 $\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-n),\frac{1-n}{2},\cos^2(c+dx)\right)(b\sec(c+dx))^{1+n}\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

output
```
hypergeom([1/2, -1/2-1/2*n],[1/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(1+n)
*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sec^2(c+dx)\right)\sec(c+dx)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(2+n)}$$

input
```
Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sec[c + d*x]^2]
*Sec[c + d*x]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(2 + n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\sec(c+dx))^{n+2}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx}{b^2}$$

$$\Big\downarrow \text{ 4259}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\cos(c+dx)}{b}\right)^{-n-2}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{-n-2}dx}{b^2}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\sin(c+dx)(b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-n-1),\frac{1-n}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

output
```
(Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c +
d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(dx+c)^2 (b\sec(dx+c))^n\,dx$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(c+dx))^n \sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\cos(c+d\,x)^2}\,dx$$

input
```
int((b/cos(c + d*x))^n/cos(c + d*x)^2,x)
```

output
```
int((b/cos(c + d*x))^n/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^n\sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**n*sec(c + d*x)**2,x)
```

3.212 $\int \sec(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

output
```
hypergeom([1/2, -1/2*n],[1-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c
)/d/n/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(1+n)}$$

input
```
Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n,x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c + d*x]^2]
*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1 + n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2030, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\sec(c+dx))^{n+1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx}{b}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\cos(c+dx)}{b}\right)^{-n-1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{-n-1}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(c+dx)(b\sec(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]*(b*Sec[c + d*x])^n,x]
```

output
```
(Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x]
)^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec (dx + c) (b \sec (dx + c))^n dx$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(c+dx))^n\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\cos(c+d\,x)}\,dx$$

input
```
int((b/cos(c + d*x))^n/cos(c + d*x),x)
```

output
```
int((b/cos(c + d*x))^n/cos(c + d*x), x)
```

Reduce [F]

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = b^n \left(\int \sec(dx+c)^n \sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**n*sec(c + d*x),x)
```

3.213 $\int (b \sec(c+dx))^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int (b \sec(c+dx))^n \, dx = -\frac{b \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(c+dx)\right) (b \sec(c+dx))^{-1+n} \sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-1
+n)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (b \sec(c+dx))^n \, dx = \frac{\cot(c+dx) \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(c+dx)\right) (b \sec(c+dx))^n \sqrt{-\tan^2(c+dx)}}{dn}$$

input
```
Integrate[(b*Sec[c + d*x])^n,x]
```

output
```
(Cot[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c + d*x]^2]*(b*Se
c[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n \int \left(\frac{\cos(c+dx)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n \int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{b\sin(c+dx)(b\sec(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Sec[c + d*x])^n,x]
```

output
```
-((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[c
 + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Maple [F]

$$\int (b\sec (dx+c))^n \, dx$$

input
```
int((b*sec(d*x+c))^n,x)
```

output
```
int((b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\sec(c+dx))^n \, dx = \int (b\sec (dx+c))^n \, dx$$

input
```
integrate((b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\sec(c+dx))^n\,dx = \int (b\sec(c+dx))^n\,dx$$

input
```
integrate((b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\,dx$$

input
```
integrate((b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n, x)
```

Giac [F]

$$\int (b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\,dx$$

input
```
integrate((b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(c+dx))^n\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

input
```
int((b/cos(c + d*x))^n,x)
```

output
```
int((b/cos(c + d*x))^n, x)
```

Reduce [F]

$$\int (b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^n\,dx\right)$$

input
```
int((b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**n,x)
```

3.214 $\int \cos(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \cos(c+dx)(b\sec(c+dx))^n\,dx = -\frac{b^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(c+dx)\right)(b\sec(c+dx))^{-2+n}\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-b^2*hypergeom([1/2, 1-1/2*n],[2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-2+n
)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \cos(c+dx)(b\sec(c+dx))^n\,dx = \frac{b\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\sec^2(c+dx)\right)(b\sec(c+dx))^{-1+n}\sqrt{-\tan^2(c+dx}}{d(-1+n)}$$

input
```
Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n,x]
```

output
```
(b*Cot[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[c + d*x]
^2]*(b*Sec[c + d*x])^(-1 + n)*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-1}dx$$

$$\Big\downarrow \text{4259}$$

$$b\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n \int \left(\frac{\cos(c+dx)}{b}\right)^{1-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n \int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{1-n}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{b^2\sin(c+dx)(b\sec(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(b*Sec[c + d*x])^n,x]
```

output
```
-((b^2*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \cos(dx+c)\,(b\sec(dx+c))^n\,dx$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos(c + dx)(b\sec(c + dx))^n \, dx = \int (b\sec(dx + c))^n \cos(dx + c) \, dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Sympy [F]

$$\int \cos(c + dx)(b\sec(c + dx))^n \, dx = \int (b\sec(c + dx))^n \cos(c + dx) \, dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c + dx)(b\sec(c + dx))^n \, dx = \int (b\sec(dx + c))^n \cos(dx + c) \, dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\sec(c+dx))^n\,dx = \int \cos(c+d\,x)\,\left(\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

input
```
int(cos(c + d*x)*(b/cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)*(b/cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos(c+dx)(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^n\cos(dx+c)\,dx\right)$$

input
```
int(cos(d*x+c)*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**n*cos(c + d*x),x)
```

3.215 $\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = -\frac{b^3 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-n}{2},\frac{5-n}{2},\cos^2(c+dx)\right)(b\sec(c+dx))^{-3+n}\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-b^3*hypergeom([1/2, 3/2-1/2*n],[5/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(
-3+n)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = \frac{\cos^2(c+dx)\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c}}{d(-2+n)}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

output
```
(Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Sec[c
 + d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-2 + n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-2}dx$$

$$\Big\downarrow \text{4259}$$

$$b^2\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\cos(c+dx)}{b}\right)^{2-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{2-n}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{b^3\sin(c+dx)(b\sec(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3-n}{2},\frac{5-n}{2},\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

output
```
-((b^3*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \cos (dx+c)^2 (b\sec (dx+c))^n \, dx$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Sympy [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(c+dx))^n \cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = \int \cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

input
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^2*(b/cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^n\cos(dx+c)^2\,dx\right)$$

input
```
int(cos(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**n*cos(c + d*x)**2,x)
```

3.216 $\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = -\frac{b^4 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-n}{2},\frac{6-n}{2},\cos^2(c+dx)\right)(b\sec(c+dx))^{-4+n}\sin(c+dx)}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-b^4*hypergeom([1/2, 2-1/2*n],[3-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-4+n
)*sin(d*x+c)/d/(4-n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = \frac{\cos^3(c+dx)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{}}{d(-3+n)}$$

input
```
Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^n,x]
```

output
```
(Cos[c + d*x]^3*Cot[c + d*x]*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2
, Sec[c + d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \left(b\csc\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-3}dx$$

$$\Big\downarrow \text{4259}$$

$$b^3\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\cos(c+dx)}{b}\right)^{3-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\int \left(\frac{\sin\left(c+dx+\frac{\pi}{2}\right)}{b}\right)^{3-n}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{b^4\sin(c+dx)(b\sec(c+dx))^{n-4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4-n}{2},\frac{6-n}{2},\cos^2(c+dx)\right)}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^n,x]
```

output
```
-((b^4*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(4 - n)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \cos (dx+c)^3 (b\sec (dx+c))^n\, dx$$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(b*sec(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = \int \cos(c+d\,x)^3 \left(\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

input
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^3*(b/cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^n\cos(dx+c)^3\,dx\right)$$

input
```
int(cos(d*x+c)^3*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**n*cos(c + d*x)**3,x)
```

3.217 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3-2n),\frac{1}{4}(1-2n),\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*hypergeom([1/2, -3/4-1/2*n],[1/4-1/2*n],cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(
b*sec(d*x+c))^n*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{5}{2}+n\right),\frac{1}{2}\left(\frac{9}{2}+n\right),\sec^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sqrt{-}}{d\left(\frac{5}{2}+n\right)}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n,x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (5/2 + n)/2, (9/2 + n)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(5/2
+ n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \sec^{n+\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \csc\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \cos^{-n-\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-n-\frac{5}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-3),\frac{1}{4}(1-2n),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n,x]
```

output
```
(2*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c
 + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d
*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(dx+c)^{\frac{5}{2}} (b\sec(dx+c))^n\, dx$$

input
```
int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^n \left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^{n+\frac{1}{2}} \sec(dx+c)^2\,dx\right)$$

input
```
int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**((2*n + 1)/2)*sec(c + d*x)**2,x)
```

3.218 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1-2n),\frac{1}{4}(3-2n),\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sin(c+dx}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*hypergeom([1/2, -1/4-1/2*n],[3/4-1/2*n],cos(d*x+c)^2)*sec(d*x+c)^(1/2)*(
b*sec(d*x+c))^n*sin(d*x+c)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{3}{2}+n\right),\frac{1}{2}\left(\frac{7}{2}+n\right),\sec^2(c+dx)\right)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\sqrt{-}}{d\left(\frac{3}{2}+n\right)}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n,x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (3/2 + n)/2, (7/2 + n)/2, Sec[c + d*x
]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(3/2
+ n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \sec^{n+\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \csc\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \cos^{-n-\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-n-\frac{3}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-2n-1),\frac{1}{4}(3-2n),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n,x]
```

output
```
(2*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt[
Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sec(dx+c)^{\frac{3}{2}} (b\sec(dx+c))^n \, dx$$

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(c+dx))^n \sec^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n*sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n \sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^n \left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^{n+\frac{1}{2}}\sec(dx+c)\,dx\right)$$

input
```
int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**((2*n + 1)/2)*sec(c + d*x),x)
```

3.219 $\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1-2n),\frac{1}{4}(5-2n),\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-2*hypergeom([1/2, 1/4-1/2*n],[5/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*s
in(d*x+c)/d/(1-2*n)/sec(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{2}+n\right),\frac{1}{2}\left(\frac{5}{2}+n\right),\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d\left(\frac{1}{2}+n\right)\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n,x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (1/2 + n)/2, (5/2 + n)/2, Sec[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1/2 + n)*Sqrt[Sec[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \sec^{n+\frac{1}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \csc\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \cos^{-n-\frac{1}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{-n-\frac{1}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1-2n),\frac{1}{4}(5-2n),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n,x]
```

output
```
(-2*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d
*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \sqrt{\sec (dx + c)}\, (b \sec (dx + c))^n\, dx$$

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^n,x)
```

Fricas [F]

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F]

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = \int (b\sec(c+dx))^n\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**n,x)
```

output
```
Integral((b*sec(c + d*x))**n*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = \int (b\sec(dx+c))^n\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = \int \left(\frac{b}{\cos(c+d\,x)}\right)^n\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec(c+dx)}(b\sec(c+dx))^n\,dx = b^n\left(\int \sec(dx+c)^{n+\frac{1}{2}}\,dx\right)$$

input
```
int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^n,x)
```

output
```
b**n*int(sec(c + d*x)**((2*n + 1)/2),x)
```

3.220 $\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3-2n),\frac{1}{4}(7-2n),\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*hypergeom([1/2, 3/4-1/2*n],[7/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*s
in(d*x+c)/d/(3-2*n)/sec(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{2}+n\right),\frac{1}{2}\left(\frac{3}{2}+n\right),\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+d}}{d\left(-\frac{1}{2}+n\right)\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^n/Sqrt[Sec[c + d*x]],x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/2 + n)/2, (3/2 + n)/2, Sec[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-1/2 + n)*Sec[c + d*x]
^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{2034}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \sec^{n-\frac{1}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \csc\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \cos^{\frac{1}{2}-n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{\frac{1}{2}-n}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3-2n),\frac{1}{4}(7-2n),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Sec[c + d*x])^n/Sqrt[Sec[c + d*x]],x]
```

output
```
(-2*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(b\sec (dx+c))^n}{\sqrt{\sec (dx+c)}}dx$$

input
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

output
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((b*sec(d*x+c))**n/sec(d*x+c)**(1/2),x)
```

output
```
Integral((b*sec(c + d*x))**n/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = b^n\left(\int \frac{\sec(dx+c)^{n+\frac{1}{2}}}{\sec(dx+c)}dx\right)$$

input
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

output
```
b**n*int(sec(c + d*x)**((2*n + 1)/2)/sec(c + d*x),x)
```

3.221 $\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5-2n),\frac{1}{4}(9-2n),\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(5-2n)\sec^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*hypergeom([1/2, 5/4-1/2*n],[9/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*s
in(d*x+c)/d/(5-2*n)/sec(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{3}{2}+n\right),\frac{1}{2}\left(\frac{1}{2}+n\right),\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+d}}{d\left(-\frac{3}{2}+n\right)\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^n/Sec[c + d*x]^(3/2),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (-3/2 + n)/2, (1/2 + n)/2, Sec[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-3/2 + n)*Sec[c + d*x]
^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \sec^{n-\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \csc\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \cos^{\frac{3}{2}-n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{\frac{3}{2}-n}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5-2n),\frac{1}{4}(9-2n),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(b*Sec[c + d*x])^n/Sec[c + d*x]^(3/2),x]
```

output
```
(-2*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d
*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(b\sec (dx+c))^n}{\sec (dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)
```

output
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((b*sec(d*x+c))**n/sec(d*x+c)**(3/2),x)
```

output
```
Integral((b*sec(c + d*x))**n/sec(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = b^n\left(\int \frac{\sec(dx+c)^{n+\frac{1}{2}}}{\sec(dx+c)^2}dx\right)$$

input
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)
```

output
```
b**n*int(sec(c + d*x)**((2*n + 1)/2)/sec(c + d*x)**2,x)
```

3.222 $\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7-2n),\frac{1}{4}(11-2n),\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(7-2n)\sec^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*hypergeom([1/2, 7/4-1/2*n],[11/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*
sin(d*x+c)/d/(7-2*n)/sec(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{5}{2}+n\right),\frac{1}{2}\left(-\frac{1}{2}+n\right),\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+}}{d\left(-\frac{5}{2}+n\right)\sec^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(b*Sec[c + d*x])^n/Sec[c + d*x]^(5/2),x]
```

output
```
(Csc[c + d*x]*Hypergeometric2F1[1/2, (-5/2 + n)/2, (-1/2 + n)/2, Sec[c + d
*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-5/2 + n)*Sec[c + d*x
]^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2034, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \sec^{n-\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec^{-n}(c+dx)(b\sec(c+dx))^n\int \csc\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx$$

$$\Big\downarrow \text{4259}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \cos^{\frac{5}{2}-n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sec(c+dx)}\cos^{n+\frac{1}{2}}(c+dx)(b\sec(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{\frac{5}{2}-n}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7-2n),\frac{1}{4}(11-2n),\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}$$

input
```
Int[(b*Sec[c + d*x])^n/Sec[c + d*x]^(5/2),x]
```

output
```
(-2*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(7 - 2*n)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c +
d*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)
```

output
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Sympy [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((b*sec(d*x+c))**n/sec(d*x+c)**(5/2),x)
```

output
```
Integral((b*sec(c + d*x))**n/sec(c + d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = b^n\left(\int \frac{\sec(dx+c)^{n+\frac{1}{2}}}{\sec(dx+c)^3}dx\right)$$

input
```
int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)
```

output
```
b**n*int(sec(c + d*x)**((2*n + 1)/2)/sec(c + d*x)**3,x)
```

3.223 $\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx=\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

output
```
2/5*d*(d*sec(b*x+a))^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx=\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

input
```
Integrate[(d*Sec[a + b*x])^(7/2)*Sin[a + b*x],x]
```

output
```
(2*d*(d*Sec[a + b*x])^(5/2))/(5*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\sec(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(a+bx))^{7/2}}{\csc(a+bx)}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{d\int (d\sec(a+bx))^{3/2}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

input
```
Int[(d*Sec[a + b*x])^(7/2)*Sin[a + b*x],x]
```

output
```
(2*d*(d*Sec[a + b*x])^(5/2))/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2d(d\sec(bx+a))^{\frac{5}{2}}}{5b}$	17
default	$\frac{2d(d\sec(bx+a))^{\frac{5}{2}}}{5b}$	17

input
```
int((d*sec(b*x+a))^(7/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*d*(d*sec(b*x+a))^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx = \frac{2\,d^3\sqrt{\frac{d}{\cos(bx+a)}}}{5\,b\cos{(bx+a)}^2}$$

input
```
integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
2/5*d^3*sqrt(d/cos(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*sec(b*x+a))**(7/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx = \frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{7}{2}}\cos(bx+a)}{5\,b}$$

input
```
integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
2/5*(d/cos(b*x + a))^(7/2)*cos(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx = \frac{2\,d^4\mathrm{sgn}(\cos(bx+a))}{5\,\sqrt{d\cos(bx+a)}b\cos(bx+a)^2}$$

input
```
integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
2/5*d^4*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b*cos(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 11.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.85

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx = \frac{8\,d^3\,\sqrt{\frac{d}{\cos(a+b\,x)}}\,\left(4\,\cos\left(2\,a+2\,b\,x\right)+\cos\left(4\,a+4\,b\,x\right)+3\right)}{5\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)+10\right)}$$

input
```
int(sin(a + b*x)*(d/cos(a + b*x))^(7/2),x)
```

output
```
(8*d^3*(d/cos(a + b*x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
/(5*b*(15*cos(2*a + 2*b*x) + 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

Reduce [F]

$$\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\sec\left(bx+a\right)}\,\sec\left(bx+a\right)^{3}\sin\left(bx+a\right)dx\right)d^{3}$$

input
```
int((d*sec(b*x+a))^(7/2)*sin(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(sec(a + b*x))*sec(a + b*x)**3*sin(a + b*x),x)*d**3
```

3.224 $\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

output
```
2/3*d*(d*sec(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

input
```
Integrate[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x],x]
```

output
```
(2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\sec(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(a+bx))^{5/2}}{\csc(a+bx)}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{d\int \sqrt{d\sec(a+bx)}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

input

```
Int[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x],x]
```

output

```
(2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2d(d\sec(bx+a))^{\frac{3}{2}}}{3b}$	17
default	$\frac{2d(d\sec(bx+a))^{\frac{3}{2}}}{3b}$	17

input
```
int((d*sec(b*x+a))^(5/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*d*(d*sec(b*x+a))^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \frac{2\,d^2\sqrt{\frac{d}{\cos(bx+a)}}}{3\,b\cos{(bx+a)}}$$

input
```
integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
2/3*d^2*sqrt(d/cos(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*sec(b*x+a))**(5/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{5}{2}}\cos(bx+a)}{3\,b}$$

input
```
integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
2/3*(d/cos(b*x + a))^(5/2)*cos(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \frac{2\,d^3\mathrm{sgn}(\cos(bx+a))}{3\sqrt{d\cos(bx+a)}b\cos(bx+a)}$$

input
```
integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
2/3*d^3*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b*cos(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \frac{4\,d^2\,\cos\left(a+b\,x\right)\,\sqrt{\frac{d}{\cos(a+b\,x)}}}{3\,b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

input
```
int(sin(a + b*x)*(d/cos(a + b*x))^(5/2),x)
```

output
```
(4*d^2*cos(a + b*x)*(d/cos(a + b*x))^(1/2))/(3*b*(cos(2*a + 2*b*x) + 1))
```

Reduce [F]

$$\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx = \sqrt{d}\left(\int\sqrt{\sec\left(bx+a\right)}\,\sec\left(bx+a\right)^2\sin\left(bx+a\right)dx\right)d^2$$

input
```
int((d*sec(b*x+a))^(5/2)*sin(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(sec(a + b*x))*sec(a + b*x)**2*sin(a + b*x),x)*d**2
```

3.225 $\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx=\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

output
```
2*d*(d*sec(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx=\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

input
```
Integrate[(d*Sec[a + b*x])^(3/2)*Sin[a + b*x],x]
```

output
```
(2*d*Sqrt[d*Sec[a + b*x]])/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\sec(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(a+bx))^{3/2}}{\csc(a+bx)}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{d\int \frac{1}{\sqrt{d\sec(a+bx)}}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

input
```
Int[(d*Sec[a + b*x])^(3/2)*Sin[a + b*x],x]
```

output
```
(2*d*Sqrt[d*Sec[a + b*x]])/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2d\sqrt{d\sec(bx+a)}}{b}$	17
default	$\frac{2d\sqrt{d\sec(bx+a)}}{b}$	17

input
```
int((d*sec(b*x+a))^(3/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*d*(d*sec(b*x+a))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx = \frac{2\,d\sqrt{\frac{d}{\cos(bx+a)}}}{b}$$

input
```
integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
2*d*sqrt(d/cos(b*x + a))/b
```

Sympy [F]

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx = \int (d\sec(a+bx))^{\frac{3}{2}}\sin(a+bx)\,dx$$

input
```
integrate((d*sec(b*x+a))**(3/2)*sin(b*x+a),x)
```

output
```
Integral((d*sec(a + b*x))**(3/2)*sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx = \frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}\cos(bx+a)}{b}$$

input
```
integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
2*(d/cos(b*x + a))^(3/2)*cos(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx = \frac{2\,d^2\mathrm{sgn}(\cos(bx+a))}{\sqrt{d\cos(bx+a)}b}$$

input
```
integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
2*d^2*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx=\frac{2\,d\,\sqrt{\frac{d}{\cos(a+b\,x)}}}{b}$$

input
```
int(sin(a + b*x)*(d/cos(a + b*x))^(3/2),x)
```

output
```
(2*d*(d/cos(a + b*x))^(1/2))/b
```

Reduce [F]

$$\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx=\sqrt{d}\left(\int\sqrt{\sec\left(bx+a\right)}\,\sec\left(bx+a\right)\sin\left(bx+a\right)dx\right)d$$

input
```
int((d*sec(b*x+a))^(3/2)*sin(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(sec(a + b*x))*sec(a + b*x)*sin(a + b*x),x)*d
```

3.226 $\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2d}{b\sqrt{d\sec(a+bx)}}$$

output
```
-2*d/b/(d*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2d}{b\sqrt{d\sec(a+bx)}}$$

input
```
Integrate[Sqrt[d*Sec[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*d)/(b*Sqrt[d*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{d\sec(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(a+bx)}}{\csc(a+bx)}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{d\int \frac{1}{(d\sec(a+bx))^{3/2}}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2d}{b\sqrt{d\sec(a+bx)}}$$

input
```
Int[Sqrt[d*Sec[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*d)/(b*Sqrt[d*Sec[a + b*x]])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{2d}{b\sqrt{d\sec(bx+a)}}$	17
default	$-\frac{2d}{b\sqrt{d\sec(bx+a)}}$	17
risch	$-\frac{2\sqrt{2}\sqrt{\frac{d\,e^{i(bx+a)}}{1+e^{2i(bx+a)}}}\cos(bx+a)}{b}$	41

input
```
int((d*sec(b*x+a))^(1/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*d/b/(d*sec(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)}{b}$$

input
```
integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-2*sqrt(d/cos(b*x + a))*cos(b*x + a)/b
```

Sympy [F]

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = \int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx$$

input
```
integrate((d*sec(b*x+a))**(1/2)*sin(b*x+a),x)
```

output
```
Integral(sqrt(d*sec(a + b*x))*sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)}{b}$$

input
```
integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-2*sqrt(d/cos(b*x + a))*cos(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2\sqrt{d\cos(bx+a)}\mathrm{sgn}(\cos(bx+a))}{b}$$

input
```
integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
-2*sqrt(d*cos(b*x + a))*sgn(cos(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2\cos(a+b\,x)\,\sqrt{\frac{d}{\cos(a+b\,x)}}}{b}$$

input
```
int(sin(a + b*x)*(d/cos(a + b*x))^(1/2),x)
```

output
```
-(2*cos(a + b*x)*(d/cos(a + b*x))^(1/2))/b
```

Reduce [F]

$$\int \sqrt{d\sec(a+bx)}\sin(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\sec(bx+a)}\,\sin(bx+a)\,dx\right)$$

input
```
int((d*sec(b*x+a))^(1/2)*sin(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(sec(a + b*x))*sin(a + b*x),x)
```

3.227 $\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = -\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

output
```
-2/3*d/b/(d*sec(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = -\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]/Sqrt[d*Sec[a + b*x]],x]
```

output
```
(-2*d)/(3*b*(d*Sec[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(a+bx)\sqrt{d\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{d\int \frac{1}{(d\sec(a+bx))^{5/2}}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]/Sqrt[d*Sec[a + b*x]],x]
```

output
```
(-2*d)/(3*b*(d*Sec[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2d}{3b(d\sec(bx+a))^{\frac{3}{2}}}$	17
default	$-\frac{2d}{3b(d\sec(bx+a))^{\frac{3}{2}}}$	17

input
```
int(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d/b/(d*sec(b*x+a))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = -\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)^2}{3\,bd}$$

input
```
integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(d/cos(b*x + a))*cos(b*x + a)^2/(b*d)
```

Sympy [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{\sin{(a+bx)}}{\sqrt{d\sec{(a+bx)}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*sec(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)/sqrt(d*sec(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = -\frac{2\cos{(bx+a)}}{3b\sqrt{\frac{d}{\cos(bx+a)}}}$$

input
```
integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/3*cos(b*x + a)/(b*sqrt(d/cos(b*x + a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = -\frac{2\sqrt{d\cos{(bx+a)}}\cos{(bx+a)}}{3\,bd\mathrm{sgn}\left(\cos{(bx+a)}\right)}$$

input
```
integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/(b*d*sgn(cos(b*x + a)))
```

Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = -\frac{2\cos\left(a+b\,x\right)^2\,\sqrt{\frac{d}{\cos(a+b\,x)}}}{3\,b\,d}$$

input
```
int(sin(a + b*x)/(d/cos(a + b*x))^(1/2),x)
```

output
```
-(2*cos(a + b*x)^2*(d/cos(a + b*x))^(1/2))/(3*b*d)
```

Reduce [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sin(bx+a)}{\sec(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(sec(a + b*x))*sin(a + b*x))/sec(a + b*x),x))/d
```

3.228 $\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx = \frac{2d^3}{b\sqrt{d\sec(a+bx)}} + \frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

output
```
2*d^3/b/(d*sec(b*x+a))^(1/2)+2/3*d*(d*sec(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx = \frac{d(5+3\cos(2(a+bx)))(d\sec(a+bx))^{3/2}}{3b}$$

input
```
Integrate[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x]^3,x]
```

output
```
(d*(5 + 3*Cos[2*(a + b*x)])*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(d\sec(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(a+bx))^{5/2}}{\csc(a+bx)^3}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{d^3\int -\frac{d^2-d^2\sec^2(a+bx)}{d^2(d\sec(a+bx))^{3/2}}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d^3\int \frac{d^2-d^2\sec^2(a+bx)}{d^2(d\sec(a+bx))^{3/2}}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{d^2-d^2\sec^2(a+bx)}{(d\sec(a+bx))^{3/2}}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{d\int \left(\frac{d^2}{(d\sec(a+bx))^{3/2}}-\sqrt{d\sec(a+bx)}\right)d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(-\frac{2d^2}{\sqrt{d\sec(a+bx)}}-\frac{2}{3}(d\sec(a+bx))^{3/2}\right)}{b}$$

input
```
Int[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x]^3,x]
```

output
```
-((d*((-2*d^2)/Sqrt[d*Sec[a + b*x]] - (2*(d*Sec[a + b*x])^(3/2))/3))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(35) = 70$.

