Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.5-Secant/235-4.5.2.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**242**]. This is test number [235].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.17 (240)	0.83 (2)
Mathematica	97.11 (235)	2.89 (7)
Maple	90.50 (219)	9.50 (23)
Fricas	60.33 (146)	39.67 (96)
Giac	50.41 (122)	49.59 (120)
Maxima	40.50 (98)	59.50 (144)
Reduce	24.79 (60)	75.21 (182)
Mupad	23.14 (56)	76.86 (186)
Sympy	2.07 (5)	97.93 (237)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

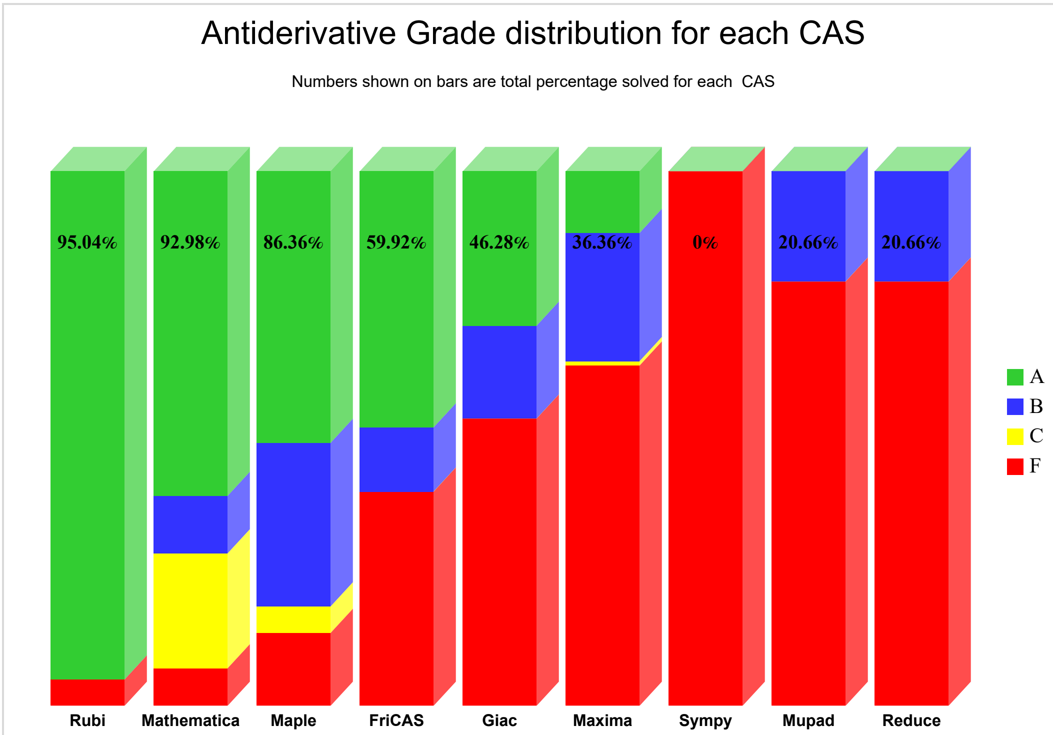
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

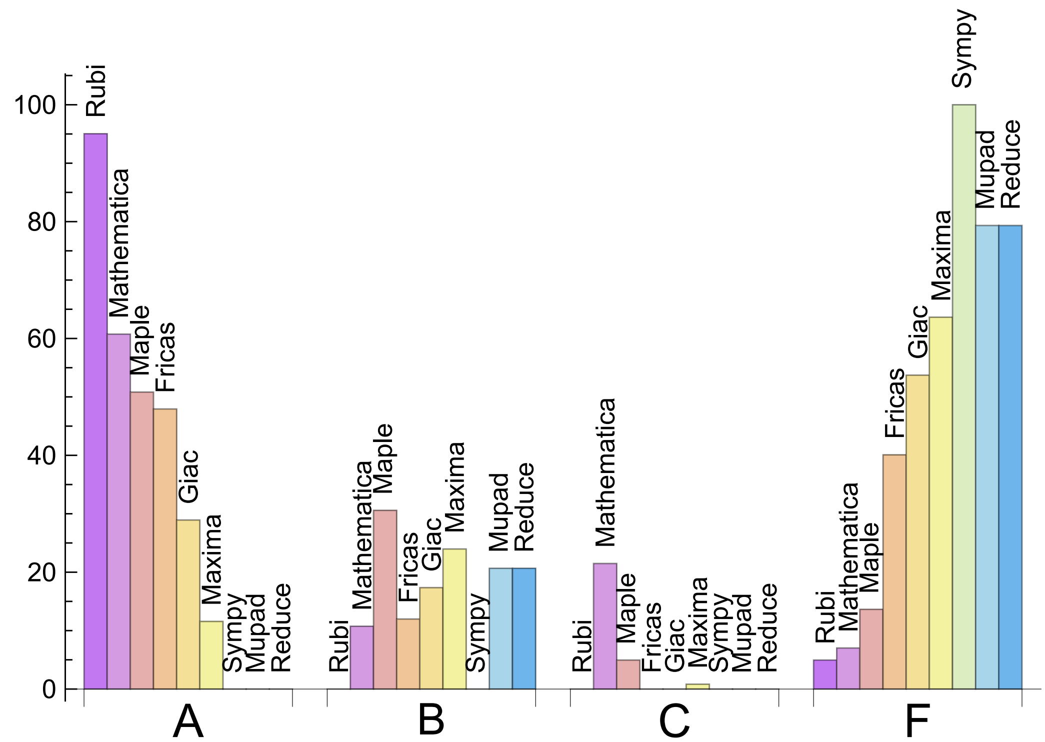
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.041	0.000	0.000	4.959
Mathematica	60.744	10.744	21.488	7.025
Maple	50.826	30.579	4.959	13.636
Fricas	47.934	11.983	0.000	40.083
Giac	28.926	17.355	0.000	53.719
Maxima	11.570	23.967	0.826	63.636
Mupad	0.000	20.661	0.000	79.339
Reduce	0.000	20.661	0.000	79.339
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	7	100.00	0.00	0.00
Maple	23	100.00	0.00	0.00
Fricas	96	66.67	33.33	0.00
Giac	120	42.50	0.00	57.50
Maxima	144	70.83	16.67	12.50
Mupad	186	0.00	100.00	0.00
Reduce	182	100.00	0.00	0.00
Sympy	237	83.54	16.46	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Reduce	0.18
Giac	0.59
Rubi	0.68
Maxima	1.12
Fricas	1.90
Maple	4.77
Mathematica	6.83
Sympy	17.24
Mupad	20.38

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	26.80	0.94	27.00	0.93
Rubi	192.12	0.93	142.00	0.97
Giac	289.06	1.71	181.50	1.44
Reduce	499.38	2.49	97.50	1.34
Fricas	559.26	3.14	407.00	2.81
Maxima	770.86	4.75	255.50	2.45
Mathematica	964.06	2.56	122.00	0.89
Maple	1589.46	3.91	224.00	1.53
Mupad	1614.84	6.20	122.00	1.09

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

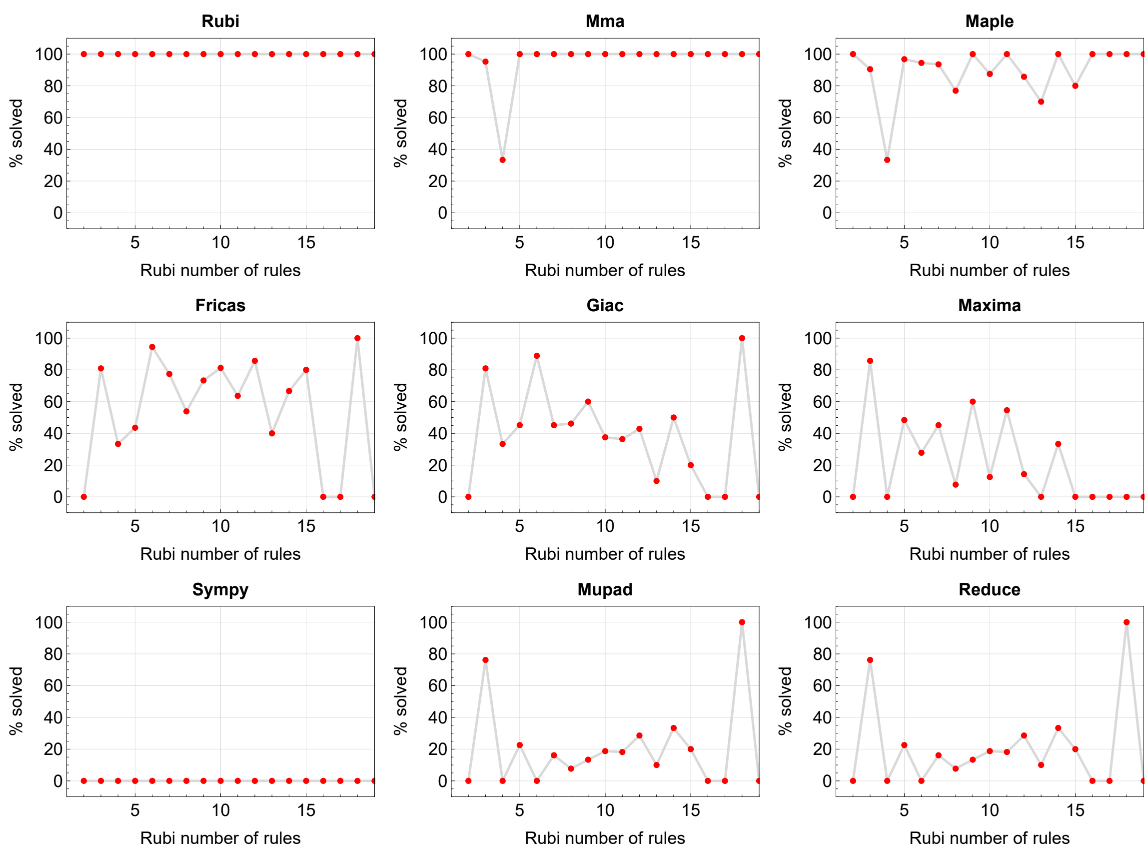


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

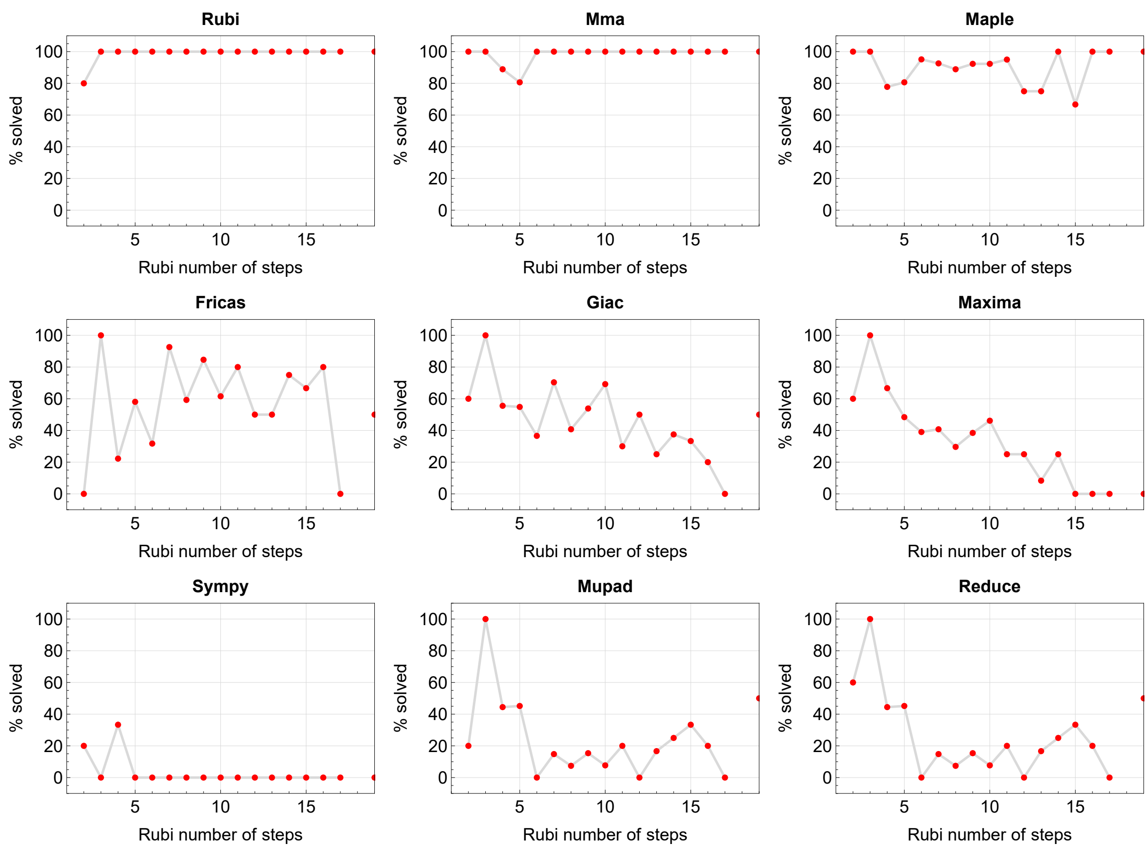


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

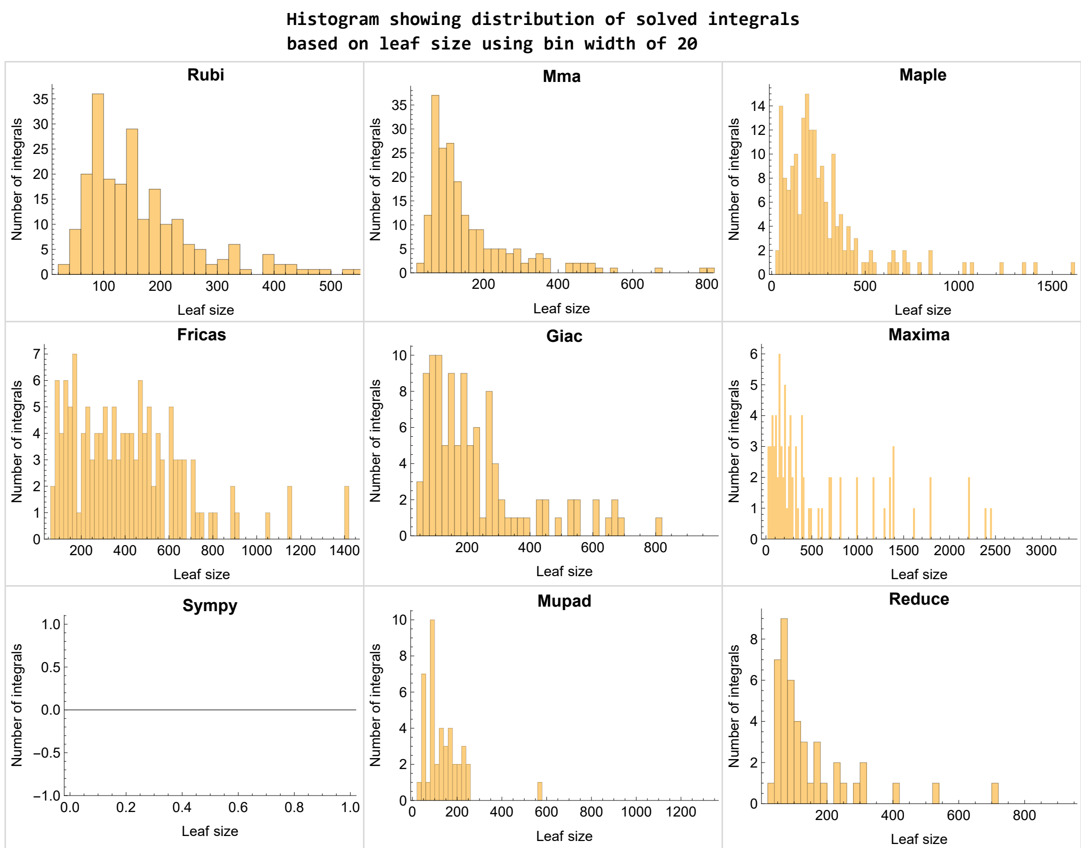


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

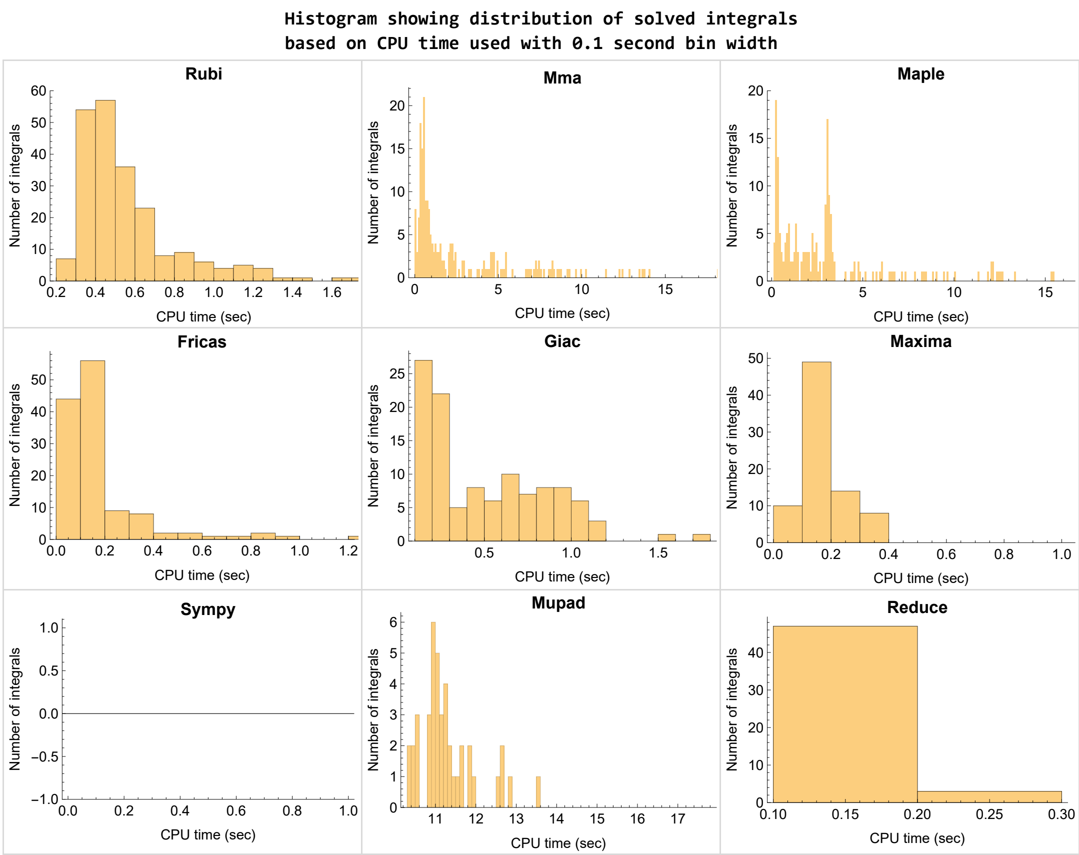


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

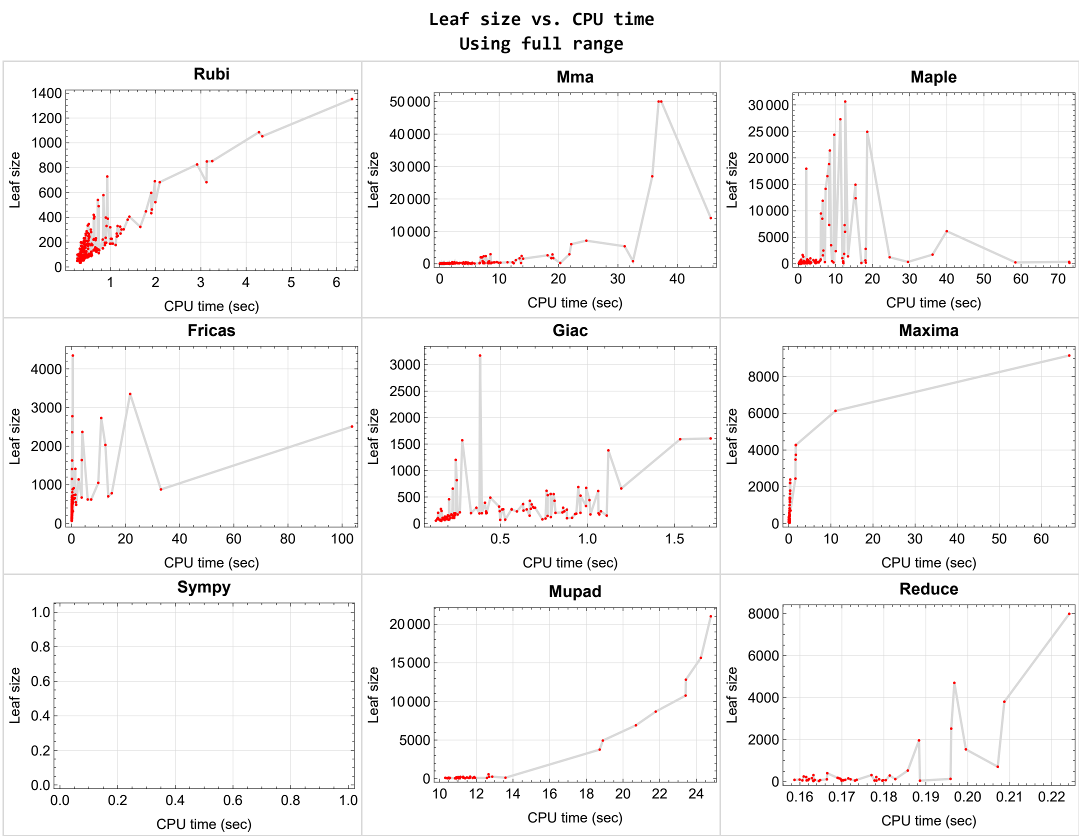


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{222, 223, 224, 225, 226, 227, 228, 229, 230, 237\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{231\}$

Mathematica $\{144, 147, 148, 149, 152, 153, 158, 159, 163, 165, 166, 167, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 200, 202, 204, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 221, 231, 241, 242\}$

Maple $\{48, 49, 55, 56, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 124, 147, 151, 152, 153, 154, 157, 158, 159, 160, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 210, 213, 214, 215, 216, 217, 221\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima **abs_integrate** was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See **https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/** for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error **Exception raised: NotImplementedError**.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function **LeafSize** was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user **slelievre** at **https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/**

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

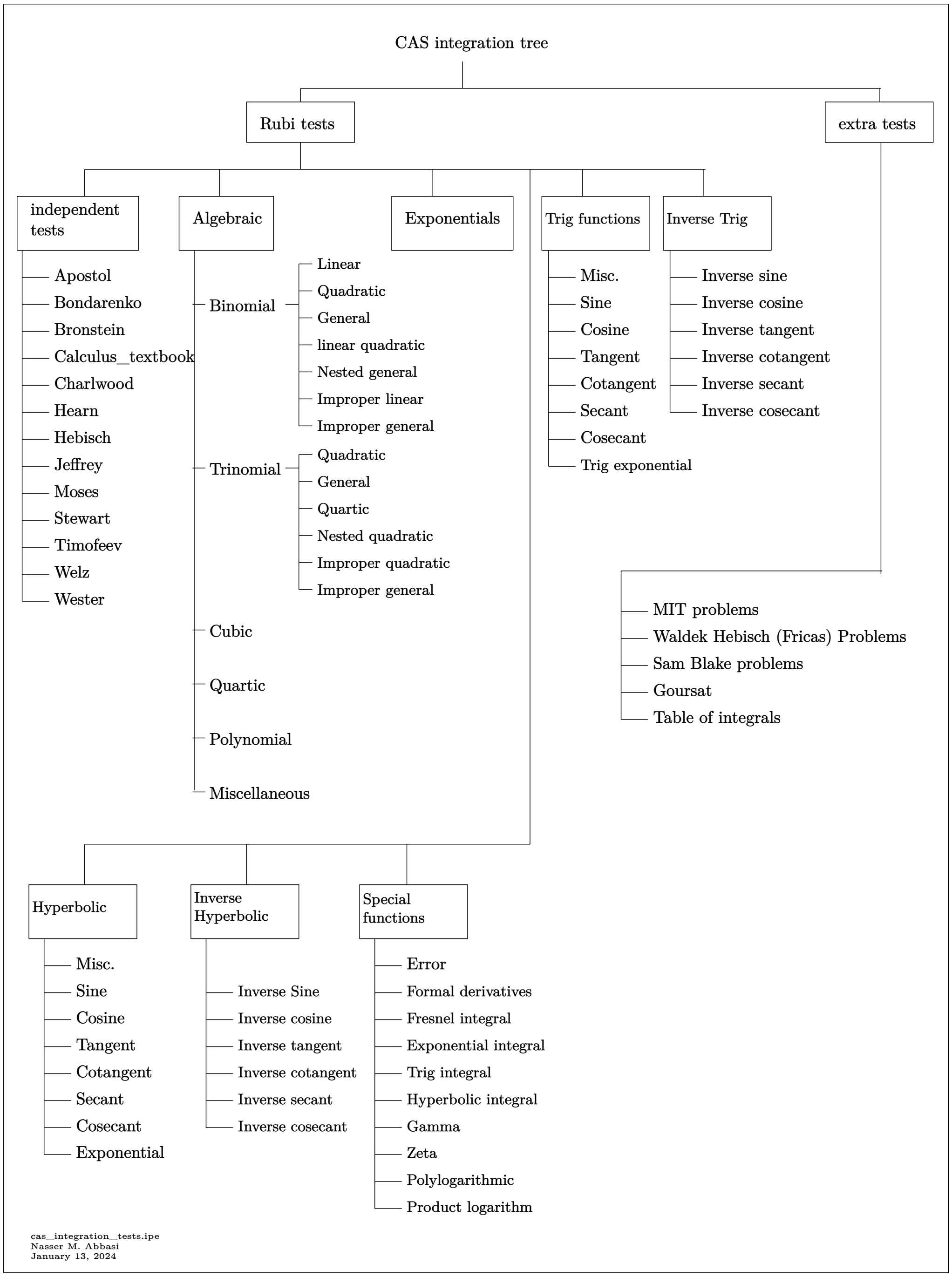
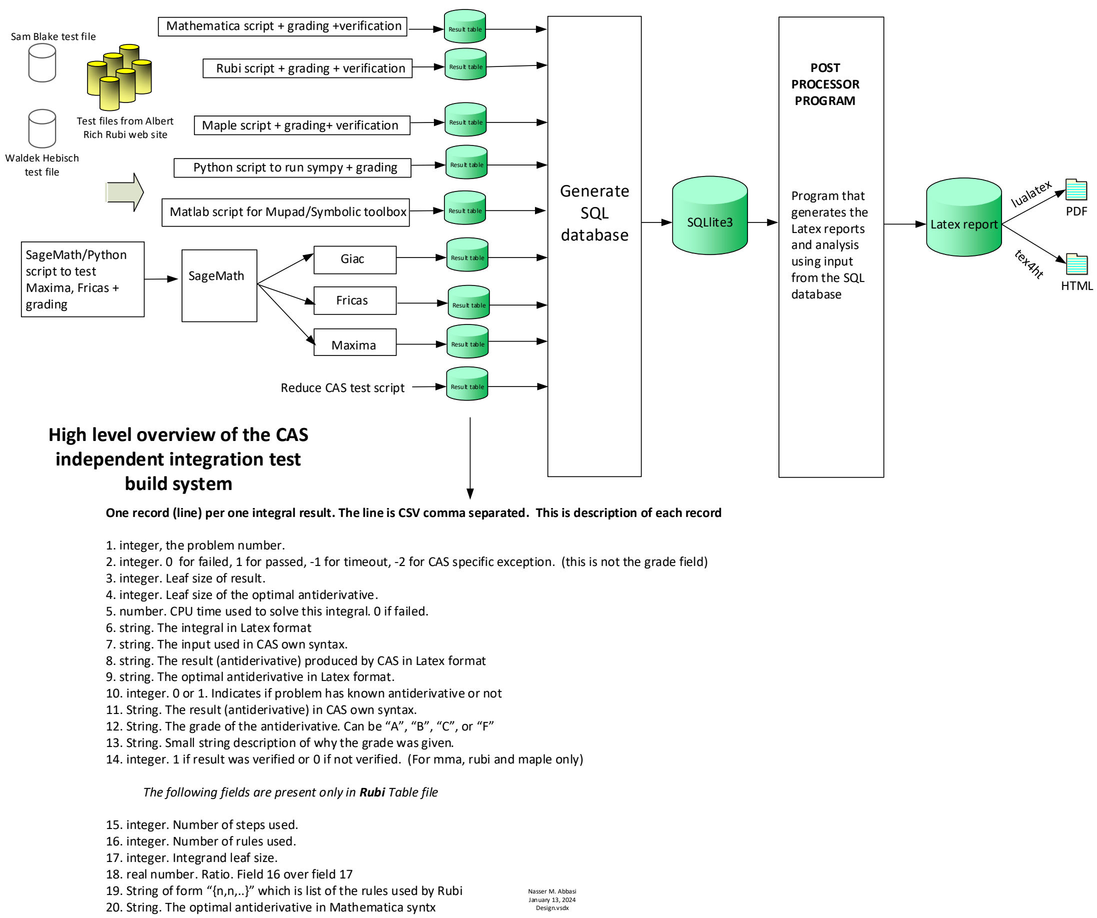


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 219, 220, 221, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242 }

B grade { }

C grade { }

F normal fail { 211, 218 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15, 19, 24, 33, 34, 41, 42, 43, 44, 45, 50, 51, 52, 53, 57, 58, 59, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 141, 142, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 168, 169, 170, 173, 174, 175, 176, 178, 179, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 197, 198, 199, 201, 203, 204, 208, 219, 232, 233, 234, 235, 236, 238, 239, 240 }

B grade { 6, 17, 25, 144, 171, 177, 180, 183, 193, 195, 200, 202, 205, 206, 209, 210, 212, 213, 214, 215, 216, 217, 221, 231, 241, 242 }

C grade { 7, 9, 10, 16, 18, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 46, 47, 48, 49, 54, 55, 56, 60, 61, 62, 63, 64, 69, 70, 71, 76, 77, 78, 84, 85, 143, 149, 163, 166, 167, 172, 207, 211, 218, 220 }

F normal fail { 131, 132, 133, 134, 135, 136, 137 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 49, 50, 51, 52, 53, 57, 58, 59, 60, 65, 66, 67, 68, 73, 74, 75, 77, 78, 81, 82, 83, 85, 89, 91, 92, 93, 98, 99, 100, 107, 108, 109, 113, 115, 116, 117, 119, 121, 123, 124, 126, 127, 128, 129, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 160, 161, 162, 166, 167, 172, 173, 178, 179, 189, 190, 191, 192, 193, 194, 195, 196, 199, 201, 203, 204, 207, 208, 211, 218, 219, 220 }

B grade { 46, 47, 48, 54, 55, 56, 61, 62, 63, 64, 69, 70, 71, 72, 76, 79, 80, 84, 88, 90, 96, 97, 104, 105, 106, 110, 111, 112, 114, 118, 120, 125, 130, 151, 152, 153, 157, 158, 159, 163, 164, 165, 168, 169, 170, 171, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 197, 198, 200, 202, 205, 206, 209, 210, 212, 213, 214, 215, 216, 217, 221 }

C grade { 1, 11, 12, 13, 86, 87, 94, 95, 101, 102, 103, 122 }

F normal fail { 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 94, 95, 101, 102, 103, 130, 143, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 178, 179, 184, 185, 187, 188, 189 }

B grade { 5, 22, 23, 61, 62, 63, 74, 75, 83, 88, 89, 96, 105, 114, 118, 122, 153, 159, 174, 180, 186, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 90, 91, 92, 93, 97, 98, 99, 100, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 198, 200, 202, 205, 206, 208, 210, 212, 221, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242 }

F(-1) timedout fail { 146, 171, 175, 176, 177, 181, 182, 183, 199, 201, 203, 204, 207, 209, 211, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 4, 5, 13, 15, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 40, 41, 89, 90, 97, 98, 106, 112, 113, 114, 119, 125 }

B grade { 3, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 45, 52, 59, 86, 87, 88, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 107, 108, 109, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 130, 150, 156, 162 }

C grade { 68, 168 }

F normal fail { 42, 43, 44, 46, 47, 48, 50, 51, 53, 58, 60, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 157, 161, 163, 166, 167, 169, 170, 172, 173, 174, 175, 176, 179, 180, 181, 184, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241 }

F(-1) timedout fail { 49, 54, 55, 56, 57, 61, 62, 63, 64, 79, 80, 81, 153, 158, 159, 160, 164, 165, 171, 177, 178, 182, 183, 242 }

F(-2) exception fail { 104, 105, 110, 111, 118, 124, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

Giac

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 50, 57, 58, 59, 90, 91, 92, 93, 98, 99, 100, 106, 107, 108, 109, 114, 115, 116, 121, 122, 123, 124, 128, 129, 130, 148, 155, 160, 161, 190, 192 }

B grade { 5, 27, 38, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 143, 147, 149, 150, 151, 152, 153, 154, 156, 158, 159, 162, 164, 165, 189, 191, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 144, 145, 146, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 231, 232, 233, 234, 238, 239, 240, 241, 242 }

F(-1) timedout fail { }

F(-2) exception fail { 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 101, 102, 103, 104, 105, 110, 111, 112, 113, 117, 118, 119, 120, 125, 126, 127, 136, 137, 157, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 235, 236 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { }

F(-1) timedout fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 227, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 96, 97, 98, 99, 112, 113, 114, 115, 116, 119, 120, 121, 122, 126, 127, 128, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242 }

F(-1) timedout fail { 62, 63, 64, 86, 87, 93, 94, 95, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 117, 118, 123, 124, 125, 129, 130, 138, 139, 165, 214, 215, 216, 217, 224, 227, 228, 229, 230 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 189, 190, 191, 192, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	176	165	206	334	179	0	191	318	228
N.S.	1	0.90	0.84	1.05	1.70	0.91	0.00	0.97	1.62	1.16
time (sec)	N/A	0.564	1.574	1.444	0.042	0.092	0.000	0.243	0.177	11.140

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	126	146	172	240	163	0	172	298	195
N.S.	1	0.90	1.04	1.23	1.71	1.16	0.00	1.23	2.13	1.39
time (sec)	N/A	0.470	1.041	1.113	0.041	0.099	0.000	0.220	0.181	11.288

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	89	122	133	203	147	0	153	226	163
N.S.	1	0.92	1.26	1.37	2.09	1.52	0.00	1.58	2.33	1.68
time (sec)	N/A	0.450	0.654	0.906	0.037	0.089	0.000	0.200	0.178	11.271

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	35	45	56	57	65	0	48	78	84
N.S.	1	0.74	0.96	1.19	1.21	1.38	0.00	1.02	1.66	1.79
time (sec)	N/A	0.326	0.025	0.469	0.035	0.072	0.000	0.162	0.170	12.581

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	52	97	84	95	103	0	103	127	91
N.S.	1	0.95	1.76	1.53	1.73	1.87	0.00	1.87	2.31	1.65
time (sec)	N/A	0.316	0.245	0.478	0.033	0.085	0.000	0.142	0.183	10.529

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	51	201	52	153	87	0	77	80	46
N.S.	1	0.91	3.59	0.93	2.73	1.55	0.00	1.38	1.43	0.82
time (sec)	N/A	0.359	1.445	0.177	0.113	0.084	0.000	0.147	0.165	11.035

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	66	53	41	174	88	0	57	52	40
N.S.	1	0.93	0.75	0.58	2.45	1.24	0.00	0.80	0.73	0.56
time (sec)	N/A	0.413	0.045	0.198	0.111	0.070	0.000	0.156	0.189	10.523

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	94	151	54	215	128	0	72	65	96
N.S.	1	0.92	1.48	0.53	2.11	1.25	0.00	0.71	0.64	0.94
time (sec)	N/A	0.488	1.171	0.221	0.115	0.075	0.000	0.202	0.161	10.563

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	122	119	67	294	172	0	88	78	124
N.S.	1	0.92	0.89	0.50	2.21	1.29	0.00	0.66	0.59	0.93
time (sec)	N/A	0.582	0.763	0.283	0.125	0.074	0.000	0.170	0.162	10.476

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	150	111	80	335	212	0	104	91	146
N.S.	1	0.91	0.68	0.49	2.04	1.29	0.00	0.63	0.55	0.89
time (sec)	N/A	0.690	0.772	0.302	0.129	0.081	0.000	0.228	0.159	11.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	169	189	217	356	195	0	210	408	259
N.S.	1	0.90	1.01	1.15	1.89	1.04	0.00	1.12	2.17	1.38
time (sec)	N/A	0.508	1.806	1.498	0.039	0.105	0.000	0.268	0.167	11.669

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	123	165	206	334	179	0	191	318	227
N.S.	1	0.93	1.25	1.56	2.53	1.36	0.00	1.45	2.41	1.72
time (sec)	N/A	0.579	1.064	1.331	0.040	0.089	0.000	0.236	0.163	11.280

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	52	61	81	94	81	0	65	116	122
N.S.	1	0.76	0.90	1.19	1.38	1.19	0.00	0.96	1.71	1.79
time (sec)	N/A	0.391	0.027	0.715	0.035	0.078	0.000	0.181	0.166	13.591

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	89	122	133	203	147	0	153	226	163
N.S.	1	0.92	1.26	1.37	2.09	1.52	0.00	1.58	2.33	1.68
time (sec)	N/A	0.438	0.424	0.914	0.037	0.090	0.000	0.225	0.161	10.917

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	76	101	77	107	118	0	104	186	104
N.S.	1	0.99	1.31	1.00	1.39	1.53	0.00	1.35	2.42	1.35
time (sec)	N/A	0.397	0.428	0.550	0.033	0.083	0.000	0.169	0.169	10.309

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	71	275	94	274	125	0	111	174	85
N.S.	1	0.91	3.53	1.21	3.51	1.60	0.00	1.42	2.23	1.09
time (sec)	N/A	0.446	3.384	0.260	0.116	0.092	0.000	0.192	0.163	10.350

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	80	226	58	274	156	0	80	89	45
N.S.	1	0.91	2.57	0.66	3.11	1.77	0.00	0.91	1.01	0.51
time (sec)	N/A	0.515	2.370	0.257	0.115	0.089	0.000	0.190	0.164	10.431

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	94	53	54	282	128	0	73	65	96
N.S.	1	0.92	0.52	0.53	2.76	1.25	0.00	0.72	0.64	0.94
time (sec)	N/A	0.592	0.051	0.226	0.119	0.078	0.000	0.206	0.162	11.041

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	122	161	67	383	172	0	88	78	122
N.S.	1	0.92	1.21	0.50	2.88	1.29	0.00	0.66	0.59	0.92
time (sec)	N/A	0.693	1.104	0.289	0.129	0.078	0.000	0.196	0.162	11.292

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	150	139	80	403	212	0	104	91	146
N.S.	1	0.91	0.85	0.49	2.46	1.29	0.00	0.63	0.55	0.89
time (sec)	N/A	0.835	1.272	0.310	0.132	0.082	0.000	0.236	0.171	11.015

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	351	137	603	242	0	153	252	145
N.S.	1	1.00	2.58	1.01	4.43	1.78	0.00	1.12	1.85	1.07
time (sec)	N/A	0.630	2.811	0.504	0.128	0.096	0.000	0.224	0.161	10.908

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	91	298	105	413	220	0	134	161	112
N.S.	1	0.89	2.92	1.03	4.05	2.16	0.00	1.31	1.58	1.10
time (sec)	N/A	0.546	2.146	0.397	0.121	0.087	0.000	0.203	0.172	10.883

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	77	189	58	268	173	0	80	57	46
N.S.	1	0.91	2.22	0.68	3.15	2.04	0.00	0.94	0.67	0.54
time (sec)	N/A	0.491	1.214	0.277	0.114	0.086	0.000	0.174	0.173	10.845

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	62	67	41	170	94	0	60	40	38
N.S.	1	0.93	1.00	0.61	2.54	1.40	0.00	0.90	0.60	0.57
time (sec)	N/A	0.404	0.032	0.209	0.116	0.073	0.000	0.158	0.163	10.904

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	56	139	37	119	86	0	53	36	41
N.S.	1	0.92	2.28	0.61	1.95	1.41	0.00	0.87	0.59	0.67
time (sec)	N/A	0.331	0.606	0.191	0.112	0.071	0.000	0.131	0.165	10.849

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	58	50	102	70	0	81	61	69
N.S.	1	0.96	0.84	0.72	1.48	1.01	0.00	1.17	0.88	1.00
time (sec)	N/A	0.422	0.449	0.205	0.112	0.081	0.000	0.138	0.178	10.905

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	34	39	38	46	81	0	95	56	58
N.S.	1	0.74	0.85	0.83	1.00	1.76	0.00	2.07	1.22	1.26
time (sec)	N/A	0.343	0.030	0.206	0.111	0.073	0.000	0.171	0.178	11.107

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	100	70	78	147	154	0	110	91	161
N.S.	1	1.02	0.71	0.80	1.50	1.57	0.00	1.12	0.93	1.64
time (sec)	N/A	0.549	0.792	0.257	0.115	0.076	0.000	0.182	0.174	11.056

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	146	81	91	167	166	0	122	104	185
N.S.	1	0.88	0.49	0.55	1.01	1.00	0.00	0.73	0.63	1.11
time (sec)	N/A	0.508	2.038	0.301	0.117	0.080	0.000	0.186	0.160	11.007

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	187	91	104	186	232	0	135	117	209
N.S.	1	0.89	0.43	0.50	0.89	1.10	0.00	0.64	0.56	1.00
time (sec)	N/A	0.583	5.232	0.339	0.119	0.083	0.000	0.232	0.179	11.119

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	143	372	118	562	289	0	154	174	134
N.S.	1	0.88	2.30	0.73	3.47	1.78	0.00	0.95	1.07	0.83
time (sec)	N/A	0.698	4.543	0.512	0.130	0.091	0.000	0.241	0.169	10.994

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	134	202	69	396	242	0	102	68	50
N.S.	1	0.91	1.36	0.47	2.68	1.64	0.00	0.69	0.46	0.34
time (sec)	N/A	0.754	2.362	0.350	0.131	0.098	0.000	0.214	0.170	11.395

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	88	90	54	277	138	0	80	53	93
N.S.	1	0.92	0.94	0.56	2.89	1.44	0.00	0.83	0.55	0.97
time (sec)	N/A	0.578	0.052	0.257	0.119	0.088	0.000	0.172	0.170	11.643

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	88	151	54	211	138	0	80	53	93
N.S.	1	0.92	1.57	0.56	2.20	1.44	0.00	0.83	0.55	0.97
time (sec)	N/A	0.486	1.134	0.248	0.116	0.079	0.000	0.169	0.180	11.936

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	80	79	50	159	124	0	71	51	85
N.S.	1	0.91	0.90	0.57	1.81	1.41	0.00	0.81	0.58	0.97
time (sec)	N/A	0.392	0.441	0.222	0.111	0.078	0.000	0.154	0.163	11.805

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	115	71	65	122	109	0	102	76	82
N.S.	1	0.91	0.56	0.52	0.97	0.87	0.00	0.81	0.60	0.65
time (sec)	N/A	0.465	0.569	0.227	0.113	0.073	0.000	0.184	0.180	11.367

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	70	78	146	154	0	116	91	161
N.S.	1	1.01	0.70	0.78	1.46	1.54	0.00	1.16	0.91	1.61
time (sec)	N/A	0.538	0.975	0.251	0.116	0.076	0.000	0.192	0.170	11.486

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	53	39	53	56	118	0	129	72	94
N.S.	1	0.79	0.58	0.79	0.84	1.76	0.00	1.93	1.07	1.40
time (sec)	N/A	0.411	0.043	0.265	0.109	0.088	0.000	0.183	0.180	11.570

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	81	104	187	232	0	142	117	209
N.S.	1	0.98	0.63	0.81	1.45	1.80	0.00	1.10	0.91	1.62
time (sec)	N/A	0.681	4.752	0.315	0.117	0.089	0.000	0.209	0.172	11.888

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	187	91	117	205	271	0	154	130	233
N.S.	1	0.89	0.43	0.56	0.98	1.29	0.00	0.73	0.62	1.11
time (sec)	N/A	0.521	5.868	0.366	0.120	0.087	0.000	0.237	0.162	12.695

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	220	499	130	227	310	0	169	143	257
N.S.	1	0.87	1.98	0.52	0.90	1.23	0.00	0.67	0.57	1.02
time (sec)	N/A	0.628	7.434	0.416	0.120	0.090	0.000	0.256	0.170	12.870