Time = 59.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 6.34

method	result
default	$\frac{\sqrt{d\sec(bx+a)}\,d^2\left(\sqrt{-\frac{\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(12\cos(bx+a)^2+12\cos(bx+a)+4+4\sec(bx+a)\right)+3\cos(bx+a)\ln\left(\frac{4\cos(bx+a)\sqrt{-\frac{\cos(bx+}{(\cos(bx+a)}}}{}\right.\right.}{6b\sqrt{-\frac{\cos(bx+a)}{(\cos(bx+a)+1}}}$

input
```
int((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/b*(d*sec(b*x+a))^(1/2)*d^2/(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(cos(b
*x+a)+1)*((-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(12*cos(b*x+a)^2+12*cos(b*x
+a)+4+4*sec(b*x+a))+3*cos(b*x+a)*ln(2*(2*cos(b*x+a)*(-cos(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)+2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(cos(b
*x+a)+1))-3*cos(b*x+a)*ln((2*cos(b*x+a)*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/
2)+2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx=\frac{2\left(3\,d^2\cos\left(bx+a\right)^2+d^2\right)\sqrt{\frac{d}{\cos(bx+a)}}}{3\,b\cos\left(bx+a\right)}$$

input
```
integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
2/3*(3*d^2*cos(b*x + a)^2 + d^2)*sqrt(d/cos(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx=\text{Timed out}$$

input
```
integrate((d*sec(b*x+a))**(5/2)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx = \frac{2\left(\frac{3\,d^2}{\sqrt{\frac{d}{\cos(bx+a)}}}+\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}\right)d}{3\,b}$$

input
```
integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
2/3*(3*d^2/sqrt(d/cos(b*x + a)) + (d/cos(b*x + a))^(3/2))*d/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx = \frac{2\left(3\,\sqrt{d\cos(bx+a)}d+\frac{d^2}{\sqrt{d\cos(bx+a)}\cos(bx+a)}\right)d\mathrm{sgn}(\cos(bx+a))}{3\,b}$$

input
```
integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
2/3*(3*sqrt(d*cos(b*x + a))*d + d^2/(sqrt(d*cos(b*x + a))*cos(b*x + a)))*d
*sgn(cos(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx=\frac{d^2\,\sqrt{\frac{d}{\cos(a+b\,x)}}\,\left(\frac{13\,\cos(a+b\,x)}{3}+\cos\left(3\,a+3\,b\,x\right)\right)}{b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

input
```
int(sin(a + b*x)^3*(d/cos(a + b*x))^(5/2),x)
```

output
```
(d^2*(d/cos(a + b*x))^(1/2)*((13*cos(a + b*x))/3 + cos(3*a + 3*b*x)))/(b*(
cos(2*a + 2*b*x) + 1))
```

Reduce [F]

$$\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx=\sqrt{d}\left(\int \sqrt{\sec\left(bx+a\right)}\,\sec\left(bx+a\right)^2\sin\left(bx+a\right)^3dx\right)d^2$$

input
```
int((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(sec(a + b*x))*sec(a + b*x)**2*sin(a + b*x)**3,x)*d**2
```

3.229 $\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = -\frac{2d^3(d\sec(a+bx))^{3/2}}{3b}+\frac{2d(d\sec(a+bx))^{7/2}}{7b}$$

output
```
-2/3*d^3*(d*sec(b*x+a))^(3/2)/b+2/7*d*(d*sec(b*x+a))^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = -\frac{d^4(1+7\cos(2(a+bx)))\sec^3(a+bx)\sqrt{d\sec(a+bx)}}{21b}$$

input
```
Integrate[(d*Sec[a + b*x])^(9/2)*Sin[a + b*x]^3,x]
```

output
```
-1/21*(d^4*(1 + 7*Cos[2*(a + b*x)])*Sec[a + b*x]^3*Sqrt[d*Sec[a + b*x]])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(d\sec(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(a+bx))^{9/2}}{\csc(a+bx)^3}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{d^3\int -\frac{\sqrt{d\sec(a+bx)}(d^2-d^2\sec^2(a+bx))}{d^2}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d^3\int \frac{\sqrt{d\sec(a+bx)}(d^2-d^2\sec^2(a+bx))}{d^2}d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \sqrt{d\sec(a+bx)}\left(d^2-d^2\sec^2(a+bx)\right)d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{d\int \left(d^2\sqrt{d\sec(a+bx)}-(d\sec(a+bx))^{5/2}\right)d(d\sec(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(\frac{2}{3}d^2(d\sec(a+bx))^{3/2}-\frac{2}{7}(d\sec(a+bx))^{7/2}\right)}{b}$$

input
```
Int[(d*Sec[a + b*x])^(9/2)*Sin[a + b*x]^3,x]
```

output
```
-((d*((2*d^2*(d*Sec[a + b*x])^(3/2))/3 - (2*(d*Sec[a + b*x])^(7/2))/7))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 58.57 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2d\left(\frac{(d\sec(bx+a))^{\frac{7}{2}}}{7}-\frac{d^{2}(d\sec(bx+a))^{\frac{3}{2}}}{3}\right)}{b}$	35
default	$\frac{2d\left(\frac{(d\sec(bx+a))^{\frac{7}{2}}}{7}-\frac{d^{2}(d\sec(bx+a))^{\frac{3}{2}}}{3}\right)}{b}$	35

input
```
int((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/b*d*(1/7*(d*sec(b*x+a))^(7/2)-1/3*d^2*(d*sec(b*x+a))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = -\frac{2\left(7\,d^4\cos\left(bx+a\right)^2-3\,d^4\right)\sqrt{\frac{d}{\cos(bx+a)}}}{21\,b\cos\left(bx+a\right)^3}$$

input
```
integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2/21*(7*d^4*cos(b*x + a)^2 - 3*d^4)*sqrt(d/cos(b*x + a))/(b*cos(b*x + a)^
3)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*sec(b*x+a))**(9/2)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = -\frac{2\left(7\,d^2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}-3\left(\frac{d}{\cos(bx+a)}\right)^{\frac{7}{2}}\right)d}{21\,b}$$

input
```
integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2/21*(7*d^2*(d/cos(b*x + a))^(3/2) - 3*(d/cos(b*x + a))^(7/2))*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = -\frac{2\left(7\,d^5\cos\left(bx+a\right)^2-3\,d^5\right)\mathrm{sgn}(\cos\left(bx+a\right))}{21\,\sqrt{d\cos\left(bx+a\right)}b\cos\left(bx+a\right)^3}$$

input
```
integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-2/21*(7*d^5*cos(b*x + a)^2 - 3*d^5)*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a
))*b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = -\frac{4\,d^4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{d}{\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}}}\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+7\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+7\right)}{21\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int(sin(a + b*x)^3*(d/cos(a + b*x))^(9/2),x)
```

output
```
-(4*d^4*exp(a*1i + b*x*1i)*(d/(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)
/2))^(1/2)*(2*exp(a*2i + b*x*2i) + 7*exp(a*4i + b*x*4i) + 7))/(21*b*(exp(a
*2i + b*x*2i) + 1)^3)
```

Reduce [F]

$$\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\sec(bx+a)}\,\sec(bx+a)^4\sin(bx+a)^3\,dx\right)d^4$$

input

```
int((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x)
```

output

```
sqrt(d)*int(sqrt(sec(a + b*x))*sec(a + b*x)**4*sin(a + b*x)**3,x)*d**4
```

3.230 $\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}} + \frac{4d^4\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{7b}$$

output
```
-4/7*c*d^3*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(1/2)-2/7*c*d*(d*csc(b*x+
a))^(7/2)/b/(c*sec(b*x+a))^(1/2)+4/7*d^4*(d*csc(b*x+a))^(1/2)*InverseJacob
iAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 16.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = \frac{2d^4\cos(2(a+bx))\cot(a+bx)\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\Big((-2+\cos(2(a}{}$$

input
```
Integrate[(d*Csc[a + b*x])^(9/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(2*d^4*Cos[2*(a + b*x)]*Cot[a + b*x]*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b
*x]]*((-2 + Cos[2*(a + b*x)])*Csc[a + b*x]^4 - 2*(-Cot[a + b*x]^2)^(3/4)*H
ypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2*Sec[a + b*x]^2))/(7*b*(-2
+ Csc[a + b*x]^2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3105, 3042, 3105, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{9/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{6}{7}d^2\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}d^2\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3105}$$

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\int\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3110

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx-\frac{2c\right.$$
$$\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3042

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx-\frac{2c\right.$$
$$\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3053

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3042

$$\frac{6}{7}d^2\left(\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3120

$$\frac{6}{7}d^2\left(\frac{2d^2\sqrt{\sin(2a+2bx)}\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(9/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*Sqrt[c*Sec[a + b*x]]) + (6*d^2*((-2*c
*d*(d*Csc[a + b*x])^(3/2))/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*d^2*Sqrt[d*Csc[
a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14

method	result
default	$\frac{\left(-\frac{2(-2\cos(bx+a)-2)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(bx+a)+\csc(b}}{7}\right.}{b}$

input
```
int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/7*(-2*cos(b*x+a)-2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a
)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2)
)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+4/7*cot(b*x+a)^3-6/7*cot(b*x+a)*csc(b*x
+a)^2)*(d*csc(b*x+a))^(1/2)*d^4*(c*sec(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.38

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx =$$

$$-\frac{2\left(\left(i\,d^4\cos{(bx+a)}^2-i\,d^4\right)\sqrt{-4i\,cd}F(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)\sin{(bx+a)}+\left(-i\,d^4\right.\right.}{}$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/7*((I*d^4*cos(b*x + a)^2 - I*d^4)*sqrt(-4*I*c*d)*elliptic_f(arcsin(cos(
b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*d^4*cos(b*x + a)^2 + I*
d^4)*sqrt(4*I*c*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*s
in(b*x + a) + (2*d^4*cos(b*x + a)^3 - 3*d^4*cos(b*x + a))*sqrt(c/cos(b*x +
 a))*sqrt(d/sin(b*x + a)))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = \int (d\csc(bx+a))^{\frac{9}{2}}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(9/2)*sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = \int (d\csc(bx+a))^{\frac{9}{2}}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(9/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = \int \sqrt{\frac{c}{\cos(a+b\,x)}}\left(\frac{d}{\sin(a+b\,x)}\right)^{9/2}dx$$

input
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(9/2),x)
```

output
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^4\,dx\right)d^4$$

input
```
int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**4,
x)*d**4
```

3.231 $\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

output
```
-8/5*c*d^3*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(1/2)-2/5*c*d*(d*csc(b*x+
a))^(5/2)/b/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2d^3\sqrt{d\csc(a+bx)}(4\cos(a+bx)+\cot(a+bx)\csc(a+bx))\sqrt{c\sec(a+bx)}}{5b}$$

input
```
Integrate[(d*Csc[a + b*x])^(7/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*d^3*Sqrt[d*Csc[a + b*x]]*(4*Cos[a + b*x] + Cot[a + b*x]*Csc[a + b*x])*
Sqrt[c*Sec[a + b*x]])/(5*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3105, 3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{4}{5}d^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}d^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3099}$$

$$-\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(7/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-8*c*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Csc
[a + b*x])^(5/2))/(5*b*Sqrt[c*Sec[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{2\left(4\cos\left(bx+a\right)^{2}-5\right)\sqrt{c\sec\left(bx+a\right)}\,\sqrt{d\csc\left(bx+a\right)}\,d^{3}\cot\left(bx+a\right)\csc\left(bx+a\right)}{5b}$	53

input
```
int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b*(4*cos(b*x+a)^2-5)*(c*sec(b*x+a))^(1/2)*(d*csc(b*x+a))^(1/2)*d^3*cot
(b*x+a)*csc(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2\left(4\,d^3\cos\left(bx+a\right)^3-5\,d^3\cos\left(bx+a\right)\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{5\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(4*d^3*cos(b*x + a)^3 - 5*d^3*cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt
(d/sin(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx = \int (d\csc\left(bx+a\right))^{\frac{7}{2}}\sqrt{c\sec\left(bx+a\right)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(7/2)*sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx=\int (d\csc(bx+a))^{\frac{7}{2}}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(7/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{4\,d^3\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,(3\,\cos(a+b\,x)-4\,\cos(3\,a+3\,b\,x)+\cos(5\,a+5\,b\,x))}{5\,b\,(\cos(4\,a+4\,b\,x)-4\,\cos(2\,a+2\,b\,x)+3)}$$

input
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(7/2),x)
```

output
```
-(4*d^3*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(3*cos(a + b*x) - 4*
cos(3*a + 3*b*x) + cos(5*a + 5*b*x)))/(5*b*(cos(4*a + 4*b*x) - 4*cos(2*a +
 2*b*x) + 3))
```

Reduce [F]

$$\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx=\sqrt{d}\,\sqrt{c}\left(\int\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^3\,dx\right)d^3$$

input
```
int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**3,
x)*d**3
```

3.232 $\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{2d^2\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3b}$$

output
```
-2/3*c*d*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(1/2)+2/3*d^2*(d*csc(b*x+a)
)^(1/2)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b
*x+2*a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.17

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{d(\cos(a+bx)+\cos(3(a+bx)))(d\csc(a+bx))^{3/2}\left(\cot^2(a+bx)+(-\cot^2(a+bx))^{3/4}\,\text{Hypergeometric2}\right.}{3b\left(-2+\csc^2(a+bx)\right)}$$

input
```
Integrate[(d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
-1/3*(d*(Cos[a + b*x] + Cos[3*(a + b*x)])*(d*Csc[a + b*x])^(3/2)*(Cot[a +
b*x]^2 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a +
b*x]^2])*Sec[a + b*x]^2*Sqrt[c*Sec[a + b*x]])/(b*(-2 + Csc[a + b*x]^2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3105, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{2}{3}d^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}d^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3110}$$

$$\frac{2}{3}d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3053

$$\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3120

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*d^2*Sqrt[d
*Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[
2*a + 2*b*x]])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(\frac{2(\cos(bx+a)+1)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\right.}{3}\right.}{b}$

input
```
int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/3*(cos(b*x+a)+1)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*c
sc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+cs
c(b*x+a)+1)^(1/2),1/2*2^(1/2))-2/3*cot(b*x+a))*(d*csc(b*x+a))^(1/2)*d^2*(c
*sec(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = \frac{-i\sqrt{-4i\,cd}d^2F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\mid -1)\sin\left(bx+a\right)+i\,\sqrt{}}{}$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(-4*I*c*d)*d^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)
), -1)*sin(b*x + a) + I*sqrt(4*I*c*d)*d^2*elliptic_f(arcsin(cos(b*x + a) -
 I*sin(b*x + a)), -1)*sin(b*x + a) - 2*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin
(b*x + a))*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = \int (d\csc(bx+a))^{\frac{5}{2}}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = \int (d\csc(bx+a))^{\frac{5}{2}}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = \int \sqrt{\frac{c}{\cos(a+b\,x)}}\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}dx$$

input
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2),x)
```

output
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx=\sqrt{d}\,\sqrt{c}\left(\int\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^2\,dx\right)d^2$$

input
```
int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**2,
x)*d**2
```

3.233 $\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

output
```
-2*c*d*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3099}$$

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\sqrt{d\csc(bx+a)}\,\sqrt{c\sec(bx+a)}\,\cos(bx+a)d}{b}$	33

input
```
int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b*(d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2)*cos(b*x+a)*d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2\,d\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)}{b}$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2*d*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = \int (d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = \int (d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2\,d\,\cos\left(a+b\,x\right)\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}}{b}$$

input
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(3/2),x)
```

output
```
-(2*d*cos(a + b*x)*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2))/b
```

Reduce [F]

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)\,dx\right)d$$

input
```
int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x),x)*
d
```

3.234 $\int \sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\,dx = \frac{\sqrt{d \csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)}}{b}$$

output
```
(d*csc(b*x+a))^(1/2)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^
(1/2)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\,dx = \frac{(-\cot^2(a+bx))^{7/4}\sqrt{d \csc(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)\sqrt{c \sec(a+bx)}\tan^3(a+b}{b}$$

input
```
Integrate[Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]],x]
```

output
```
((-Cot[a + b*x]^2)^(7/4)*Sqrt[d*Csc[a + b*x]]*Hypergeometric2F1[1/2, 3/4,
3/2, Csc[a + b*x]^2]*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^3)/b
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}dx$$

$$\Big\downarrow \text{3110}$$

$$\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3053}$$

$$\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

input
```
Int[Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]],x]
```

output
```
(Sqrt[d*Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sq
rt[Sin[2*a + 2*b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.

Time = 1.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

method	result
default	$\frac{(\cos(bx+a)+1)\sqrt{c\sec(bx+a)}\,\sqrt{d\csc(bx+a)}\,\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a}}{b}$

input
```
int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cos(b*x+a)+1)*(c*sec(b*x+a))^(1/2)*(d*csc(b*x+a))^(1/2)*(-cot(b*x+a)+
csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*
x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx = \frac{-i\sqrt{-4i\,cd}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+i\sqrt{4i\,cd}F(\arcsin\left(\cos\left(bx+a\right)-i\,\sin\left(bx+\right.\right.}{2\,b}$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*sqrt(-4*I*c*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -
1) + I*sqrt(4*I*c*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)
)/b
```

Sympy [F]

$$\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx = \int \sqrt{c\sec\left(a+bx\right)}\sqrt{d\csc\left(a+bx\right)}\,dx$$

input
```
integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sec(a + b*x))*sqrt(d*csc(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx = \int \sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx = \int \sqrt{d\csc(bx+a)}\sqrt{c\sec(bx+a)}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx = \int \sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,dx$$

input
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2),x)
```

output
```
int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec\left(bx+a\right)}\,\sqrt{\csc\left(bx+a\right)}dx\right)$$

input
```
int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x)),x)
```

3.235 $\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 198

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

output
```
1/2*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/(d*
csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*
(c*sec(b*x+a))^(1/2)*2^(1/2)/b/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)-1/2*a
rctanh(2^(1/2)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(c*sec(b*x+a))^(1/2)*2^(1/
2)/b/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = -\frac{\left(\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\right)\cot(a+bx)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt[4]{\cot^2(a+bx)}\sqrt{d\csc(a+bx)}}$$

input

```
Integrate[Sqrt[c*Sec[a + b*x]]/Sqrt[d*Csc[a + b*x]],x]
```

output

```
-(((ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))] +
 ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])])*Cot
[a + b*x]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*(Cot[a + b*x]^2)^(1/4)*Sqrt[d*C
sc[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 3109, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \; 3109$$

$$\frac{\sqrt{c\sec(a+bx)}\int\sqrt{\tan(a+bx)}dx}{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{c\sec(a+bx)}\int\sqrt{\tan(a+bx)}dx}{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 3957

$$\frac{\sqrt{c\sec(a+bx)}\int\frac{\sqrt{\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 266

$$\frac{2\sqrt{c\sec(a+bx)}\int\frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 826

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\int\frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 1476

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right.\right.}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 1082

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{t}\right.}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 217

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 1479

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 25

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\cdots}\right.}{\cdots}\right.\right.}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)+\frac{1}{2}\left(\cdots\right.\right.}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 1103

$$\frac{2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}-\frac{\log\cdots}{\cdots}\right.\right.}{b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

input

```
Int[Sqrt[c*Sec[a + b*x]]/Sqrt[d*Csc[a + b*x]],x]
```

output

```
(2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2
]*Sqrt[Tan[a + b*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + T
an[a + b*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*
x]]/(2*Sqrt[2]))/2)*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan
[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(162) = 324$.

Time = 5.48 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{2}\,\sqrt{c\sec(bx+a)}\left((1-\cos(bx+a))^2\csc(bx+a)^2-1\right)\left(\ln\left(\frac{(1-\cos(bx+a))^2\csc(bx+a)+2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)+2-2\cos(bx\cdots}{1-\cos(bx+a)}\right.\right.}{\cdots}$

input
```
int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*2^(1/2)*(c*sec(b*x+a))^(1/2)*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)/(d*cs
c(b*x+a))^(1/2)*(ln(1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*(-2*si
n(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)-sin(
b*x+a)))-2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b
*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1))-ln(-1/(1-cos(b*x+a))*(-(1-cos(b*x+a))^
2*csc(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a
)-2+2*cos(b*x+a)+sin(b*x+a)))-2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1)))/(-sin(b*x+a)*cos
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(162) = 324$.

Time = 0.13 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{2\sqrt{2}\sqrt{\frac{c}{d}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{d}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2c}\right)+\sqrt{2}\sqrt{\frac{c}{d}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{d}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx}}}{2(c\cos(bx+}}{}$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(2)*sqrt(c/d)*arctan(-1/2*sqrt(2)*sqrt(c/d)*sqrt(c/cos(b*x + a)
)*sqrt(d/sin(b*x + a))*(cos(b*x + a) - sin(b*x + a))/c) + sqrt(2)*sqrt(c/d
)*arctan(-1/2*(sqrt(2)*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))
 + 2*c*cos(b*x + a) + 2*c*sin(b*x + a))/(c*cos(b*x + a) - c*sin(b*x + a)))
 + sqrt(2)*sqrt(c/d)*arctan(-1/2*(sqrt(2)*sqrt(c/d)*sqrt(c/cos(b*x + a))*s
qrt(d/sin(b*x + a)) - 2*c*cos(b*x + a) - 2*c*sin(b*x + a))/(c*cos(b*x + a)
 - c*sin(b*x + a))) + sqrt(2)*sqrt(c/d)*log(2*sqrt(2)*(cos(b*x + a)^3 - co
s(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/d)*sqrt(c/cos(b*x + a))*s
qrt(d/sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) - sqrt(2)*sqrt(c/
d)*log(-2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x
+ a))*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) + 4*c*cos(b*x +
a)*sin(b*x + a) + c))/b
```

Sympy [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{\sqrt{c\sec{(a+bx)}}}{\sqrt{d\csc{(a+bx)}}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sec(a + b*x))/sqrt(d*csc(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{\sqrt{c\sec{(bx+a)}}}{\sqrt{d\csc{(bx+a)}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sec(b*x + a))/sqrt(d*csc(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{\sqrt{c\sec\left(bx+a\right)}}{\sqrt{d\csc\left(bx+a\right)}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sec(b*x + a))/sqrt(d*csc(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{\sqrt{\frac{c}{\cos(a+b\,x)}}}{\sqrt{\frac{d}{\sin(a+b\,x)}}}\,dx$$

input
```
int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(1/2),x)
```

output
```
int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)}dx\right)}{d}$$

input
```
int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/csc(a + b*x),
x))/d
```

3.236 $\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = -\frac{c}{bd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{2bd^2}$$

output
```
-c/b/d/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)+1/2*(d*csc(b*x+a))^(1/2)*
InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^
(1/2)/b/d^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = -\frac{\left(1+\cos(2(a+bx))+(-\cot^2(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)\right)(c\sec(a+bx))^{3/2}}{2bcd\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(3/2),x]
```

output
```
-1/2*((1 + Cos[2*(a + b*x)] + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/
2, 3/4, 3/2, Csc[a + b*x]^2])*(c*Sec[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Csc[a
+ b*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3107, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c \sec(a+bx)}}{(d \csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c \sec(a+bx)}}{(d \csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{\int \sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}dx}{2d^2}-\frac{c}{bd\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}dx}{2d^2}-\frac{c}{bd\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}}$$

$$\Big\downarrow \text{3110}$$

$$\frac{\sqrt{c \cos(a+bx)}\sqrt{c \sec(a+bx)}\sqrt{d \sin(a+bx)}\sqrt{d \csc(a+bx)}\int \frac{1}{\sqrt{c \cos(a+bx)}\sqrt{d \sin(a+bx)}}dx}{2d^2}-\frac{c}{bd\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{2d^2}-\frac{c}{bd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2d^2}-\frac{c}{bd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2d^2}-\frac{c}{bd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bd^2}-\frac{c}{bd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

input
```
Int[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(3/2),x]
```

output
```
-(c/(b*d*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])) + (Sqrt[d*Csc[a + b*x
]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]
])/(2*b*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

method	result
default	$\frac{\left(-\frac{\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)(-\cot(bx+}{2}}{bd\sqrt{d\csc(bx+a)}}$

input
```
int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(
1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1
/2),1/2*2^(1/2))*(-cot(b*x+a)-csc(b*x+a))-cos(b*x+a))*(c*sec(b*x+a))^(1/2)
/d/(d*csc(b*x+a))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sec\left(bx+a\right)}}{\left(d\csc\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(d^2*csc(b*x + a)^2), x
)
```

Sympy [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sec\left(a+bx\right)}}{\left(d\csc\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(3/2),x)
```

output
```
Integral(sqrt(c*sec(a + b*x))/(d*csc(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sec(bx+a)}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sec(bx+a)}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{c}{\cos(a+b\,x)}}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(3/2),x)
```

output
```
int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^2}dx\right)}{d^2}$$

input
```
int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/csc(a + b*x)*
*2,x))/d**2
```

3.237 $\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx &= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} \\ &\quad -\frac{3\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} \\ &\quad +\frac{3\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} \\ &\quad -\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\end{aligned}$$

output

```
-1/2*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2)+3/8*arctan(-1+2^(1/2)
*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/d^2/(d*csc(b*x+a))^(1/2)
/tan(b*x+a)^(1/2)+3/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1
/2)*2^(1/2)/b/d^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)-3/8*arctanh(2^(1/2
)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/d^2/(d*c
sc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx =$$

$$-\frac{\left(4\cos^2(a+bx)+3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right.\right.}{8bd^2\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(5/2),x]
```

output
```
-1/8*((4*Cos[a + b*x]^2 + 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sq
rt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) + 3*Sqrt[2]*ArcTanh[
(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]
^2)^(3/4))*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x])/(b*d^2*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3107, 3042, 3109, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3107}$$

$$\frac{3\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{4d^2}-\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{3\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{4d^2}-\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 3109

$$\frac{3\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{4d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{3\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{4d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 3957

$$\frac{3\sqrt{c\sec(a+bx)}\int \frac{\sqrt{\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{4bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 266

$$\frac{3\sqrt{c\sec(a+bx)}\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 826

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 1476

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int \frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right.\right.}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 1082

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{t}\right.}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\big\downarrow$ 217

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-$$

$$\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\big\downarrow$ 1479

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan(\cdots}{}\right.\right.}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\big\downarrow$ 25

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan(\sqrt{\cdots}}{}\right.\right.}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\big\downarrow$ 27

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)+\frac{1}{2}\left(\right.\right.}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\big\downarrow$ 1103

$$\frac{3\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}-\text{lo}\right.\right.}{2bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{c}{2bd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

input
```
Int[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(5/2),x]
```

output
```
-1/2*c/(b*d*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) + (3*((-(ArcTan[1
 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a +
b*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2
*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2])
)/2)*Sqrt[c*Sec[a + b*x]])/(2*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(200) = 400$.

Time = 5.77 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.76

method	result
default	$-\frac{\sqrt{2}\left(-3\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)-2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)-2\cos(bx+a)-\sin(bx+a)+\csc(bx+a)+2}{\cos(bx+a)-1}\right)+3\ln\left(\right.\right.}{}$

input
```
int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/256/b*2^(1/2)/d^2*(-3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin
(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x
+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a
)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x
+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+6*arctan((-(-2*sin(b*x+a)*cos
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))-6
*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(
b*x+a)-1)/(cos(b*x+a)-1))+(4*cos(b*x+a)+4)*sin(b*x+a)*(-2*sin(b*x+a)*cos(b
*x+a)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*sin(b*x+a)^3*(c*sec(b*x+a))^(1/2
)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*csc(b*x+a))^(1/2)*sec
(1/2*b*x+1/2*a)^5*csc(1/2*b*x+1/2*a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(200) = 400$.

Time = 0.13 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.01

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}d\sqrt{\frac{c}{d}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{d}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2c}\right)+3\sqrt{2}d\sqrt{\frac{c}{d}}\mathrm{arc}}{}$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(6*sqrt(2)*d*sqrt(c/d)*arctan(-1/2*sqrt(2)*sqrt(c/d)*sqrt(c/cos(b*x +
 a))*sqrt(d/sin(b*x + a))*(cos(b*x + a) - sin(b*x + a))/c) + 3*sqrt(2)*d*s
qrt(c/d)*arctan(-1/2*(sqrt(2)*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*
x + a)) + 2*c*cos(b*x + a) + 2*c*sin(b*x + a))/(c*cos(b*x + a) - c*sin(b*x
 + a))) + 3*sqrt(2)*d*sqrt(c/d)*arctan(-1/2*(sqrt(2)*sqrt(c/d)*sqrt(c/cos(
b*x + a))*sqrt(d/sin(b*x + a)) - 2*c*cos(b*x + a) - 2*c*sin(b*x + a))/(c*c
os(b*x + a) - c*sin(b*x + a))) + 3*sqrt(2)*d*sqrt(c/d)*log(2*sqrt(2)*(cos(
b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/d)*sqrt(c/
cos(b*x + a))*sqrt(d/sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) -
3*sqrt(2)*d*sqrt(c/d)*log(-2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(
b*x + a) - cos(b*x + a))*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a
)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) + 16*(cos(b*x + a)^3 - cos(b*x + a
))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)))/(b*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{c\sec\left(bx+a\right)}}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{c\sec\left(bx+a\right)}}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{c}{\cos(a+b\,x)}}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(5/2),x)
```

output
```
int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^3}dx\right)}{d^3}$$

input
```
int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/csc(a + b*x)*
*3,x))/d**3
```

3.238 $\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = \frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}} - \frac{16cd^3(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{21b} - \frac{2cd(d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}}{7b}$$

output
```
64/21*c*d^5*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(1/2)-16/21*c*d^3*(d*csc
(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2)/b-2/7*c*d*(d*csc(b*x+a))^(7/2)*(c*sec(
b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2cd^5(-32+8\csc^2(a+bx)+3\csc^4(a+bx))\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d^5*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[c*Sec[a + b*x]]
)/(21*b*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3105, 3042, 3105, 3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{9/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{8}{7}d^2\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}d^2\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

$$\Big\downarrow \text{3105}$$

$$\frac{8}{7}d^2\left(\frac{4}{3}d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}d^2\left(\frac{4}{3}d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

$$\Big\downarrow \text{3099}$$

$$\frac{8}{7}d^2\left(\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

input
```
Int[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(7/2)*Sqrt[c*Sec[a + b*x]])/(7*b) + (8*d^2*((8*c*
d^3*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc[a + b
*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(3*b)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\left(32\cos(bx+a)^4-56\cos(bx+a)^2+21\right)\sqrt{c\sec(bx+a)}\,c\,d^4\sqrt{d\csc(bx+a)}\,\csc(bx+a)^3}{21b}$	60

input
```
int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/b*(32*cos(b*x+a)^4-56*cos(b*x+a)^2+21)*(c*sec(b*x+a))^(1/2)*c*d^4*(d*
csc(b*x+a))^(1/2)*csc(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2\left(32\,cd^4\cos\left(bx+a\right)^4-56\,cd^4\cos\left(bx+a\right)^2+21\,cd^4\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{21\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/21*(32*c*d^4*cos(b*x + a)^4 - 56*c*d^4*cos(b*x + a)^2 + 21*c*d^4)*sqrt(
c/cos(b*x + a))*sqrt(d/sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = \int (d\csc(bx+a))^{\frac{9}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = \int (d\csc(bx+a))^{\frac{9}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx =$$
$$-\frac{16\,c\,d^4\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,\left(41\,\sin\left(a+b\,x\right)-29\,\sin\left(3\,a+3\,b\,x\right)+12\,\sin\left(5\,a+5\,b\,x\right)-2\,\sin\left(7\,a+7\right.\right.}{21\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-10\right)}$$

input
```
int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(9/2),x)
```

output
```
-(16*c*d^4*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(41*sin(a + b*x)
- 29*sin(3*a + 3*b*x) + 12*sin(5*a + 5*b*x) - 2*sin(7*a + 7*b*x)))/(21*b*(
15*cos(2*a + 2*b*x) - 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

Reduce [F]

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^4\sec(bx+a)\,dx\right)c\,d^4$$

input
```
int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**4*
sec(a + b*x),x)*c*d**4
```

3.239 $\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}} - \frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b} - \frac{2cd(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}{5b} - \frac{24c^2d^4E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
24/5*c*d^5*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(3/2)-12/5*c*d^3*(d*csc(b
*x+a))^(1/2)*(c*sec(b*x+a))^(1/2)/b-2/5*c*d*(d*csc(b*x+a))^(5/2)*(c*sec(b*
x+a))^(1/2)/b+24/5*c^2*d^4*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/(d*csc(b
*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx =$$
$$-\frac{2cd^3\sqrt{d\csc(a+bx)}\left(\cot^2(a+bx)\left(6\cos(2(a+bx))+\csc^2(a+bx)\right)+12\cos^2(a+bx)\sqrt[4]{-\cot^2(a+bx)}\,\text{H}\right.}{5b}$$

input
```
Integrate[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d^3*Sqrt[d*Csc[a + b*x]]*(Cot[a + b*x]^2*(6*Cos[2*(a + b*x)] + Csc[a
 + b*x]^2) + 12*Cos[a + b*x]^2*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-
1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^2)/(5*b)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3105, 3042, 3105, 3042, 3106, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{6}{5}d^2\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}d^2\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$\big\downarrow$ 3105

$$\frac{6}{5}d^2\left(2d^2\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}dx-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$\big\downarrow$ 3042

$$\frac{6}{5}d^2\left(2d^2\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}dx-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$\big\downarrow$ 3106

$$\frac{6}{5}d^2\left(2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-2c^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$\big\downarrow$ 3042

$$\frac{6}{5}d^2\left(2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-2c^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$\big\downarrow$ 3110

$$\frac{6}{5}d^2\left(2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a}{}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$\big\downarrow$ 3042

$$\frac{6}{5}d^2\left(2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a}{}\right)-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3052}$$

$$\frac{6}{5}d^2\left(2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a}{b}\right.$$
$$\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}d^2\left(2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a}{b}\right.$$
$$\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6}{5}d^2\left(2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(}{b}\right.$$
$$\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

input
```
Int[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]])/(5*b) + (6*d^2*((-2*c
*d*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/b + 2*d^2*((2*c*d*Sqrt[c*Sec
[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c^2*EllipticE[a - Pi/4 + b*x,
2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])))
)/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.39

method	result
default	$\frac{\left(-\frac{2(-12\cos(bx+a)-12)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(bx+a)+\csc}}{5}\right.}{}$

input
```
int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/5*(-12*cos(b*x+a)-12)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x
+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/
2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)-2/5*(6*cos(b*x+a)+6)*(2*cot(b*x+a)-2*
csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+c
sc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)-24/5*cos(
b*x+a)+12/5-2/5*csc(b*x+a)^2)*(c*sec(b*x+a))^(1/2)*c*d^3*(d*csc(b*x+a))^(1
/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.59

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = \frac{2\left(3\left(cd^3\cos(bx+a)^2-cd^3\right)\sqrt{-4i\,cd}E(\arcsin(\cos(bx+a)+i\,\sin(bx+a

output
```
2/5*(3*(c*d^3*cos(b*x + a)^2 - c*d^3)*sqrt(-4*I*c*d)*elliptic_e(arcsin(cos
(b*x + a) + I*sin(b*x + a)), -1) + 3*(c*d^3*cos(b*x + a)^2 - c*d^3)*sqrt(4
*I*c*d)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - 3*(c*d^3*c
os(b*x + a)^2 - c*d^3)*sqrt(-4*I*c*d)*elliptic_f(arcsin(cos(b*x + a) + I*s
in(b*x + a)), -1) - 3*(c*d^3*cos(b*x + a)^2 - c*d^3)*sqrt(4*I*c*d)*ellipti
c_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (12*c*d^3*cos(b*x + a)^4
- 18*c*d^3*cos(b*x + a)^2 + 5*c*d^3)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x +
 a)))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = \int (d\csc(bx+a))^{\frac{7}{2}}\,(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = \int (d\csc(bx+a))^{\frac{7}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{7/2}\,dx$$

input
```
int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(7/2),x)
```

output
```
int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^3\sec(bx+a)\,dx\right)c\,d^3$$

input
```
int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**3*
sec(a + b*x),x)*c*d**3
```

3.240 $\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = \frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{3b}$$

output
```
8/3*c*d^3*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(1/2)-2/3*c*d*(d*csc(b*x+a
))^(3/2)*(c*sec(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2cd^3(-4+\csc^2(a+bx))\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d^3*(-4 + Csc[a + b*x]^2)*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a +
b*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3105, 3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{4}{3}d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}dx-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3099}$$

$$\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}$$

input
```
Int[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(8*c*d^3*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc[
a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{2\left(4\cos(bx+a)^{2}-3\right)\sqrt{c\sec(bx+a)}\,c\,d^{2}\sqrt{d\csc(bx+a)}\,\csc(bx+a)}{3b}$	48

input
```
int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*(4*cos(b*x+a)^2-3)*(c*sec(b*x+a))^(1/2)*c*d^2*(d*csc(b*x+a))^(1/2)*
csc(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2\,\left(4\,cd^2\cos\left(bx+a\right)^2-3\,cd^2\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{3\,b\sin\left(bx+a\right)}$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(4*c*d^2*cos(b*x + a)^2 - 3*c*d^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*
x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = \int \left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = \int (d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = \frac{2\,c\,d^2\,(2\,\sin(a+b\,x)-\sin(3\,a+3\,b\,x))\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}}{3\,b\sin(a+b\,x)^2}$$

input
```
int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(5/2),x)
```

output
```
(2*c*d^2*(2*sin(a + b*x) - sin(3*a + 3*b*x))*(c/cos(a + b*x))^(1/2)*(d/sin
(a + b*x))^(1/2))/(3*b*sin(a + b*x)^2)
```

Reduce [F]

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^2\sec(bx+a)\,dx\right)c\,d^2$$

input
```
int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**2*
sec(a + b*x),x)*c*d**2
```

3.241 $\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 125

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx = \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{b} - \frac{4c^2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
4*c*d^3*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(3/2)-2*c*d*(d*csc(b*x+a))^(
1/2)*(c*sec(b*x+a))^(1/2)/b+4*c^2*d^2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))
/b/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}\left(\cos(2(a+bx))\cot^2(a+bx)+2\cos^2(a+bx)\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\right.\right.}{b}$$

input
```
Integrate[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d*Sqrt[d*Csc[a + b*x]]*(Cos[2*(a + b*x)]*Cot[a + b*x]^2 + 2*Cos[a +
b*x]^2*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b
*x]^2])*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^2)/b
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3105, 3042, 3106, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3105}$$

$$2d^2\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}dx-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}dx-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{3106}$$

$$2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-2c^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-2c^2\int\frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$\downarrow$ 3110

$$2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$\downarrow$ 3042

$$2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$\downarrow$ 3052

$$2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$\downarrow$ 3042

$$2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

$\downarrow$ 3119

$$2d^2\left(\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}\right)-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

input
```
Int[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/b + 2*d^2*((2*c*d*Sqrt[
c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c^2*EllipticE[a - Pi/4 +
b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x
]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.74

method	result
default	$\frac{\left(2-4\cos(bx+a)+(4\cos(bx+a)+4)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{Elliptic}\right.}{}$

input
```
int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2-4*cos(b*x+a)+(4*cos(b*x+a)+4)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*c
ot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-
cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-2*cos(b*x+a)-2)*(2*cot(b*x+a
)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+
a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2))*(c*s
ec(b*x+a))^(1/2)*c*d*(d*csc(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.26

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx = \frac{\sqrt{-4i\,cd}cdE(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{4i\,cd}cdE(\mathrm{arcsi}}{}$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(-4*I*c*d)*c*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)
+ sqrt(4*I*c*d)*c*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)
- sqrt(-4*I*c*d)*c*d*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)
 - sqrt(4*I*c*d)*c*d*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)
 - 2*(2*c*d*cos(b*x + a)^2 - c*d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)
))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx = \int (d\csc(bx+a))^{\frac{3}{2}}\,(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx=\int (d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx=\int \left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}dx$$

input
```
int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2),x)
```

output
```
int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx=\sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)\sec(bx+a)\,dx\right)cd$$

input
```
int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)*sec
(a + b*x),x)*c*d
```

3.242 $\int \sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int \sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2} \, dx = \frac{2cd\sqrt{c \sec(a+bx)}}{b\sqrt{d \csc(a+bx)}}$$

output
```
2*c*d*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2} \, dx = \frac{2cd\sqrt{c \sec(a+bx)}}{b\sqrt{d \csc(a+bx)}}$$

input
```
Integrate[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(2*c*d*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}dx$$

$$\Big\downarrow \text{3099}$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

input
```
Int[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2),x]
```

output
```
(2*c*d*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{2\sin(bx+a)\sqrt{c\sec(bx+a)}\,c\sqrt{d\csc(bx+a)}}{b}$	33

input
```
int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*sin(b*x+a)*(c*sec(b*x+a))^(1/2)*c*(d*csc(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}\,dx = \frac{2\,c\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sin(bx+a)}{b}$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2*c*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sin(b*x + a)/b
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2}\,dx = \int \sqrt{d \csc(bx+a)}(c \sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2}\,dx = \int \sqrt{d \csc(bx+a)}(c \sec(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2}\,dx = \frac{2\,c\,\sin\left(a+b\,x\right)\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}}{b}$$

input
```
int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(1/2),x)
```

output
```
(2*c*sin(a + b*x)*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2))/b
```

Reduce [F]

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\sec(bx+a)\,dx\right)c$$

input
```
int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x),x)*
c
```

3.243 $\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2c^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
2*c*d*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(3/2)+2*c^2*EllipticE(cos(a+1/
4*Pi+b*x),2^(1/2))/b/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2
*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = -\frac{2cd\left(-1+\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sec[a + b*x])^(3/2)/Sqrt[d*Csc[a + b*x]],x]
```

output
```
(-2*c*d*(-1 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Cs
c[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3106, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \ 3106$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-2c^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-2c^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \ 3110$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \ 3052$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(3/2)/Sqrt[d*Csc[a + b*x]],x]
```

output
```
(2*c*d*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin
[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(80) = 160$.

Time = 1.61 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.49

method	result
default	$\left(2-2\cos(bx+a)+\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,(-\cos(bx+a)-1)\,\mathrm{Ellipti}$

input
```
int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2-2*cos(b*x+a)+(cot(b*x+a)-csc(b*x+a))^(1/2)*(-cot(b*x+a)+csc(b*x+a)+
1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(-cos(b*x+a)-1)*EllipticF((-c
ot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(2*cos(b*x+a)+2)*(cot(b*x+a)-cs
c(b*x+a))^(1/2)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a
)+2)^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2)))*(c*sec
(b*x+a))^(1/2)*c/(d*csc(b*x+a))^(1/2)*csc(b*x+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.74

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{\sqrt{-4i\,cd}cE(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+\sqrt{4i\,cd}cE(\arcsin\left(\cos\left(\right.\right.}{}$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-4*I*c*d)*c*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1
) + sqrt(4*I*c*d)*c*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)
- sqrt(-4*I*c*d)*c*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) -
 sqrt(4*I*c*d)*c*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - 4
*(c*cos(b*x + a)^2 - c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)))/(b*d)
```

Sympy [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{(c\sec\left(a+bx\right))^{\frac{3}{2}}}{\sqrt{d\csc\left(a+bx\right)}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(1/2),x)
```

output
```
Integral((c*sec(a + b*x))**(3/2)/sqrt(d*csc(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{\sqrt{d\csc(bx+a)}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{\sqrt{d\csc(bx+a)}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}{\sqrt{\frac{d}{\sin(a+b\,x)}}}\,dx$$

input
```
int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(1/2),x)
```

output
```
int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\sec(bx+a)}{\csc(bx+a)}dx\right)c}{d}$$

input
```
int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x))/
csc(a + b*x),x)*c)/d
```

3.244 $\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} + \frac{c^2\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}} - \frac{c^2\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}} - \frac{c^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}$$

output
```
2*c*(c*sec(b*x+a))^(1/2)/b/d/(d*csc(b*x+a))^(1/2)-1/2*c^2*arctan(-1+2^(1/2
)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/d^2/(c
*sec(b*x+a))^(1/2)-1/2*c^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a
))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/d^2/(c*sec(b*x+a))^(1/2)-1/2*c^2*arcta
nh(2^(1/2)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(d*csc(b*x+a))^(1/2)*tan(b*x+a
)^(1/2)*2^(1/2)/b/d^2/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.57

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{c\left(4+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}-\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cot\cdots}}{1+\sqrt{\text{co}\cdots}}\right.\right.\cdots}{2bd\sqrt{d\csc(a+bx)}\cdots}$$

input
```
Integrate[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(3/2),x]
```

output
```
(c*(4 + Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^
2)^(1/4))]*(Cot[a + b*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*x]
^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(1/4))*Sqrt[c*Sec[
a + b*x]])/(2*b*d*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3104, 3042, 3109, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{c^2\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{c^2\int\frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{ 3109}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\tan(a+bx)}}dx}{d^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\tan(a+bx)}}dx}{d^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\tan(a+bx)}(\tan^2(a+bx)+1)}d\tan(a+bx)}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+\right.\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{}{-\tan}\right.\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 217$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{ta\right.}}{\sqrt{2}}\right.\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 1479$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+\right.}}{2\sqrt{2}}\right.\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(\right.\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 1103$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}-\frac{2c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\right.}\right.}{2\sqrt{2}}\right.\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(3/2),x]
```

output
```
(2*c*Sqrt[c*Sec[a + b*x]])/(b*d*Sqrt[d*Csc[a + b*x]]) - (2*c^2*Sqrt[d*Csc[
a + b*x]]*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[a
 + b*x]] + Tan[a + b*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Ta
n[a + b*x]]/(2*Sqrt[2]))/2)*Sqrt[Tan[a + b*x]])/(b*d^2*Sqrt[c*Sec[a + b*x]
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(209) = 418$.

Time = 6.46 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.51

method	result	size
default	Expression too large to display	873

input
```
int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cos(b*x+a)*
cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))*cos(b*x
+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-(-2*sin(b
*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x
+a)-1))*cos(b*x+a)-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(
cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-
1))*cos(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan(
((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1
)/(cos(b*x+a)-1))*cos(b*x+a)+(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(c
os(b*x+a)-1))-2*arctan((-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b
*x+a)+1)^2)^(1/2)-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(c
os(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1
))+2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+
cos(b*x+a)-1)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(209) = 418$.

Time = 0.14 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.04

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{2\sqrt{2}cd\sqrt{\frac{c}{d}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{d}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)\sin(bx+a)}{c\cos(bx+a)-c\sin(bx+a)}\right)-\sqrt{2}cd\sqrt{\frac{c}{d}}\operatorname{arcta}}{}$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(2)*c*d*sqrt(c/d)*arctan(-sqrt(2)*sqrt(c/d)*sqrt(c/cos(b*x + a)
)*sqrt(d/sin(b*x + a))*cos(b*x + a)*sin(b*x + a)/(c*cos(b*x + a) - c*sin(b
*x + a))) - sqrt(2)*c*d*sqrt(c/d)*arctan(-1/2*(sqrt(2)*sqrt(c/d)*sqrt(c/co
s(b*x + a))*sqrt(d/sin(b*x + a)) + 2*c*cos(b*x + a) + 2*c*sin(b*x + a))/(c
*cos(b*x + a) - c*sin(b*x + a))) - sqrt(2)*c*d*sqrt(c/d)*arctan(-1/2*(sqrt
(2)*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) - 2*c*cos(b*x + a)
 - 2*c*sin(b*x + a))/(c*cos(b*x + a) - c*sin(b*x + a))) + sqrt(2)*c*d*sqrt
(c/d)*log(2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*
x + a))*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) + 4*c*cos(b*x
+ a)*sin(b*x + a) + c) - sqrt(2)*c*d*sqrt(c/d)*log(-2*sqrt(2)*(cos(b*x + a
)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/d)*sqrt(c/cos(b*x
 + a))*sqrt(d/sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) + 16*c*sq
rt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sin(b*x + a))/(b*d^2)
```

Sympy [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{(c\sec{(a+bx)})^{\frac{3}{2}}}{(d\csc{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(3/2),x)
```

output
```
Integral((c*sec(a + b*x))**(3/2)/(d*csc(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{(c\sec{(bx+a)})^{\frac{3}{2}}}{(d\csc{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(3/2),x)
```

output
```
int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\sec(bx+a)}{\csc(bx+a)^2}dx\right)c}{d^2}$$

input
```
int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x))/
csc(a + b*x)**2,x)*c)/d**2
```

3.245 $\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{3c^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{bd^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
2*c*(c*sec(b*x+a))^(1/2)/b/d/(d*csc(b*x+a))^(3/2)+3*c^2*EllipticE(cos(a+1/
4*Pi+b*x),2^(1/2))/b/d^2/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b
*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.71 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx = -\frac{c\left(-2+3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(5/2),x]
```

output
```
-((c*(-2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc
[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2)))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3104, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3110}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(5/2),x]
```

output
```
(2*c*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2)) - (3*c^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt
[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(85) = 170$.

Time = 1.71 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.51

method	result
default	$\frac{\left((6\cos(bx+a)+6)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\ \text{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\ \sqrt{c}\right.}{}$

input
```
int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*((6*cos(b*x+a)+6)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*EllipticE((-cot(b
*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)
*(cot(b*x+a)-csc(b*x+a))^(1/2)+(-3*cos(b*x+a)-3)*(-cot(b*x+a)+csc(b*x+a)+1
)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*
EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*cos(b*x+a)^2-6*c
os(b*x+a)+4)*(c*sec(b*x+a))^(1/2)*c/d^2/(d*csc(b*x+a))^(1/2)*csc(b*x+a)
```

Fricas [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*sec(b*x + a)/(d^3*csc
(b*x + a)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{3}{2}}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c \sec(a+bx))^{3/2}}{(d \csc(a+bx))^{5/2}} \, dx = \int \frac{(c \sec(bx+a))^{\frac{3}{2}}}{(d \csc(bx+a))^{\frac{5}{2}}} \, dx$$

input
```
integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c \sec(a+bx))^{3/2}}{(d \csc(a+bx))^{5/2}} \, dx = \int \frac{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}} \, dx$$

input
```
int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(5/2),x)
```

output
```
int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c \sec(a+bx))^{3/2}}{(d \csc(a+bx))^{5/2}} \, dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\sec(bx+a)}{\csc(bx+a)^3} dx\right) c}{d^3}$$

input
```
int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x))/
csc(a + b*x)**3,x)*c)/d**3
```

3.246 $\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx = \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}} - \frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b} - \frac{2cd(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}}{7b} + \frac{40c^2d^4\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{21b}$$

output
```
40/21*c*d^5*(c*sec(b*x+a))^(3/2)/b/(d*csc(b*x+a))^(1/2)-20/21*c*d^3*(d*csc
(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2)/b-2/7*c*d*(d*csc(b*x+a))^(7/2)*(c*sec(
b*x+a))^(3/2)/b+40/21*c^2*d^4*(d*csc(b*x+a))^(1/2)*InverseJacobiAM(a-1/4*P
i+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.55

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx =$$
$$-\frac{2cd^5\left(-7+\cot^2(a+bx)\left(13+3\csc^2(a+bx)\right)+20(-\cot^2(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(\right.\right.}{21b\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d^5*(-7 + Cot[a + b*x]^2*(13 + 3*Csc[a + b*x]^2) + 20*(-Cot[a + b*x]
^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*(c*Sec[a + b*x
])^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3105, 3042, 3105, 3042, 3106, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{9/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{9/2}dx$$
$$\Big\downarrow \text{3105}$$
$$\frac{10}{7}d^2\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}d^2\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

$\downarrow$ 3105

$$\frac{10}{7}d^2\left(2d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

$\downarrow$ 3042

$$\frac{10}{7}d^2\left(2d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

$\downarrow$ 3106

$$\frac{10}{7}d^2\left(2d^2\left(\frac{2}{3}c^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+}{3b}\right.$$
$$\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

$\downarrow$ 3042

$$\frac{10}{7}d^2\left(2d^2\left(\frac{2}{3}c^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+}{3b}\right.$$
$$\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

$\downarrow$ 3110

$$\frac{10}{7}d^2\left(2d^2\left(\frac{2}{3}c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}d\right.\right.$$
$$\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

$\downarrow$ 3042

$$\frac{10}{7}d^2\left(2d^2\left(\frac{2}{3}c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}d\right.\right.$$
$$\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

$$\Big\downarrow \text{3053}$$

$$\frac{10}{7}d^2\left(2d^2\left(\frac{2}{3}c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{7}d^2\left(2d^2\left(\frac{2}{3}c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}\right.$$

$$\Big\downarrow \text{3120}$$

$$\frac{10}{7}d^2\left(2d^2\left(\frac{2c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right.\right.$$
$$\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

input
```
Int[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2))/(7*b) + (10*d^2*((-
2*c*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(3*b) + 2*d^2*((2*c*d
*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) + (2*c^2*Sqrt[d*Csc[a
+ b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2
*b*x]])/(3*b))))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\frac{\left(-\frac{2(-20\cos(bx+a)-20)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(bx+a)+\cdots}}{21}\right.}{b\cdots}$$

input
```
int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x)
```

output
```
1/b*(-2/21*(-20*cos(b*x+a)-20)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*
x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1
/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+40/21*cot(b*x+a)^3-20/7*cot(b*x+a)*c
sc(b*x+a)^2+2/3*sec(b*x+a)*csc(b*x+a)^3)*(c*sec(b*x+a))^(1/2)*c^2*d^4*(d*c
sc(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.36

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx =$$

$$-\frac{2\left(10\left(i\,c^2d^4\cos(bx+a)^3-i\,c^2d^4\cos(bx+a)\right)\sqrt{-4i\,cd}\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\mid-1)\sin\cdots\right.}{\cdots}$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/21*(10*(I*c^2*d^4*cos(b*x + a)^3 - I*c^2*d^4*cos(b*x + a))*sqrt(-4*I*c*
d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 10
*(-I*c^2*d^4*cos(b*x + a)^3 + I*c^2*d^4*cos(b*x + a))*sqrt(4*I*c*d)*ellipt
ic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (20*c^2*d^4
*cos(b*x + a)^4 - 30*c^2*d^4*cos(b*x + a)^2 + 7*c^2*d^4)*sqrt(c/cos(b*x +
a))*sqrt(d/sin(b*x + a)))/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a
))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx = \int (d\csc(bx+a))^{\frac{9}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx = \int (d\csc(bx+a))^{\frac{9}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{9/2}dx$$

input
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(9/2),x)
```

output
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^4\sec(bx+a)^2\,dx\right)c^2d^4$$

input
```
int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**4*
sec(a + b*x)**2,x)*c**2*d**4
```

3.247 $\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{64c^3d^3\sqrt{d\csc(a+bx)}}{15b\sqrt{c\sec(a+bx)}} + \frac{16cd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{15b} - \frac{2cd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}{5b}$$

output
```
-64/15*c^3*d^3*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(1/2)+16/15*c*d^3*(d*
csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2)/b-2/5*c*d*(d*csc(b*x+a))^(5/2)*(c*s
ec(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2cd^3\left(-5+32\cos^2(a+bx)+3\cot^2(a+bx)\right)\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{15b}$$

input
```
Integrate[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d^3*(-5 + 32*Cos[a + b*x]^2 + 3*Cot[a + b*x]^2)*Sqrt[d*Csc[a + b*x]]
*(c*Sec[a + b*x])^(3/2))/(15*b)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3105, 3042, 3106, 3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{7/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{8}{5}d^2\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}d^2\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3106}$$

$$\frac{8}{5}d^2\left(\frac{4}{3}c^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}d^2\left(\frac{4}{3}c^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

$$\Big\downarrow \text{3099}$$

$$\frac{8}{5}d^2\left(\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}-\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

input
```
Int[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2))/(5*b) + (8*d^2*((-8
*c^3*d*Sqrt[d*Csc[a + b*x]])/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*c*d*Sqrt[d*Cs
c[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(3*b)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\frac{2\left(32\cos\left(bx+a\right)^{4}-40\cos\left(bx+a\right)^{2}+5\right)\sqrt{c\sec\left(bx+a\right)}\,c^{2}d^{3}\sqrt{d\csc\left(bx+a\right)}\,\sec\left(bx+a\right)\csc\left(bx+a\right)^{2}}{15b}$$

input
```
int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x)
```

output
```
2/15/b*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+5)*(c*sec(b*x+a))^(1/2)*c^2*d^3*(d
*csc(b*x+a))^(1/2)*sec(b*x+a)*csc(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2\left(32\,c^{2}d^{3}\cos\left(bx+a\right)^{4}-40\,c^{2}d^{3}\cos\left(bx+a\right)^{2}+5\,c^{2}d^{3}\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{15\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)}$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(32*c^2*d^3*cos(b*x + a)^4 - 40*c^2*d^3*cos(b*x + a)^2 + 5*c^2*d^3)*
sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x +
a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = \int (d\csc(bx+a))^{\frac{7}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = \int (d\csc(bx+a))^{\frac{7}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = \frac{16\,c^2\,d^3\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,\left(5\,\cos\left(a+b\,x\right)-3\,\cos\left(3\,a+3\,b\,x\right)-4\,\cos\left(5\right.\right.}{15\,b\,\left(\cos\left(2\,a+2\,b\,x\right)+2\,\cos\left(4\,a+4\,b\,x\right)-\cos\left(6\,a+\right.\right.}$$

input
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(7/2),x)
```

output
```
(16*c^2*d^3*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(5*cos(a + b*x)
- 3*cos(3*a + 3*b*x) - 4*cos(5*a + 5*b*x) + 2*cos(7*a + 7*b*x)))/(15*b*(co
s(2*a + 2*b*x) + 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) - 2))
```

Reduce [F]

$$\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^3\sec(bx+a)^2\,dx\right)c^2d^3$$

input
```
int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**3*
sec(a + b*x)**2,x)*c**2*d**3
```

3.248 $\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx = \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b} + \frac{4c^2d^2\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3b}$$

output
```
4/3*c*d^3*(c*sec(b*x+a))^(3/2)/b/(d*csc(b*x+a))^(1/2)-2/3*c*d*(d*csc(b*x+a
))^(3/2)*(c*sec(b*x+a))^(3/2)/b+4/3*c^2*d^2*(d*csc(b*x+a))^(1/2)*InverseJa
cobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.85 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2c^3d(d\csc(a+bx))^{3/2}\left(-1+\cot^2(a+bx)+2(-\cot^2(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+\right.\right.}{3b\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c^3*d*(d*Csc[a + b*x])^(3/2)*(-1 + Cot[a + b*x]^2 + 2*(-Cot[a + b*x]^2
)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Tan[a + b*x]^2)/
(3*b*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3105, 3042, 3106, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3105}$$

$$2d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}dx-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3106}$$

$$2d^2\left(\frac{2}{3}c^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\left(\frac{2}{3}c^2\int\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$\big\downarrow$ 3110

$$2d^2\left(\frac{2}{3}c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx+\frac{2c}{3}\right.$$
$$-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$\big\downarrow$ 3042

$$2d^2\left(\frac{2}{3}c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx+\frac{2c}{3}\right.$$
$$-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$\big\downarrow$ 3053

$$2d^2\left(\frac{2}{3}c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$\big\downarrow$ 3042

$$2d^2\left(\frac{2}{3}c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

$\big\downarrow$ 3120

$$2d^2\left(\frac{2c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}\right)-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

input
```
Int[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(3*b) + 2*d^2*((2*c
*d*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) + (2*c^2*Sqrt[d*Csc[
a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\frac{\left(\frac{2(2\cos(bx+a)+2)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\ \operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)}\right.}{3}\right.}{b}$$

input
```
int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x)
```

output
```
1/b*(2/3*(2*cos(b*x+a)+2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-
csc(b*x+a))^(1/2)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*EllipticF((-cot(b*x+a)+
csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-4/3*cot(b*x+a)+2/3*sec(b*x+a)*csc(b*x+a))
*(c*sec(b*x+a))^(1/2)*c^2*d^2*(d*csc(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.22

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx =$$

$$-\frac{2\left(i\sqrt{-4i\,cd}c^2d^2\cos\left(bx+a\right)F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)\sin\left(bx+a\right)-i\sqrt{4i\,cd}c^2d^2\,\mathrm{co}\right.}{3\,b\,\mathrm{co}}$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(-4*I*c*d)*c^2*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a)
 + I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(4*I*c*d)*c^2*d^2*cos(b*x + a
)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*
c^2*d^2*cos(b*x + a)^2 - c^2*d^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)
))/(b*cos(b*x + a)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx = \int (d\csc(bx+a))^{\frac{5}{2}}\,(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx=\int (d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx=\int \left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}\,dx$$

input
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2),x)
```

output
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx=\sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^2\sec(bx+a)^2\,dx\right)c^2d^2$$

input
```
int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**2*
sec(a + b*x)**2,x)*c**2*d**2
```

3.249 $\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}+\frac{2cd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{3b}$$

output
```
-8/3*c^3*d*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(1/2)+2/3*c*d*(d*csc(b*x+
a))^(1/2)*(c*sec(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2cd(1+2\cos(2(a+bx)))\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}{3b}$$

input
```
Integrate[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d*(1 + 2*Cos[2*(a + b*x)])*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/
2))/(3*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3106, 3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3106}$$

$$\frac{4}{3}c^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}c^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}$$

$$\Big\downarrow \text{3099}$$

$$\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}-\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-8*c^3*d*Sqrt[d*Csc[a + b*x]])/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*c*d*Sqrt[d
*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\left(-\frac{8\cos(bx+a)}{3}+\frac{2\sec(bx+a)}{3}\right)\sqrt{c\sec(bx+a)}\,c^{2}d\sqrt{d\csc(bx+a)}}{b}$	46

input
```
int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-8/3*cos(b*x+a)+2/3*sec(b*x+a))*(c*sec(b*x+a))^(1/2)*c^2*d*(d*csc(b*x
+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{2\left(4\,c^2d\cos\left(bx+a\right)^2-c^2d\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{3\,b\cos\left(bx+a\right)}$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(4*c^2*d*cos(b*x + a)^2 - c^2*d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x
+ a))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = \int \left(d\csc\left(bx+a\right)\right)^{\frac{3}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = \int (d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = -\frac{4\,c^2\,d\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,(2\,\cos(a+b\,x)+\cos(3\,a+3\,b\,x))}{3\,b\,(\cos(2\,a+2\,b\,x)+1)}$$

input
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(3/2),x)
```

output
```
-(4*c^2*d*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(2*cos(a + b*x) +
cos(3*a + 3*b*x)))/(3*b*(cos(2*a + 2*b*x) + 1))
```

Reduce [F]

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)\sec(bx+a)^2\,dx\right)c^2 d$$

input
```
int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)*sec
(a + b*x)**2,x)*c**2*d
```

3.250 $\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx=\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}+\frac{2c^2\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3b}$$

output
```
2/3*c*d*(c*sec(b*x+a))^(3/2)/b/(d*csc(b*x+a))^(1/2)+2/3*c^2*(d*csc(b*x+a))
^(1/2)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*
x+2*a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx=-\frac{2cd\left(-1+(-\cot^2(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)\right)(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d*(-1 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc
[a + b*x]^2])*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3106, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}dx$$

$$\Big\downarrow \text{3106}$$

$$\frac{2}{3}c^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}c^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3110}$$

$$\frac{2}{3}c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3053

$$\frac{2}{3}c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{2}{3}c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3120

$$\frac{2c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

input
```
Int[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2),x]
```

output
```
(2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) + (2*c^2*Sqrt[d*
Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2
*a + 2*b*x]])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(\frac{2(\cos(bx+a)+1)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\right.}{3}\right.}{b}$

input
```
int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/3*(cos(b*x+a)+1)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*c
sc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+cs
c(b*x+a)+1)^(1/2),1/2*2^(1/2))+2/3*tan(b*x+a))*(c*sec(b*x+a))^(1/2)*c^2*(d
*csc(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx = \frac{-i\sqrt{-4i\,cd}c^2\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|\,-1)+i\sqrt{4i\,cd}c^2\cos(bx \cdots}{3\,b\cos(bx \cdots}$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(-4*I*c*d)*c^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I
*sin(b*x + a)), -1) + I*sqrt(4*I*c*d)*c^2*cos(b*x + a)*elliptic_f(arcsin(c
os(b*x + a) - I*sin(b*x + a)), -1) + 2*c^2*sqrt(c/cos(b*x + a))*sqrt(d/sin
(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx = \int \sqrt{d\csc(bx+a)}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx = \int \sqrt{d\csc(bx+a)}(c\sec(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}\,dx = \int \left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\sqrt{\frac{d}{\sin(a+b\,x)}}\,dx$$

input
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2),x)
```

output
```
int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d \csc(a+bx)}(c \sec(a+bx))^{5/2} \, dx = \sqrt{d}\sqrt{c}\left(\int \sqrt{\sec(bx+a)}\sqrt{\csc(bx+a)}\sec(bx+a)^2 \, dx\right) c^2$$

input

```
int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x)
```

output

```
sqrt(d)*sqrt(c)*int(sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x)**2,
x)*c**2
```