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	157	122	162	0	373	0	295	102	0
N.S.	1	0.90	0.70	0.93	0.00	2.13	0.00	1.69	0.58	0.00
time (sec)	N/A	0.449	5.325	2.286	0.000	0.107	0.000	0.694	0.209	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	128	112	146	0	347	0	263	82	0
N.S.	1	0.91	0.80	1.04	0.00	2.48	0.00	1.88	0.59	0.00
time (sec)	N/A	0.427	1.204	1.887	0.000	0.103	0.000	0.566	0.208	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	99	100	135	0	313	0	232	58	0
N.S.	1	0.94	0.95	1.29	0.00	2.98	0.00	2.21	0.55	0.00
time (sec)	N/A	0.424	0.740	1.730	0.000	0.100	0.000	0.497	0.200	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	91	104	147	234	0	193	36	0
N.S.	1	1.03	1.38	1.58	2.23	3.55	0.00	2.92	0.55	0.00
time (sec)	N/A	0.357	0.441	1.399	0.164	0.095	0.000	0.395	0.199	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	63	46	193	0	266	0	196	29	0
N.S.	1	0.91	0.67	2.80	0.00	3.86	0.00	2.84	0.42	0.00
time (sec)	N/A	0.370	0.326	0.740	0.000	0.136	0.000	0.332	0.195	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	96	59	214	0	339	0	296	38	0
N.S.	1	0.92	0.57	2.06	0.00	3.26	0.00	2.85	0.37	0.00
time (sec)	N/A	0.384	0.412	0.825	0.000	0.127	0.000	0.363	0.202	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	139	126	59	255	0	405	0	391	49	0
N.S.	1	0.91	0.42	1.83	0.00	2.91	0.00	2.81	0.35	0.00
time (sec)	N/A	0.388	0.599	0.898	0.000	0.137	0.000	0.412	0.190	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	174	156	59	294	0	475	0	487	58	0
N.S.	1	0.90	0.34	1.69	0.00	2.73	0.00	2.80	0.33	0.00
time (sec)	N/A	0.392	3.859	0.963	0.000	0.146	0.000	0.443	0.208	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	154	124	162	0	385	0	297	83	0
N.S.	1	0.87	0.70	0.92	0.00	2.18	0.00	1.68	0.47	0.00
time (sec)	N/A	0.407	1.545	3.409	0.000	0.110	0.000	0.861	0.211	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	125	114	145	0	355	0	265	81	0
N.S.	1	0.88	0.80	1.02	0.00	2.50	0.00	1.87	0.57	0.00
time (sec)	N/A	0.398	0.857	2.536	0.000	0.102	0.000	0.652	0.213	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	94	100	129	998	303	0	227	39	0
N.S.	1	0.93	0.99	1.28	9.88	3.00	0.00	2.25	0.39	0.00
time (sec)	N/A	0.357	0.529	2.257	0.211	0.092	0.000	0.419	0.210	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	65	78	109	0	269	0	198	59	0
N.S.	1	0.93	1.11	1.56	0.00	3.84	0.00	2.83	0.84	0.00
time (sec)	N/A	0.382	0.422	1.047	0.000	0.128	0.000	0.422	0.198	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	93	73	215	0	351	0	318	78	0
N.S.	1	0.91	0.72	2.11	0.00	3.44	0.00	3.12	0.76	0.00
time (sec)	N/A	0.388	0.584	1.216	0.000	0.145	0.000	0.494	0.212	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	137	126	73	266	0	417	0	428	99	0
N.S.	1	0.92	0.53	1.94	0.00	3.04	0.00	3.12	0.72	0.00
time (sec)	N/A	0.384	0.898	1.285	0.000	0.140	0.000	0.670	0.215	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	156	73	314	0	495	0	555	118	0
N.S.	1	0.91	0.42	1.83	0.00	2.88	0.00	3.23	0.69	0.00
time (sec)	N/A	0.403	4.199	1.349	0.000	0.145	0.000	0.806	0.233	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	181	134	174	0	441	0	329	127	0
N.S.	1	0.85	0.63	0.82	0.00	2.08	0.00	1.55	0.60	0.00
time (sec)	N/A	0.413	2.246	73.039	0.000	0.108	0.000	0.992	0.238	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	152	124	165	0	409	0	297	63	0
N.S.	1	0.86	0.70	0.93	0.00	2.31	0.00	1.68	0.36	0.00
time (sec)	N/A	0.391	0.972	16.981	0.000	0.105	0.000	0.700	0.223	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	119	110	144	1396	353	0	227	81	0
N.S.	1	0.90	0.83	1.09	10.58	2.67	0.00	1.72	0.61	0.00
time (sec)	N/A	0.421	0.578	4.588	0.230	0.106	0.000	0.596	0.240	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	90	66	119	0	291	0	268	96	0
N.S.	1	0.87	0.64	1.16	0.00	2.83	0.00	2.60	0.93	0.00
time (sec)	N/A	0.406	0.355	3.063	0.000	0.124	0.000	0.565	0.208	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	70	70	136	0	339	0	243	122	0
N.S.	1	0.95	0.95	1.84	0.00	4.58	0.00	3.28	1.65	0.00
time (sec)	N/A	0.403	0.527	12.149	0.000	0.134	0.000	0.633	0.231	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	104	92	73	268	0	441	0	428	156	0
N.S.	1	0.88	0.70	2.58	0.00	4.24	0.00	4.12	1.50	0.00
time (sec)	N/A	0.401	1.010	58.477	0.000	0.140	0.000	0.812	0.226	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	140	126	73	308	0	527	0	556	182	0
N.S.	1	0.90	0.52	2.20	0.00	3.76	0.00	3.97	1.30	0.00
time (sec)	N/A	0.408	4.265	3.087	0.000	0.147	0.000	0.789	0.240	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	153	73	356	0	601	0	672	216	0
N.S.	1	0.89	0.42	2.07	0.00	3.49	0.00	3.91	1.26	0.00
time (sec)	N/A	0.413	5.192	0.994	0.000	0.158	0.000	0.993	0.243	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	185	188	153	218	0	552	0	0	156	0
N.S.	1	1.02	0.83	1.18	0.00	2.98	0.00	0.00	0.84	0.00
time (sec)	N/A	0.507	4.980	2.365	0.000	0.516	0.000	0.000	0.240	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	152	155	166	204	0	518	0	0	126	0
N.S.	1	1.02	1.09	1.34	0.00	3.41	0.00	0.00	0.83	0.00
time (sec)	N/A	0.440	2.298	2.015	0.000	0.417	0.000	0.000	0.218	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	120	100	194	0	438	0	0	92	0
N.S.	1	1.01	0.84	1.63	0.00	3.68	0.00	0.00	0.77	0.00
time (sec)	N/A	0.398	0.604	1.784	0.000	0.202	0.000	0.000	0.196	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	85	102	132	699	298	0	0	60	0
N.S.	1	0.98	1.17	1.52	8.03	3.43	0.00	0.00	0.69	0.00
time (sec)	N/A	0.350	0.324	0.970	0.387	0.149	0.000	0.000	0.186	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	119	75	263	0	436	0	0	34	0
N.S.	1	0.98	0.62	2.17	0.00	3.60	0.00	0.00	0.28	0.00
time (sec)	N/A	0.397	0.371	0.754	0.000	0.193	0.000	0.000	0.174	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	151	83	321	0	520	0	0	51	0
N.S.	1	0.94	0.52	1.99	0.00	3.23	0.00	0.00	0.32	0.00
time (sec)	N/A	0.446	0.359	0.862	0.000	0.175	0.000	0.000	0.183	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	196	183	83	361	0	608	0	0	52	0
N.S.	1	0.93	0.42	1.84	0.00	3.10	0.00	0.00	0.27	0.00
time (sec)	N/A	0.467	0.479	0.967	0.000	0.183	0.000	0.000	0.178	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	208	213	196	676	0	634	0	0	206	0
N.S.	1	1.02	0.94	3.25	0.00	3.05	0.00	0.00	0.99	0.00
time (sec)	N/A	0.518	4.144	2.664	0.000	0.872	0.000	0.000	0.210	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	175	178	132	192	0	550	0	0	166	0
N.S.	1	1.02	0.75	1.10	0.00	3.14	0.00	0.00	0.95	0.00
time (sec)	N/A	0.446	2.617	3.004	0.000	0.390	0.000	0.000	0.209	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	142	119	170	0	542	0	0	122	0
N.S.	1	1.19	1.00	1.43	0.00	4.55	0.00	0.00	1.03	0.00
time (sec)	N/A	0.410	0.753	2.083	0.000	0.303	0.000	0.000	0.206	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	146	145	170	0	505	0	0	80	0
N.S.	1	1.29	1.28	1.50	0.00	4.47	0.00	0.00	0.71	0.00
time (sec)	N/A	0.381	0.593	1.643	0.000	0.200	0.000	0.000	0.178	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	182	175	102	702	0	514	0	0	50	0
N.S.	1	0.96	0.56	3.86	0.00	2.82	0.00	0.00	0.27	0.00
time (sec)	N/A	0.462	0.401	4.049	0.000	0.178	0.000	0.000	0.177	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	219	209	102	363	0	560	0	0	43	0
N.S.	1	0.95	0.47	1.66	0.00	2.56	0.00	0.00	0.20	0.00
time (sec)	N/A	0.496	0.397	1.230	0.000	0.187	0.000	0.000	0.161	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	254	241	102	403	0	714	0	0	70	0
N.S.	1	0.95	0.40	1.59	0.00	2.81	0.00	0.00	0.28	0.00
time (sec)	N/A	0.535	0.497	1.352	0.000	0.201	0.000	0.000	0.181	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	265	281	180	1033	0	742	0	0	310	0
N.S.	1	1.06	0.68	3.90	0.00	2.80	0.00	0.00	1.17	0.00
time (sec)	N/A	0.575	8.264	3.246	0.000	1.439	0.000	0.000	0.219	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	235	247	164	851	0	655	0	0	256	0
N.S.	1	1.05	0.70	3.62	0.00	2.79	0.00	0.00	1.09	0.00
time (sec)	N/A	0.535	4.997	2.399	0.000	1.209	0.000	0.000	0.216	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	204	212	136	215	0	645	0	0	206	0
N.S.	1	1.04	0.67	1.05	0.00	3.16	0.00	0.00	1.01	0.00
time (sec)	N/A	0.456	2.491	2.684	0.000	0.922	0.000	0.000	0.194	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	145	222	0	645	0	0	152	0
N.S.	1	1.00	0.71	1.09	0.00	3.18	0.00	0.00	0.75	0.00
time (sec)	N/A	0.439	1.374	2.266	0.000	0.654	0.000	0.000	0.183	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	148	204	178	220	0	605	0	0	100	0
N.S.	1	1.38	1.20	1.49	0.00	4.09	0.00	0.00	0.68	0.00
time (sec)	N/A	0.424	1.365	1.852	0.000	0.386	0.000	0.000	0.175	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	238	230	117	737	0	608	0	0	52	0
N.S.	1	0.97	0.49	3.10	0.00	2.55	0.00	0.00	0.22	0.00
time (sec)	N/A	0.524	0.439	4.756	0.000	0.186	0.000	0.000	0.160	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	277	266	112	423	0	706	0	0	67	0
N.S.	1	0.96	0.40	1.53	0.00	2.55	0.00	0.00	0.24	0.00
time (sec)	N/A	0.526	0.511	1.310	0.000	0.207	0.000	0.000	0.160	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	187	84	338	1289	471	0	0	129	0
N.S.	1	1.01	0.45	1.83	6.97	2.55	0.00	0.00	0.70	0.00
time (sec)	N/A	0.992	1.682	3.263	0.251	0.158	0.000	0.000	0.246	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	140	72	263	710	437	0	0	94	0
N.S.	1	1.01	0.52	1.89	5.11	3.14	0.00	0.00	0.68	0.00
time (sec)	N/A	0.741	0.581	3.173	0.204	0.150	0.000	0.000	0.219	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	57	239	243	361	0	0	61	0
N.S.	1	1.00	0.61	2.57	2.61	3.88	0.00	0.00	0.66	0.00
time (sec)	N/A	0.518	0.461	3.173	0.184	0.153	0.000	0.000	0.183	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	87	39	213	0	0	28	0
N.S.	1	1.00	1.06	1.81	0.81	4.44	0.00	0.00	0.58	0.00
time (sec)	N/A	0.330	0.244	0.839	0.183	0.134	0.000	0.000	0.168	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	62	121	65	0	0	72	42	0
N.S.	1	1.00	1.22	2.37	1.27	0.00	0.00	1.41	0.82	0.00
time (sec)	N/A	0.288	0.314	3.138	0.126	0.000	0.000	0.528	0.154	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	72	166	399	0	0	147	51	0
N.S.	1	1.00	0.75	1.73	4.16	0.00	0.00	1.53	0.53	0.00
time (sec)	N/A	0.464	0.619	3.209	0.188	0.000	0.000	0.669	0.174	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	144	90	186	1173	0	0	178	62	0
N.S.	1	1.01	0.63	1.31	8.26	0.00	0.00	1.25	0.44	0.00
time (sec)	N/A	0.672	0.769	3.319	0.277	0.000	0.000	0.929	0.164	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	192	100	199	2444	0	0	208	71	0
N.S.	1	1.02	0.53	1.06	13.00	0.00	0.00	1.11	0.38	0.00
time (sec)	N/A	0.887	1.760	3.348	1.587	0.000	0.000	0.857	0.177	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	188	87	339	1356	479	0	0	128	0
N.S.	1	0.99	0.46	1.78	7.14	2.52	0.00	0.00	0.67	0.00
time (sec)	N/A	1.012	0.753	2.997	0.246	0.155	0.000	0.000	0.256	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	69	66	238	477	357	0	0	64	0
N.S.	1	0.67	0.64	2.31	4.63	3.47	0.00	0.00	0.62	0.00
time (sec)	N/A	0.429	0.529	3.038	0.187	0.151	0.000	0.000	0.196	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	68	243	243	360	0	0	59	0
N.S.	1	1.00	0.73	2.61	2.61	3.87	0.00	0.00	0.63	0.00
time (sec)	N/A	0.507	0.325	3.010	0.182	0.150	0.000	0.000	0.191	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	64	65	196	60	0	0	0	83	0
N.S.	1	0.62	0.62	1.88	0.58	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.317	0.337	2.861	0.182	0.000	0.000	0.000	0.168	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	76	164	95	0	0	120	102	0
N.S.	1	1.00	0.76	1.64	0.95	0.00	0.00	1.20	1.02	0.00
time (sec)	N/A	0.473	0.663	3.022	0.126	0.000	0.000	0.790	0.181	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	145	83	187	1786	0	0	170	123	0
N.S.	1	0.99	0.57	1.28	12.23	0.00	0.00	1.16	0.84	0.00
time (sec)	N/A	0.662	0.928	3.102	0.319	0.000	0.000	1.017	0.176	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	195	102	200	3480	0	0	200	142	0
N.S.	1	0.99	0.52	1.02	17.76	0.00	0.00	1.02	0.72	0.00
time (sec)	N/A	0.908	2.291	3.163	1.603	0.000	0.000	0.817	0.197	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	86	76	364	1619	416	0	0	99	0
N.S.	1	0.56	0.50	2.38	10.58	2.72	0.00	0.00	0.65	0.00
time (sec)	N/A	0.509	0.983	29.534	0.296	0.152	0.000	0.000	0.231	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	188	87	339	1356	479	0	0	128	0
N.S.	1	0.99	0.46	1.78	7.14	2.52	0.00	0.00	0.67	0.00
time (sec)	N/A	1.054	0.504	3.075	0.245	0.149	0.000	0.000	0.246	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	140	72	263	710	432	0	0	94	0
N.S.	1	1.01	0.52	1.89	5.11	3.11	0.00	0.00	0.68	0.00
time (sec)	N/A	0.729	0.278	3.124	0.209	0.149	0.000	0.000	0.229	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	72	69	288	0	0	0	0	131	0
N.S.	1	0.47	0.45	1.89	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.317	0.319	2.993	0.000	0.000	0.000	0.000	0.178	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	65	224	0	456	0	0	157	0
N.S.	1	1.00	0.68	2.33	0.00	4.75	0.00	0.00	1.64	0.00
time (sec)	N/A	0.534	0.539	3.005	0.000	0.154	0.000	0.000	0.196	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	76	189	139	0	0	170	191	0
N.S.	1	1.00	0.76	1.89	1.39	0.00	0.00	1.70	1.91	0.00
time (sec)	N/A	0.470	0.882	3.108	0.130	0.000	0.000	1.068	0.196	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	149	96	202	3738	0	0	200	217	0
N.S.	1	1.01	0.65	1.36	25.26	0.00	0.00	1.35	1.47	0.00
time (sec)	N/A	0.685	2.123	3.213	1.680	0.000	0.000	0.960	0.201	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	195	106	215	6134	0	0	230	251	0
N.S.	1	1.01	0.55	1.11	31.62	0.00	0.00	1.19	1.29	0.00
time (sec)	N/A	0.880	4.650	3.283	11.093	0.000	0.000	1.078	0.215	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	245	120	228	9150	0	0	260	277	0
N.S.	1	1.00	0.49	0.93	37.50	0.00	0.00	1.07	1.14	0.00
time (sec)	N/A	1.143	5.451	3.408	66.606	0.000	0.000	0.882	0.214	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	82	83	419	0	0	0	0	172	0
N.S.	1	0.40	0.41	2.05	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.319	1.734	2.972	0.000	0.000	0.000	0.000	0.200	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	68	69	327	0	0	0	0	127	0
N.S.	1	0.45	0.46	2.17	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.310	0.505	2.959	0.000	0.000	0.000	0.000	0.188	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	62	61	221	60	0	0	0	84	0
N.S.	1	0.61	0.60	2.17	0.59	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.310	0.311	2.934	0.187	0.000	0.000	0.000	0.167	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	57	52	34	0	0	0	41	0
N.S.	1	1.00	1.16	1.06	0.69	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.269	0.227	0.826	0.127	0.000	0.000	0.000	0.171	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	54	199	39	272	0	67	47	0
N.S.	1	1.04	1.17	4.33	0.85	5.91	0.00	1.46	1.02	0.00
time (sec)	N/A	0.339	0.268	2.822	0.180	0.245	0.000	0.500	0.160	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	95	89	249	818	0	0	106	64	0
N.S.	1	0.57	0.53	1.48	4.87	0.00	0.00	0.63	0.38	0.00
time (sec)	N/A	0.344	0.531	2.906	0.205	0.000	0.000	0.912	0.170	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	111	101	275	2206	0	0	141	65	0
N.S.	1	0.41	0.37	1.00	8.05	0.00	0.00	0.51	0.24	0.00
time (sec)	N/A	0.350	1.441	3.008	0.308	0.000	0.000	0.772	0.168	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	85	97	398	2393	0	0	0	212	0
N.S.	1	0.40	0.45	1.85	11.13	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.323	1.419	3.023	0.346	0.000	0.000	0.000	0.209	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	65	208	0	465	0	0	157	0
N.S.	1	1.00	0.68	2.17	0.00	4.84	0.00	0.00	1.64	0.00
time (sec)	N/A	0.523	0.512	3.010	0.000	0.149	0.000	0.000	0.186	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	181	70	0	0	0	104	0
N.S.	1	1.00	0.82	1.85	0.71	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.461	0.458	3.127	0.128	0.000	0.000	0.000	0.183	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	184	395	0	0	0	51	0
N.S.	1	1.00	0.80	1.96	4.20	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.447	0.256	3.199	0.196	0.000	0.000	0.000	0.162	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	93	108	248	818	0	0	99	63	0
N.S.	1	0.43	0.50	1.15	3.80	0.00	0.00	0.46	0.29	0.00
time (sec)	N/A	0.332	0.297	2.990	0.198	0.000	0.000	0.883	0.180	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	71	72	242	486	492	0	149	56	0
N.S.	1	0.70	0.71	2.40	4.81	4.87	0.00	1.48	0.55	0.00
time (sec)	N/A	0.428	0.398	3.052	0.207	0.312	0.000	1.111	0.162	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	128	118	335	4272	0	0	185	83	0
N.S.	1	0.37	0.34	0.97	12.31	0.00	0.00	0.53	0.24	0.00
time (sec)	N/A	0.371	0.872	3.099	1.665	0.000	0.000	0.936	0.187	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	220	89	85	275	0	0	0	93	252	0
N.S.	1	0.40	0.39	1.25	0.00	0.00	0.00	0.42	1.15	0.00
time (sec)	N/A	0.331	1.597	3.042	0.000	0.000	0.000	0.760	0.216	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	79	226	102	0	0	0	187	0
N.S.	1	1.00	0.81	2.31	1.04	0.00	0.00	0.00	1.91	0.00
time (sec)	N/A	0.460	0.661	3.204	0.133	0.000	0.000	0.000	0.189	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	143	86	224	1786	0	0	0	124	0
N.S.	1	0.99	0.60	1.56	12.40	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.676	0.510	3.317	0.342	0.000	0.000	0.000	0.188	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	142	89	226	1165	0	0	0	61	0
N.S.	1	1.01	0.64	1.61	8.32	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.649	0.374	3.267	0.283	0.000	0.000	0.000	0.164	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	107	101	275	2206	0	0	77	65	0
N.S.	1	0.40	0.37	1.02	8.17	0.00	0.00	0.29	0.24	0.00
time (sec)	N/A	0.335	0.510	3.017	0.321	0.000	0.000	0.742	0.177	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	126	118	335	4272	0	0	193	80	0
N.S.	1	0.37	0.34	0.97	12.38	0.00	0.00	0.56	0.23	0.00
time (sec)	N/A	0.356	0.807	3.011	1.693	0.000	0.000	1.064	0.171	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	88	84	350	1386	564	0	228	67	0
N.S.	1	0.58	0.56	2.32	9.18	3.74	0.00	1.51	0.44	0.00
time (sec)	N/A	0.510	0.688	3.000	0.310	0.391	0.000	0.868	0.186	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	26	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.290	0.000	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	28	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.323	0.000	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	94	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.303	0.000	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	68	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.293	0.000	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	40	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.273	0.000	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	30	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	40	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.304	0.000	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	154	101	0	0	0	0	0	98	0
N.S.	1	0.90	0.59	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.379	0.507	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	96	0	0	0	0	0	61	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.452	0.464	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	0	0	0	0	0	28	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.274	0.307	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	147	94	0	0	0	0	0	41	0
N.S.	1	1.06	0.68	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.325	0.320	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	194	122	0	0	0	0	0	51	0
N.S.	1	0.95	0.60	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.367	0.346	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	133	131	0	293	0	189	28	0
N.S.	1	0.99	1.46	1.44	0.00	3.22	0.00	2.08	0.31	0.00
time (sec)	N/A	0.419	0.885	0.678	0.000	0.141	0.000	0.381	0.170	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	810	257	0	0	0	0	63	0
N.S.	1	1.00	3.51	1.11	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.680	32.527	5.502	0.000	0.000	0.000	0.000	0.169	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	178	247	0	0	0	0	28	0
N.S.	1	1.00	0.79	1.10	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.637	6.700	1.995	0.000	0.000	0.000	0.000	0.170	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	325	182	287	0	0	0	0	48	0
N.S.	1	1.02	0.57	0.90	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.216	7.312	4.760	0.000	0.000	0.000	0.000	0.172	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	271	212	229	237	0	472	0	535	121	0
N.S.	1	0.78	0.85	0.87	0.00	1.74	0.00	1.97	0.45	0.00
time (sec)	N/A	0.423	8.057	2.425	0.000	0.115	0.000	0.772	0.186	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	167	192	185	0	392	0	365	93	0
N.S.	1	0.81	0.94	0.90	0.00	1.91	0.00	1.78	0.45	0.00
time (sec)	N/A	0.373	8.645	2.206	0.000	0.107	0.000	0.631	0.187	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	127	444	147	0	320	0	268	65	0
N.S.	1	0.88	3.08	1.02	0.00	2.22	0.00	1.86	0.45	0.00
time (sec)	N/A	0.341	9.231	1.963	0.000	0.104	0.000	0.517	0.176	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	76	110	147	235	0	193	37	0
N.S.	1	1.00	1.15	1.67	2.23	3.56	0.00	2.92	0.56	0.00
time (sec)	N/A	0.392	0.364	1.684	0.169	0.097	0.000	0.420	0.175	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	144	499	0	669	0	277	27	0
N.S.	1	1.00	1.37	4.75	0.00	6.37	0.00	2.64	0.26	0.00
time (sec)	N/A	0.583	2.088	5.725	0.000	0.312	0.000	0.676	0.159	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	219	195	244	9498	0	1413	0	617	43	0
N.S.	1	0.89	1.11	43.37	0.00	6.45	0.00	2.82	0.20	0.00
time (sec)	N/A	0.390	4.584	6.076	0.000	1.424	0.000	0.766	0.174	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	287	268	311	21396	0	2368	0	1381	59	0
N.S.	1	0.93	1.08	74.55	0.00	8.25	0.00	4.81	0.21	0.00
time (sec)	N/A	0.454	6.764	8.490	0.000	4.004	0.000	1.120	0.170	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	267	232	219	318	0	482	0	525	192	0
N.S.	1	0.87	0.82	1.19	0.00	1.81	0.00	1.97	0.72	0.00
time (sec)	N/A	0.416	2.973	2.365	0.000	0.113	0.000	0.961	0.222	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	170	145	183	0	398	0	359	136	0
N.S.	1	0.84	0.72	0.91	0.00	1.97	0.00	1.78	0.67	0.00
time (sec)	N/A	0.355	1.354	2.938	0.000	0.112	0.000	0.681	0.209	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	102	141	998	316	0	263	80	0
N.S.	1	1.03	0.97	1.34	9.50	3.01	0.00	2.50	0.76	0.00
time (sec)	N/A	0.564	0.524	2.418	0.201	0.107	0.000	0.510	0.187	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	135	840	0	731	0	0	59	0
N.S.	1	1.00	1.23	7.64	0.00	6.65	0.00	0.00	0.54	0.00
time (sec)	N/A	0.579	0.750	6.013	0.000	0.725	0.000	0.000	0.210	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	229	206	293	8518	0	1640	0	688	91	0
N.S.	1	0.90	1.28	37.20	0.00	7.16	0.00	3.00	0.40	0.00
time (sec)	N/A	0.411	3.789	6.450	0.000	3.786	0.000	0.948	0.211	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	310	288	359	18832	0	2729	0	1592	123	0
N.S.	1	0.93	1.16	60.75	0.00	8.80	0.00	5.14	0.40	0.00
time (sec)	N/A	0.475	4.768	8.275	0.000	10.979	0.000	1.532	0.213	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	336	255	286	371	0	620	0	613	296	0
N.S.	1	0.76	0.85	1.10	0.00	1.85	0.00	1.82	0.88	0.00
time (sec)	N/A	0.459	4.482	72.960	0.000	0.121	0.000	1.062	0.258	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	198	191	222	0	500	0	441	212	0
N.S.	1	0.77	0.74	0.86	0.00	1.94	0.00	1.71	0.82	0.00
time (sec)	N/A	0.421	2.119	18.178	0.000	0.107	0.000	1.012	0.223	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	150	128	169	1396	390	0	309	128	0
N.S.	1	1.06	0.90	1.19	9.83	2.75	0.00	2.18	0.90	0.00
time (sec)	N/A	0.833	0.802	4.859	0.229	0.105	0.000	0.688	0.210	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	203	162	343	1406	0	1140	0	0	95	0
N.S.	1	0.80	1.69	6.93	0.00	5.62	0.00	0.00	0.47	0.00
time (sec)	N/A	0.429	4.800	13.375	0.000	2.617	0.000	0.000	0.195	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	329	248	280	6167	0	2031	0	661	143	0
N.S.	1	0.75	0.85	18.74	0.00	6.17	0.00	2.01	0.43	0.00
time (sec)	N/A	0.513	2.896	39.979	0.000	12.520	0.000	1.194	0.230	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	536	405	551	17961	0	3351	0	1606	191	0
N.S.	1	0.76	1.03	33.51	0.00	6.25	0.00	3.00	0.36	0.00
time (sec)	N/A	0.648	9.610	2.141	0.000	21.661	0.000	1.707	0.237	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	258	197	787	430	0	619	0	0	137	0
N.S.	1	0.76	3.05	1.67	0.00	2.40	0.00	0.00	0.53	0.00
time (sec)	N/A	0.418	8.344	2.421	0.000	7.256	0.000	0.000	0.210	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	183	142	295	299	0	481	0	0	99	0
N.S.	1	0.78	1.61	1.63	0.00	2.63	0.00	0.00	0.54	0.00
time (sec)	N/A	0.358	2.468	1.962	0.000	1.717	0.000	0.000	0.222	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	92	173	699	314	0	0	61	0
N.S.	1	1.00	1.01	1.90	7.68	3.45	0.00	0.00	0.67	0.00
time (sec)	N/A	0.413	0.441	1.067	0.391	0.517	0.000	0.000	0.207	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	166	177	243	659	0	1050	0	0	48	0
N.S.	1	1.07	1.46	3.97	0.00	6.33	0.00	0.00	0.29	0.00
time (sec)	N/A	1.117	2.973	5.944	0.000	9.888	0.000	0.000	0.195	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	416	301	337	11907	0	2508	0	0	80	0
N.S.	1	0.72	0.81	28.62	0.00	6.03	0.00	0.00	0.19	0.00
time (sec)	N/A	0.569	9.908	6.558	0.000	103.665	0.000	0.000	0.204	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	653	490	2940	24369	0	0	0	0	112	0
N.S.	1	0.75	4.50	37.32	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.741	21.827	9.670	0.000	0.000	0.000	0.000	0.224	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	324	233	856	527	0	701	0	0	177	0
N.S.	1	0.72	2.64	1.63	0.00	2.16	0.00	0.00	0.55	0.00
time (sec)	N/A	0.441	7.220	2.297	0.000	13.575	0.000	0.000	0.222	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	290	207	177	337	0	620	0	0	129	0
N.S.	1	0.71	0.61	1.16	0.00	2.14	0.00	0.00	0.44	0.00
time (sec)	N/A	0.429	4.573	2.065	0.000	5.974	0.000	0.000	0.219	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	127	133	230	248	0	548	0	0	81	0
N.S.	1	1.05	1.81	1.95	0.00	4.31	0.00	0.00	0.64	0.00
time (sec)	N/A	0.634	2.178	1.743	0.000	1.593	0.000	0.000	0.205	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	394	273	302	2505	0	0	0	0	70	0
N.S.	1	0.69	0.77	6.36	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.501	5.498	6.745	0.000	0.000	0.000	0.000	0.202	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	560	391	470	14151	0	0	0	0	118	0
N.S.	1	0.70	0.84	25.27	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.638	11.406	7.340	0.000	0.000	0.000	0.000	0.229	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	802	579	2632	27299	0	0	0	0	166	0
N.S.	1	0.72	3.28	34.04	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.847	18.151	11.325	0.000	0.000	0.000	0.000	0.237	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	480	346	436	712	0	880	0	0	217	0
N.S.	1	0.72	0.91	1.48	0.00	1.83	0.00	0.00	0.45	0.00
time (sec)	N/A	0.523	9.108	2.478	0.000	33.051	0.000	0.000	0.220	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	468	334	262	548	0	782	0	0	159	0
N.S.	1	0.71	0.56	1.17	0.00	1.67	0.00	0.00	0.34	0.00
time (sec)	N/A	0.500	5.431	1.490	0.000	14.877	0.000	0.000	0.211	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	164	176	343	376	0	670	0	0	101	0
N.S.	1	1.07	2.09	2.29	0.00	4.09	0.00	0.00	0.62	0.00
time (sec)	N/A	0.879	7.275	1.852	0.000	3.744	0.000	0.000	0.208	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	592	420	360	3510	0	0	0	0	92	0
N.S.	1	0.71	0.61	5.93	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.632	10.233	8.823	0.000	0.000	0.000	0.000	0.205	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	756	540	465	16553	0	0	0	0	156	0
N.S.	1	0.71	0.62	21.90	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.725	12.267	7.825	0.000	0.000	0.000	0.000	0.241	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	999	729	2904	30639	0	0	0	0	220	0
N.S.	1	0.73	2.91	30.67	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.932	18.980	12.627	0.000	0.000	0.000	0.000	0.246	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	240	1680	0	806	0	0	26	0
N.S.	1	1.00	1.95	13.66	0.00	6.55	0.00	0.00	0.21	0.00
time (sec)	N/A	0.693	13.685	1.143	0.000	0.270	0.000	0.000	0.191	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	102	165	0	206	0	0	38	0
N.S.	1	1.00	1.67	2.70	0.00	3.38	0.00	0.00	0.62	0.00
time (sec)	N/A	0.280	0.599	0.499	0.000	0.139	0.000	0.000	0.198	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	135	344	0	517	0	0	54	0
N.S.	1	1.00	1.22	3.10	0.00	4.66	0.00	0.00	0.49	0.00
time (sec)	N/A	0.745	1.018	0.384	0.000	0.203	0.000	0.000	0.201	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	184	404	0	883	0	0	39	0
N.S.	1	1.00	1.30	2.87	0.00	6.26	0.00	0.00	0.28	0.00
time (sec)	N/A	0.716	12.452	0.358	0.000	0.381	0.000	0.000	0.192	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	171	354	0	913	0	0	59	0
N.S.	1	1.00	1.21	2.51	0.00	6.48	0.00	0.00	0.42	0.00
time (sec)	N/A	0.713	0.680	0.363	0.000	0.804	0.000	0.000	0.204	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	250	0	274	138	573
N.S.	1	1.00	1.01	1.09	0.00	3.73	0.00	4.09	2.06	8.55
time (sec)	N/A	0.391	0.175	0.168	0.000	0.098	0.000	0.160	0.196	12.668

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	151	155	168	0	561	0	201	534	3763
N.S.	1	1.23	1.26	1.37	0.00	4.56	0.00	1.63	4.34	30.59
time (sec)	N/A	0.693	0.963	0.200	0.000	0.117	0.000	0.144	0.186	18.728

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	250	267	287	0	1152	0	457	1542	6909
N.S.	1	1.23	1.31	1.41	0.00	5.65	0.00	2.24	7.56	33.87
time (sec)	N/A	1.163	1.627	0.337	0.000	0.156	0.000	0.206	0.200	20.703

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	158	136	195	0	672	0	237	715	4934
N.S.	1	1.19	1.02	1.47	0.00	5.05	0.00	1.78	5.38	37.10
time (sec)	N/A	0.666	0.847	0.214	0.000	0.122	0.000	0.163	0.207	18.900

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	269	493	386	0	1409	0	658	1969	8682
N.S.	1	1.14	2.08	1.63	0.00	5.95	0.00	2.78	8.31	36.63
time (sec)	N/A	1.140	2.460	0.364	0.000	0.174	0.000	0.228	0.188	21.783

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	433	438	635	0	2362	0	1201	3809	12818
N.S.	1	1.15	1.16	1.68	0.00	6.27	0.00	3.19	10.10	34.00
time (sec)	N/A	1.905	4.018	0.631	0.000	0.247	0.000	0.245	0.209	23.429

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	302	517	458	0	1629	0	818	2529	10759
N.S.	1	1.19	2.04	1.80	0.00	6.41	0.00	3.22	9.96	42.36
time (sec)	N/A	1.246	2.100	0.421	0.000	0.208	0.000	0.250	0.196	23.413

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	463	459	785	0	2776	0	1572	4706	15647
N.S.	1	1.12	1.11	1.91	0.00	6.74	0.00	3.82	11.42	37.98
time (sec)	N/A	1.917	4.383	0.723	0.000	0.314	0.000	0.282	0.197	24.246

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	622	683	668	1345	0	4346	0	3173	7989	21021
N.S.	1	1.10	1.07	2.16	0.00	6.99	0.00	5.10	12.84	33.80
time (sec)	N/A	3.124	8.196	1.336	0.000	0.486	0.000	0.384	0.224	24.788

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	379	647	0	0	0	0	38	0
N.S.	1	1.00	1.18	2.02	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.998	8.787	18.061	0.000	0.000	0.000	0.000	0.202	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	225	400	0	0	0	0	26	0
N.S.	1	1.00	1.02	1.82	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.671	7.546	4.352	0.000	0.000	0.000	0.000	0.196	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	381	6063	1239	0	0	0	0	88	0
N.S.	1	1.00	15.96	3.26	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.382	22.142	24.595	0.000	0.000	0.000	0.000	0.205	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	330	230	536	0	0	0	0	63	0
N.S.	1	1.01	0.71	1.64	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.160	8.237	4.549	0.000	0.000	0.000	0.000	0.215	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	448	7138	1739	0	0	0	0	150	0
N.S.	1	1.01	16.15	3.93	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.785	24.681	36.161	0.000	0.000	0.000	0.000	0.240	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	145	182	0	0	0	0	63	0
N.S.	1	1.00	0.70	0.88	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.480	3.258	12.102	0.000	0.000	0.000	0.000	0.196	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	251	283	0	0	0	0	49	0
N.S.	1	1.00	1.16	1.31	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.673	20.286	4.478	0.000	0.000	0.000	0.000	0.189	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	405	1138	1070	0	0	0	0	95	0
N.S.	1	1.08	3.03	2.85	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.421	12.809	11.857	0.000	0.000	0.000	0.000	0.212	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	522	1589	2819	0	0	0	0	127	0
N.S.	1	1.05	3.21	5.69	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.994	14.038	18.132	0.000	0.000	0.000	0.000	0.216	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	26982	510	0	0	0	0	25	0
N.S.	1	1.00	69.36	1.31	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.943	35.781	9.065	0.000	0.000	0.000	0.000	0.188	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	336	322	0	0	0	0	37	0
N.S.	1	1.00	1.70	1.63	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.349	4.670	5.176	0.000	0.000	0.000	0.000	0.194	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	597	1708	1600	0	0	0	0	53	0
N.S.	1	1.00	2.86	2.68	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.902	13.436	6.610	0.000	0.000	0.000	0.000	0.195	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	899	826	1990	7332	0	0	0	0	69	0
N.S.	1	0.92	2.21	8.16	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.916	7.134	8.311	0.000	0.000	0.000	0.000	0.163	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	593	0	50041	459	0	0	0	0	85	0
N.S.	1	0.00	84.39	0.77	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	37.296	12.068	0.000	0.000	0.000	0.000	0.199	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	744	691	1750	2391	0	0	0	0	117	0
N.S.	1	0.93	2.35	3.21	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.983	18.779	10.082	0.000	0.000	0.000	0.000	0.201	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	919	850	1960	6042	0	0	0	0	149	0
N.S.	1	0.92	2.13	6.57	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	3.131	6.990	12.577	0.000	0.000	0.000	0.000	0.199	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1122	1053	2385	14939	0	0	0	0	181	0
N.S.	1	0.94	2.13	13.31	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	4.357	7.751	15.390	0.000	0.000	0.000	0.000	0.222	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	891	853	2026	7310	0	0	0	0	234	0
N.S.	1	0.96	2.27	8.20	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	3.251	6.895	12.458	0.000	0.000	0.000	0.000	0.244	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1150	1086	2344	12381	0	0	0	0	282	0
N.S.	1	0.94	2.04	10.77	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	4.284	7.404	15.427	0.000	0.000	0.000	0.000	0.258	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1428	1353	2979	24930	0	0	0	0	330	0
N.S.	1	0.95	2.09	17.46	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	6.339	8.546	18.562	0.000	0.000	0.000	0.000	0.279	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	0	50041	457	0	0	0	0	85	0
N.S.	1	0.00	76.75	0.70	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	36.846	12.060	0.000	0.000	0.000	0.000	0.186	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	325	322	0	0	0	0	37	0
N.S.	1	1.00	1.64	1.63	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.354	4.661	7.153	0.000	0.000	0.000	0.000	0.165	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	249	263	0	0	0	0	60	0
N.S.	1	1.00	0.63	0.66	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.899	2.213	9.471	0.000	0.000	0.000	0.000	0.179	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	622	683	1761	1871	0	0	0	0	94	0
N.S.	1	1.10	2.83	3.01	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.091	19.116	12.338	0.000	0.000	0.000	0.000	0.166	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	0.93	1.07
time (sec)	N/A	0.429	14.068	0.155	0.453	0.000	1.229	0.588	0.177	90.973

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	52	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.79	1.07
time (sec)	N/A	0.247	83.865	0.144	0.739	0.000	11.391	0.801	0.173	94.499

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	80	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	2.76	0.00
time (sec)	N/A	0.254	80.276	0.146	1.018	0.000	0.000	0.951	0.166	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	27	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	0.93	1.07
time (sec)	N/A	0.460	14.353	0.145	0.460	0.000	4.142	0.726	0.178	97.391

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	27	27	52	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.93	0.93	1.79	1.07
time (sec)	N/A	0.263	84.492	0.148	0.958	0.000	63.081	0.995	0.166	95.283

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	80	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	2.76	0.00
time (sec)	N/A	0.260	81.959	0.155	0.988	0.000	0.000	1.265	0.183	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	115	31
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	3.97	1.07
time (sec)	N/A	0.480	71.951	0.148	0.555	0.000	0.000	1.870	0.232	100.280

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	171	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	5.90	0.00
time (sec)	N/A	0.260	92.562	0.148	0.756	0.000	0.000	2.343	0.239	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A	N/A	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	25	27	0	0	27	227	0
N.S.	1	1.00	1.07	0.86	0.93	0.00	0.00	0.93	7.83	0.00
time (sec)	N/A	0.260	121.310	0.153	1.760	0.000	0.000	3.336	0.278	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	106	2425	0	0	0	0	0	34	0
N.S.	1	0.91	20.91	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.527	13.785	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	303	191	0	0	0	0	0	90	0
N.S.	1	1.10	0.69	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.295	1.549	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	227	152	0	0	0	0	0	67	0
N.S.	1	1.11	0.74	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.029	0.635	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	177	124	0	0	0	0	0	42	0
N.S.	1	1.13	0.79	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.603	0.124	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	235	159	0	0	0	0	0	35	0
N.S.	1	1.13	0.76	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.896	0.570	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	269	215	0	0	0	0	0	45	0
N.S.	1	1.08	0.87	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.225	1.284	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	34	33
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.26	1.22
time (sec)	N/A	0.364	5.678	0.345	3.222	0.163	6.351	0.747	0.180	12.798

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	323	278	0	0	0	0	0	103	0
N.S.	1	1.09	0.94	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.661	0.716	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	227	200	0	0	0	0	0	74	0
N.S.	1	1.08	0.95	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.011	0.344	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	177	125	0	0	0	0	0	45	0
N.S.	1	1.13	0.80	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.595	0.151	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	221	5411	0	0	0	0	0	34	0
N.S.	1	1.07	26.27	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.831	31.132	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	330	14108	0	0	0	0	0	50	0
N.S.	1	1.02	43.81	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.916	45.636	0.000	0.000	0.000	0.000	0.000	0.184	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [197] had the largest ratio of [.7199999999999999973]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	0.90	26	0.192
2	A	5	5	0.90	26	0.192
3	A	7	7	0.92	26	0.269
4	A	7	7	0.74	26	0.269
5	A	5	5	0.95	24	0.208
6	A	3	3	0.91	26	0.115
7	A	3	3	0.93	26	0.115
8	A	3	3	0.92	26	0.115
9	A	3	3	0.92	26	0.115
10	A	3	3	0.91	26	0.115
11	A	5	5	0.90	26	0.192
12	A	10	10	0.93	26	0.385
13	A	9	9	0.76	26	0.346
14	A	7	7	0.92	26	0.269
15	A	5	5	0.99	24	0.208
16	A	5	5	0.91	26	0.192
17	A	3	3	0.91	26	0.115
18	A	3	3	0.92	26	0.115
19	A	3	3	0.92	26	0.115
20	A	3	3	0.91	26	0.115
21	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	5	0.89	26	0.192
23	A	3	3	0.91	26	0.115
24	A	3	3	0.93	26	0.115
25	A	3	3	0.92	24	0.125
26	A	8	8	0.96	26	0.308
27	A	7	7	0.74	26	0.269
28	A	11	11	1.02	26	0.423
29	A	5	5	0.88	26	0.192
30	A	5	5	0.89	26	0.192
31	A	5	5	0.88	26	0.192
32	A	3	3	0.91	26	0.115
33	A	3	3	0.92	26	0.115
34	A	3	3	0.92	26	0.115
35	A	3	3	0.91	24	0.125
36	A	5	5	0.91	26	0.192
37	A	11	11	1.01	26	0.423
38	A	9	9	0.79	26	0.346
39	A	14	14	0.98	26	0.538
40	A	5	5	0.89	26	0.192
41	A	5	5	0.87	26	0.192
42	A	7	6	0.90	28	0.214
43	A	7	6	0.91	28	0.214
44	A	7	6	0.94	28	0.214
45	A	7	6	1.03	26	0.231
46	A	7	6	0.91	28	0.214
47	A	8	7	0.92	28	0.250
48	A	9	8	0.91	28	0.286
49	A	10	9	0.90	28	0.321
50	A	8	7	0.87	28	0.250
51	A	8	7	0.88	28	0.250
52	A	8	7	0.93	26	0.269
53	A	7	6	0.93	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	7	0.91	28	0.250
55	A	9	8	0.92	28	0.286
56	A	10	9	0.91	28	0.321
57	A	7	6	0.85	28	0.214
58	A	7	6	0.86	28	0.214
59	A	7	6	0.90	26	0.231
60	A	7	6	0.87	28	0.214
61	A	7	6	0.95	28	0.214
62	A	7	6	0.88	28	0.214
63	A	7	6	0.90	28	0.214
64	A	7	6	0.89	28	0.214
65	A	13	12	1.02	28	0.429
66	A	11	10	1.02	28	0.357
67	A	8	7	1.01	28	0.250
68	A	7	6	0.98	26	0.231
69	A	10	9	0.98	28	0.321
70	A	11	10	0.94	28	0.357
71	A	13	12	0.93	28	0.429
72	A	13	12	1.02	28	0.429
73	A	11	10	1.02	28	0.357
74	A	9	8	1.19	28	0.286
75	A	8	7	1.29	26	0.269
76	A	11	10	0.96	28	0.357
77	A	14	13	0.95	28	0.464
78	A	16	15	0.95	28	0.536
79	A	16	15	1.06	28	0.536
80	A	14	13	1.05	28	0.464
81	A	11	10	1.04	28	0.357
82	A	11	10	1.00	28	0.357
83	A	10	9	1.38	26	0.346
84	A	15	14	0.97	28	0.500
85	A	16	15	0.96	28	0.536

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	11	1.01	30	0.367
87	A	9	9	1.01	30	0.300
88	A	7	7	1.00	30	0.233
89	A	5	5	1.00	30	0.167
90	A	4	3	1.00	30	0.100
91	A	6	5	1.00	30	0.167
92	A	8	7	1.01	30	0.233
93	A	10	9	1.02	30	0.300
94	A	11	11	0.99	30	0.367
95	A	7	7	0.67	30	0.233
96	A	7	7	1.00	30	0.233
97	A	6	5	0.62	30	0.167
98	A	6	5	1.00	30	0.167
99	A	8	7	0.99	30	0.233
100	A	10	9	0.99	30	0.300
101	A	9	9	0.56	30	0.300
102	A	11	11	0.99	30	0.367
103	A	9	9	1.01	30	0.300
104	A	6	5	0.47	30	0.167
105	A	7	7	1.00	30	0.233
106	A	6	5	1.00	30	0.167
107	A	8	7	1.01	30	0.233
108	A	10	9	1.01	30	0.300
109	A	12	11	1.00	30	0.367
110	A	6	5	0.40	30	0.167
111	A	6	5	0.45	30	0.167
112	A	6	5	0.61	30	0.167
113	A	4	3	1.00	30	0.100
114	A	6	6	1.04	30	0.200
115	A	6	5	0.57	30	0.167
116	A	6	5	0.41	30	0.167
117	A	6	5	0.40	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	7	1.00	30	0.233
119	A	6	5	1.00	30	0.167
120	A	6	5	1.00	30	0.167
121	A	6	5	0.43	30	0.167
122	A	10	10	0.70	30	0.333
123	A	6	5	0.37	30	0.167
124	A	6	5	0.40	30	0.167
125	A	6	5	1.00	30	0.167
126	A	8	7	0.99	30	0.233
127	A	8	7	1.01	30	0.233
128	A	6	5	0.40	30	0.167
129	A	6	5	0.37	30	0.167
130	A	14	14	0.58	30	0.467
131	A	4	3	1.00	24	0.125
132	A	5	4	1.00	26	0.154
133	A	5	4	1.00	26	0.154
134	A	5	4	1.00	26	0.154
135	A	5	4	1.00	24	0.167
136	A	5	4	1.00	26	0.154
137	A	5	4	1.00	26	0.154
138	A	6	5	0.90	28	0.179
139	A	6	5	1.00	28	0.179
140	A	4	3	1.00	28	0.107
141	A	7	6	1.06	28	0.214
142	A	9	8	0.95	28	0.286
143	A	9	8	0.99	27	0.296
144	A	5	5	1.00	27	0.185
145	A	5	5	1.00	27	0.185
146	A	9	9	1.02	27	0.333
147	A	5	4	0.78	27	0.148
148	A	5	4	0.81	27	0.148
149	A	5	4	0.88	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	7	6	1.00	25	0.240
151	A	8	7	1.00	27	0.259
152	A	9	8	0.89	27	0.296
153	A	11	10	0.93	27	0.370
154	A	11	10	0.87	27	0.370
155	A	9	8	0.84	27	0.296
156	A	10	9	1.03	25	0.360
157	A	8	7	1.00	27	0.259
158	A	10	9	0.90	27	0.333
159	A	12	11	0.93	27	0.407
160	A	6	5	0.76	27	0.185
161	A	6	5	0.77	27	0.185
162	A	13	12	1.06	25	0.480
163	A	6	5	0.80	27	0.185
164	A	6	5	0.75	27	0.185
165	A	6	5	0.76	27	0.185
166	A	6	5	0.76	27	0.185
167	A	6	5	0.78	27	0.185
168	A	8	7	1.00	25	0.280
169	A	12	11	1.07	27	0.407
170	A	6	5	0.72	27	0.185
171	A	6	5	0.75	27	0.185
172	A	6	5	0.72	27	0.185
173	A	6	5	0.71	27	0.185
174	A	11	10	1.05	25	0.400
175	A	6	5	0.69	27	0.185
176	A	6	5	0.70	27	0.185
177	A	6	5	0.72	27	0.185
178	A	6	5	0.72	27	0.185
179	A	6	5	0.71	27	0.185
180	A	14	13	1.07	25	0.520
181	A	6	5	0.71	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	6	5	0.71	27	0.185
183	A	6	5	0.73	27	0.185
184	A	8	7	1.00	29	0.241
185	A	4	3	1.00	29	0.103
186	A	8	7	1.00	29	0.241
187	A	8	7	1.00	29	0.241
188	A	8	7	1.00	29	0.241
189	A	8	7	1.00	23	0.304
190	A	11	10	1.23	23	0.435
191	A	14	13	1.23	23	0.565
192	A	11	10	1.19	25	0.400
193	A	13	12	1.14	25	0.480
194	A	16	15	1.15	25	0.600
195	A	13	12	1.19	25	0.480
196	A	15	14	1.12	25	0.560
197	A	19	18	1.10	25	0.720
198	A	8	8	1.00	25	0.320
199	A	5	5	1.00	27	0.185
200	A	11	11	1.00	25	0.440
201	A	8	8	1.01	27	0.296
202	A	14	14	1.01	25	0.560
203	A	5	5	1.00	25	0.200
204	A	5	5	1.00	27	0.185
205	A	11	11	1.08	25	0.440
206	A	14	14	1.05	25	0.560
207	A	5	5	1.00	29	0.172
208	A	2	2	1.00	29	0.069
209	A	8	8	1.00	29	0.276
210	A	13	13	0.92	29	0.448
211	F	0	0	N/A	0.000	N/A
212	A	10	10	0.93	29	0.345
213	A	13	13	0.92	29	0.448

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	17	17	0.94	29	0.586
215	A	13	13	0.96	29	0.448
216	A	16	16	0.94	29	0.552
217	A	19	19	0.95	29	0.655
218	F	0	0	N/A	0.000	N/A
219	A	2	2	1.00	29	0.069
220	A	5	5	1.00	29	0.172
221	A	11	11	1.10	29	0.379
222	N/A	4	0	1.00	29	0.000
223	N/A	2	0	1.00	29	0.000
224	N/A	2	0	1.00	29	0.000
225	N/A	4	0	1.00	29	0.000
226	N/A	2	0	1.00	29	0.000
227	N/A	2	0	1.00	29	0.000
228	N/A	4	0	1.00	29	0.000
229	N/A	2	0	1.00	29	0.000
230	N/A	2	0	1.00	29	0.000
231	A	8	7	0.91	27	0.259
232	A	12	12	1.10	27	0.444
233	A	13	13	1.11	27	0.481
234	A	8	8	1.13	25	0.320
235	A	10	10	1.13	27	0.370
236	A	13	13	1.08	27	0.481
237	N/A	4	0	1.00	27	0.000
238	A	15	15	1.09	27	0.556
239	A	13	13	1.08	27	0.481
240	A	8	8	1.13	25	0.320
241	A	11	10	1.07	27	0.370
242	A	7	7	1.02	27	0.259

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = a^2c^5x-\frac{19a^2c^5\text{arctanh}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan(e+fx)}{f}+\frac{17a^2c^5\sec(e+fx)\tan(e+fx)}{16f}+\frac{a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^5\tan^3(e+fx)}{3f}-\frac{3a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{6f}+\frac{3a^2c^5\tan^5(e+fx)}{5f}$$

output
```
a^2*c^5*x-19/16*a^2*c^5*arctanh(sin(f*x+e))/f-a^2*c^5*tan(f*x+e)/f+17/16*a
^2*c^5*sec(f*x+e)*tan(f*x+e)/f+1/8*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/3*a
^2*c^5*tan(f*x+e)^3/f-3/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^2*c^5*se
c(f*x+e)^3*tan(f*x+e)^3/f+3/5*a^2*c^5*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\int (a + a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$= \frac{a^2c^5\sec^6(e+fx)\,(1200e+1200fx-4560\text{arctanh}(\sin(e+fx))\cos^6(e+fx)+1800(e+fx)\cos(2(e+}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^2*c^5*Sec[e + f*x]^6*(1200*e + 1200*f*x - 4560*ArcTanh[Sin[e + f*x]]*Co
s[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] +
720*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x
)] - 210*Sin[e + f*x] - 120*Sin[2*(e + f*x)] + 865*Sin[3*(e + f*x)] - 768*
Sin[4*(e + f*x)] + 435*Sin[5*(e + f*x)] - 88*Sin[6*(e + f*x)]))/(3840*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int (c-c\sec(e+fx))^3\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4374}$$

$$a^2c^2\int\left(c^3\tan^4(e+fx)-c^3\sec^3(e+fx)\tan^4(e+fx)+3c^3\sec^2(e+fx)\tan^4(e+fx)-3c^3\sec(e+fx)\tan^4(e$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(-\frac{19c^3\text{arctanh}(\sin(e+fx))}{16f}+\frac{3c^3\tan^5(e+fx)}{5f}+\frac{c^3\tan^3(e+fx)}{3f}-\frac{c^3\tan(e+fx)}{f}-\frac{c^3\tan^3(e+fx)\sec}{6f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

output
```
a^2*c^2*(c^3*x - (19*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (c^3*Tan[e + f*x]
)/f + (17*c^3*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (c^3*Sec[e + f*x]^3*Tan[
e + f*x])/(8*f) + (c^3*Tan[e + f*x]^3)/(3*f) - (3*c^3*Sec[e + f*x]*Tan[e +
 f*x]^3)/(4*f) - (c^3*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*f) + (3*c^3*Tan[e
+ f*x]^5)/(5*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05

method	result
risch	$a^2c^5x-\frac{ic^5a^2\left(435\,\mathrm{e}^{11i(fx+e)}-240\,\mathrm{e}^{10i(fx+e)}+865\,\mathrm{e}^{9i(fx+e)}+1200\,\mathrm{e}^{8i(fx+e)}-210\,\mathrm{e}^{7i(fx+e)}+1760\,\mathrm{e}^{6i(fx+e)}+210\right.}{120f\left(\mathrm{e}^{2i(fx+e)}+1\right)^6}$
parallelrisch	$-\frac{c^5a^2\left(\frac{19\left(-5-\frac{15\cos(2fx+2e)}{2}-3\cos(4fx+4e)-\frac{\cos(6fx+6e)}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8}+\frac{19\left(5+\frac{\cos(6fx+6e)}{2}+3\cos(4fx+4e)+\frac{15}{}\right.}{8}\right.}{}$
derivativedivides	$\frac{c^5a^2(fx+e)-3c^5a^2\ln(\sec(fx+e)+\tan(fx+e))+c^5a^2\tan(fx+e)+5c^5a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
default	$\frac{c^5a^2(fx+e)-3c^5a^2\ln(\sec(fx+e)+\tan(fx+e))+c^5a^2\tan(fx+e)+5c^5a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
parts	$a^2c^5x+\frac{a^2c^5\tan(fx+e)}{f}-\frac{3c^5a^2\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{5c^5a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(f}{2}\right.}{f}$
norman	$\frac{a^2c^5x+a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-6a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+15a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-20a^2c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+15a^2c^5x\tan\left(\right.}{}$

input

```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output

```
a^2*c^5*x-1/120*I*c^5*a^2*(435*exp(11*I*(f*x+e))-240*exp(10*I*(f*x+e))+865
*exp(9*I*(f*x+e))+1200*exp(8*I*(f*x+e))-210*exp(7*I*(f*x+e))+1760*exp(6*I*
(f*x+e))+210*exp(5*I*(f*x+e))+1440*exp(4*I*(f*x+e))-865*exp(3*I*(f*x+e))+1
296*exp(2*I*(f*x+e))-435*exp(I*(f*x+e))+176)/f/(exp(2*I*(f*x+e))+1)^6+19/1
6*c^5*a^2/f*ln(exp(I*(f*x+e))-I)-19/16*c^5*a^2/f*ln(exp(I*(f*x+e))+I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx$$
$$= \frac{480\, a^2 c^5 f x \cos\left(f x + e\right)^6 - 285\, a^2 c^5 \cos\left(f x + e\right)^6 \log\left(\sin\left(f x + e\right) + 1\right) + 285\, a^2 c^5 \cos\left(f x + e\right)^6 \log\left(-\mathrm{si}\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/480*(480*a^2*c^5*f*x*cos(f*x + e)^6 - 285*a^2*c^5*cos(f*x + e)^6*log(sin
(f*x + e) + 1) + 285*a^2*c^5*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(17
6*a^2*c^5*cos(f*x + e)^5 - 435*a^2*c^5*cos(f*x + e)^4 + 208*a^2*c^5*cos(f*
x + e)^3 + 110*a^2*c^5*cos(f*x + e)^2 - 144*a^2*c^5*cos(f*x + e) + 40*a^2*
c^5)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx$$
$$= -a^2 c^5 \left(\int (-1)\, dx + \int 3 \sec(e + fx)\, dx + \int \left(-\sec^2(e + fx)\right) dx + \int \left(-5 \sec^3(e + fx)\right) dx + \int 5 \sec^4(e + fx)\, dx + \int \sec^5(e + fx)\, dx + \int \left(-3 \sec^6(e + fx)\right) dx + \int \sec^7(e + fx)\, dx \right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

output
```
-a**2*c**5*(Integral(-1, x) + Integral(3*sec(e + f*x), x) + Integral(-sec(
e + f*x)**2, x) + Integral(-5*sec(e + f*x)**3, x) + Integral(5*sec(e + f*x
)**4, x) + Integral(sec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x)
+ Integral(sec(e + f*x)**7, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$

$$= \frac{96\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^5-800\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
1/480*(96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^5
 - 800*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 480*(f*x + e)*a^2*c^5 +
 5*a^2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(s
in(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
+ e) + 1) + 15*log(sin(f*x + e) - 1)) + 30*a^2*c^5*(2*(3*sin(f*x + e)^3 -
5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
e) + 1) + 3*log(sin(f*x + e) - 1)) - 600*a^2*c^5*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 1440*a^2*c^
5*log(sec(f*x + e) + tan(f*x + e)) + 480*a^2*c^5*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.97

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$

$$= \frac{240\,(fx+e)a^2c^5-285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+285\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(525\,a\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)*a^2*c^5 - 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) +
1)) + 285*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(525*a^2*c^5*tan(
1/2*f*x + 1/2*e)^11 - 3135*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 1746*a^2*c^5*t
an(1/2*f*x + 1/2*e)^7 - 366*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 - 95*a^2*c^5*ta
n(1/2*f*x + 1/2*e)^3 + 45*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2
*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.16

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = a^2\,c^5\,x - \frac{-\frac{35\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}+\frac{209\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{8}-\frac{291\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20}+\frac{61\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20}+\frac{19\,a^2\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-6} - \frac{19\,a^2\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5,x)
```

output
```
a^2*c^5*x - ((19*a^2*c^5*tan(e/2 + (f*x)/2)^3)/24 + (61*a^2*c^5*tan(e/2 +
(f*x)/2)^5)/20 - (291*a^2*c^5*tan(e/2 + (f*x)/2)^7)/20 + (209*a^2*c^5*tan(
e/2 + (f*x)/2)^9)/8 - (35*a^2*c^5*tan(e/2 + (f*x)/2)^11)/8 - (3*a^2*c^5*ta
n(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2
- 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)
^10 + tan(e/2 + (f*x)/2)^12 + 1)) - (19*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))
/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.62

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^5 \, dx$$
$$= \frac{a^2c^5 \left(176\cos(fx + e)\sin(fx + e)^5 - 560\cos(fx + e)\sin(fx + e)^3 + 240\cos(fx + e)\sin(fx + e) + 28\right.}{}$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x)
```

output
```
(a**2*c**5*(176*cos(e + f*x)*sin(e + f*x)**5 - 560*cos(e + f*x)*sin(e + f*
x)**3 + 240*cos(e + f*x)*sin(e + f*x) + 285*log(tan((e + f*x)/2) - 1)*sin(
e + f*x)**6 - 855*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 855*log(tan(
(e + f*x)/2) - 1)*sin(e + f*x)**2 - 285*log(tan((e + f*x)/2) - 1) - 285*lo
g(tan((e + f*x)/2) + 1)*sin(e + f*x)**6 + 855*log(tan((e + f*x)/2) + 1)*si
n(e + f*x)**4 - 855*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 + 285*log(ta
n((e + f*x)/2) + 1) + 240*sin(e + f*x)**6*f*x - 435*sin(e + f*x)**5 - 720*
sin(e + f*x)**4*f*x + 760*sin(e + f*x)**3 + 720*sin(e + f*x)**2*f*x - 285*
sin(e + f*x) - 240*f*x))/(240*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*s
in(e + f*x)**2 - 1))
```

3.2 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 26, antiderivative size = 140

$$\begin{aligned}\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx &= a^2c^4x-\frac{3a^2c^4\operatorname{arctanh}(\sin(e+fx))}{4f}-\frac{a^2c^4\tan(e+fx)}{f}\\ &\quad+\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{4f}+\frac{a^2c^4\tan^3(e+fx)}{3f}\\ &\quad-\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{2f}+\frac{a^2c^4\tan^5(e+fx)}{5f}\end{aligned}$$

output
```
a^2*c^4*x-3/4*a^2*c^4*arctanh(sin(f*x+e))/f-a^2*c^4*tan(f*x+e)/f+3/4*a^2*c
^4*sec(f*x+e)*tan(f*x+e)/f+1/3*a^2*c^4*tan(f*x+e)^3/f-1/2*a^2*c^4*sec(f*x+
e)*tan(f*x+e)^3/f+1/5*a^2*c^4*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int (a + a\sec(e+fx))^2 (c - c\sec(e+fx))^4 \, dx$$
$$= \frac{a^2c^4\sec^5(e+fx)\,(600(e+fx)\cos(e+fx) - 720\text{arctanh}(\sin(e+fx))\cos^5(e+fx) + 300e\cos(3(e+f$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^2*c^4*Sec[e + f*x]^5*(600*(e + f*x)*Cos[e + f*x] - 720*ArcTanh[Sin[e +
f*x]]*Cos[e + f*x]^5 + 300*e*Cos[3*(e + f*x)] + 300*f*x*Cos[3*(e + f*x)] +
 60*e*Cos[5*(e + f*x)] + 60*f*x*Cos[5*(e + f*x)] + 40*Sin[e + f*x] + 60*Si
n[2*(e + f*x)] - 220*Sin[3*(e + f*x)] + 150*Sin[4*(e + f*x)] - 68*Sin[5*(e
 + f*x)]))/(960*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^4dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{ 4392}$$
$$a^2c^2\int (c-c\sec(e+fx))^2\tan^4(e+fx)dx$$
$$\Big\downarrow \text{ 3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4374}$$

$$a^2c^2\int \left(c^2\tan^4(e+fx)+c^2\sec^2(e+fx)\tan^4(e+fx)-2c^2\sec(e+fx)\tan^4(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(-\frac{3c^2\text{arctanh}(\sin(e+fx))}{4f}+\frac{c^2\tan^5(e+fx)}{5f}+\frac{c^2\tan^3(e+fx)}{3f}-\frac{c^2\tan(e+fx)}{f}-\frac{c^2\tan^3(e+fx)\sec(e}{2f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

output
```
a^2*c^2*(c^2*x - (3*c^2*ArcTanh[Sin[e + f*x]])/(4*f) - (c^2*Tan[e + f*x])/
f + (3*c^2*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (c^2*Tan[e + f*x]^3)/(3*f) -
 (c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(2*f) + (c^2*Tan[e + f*x]^5)/(5*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

method	result
parts	$a^2c^4x - \frac{c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f} - \frac{a^2c^4\tan(fx+e)}{f} + \frac{2a^2c^4\sec(fx+e)\tan(fx+e)}{f}$
risch	$a^2c^4x - \frac{ic^4a^2\left(75\,e^{9i(fx+e)}+60\,e^{8i(fx+e)}+30\,e^{7i(fx+e)}+360\,e^{6i(fx+e)}+320\,e^{4i(fx+e)}-30\,e^{3i(fx+e)}+280\,e^{2i(fx+e}\right.}{30f\left(e^{2i(fx+e)}+1\right)^5}$
derivativedivides	$\frac{c^4a^2(fx+e)-2c^4a^2\ln(\sec(fx+e)+\tan(fx+e))-c^4a^2\tan(fx+e)+4c^4a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
default	$\frac{c^4a^2(fx+e)-2c^4a^2\ln(\sec(fx+e)+\tan(fx+e))-c^4a^2\tan(fx+e)+4c^4a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
parallelrisch	$\frac{c^4a^2\left(\left(\frac{15\cos(fx+e)}{2}+\frac{15\cos(3fx+3e)}{4}+\frac{3\cos(5fx+5e)}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{15\cos(fx+e)}{2}-\frac{15\cos(3fx+3e)}{4}-\frac{3\cos(5fx}{4}\right.\right.}{f(c}$
norman	$\frac{a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-a^2c^4x+5a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-10a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10a^2c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-5a^2c^4x\tan\left(\frac{f}{2}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^4*x-c^4*a^2/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-
a^2*c^4*tan(f*x+e)/f+2*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f+c^4*a^2/f*(-2/3-1/3
*sec(f*x+e)^2)*tan(f*x+e)-2*c^4*a^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))
*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{120\,a^2c^4fx\cos(fx+e)^5 - 45\,a^2c^4\cos(fx+e)^5\log(\sin(fx+e)+1) + 45\,a^2c^4\cos(fx+e)^5\log(-\sin}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/120*(120*a^2*c^4*f*x*cos(f*x + e)^5 - 45*a^2*c^4*cos(f*x + e)^5*log(sin(
f*x + e) + 1) + 45*a^2*c^4*cos(f*x + e)^5*log(-sin(f*x + e) + 1) - 2*(68*a
^2*c^4*cos(f*x + e)^4 - 75*a^2*c^4*cos(f*x + e)^3 + 4*a^2*c^4*cos(f*x + e)
^2 + 30*a^2*c^4*cos(f*x + e) - 12*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^5
)
```

Sympy [F]

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^4\,dx = a^2c^4\left(\int 1\,dx + \int (-2\sec(e + fx))\,dx + \int (-\sec^2(e + fx))\,dx + \int 4\sec^3(e + fx)\,dx + \int (-\sec^4(e + fx))\,dx + \int (-2\sec^5(e + fx))\,dx + \int \sec^6(e + fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

output
```
a**2*c**4*(Integral(1, x) + Integral(-2*sec(e + f*x), x) + Integral(-sec(e
 + f*x)**2, x) + Integral(4*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)**
4, x) + Integral(-2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.71

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))^4\,dx = \frac{8\left(3\tan(fx + e)^5 + 10\tan(fx + e)^3 + 15\tan(fx + e)\right)a^2c^4 - 40\left(\tan(fx + e)^3 + 3\tan(fx + e)\right)a^2c^4}{}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4
- 40*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 120*(f*x + e)*a^2*c^4 + 1
5*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f
*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*
a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) - 240*a^2*c^4*log(sec(f*x + e) + tan(f*x + e)) - 120*a
^2*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = \frac{60\,(fx+e)a^2c^4 - 45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 45\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(105\,a^2c^4\,\mathrm{t}\right.}{}}{60\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/60*(60*(f*x + e)*a^2*c^4 - 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))
 + 45*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(105*a^2*c^4*tan(1/2*
f*x + 1/2*e)^9 - 530*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 328*a^2*c^4*tan(1/2*
f*x + 1/2*e)^5 - 110*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^4*tan(1/2*f
*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 11.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = a^2\,c^4\,x + \frac{\frac{7\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2} - \frac{53\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3} + \frac{164\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15} - \frac{11\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3} + \frac{a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10} - 5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8 + 10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6 - 10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4 + 5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2 - 1\right)} - \frac{3\,a^2\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4,x)
```

output
```
a^2*c^4*x + ((164*a^2*c^4*tan(e/2 + (f*x)/2)^5)/15 - (11*a^2*c^4*tan(e/2 +
 (f*x)/2)^3)/3 - (53*a^2*c^4*tan(e/2 + (f*x)/2)^7)/3 + (7*a^2*c^4*tan(e/2
+ (f*x)/2)^9)/2 + (a^2*c^4*tan(e/2 + (f*x)/2))/2)/(f*(5*tan(e/2 + (f*x)/2)
^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)
/2)^8 + tan(e/2 + (f*x)/2)^10 - 1)) - (3*a^2*c^4*atanh(tan(e/2 + (f*x)/2))
)/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.13

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^4 \, dx$$
$$= \frac{a^2c^4\left(45\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx+e\right)^4 - 90\cos\left(fx+e\right)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(fx\right.\right.}{}$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x)
```

output
```
(a**2*c**4*(45*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 - 90
*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + 45*cos(e + f*x)*
log(tan((e + f*x)/2) - 1) - 45*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(
e + f*x)**4 + 90*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 -
45*cos(e + f*x)*log(tan((e + f*x)/2) + 1) + 60*cos(e + f*x)*sin(e + f*x)**
4*f*x - 75*cos(e + f*x)*sin(e + f*x)**3 - 120*cos(e + f*x)*sin(e + f*x)**2
*f*x + 45*cos(e + f*x)*sin(e + f*x) + 60*cos(e + f*x)*f*x - 68*sin(e + f*x
)**5 + 140*sin(e + f*x)**3 - 60*sin(e + f*x)))/(60*cos(e + f*x)*f*(sin(e +
 f*x)**4 - 2*sin(e + f*x)**2 + 1))
```

3.3 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\begin{aligned}\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx &= a^2c^3x-\frac{3a^2c^3\text{arctanh}(\sin(e+fx))}{8f}-\frac{a^2(8c^3-3c^3\sec(e+fx))\tan(e+fx)}{8f}\\&\quad+\frac{a^2(4c^3-3c^3\sec(e+fx))\tan^3(e+fx)}{12f}\end{aligned}$$

output
```
a^2*c^3*x-3/8*a^2*c^3*arctanh(sin(f*x+e))/f-1/8*a^2*(8*c^3-3*c^3*sec(f*x+e
))*tan(f*x+e)/f+1/12*a^2*(4*c^3-3*c^3*sec(f*x+e))*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \frac{a^2c^3\sec^4(e+fx)\,(72e+72fx-72\text{arctanh}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^2*c^3*Sec[e + f*x]^4*(72*e + 72*f*x - 72*ArcTanh[Sin[e + f*x]]*Cos[e +
f*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Co
s[4*(e + f*x)] - 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] + 30*Sin[3*(e + f*x
)] - 32*Sin[4*(e + f*x)]))/(192*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4392, 3042, 4369, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int (c-c\sec(e+fx))\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4369}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(4c-3c\sec(e+fx))}{12f}-\frac{1}{4}\int (4c-3c\sec(e+fx))\tan^2(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(4c-3c\sec(e+fx))}{12f}-\frac{1}{4}\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(4c-3c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx\right)$$

$$\Big\downarrow \text{4369}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\int (8c-3c\sec(e+fx))dx-\frac{\tan(e+fx)(8c-3c\sec(e+fx))}{2f}\right)+\frac{\tan^3(e+fx)(4c-3c\sec(e+fx)}{12f}\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\left(8cx-\frac{3c\text{arctanh}(\sin(e+fx))}{f}\right)-\frac{\tan(e+fx)(8c-3c\sec(e+fx))}{2f}\right)+\frac{\tan^3(e+fx)(4c-3c\sec(e}{12f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

output
```
a^2*c^2*(((4*c - 3*c*Sec[e + f*x])*Tan[e + f*x]^3)/(12*f) + ((8*c*x - (3*c
*ArcTanh[Sin[e + f*x]])/f)/2 - ((8*c - 3*c*Sec[e + f*x])*Tan[e + f*x])/(2*
f))/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

method	result
parts	$a^{2} c^{3} x-\frac{c^{3} a^{2} \left(-\frac{2}{3}-\frac{\sec \left(f x+e\right)^{2}}{3}\right) \tan \left(f x+e\right)}{f}-\frac{2 c^{3} a^{2} \tan \left(f x+e\right)}{f}+\frac{c^{3} a^{2} \sec \left(f x+e\right) \tan \left(f x+e\right)}{f}-\frac{c^{3} a^{2} \left(-\left(-\right.\right.}{}$
risch	$a^{2} c^{3} x-\frac{i c^{3} a^{2} \left(15 \,{\mathrm e}^{7 i \left(f x+e\right)}+48 \,{\mathrm e}^{6 i \left(f x+e\right)}-9 \,{\mathrm e}^{5 i \left(f x+e\right)}+96 \,{\mathrm e}^{4 i \left(f x+e\right)}+9 \,{\mathrm e}^{3 i \left(f x+e\right)}+80 \,{\mathrm e}^{2 i \left(f x+e\right)}-15 \,{\mathrm e}^{i \left(f x+e\right)}+32\right)}{12 f \left({\mathrm e}^{2 i \left(f x+e\right)}+1\right)^{4}}$
derivativedivides	$\frac{c^{3} a^{2} \left(f x+e\right)-c^{3} a^{2} \ln \left(\sec \left(f x+e\right)+\tan \left(f x+e\right)\right)-2 c^{3} a^{2} \tan \left(f x+e\right)+2 c^{3} a^{2} \left(\frac{\sec \left(f x+e\right) \tan \left(f x+e\right)}{2}+\frac{\ln \left(\sec \left(f x+e\right)+\tan \left(f x+e\right.\right.}{2}\right.}{}$
default	$\frac{c^{3} a^{2} \left(f x+e\right)-c^{3} a^{2} \ln \left(\sec \left(f x+e\right)+\tan \left(f x+e\right)\right)-2 c^{3} a^{2} \tan \left(f x+e\right)+2 c^{3} a^{2} \left(\frac{\sec \left(f x+e\right) \tan \left(f x+e\right)}{2}+\frac{\ln \left(\sec \left(f x+e\right)+\tan \left(f x+e\right.\right.}{2}\right.}{}$
parallelrisch	$\frac{a^{2} c^{3} \left(9 \left(\cos \left(4 f x+4 e\right)+4 \cos \left(2 f x+2 e\right)+3\right) \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)+9 \left(-3-\cos \left(4 f x+4 e\right)-4 \cos \left(2 f x+2 e\right)\right) \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right.\right.}{24 f \left(\cos \left(4 f x+4 e\right)\right.}$
norman	$\frac{a^{2} c^{3} x+a^{2} c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{8}-4 a^{2} c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+6 a^{2} c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-4 a^{2} c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-\frac{5 c^{3} a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{4 f}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{4}}$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^3*x-c^3*a^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*c^3*a^2/f*tan(f*x
+e)+c^3*a^2/f*sec(f*x+e)*tan(f*x+e)-c^3*a^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec
(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \frac{48\,a^{2} c^{3} f x \cos\left(f x + e\right)^{4} - 9\,a^{2} c^{3} \cos\left(f x + e\right)^{4} \log\left(\sin\left(f x + e\right) + 1\right) + 9\,a^{2} c^{3} \cos\left(f x + e\right)^{4} \log\left(-\sin\left(f x\right.\right.}{48\,f \cos\left(\right.}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/48*(48*a^2*c^3*f*x*cos(f*x + e)^4 - 9*a^2*c^3*cos(f*x + e)^4*log(sin(f*x
 + e) + 1) + 9*a^2*c^3*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^2*c
^3*cos(f*x + e)^3 - 15*a^2*c^3*cos(f*x + e)^2 - 8*a^2*c^3*cos(f*x + e) + 6
*a^2*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^3 \, dx = -a^2c^3\left(\int (-1)\, dx + \int \sec(e + fx)\, dx + \int 2\sec^2(e + fx)\, dx + \int \left(-2\sec^3(e + fx)\right) dx + \int \left(-\sec^4(e + fx)\right) dx + \int \sec^5(e + fx)\, dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

output
```
-a**2*c**3*(Integral(-1, x) + Integral(sec(e + f*x), x) + Integral(2*sec(e
 + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(-sec(e + f*x)*
*4, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(91) = 182.

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.09

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^3 \, dx = \frac{16\left(\tan(fx + e)^3 + 3\tan(fx + e)\right)a^2c^3 + 48(fx + e)a^2c^3 + 3a^2c^3\left(\frac{2\left(3\sin(fx+e)^3 - 5\sin(fx+e)\right)}{\sin(fx+e)^4 - 2\sin(fx+e)^2 + 1} - 3\log(\sin\cdots\right.}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 48*(f*x + e)*a^2*c^3
+ 3*a^2*c^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin
(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 24
*a^2*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 48*a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) - 96*a^
2*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$=\frac{24\,(fx+e)a^2c^3-9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)+9\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(33\,a^2c^3\tan(\right.}{}}{24\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/24*(24*(f*x + e)*a^2*c^3 - 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))
+ 9*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(33*a^2*c^3*tan(1/2*f*x
 + 1/2*e)^7 - 137*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 71*a^2*c^3*tan(1/2*f*x
+ 1/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)
^4)/f
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$=\frac{\frac{11\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{137\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{71\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}-\frac{5\,a^2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$
$$+a^2\,c^3\,x-\frac{3\,a^2\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3,x)
```

output
```
((71*a^2*c^3*tan(e/2 + (f*x)/2)^3)/12 - (137*a^2*c^3*tan(e/2 + (f*x)/2)^5)
/12 + (11*a^2*c^3*tan(e/2 + (f*x)/2)^7)/4 - (5*a^2*c^3*tan(e/2 + (f*x)/2))
/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x
)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^2*c^3*x - (3*a^2*c^3*atanh(tan(e/2
 + (f*x)/2)))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.33

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^3 \, dx$$
$$= \frac{a^2c^3 \big(32\cos(fx+e)\sin(fx+e)^3 - 24\cos(fx+e)\sin(fx+e) + 9\log\big(\tan\big(\frac{fx}{2}+\frac{e}{2}\big)-1\big)\sin(fx+e)^4}{}$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x)
```

output
```
(a**2*c**3*(32*cos(e + f*x)*sin(e + f*x)**3 - 24*cos(e + f*x)*sin(e + f*x)
 + 9*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 - 18*log(tan((e + f*x)/2) -
 1)*sin(e + f*x)**2 + 9*log(tan((e + f*x)/2) - 1) - 9*log(tan((e + f*x)/2)
 + 1)*sin(e + f*x)**4 + 18*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 - 9*l
og(tan((e + f*x)/2) + 1) + 24*sin(e + f*x)**4*f*x - 15*sin(e + f*x)**3 - 4
8*sin(e + f*x)**2*f*x + 9*sin(e + f*x) + 24*f*x))/(24*f*(sin(e + f*x)**4 -
 2*sin(e + f*x)**2 + 1))
```

3.4 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2x-\frac{a^2c^2\tan(e+fx)}{f}+\frac{a^2c^2\tan^3(e+fx)}{3f}$$

output
```
a^2*c^2*x-a^2*c^2*tan(f*x+e)/f+1/3*a^2*c^2*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\frac{\arctan(\tan(e+fx))}{f}-\frac{\tan(e+fx)}{f}+\frac{\tan^3(e+fx)}{3f}\right)$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*(ArcTan[Tan[e + f*x]]/f - Tan[e + f*x]/f + Tan[e + f*x]^3/(3*f))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \tan(e+fx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)}{3f}-\int \tan^2(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)}{3f}-\int \tan(e+fx)^2dx\right)$$

$$\Big\downarrow \text{3954}$$

$$a^2c^2\left(\int 1dx+\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)}{3f}-\frac{\tan(e+fx)}{f}+x\right)$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*(x - Tan[e + f*x]/f + Tan[e + f*x]^3/(3*f))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

method	result
parts	$a^{2}c^{2}x-\frac{c^{2}a^{2}\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}-\frac{2a^{2}c^{2}\tan(fx+e)}{f}$
derivativedivides	$\frac{c^{2}a^{2}(fx+e)-2c^{2}a^{2}\tan(fx+e)-c^{2}a^{2}\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$
default	$\frac{c^{2}a^{2}(fx+e)-2c^{2}a^{2}\tan(fx+e)-c^{2}a^{2}\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$
risch	$a^{2}c^{2}x-\frac{4ic^{2}a^{2}\left(3\,\mathrm{e}^{4i(fx+e)}+3\,\mathrm{e}^{2i(fx+e)}+2\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^{3}}$
parallelrisch	$\frac{c^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}xf-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}xf+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}xf-\frac{20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-fx+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$
norman	$\frac{a^{2}c^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-a^{2}c^{2}x+3a^{2}c^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3a^{2}c^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+\frac{2c^{2}a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{20c^{2}a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f}+}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^2*x-c^2*a^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a^2*c^2*tan(f*x+e
)/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{3\,a^{2}c^{2}fx\cos\left(fx+e\right)^{3}-\left(4\,a^{2}c^{2}\cos\left(fx+e\right)^{2}-a^{2}c^{2}\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^{3}}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*c^2*f*x*cos(f*x + e)^3 - (4*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*s
in(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\int 1\,dx+\int \left(-2\sec^2{(e+fx)}\right)\,dx+\int \sec^4{(e+fx)}\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

output
```
a**2*c**2*(Integral(1, x) + Integral(-2*sec(e + f*x)**2, x) + Integral(sec
(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{(\tan{(fx+e)}^3+3\,\tan{(fx+e)})a^2c^2+3\,(fx+e)a^2c^2-6\,a^2c^2\tan{(fx+e)}}{3\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2 + 3*(f*x + e)*a^2*c^2 - 6*a
^2*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{a^2c^2\tan{(fx+e)}^3+3\,(fx+e)a^2c^2-3\,a^2c^2\tan{(fx+e)}}{3\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(a^2*c^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2*c^2 - 3*a^2*c^2*tan(f*x + e)
)/f
```

Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2\,c^2\,x+\frac{2\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{20\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+2\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)^3}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2,x)
```

output
```
a^2*c^2*x + (2*a^2*c^2*tan(e/2 + (f*x)/2)^5 - (20*a^2*c^2*tan(e/2 + (f*x)/
2)^3)/3 + 2*a^2*c^2*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{a^2c^2\left(3\cos\left(fx+e\right)\sin\left(fx+e\right)^2fx-3\cos\left(fx+e\right)fx-4\sin\left(fx+e\right)^3+3\sin\left(fx+e\right)\right)}{3\cos\left(fx+e\right)f\left(\sin\left(fx+e\right)^2-1\right)}$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x)
```

output
```
(a**2*c**2*(3*cos(e + f*x)*sin(e + f*x)**2*f*x - 3*cos(e + f*x)*f*x - 4*si
n(e + f*x)**3 + 3*sin(e + f*x)))/(3*cos(e + f*x)*f*(sin(e + f*x)**2 - 1))
```

3.5 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = a^2cx+\frac{a^2c\text{arctanh}(\sin(e+fx))}{2f}-\frac{c(2a^2+a^2\sec(e+fx))\tan(e+fx)}{2f}$$

output
```
a^2*c*x+1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*c*(2*a^2+a^2*sec(f*x+e))*tan(f
*x+e)/f
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\sec^2(e+fx)\left(e+fx+6\arctan(\tan(e+fx))\cos^2(e+fx)+4\text{arctanh}(\sin(e+fx))\cos^2(e+fx)+\right.}{8f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
(a^2*c*Sec[e + f*x]^2*(e + f*x + 6*ArcTan[Tan[e + f*x]]*Cos[e + f*x]^2 + 4
*ArcTanh[Sin[e + f*x]]*Cos[e + f*x]^2 + e*Cos[2*(e + f*x)] + f*x*Cos[2*(e
+ f*x)] - 4*Sin[e + f*x] - 4*Sin[2*(e + f*x)]))/(8*f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 4392, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int (\sec(e+fx)a+a)\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx$$

$$\Big\downarrow \text{4369}$$

$$-ac\left(\frac{\tan(e+fx)(a\sec(e+fx)+2a)}{2f}-\frac{1}{2}\int (\sec(e+fx)a+2a)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$-ac\left(\frac{1}{2}\left(-\frac{a\text{arctanh}(\sin(e+fx))}{f}-2ax\right)+\frac{\tan(e+fx)(a\sec(e+fx)+2a)}{2f}\right)$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*((-2*a*x - (a*ArcTanh[Sin[e + f*x]])/f)/2 + ((2*a + a*Sec[e + f*x])*
Tan[e + f*x])/(2*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{a^2c(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))-a^2c\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{a^2c(fx+e)+a^2c\ln(\sec(fx+e)+\tan(fx+e))-a^2c\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parts	$a^2cx+\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^2c\tan(fx+e)}{f}-\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
risch	$a^2cx+\frac{ia^2c(e^{3i(fx+e)}-2e^{2i(fx+e)}-e^{i(fx+e)}-2)}{f(e^{2i(fx+e)}+1)^2}+\frac{a^2c\ln(e^{i(fx+e)}+i)}{2f}-\frac{a^2c\ln(e^{i(fx+e)}-i)}{2f}$
parallelrisch	$-\frac{c\left((1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2fx\cos(2fx+2e)-2fx+2\sin\right.}{2f(1+\cos(2fx+2e))}$
norman	$\frac{a^2cx+a^2cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-2a^2cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{3a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*c*(f*x+e)+a^2*c*ln(sec(f*x+e)+tan(f*x+e))-a^2*c*tan(f*x+e)-a^2*c*
(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{4\,a^2cfx\cos(fx+e)^2+a^2c\cos(fx+e)^2\log(\sin(fx+e)+1)-a^2c\cos(fx+e)^2\log(-\sin(fx+e)+}{4\,f\cos(fx+e)^2}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(4*a^2*c*f*x*cos(f*x + e)^2 + a^2*c*cos(f*x + e)^2*log(sin(f*x + e) +
1) - a^2*c*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*(2*a^2*c*cos(f*x + e)
 + a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= -a^2c\left(\int (-1)\;dx + \int (-\sec{(e+fx)})\;dx + \int \sec^2{(e+fx)}\,dx + \int \sec^3{(e+fx)}\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

output
```
-a**2*c*(Integral(-1, x) + Integral(-sec(e + f*x), x) + Integral(sec(e + f
*x)**2, x) + Integral(sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{4\,(fx+e)a^2c + a^2c\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1} - \log{(\sin{(fx+e)}+1)} + \log{(\sin{(fx+e)}-1)}\right) + 4\,a^2c\log{(\sec{(fx+}}{4\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*(4*(f*x + e)*a^2*c + a^2*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(
sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 4*a^2*c*log(sec(f*x + e) + ta
n(f*x + e)) - 4*a^2*c*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))\,dx$$
$$= \frac{2\,(fx+e)a^2c + a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) - a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) + \frac{2\left(a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}}{2\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/2*(2*(f*x + e)*a^2*c + a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - a^2*c*
log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a
^2*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))\,dx$$
$$= a^2\,c\,x - \frac{3\,a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) - a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - 2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + 1\right)} + \frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)),x)
```

output
```
a^2*c*x - (3*a^2*c*tan(e/2 + (f*x)/2) - a^2*c*tan(e/2 + (f*x)/2)^3)/(f*(ta
n(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1)) + (a^2*c*atanh(tan(e/2 +
 (f*x)/2)))/f
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.31

$$\int (a + a\sec(e + fx))^2(c - c\sec(e + fx))\,dx$$
$$= \frac{a^2c\big(2\cos(fx+e)\sin(fx+e) - \log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)\sin(fx+e)^2 + \log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big) + \log\big(\tan}{2f\left(\sin(fx+e\right.}$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

output
```
(a**2*c*(2*cos(e + f*x)*sin(e + f*x) - log(tan((e + f*x)/2) - 1)*sin(e + f
*x)**2 + log(tan((e + f*x)/2) - 1) + log(tan((e + f*x)/2) + 1)*sin(e + f*x
)**2 - log(tan((e + f*x)/2) + 1) + 2*sin(e + f*x)**2*f*x + sin(e + f*x) -
2*f*x))/(2*f*(sin(e + f*x)**2 - 1))
```

3.6 $\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx=\frac{a^2x}{c}-\frac{a^2\text{arctanh}(\sin(e+fx))}{cf}-\frac{4a^2\tan(e+fx)}{cf(1-\sec(e+fx))}$$

output
```
a^2*x/c-a^2*arctanh(sin(f*x+e))/c/f-4*a^2*tan(f*x+e)/c/f/(1-sec(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(56) = 112.

Time = 1.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.59

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^{3/2}\tan(e+fx)\left(4\sqrt{c}\left(\sqrt{a}\sqrt{1-\sec(e+fx)}(1+\sec(e+fx))+\arcsin\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec(e+fx)\right.\right.}{c^{3/2}f(1-\sec(e+}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

output
```
-((a^(3/2)*Tan[e + f*x]*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 + Se
c[e + f*x]) + ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e +
 f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^2) - ArcTanh[Sqrt[-(a*c*
Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sqrt[-(a*c*Tan[
e + f*x]^2)]))/(c^(3/2)*f*(1 - Sec[e + f*x])^(3/2)*(1 + Sec[e + f*x])))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{1-\sec(e+fx)}+\frac{2\sec(e+fx)a^2}{1-\sec(e+fx)}+\frac{a^2}{1-\sec(e+fx)}\right)dx}{c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{a^2\text{arctanh}(\sin(e+fx))}{f}-\frac{4a^2\tan(e+fx)}{f(1-\sec(e+fx))}+a^2x}{c}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x]),x]
```

output
```
(a^2*x - (a^2*ArcTanh[Sin[e + f*x]])/f - (4*a^2*Tan[e + f*x])/(f*(1 - Sec[
e + f*x])))/c
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

method	result
parallelrisch	$\frac{a^{2}\left(fx+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+4\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}$
derivativedivides	$\frac{4a^{2}\left(\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$
default	$\frac{4a^{2}\left(\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$
risch	$\frac{a^{2}x}{c}+\frac{8ia^{2}}{fc\left(e^{i(fx+e)}-1\right)}+\frac{a^{2}\ln\left(e^{i(fx+e)}-i\right)}{cf}-\frac{a^{2}\ln\left(e^{i(fx+e)}+i\right)}{cf}$
norman	$\frac{\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{c}-\frac{4a^{2}}{cf}+\frac{4a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{cf}-\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a^{2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{a^{2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/c/f*a^2*(f*x+ln(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)+1)+4*cot(1/2
*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$$
$$= \frac{2\,a^2fx\sin(fx+e) - a^2\log(\sin(fx+e)+1)\sin(fx+e) + a^2\log(-\sin(fx+e)+1)\sin(fx+e) + 8}{2\,cf\sin(fx+e)}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(2*a^2*f*x*sin(f*x + e) - a^2*log(sin(f*x + e) + 1)*sin(f*x + e) + a^2
*log(-sin(f*x + e) + 1)*sin(f*x + e) + 8*a^2*cos(f*x + e) + 8*a^2)/(c*f*si
n(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{\sec^2(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{1}{\sec(e+fx)-1}\,dx\right)}{c}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

output
```
-a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f
*x)**2/(sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(54) = 108.

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.73

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$$
$$= \frac{a^2\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} + \frac{\cos(fx+e)+1}{c\sin(fx+e)}\right) - a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c} - \frac{\cos(fx+e)+1}{c\sin(fx+e)}\right) + \frac{2\,a^2(\cos(}{c\sin(}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
(a^2*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*
sin(f*x + e))) - a^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin
(f*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e)))
 + 2*a^2*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{\frac{(fx+e)a^2}{c} - \frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c} + \frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c} + \frac{4\,a^2}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
((f*x + e)*a^2/c - a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + a^2*log(abs(
tan(1/2*f*x + 1/2*e) - 1))/c + 4*a^2/(c*tan(1/2*f*x + 1/2*e)))/f
```

Mupad [B] (verification not implemented)

Time = 11.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{a^2\,x}{c} - \frac{a^2\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right) - \frac{4}{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{c\,f}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x)),x)
```

output
```
(a^2*x)/c - (a^2*(2*atanh(tan(e/2 + (f*x)/2)) - 4/tan(e/2 + (f*x)/2)))/(c*
f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$$
$$= \frac{a^2\left(\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)fx+4\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cf}$$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

output
```
(a**2*(log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2) - log(tan((e + f*x)/2) +
 1)*tan((e + f*x)/2) + tan((e + f*x)/2)*f*x + 4))/(tan((e + f*x)/2)*c*f)
```

3.7 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2} \, dx = \frac{a^2x}{c^2} - \frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2} - \frac{4a^2\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

output
```
a^2*x/c^2-4/3*a^2*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2-4/3*a^2*tan(f*x+e)/c^2
/f/(1-sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2} \, dx = -\frac{2a^2\cot^3\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3c^2f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^2*Cot[e/2 + (f*x)/2]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e/2 + (
f*x)/2]^2])/(3*c^2*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^2}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^2}+\frac{a^2}{(1-\sec(e+fx))^2}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4a^2\tan(e+fx)}{3f(1-\sec(e+fx))}-\frac{4a^2\tan(e+fx)}{3f(1-\sec(e+fx))^2}+a^2x}{c^2}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(3*f*(1 - Sec[e + f*x])^2) - (4*a^2*Tan[e +
f*x])/(3*f*(1 - Sec[e + f*x])))/c^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$-\frac{a^{2} \left(2 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}-3 f x -6 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 c^{2} f}$	41
derivativedivides	$\frac{2 a^{2} \left(-\frac{1}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,c^{2}}$	47
default	$\frac{2 a^{2} \left(-\frac{1}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,c^{2}}$	47
risch	$\frac{a^{2} x}{c^{2}}+\frac{8 i a^{2} \left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}-3 \,{\mathrm e}^{i \left(f x +e \right)}+2\right)}{3 f \,c^{2} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{3}}$	59
norman	$\frac{\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{c}+\frac{2 a^{2}}{3 c f}-\frac{8 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{3 c f}+\frac{2 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{c f}-\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{c}}{c \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	126

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*(2*cot(1/2*f*x+1/2*e)^3-3*f*x-6*cot(1/2*f*x+1/2*e))/c^2/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{8\,a^2\cos\left(fx+e\right)^2+4\,a^2\cos\left(fx+e\right)-4\,a^2+3\left(a^2fx\cos\left(fx+e\right)-a^2fx\right)\sin\left(fx+e\right)}{3\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(8*a^2*cos(f*x + e)^2 + 4*a^2*cos(f*x + e) - 4*a^2 + 3*(a^2*f*x*cos(f*
x + e) - a^2*f*x)*sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)
)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{1}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Int
egral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(63) = 126$.

Time $= 0.11$ (sec) , antiderivative size $= 174$, normalized size of antiderivative $= 2.45$

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^2}\,dx$$

$$= \frac{a^2\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} + \frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right) - \frac{a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3} + \frac{2a^2\left(\frac{3\sin(fx+}{(\cos(fx+e)}\right.}{c^2}}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(a^2*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) -
a^2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*
sin(f*x + e)^3) + 2*a^2*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f
*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time $= 0.16$ (sec) , antiderivative size $= 57$, normalized size of antiderivative $= 0.80$

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^2}\,dx = \frac{\frac{3(fx+e)a^2}{c^2} + \frac{2\left(3a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - a^2\right)}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}}{3f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*a^2/c^2 + 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - a^2)/(c^2*tan
(1/2*f*x + 1/2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(-2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+6\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)+3\,f\,x\right)}{3\,c^2\,f}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^2,x)
```

output
```
(a^2*(6*cot(e/2 + (f*x)/2) - 2*cot(e/2 + (f*x)/2)^3 + 3*f*x))/(3*c^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3fx+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2\right)}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2f}$$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

output
```
(a**2*(3*tan((e + f*x)/2)**3*f*x + 6*tan((e + f*x)/2)**2 - 2))/(3*tan((e +
 f*x)/2)**3*c**2*f)
```

3.8 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2x}{c^3} - \frac{4a^2\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} - \frac{8a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{23a^2\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

output
```
a^2*x/c^3-4/5*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3-8/15*a^2*tan(f*x+e)/c^
3/f/(1-sec(f*x+e))^2-23/15*a^2*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.48

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^{3/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(43-11\sec(e+fx)-31\sec^2(e+fx)+23\sec^3(e+fx))-60\operatorname{arctanh}\left(\frac{\sqrt{-ac\tan\ldots}}{\sqrt{\ldots}}\right.\right.}{15c^{7/2}f(-1+\sec(e+fx))^3(1+\sec(e+fx)\ldots}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^(3/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(43 - 11*Sec[e + f*x] - 31*Sec[e +
f*x]^2 + 23*Sec[e + f*x]^3) - 60*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt
[a]*Sqrt[c])]*Sec[e + f*x]^2*Sin[(e + f*x)/2]^4*Sqrt[-(a*c*Tan[e + f*x]^2)
]))/(15*c^(7/2)*f*(-1 + Sec[e + f*x])^3*(1 + Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^3}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^3}+\frac{a^2}{(1-\sec(e+fx))^3}\right)dx}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{23a^2\tan(e+fx)}{15f(1-\sec(e+fx))}-\frac{8a^2\tan(e+fx)}{15f(1-\sec(e+fx))^2}-\frac{4a^2\tan(e+fx)}{5f(1-\sec(e+fx))^3}+a^2x}{c^3}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(5*f*(1 - Sec[e + f*x])^3) - (8*a^2*Tan[e +
f*x])/(15*f*(1 - Sec[e + f*x])^2) - (23*a^2*Tan[e + f*x])/(15*f*(1 - Sec[e
 + f*x])))/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

method	result	size
parallelrisch	$\frac{a^{2} \left(3 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-10 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+15 f x+30 \cot \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 c^{3} f}$	54
derivativedivides	$\frac{a^{2} \left(\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+2 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,c^{3}}$	63
default	$\frac{a^{2} \left(\frac{1}{5 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)}+2 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,c^{3}}$	63
risch	$\frac{a^{2} x}{c^{3}}+\frac{2 i a^{2} \left(75 \,{\mathrm e}^{4 i \left(f x +e \right)}-180 \,{\mathrm e}^{3 i \left(f x +e \right)}+250 \,{\mathrm e}^{2 i \left(f x +e \right)}-140 \,{\mathrm e}^{i \left(f x +e \right)}+43\right)}{15 f \,c^{3} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{5}}$	81
norman	$\frac{\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{c}-\frac{a^{2}}{5 c f}+\frac{13 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{15 c f}-\frac{8 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{3 c f}+\frac{2 a^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{c f}-\frac{a^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{c}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	148

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*a^2*(3*cot(1/2*f*x+1/2*e)^5-10*cot(1/2*f*x+1/2*e)^3+15*f*x+30*cot(1/2
*f*x+1/2*e))/c^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{43\,a^2\cos\left(fx+e\right)^3 - 11\,a^2\cos\left(fx+e\right)^2 - 31\,a^2\cos\left(fx+e\right) + 23\,a^2 + 15\left(a^2fx\cos\left(fx+e\right)^2 - 2\,a^2fx\right.}{15\left(c^3f\cos\left(fx+e\right)^2 - 2\,c^3f\cos\left(fx+e\right) + c^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(43*a^2*cos(f*x + e)^3 - 11*a^2*cos(f*x + e)^2 - 31*a^2*cos(f*x + e)
+ 23*a^2 + 15*(a^2*f*x*cos(f*x + e)^2 - 2*a^2*f*x*cos(f*x + e) + a^2*f*x)*
sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f
*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx + \int \frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx + \int \frac{}{\sec^3\left(e+fx\right)-3\sec}\right.}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

output
```
-a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*se
c(e + f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**3 - 3*sec(
e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(90) = 180$.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.11

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^3}\,dx$$
$$= \frac{a^2\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3} - \frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right) - \frac{2a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(}{c^3\sin(fx+e)^5}}{60f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/60*(a^2*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*
(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) - 2*a^2*(10*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x +
e) + 1)^5/(c^3*sin(f*x + e)^5) - 3*a^2*(5*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^3}\,dx = \frac{\frac{15\,(fx+e)a^2}{c^3} + \frac{30\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 10\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 3\,a^2}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*a^2/c^3 + (30*a^2*tan(1/2*f*x + 1/2*e)^4 - 10*a^2*tan(1
/2*f*x + 1/2*e)^2 + 3*a^2)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 10.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2\,x}{c^3}+\frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^3,x)
```

output
```
(a^2*x)/c^3 + ((a^2*cos(e/2 + (f*x)/2)^5)/5 + 2*a^2*cos(e/2 + (f*x)/2)*sin
(e/2 + (f*x)/2)^4 - (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)/(
c^3*f*sin(e/2 + (f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2\left(15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 fx+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+3\right)}{15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^3 f}$$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

output
```
(a**2*(15*tan((e + f*x)/2)**5*f*x + 30*tan((e + f*x)/2)**4 - 10*tan((e + f
*x)/2)**2 + 3))/(15*tan((e + f*x)/2)**5*c**3*f)
```

3.9 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2x}{c^4} - \frac{4a^2\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4} - \frac{12a^2\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3} - \frac{59a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2} - \frac{164a^2\tan(e+fx)}{105c^4f(1-\sec(e+fx))}$$

output
```
a^2*x/c^4-4/7*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4-12/35*a^2*tan(f*x+e)/c
^4/f/(1-sec(f*x+e))^3-59/105*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-164/105
*a^2*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = -\frac{a^2\csc^7(e+fx)\left(7032+18165\cos(e+fx)+19348\cos(2(e+fx))+9303\cos(3(e+fx))+3080\cos(\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

output
```
-1/6720*(a^2*Csc[e + f*x]^7*(7032 + 18165*Cos[e + f*x] + 19348*Cos[2*(e +
f*x)] + 9303*Cos[3*(e + f*x)] + 3080*Cos[4*(e + f*x)] + 2149*Cos[5*(e + f*
x)] + 1260*Cos[6*(e + f*x)] + 143*Cos[7*(e + f*x)] + 960*Cos[e + f*x]^7*Hy
pergeometric2F1[-7/2, 1, -5/2, -Tan[e + f*x]^2]))/(c^4*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^4}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^4}+\frac{a^2}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{164a^2\tan(e+fx)}{105f(1-\sec(e+fx))}-\frac{59a^2\tan(e+fx)}{105f(1-\sec(e+fx))^2}-\frac{12a^2\tan(e+fx)}{35f(1-\sec(e+fx))^3}-\frac{4a^2\tan(e+fx)}{7f(1-\sec(e+fx))^4}+a^2x}{c^4}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(7*f*(1 - Sec[e + f*x])^4) - (12*a^2*Tan[e +
 f*x])/(35*f*(1 - Sec[e + f*x])^3) - (59*a^2*Tan[e + f*x])/(105*f*(1 - Sec
[e + f*x])^2) - (164*a^2*Tan[e + f*x])/(105*f*(1 - Sec[e + f*x])))/c^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{a^{2}\left(15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-63\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+140\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-210fx-420\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{210c^{4}f}$
derivativedivides	$\frac{a^{2}\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,c^{4}}$
default	$\frac{a^{2}\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,c^{4}}$
risch	$\frac{a^{2}x}{c^{4}}+\frac{2ia^{2}\left(630\,{\mathrm e}^{6i(fx+e)}-2415\,{\mathrm e}^{5i(fx+e)}+5215\,{\mathrm e}^{4i(fx+e)}-5950\,{\mathrm e}^{3i(fx+e)}+4284\,{\mathrm e}^{2i(fx+e)}-1603\,{\mathrm e}^{i(fx+e)}+319\right)}{105f\,c^{4}\left({\mathrm e}^{i(fx+e)}-1\right)^{7}}$
norman	$\frac{\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{c}+\frac{a^{2}}{14cf}-\frac{13a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{35cf}+\frac{29a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{30cf}-\frac{8a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{3cf}+\frac{2a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{cf}-\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/210*a^2*(15*cot(1/2*f*x+1/2*e)^7-63*cot(1/2*f*x+1/2*e)^5+140*cot(1/2*f*
x+1/2*e)^3-210*f*x-420*cot(1/2*f*x+1/2*e))/c^4/f
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{319\,a^2\cos\left(fx+e\right)^4-327\,a^2\cos\left(fx+e\right)^3-95\,a^2\cos\left(fx+e\right)^2+387\,a^2\cos\left(fx+e\right)-164\,a^2+105\,(a}{105\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/105*(319*a^2*cos(f*x + e)^4 - 327*a^2*cos(f*x + e)^3 - 95*a^2*cos(f*x +
e)^2 + 387*a^2*cos(f*x + e) - 164*a^2 + 105*(a^2*f*x*cos(f*x + e)^3 - 3*a^
2*f*x*cos(f*x + e)^2 + 3*a^2*f*x*cos(f*x + e) - a^2*f*x)*sin(f*x + e))/((c
^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*
f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\left(\int \frac{2\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,d}{c^4}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

output
```
a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e +
 f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)
 + Integral(1/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4
*sec(e + f*x) + 1), x))/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.21

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{5\,a^2\left(\frac{336\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4} + \frac{\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{77\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{315\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right) + a^2\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{35}{(\cos} \cdots\right.}{840\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
1/840*(5*a^2*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(
f*x + e)^7)) + a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*
(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 6*a^2*(21*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^4}\,dx = \frac{\frac{210\,(fx+e)a^2}{c^4} + \frac{420\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 140\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 15\,a^2}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{210\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/210*(210*(f*x + e)*a^2/c^4 + (420*a^2*tan(1/2*f*x + 1/2*e)^6 - 140*a^2*t
an(1/2*f*x + 1/2*e)^4 + 63*a^2*tan(1/2*f*x + 1/2*e)^2 - 15*a^2)/(c^4*tan(1
/2*f*x + 1/2*e)^7))/f
```

Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{(a + a\sec(e + fx))^2}{(c - c\sec(e + fx))^4}\,dx = \frac{a^2\,x}{c^4} - \frac{\frac{a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{14} - \frac{3\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{10} + \frac{2\,a^2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3} - 2\,a^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{c^4\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^4,x)
```

output
```
(a^2*x)/c^4 - ((a^2*cos(e/2 + (f*x)/2)^7)/14 - 2*a^2*cos(e/2 + (f*x)/2)*si
n(e/2 + (f*x)/2)^6 + (2*a^2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4)/3 -
 (3*a^2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/10)/(c^4*f*sin(e/2 + (f
*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\left(210\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 fx+420\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-140\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-15\right)}{210\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 c^4 f}$$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

output
```
(a**2*(210*tan((e + f*x)/2)**7*f*x + 420*tan((e + f*x)/2)**6 - 140*tan((e
+ f*x)/2)**4 + 63*tan((e + f*x)/2)**2 - 15))/(210*tan((e + f*x)/2)**7*c**4
*f)
```

3.10 $\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2x}{c^5} - \frac{4a^2\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} - \frac{16a^2\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{37a^2\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{179a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{494a^2\tan(e+fx)}{315c^5f(1-\sec(e+fx))}$$

output
```
a^2*x/c^5-4/9*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5-16/63*a^2*tan(f*x+e)/c
^5/f/(1-sec(f*x+e))^4-37/105*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-179/315
*a^2*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-494/315*a^2*tan(f*x+e)/c^5/f/(1-sec
(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.77 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a^2\cot^9(e+fx)\left(441+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+2205\sec(e+fx)+1323\,\text{s}\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^2*Cot[e + f*x]^9*(441 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*
x]^2] + 2205*Sec[e + f*x] + 1323*Sec[e + f*x]^2 - 2205*Sec[e + f*x]^3 + 44
1*Sec[e + f*x]^4 + 3969*Sec[e + f*x]^5 - 2223*Sec[e + f*x]^7 + 494*Sec[e +
 f*x]^9))/(315*c^5*f)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^2}{(c-c\sec(e+fx))^5}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$
$$\Big\downarrow \ 4391$$
$$\frac{\int \left(\frac{\sec^2(e+fx)a^2}{(1-\sec(e+fx))^5}+\frac{2\sec(e+fx)a^2}{(1-\sec(e+fx))^5}+\frac{a^2}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{494a^2\tan(e+fx)}{315f(1-\sec(e+fx))}-\frac{179a^2\tan(e+fx)}{315f(1-\sec(e+fx))^2}-\frac{37a^2\tan(e+fx)}{105f(1-\sec(e+fx))^3}-\frac{16a^2\tan(e+fx)}{63f(1-\sec(e+fx))^4}-\frac{4a^2\tan(e+fx)}{9f(1-\sec(e+fx))^5}+a^2x}{c^5}$$

input
```
Int[(a + a*Sec[e + f*x])^2/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^2*x - (4*a^2*Tan[e + f*x])/(9*f*(1 - Sec[e + f*x])^5) - (16*a^2*Tan[e +
 f*x])/(63*f*(1 - Sec[e + f*x])^4) - (37*a^2*Tan[e + f*x])/(105*f*(1 - Sec
[e + f*x])^3) - (179*a^2*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])^2) - (494
*a^2*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])))/c^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result
parallelrisch	$\frac{a^{2}\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-180\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+441\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-840\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+1260fx+2520\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1260c^{5}f}$
derivativedivides	$\frac{a^{2}\left(8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{4}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{8}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4f\,c^{5}}$
default	$\frac{a^{2}\left(8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{4}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{8}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{8}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4f\,c^{5}}$
risch	$\frac{a^{2}x}{c^{5}}+\frac{2ia^{2}\left(2205\,e^{8i(fx+e)}-11655\,e^{7i(fx+e)}+34335\,e^{6i(fx+e)}-58905\,e^{5i(fx+e)}+67599\,e^{4i(fx+e)}-50001\,e^{3i(fx+e)}-\right.}{315f\,c^{5}\left(e^{i(fx+e)}-1\right)^{9}}$
norman	$\frac{\frac{a^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{c}-\frac{a^{2}}{36cf}+\frac{43a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{252cf}-\frac{69a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{140cf}+\frac{61a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{60cf}-\frac{8a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{3cf}+\frac{2a^{2}\tan\left(\frac{fx}{2}+\right.}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/1260*a^2*(35*cot(1/2*f*x+1/2*e)^9-180*cot(1/2*f*x+1/2*e)^7+441*cot(1/2*f
*x+1/2*e)^5-840*cot(1/2*f*x+1/2*e)^3+1260*f*x+2520*cot(1/2*f*x+1/2*e))/c^5
/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$$
$$=\frac{1004\,a^{2}\cos\left(fx+e\right)^{5}-1811\,a^{2}\cos\left(fx+e\right)^{4}+797\,a^{2}\cos\left(fx+e\right)^{3}+1457\,a^{2}\cos\left(fx+e\right)^{2}-1661\,a^{2}\cos}{315\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx\right.\right.}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(1004*a^2*cos(f*x + e)^5 - 1811*a^2*cos(f*x + e)^4 + 797*a^2*cos(f*x
 + e)^3 + 1457*a^2*cos(f*x + e)^2 - 1661*a^2*cos(f*x + e) + 494*a^2 + 315*
(a^2*f*x*cos(f*x + e)^4 - 4*a^2*f*x*cos(f*x + e)^3 + 6*a^2*f*x*cos(f*x + e
)^2 - 4*a^2*f*x*cos(f*x + e) + a^2*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)
^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e
) + c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^2\left(\int \frac{2\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int \frac{\sec^2(e+}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+f}\right.}{c^5}$$

input
```
integrate((a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

output
```
-a**2*(Integral(2*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*s
ec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(s
ec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 -
 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**
5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e
+ f*x) - 1), x))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(144) = 288.

Time = 0.13 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.04

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5}-\frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right)-\frac{2a^2}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
1/5040*(a^2*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 2*a^2*(180
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^2*(18
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c^
5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{\frac{1260\,(fx+e)a^2}{c^5} + \frac{2520\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-840\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+441\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-180\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}}{1260\,f}$$

input
```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/1260*(1260*(f*x + e)*a^2/c^5 + (2520*a^2*tan(1/2*f*x + 1/2*e)^8 - 840*a^
2*tan(1/2*f*x + 1/2*e)^6 + 441*a^2*tan(1/2*f*x + 1/2*e)^4 - 180*a^2*tan(1/
2*f*x + 1/2*e)^2 + 35*a^2)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

Mupad [B] (verification not implemented)

Time = 11.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\left(\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{36}-\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{7}+\frac{7\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{20}-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{3}+2\cos\left(\frac{e}{2}+ \right.\right.}{c^5\,f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

input
```
int((a + a/cos(e + f*x))^2/(c - c/cos(e + f*x))^5,x)
```

output
```
(a^2*(cos(e/2 + (f*x)/2)^9/36 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8
+ sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)
/2)^6)/3 + (7*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/20 - (cos(e/2 + (
f*x)/2)^7*sin(e/2 + (f*x)/2)^2)/7))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

$$\int \frac{(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\left(1260\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9fx+2520\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-840\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+441\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-180\tan\left(\frac{fx}{2} \right.\right.}{1260\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9c^5f}$$

input
```
int((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

output
```
(a**2*(1260*tan((e + f*x)/2)**9*f*x + 2520*tan((e + f*x)/2)**8 - 840*tan((
e + f*x)/2)**6 + 441*tan((e + f*x)/2)**4 - 180*tan((e + f*x)/2)**2 + 35))/
(1260*tan((e + f*x)/2)**9*c**5*f)
```

3.11 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\begin{aligned}\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx &= a^3c^5x-\frac{5a^3c^5\operatorname{arctanh}(\sin(e+fx))}{8f}-\frac{a^3c^5\tan(e+fx)}{f}\\ &\quad+\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^3c^5\tan^3(e+fx)}{3f}\\ &\quad-\frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{12f}-\frac{a^3c^5\tan^5(e+fx)}{5f}\\ &\quad+\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{3f}-\frac{a^3c^5\tan^7(e+fx)}{7f}\end{aligned}$$

output
```
a^3*c^5*x-5/8*a^3*c^5*arctanh(sin(f*x+e))/f-a^3*c^5*tan(f*x+e)/f+5/8*a^3*c
^5*sec(f*x+e)*tan(f*x+e)/f+1/3*a^3*c^5*tan(f*x+e)^3/f-5/12*a^3*c^5*sec(f*x
+e)*tan(f*x+e)^3/f-1/5*a^3*c^5*tan(f*x+e)^5/f+1/3*a^3*c^5*sec(f*x+e)*tan(f
*x+e)^5/f-1/7*a^3*c^5*tan(f*x+e)^7/f
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^5 \, dx$$
$$= \frac{a^3c^5\sec^7(e + fx)\,(14700(e + fx)\cos(e + fx) - 16800\text{arctanh}(\sin(e + fx))\cos^7(e + fx) + 8820e\cos(3(}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^3*c^5*Sec[e + f*x]^7*(14700*(e + f*x)*Cos[e + f*x] - 16800*ArcTanh[Sin[
e + f*x]]*Cos[e + f*x]^7 + 8820*e*Cos[3*(e + f*x)] + 8820*f*x*Cos[3*(e + f
*x)] + 2940*e*Cos[5*(e + f*x)] + 2940*f*x*Cos[5*(e + f*x)] + 420*e*Cos[7*(
e + f*x)] + 420*f*x*Cos[7*(e + f*x)] - 4200*Sin[e + f*x] + 2975*Sin[2*(e +
 f*x)] - 2184*Sin[3*(e + f*x)] + 980*Sin[4*(e + f*x)] - 2408*Sin[5*(e + f*
x)] + 1155*Sin[6*(e + f*x)] - 584*Sin[7*(e + f*x)]))/(26880*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e + fx) + a)^3(c - c\sec(e + fx))^5 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e + fx + \frac{\pi}{2}\right) + a\right)^3 \left(c - c\csc\left(e + fx + \frac{\pi}{2}\right)\right)^5 dx$$

$$\Big\downarrow \text{ 4392}$$

$$-a^3c^3\int (c - c\sec(e + fx))^2\tan^6(e + fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$-a^3c^3\int \cot\left(e+fx+\frac{\pi}{2}\right)^6\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \; 4374$$

$$-a^3c^3\int \left(c^2\tan^6(e+fx)+c^2\sec^2(e+fx)\tan^6(e+fx)-2c^2\sec(e+fx)\tan^6(e+fx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-a^3c^3\left(\frac{5c^2\text{arctanh}(\sin(e+fx))}{8f}+\frac{c^2\tan^7(e+fx)}{7f}+\frac{c^2\tan^5(e+fx)}{5f}-\frac{c^2\tan^3(e+fx)}{3f}+\frac{c^2\tan(e+fx)}{f}-\frac{c^2}{}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

output
```
-(a^3*c^3*(-(c^2*x) + (5*c^2*ArcTanh[Sin[e + f*x]])/(8*f) + (c^2*Tan[e + f
*x])/f - (5*c^2*Sec[e + f*x]*Tan[e + f*x])/(8*f) - (c^2*Tan[e + f*x]^3)/(3
*f) + (5*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(12*f) + (c^2*Tan[e + f*x]^5)/(5
*f) - (c^2*Sec[e + f*x]*Tan[e + f*x]^5)/(3*f) + (c^2*Tan[e + f*x]^7)/(7*f)
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.15

method	result
risch	$a^3c^5x - \frac{ic^5a^3\left(1155\,e^{13i(fx+e)}+1680\,e^{12i(fx+e)}+980\,e^{11i(fx+e)}+10080\,e^{10i(fx+e)}+2975\,e^{9i(fx+e)}+16240\,e^{8i(fx\ldots}\right.}{420f\left(e^{2i\ldots}\right.}$
parts	$a^3c^5x+\frac{c^5a^3\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{2a^3c^5\tan(fx+e)}{f}+\frac{3a^3c^5\sec(fx+e)\tan(fx+e)}{f}-\frac{6c^5a^3\left(-\left(-\ldots\right.\right.}{}$
parallelrisch	$-\frac{10c^5\left(\frac{(-\cos(7fx+7e)-7\cos(5fx+5e)-21\cos(3fx+3e)-35\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{(\cos(7fx+7e)+7\cos(5fx+5e)+2\ldots}{}\right.}{}$
derivativedivides	$\frac{c^5a^3(fx+e)-2c^5a^3\ln(\sec(fx+e)+\tan(fx+e))-2c^5a^3\tan(fx+e)+6c^5a^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+\ldots}{2}\right.}{}$
default	$\frac{c^5a^3(fx+e)-2c^5a^3\ln(\sec(fx+e)+\tan(fx+e))-2c^5a^3\tan(fx+e)+6c^5a^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+\ldots}{2}\right.}{}$
norman	$\frac{a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}-a^3c^5x+7a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-21a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+35a^3c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-35a^3c^5x\tan\left(\ldots\right.}{}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^5*x-1/420*I*c^5*a^3*(1155*exp(13*I*(f*x+e))+1680*exp(12*I*(f*x+e))+9
80*exp(11*I*(f*x+e))+10080*exp(10*I*(f*x+e))+2975*exp(9*I*(f*x+e))+16240*e
xp(8*I*(f*x+e))+24640*exp(6*I*(f*x+e))-2975*exp(5*I*(f*x+e))+14448*exp(4*I
*(f*x+e))-980*exp(3*I*(f*x+e))+6496*exp(2*I*(f*x+e))-1155*exp(I*(f*x+e))+1
168)/f/(exp(2*I*(f*x+e))+1)^7+5/8*c^5*a^3/f*ln(exp(I*(f*x+e))-I)-5/8*c^5*a
^3/f*ln(exp(I*(f*x+e))+I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= \frac{1680\,a^3c^5fx\cos{(fx+e)}^7 - 525\,a^3c^5\cos{(fx+e)}^7\log{(\sin{(fx+e)}+1)} + 525\,a^3c^5\cos{(fx+e)}^7\log{(-}}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/1680*(1680*a^3*c^5*f*x*cos(f*x + e)^7 - 525*a^3*c^5*cos(f*x + e)^7*log(s
in(f*x + e) + 1) + 525*a^3*c^5*cos(f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(
1168*a^3*c^5*cos(f*x + e)^6 - 1155*a^3*c^5*cos(f*x + e)^5 - 256*a^3*c^5*co
s(f*x + e)^4 + 910*a^3*c^5*cos(f*x + e)^3 - 192*a^3*c^5*cos(f*x + e)^2 - 2
80*a^3*c^5*cos(f*x + e) + 120*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= -a^3c^5\left(\int (-1)\,dx + \int 2\sec{(e+fx)}\,dx + \int 2\sec^2{(e+fx)}\,dx + \int \left(-6\sec^3{(e+fx)}\right)dx + \int 6\sec^5{(e+fx)}\,dx + \int \left(-2\sec^6{(e+fx)}\right)dx + \int \left(-2\sec^7{(e+fx)}\right)dx + \int \sec^8{(e+fx)}\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

output
```
-a**3*c**5*(Integral(-1, x) + Integral(2*sec(e + f*x), x) + Integral(2*sec
(e + f*x)**2, x) + Integral(-6*sec(e + f*x)**3, x) + Integral(6*sec(e + f*
x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7,
 x) + Integral(sec(e + f*x)**8, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(174) = 348$.

Time = 0.04 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.89

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^5 \, dx =$$

$$-\frac{48\left(5\tan\left(fx+e\right)^7 + 21\tan\left(fx+e\right)^5 + 35\tan\left(fx+e\right)^3 + 35\tan\left(fx+e\right)\right)a^3c^5 - 224\left(3\tan\left(fx+\right.\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
-1/1680*(48*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35
*tan(f*x + e))*a^3*c^5 - 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*ta
n(f*x + e))*a^3*c^5 - 1680*(f*x + e)*a^3*c^5 + 35*a^3*c^5*(2*(15*sin(f*x +
 e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x +
 e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x
+ e) - 1)) - 630*a^3*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x +
 e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) + 2520*a^3*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x
 + e) + 1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^5*log(sec(f*x + e) + tan(
f*x + e)) + 3360*a^3*c^5*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^5 \, dx$$

$$= \frac{840\left(fx+e\right)a^3c^5 - 525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 525\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(1365\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/840*(840*(f*x + e)*a^3*c^5 - 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) +
1)) + 525*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(1365*a^3*c^5*tan
(1/2*f*x + 1/2*e)^13 - 9660*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 29673*a^3*c^
5*tan(1/2*f*x + 1/2*e)^9 - 21216*a^3*c^5*tan(1/2*f*x + 1/2*e)^7 + 9863*a^3
*c^5*tan(1/2*f*x + 1/2*e)^5 - 2660*a^3*c^5*tan(1/2*f*x + 1/2*e)^3 + 315*a^
3*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 11.67 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= \frac{\frac{13\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{4}-23\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\frac{1413\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{20}-\frac{1768\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{1409\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right.}{60}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-2\right.}$$
$$+a^3\,c^5\,x-\frac{5\,a^3\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5,x)
```

output
```
((1409*a^3*c^5*tan(e/2 + (f*x)/2)^5)/60 - (19*a^3*c^5*tan(e/2 + (f*x)/2)^3
)/3 - (1768*a^3*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1413*a^3*c^5*tan(e/2 + (f*
x)/2)^9)/20 - 23*a^3*c^5*tan(e/2 + (f*x)/2)^11 + (13*a^3*c^5*tan(e/2 + (f*
x)/2)^13)/4 + (3*a^3*c^5*tan(e/2 + (f*x)/2))/4)/(f*(7*tan(e/2 + (f*x)/2)^2
 - 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/
2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x
)/2)^14 - 1)) + a^3*c^5*x - (5*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.17

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$= \frac{a^3c^5\left(525\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^6-1575\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\right.}{}$$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x)
```

output
```
(a**3*c**5*(525*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**6 - 1
575*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 1575*cos(e +
f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 - 525*cos(e + f*x)*log(tan(
(e + f*x)/2) - 1) - 525*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x
)**6 + 1575*cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**4 - 1575*
cos(e + f*x)*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 + 525*cos(e + f*x)*
log(tan((e + f*x)/2) + 1) + 840*cos(e + f*x)*sin(e + f*x)**6*f*x - 1155*co
s(e + f*x)*sin(e + f*x)**5 - 2520*cos(e + f*x)*sin(e + f*x)**4*f*x + 1400*
cos(e + f*x)*sin(e + f*x)**3 + 2520*cos(e + f*x)*sin(e + f*x)**2*f*x - 525
*cos(e + f*x)*sin(e + f*x) - 840*cos(e + f*x)*f*x - 1168*sin(e + f*x)**7 +
 3248*sin(e + f*x)**5 - 2800*sin(e + f*x)**3 + 840*sin(e + f*x)))/(840*cos
(e + f*x)*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x)**2 - 1))
```

3.12 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 26, antiderivative size = 132

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = a^3c^4x-\frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f}-\frac{a^3(16c^4-5c^4\sec(e+fx))\tan(e+fx)}{16f} + \frac{a^3(8c^4-5c^4\sec(e+fx))\tan^3(e+fx)}{24f}-\frac{a^3(6c^4-5c^4\sec(e+fx))\tan^5(e+fx)}{30f}$$

output
```
a^3*c^4*x-5/16*a^3*c^4*arctanh(sin(f*x+e))/f-1/16*a^3*(16*c^4-5*c^4*sec(f*
x+e))*tan(f*x+e)/f+1/24*a^3*(8*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^3/f-1/30*a
^3*(6*c^4-5*c^4*sec(f*x+e))*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.25

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = \frac{a^3c^4\sec^6(e+fx)\,(1200e+1200fx-1200\text{arctanh}(\sin(e+fx))\cos^6(e+fx)+1800(e+fx)\cos(2(e+$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^3*c^4*Sec[e + f*x]^6*(1200*e + 1200*f*x - 1200*ArcTanh[Sin[e + f*x]]*Co
s[e + f*x]^6 + 1800*(e + f*x)*Cos[2*(e + f*x)] + 720*e*Cos[4*(e + f*x)] +
720*f*x*Cos[4*(e + f*x)] + 120*e*Cos[6*(e + f*x)] + 120*f*x*Cos[6*(e + f*x
)] + 450*Sin[e + f*x] - 600*Sin[2*(e + f*x)] - 25*Sin[3*(e + f*x)] - 384*S
in[4*(e + f*x)] + 165*Sin[5*(e + f*x)] - 184*Sin[6*(e + f*x)]))/(3840*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4392, 3042, 4369, 3042, 4369, 27, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int (c-c\sec(e+fx))\tan^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \cot\left(e+fx+\frac{\pi}{2}\right)^6\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4369}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)(6c-5c\sec(e+fx))}{30f}-\frac{1}{6}\int (6c-5c\sec(e+fx))\tan^4(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)(6c-5c\sec(e+fx))}{30f}-\frac{1}{6}\int\cot\left(e+fx+\frac{\pi}{2}\right)^4\left(6c-5c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx\right)$$

$\big\downarrow$ 4369

$$-a^3c^3\left(\frac{1}{6}\left(\frac{1}{4}\int3(8c-5c\sec(e+fx))\tan^2(e+fx)dx-\frac{\tan^3(e+fx)(8c-5c\sec(e+fx))}{4f}\right)+\frac{\tan^5(e+fx)(6c$$

$\big\downarrow$ 27

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\int(8c-5c\sec(e+fx))\tan^2(e+fx)dx-\frac{\tan^3(e+fx)(8c-5c\sec(e+fx))}{4f}\right)+\frac{\tan^5(e+fx)(6c}{30}$$

$\big\downarrow$ 3042

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\int\cot\left(e+fx+\frac{\pi}{2}\right)^2\left(8c-5c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx-\frac{\tan^3(e+fx)(8c-5c\sec(e+fx))}{4f}\right)+\tan$$

$\big\downarrow$ 4369

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\tan(e+fx)(16c-5c\sec(e+fx))}{2f}-\frac{1}{2}\int(16c-5c\sec(e+fx))dx\right)-\frac{\tan^3(e+fx)(8c-5c\sec(}{4f}$$

$\big\downarrow$ 2009

$$-a^3c^3\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{5c\text{arctanh}(\sin(e+fx))}{f}-16cx\right)+\frac{\tan(e+fx)(16c-5c\sec(e+fx))}{2f}\right)-\frac{\tan^3(e+fx)(8c-}{4f}$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

output
```
-(a^3*c^3*(((6*c - 5*c*Sec[e + f*x])*Tan[e + f*x]^5)/(30*f) + (-1/4*((8*c
- 5*c*Sec[e + f*x])*Tan[e + f*x]^3)/f + (3*((-16*c*x + (5*c*ArcTanh[Sin[e
+ f*x]])/f)/2 + ((16*c - 5*c*Sec[e + f*x])*Tan[e + f*x])/(2*f)))/4)/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.56

method	result
risch	$a^3c^4x - \frac{ic^4a^3(165\,\mathrm{e}^{11i(fx+e)}+720\,\mathrm{e}^{10i(fx+e)}-25\,\mathrm{e}^{9i(fx+e)}+2160\,\mathrm{e}^{8i(fx+e)}+450\,\mathrm{e}^{7i(fx+e)}+3680\,\mathrm{e}^{6i(fx+e)}-450\,\mathrm{e}}{120f(\mathrm{e}^{2i(fx+e)}+1)^6}$
parallelrisch	$-\frac{5\left(\frac{\left(-5-\frac{15\cos(2fx+2e)}{2}-3\cos(4fx+4e)-\frac{\cos(6fx+6e)}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8}+\frac{\left(5+\frac{\cos(6fx+6e)}{2}+3\cos(4fx+4e)+\frac{15\cos(2fx}{2}\right.}{8}\right.}{}$
derivativedivides	$\frac{c^4a^3(fx+e)-c^4a^3\ln(\sec(fx+e)+\tan(fx+e))-3c^4a^3\tan(fx+e)+3c^4a^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
default	$\frac{c^4a^3(fx+e)-c^4a^3\ln(\sec(fx+e)+\tan(fx+e))-3c^4a^3\tan(fx+e)+3c^4a^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
parts	$a^3c^4x+\frac{c^4a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)}{f}-\frac{c^4a^3\ln(s}{}$
norman	$\frac{a^3c^4x+a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-6a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+15a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-20a^3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+15a^3c^4x\tan\left(\right.}{}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^4*x-1/120*I*c^4*a^3*(165*exp(11*I*(f*x+e))+720*exp(10*I*(f*x+e))-25*
exp(9*I*(f*x+e))+2160*exp(8*I*(f*x+e))+450*exp(7*I*(f*x+e))+3680*exp(6*I*(
f*x+e))-450*exp(5*I*(f*x+e))+3360*exp(4*I*(f*x+e))+25*exp(3*I*(f*x+e))+148
8*exp(2*I*(f*x+e))-165*exp(I*(f*x+e))+368)/f/(exp(2*I*(f*x+e))+1)^6-5/16*c
^4*a^3/f*ln(exp(I*(f*x+e))+I)+5/16*c^4*a^3/f*ln(exp(I*(f*x+e))-I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$=\frac{480\,a^3c^4fx\cos(fx+e)^6-75\,a^3c^4\cos(fx+e)^6\log(\sin(fx+e)+1)+75\,a^3c^4\cos(fx+e)^6\log(-\sin(}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/480*(480*a^3*c^4*f*x*cos(f*x + e)^6 - 75*a^3*c^4*cos(f*x + e)^6*log(sin(
f*x + e) + 1) + 75*a^3*c^4*cos(f*x + e)^6*log(-sin(f*x + e) + 1) - 2*(368*
a^3*c^4*cos(f*x + e)^5 - 165*a^3*c^4*cos(f*x + e)^4 - 176*a^3*c^4*cos(f*x
+ e)^3 + 130*a^3*c^4*cos(f*x + e)^2 + 48*a^3*c^4*cos(f*x + e) - 40*a^3*c^4
)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = a^3c^4\left(\int 1\,dx+\int(-\sec(e+fx))\,dx+\int\left(-3\sec^2(e+fx)\right)dx+\int 3\sec^3(e+fx)\,dx+\int 3\sec^4(e+fx)\,dx+\int\left(-3\sec^5(e+fx)\right)dx+\int\left(-\sec^6(e+fx)\right)dx+\int\sec^7(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

output
```
a**3*c**4*(Integral(1, x) + Integral(-sec(e + f*x), x) + Integral(-3*sec(e
 + f*x)**2, x) + Integral(3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)*
*4, x) + Integral(-3*sec(e + f*x)**5, x) + Integral(-sec(e + f*x)**6, x) +
 Integral(sec(e + f*x)**7, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(124) = 248$.