3.251 $\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

output
```
2/3*c*d*(c*sec(b*x+a))^(3/2)/b/(d*csc(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sec[a + b*x])^(5/2)/Sqrt[d*Csc[a + b*x]],x]
```

output
```
(2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*(d*Csc[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}dx$$
$$\Big\downarrow \text{3099}$$
$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

input
```
Int[(c*Sec[a + b*x])^(5/2)/Sqrt[d*Csc[a + b*x]],x]
```

output
```
(2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*(d*Csc[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{2\sqrt{c\sec(bx+a)}\, c^{2}\tan(bx+a)}{3b\sqrt{d\csc(bx+a)}}$	35

input
```
int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*(c*sec(b*x+a))^(1/2)*c^2/(d*csc(b*x+a))^(1/2)*tan(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = -\frac{2\left(c^{2}\cos\left(bx+a\right)^{2}-c^{2}\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{3\,bd\cos\left(bx+a\right)}$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(c^2*cos(b*x + a)^2 - c^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))/
(b*d*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{\sqrt{d\csc(bx+a)}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(5/2)/sqrt(d*csc(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{\sqrt{d\csc(bx+a)}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(5/2)/sqrt(d*csc(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{c^2\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,(\cos(a+b\,x)-\cos(3\,a+3\,b\,x))}{3\,b\,d\,(\cos(2\,a+2\,b\,x)+1)}$$

input
```
int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(1/2),x)
```

output
```
(c^2*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(cos(a + b*x) - cos(3*a
 + 3*b*x)))/(3*b*d*(cos(2*a + 2*b*x) + 1))
```

Reduce [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\sec(bx+a)^2}{\csc(bx+a)}dx\right)c^2}{d}$$

input
```
int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x)**
2)/csc(a + b*x),x)*c**2)/d
```

3.252 $\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{c^2\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3bd^2}$$

output
```
2/3*c*(c*sec(b*x+a))^(3/2)/b/d/(d*csc(b*x+a))^(1/2)-1/3*c^2*(d*csc(b*x+a))
^(1/2)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*
x+2*a)^(1/2)/b/d^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{c\left(2+(-\cot^2(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)\right)(c\sec(a+}{3bd\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(3/2),x]
```

output
```
(c*(2 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b
*x]^2])*(c*Sec[a + b*x])^(3/2))/(3*b*d*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3104, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{3d^2}$$

$$\Big\downarrow \text{3110}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{3d^2}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3bd^2}$$

input
```
Int[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(3/2),x]
```

output
```
(2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*Sqrt[d*Csc[a + b*x]]) - (c^2*Sqrt[d*Cs
c[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
 + 2*b*x]])/(3*b*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

method	result
default	$-\frac{\sqrt{c\sec(bx+a)}\, c^{2} \left(\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\, \sqrt{\cot(bx+a)-\csc(bx+a)}\, \operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right) \sqrt{-\mathrm{co}}\right.}{3b\sqrt{d\csc(bx+a)}\, d}$

input
```
int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(c*sec(b*x+a))^(1/2)*c^2/(d*csc(b*x+a))^(1/2)/d*((2*cot(b*x+a)-2*cs
c(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc
(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)
+csc(b*x+a))-2*sec(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{i\sqrt{-4i\,cd}c^2\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)-i\sqrt{4i\,c\ldots}}{}$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(I*sqrt(-4*I*c*d)*c^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*
sin(b*x + a)), -1) - I*sqrt(4*I*c*d)*c^2*cos(b*x + a)*elliptic_f(arcsin(co
s(b*x + a) - I*sin(b*x + a)), -1) + 4*c^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(
b*x + a))*sin(b*x + a))/(b*d^2*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{(d\csc(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(3/2),x)
```

output
```
int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\sec(bx+a)^2}{\csc(bx+a)^2}dx\right)c^2}{d^2}$$

input
```
int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x)**
2)/csc(a + b*x)**2,x)*c**2)/d**2
```

3.253 $\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} + \frac{c^2\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{c^2\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{c^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

output
```
2/3*c*(c*sec(b*x+a))^(3/2)/b/d/(d*csc(b*x+a))^(3/2)-1/2*c^2*arctan(-1+2^(1
/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/d^2/(d*csc(b*x+a))^(1
/2)/tan(b*x+a)^(1/2)-1/2*c^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x
+a))^(1/2)*2^(1/2)/b/d^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/2*c^2*arc
tanh(2^(1/2)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(c*sec(b*x+a))^(1/2)*2^(1/2)
/b/d^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.57

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{c\left(4+3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{1+\cdots}\right.\cdots\right.}{6bd(d\csc(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(5/2),x]
```

output
```
(c*(4 + 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x
]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a +
b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(3/4))*(c*Sec[
a + b*x])^(3/2))/(6*b*d*(d*Csc[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3104, 3042, 3109, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{c^2\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{d^2}$$

$$\downarrow \text{3042}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{c^2\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{d^2}$$

$$\downarrow \text{3109}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{c^2\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\downarrow \text{3042}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{c^2\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\downarrow \text{3957}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{c^2\sqrt{c\sec(a+bx)}\int \frac{\sqrt{\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\downarrow \text{266}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\downarrow \text{826}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\downarrow \text{1476}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int \frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+b}\right.\right.}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\downarrow \text{1082}$$

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 217

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 1479

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\ldots}{\ldots}\right.\right.}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 25

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\ldots}{\ldots}\right.\right.}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 27

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}-\frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)+\ldots\right.}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 1103

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{2c^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}-\right.\right.}{bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

input
```
Int[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(5/2),x]
```

output
```
(2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*(d*Csc[a + b*x])^(3/2)) - (2*c^2*((-(A
rcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[T
an[a + b*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b
*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*S
qrt[2]))/2)*Sqrt[c*Sec[a + b*x]])/(b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(209) = 418$.

Time = 5.88 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.05

method	result
default	$\left(\frac{\ln\left(\frac{2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)-\cos(bx+a)\cot(bx+a)+\sin(bx+a)+2\cos(bx+a)-\csc(bx+a)+2\cot(bx+a)-2}{\cos(bx+a)-1}\right)\sqrt{-\frac{2\sin(bx+a)\cos(bx}{(\cos(bx+a)+1)}}}{12}\right.$

input
```
int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*ln((2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+
a)-cos(b*x+a)*cot(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a)-2
)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(3*cot
(b*x+a)+3*csc(b*x+a))+1/12*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x
+a)/(cos(b*x+a)+1)^2)^(1/2)*(6*cot(b*x+a)+6*csc(b*x+a))+1/12*ln(-(cos(b*x+
a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))*(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-3*cot(b*x+a)-3*csc(b*x+a))
+1/12*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)
-cos(b*x+a)+1)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)*(6*cot(b*x+a)+6*csc(b*x+a))+2/3*tan(b*x+a))*c^2*(c*sec(b*x+a))^(1/2
)/(d*csc(b*x+a))^(1/2)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(209) = 418$.

Time = 0.14 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.22

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx =$$

$$-\frac{6\sqrt{2}c^2d\sqrt{\frac{c}{d}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{d}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\left(\cos(bx+a)-\sin(bx+a)\right)}{2c}\right)\cos(bx+a)+3\sqrt{2}c^2d\sqrt{\frac{c}{d}}\arctan\left(-\cdots\right.}{\cdots}$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-1/24*(6*sqrt(2)*c^2*d*sqrt(c/d)*arctan(-1/2*sqrt(2)*sqrt(c/d)*sqrt(c/cos(
b*x + a))*sqrt(d/sin(b*x + a))*(cos(b*x + a) - sin(b*x + a))/c)*cos(b*x +
a) + 3*sqrt(2)*c^2*d*sqrt(c/d)*arctan(-1/2*(sqrt(2)*sqrt(c/d)*sqrt(c/cos(b
*x + a))*sqrt(d/sin(b*x + a)) + 2*c*cos(b*x + a) + 2*c*sin(b*x + a))/(c*co
s(b*x + a) - c*sin(b*x + a)))*cos(b*x + a) + 3*sqrt(2)*c^2*d*sqrt(c/d)*arc
tan(-1/2*(sqrt(2)*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) - 2*
c*cos(b*x + a) - 2*c*sin(b*x + a))/(c*cos(b*x + a) - c*sin(b*x + a)))*cos(
b*x + a) + 3*sqrt(2)*c^2*d*sqrt(c/d)*cos(b*x + a)*log(2*sqrt(2)*(cos(b*x +
 a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/d)*sqrt(c/cos(b
*x + a))*sqrt(d/sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) - 3*sqr
t(2)*c^2*d*sqrt(c/d)*cos(b*x + a)*log(-2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x
 + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/d)*sqrt(c/cos(b*x + a))*sqrt(d
/sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) + 16*(c^2*cos(b*x + a)
^2 - c^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)))/(b*d^3*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{(c\sec{(bx+a)})^{\frac{5}{2}}}{(d\csc{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{(c\sec(bx+a))^{\frac{5}{2}}}{(d\csc(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \int \frac{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(5/2),x)
```

output
```
int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\sec(bx+a)^2}{\csc(bx+a)^3}dx\right)c^2}{d^3}$$

input
```
int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*sec(a + b*x)**
2)/csc(a + b*x)**3,x)*c**2)/d**3
```

3.254 $\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}} - \frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

output
```
-8/21*c*d^3*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(3/2)-2/7*c*d*(d*csc(b*x
+a))^(7/2)/b/(c*sec(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{2cd(-5+2\cos(2(a+bx)))(d\csc(a+bx))^{7/2}}{21b(c\sec(a+bx))^{3/2}}$$

input
```
Integrate[(d*Csc[a + b*x])^(9/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(2*c*d*(-5 + 2*Cos[2*(a + b*x)])*(d*Csc[a + b*x])^(7/2))/(21*b*(c*Sec[a +
b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3105, 3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{4}{7}d^2\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}dx-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{7}d^2\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}dx-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3099}$$

$$-\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

input
```
Int[(d*Csc[a + b*x])^(9/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-8*c*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(
d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2\sqrt{d\csc(bx+a)}\, d^{4} \left(4\cot(bx+a)^{3}-7\cot(bx+a)\csc(bx+a)^{2}\right)}{21b\sqrt{c\sec(bx+a)}}$	56

input
```
int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/b*(d*csc(b*x+a))^(1/2)*d^4/(c*sec(b*x+a))^(1/2)*(4*cot(b*x+a)^3-7*cot
(b*x+a)*csc(b*x+a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{(d \csc(a+bx))^{9/2}}{\sqrt{c \sec(a+bx)}}\,dx = -\frac{2\left(4\,d^4\cos\left(bx+a\right)^4-7\,d^4\cos\left(bx+a\right)^2\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{21\left(bc\cos\left(bx+a\right)^2-bc\right)\sin\left(bx+a\right)}$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/21*(4*d^4*cos(b*x + a)^4 - 7*d^4*cos(b*x + a)^2)*sqrt(c/cos(b*x + a))*s
qrt(d/sin(b*x + a))/((b*c*cos(b*x + a)^2 - b*c)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d \csc(a+bx))^{9/2}}{\sqrt{c \sec(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d \csc(a+bx))^{9/2}}{\sqrt{c \sec(a+bx)}}\,dx = \int \frac{\left(d\csc\left(bx+a\right)\right)^{\frac{9}{2}}}{\sqrt{c\sec\left(bx+a\right)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(9/2)/sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(9/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{8\,d^4\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,(11\,\sin(2\,a+2\,b\,x)-7\,\sin(4\,a+4\,b\,x)+\sin(6\,a+6\,b\,x))}{21\,b\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,(15\,\cos(2\,a+2\,b\,x)-6\,\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)-10}$$

input
```
int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(1/2),x)
```

output
```
(8*d^4*(d/sin(a + b*x))^(1/2)*(11*sin(2*a + 2*b*x) - 7*sin(4*a + 4*b*x) +
sin(6*a + 6*b*x)))/(21*b*(c/cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) - 6*c
os(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^4}{\sec(bx+a)}dx\right)d^4}{c}$$

input
```
int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
4)/sec(a + b*x),x)*d**4)/c
```

3.255 $\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}} - \frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}} - \frac{4d^4E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
-4/5*c*d^3*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(3/2)-2/5*c*d*(d*csc(b*x+
a))^(5/2)/b/(c*sec(b*x+a))^(3/2)+4/5*d^4*EllipticE(cos(a+1/4*Pi+b*x),2^(1/
2))/b/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2d^2(d\csc(a+bx))^{3/2}\left(-((-2+\cos(2(a+bx)))\cot^3(a+bx)) + \sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\right.\right.}{5b\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(7/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*d^2*(d*Csc[a + b*x])^(3/2)*(-((-2 + Cos[2*(a + b*x)])*Cot[a + b*x]^3)
+ (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2
]*Sin[2*(a + b*x)])*Tan[a + b*x]^2)/(5*b*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3105, 3042, 3105, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{2}{5}d^2\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}dx-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}d^2\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}dx-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3105}$$

$$\frac{2}{5}d^2\left(-2d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}d^2\left(-2d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3110}$$

$$\frac{2}{5}d^2\left(-\frac{2d^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}d^2\left(-\frac{2d^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2}{5}d^2\left(-\frac{2d^2\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}d^2\left(-\frac{2d^2\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{5}d^2\left(-\frac{2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

input
```
Int[(d*Csc[a + b*x])^(7/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(3/2)) + (2*d^2*((-2
*c*d*Sqrt[d*Csc[a + b*x]])/(b*(c*Sec[a + b*x])^(3/2)) - (2*d^2*EllipticE[a
 - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2
*a + 2*b*x]])))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(109) = 218.

Time = 1.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.73

method	result
default	$-\frac{2\sqrt{d\csc(bx+a)}\,d^3\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\text{EllipticE}\left(\sqrt{-\cot(b\right.\right.}{}$

input
```
int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/b*(d*csc(b*x+a))^(1/2)*d^3/(c*sec(b*x+a))^(1/2)*((-cot(b*x+a)+csc(b*x
+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-2-2*sec(b*x
+a))+(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*
(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1
/2*2^(1/2))*(1+sec(b*x+a))+2+csc(b*x+a)*cot(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.94

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{-4i\,cd}E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+}{}$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/5*((d^3*cos(b*x + a)^2 - d^3)*sqrt(-4*I*c*d)*elliptic_e(arcsin(cos(b*x +
 a) + I*sin(b*x + a)), -1) + (d^3*cos(b*x + a)^2 - d^3)*sqrt(4*I*c*d)*elli
ptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (d^3*cos(b*x + a)^2 -
d^3)*sqrt(-4*I*c*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)
- (d^3*cos(b*x + a)^2 - d^3)*sqrt(4*I*c*d)*elliptic_f(arcsin(cos(b*x + a)
- I*sin(b*x + a)), -1) - 2*(2*d^3*cos(b*x + a)^4 - 3*d^3*cos(b*x + a)^2)*s
qrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)))/(b*c*cos(b*x + a)^2 - b*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{7/2}}{\sqrt{\frac{c}{\cos(a+b\,x)}}}\,dx$$

input
```
int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(1/2),x)
```

output
```
int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^3}{\sec(bx+a)}dx\right)d^3}{c}$$

input
```
int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
3)/sec(a + b*x),x)*d**3)/c
```

3.256 $\int \frac{(d \csc(a+bx))^{5/2}}{\sqrt{c \sec(a+bx)}} dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{(d \csc(a+bx))^{5/2}}{\sqrt{c \sec(a+bx)}} dx = -\frac{2cd(d \csc(a+bx))^{3/2}}{3b(c \sec(a+bx))^{3/2}}$$

output
```
-2/3*c*d*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(d \csc(a+bx))^{5/2}}{\sqrt{c \sec(a+bx)}} dx = -\frac{2cd(d \csc(a+bx))^{3/2}}{3b(c \sec(a+bx))^{3/2}}$$

input
```
Integrate[(d*Csc[a + b*x])^(5/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}dx$$
$$\Big\downarrow \text{3099}$$
$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

input
```
Int[(d*Csc[a + b*x])^(5/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\sqrt{d\csc(bx+a)}\, d^2\cot(bx+a)}{3b\sqrt{c\sec(bx+a)}}$	35

input
```
int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*(d*csc(b*x+a))^(1/2)*d^2/(c*sec(b*x+a))^(1/2)*cot(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2\,d^2\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos{(bx+a)}^2}{3\,bc\sin{(bx+a)}}$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^2/(b*c*sin
(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx=\int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(5/2)/sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx=\int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(5/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx=-\frac{d^2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d}{\sin(a+b\,x)}}}{3\,b\sin\left(a+b\,x\right)^2\,\sqrt{\frac{c}{\cos(a+b\,x)}}}$$

input
```
int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(1/2),x)
```

output
```
-(d^2*sin(2*a + 2*b*x)*(d/sin(a + b*x))^(1/2))/(3*b*sin(a + b*x)^2*(c/cos(
a + b*x))^(1/2))
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^2}{\sec(bx+a)}dx\right)d^2}{c}$$

input
```
int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
2)/sec(a + b*x),x)*d**2)/c
```

3.257 $\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
-2*c*d*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(3/2)+2*d^2*EllipticE(cos(a+1
/4*Pi+b*x),2^(1/2))/b/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+
2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2d^2\left(\cot^2(a+bx)+\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)\right)\tan(a+bx)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(3/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*d^2*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2,
1/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Se
c[a + b*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3105, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3105}$$

$$-2d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-2d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3110}$$

$$-\frac{2d^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{2d^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

input
```
Int[(d*Csc[a + b*x])^(3/2)/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*(c*Sec[a + b*x])^(3/2)) - (2*d^2*Elliptic
E[a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Si
n[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] && !GtQ[n, m]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(80) = 160$.

Time = 1.50 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.09

method	result
default	$\frac{\sqrt{d \csc(bx+a)}\, d \left(2\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\, \sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\, \sqrt{\cot(bx+a)-\csc(bx+a)}\, \operatorname{EllipticE}\left(\sqrt{-\cot(bx+a}\right.\right.}{}$

input
```
int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(d*csc(b*x+a))^(1/2)*d/(c*sec(b*x+a))^(1/2)*(2*(-cot(b*x+a)+csc(b*x+a)
+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2
)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-(-cot(b*x+a)+csc
(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*(-cot(b
*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-
csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*
sec(b*x+a)-(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^
(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(
1/2),1/2*2^(1/2))*sec(b*x+a)-2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.69

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx =$$
$$-\frac{4\,d\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos{(bx+a)}^2 - \sqrt{-4i\,cd}dE(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1) - \sqrt{4i\,cd}}{}$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(4*d*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^2 - sqrt(
-4*I*c*d)*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) - sqrt(4
*I*c*d)*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + sqrt(-4*
I*c*d)*d*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + sqrt(4*I*
c*d)*d*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1))/(b*c)
```

Sympy [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{(d\csc{(a+bx)})^{\frac{3}{2}}}{\sqrt{c\sec{(a+bx)}}}\,dx$$

input
```
integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Integral((d*csc(a + b*x))**(3/2)/sqrt(c*sec(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(3/2)/sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(3/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}{\sqrt{\frac{c}{\cos(a+b\,x)}}}\,dx$$

input
```
int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(1/2),x)
```

output
```
int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)}{\sec(bx+a)}dx\right)d}{c}$$

input
```
int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x))/
sec(a + b*x),x)*d)/c
```

3.258 $\int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} \, dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d \csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}b\sqrt{c \sec(a+bx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d \csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}b\sqrt{c \sec(a+bx)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{d \csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}b\sqrt{c \sec(a+bx)}}$$

output
```
1/2*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1
/2)*2^(1/2)/b/(c*sec(b*x+a))^(1/2)+1/2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*
(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/(c*sec(b*x+a))^(1/2)+1/2*a
rctanh(2^(1/2)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(d*csc(b*x+a))^(1/2)*tan(b
*x+a)^(1/2)*2^(1/2)/b/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec), antiderivative size = 123, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx = -\frac{\left(\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\right)\cot(a+bx)\sqrt{d \csc(a+bx)}}{\sqrt{2}b\cot^2(a+bx)^{3/4}\sqrt{c \sec(a+bx)}}$$

input
```
Integrate[Sqrt[d*Csc[a + b*x]]/Sqrt[c*Sec[a + b*x]],x]
```

output
```
-(((ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))] -
 ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])])*Cot
[a + b*x]*Sqrt[d*Csc[a + b*x]])/(Sqrt[2]*b*(Cot[a + b*x]^2)^(3/4)*Sqrt[c*S
ec[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.45 (sec), antiderivative size = 171, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3109, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx$$

$$\Big\downarrow \text{3109}$$

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\tan(a+bx)}}dx}{\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\tan(a+bx)}}dx}{\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3957

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\tan(a+bx)}(\tan^2(a+bx)+1)}d\tan(a+bx)}{b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 266

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 755

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 1476

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right.\right.}{b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 1082

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+}\right.\right.}{b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 217

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}}\right.\right.}{b\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 1479

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx}}{2\sqrt{2}}\right.\right.}{b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 25

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)-\right.}{b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 27

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a\right.\right.}{b\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 1103

$$\frac{2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan\right.}{2\sqrt{2}}\right.\right.}{b\sqrt{c\sec(a+bx)}}$$

input
```
Int[Sqrt[d*Csc[a + b*x]]/Sqrt[c*Sec[a + b*x]],x]
```

output
```
(2*Sqrt[d*Csc[a + b*x]]*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt
[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[
a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2]))/2)*Sqrt[Tan[a + b*x]])/(b*Sqrt[c*Se
c[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(162) = 324$.

Time = 6.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{(1-\cos(bx+a))^2\csc(bx+a)+2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)+2-2\cos(bx+a)-\sin(bx+a)}{1-\cos(bx+a)}\right)+2\arctan\left(\frac{\sqrt{-\frac{2\sin(bx+a)\cos(}{(\cos(bx+a)+}}}{}\right.\right.}{}$

input
```
int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*2^(1/2)*(ln(1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*(-2*sin
(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)-sin(b
*x+a)))+2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*
x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1))-ln(-1/(1-cos(b*x+a))*(-(1-cos(b*x+a))^2
*csc(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)
-2+2*cos(b*x+a)+sin(b*x+a)))+2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1)))*(d*csc(b*x+a))^(1
/2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(c*sec(b*x+a))^(1/2)*(
-cot(b*x+a)+csc(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(162) = 324$.

Time = 0.12 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2\sqrt{2}\sqrt{\frac{d}{c}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d}{c}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)\sin(bx+a)}{d\cos(bx+a)-d\sin(bx+a)}\right)-\sqrt{2}\sqrt{\frac{d}{c}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d}{c}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx}}}{2\,(d\cos(bx}\right.}{}$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2)*sqrt(d/c)*arctan(-sqrt(2)*sqrt(d/c)*sqrt(c/cos(b*x + a))*s
qrt(d/sin(b*x + a))*cos(b*x + a)*sin(b*x + a)/(d*cos(b*x + a) - d*sin(b*x
+ a))) - sqrt(2)*sqrt(d/c)*arctan(-1/2*(sqrt(2)*sqrt(d/c)*sqrt(c/cos(b*x +
 a))*sqrt(d/sin(b*x + a)) + 2*d*cos(b*x + a) + 2*d*sin(b*x + a))/(d*cos(b*
x + a) - d*sin(b*x + a))) - sqrt(2)*sqrt(d/c)*arctan(-1/2*(sqrt(2)*sqrt(d/
c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) - 2*d*cos(b*x + a) - 2*d*sin(
b*x + a))/(d*cos(b*x + a) - d*sin(b*x + a))) + sqrt(2)*sqrt(d/c)*log(2*sqr
t(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(d/
c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) + 4*d*cos(b*x + a)*sin(b*x +
a) + d) - sqrt(2)*sqrt(d/c)*log(-2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^
2*sin(b*x + a) - cos(b*x + a))*sqrt(d/c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b
*x + a)) + 4*d*cos(b*x + a)*sin(b*x + a) + d))/b
```

Sympy [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{\sqrt{d\csc{(a+bx)}}}{\sqrt{c\sec{(a+bx)}}}\,dx$$

input
```
integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*csc(a + b*x))/sqrt(c*sec(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{\sqrt{d\csc{(bx+a)}}}{\sqrt{c\sec{(bx+a)}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(b*x + a))/sqrt(c*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{\sqrt{d\csc\left(bx+a\right)}}{\sqrt{c\sec\left(bx+a\right)}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(b*x + a))/sqrt(c*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = \int \frac{\sqrt{\frac{d}{\sin(a+b\,x)}}}{\sqrt{\frac{c}{\cos(a+b\,x)}}}\,dx$$

input
```
int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(1/2),x)
```

output
```
int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\sec(bx+a)}dx\right)}{c}$$

input
```
int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/sec(a + b*x),
x))/c
```

3.259 $\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
-EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a)
)^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)\tan(a+bx)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[1/(Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]),x]
```

output
```
((-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2]
*Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \text{ 3110}$$

$$\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{ 3052}$$

$$\frac{\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

input
```
Int[1/(Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]),x]
```

output
```
EllipticE[a - Pi/4 + b*x, 2]/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*
Sqrt[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(49) = 98$.

Time = 1.37 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.45

method	result
default	$-\frac{\left((2\cos(bx+a)+2)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\ \text{EllipticE}\left(\sqrt{-\cot(bx\right.\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*((2*cos(b*x+a)+2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-cot(b*x+a)+csc(b*
x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*EllipticE((-cot(b*x+a)+c
sc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-cos(b*x+a)-1)*(-cot(b*x+a)+csc(b*x+a)+1)
^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*E
llipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+cos(b*x+a)*(-2+2*co
s(b*x+a)))/(c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2)*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c*d*csc(b*x + a)*sec(b
*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sec\left(a+bx\right)}\sqrt{d\csc\left(a+bx\right)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Integral(1/(sqrt(c*sec(a + b*x))*sqrt(d*csc(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\sqrt{c\sec\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)\sec(bx+a)}dx\right)}{cd}$$

input
```
int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
*sec(a + b*x)),x))/(c*d)
```

3.260 $\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx = -\frac{c}{2bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{4\sqrt{2}bd^2\sqrt{c\sec(a+bx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{4\sqrt{2}bd^2\sqrt{c\sec(a+bx)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{4\sqrt{2}bd^2\sqrt{c\sec(a+bx)}}$$

output
```
-1/2*c/b/d/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2)+1/8*arctan(-1+2^(1/2)
*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/d^2/(c*
sec(b*x+a))^(1/2)+1/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1
/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/d^2/(c*sec(b*x+a))^(1/2)+1/8*arctanh(2^(1/2
)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2
^(1/2)/b/d^2/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.63

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx =$$

$$-\frac{\left(4\cos^2(a+bx)+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\right.}{8bd^2\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[1/((d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]),x]
```

output
```
-1/8*((4*Cos[a + b*x]^2 + Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt
[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqr
t[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^
(1/4))*Sqrt[d*Csc[a + b*x]]*Tan[a + b*x])/(b*d^2*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3107, 3042, 3109, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4d^2}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4d^2}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3109}$$

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4d^2\sqrt{c\sec(a+bx)}}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4d^2\sqrt{c\sec(a+bx)}}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}(\tan^2(a+bx)+1)}d\tan(a+bx)}{4bd^2\sqrt{c\sec(a+bx)}}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{2bd^2\sqrt{c\sec(a+bx)}}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bd^2\sqrt{c\sec(a+bx)}}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right.\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 1082

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+b}}{}\right.\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 217

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a-}\right.}{\sqrt{2}}\right.\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 1479

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right.\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 25