Time = 0.04 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.53

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = -\frac{32\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^4-480\left(\tan(fx+e)^3+3\tan(fx+e)\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/480*(32*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^
4 - 480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 - 480*(f*x + e)*a^3*c^4
+ 5*a^3*c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(
sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
 + e) + 1) + 15*log(sin(f*x + e) - 1)) - 90*a^3*c^4*(2*(3*sin(f*x + e)^3 -
 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
 e) + 1) + 3*log(sin(f*x + e) - 1)) + 360*a^3*c^4*(2*sin(f*x + e)/(sin(f*x
 + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^3*c^
4*log(sec(f*x + e) + tan(f*x + e)) + 1440*a^3*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$

$$= \frac{240\,(fx+e)a^3c^4 - 75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) + 75\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) + \frac{2\left(315\,a^3c^4 \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)*a^3*c^4 - 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1
)) + 75*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(315*a^3*c^4*tan(1/
2*f*x + 1/2*e)^11 - 1945*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 5118*a^3*c^4*tan
(1/2*f*x + 1/2*e)^7 - 3138*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 1095*a^3*c^4*t
an(1/2*f*x + 1/2*e)^3 - 165*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1
/2*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 11.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.72

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^4\,dx = a^3\,c^4\,x$$
$$+ \frac{\frac{21\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8} - \frac{389\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24} + \frac{853\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20} - \frac{523\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20} + \frac{73\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{8}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12} - 6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8} - 20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6} + 15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4} - 6\right.}$$
$$- \frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4,x)
```

output
```
a^3*c^4*x + ((73*a^3*c^4*tan(e/2 + (f*x)/2)^3)/8 - (523*a^3*c^4*tan(e/2 +
(f*x)/2)^5)/20 + (853*a^3*c^4*tan(e/2 + (f*x)/2)^7)/20 - (389*a^3*c^4*tan(
e/2 + (f*x)/2)^9)/24 + (21*a^3*c^4*tan(e/2 + (f*x)/2)^11)/8 - (11*a^3*c^4*
tan(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^
2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/
2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) - (5*a^3*c^4*atanh(tan(e/2 + (f*x)/2))
)/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.41

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^4\,dx$$
$$= \frac{a^3c^4\left(368\cos(fx+e)\sin(fx+e)^5 - 560\cos(fx+e)\sin(fx+e)^3 + 240\cos(fx+e)\sin(fx+e) + 75\right.}{}$$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x)
```

output
```
(a**3*c**4*(368*cos(e + f*x)*sin(e + f*x)**5 - 560*cos(e + f*x)*sin(e + f*
x)**3 + 240*cos(e + f*x)*sin(e + f*x) + 75*log(tan((e + f*x)/2) - 1)*sin(e
 + f*x)**6 - 225*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 225*log(tan((
e + f*x)/2) - 1)*sin(e + f*x)**2 - 75*log(tan((e + f*x)/2) - 1) - 75*log(t
an((e + f*x)/2) + 1)*sin(e + f*x)**6 + 225*log(tan((e + f*x)/2) + 1)*sin(e
 + f*x)**4 - 225*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 + 75*log(tan((e
 + f*x)/2) + 1) + 240*sin(e + f*x)**6*f*x - 165*sin(e + f*x)**5 - 720*sin(
e + f*x)**4*f*x + 200*sin(e + f*x)**3 + 720*sin(e + f*x)**2*f*x - 75*sin(e
 + f*x) - 240*f*x))/(240*f*(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e
+ f*x)**2 - 1))
```

3.13 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 68

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = a^3c^3x-\frac{a^3c^3\tan(e+fx)}{f}+\frac{a^3c^3\tan^3(e+fx)}{3f}-\frac{a^3c^3\tan^5(e+fx)}{5f}$$

output
```
a^3*c^3*x-a^3*c^3*tan(f*x+e)/f+1/3*a^3*c^3*tan(f*x+e)^3/f-1/5*a^3*c^3*tan(
f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -a^3c^3\left(-\frac{\arctan(\tan(e+fx))}{f}+\frac{\tan(e+fx)}{f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan^5(e+fx)}{5f}\right)$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

output
```
-(a^3*c^3*(-(ArcTan[Tan[e + f*x]]/f) + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*
f) + Tan[e + f*x]^5/(5*f)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \tan^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \tan(e+fx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)}{5f}-\int \tan^4(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)}{5f}-\int \tan(e+fx)^4dx\right)$$

$$\Big\downarrow \text{3954}$$

$$-a^3c^3\left(\int \tan^2(e+fx)dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\left(\int \tan(e+fx)^2dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \; 3954$$

$$-a^3c^3\left(-\int 1dx+\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}\right)$$

$$\Big\downarrow \; 24$$

$$-a^3c^3\left(\frac{\tan^5(e+fx)}{5f}-\frac{\tan^3(e+fx)}{3f}+\frac{\tan(e+fx)}{f}-x\right)$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

output
```
-(a^3*c^3*(-x + Tan[e + f*x]/f - Tan[e + f*x]^3/(3*f) + Tan[e + f*x]^5/(5*
f)))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result
risch	$a^3c^3x - \frac{2ic^3a^3(45\,\mathrm{e}^{8i(fx+e)}+90\,\mathrm{e}^{6i(fx+e)}+140\,\mathrm{e}^{4i(fx+e)}+70\,\mathrm{e}^{2i(fx+e)}+23)}{15f(\mathrm{e}^{2i(fx+e)}+1)^5}$
derivativedivides	$\frac{c^3a^3(fx+e)-3c^3a^3\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(f}{f}$
default	$\frac{c^3a^3(fx+e)-3c^3a^3\tan(fx+e)-3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(f}{f}$
parts	$a^3c^3x - \frac{3a^3c^3\tan(fx+e)}{f} - \frac{3c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f} + \frac{c^3a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan}{f}$
parallelrisch	$\frac{c^3a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}xf-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8xf+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6xf-\frac{32\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3}-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}$
norman	$\frac{a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-a^3c^3x+5a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-10a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10a^3c^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-5a^3c^3x\tan\left(\frac{f}{}\right.}{\left(\tan\left(\frac{fx}{2}+\right.\right.}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^3*x-2/15*I*c^3*a^3*(45*exp(8*I*(f*x+e))+90*exp(6*I*(f*x+e))+140*exp(
4*I*(f*x+e))+70*exp(2*I*(f*x+e))+23)/f/(exp(2*I*(f*x+e))+1)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = \frac{15\,a^3c^3fx\cos(fx+e)^5-\left(23\,a^3c^3\cos(fx+e)^4-11\,a^3c^3\cos(fx+e)^2+3\,a^3c^3\right)\sin(fx+e)}{15\,f\cos(fx+e)^5}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^3*c^3*f*x*cos(f*x + e)^5 - (23*a^3*c^3*cos(f*x + e)^4 - 11*a^3*
c^3*cos(f*x + e)^2 + 3*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$= -a^3c^3\left(\int(-1)\,dx+\int 3\sec^2(e+fx)\,dx+\int\left(-3\sec^4(e+fx)\right)\,dx + \int \sec^6(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

output
```
-a**3*c**3*(Integral(-1, x) + Integral(3*sec(e + f*x)**2, x) + Integral(-3
*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx =$$
$$-\frac{\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^3-15\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c}{15\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^3 -
15*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 - 15*(f*x + e)*a^3*c^3 + 45*a
^3*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\frac{3\,a^3c^3\tan\left(fx+e\right)^5-5\,a^3c^3\tan\left(fx+e\right)^3-15\left(fx+e\right)a^3c^3+15\,a^3c^3\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/15*(3*a^3*c^3*tan(f*x + e)^5 - 5*a^3*c^3*tan(f*x + e)^3 - 15*(f*x + e)*
a^3*c^3 + 15*a^3*c^3*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = a^3\,c^3\,x + \frac{2\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9-\frac{32\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}+\frac{356\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}-\frac{32\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+2\,a^3\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^5}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3,x)
```

output
```
a^3*c^3*x + ((356*a^3*c^3*tan(e/2 + (f*x)/2)^5)/15 - (32*a^3*c^3*tan(e/2 +
 (f*x)/2)^3)/3 - (32*a^3*c^3*tan(e/2 + (f*x)/2)^7)/3 + 2*a^3*c^3*tan(e/2 +
 (f*x)/2)^9 + 2*a^3*c^3*tan(e/2 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^2 - 1)^
5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$= \frac{a^3c^3\left(15\cos\left(fx+e\right)\sin\left(fx+e\right)^4 fx-30\cos\left(fx+e\right)\sin\left(fx+e\right)^2 fx+15\cos\left(fx+e\right)fx-23\sin\left(f\right.\right.}{15\cos\left(fx+e\right)f\left(\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1\right)}$$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x)
```

output
```
(a**3*c**3*(15*cos(e + f*x)*sin(e + f*x)**4*f*x - 30*cos(e + f*x)*sin(e +
f*x)**2*f*x + 15*cos(e + f*x)*f*x - 23*sin(e + f*x)**5 + 35*sin(e + f*x)**
3 - 15*sin(e + f*x)))/(15*cos(e + f*x)*f*(sin(e + f*x)**4 - 2*sin(e + f*x)
**2 + 1))
```

3.14 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = a^3c^2x+\frac{3a^3c^2\text{arctanh}(\sin(e+fx))}{8f}-\frac{c^2(8a^3+3a^3\sec(e+fx))\tan(e+fx)}{8f}+\frac{c^2(4a^3+3a^3\sec(e+fx))\tan^3(e+fx)}{12f}$$

output
```
a^3*c^2*x+3/8*a^3*c^2*arctanh(sin(f*x+e))/f-1/8*c^2*(8*a^3+3*a^3*sec(f*x+e
))*tan(f*x+e)/f+1/12*c^2*(4*a^3+3*a^3*sec(f*x+e))*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{a^3c^2\sec^4(e+fx)\,(72e+72fx+72\text{arctanh}(\sin(e+fx))\cos^4(e+fx)+96(e+fx)\cos(2(e+fx))+24}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^3*c^2*Sec[e + f*x]^4*(72*e + 72*f*x + 72*ArcTanh[Sin[e + f*x]]*Cos[e +
f*x]^4 + 96*(e + f*x)*Cos[2*(e + f*x)] + 24*e*Cos[4*(e + f*x)] + 24*f*x*Co
s[4*(e + f*x)] + 18*Sin[e + f*x] - 32*Sin[2*(e + f*x)] - 30*Sin[3*(e + f*x
)] - 32*Sin[4*(e + f*x)]))/(192*f)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4392, 3042, 4369, 3042, 4369, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int (\sec(e+fx)a+a)\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx$$

$$\Big\downarrow \text{4369}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(3a\sec(e+fx)+4a)}{12f}-\frac{1}{4}\int (3\sec(e+fx)a+4a)\tan^2(e+fx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{\tan^3(e+fx)(3a\sec(e+fx)+4a)}{12f}-\frac{1}{4}\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(3\csc\left(e+fx+\frac{\pi}{2}\right)a+4a\right)dx\right)$$

$$\Big\downarrow \text{4369}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\int(3\sec(e+fx)a+8a)dx-\frac{\tan(e+fx)(3a\sec(e+fx)+8a)}{2f}\right)+\frac{\tan^3(e+fx)(3a\sec(e+fx)+4}{12f}\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2c^2\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{3a\text{arctanh}(\sin(e+fx))}{f}+8ax\right)-\frac{\tan(e+fx)(3a\sec(e+fx)+8a)}{2f}\right)+\frac{\tan^3(e+fx)(3a\sec(e+}{12f}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

output
```
a^2*c^2*(((4*a + 3*a*Sec[e + f*x])*Tan[e + f*x]^3)/(12*f) + ((8*a*x + (3*a
*ArcTanh[Sin[e + f*x]])/f)/2 - ((8*a + 3*a*Sec[e + f*x])*Tan[e + f*x])/(2*
f))/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4369
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Simp[e^2/m   Int[(e*Cot[c + d*x])^(m - 2)*(
a*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m
, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

method	result
parts	$a^3c^2x - \frac{c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f} + \frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+}{8}\right.}{f}$
risch	$a^3c^2x + \frac{ic^2a^3\left(15\,\mathrm{e}^{7i(fx+e)}-48\,\mathrm{e}^{6i(fx+e)}-9\,\mathrm{e}^{5i(fx+e)}-96\,\mathrm{e}^{4i(fx+e)}+9\,\mathrm{e}^{3i(fx+e)}-80\,\mathrm{e}^{2i(fx+e)}-15\,\mathrm{e}^{i(fx+e)}-32\right)}{12f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}$
derivativedivides	$\frac{c^2a^3(fx+e)+c^2a^3\ln(\sec(fx+e)+\tan(fx+e))-2c^2a^3\tan(fx+e)-2c^2a^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
default	$\frac{c^2a^3(fx+e)+c^2a^3\ln(\sec(fx+e)+\tan(fx+e))-2c^2a^3\tan(fx+e)-2c^2a^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e}{2}\right.}{}$
parallelrisch	$\frac{a^3c^2\left(-9(\cos(4fx+4e)+4\cos(2fx+2e)+3)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-9(-3-\cos(4fx+4e)-4\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\right.\right.\right.}{24f(\cos(4fx+4e}$
norman	$\frac{a^3c^2x+a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-4a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+6a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-4a^3c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-\frac{11c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^2*x-c^2*a^3/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+c^2*a^3/f*(-(-1/4*s
ec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-2*c^
2*a^3/f*tan(f*x+e)-c^2*a^3/f*tan(f*x+e)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{48\,a^3c^2fx\cos(fx+e)^4+9\,a^3c^2\cos(fx+e)^4\log(\sin(fx+e)+1)-9\,a^3c^2\cos(fx+e)^4\log(-\sin(fx}{48\,f\cos(}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/48*(48*a^3*c^2*f*x*cos(f*x + e)^4 + 9*a^3*c^2*cos(f*x + e)^4*log(sin(f*x
 + e) + 1) - 9*a^3*c^2*cos(f*x + e)^4*log(-sin(f*x + e) + 1) - 2*(32*a^3*c
^2*cos(f*x + e)^3 + 15*a^3*c^2*cos(f*x + e)^2 - 8*a^3*c^2*cos(f*x + e) - 6
*a^3*c^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= a^3c^2\left(\int 1\,dx+\int \sec(e+fx)\,dx+\int \left(-2\sec^2(e+fx)\right)\,dx+\int \left(-2\sec^3(e+fx)\right)\,dx+\int \sec^4(e+fx)\,dx+\int \sec^5(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

output
```
a**3*c**2*(Integral(1, x) + Integral(sec(e + f*x), x) + Integral(-2*sec(e
+ f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4
, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(91) = 182$.

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.09

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{16\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c^2+48(fx+e)a^3c^2-3a^3c^2\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 48*(f*x + e)*a^3*c^2
- 3*a^3*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin
(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 24
*a^3*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) + 48*a^3*c^2*log(sec(f*x + e) + tan(f*x + e)) - 96*a^
3*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{24\,(fx+e)a^3c^2+9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-9\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)+\frac{2\left(15\,a^3c^2\tan\left(\cdots\right.\right.}{\cdots}}{24\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/24*(24*(f*x + e)*a^3*c^2 + 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))
- 9*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) - 1)) + 2*(15*a^3*c^2*tan(1/2*f*x
 + 1/2*e)^7 - 71*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 137*a^3*c^2*tan(1/2*f*x
+ 1/2*e)^3 - 33*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)
^4)/f
```

Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{\frac{5\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{71\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{137\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}-\frac{11\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$
$$+a^3\,c^2\,x+\frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2,x)
```

output
```
((137*a^3*c^2*tan(e/2 + (f*x)/2)^3)/12 - (71*a^3*c^2*tan(e/2 + (f*x)/2)^5)
/12 + (5*a^3*c^2*tan(e/2 + (f*x)/2)^7)/4 - (11*a^3*c^2*tan(e/2 + (f*x)/2))
/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x
)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + a^3*c^2*x + (3*a^3*c^2*atanh(tan(e/2
 + (f*x)/2)))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.33

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{a^3c^2\left(32\cos(fx+e)\sin(fx+e)^3-24\cos(fx+e)\sin(fx+e)-9\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^4\right.}{}$$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x)
```

output
```
(a**3*c**2*(32*cos(e + f*x)*sin(e + f*x)**3 - 24*cos(e + f*x)*sin(e + f*x)
 - 9*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**4 + 18*log(tan((e + f*x)/2) -
 1)*sin(e + f*x)**2 - 9*log(tan((e + f*x)/2) - 1) + 9*log(tan((e + f*x)/2)
 + 1)*sin(e + f*x)**4 - 18*log(tan((e + f*x)/2) + 1)*sin(e + f*x)**2 + 9*l
og(tan((e + f*x)/2) + 1) + 24*sin(e + f*x)**4*f*x + 15*sin(e + f*x)**3 - 4
8*sin(e + f*x)**2*f*x - 9*sin(e + f*x) + 24*f*x))/(24*f*(sin(e + f*x)**4 -
 2*sin(e + f*x)**2 + 1))
```

3.15 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = a^3cx+\frac{a^3c\text{arctanh}(\sin(e+fx))}{f}-\frac{a^3c\tan(e+fx)}{f} - \frac{a^3c\sec(e+fx)\tan(e+fx)}{f}-\frac{a^3c\tan^3(e+fx)}{3f}$$

output
```
a^3*c*x+a^3*c*arctanh(sin(f*x+e))/f-a^3*c*tan(f*x+e)/f-a^3*c*sec(f*x+e)*ta
n(f*x+e)/f-1/3*a^3*c*tan(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{a^3c\sec^3(e+fx)\,(9(e+fx)\cos(e+fx)+12\text{arctanh}(\sin(e+fx))\cos^3(e+fx)+3e\cos(3(e+fx))+3}{12f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output
```
(a^3*c*Sec[e + f*x]^3*(9*(e + f*x)*Cos[e + f*x] + 12*ArcTanh[Sin[e + f*x]]
*Cos[e + f*x]^3 + 3*e*Cos[3*(e + f*x)] + 3*f*x*Cos[3*(e + f*x)] - 6*Sin[e
+ f*x] - 6*Sin[2*(e + f*x)] - 2*Sin[3*(e + f*x)]))/(12*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 4392}$$

$$-ac\int (\sec(e+fx)a+a)^2\tan^2(e+fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$$\Big\downarrow \text{ 4374}$$

$$-ac\int \left(a^2\tan^2(e+fx)+a^2\sec^2(e+fx)\tan^2(e+fx)+2a^2\sec(e+fx)\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-ac\left(-\frac{a^2\text{arctanh}(\sin(e+fx))}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan(e+fx)\sec(e+fx)}{f}-a^2x\right)$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

output
```
-(a*c*(-(a^2*x) - (a^2*ArcTanh[Sin[e + f*x]])/f + (a^2*Tan[e + f*x])/f + (
a^2*Sec[e + f*x]*Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
parts	$a^3cx+\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^3c\sec(fx+e)\tan(fx+e)}{f}+\frac{a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
derivativedivides	$\frac{a^3c(fx+e)+2a^3c\ln(\sec(fx+e)+\tan(fx+e))-2a^3c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^3c\left(-\frac{2}{3}-\frac{\sec(f}{}\right.}{f}$
default	$\frac{a^3c(fx+e)+2a^3c\ln(\sec(fx+e)+\tan(fx+e))-2a^3c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^3c\left(-\frac{2}{3}-\frac{\sec(f}{}\right.}{f}$
risch	$a^3cx+\frac{2ia^3c\left(3\,\mathrm{e}^{5i(fx+e)}-6\,\mathrm{e}^{2i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}-2\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}+\frac{a^3c\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{f}-\frac{a^3c\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{f}$
parallelrisch	$-\frac{2c\left(\frac{3(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}-\frac{9fx\cos(fx+e)}{2}\right.}{3f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{a^3cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-a^3cx+\frac{4a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{4a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+3a^3cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3a^3cx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}+\frac{a^3c\,l}{}$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+a^3*c/f*ln(sec(f*x+e)+tan(f*x+e))-a^3*c*sec(f*x+e)*tan(f*x+e)/f+a^
3*c/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{6\,a^3cfx\cos(fx+e)^3+3\,a^3c\cos(fx+e)^3\log(\sin(fx+e)+1)-3\,a^3c\cos(fx+e)^3\log(-\sin(fx+e}{6\,f\cos(fx+e)^3}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/6*(6*a^3*c*f*x*cos(f*x + e)^3 + 3*a^3*c*cos(f*x + e)^3*log(sin(f*x + e)
+ 1) - 3*a^3*c*cos(f*x + e)^3*log(-sin(f*x + e) + 1) - 2*(2*a^3*c*cos(f*x
+ e)^2 + 3*a^3*c*cos(f*x + e) + a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -a^3c\left(\int(-1)\,dx + \int(-2\sec(e+fx))\,dx + \int 2\sec^3(e+fx)\,dx + \int\sec^4(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

output
```
-a**3*c*(Integral(-1, x) + Integral(-2*sec(e + f*x), x) + Integral(2*sec(e
 + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\frac{2\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c-6(fx+e)a^3c-3a^3c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\right.}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-1/6*(2*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 6*(f*x + e)*a^3*c - 3*a^
3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin
(f*x + e) - 1)) - 12*a^3*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))\,dx$$

$$= \frac{3\,(fx+e)a^3c + 3\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) - 3\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) - \frac{4\left(a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}{\left(\tan\left(\frac{1}{2}\,f\right.\right.}}{3\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*a^3*c + 3*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^
3*c*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 4*(a^3*c*tan(1/2*f*x + 1/2*e)^3 -
 3*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 10.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))\,dx$$

$$= \frac{4\,a^3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) - \frac{4\,a^3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6 - 3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 + 3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - 1\right)} + a^3\,c\,x + \frac{2\,a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)),x)
```

output
```
(4*a^3*c*tan(e/2 + (f*x)/2) - (4*a^3*c*tan(e/2 + (f*x)/2)^3)/3)/(f*(3*tan(
e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1)) + a
^3*c*x + (2*a^3*c*atanh(tan(e/2 + (f*x)/2)))/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.42

$$\int (a + a\sec(e+fx))^3(c - c\sec(e+fx))\,dx$$
$$= \frac{a^3c\left(-3\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin(fx+e)^2 + 3\cos(fx+e)\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right) + 3\cos(\right.}{}$$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

output
```
(a**3*c*( - 3*cos(e + f*x)*log(tan((e + f*x)/2) - 1)*sin(e + f*x)**2 + 3*c
os(e + f*x)*log(tan((e + f*x)/2) - 1) + 3*cos(e + f*x)*log(tan((e + f*x)/2
) + 1)*sin(e + f*x)**2 - 3*cos(e + f*x)*log(tan((e + f*x)/2) + 1) + 3*cos(
e + f*x)*sin(e + f*x)**2*f*x + 3*cos(e + f*x)*sin(e + f*x) - 3*cos(e + f*x
)*f*x - 2*sin(e + f*x)**3 + 3*sin(e + f*x)))/(3*cos(e + f*x)*f*(sin(e + f*
x)**2 - 1))
```

3.16 $\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx=\frac{a^3x}{c}-\frac{4a^3\operatorname{arctanh}(\sin(e+fx))}{cf}+\frac{8a^3\cot(e+fx)}{cf}+\frac{8a^3\csc(e+fx)}{cf}-\frac{a^3\tan(e+fx)}{cf}$$

output
```
a^3*x/c-4*a^3*arctanh(sin(f*x+e))/c/f+8*a^3*cot(f*x+e)/c/f+8*a^3*csc(f*x+e
)/c/f-a^3*tan(f*x+e)/c/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.38 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.53

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{\tan(e+fx)\left(-8\sqrt{2}a^3\sqrt{c}\cos^6\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sec^4(e+\cdots}{\cdots}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

output
```
-1/5*(Tan[e + f*x]*(-8*Sqrt[2]*a^3*Sqrt[c]*Cos[(e + f*x)/2]^6*Hypergeometr
ic2F1[3/2, 5/2, 7/2, (1 + Sec[e + f*x])/2]*Sec[e + f*x]^4*Sin[(e + f*x)/2]
^2 + 5*a^(5/2)*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 + Sec[e + f*x
]) + ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e + f*x]*Sqr
t[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^2) - ArcTanh[Sqrt[-(a*c*Tan[e + f
*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sqrt[-(a*c*Tan[e + f*x]^
2)])))/(c^(3/2)*f*(1 - Sec[e + f*x])^(3/2)*(1 + Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^4dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^4}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(\cot^2(e+fx)a^4+6\csc^2(e+fx)a^4+\csc^2(e+fx)\sec^2(e+fx)a^4+4\cot(e+fx)\csc(e+fx)a^4+4\csc^2(e\ldots\right.}{ac}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{4a^4\text{arctanh}(\sin(e+fx))}{f}+\frac{a^4\tan(e+fx)}{f}-\frac{8a^4\cot(e+fx)}{f}-\frac{8a^4\csc(e+fx)}{f}+a^4(-x)}{ac}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x]),x]
```

output
```
-((-(a^4*x) + (4*a^4*ArcTanh[Sin[e + f*x]])/f - (8*a^4*Cot[e + f*x])/f - (
8*a^4*Csc[e + f*x])/f + (a^4*Tan[e + f*x])/f)/(a*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{fc}$
default	$\frac{8a^3\left(\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{fc}$
parallelrisch	$\frac{a^3\left(fx\cos(fx+e)+9\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)+4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)-c\right.}{cf\cos(fx+e)}$
risch	$\frac{a^3x}{c}+\frac{2ia^3\left(8\,\mathrm{e}^{2i(fx+e)}-\mathrm{e}^{i(fx+e)}+9\right)}{fc\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)}-\frac{4a^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{cf}+\frac{4a^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{cf}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{c}+\frac{8a^3}{cf}-\frac{18a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}+\frac{10a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}-\frac{2a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{4a^3\ln\left(\tan\left(\right.\right.}{c}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
8/f*a^3/c*(1/8/(tan(1/2*f*x+1/2*e)-1)+1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/8/(ta
n(1/2*f*x+1/2*e)+1)-1/2*ln(tan(1/2*f*x+1/2*e)+1)+1/4*arctan(tan(1/2*f*x+1/
2*e))+1/tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$$
$$=\frac{a^3fx\cos(fx+e)\sin(fx+e)-2\,a^3\cos(fx+e)\log(\sin(fx+e)+1)\sin(fx+e)+2\,a^3\cos(fx+e)\,\mathrm{l}}{cf\cos(fx+e)\sin(fx+}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
(a^3*f*x*cos(f*x + e)*sin(f*x + e) - 2*a^3*cos(f*x + e)*log(sin(f*x + e) +
 1)*sin(f*x + e) + 2*a^3*cos(f*x + e)*log(-sin(f*x + e) + 1)*sin(f*x + e)
+ 9*a^3*cos(f*x + e)^2 + 8*a^3*cos(f*x + e) - a^3)/(c*f*cos(f*x + e)*sin(f
*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{3\sec^2(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{\sec^3(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{1}{\sec(e+fx)-1}\,dx\right)}{c}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

output
```
-a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e +
 f*x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) -
 1), x) + Integral(1/(sec(e + f*x) - 1), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(78) = 156$.

Time = 0.12 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.51

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)-a^3\left(\frac{2\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}+\frac{\cos(fx+e)}{c\sin(fx+}\right.}{f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-(a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*
x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(c
os(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) - a^
3*(2*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c + (cos(f*x + e) + 1)/(c*sin
(f*x + e))) + 3*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e)))
- 3*a^3*(cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{\frac{(fx+e)a^3}{c} - \frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c} + \frac{4\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c} + \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)c}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
((f*x + e)*a^3/c - 4*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c + 4*a^3*log(
abs(tan(1/2*f*x + 1/2*e) - 1))/c + 2*(5*a^3*tan(1/2*f*x + 1/2*e)^2 - 4*a^3
)/((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int \frac{(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{a^3\,x}{c} - \frac{10\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-8\,a^3}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)} - \frac{8\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x)),x)
```

output
```
(a^3*x)/c - (10*a^3*tan(e/2 + (f*x)/2)^2 - 8*a^3)/(f*(c*tan(e/2 + (f*x)/2)
 - c*tan(e/2 + (f*x)/2)^3)) - (8*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.23

$$\int \frac{(a + a\sec(e + fx))^3}{c - c\sec(e + fx)}\,dx$$
$$= \frac{a^3\Big(4\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 - 4\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\big)\tan\left(\frac{fx}{2}+\frac{e}{2}\right) - 4\log\big(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\ \cdots}{\tan\left(\frac{fx}{2}+\ \cdots\right.}$$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

output
```
(a**3*(4*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**3 - 4*log(tan((e + f*
x)/2) - 1)*tan((e + f*x)/2) - 4*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)
**3 + 4*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2) + tan((e + f*x)/2)**3*f
*x + 10*tan((e + f*x)/2)**2 - tan((e + f*x)/2)*f*x - 8))/(tan((e + f*x)/2)
*c*f*(tan((e + f*x)/2)**2 - 1))
```

3.17 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3x}{c^2}+\frac{a^3\operatorname{arctanh}(\sin(e+fx))}{c^2f} - \frac{8a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{3c^2f(1-\sec(e+fx))}$$

output
```
a^3*x/c^2+a^3*arctanh(sin(f*x+e))/c^2/f-8/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*
x+e))^2+4/3*a^3*tan(f*x+e)/c^2/f/(1-sec(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(88) = 176.

Time = 2.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{a^{5/2}\tan(e+fx)\left(4\sqrt{c}\left(\sqrt{a}\sqrt{1-\sec(e+fx)}(1+\sec(e+fx))^2+6\arcsin\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec^2(e+\right.\right.}{3c}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

output
```
-1/3*(a^(5/2)*Tan[e + f*x]*(4*Sqrt[c]*(Sqrt[a]*Sqrt[1 - Sec[e + f*x]]*(1 +
 Sec[e + f*x])^2 + 6*ArcSin[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*
Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^4) + 6*ArcTanh[
Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[1 - Sec[e + f*x]]*Sec[
e + f*x]*Sin[(e + f*x)/2]^2*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(c^(5/2)*f*(1 -
Sec[e + f*x])^(5/2)*(1 + Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^2}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^2}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^2}+\frac{a^3}{(1-\sec(e+fx))^2}\right)dx}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3\text{arctanh}(\sin(e+fx))}{f}+\frac{4a^3\tan(e+fx)}{3f(1-\sec(e+fx))}-\frac{8a^3\tan(e+fx)}{3f(1-\sec(e+fx))^2}+a^3x}{c^2}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^2,x]
```

output
```
(a^3*x + (a^3*ArcTanh[Sin[e + f*x]])/f - (8*a^3*Tan[e + f*x])/(3*f*(1 - Se
c[e + f*x])^2) + (4*a^3*Tan[e + f*x])/(3*f*(1 - Sec[e + f*x])))/c^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
 + f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result
parallelrisch	$-\frac{a^3\left(4\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3-3fx-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\right)}{3c^2f}$
derivativedivides	$\frac{4a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}\right)}{f\,c^2}$
default	$\frac{4a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}\right)}{f\,c^2}$
risch	$\frac{a^3x}{c^2}+\frac{8ia^3\left(3\,\mathrm{e}^{2i(fx+e)}+1\right)}{3f\,c^2\left(\mathrm{e}^{i(fx+e)}-1\right)^3}+\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{c^2f}-\frac{a^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{c^2f}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{c}-\frac{4a^3}{3cf}+\frac{8a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3cf}-\frac{4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3cf}-\frac{2a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{a^3\ln\left(\tan\left(\frac{f}{2}\right.\right.}{c^2f}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3*(4*cot(1/2*f*x+1/2*e)^3-3*f*x-3*ln(tan(1/2*f*x+1/2*e)+1)+3*ln(tan
(1/2*f*x+1/2*e)-1))/c^2/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{8\,a^3\cos\left(fx+e\right)^2+16\,a^3\cos\left(fx+e\right)+8\,a^3+3\left(a^3\cos\left(fx+e\right)-a^3\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right.}{6\left(c^2f\cos\left(fx+e\right.\right.}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/6*(8*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 8*a^3 + 3*(a^3*cos(f*x +
 e) - a^3)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*(a^3*cos(f*x + e) - a^3)
*log(-sin(f*x + e) + 1)*sin(f*x + e) + 6*(a^3*f*x*cos(f*x + e) - a^3*f*x)*
sin(f*x + e))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\sec^2\left(e+fx\right)}\right.}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

output
```
a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) +
 Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + I
ntegral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(1/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(80) = 160$.

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.11

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^2}\,dx$$
$$= \frac{a^3\left(\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} + \frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right) + a^3\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2} - \right.}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(a^3*(12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2 + (9*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) +
a^3*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(
cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 3*a^3*(3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + 3*a^3*(3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x
+ e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^2}\,dx$$
$$= \frac{\frac{3(fx+e)a^3}{c^2} + \frac{3a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c^2} - \frac{3a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c^2} - \frac{4a^3}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}}{3f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*a^3/c^2 + 3*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 -
3*a^3*log(abs(tan(1/2*f*x + 1/2*e) - 1))/c^2 - 4*a^3/(c^2*tan(1/2*f*x + 1/
2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\,x}{c^2} + \frac{a^3\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right) - \frac{4\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}\right)}{c^2\,f}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^2,x)
```

output
```
(a^3*x)/c^2 + (a^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*cot(e/2 + (f*x)/2)^3)
/3))/(c^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\left(-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 + 3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 + 3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 f}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 c^2 f}$$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

output
```
(a**3*( - 3*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**3 + 3*log(tan((e +
 f*x)/2) + 1)*tan((e + f*x)/2)**3 + 3*tan((e + f*x)/2)**3*f*x - 4))/(3*tan
((e + f*x)/2)**3*c**2*f)
```

3.18 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} dx = \frac{a^3x}{c^3} - \frac{8a^3\tan(e+fx)}{5c^3f(1-\sec(e+fx))^3} + \frac{4a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))^2} - \frac{26a^3\tan(e+fx)}{15c^3f(1-\sec(e+fx))}$$

output
```
a^3*x/c^3-8/5*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))^3+4/15*a^3*tan(f*x+e)/c^
3/f/(1-sec(f*x+e))^2-26/15*a^3*tan(f*x+e)/c^3/f/(1-sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} dx = \frac{2a^3\cot^5\left(\frac{e}{2}+\frac{fx}{2}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{5c^3f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^3*Cot[e/2 + (f*x)/2]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e/2 + (f
*x)/2]^2])/(5*c^3*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4391}$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^3}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^3}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^3}+\frac{a^3}{(1-\sec(e+fx))^3}\right)dx}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{26a^3\tan(e+fx)}{15f(1-\sec(e+fx))}+\frac{4a^3\tan(e+fx)}{15f(1-\sec(e+fx))^2}-\frac{8a^3\tan(e+fx)}{5f(1-\sec(e+fx))^3}+a^3x}{c^3}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^3,x]
```

output
```
(a^3*x - (8*a^3*Tan[e + f*x])/(5*f*(1 - Sec[e + f*x])^3) + (4*a^3*Tan[e +
f*x])/(15*f*(1 - Sec[e + f*x])^2) - (26*a^3*Tan[e + f*x])/(15*f*(1 - Sec[e
 + f*x])))/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{a^{3}\left(6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-10\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+15fx+30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15c^{3}f}$
derivativedivides	$\frac{2a^{3}\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^{3}}$
default	$\frac{2a^{3}\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,c^{3}}$
risch	$\frac{a^{3}x}{c^{3}}+\frac{4ia^{3}\left(45\,{\rm e}^{4i(fx+e)}-90\,{\rm e}^{3i(fx+e)}+140\,{\rm e}^{2i(fx+e)}-70\,{\rm e}^{i(fx+e)}+23\right)}{15f\,c^{3}\left({\rm e}^{i(fx+e)}-1\right)^{5}}$
norman	$\frac{\frac{a^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{c}+\frac{a^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{c}+\frac{2a^{3}}{5cf}-\frac{22a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{15cf}+\frac{56a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{15cf}-\frac{14a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{3cf}+\frac{2a^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*a^3*(6*cot(1/2*f*x+1/2*e)^5-10*cot(1/2*f*x+1/2*e)^3+15*f*x+30*cot(1/2
*f*x+1/2*e))/c^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{46\,a^3\cos\left(fx+e\right)^3-2\,a^3\cos\left(fx+e\right)^2-22\,a^3\cos\left(fx+e\right)+26\,a^3+15\,\left(a^3fx\cos\left(fx+e\right)^2-2\,a^3fx\cos\cdots\right.}{15\,\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(46*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e)^2 - 22*a^3*cos(f*x + e) +
 26*a^3 + 15*(a^3*f*x*cos(f*x + e)^2 - 2*a^3*f*x*cos(f*x + e) + a^3*f*x)*s
in(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*
x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx =$$
$$-\frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec\cdots}{\sec^3\left(e+fx\right)-3\sec\cdots}\cdots\right.}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

output
```
-a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*se
c(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(
e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e +
f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(1/(sec(e
+ f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(90) = 180$.

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.76

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{a^3\left(\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^3} - \frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right) + \frac{a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 3\right)(\cos}{c^3\sin(fx+e)^5}}{60f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/60*(a^3*(120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3 - (20*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*
(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5)) + a^3*(10*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e)
 + 1)^5/(c^3*sin(f*x + e)^5) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3
*sin(f*x + e)^5) - 9*a^3*(5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(
f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)a^3}{c^3} + \frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*a^3/c^3 + 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 - 5*a^3*tan(
1/2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{a^3\,x}{c^3}+\frac{\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+2\,a^3\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{c^3\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^3,x)
```

output
```
(a^3*x)/c^3 + ((2*a^3*cos(e/2 + (f*x)/2)^5)/5 + 2*a^3*cos(e/2 + (f*x)/2)*s
in(e/2 + (f*x)/2)^4 - (2*a^3*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^2)/3)
/(c^3*f*sin(e/2 + (f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{a^3\left(15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 fx+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+6\right)}{15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^3 f}$$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

output
```
(a**3*(15*tan((e + f*x)/2)**5*f*x + 30*tan((e + f*x)/2)**4 - 10*tan((e + f
*x)/2)**2 + 6))/(15*tan((e + f*x)/2)**5*c**3*f)
```

3.19 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3x}{c^4}-\frac{8a^3\tan(e+fx)}{7c^4f(1-\sec(e+fx))^4}+\frac{4a^3\tan(e+fx)}{35c^4f(1-\sec(e+fx))^3}-\frac{62a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))^2}-\frac{167a^3\tan(e+fx)}{105c^4f(1-\sec(e+fx))}$$

output
```
a^3*x/c^4-8/7*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^4+4/35*a^3*tan(f*x+e)/c^
4/f/(1-sec(f*x+e))^3-62/105*a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))^2-167/105*
a^3*tan(f*x+e)/c^4/f/(1-sec(f*x+e))
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^{5/2}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-337+276\sec(e+fx)+50\sec^2(e+fx)-396\sec^3(e+fx)+167\sec^4(e+f\right.}{105c^{9/2}f(-1+\sec(e+fx))^4(1}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^(5/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-337 + 276*Sec[e + f*x] + 50*Sec[e
 + f*x]^2 - 396*Sec[e + f*x]^3 + 167*Sec[e + f*x]^4) - 840*ArcTanh[Sqrt[-(
a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^6*
Sqrt[-(a*c*Tan[e + f*x]^2)]))/(105*c^(9/2)*f*(-1 + Sec[e + f*x])^4*(1 + Se
c[e + f*x]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^4}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^4}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^4}+\frac{a^3}{(1-\sec(e+fx))^4}\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{167a^3\tan(e+fx)}{105f(1-\sec(e+fx))}-\frac{62a^3\tan(e+fx)}{105f(1-\sec(e+fx))^2}+\frac{4a^3\tan(e+fx)}{35f(1-\sec(e+fx))^3}-\frac{8a^3\tan(e+fx)}{7f(1-\sec(e+fx))^4}+a^3x}{c^4}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^4,x]
```

output
```
(a^3*x - (8*a^3*Tan[e + f*x])/(7*f*(1 - Sec[e + f*x])^4) + (4*a^3*Tan[e +
f*x])/(35*f*(1 - Sec[e + f*x])^3) - (62*a^3*Tan[e + f*x])/(105*f*(1 - Sec[
e + f*x])^2) - (167*a^3*Tan[e + f*x])/(105*f*(1 - Sec[e + f*x])))/c^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{a^{3}\left(15 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}-42 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}+70 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}-105 f x-210 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{105 c^{4} f}$
derivativedivides	$\frac{a^{3}\left(2 \arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}\right)}{f\, c^{4}}$
default	$\frac{a^{3}\left(2 \arctan\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)\right)-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}\right)}{f\, c^{4}}$
risch	$\frac{a^{3} x}{c^{4}}+\frac{2 i a^{3}\left(735 \,{\mathrm e}^{6 i \left(f x +e \right)}-2520 \,{\mathrm e}^{5 i \left(f x +e \right)}+5635 \,{\mathrm e}^{4 i \left(f x +e \right)}-6160 \,{\mathrm e}^{3 i \left(f x +e \right)}+4557 \,{\mathrm e}^{2 i \left(f x +e \right)}-1624 \,{\mathrm e}^{i \left(f x +e \right)}+337\right)}{105 f \,c^{4} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{7}}$
norman	$\frac{\frac{a^{3} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{c}+\frac{a^{3} x \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{11}}{c}-\frac{a^{3}}{7 c f}+\frac{24 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{35 c f}-\frac{169 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{105 c f}+\frac{56 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{15 c f}-\frac{14 a^{3} \tan\left(\frac{f x}{2}\right.}{3 c f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/105*a^3*(15*cot(1/2*f*x+1/2*e)^7-42*cot(1/2*f*x+1/2*e)^5+70*cot(1/2*f*x
+1/2*e)^3-105*f*x-210*cot(1/2*f*x+1/2*e))/c^4/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{337\,a^3\cos\left(fx+e\right)^4-276\,a^3\cos\left(fx+e\right)^3-50\,a^3\cos\left(fx+e\right)^2+396\,a^3\cos\left(fx+e\right)-167\,a^3+105\,(a}{105\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/105*(337*a^3*cos(f*x + e)^4 - 276*a^3*cos(f*x + e)^3 - 50*a^3*cos(f*x +
e)^2 + 396*a^3*cos(f*x + e) - 167*a^3 + 105*(a^3*f*x*cos(f*x + e)^3 - 3*a^
3*f*x*cos(f*x + e)^2 + 3*a^3*f*x*cos(f*x + e) - a^3*f*x)*sin(f*x + e))/((c
^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*
f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,d}{}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

output
```
a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e
 + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1),
x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec
(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**4 - 4*s
ec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.88

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^4}\,dx = \frac{5\,a^3\left(\frac{336\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^4} + \frac{\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{77\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{315\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 3\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}\right) + \frac{3\,a^3\left(\frac{21\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
1/840*(5*a^3*(336*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + (21*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 77*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(cos(f*x + e) + 1)^7/(c^4*sin(
f*x + e)^7)) + 3*a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15
)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + 9*a^3*(21*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) -
 a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e)
+ 1)^7/(c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^4}\,dx = \frac{\frac{105\,(fx+e)a^3}{c^4} + \frac{210\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 70\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 42\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 15\,a^3}{c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{105\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/105*(105*(f*x + e)*a^3/c^4 + (210*a^3*tan(1/2*f*x + 1/2*e)^6 - 70*a^3*ta
n(1/2*f*x + 1/2*e)^4 + 42*a^3*tan(1/2*f*x + 1/2*e)^2 - 15*a^3)/(c^4*tan(1/
2*f*x + 1/2*e)^7))/f
```

Mupad [B] (verification not implemented)

Time = 11.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3\left(-\frac{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{7}+\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{5}-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}+2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+(e\right.}{c^4\,f\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^4,x)
```

output
```
(a^3*(2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 - cos(e/2 + (f*x)/2)^7/7 +
 sin(e/2 + (f*x)/2)^7*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/
2)^4)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2)/5))/(c^4*f*sin(e/2
 + (f*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3\left(105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 fx+210\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-70\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+42\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-15\right)}{105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 c^4 f}$$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

output
```
(a**3*(105*tan((e + f*x)/2)**7*f*x + 210*tan((e + f*x)/2)**6 - 70*tan((e +
 f*x)/2)**4 + 42*tan((e + f*x)/2)**2 - 15))/(105*tan((e + f*x)/2)**7*c**4*
f)
```

3.20 $\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3x}{c^5} - \frac{8a^3\tan(e+fx)}{9c^5f(1-\sec(e+fx))^5} + \frac{4a^3\tan(e+fx)}{63c^5f(1-\sec(e+fx))^4} - \frac{38a^3\tan(e+fx)}{105c^5f(1-\sec(e+fx))^3} - \frac{181a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))^2} - \frac{496a^3\tan(e+fx)}{315c^5f(1-\sec(e+fx))}$$

output
```
a^3*x/c^5-8/9*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^5+4/63*a^3*tan(f*x+e)/c^
5/f/(1-sec(f*x+e))^4-38/105*a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^3-181/315*
a^3*tan(f*x+e)/c^5/f/(1-sec(f*x+e))^2-496/315*a^3*tan(f*x+e)/c^5/f/(1-sec(
f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\csc^9(e+fx)\left(48242+81711\cos(e+fx)+59544\cos(2(e+fx))+45591\cos(3(e+fx))+30744\cos\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^3*Csc[e + f*x]^9*(48242 + 81711*Cos[e + f*x] + 59544*Cos[2*(e + f*x)] +
 45591*Cos[3*(e + f*x)] + 30744*Cos[4*(e + f*x)] + 13221*Cos[5*(e + f*x)]
+ 4200*Cos[6*(e + f*x)] + 1656*Cos[7*(e + f*x)] + 630*Cos[8*(e + f*x)] + 6
1*Cos[9*(e + f*x)] + 1120*Cos[e + f*x]^9*Hypergeometric2F1[-9/2, 1, -7/2,
-Tan[e + f*x]^2]))/(10080*c^5*f)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^3}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \; 4391$$

$$\frac{\int \left(\frac{\sec^3(e+fx)a^3}{(1-\sec(e+fx))^5}+\frac{3\sec^2(e+fx)a^3}{(1-\sec(e+fx))^5}+\frac{3\sec(e+fx)a^3}{(1-\sec(e+fx))^5}+\frac{a^3}{(1-\sec(e+fx))^5}\right)dx}{c^5}$$

$\downarrow$ 2009

$$\frac{-\frac{496a^3\tan(e+fx)}{315f(1-\sec(e+fx))}-\frac{181a^3\tan(e+fx)}{315f(1-\sec(e+fx))^2}-\frac{38a^3\tan(e+fx)}{105f(1-\sec(e+fx))^3}+\frac{4a^3\tan(e+fx)}{63f(1-\sec(e+fx))^4}-\frac{8a^3\tan(e+fx)}{9f(1-\sec(e+fx))^5}+a^3x}{c^5}$$

input
```
Int[(a + a*Sec[e + f*x])^3/(c - c*Sec[e + f*x])^5,x]
```

output
```
(a^3*x - (8*a^3*Tan[e + f*x])/(9*f*(1 - Sec[e + f*x])^5) + (4*a^3*Tan[e +
f*x])/(63*f*(1 - Sec[e + f*x])^4) - (38*a^3*Tan[e + f*x])/(105*f*(1 - Sec[
e + f*x])^3) - (181*a^3*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])^2) - (496*
a^3*Tan[e + f*x])/(315*f*(1 - Sec[e + f*x])))/c^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result
parallelrisch	$\frac{a^3\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^9-135\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7+252\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-420\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+630fx+1260\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{630c^5f}$
derivativedivides	$\frac{a^3\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,c^5}$
default	$\frac{a^3\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,c^5}$
risch	$\frac{a^3x}{c^5}+\frac{2ia^3\left(2520\,\mathrm{e}^{8i(fx+e)}-12285\,\mathrm{e}^{7i(fx+e)}+36645\,\mathrm{e}^{6i(fx+e)}-61425\,\mathrm{e}^{5i(fx+e)}+71001\,\mathrm{e}^{4i(fx+e)}-51639\,\mathrm{e}^{3i(fx+e)}-\right.}{315f\,c^5\left(\mathrm{e}^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{c}+\frac{a^3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{c}+\frac{a^3}{18cf}-\frac{41a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{126cf}+\frac{557a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{630cf}-\frac{353a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{210cf}+\frac{56a^3\tan\left(\right.}{15c}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/630*a^3*(35*cot(1/2*f*x+1/2*e)^9-135*cot(1/2*f*x+1/2*e)^7+252*cot(1/2*f*
x+1/2*e)^5-420*cot(1/2*f*x+1/2*e)^3+630*f*x+1260*cot(1/2*f*x+1/2*e))/c^5/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$$
$$=\frac{1051\,a^3\cos\left(fx+e\right)^5-1684\,a^3\cos\left(fx+e\right)^4+898\,a^3\cos\left(fx+e\right)^3+1468\,a^3\cos\left(fx+e\right)^2-1669\,a^3\cos}{315\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx\right.\right.}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(1051*a^3*cos(f*x + e)^5 - 1684*a^3*cos(f*x + e)^4 + 898*a^3*cos(f*x
 + e)^3 + 1468*a^3*cos(f*x + e)^2 - 1669*a^3*cos(f*x + e) + 496*a^3 + 315*
(a^3*f*x*cos(f*x + e)^4 - 4*a^3*f*x*cos(f*x + e)^3 + 6*a^3*f*x*cos(f*x + e
)^2 - 4*a^3*f*x*cos(f*x + e) + a^3*f*x)*sin(f*x + e))/((c^5*f*cos(f*x + e)
^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e
) + c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^5}\,dx = -\frac{a^3\left(\int \frac{3\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx + \int \frac{3\sec^2(e+}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+f}\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

output
```
-a**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*s
ec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3
*sec(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3
 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3
/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*
x)**2 + 5*sec(e + f*x) - 1), x) + Integral(1/(sec(e + f*x)**5 - 5*sec(e +
f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x
))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(144) = 288$.

Time = 0.13 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.46

$$\int \frac{(a + a\sec(e + fx))^3}{(c - c\sec(e + fx))^5}\,dx = \frac{a^3\left(\frac{10080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^5} - \frac{\left(\frac{270\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{1008\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{2730\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{9765\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - 35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}\right) - \frac{3a^3}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
1/5040*(a^3*(10080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^5 - (270*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 1008*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 2730*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9765*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)) - 3*a^3*(180
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) - 15*a^3*(1
8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(c
^5*sin(f*x + e)^9) - 7*a^3*(18*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x +
 e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{\frac{630\,(fx+e)a^3}{c^5} + \frac{1260\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-420\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+252\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-135\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^3}{c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}}{630\,f}$$

input
```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/630*(630*(f*x + e)*a^3/c^5 + (1260*a^3*tan(1/2*f*x + 1/2*e)^8 - 420*a^3*
tan(1/2*f*x + 1/2*e)^6 + 252*a^3*tan(1/2*f*x + 1/2*e)^4 - 135*a^3*tan(1/2*
f*x + 1/2*e)^2 + 35*a^3)/(c^5*tan(1/2*f*x + 1/2*e)^9))/f
```

Mupad [B] (verification not implemented)

Time = 11.02 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\left(\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{18}-\frac{3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{14}+\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{5}-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{3}+2\cos\left(\frac{e}{2}+\right.\right.}{c^5 f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

input
```
int((a + a/cos(e + f*x))^3/(c - c/cos(e + f*x))^5,x)
```

output
```
(a^3*(cos(e/2 + (f*x)/2)^9/18 + 2*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^8
+ sin(e/2 + (f*x)/2)^9*(e + f*x) - (2*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)
/2)^6)/3 + (2*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^4)/5 - (3*cos(e/2 +
(f*x)/2)^7*sin(e/2 + (f*x)/2)^2)/14))/(c^5*f*sin(e/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

$$\int \frac{(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\left(630\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9fx+1260\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-420\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+252\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-135\tan\left(\frac{fx}{2}+\right.\right.}{630\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9c^5f}$$

input
```
int((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

output
```
(a**3*(630*tan((e + f*x)/2)**9*f*x + 1260*tan((e + f*x)/2)**8 - 420*tan((e
 + f*x)/2)**6 + 252*tan((e + f*x)/2)**4 - 135*tan((e + f*x)/2)**2 + 35))/(
630*tan((e + f*x)/2)**9*c**5*f)
```

3.21 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2} dx = \frac{c^5x}{a^2} - \frac{47c^5\text{arctanh}(\sin(e+fx))}{2a^2f} + \frac{13c^5\tan(e+fx)}{2a^2f} + \frac{112c^5\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{32c^5\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^5-c^5\sec(e+fx))\tan(e+fx)}{2a^2f}$$

output
```
c^5*x/a^2-47/2*c^5*arctanh(sin(f*x+e))/a^2/f+13/2*c^5*tan(f*x+e)/a^2/f+112
/3*c^5*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-32/3*c^5*tan(f*x+e)/f/(a+a*sec(f*x+
e))^2+1/2*(c^5-c^5*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.81 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.58

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2} dx$$

$$= \frac{c^{9/2}\tan(e+fx)\left(8\sqrt{a}\sqrt{c}+16\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\text{se}\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^(9/2)*Tan[e + f*x]*(8*Sqrt[a]*Sqrt[c] + 16*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hype
rgeometric2F1[-7/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x
]] + 8*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Se
c[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeo
metric2F1[-3/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] -
 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x] - 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 - 3*Arc
Tanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c*Tan[e + f*x
]^2)] - 3*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f
*x]*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(3*a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Se
c[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^7dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^7}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4374}$$

$$\int \left(\cot^4(e+fx)c^7 + 35\csc^4(e+fx)c^7 - 35\cot(e+fx)\csc^3(e+fx)c^7 - \csc^4(e+fx)\sec^3(e+fx)c^7 + 21\cot^2(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{47c^7\text{arctanh}(\sin(e+fx))}{2f} + \frac{7c^7\tan(e+fx)}{f} - \frac{64c^7\cot^3(e+fx)}{3f} - \frac{48c^7\cot(e+fx)}{f} + \frac{131c^7\csc^3(e+fx)}{6f} + \frac{33c^7\csc(e+fx)}{2f} - \frac{c^7\csc^3(e}{}}{a^2c^2}$$

input
```
Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^7*x - (47*c^7*ArcTanh[Sin[e + f*x]])/(2*f) - (48*c^7*Cot[e + f*x])/f -
(64*c^7*Cot[e + f*x]^3)/(3*f) + (33*c^7*Csc[e + f*x])/(2*f) + (131*c^7*Csc
[e + f*x]^3)/(6*f) - (c^7*Csc[e + f*x]^3*Sec[e + f*x]^2)/(2*f) + (7*c^7*Ta
n[e + f*x])/f)/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{16c^5\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}+\frac{\arctan\left(\tan\left(\frac{f}{2}\right.\right.}{8}\right.}{f\,a^2}$
default	$\frac{16c^5\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{15}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{47\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}+\frac{\arctan\left(\tan\left(\frac{f}{2}\right.\right.}{8}\right.}{f\,a^2}$
parallelrisch	$\frac{125c^5\left(\frac{94(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{125}+\frac{94(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{125}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{61\,\mathrm{co}}{}\right.\right.}{4f\,a^2(1+\cos(2fx+2e))}$
risch	$\frac{c^5x}{a^2}+\frac{ic^5\left(99\,\mathrm{e}^{6i(fx+e)}+435\,\mathrm{e}^{5i(fx+e)}+484\,\mathrm{e}^{4i(fx+e)}+930\,\mathrm{e}^{3i(fx+e)}+575\,\mathrm{e}^{2i(fx+e)}+507\,\mathrm{e}^{i(fx+e)}+202\right)}{3f\,a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}-\frac{47c^5\ln}{}$
norman	$\frac{\frac{c^5x}{a}+\frac{c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}+\frac{6c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}+\frac{45c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{491c^5\tan\left(\frac{fx}{2}+\right.}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4a}$

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
16/f*c^5/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)-1/32/(tan(1/2*
f*x+1/2*e)-1)^2-15/32/(tan(1/2*f*x+1/2*e)-1)+47/32*ln(tan(1/2*f*x+1/2*e)-1
)+1/8*arctan(tan(1/2*f*x+1/2*e))+1/32/(tan(1/2*f*x+1/2*e)+1)^2-15/32/(tan(
1/2*f*x+1/2*e)+1)-47/32*ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.78

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{12\,c^5fx\cos\left(fx+e\right)^4+24\,c^5fx\cos\left(fx+e\right)^3+12\,c^5fx\cos\left(fx+e\right)^2-141\left(c^5\cos\left(fx+e\right)^4+2\,c^5\cos\left(\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/12*(12*c^5*f*x*cos(f*x + e)^4 + 24*c^5*f*x*cos(f*x + e)^3 + 12*c^5*f*x*c
os(f*x + e)^2 - 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x + e)^3 + c^5*cos(f
*x + e)^2)*log(sin(f*x + e) + 1) + 141*(c^5*cos(f*x + e)^4 + 2*c^5*cos(f*x
 + e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(202*c^5*cos(f*x
+ e)^3 + 305*c^5*cos(f*x + e)^2 + 36*c^5*cos(f*x + e) - 3*c^5)*sin(f*x + e
))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^5\left(\int \frac{5\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \left(-\frac{10\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{10\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \left(\right.}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

output
```
-c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(-5*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(127) = 254$.

Time = 0.13 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.43

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e)
 + 1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 5*c^5*
((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(c
os(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^2) - c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos
(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) +
10*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - 5*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{6(fx+e)c^5}{a^2} - \frac{141c^5\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} + \frac{141c^5\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{6\left(15c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-13c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^2} + \cdots}{6f}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/6*(6*(f*x + e)*c^5/a^2 - 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2
+ 141*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*(15*c^5*tan(1/2*f*x +
 1/2*e)^3 - 13*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a
^2) + 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^5*tan(1/2*f*x + 1/2*e))
/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5\,x}{a^2} - \frac{15\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3 - 13\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - 2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + a^2\right)} + \frac{32\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} + \frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} - \frac{47\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

input
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^2,x)
```

output
```
(c^5*x)/a^2 - (15*c^5*tan(e/2 + (f*x)/2)^3 - 13*c^5*tan(e/2 + (f*x)/2))/(f
*(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) + (32*c^5*
tan(e/2 + (f*x)/2))/(a^2*f) + (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) - (4
7*c^5*atanh(tan(e/2 + (f*x)/2)))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.85

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{c^5\left(141\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4 - 282\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 + 141\log\left(\tan\left(\frac{f}{2}\right.\right.\right.}{}$$

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

output
```
(c**5*(141*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**4 - 282*log(tan((e
+ f*x)/2) - 1)*tan((e + f*x)/2)**2 + 141*log(tan((e + f*x)/2) - 1) - 141*l
og(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**4 + 282*log(tan((e + f*x)/2) +
1)*tan((e + f*x)/2)**2 - 141*log(tan((e + f*x)/2) + 1) + 32*tan((e + f*x)/
2)**7 + 128*tan((e + f*x)/2)**5 + 6*tan((e + f*x)/2)**4*f*x - 442*tan((e +
 f*x)/2)**3 - 12*tan((e + f*x)/2)**2*f*x + 270*tan((e + f*x)/2) + 6*f*x))/
(6*a**2*f*(tan((e + f*x)/2)**4 - 2*tan((e + f*x)/2)**2 + 1))
```

3.22 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4x}{a^2} - \frac{6c^4\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{16c^4\cot(e+fx)}{a^2f} - \frac{32c^4\cot^3(e+fx)}{3a^2f} + \frac{32c^4\csc^3(e+fx)}{3a^2f} + \frac{c^4\tan(e+fx)}{a^2f}$$

output
```
c^4*x/a^2-6*c^4*arctanh(sin(f*x+e))/a^2/f-16*c^4*cot(f*x+e)/a^2/f-32/3*c^4
*cot(f*x+e)^3/a^2/f+32/3*c^4*csc(f*x+e)^3/a^2/f+c^4*tan(f*x+e)/a^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.92

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^{7/2}\tan(e+fx)\left(-8\sqrt{a}\sqrt{c}-8\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\cdots}\right.}{\cdots}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(c^(7/2)*Tan[e + f*x]*(-8*Sqrt[a]*Sqrt[c] - 8*Sqrt[2]*Sqrt[a]*Sqrt[c]
*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e
+ f*x]] - 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1
 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x
] + 4*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 + 3*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2
)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c*Tan[e + f*x]^2)] + 3*ArcTanh[Sqrt[-(a*c*T
an[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)
]))/(a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(c-c\sec(e+fx))^6dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{4374}$$

$$\frac{\int \left(\cot^4(e+fx)c^6+15\csc^4(e+fx)c^6-20\cot(e+fx)\csc^3(e+fx)c^6+15\cot^2(e+fx)\csc^2(e+fx)c^6+\csc^4(\right.}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{6c^6\text{arctanh}(\sin(e+fx))}{f}+\frac{c^6\tan(e+fx)}{f}-\frac{32c^6\cot^3(e+fx)}{3f}-\frac{16c^6\cot(e+fx)}{f}+\frac{32c^6\csc^3(e+fx)}{3f}+c^6x}{a^2c^2}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^6*x - (6*c^6*ArcTanh[Sin[e + f*x]])/f - (16*c^6*Cot[e + f*x])/f - (32*c
^6*Cot[e + f*x]^3)/(3*f) + (32*c^6*Csc[e + f*x]^3)/(3*f) + (c^6*Tan[e + f*
x])/f)/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{8c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}\right.}{f\,a^2}$
default	$\frac{8c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}-\frac{1}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}\right.}{f\,a^2}$
parallelrisch	$\frac{6\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+\frac{19\left(\cos(fx+e)+\frac{\cos(2fx+2e)}{4}+\frac{25}{76}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec}{18}\right.}{f\,a^2\cos(fx+e)}$
risch	$\frac{c^4x}{a^2}+\frac{2ic^4\left(51\,\mathrm{e}^{3i(fx+e)}+25\,\mathrm{e}^{2i(fx+e)}+57\,\mathrm{e}^{i(fx+e)}+19\right)}{3f\,a^2\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^3}+\frac{6c^4\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{f\,a^2}-\frac{6c^4\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{f\,a^2}$
norman	$\frac{\frac{c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}-\frac{c^4x}{a}-\frac{10c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{76c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{18c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}+\frac{8c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}+\frac{3c^4x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a}$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
8/f*c^4/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+tan(1/2*f*x+1/2*e)-1/8/(tan(1/2*f*x+
1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1)+1/4*arctan(tan(1/2*f*x+1/2*e))-1/8/
(tan(1/2*f*x+1/2*e)-1)+3/4*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(98) = 196.

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.16

$$\int\frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{3\,c^4fx\cos\left(fx+e\right)^3+6\,c^4fx\cos\left(fx+e\right)^2+3\,c^4fx\cos\left(fx+e\right)-9\left(c^4\cos\left(fx+e\right)^3+2\,c^4\cos\left(fx+e\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*c^4*f*x*cos(f*x + e)^3 + 6*c^4*f*x*cos(f*x + e)^2 + 3*c^4*f*x*cos(f
*x + e) - 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))
*log(sin(f*x + e) + 1) + 9*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^
4*cos(f*x + e))*log(-sin(f*x + e) + 1) + (19*c^4*cos(f*x + e)^2 + 38*c^4*c
os(f*x + e) + 3*c^4)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x
 + e)^2 + a^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4\left(\int\left(-\frac{4\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{6\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{4\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\right.}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(6*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(-4*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + I
ntegral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(98) = 196.

Time = 0.12 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.05

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+}{}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(c^4*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 4*c^4*((9*sin(f*x
 + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x +
 e) + 1) - 1)/a^2) - c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))
/a^2) + 6*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/a^2 - 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3(fx+e)c^4}{a^2} - \frac{18c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} + \frac{18c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{6c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2} + \frac{8\left(a^4c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3\cdots\right.}{a^6}}{3f}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*c^4/a^2 - 18*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 +
 18*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 - 6*c^4*tan(1/2*f*x + 1/2*e
)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) + 8*(a^4*c^4*tan(1/2*f*x + 1/2*e)^3 +
 3*a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4\,x}{a^2}+\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}+\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} - \frac{12\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}-\frac{2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^2,x)
```

output
```
(c^4*x)/a^2 + (8*c^4*tan(e/2 + (f*x)/2))/(a^2*f) + (8*c^4*tan(e/2 + (f*x)/
2)^3)/(3*a^2*f) - (12*c^4*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (2*c^4*tan(
e/2 + (f*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.58

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4\left(18\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-18\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-18\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\right.\right.}{3a^2}$$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

output
```
(c**4*(18*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2 - 18*log(tan((e +
f*x)/2) - 1) - 18*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2 + 18*log(t
an((e + f*x)/2) + 1) + 8*tan((e + f*x)/2)**5 + 16*tan((e + f*x)/2)**3 + 3*
tan((e + f*x)/2)**2*f*x - 30*tan((e + f*x)/2) - 3*f*x))/(3*a**2*f*(tan((e
+ f*x)/2)**2 - 1))
```

3.23 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^3x}{a^2} - \frac{c^3\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{8c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2} + \frac{4c^3\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

output
```
c^3*x/a^2-c^3*arctanh(sin(f*x+e))/a^2/f-8/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*
x+e))^2+4/3*c^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.22

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^{5/2}\tan(e+fx)\left(4\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec(e+fx)} -}{3a^{5/2}f(-1+}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^(5/2)*Tan[e + f*x]*(4*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-3/2, -
3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] - 4*Sqrt[a]*Sqrt[c
]*(-2 + Sec[e + f*x] + Sec[e + f*x]^2) - 6*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]
^2)]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^2*Sec[e + f*x]*Sqrt[-(a*c*Tan[e +
 f*x]^2)]))/(3*a^(5/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(-\frac{\sec^3(e+fx)c^3}{(\sec(e+fx)+1)^2}+\frac{3\sec^2(e+fx)c^3}{(\sec(e+fx)+1)^2}-\frac{3\sec(e+fx)c^3}{(\sec(e+fx)+1)^2}+\frac{c^3}{(\sec(e+fx)+1)^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{c^3\text{arctanh}(\sin(e+fx))}{f}+\frac{4c^3\tan(e+fx)}{3f(\sec(e+fx)+1)}-\frac{8c^3\tan(e+fx)}{3f(\sec(e+fx)+1)^2}+c^3x}{a^2}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^3*x - (c^3*ArcTanh[Sin[e + f*x]])/f - (8*c^3*Tan[e + f*x])/(3*f*(1 + Se
c[e + f*x])^2) + (4*c^3*Tan[e + f*x])/(3*f*(1 + Sec[e + f*x])))/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{c^{3} \left(4 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+3 f x +3 \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)-3 \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)\right)}{3 f \,a^{2}}$
derivativedivides	$\frac{4 c^{3} \left(\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f \,a^{2}}$
default	$\frac{4 c^{3} \left(\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2}-\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f \,a^{2}}$
risch	$\frac{c^{3} x}{a^{2}}-\frac{8 i c^{3} \left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{3 f \,a^{2} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{3}}+\frac{c^{3} \ln \left({\mathrm e}^{i \left(f x +e \right)}-i\right)}{f \,a^{2}}-\frac{c^{3} \ln \left({\mathrm e}^{i \left(f x +e \right)}+i\right)}{f \,a^{2}}$
norman	$\frac{\frac{c^{3} x}{a}+\frac{c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{a}+\frac{4 c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 a f}-\frac{8 c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{3 a f}+\frac{4 c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{3 a f}-\frac{2 c^{3} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{a}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} a}+\frac{c^{3} \ln \left(\tan \left(\frac{f x}{2}\right.\right.}{f \,a^{2}}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a^2*c^3*(4*tan(1/2*f*x+1/2*e)^3+3*f*x+3*ln(tan(1/2*f*x+1/2*e)-1)-3*l
n(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(81) = 162.

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.04

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^2}\,dx = \frac{6\,c^3fx\cos\left(fx+e\right)^2 + 12\,c^3fx\cos\left(fx+e\right) + 6\,c^3fx - 3\left(c^3\cos\left(fx+e\right)^2 + 2\,c^3\cos\left(fx+e\right) + c^3\right)\log\left(s\right.}{6\left(a^2f\cos\left(fx\right.\right.}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/6*(6*c^3*f*x*cos(f*x + e)^2 + 12*c^3*f*x*cos(f*x + e) + 6*c^3*f*x - 3*(c
^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(sin(f*x + e) + 1) + 3*(c
^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 8*(
c^3*cos(f*x + e) - c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(
f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{(c - c\sec(e+fx))^3}{(a + a\sec(e+fx))^2}\,dx = -\frac{c^3\left(\int \frac{3\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx + \int\left(-\frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx + \int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx + \int\left(\right.\right.}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
 Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(81) = 162$.

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.15

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^2}\,dx = \frac{c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right) - c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12\,\mathrm{ar}\cdots}{}\right.}{6f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin
(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - c^3*((9*sin(f*x + e)/(cos(f*x + e
) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 3*c^3*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^2}\,dx = \frac{\frac{4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{a^2} + \frac{3(fx+e)c^3}{a^2} - \frac{3c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} + \frac{3c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}}{3f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(4*c^3*tan(1/2*f*x + 1/2*e)^3/a^2 + 3*(f*x + e)*c^3/a^2 - 3*c^3*log(ab
s(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 3*c^3*log(abs(tan(1/2*f*x + 1/2*e) - 1)
)/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.54

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^3\,x}{a^2} - \frac{c^3\,\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3}\right)}{a^2\,f}$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^2,x)
```

output
```
(c^3*x)/a^2 - (c^3*(2*atanh(tan(e/2 + (f*x)/2)) - (4*tan(e/2 + (f*x)/2)^3)
/3))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{c^3\left(3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-3\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3fx\right)}{3a^2f}$$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

output
```
(c**3*(3*log(tan((e + f*x)/2) - 1) - 3*log(tan((e + f*x)/2) + 1) + 4*tan((
e + f*x)/2)**3 + 3*f*x))/(3*a**2*f)
```

3.24 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx=\frac{c^2x}{a^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{4c^2\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

output
```
c^2*x/a^2-4/3*c^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-4/3*c^2*tan(f*x+e)/a^2
/f/(1+sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx=\frac{c^2\left(\frac{2\arctan\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}\right)}{a^2}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^2*((2*ArcTan[Tan[e/2 + (f*x)/2]])/f - (2*Tan[e/2 + (f*x)/2])/f + (2*Tan
[e/2 + (f*x)/2]^3)/(3*f)))/a^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^2(e+fx)c^2}{(\sec(e+fx)+1)^2}-\frac{2\sec(e+fx)c^2}{(\sec(e+fx)+1)^2}+\frac{c^2}{(\sec(e+fx)+1)^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4c^2\tan(e+fx)}{3f(\sec(e+fx)+1)}-\frac{4c^2\tan(e+fx)}{3f(\sec(e+fx)+1)^2}+c^2x}{a^2}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c^2*x - (4*c^2*Tan[e + f*x])/(3*f*(1 + Sec[e + f*x])^2) - (4*c^2*Tan[e +
f*x])/(3*f*(1 + Sec[e + f*x]))))/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{c^{2}\left(2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3fx-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f\,a^{2}}$	41
derivativedivides	$\frac{2c^{2}\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$	47
default	$\frac{2c^{2}\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$	47
risch	$\frac{c^{2}x}{a^{2}}-\frac{8ic^{2}\left(3\,{\mathrm e}^{2i(fx+e)}+3\,{\mathrm e}^{i(fx+e)}+2\right)}{3f\,a^{2}\left({\mathrm e}^{i(fx+e)}+1\right)^{3}}$	59
norman	$\frac{\frac{c^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{a}-\frac{c^{2}x}{a}+\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{8c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}+\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3af}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	113

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^2*(2*tan(1/2*f*x+1/2*e)^3+3*f*x-6*tan(1/2*f*x+1/2*e))/f/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{3\,c^{2}fx\cos\left(fx+e\right)^{2}+6\,c^{2}fx\cos\left(fx+e\right)+3\,c^{2}fx-4\left(2\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\sin\left(fx+e\right)}{3\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*c^2*f*x*cos(f*x + e)^2 + 6*c^2*f*x*cos(f*x + e) + 3*c^2*f*x - 4*(2*
c^2*cos(f*x + e) + c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(
f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(\int\left(-\frac{2\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\frac{1}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(63) = 126$.

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{2c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx}{(\cos(fx-}\right.}{a^2}}{6f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - c^2*(3*
sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
 + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^2}\,dx = \frac{\frac{3\,(fx+e)c^2}{a^2} + \frac{2\left(a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*c^2/a^2 + 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*t
an(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^2}\,dx = \frac{2\,c^2\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^3 - 3\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) + \frac{3\,f\,x}{2}\right)}{3\,a^2\,f}$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^2,x)
```

output
```
(2*c^2*(tan(e/2 + (f*x)/2)^3 - 3*tan(e/2 + (f*x)/2) + (3*f*x)/2))/(3*a^2*f
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3fx\right)}{3a^2f}$$

input

```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

output

```
(c**2*(2*tan((e + f*x)/2)**3 - 6*tan((e + f*x)/2) + 3*f*x))/(3*a**2*f)
```

3.25 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \frac{cx}{a^2}-\frac{2c\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2}-\frac{5c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}$$

output
```
c*x/a^2-2/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2-5/3*c*tan(f*x+e)/a^2/f/(1+
sec(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{\sqrt{c}\tan(e+fx)\left(\sqrt{a}\sqrt{c}(-7+2\sec(e+fx)+5\sec^2(e+fx))+6\text{arctanh}\left(\frac{\sqrt{-ac\tan^2(e+fx)}}{\sqrt{a}\sqrt{c}}\right)\cos^2\left(\frac{1}{2}(e\right.\right.}{3a^{5/2}f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(Sqrt[c]*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-7 + 2*Sec[e + f*x] + 5*Sec[e
 + f*x]^2) + 6*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Cos[
(e + f*x)/2]^2*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(a^(5/2)*f*(-1 +
 Sec[e + f*x])*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{c}{(\sec(e+fx)+1)^2}-\frac{c\sec(e+fx)}{(\sec(e+fx)+1)^2}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{5c\tan(e+fx)}{3f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(\sec(e+fx)+1)^2}+cx}{a^2}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^2,x]
```

output
```
(c*x - (2*c*Tan[e + f*x])/(3*f*(1 + Sec[e + f*x])^2) - (5*c*Tan[e + f*x])/
(3*f*(1 + Sec[e + f*x])))/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3fx-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f\,a^{2}}$	37
derivativedivides	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$	46
default	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$	46
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}}{a}$	50
risch	$\frac{cx}{a^{2}}-\frac{2ic\left(9\,{\mathrm e}^{2i(fx+e)}+12\,{\mathrm e}^{i(fx+e)}+7\right)}{3f\,a^{2}\left({\mathrm e}^{i(fx+e)}+1\right)^{3}}$	55

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*(tan(1/2*f*x+1/2*e)^3+3*f*x-6*tan(1/2*f*x+1/2*e))/f/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\,dx = \frac{3\,cfx\cos\left(fx + e\right)^2 + 6\,cfx\cos\left(fx + e\right) + 3\,cfx - \left(7\,c\cos\left(fx + e\right) + 5\,c\right)\sin\left(fx + e\right)}{3\,\left(a^2f\cos\left(fx + e\right)^2 + 2\,a^2f\cos\left(fx + e\right) + a^2f\right)}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(3*c*f*x*cos(f*x + e)^2 + 6*c*f*x*cos(f*x + e) + 3*c*f*x - (7*c*cos(f*
x + e) + 5*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) +
 a^2*f)
```

Sympy [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\,dx = -\frac{c\left(\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx + \int \left(-\frac{1}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx\right)}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

output
```
-c*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-1/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\,dx = -\frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right) + \frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2 - 12*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + c*(3*sin(
f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^2}\,dx = \frac{\frac{3(fx+e)c}{a^2} + \frac{a^4 c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 6a^4 c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6}}{3f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*(3*(f*x + e)*c/a^2 + (a^4*c*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*tan(1/2*f
*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx=\frac{c\,x}{a^2}-\frac{c\left(6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^2,x)
```

output
```
(c*x)/a^2 - (c*(6*tan(e/2 + (f*x)/2) - tan(e/2 + (f*x)/2)^3))/(3*a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx=\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3fx\right)}{3a^2f}$$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

output
```
(c*(tan((e + f*x)/2)**3 - 6*tan((e + f*x)/2) + 3*f*x))/(3*a**2*f)
```

3.26 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{x}{a^2c}+\frac{\cot(e+fx)(3-2\sec(e+fx))}{3a^2cf}-\frac{\cot^3(e+fx)(1-\sec(e+fx))}{3a^2cf}$$

output
```
x/a^2/c+1/3*cot(f*x+e)*(3-2*sec(f*x+e))/a^2/c/f-1/3*cot(f*x+e)^3*(1-sec(f*
x+e))/a^2/c/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\cot^3(e+fx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)-3\sec(e+fx)+2\sec^3(e+fx)\right)}{3a^2cf}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

output
```
-1/3*(Cot[e + f*x]^3*(Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2] -
3*Sec[e + f*x] + 2*Sec[e + f*x]^3))/(a^2*c*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4392$$

$$\frac{\int \cot^4(e+fx)(c-c\sec(e+fx))dx}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \; 4370$$

$$\frac{\frac{1}{3}\int -\cot^2(e+fx)(3c-2c\sec(e+fx))dx-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{1}{3}\int \cot^2(e+fx)(3c-2c\sec(e+fx))dx-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{1}{3}\int \frac{3c-2c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \ 4370$$

$$\frac{\frac{1}{3}\left(\frac{\cot(e+fx)(3c-2c\sec(e+fx))}{f}-\int -3cdx\right)-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

$$\Big\downarrow \ 24$$

$$\frac{\frac{1}{3}\left(\frac{\cot(e+fx)(3c-2c\sec(e+fx))}{f}+3cx\right)-\frac{\cot^3(e+fx)(c-c\sec(e+fx))}{3f}}{a^2c^2}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

output
```
(-1/3*(Cot[e + f*x]^3*(c - c*Sec[e + f*x]))/f + (3*c*x + (Cot[e + f*x]*(3*
c - 2*c*Sec[e + f*x]))/f)/3)/(a^2*c^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+12fx+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{12a^{2}cf}$	50
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	60
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	60
risch	$\frac{x}{a^{2}c}-\frac{2i\left(3\,\mathrm{e}^{3i(fx+e)}-5\,\mathrm{e}^{i(fx+e)}-4\right)}{3f\,a^{2}c\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)}$	72
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{4acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{12acf}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	89

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(tan(1/2*f*x+1/2*e)^3+12*f*x+3*cot(1/2*f*x+1/2*e)-12*tan(1/2*f*x+1/2*
e))/a^2/c/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{4\cos(fx+e)^2+3\,(fx\cos(fx+e)+fx)\sin(fx+e)+\cos(fx+e)-2}{3\,(a^2cf\cos(fx+e)+a^2cf)\sin(fx+e)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(4*cos(f*x + e)^2 + 3*(f*x*cos(f*x + e) + f*x)*sin(f*x + e) + cos(f*x
+ e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sec^3(e+fx)+\sec^2(e+fx)-\sec(e+fx)-1}\,dx}{a^2c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1), x)/(a*
*2*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\frac{\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}-\frac{24\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c}-\frac{3\,(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-1/12*((12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3)/(a^2*c) - 24*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c) - 3*(
cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\frac{12\,(fx+e)}{a^2c}+\frac{3}{a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\frac{a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-12\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6c^3}}{12\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/12*(12*(f*x + e)/(a^2*c) + 3/(a^2*c*tan(1/2*f*x + 1/2*e)) + (a^4*c^2*tan
(1/2*f*x + 1/2*e)^3 - 12*a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{x}{a^2\,c}+\frac{\frac{4\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{3}-\frac{7\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{6}+\frac{1}{12}}{a^2\,c\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
```

output
```
x/(a^2*c) + ((4*cos(e/2 + (f*x)/2)^4)/3 - (7*cos(e/2 + (f*x)/2)^2)/6 + 1/1
2)/(a^2*c*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)fx+3}{12\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a^2cf}$$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

output
```
(tan((e + f*x)/2)**4 - 12*tan((e + f*x)/2)**2 + 12*tan((e + f*x)/2)*f*x +
3)/(12*tan((e + f*x)/2)*a**2*c*f)
```

3.27 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx=\frac{x}{a^2c^2}+\frac{\cot(e+fx)}{a^2c^2f}-\frac{\cot^3(e+fx)}{3a^2c^2f}$$

output
```
x/a^2/c^2+cot(f*x+e)/a^2/c^2/f-1/3*cot(f*x+e)^3/a^2/c^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3a^2c^2f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

output
```
-1/3*(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[e + f*x]^2])/(a
^2*c^2*f)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan\left(e+fx+\frac{\pi}{2}\right)^4dx}{a^2c^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-\int \cot^2(e+fx)dx-\frac{\cot^3(e+fx)}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{\cot^3(e+fx)}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int 1dx-\frac{\cot^3(e+fx)}{3f}+\frac{\cot(e+fx)}{f}}{a^2c^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{-\frac{\cot^3(e+fx)}{3f}+\frac{\cot(e+fx)}{f}+x}{a^2c^2}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

output
```
(x + Cot[e + f*x]/f - Cot[e + f*x]^3/(3*f))/(a^2*c^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)-\frac{\pi}{2}+\operatorname{arccot}(\cot(fx+e))}{c^2a^2f}$	38
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+24fx-15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{24f\,c^2a^2}$	63
risch	$\frac{x}{a^2c^2}+\frac{4i\left(3\,\mathrm{e}^{4i(fx+e)}-3\,\mathrm{e}^{2i(fx+e)}+2\right)}{3f\,c^2a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3\left(\mathrm{e}^{i(fx+e)}-1\right)^3}$	72
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{ca}-\frac{1}{24acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{24acf}}{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	116

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2/a^2/f*(-1/3*cot(f*x+e)^3+cot(f*x+e)-1/2*Pi+arccot(cot(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{4\,\cos\left(fx+e\right)^3+3\,\left(fx\cos\left(fx+e\right)^2-fx\right)\sin\left(fx+e\right)-3\,\cos\left(fx+e\right)}{3\,\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(4*cos(f*x + e)^3 + 3*(f*x*cos(f*x + e)^2 - f*x)*sin(f*x + e) - 3*cos(
f*x + e))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a + a\sec(e + fx))^2(c - c\sec(e + fx))^2}\,dx = \frac{\int \frac{1}{\sec^4(e+fx)-2\sec^2(e+fx)+1}\,dx}{a^2c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a + a\sec(e + fx))^2(c - c\sec(e + fx))^2}\,dx = \frac{\frac{3(fx+e)}{a^2c^2} + \frac{3\tan(fx+e)^2-1}{a^2c^2\tan(fx+e)^3}}{3f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)/(a^2*c^2) + (3*tan(f*x + e)^2 - 1)/(a^2*c^2*tan(f*x + e)^
3))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(44) = 88.

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a + a\sec(e + fx))^2(c - c\sec(e + fx))^2}\,dx = \frac{\frac{24(fx+e)}{a^2c^2} + \frac{15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1}{a^2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3} + \frac{a^4c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-15a^4c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^6}}{24f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/24*(24*(f*x + e)/(a^2*c^2) + (15*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^2*c^2*ta
n(1/2*f*x + 1/2*e)^3) + (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 15*a^4*c^4*tan(1
/2*f*x + 1/2*e))/(a^6*c^6))/f
```

Mupad [B] (verification not implemented)

Time = 11.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = -\frac{\cos(3\,e+3\,f\,x)+\frac{3\,\sin(3\,e+3\,f\,x)\,(e+f\,x)}{4}-\frac{9\,\sin(e+f\,x)\,(e+f\,x)}{4}}{3\,a^2\,c^2\,f\,\sin(e+f\,x)^3}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
```

output
```
-(cos(3*e + 3*f*x) + (3*sin(3*e + 3*f*x)*(e + f*x))/4 - (9*sin(e + f*x)*(e
 + f*x))/4)/(3*a^2*c^2*f*sin(e + f*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{4\cos(fx+e)\sin(fx+e)^2-\cos(fx+e)+3\sin(fx+e)^3fx}{3\sin(fx+e)^3a^2c^2f}$$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

output
```
(4*cos(e + f*x)*sin(e + f*x)**2 - cos(e + f*x) + 3*sin(e + f*x)**3*f*x)/(3
*sin(e + f*x)**3*a**2*c**2*f)
```

3.28 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 98

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{x}{a^2c^3}+\frac{\cot^5(e+fx)(1+\sec(e+fx))}{5a^2c^3f} - \frac{\cot^3(e+fx)(5+4\sec(e+fx))}{15a^2c^3f}+\frac{\cot(e+fx)(15+8\sec(e+fx))}{15a^2c^3f}$$

output
```
x/a^2/c^3+1/5*cot(f*x+e)^5*(1+sec(f*x+e))/a^2/c^3/f-1/15*cot(f*x+e)^3*(5+4
*sec(f*x+e))/a^2/c^3/f+1/15*cot(f*x+e)*(15+8*sec(f*x+e))/a^2/c^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)+15\sec(e+fx)-20\sec^3(e+fx)+8\right.}{15a^2c^3f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

output
```
(Cot[e + f*x]^5*(3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] + 15*
Sec[e + f*x] - 20*Sec[e + f*x]^3 + 8*Sec[e + f*x]^5))/(15*a^2*c^3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\int -\cot^4(e+fx)(4\sec(e+fx)a+5a)dx-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{1}{5}\int \cot^4(e+fx)(4\sec(e+fx)a+5a)dx-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{5}\int \frac{4\csc\left(e+fx+\frac{\pi}{2}\right)a+5a}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}-\frac{1}{3}\int -\cot^2(e+fx)(8\sec(e+fx)a+15a)dx\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \cot^2(e+fx)(8\sec(e+fx)a+15a)dx+\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{8\csc\left(e+fx+\frac{\pi}{2}\right)a+15a}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx+\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\int -15adx-\frac{\cot(e+fx)(8a\sec(e+fx)+15a)}{f}\right)+\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{24}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(4a\sec(e+fx)+5a)}{3f}+\frac{1}{3}\left(-\frac{\cot(e+fx)(8a\sec(e+fx)+15a)}{f}-15ax\right)\right)-\frac{\cot^5(e+fx)(a\sec(e+fx)+a)}{5f}}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

output
```
-((-1/5*(Cot[e + f*x]^5*(a + a*Sec[e + f*x]))/f + ((Cot[e + f*x]^3*(5*a +
4*a*Sec[e + f*x]))/(3*f) + (-15*a*x - (Cot[e + f*x]*(15*a + 8*a*Sec[e + f*
x]))/f)/3)/5)/(a^3*c^3))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+240fx+240\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{240f\,a^{2}c^{3}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^{3}a^{2}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{16}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^{3}a^{2}}$
risch	$\frac{x}{a^{2}c^{3}}+\frac{2i\left(15\,e^{7i(fx+e)}+15\,e^{6i(fx+e)}-65\,e^{5i(fx+e)}+25\,e^{4i(fx+e)}+73\,e^{3i(fx+e)}-31\,e^{2i(fx+e)}-31\,e^{i(fx+e)}+23\right)}{15f\,c^{3}a^{2}\left(e^{i(fx+e)}-1\right)^{5}\left(e^{i(fx+e)}+1\right)^{3}}$
norman	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{acf}+\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{ca}+\frac{1}{80acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8acf}-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{48acf}}{a\,c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/240*(3*cot(1/2*f*x+1/2*e)^5-30*cot(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)
^3+240*f*x+240*cot(1/2*f*x+1/2*e)-90*tan(1/2*f*x+1/2*e))/f/a^2/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$$
$$= \frac{23\cos(fx+e)^4-8\cos(fx+e)^3-27\cos(fx+e)^2+15\left(fx\cos(fx+e)^3-fx\cos(fx+e)^2-fx\cos\ldots\right.}{15\left(a^2c^3f\cos(fx+e)^3-a^2c^3f\cos(fx+e)^2-a^2c^3f\cos(fx+e)+a\ldots\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(23*cos(f*x + e)^4 - 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*c
os(f*x + e)^3 - f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) + f*x)*sin(f*x + e)
+ 7*cos(f*x + e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^
2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sec^5(e+fx)-\sec^4(e+fx)-2\sec^3(e+fx)+2\sec^2(e+fx)+\sec(e+fx)-1}\,dx}{a^2c^3}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(1/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec
(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{5\left(\frac{18\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^3}-\frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^3}+\frac{3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{80\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{a^2c^3\sin(fx+e)^5}}{240f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/240*(5*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2*c^3) - 480*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2*c^
3) + 3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 80*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\frac{240\,(fx+e)}{a^2c^3}+\frac{3\left(80\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-10\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}+\frac{5\left(a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-18\,a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6c^9}}{240\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)/(a^2*c^3) + 3*(80*tan(1/2*f*x + 1/2*e)^4 - 10*tan(1/2
*f*x + 1/2*e)^2 + 1)/(a^2*c^3*tan(1/2*f*x + 1/2*e)^5) + 5*(a^4*c^6*tan(1/2
*f*x + 1/2*e)^3 - 18*a^4*c^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{3\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8+5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-90\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-}{240\,a^2\,c^3\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\right.}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

output
```
(3*cos(e/2 + (f*x)/2)^8 + 5*sin(e/2 + (f*x)/2)^8 - 90*cos(e/2 + (f*x)/2)^2
*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 30
*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 + 240*cos(e/2 + (f*x)/2)^3*sin(
e/2 + (f*x)/2)^5*(e + f*x))/(240*a^2*c^3*f*cos(e/2 + (f*x)/2)^3*sin(e/2 +
(f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5fx+240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+3}{240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5a^2c^3f}$$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

output
```
(5*tan((e + f*x)/2)**8 - 90*tan((e + f*x)/2)**6 + 240*tan((e + f*x)/2)**5*
f*x + 240*tan((e + f*x)/2)**4 - 30*tan((e + f*x)/2)**2 + 3)/(240*tan((e +
f*x)/2)**5*a**2*c**3*f)
```

3.29 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 166

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{x}{a^2c^4}+\frac{\cot(e+fx)}{a^2c^4f}-\frac{\cot^3(e+fx)}{3a^2c^4f}+\frac{\cot^5(e+fx)}{5a^2c^4f}-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{2\csc(e+fx)}{a^2c^4f}-\frac{2\csc^3(e+fx)}{a^2c^4f}+\frac{6\csc^5(e+fx)}{5a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}$$

output
```
x/a^2/c^4+cot(f*x+e)/a^2/c^4/f-1/3*cot(f*x+e)^3/a^2/c^4/f+1/5*cot(f*x+e)^5
/a^2/c^4/f-2/7*cot(f*x+e)^7/a^2/c^4/f+2*csc(f*x+e)/a^2/c^4/f-2*csc(f*x+e)^
3/a^2/c^4/f+6/5*csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^7(e+fx)\left(5+5\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(e+fx)\right)+70\sec(e+fx)-140\sec^3(e+\right.}{35a^2c^4f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

output
```
-1/35*(Cot[e + f*x]^7*(5 + 5*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[e + f*x
]^2] + 70*Sec[e + f*x] - 140*Sec[e + f*x]^3 + 112*Sec[e + f*x]^5 - 32*Sec[
e + f*x]^7))/(a^2*c^4*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^2dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{4374}$$

$$\frac{\int \left(a^2\cot^8(e+fx)+2a^2\csc(e+fx)\cot^7(e+fx)+a^2\csc^2(e+fx)\cot^6(e+fx)\right)dx}{a^4c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2a^2\cot^7(e+fx)}{7f}+\frac{a^2\cot^5(e+fx)}{5f}-\frac{a^2\cot^3(e+fx)}{3f}+\frac{a^2\cot(e+fx)}{f}-\frac{2a^2\csc^7(e+fx)}{7f}+\frac{6a^2\csc^5(e+fx)}{5f}-\frac{2a^2\csc^3(e+fx)}{f}+2a^2\text{cs}}{a^4c^4}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

output
```
(a^2*x + (a^2*Cot[e + f*x])/f - (a^2*Cot[e + f*x]^3)/(3*f) + (a^2*Cot[e +
f*x]^5)/(5*f) - (2*a^2*Cot[e + f*x]^7)/(7*f) + (2*a^2*Csc[e + f*x])/f - (2
*a^2*Csc[e + f*x]^3)/f + (6*a^2*Csc[e + f*x]^5)/(5*f) - (2*a^2*Csc[e + f*x
]^7)/(7*f))/(a^4*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{-15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+147\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-770\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3360fx+4410\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-735\tan\left(\frac{fx}{2}\right.}{3360f\,a^{2}c^{4}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{42}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{32f\,c^{4}a^{2}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{7}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{22}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{42}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+64\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{32f\,c^{4}a^{2}}$
risch	$\frac{x}{a^{2}c^{4}}+\frac{2i\left(210\,e^{9i(fx+e)}-315\,e^{8i(fx+e)}-420\,e^{7i(fx+e)}+1470\,e^{6i(fx+e)}-504\,e^{5i(fx+e)}-1204\,e^{4i(fx+e)}+1108\,e^{3i(fx\right.}{105f\,c^{4}a^{2}\left(e^{i(fx+e)}-1\right)^{7}\left(e^{i(fx+e)}+1\right)^{3}}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{ca}-\frac{1}{224acf}+\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{160acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{48acf}+\frac{21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16acf}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{32acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{96acf}}{a\,c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3360*(-15*cot(1/2*f*x+1/2*e)^7+147*cot(1/2*f*x+1/2*e)^5-770*cot(1/2*f*x+
1/2*e)^3+35*tan(1/2*f*x+1/2*e)^3+3360*f*x+4410*cot(1/2*f*x+1/2*e)-735*tan(
1/2*f*x+1/2*e))/f/a^2/c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{191\cos(fx+e)^5-172\cos(fx+e)^4-253\cos(fx+e)^3+258\cos(fx+e)^2+105\left(fx\cos(fx+e)^4\right.}{105\left(a^2c^4f\cos(fx+e)^4-2\,a^2c^4f\cos(fx+e)^3+2\,a^2c^4\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/105*(191*cos(f*x + e)^5 - 172*cos(f*x + e)^4 - 253*cos(f*x + e)^3 + 258*
cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*co
s(f*x + e) - f*x)*sin(f*x + e) + 87*cos(f*x + e) - 96)/((a^2*c^4*f*cos(f*x
 + e)^4 - 2*a^2*c^4*f*cos(f*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*
f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{1}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(1/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**4 + 4*sec(
e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{\frac{35\left(\frac{21\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4}-\frac{6720\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^4}-\frac{\left(\frac{147\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{770\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4410\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+\cdots}{a^2c^4\sin(fx+e)^7}}{3360\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/3360*(35*(21*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/(a^2*c^4) - 6720*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^2
*c^4) - (147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 770*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 4410*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x
 + e) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{\frac{3360\,(fx+e)}{a^2c^4} + \frac{4410\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-770\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+147\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15}{a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7} + \frac{35\left(a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-21\,a^4c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{a^6c^{12}}}{3360\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/3360*(3360*(f*x + e)/(a^2*c^4) + (4410*tan(1/2*f*x + 1/2*e)^6 - 770*tan(
1/2*f*x + 1/2*e)^4 + 147*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^2*c^4*tan(1/2*f*x
 + 1/2*e)^7) + 35*(a^4*c^8*tan(1/2*f*x + 1/2*e)^3 - 21*a^4*c^8*tan(1/2*f*x
 + 1/2*e))/(a^6*c^12))/f
```

Mupad [B] (verification not implemented)

Time = 11.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{35\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-15\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-735\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8+4410\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\right.}{3360\,a^2\,c^4\,f\,c}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

output
```
(35*sin(e/2 + (f*x)/2)^10 - 15*cos(e/2 + (f*x)/2)^10 - 735*cos(e/2 + (f*x)
/2)^2*sin(e/2 + (f*x)/2)^8 + 4410*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^
6 - 770*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^4 + 147*cos(e/2 + (f*x)/2)
^8*sin(e/2 + (f*x)/2)^2 + 3360*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7*(
e + f*x))/(3360*a^2*c^4*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10} - 735\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8} + 3360\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7} fx + 4410\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6} - 770\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3360\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7} a^2c^4f}$$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

output
```
(35*tan((e + f*x)/2)**10 - 735*tan((e + f*x)/2)**8 + 3360*tan((e + f*x)/2)
**7*f*x + 4410*tan((e + f*x)/2)**6 - 770*tan((e + f*x)/2)**4 + 147*tan((e
+ f*x)/2)**2 - 15)/(3360*tan((e + f*x)/2)**7*a**2*c**4*f)
```

3.30 $\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 210

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx &= \frac{x}{a^2c^5}+\frac{\cot(e+fx)}{a^2c^5f}-\frac{\cot^3(e+fx)}{3a^2c^5f}+\frac{\cot^5(e+fx)}{5a^2c^5f}-\frac{\cot^7(e+fx)}{7a^2c^5f}\\ &\quad+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{3\csc(e+fx)}{a^2c^5f}-\frac{13\csc^3(e+fx)}{3a^2c^5f}\\ &\quad+\frac{21\csc^5(e+fx)}{5a^2c^5f}-\frac{15\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}\end{aligned}$$

output
```
x/a^2/c^5+cot(f*x+e)/a^2/c^5/f-1/3*cot(f*x+e)^3/a^2/c^5/f+1/5*cot(f*x+e)^5
/a^2/c^5/f-1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+3*csc(f*x
+e)/a^2/c^5/f-13/3*csc(f*x+e)^3/a^2/c^5/f+21/5*csc(f*x+e)^5/a^2/c^5/f-15/7
*csc(f*x+e)^7/a^2/c^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\cot^9(e+fx)\left(105+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+945\sec(e+fx)-2415\sec^3(\right.}{315a^2c^5f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

output
```
(Cot[e + f*x]^9*(105 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2
] + 945*Sec[e + f*x] - 2415*Sec[e + f*x]^3 + 2898*Sec[e + f*x]^5 - 1656*Se
c[e + f*x]^7 + 368*Sec[e + f*x]^9))/(315*a^2*c^5*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^{10}(e+fx)(\sec(e+fx)a+a)^3dx}{a^5c^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}dx}{a^5c^5}$$

$\Big\downarrow$ 4374

$$-\frac{\int \left(a^3\cot^{10}(e+fx)+3a^3\csc(e+fx)\cot^9(e+fx)+3a^3\csc^2(e+fx)\cot^8(e+fx)+a^3\csc^3(e+fx)\cot^7(e+\right.}{a^5c^5}$$

$\Big\downarrow$ 2009

$$-\frac{-\frac{4a^3\cot^9(e+fx)}{9f}+\frac{a^3\cot^7(e+fx)}{7f}-\frac{a^3\cot^5(e+fx)}{5f}+\frac{a^3\cot^3(e+fx)}{3f}-\frac{a^3\cot(e+fx)}{f}-\frac{4a^3\csc^9(e+fx)}{9f}+\frac{15a^3\csc^7(e+fx)}{7f}-\frac{21a}{}}{a^5c^5}$$

input
```
Int[1/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

output
```
-((-(a^3*x) - (a^3*Cot[e + f*x])/f + (a^3*Cot[e + f*x]^3)/(3*f) - (a^3*Cot
[e + f*x]^5)/(5*f) + (a^3*Cot[e + f*x]^7)/(7*f) - (4*a^3*Cot[e + f*x]^9)/(
9*f) - (3*a^3*Csc[e + f*x])/f + (13*a^3*Csc[e + f*x]^3)/(3*f) - (21*a^3*Cs
c[e + f*x]^5)/(5*f) + (15*a^3*Csc[e + f*x]^7)/(7*f) - (4*a^3*Csc[e + f*x]^
9)/(9*f))/(a^5*c^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.50

method	result
parallelrisch	$\frac{35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-360\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+1827\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-6720\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+20160fx-2520\tan}{20160f\,a^{2}c^{5}}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{64}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{99}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+128\,\mathrm{ar}}{64f\,c^{5}a^{2}}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{8}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{29}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{64}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{99}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+128\,\mathrm{ar}}{64f\,c^{5}a^{2}}$
risch	$\frac{x}{a^{2}c^{5}}+\frac{2i\left(945\,\mathrm{e}^{11i(fx+e)}-3150\,\mathrm{e}^{10i(fx+e)}+2625\,\mathrm{e}^{9i(fx+e)}+6300\,\mathrm{e}^{8i(fx+e)}-13482\,\mathrm{e}^{7i(fx+e)}+5292\,\mathrm{e}^{6i(fx+e)}+1056\right.}{315f\,c^{5}a^{2}\left(\mathrm{e}^{i(fx+e)}-1\right)^{9}\left(\mathrm{e}^{i(fx+\right.}}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{ca}+\frac{1}{576acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{56acf}+\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{320acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{3acf}+\frac{99\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{192acf}}{a\,c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/20160*(35*cot(1/2*f*x+1/2*e)^9-360*cot(1/2*f*x+1/2*e)^7+1827*cot(1/2*f*x
+1/2*e)^5+105*tan(1/2*f*x+1/2*e)^3-6720*cot(1/2*f*x+1/2*e)^3+20160*f*x-252
0*tan(1/2*f*x+1/2*e)+31185*cot(1/2*f*x+1/2*e))/f/a^2/c^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{668\cos(fx+e)^6 - 1059\cos(fx+e)^5 - 573\cos(fx+e)^4 + 1813\cos(fx+e)^3 - 393\cos(fx+e)^2 + \cdots}{315\left(a^2c^5f\cos(fx+e)^5 - 3a^2c^5f\cos(fx+e)\cdots\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(668*cos(f*x + e)^6 - 1059*cos(f*x + e)^5 - 573*cos(f*x + e)^4 + 181
3*cos(f*x + e)^3 - 393*cos(f*x + e)^2 + 315*(f*x*cos(f*x + e)^5 - 3*f*x*co
s(f*x + e)^4 + 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 - 3*f*x*cos(f*x
 + e) + f*x)*sin(f*x + e) - 789*cos(f*x + e) + 368)/((a^2*c^5*f*cos(f*x +
e)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5
*f*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$$
$$= -\frac{\int \frac{1}{\sec^7(e+fx)-3\sec^6(e+fx)+\sec^5(e+fx)+5\sec^4(e+fx)-5\sec^3(e+fx)-\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{a^2c^5}$$

input
```
integrate(1/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

output
```
-Integral(1/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)**5 + 5*sec
(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*x) - 1),
x)/(a**2*c**5)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = -\frac{\frac{105\left(\frac{24\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^5} - \frac{40320\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2c^5} + \frac{\left(\frac{360\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1827\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{31185\sin(f}{(\cos(fx+e)}\right.}{a^2c^5\sin(fx+e)^9}}{20160\,f}$$

input

```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output

```
-1/20160*(105*(24*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^2*c^5) - 40320*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(
a^2*c^5) + (360*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1827*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 6720*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 31185*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(
f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\frac{20160\,(fx+e)}{a^2c^5} + \frac{31185\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-6720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+1827\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-360\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+35}{a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9} + \frac{105\left(a^4c^{10}\tan\left(\frac{1}{2}fx+\right.\right.}{}}{20160\,f}$$

input

```
integrate(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output

```
1/20160*(20160*(f*x + e)/(a^2*c^5) + (31185*tan(1/2*f*x + 1/2*e)^8 - 6720*
tan(1/2*f*x + 1/2*e)^6 + 1827*tan(1/2*f*x + 1/2*e)^4 - 360*tan(1/2*f*x + 1
/2*e)^2 + 35)/(a^2*c^5*tan(1/2*f*x + 1/2*e)^9) + 105*(a^4*c^10*tan(1/2*f*x
 + 1/2*e)^3 - 24*a^4*c^10*tan(1/2*f*x + 1/2*e))/(a^6*c^15))/f
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+105\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-2520\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+31185\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}\right.}{}$$

input
```
int(1/((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

output
```
(35*cos(e/2 + (f*x)/2)^12 + 105*sin(e/2 + (f*x)/2)^12 - 2520*cos(e/2 + (f*
x)/2)^2*sin(e/2 + (f*x)/2)^10 + 31185*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)
/2)^8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1827*cos(e/2 + (f
*x)/2)^8*sin(e/2 + (f*x)/2)^4 - 360*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/
2)^2 + 20160*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9*(e + f*x))/(20160*a
^2*c^5*f*cos(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-2520\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+20160\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}fx+31185\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-6720\tan\left(\right.}{20160\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}a^2c^5f}$$

input
```
int(1/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

output
```
(105*tan((e + f*x)/2)**12 - 2520*tan((e + f*x)/2)**10 + 20160*tan((e + f*x
)/2)**9*f*x + 31185*tan((e + f*x)/2)**8 - 6720*tan((e + f*x)/2)**6 + 1827*
tan((e + f*x)/2)**4 - 360*tan((e + f*x)/2)**2 + 35)/(20160*tan((e + f*x)/2
)**9*a**2*c**5*f)
```

3.31 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 162

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx &= \frac{c^5x}{a^3}+\frac{8c^5\operatorname{arctanh}(\sin(e+fx))}{a^3f}+\frac{32c^5\cot(e+fx)}{a^3f}\\&\quad+\frac{128c^5\cot^3(e+fx)}{3a^3f}+\frac{128c^5\cot^5(e+fx)}{5a^3f}\\&\quad-\frac{16c^5\csc(e+fx)}{a^3f}+\frac{64c^5\csc^3(e+fx)}{3a^3f}\\&\quad-\frac{128c^5\csc^5(e+fx)}{5a^3f}-\frac{c^5\tan(e+fx)}{a^3f}\end{aligned}$$

output

```
c^5*x/a^3+8*c^5*arctanh(sin(f*x+e))/a^3/f+32*c^5*cot(f*x+e)/a^3/f+128/3*c^
5*cot(f*x+e)^3/a^3/f+128/5*c^5*cot(f*x+e)^5/a^3/f-16*c^5*csc(f*x+e)/a^3/f+
64/3*c^5*csc(f*x+e)^3/a^3/f-128/5*c^5*csc(f*x+e)^5/a^3/f-c^5*tan(f*x+e)/a^
3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.54 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.30

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{c^{9/2}\tan(e+fx)\left(-46\sqrt{a}\sqrt{c}-48\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{}\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

output
```
-1/15*(c^(9/2)*Tan[e + f*x]*(-46*Sqrt[a]*Sqrt[c] - 48*Sqrt[2]*Sqrt[a]*Sqrt
[c]*Hypergeometric2F1[-7/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec
[e + f*x]] - 24*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2, -5/2, -3/2
, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] - 2*Sqrt[a]*Sqrt[c]*Sec[e +
 f*x] + 22*Sqrt[a]*Sqrt[c]*Sec[e + f*x]^2 + 26*Sqrt[a]*Sqrt[c]*Sec[e + f*x
]^3 + 15*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sqrt[-(a*c
*Tan[e + f*x]^2)] + 30*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c
])]*Sec[e + f*x]*Sqrt[-(a*c*Tan[e + f*x]^2)] + 15*ArcTanh[Sqrt[-(a*c*Tan[e
 + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Sec[e + f*x]^2*Sqrt[-(a*c*Tan[e + f*x]^2)])
)/(a^(7/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^8dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^8}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(\cot^6(e+fx)c^8+28\csc^6(e+fx)c^8-56\cot(e+fx)\csc^5(e+fx)c^8+70\cot^2(e+fx)\csc^4(e+fx)c^8-56\,c\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{8c^8\text{arctanh}(\sin(e+fx))}{f}+\frac{c^8\tan(e+fx)}{f}-\frac{128c^8\cot^5(e+fx)}{5f}-\frac{128c^8\cot^3(e+fx)}{3f}-\frac{32c^8\cot(e+fx)}{f}+\frac{128c^8\csc^5(e+fx)}{5f}-\frac{64c^8}{}}{a^3c^3}$$

input
```
Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((-(c^8*x) - (8*c^8*ArcTanh[Sin[e + f*x]])/f - (32*c^8*Cot[e + f*x])/f -
(128*c^8*Cot[e + f*x]^3)/(3*f) - (128*c^8*Cot[e + f*x]^5)/(5*f) + (16*c^8*
Csc[e + f*x])/f - (64*c^8*Csc[e + f*x]^3)/(3*f) + (128*c^8*Csc[e + f*x]^5)
/(5*f) + (c^8*Tan[e + f*x])/f)/(a^3*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{8c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}-\ln\left(\tan\ldots\right.\right.}{f\,a^3}$
default	$\frac{8c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+8}+\frac{1}{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8}-\ln\left(\tan\ldots\right.\right.}{f\,a^3}$
parallelrisch	$-\frac{2113c^5\left(\frac{1920\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)}{2113}-\frac{1920\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)}{2113}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{954\cos(2f\ldots}{2113}\right.\right.}{240\cos(fx+e)a^3f}$
risch	$\frac{c^5x}{a^3}-\frac{2ic^5\left(240\,\mathrm{e}^{6i(fx+e)}+735\,\mathrm{e}^{5i(fx+e)}+1835\,\mathrm{e}^{4i(fx+e)}+1750\,\mathrm{e}^{3i(fx+e)}+1894\,\mathrm{e}^{2i(fx+e)}+955\,\mathrm{e}^{i(fx+e)}+239\right)}{15fa^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{2i(fx+e)}+1\right)}+\ldots$
norman	$\frac{\frac{c^5x}{a}+\frac{c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{a}+\frac{6c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}-\frac{4c^5x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{a}-\frac{18c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{202c^5\tan\left(\frac{fx}{2}+\ldots\right.}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\ldots\right.}$

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
8/f*c^5/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*
f*x+1/2*e)+ln(tan(1/2*f*x+1/2*e)+1)+1/8/(tan(1/2*f*x+1/2*e)+1)+1/8/(tan(1/
2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)-1)+1/4*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.78

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{15\,c^5fx\cos\left(fx+e\right)^4+45\,c^5fx\cos\left(fx+e\right)^3+45\,c^5fx\cos\left(fx+e\right)^2+15\,c^5fx\cos\left(fx+e\right)+60\,\left(c^5\cos\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*c^5*f*x*cos(f*x + e)^4 + 45*c^5*f*x*cos(f*x + e)^3 + 45*c^5*f*x*c
os(f*x + e)^2 + 15*c^5*f*x*cos(f*x + e) + 60*(c^5*cos(f*x + e)^4 + 3*c^5*c
os(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*log(sin(f*x + e)
+ 1) - 60*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^
2 + c^5*cos(f*x + e))*log(-sin(f*x + e) + 1) - (239*c^5*cos(f*x + e)^3 + 4
77*c^5*cos(f*x + e)^2 + 349*c^5*cos(f*x + e) + 15*c^5)*sin(f*x + e))/(a^3*
f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f
*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^5\left(\int \frac{5\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{10\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{}{\sec^3\left(e+fx\right)}\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

output
```
-c**5*(Integral(5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*se
c(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e + f*x)**3/(sec
(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-5*s
ec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),
 x) + Integral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3
*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(154) = 308$.