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right.\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 1103

$$\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(\right.}{2\sqrt{2}}\right.\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

input
```
Int[1/((d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]),x]
```

output
```
-1/2*c/(b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) + (Sqrt[d*Csc[a +
 b*x]]*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[a +
b*x]] + Tan[a + b*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a
 + b*x]]/(2*Sqrt[2]))/2)*Sqrt[Tan[a + b*x]])/(2*b*d^2*Sqrt[c*Sec[a + b*x]]
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(200) = 400$.

Time = 6.51 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{2}\left(\cos(bx+a)(4\cos(bx+a)+4)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}+2\arctan\left(\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)+\cos(bx+a)-1}{\cos(bx+a)-1}\right)+2\,\mathrm{arcta}\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/256/b*2^(1/2)/d*(cos(b*x+a)*(4*cos(b*x+a)+4)*(-2*sin(b*x+a)*cos(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)+2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))+2*arctan(((-2*sin(b*x+a
)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-
1))+ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(co
s(b*x+a)-1))-ln((2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b
*x+a)-cos(b*x+a)*cot(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a
)-2)/(cos(b*x+a)-1)))*sin(b*x+a)^3/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)*sec(1/2*b*x+1/2*a)^5*cs
c(1/2*b*x+1/2*a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(200) = 400$.

Time = 0.17 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.10

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx =$$

$$-\frac{2\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{\frac{1}{cd}}\cos(bx+a)\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{}{\sin(b}}}{2\,(\mathrm{co}}\right.}{}$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
-1/32*(2*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-sqrt(2)*sqrt(c/cos(b*x + a))*sq
rt(d/sin(b*x + a))*sqrt(1/(c*d))*cos(b*x + a)*sin(b*x + a)/(cos(b*x + a) -
 sin(b*x + a))) - sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*(sqrt(2)*sqrt(c/co
s(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 2*cos(b*x + a) + 2*sin(b*
x + a))/(cos(b*x + a) - sin(b*x + a))) - sqrt(2)*c*d*sqrt(1/(c*d))*arctan(
-1/2*(sqrt(2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) - 2*
cos(b*x + a) - 2*sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + sqrt(2)*c*
d*sqrt(1/(c*d))*log(2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a
) - cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d))
+ 4*cos(b*x + a)*sin(b*x + a) + 1) - sqrt(2)*c*d*sqrt(1/(c*d))*log(-2*sqrt
(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/c
os(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 4*cos(b*x + a)*sin(b*x +
 a) + 1) + 16*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^2*sin
(b*x + a))/(b*c*d^2)
```

Sympy [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sec(a+bx)}\,(d\csc(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Integral(1/(sqrt(c*sec(a + b*x))*(d*csc(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{c}{\cos(a+b\,x)}}\,\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(3/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^2\sec(bx+a)}dx\right)}{c\,d^2}$$

input
```
int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
**2*sec(a + b*x)),x))/(c*d**2)
```

3.261 $\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2bd^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
-1/3*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2)-1/2*EllipticE(cos(a+1
/4*Pi+b*x),2^(1/2))/b/d^2/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*
b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = -\frac{\left(1+\cos(2(a+bx))-3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)\right)\tan(a+bx)}{6bd^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[1/((d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]]),x]
```

output
```
-1/6*((1 + Cos[2*(a + b*x)] - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[
-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(b*d^2*Sqrt[d*Csc[a + b*x]]
*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3107, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{2d^2}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{2d^2}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3110}$$

$$\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{2d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{2d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\int \sqrt{\sin(2a+2bx)}dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(2a+2bx)}dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

input
```
Int[1/((d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]]),x]
```

output
```
-1/3*c/(b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a -
 Pi/4 + b*x, 2]/(2*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Si
n[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(82) = 164$.

Time = 2.64 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.62

method	result
default	$\frac{\left((-6\cos(bx+a)-6)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \text{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\text{co}\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/b*((-6*cos(b*x+a)-6)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-
csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*
(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+(3*cos(b*x+a)+3)*(2*cot(b*x+a)-2*csc(b*x+
a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a
)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+cos(b*x+a)*(4*cos
(b*x+a)^3-10*cos(b*x+a)+6))/(c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2)/d^2*
sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\sqrt{c\sec\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c*d^3*csc(b*x + a)^3*s
ec(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\sqrt{c\sec\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{5}{2}}\sqrt{c\sec(bx+a)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{c}{\cos(a+b\,x)}}\,\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^3\sec(bx+a)}dx\right)}{c\,d^3}$$

input
```
int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
**3*sec(a + b*x)),x))/(c*d**3)
```

3.262 $\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{8d^5\sqrt{d\csc(a+bx)}}{45bc\sqrt{c\sec(a+bx)}} + \frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

output
```
8/45*d^5*(d*csc(b*x+a))^(1/2)/b/c/(c*sec(b*x+a))^(1/2)+2/45*d^3*(d*csc(b*x
+a))^(5/2)/b/c/(c*sec(b*x+a))^(1/2)-2/9*d*(d*csc(b*x+a))^(9/2)/b/c/(c*sec(
b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{2d^3(-7+2\cos(2(a+bx)))\cot^2(a+bx)(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(11/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(2*d^3*(-7 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^2*(d*Csc[a + b*x])^(5/2))/(4
5*b*c*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3103, 3042, 3105, 3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3103}$$

$$-\frac{d^2\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}dx}{9c^2}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{d^2\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}dx}{9c^2}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 3105}$$

$$-\frac{d^2\left(\frac{4}{5}d^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}\right)}{9c^2}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{d^2\left(\frac{4}{5}d^2\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}\right)}{9c^2}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{ 3099}$$

$$-\frac{d^2\left(-\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}\right)}{9c^2}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(11/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*d*(d*Csc[a + b*x])^(9/2))/(9*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*((-8*c*d
^3*Sqrt[d*Csc[a + b*x]])/(5*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Csc[a + b*
x])^(5/2))/(5*b*Sqrt[c*Sec[a + b*x]])))/(9*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{2\sqrt{d\csc(bx+a)}\, d^5\left(4\cot(bx+a)^4-9\csc(bx+a)^2\cot(bx+a)^2\right)}{45bc\sqrt{c\sec(bx+a)}}$	61

input
```
int((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/b*(d*csc(b*x+a))^(1/2)*d^5/c/(c*sec(b*x+a))^(1/2)*(4*cot(b*x+a)^4-9*c
sc(b*x+a)^2*cot(b*x+a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.80

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{2\left(4\,d^5\cos\left(bx+a\right)^5-9\,d^5\cos\left(bx+a\right)^3\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}}{45\left(bc^2\cos\left(bx+a\right)^4-2\,bc^2\cos\left(bx+a\right)^2+bc^2\right)}$$

input
```
integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas"
)
```

output
```
2/45*(4*d^5*cos(b*x + a)^5 - 9*d^5*cos(b*x + a)^3)*sqrt(c/cos(b*x + a))*sq
rt(d/sin(b*x + a))/(b*c^2*cos(b*x + a)^4 - 2*b*c^2*cos(b*x + a)^2 + b*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(11/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx=\int \frac{(d\csc(bx+a))^{\frac{11}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((d*csc(b*x + a))^(11/2)/(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx=\int \frac{(d\csc(bx+a))^{\frac{11}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(11/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx=\frac{8\,d^5\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,\left(9\,\cos\left(2\,a+2\,b\,x\right)+14\,\cos\left(4\,a+4\,b\,x\right)-9\,\cos\left(6\,a+6\,b\,x\right)+\right.}{45\,b\,c\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\left(28\,\cos\left(4\,a+4\,b\,x\right)-56\,\cos\left(2\,a+2\,b\,x\right)-8\,\cos\left(6\,a+6\,b\,x\right)\right.}$$

input
```
int((d/sin(a + b*x))^(11/2)/(c/cos(a + b*x))^(3/2),x)
```

output
```
(8*d^5*(d/sin(a + b*x))^(1/2)*(9*cos(2*a + 2*b*x) + 14*cos(4*a + 4*b*x) -
9*cos(6*a + 6*b*x) + cos(8*a + 8*b*x) - 15))/(45*b*c*(c/cos(a + b*x))^(1/2
)*(28*cos(4*a + 4*b*x) - 56*cos(2*a + 2*b*x) - 8*cos(6*a + 6*b*x) + cos(8*
a + 8*b*x) + 35))
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^{5}}{\sec(bx+a)^{2}}dx\right)d^{5}}{c^{2}}$$

input
```
int((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
5)/sec(a + b*x)**2,x)*d**5)/c**2
```

3.263 $\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{2d^4\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{21bc^2}$$

output
```
2/21*d^3*(d*csc(b*x+a))^(3/2)/b/c/(c*sec(b*x+a))^(1/2)-2/7*d*(d*csc(b*x+a)
)^(7/2)/b/c/(c*sec(b*x+a))^(1/2)-2/21*d^4*(d*csc(b*x+a))^(1/2)*InverseJaco
biAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/c^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{d^3\cos(2(a+bx))(d\csc(a+bx))^{3/2}\left((5+\cos(2(a+bx)))\csc^4(a+bx)-2(-\cot^2(a+bx))^{3/4}\,\text{Hypergeor}\right.}{21bc\left(-2+\csc^2(a+bx)\right)\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
-1/21*(d^3*Cos[2*(a + b*x)]*(d*Csc[a + b*x])^(3/2)*((5 + Cos[2*(a + b*x)])
*Csc[a + b*x]^4 - 2*(-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/
2, Csc[a + b*x]^2]*Sec[a + b*x]^2))/(b*c*(-2 + Csc[a + b*x]^2)*Sqrt[c*Sec[
a + b*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3103, 3042, 3105, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{d^2\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}dx}{7c^2}-\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}dx}{7c^2}-\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3105}$$

$$-\frac{d^2\left(\frac{2}{3}d^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)}{7c^2}-\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\left(\frac{2}{3}d^2\int\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)}{7c^2}-\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3110

$$-\frac{d^2\left(\frac{2}{3}d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx-\frac{2cd(d\csc(a+bx}{3b\sqrt{c\sec(a+}}\right.}{7c^2}$$
$$\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3042

$$-\frac{d^2\left(\frac{2}{3}d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx-\frac{2cd(d\csc(a+bx}{3b\sqrt{c\sec(a+}}\right.}{7c^2}$$
$$\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3053

$$-\frac{d^2\left(\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)}{7c^2}-$$
$$\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3042

$$-\frac{d^2\left(\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)}{7c^2}-$$
$$\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

$\downarrow$ 3120

$$-\frac{d^2\left(\frac{2d^2\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}\right)}{7c^2}-$$
$$\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*d*(d*Csc[a + b*x])^(7/2))/(7*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*((-2*c*d
*(d*Csc[a + b*x])^(3/2))/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*d^2*Sqrt[d*Csc[a
+ b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2
*b*x]])/(3*b)))/(7*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(\frac{2\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)(-1-\sec(bx}{21}}{b\sqrt{c\sec(bx+a)}\,c}$

input
```
int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/21*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(
1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1
/2),1/2*2^(1/2))*(-1-sec(b*x+a))+2/21*(-cos(b*x+a)^2-2)*csc(b*x+a)^3)*(d*c
sc(b*x+a))^(1/2)*d^4/(c*sec(b*x+a))^(1/2)/c
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.36

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\left(i\,d^4\cos\left(bx+a\right)^2-i\,d^4\right)\sqrt{-4i\,cd}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)\,\mathrm{s}}{}$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/21*((I*d^4*cos(b*x + a)^2 - I*d^4)*sqrt(-4*I*c*d)*elliptic_f(arcsin(cos(
b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*d^4*cos(b*x + a)^2 + I*
d^4)*sqrt(4*I*c*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*s
in(b*x + a) + 2*(d^4*cos(b*x + a)^3 + 2*d^4*cos(b*x + a))*sqrt(c/cos(b*x +
 a))*sqrt(d/sin(b*x + a)))/((b*c^2*cos(b*x + a)^2 - b*c^2)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{9/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(3/2),x)
```

output
```
int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^4}{\sec(bx+a)^2}dx\right)d^4}{c^2}$$

input
```
int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
4)/sec(a + b*x)**2,x)*d**4)/c**2
```

3.264 $\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

output
```
-2/5*c*d*(d*csc(b*x+a))^(5/2)/b/(c*sec(b*x+a))^(5/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2d^3\cot^2(a+bx)\sqrt{d\csc(a+bx)}}{5bc\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*d^3*Cot[a + b*x]^2*Sqrt[d*Csc[a + b*x]])/(5*b*c*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3099}$$

$$-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

input
```
Int[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{2\sqrt{d\csc(bx+a)}\, d^{3} \cot(bx+a)^{2}}{5bc\sqrt{c\sec(bx+a)}}$	40

input
```
int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/b*(d*csc(b*x+a))^(1/2)*d^3/c/(c*sec(b*x+a))^(1/2)*cot(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.79

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{2\,d^3\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos\left(bx+a\right)^3}{5\left(bc^2\cos\left(bx+a\right)^2-bc^2\right)}$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/5*d^3*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^3/(b*c^2*co
s(b*x + a)^2 - b*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{2\,d^3\,(\cos(4\,a+4\,b\,x)-1)\,\sqrt{\frac{d}{\sin(a+b\,x)}}}{5\,b\,c\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,(\cos(4\,a+4\,b\,x)-4\,\cos(2\,a+2\,b\,x)+3)}$$

input
```
int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(3/2),x)
```

output
```
(2*d^3*(cos(4*a + 4*b*x) - 1)*(d/sin(a + b*x))^(1/2))/(5*b*c*(c/cos(a + b*
x))^(1/2)*(cos(4*a + 4*b*x) - 4*cos(2*a + 2*b*x) + 3))
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^3}{\sec(bx+a)^2}dx\right)d^3}{c^2}$$

input
```
int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
3)/sec(a + b*x)**2,x)*d**3)/c**2
```

3.265 $\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{d^2\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3bc^2}$$

output
```
-2/3*d*(d*csc(b*x+a))^(3/2)/b/c/(c*sec(b*x+a))^(1/2)-1/3*d^2*(d*csc(b*x+a)
)^(1/2)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b
*x+2*a)^(1/2)/b/c^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.95 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{d\cos(2(a+bx))(d\csc(a+bx))^{3/2}\left(2\cot^2(a+bx)-(-\cot^2(a+bx))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc\right.\right.}{3b\left(-2+\csc^2(a+bx)\right)(c\sec(a+bx))^{3/2}}$$

input
```
Integrate[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
-1/3*(d*Cos[2*(a + b*x)]*(d*Csc[a + b*x])^(3/2)*(2*Cot[a + b*x]^2 - (-Cot[
a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Sec[a
+ b*x]^3)/(b*(-2 + Csc[a + b*x]^2)*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3103, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{d^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{3c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{3c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3110}$$

$$-\frac{d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{3c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{3c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3053

$$-\frac{d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3042

$$-\frac{d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

$\Big\downarrow$ 3120

$$-\frac{d^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3bc^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*Sqrt[d*C
sc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*
a + 2*b*x]])/(3*b*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\sqrt{d\csc(bx+a)}\, d^{2}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\;\text{EllipticF}\left(\sqrt{-\cot(bx}\right.\right.}{3b\sqrt{c\sec(bx+a)}\,c}$

input
```
int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(d*csc(b*x+a))^(1/2)*d^2/(c*sec(b*x+a))^(1/2)/c*((-cot(b*x+a)+csc(b
*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))
^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(1+sec(b*x+
a))+2*csc(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{i\sqrt{-4i\,cd}d^2F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)\sin\left(bx+a\right)-i\sqrt{4i\,cd}}{}$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(I*sqrt(-4*I*c*d)*d^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a))
, -1)*sin(b*x + a) - I*sqrt(4*I*c*d)*d^2*elliptic_f(arcsin(cos(b*x + a) -
I*sin(b*x + a)), -1)*sin(b*x + a) - 4*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(
b*x + a))*cos(b*x + a))/(b*c^2*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}}{\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(3/2),x)
```

output
```
int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^2}{\sec(bx+a)^2}dx\right)d^2}{c^2}$$

input
```
int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
2)/sec(a + b*x)**2,x)*d**2)/c**2
```

3.266 $\int \frac{(d \csc(a+bx))^{3/2}}{(c \sec(a+bx))^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 248

$$\begin{aligned}\int \frac{(d \csc(a+bx))^{3/2}}{(c \sec(a+bx))^{3/2}} \, dx = & -\frac{2d\sqrt{d \csc(a+bx)}}{bc\sqrt{c \sec(a+bx)}} \\ & + \frac{d^2 \arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c \sec(a+bx)}}{\sqrt{2}bc^2\sqrt{d \csc(a+bx)}\sqrt{\tan(a+bx)}} \\ & - \frac{d^2 \arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c \sec(a+bx)}}{\sqrt{2}bc^2\sqrt{d \csc(a+bx)}\sqrt{\tan(a+bx)}} \\ & + \frac{d^2 \text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{c \sec(a+bx)}}{\sqrt{2}bc^2\sqrt{d \csc(a+bx)}\sqrt{\tan(a+bx)}}\end{aligned}$$

output
```
-2*d*(d*csc(b*x+a))^(1/2)/b/c/(c*sec(b*x+a))^(1/2)-1/2*d^2*arctan(-1+2^(1/
2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/c^2/(d*csc(b*x+a))^(1/
2)/tan(b*x+a)^(1/2)-1/2*d^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+
a))^(1/2)*2^(1/2)/b/c^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/2*d^2*arct
anh(2^(1/2)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(c*sec(b*x+a))^(1/2)*2^(1/2)/
b/c^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.65

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx =$$

$$-\frac{d\left(4\cot^2(a+bx)-\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\right.}{2bc\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
-1/2*(d*(4*Cot[a + b*x]^2 - Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sq
rt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) - Sqrt[2]*ArcTanh[(S
qrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2
)^(3/4))*Sqrt[d*Csc[a + b*x]]*Tan[a + b*x]^2)/(b*c*Sqrt[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3103, 3042, 3109, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{d^2\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{c^2}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{c^2}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3109}$$

$$-\frac{d^2\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^2\sqrt{c\sec(a+bx)}\int \frac{\sqrt{\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int \frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+\right.\right.}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\right.}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 217$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 1479$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arc}}{}\right.\right.}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)\right.}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2d^2\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}\right.\right.}{bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

input
```
Int[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*Sqrt[c*Sec[a + b*x]]) - (2*d^2*((-(ArcTan
[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/
(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2
]))/2)*Sqrt[c*Sec[a + b*x]])/(b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(209) = 418$.

Time = 10.62 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{2}\left(4\cos(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^{2}}}+2\arctan\left(\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^{2}}}\sin(bx+a)+\cos(bx+a)-1}{\cos(bx+a)-1}\right)\sin(bx+a)-\ln\left(-\frac{\cos(bx}{}\right.\right.}{}$

input
```
int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*2^(1/2)*(4*cos(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)+2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a
)+cos(b*x+a)-1)/(cos(b*x+a)-1))*sin(b*x+a)-ln(-(cos(b*x+a)*cot(b*x+a)-2*co
t(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*
cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))*sin(b*x+a)+2*arctan(((
-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/
(cos(b*x+a)-1))*sin(b*x+a)+ln((2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)*sin(b*x+a)-cos(b*x+a)*cot(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+
a)+2*cot(b*x+a)-2)/(cos(b*x+a)-1))*sin(b*x+a)+4*(-2*sin(b*x+a)*cos(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2))*(d*csc(b*x+a))^(1/2)*d/(cos(b*x+a)+1)/(-sin(b*x+a
)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/c/(c*sec(b*x+a))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(209) = 418$.

Time = 0.13 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.98

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}cd\sqrt{\frac{d}{c}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d}{c}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2d}\right)+\sqrt{2}cd\sqrt{\frac{d}{c}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d}{c}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{}}{2(d\,c}\right.}{}$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2)*c*d*sqrt(d/c)*arctan(-1/2*sqrt(2)*sqrt(d/c)*sqrt(c/cos(b*x
 + a))*sqrt(d/sin(b*x + a))*(cos(b*x + a) - sin(b*x + a))/d) + sqrt(2)*c*d
*sqrt(d/c)*arctan(-1/2*(sqrt(2)*sqrt(d/c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(
b*x + a)) + 2*d*cos(b*x + a) + 2*d*sin(b*x + a))/(d*cos(b*x + a) - d*sin(b
*x + a))) + sqrt(2)*c*d*sqrt(d/c)*arctan(-1/2*(sqrt(2)*sqrt(d/c)*sqrt(c/co
s(b*x + a))*sqrt(d/sin(b*x + a)) - 2*d*cos(b*x + a) - 2*d*sin(b*x + a))/(d
*cos(b*x + a) - d*sin(b*x + a))) + sqrt(2)*c*d*sqrt(d/c)*log(2*sqrt(2)*(co
s(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(d/c)*sqrt(
c/cos(b*x + a))*sqrt(d/sin(b*x + a)) + 4*d*cos(b*x + a)*sin(b*x + a) + d)
- sqrt(2)*c*d*sqrt(d/c)*log(-2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*si
n(b*x + a) - cos(b*x + a))*sqrt(d/c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x +
 a)) + 4*d*cos(b*x + a)*sin(b*x + a) + d) + 16*d*sqrt(c/cos(b*x + a))*sqrt
(d/sin(b*x + a))*cos(b*x + a))/(b*c^2)
```

Sympy [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(a+bx))^{\frac{3}{2}}}{(c\sec(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Integral((d*csc(a + b*x))**(3/2)/(c*sec(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(3/2),x)
```

output
```
int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)}{\sec(bx+a)^2}dx\right)d}{c^2}$$

input
```
int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x))/
sec(a + b*x)**2,x)*d)/c**2
```

3.267 $\int \frac{\sqrt{d \csc(a+bx)}}{(c \sec(a+bx))^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{\sqrt{d \csc(a+bx)}}{(c \sec(a+bx))^{3/2}} \, dx = \frac{d}{bc\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{\sqrt{d \csc(a+bx)} \operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)}}{2bc^2}$$

output
```
d/b/c/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)+1/2*(d*csc(b*x+a))^(1/2)*I
nverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(
1/2)/b/c^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.73 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d \csc(a+bx)}}{(c \sec(a+bx))^{3/2}} \, dx = \frac{d\left(1+\cos(2(a+bx))-\left(-\cot^2(a+bx)\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a\right.\right.}{2b\sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2}}$$

input
```
Integrate[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(3/2),x]
```

output
```
(d*(1 + Cos[2*(a + b*x)] - (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2,
3/4, 3/2, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(2*b*Sqrt[d*Csc[a + b*x]]*(c*Se
c[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3108, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \ 3108$$

$$\frac{\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \ 3110$$

$$\frac{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bc^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

input
```
Int[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(3/2),x]
```

output
```
d/(b*c*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*
EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/
(2*b*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(\frac{\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)(1+\sec(bx+a}{2}\right.}{b\sqrt{c\sec(bx+a)}\,c}$

input
```
int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1
/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/
2),1/2*2^(1/2))*(1+sec(b*x+a))+sin(b*x+a))*(d*csc(b*x+a))^(1/2)/(c*sec(b*x
+a))^(1/2)/c
```

Fricas [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{d\csc\left(bx+a\right)}}{\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^2*sec(b*x + a)^2), x
)
```

Sympy [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{d\csc\left(a+bx\right)}}{\left(c\sec\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Integral(sqrt(d*csc(a + b*x))/(c*sec(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{d\csc\left(bx+a\right)}}{\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{d\csc(bx+a)}}{(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{d}{\sin(a+b\,x)}}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(3/2),x)
```

output
```
int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\sec(bx+a)^2}dx\right)}{c^2}$$

input
```
int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/sec(a + b*x)*
*2,x))/c**2
```

3.268 $\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

output
```
1/2*d/b/c/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2)+1/8*arctan(-1+2^(1/2)*
tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/c^2/(d*csc(b*x+a))^(1/2)/
tan(b*x+a)^(1/2)+1/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/
2)*2^(1/2)/b/c^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)-1/8*arctanh(2^(1/2)
*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/c^2/(d*cs
c(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \frac{d\left(4\cos^2(a+bx)-\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx \ldots\right.}{8b(d\csc(a+bx \ldots}$$

input
```
Integrate[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)),x]
```

output
```
(d*(4*Cos[a + b*x]^2 - Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]
*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) - Sqrt[2]*ArcTanh[(Sqrt[2
]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(3/
4))*Sec[a + b*x]^3)/(8*b*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3108, 3042, 3109, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3108}$$

$$\frac{\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{4c^2}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{4c^2}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3109}$$

$$\frac{\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{4c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{4c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\sqrt{c\sec(a+bx)}\int \frac{\sqrt{\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{4bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{c\sec(a+bx)}\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int \frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 217

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+$$
$$\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 1479

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 25

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)+\frac{1}{2}\left(\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$
$$\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 1103

$$\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}-\frac{\log(}{}\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

$$\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}$$

input
```
Int[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)),x]
```

output
```
d/(2*b*c*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) + (((-(ArcTan[1 - Sq
rt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]
]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt
[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2]))/2)*
Sqrt[c*Sec[a + b*x]])/(2*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(200) = 400$.

Time = 10.82 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.73

method	result
default	$\frac{\sqrt{2}\left(\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)-2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)-2\cos(bx+a)-\sin(bx+a)+\csc(bx+a)+2}{\cos(bx+a)-1}\right)-\ln\left(-\frac{\cos(bx}{}\right.\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/256/b*2^(1/2)/c*(ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a
)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+cs
c(b*x+a)+2)/(cos(b*x+a)-1))-ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(
b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1))+2*arctan((
(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)
/(cos(b*x+a)-1))+(4*cos(b*x+a)+4)*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2))*sin(b*x+a)^3/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)*sec(1/2*b*x+1/2*a)^5*cs
c(1/2*b*x+1/2*a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(200) = 400$.

Time = 0.12 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \frac{2\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{1}{2}\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{\frac{1}{cd}}(\cos(bx+a\right.}{}$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas
")
```

output
```
1/32*(2*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*sqrt(2)*sqrt(c/cos(b*x + a))
*sqrt(d/sin(b*x + a))*sqrt(1/(c*d))*(cos(b*x + a) - sin(b*x + a))) + sqrt(
2)*c*d*sqrt(1/(c*d))*arctan(-1/2*(sqrt(2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(
b*x + a))*sqrt(1/(c*d)) + 2*cos(b*x + a) + 2*sin(b*x + a))/(cos(b*x + a) -
 sin(b*x + a))) + sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*(sqrt(2)*sqrt(c/co
s(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) - 2*cos(b*x + a) - 2*sin(b*
x + a))/(cos(b*x + a) - sin(b*x + a))) + sqrt(2)*c*d*sqrt(1/(c*d))*log(2*s
qrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(
c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 4*cos(b*x + a)*sin(b*
x + a) + 1) - sqrt(2)*c*d*sqrt(1/(c*d))*log(-2*sqrt(2)*(cos(b*x + a)^3 - c
os(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin
(b*x + a))*sqrt(1/(c*d)) + 4*cos(b*x + a)*sin(b*x + a) + 1) - 16*(cos(b*x
+ a)^3 - cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)))/(b*c^2*d
)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sec(a+bx))^{\frac{3}{2}}\sqrt{d\csc(a+bx)}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Integral(1/((c*sec(a + b*x))**(3/2)*sqrt(d*csc(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\sqrt{\frac{d}{\sin(a+b\,x)}}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(1/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)\sec(bx+a)^2}dx\right)}{c^2d}$$

input
```
int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
*sec(a + b*x)**2),x))/(c**2*d)
```

3.269 $\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = -\frac{c}{3bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}+\frac{1}{6bcd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{\sqrt{d\csc(a+bx)}\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{12bc^2d^2}$$

output
```
-1/3*c/b/d/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2)+1/6/b/c/d/(d*csc(b*x+
a))^(1/2)/(c*sec(b*x+a))^(1/2)+1/12*(d*csc(b*x+a))^(1/2)*InverseJacobiAM(a
-1/4*Pi+b*x,2^(1/2))*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/c^2/d^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.71 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = \frac{-2\cos(2(a+bx))+\frac{\csc^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\csc^2(a+bx)\right)}{\sqrt[4]{-\cot^2(a+bx)}}}{12bcd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)),x]
```

output
```
(-2*Cos[2*(a + b*x)] + (Csc[a + b*x]^2*Hypergeometric2F1[1/2, 3/4, 3/2, Cs
c[a + b*x]^2])/(-Cot[a + b*x]^2)^(1/4))/(12*b*c*d*Sqrt[d*Csc[a + b*x]]*Sqr
t[c*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3107, 3042, 3108, 3042, 3110, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}dx}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}dx}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3108}$$

$$\frac{\frac{\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 3110

$$\frac{\frac{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 3053

$$\frac{\frac{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2c^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 3120

$$\frac{\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bc^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}}{6d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}$$

input
```
Int[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)),x]
```

output
```
-1/3*c/(b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)) + (d/(b*c*Sqrt[d*
Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*EllipticF[a -
Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*c^2))/(6*
d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.86 (sec) , antiderivative size = 998, normalized size of antiderivative = 7.39

method	result	size
default	Expression too large to display	998

input
```
int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/b/c/d*(sin(b*x+a)*cos(b*x+a)*(16*cos(b*x+a)^2-8)+I*(-6*cos(b*x+a)-6
)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(co
t(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2
-1/2*I,1/2*2^(1/2))+I*(6*cos(b*x+a)+6)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2
*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi
((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+(-6*cos(b*x+a)-6)
*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot
(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-
1/2*I,1/2*2^(1/2))+(-6*cos(b*x+a)-6)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*c
ot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((
-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+(8*cos(b*x+a)+8)*(-
cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*
x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1
/2))+(3*cos(b*x+a)+3)*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*
x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)
+csc(b*x+a)+2)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)+(6*cos(b*x+a)+6)*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)
/(cos(b*x+a)+1)^2)^(1/2)+(6*cos(b*x+a)+6)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b
*x+a)+1)^2)^(1/2)*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(...
```

Fricas [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^2*d^2*csc(b*x + a)^2
*sec(b*x + a)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{3}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^2\sec(bx+a)^2}dx\right)}{c^2d^2}$$

input
```
int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
**2*sec(a + b*x)**2),x))/(c**2*d**2)
```

3.270 $\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 295

$$\begin{aligned}
\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx =
&-\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} \\
&+\frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} \\
&-\frac{3\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} \\
&+\frac{3\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} \\
&-\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}
\end{aligned}$$

output
```
-1/4*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2)+3/16/b/c/d/(d*csc(b*x
+a))^(3/2)/(c*sec(b*x+a))^(1/2)+3/64*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(
c*sec(b*x+a))^(1/2)*2^(1/2)/b/c^2/d^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2
)+3/64*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/c
^2/d^2/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)-3/64*arctanh(2^(1/2)*tan(b*x+
a)^(1/2)/(1+tan(b*x+a)))*(c*sec(b*x+a))^(1/2)*2^(1/2)/b/c^2/d^2/(d*csc(b*x
+a))^(1/2)/tan(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.57

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = -\frac{\left(2(\cos(2(a+bx))+\cos(4(a+bx)))+3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}+3\sqrt{2}\text{arctanh}\right.}{64bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}$$

input
```
Integrate[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2)),x]
```

output
```
-1/64*((2*(Cos[2*(a + b*x)] + Cos[4*(a + b*x)]) + 3*Sqrt[2]*ArcTan[(-1 + S
qrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3
/4) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a +
 b*x]^2])]*(Cot[a + b*x]^2)^(3/4))*Sec[a + b*x]^3)/(b*d*(d*Csc[a + b*x])^(
3/2)*(c*Sec[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.86, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3107, 3042, 3108, 3042, 3109, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}dx$$

$\Big\downarrow$ 3107

$$\frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}dx}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}dx}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3108

$$\frac{3\left(\frac{\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{4c^2}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}\right)}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}dx}{4c^2}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}\right)}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3109

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{4c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}\right)}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\int \sqrt{\tan(a+bx)}dx}{4c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}\right)}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3957

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\int\frac{\sqrt{\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{4bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}\right)}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 266

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\int\frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}\right)}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 826

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\int\frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}\right)}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 1476

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\mathrm{ta}}\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}\right.}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 1082

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{}{2bc}\right.}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 217

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}(d\csc(a+\dots}\right.}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 1479

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+\dots\right.}{\sqrt{2}}\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}\right.}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 25

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\dots\right.}{\sqrt{2}}\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}\right.}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 27

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\dots}\right.}{\sqrt{2}}\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}\right.}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

$\downarrow$ 1103

$$\frac{3\left(\frac{\sqrt{c\sec(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+b\right.}{2\sqrt{2}}\right.\right.}{2bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}\right.}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}$$

input
```
Int[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2)),x]
```

output
```
-1/4*c/(b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)) + (3*(d/(2*b*c*
(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) + (((-(ArcTan[1 - Sqrt[2]*Sqr
t[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]
)/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2]) - Lo
g[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2]))/2)*Sqrt[c*Se
c[a + b*x]])/(2*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])))/(8*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (warning: unable to verify)

Time = 11.24 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{2}\left(-3\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)-2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)-2\cos(bx+a)-\sin(bx+a)+\csc(bx+a)+2}{\cos(bx+a)-1}\right)+3\ln\left(\right.\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16384/b*2^(1/2)/d^2/c*(-3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2
*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin
(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b
*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos
(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))-6*arctan(((-2*sin(b*x+a)*
cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1)
)-6*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+c
os(b*x+a)-1)/(cos(b*x+a)-1))+(16*cos(b*x+a)^3+16*cos(b*x+a)^2-12*cos(b*x+a
)-12)*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2))*sin(b*
x+a)^6/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*csc(b*x+a))^(1/2
)/(c*sec(b*x+a))^(1/2)*sec(1/2*b*x+1/2*a)^8*csc(1/2*b*x+1/2*a)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(240) = 480$.