Time = 0.13 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.47

$$\int \frac{(c - c\sec(e + fx))^5}{(a + a\sec(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(3*c^5*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 -
 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)/a^3) + 5*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) +
20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f
*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + c^5*((105*sin(f*x + e)/(cos(f*x + e
) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 120*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 10
*c^5*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 5*c^5*(15*sin(f*x +
 e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 30*c^5*(5*sin(f*x + e)/(cos(f*x + e)
+ 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)c^5}{a^3}+\frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{120\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{30\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3}-\frac{8\left(3\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{15\,f}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*c^5/a^3 + 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^
3 - 120*c^5*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^5*tan(1/2*f*x +
1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 8*(3*a^12*c^5*tan(1/2*f*x + 1/
2*e)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1/2*f*x + 1/2
*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 10.99 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{c^5\,x}{a^3}-\frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}-\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f}-\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}+\frac{16\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}+\frac{2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

input
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^5*x)/a^3 - (16*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (8*c^5*tan(e/2 + (f*x)
/2)^3)/(3*a^3*f) - (8*c^5*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) + (16*c^5*atanh(
tan(e/2 + (f*x)/2)))/(a^3*f) + (2*c^5*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2
+ (f*x)/2)^2 - a^3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.07

$$\int \frac{(c - c\sec(e + fx))^5}{(a + a\sec(e + fx))^3}\,dx$$
$$= \frac{c^5\left(-120\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+120\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+120\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right.}{}$$

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

output
```
(c**5*( - 120*log(tan((e + f*x)/2) - 1)*tan((e + f*x)/2)**2 + 120*log(tan(
(e + f*x)/2) - 1) + 120*log(tan((e + f*x)/2) + 1)*tan((e + f*x)/2)**2 - 12
0*log(tan((e + f*x)/2) + 1) - 24*tan((e + f*x)/2)**7 - 16*tan((e + f*x)/2)
**5 - 200*tan((e + f*x)/2)**3 + 15*tan((e + f*x)/2)**2*f*x + 270*tan((e +
f*x)/2) - 15*f*x))/(15*a**3*f*(tan((e + f*x)/2)**2 - 1))
```

3.32 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4x}{a^3}+\frac{c^4\text{arctanh}(\sin(e+fx))}{a^3f} - \frac{3c^4\tan(e+fx)}{a^3f(1+\sec(e+fx))^3}-\frac{c^4\sec^2(e+fx)\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{14c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2}-\frac{23c^4\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

output
```
c^4*x/a^3+c^4*arctanh(sin(f*x+e))/a^3/f-3*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+
e))^3-1/5*c^4*sec(f*x+e)^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+14/5*c^4*tan(
f*x+e)/a^3/f/(1+sec(f*x+e))^2-23/5*c^4*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{2c^{7/2}\tan(e+fx)\left(12\sqrt{2}\sqrt{a}\sqrt{c}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{1-\sec(e+fx}\right.}{1}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

output
```
(2*c^(7/2)*Tan[e + f*x]*(12*Sqrt[2]*Sqrt[a]*Sqrt[c]*Hypergeometric2F1[-5/2
, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[1 - Sec[e + f*x]] + Sqrt[a]*Sqrt[
c]*(23 + Sec[e + f*x] - 11*Sec[e + f*x]^2 - 13*Sec[e + f*x]^3) - 30*ArcTan
h[Sqrt[-(a*c*Tan[e + f*x]^2)]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e
+ f*x]^2*Sqrt[-(a*c*Tan[e + f*x]^2)]))/(15*a^(7/2)*f*(-1 + Sec[e + f*x])*(
1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{\sec^4(e+fx)c^4}{(\sec(e+fx)+1)^3}-\frac{4\sec^3(e+fx)c^4}{(\sec(e+fx)+1)^3}+\frac{6\sec^2(e+fx)c^4}{(\sec(e+fx)+1)^3}-\frac{4\sec(e+fx)c^4}{(\sec(e+fx)+1)^3}+\frac{c^4}{(\sec(e+fx)+1)^3}\right)dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c^4\text{arctanh}(\sin(e+fx))}{f}-\frac{c^4\tan(e+fx)\sec^2(e+fx)}{5f(\sec(e+fx)+1)^3}-\frac{23c^4\tan(e+fx)}{5f(\sec(e+fx)+1)}+\frac{14c^4\tan(e+fx)}{5f(\sec(e+fx)+1)^2}-\frac{3c^4\tan(e+fx)}{f(\sec(e+fx)+1)^3}+c^4x}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c^4*x + (c^4*ArcTanh[Sin[e + f*x]])/f - (3*c^4*Tan[e + f*x])/(f*(1 + Sec[
e + f*x])^3) - (c^4*Sec[e + f*x]^2*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3
) + (14*c^4*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^2) - (23*c^4*Tan[e + f*x
])/(5*f*(1 + Sec[e + f*x])))/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

method	result
parallelrisch	$-\frac{c^{4}\left(4 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-5 f x+20 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)-5 \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)+5 \ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)\right)}{5 a^{3} f}$
derivativedivides	$\frac{4 c^{4}\left(-\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}-\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f a^{3}}$
default	$\frac{4 c^{4}\left(-\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}-\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{\arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2}+\frac{\ln \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{f a^{3}}$
risch	$\frac{c^{4} x}{a^{3}}-\frac{16 i c^{4}\left(5 \mathrm{e}^{4 i (f x+e)}+10 \mathrm{e}^{3 i (f x+e)}+20 \mathrm{e}^{2 i (f x+e)}+10 \mathrm{e}^{i (f x+e)}+3\right)}{5 f a^{3}\left(\mathrm{e}^{i (f x+e)}+1\right)^{5}}+\frac{c^{4} \ln \left(\mathrm{e}^{i (f x+e)}+i\right)}{a^{3} f}-\frac{c^{4} \ln \left(\mathrm{e}^{i (f x+e)}-i\right)}{a^{3} f}$
norman	$\frac{\frac{c^{4} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{a}-\frac{c^{4} x}{a}+\frac{4 c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{a f}-\frac{12 c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{a f}+\frac{64 c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5 a f}-\frac{32 c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{5 a f}+\frac{12 c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{5 a f} + \cdots}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3} a^{2}}$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*c^4*(4*tan(1/2*f*x+1/2*e)^5-5*f*x+20*tan(1/2*f*x+1/2*e)-5*ln(tan(1/2*
f*x+1/2*e)+1)+5*ln(tan(1/2*f*x+1/2*e)-1))/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.64

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{10\,c^4 fx\cos{(fx+e)}^3 + 30\,c^4 fx\cos{(fx+e)}^2 + 30\,c^4 fx\cos{(fx+e)} + 10\,c^4 fx + 5\left(c^4\cos{(fx+e)}^3 + 3\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/10*(10*c^4*f*x*cos(f*x + e)^3 + 30*c^4*f*x*cos(f*x + e)^2 + 30*c^4*f*x*c
os(f*x + e) + 10*c^4*f*x + 5*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 +
3*c^4*cos(f*x + e) + c^4)*log(sin(f*x + e) + 1) - 5*(c^4*cos(f*x + e)^3 +
3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*log(-sin(f*x + e) + 1) -
16*(3*c^4*cos(f*x + e)^2 + 4*c^4*cos(f*x + e) + 3*c^4)*sin(f*x + e))/(a^3*
f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^4\left(\int\left(-\frac{4\sec{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\right)dx + \int\frac{6\sec^2{(e+fx)}}{\sec^3{(e+fx)}+3\sec^2{(e+fx)}+3\sec{(e+fx)}+1}\,dx + \int\left(-\frac{}{\sec^3{(e+fx}}\right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*x)**3/(sec(e
 + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e
+ f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) +
 Integral(1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x)
)/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(142) = 284$.

Time = 0.13 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.68

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^3}\,dx =$$

$$-\frac{c^4\left(\frac{\frac{105\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+c^4\left(\frac{\frac{105\,\sin(fx+e)}{\cos(fx+e)+1}-\cdots}{}\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(
f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/a^3) + c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 12
0*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 4*c^4*(15*sin(f*x + e)/(c
os(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5)/a^3 + 4*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) -
 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5)/a^3 - 18*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(c
os(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{5\,(fx+e)c^4}{a^3}+\frac{5\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{5\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{4\left(a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{5\,f}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/5*(5*(f*x + e)*c^4/a^3 + 5*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 -
5*c^4*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 4*(a^12*c^4*tan(1/2*f*x + 1
/2*e)^5 + 5*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 11.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.34

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4\left(2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}+f\,x\right)}{a^3\,f}$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^4*(2*atanh(tan(e/2 + (f*x)/2)) - 4*tan(e/2 + (f*x)/2) - (4*tan(e/2 + (f
*x)/2)^5)/5 + f*x))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.46

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4\left(-5\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+5\log\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5fx\right)}{5a^3f}$$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

output
```
(c**4*( - 5*log(tan((e + f*x)/2) - 1) + 5*log(tan((e + f*x)/2) + 1) - 4*ta
n((e + f*x)/2)**5 - 20*tan((e + f*x)/2) + 5*f*x))/(5*a**3*f)
```

3.33 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{c^3x}{a^3} - \frac{8c^3\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} + \frac{4c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))^2} - \frac{26c^3\tan(e+fx)}{15a^3f(1+\sec(e+fx))}$$

output
```
c^3*x/a^3-8/5*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3+4/15*c^3*tan(f*x+e)/a^
3/f/(1+sec(f*x+e))^2-26/15*c^3*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(-\frac{2\arctan\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} + \frac{2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f} - \frac{2\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f} + \frac{2\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}\right)}{a^3}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

output
```
-((c^3*((-2*ArcTan[Tan[e/2 + (f*x)/2]])/f + (2*Tan[e/2 + (f*x)/2])/f - (2*
Tan[e/2 + (f*x)/2]^3)/(3*f) + (2*Tan[e/2 + (f*x)/2]^5)/(5*f)))/a^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sec(e+fx))^3}{(a\sec(e+fx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(c - c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \text{4391}$$
$$\frac{\int \left(-\frac{\sec^3(e+fx)c^3}{(\sec(e+fx)+1)^3} + \frac{3\sec^2(e+fx)c^3}{(\sec(e+fx)+1)^3} - \frac{3\sec(e+fx)c^3}{(\sec(e+fx)+1)^3} + \frac{c^3}{(\sec(e+fx)+1)^3}\right)dx}{a^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{26c^3\tan(e+fx)}{15f(\sec(e+fx)+1)} + \frac{4c^3\tan(e+fx)}{15f(\sec(e+fx)+1)^2} - \frac{8c^3\tan(e+fx)}{5f(\sec(e+fx)+1)^3} + c^3x}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c^3*x - (8*c^3*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3) + (4*c^3*Tan[e +
f*x])/(15*f*(1 + Sec[e + f*x])^2) - (26*c^3*Tan[e + f*x])/(15*f*(1 + Sec[e
 + f*x])))/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

method	result
parallelrisch	$-\frac{c^{3}\left(6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-15fx+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15a^{3}f}$
derivativedivides	$\frac{2c^{3}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{3}}$
default	$\frac{2c^{3}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{3}}$
risch	$\frac{c^{3}x}{a^{3}}-\frac{4ic^{3}\left(45\,{\rm e}^{4i(fx+e)}+90\,{\rm e}^{3i(fx+e)}+140\,{\rm e}^{2i(fx+e)}+70\,{\rm e}^{i(fx+e)}+23\right)}{15f\,a^{3}\left({\rm e}^{i(fx+e)}+1\right)^{5}}$
norman	$\frac{\frac{c^{3}x}{a}+\frac{c^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{a}+\frac{14c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}+\frac{22c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{15af}-\frac{2c^{3}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{a}-\frac{2c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa}-\frac{56c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15fa}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}a^{2}}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*c^3*(6*tan(1/2*f*x+1/2*e)^5-10*tan(1/2*f*x+1/2*e)^3-15*f*x+30*tan(1/
2*f*x+1/2*e))/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{15\,c^3fx\cos\left(fx+e\right)^3+45\,c^3fx\cos\left(fx+e\right)^2+45\,c^3fx\cos\left(fx+e\right)+15\,c^3fx-2\left(23\,c^3\cos\left(fx+e\right)^2-\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a\right.}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*c^3*f*x*cos(f*x + e)^3 + 45*c^3*f*x*cos(f*x + e)^2 + 45*c^3*f*x*c
os(f*x + e) + 15*c^3*f*x - 2*(23*c^3*cos(f*x + e)^2 + 24*c^3*cos(f*x + e)
+ 13*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3
*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{c^3\left(\int\frac{3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{}{\sec^3\left(e+fx\right)}\right.}{a^3}$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec
(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e +
 f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-1/(sec(
e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(90) = 180$.

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.89

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^3}\,dx =$$

$$-\frac{c^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right)+\frac{c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1}\right)}{a^3}}{60f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + c^3*(15*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5)/a^3 + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3
- 9*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^3}\,dx = \frac{\frac{15(fx+e)c^3}{a^3} - \frac{2\left(3a^{12}c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5 - 5a^{12}c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 15a^{12}c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}}}{15f}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*c^3/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 5*a^12
*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 11.64 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{c^3\,x}{a^3}-\frac{\frac{46\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15}-\frac{22\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15}+\frac{2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3}{5}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^3*x)/a^3 - ((2*c^3*sin(e/2 + (f*x)/2))/5 - (22*c^3*cos(e/2 + (f*x)/2)^2
*sin(e/2 + (f*x)/2))/15 + (46*c^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))
/15)/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.55

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{c^3\left(-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15fx\right)}{15a^3f}$$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

output
```
(c**3*( - 6*tan((e + f*x)/2)**5 + 10*tan((e + f*x)/2)**3 - 30*tan((e + f*x
)/2) + 15*f*x))/(15*a**3*f)
```

3.34 $\int \frac{(c-c \sec (e+f x))^2}{(a+a \sec (e+f x))^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(c-c \sec (e+f x))^2}{(a+a \sec (e+f x))^3} \, dx = \frac{c^2 x}{a^3}-\frac{4 c^2 \tan (e+f x)}{5 a^3 f (1+\sec (e+f x))^3}-\frac{8 c^2 \tan (e+f x)}{15 a^3 f (1+\sec (e+f x))^2}-\frac{23 c^2 \tan (e+f x)}{15 a^3 f (1+\sec (e+f x))}$$

output
```
c^2*x/a^3-4/5*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-8/15*c^2*tan(f*x+e)/a^
3/f/(1+sec(f*x+e))^2-23/15*c^2*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.57

$$\int \frac{(c-c \sec (e+f x))^2}{(a+a \sec (e+f x))^3} \, dx = -\frac{c^{3/2} \tan (e+f x) \left(\sqrt{a} \sqrt{c} (-43-11 \sec (e+f x)+31 \sec ^2(e+f x)+23 \sec ^3(e+f x))+60 \text{arctanh}\left(\frac{\sqrt{\cdot}}{\cdot}\right.\right.}{15 a^{7/2} f (-1+\sec (e+f x)) (1+\sec (e+f}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

output
```
-1/15*(c^(3/2)*Tan[e + f*x]*(Sqrt[a]*Sqrt[c]*(-43 - 11*Sec[e + f*x] + 31*S
ec[e + f*x]^2 + 23*Sec[e + f*x]^3) + 60*ArcTanh[Sqrt[-(a*c*Tan[e + f*x]^2)
]/(Sqrt[a]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^2*Sqrt[-(a*c*Tan[e +
f*x]^2)]))/(a^(7/2)*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \ 4391$$

$$\frac{\int \left(\frac{\sec^2(e+fx)c^2}{(\sec(e+fx)+1)^3}-\frac{2\sec(e+fx)c^2}{(\sec(e+fx)+1)^3}+\frac{c^2}{(\sec(e+fx)+1)^3}\right)dx}{a^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{23c^2\tan(e+fx)}{15f(\sec(e+fx)+1)}-\frac{8c^2\tan(e+fx)}{15f(\sec(e+fx)+1)^2}-\frac{4c^2\tan(e+fx)}{5f(\sec(e+fx)+1)^3}+c^2x}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c^2*x - (4*c^2*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3) - (8*c^2*Tan[e +
f*x])/(15*f*(1 + Sec[e + f*x])^2) - (23*c^2*Tan[e + f*x])/(15*f*(1 + Sec[e
 + f*x]))/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{c^{2} \left(3 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}-10 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}-15 f x +30 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{15 a^{3} f}$	54
derivativedivides	$\frac{c^{2} \left(-\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)+2 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,a^{3}}$	61
default	$\frac{c^{2} \left(-\frac{\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3}-2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)+2 \arctan \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)\right)\right)}{f \,a^{3}}$	61
risch	$\frac{c^{2} x}{a^{3}}-\frac{2 i c^{2} \left(75 \,{\mathrm e}^{4 i \left(f x +e \right)}+180 \,{\mathrm e}^{3 i \left(f x +e \right)}+250 \,{\mathrm e}^{2 i \left(f x +e \right)}+140 \,{\mathrm e}^{i \left(f x +e \right)}+43\right)}{15 f \,a^{3} \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{5}}$	81
norman	$\frac{\frac{c^{2} x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{a}-\frac{c^{2} x}{a}+\frac{2 c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{a f}-\frac{8 c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 a f}+\frac{13 c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}{15 a f}-\frac{c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}{5 a f}}{\left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) a^{2}}$	135

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*c^2*(3*tan(1/2*f*x+1/2*e)^5-10*tan(1/2*f*x+1/2*e)^3-15*f*x+30*tan(1/
2*f*x+1/2*e))/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{15\,c^2fx\cos\left(fx+e\right)^3+45\,c^2fx\cos\left(fx+e\right)^2+45\,c^2fx\cos\left(fx+e\right)+15\,c^2fx-\left(43\,c^2\cos\left(fx+e\right)^2+\right.}{15\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3\right.}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(15*c^2*f*x*cos(f*x + e)^3 + 45*c^2*f*x*cos(f*x + e)^2 + 45*c^2*f*x*c
os(f*x + e) + 15*c^2*f*x - (43*c^2*cos(f*x + e)^2 + 54*c^2*cos(f*x + e) +
23*c^2)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a
^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^2\left(\int\left(-\frac{2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{}{\sec^3\left(e+fx\right)+3}\right.}{a^3}$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*se
c(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(1/(sec(e + f*x)**3 + 3*sec(
e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(90) = 180$.

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.20

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^3}\,dx =$$

$$-\frac{c^2\left(\frac{\frac{105\,\sin(fx+e)}{\cos(fx+e)+1} - \frac{20\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{120\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right) + \frac{2\,c^2\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,\sin(fx+e}{(\cos(fx+e)+}\right.}{a^3}}{60\,f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c^2*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 2*c^2*(15*sin(f*x + e)/(cos(f*x +
e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5)/a^3 - 3*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^3}\,dx$$
$$= \frac{\frac{15\,(fx+e)c^2}{a^3} - \frac{3\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,a^{12}c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{15\,f}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/15*(15*(f*x + e)*c^2/a^3 - (3*a^12*c^2*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*
c^2*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^2*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{c^2\,x}{a^3}-\frac{\frac{43\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15}-\frac{16\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{15}+\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2}{5}}{a^3\,f\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^3,x)
```

output
```
(c^2*x)/a^3 - ((c^2*sin(e/2 + (f*x)/2))/5 - (16*c^2*cos(e/2 + (f*x)/2)^2*s
in(e/2 + (f*x)/2))/15 + (43*c^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/1
5)/(a^3*f*cos(e/2 + (f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.55

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{c^2\left(-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15fx\right)}{15a^3f}$$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

output
```
(c**2*( - 3*tan((e + f*x)/2)**5 + 10*tan((e + f*x)/2)**3 - 30*tan((e + f*x
)/2) + 15*f*x))/(15*a**3*f)
```

3.35 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{cx}{a^3} - \frac{2c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3} - \frac{3c\tan(e+fx)}{5a^3f(1+\sec(e+fx))^2} - \frac{8c\tan(e+fx)}{5a^3f(1+\sec(e+fx))}$$

output
```
c*x/a^3-2/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))^3-3/5*c*tan(f*x+e)/a^3/f/(1+
sec(f*x+e))^2-8/5*c*tan(f*x+e)/a^3/f/(1+sec(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{c\cot^5(e+fx)\left(16+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-60\sec(e+fx)+5\sec^2(e+f}{15a^3f}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c*Cot[e + f*x]^5*(16 + 3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2
] - 60*Sec[e + f*x] + 5*Sec[e + f*x]^2 + 60*Sec[e + f*x]^3 - 24*Sec[e + f*
x]^5))/(15*a^3*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4391, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c - c\sec(e + fx)}{(a\sec(e + fx) + a)^3} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c - c\csc\left(e + fx + \frac{\pi}{2}\right)}{\left(a\csc\left(e + fx + \frac{\pi}{2}\right) + a\right)^3} dx$$

$$\Big\downarrow \text{4391}$$

$$\frac{\int \left(\frac{c}{(\sec(e+fx)+1)^3} - \frac{c\sec(e+fx)}{(\sec(e+fx)+1)^3}\right) dx}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{8c\tan(e+fx)}{5f(\sec(e+fx)+1)} - \frac{3c\tan(e+fx)}{5f(\sec(e+fx)+1)^2} - \frac{2c\tan(e+fx)}{5f(\sec(e+fx)+1)^3} + cx}{a^3}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^3,x]
```

output
```
(c*x - (2*c*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])^3) - (3*c*Tan[e + f*x])/
(5*f*(1 + Sec[e + f*x])^2) - (8*c*Tan[e + f*x])/(5*f*(1 + Sec[e + f*x])))/
a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4391
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_), x_Symbol] :> Simp[c^n   Int[ExpandTrig[(1 + (d/c)*csc[e
+ f*x])^n, (a + b*csc[e + f*x])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
 && LtQ[m + n, 2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

method	result	size
parallelrisch	$-\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-10fx+20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10a^{3}f}$	50
derivativedivides	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,a^{3}}$	58
default	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{2f\,a^{3}}$	58
norman	$\frac{\frac{cx}{a}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2af}-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{10af}}{a^{2}}$	70
risch	$\frac{cx}{a^{3}}-\frac{2ic\left(20\,e^{4i(fx+e)}+55\,e^{3i(fx+e)}+75\,e^{2i(fx+e)}+45\,e^{i(fx+e)}+13\right)}{5f\,a^{3}\left(e^{i(fx+e)}+1\right)^{5}}$	77

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*c*(tan(1/2*f*x+1/2*e)^5-5*tan(1/2*f*x+1/2*e)^3-10*f*x+20*tan(1/2*f*x
+1/2*e))/a^3/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^3}\,dx$$
$$= \frac{5\,cfx\cos\left(fx+e\right)^3 + 15\,cfx\cos\left(fx+e\right)^2 + 15\,cfx\cos\left(fx+e\right) + 5\,cfx - \left(13\,c\cos\left(fx+e\right)^2 + 19\,c\cos\right.}{5\left(a^3 f\cos\left(fx+e\right)^3 + 3\,a^3 f\cos\left(fx+e\right)^2 + 3\,a^3 f\cos\left(fx+e\right) + a^3 f\right)}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/5*(5*c*f*x*cos(f*x + e)^3 + 15*c*f*x*cos(f*x + e)^2 + 15*c*f*x*cos(f*x +
 e) + 5*c*f*x - (13*c*cos(f*x + e)^2 + 19*c*cos(f*x + e) + 8*c)*sin(f*x +
e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e)
+ a^3*f)
```

Sympy [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^3}\,dx =$$
$$-\frac{c\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx + \int \left(-\frac{1}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx\right)}{a^3}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

output
```
-c*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(-1/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.81

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^3}\,dx = -\frac{c\left(\frac{\frac{105\,\sin(fx+e)}{\cos(fx+e)+1} - \frac{20\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{120\,\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3}\right) + \frac{c\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right.}{a^3}}{60\,f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/60*(c*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 120*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) + c*(15*sin(f*x + e)/(cos(f*x + e) + 1
) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^3}\,dx = \frac{\frac{10\,(fx+e)c}{a^3} - \frac{a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 5\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 20\,a^{12}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{10\,f}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/10*(10*(f*x + e)*c/a^3 - (a^12*c*tan(1/2*f*x + 1/2*e)^5 - 5*a^12*c*tan(1
/2*f*x + 1/2*e)^3 + 20*a^12*c*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 11.80 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{c\,x}{a^3} - \frac{\frac{13\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{5} - \frac{7\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{10} + \frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{10}}{a^3\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^3,x)
```

output
```
(c*x)/a^3 - ((c*sin(e/2 + (f*x)/2))/10 - (7*c*cos(e/2 + (f*x)/2)^2*sin(e/2
 + (f*x)/2))/10 + (13*c*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2))/5)/(a^3*f
*cos(e/2 + (f*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^3}\,dx = \frac{c\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+10fx\right)}{10a^3f}$$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

output
```
(c*( - tan((e + f*x)/2)**5 + 5*tan((e + f*x)/2)**3 - 20*tan((e + f*x)/2) +
 10*f*x))/(10*a**3*f)
```

3.36 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{x}{a^3c}+\frac{\cot(e+fx)}{a^3cf}-\frac{\cot^3(e+fx)}{3a^3cf}+\frac{2\cot^5(e+fx)}{5a^3cf}-\frac{2\csc(e+fx)}{a^3cf}+\frac{4\csc^3(e+fx)}{3a^3cf}-\frac{2\csc^5(e+fx)}{5a^3cf}$$

output
```
x/a^3/c+cot(f*x+e)/a^3/c/f-1/3*cot(f*x+e)^3/a^3/c/f+2/5*cot(f*x+e)^5/a^3/c
/f-2*csc(f*x+e)/a^3/c/f+4/3*csc(f*x+e)^3/a^3/c/f-2/5*csc(f*x+e)^5/a^3/c/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.57 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\cot^5(e+fx)\left(3+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-30\sec(e+fx)+40\sec^3(e+fx\right.}{15a^3cf}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

output
```
(Cot[e + f*x]^5*(3 + 3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] -
 30*Sec[e + f*x] + 40*Sec[e + f*x]^3 - 16*Sec[e + f*x]^5))/(15*a^3*c*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))^2dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(c^2\cot^6(e+fx)-2c^2\csc(e+fx)\cot^5(e+fx)+c^2\csc^2(e+fx)\cot^4(e+fx)\right)dx}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2c^2\cot^5(e+fx)}{5f}+\frac{c^2\cot^3(e+fx)}{3f}-\frac{c^2\cot(e+fx)}{f}+\frac{2c^2\csc^5(e+fx)}{5f}-\frac{4c^2\csc^3(e+fx)}{3f}+\frac{2c^2\csc(e+fx)}{f}-c^2x}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

output
```
-((-(c^2*x) - (c^2*Cot[e + f*x])/f + (c^2*Cot[e + f*x]^3)/(3*f) - (2*c^2*C
ot[e + f*x]^5)/(5*f) + (2*c^2*Csc[e + f*x])/f - (4*c^2*Csc[e + f*x]^3)/(3*
f) + (2*c^2*Csc[e + f*x]^5)/(5*f))/(a^3*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+25\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+120fx+15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-165\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{120f\,a^{3}c}$	65
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f\,a^{3}c}$	73
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+16\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f\,a^{3}c}$	73
risch	$\frac{x}{a^{3}c}-\frac{4i\left(15\,\mathrm{e}^{5i(fx+e)}+30\,\mathrm{e}^{4i(fx+e)}+10\,\mathrm{e}^{3i(fx+e)}-35\,\mathrm{e}^{2i(fx+e)}-37\,\mathrm{e}^{i(fx+e)}-13\right)}{15f\,a^{3}c\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}\left(\mathrm{e}^{i(fx+e)}-1\right)}$	105
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{ca}+\frac{1}{8acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{24acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{40acf}}{a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	111

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/120*(-3*tan(1/2*f*x+1/2*e)^5+25*tan(1/2*f*x+1/2*e)^3+120*f*x+15*cot(1/2*
f*x+1/2*e)-165*tan(1/2*f*x+1/2*e))/f/a^3/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$=\frac{26\cos(fx+e)^{3}+22\cos(fx+e)^{2}+15\left(fx\cos(fx+e)^{2}+2\,fx\cos(fx+e)+fx\right)\sin(fx+e)-17}{15\left(a^{3}cf\cos(fx+e)^{2}+2\,a^{3}cf\cos(fx+e)+a^{3}cf\right)\sin(fx+e)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
1/15*(26*cos(f*x + e)^3 + 22*cos(f*x + e)^2 + 15*(f*x*cos(f*x + e)^2 + 2*f
*x*cos(f*x + e) + f*x)*sin(f*x + e) - 17*cos(f*x + e) - 16)/((a^3*c*f*cos(
f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1}\,dx}{a^3c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1), x)
/(a**3*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\frac{\frac{165\sin(fx+e)}{\cos(fx+e)+1}-\frac{25\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}-\frac{240\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c}-\frac{15(\cos(fx+e)+1)}{a^3c\sin(fx+e)}}{120f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-1/120*((165*sin(f*x + e)/(cos(f*x + e) + 1) - 25*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) - 240*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c) - 15*(cos(f*x + e) + 1)/(a^3*c*si
n(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\frac{120\,(fx+e)}{a^3c}+\frac{15}{a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-\frac{3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-25\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+165\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}c^5}}{120\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
1/120*(120*(f*x + e)/(a^3*c) + 15/(a^3*c*tan(1/2*f*x + 1/2*e)) - (3*a^12*c
^4*tan(1/2*f*x + 1/2*e)^5 - 25*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*a^12*
c^4*tan(1/2*f*x + 1/2*e))/(a^15*c^5))/f
```

Mupad [B] (verification not implemented)

Time = 11.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{x}{a^3\,c}+\frac{\frac{26\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{15}-\frac{28\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{15}+\frac{17\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{60}-\frac{1}{40}}{a^3\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

output
```
x/(a^3*c) + ((17*cos(e/2 + (f*x)/2)^2)/60 - (28*cos(e/2 + (f*x)/2)^4)/15 +
 (26*cos(e/2 + (f*x)/2)^6)/15 - 1/40)/(a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/
2 + (f*x)/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+25\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-165\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+120\tan\left(\frac{fx}{2}+\frac{e}{2}\right)fx+15}{120\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a^3cf}$$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

output
```
( - 3*tan((e + f*x)/2)**6 + 25*tan((e + f*x)/2)**4 - 165*tan((e + f*x)/2)*
*2 + 120*tan((e + f*x)/2)*f*x + 15)/(120*tan((e + f*x)/2)*a**3*c*f)
```

3.37 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 100

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{x}{a^3c^2}+\frac{\cot(e+fx)(15-8\sec(e+fx))}{15a^3c^2f} - \frac{\cot^3(e+fx)(5-4\sec(e+fx))}{15a^3c^2f}+\frac{\cot^5(e+fx)(1-\sec(e+fx))}{5a^3c^2f}$$

output
```
x/a^3/c^2+1/15*cot(f*x+e)*(15-8*sec(f*x+e))/a^3/c^2/f-1/15*cot(f*x+e)^3*(5
-4*sec(f*x+e))/a^3/c^2/f+1/5*cot(f*x+e)^5*(1-sec(f*x+e))/a^3/c^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\cot^5(e+fx)\left(3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)-15\sec(e+fx)+20\sec^3(e+fx)-8\right.}{15a^3c^2f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

output
```
(Cot[e + f*x]^5*(3*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2] - 15*
Sec[e + f*x] + 20*Sec[e + f*x]^3 - 8*Sec[e + f*x]^5))/(15*a^3*c^2*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(c-c\sec(e+fx))dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\int -\cot^4(e+fx)(5c-4c\sec(e+fx))dx-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{1}{5}\int \cot^4(e+fx)(5c-4c\sec(e+fx))dx-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{5}\int \frac{5c-4c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}-\frac{1}{3}\int -\cot^2(e+fx)(15c-8c\sec(e+fx))dx\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \cot^2(e+fx)(15c-8c\sec(e+fx))dx+\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{15c-8c\csc\left(e+fx+\frac{\pi}{2}\right)}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx+\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{4370}$$

$$-\frac{\frac{1}{5}\left(\frac{1}{3}\left(\int -15cdx-\frac{\cot(e+fx)(15c-8c\sec(e+fx))}{f}\right)+\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{1}{5}\left(\frac{\cot^3(e+fx)(5c-4c\sec(e+fx))}{3f}+\frac{1}{3}\left(-\frac{\cot(e+fx)(15c-8c\sec(e+fx))}{f}-15cx\right)\right)-\frac{\cot^5(e+fx)(c-c\sec(e+fx))}{5f}}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

output
```
-((-1/5*(Cot[e + f*x]^5*(c - c*Sec[e + f*x]))/f + ((Cot[e + f*x]^3*(5*c -
4*c*Sec[e + f*x]))/(3*f) + (-15*c*x - (Cot[e + f*x]*(15*c - 8*c*Sec[e + f*
x]))/f)/3)/5)/(a^3*c^3))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+240fx-240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+90\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{240f\,a^3c^2}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-16\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+32\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$
risch	$\frac{x}{a^3c^2}-\frac{2i\left(15\,e^{7i(fx+e)}-15\,e^{6i(fx+e)}-65\,e^{5i(fx+e)}-25\,e^{4i(fx+e)}+73\,e^{3i(fx+e)}+31\,e^{2i(fx+e)}-31\,e^{i(fx+e)}-23\right)}{15f\,c^2a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^3}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{ca}-\frac{1}{48acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{80acf}}{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/240*(-3*tan(1/2*f*x+1/2*e)^5+30*tan(1/2*f*x+1/2*e)^3-5*cot(1/2*f*x+1/2*e
)^3+240*f*x-240*tan(1/2*f*x+1/2*e)+90*cot(1/2*f*x+1/2*e))/f/a^3/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$$
$$= \frac{23\cos(fx+e)^4+8\cos(fx+e)^3-27\cos(fx+e)^2+15\left(fx\cos(fx+e)^3+fx\cos(fx+e)^2-fx\cos\right.}{15\left(a^3c^2f\cos(fx+e)^3+a^3c^2f\cos(fx+e)^2-a^3c^2f\cos(fx+e)-a\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/15*(23*cos(f*x + e)^4 + 8*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + 15*(f*x*c
os(f*x + e)^3 + f*x*cos(f*x + e)^2 - f*x*cos(f*x + e) - f*x)*sin(f*x + e)
- 7*cos(f*x + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^
2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{\sec^5(e+fx)+\sec^4(e+fx)-2\sec^3(e+fx)-2\sec^2(e+fx)+\sec(e+fx)+1}\,dx}{a^3c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(
e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\frac{3\left(\frac{80\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^2}-\frac{480\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^2}-\frac{5\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/240*(3*(80*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) - 480*arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^2) - 5*(18*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\frac{240\,(fx+e)}{a^3c^2} + \frac{5\left(18\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3} - \frac{3\left(a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+80\,a^{12}c^8\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}c^{10}}}{240\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/240*(240*(f*x + e)/(a^3*c^2) + 5*(18*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^
2*tan(1/2*f*x + 1/2*e)^3) - 3*(a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c
^8*tan(1/2*f*x + 1/2*e)^3 + 80*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))
/f
```

Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{5\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8+3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^8-30\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+240\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{240\,a^3\,c^2\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}\right.}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

output
```
-(5*cos(e/2 + (f*x)/2)^8 + 3*sin(e/2 + (f*x)/2)^8 - 30*cos(e/2 + (f*x)/2)^
2*sin(e/2 + (f*x)/2)^6 + 240*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2)^4 - 9
0*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^2 - 240*cos(e/2 + (f*x)/2)^5*sin
(e/2 + (f*x)/2)^3*(e + f*x))/(240*a^3*c^2*f*cos(e/2 + (f*x)/2)^5*sin(e/2 +
 (f*x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$$
$$=\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3fx+90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-5}{240\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3a^3c^2f}$$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

output
```
( - 3*tan((e + f*x)/2)**8 + 30*tan((e + f*x)/2)**6 - 240*tan((e + f*x)/2)*
*4 + 240*tan((e + f*x)/2)**3*f*x + 90*tan((e + f*x)/2)**2 - 5)/(240*tan((e
 + f*x)/2)**3*a**3*c**2*f)
```

3.38 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{x}{a^3c^3}+\frac{\cot(e+fx)}{a^3c^3f}-\frac{\cot^3(e+fx)}{3a^3c^3f}+\frac{\cot^5(e+fx)}{5a^3c^3f}$$

output
```
x/a^3/c^3+cot(f*x+e)/a^3/c^3/f-1/3*cot(f*x+e)^3/a^3/c^3/f+1/5*cot(f*x+e)^5
/a^3/c^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5a^3c^3f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

output
```
(Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[e + f*x]^2])/(5*a^3*
c^3*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 4392, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \tan\left(e+fx+\frac{\pi}{2}\right)^6dx}{a^3c^3}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{-\int \cot^4(e+fx)dx-\frac{\cot^5(e+fx)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\int \tan\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{\cot^5(e+fx)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\int \cot^2(e+fx)dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \tan\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}}{a^3c^3}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{-\int 1dx-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}}{a^3c^3}$$

$$\Big\downarrow \text{24}$$

$$-\frac{-\frac{\cot^5(e+fx)}{5f}+\frac{\cot^3(e+fx)}{3f}-\frac{\cot(e+fx)}{f}-x}{a^3c^3}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

output
```
-((-x - Cot[e + f*x]/f + Cot[e + f*x]^3/(3*f) - Cot[e + f*x]^5/(5*f))/(a^3
*c^3))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

method	result	siz
default	$-\frac{-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)+\frac{\pi}{2}-\operatorname{arccot}(\cot(fx+e))}{c^3a^3f}$	53
parallelrisch	$\frac{-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+480fx-330\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+330\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{480f\,c^3a^3}$	91
risch	$\frac{x}{a^3c^3}+\frac{2i\left(45\,e^{8i(fx+e)}-90\,e^{6i(fx+e)}+140\,e^{4i(fx+e)}-70\,e^{2i(fx+e)}+23\right)}{15f\,c^3a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^5}$	94
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{ca}+\frac{1}{160acf}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{96acf}+\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{16acf}-\frac{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{16acf}+\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{96acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{160acf}}{c^2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	160

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3/a^3/f*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)+1/2*Pi-arccot(
cot(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$$
$$=\frac{23\cos(fx+e)^5-35\cos(fx+e)^3+15\left(fx\cos(fx+e)^4-2fx\cos(fx+e)^2+fx\right)\sin(fx+e)+15}{15\left(a^3c^3f\cos(fx+e)^4-2a^3c^3f\cos(fx+e)^2+a^3c^3f\right)\sin(fx+e)}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/15*(23*cos(f*x + e)^5 - 35*cos(f*x + e)^3 + 15*(f*x*cos(f*x + e)^4 - 2*f
*x*cos(f*x + e)^2 + f*x)*sin(f*x + e) + 15*cos(f*x + e))/((a^3*c^3*f*cos(f
*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sec^6(e+fx)-3\sec^4(e+fx)+3\sec^2(e+fx)-1}\,dx}{a^3c^3}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(1/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1),
 x)/(a**3*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)}{a^3c^3} + \frac{15\,\tan(fx+e)^4-5\,\tan(fx+e)^2+3}{a^3c^3\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/15*(15*(f*x + e)/(a^3*c^3) + (15*tan(f*x + e)^4 - 5*tan(f*x + e)^2 + 3)/
(a^3*c^3*tan(f*x + e)^5))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{480\,(fx+e)}{a^3c^3} + \frac{330\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-35\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3}{a^3c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5} - \frac{3\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+330\,a^{12}c^{12}\tan\left(\frac{1}{2}\,fx+\right.}{a^{15}c^{15}}}{480\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
1/480*(480*(f*x + e)/(a^3*c^3) + (330*tan(1/2*f*x + 1/2*e)^4 - 35*tan(1/2*
f*x + 1/2*e)^2 + 3)/(a^3*c^3*tan(1/2*f*x + 1/2*e)^5) - (3*a^12*c^12*tan(1/
2*f*x + 1/2*e)^5 - 35*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 330*a^12*c^12*tan
(1/2*f*x + 1/2*e))/(a^15*c^15))/f
```

Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\frac{5\cos(e+fx)}{24}-\frac{5\cos(3e+3fx)}{48}+\frac{23\cos(5e+5fx)}{240}-\frac{5\sin(3e+3fx)(e+fx)}{16}+\frac{\sin(5e+5fx)(e+fx)}{16}+\frac{5\sin(e+fx)(e+fx)}{8}}{a^3c^3f\sin(e+fx)^5}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
```

output
```
((5*cos(e + f*x))/24 - (5*cos(3*e + 3*f*x))/48 + (23*cos(5*e + 5*f*x))/240
 - (5*sin(3*e + 3*f*x)*(e + f*x))/16 + (sin(5*e + 5*f*x)*(e + f*x))/16 + (
5*sin(e + f*x)*(e + f*x))/8)/(a^3*c^3*f*sin(e + f*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{23\cos(fx+e)\sin(fx+e)^4-11\cos(fx+e)\sin(fx+e)^2+3\cos(fx+e)+15\sin(fx+e)^5fx}{15\sin(fx+e)^5a^3c^3f}$$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

output
```
(23*cos(e + f*x)*sin(e + f*x)**4 - 11*cos(e + f*x)*sin(e + f*x)**2 + 3*cos
(e + f*x) + 15*sin(e + f*x)**5*f*x)/(15*sin(e + f*x)**5*a**3*c**3*f)
```

3.39 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4} dx = \frac{x}{a^3c^4} - \frac{\cot^7(e+fx)(1+\sec(e+fx))}{7a^3c^4f} + \frac{\cot^5(e+fx)(7+6\sec(e+fx))}{35a^3c^4f} + \frac{\cot(e+fx)(35+16\sec(e+fx))}{35a^3c^4f} - \frac{\cot^3(e+fx)(35+24\sec(e+fx))}{105a^3c^4f}$$

output
```
x/a^3/c^4-1/7*cot(f*x+e)^7*(1+sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)^5*(7+6
*sec(f*x+e))/a^3/c^4/f+1/35*cot(f*x+e)*(35+16*sec(f*x+e))/a^3/c^4/f-1/105*
cot(f*x+e)^3*(35+24*sec(f*x+e))/a^3/c^4/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.75 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4} dx = -\frac{\csc^7(e+fx)\left(-106+301\cos(2(e+fx))-70\cos(4(e+fx))+35\cos(6(e+fx))+160\cos^7(e+fx)\right.}{1120a^3c^4f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

output
```
-1/1120*(Csc[e + f*x]^7*(-106 + 301*Cos[2*(e + f*x)] - 70*Cos[4*(e + f*x)]
 + 35*Cos[6*(e + f*x)] + 160*Cos[e + f*x]^7*Hypergeometric2F1[-7/2, 1, -5/
2, -Tan[e + f*x]^2]))/(a^3*c^4*f)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 4392, 3042, 4370, 25, 3042, 4370, 25, 3042, 4370, 27, 3042, 4370, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{4370}$$

$$\frac{\frac{1}{7}\int -\cot^6(e+fx)(6\sec(e+fx)a+7a)dx-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{7}\int \cot^6(e+fx)(6\sec(e+fx)a+7a)dx-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{7}\int \frac{6\csc\left(e+fx+\frac{\pi}{2}\right)a+7a}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 4370

$$\frac{\frac{1}{7}\left(\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}-\frac{1}{5}\int -\cot^4(e+fx)(24\sec(e+fx)a+35a)dx\right)-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{7}\left(\frac{1}{5}\int \cot^4(e+fx)(24\sec(e+fx)a+35a)dx+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{1}{5}\int \frac{24\csc\left(e+fx+\frac{\pi}{2}\right)a+35a}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\frac{\cot^7(e+fx)(a\sec(e+fx)+a)}{7f}}{a^4c^4}$$

$\Big\downarrow$ 4370

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int -3\cot^2(e+fx)(16\sec(e+fx)a+35a)dx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)}{a^4c^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int \cot^2(e+fx)(16\sec(e+fx)a+35a)dx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-}{a^4c^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int \frac{16\csc\left(e+fx+\frac{\pi}{2}\right)a+35a}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\frac{\cot^7(e+fx)(a\sec(e+}{7f}}{a^4c^4}$$

$\Big\downarrow$ 4370

$$\frac{\frac{1}{7}\left(\frac{1}{5}\left(-\int -35adx-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}+\frac{\cot(e+fx)(16a\sec(e+fx)+35a)}{f}\right)+\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}\right)-\cot}{a^4c^4}$$

$\Big\downarrow$ 24

$$\frac{\frac{1}{7}\left(\frac{\cot^5(e+fx)(6a\sec(e+fx)+7a)}{5f}+\frac{1}{5}\left(-\frac{\cot^3(e+fx)(24a\sec(e+fx)+35a)}{3f}+\frac{\cot(e+fx)(16a\sec(e+fx)+35a)}{f}+35ax\right)\right)-\frac{\cot^7(e+fx}{}}{a^4c^4}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

output
```
(-1/7*(Cot[e + f*x]^7*(a + a*Sec[e + f*x]))/f + ((Cot[e + f*x]^5*(7*a + 6*
a*Sec[e + f*x]))/(5*f) + (35*a*x + (Cot[e + f*x]*(35*a + 16*a*Sec[e + f*x]
))/f - (Cot[e + f*x]^3*(35*a + 24*a*Sec[e + f*x]))/(3*f))/5)/7)/(a^4*c^4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4370
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Simp[1/(e^2*(m + 1))   Int[(e*Cot[c + d*x])^(m + 2)*(a*
(m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && L
tQ[m, -1]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{-15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+168\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+280\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-1015\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+6720fx-3045\tan(}{6720 f\, a^{3} c^{4}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{29}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{64}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+12}{64 f\, c^{4} a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{8}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{29}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{64}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+12}{64 f\, c^{4} a^{3}}$
risch	$\frac{x}{a^{3} c^{4}}+\frac{2i\left(105\,\mathrm{e}^{11i(fx+e)}+210\,\mathrm{e}^{10i(fx+e)}-735\,\mathrm{e}^{9i(fx+e)}+1638\,\mathrm{e}^{7i(fx+e)}-196\,\mathrm{e}^{6i(fx+e)}-1882\,\mathrm{e}^{5i(fx+e)}+880\,\mathrm{e}^{4i(f}\right.}{105 f\, c^{4} a^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}}$
norman	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{acf}+\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{ca}-\frac{1}{448acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{40acf}-\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{192acf}-\frac{29\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{24acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{320acf}}{c^{3} a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/6720*(-15*cot(1/2*f*x+1/2*e)^7-21*tan(1/2*f*x+1/2*e)^5+168*cot(1/2*f*x+1
/2*e)^5+280*tan(1/2*f*x+1/2*e)^3-1015*cot(1/2*f*x+1/2*e)^3+6720*f*x-3045*t
an(1/2*f*x+1/2*e)+6720*cot(1/2*f*x+1/2*e))/f/a^3/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{176\,\cos\left(fx+e\right)^{6}-71\,\cos\left(fx+e\right)^{5}-335\,\cos\left(fx+e\right)^{4}+125\,\cos\left(fx+e\right)^{3}+225\,\cos\left(fx+e\right)^{2}+105}{105\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}-a^{3}c^{4}f\cos\left(fx+e\right)^{4}-2\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
1/105*(176*cos(f*x + e)^6 - 71*cos(f*x + e)^5 - 335*cos(f*x + e)^4 + 125*c
os(f*x + e)^3 + 225*cos(f*x + e)^2 + 105*(f*x*cos(f*x + e)^5 - f*x*cos(f*x
 + e)^4 - 2*f*x*cos(f*x + e)^3 + 2*f*x*cos(f*x + e)^2 + f*x*cos(f*x + e) -
 f*x)*sin(f*x + e) - 57*cos(f*x + e) - 48)/((a^3*c^4*f*cos(f*x + e)^5 - a^
3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*cos(f*x
+ e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{\int \frac{1}{\sec^{7}(e+fx)-\sec^{6}(e+fx)-3\sec^{5}(e+fx)+3\sec^{4}(e+fx)+3\sec^{3}(e+fx)-3\sec^{2}(e+fx)-\sec(e+fx)+1}\,dx}{a^{3}c^{4}}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(1/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**5 + 3*sec(
e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x) + 1), x
)/(a**3*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\frac{7\left(\frac{435\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^4}-\frac{13440\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^4}-\frac{\left(\frac{168\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1015\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{6720\sin(fx+}{(\cos(fx+e)+}\right.}{a^3c^4\sin(fx+e)^7}}{6720\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/6720*(7*(435*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) - 13440*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^4) - (168*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 1015*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6720*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin(f*x
 + e)^7))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\frac{6720\,(fx+e)}{a^3c^4}+\frac{6720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-1015\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+168\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-15}{a^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}-\frac{7\left(3a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-40a^{12}c^{16}\tan\left(\frac{1}{2}f\right.\right.}{a^{15}c^{20}}}{6720\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/6720*(6720*(f*x + e)/(a^3*c^4) + (6720*tan(1/2*f*x + 1/2*e)^6 - 1015*tan
(1/2*f*x + 1/2*e)^4 + 168*tan(1/2*f*x + 1/2*e)^2 - 15)/(a^3*c^4*tan(1/2*f*
x + 1/2*e)^7) - 7*(3*a^12*c^16*tan(1/2*f*x + 1/2*e)^5 - 40*a^12*c^16*tan(1
/2*f*x + 1/2*e)^3 + 435*a^12*c^16*tan(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

Mupad [B] (verification not implemented)

Time = 11.89 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{15\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+21\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-280\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+3045\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}\right.}{}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

output
```
-(15*cos(e/2 + (f*x)/2)^12 + 21*sin(e/2 + (f*x)/2)^12 - 280*cos(e/2 + (f*x
)/2)^2*sin(e/2 + (f*x)/2)^10 + 3045*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2
)^8 - 6720*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^6 + 1015*cos(e/2 + (f*x
)/2)^8*sin(e/2 + (f*x)/2)^4 - 168*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)/2)
^2 - 6720*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7*(e + f*x))/(6720*a^3*c
^4*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{-21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}+280\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-3045\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+6720\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7fx+6720\tan\left(\frac{fx}{2}\right.}{6720\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7a^3c^4f}$$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

output
```
( - 21*tan((e + f*x)/2)**12 + 280*tan((e + f*x)/2)**10 - 3045*tan((e + f*x
)/2)**8 + 6720*tan((e + f*x)/2)**7*f*x + 6720*tan((e + f*x)/2)**6 - 1015*t
an((e + f*x)/2)**4 + 168*tan((e + f*x)/2)**2 - 15)/(6720*tan((e + f*x)/2)*
*7*a**3*c**4*f)
```

3.40 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{x}{a^3c^5}+\frac{\cot(e+fx)}{a^3c^5f}-\frac{\cot^3(e+fx)}{3a^3c^5f}+\frac{\cot^5(e+fx)}{5a^3c^5f}-\frac{\cot^7(e+fx)}{7a^3c^5f}+\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc(e+fx)}{a^3c^5f}-\frac{8\csc^3(e+fx)}{3a^3c^5f}+\frac{12\csc^5(e+fx)}{5a^3c^5f}-\frac{8\csc^7(e+fx)}{7a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}$$

output
```
x/a^3/c^5+cot(f*x+e)/a^3/c^5/f-1/3*cot(f*x+e)^3/a^3/c^5/f+1/5*cot(f*x+e)^5
/a^3/c^5/f-1/7*cot(f*x+e)^7/a^3/c^5/f+2/9*cot(f*x+e)^9/a^3/c^5/f+2*csc(f*x
+e)/a^3/c^5/f-8/3*csc(f*x+e)^3/a^3/c^5/f+12/5*csc(f*x+e)^5/a^3/c^5/f-8/7*c
sc(f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^5/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.87 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\cot^9(e+fx)\left(35+35\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(e+fx)\right)+630\sec(e+fx)-1680\sec^3(e}{315a^3c^5f}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

output
```
(Cot[e + f*x]^9*(35 + 35*Hypergeometric2F1[-9/2, 1, -7/2, -Tan[e + f*x]^2]
 + 630*Sec[e + f*x] - 1680*Sec[e + f*x]^3 + 2016*Sec[e + f*x]^5 - 1152*Sec
[e + f*x]^7 + 256*Sec[e + f*x]^9))/(315*a^3*c^5*f)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^5}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$
$$\Big\downarrow \text{4392}$$
$$-\frac{\int \cot^{10}(e+fx)(\sec(e+fx)a+a)^2dx}{a^5c^5}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}dx}{a^5c^5}$$

$$\Big\downarrow \text{4374}$$

$$-\frac{\int \left(a^2\cot^{10}(e+fx)+2a^2\csc(e+fx)\cot^9(e+fx)+a^2\csc^2(e+fx)\cot^8(e+fx)\right)dx}{a^5c^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2a^2\cot^9(e+fx)}{9f}+\frac{a^2\cot^7(e+fx)}{7f}-\frac{a^2\cot^5(e+fx)}{5f}+\frac{a^2\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-\frac{2a^2\csc^9(e+fx)}{9f}+\frac{8a^2\csc^7(e+fx)}{7f}-12a^2}{a^5c^5}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

output
```
-((-(a^2*x) - (a^2*Cot[e + f*x])/f + (a^2*Cot[e + f*x]^3)/(3*f) - (a^2*Cot
[e + f*x]^5)/(5*f) + (a^2*Cot[e + f*x]^7)/(7*f) - (2*a^2*Cot[e + f*x]^9)/(
9*f) - (2*a^2*Csc[e + f*x])/f + (8*a^2*Csc[e + f*x]^3)/(3*f) - (12*a^2*Csc
[e + f*x]^5)/(5*f) + (8*a^2*Csc[e + f*x]^7)/(7*f) - (2*a^2*Csc[e + f*x]^9)
/(9*f))/(a^5*c^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

method	result
parallelrisch	$\frac{35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-405\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+2331\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+945\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-9765\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+}{40320f\,a^{3}c^{5}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{}{5\tan}}{128f\,c^{5}a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+256\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{9}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{}{5\tan}}{128f\,c^{5}a^{3}}$
risch	$\frac{x}{a^{3}c^{5}}+\frac{4i\left(315\,\mathrm{e}^{13i(fx+e)}-315\,\mathrm{e}^{12i(fx+e)}-1470\,\mathrm{e}^{11i(fx+e)}+3360\,\mathrm{e}^{10i(fx+e)}+1113\,\mathrm{e}^{9i(fx+e)}-6447\,\mathrm{e}^{8i(fx+e)}+2028\right.}{315f\,c^{5}a^{3}\left(\mathrm{e}^{i(fx+e}\right.}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{ca}+\frac{1}{1152acf}-\frac{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{896acf}+\frac{37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{640acf}-\frac{31\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{128acf}+\frac{163\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{128acf}-\frac{37\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128acf}+\frac{3}{}}{c^{4}a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/40320*(35*cot(1/2*f*x+1/2*e)^9-405*cot(1/2*f*x+1/2*e)^7-63*tan(1/2*f*x+1
/2*e)^5+2331*cot(1/2*f*x+1/2*e)^5+945*tan(1/2*f*x+1/2*e)^3-9765*cot(1/2*f*
x+1/2*e)^3+40320*f*x-11655*tan(1/2*f*x+1/2*e)+51345*cot(1/2*f*x+1/2*e))/f/
a^3/c^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{598\,\cos\left(fx+e\right)^7-566\,\cos\left(fx+e\right)^6-1212\,\cos\left(fx+e\right)^5+1310\,\cos\left(fx+e\right)^4+860\,\cos\left(fx+e\right)^3-}{315\,\left(a^3c^5f\cos\left(fx+e\right)^6-2\,a^3c^5f\cos\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
1/315*(598*cos(f*x + e)^7 - 566*cos(f*x + e)^6 - 1212*cos(f*x + e)^5 + 131
0*cos(f*x + e)^4 + 860*cos(f*x + e)^3 - 1014*cos(f*x + e)^2 + 315*(f*x*cos
(f*x + e)^6 - 2*f*x*cos(f*x + e)^5 - f*x*cos(f*x + e)^4 + 4*f*x*cos(f*x +
e)^3 - f*x*cos(f*x + e)^2 - 2*f*x*cos(f*x + e) + f*x)*sin(f*x + e) - 197*c
os(f*x + e) + 256)/((a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5
 - a^3*c^5*f*cos(f*x + e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f
*x + e)^2 - 2*a^3*c^5*f*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\int \frac{1}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1}\,dx}{a^3c^5}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

output
```
-Integral(1/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x)**6 + 6*s
ec(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e + f*x) -
1), x)/(a**3*c**5)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\frac{63\left(\frac{185\sin(fx+e)}{\cos(fx+e)+1}-\frac{15\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{80640\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^5}+\frac{\left(\frac{405\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2331\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9765\sin(fx\ldots}{(\cos(fx+e)\ldots}\right.}{a^3c^5\sin(\ldots}}{40320\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
-1/40320*(63*(185*sin(f*x + e)/(cos(f*x + e) + 1) - 15*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - 80640*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^5) + (405*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 2331*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9765*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 51345*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 35)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{\frac{40320\,(fx+e)}{a^3c^5}+\frac{51345\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-9765\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+2331\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-405\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+35}{a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}-\frac{63\left(a^{12}c^{20}\tan\left(\frac{1}{2}fx+\ldots\right.\right.}{\ldots}}{40320\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/40320*(40320*(f*x + e)/(a^3*c^5) + (51345*tan(1/2*f*x + 1/2*e)^8 - 9765*
tan(1/2*f*x + 1/2*e)^6 + 2331*tan(1/2*f*x + 1/2*e)^4 - 405*tan(1/2*f*x + 1
/2*e)^2 + 35)/(a^3*c^5*tan(1/2*f*x + 1/2*e)^9) - 63*(a^12*c^20*tan(1/2*f*x
 + 1/2*e)^5 - 15*a^12*c^20*tan(1/2*f*x + 1/2*e)^3 + 185*a^12*c^20*tan(1/2*
f*x + 1/2*e))/(a^15*c^25))/f
```

Mupad [B] (verification not implemented)