Time = 0.14 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.78

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = \frac{6\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{1}{2}\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{\frac{1}{cd}}(\cos(bx+\right.}{}$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas
")
```

output
```
1/256*(6*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*sqrt(2)*sqrt(c/cos(b*x + a)
)*sqrt(d/sin(b*x + a))*sqrt(1/(c*d))*(cos(b*x + a) - sin(b*x + a))) + 3*sq
rt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*(sqrt(2)*sqrt(c/cos(b*x + a))*sqrt(d/s
in(b*x + a))*sqrt(1/(c*d)) + 2*cos(b*x + a) + 2*sin(b*x + a))/(cos(b*x + a
) - sin(b*x + a))) + 3*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*(sqrt(2)*sqrt
(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) - 2*cos(b*x + a) - 2*s
in(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*sqrt(2)*c*d*sqrt(1/(c*d))*
log(2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a)
)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 4*cos(b*x + a)
*sin(b*x + a) + 1) - 3*sqrt(2)*c*d*sqrt(1/(c*d))*log(-2*sqrt(2)*(cos(b*x +
 a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(c/cos(b*x + a))*s
qrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 4*cos(b*x + a)*sin(b*x + a) + 1) + 16*
(4*cos(b*x + a)^5 - 7*cos(b*x + a)^3 + 3*cos(b*x + a))*sqrt(c/cos(b*x + a)
)*sqrt(d/sin(b*x + a)))/(b*c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{5}{2}}(c\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(5/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^3\sec(bx+a)^2}dx\right)}{c^2d^3}$$

input
```
int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
**3*sec(a + b*x)**2),x))/(c**2*d**3)
```

3.271 $\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

output
```
-2/7*c*d*(d*csc(b*x+a))^(7/2)/b/(c*sec(b*x+a))^(7/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2d^4\cot^3(a+bx)\sqrt{d\csc(a+bx)}}{7bc^2\sqrt{c\sec(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*d^4*Cot[a + b*x]^3*Sqrt[d*Csc[a + b*x]])/(7*b*c^2*Sqrt[c*Sec[a + b*x]]
)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3099}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3099}$$

$$-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

input
```
Int[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3099
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{2\sqrt{d\csc(bx+a)}\, d^{4} \cot(bx+a)^{3}}{7b\, c^{2} \sqrt{c\sec(bx+a)}}$	40

input
```
int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/b*(d*csc(b*x+a))^(1/2)*d^4/c^2/(c*sec(b*x+a))^(1/2)*cot(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{2\,d^{4}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos\left(bx+a\right)^{4}}{7\left(bc^{3}\cos\left(bx+a\right)^{2}-bc^{3}\right)\sin\left(bx+a\right)}$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/7*d^4*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^4/((b*c^3*c
os(b*x + a)^2 - b*c^3)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{9}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.82

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{2\,d^4\,\sqrt{\frac{d}{\sin(a+b\,x)}}\,\left(3\,\sin\left(2\,a+2\,b\,x\right)-\sin\left(6\,a+6\,b\,x\right)\right)}{7\,b\,c^2\,\sqrt{\frac{c}{\cos(a+b\,x)}}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-1\text{[illegible]}\right.}$$

input
```
int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(5/2),x)
```

output
```
(2*d^4*(d/sin(a + b*x))^(1/2)*(3*sin(2*a + 2*b*x) - sin(6*a + 6*b*x)))/(7*
b*c^2*(c/cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) - 6*cos(4*a + 4*b*x) + c
os(6*a + 6*b*x) - 10))
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^4}{\sec(bx+a)^3}dx\right)d^4}{c^3}$$

input
```
int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
4)/sec(a + b*x)**3,x)*d**4)/c**3
```

3.272 $\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx=\frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}+\frac{6d^4E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{5bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
6/5*d^3*(d*csc(b*x+a))^(1/2)/b/c/(c*sec(b*x+a))^(3/2)-2/5*d*(d*csc(b*x+a))
^(5/2)/b/c/(c*sec(b*x+a))^(3/2)-6/5*d^4*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2
))/b/c^2/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.74 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.75

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx=\frac{d^5\Big((1-3\cos(2(a+bx)))\cot^2(a+bx)\csc^2(a+bx)+6\sqrt[4]{-\cot^2(a+bx)}\,\text{Hyper}}{5bc^3(d\csc(a+bx))^{3/2}}$$

input
```
Integrate[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
(d^5*((1 - 3*Cos[2*(a + b*x)])*Cot[a + b*x]^2*Csc[a + b*x]^2 + 6*(-Cot[a +
 b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*S
ec[a + b*x]])/(5*b*c^3*(d*Csc[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3103, 3042, 3105, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3103}$$

$$-\frac{3d^2\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}dx}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3d^2\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}dx}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3105}$$

$$-\frac{3d^2\left(-2d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3d^2\left(-2d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3110}$$

$$-\frac{3d^2\left(-\frac{2d^2\int\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{3d^2\left(-\frac{2d^2\int\sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$\downarrow$ 3052

$$-\frac{3d^2\left(-\frac{2d^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{3d^2\left(-\frac{2d^2\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

$\downarrow$ 3119

$$-\frac{3d^2\left(-\frac{2d^2E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}\right)}{5c^2}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

input
```
Int[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*d*(d*Csc[a + b*x])^(5/2))/(5*b*c*(c*Sec[a + b*x])^(3/2)) - (3*d^2*((-2
*c*d*Sqrt[d*Csc[a + b*x]])/(b*(c*Sec[a + b*x])^(3/2)) - (2*d^2*EllipticE[a
 - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2
*a + 2*b*x]])))/(5*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{d\csc(bx+a)}\, d^3\left(\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{-\cot(}$

input
```
int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/b*(d*csc(b*x+a))^(1/2)*d^3/(c*sec(b*x+a))^(1/2)/c^2*((2*cot(b*x+a)-2*c
sc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+cs
c(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(-6-6*sec(
b*x+a))+(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*
EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b
*x+a)+1)^(1/2)*(3+3*sec(b*x+a))+6-2*csc(b*x+a)*cot(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.88

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx =$$

$$-\frac{3\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{-4i\,cd}E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+3\left(d^3\cos\left(bx+a\right)^2-d^3\right.}{}$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(3*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-4*I*c*d)*elliptic_e(arcsin(cos(b
*x + a) + I*sin(b*x + a)), -1) + 3*(d^3*cos(b*x + a)^2 - d^3)*sqrt(4*I*c*d
)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - 3*(d^3*cos(b*x +
 a)^2 - d^3)*sqrt(-4*I*c*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a
)), -1) - 3*(d^3*cos(b*x + a)^2 - d^3)*sqrt(4*I*c*d)*elliptic_f(arcsin(cos
(b*x + a) - I*sin(b*x + a)), -1) - 4*(3*d^3*cos(b*x + a)^4 - 2*d^3*cos(b*x
 + a)^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)))/(b*c^3*cos(b*x + a)^2
- b*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{7}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{7/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(5/2),x)
```

output
```
int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^3}{\sec(bx+a)^3}dx\right)d^3}{c^3}$$

input
```
int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
3)/sec(a + b*x)**3,x)*d**3)/c**3
```

3.273 $\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 251

$$\begin{aligned}\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx &= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}\\
&+\frac{d^2\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}\\
&-\frac{d^2\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}\\
&-\frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}\end{aligned}$$

output
```
-2/3*d*(d*csc(b*x+a))^(3/2)/b/c/(c*sec(b*x+a))^(3/2)-1/2*d^2*arctan(-1+2^(
1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c^2
/(c*sec(b*x+a))^(1/2)-1/2*d^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*
x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c^2/(c*sec(b*x+a))^(1/2)-1/2*d^2*ar
ctanh(2^(1/2)*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(d*csc(b*x+a))^(1/2)*tan(b*
x+a)^(1/2)*2^(1/2)/b/c^2/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.61

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx =$$
$$-\frac{d^3\left(4\cot^2(a+bx)-3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right.\right.}{6bc^3\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
-1/6*(d^3*(4*Cot[a + b*x]^2 - 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])
/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(1/4) + 3*Sqrt[2]*ArcT
anh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a +
b*x]^2)^(1/4))*Sqrt[c*Sec[a + b*x]])/(b*c^3*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3103, 3042, 3109, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{d^2\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{c^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3109}$$

$$-\frac{d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{c^2\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{c^2\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}(\tan^2(a+bx)+1)}d\tan(a+bx)}{bc^2\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{bc^2\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{bc^2\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+\right.\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \ 1082$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\int\frac{}{-\mathrm{t}}\right.\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \ 217$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{}\right.}{\sqrt{2}}\right.\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \ 1479$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(}}{2\sqrt{2}}\right.\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right.\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\mathrm{ta}}\right.\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)+\sqrt{2}\ldots\right.}{2\sqrt{\ldots}}\right.\right.}{bc^2\sqrt{c\sec(a+bx)}}$$
$$\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

input
```
Int[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*(c*Sec[a + b*x])^(3/2)) - (2*d^2*Sqrt
[d*Csc[a + b*x]]*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + Arc
Tan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqr
t[Tan[a + b*x]] + Tan[a + b*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x
]] + Tan[a + b*x]]/(2*Sqrt[2]))/2)*Sqrt[Tan[a + b*x]])/(b*c^2*Sqrt[c*Sec[a
 + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(209) = 418.

Time = 6.99 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{2}\left((3\cos(bx+a)-3)\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)+2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)-2\cos(bx+a)-\sin(bx+a)+\csc(bx+a}{\cos(bx+a)-1}\right.\right.}{}$

input
```
int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b*2^(1/2)*((3*cos(b*x+a)-3)*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+
2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a
)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+(-3*cos(b*x+a)+3)*ln(-(cos(b*x+
a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+(-6*
cos(b*x+a)+6)*arctan((-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*s
in(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))+(6*cos(b*x+a)-6)*arctan(((-2*sin(b
*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x
+a)-1))+4*cos(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2))*d^
2*(d*csc(b*x+a))^(1/2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/c^2
/(c*sec(b*x+a))^(1/2)*csc(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(209) = 418$.

Time = 0.15 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.24

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}cd^2\sqrt{\frac{d}{c}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d}{c}}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)\sin(bx+a)}{d\cos(bx+a)-d\sin(bx+a)}\right)\sin(bx+a) - 3}{}$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(6*sqrt(2)*c*d^2*sqrt(d/c)*arctan(-sqrt(2)*sqrt(d/c)*sqrt(c/cos(b*x +
 a))*sqrt(d/sin(b*x + a))*cos(b*x + a)*sin(b*x + a)/(d*cos(b*x + a) - d*si
n(b*x + a)))*sin(b*x + a) - 3*sqrt(2)*c*d^2*sqrt(d/c)*arctan(-1/2*(sqrt(2)
*sqrt(d/c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) + 2*d*cos(b*x + a) +
2*d*sin(b*x + a))/(d*cos(b*x + a) - d*sin(b*x + a)))*sin(b*x + a) - 3*sqrt
(2)*c*d^2*sqrt(d/c)*arctan(-1/2*(sqrt(2)*sqrt(d/c)*sqrt(c/cos(b*x + a))*sq
rt(d/sin(b*x + a)) - 2*d*cos(b*x + a) - 2*d*sin(b*x + a))/(d*cos(b*x + a)
- d*sin(b*x + a)))*sin(b*x + a) + 3*sqrt(2)*c*d^2*sqrt(d/c)*log(2*sqrt(2)*
(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(d/c)*sq
rt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) + 4*d*cos(b*x + a)*sin(b*x + a) +
d)*sin(b*x + a) - 3*sqrt(2)*c*d^2*sqrt(d/c)*log(-2*sqrt(2)*(cos(b*x + a)^3
 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(d/c)*sqrt(c/cos(b*x +
a))*sqrt(d/sin(b*x + a)) + 4*d*cos(b*x + a)*sin(b*x + a) + d)*sin(b*x + a)
 - 16*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^2)/(b*c^3
*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{5}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(5/2),x)
```

output
```
int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)^2}{\sec(bx+a)^3}dx\right)d^2}{c^3}$$

input

```
int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x)
```

output

```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x)**
2)/sec(a + b*x)**3,x)*d**2)/c**3
```

3.274 $\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
-2*d*(d*csc(b*x+a))^(1/2)/b/c/(c*sec(b*x+a))^(3/2)+3*d^2*EllipticE(cos(a+1
/4*Pi+b*x),2^(1/2))/b/c^2/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*
b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{d^3\left(2\cot^2(a+bx)+3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{bc^3(d\csc(a+bx))^{3/2}}$$

input
```
Integrate[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
-((d^3*(2*Cot[a + b*x]^2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/
2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*c^3*(d*Csc[a + b*x]
)^(3/2)))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3103, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{3d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{c^2}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{c^2}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3110}$$

$$-\frac{3d^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3052$$

$$-\frac{3d^2\int\sqrt{\sin(2a+2bx)}dx}{c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3d^2\int\sqrt{\sin(2a+2bx)}dx}{c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{3d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

input
```
Int[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(5/2),x]
```

output
```
(-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*(c*Sec[a + b*x])^(3/2)) - (3*d^2*Elliptic
E[a - Pi/4 + b*x, 2])/(b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqr
t[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(85) = 170$.

Time = 1.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.33

method	result
default	$\frac{\left(\frac{\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)(6+6\sec(bx+\ldots}{2}\right.}{}$

input
```
int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1
/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/
2),1/2*2^(1/2))*(6+6*sec(b*x+a))+1/2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*c
ot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-
cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-3-3*sec(b*x+a))+cos(b*x+a)-3
)*d*(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/c^2
```

Fricas [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d*csc(b*x + a)/(c^3*sec
(b*x + a)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{(d\csc(bx+a))^{\frac{3}{2}}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(5/2),x)
```

output
```
int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}\,\csc(bx+a)}{\sec(bx+a)^3}dx\right)d}{c^3}$$

input
```
int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x))*csc(a + b*x))/
sec(a + b*x)**3,x)*d)/c**3
```

3.275 $\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx &= \frac{d}{2bc\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} \\ &- \frac{3\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}} \\ &+ \frac{3\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}} \\ &+ \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{4\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}\end{aligned}$$

output
```
1/2*d/b/c/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2)+3/8*arctan(-1+2^(1/2)*
tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c^2/(c*s
ec(b*x+a))^(1/2)+3/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/
2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c^2/(c*sec(b*x+a))^(1/2)+3/8*arctanh(2^(1/2)
*tan(b*x+a)^(1/2)/(1+tan(b*x+a)))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^
(1/2)/b/c^2/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{d\left(4\cos^2(a+bx)-3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}+3\sqrt{2}\text{arc}\right.}{8bc^3\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(5/2),x]
```

output
```
(d*(4*Cos[a + b*x]^2 - 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[
2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(1/4) + 3*Sqrt[2]*ArcTanh[(Sq
rt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)
^(1/4))*Sqrt[c*Sec[a + b*x]])/(8*b*c^3*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.85, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3108, 3042, 3109, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3108}$$

$$\frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4c^2}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4c^2}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3109}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4c^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4c^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}(\tan^2(a+bx)+1)}d\tan(a+bx)}{4bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{755}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(a+ \cdots}}{\cdots}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(\cdots}}{\sqrt{2}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+b \cdots}}{2\sqrt{2}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)- \cdots\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a \cdots}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan\cdots}\right.}{2\sqrt{2}}\cdots\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

input
```
Int[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(5/2),x]
```

output
```
d/(2*b*c*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) + (3*Sqrt[d*Csc[a +
b*x]]*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b
*x]] + Tan[a + b*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a
+ b*x]]/(2*Sqrt[2]))/2)*Sqrt[Tan[a + b*x]])/(2*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(200) = 400$.

Time = 7.90 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.73

method	result
default	$\sqrt{2}\,\sin(bx+a)^4 \Bigg(\cos(bx+a)(4\cos(bx+a)+4)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}+3\ln\Bigg(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)-2\sqrt{-\frac{2\sin(bx+a)\cos(}{(\cos(bx+a)+}}}{\cos(bx-}$

input
```
int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/256/b*2^(1/2)/c^2*sin(b*x+a)^4*(cos(b*x+a)*(4*cos(b*x+a)+4)*(-2*sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b
*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos
(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))-6*arctan(((-2*sin(b*x+a)*
cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1)
)-3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(co
s(b*x+a)-1))-6*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*s
in(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1)))*(d*csc(b*x+a))^(1/2)/(-sin(b*x+a)
*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(c*sec(b*x+a))^(1/2)*sec(1/2*b*x+1/2*a
)^5*csc(1/2*b*x+1/2*a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(200) = 400$.

Time = 0.13 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{16\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)^2\sin(bx+a) - 6\sqrt{2}c\sqrt{\frac{d}{c}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d}{c}}\sqrt{\cdots}}{\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(16*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^2*sin(b*x
+ a) - 6*sqrt(2)*c*sqrt(d/c)*arctan(-sqrt(2)*sqrt(d/c)*sqrt(c/cos(b*x + a)
)*sqrt(d/sin(b*x + a))*cos(b*x + a)*sin(b*x + a)/(d*cos(b*x + a) - d*sin(b
*x + a))) + 3*sqrt(2)*c*sqrt(d/c)*arctan(-1/2*(sqrt(2)*sqrt(d/c)*sqrt(c/co
s(b*x + a))*sqrt(d/sin(b*x + a)) + 2*d*cos(b*x + a) + 2*d*sin(b*x + a))/(d
*cos(b*x + a) - d*sin(b*x + a))) + 3*sqrt(2)*c*sqrt(d/c)*arctan(-1/2*(sqrt
(2)*sqrt(d/c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) - 2*d*cos(b*x + a)
 - 2*d*sin(b*x + a))/(d*cos(b*x + a) - d*sin(b*x + a))) - 3*sqrt(2)*c*sqrt
(d/c)*log(2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*
x + a))*sqrt(d/c)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)) + 4*d*cos(b*x
+ a)*sin(b*x + a) + d) + 3*sqrt(2)*c*sqrt(d/c)*log(-2*sqrt(2)*(cos(b*x + a
)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))*sqrt(d/c)*sqrt(c/cos(b*x
 + a))*sqrt(d/sin(b*x + a)) + 4*d*cos(b*x + a)*sin(b*x + a) + d))/(b*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{d\csc(bx+a)}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{d\csc(bx+a)}}{(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{d}{\sin(a+b\,x)}}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(5/2),x)
```

output
```
int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\sec(bx+a)^3}dx\right)}{c^3}$$

input
```
int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/sec(a + b*x)*
*3,x))/c**3
```

3.276 $\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
1/3*d/b/c/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2)-1/2*EllipticE(cos(a+1/
4*Pi+b*x),2^(1/2))/b/c^2/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b
*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \frac{d\left(1+\cos(2(a+bx))+3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\right.}{6bc^3(d\csc(a+bx))^{3/2}}$$

input
```
Integrate[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
(d*(1 + Cos[2*(a + b*x)] + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/
2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(6*b*c^3*(d*Csc[a + b*
x])^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3108, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3108}$$

$$\frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{2c^2}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{2c^2}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3110}$$

$$\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{2c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{2c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\int \sqrt{\sin(2a+2bx)}dx}{2c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(2a+2bx)}dx}{2c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

input
```
Int[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
d/(3*b*c*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a - Pi
/4 + b*x, 2]/(2*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2
*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(82) = 164$.

Time = 1.78 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.62

method	result
default	$-\frac{\left((6\cos(bx+a)+6)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\ \operatorname{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\ \sqrt{}\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b*((6*cos(b*x+a)+6)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*EllipticE((-cot
(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/
2)*(cot(b*x+a)-csc(b*x+a))^(1/2)+(-3*cos(b*x+a)-3)*(-cot(b*x+a)+csc(b*x+a)
+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2
)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+cos(b*x+a)*(4*co
s(b*x+a)^3+2*cos(b*x+a)-6))/(c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2)/c^2*
sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^3*d*csc(b*x + a)*sec
(b*x + a)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\csc\left(bx+a\right)}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\csc(bx+a)}\,(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\sqrt{\frac{d}{\sin(a+b\,x)}}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)\sec(bx+a)^3}dx\right)}{c^3 d}$$

input
```
int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
*sec(a + b*x)**3),x))/(c**3*d)
```

3.277 $\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 295

$$\begin{aligned}\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx &= -\frac{c}{4bd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}}+\frac{1}{16bcd\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}}\\
&\quad-\frac{3\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{c\sec(a+bx)}}\\
&\quad+\frac{3\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{c\sec(a+bx)}}\\
&\quad+\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{32\sqrt{2}bc^2d^2\sqrt{c\sec(a+bx)}}\end{aligned}$$

output
```
-1/4*c/b/d/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(7/2)+1/16/b/c/d/(d*csc(b*x
+a))^(1/2)/(c*sec(b*x+a))^(3/2)+3/64*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(
d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c^2/d^2/(c*sec(b*x+a))^(1/2
)+3/64*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^
(1/2)*2^(1/2)/b/c^2/d^2/(c*sec(b*x+a))^(1/2)+3/64*arctanh(2^(1/2)*tan(b*x+
a)^(1/2)/(1+tan(b*x+a)))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c
^2/d^2/(c*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.56

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{\left(4+6\cos(2(a+bx))+2\cos(4(a+bx))+3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}-3\sqrt{2}\text{arc}\ldots}{64bc^3d\sqrt{d\csc(a+bx)}}$$

input
```
Integrate[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
-1/64*((4 + 6*Cos[2*(a + b*x)] + 2*Cos[4*(a + b*x)] + 3*Sqrt[2]*ArcTan[(-1
 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2
)^(1/4) - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot
[a + b*x]^2])]*(Cot[a + b*x]^2)^(1/4))*Sqrt[c*Sec[a + b*x]])/(b*c^3*d*Sqrt
[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.86, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3107, 3042, 3108, 3042, 3109, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}dx}{8d^2} - \frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}dx}{8d^2} - \frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3108

$$\frac{\frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4c^2} + \frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2} - \frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4c^2} + \frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2} - \frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3109

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4c^2\sqrt{c\sec(a+bx)}} + \frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2} - \frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4c^2\sqrt{c\sec(a+bx)}} + \frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2} - \frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 3957

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}\left(\tan^2(a+bx)+1\right)}d\tan(a+bx)}{4bc^2\sqrt{c\sec(a+bx)}} + \frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2} - \frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\Big\downarrow$ 266

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int\frac{1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 755

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 1476

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx}\cdots}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}\cdots}{8d^2}\cdots\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 1082

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)\cdots\right.}{2bc^2\sqrt{c\sec(a+bx)}}\cdots}{8d^2}\cdots\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 217

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)\right)}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{\cdots}{2bc(c\sec(a+bx))\cdots}}{8d^2}\cdots\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\big\downarrow$ 1479

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 25

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 27

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}{}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

$\downarrow$ 1103

$$\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(a+bx)-\sqrt{\right.}{2\sqrt{}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2}$$

$$\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}$$

input
```
Int[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
-1/4*c/(b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + (d/(2*b*c*Sqrt[
d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) + (3*Sqrt[d*Csc[a + b*x]]*((-(ArcT
an[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[
a + b*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a +
 b*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqr
t[2]))/2)*Sqrt[Tan[a + b*x]])/(2*b*c^2*Sqrt[c*Sec[a + b*x]]))/(8*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (warning: unable to verify)

Time = 9.57 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{2}\left(3\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)+2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^{2}}}\,\sin(bx+a)-2\cos(bx+a)-\sin(bx+a)+\csc(bx+a)+2}{\cos(bx+a)-1}\right)-3\ln\left(\frac{2}{}\right.\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16384/b*2^(1/2)/d/c^2*(3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(
b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))-3*ln((2*(-2*sin(b*x+a)*cos(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)*cot(b*x+a)+sin(b*x+a)+2*cos(b*x
+a)-csc(b*x+a)+2*cot(b*x+a)-2)/(cos(b*x+a)-1))+6*arctan(((-2*sin(b*x+a)*co
s(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))+
6*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos
(b*x+a)+1)/(cos(b*x+a)-1))+cos(b*x+a)*(16*cos(b*x+a)^3+16*cos(b*x+a)^2-4*c
os(b*x+a)-4)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2))*sin(b*x+a)
^6/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*csc(b*x+a))^(1/2)/(c
*sec(b*x+a))^(1/2)*sec(1/2*b*x+1/2*a)^8*csc(1/2*b*x+1/2*a)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(240) = 480$.