Time = 12.70 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{35\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-63\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}+945\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-11655\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\sin\left(\frac{e}{2}+\right.}{}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

output
```
(35*cos(e/2 + (f*x)/2)^14 - 63*sin(e/2 + (f*x)/2)^14 + 945*cos(e/2 + (f*x)
/2)^2*sin(e/2 + (f*x)/2)^12 - 11655*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2
)^10 + 51345*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^8 - 9765*cos(e/2 + (f
*x)/2)^8*sin(e/2 + (f*x)/2)^6 + 2331*cos(e/2 + (f*x)/2)^10*sin(e/2 + (f*x)
/2)^4 - 405*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^2 + 40320*cos(e/2 + (
f*x)/2)^5*sin(e/2 + (f*x)/2)^9*(e + f*x))/(40320*a^3*c^5*f*cos(e/2 + (f*x)
/2)^5*sin(e/2 + (f*x)/2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{-63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}+945\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}-11655\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+40320\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}fx+51345\tan}{40320\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}a^3c^5 f}$$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

output
```
( - 63*tan((e + f*x)/2)**14 + 945*tan((e + f*x)/2)**12 - 11655*tan((e + f*
x)/2)**10 + 40320*tan((e + f*x)/2)**9*f*x + 51345*tan((e + f*x)/2)**8 - 97
65*tan((e + f*x)/2)**6 + 2331*tan((e + f*x)/2)**4 - 405*tan((e + f*x)/2)**
2 + 35)/(40320*tan((e + f*x)/2)**9*a**3*c**5*f)
```

3.41 $\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx &= \frac{x}{a^3c^6}+\frac{\cot(e+fx)}{a^3c^6f}-\frac{\cot^3(e+fx)}{3a^3c^6f}+\frac{\cot^5(e+fx)}{5a^3c^6f}-\frac{\cot^7(e+fx)}{7a^3c^6f}\\ &\quad+\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{3\csc(e+fx)}{a^3c^6f}-\frac{16\csc^3(e+fx)}{3a^3c^6f}\\ &\quad+\frac{34\csc^5(e+fx)}{5a^3c^6f}-\frac{36\csc^7(e+fx)}{7a^3c^6f}+\frac{19\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}\end{aligned}$$

output
```
x/a^3/c^6+cot(f*x+e)/a^3/c^6/f-1/3*cot(f*x+e)^3/a^3/c^6/f+1/5*cot(f*x+e)^5
/a^3/c^6/f-1/7*cot(f*x+e)^7/a^3/c^6/f+1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(
f*x+e)^11/a^3/c^6/f+3*csc(f*x+e)/a^3/c^6/f-16/3*csc(f*x+e)^3/a^3/c^6/f+34/
5*csc(f*x+e)^5/a^3/c^6/f-36/7*csc(f*x+e)^7/a^3/c^6/f+19/9*csc(f*x+e)^9/a^3
/c^6/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

Mathematica [A] (verified)

Time = 7.43 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\sec^8(e+fx)(24393600fx\cos(fx)-24393600fx\cos(2e+fx)-14636160fx\cos(e+2f}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

output
```
(Csc[e/2]*Sec[e/2]*Sec[e + f*x]^8*(24393600*f*x*Cos[f*x] - 24393600*f*x*Co
s[2*e + f*x] - 14636160*f*x*Cos[e + 2*f*x] + 14636160*f*x*Cos[3*e + 2*f*x]
 - 7539840*f*x*Cos[2*e + 3*f*x] + 7539840*f*x*Cos[4*e + 3*f*x] + 11088000*
f*x*Cos[3*e + 4*f*x] - 11088000*f*x*Cos[5*e + 4*f*x] - 2217600*f*x*Cos[4*e
 + 5*f*x] + 2217600*f*x*Cos[6*e + 5*f*x] - 2217600*f*x*Cos[5*e + 6*f*x] +
2217600*f*x*Cos[7*e + 6*f*x] + 1330560*f*x*Cos[6*e + 7*f*x] - 1330560*f*x*
Cos[8*e + 7*f*x] - 221760*f*x*Cos[7*e + 8*f*x] + 221760*f*x*Cos[9*e + 8*f*
x] + 17677440*Sin[e] - 49287040*Sin[f*x] - 86058610*Sin[e + f*x] + 5163516
6*Sin[2*(e + f*x)] + 26599934*Sin[3*(e + f*x)] - 39117550*Sin[4*(e + f*x)]
 + 7823510*Sin[5*(e + f*x)] + 7823510*Sin[6*(e + f*x)] - 4694106*Sin[7*(e
+ f*x)] + 782351*Sin[8*(e + f*x)] - 55651200*Sin[2*e + f*x] + 47971968*Sin
[e + 2*f*x] + 14990976*Sin[3*e + 2*f*x] + 8100992*Sin[2*e + 3*f*x] + 24334
464*Sin[4*e + 3*f*x] - 28627840*Sin[3*e + 4*f*x] - 19071360*Sin[5*e + 4*f*
x] + 9687680*Sin[4*e + 5*f*x] - 147840*Sin[6*e + 5*f*x] + 5548160*Sin[5*e
+ 6*f*x] + 3991680*Sin[7*e + 6*f*x] - 4393344*Sin[6*e + 7*f*x] - 1330560*S
in[8*e + 7*f*x] + 953984*Sin[7*e + 8*f*x])*Tan[e + f*x])/(113541120*a^3*c^
6*f*(-1 + Sec[e + f*x])^6*(1 + Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4392, 3042, 4374, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^6}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^6}dx$$

$\Big\downarrow$ 4392

$$\frac{\int \cot^{12}(e+fx)(\sec(e+fx)a+a)^3dx}{a^6c^6}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3}{\cot\left(e+fx+\frac{\pi}{2}\right)^{12}}dx}{a^6c^6}$$

$\Big\downarrow$ 4374

$$\frac{\int \left(a^3\cot^{12}(e+fx)+3a^3\csc(e+fx)\cot^{11}(e+fx)+3a^3\csc^2(e+fx)\cot^{10}(e+fx)+a^3\csc^3(e+fx)\cot^9(e+\right.}{a^6c^6}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{4a^3\cot^{11}(e+fx)}{11f}+\frac{a^3\cot^9(e+fx)}{9f}-\frac{a^3\cot^7(e+fx)}{7f}+\frac{a^3\cot^5(e+fx)}{5f}-\frac{a^3\cot^3(e+fx)}{3f}+\frac{a^3\cot(e+fx)}{f}-\frac{4a^3\csc^{11}(e+fx)}{11f}+\frac{19a^3 c}{}}{a^6c^6}$$

input
```
Int[1/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

output
```
(a^3*x + (a^3*Cot[e + f*x])/f - (a^3*Cot[e + f*x]^3)/(3*f) + (a^3*Cot[e +
f*x]^5)/(5*f) - (a^3*Cot[e + f*x]^7)/(7*f) + (a^3*Cot[e + f*x]^9)/(9*f) -
(4*a^3*Cot[e + f*x]^11)/(11*f) + (3*a^3*Csc[e + f*x])/f - (16*a^3*Csc[e +
f*x]^3)/(3*f) + (34*a^3*Csc[e + f*x]^5)/(5*f) - (36*a^3*Csc[e + f*x]^7)/(7
*f) + (19*a^3*Csc[e + f*x]^9)/(9*f) - (4*a^3*Csc[e + f*x]^11)/(11*f))/(a^6
*c^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4374
```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Int[ExpandIntegrand[(e*Cot[c + d*x])^m, (a + b*Csc[
c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.52

method	result
parallelrisch	$\frac{-315\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+3850\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-22770\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+90090\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-693\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-295680\cot \cdots}{887040 f\, a^{3} c^{6}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-46\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{\cdots}{7\,\mathrm{t}\cdots}}{256 f\, a^{3} c^{6}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}+\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-46\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+512\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{10}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{\cdots}{7\,\mathrm{t}\cdots}}{256 f\, a^{3} c^{6}}$
risch	$\frac{x}{a^{3} c^{6}}+\frac{2i\left(10395\,e^{15i(fx+e)}-31185\,e^{14i(fx+e)}+1155\,e^{13i(fx+e)}+148995\,e^{12i(fx+e)}-190113\,e^{11i(fx+e)}-117117\,e^{10\cdots}\right.}{\cdots}$
norman	$\frac{\frac{x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{ca}-\frac{1}{2816acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{1152acf}-\frac{23\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{896acf}+\frac{13\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{128acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{3acf}+\frac{191\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128acf}-\frac{23\,\mathrm{t}\cdots}{\cdots}}{c^{5} a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOSE)
```

output
```
1/887040*(-315*cot(1/2*f*x+1/2*e)^11+3850*cot(1/2*f*x+1/2*e)^9-22770*cot(1
/2*f*x+1/2*e)^7+90090*cot(1/2*f*x+1/2*e)^5-693*tan(1/2*f*x+1/2*e)^5-295680
*cot(1/2*f*x+1/2*e)^3+11550*tan(1/2*f*x+1/2*e)^3+887040*f*x+1323630*cot(1/
2*f*x+1/2*e)-159390*tan(1/2*f*x+1/2*e))/f/a^3/c^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{7453\cos(fx+e)^8 - 11964\cos(fx+e)^7 - 11866\cos(fx+e)^6 + 30542\cos(fx+e)^5 + 90\cos(fx+e}{3465\left(a^3c^6f\cos(fx+e)\right.}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fricas")
```

output
```
1/3465*(7453*cos(f*x + e)^8 - 11964*cos(f*x + e)^7 - 11866*cos(f*x + e)^6
+ 30542*cos(f*x + e)^5 + 90*cos(f*x + e)^4 - 26438*cos(f*x + e)^3 + 8539*c
os(f*x + e)^2 + 3465*(f*x*cos(f*x + e)^7 - 3*f*x*cos(f*x + e)^6 + f*x*cos(
f*x + e)^5 + 5*f*x*cos(f*x + e)^4 - 5*f*x*cos(f*x + e)^3 - f*x*cos(f*x + e
)^2 + 3*f*x*cos(f*x + e) - f*x)*sin(f*x + e) + 7671*cos(f*x + e) - 3712)/(
(a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x + e)^6 + a^3*c^6*f*cos(f*x
 + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6*f*cos(f*x + e)^3 - a^3*c^
6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a^3*c^6*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \frac{1}{\sec^9(e+fx)-3\sec^8(e+fx)+8\sec^6(e+fx)-6\sec^5(e+fx)-6\sec^4(e+fx)+8\sec^3(e+fx)-3\sec(e+fx)+1}\,dx}{a^3c^6}$$

input
```
integrate(1/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

output
```
Integral(1/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)**6 - 6*se
c(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e + f*x) + 1
), x)/(a**3*c**6)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{\frac{231\left(\frac{690\sin(fx+e)}{\cos(fx+e)+1}-\frac{50\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6}-\frac{1774080\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^3c^6}-\frac{5\left(\frac{770\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4554\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{18018}{(\cos(f}\right.}{}}{887040\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="maxima")
```

output
```
-1/887040*(231*(690*sin(f*x + e)/(cos(f*x + e) + 1) - 50*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) - 17
74080*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/(a^3*c^6) - 5*(770*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 4554*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 18
018*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 59136*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 264726*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x +
e) + 1)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\frac{887040(fx+e)}{a^3c^6}+\frac{5\left(264726\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-59136\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}+18018\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-4554\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+770\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.\right.}{a^3c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}}}{887040\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="giac")
```

output
```
1/887040*(887040*(f*x + e)/(a^3*c^6) + 5*(264726*tan(1/2*f*x + 1/2*e)^10 -
 59136*tan(1/2*f*x + 1/2*e)^8 + 18018*tan(1/2*f*x + 1/2*e)^6 - 4554*tan(1/
2*f*x + 1/2*e)^4 + 770*tan(1/2*f*x + 1/2*e)^2 - 63)/(a^3*c^6*tan(1/2*f*x +
 1/2*e)^11) - 231*(3*a^12*c^24*tan(1/2*f*x + 1/2*e)^5 - 50*a^12*c^24*tan(1
/2*f*x + 1/2*e)^3 + 690*a^12*c^24*tan(1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{315\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}+693\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}-11550\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}+159390\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}}{}$$

input
```
int(1/((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

output
```
-(315*cos(e/2 + (f*x)/2)^16 + 693*sin(e/2 + (f*x)/2)^16 - 11550*cos(e/2 +
(f*x)/2)^2*sin(e/2 + (f*x)/2)^14 + 159390*cos(e/2 + (f*x)/2)^4*sin(e/2 + (
f*x)/2)^12 - 1323630*cos(e/2 + (f*x)/2)^6*sin(e/2 + (f*x)/2)^10 + 295680*c
os(e/2 + (f*x)/2)^8*sin(e/2 + (f*x)/2)^8 - 90090*cos(e/2 + (f*x)/2)^10*sin
(e/2 + (f*x)/2)^6 + 22770*cos(e/2 + (f*x)/2)^12*sin(e/2 + (f*x)/2)^4 - 385
0*cos(e/2 + (f*x)/2)^14*sin(e/2 + (f*x)/2)^2 - 887040*cos(e/2 + (f*x)/2)^5
*sin(e/2 + (f*x)/2)^11*(e + f*x))/(887040*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*s
in(e/2 + (f*x)/2)^11)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{-693\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{16}+11550\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}-159390\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}+887040\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}fx+132}{887040}$$

input
```
int(1/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

output
```
( - 693*tan((e + f*x)/2)**16 + 11550*tan((e + f*x)/2)**14 - 159390*tan((e
+ f*x)/2)**12 + 887040*tan((e + f*x)/2)**11*f*x + 1323630*tan((e + f*x)/2)
**10 - 295680*tan((e + f*x)/2)**8 + 90090*tan((e + f*x)/2)**6 - 22770*tan(
(e + f*x)/2)**4 + 3850*tan((e + f*x)/2)**2 - 315)/(887040*tan((e + f*x)/2)
**11*a**3*c**6*f)
```

3.42 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \frac{2\sqrt{a}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2ac^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^3c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^4c^4\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

output

```
2*a^(1/2)*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a*c^4*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^4*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^3*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^4*c^4*
tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 5.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$$
$$= \frac{2ac^4\left(105\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-176+122\sec(e+fx)-66\sec^2(e+fx)+\right.}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

output
```
(2*a*c^4*(105*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
 c*Sec[e + f*x]]*(-176 + 122*Sec[e + f*x] - 66*Sec[e + f*x]^2 + 15*Sec[e +
 f*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
 + f*x]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4392}$$

$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \textbf{4375}$$

$$-\frac{2a^5c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \textbf{254}$$

$$-\frac{2a^5c^4\int \left(\frac{\tan^6(e+fx)}{a(\sec(e+fx)a+a)^3}-\frac{\tan^4(e+fx)}{a^2(\sec(e+fx)a+a)^2}+\frac{\tan^2(e+fx)}{a^3(\sec(e+fx)a+a)}+\frac{1}{a^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^4}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{2a^5c^4\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{9/2}}+\frac{\tan(e+fx)}{a^4\sqrt{a\sec(e+fx)+a}}-\frac{\tan^3(e+fx)}{3a^3(a\sec(e+fx)+a)^{3/2}}+\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}-\frac{\tan^7(e+fx)}{7a(a\sec(e+fx)+a}\right.}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*a^5*c^4*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
9/2)) + Tan[e + f*x]/(a^4*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a^
3*(a + a*Sec[e + f*x])^(3/2)) + Tan[e + f*x]^5/(5*a^2*(a + a*Sec[e + f*x])
^(5/2)) - Tan[e + f*x]^7/(7*a*(a + a*Sec[e + f*x])^(7/2))))/f
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2c^4\sqrt{a(1+\sec(fx+e))}\left(\left(176\cos(fx+e)^3-122\cos(fx+e)^2+66\cos(fx+e)-15\right)\tan(fx+e)\sec(fx+e)^2+(-105\cos(fx+e)-105\right.}{105f(\cos(fx+e)+1)}$
parts	$\frac{c^4\sqrt{2}\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}+\frac{2c^4\left(16\cos(fx+e)^3+8\cos(fx+e)^2+6\cos\right.}{35}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/105*c^4/f*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*((176*cos(f*x+e)^3-12
2*cos(f*x+e)^2+66*cos(f*x+e)-15)*tan(f*x+e)*sec(f*x+e)^2+(-105*cos(f*x+e)-
105)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*cs
c(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.13

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx$$

$$= \left[\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(f}{\cos(fx+e)+1}\right.}{105\left(f\cos\left(fx\right.\right.}\right.$$

$$-\frac{2\left(105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(176\,c^4\cos\left(fx+e\right)^3\right.\right.}{105\left(f\cos\left(fx+e\right)^4+f\cos\left(\right.\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(176*c^4*c
os(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*cos(f*x + e) - 15*c^4)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*c
os(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt
(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s
in(f*x + e))) + (176*c^4*cos(f*x + e)^3 - 122*c^4*cos(f*x + e)^2 + 66*c^4*
cos(f*x + e) - 15*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = c^4\left(\int \left(-4\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int 6\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx + \int \left(-4\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\right)dx + \int \sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\,dx + \int \sqrt{a\sec(e+fx)+a}\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**4,x)
```

output
```
c**4*(Integral(-4*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(6*s
qrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(-4*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**3, x) + Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)**4, x) + Integral(sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \int \sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^4\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/210*(105*((c^4*cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(
2*f*x + 2*e) + c^4)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) -
(c^4*cos(2*f*x + 2*e)^2 + c^4*sin(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e)
+ c^4)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^4*f*cos(
2*f*x + 2*e)^2 + c^4*f*sin(2*f*x + 2*e)^2 + 2*c^4*f*cos(2*f*x + 2*e) + c^4
*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*
cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e
)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2
*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x +
 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*
x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6
*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin
(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e...
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.69

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = -\frac{\frac{105\sqrt{-a}ac^4\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(105\sqrt{2}a^4c^4\mathrm{sgn}(\cos(fx+e))-\left(38\right.\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
-1/105*(105*sqrt(-a)*a*c^4*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(105*sqrt(2)*a^4*c^4*sgn(co
s(f*x + e)) - (385*sqrt(2)*a^4*c^4*sgn(cos(f*x + e)) + (379*sqrt(2)*a^4*c^
4*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 539*sqrt(2)*a^4*c^4*sgn(cos(f
*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/
2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)
))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^4\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^4, x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^4\,dx &= \sqrt{a}\,c^4\bigg(\int \sqrt{\sec(fx+e)+1}dx+\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4\,dx \\ &\quad -4\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right) \\ &\quad +6\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right) \\ &\quad -4\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\bigg)\end{aligned}$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^4,x)
```

output
```
sqrt(a)*c**4*(int(sqrt(sec(e + f*x) + 1),x) + int(sqrt(sec(e + f*x) + 1)*s
ec(e + f*x)**4,x) - 4*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x) + 6*in
t(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x) - 4*int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x),x))
```

3.43 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 140

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \frac{2\sqrt{a}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}-\frac{2a^3c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*a^(1/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a*c^3*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^3*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)-2/5*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx = \frac{2ac^3\left(15\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-23+11\sec(e+fx)-3\sec^2(e+fx))\right)\tan(}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a*c^3*(15*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
c*Sec[e + f*x]]*(-23 + 11*Sec[e + f*x] - 3*Sec[e + f*x]^2))*Tan[e + f*x])/
(15*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^4c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{254}$$

$$\frac{2a^4c^3\int\left(\frac{\tan^4(e+fx)}{a(\sec(e+fx)a+a)^2}-\frac{\tan^2(e+fx)}{a^2(\sec(e+fx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4c^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{7/2}}-\frac{\tan(e+fx)}{a^3\sqrt{a\sec(e+fx)+a}}+\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}-\frac{\tan^5(e+fx)}{5a(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^4*c^3*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(7/2
) - Tan[e + f*x]/(a^3*Sqrt[a + a*Sec[e + f*x]]) + Tan[e + f*x]^3/(3*a^2*(a
 + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*a*(a + a*Sec[e + f*x])^(5/2)
)))/f
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)    Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

method	result
default	$-\frac{2c^3\sqrt{a(1+\sec(fx+e))}\left(23\sin(fx+e)-11\tan(fx+e)+3\sec(fx+e)\tan(fx+e)-15(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\ }{\sqrt{\ }}\right.\right.}{15f(\cos(fx+e)+1)}$
parts	$\frac{c^3\sqrt{2}\,\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}+\frac{6c^3\sqrt{a(1+\sec(fx+e))}\,(\cot(fx+e)-\csc(f}{f}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15*c^3/f*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(23*sin(f*x+e)-11*tan(
f*x+e)+3*sec(f*x+e)*tan(f*x+e)-15*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)
^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.48

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$
$$= \left[\frac{15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx\right.}{\cos(fx+e)+1}}{15\left(f\cos(fx+e)^3+f\cos\right.}\right.$$
$$-\frac{2\left(15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(23\,c^3\cos(fx+e)^2-\right.}{15\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(
f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(23*c^3*cos(
f*x + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(c^
3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (23*c^3*cos(f*x
 + e)^2 - 11*c^3*cos(f*x + e) + 3*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$
$$= -c^3\left(\int 3\sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx + \int\left(-3\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx + \int\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx + \int\left(-\sqrt{a\sec(e+fx)+a}\right)dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-3*
sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x
) + a)*sec(e + f*x)**3, x) + Integral(-sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$
$$=\int -\sqrt{a\sec{(fx+e)}+a}(c\sec{(fx+e)}-c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*
f*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c
^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) +
c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*
f*x + 2*e)^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f
)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e)
+ 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e
) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(124) = 248$.

Time = 0.57 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.88

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx =$$

$$-\frac{\frac{15\sqrt{-a}ac^3\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)\operatorname{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(15\sqrt{2}a^3c^3\operatorname{sgn}(\cos(fx+e))+\left(37\sqrt{\ }\right.\right.}{}}{15f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(-a)*a*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-
a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(15*sqrt(2)*a^3*c^3*sgn(cos(f
*x + e)) + (37*sqrt(2)*a^3*c^3*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 -
40*sqrt(2)*a^3*c^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x
+ 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2
+ a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$
$$=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3\,dx$$
$$= \sqrt{a}\,c^3\left(\int \sqrt{\sec(fx+e)+1}dx - \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right) + 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right) - 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*c**3*(int(sqrt(sec(e + f*x) + 1),x) - int(sqrt(sec(e + f*x) + 1)*s
ec(e + f*x)**3,x) + 3*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x) - 3*in
t(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.44 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \frac{2\sqrt{a}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*a^(1/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a*c^2*
tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*
x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \frac{2ac^2\left(3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+(-4+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a*c^2*(3*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + (-4 + Sec[
e + f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(3*f*Sqrt[a*(1 + Sec[e +
 f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^3c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{254}$$

$$-\frac{2a^3c^2\int \left(\frac{\tan^2(e+fx)}{a(\sec(e+fx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3c^2\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}-\frac{\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^3*c^2*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
5/2)) + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a*
(a + a*Sec[e + f*x])^(3/2))))/f
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29

method	result
default	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\left(-8\sin(fx+e)+2\tan(fx+e)+3\sqrt{2}\left(\cos(fx+e)+1\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\left(-\cot(fx+e)+\mathrm{cs}\right.}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(f}}\right.\right.}{3f(\cos(fx+e)+1)}$
parts	$\frac{c^2\sqrt{2}\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\left(-\cot(fx+e)+\csc(fx+e)\right)}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}+\frac{c^2(4\sin(fx+e)+2\tan(fx+e))\sqrt{a(1+\sec}}{f(3\cos(fx+e)+3)}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^2/f*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(-8*sin(f*x+e)+2*tan(f*x
+e)+3*2^(1/2)*(cos(f*x+e)+1)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(
2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(
f*x+e)+csc(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx$$

$$=\left[\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e}{\cos(fx+e)+1}\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}\right.$$

$$-\frac{2\left(3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(4\,c^2\cos\left(fx+e\right)-c^2\right)\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(4*c^2*cos(f*x +
 e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*
x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e)
)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))) + (4*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = c^2\left(\int\left(-2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx + \int\sqrt{a\sec(e+fx)+a}\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral(-2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(sqr
t(a*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(sqrt(a*sec(e + f*x) +
 a), x))
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \int\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(3*(2*c^2*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f
*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f
*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6
*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2
*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*s
in(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (c
os(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + c
os(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*
f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(93) = 186$.

Time = 0.50 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.21

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = -\frac{\frac{3\sqrt{-a}ac^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(5\sqrt{2}a^2c^2\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx\right.\right.}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}}{3f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(-a)*a*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(5*sqrt(2)*a^2*c^2*sgn(cos(f*x
+ e))*tan(1/2*f*x + 1/2*e)^2 - 3*sqrt(2)*a^2*c^2*sgn(cos(f*x + e)))*tan(1/
2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2*e
)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2\,dx = \sqrt{a}\,c^2\left(\int \sqrt{\sec(fx+e)+1}dx+\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx - 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*c**2*(int(sqrt(sec(e + f*x) + 1),x) + int(sqrt(sec(e + f*x) + 1)*s
ec(e + f*x)**2,x) - 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.45 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2ac\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^(1/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a*c*tan(
f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \frac{2c\sqrt{a(1+\sec(e+fx))}\left(\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)-\sqrt{c-c\sec(e+fx)}\right)\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

output
```
(2*c*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/
Sqrt[c]] - Sqrt[c - c*Sec[e + f*x]])*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e
 + f*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4392, 3042, 4375, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \frac{\tan^2(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^2c\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{2a^2c\left(-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow 216$$

$$\frac{2a^2c\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^2*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2)
- Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]])))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.58

method	result
default	$\frac{c\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}+2\cot(fx+e)-2\csc(fx+e)\right)}{f}$
parts	$\frac{c\sqrt{2}\,\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}+\frac{2c\sqrt{a(1+\sec(fx+e))}\,(\cot(fx+e)-\csc(fx-}{f}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
c/f*(a*(1+sec(f*x+e)))^(1/2)*(2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)+2*cot(f*x+e)-2*csc(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.55

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$=\left[\frac{(c\cos(fx+e)+c)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)-2\,c\sqrt{\frac{a\cos(}{\cos(}}}{f\cos(fx+e)+f}\right.$$
$$\left.-\frac{2\left((c\cos(fx+e)+c)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{f\cos(fx+e)+f}\right]$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x +
 e) - a)/(cos(f*x + e) + 1)) - 2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) + c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e) + f)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = -c\left(\int \sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}\,dx+\int\left(-\sqrt{a\sec{(e+fx)}+a}\right)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e)),x)
```

output
```
-c*(Integral(sqrt(a*sec(e + f*x) + a)*sec(e + f*x), x) + Integral(-sqrt(a*
sec(e + f*x) + a), x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(58) = 116$.

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx = \frac{\sqrt{a}c\arctan\left(\left(\cos\left(2fx+2e\right)^2+\sin\left(2fx+2e\right)^2+2\cos\left(2fx+2e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
+ sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
cos(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(58) = 116$.

Time = 0.39 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.92

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$

$$= \frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\,ac\,\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a} - \frac{\sqrt{-a}\,ac\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6\right.}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6\right.}\right.}{|a|}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
(2*sqrt(2)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*a*c*sgn(cos(f*x + e))*tan(1
/2*f*x + 1/2*e)/(a*tan(1/2*f*x + 1/2*e)^2 - a) - sqrt(-a)*a*c*log(abs(2*(s
qrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*
sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1
/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs
(a))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$= \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x)), x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))\,dx$$
$$=\sqrt{a}\,c\left(\int \sqrt{\sec(fx+e)+1}dx-\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e)),x)
```

output
```
sqrt(a)*c*(int(sqrt(sec(e + f*x) + 1),x) - int(sqrt(sec(e + f*x) + 1)*sec(
e + f*x),x))
```

3.46 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+2*cot(f*x+
e)*(a+a*sec(f*x+e))^(1/2)/c/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \frac{2\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{cf}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

output
```
(2*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*Sqrt[a*(
1 + Sec[e + f*x])])/(c*f)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^{3/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{264}$$

$$\frac{2\left(\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{cf}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)}{cf}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x]),x]
```

output
```
(2*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + Cot[
e + f*x]*Sqrt[a + a*Sec[e + f*x]]))/(c*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(61) = 122$.

Time = 0.74 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.80

method	result
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)-2\sqrt{2}\,(\cos(}{cf(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/c/f*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(2^(1/2)*(cos(f*x+e)+1)*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e
)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-2*2^(1/2)*(co
s(f*x+e)+1)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*cot(f*x+e)+(-2*cos(f*x+e)+2)*cot(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.86

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sin(fx+e)-7\,a\cos(fx+e)+a}{\cos(fx+e)+1}\right)\sin\left(fx+e\right)+4}{2\,cf\sin\left(fx+e\right)}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*co
s(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (sqrt(a)*arctan(2*sqr
t(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*
a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\,dx = -\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c - c\sec(e + fx)}\,dx = \int -\frac{\sqrt{a\sec(fx + e) + a}}{c\sec(fx + e) - c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(61) = 122$.

Time = 0.33 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c|a|}+\frac{4\sqrt{-a}a}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.\right.}\right.}{2f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
- sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/(c*abs(a)) + 4*sqrt(-a)*a/(((sqrt(-a)*tan(1/2*f*x
+ 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c))*sgn(cos(f*x + e
))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)-1}dx\right)}{c}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) - 1),x))/c
```

3.47 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^2f}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f+2*cot(f*
x+e)*(a+a*sec(f*x+e))^(1/2)/c^2/f-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/
a/c^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^2}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(3*
f*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4392, 3042, 4375, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4392}$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{5/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \; 4375$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{ac^2f}$$

$$\Big\downarrow \; 264$$

$$-\frac{2\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{ac^2f}$$

$$\Big\downarrow \; 264$$

$$-\frac{2\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)\right)}{ac^2f}$$

$$\Big\downarrow \; 216$$

$$-\frac{2\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)\right)}{ac^2f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(Sq
rt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*Se
c[e + f*x]])))/(a*c^2*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(92) = 184$.

Time = 0.82 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.06

method	result
default	$-\frac{\sqrt{a(1+\sec(fx+e))}\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}+\left(-7\cos(fx+e)^2-2\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2/f*(a*(1+sec(f*x+e)))^(1/2)*(-3*2^(1/2)*arctanh(2^(1/2)/(cot(f*x+e
)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+(-7*cos(f*x+e)^2-2*cos(f*x+e)+5)*2^
(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*cot(f*x+e)*csc(f*x+e)^2+(-6*cos(f*x+e)^2+12*cos(f*x+e)-6)*cot(f*x+e)*c
sc(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx = \left[\frac{3\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-7\,a\cos(fx+e)-}{\cos(fx+e)+1}\right.}{6\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*
x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*
(4*cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*sqrt(a)*(cos(f*x +
e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) +
 2*(4*cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^2}\,dx = \frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),
x)/c**2
```

Maxima [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^2}\,dx = \int \frac{\sqrt{a\sec(fx+e)+a}}{(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(92) = 184$.

Time = 0.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^2}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{3\sqrt{2}\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c^2|a|}+\frac{2\left(9\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\cdots\right.\right.\right.}{\left(\left(\sqrt{\cdots}\right.\right.}\right.}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*(3*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e
) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 +
 4*sqrt(2)*abs(a) - 6*a))/(c^2*abs(a)) + 2*(9*(sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a - 12*(sqrt(-a)*tan(
1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^2 + 7
*sqrt(-a)*a^3)/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/
2*e)^2 + a))^2 - a)^3*c^2))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^2}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2-2\sec(fx+e)+1}dx\right)}{c^2}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1)
,x))/c**2
```

3.48 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^3f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^3f}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f+2*cot(f*
x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/
a/c^3/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/a^2/c^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{5c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(5*c
^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4392, 3042, 4375, 264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{7/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^3f}$$

$$\Big\downarrow \text{264}$$

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^2c^3f}$$

$$\Big\downarrow \text{264}$$

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\right.\right.}{a^2c^3f}$$

$$\Big\downarrow \text{264}$$

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+\right.\right.\right.}{a^2c^3f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+}}\right.\right.\right.\right.}{a^2c^3f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*((Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(a
+ a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt
[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]]))))/(a^2*c^3
*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(123) = 246$.

Time = 0.90 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.83

method	result
default	$\left(-\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)-\frac{\sqrt{2}\left(289\cos(fx+e)^4+182\cos(fx+e)^3-408\cos(\ldots\right.}{\ldots}\right.$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3/f*(-2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(co
t(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)
^(1/2))-1/120*2^(1/2)*(289*cos(f*x+e)^4+182*cos(f*x+e)^3-408*cos(f*x+e)^2-
86*cos(f*x+e)+215)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*cot(f*x+e)*csc(f*x+e)^4-1/120*(210*cos(f*x+e)^4-390*cos(f
*x+e)^3-90*cos(f*x+e)^2+510*cos(f*x+e)-240)*cot(f*x+e)*csc(f*x+e)^4)*(a*(1
+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^3}\,dx$$
$$= \left[\frac{15\left(\cos\left(fx + e\right)^2 - 2\cos\left(fx + e\right) + 1\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3 - 4\left(2\cos(fx+e)^2 - \cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx+e)+1}\right.}{30\left(c^3 f\cos\left(fx + e\right)^2 - 2\,c\right.}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/30*(15*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x
 + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) +
1))*sin(f*x + e) + 4*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c
^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(cos(f*x + e)^2 - 2*cos
(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))
*sin(f*x + e) + 2*(23*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 15*cos(f*x + e)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*
f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 +
3*sec(e + f*x) - 1), x)/c**3
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx = \int -\frac{\sqrt{a\sec(fx+e)+a}}{(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) - c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(123) = 246$.

Time = 0.41 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.81

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{15\sqrt{2}\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c^3|a|}+\frac{105\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx-\right.}\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/30*sqrt(2)*(15*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2
*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
 + 4*sqrt(2)*abs(a) - 6*a))/(c^3*abs(a)) + (105*(sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-a)*a - 300*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2
+ 430*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)
)^4*sqrt(-a)*a^3 - 260*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*
x + 1/2*e)^2 + a))^2*sqrt(-a)*a^4 + 73*sqrt(-a)*a^5)/(((sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^5*c^3))*sgn(cos(
f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^3}\,dx=-\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)}{c^3}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x)
```

output
```
( - sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 - 3*sec(e + f*x)**
2 + 3*sec(e + f*x) - 1),x))/c**3
```

3.49 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3ac^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^2c^4f} - \frac{2\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^3c^4f}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f+2*cot(f*
x+e)*(a+a*sec(f*x+e))^(1/2)/c^4/f-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/
a/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/a^2/c^4/f-2/7*cot(f*x+e)^7
*(a+a*sec(f*x+e))^(7/2)/a^3/c^4/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.86 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = -\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\sec(e+fx)\right)\tan(e+fx)}{7f\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^4}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*a*Hypergeometric2F1[-7/2, 1, -5/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(7*
f*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4392, 3042, 4375, 264, 264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{ 4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^{9/2}dx}{a^4c^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \ 4375$$

$$-\frac{2\int \frac{\cot^8(e+fx)(\sec(e+fx)a+a)^4}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3c^4f}$$

$$\Big\downarrow \ 264$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^3c^4f}$$

$$\Big\downarrow \ 264$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right.\right.}{a^3c^4f}$$

$$\Big\downarrow \ 264$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e\right.\right.\right.}{a^3c^4f}$$

$$\Big\downarrow \ 264$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e\right.\right.\right.}{a^3c^4f}$$

$$\Big\downarrow \ 216$$

$$-\frac{2\left(\frac{1}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\left(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e-\right.\right.\right.}{a^3c^4f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*((Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/7 - a*((Cot[e + f*x]^5*(a
 + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2
))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]
+ Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])))))/(a^3*c^4*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.69

method	result
default	$\frac{\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}+\frac{\sqrt{2}\left(8117\cos(fx+e)^6+8206\cos(fx+e)^5-17021\cos\right.}{}\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/c^4/f*(2^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc
(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)+1/3360*2^(1/2)*(8117*cos(f*x+e)^6+8206*cos(f*x+e)^5-17021*cos(f*x+
e)^4-6844*cos(f*x+e)^3+18939*cos(f*x+e)^2+2478*cos(f*x+e)-6195)*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)*c
sc(f*x+e)^6+1/3360*(4970*cos(f*x+e)^6-7630*cos(f*x+e)^5-12460*cos(f*x+e)^4
+26740*cos(f*x+e)^3-3710*cos(f*x+e)^2-14630*cos(f*x+e)+6720)*cot(f*x+e)*cs
c(f*x+e)^6)*(a*(1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \left[\frac{105\left(\cos(fx+e)^3-3\cos(fx+e)^2+3\cos(fx+e)-1\right)\sqrt{-a}\log\left(-\frac{8a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx\right.}{}\right.}{210\left(c^4 f\cos(fx\right.}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/210*(105*(cos(f*x + e)^3 - 3*cos(f*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(
-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e)
+ a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(176*cos(f*x + e)^4 - 406*cos(f*
x + e)^3 + 350*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f
*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(cos(f*x + e)^3 - 3*cos(f
*x + e)^2 + 3*cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*c
os(f*x + e) - a))*sin(f*x + e) + 2*(176*cos(f*x + e)^4 - 406*cos(f*x + e)^
3 + 350*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*
x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{\sqrt{a\sec{(e+fx)}+a}}{\sec^4{(e+fx)}-4\sec^3{(e+fx)}+6\sec^2{(e+fx)}-4\sec{(e+fx)}+1}\,dx}{c^4}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**4,x)
```

output
```
Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6
*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x)/c**4
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(154) = 308$.

Time = 0.44 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{210\sqrt{2}\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c^4|a|}+\frac{1575\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}f\right.}\right.}{}\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
-1/420*sqrt(2)*(210*sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1
/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/a
bs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))
^2 + 4*sqrt(2)*abs(a) - 6*a))/(c^4*abs(a)) + (1575*(sqrt(-a)*tan(1/2*f*x +
 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*sqrt(-a)*a - 7140*(sqrt(
-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^10*sqrt(-a
)*a^2 + 16415*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e
)^2 + a))^8*sqrt(-a)*a^3 - 19880*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^4 + 14637*(sqrt(-a)*tan(1/2*f*x
+ 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^5 - 5684*(sqr
t(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-
a)*a^6 + 1037*sqrt(-a)*a^7)/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^2 - a)^7*c^4))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^4}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4-4\sec(fx+e)^3+6\sec(fx+e)^2-4\sec(fx+e)+1}dx\right)}{c^4}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 +
 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1),x))/c**4
```

3.50 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^{3/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^4c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{2a^5c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

output
```
2*a^(3/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^
3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)-2/5*a^4*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-2/7*a^5*c^
3*tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx =$$

$$-\frac{2a^2\left(-105c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+c^3\sqrt{c-c\sec(e+fx)}(146-32\sec(e+fx)-24\sec^2(e+fx)+}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(-2*a^2*(-105*c^(7/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + c^3*Sqrt
[c - c*Sec[e + f*x]]*(146 - 32*Sec[e + f*x] - 24*Sec[e + f*x]^2 + 15*Sec[e
 + f*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec
[e + f*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4392, 3042, 4375, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^5c^3\int \frac{\tan^6(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{2a^5c^3\left(\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}\right)}{f}$$

$$\Big\downarrow \text{254}$$

$$\frac{2a^5c^3\left(\int \left(\frac{\tan^4(e+fx)}{a(\sec(e+fx)a+a)^2}-\frac{\tan^2(e+fx)}{a^2(\sec(e+fx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^5c^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{7/2}}-\frac{\tan(e+fx)}{a^3\sqrt{a\sec(e+fx)+a}}+\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}-\frac{\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}-\frac{\tan^5(e+fx)}{5a(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^5*c^3*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(7/2
) - Tan[e + f*x]/(a^3*Sqrt[a + a*Sec[e + f*x]]) + Tan[e + f*x]^3/(3*a^2*(a
 + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*a*(a + a*Sec[e + f*x])^(5/2)
) - Tan[e + f*x]^7/(7*(a + a*Sec[e + f*x])^(7/2))))/f
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

method	result
default	$-\frac{2c^3a\sqrt{a(1+\sec(fx+e))}\left(\left(146\cos(fx+e)^3-32\cos(fx+e)^2-24\cos(fx+e)+15\right)\tan(fx+e)\sec(fx+e)^2+105(\cos(fx+e)+1)\sqrt{-}\right.}{105f(\cos(fx+e)+1)}$
parts	$\frac{c^3a\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)+2\sin(f\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*c^3/f*a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*((146*cos(f*x+e)^3-
32*cos(f*x+e)^2-24*cos(f*x+e)+15)*tan(f*x+e)*sec(f*x+e)^2+105*(cos(f*x+e)+
1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+
e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.18

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \left[\frac{105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{\cos(fx-}\right.}{}\right.$$

$$-\frac{2\left(105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(146\,ac^3\cos(fx+e)\right.\right.}{105\left(f\cos(fx+e)^4+f\cos(\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*
a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(146*a
*c^3*cos(f*x + e)^3 - 32*a*c^3*cos(f*x + e)^2 - 24*a*c^3*cos(f*x + e) + 15
*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x +
 e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f
*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)/(sqrt(a)*sin(f*x + e))) + (146*a*c^3*cos(f*x + e)^3 - 32*a*c^3*cos(f*
x + e)^2 - 24*a*c^3*cos(f*x + e) + 15*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^3\,dx = -c^3\left(\int \left(-a\sqrt{a\sec(e + fx) + a}\right)dx + \int 2a\sqrt{a\sec(e + fx) + a}\sec(e + fx)\,dx + \int \left(-2a\sqrt{a\sec(e + fx) + a}\sec^3(e + fx)\right)dx + \int a\sqrt{a\sec(e + fx) + a}\sec^4(e + fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(2*a*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x), x) + Integral(-2*a*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**3, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x
))
```

Maxima [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^3\,dx = \int -(a\sec(fx + e) + a)^{\frac{3}{2}}(c\sec(fx + e) - c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/210*(105*((a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^
3*cos(2*f*x + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 + 1) - (a*c^3*cos(2*f*x + 2*e)^2 + a*c^3*sin(2*f*x + 2*e)^2 + 2*a*c^3*cos
(2*f*x + 2*e) + a*c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1)
 - 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + a*c^3*f*sin(2*f*x + 2*e)^2 + 2*a*c^3*f*
cos(2*f*x + 2*e) + a*c^3*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
+ 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e
) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x +
 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x
 + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f
*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f
*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9...
```

Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx =$$

$$-\frac{\frac{105\sqrt{-a}a^2c^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\text{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(105\sqrt{2}a^5c^3\text{sgn}(\cos(fx+e))-\left(385\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/105*(105*sqrt(-a)*a^2*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqr
t(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(105*sqrt(2)*a^5*c^3*sgn(
cos(f*x + e)) - (385*sqrt(2)*a^5*c^3*sgn(cos(f*x + e)) + (139*sqrt(2)*a^5*
c^3*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 539*sqrt(2)*a^5*c^3*sgn(cos
(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x +
1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3\,dx = \sqrt{a}\,a\,c^3\left(\int \sqrt{\sec(fx+e)+1}dx - \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right) + 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right) - 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\right)$$

input

```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x)
```

output

```
sqrt(a)*a*c**3*(int(sqrt(sec(e + f*x) + 1),x) - int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x)**4,x) + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x) - 2*
int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.51 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 142

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^{3/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^3c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{2a^4c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*a^(3/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c^
2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^3*c^2*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)+2/5*a^4*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^2\left(15c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+c^2\sqrt{c-c\sec(e+fx)}(-17-\sec(e+fx)+3\,\mathrm{s}\right.}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a^2*(15*c^(5/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + c^2*Sqrt[c
- c*Sec[e + f*x]]*(-17 - Sec[e + f*x] + 3*Sec[e + f*x]^2))*Tan[e + f*x])/(
15*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4392, 3042, 4375, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^4c^2\int \frac{\tan^4(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{363}$$

$$-\frac{2a^4c^2\left(\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \ 254$$

$$-\frac{2a^4c^2\left(\int \left(\frac{\tan^2(e+fx)}{a(\sec(e+fx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^4c^2\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}-\frac{\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}-\frac{\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^4*c^2*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
5/2)) + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a*
(a + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*(a + a*Sec[e + f*x])^(5/2)
)))/f
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

method	result
default	$-\frac{2c^2 a\sqrt{a(1+\sec(fx+e))}\left(17\sin(fx+e)+\tan(fx+e)-3\sec(fx+e)\tan(fx+e)+15(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{}{\sqrt{co}}\right.\right.}{15f(\cos(fx+e)+1)}$
parts	$\frac{c^2 a\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)+2\sin(f\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*c^2/f*a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(17*sin(f*x+e)+tan(f
*x+e)-3*sec(f*x+e)*tan(f*x+e)+15*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e
)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.50

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e+fx))^2\,dx = \left[\frac{15\left(ac^2\cos(fx+e)^3 + ac^2\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\ldots}{\cos(fx+\ldots}\right.}{15\,(f\ldots}\right.$$

$$-\frac{2\left(15\left(ac^2\cos(fx+e)^3 + ac^2\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) + \left(17\,ac^2\cos(fx+e)^2 - \ldots\right.}{15\left(f\cos(fx+e)^3 + f\cos(fx+e)^2\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*
cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(17*a*c^
2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*
(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (17*
a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 3*a*c^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^2\,dx = c^2\left(\int a\sqrt{a\sec(e+fx)+a}\,dx + \int\left(-a\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int\left(-a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\right)dx + \int a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral(a*sqrt(a*sec(e + f*x) + a), x) + Integral(-a*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x), x) + Integral(-a*sqrt(a*sec(e + f*x) + a)*sec(e +
 f*x)**2, x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*
cos(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
 1) - (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2
*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) -
 2*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*co
s(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(126) = 252$.

Time = 0.65 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.87

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^2\,dx =$$

$$-\frac{\frac{15\sqrt{-a}a^2c^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{|a|} + \frac{2\left(15\sqrt{2}a^4c^2\operatorname{sgn}(\cos(fx+e))+\left(13\sqrt{2}a\right.\right.}{}}{15f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(-a)*a^2*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(15*sqrt(2)*a^4*c^2*sgn(cos
(f*x + e)) + (13*sqrt(2)*a^4*c^2*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2
- 40*sqrt(2)*a^4*c^2*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*
x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^
2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^2\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{3/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)^2 dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2\,dx = \sqrt{a}\,a\,c^2\left(\int\sqrt{\sec(fx+e)+1}dx + \int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx - \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right) - \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*a*c**2*(int(sqrt(sec(e + f*x) + 1),x) + int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x)**3,x) - int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x) - int(
sqrt(sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.52 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^2c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2a^3c\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^2*c*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a^3*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e)
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -\frac{2a^2c\left(-3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+(2+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(-2*a^2*c*(-3*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + (2 + Sec
[e + f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(3*f*Sqrt[a*(1 + Sec[e
+ f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4392, 3042, 4375, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \sqrt{\sec(e+fx)a+a}\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^3c\int \frac{\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{363}$$

$$\frac{2a^3c\left(\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{\tan^3(e+fx)}{3(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{262}$$

$$\frac{2a^3c\left(-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\tan^3(e+fx)}{3(a\sec(e+fx)+a)^{3/2}}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2a^3c\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}-\frac{\tan^3(e+fx)}{3(a\sec(e+fx)+a)^{3/2}}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^3*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2)
- Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*(a + a*Sec
[e + f*x])^(3/2))))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

method	result
default	$-\frac{2ca\sqrt{a(1+\sec(fx+e))}\left(2\sin(fx+e)+\tan(fx+e)+3(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx}{\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(fx+}}\right.\right.}{3f(\cos(fx+e)+1)}$
parts	$\frac{ca\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^{2}-1}}\right)+2\sin(fa\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c/f*a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(2*sin(f*x+e)+tan(f*x+e
)+3*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot
(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.00

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = \left[\frac{3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+\ldots}{\cos(fx+e)+1}\right.}{3\left(f\cos\left(fx+e\right)^2+\ldots\right.}\right.$$

$$-\frac{2\left(3\left(ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(2\,ac\cos\left(fx+e\right)+ac\right)\sqrt{\ldots}\right.}{3\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(2*a*c*cos(f*x +
 e) + a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*
x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e)
)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))) + (2*a*c*cos(f*x + e) + a*c)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))\,dx = -c\left(\int\left(-a\sqrt{a\sec\left(e+fx\right)+a}\right)dx+\int a\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e)),x)
```

output
```
-c*(Integral(-a*sqrt(a*sec(e + f*x) + a), x) + Integral(a*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**2, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(89) = 178$.

Time = 0.21 (sec) , antiderivative size = 998, normalized size of antiderivative = 9.88

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e),...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(89) = 178$.

Time = 0.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.25

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))\,dx =$$

$$-\frac{\frac{3\sqrt{-a}a^2c\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(\sqrt{2}a^3c\,\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right.}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)}}{3f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(-a)*a^2*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(sqrt(2)*a^3*c*sgn(cos(f*x + e)
)*tan(1/2*f*x + 1/2*e)^2 - 3*sqrt(2)*a^3*c*sgn(cos(f*x + e)))*tan(1/2*f*x
+ 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{3/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x)), x)
```

Reduce [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))\,dx = \sqrt{a}\,ac\left(\int \sqrt{\sec(fx + e) + 1}dx - \left(\int \sqrt{\sec(fx + e) + 1}\,\sec(fx + e)^2\,dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x)
```

output
```
sqrt(a)*a*c*(int(sqrt(sec(e + f*x) + 1),x) - int(sqrt(sec(e + f*x) + 1)*se
c(e + f*x)**2,x))
```

3.53 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{4a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+4*a*cot(f*
x+e)*(a+a*sec(f*x+e))^(1/2)/c/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \frac{2a\cot(e+fx)\sqrt{a(1+\sec(e+fx))}\left(2\sqrt{c}-\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)\sqrt{c-\cdots}\right.}{c^{3/2}f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(2*Sqrt[c] - ArcTanh[Sqrt[c -
 c*Sec[e + f*x]]/Sqrt[c]]*Sqrt[c - c*Sec[e + f*x]]))/(c^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^{5/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{359}$$

$$\frac{2a\left(2\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{cf}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+2\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)}{cf}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + 2*
Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]]))/(c*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{\left(-\sqrt{2}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^{2}-1}}\right)+4\cot(fx+e)\right)a\sqrt{a(1+\sec(fx+e))}}{cf}$	109

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/c/f*(-2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(
f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(
1/2))+4*cot(f*x+e))*a*(a*(1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.84

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{-a}a\log\left(-\frac{8\,a\cos(fx+e)^{3}-4\left(2\cos(fx+e)^{2}-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-}{\cos(fx+e)+1}\right.}{2\,cf\sin\left(fx+e\right.}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*a*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x
+ e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*
cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), (a^(3/2)*arctan(2
*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 4*a*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = -\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

output
```
-(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral(a*
sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x))/c
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{c\sec(fx+e)-c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate((a*sec(f*x + e) + a)^(3/2)/(c*sec(f*x + e) - c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(62) = 124$.

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.83

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{c - c\sec(e + fx)}\, dx =$$

$$-\frac{\sqrt{2}\sqrt{-a}a^3\left(\frac{\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{ac|a|}+\frac{8}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.\right.}\right.}{2f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*sqrt(-a)*a^3*(sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e
) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2
*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 +
 4*sqrt(2)*abs(a) - 6*a))/(a*c*abs(a)) + 8/(((sqrt(-a)*tan(1/2*f*x + 1/2*e
) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*a*c))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{c - c\sec(e + fx)}\, dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}{c - \frac{c}{\cos(e+f\,x)}}\, dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c-c\sec(e+fx)}\,dx = -\frac{\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)-1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right)}{c}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) - 1),x) + int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) - 1),x)))/c
```

3.54 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 102

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^2f} - \frac{4\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f+2*a*cot(
f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^2/f-4/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2
)/c^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^2\left(-2+3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\sec(e+fx)\right)\left(-1+\sec(e+\right.\right.}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a^2*(-2 + 3*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*(-1 + Sec
[e + f*x]))*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e
 + f*x])])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4392, 3042, 4375, 359, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4392}$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{7/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 4375}$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^2f}$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{2\left(\frac{2}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{c^2f}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2\left(\frac{2}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+fx)+a}-a\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+\right.\right.\right.}{\sqrt{\sec(e+f}}}{c^2f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\left(\frac{2}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)\right)}{c^2f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*((2*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(
Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*
Sec[e + f*x]])))/(c^2*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(90) = 180$.

Time = 1.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.11

method	result
default	$\frac{a\sqrt{a(1+\sec(fx+e))}\left(-3\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)+\left(8\cos(fx+e)^2+4\cos\right.\right.}{3}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/c^2/f*a*(a*(1+sec(f*x+e)))^(1/2)*(-3*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*
x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+(8*cos(f*x+e)^2+4*cos(f*x+e)-4)*2^(
1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*cot(f*x+e)*csc(f*x+e)^2+(6*cos(f*x+e)^2-12*cos(f*x+e)+6)*cot(f*x+e)*csc
(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.44

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^2}\,dx = \left[\frac{3\,(a\cos(fx + e) - a)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3 - 4\left(2\cos(fx+e)^2 - \cos(fx+e)\right)\sqrt{-a}\sqrt{}}{\cos(fx+e)+}\right.}{6\,(c^2} \right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/6*(3*(a*cos(f*x + e) - a)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(
f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) +
4*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(3*(a*cos(f*x +
e) - a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*
x + e) + 2*(5*a*cos(f*x + e)^2 - 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^2}\,dx = \frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1
), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x)**2
- 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(90) = 180$.