Time = 0.15 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.82

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{6\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{\frac{1}{cd}}\cos(bx+a)\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-3\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\mathrm{si}}}{2(}\right.}{}$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas
")
```

output
```
-1/256*(6*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-sqrt(2)*sqrt(c/cos(b*x + a))*s
qrt(d/sin(b*x + a))*sqrt(1/(c*d))*cos(b*x + a)*sin(b*x + a)/(cos(b*x + a)
- sin(b*x + a))) - 3*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*(sqrt(2)*sqrt(c
/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 2*cos(b*x + a) + 2*sin
(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*sqrt(2)*c*d*sqrt(1/(c*d))*ar
ctan(-1/2*(sqrt(2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d))
 - 2*cos(b*x + a) - 2*sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*sqr
t(2)*c*d*sqrt(1/(c*d))*log(2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(
b*x + a) - cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/
(c*d)) + 4*cos(b*x + a)*sin(b*x + a) + 1) - 3*sqrt(2)*c*d*sqrt(1/(c*d))*lo
g(-2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))
*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 4*cos(b*x + a)*
sin(b*x + a) + 1) + 16*(4*cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(c/cos(b*x
+ a))*sqrt(d/sin(b*x + a))*sin(b*x + a))/(b*c^3*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{3}{2}}(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(3/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^2\sec(bx+a)^3}dx\right)}{c^3d^2}$$

input
```
int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
**2*sec(a + b*x)**3),x))/(c**3*d**2)
```

3.278 $\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{20bc^2d^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

output
```
-1/5*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(7/2)+1/10/b/c/d/(d*csc(b*x
+a))^(3/2)/(c*sec(b*x+a))^(3/2)-3/20*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/
b/c^2/d^2/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.86 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = \frac{\left(-2\cos^2(a+bx)\cos(2(a+bx))+3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hyperge}\right.}{20bc^3d(d\csc(a+}$$

input
```
Integrate[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
((-2*Cos[a + b*x]^2*Cos[2*(a + b*x)] + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeom
etric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(20*b*c^3*
d*(d*Csc[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3107, 3042, 3108, 3042, 3110, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}dx}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{5/2}}dx}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3108

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{2c^2}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}\right)}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}dx}{2c^2}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}\right)}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3110

$$\frac{3\left(\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{2c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}\right)}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}dx}{2c^2\sqrt{c\cos(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}\right)}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3052

$$\frac{3\left(\frac{\int \sqrt{\sin(2a+2bx)}dx}{2c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}\right)}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{\int \sqrt{\sin(2a+2bx)}dx}{2c^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}\right)}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

$\Big\downarrow$ 3119

$$\frac{3\left(\frac{E\left(a+bx-\frac{\pi}{4}|2\right)}{2bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}\right)}{10d^2}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}$$

input
```
Int[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
-1/5*c/(b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(7/2)) + (3*(d/(3*b*c*
(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a - Pi/4 + b*x,
 2]/(2*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*
x]])))/(10*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3110
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n   Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
 x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/
2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(116) = 232$.

Time = 1.94 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.94

method	result
default	$\frac{\left((-6\cos(bx+a)-6)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{-\cos}\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/40/b*((-6*cos(b*x+a)-6)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-
csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*
(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+(3*cos(b*x+a)+3)*(2*cot(b*x+a)-2*csc(b*x+
a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a
)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+cos(b*x+a)*(8*cos
(b*x+a)^5-12*cos(b*x+a)^3-2*cos(b*x+a)+6))/(c*sec(b*x+a))^(1/2)/(d*csc(b*x
+a))^(1/2)/c^2/d^2*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^3*d^3*csc(b*x + a)^3
*sec(b*x + a)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(d\csc\left(bx+a\right)\right)^{\frac{5}{2}}\left(c\sec\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^{3}\sec(bx+a)^{3}}dx\right)}{c^{3}d^{3}}$$

input
```
int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
**3*sec(a + b*x)**3),x))/(c**3*d**3)
```

3.279 $\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 330

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{7/2}} + \frac{5}{192bcd^3\sqrt{d\csc(a+bx)}(c\sec(a+bx))^{3/2}} - \frac{5\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{128\sqrt{2}bc^2d^4\sqrt{c\sec(a+bx)}} + \frac{5\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{128\sqrt{2}bc^2d^4\sqrt{c\sec(a+bx)}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)}}{1+\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{128\sqrt{2}bc^2d^4\sqrt{c\sec(a+bx)}}$$

output
```
-1/6*c/b/d/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(7/2)-5/48*c/b/d^3/(d*csc(b
*x+a))^(1/2)/(c*sec(b*x+a))^(7/2)+5/192/b/c/d^3/(d*csc(b*x+a))^(1/2)/(c*se
c(b*x+a))^(3/2)+5/256*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(
1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c^2/d^4/(c*sec(b*x+a))^(1/2)+5/256*arctan(
1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/
b/c^2/d^4/(c*sec(b*x+a))^(1/2)+5/256*arctanh(2^(1/2)*tan(b*x+a)^(1/2)/(1+t
an(b*x+a)))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)*2^(1/2)/b/c^2/d^4/(c*sec
(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx =$$

$$-\frac{\left(28+34\cos(2(a+bx))+2\cos(4(a+bx))-4\cos(6(a+bx))+15\sqrt{2}\arctan\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{c\cdots}\right.}{768bc^3d^3\sqrt{d\csc(a+b\cdots}}$$

input
```
Integrate[1/((d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
-1/768*((28 + 34*Cos[2*(a + b*x)] + 2*Cos[4*(a + b*x)] - 4*Cos[6*(a + b*x)
] + 15*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2
)^(1/4))]*(Cot[a + b*x]^2)^(1/4) - 15*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*
x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(1/4))*Sqrt[c*Se
c[a + b*x]])/(b*c^3*d^3*Sqrt[d*Csc[a + b*x]])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.90, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 3107, 3042, 3107, 3042, 3108, 3042, 3109, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{7/2}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{7/2}}dx$$

$\Big\downarrow$ 3107

$$\frac{5\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}dx}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{5\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}dx}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 3107

$$\frac{5\left(\frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}dx}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}dx}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 3108

$$\frac{5\left(\frac{\frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4c^2}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{\frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}dx}{4c^2}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\big\downarrow$ 3109

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4c^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}}dx}{4c^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\big\downarrow$ 3957

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\sqrt{\tan(a+bx)}\left(\tan^2(a+bx)+1\right)}d\tan(a+bx)}{4bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\big\downarrow$ 266

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\int \frac{1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}\right)}{12d^2}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\big\downarrow$ 755

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}}{8d^2}-\frac{\cdots}{4bd(c\sec(a+b\cdots}\cdots\right.}{12d^2}\cdots$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$$\Big\downarrow \ 1476$$

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\int\frac{1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\cdots}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}\cdots}{8d^2}\cdots\right.}{12d^2}\cdots$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$$\Big\downarrow \ 1082$$

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+bx)-1}d\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}\right)\right)}{2bc^2\sqrt{c\sec(a+bx)}}\cdots}{8d^2}\cdots\right.}{12d^2}\cdots$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$$\Big\downarrow \ 217$$

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\sqrt{\tan(a+bx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)\right)}{2bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\cdots}}}{8d^2}\cdots\right.}{12d^2}\cdots$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$$\Big\downarrow \ 1479$$

$$5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{t\ldots}\right.}{\sqrt{\ldots}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2} \ldots}{12d^2}\right.$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 25

$$5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a\ldots}\right.}{\sqrt{2}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2} \ldots}{12d^2}\right.$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 27

$$5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+bx)}}{\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+bx)}+1}{\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1}d\sqrt{\tan(a+bx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(\ldots}\right.}{\sqrt{2}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2} \ldots}{12d^2}\right.$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(a+bx)-\sqrt{2}\right.}{2\sqrt{}}\right.\right.}{2bc^2\sqrt{c\sec(a+bx)}}}{8d^2}\right.}{12d^2}$$

$$\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

input
```
Int[1/((d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2)),x]
```

output
```
-1/6*c/(b*d*(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(7/2)) + (5*(-1/4*c/(b
*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + (d/(2*b*c*Sqrt[d*Csc[a +
 b*x]]*(c*Sec[a + b*x])^(3/2)) + (3*Sqrt[d*Csc[a + b*x]]*((-(ArcTan[1 - Sq
rt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]
]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/Sq
rt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqrt[2]))/2)
*Sqrt[Tan[a + b*x]])/(2*b*c^2*Sqrt[c*Sec[a + b*x]]))/(8*d^2)))/(12*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3109
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
)   Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !Integer
Q[n] && EqQ[m + n, 0]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 9.95 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.36

method	result
default	$\frac{\sqrt{2}\left(15\ln\left(-2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\cot(bx+a)+2-2\cot(bx+a)-2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\csc(bx+a)\right)-15\ln\left(2\sqrt{-\frac{2\sin(bx+}{(\cos(b}}\right.\right.}{}$

input
```
int(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1572864/b*2^(1/2)/d^3/c^2*(15*ln(-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a
)+1)^2)^(1/2)*cot(b*x+a)+2-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b
*x+a)+1)^2)^(1/2)*csc(b*x+a))-15*ln(2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a
)+1)^2)^(1/2)*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/
2)*csc(b*x+a)-2*cot(b*x+a)+2)-30*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*
x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))+30*arctan((-(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(co
s(b*x+a)-1))+4*cos(b*x+a)*(5+32*cos(b*x+a)^5+32*cos(b*x+a)^4-52*cos(b*x+a)
^3-52*cos(b*x+a)^2+5*cos(b*x+a))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2))*sin(b*x+a)^9/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*
csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)*sec(1/2*b*x+1/2*a)^11*csc(1/2*b*x+1
/2*a)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(269) = 538.

Time = 0.16 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.66

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx =$$

$$-\frac{30\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{\frac{1}{cd}}\cos(bx+a)\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-15\sqrt{2}cd\sqrt{\frac{1}{cd}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{c}{\cos(bx+a)}}\cdots}{}\right.}{}$$

input
```
integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas
")
```

output
```
-1/3072*(30*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-sqrt(2)*sqrt(c/cos(b*x + a))
*sqrt(d/sin(b*x + a))*sqrt(1/(c*d))*cos(b*x + a)*sin(b*x + a)/(cos(b*x + a
) - sin(b*x + a))) - 15*sqrt(2)*c*d*sqrt(1/(c*d))*arctan(-1/2*(sqrt(2)*sqr
t(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 2*cos(b*x + a) + 2*
sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 15*sqrt(2)*c*d*sqrt(1/(c*d)
)*arctan(-1/2*(sqrt(2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c
*d)) - 2*cos(b*x + a) - 2*sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 1
5*sqrt(2)*c*d*sqrt(1/(c*d))*log(2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2
*sin(b*x + a) - cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sq
rt(1/(c*d)) + 4*cos(b*x + a)*sin(b*x + a) + 1) - 15*sqrt(2)*c*d*sqrt(1/(c*
d))*log(-2*sqrt(2)*(cos(b*x + a)^3 - cos(b*x + a)^2*sin(b*x + a) - cos(b*x
 + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sqrt(1/(c*d)) + 4*cos(b*x
 + a)*sin(b*x + a) + 1) - 16*(32*cos(b*x + a)^6 - 52*cos(b*x + a)^4 + 5*co
s(b*x + a)^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sin(b*x + a))/(b*c
^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{7}{2}}\,(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{(d\csc(bx+a))^{\frac{7}{2}}(c\sec(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\left(\frac{d}{\sin(a+b\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(7/2)),x)
```

output
```
int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sec(bx+a)}\,\sqrt{\csc(bx+a)}}{\csc(bx+a)^4\sec(bx+a)^3}dx\right)}{c^3d^4}$$

input
```
int(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sec(a + b*x))*sqrt(csc(a + b*x)))/(csc(a + b*x)
**4*sec(a + b*x)**3),x))/(c**3*d**4)
```

3.280 $\int \csc^n(e+fx)\sec^m(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)\sec^{1+m}(e+fx)}{f(1-n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*m)*csc(f*x+e)^(-1+n)*hypergeom([1/2+1/2*m, 1/2-1/2
*n],[3/2-1/2*n],sin(f*x+e)^2)*sec(f*x+e)^(1+m)/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.76 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.43

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = -\frac{(-3+n}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1}{2}-\frac{n}{2},m,1-m-n,\frac{3}{2}-\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}$$

input
```
Integrate[Csc[e + f*x]^n*Sec[e + f*x]^m,x]
```

output
```
-(((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Csc[e + f*x]^(-1 + n)*Sec[e + f*x]^m)/(f*(-1 + n)*
((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[3/2 - n/2, m, 2 - m - n,
5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2 - n/2
, 1 + m, 1 - m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^m(e+fx)\csc^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^m\csc(e+fx)^n dx$$

$$\Big\downarrow \text{3111}$$

$$\cos^{m+1}(e+fx)\sec^{m+1}(e+fx)\sin^{n-1}(e+fx)\csc^{n-1}(e+fx)\int \cos^{-m}(e+fx)\sin^{-n}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{m+1}(e+fx)\sec^{m+1}(e+fx)\sin^{n-1}(e+fx)\csc^{n-1}(e+fx)\int \cos(e+fx)^{-m}\sin(e+fx)^{-n}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)\csc^{n-1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^n*Sec[e + f*x]^m,x]
```

output
```
((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
 m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(f*(1 -
 n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int \csc (fx+e)^n \sec (fx+e)^m \, dx$$

input
```
int(csc(f*x+e)^n*sec(f*x+e)^m,x)
```

output
```
int(csc(f*x+e)^n*sec(f*x+e)^m,x)
```

Fricas [F]

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \int \csc(fx+e)^n \sec(fx+e)^m \,dx$$

input
```
integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="fricas")
```

output
```
integral(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Sympy [F]

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \int \csc^n(e+fx)\sec^m(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**n*sec(f*x+e)**m,x)
```

output
```
Integral(csc(e + f*x)**n*sec(e + f*x)**m, x)
```

Maxima [F]

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \int \csc(fx+e)^n \sec(fx+e)^m \,dx$$

input
```
integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Giac [F]

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \int \csc(fx+e)^n \sec(fx+e)^m\,dx$$

input
```
integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \int \left(\frac{1}{\cos(e+fx)}\right)^m \left(\frac{1}{\sin(e+fx)}\right)^n dx$$

input
```
int((1/cos(e + f*x))^m*(1/sin(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x))^m*(1/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \int \sec(fx+e)^m \csc(fx+e)^n\,dx$$

input
```
int(csc(f*x+e)^n*sec(f*x+e)^m,x)
```

output
```
int(sec(e + f*x)**m*csc(e + f*x)**n,x)
```

3.281 $\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx = \frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)(a\sec(e+fx))^{1+m}}{af(1-n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*m)*csc(f*x+e)^(-1+n)*hypergeom([1/2+1/2*m, 1/2-1/2
*n],[3/2-1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^(1+m)/a/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.84 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.26

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx = -\frac{(-3+n)}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1}{2}-\frac{n}{2},m,1-m-n,\frac{3}{2}-\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}$$

input
```
Integrate[Csc[e + f*x]^n*(a*Sec[e + f*x])^m,x]
```

output
```
-(((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Csc[e + f*x]^(-1 + n)*(a*Sec[e + f*x])^m)/(f*(-1 +
 n)*((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[3/2 - n/2, m, 2 - m -
 n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2 -
 n/2, 1 + m, 1 - m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^n(a\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3111}$$

$$\frac{\sin^{n-1}(e+fx)\csc^{n-1}(e+fx)(a\cos(e+fx))^{m+1}(a\sec(e+fx))^{m+1}\int(a\cos(e+fx))^{-m}\sin^{-n}(e+fx)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{n-1}(e+fx)\csc^{n-1}(e+fx)(a\cos(e+fx))^{m+1}(a\sec(e+fx))^{m+1}\int(a\cos(e+fx))^{-m}\sin(e+fx)^{-n}dx}{a^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\csc^{n-1}(e+fx)(a\sec(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{af(1-n)}$$

input
```
Int[Csc[e + f*x]^n*(a*Sec[e + f*x])^m,x]
```

output
```
((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
 m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m))/(a*
f*(1 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int \csc (fx+e)^n (a\sec (fx+e))^m\,dx$$

input
```
int(csc(f*x+e)^n*(a*sec(f*x+e))^m,x)
```

output
```
int(csc(f*x+e)^n*(a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx=\int (a\sec(fx+e))^m\csc(fx+e)^n\,dx$$

input
```
integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Sympy [F]

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx=\int (a\sec(e+fx))^m\csc^n(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**n*(a*sec(f*x+e))**m,x)
```

output
```
Integral((a*sec(e + f*x))**m*csc(e + f*x)**n, x)
```

Maxima [F]

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx=\int (a\sec(fx+e))^m\csc(fx+e)^n\,dx$$

input
```
integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Giac [F]

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx = \int (a\sec(fx+e))^m\csc(fx+e)^n\,dx$$

input
```
integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx = \int \left(\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{1}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int((a/cos(e + f*x))^m*(1/sin(e + f*x))^n,x)
```

output
```
int((a/cos(e + f*x))^m*(1/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx = a^m\left(\int \sec(fx+e)^m\csc(fx+e)^n\,dx\right)$$

input
```
int(csc(f*x+e)^n*(a*sec(f*x+e))^m,x)
```

output
```
a**m*int(sec(e + f*x)**m*csc(e + f*x)**n,x)
```

3.282 $\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = \frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)\sec^{1+m}(e+fx}{f(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(1/2+1/2*m)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/2+1/2*m, 1
/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^2)*sec(f*x+e)^(1+m)/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.68 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.35

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = -\frac{b(-3+n)\,\text{Ap}}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1-n}{2},m,1-m-n,\frac{3-n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\,(}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^m,x]
```

output
```
-((b*(-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*(b*Csc[e + f*x])^(-1 + n)*Sec[e + f*x]^m)/(f*(-1
 + n)*((-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[(3 - n)/2, m, 2 - m
 - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[(3
- n)/2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^m(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^m(b\csc(e+fx))^ndx$$

$$\Big\downarrow \text{3111}$$

$$b^2\cos^{m+1}(e+fx)\sec^{m+1}(e+fx)(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos^{-m}(e+fx)(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\cos^{m+1}(e+fx)\sec^{m+1}(e+fx)(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos(e+fx)^{-m}(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^m,x]
```

output
```
(b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F
1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(
f*(1 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int (b\csc(fx+e))^n \sec(fx+e)^m \, dx$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^m,x)
```

output
```
int((b*csc(f*x+e))^n*sec(f*x+e)^m,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)^m\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = \int (b\csc(e+fx))^n \sec^m(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**n*sec(f*x+e)**m,x)
```

output
```
Integral((b*csc(e + f*x))**n*sec(e + f*x)**m, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)^m\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)^m\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = \int \left(\frac{b}{\sin(e+f\,x)}\right)^n \left(\frac{1}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int((b/sin(e + f*x))^n*(1/cos(e + f*x))^m,x)
```

output
```
int((b/sin(e + f*x))^n*(1/cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = b^n\left(\int \sec(fx+e)^m \csc(fx+e)^n\,dx\right)$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^m,x)
```

output
```
b**n*int(sec(e + f*x)**m*csc(e + f*x)**n,x)
```

3.283 $\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx = \frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)(a\sec(e+fx))}{af(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(1/2+1/2*m)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/2+1/2*m, 1
/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^(1+m)/a/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.75 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.18

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx = -\frac{b(-3+n)\,\text{App}}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1-n}{2},m,1-m-n,\frac{3-n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\,(\right.}$$

input
```
Integrate[(b*Csc[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```

output
```
-((b*(-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*(b*Csc[e + f*x])^(-1 + n)*(a*Sec[e + f*x])^m)/(f
*(-1 + n)*((-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[(3 - n)/2, m, 2
 - m - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1
[(3 - n)/2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx))^m(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(e+fx))^m(b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{3111}$$

$$\frac{b^2(a\cos(e+fx))^{m+1}(a\sec(e+fx))^{m+1}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int(a\cos(e+fx))^{-m}(b\sin(e+fx))}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2(a\cos(e+fx))^{m+1}(a\sec(e+fx))^{m+1}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int(a\cos(e+fx))^{-m}(b\sin(e+fx))}{a^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}(a\sec(e+fx))^{m+1}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{m+1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{af(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```

output
```
(b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F
1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m
))/(a*f*(1 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int (b\csc(fx+e))^n (a\sec(fx+e))^m \, dx$$

input
```
int((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

output
```
int((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx=\int (b\csc(fx+e))^n\,(a\sec(fx+e))^m\;dx$$

input
```
integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx=\int (a\sec(e+fx))^m\,(b\csc(e+fx))^n\;dx$$

input
```
integrate((b*csc(f*x+e))**n*(a*sec(f*x+e))**m,x)
```

output
```
Integral((a*sec(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx=\int (b\csc(fx+e))^n\,(a\sec(fx+e))^m\;dx$$

input
```
integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx=\int (b\csc(fx+e))^n\,(a\sec(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx=\int \left(\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int((a/cos(e + f*x))^m*(b/sin(e + f*x))^n,x)
```

output
```
int((a/cos(e + f*x))^m*(b/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx=b^na^m\left(\int \sec(fx+e)^m\csc(fx+e)^n\,dx\right)$$

input
```
int((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

output
```
b**n*a**m*int(sec(e + f*x)**m*csc(e + f*x)**n,x)
```

3.284 $\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = \frac{(b\csc(e+fx))^{5+n}\,\text{Hypergeometric2F1}\left(3,\frac{5+n}{2},\frac{7+n}{2},\csc^2(e+fx)\right)}{b^5f(5+n)}$$

output
```
(b*csc(f*x+e))^(5+n)*hypergeom([3, 5/2+1/2*n],[7/2+1/2*n],csc(f*x+e)^2)/b^
5/f/(5+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = -\frac{b(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(3,\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(-1+n)}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^5,x]
```

output
```
-((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3, (1 - n)/2, (3 - n)/2,
Sin[e + f*x]^2])/(f*(-1 + n)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3101, 25, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(e+fx)^5(b\csc(e+fx))^ndx$$

$$\Big\downarrow \ 3101$$

$$-\frac{\int -\frac{b^6(b\csc(e+fx))^{n+4}}{(b^2-b^2\csc^2(e+fx))^3}d(b\csc(e+fx))}{b^5f}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{b^6(b\csc(e+fx))^{n+4}}{(b^2-b^2\csc^2(e+fx))^3}d(b\csc(e+fx))}{b^5f}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{(b\csc(e+fx))^{n+4}}{(b^2-b^2\csc^2(e+fx))^3}d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \ 278$$

$$\frac{(b\csc(e+fx))^{n+5}\,\text{Hypergeometric2F1}\left(3,\frac{n+5}{2},\frac{n+7}{2},\csc^2(e+fx)\right)}{b^5f(n+5)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^5,x]
```

output
```
((b*Csc[e + f*x])^(5 + n)*Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, Csc[e
 + f*x]^2])/(b^5*f*(5 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int (b\csc(fx+e))^n \sec(fx+e)^5\,dx$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^5,x)
```

output
```
int((b*csc(f*x+e))^n*sec(f*x+e)^5,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)^5\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = \int (b\csc(e+fx))^n \sec^5(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**n*sec(f*x+e)**5,x)
```

output
```
Integral((b*csc(e + f*x))**n*sec(e + f*x)**5, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)^5\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)^5\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos(e+f\,x)^5}\,dx$$

input
```
int((b/sin(e + f*x))^n/cos(e + f*x)^5,x)
```

output
```
int((b/sin(e + f*x))^n/cos(e + f*x)^5, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx = b^n\left(\int \csc(fx+e)^n \sec(fx+e)^5\,dx\right)$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^5,x)
```

output
```
b**n*int(csc(e + f*x)**n*sec(e + f*x)**5,x)
```

3.285 $\int (b\csc(e+fx))^n \sec^3(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int (b\csc(e+fx))^n \sec^3(e+fx)\,dx = -\frac{(b\csc(e+fx))^{3+n}\,\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},\csc^2(e+fx)\right)}{b^3 f(3+n)}$$

output
```
-(b*csc(f*x+e))^(3+n)*hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],csc(f*x+e)^2)/b
^3/f/(3+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int (b\csc(e+fx))^n \sec^3(e+fx)\,dx = -\frac{b(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(2,\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(-1+n)}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^3,x]
```

output
```
-((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[2, (1 - n)/2, (3 - n)/2,
Sin[e + f*x]^2])/(f*(-1 + n)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3101, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(e+fx)^3(b\csc(e+fx))^n dx$$

$$\Big\downarrow \; 3101$$

$$-\frac{\int \frac{b^4(b\csc(e+fx))^{n+2}}{(b^2-b^2\csc^2(e+fx))^2}d(b\csc(e+fx))}{b^3f}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\int \frac{(b\csc(e+fx))^{n+2}}{(b^2-b^2\csc^2(e+fx))^2}d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \; 278$$

$$-\frac{(b\csc(e+fx))^{n+3}\,\text{Hypergeometric2F1}\left(2,\frac{n+3}{2},\frac{n+5}{2},\csc^2(e+fx)\right)}{b^3f(n+3)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^3,x]
```

output
```
-(((b*Csc[e + f*x])^(3 + n)*Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Csc
[e + f*x]^2])/(b^3*f*(3 + n)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int (b\csc(fx+e))^n \sec(fx+e)^3 \, dx$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^3,x)
```

output
```
int((b*csc(f*x+e))^n*sec(f*x+e)^3,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx=\int (b\csc(fx+e))^n\sec(fx+e)^3\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx=\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**n*sec(f*x+e)**3,x)
```

output
```
Integral((b*csc(e + f*x))**n*sec(e + f*x)**3, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx=\int (b\csc(fx+e))^n\sec(fx+e)^3\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx=\int (b\csc(fx+e))^n\sec(fx+e)^3\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx=\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos(e+f\,x)^3}\,dx$$

input
```
int((b/sin(e + f*x))^n/cos(e + f*x)^3,x)
```

output
```
int((b/sin(e + f*x))^n/cos(e + f*x)^3, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx=b^n\left(\int \csc(fx+e)^n\sec(fx+e)^3\,dx\right)$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^3,x)
```

output
```
b**n*int(csc(e + f*x)**n*sec(e + f*x)**3,x)
```

3.286 $\int (b\csc(e+fx))^n \sec(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = \frac{(b\csc(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\csc^2(e+fx)\right)}{bf(1+n)}$$

output
```
(b*csc(f*x+e))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],csc(f*x+e)^2)/b/
f/(1+n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = -\frac{b(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(-1+n)}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x],x]
```

output
```
-((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
Sin[e + f*x]^2])/(f*(-1 + n)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3101, 25, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)(b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{b^2(b\csc(e+fx))^n}{b^2-b^2\csc^2(e+fx)}d(b\csc(e+fx))}{bf}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2(b\csc(e+fx))^n}{b^2-b^2\csc^2(e+fx)}d(b\csc(e+fx))}{bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{(b\csc(e+fx))^n}{b^2-b^2\csc^2(e+fx)}d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\csc(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\csc^2(e+fx)\right)}{bf(n+1)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sec[e + f*x],x]
```

output
```
((b*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Csc[e
 + f*x]^2])/(b*f*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int (b\csc (fx+e))^n \sec (fx+e)\, dx$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e),x)
```

output
```
int((b*csc(f*x+e))^n*sec(f*x+e),x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = \int (b\csc(e+fx))^n \sec(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**n*sec(f*x+e),x)
```

output
```
Integral((b*csc(e + f*x))**n*sec(e + f*x), x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Giac [F]

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = \int (b\csc(fx+e))^n \sec(fx+e)\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos(e+f\,x)}\,dx$$

input
```
int((b/sin(e + f*x))^n/cos(e + f*x),x)
```

output
```
int((b/sin(e + f*x))^n/cos(e + f*x), x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n \sec(e+fx)\,dx = b^n\left(\int \csc(fx+e)^n \sec(fx+e)\,dx\right)$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e),x)
```

output
```
b**n*int(csc(e + f*x)**n*sec(e + f*x),x)
```

3.287 $\int \cos(e+fx)(b\csc(e+fx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = \frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}$$

output
```
b*(b*csc(f*x+e))^(-1+n)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = -\frac{b(b\csc(e+fx))^{-1+n}}{f(-1+n)}$$

input
```
Integrate[Cos[e + f*x]*(b*Csc[e + f*x])^n,x]
```

output
```
-((b*(b*Csc[e + f*x])^(-1 + n))/(f*(-1 + n)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^n}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{b\int (b\csc(e+fx))^{n-2}d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{15}$$

$$\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}$$

input
```
Int[Cos[e + f*x]*(b*Csc[e + f*x])^n,x]
```

output
```
(b*(b*Csc[e + f*x])^(-1 + n))/(f*(1 - n))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-\frac{e^{n \ln(b \csc(fx+e))}}{f(-1+n)\csc(fx+e)}$
default	$-\frac{e^{n \ln(b \csc(fx+e))}}{f(-1+n)\csc(fx+e)}$
parallelrisch	$-\frac{2^{-n}\left(b \sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{n}\sin(fx+e)}{f(-1+n)}$
norman	$-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)e^{n \ln\left(\frac{b\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}}{f(-1+n)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}$
risch	$-\frac{\left(e^{2i(fx+e)}-1\right)^{-n}\sin(fx+e)2^{n}b^{n}\left(e^{i(fx+e)}\right)^{n}e^{\frac{i\pi n\left(-\operatorname{csgn}\left(\frac{ib\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^{3}+\operatorname{csgn}\left(\frac{ib\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^{2}\operatorname{csgn}(ib)+\operatorname{csgn}\left(\frac{ib\,e^{i(\cdot}}{e^{2i(\cdot}}\right.}{}}}{}$

input
```
int(cos(f*x+e)*(b*csc(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/f/(-1+n)*exp(n*ln(b*csc(f*x+e)))/csc(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = -\frac{\left(\frac{b}{\sin(fx+e)}\right)^{n}\sin\left(fx+e\right)}{fn-f}$$

input
```
integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
-(b/sin(f*x + e))^n*sin(f*x + e)/(f*n - f)
```

Sympy [F]

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(e+fx))^n\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(b*csc(f*x+e))**n,x)
```

output
```
Integral((b*csc(e + f*x))**n*cos(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = -\frac{b^n\sin(fx+e)^{-n}\sin(fx+e)}{f(n-1)}$$

input
```
integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
-b^n*sin(f*x + e)^(-n)*sin(f*x + e)/(f*(n - 1))
```

Giac [F]

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = -\frac{\sin\left(e+f\,x\right)\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^n}{f\,\left(n-1\right)}$$

input
```
int(cos(e + f*x)*(b/sin(e + f*x))^n,x)
```

output
```
-(sin(e + f*x)*(b/sin(e + f*x))^n)/(f*(n - 1))
```

Reduce [F]

$$\int \cos(e+fx)(b\csc(e+fx))^n\,dx = b^n\left(\int \csc\left(fx+e\right)^n\cos\left(fx+e\right)dx\right)$$

input
```
int(cos(f*x+e)*(b*csc(f*x+e))^n,x)
```

output
```
b**n*int(csc(e + f*x)**n*cos(e + f*x),x)
```

3.288 $\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx = -\frac{b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b^3*(b*csc(f*x+e))^(-3+n)/f/(3-n)+b*(b*csc(f*x+e))^(-1+n)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx = -\frac{b(-5+n+(-1+n)\cos(2(e+fx)))(b\csc(e+fx))^{-1+n}}{2f(-3+n)(-1+n)}$$

input
```
Integrate[Cos[e + f*x]^3*(b*Csc[e + f*x])^n,x]
```

output
```
-1/2*(b*(-5 + n + (-1 + n)*Cos[2*(e + f*x)])*(b*Csc[e + f*x])^(-1 + n))/(f
*(-3 + n)*(-1 + n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3101, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^n}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{b^3\int -\frac{(b\csc(e+fx))^{n-4}\left(b^2-b^2\csc^2(e+fx)\right)}{b^2}d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\int \frac{(b\csc(e+fx))^{n-4}\left(b^2-b^2\csc^2(e+fx)\right)}{b^2}d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int (b\csc(e+fx))^{n-4}\left(b^2-b^2\csc^2(e+fx)\right)d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(b^2(b\csc(e+fx))^{n-4}-(b\csc(e+fx))^{n-2}\right)d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(b\csc(e+fx))^{n-1}}{1-n}-\frac{b^2(b\csc(e+fx))^{n-3}}{3-n}\right)}{f}$$

input

```
Int[Cos[e + f*x]^3*(b*Csc[e + f*x])^n,x]
```

output
```
(b*(-((b^2*(b*Csc[e + f*x])^(-3 + n))/(3 - n)) + (b*Csc[e + f*x])^(-1 + n)
/(1 - n)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

method	result	size
parallelrisch	$-\frac{\left(b\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{n}2^{-n}\left(\left(-1+n\right)\sin\left(3fx+3e\right)+\sin\left(fx+e\right)\left(n-9\right)\right)}{4f\left(n^{2}-4n+3\right)}$	67

input
```
int(cos(f*x+e)^3*(b*csc(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e))^n*2^(-n)*((-1+n)*sin(3*f*x+
3*e)+sin(f*x+e)*(n-9))/f/(n^2-4*n+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx = -\frac{\left((n-1)\cos\left(fx+e\right)^2-2\right)\left(\frac{b}{\sin(fx+e)}\right)^n\sin\left(fx+e\right)}{fn^2-4\,fn+3\,f}$$

input
```
integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
-((n - 1)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^n*sin(f*x + e)/(f*n^2 - 4*f
*n + 3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(b*csc(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx = \frac{\frac{b^n\sin(fx+e)^{-n}\sin(fx+e)^3}{n-3} - \frac{b^n\sin(fx+e)^{-n}\sin(fx+e)}{n-1}}{f}$$

input
```
integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
(b^n*sin(f*x + e)^(-n)*sin(f*x + e)^3/(n - 3) - b^n*sin(f*x + e)^(-n)*sin(
f*x + e)/(n - 1))/f
```

Giac [F]

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx$$
$$= \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n (9\sin(e+f\,x)+\sin(3\,e+3\,f\,x)-n\sin(e+f\,x)-n\sin(3\,e+3\,f\,x))}{4\,f\,(n^2-4\,n+3)}$$

input
```
int(cos(e + f*x)^3*(b/sin(e + f*x))^n,x)
```

output
```
((b/sin(e + f*x))^n*(9*sin(e + f*x) + sin(3*e + 3*f*x) - n*sin(e + f*x) -
n*sin(3*e + 3*f*x)))/(4*f*(n^2 - 4*n + 3))
```

Reduce [F]

$$\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx = b^n\left(\int \csc(fx+e)^n\cos(fx+e)^3\,dx\right)$$

input
```
int(cos(f*x+e)^3*(b*csc(f*x+e))^n,x)
```

output
```
b**n*int(csc(e + f*x)**n*cos(e + f*x)**3,x)
```

3.289 $\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx=\frac{b^5(b\csc(e+fx))^{-5+n}}{f(5-n)}-\frac{2b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}$$

output
```
b^5*(b*csc(f*x+e))^(-5+n)/f/(5-n)-2*b^3*(b*csc(f*x+e))^(-3+n)/f/(3-n)+b*(b
*csc(f*x+e))^(-1+n)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx = -\frac{(b\csc(e+fx))^n\,(3-4n+n^2-2(5-6n+n^2)\csc^2(e+fx)+(15-8n+n^2)\csc^4(e+fx))\sin^5(e+}{f(-5+n)(-3+n)(-1+n)}$$

input
```
Integrate[Cos[e + f*x]^5*(b*Csc[e + f*x])^n,x]
```

output
```
-(((b*Csc[e + f*x])^n*(3 - 4*n + n^2 - 2*(5 - 6*n + n^2)*Csc[e + f*x]^2 +
(15 - 8*n + n^2)*Csc[e + f*x]^4)*Sin[e + f*x]^5)/(f*(-5 + n)*(-3 + n)*(-1
+ n)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3101, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^n}{\sec(e+fx)^5}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{b^5\int \frac{(b\csc(e+fx))^{n-6}\left(b^2-b^2\csc^2(e+fx)\right)^2}{b^4}d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int (b\csc(e+fx))^{n-6}\left(b^2-b^2\csc^2(e+fx)\right)^2d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(b^4(b\csc(e+fx))^{n-6}-2b^2(b\csc(e+fx))^{n-4}+(b\csc(e+fx))^{n-2}\right)d(b\csc(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{b^4(b\csc(e+fx))^{n-5}}{5-n}+\frac{2b^2(b\csc(e+fx))^{n-3}}{3-n}-\frac{(b\csc(e+fx))^{n-1}}{1-n}\right)}{f}$$

input
```
Int[Cos[e + f*x]^5*(b*Csc[e + f*x])^n,x]
```

output
```
-((b*(-((b^4*(b*Csc[e + f*x])^(-5 + n))/(5 - n)) + (2*b^2*(b*Csc[e + f*x])
^(-3 + n))/(3 - n) - (b*Csc[e + f*x])^(-1 + n)/(1 - n)))/f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

method	result
parallelrisch	$-\frac{\left(\left(\frac{3}{2}n^{2}-14n+\frac{25}{2}\right)\sin\left(3fx+3e\right)+\left(\frac{1}{2}n^{2}-2n+\frac{3}{2}\right)\sin\left(5fx+5e\right)+\sin\left(fx+e\right)\left(n^{2}-12n+75\right)\right)2^{-n}\left(b\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f\left(n^{3}-9n^{2}+23n-15\right)}$

input
```
int(cos(f*x+e)^5*(b*csc(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/8*((3/2*n^2-14*n+25/2)*sin(3*f*x+3*e)+(1/2*n^2-2*n+3/2)*sin(5*f*x+5*e)+
sin(f*x+e)*(n^2-12*n+75))*2^(-n)*(b*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e))
^n/f/(n^3-9*n^2+23*n-15)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx = -\frac{\left((n^2-4\,n+3)\cos\left(fx+e\right)^4-4\,(n-1)\cos\left(fx+e\right)^2+8\right)\left(\frac{b}{\sin(fx+e)}\right)^n\sin\left(fx+e\right)}{fn^3-9\,fn^2+23\,fn-15\,f}$$

input
```
integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
-((n^2 - 4*n + 3)*cos(f*x + e)^4 - 4*(n - 1)*cos(f*x + e)^2 + 8)*(b/sin(f*
x + e))^n*sin(f*x + e)/(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(b*csc(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx = -\frac{\frac{b^n\sin(fx+e)^{-n}\sin(fx+e)^5}{n-5} - \frac{2\,b^n\sin(fx+e)^{-n}\sin(fx+e)^3}{n-3} + \frac{b^n\sin(fx+e)^{-n}\sin(fx+e)}{n-1}}{f}$$

input
```
integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
-(b^n*sin(f*x + e)^(-n)*sin(f*x + e)^5/(n - 5) - 2*b^n*sin(f*x + e)^(-n)*s
in(f*x + e)^3/(n - 3) + b^n*sin(f*x + e)^(-n)*sin(f*x + e)/(n - 1))/f
```

Giac [F]

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^5, x)
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.72

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx =$$

$$-\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n\left(150\sin\left(e+f\,x\right)+25\sin\left(3\,e+3\,f\,x\right)+3\sin\left(5\,e+5\,f\,x\right)+3\,n^2\sin\left(3\,e+3\,f\,x\right)+n^2\right.}{16\,f\,\left(n^3-9\,n\right.}$$

input
```
int(cos(e + f*x)^5*(b/sin(e + f*x))^n,x)
```

output
```
-((b/sin(e + f*x))^n*(150*sin(e + f*x) + 25*sin(3*e + 3*f*x) + 3*sin(5*e +
 5*f*x) + 3*n^2*sin(3*e + 3*f*x) + n^2*sin(5*e + 5*f*x) - 24*n*sin(e + f*x
) - 28*n*sin(3*e + 3*f*x) - 4*n*sin(5*e + 5*f*x) + 2*n^2*sin(e + f*x)))/(1
6*f*(23*n - 9*n^2 + n^3 - 15))
```

Reduce [F]

$$\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx = b^n\left(\int \csc(fx+e)^n \cos(fx+e)^5\,dx\right)$$

input
```
int(cos(f*x+e)^5*(b*csc(f*x+e))^n,x)
```

output
```
b**n*int(csc(e + f*x)**n*cos(e + f*x)**5,x)
```

3.290 $\int (b\csc(e+fx))^n \sec^6(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int (b\csc(e+fx))^n \sec^6(e+fx)\,dx = \frac{b\sqrt{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(1/2)*(b*csc(f*x+e))^(-1+n)*hypergeom([7/2, 1/2-1/2*n],[3
/2-1/2*n],sin(f*x+e)^2)*sec(f*x+e)/f/(1-n)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int (b\csc(e+fx))^n \sec^6(e+fx)\,dx = \frac{(b\csc(e+fx))^n\,\text{Hypergeometric2F1}\left(-2-\frac{n}{2},\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{-n/2}\tan(e+f}{f(1-n)}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^6,x]
```

output
```
((b*Csc[e + f*x])^n*Hypergeometric2F1[-2 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan
[e + f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6(b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{3111}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \sec^6(e+fx)(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \frac{(b\sin(e+fx))^{-n}}{\cos(e+fx)^6}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^6,x]
```

output
```
(b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[7/2, (
1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int (b\csc (fx+e))^n \sec (fx+e)^6\, dx$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^6,x)
```

output
```
int((b*csc(f*x+e))^n*sec(f*x+e)^6,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n \sec^6(e+fx)\, dx = \int (b\csc (fx+e))^n \sec (fx+e)^6\, dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n\sec^6(e+fx)\,dx=\text{Timed out}$$

input
```
integrate((b*csc(f*x+e))**n*sec(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\csc(e+fx))^n\sec^6(e+fx)\,dx=\int (b\csc(fx+e))^n\sec(fx+e)^6\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n\sec^6(e+fx)\,dx=\int (b\csc(fx+e))^n\sec(fx+e)^6\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n\sec^6(e+fx)\,dx=\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)^6}\,dx$$

input
```
int((b/sin(e + f*x))^n/cos(e + f*x)^6,x)
```

output
```
int((b/sin(e + f*x))^n/cos(e + f*x)^6, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n\sec^6(e+fx)\,dx=b^n\left(\int \csc\left(fx+e\right)^n\sec\left(fx+e\right)^6dx\right)$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^6,x)
```

output
```
b**n*int(csc(e + f*x)**n*sec(e + f*x)**6,x)
```

3.291 $\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx = \frac{b\sqrt{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(1/2)*(b*csc(f*x+e))^(-1+n)*hypergeom([5/2, 1/2-1/2*n],[3
/2-1/2*n],sin(f*x+e)^2)*sec(f*x+e)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx = \frac{(b\csc(e+fx))^n\,\text{Hypergeometric2F1}\left(-1-\frac{n}{2},\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{-n/2}\tan(e+f}{f(1-n)}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^4,x]
```

output
```
((b*Csc[e + f*x])^n*Hypergeometric2F1[-1 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan
[e + f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4(b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{3111}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \sec^4(e+fx)(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \frac{(b\sin(e+fx))^{-n}}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^4,x]
```

output
```
(b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/2, (
1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int (b\csc (fx+e))^n \sec (fx+e)^4\,dx$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^4,x)
```

output
```
int((b*csc(f*x+e))^n*sec(f*x+e)^4,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx = \int (b\csc (fx+e))^n \sec (fx+e)^4\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx=\int (b\csc{(e+fx)})^n\sec^4{(e+fx)}\,dx$$

input
```
integrate((b*csc(f*x+e))**n*sec(f*x+e)**4,x)
```

output
```
Integral((b*csc(e + f*x))**n*sec(e + f*x)**4, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx=\int (b\csc{(fx+e)})^n\sec{(fx+e)}^4\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx=\int (b\csc{(fx+e)})^n\sec{(fx+e)}^4\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((b/sin(e + f*x))^n/cos(e + f*x)^4,x)
```

output
```
int((b/sin(e + f*x))^n/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx = b^n\left(\int \csc\left(fx+e\right)^n \sec\left(fx+e\right)^4 dx\right)$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^4,x)
```

output
```
b**n*int(csc(e + f*x)**n*sec(e + f*x)**4,x)
```

3.292 $\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx = \frac{b\sqrt{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(1/2)*(b*csc(f*x+e))^(-1+n)*hypergeom([3/2, 1/2-1/2*n],[3
/2-1/2*n],sin(f*x+e)^2)*sec(f*x+e)/f/(1-n)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx = \frac{2(b\csc(e+fx))^n\left(-\frac{n\,\text{Hypergeometric2F1}\left(1-n,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}\tan\left(\frac{1}{2}(e+fx)\right)}{-1+n}+\frac{1}{2}\tan(e+f}{f}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^2,x]
```

output
```
(2*(b*Csc[e + f*x])^n*(-((n*Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2,
 -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/((-1 + n)*(Sec[(e + f*x)/2]^2)^n))
 + Tan[e + f*x]/2))/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^2(b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{3111}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \sec^2(e+fx)(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \frac{(b\sin(e+fx))^{-n}}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^2,x]
```

output
```
(b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/2, (
1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int (b\csc (fx+e))^n \sec (fx+e)^2 \, dx$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^2,x)
```

output
```
int((b*csc(f*x+e))^n*sec(f*x+e)^2,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n \sec^2(e+fx)\, dx = \int (b\csc (fx+e))^n \sec (fx+e)^2 \, dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n\sec^2(e+fx)\,dx=\int (b\csc\left(e+fx\right))^n\sec^2\left(e+fx\right)dx$$

input
```
integrate((b*csc(f*x+e))**n*sec(f*x+e)**2,x)
```

output
```
Integral((b*csc(e + f*x))**n*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n\sec^2(e+fx)\,dx=\int (b\csc\left(fx+e\right))^n\sec\left(fx+e\right)^2\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n\sec^2(e+fx)\,dx=\int (b\csc\left(fx+e\right))^n\sec\left(fx+e\right)^2\,dx$$

input
```
integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((b/sin(e + f*x))^n/cos(e + f*x)^2,x)
```

output
```
int((b/sin(e + f*x))^n/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx = b^n\left(\int \csc\left(fx+e\right)^n \sec\left(fx+e\right)^2 dx\right)$$

input
```
int((b*csc(f*x+e))^n*sec(f*x+e)^2,x)
```

output
```
b**n*int(csc(e + f*x)**n*sec(e + f*x)**2,x)
```

3.293 $\int (b \csc(e+fx))^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 72

$$\int (b \csc(e+fx))^n \, dx = \frac{b \cos(e+fx)(b \csc(e+fx))^{-1+n} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

output
```
b*cos(f*x+e)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],
sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (b \csc(e+fx))^n \, dx = -\frac{\cos(e+fx)(b \csc(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3}{2},\cos^2(e+fx)\right) \sin(e+fx) \sin^2(e+fx)^{\frac{1}{2}(-}}{f}$$

input
```
Integrate[(b*Csc[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, 3/2,
Cos[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 + n)/2))/f)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^n (b\csc(e+fx))^n \int \left(\frac{\sin(e+fx)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^n (b\csc(e+fx))^n \int \left(\frac{\sin(e+fx)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[(b*Csc[e + f*x])^n,x]
```

output
```
(b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2
, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (b\csc (fx+e))^{n} dx$$

input
```
int((b*csc(f*x+e))^n,x)
```

output
```
int((b*csc(f*x+e))^n,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n dx = \int (b\csc (fx+e))^{n} dx$$

input
```
integrate((b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n\,dx = \int (b\csc{(e+fx)})^n\,dx$$

input
```
integrate((b*csc(f*x+e))**n,x)
```

output
```
Integral((b*csc(e + f*x))**n, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n\,dx = \int (b\csc{(fx+e)})^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n\,dx = \int (b\csc{(fx+e)})^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n\,dx = \int \left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int((b/sin(e + f*x))^n,x)
```

output
```
int((b/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n\,dx = b^n\left(\int \csc(fx+e)^n\,dx\right)$$

input
```
int((b*csc(f*x+e))^n,x)
```

output
```
b**n*int(csc(e + f*x)**n,x)
```

3.294 $\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx = \frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

output
```
b*cos(f*x+e)*(b*csc(f*x+e))^(-1+n)*hypergeom([-1/2, 1/2-1/2*n],[3/2-1/2*n]
,sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. $2(72) = 144$.

Time = 0.61 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.29

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx = -\frac{2(b\csc(e+fx))^n\left(\text{Hypergeometric2F1}\left(1-n,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\,\text{Hypergeometri}\right.}{}$$

input
```
Integrate[Cos[e + f*x]^2*(b*Csc[e + f*x])^n,x]
```

output
```
(-2*(b*Csc[e + f*x])^n*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Ta
n[(e + f*x)/2]^2] - 4*Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[
(e + f*x)/2]^2] + 4*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2])/(f*(-1 + n)*(Sec[(e + f*x)/2]^2)^n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^n}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{3111}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos^2(e+fx)(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos(e+fx)^2(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^2*(b*Csc[e + f*x])^n,x]
```

output
```
(b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int \cos(fx+e)^2 (b\csc(fx+e))^n \, dx$$

input
```
int(cos(f*x+e)^2*(b*csc(f*x+e))^n,x)
```

output
```
int(cos(f*x+e)^2*(b*csc(f*x+e))^n,x)
```

Fricas [F]

$$\int \cos^2(e+fx)(b\csc(e+fx))^n \, dx = \int (b\csc(fx+e))^n \cos(fx+e)^2 \, dx$$

input
```
integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Sympy [F]

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(e+fx))^n\cos^2(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**2*(b*csc(f*x+e))**n,x)
```

output
```
Integral((b*csc(e + f*x))**n*cos(e + f*x)**2, x)
```

Maxima [F]

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx=\int \cos\left(e+f\,x\right)^2\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^n\,dx$$

input
```
int(cos(e + f*x)^2*(b/sin(e + f*x))^n,x)
```

output
```
int(cos(e + f*x)^2*(b/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx=b^n\left(\int \csc\left(fx+e\right)^n\cos\left(fx+e\right)^2dx\right)$$

input
```
int(cos(f*x+e)^2*(b*csc(f*x+e))^n,x)
```

output
```
b**n*int(csc(e + f*x)**n*cos(e + f*x)**2,x)
```

3.295 $\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx = \frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

output
```
b*cos(f*x+e)*(b*csc(f*x+e))^(-1+n)*hypergeom([-3/2, 1/2-1/2*n],[3/2-1/2*n]
,sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 246 vs. 2(72) = 144.

Time = 0.86 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.42

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx = -\frac{2(b\csc(e+fx))^n\left(\text{Hypergeometric2F1}\left(1-n,\frac{1}{2}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-8(\text{Hypergeometr}\right.}{}$$

input
```
Integrate[Cos[e + f*x]^4*(b*Csc[e + f*x])^n,x]
```

output
```
(-2*(b*Csc[e + f*x])^n*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Ta
n[(e + f*x)/2]^2] - 8*(Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan
[(e + f*x)/2]^2] - 3*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(
e + f*x)/2]^2] + 4*Hypergeometric2F1[4 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e
+ f*x)/2]^2] - 2*Hypergeometric2F1[5 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e +
f*x)/2]^2]))*Tan[(e + f*x)/2])/(f*(-1 + n)*(Sec[(e + f*x)/2]^2)^n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^n}{\sec(e+fx)^4}dx$$

$$\Big\downarrow \text{3111}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos^4(e+fx)(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos(e+fx)^4(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^4*(b*Csc[e + f*x])^n,x]
```

output
```
(b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-3/2, (1 - n)/
2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int \cos (fx+e)^4 (b\csc (fx+e))^n \, dx$$

input
```
int(cos(f*x+e)^4*(b*csc(f*x+e))^n,x)
```

output
```
int(cos(f*x+e)^4*(b*csc(f*x+e))^n,x)
```

Fricas [F]

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Sympy [F]

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(e+fx))^n\cos^4(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**4*(b*csc(f*x+e))**n,x)
```

output
```
Integral((b*csc(e + f*x))**n*cos(e + f*x)**4, x)
```

Maxima [F]

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx=\int (b\csc(fx+e))^n\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx=\int \cos(e+f\,x)^4\left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int(cos(e + f*x)^4*(b/sin(e + f*x))^n,x)
```

output
```
int(cos(e + f*x)^4*(b/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx=b^n\left(\int \csc(fx+e)^n\cos(fx+e)^4\,dx\right)$$

input
```
int(cos(f*x+e)^4*(b*csc(f*x+e))^n,x)
```

output
```
b**n*int(csc(e + f*x)**n*cos(e + f*x)**4,x)
```

3.296 $\int (b\csc(e+fx))^n (c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int (b\csc(e+fx))^n(c\sec(e+fx))^{3/2}\,dx = \frac{b\cos^2(e+fx)^{5/4}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)(c\sec(e}{cf(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(5/4)*(b*csc(f*x+e))^(-1+n)*hypergeom([5/4, 1/2-1/2*n],[3
/2-1/2*n],sin(f*x+e)^2)*(c*sec(f*x+e))^(5/2)/c/f/(1-n)
```

Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int (b\csc(e+fx))^n(c\sec(e+fx))^{3/2}\,dx = \frac{2\cot(e+fx)(b\csc(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\sec^2(e+fx)\right)}{f(3+2n)}$$

input
```
Integrate[(b*Csc[e + f*x])^n*(c*Sec[e + f*x])^(3/2),x]
```

output
```
(2*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/
4, (7 + 2*n)/4, Sec[e + f*x]^2]*(c*Sec[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(
(1 + n)/2))/(f*(3 + 2*n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sec(e+fx))^{3/2}(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sec(e+fx))^{3/2}(b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{3111}$$

$$\frac{b^2(c\cos(e+fx))^{5/2}(c\sec(e+fx))^{5/2}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \frac{(b\sin(e+fx))^{-n}}{(c\cos(e+fx))^{3/2}}dx}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2(c\cos(e+fx))^{5/2}(c\sec(e+fx))^{5/2}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \frac{(b\sin(e+fx))^{-n}}{(c\cos(e+fx))^{3/2}}dx}{c^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\cos^2(e+fx)^{5/4}(c\sec(e+fx))^{5/2}(b\csc(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n*(c*Sec[e + f*x])^(3/2),x]
```

output
```
(b*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/4,
 (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(5/2))/(c*f*(1 - n
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int (b\csc(fx+e))^n (c\sec(fx+e))^{\frac{3}{2}} dx$$

input
```
int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x)
```

output
```
int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n (c\sec(e+fx))^{3/2}\,dx = \int (c\sec(fx+e))^{\frac{3}{2}} (b\csc(fx+e))^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n*c*sec(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n (c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*csc(f*x+e))**n*(c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\csc(e+fx))^n (c\sec(e+fx))^{3/2}\,dx = \int (c\sec(fx+e))^{\frac{3}{2}} (b\csc(fx+e))^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e))^(3/2)*(b*csc(f*x + e))^n, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n(c\sec(e+fx))^{3/2}\,dx=\int (c\sec(fx+e))^{\frac{3}{2}}(b\csc(fx+e))^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sec(f*x + e))^(3/2)*(b*csc(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n(c\sec(e+fx))^{3/2}\,dx=\int \left(\frac{c}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int((c/cos(e + f*x))^(3/2)*(b/sin(e + f*x))^n,x)
```

output
```
int((c/cos(e + f*x))^(3/2)*(b/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n(c\sec(e+fx))^{3/2}\,dx=\sqrt{c}\,b^n\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)^n\sec(fx+e)\,dx\right)c$$

input
```
int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*b**n*int(sqrt(sec(e + f*x))*csc(e + f*x)**n*sec(e + f*x),x)*c
```

3.297 $\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx = \frac{b\cos^2(e+fx)^{3/4}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)(c\sec(e+fx))^{3/2}}{cf(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(3/4)*(b*csc(f*x+e))^(-1+n)*hypergeom([3/4, 1/2-1/2*n],[3
/2-1/2*n],sin(f*x+e)^2)*(c*sec(f*x+e))^(3/2)/c/f/(1-n)
```

Mathematica [A] (verified)

Time = 9.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx = \frac{2\cot(e+fx)(b\csc(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sec^2(e+fx)\right)\sqrt{c\sec(e+}}{f+2fn}$$

input
```
Integrate[(b*Csc[e + f*x])^n*Sqrt[c*Sec[e + f*x]],x]
```

output
```
(2*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/
4, (5 + 2*n)/4, Sec[e + f*x]^2]*Sqrt[c*Sec[e + f*x]]*(-Tan[e + f*x]^2)^((1
 + n)/2))/(f + 2*f*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sec(e+fx)}(b\csc(e+fx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{c\sec(e+fx)}(b\csc(e+fx))^n dx$$
$$\Big\downarrow \text{3111}$$
$$\frac{b^2(c\cos(e+fx))^{3/2}(c\sec(e+fx))^{3/2}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \frac{(b\sin(e+fx))^{-n}}{\sqrt{c\cos(e+fx)}}dx}{c^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{b^2(c\cos(e+fx))^{3/2}(c\sec(e+fx))^{3/2}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \frac{(b\sin(e+fx))^{-n}}{\sqrt{c\cos(e+fx)}}dx}{c^2}$$
$$\Big\downarrow \text{3057}$$
$$\frac{b\cos^2(e+fx)^{3/4}(c\sec(e+fx))^{3/2}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n*Sqrt[c*Sec[e + f*x]],x]
```

output
```
(b*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/4,
 (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(3/2))/(c*f*(1 - n
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int (b\csc(fx+e))^n \sqrt{c\sec(fx+e)}dx$$

input
```
int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)
```

output
```
int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx=\int \sqrt{c\sec\left(fx+e\right)}(b\csc\left(fx+e\right))^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx=\int (b\csc\left(e+fx\right))^n\sqrt{c\sec\left(e+fx\right)}\,dx$$

input
```
integrate((b*csc(f*x+e))**n*(c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((b*csc(e + f*x))**n*sqrt(c*sec(e + f*x)), x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx=\int \sqrt{c\sec\left(fx+e\right)}(b\csc\left(fx+e\right))^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx=\int \sqrt{c\sec(fx+e)}(b\csc(fx+e))^n\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx=\int \sqrt{\frac{c}{\cos(e+f\,x)}}\left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int((c/cos(e + f*x))^(1/2)*(b/sin(e + f*x))^n,x)
```

output
```
int((c/cos(e + f*x))^(1/2)*(b/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx=\sqrt{c}\,b^n\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)^n\,dx\right)$$

input
```
int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*b**n*int(sqrt(sec(e + f*x))*csc(e + f*x)**n,x)
```

3.298 $\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \frac{b\sqrt[4]{\cos^2(e+fx)}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)\sqrt{c\sec(e+fx)}}{cf(1-n)}$$

output
```
b*(cos(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/4, 1/2-1/2*n],[3
/2-1/2*n],sin(f*x+e)^2)*(c*sec(f*x+e))^(1/2)/c/f/(1-n)
```

Mathematica [A] (verified)

Time = 27.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \frac{(b\csc(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{n}{2},\frac{5}{4}-\frac{n}{2},\frac{3}{2}-\frac{n}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{\frac{1}{4}-\frac{n}{2}}\tan(e+fx}{f(1-n)\sqrt{c\sec(e+fx)}}$$

input
```
Integrate[(b*Csc[e + f*x])^n/Sqrt[c*Sec[e + f*x]],x]
```

output
```
((b*Csc[e + f*x])^n*Hypergeometric2F1[1/2 - n/2, 5/4 - n/2, 3/2 - n/2, -Ta
n[e + f*x]^2]*(Sec[e + f*x]^2)^(1/4 - n/2)*Tan[e + f*x])/(f*(1 - n)*Sqrt[c
*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3111}$$

$$\frac{b^2\sqrt{c\cos(e+fx)}\sqrt{c\sec(e+fx)}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \sqrt{c\cos(e+fx)}(b\sin(e+fx))^{-n}dx}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{c\cos(e+fx)}\sqrt{c\sec(e+fx)}(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \sqrt{c\cos(e+fx)}(b\sin(e+fx))^{-n}dx}{c^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)}$$

input
```
Int[(b*Csc[e + f*x])^n/Sqrt[c*Sec[e + f*x]],x]
```

output
```
(b*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/4,
 (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sqrt[c*Sec[e + f*x]])/(c*f*(1 - n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int \frac{(b\csc(fx+e))^n}{\sqrt{c\sec(fx+e)}}dx$$

input
```
int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x)
```

output
```
int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^n}{\sqrt{c\sec(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n/(c*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx$$

input
```
integrate((b*csc(f*x+e))**n/(c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((b*csc(e + f*x))**n/sqrt(c*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^n}{\sqrt{c\sec(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n/sqrt(c*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^n}{\sqrt{c\sec(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n/sqrt(c*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \int \frac{\left(\frac{b}{\sin(e+fx)}\right)^n}{\sqrt{\frac{c}{\cos(e+fx)}}}\,dx$$

input
```
int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(1/2),x)
```

output
```
int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \frac{\sqrt{c}\,b^n\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^n}{\sec(fx+e)}dx\right)}{c}$$

input
```
int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*b**n*int((sqrt(sec(e + f*x))*csc(e + f*x)**n)/sec(e + f*x),x))/c
```

3.299 $\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx=\frac{b(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}}$$

output
```
b*(b*csc(f*x+e))^(-1+n)*hypergeom([-1/4, 1/2-1/2*n],[3/2-1/2*n],sin(f*x+e)
^2)/c/f/(1-n)/(cos(f*x+e)^2)^(1/4)/(c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.42

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx=$$

$$-\frac{2\cos(2(e+fx))\cot(e+fx)(b\csc(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\sec^2(e+\right.}{c^2f(-3+2n)\left(-2+\sec^2(e+fx)\right)}$$

input
```
Integrate[(b*Csc[e + f*x])^n/(c*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*Cos[2*(e + f*x)]*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1
+ n)/2, (-3 + 2*n)/4, (1 + 2*n)/4, Sec[e + f*x]^2]*Sqrt[c*Sec[e + f*x]]*(-
Tan[e + f*x]^2)^((1 + n)/2))/(c^2*f*(-3 + 2*n)*(-2 + Sec[e + f*x]^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3111, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}dx$$
$$\Big\downarrow \ 3111$$
$$\frac{b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int(c\cos(e+fx))^{3/2}(b\sin(e+fx))^{-n}dx}{c^2\sqrt{c\cos(e+fx)}\sqrt{c\sec(e+fx)}}$$
$$\Big\downarrow \ 3042$$
$$\frac{b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int(c\cos(e+fx))^{3/2}(b\sin(e+fx))^{-n}dx}{c^2\sqrt{c\cos(e+fx)}\sqrt{c\sec(e+fx)}}$$
$$\Big\downarrow \ 3057$$
$$\frac{b(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\sqrt{c\sec(e+fx)}}$$

input
```
Int[(b*Csc[e + f*x])^n/(c*Sec[e + f*x])^(3/2),x]
```

output
```
(b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2,
 Sin[e + f*x]^2])/(c*f*(1 - n)*(Cos[e + f*x]^2)^(1/4)*Sqrt[c*Sec[e + f*x]]
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3111
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)   Int[1/((a*Sin[e
+ f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !
SimplerQ[-m, -n]
```

Maple [F]

$$\int \frac{(b\csc(fx+e))^n}{(c\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x)
```

output
```
int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(fx+e))^n}{(c\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n/(c^2*sec(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\csc{(e+fx)})^n}{(c\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))**n/(c*sec(f*x+e))**(3/2),x)
```

output
```
Integral((b*csc(e + f*x))**n/(c*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\csc{(fx+e)})^n}{(c\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n/(c*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\csc{(fx+e)})^n}{(c\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n/(c*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\left(\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(3/2),x)
```

output
```
int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,b^n\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^n}{\sec(fx+e)^2}dx\right)}{c^2}$$

input
```
int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*b**n*int((sqrt(sec(e + f*x))*csc(e + f*x)**n)/sec(e + f*x)**2,x))
/c**2
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file