Time = 0.49 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.12

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx =$$

$$-\frac{\frac{3\sqrt{-a}a^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{c^2|a|}+\frac{4\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan(}\right.\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs
(a) - 6*a))*sgn(cos(f*x + e))/(c^2*abs(a)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/
2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*sgn(c
os(f*x + e)) - 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/
2*e)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(f*x + e)) + 2*sqrt(-a)*a^4*sgn(cos(f*x
 + e)))/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2
+ a))^2 - a)^3*c^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^2}\,dx=\frac{\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-2\sec(fx+e)+1}dx\right)}{c^2}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 - 2*sec(e + f*x) +
 1),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 - 2*se
c(e + f*x) + 1),x)))/c**2
```

3.55 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^3f} + \frac{4\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^3f}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f+2*a*cot(
f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2
)/c^3/f+4/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/a/c^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.90 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{2a^2\left(-6+5\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)(-1+\sec(e+fx))\right)\tan(e+fx)}{15c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(-2*a^2*(-6 + 5*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*(-1 + S
ec[e + f*x]))*Tan[e + f*x])/(15*c^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Se
c[e + f*x])])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4392, 3042, 4375, 359, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 4392$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{9/2}dx}{a^3c^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \; 4375$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{ac^3f}$$

$$\Big\downarrow \; 359$$

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{ac^3f}$$

$$\Big\downarrow \; 264$$

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\right.\right.}{ac^3f}$$

$$\Big\downarrow \; 264$$

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\cot(e+fx)\sqrt{a\sec(e+\right.\right.\right.}{ac^3f}$$

$$\Big\downarrow \; 216$$

$$\frac{2\left(\frac{2}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\left(\frac{1}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right.\right.\right.\right.}{ac^3f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*((2*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(
a + a*Sec[e + f*x])^(3/2))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + a*Sec[e + f*x]]] + Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]]))))/(a*c^3
*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(121) = 242$.

Time = 1.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.94

method	result
default	$-\frac{a\sqrt{a(1+\sec(fx+e))}\left(\sin(fx+e)^2(-15\cos(fx+e)+15)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)\sqrt{-\frac{2\,\mathrm{c}}{\mathrm{cos}(}}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15/c^3/f*a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)/(cos(f*x+e)^2-2*cos(
f*x+e)+1)*(sin(f*x+e)^2*(-15*cos(f*x+e)+15)*2^(1/2)*arctanh(2^(1/2)*(cot(f
*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1
/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2^(1/2)*(41*cos(f*x+e)^3-13*cos(
f*x+e)^2-29*cos(f*x+e)+25)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)+(30*cos(f*x+e)^3-90*cos(f*x+e)^2+90*co
s(f*x+e)-30)*cot(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.04

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \left[\frac{15\left(a\cos(fx+e)^2-2\,a\cos(fx+e)+a\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos\right.}{}\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/30*(15*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-a)*log(-(8*a*cos
(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e
) + 1))*sin(f*x + e) + 4*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2 + 15*a
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos(f*x +
e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/15*(15*(a*cos(f*x +
e)^2 - 2*a*cos(f*x + e) + a)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(
f*x + e) - a))*sin(f*x + e) + 2*(26*a*cos(f*x + e)^3 - 35*a*cos(f*x + e)^2
 + 15*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*cos
(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^3}\, dx = -\frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\, dx + \int \frac{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\, dx}{c^3}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2
 + 3*sec(e + f*x) - 1), x) + Integral(a*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^3}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(121) = 242$.

Time = 0.67 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.12

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx =$$

$$-\frac{\frac{15\sqrt{-a}a^2\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)\operatorname{sgn}(\cos(fx+e))}{c^3|a|}+\frac{2\sqrt{2}\left(30\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan}\right.\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) + 2*sqrt(2)*(30*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-a)*a^2*sg
n(cos(f*x + e)) - 75*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x
+ 1/2*e)^2 + a))^6*sqrt(-a)*a^3*sgn(cos(f*x + e)) + 115*(sqrt(-a)*tan(1/2*
f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^4*sgn(cos
(f*x + e)) - 65*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^2*sqrt(-a)*a^5*sgn(cos(f*x + e)) + 19*sqrt(-a)*a^6*sgn(cos(f*x
 + e)))/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2
+ a))^2 - a)^5*c^3))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^3}\,dx = -\frac{\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)}{c^3}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x)
```

output
```
( - sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 - 3*sec(e + f*x
)**2 + 3*sec(e + f*x) - 1),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/
(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x)))/c**3
```

3.56 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5ac^4f} - \frac{4\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7a^2c^4f}$$

output

```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f+2*a*cot(
f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^4/f-2/3*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2
)/c^4/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/a/c^4/f-4/7*cot(f*x+e)^7*(
a+a*sec(f*x+e))^(7/2)/a^2/c^4/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^2\left(-10+7\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)\left(-1+\sec(e\right.\right.}{35c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(2*a^2*(-10 + 7*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*(-1 + S
ec[e + f*x]))*Tan[e + f*x])/(35*c^4*f*(-1 + Sec[e + f*x])^4*Sqrt[a*(1 + Se
c[e + f*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4392, 3042, 4375, 359, 264, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^{11/2}dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(\csc(e+fx+\frac{\pi}{2})a+a)^{11/2}}{\cot(e+fx+\frac{\pi}{2})^8}dx}{a^4c^4}$$

$$\Big\downarrow \; 4375$$

$$-\frac{2\int \frac{\cot^8(e+fx)(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^4f}$$

$$\Big\downarrow \; 359$$

$$-\frac{2\left(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^2c^4f}$$

$$\Big\downarrow \; 264$$

$$-\frac{2\Bigg(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\Bigg(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}}{a^2c^4f}$$

$$\Big\downarrow \; 264$$

$$-\frac{2\Bigg(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\Bigg(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\Bigg(\frac{1}{3}\cot^3(e+fx)(a\sec(e}{a^2c^4f}$$

$$\Big\downarrow \; 264$$

$$-\frac{2\Bigg(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\Bigg(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\Bigg(\frac{1}{3}\cot^3(e+fx)(a\sec(e}{a^2c^4f}$$

$$\Big\downarrow \; 216$$

$$-\frac{2\Big(\frac{2}{7}\cot^7(e+fx)(a\sec(e+fx)+a)^{7/2}-a\Big(\frac{1}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-a\Big(\frac{1}{3}\cot^3(e+fx)(a\sec(e-}{a^2c^4f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*((2*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/7 - a*((Cot[e + f*x]^5*
(a + a*Sec[e + f*x])^(5/2))/5 - a*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3
/2))/3 - a*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]
] + Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])))))/(a^2*c^4*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(152) = 304.

Time = 1.35 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.83

method	result
default	$\frac{a\left(\sin(fx+e)^{5}(840\cos(fx+e)-840)\sqrt{2}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^{2}-1}}\right)+\sqrt{2}\,\cos(fx\right.}{}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/840/c^4/f*a*(sin(f*x+e)^5*(840*cos(f*x+e)-840)*2^(1/2)*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-
2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+2^(1/2)*cos(f*x+e)*(2263*co
s(f*x+e)^5-29*cos(f*x+e)^4-3410*cos(f*x+e)^3+2494*cos(f*x+e)^2+2107*cos(f*
x+e)-1505)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)+cos(f*x+e)*(1470*cos(f*x+e)^5-4200*cos(f*x+e)^4+2100*cos(f*x+e)^3
+4200*cos(f*x+e)^2-5250*cos(f*x+e)+1680))*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*
x+e)+1)/(cos(f*x+e)^2-2*cos(f*x+e)+1)*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.88

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \left[\frac{105\left(a\cos(fx+e)^{3}-3\,a\cos(fx+e)^{2}+3\,a\cos(fx+e)-a\right)\sqrt{-a}\log\left(\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/210*(105*(a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)
*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x
 + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(191*a*cos(f*x + e)^4 - 40
6*a*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x +
e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/105*(105*(a*cos(f*x
+ e)^3 - 3*a*cos(f*x + e)^2 + 3*a*cos(f*x + e) - a)*sqrt(a)*arctan(2*sqrt(
a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*
cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(191*a*cos(f*x + e)
^4 - 406*a*cos(f*x + e)^3 + 350*a*cos(f*x + e)^2 - 105*a*cos(f*x + e))*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos
(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{a\sqrt{a\sec(e+fx)+a}}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{a\sqrt{a\sec(e+f}}{\sec^4(e+fx)-4\sec^3(e+fx)+}}{c^4}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**4,x)
```

output
```
(Integral(a*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3
+ 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(a*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*
x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(152) = 304$.

Time = 0.81 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.23

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
-1/105*(105*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-
a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(f*x + e))/(c^4*abs(a)) + (420*sqrt(2)*(sqrt(-a)*ta
n(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*sqrt(-a)*a^2*
sgn(cos(f*x + e)) - 1785*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^10*sqrt(-a)*a^3*sgn(cos(f*x + e)) + 4235*sqrt
(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^
8*sqrt(-a)*a^4*sgn(cos(f*x + e)) - 4970*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^5*sgn(cos(f*x + e
)) + 3738*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1
/2*e)^2 + a))^4*sqrt(-a)*a^6*sgn(cos(f*x + e)) - 1421*sqrt(2)*(sqrt(-a)*ta
n(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^7*s
gn(cos(f*x + e)) + 263*sqrt(2)*sqrt(-a)*a^8*sgn(cos(f*x + e)))/(((sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^7*c^4)
)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^4}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^4, x)
```

Reduce [F]

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^4}\,dx = \frac{\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4 - 4\sec(fx+e)^3 + 6\sec(fx+e)^2 - 4\sec(fx+e) + 1}dx + \int \frac{\sqrt{\sec(\cdots}}{\sec(fx+e)^4 - 4\sec(fx\cdots}\right.}{c^4}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x)
```

output
```
(sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4 - 4*sec(e + f*x)**
3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) +
1)*sec(e + f*x))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2
- 4*sec(e + f*x) + 1),x)))/c**4
```

3.57 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 212

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^{5/2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2a^5c^3\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} - \frac{6a^6c^3\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}} - \frac{2a^7c^3\tan^9(e+fx)}{9f(a+a\sec(e+fx))^{9/2}}$$

output

```
2*a^(5/2)*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^
3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^3*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)-2/5*a^5*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)-6/7*a^6*c^
3*tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)-2/9*a^7*c^3*tan(f*x+e)^9/f/(a+a*se
c(f*x+e))^(9/2)
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.63

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \frac{2a^3c^3\left(315\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(-383-34\sec(e+fx)\cdots}{315f\sqrt{a(1+\sec(e+fx))}\sqrt{c\cdots}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^3*c^3*(315*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c
 - c*Sec[e + f*x]]*(-383 - 34*Sec[e + f*x] + 132*Sec[e + f*x]^2 + 5*Sec[e
+ f*x]^3 - 35*Sec[e + f*x]^4))*Tan[e + f*x])/(315*f*Sqrt[a*(1 + Sec[e + f*
x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^6c^3\int \frac{\tan^6(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{2a^6c^3\int \left(\frac{a\tan^8(e+fx)}{(\sec(e+fx)a+a)^4}+\frac{3\tan^6(e+fx)}{(\sec(e+fx)a+a)^3}+\frac{\tan^4(e+fx)}{a(\sec(e+fx)a+a)^2}-\frac{\tan^2(e+fx)}{a^2(\sec(e+fx)a+a)}-\frac{1}{a^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}+\frac{1}{a^3}\right)d\left(-\frac{}{\sqrt{}}\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^6c^3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{7/2}}-\frac{\tan(e+fx)}{a^3\sqrt{a\sec(e+fx)+a}}+\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}-\frac{a\tan^9(e+fx)}{9(a\sec(e+fx)+a)^{9/2}}-\frac{3\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}-\right.}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^6*c^3*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(7/2
) - Tan[e + f*x]/(a^3*Sqrt[a + a*Sec[e + f*x]]) + Tan[e + f*x]^3/(3*a^2*(a
 + a*Sec[e + f*x])^(3/2)) - Tan[e + f*x]^5/(5*a*(a + a*Sec[e + f*x])^(5/2)
) - (3*Tan[e + f*x]^7)/(7*(a + a*Sec[e + f*x])^(7/2)) - (a*Tan[e + f*x]^9)
/(9*(a + a*Sec[e + f*x])^(9/2))))/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 73.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

method	result
default	$\frac{2c^3a^2\sqrt{a(1+\sec(fx+e))}\left(\left(-383\cos(fx+e)^4-34\cos(fx+e)^3+132\cos(fx+e)^2+5\cos(fx+e)-35\right)\tan(fx+e)\sec(fx+e)^3+315(c\right.}{315f(\cos(fx+e)+1)}$
parts	$\frac{c^3a^2\sqrt{a(1+\sec(fx+e))}\left(16\sin(fx+e)+2\tan(fx+e)+3\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/315*c^3/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*((-383*cos(f*x+e)^
4-34*cos(f*x+e)^3+132*cos(f*x+e)^2+5*cos(f*x+e)-35)*tan(f*x+e)*sec(f*x+e)^
3+315*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(c
ot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc
(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.08

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \left[\frac{315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)\ldots}{\cos(fx+e)}}\ldots}{\cos(\ldots}\right.\ldots}{\ldots}\right.$$

$$-\frac{2\left(315\left(a^2c^3\cos(fx+e)^5+a^2c^3\cos(fx+e)^4\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(383\,a^2c^3\cos(fx+\ldots\right.\right.}{315\left(f\cos(fx\ldots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(3
83*a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos(f*
x + e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -2/315*
(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arctan(sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) +
 (383*a^2*c^3*cos(f*x + e)^4 + 34*a^2*c^3*cos(f*x + e)^3 - 132*a^2*c^3*cos
(f*x + e)^2 - 5*a^2*c^3*cos(f*x + e) + 35*a^2*c^3)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4)]
```

Sympy [F]

$$
\begin{aligned}
\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \\
-c^3\left(\int\left(-a^2\sqrt{a\sec(e+fx)+a}\right)\,dx\right. \\
+\int a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\,dx \\
+\int 2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx \\
+\int\left(-2a^2\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\right)\,dx \\
+\int\left(-a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)\right)\,dx \\
\left.+\int a^2\sqrt{a\sec(e+fx)+a}\sec^5(e+fx)\,dx\right)
\end{aligned}
$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**3,x)
```

output
```
-c**3*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(a**2*sqrt(a*
sec(e + f*x) + a)*sec(e + f*x), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x)**2, x) + Integral(-2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e
+ f*x)**3, x) + Integral(-a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4, x
) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**5, x))
```

Maxima [F(-1)]

Timed out.

$$
\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \text{Timed out}
$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.55

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^3\,dx =$$

$$-\frac{\frac{315\sqrt{-a}a^3c^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(315\sqrt{2}a^7c^3\mathrm{sgn}(\cos(fx+e))-\left(1470\right.\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/315*(315*sqrt(-a)*a^3*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqr
t(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(315*sqrt(2)*a^7*c^3*sgn(
cos(f*x + e)) - (1470*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)) - (2772*sqrt(2)*a^
7*c^3*sgn(cos(f*x + e)) + (257*sqrt(2)*a^7*c^3*sgn(cos(f*x + e))*tan(1/2*f
*x + 1/2*e)^2 - 1314*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*
e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)
/((a*tan(1/2*f*x + 1/2*e)^2 - a)^4*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^3\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{5/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)^3 dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^3\,dx = \sqrt{a}\,a^2c^3\left(\int \sqrt{\sec(fx+e)+1}dx - \left(\int \sqrt{\sec(fx+e)+1}\sec(fx+e)^5\,dx\right) + \int \sqrt{\sec(fx+e)+1}\sec(fx+e)^4\,dx + 2\left(\int \sqrt{\sec(fx+e)+1}\sec(fx+e)^3\,dx\right) - 2\left(\int \sqrt{\sec(fx+e)+1}\sec(fx+e)^2\,dx\right) - \left(\int \sqrt{\sec(fx+e)+1}\sec(fx+e)\,dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*a**2*c**3*(int(sqrt(sec(e + f*x) + 1),x) - int(sqrt(sec(e + f*x) +
 1)*sec(e + f*x)**5,x) + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**4,x) + 2
*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x) - 2*int(sqrt(sec(e + f*x) +
 1)*sec(e + f*x)**2,x) - int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.58 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \frac{2a^{5/2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} - \frac{2a^3c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^4c^2\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} + \frac{6a^5c^2\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}} + \frac{2a^6c^2\tan^7(e+fx)}{7f(a+a\sec(e+fx))^{7/2}}$$

output
```
2*a^(5/2)*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c^
2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a^4*c^2*tan(f*x+e)^3/f/(a+a*sec(
f*x+e))^(3/2)+6/5*a^5*c^2*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)+2/7*a^6*c^
2*tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e+fx))^2\,dx = \frac{2a^3c^2\left(105\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right) + \sqrt{c - c\sec(e+fx)}(-92 - 46\sec(e+fx) + \right.}{105f\sqrt{a(1+\sec(e+fx))}\sqrt{c - c\sec(e -}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(2*a^3*c^2*(105*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c
 - c*Sec[e + f*x]]*(-92 - 46*Sec[e + f*x] + 18*Sec[e + f*x]^2 + 15*Sec[e +
 f*x]^3))*Tan[e + f*x])/(105*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
 + f*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c - c\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c - c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \sqrt{\sec(e+fx)a+a}\tan^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cot\left(e+fx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^5c^2\int \frac{\tan^4(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{364}$$

$$-\frac{2a^5c^2\int \left(\frac{a\tan^6(e+fx)}{(\sec(e+fx)a+a)^3}+\frac{3\tan^4(e+fx)}{(\sec(e+fx)a+a)^2}+\frac{\tan^2(e+fx)}{a(\sec(e+fx)a+a)}+\frac{1}{a^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}-\frac{1}{a^2}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^5c^2\left(-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{5/2}}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}-\frac{a\tan^7(e+fx)}{7(a\sec(e+fx)+a)^{7/2}}-\frac{3\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}-\frac{\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}\right.}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^5*c^2*(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(
5/2)) + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(3*a*
(a + a*Sec[e + f*x])^(3/2)) - (3*Tan[e + f*x]^5)/(5*(a + a*Sec[e + f*x])^(
5/2)) - (a*Tan[e + f*x]^7)/(7*(a + a*Sec[e + f*x])^(7/2))))/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 16.98 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

method	result
default	$\frac{2c^2a^2\sqrt{a(1+\sec(fx+e))}\left(\left(-92\cos(fx+e)^3-46\cos(fx+e)^2+18\cos(fx+e)+15\right)\tan(fx+e)\sec(fx+e)^2+(105\cos(fx+e)+105)\right.}{105f(\cos(fx+e)+1)}$
parts	$\frac{c^2a^2\sqrt{a(1+\sec(fx+e))}\left(16\sin(fx+e)+2\tan(fx+e)+3\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/105*c^2/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*((-92*cos(f*x+e)^3
-46*cos(f*x+e)^2+18*cos(f*x+e)+15)*tan(f*x+e)*sec(f*x+e)^2+(105*cos(f*x+e)
+105)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*c
sc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.31

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e+fx))^2\,dx = \left[\frac{105\left(a^2c^2\cos\left(fx+e\right)^4 + a^2c^2\cos\left(fx+e\right)^3\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2 - 2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)\cdots}{\cos(fx+e)}}}{\cos(\cdots}\right.}{}\right.$$

$$-\frac{2\left(105\left(a^2c^2\cos\left(fx+e\right)^4 + a^2c^2\cos\left(fx+e\right)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) + \left(92\,a^2c^2\cos\left(fx+\cdots\right.\right.}{105\left(f\cos\left(fx+e\right)^4 + f\cos\cdots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(9
2*a^2*c^2*cos(f*x + e)^3 + 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x
+ e) - 15*a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(
f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x + e)^4
+ a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (92*a^2*c^2*cos(f*x + e)^3 +
 46*a^2*c^2*cos(f*x + e)^2 - 18*a^2*c^2*cos(f*x + e) - 15*a^2*c^2)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f
*x + e)^3)]
```

Sympy [F]

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^2\,dx = c^2\left(\int a^2\sqrt{a\sec\left(e + fx\right) + a}\,dx + \int\left(-2a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^2\left(e+fx\right)\right)dx + \int a^2\sqrt{a\sec\left(e+fx\right)+a}\sec^4\left(e+fx\right)dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**2,x)
```

output
```
c**2*(Integral(a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-2*a**2*sqrt(a
*sec(e + f*x) + a)*sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x)
 + a)*sec(e + f*x)**4, x))
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*
a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 +
 2*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x +
 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x
 + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin
(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*...
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.68

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx =$$

$$-\frac{\frac{105\sqrt{-a}a^3c^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(105\sqrt{2}a^6c^2\operatorname{sgn}(\cos(fx+e))-\left(385\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/105*(105*sqrt(-a)*a^3*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqr
t(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(105*sqrt(2)*a^6*c^2*sgn(
cos(f*x + e)) - (385*sqrt(2)*a^6*c^2*sgn(cos(f*x + e)) + (43*sqrt(2)*a^6*c
^2*sgn(cos(f*x + e))*tan(1/2*f*x + 1/2*e)^2 - 203*sqrt(2)*a^6*c^2*sgn(cos(
f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1
/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a
)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2 dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2\,dx = \sqrt{a}\,a^2c^2\left(\int\sqrt{\sec(fx+e)+1}dx + \int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4\,dx - 2\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*a**2*c**2*(int(sqrt(sec(e + f*x) + 1),x) + int(sqrt(sec(e + f*x) +
 1)*sec(e + f*x)**4,x) - 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x))
```

3.59 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 132

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=\frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}-\frac{2a^3c\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{2a^4c\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}}-\frac{2a^5c\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output
```
2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f-2*a^3*c*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a^4*c*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^
(3/2)-2/5*a^5*c*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=$$

$$-\frac{2a^3c\left(-5\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)+\sqrt{c-c\sec(e+fx)}(1+3\sec(e+fx)+\sec^2(e+fx))\right)\tan(e+f}{5f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(-2*a^3*c*(-5*Sqrt[c]*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] + Sqrt[c -
 c*Sec[e + f*x]]*(1 + 3*Sec[e + f*x] + Sec[e + f*x]^2))*Tan[e + f*x])/(5*f
*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int (\sec(e+fx)a+a)^{3/2}\tan^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$-ac\int \cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^4c\int \frac{\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{364}$$

$$\frac{2a^4c\int \left(\frac{a\tan^4(e+fx)}{(\sec(e+fx)a+a)^2}+\frac{3\tan^2(e+fx)}{\sec(e+fx)a+a}+\frac{1}{a}-\frac{1}{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4c\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}-\frac{a\tan^5(e+fx)}{5(a\sec(e+fx)+a)^{5/2}}-\frac{\tan^3(e+fx)}{(a\sec(e+fx)+a)^{3/2}}-\frac{\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^4*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2)
- Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]]) - Tan[e + f*x]^3/(a + a*Sec[e
+ f*x])^(3/2) - (a*Tan[e + f*x]^5)/(5*(a + a*Sec[e + f*x])^(5/2))))/f
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

method	result
default	$-\frac{2c\,a^{2}\sqrt{a(1+\sec(fx+e))}\left(\sin(fx+e)+3\tan(fx+e)+\sec(fx+e)\tan(fx+e)+5(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{}{\sqrt{\cot(f}}\right.\right.}{5f(\cos(fx+e)+1)}$
parts	$\frac{c\,a^{2}\sqrt{a(1+\sec(fx+e))}\left(16\sin(fx+e)+2\tan(fx+e)+3\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+c}{\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/5*c/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(sin(f*x+e)+3*tan(f*x
+e)+sec(f*x+e)*tan(f*x+e)+5*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot
(f*x+e)+csc(f*x+e)^2-1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.67

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \left[\frac{5\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(\ldots}{\cos(fx+e)+\ldots}\right.}{5\left(f\cos(\ldots\right.}\right.$$

$$-\frac{2\left(5\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(a^2c\cos(fx+e)^2+3\,a\ldots\right.}{5\left(f\cos(fx+e)^3+f\cos(fx+e)^2\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/5*(5*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(a^2*c*cos
(f*x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -2/5*(5*(a^
2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c*cos(
f*x + e)^2 + 3*a^2*c*cos(f*x + e) + a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = -c\left(\int \left(-a^2\sqrt{a\sec(e+fx)+a}\right)dx + \int \left(-a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)\right)dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)\,dx + \int a^2\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e)),x)
```

output
```
-c*(Integral(-a**2*sqrt(a*sec(e + f*x) + a), x) + Integral(-a**2*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*
sec(e + f*x)**2, x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)*
*3, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(118) = 236$.

Time = 0.23 (sec) , antiderivative size = 1396, normalized size of antiderivative = 10.58

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x +
 2*e) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3
*a^2*cos(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(
a) + 3*((a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x
 + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*...
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.72

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx =$$

$$-\frac{\frac{5\sqrt{-a}a^3c\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(\sqrt{2}a^5c\operatorname{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}\cdots\right.}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^{\cdots}}\cdots}{5f}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/5*(5*sqrt(-a)*a^3*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(sqrt(2)*a^5*c*sgn(cos(f*x + e)
)*tan(1/2*f*x + 1/2*e)^4 - 5*sqrt(2)*a^5*c*sgn(cos(f*x + e)))*tan(1/2*f*x
+ 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2
+ a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x)), x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))\,dx=\sqrt{a}\,a^2c\left(\int \sqrt{\sec(fx+e)+1}dx-\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)-\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)+\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x)
```

output
```
sqrt(a)*a**2*c*(int(sqrt(sec(e + f*x) + 1),x) - int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x)**3,x) - int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x) + int(
sqrt(sec(e + f*x) + 1)*sec(e + f*x),x))
```

3.60 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{8a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{cf} - \frac{2a^3\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+8*a^2*cot(
f*x+e)*(a+a*sec(f*x+e))^(1/2)/c/f-2*a^3*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \frac{2a^2\csc(e+fx)\left(-1+4\cos(e+fx)+\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\right.\right.}{cf}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^2*Csc[e + f*x]*(-1 + 4*Cos[e + f*x] + Cos[e + f*x]*Hypergeometric2F1[
-1/2, 1, 1/2, 1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])])/(c*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{c-c\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^2(e+fx)(\sec(e+fx)a+a)^{7/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{364}$$

$$\frac{2a^2\int \left(4(\sec(e+fx)a+a)\cot^2(e+fx)+a-\frac{a}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+4\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{a\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x]),x]
```

output
```
(2*a^2*(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] +
4*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] - (a*Tan[e + f*x])/Sqrt[a + a*Sec[
e + f*x]]))/(c*f)
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(-\sqrt{2}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)+10\cot(fx+e)-2\csc(fx+e)\right)a^2\sqrt{a(1+\sec(}}{cf}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/c/f*(-2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(
f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(
1/2))+10*cot(f*x+e)-2*csc(f*x+e))*a^2*(a*(1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.83

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{-a}a^2\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-}{\cos(fx+e)+1}\right.}{2\,cf\sin(f}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*a^2*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*
a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(5*a^2*cos(f*x +
e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*sin(f*x + e)), (a^
(5/2)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2
*(5*a^2*cos(f*x + e) - a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(c*f*
sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = -\frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

output
```
-(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) - 1), x) + Integral
(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e + f*x) - 1), x) + Int
egral(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2/(sec(e + f*x) - 1), x)
)/c
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \int -\frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{c\sec(fx+e)-c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate((a*sec(f*x + e) + a)^(5/2)/(c*sec(f*x + e) - c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(93) = 186.

Time = 0.57 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.60

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx = \frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\,a^3\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)c} - \frac{\sqrt{-a}\,a^3\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.\right.}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.\right.}\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
(2*sqrt(2)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*a^3*sgn(cos(f*x + e))*tan(1
/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*c) - sqrt(-a)*a^3*log(abs(
2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
- 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))
/(c*abs(a)) - 8*sqrt(2)*sqrt(-a)*a^3*sgn(cos(f*x + e))/(((sqrt(-a)*tan(1/2
*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c-c\sec(e+fx)}\,dx=\frac{\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)-1}dx\right)-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)-1}dx\right)-2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx\right)\right)}{c}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x)
```

output
```
(sqrt(a)*a**2*( - int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) - 1),x) - int((
sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) - 1),x) - 2*int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) - 1),x)))/c
```

3.61 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 74

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^2f} - \frac{8a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^2f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^2/f-8/3*a*co
t(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/c^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a^3\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+fx)\right)+3\sec(e+fx)\right)\tan(e+fx)}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a^3*(Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]] + 3*Sec[e + f*
x])*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])
])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4392$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{9/2}dx}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \; 4375$$

$$-\frac{2a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^2f}$$

$$\Big\downarrow \; 364$$

$$-\frac{2a\int \left(4(\sec(e+fx)a+a)^2\cot^4(e+fx)+\frac{a^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^2f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a\left(\frac{4}{3}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)\right)}{c^2f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^2,x]
```

output
```
(-2*a*(-(a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])
+ (4*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/3))/(c^2*f)
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(64) = 128.

Time = 12.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

method	result
default	$\frac{a^2\sqrt{a(1+\sec(fx+e))}\left(8\cos(fx+e)\cot(fx+e)-3\sqrt{2}\,(\cos(fx+e)-1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)}}\right.\right.}{3c^2f(\cos(fx+e)-1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/c^2/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)-1)*(8*cos(f*x+e)*cot(f*
x+e)-3*2^(1/2)*(cos(f*x+e)-1)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh
(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc
(f*x+e)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(64) = 128.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.58

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \left[\frac{16\,a^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)^2+3\,(a^2\cos(fx+e)-a^2)\sqrt{-a}\log\left(-\frac{8\,a}{}\right.}{6\,(c^2f\cos(fx}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/6*(16*a^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)^2 + 3*(a
^2*cos(f*x + e) - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x +
e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(
f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e))/((c^2*f
*cos(f*x + e) - c^2*f)*sin(f*x + e)), 1/3*(8*a^2*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)^2 + 3*(a^2*cos(f*x + e) - a^2)*sqrt(a)*arctan(
2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e))/((c^2*f*cos(f*x
 + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{a^2\sqrt{a\sec(e+fx)+a}}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{a^2\sqrt{a\sec(e+f}{\sec^2(e+fx)-}}{c^2}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
(Integral(a**2*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x)**2 - 2*sec(e + f*x)
+ 1), x) + Integral(2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)/(sec(e +
f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(a**2*sqrt(a*sec(e + f*x) + a)
*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(64) = 128$.

Time = 0.63 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.28

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\, dx =$$

$$-\frac{\sqrt{2}\sqrt{-a}a^5\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{a^2c^2|a|} + \frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx\right.}\right.\right.}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}\right.\right.}\right.}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*sqrt(-a)*a^5*(3*sqrt(2)*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2
*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs
(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
 + 4*sqrt(2)*abs(a) - 6*a))/(a^2*c^2*abs(a)) + 8*(3*(sqrt(-a)*tan(1/2*f*x
+ 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4 + a^2)/(((sqrt(-a)*tan(1
/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3*a^2*c^2))*
sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^2}\, dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^2}\, dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^2}\,dx = \frac{\sqrt{a}\,a^2\Big(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+2\Big(\int \frac{\sqrt{\sec(}}{\sec(fx+}}{c^2}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 - 2*sec(e + f*x
) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**2
- 2*sec(e + f*x) + 1),x) + 2*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(se
c(e + f*x)**2 - 2*sec(e + f*x) + 1),x)))/c**2
```

3.62 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^3 f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^3 f} + \frac{8\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^3 f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^3/f+2*a^2*co
t(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^3/f+8/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5
/2)/c^3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{2a^3\left(4+3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e+fx)\right)+5\sec(e+fx\right.}{15c^3 f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*a^3*(4 + 3*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]] + 5*Sec[e
 + f*x])*Tan[e + f*x])/(15*c^3*f*(-1 + Sec[e + f*x])^3*Sqrt[a*(1 + Sec[e +
 f*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{11/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{11/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{c^3f}$$

$$\Big\downarrow \text{364}$$

$$\frac{2\int \left(4(\sec(e+fx)a+a)^3\cot^6(e+fx)+a^2(\sec(e+fx)a+a)\cot^2(e+fx)-\frac{a^3}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}\right)d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+}}\right.}{c^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+a^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}+\frac{4}{5}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}\right)}{c^3f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^3,x]
```

output
```
(2*(a^(5/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + a^2*
Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] + (4*Cot[e + f*x]^5*(a + a*Sec[e + f
*x])^(5/2))/5))/(c^3*f)
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)    Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(92) = 184$.

Time = 58.48 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.58

method	result
default	$-\frac{a^2\sqrt{a(1+\sec(fx+e))}\left(\sin(fx+e)^2(-5\cos(fx+e)+5)\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+}}\right.\right.}{5}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5/c^3/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)/(cos(f*x+e)^2-2*cos
(f*x+e)+1)*(sin(f*x+e)^2*(-5*cos(f*x+e)+5)*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(
f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+(14*cos(f*x+e)^3-2*cos(f*x+e)^2-6
*cos(f*x+e)+10)*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*cot(f*x+e)+(10*cos(f*x+e)^3-30*cos(f*x+e)^2+30*cos(f
*x+e)-10)*cot(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(92) = 184$.

Time = 0.14 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.24

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \left[\frac{5\left(a^2\cos(fx+e)^2-2\,a^2\cos(fx+e)+a^2\right)\sqrt{-a}\log\left(-\frac{8\,a\cos(fx+e)^3-4\left(2\,c\right.}{}\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/10*(5*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*
a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*
x + e) + 1))*sin(f*x + e) + 4*(9*a^2*cos(f*x + e)^3 - 10*a^2*cos(f*x + e)^
2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^3*f*c
os(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/5*(5*(a^2*c
os(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x +
e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(9*a^2*cos(f*x + e)^3 - 10*a^
2*cos(f*x + e)^2 + 5*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(92) = 184$.

Time = 0.81 (sec) , antiderivative size = 428, normalized size of antiderivative = 4.12

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^3}\,dx =$$

$$-\frac{\frac{5\sqrt{-a}a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{c^3|a|}+\frac{4\sqrt{2}\left(5\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan(\cdots}\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/5*(5*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs
(a) - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) + 4*sqrt(2)*(5*(sqrt(-a)*tan(1/
2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sqrt(-a)*a^3*sgn(c
os(f*x + e)) - 10*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1
/2*e)^2 + a))^6*sqrt(-a)*a^4*sgn(cos(f*x + e)) + 20*(sqrt(-a)*tan(1/2*f*x
+ 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^5*sgn(cos(f*x
 + e)) - 10*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^
2 + a))^2*sqrt(-a)*a^6*sgn(cos(f*x + e)) + 3*sqrt(-a)*a^7*sgn(cos(f*x + e)
))/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))
^2 - a)^5*c^3))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^3}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^3}\,dx = \frac{\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec}{\sec(fx+e)^3-3\sec(fx+e)^2\cdots}\right.\right.}{c^3}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x)
```

output
```
(sqrt(a)*a**2*( - int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1),x) - int((sqrt(sec(e + f*x) + 1)*sec(e + f*x
)**2)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x) - 2*in
t((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 - 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) - 1),x)))/c**3
```

3.63 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 140

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^4f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^4f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^4f} - \frac{8\cot^7(e+fx)(a+a\sec(e+fx))^{7/2}}{7ac^4f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^4/f+2*a^2*co
t(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^4/f-2/3*a*cot(f*x+e)^3*(a+a*sec(f*x+e))^
(3/2)/c^4/f-8/7*cot(f*x+e)^7*(a+a*sec(f*x+e))^(7/2)/a/c^4/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = -\frac{2a^3\left(8+5\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},1-\sec(e+fx)\right)+7\sec(e+fx)\right)\tan(e+fx)}{35c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*a^3*(8 + 5*Hypergeometric2F1[-7/2, 1, -5/2, 1 - Sec[e + f*x]] + 7*Sec[
e + f*x])*Tan[e + f*x])/(35*c^4*f*(-1 + Sec[e + f*x])^4*Sqrt[a*(1 + Sec[e
+ f*x])])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^8(e+fx)(\sec(e+fx)a+a)^{13/2}dx}{a^4c^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{13/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^8}dx}{a^4c^4}$$

$$\Big\downarrow \text{ 4375}$$

$$-\frac{2\int \frac{\cot^8(e+fx)(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{ac^4f}$$

$$\Big\downarrow \text{ 364}$$

$$-\frac{2\int \Big(4(\sec(e+fx)a+a)^4\cot^8(e+fx)+a^2(\sec(e+fx)a+a)^2\cot^4(e+fx)-a^3(\sec(e+fx)a+a)\cot^2(e+}{ac^4f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\left(-a^{7/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)-a^3\cot(e+fx)\sqrt{a\sec(e+fx)+a}+\frac{1}{3}a^2\cot^3(e+fx)(a\sec(e+fx)+a)^{3/}\right.}{ac^4f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^4,x]
```

output
```
(-2*(-(a^(7/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]) -
a^3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] + (a^2*Cot[e + f*x]^3*(a + a*Sec
[e + f*x])^(3/2))/3 + (4*Cot[e + f*x]^7*(a + a*Sec[e + f*x])^(7/2))/7))/(a
*c^4*f)
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(124) = 248$.

Time = 3.09 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.20

$$-\frac{a^2\sqrt{a\left(1+\sec\left(fx+e\right)\right)}\left(\sin\left(fx+e\right)^2\left(21\cos\left(fx+e\right)^2-42\cos\left(fx+e\right)+21\right)\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \text{arct}\right.}{}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x)
```

output
```
-1/21/c^4/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)/(cos(f*x+e)^3-3*co
s(f*x+e)^2+3*cos(f*x+e)-1)*(sin(f*x+e)^2*(21*cos(f*x+e)^2-42*cos(f*x+e)+21
)*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)
^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))
+2^(1/2)*(61*cos(f*x+e)^4-60*cos(f*x+e)^3-2*cos(f*x+e)^2+84*cos(f*x+e)-35)
*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
ot(f*x+e)+(42*cos(f*x+e)^4-168*cos(f*x+e)^3+252*cos(f*x+e)^2-168*cos(f*x+e
)+42)*cot(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(124) = 248$.

Time = 0.15 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.76

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^4}\, dx = \left[\frac{21\left(a^2\cos\left(fx+e\right)^3 - 3\,a^2\cos\left(fx+e\right)^2 + 3\,a^2\cos\left(fx+e\right) - a^2\right)\sqrt{-a}\log}{} \right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/42*(21*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e)
 - a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos(f*x + e)^2 - cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*c
os(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*(40*a^2*cos(f*x + e)
^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a^2*cos(f*x + e))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*
cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/21*(21*(a^
2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)*sqrt(a
)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(40
*a^2*cos(f*x + e)^4 - 77*a^2*cos(f*x + e)^3 + 70*a^2*cos(f*x + e)^2 - 21*a
^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((c^4*f*cos(f*x
+ e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x +
e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^4}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**4,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(124) = 248.

Time = 0.79 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.97

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
-1/21*(21*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/(c^4*abs(a)) + 4*(21*sqrt(2)*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*sqrt(-a)*a^3*s
gn(cos(f*x + e)) - 84*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^10*sqrt(-a)*a^4*sgn(cos(f*x + e)) + 217*sqrt(2)*
(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^8*sq
rt(-a)*a^5*sgn(cos(f*x + e)) - 238*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
- sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^6*sgn(cos(f*x + e)) +
189*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^
2 + a))^4*sqrt(-a)*a^7*sgn(cos(f*x + e)) - 70*sqrt(2)*(sqrt(-a)*tan(1/2*f*
x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^8*sgn(cos(f
*x + e)) + 13*sqrt(2)*sqrt(-a)*a^9*sgn(cos(f*x + e)))/(((sqrt(-a)*tan(1/2*
f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^7*c^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^4, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^4}\,dx=\frac{\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4-4\sec(fx+e)^3+6\sec(fx+e)^2-4\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec}}{\sec(fx+e)^4-4\sec(f}\right.}{}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x)
```

output
```
(sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4 - 4*sec(e + f*x
)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x)
 + 1)*sec(e + f*x)**2)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*
x)**2 - 4*sec(e + f*x) + 1),x) + 2*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x
))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*
x) + 1),x)))/c**4
```

3.64 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{c^5f} + \frac{2a^2\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{c^5f} - \frac{2a\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3c^5f} + \frac{2\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5c^5f} + \frac{8\cot^9(e+fx)(a+a\sec(e+fx))^{9/2}}{9a^2c^5f}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c^5/f+2*a^2*co
t(f*x+e)*(a+a*sec(f*x+e))^(1/2)/c^5/f-2/3*a*cot(f*x+e)^3*(a+a*sec(f*x+e))^
(3/2)/c^5/f+2/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/c^5/f+8/9*cot(f*x+e)^9
*(a+a*sec(f*x+e))^(9/2)/a^2/c^5/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.42

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \frac{2a^3\left(12+7\,\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},1-\sec(e+fx)\right)+9\sec(e+f}{63c^5f(-1+\sec(e+fx))^5\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

output
```
(2*a^3*(12 + 7*Hypergeometric2F1[-9/2, 1, -7/2, 1 - Sec[e + f*x]] + 9*Sec[
e + f*x])*Tan[e + f*x])/(63*c^5*f*(-1 + Sec[e + f*x])^5*Sqrt[a*(1 + Sec[e
+ f*x])])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4392, 3042, 4375, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^{10}(e+fx)(\sec(e+fx)a+a)^{15/2}dx}{a^5c^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{15/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}dx}{a^5c^5}$$

$\big\downarrow$ 4375

$$\frac{2\int \frac{\cot^{10}(e+fx)(\sec(e+fx)a+a)^5\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^5f}$$

$\big\downarrow$ 364

$$\frac{2\int \Big(4(\sec(e+fx)a+a)^5\cot^{10}(e+fx)+a^2(\sec(e+fx)a+a)^3\cot^6(e+fx)-a^3(\sec(e+fx)a+a)^2\cot^4(e+}{a^2c^5f}$$

$\big\downarrow$ 2009

$$\frac{2\Big(a^{9/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)+a^4\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{3}a^3\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}+}{a^2c^5f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^5,x]
```

output
```
(2*(a^(9/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]] + a^4*
Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]] - (a^3*Cot[e + f*x]^3*(a + a*Sec[e +
 f*x])^(3/2))/3 + (a^2*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/5 + (4*C
ot[e + f*x]^9*(a + a*Sec[e + f*x])^(9/2))/9))/(a^2*c^5*f)
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(152) = 304$.

Time = 0.99 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.07

$$a^2\left(\sin(fx+e)^5\left(-360\cos(fx+e)^2+720\cos(fx+e)-360\right)\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}(}{\sqrt{\cot(fx+e)^2-2}}\right.\right.$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x)
```

output
```
1/360/c^5/f*a^2*(sin(f*x+e)^5*(-360*cos(f*x+e)^2+720*cos(f*x+e)-360)*2^(1/
2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*cs
c(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+2^(1/2
)*cos(f*x+e)*(1027*cos(f*x+e)^6-708*cos(f*x+e)^5-969*cos(f*x+e)^4+2896*cos
(f*x+e)^3-63*cos(f*x+e)^2-1548*cos(f*x+e)+645)*(-cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)*(630*cos(f*x+e)^6-
2430*cos(f*x+e)^5+2700*cos(f*x+e)^4+900*cos(f*x+e)^3-4050*cos(f*x+e)^2+297
0*cos(f*x+e)-720))*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)/(cos(f*x+e)^3-3
*cos(f*x+e)^2+3*cos(f*x+e)-1)*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.49

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^5} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
[1/90*(45*(a^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^
2 - 4*a^2*cos(f*x + e) + a^2)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 - 4*(2*cos
(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) +
 4*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324*a^2*cos(f*x + e)^
3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f
*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e)), 1/45*(45*(a
^2*cos(f*x + e)^4 - 4*a^2*cos(f*x + e)^3 + 6*a^2*cos(f*x + e)^2 - 4*a^2*co
s(f*x + e) + a^2)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) -
a))*sin(f*x + e) + 2*(89*a^2*cos(f*x + e)^5 - 243*a^2*cos(f*x + e)^4 + 324
*a^2*cos(f*x + e)^3 - 195*a^2*cos(f*x + e)^2 + 45*a^2*cos(f*x + e))*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e)))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*
x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x
+ e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^5} \, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**5,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^5}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(152) = 304$.

Time $= 0.99$ (sec) , antiderivative size $= 672$, normalized size of antiderivative $= 3.91$

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
-1/45*(45*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/(c^5*abs(a)) + 4*(45*sqrt(2)*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^16*sqrt(-a)*a^3*s
gn(cos(f*x + e)) - 270*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^14*sqrt(-a)*a^4*sgn(cos(f*x + e)) + 900*sqrt(2)
*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^12*
sqrt(-a)*a^5*sgn(cos(f*x + e)) - 1575*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^10*sqrt(-a)*a^6*sgn(cos(f*x + e)
) + 1953*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/
2*e)^2 + a))^8*sqrt(-a)*a^7*sgn(cos(f*x + e)) - 1452*sqrt(2)*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^8*sg
n(cos(f*x + e)) + 738*sqrt(2)*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^9*sgn(cos(f*x + e)) - 207*sqrt(2)*(
sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqr
t(-a)*a^10*sgn(cos(f*x + e)) + 28*sqrt(2)*sqrt(-a)*a^11*sgn(cos(f*x + e)))
/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
 - a)^9*c^5))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^5}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^5, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^5}\,dx = \frac{\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^5-5\sec(fx+e)^4+10\sec(fx+e)^3-10\sec(fx+e)^2+5\sec(fx+e)-1}dx\right)-\right.}{}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x)
```

output
```
(sqrt(a)*a**2*( - int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**5 - 5*sec(e +
f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1),x)
 - int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**5 - 5*sec(e
 + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1)
,x) - 2*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**5 - 5*sec
(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) -
1),x)))/c**5
```

3.65 $\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{16\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{14c^4\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^4\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}} + \frac{2a^2c^4\tan^5(e+fx)}{5f(a+a\sec(e+fx))^{5/2}}$$

output

```
2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/f-16*2^(1/
2)*c^4*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/
2)/f+14*c^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2*a*c^4*tan(f*x+e)^3/f/(a+
a*sec(f*x+e))^(3/2)+2/5*a^2*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 4.98 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\left(100-155\cos(e+fx)+96\cos(2(e+fx))-41\cos(3(e+fx))+20\arctan\left(\sqrt{-1+\right.\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^4*Cot[(e + f*x)/2]*(100 - 155*Cos[e + f*x] + 96*Cos[2*(e + f*x)] - 41*C
os[3*(e + f*x)] + 20*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^3*Sqrt[-
1 + Sec[e + f*x]] - 160*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Co
s[e + f*x]^3*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^3)/(10*f*Sqrt[a*(1 + Se
c[e + f*x])])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4392, 3042, 4375, 381, 27, 444, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a\sec(e+fx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{381}$$

$$-\frac{2a^4c^4\left(-\frac{\int \frac{5\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{5a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a^4c^4\left(-\frac{\int \frac{\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{444}$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{\int \frac{3a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{\int \frac{\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5/2}}\right)}{f}$$

$$\downarrow 444$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{\int \frac{a\left(\frac{15a\tan^2(e+fx)}{\sec(e+fx)a+a}+14\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+\ldots}\right)}{f}$$

$$\downarrow 27$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{\int \frac{\frac{15a\tan^2(e+fx)}{\sec(e+fx)a+a}+14}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+\ldots}\right)}{f}$$

$$\downarrow 397$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{16\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\ldots\right)}{f}$$

$$\downarrow 216$$

$$-\frac{2a^4c^4\left(-\frac{-\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{8\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{\tan^3(e+fx)}{a(a\sec(e+fx)+a)^{3/2}}}{a^2}-\frac{\tan^5(e+fx)}{5a^2(a\sec(e+fx)+a)^{5}}\right.}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^4/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(-2*a^4*c^4*(-1/5*Tan[e + f*x]^5/(a^2*(a + a*Sec[e + f*x])^(5/2)) - (-(Tan
[e + f*x]^3/(a*(a + a*Sec[e + f*x])^(3/2))) - (-((ArcTan[(Sqrt[a]*Tan[e +
f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt[a] - (8*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e
 + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/Sqrt[a])/a) - (7*Tan[e + f*x
])/(a*Sqrt[a + a*Sec[e + f*x]]))/a)/a^2))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 2.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

method	result
default	$-\frac{2c^4\sqrt{a(1+\sec(fx+e))}\left(-41\sin(fx+e)+7\tan(fx+e)-\sec(fx+e)\tan(fx+e)-5(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\cdots}{\sqrt{\cot\cdots}}\cdots\right.\right.}{5fa(c\cdots}$
parts	$-\frac{c^4\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)-\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+\cdots}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e\cdots}}\cdots\right.\right.}{fa}$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*c^4/f/a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(-41*sin(f*x+e)+7*tan
(f*x+e)-sec(f*x+e)*tan(f*x+e)-5*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2
-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+40*2^(1/2)*(cos(f*x+e)+1)*(-cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+
csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.98

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{40\sqrt{2}\left(ac^4\cos(fx+e)^3+ac^4\cos(fx+e)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{}\right.$$

$$-\frac{2\left(5\left(c^4\cos(fx+e)^3+c^4\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(41c^4\cos(fx+e)^2-\right.\right.}{5\left(af\cos(fx\right.}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/5*(40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4*cos(f*x + e)^2)*sqrt(-1/a)*
log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x
+ e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2
 + 2*cos(f*x + e) + 1)) - 5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt
(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)
) + 2*(41*c^4*cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e
)^2), -2/5*(5*(c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*sqrt(a)*arctan(sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
- (41*c^4*cos(f*x + e)^2 - 7*c^4*cos(f*x + e) + c^4)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e) - 40*sqrt(2)*(a*c^4*cos(f*x + e)^3 + a*c^4
*cos(f*x + e)^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*co
s(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x + e)^3 + a*f*cos(
f*x + e)^2)]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = c^4\left(\int \left(-\frac{4\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{6\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int \left(-\frac{4\sec^3(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{\sec^4(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int \frac{1}{\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(6*s
ec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(-4*sec(e + f*x)**3/
sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**4/sqrt(a*sec(e + f*x
) + a), x) + Integral(1/sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c\sec(fx+e)-c)^4}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^4/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^4}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \frac{\sqrt{a}\,c^4\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx + \int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)+1}dx - 4\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)+1}dx\right) + 6\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+}\right.\right.}{a}$$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*c**4*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x) + int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e + f*x) + 1),x) - 4*int((sqrt(s
ec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x) + 1),x) + 6*int((sqrt(sec(
e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x) - 4*int((sqrt(sec(e +
 f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)))/a
```

3.66 $\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{8\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{6c^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2ac^3\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/f-8*2^(1/2
)*c^3*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2
)/f+6*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-2/3*a*c^3*tan(f*x+e)^3/f/(a+
a*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.09

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \frac{4c^3\cos\left(\frac{e}{2}\right)\cos(e)\cot\left(\frac{1}{2}(e+fx)\right)\left(-6+11\cos(e+fx)-5\cos(2(e+fx))+3\arctan\left(\sqrt{-1+\sec(e+\right.\right.}{3f\left(\cos\left(\frac{e}{2}\right)+c\right.}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(4*c^3*Cos[e/2]*Cos[e]*Cot[(e + f*x)/2]*(-6 + 11*Cos[e + f*x] - 5*Cos[2*(e
 + f*x)] + 3*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[
e + f*x]] - 12*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x
]^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(3*f*(Cos[e/2] + Cos[(3*e)/2]
)*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 381, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a\sec(e+fx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4392}$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$\frac{2a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{381}$$

$$\frac{2a^3c^3\left(-\frac{\int \frac{3\tan^2(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^3c^3\left(-\frac{\int \frac{\tan^2(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{444}$$

$$\frac{2a^3c^3\left(-\frac{-\frac{\int \frac{a\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^3c^3\left(-\frac{-\frac{\int \frac{\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+6}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{2a^3c^3\left(-\frac{-\frac{8\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{2a^3c^3\left(-\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{4\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{3\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}-\frac{\tan^3(e+fx)}{3a^2(a\sec(e+fx)+a)^{3/2}}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a^3*c^3*(-1/3*Tan[e + f*x]^3/(a^2*(a + a*Sec[e + f*x])^(3/2)) - (-((Arc
Tan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt[a] - (4*Sqrt[2]*
ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/Sqrt[a]
)/a) - (3*Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f*x]]))/a^2))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 2.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.34

method	result
default	$\frac{2c^3\sqrt{a(1+\sec(fx+e))}\left(10\sin(fx+e)-\tan(fx+e)+3(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)}}\right.\right.}{3fa(\cos(fx+e)+1)}$
parts	$-\frac{c^3\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e}}\right.\right.}{fa}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*c^3/f/a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(10*sin(f*x+e)-tan(f*x
+e)+3*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(c
ot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc
(f*x+e)))-12*2^(1/2)*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln(
(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.41

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{12\sqrt{2}\left(ac^3\cos(fx+e)^2+ac^3\cos(fx+e)\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2\cdots}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(3\left(c^3\cos(fx+e)^2+c^3\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(10\,c^3\cos(fx+e)-c^3\cdots\right.\right.}{3\left(af\cos(fx+e)^2+a\cdots\right.}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(12*sqrt(2)*(a*c^3*cos(f*x + e)^2 + a*c^3*cos(f*x + e))*sqrt(-1/a)*lo
g((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x +
e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 +
 2*cos(f*x + e) + 1)) - 3*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)
*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) +
2*(10*c^3*cos(f*x + e) - c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(
f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -2/3*(3*(c^3*cos(f*x +
e)^2 + c^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (10*c^3*cos(f*x + e) - c^3)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 12*sqrt(2)*(a*c^3*co
s(f*x + e)^2 + a*c^3*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f*cos(f*x
 + e)^2 + a*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = -c^3\left(\int \frac{3\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{3\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int \frac{\sec^3(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{1}{\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-3*
sec(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(sec(e + f*x)**3/sq
rt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(a*sec(e + f*x) + a), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int -\frac{(c\sec(fx+e)-c)^3}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)^3/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^3}{\sqrt{a + a\sec(e + fx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^3}{\sqrt{a + a\sec(e + fx)}}\, dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^3}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\, dx$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c - c\sec(e + fx))^3}{\sqrt{a + a\sec(e + fx)}}\, dx$$
$$= \frac{\sqrt{a}\,c^3\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx - \left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)+1}dx\right) + 3\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}dx\right) - 3\left(\int \frac{\sqrt{\sec(fx+e}}{\sec(}\right.\right.}{a}$$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*c**3*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x) - int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x) + 1),x) + 3*int((sqrt(s
ec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x) - 3*int((sqrt(sec(
e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)))/a
```

3.67 $\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{2c^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/f-4*2^(1/2
)*c^2*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2
)/f+2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2c^2\left(\operatorname{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)-2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)+\sqrt{1-\sec(e+fx)}\right)\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*c^2*(ArcTanh[Sqrt[1 - Sec[e + f*x]]] - 2*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e
 + f*x]]/Sqrt[2]] + Sqrt[1 - Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[1 - Sec[
e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4392, 3042, 4375, 381, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{381}$$

$$-\frac{2a^2c^2\left(-\frac{\int \frac{\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$\downarrow$ 397

$$-\frac{2a^2c^2\left(-\frac{4\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$\downarrow$ 216

$$-\frac{2a^2c^2\left(-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a^2}-\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(-2*a^2*c^2*(-((ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sq
rt[a] - (2*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
 + f*x]])])/Sqrt[a])/a^2) - Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]])))/
f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 1.78 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.63

method	result
default	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)-4\sqrt{-\frac{2}{\cos}}\right.}{fa(\cos(fx+e)+1)}$
parts	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e}}\right.\right.}{fa}$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c^2/f/a*(a*(1+sec(f*x+e)))^(1/2)*(2^(1/2)*(cos(f*x+e)+1)*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+
csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))-4*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)+1)*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*
x+e)+csc(f*x+e))+2*sin(f*x+e))/(cos(f*x+e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.68

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{2\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)+2\sqrt{2}(ac^2\cos(fx+e)+ac^2)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin}{\cos(fx+e)^2+}\right.}{}\right.$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[(2*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 2*sqrt(2)*(
a*c^2*cos(f*x + e) + a*c^2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^
2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - (c^2*cos(
f*x + e) + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)))/(a*f*cos(f*x + e) + a*f), 2*(c^2*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e) + c^2)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) + 2*sqrt(2)*(a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/
(a*f*cos(f*x + e) + a*f)]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = c^2\left(\int\left(-\frac{2\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)dx + \int\frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}}\,dx + \int\frac{1}{\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(sec
(e + f*x)**2/sqrt(a*sec(e + f*x) + a), x) + Integral(1/sqrt(a*sec(e + f*x)
 + a), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c\sec(fx+e)-c)^2}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^2/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}dx-2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)\right)}{a}$$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*c**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x) + int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x) - 2*int((sqrt(s
ec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)))/a
```

3.68 $\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/f-2*2^(1/2)*
c*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx =\frac{2c^{3/2}\left(\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\right)\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*c^(3/2)*(ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]] - Sqrt[2]*ArcTanh[Sq
rt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])])*Tan[e + f*x])/(f*Sqrt[a*(1 + Se
c[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4392, 3042, 4375, 383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \ 4392$$

$$-ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \ 4375$$

$$\frac{2ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \ 383$$

$$\frac{2ac\left(\frac{2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}\right)}{f}$$

$$\downarrow 216$$

$$\frac{2ac\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{a^{3/2}}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*c*(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/a^(3/2) -
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]
)/a^(3/2)))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.52

method	result
default	$\frac{c\sqrt{-a(-1-\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)-2\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(\right.\right.}{fa}$
parts	$\frac{c\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)+\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2}}\right.\right.}{fa}$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/f/a*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2^(
1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+
csc(f*x+e)))-2*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+
e)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.43

$$\int \frac{c-c\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$=\left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)-\sqrt{-ac}\log\left(\frac{2a\cos(fx+\right.}{af}\right.$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*a*c*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a)*c*log((2*a*cos
(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f), 2*(sqrt
(2)*sqrt(a)*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a*f)]
```

Sympy [F]

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = -c\left(\int \frac{\sec(e + fx)}{\sqrt{a\sec(e + fx) + a}}\,dx + \int \left(-\frac{1}{\sqrt{a\sec(e + fx) + a}}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
-c*(Integral(sec(e + f*x)/sqrt(a*sec(e + f*x) + a), x) + Integral(-1/sqrt(
a*sec(e + f*x) + a), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 699, normalized size of antiderivative = 8.03

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(f*x + e)
^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos(f*x +
e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos(f*x
+ e) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs(2*e
^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2 -
4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) + 2*
sin(f*x + e))/abs(2*e^(I*f*x + I*e) + 2), ((abs(2*e^(I*f*x + I*e) + 2)^4 +
 16*cos(f*x + e)^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^
2 - 2*cos(f*x + e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 +
 32*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)
^2 - 64*cos(f*x + e) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(
f*x + e)/abs(2*e^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*c
os(f*x + e)^2 - 4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*
e) + 2)^2)) + 2*cos(f*x + e) - 2)/abs(2*e^(I*f*x + I*e) + 2)) - sqrt(a)*ar
ctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x +
e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*x + e)
))*c/(a*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{c - \frac{c}{\cos(e+f\,x)}}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{c - c\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = \frac{\sqrt{a}\,c\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx - \left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)\right)}{a}$$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*c*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x) - int((sqrt(s
ec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)))/a
```

3.69 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/c/f-1/2*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(1/2)/c/
f+cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\right.\right.}{cf\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

output
```
-((Cot[(e + f*x)/2]*(Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]
 - 2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]))/(c*f*Sqrt[a*(1 +
Sec[e + f*x])]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4392, 3042, 4375, 382, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4392$$

$$-\frac{\int \cot^2(e+fx)\sqrt{\sec(e+fx)a+a}dx}{ac}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\cot\left(e+fx+\frac{\pi}{2}\right)^2}dx}{ac}$$

$$\Big\downarrow \text{ 4375}$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{acf}$$

$$\Big\downarrow \text{ 382}$$

$$\frac{2\left(\frac{1}{2}\int -\frac{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)}{acf}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{acf}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int \frac{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{acf}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+\cdots}{\sqrt{\sec(e+f\cdots}}\right.\right.\right.}{acf}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)\right)}{acf}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

output
```
(2*(-1/2*(a*((-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/
Sqrt[a] + ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
]/(Sqrt[2]*Sqrt[a]))) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/(a*c*f
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(104) = 208$.

Time = 0.75 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\sqrt{a(1+\sec(fx+e))}\left(2\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cot(fx+e)+(2\cos(fx+e)-2)\cot(fx+e)+2\sqrt{2}\,(\cos(\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/f/a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(2*2^(1/2)*(cos(f*x+e)+
1)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*cot(f*x+e)+(2*cos(f*x+e)-2)*cot(f*x+e)+2*2^(1/2)*(cos(f*x+e)+1)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f
*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-c
ot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.60

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e) - 2\sqrt{-a}\log \right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*
x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) - 2*sqrt(-
a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) +
 a)/(cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x
 + e)))*sin(f*x + e) + 2*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x
+ e) - a))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$= -\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*sec(e + f*x) +
 a)), x)/c
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int -\frac{1}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2-1}dx\right)}{ac}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 - 1),x))/(a*c)
```

3.70 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^2f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}\sqrt{a}c^2f} + \frac{3\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2ac^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{3a^2c^2f}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/c^2/f-1/4*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(1/2)/
c^2/f+3/2*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c^2/f-1/3*cot(f*x+e)^3*(a+a*
sec(f*x+e))^(3/2)/a^2/c^2/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{\left(\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\sec(e+f\right.\right.}{3c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

output
```
((Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2] - 2*Hypergeometri
c2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]])*Tan[e + f*x])/(3*c^2*f*(-1 + Sec[e
+ f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 382, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)(\sec(e+fx)a+a)^{3/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$\downarrow$ 4375

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2c^2f}$$

$\downarrow$ 382

$$-\frac{2\left(\frac{1}{6}\int -\frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}\right)}{a^2c^2f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^2c^2f}$$

$\downarrow$ 445

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right.}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e}{\sec(e+fx}\right.}\right.\right.}{a^2c^2f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int \frac{\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+7}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(}{\sec(e+f}\right.}\right.\right.}{a^2c^2f}$$

$\downarrow$ 397

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(4\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\right.\right.\right.\right.}{a^2c^2f}$$

$\downarrow$ 216

$$-\frac{2\left(\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{3}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}\right.\right.\right.}{a^2c^2f}$$

input

```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^2),x]
```

output

```
(-2*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/6 - (a*(-1/2*(a*((-4*ArcT
an[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + ArcTan[(Sqr
t[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]*Sqrt[a])))
 + (3*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/2))/(a^2*c^2*f)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397

```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(136) = 272$.

Time = 0.86 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.99

method	result
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\left(37\cos(fx+e)^3+51\cos(fx+e)^2-21\cos(fx+e)-35\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cot(fx+e)\csc(fx}{}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/c^2/f/a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(2^(1/2)*(37*cos(f*x+
e)^3+51*cos(f*x+e)^2-21*cos(f*x+e)-35)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)*csc(f*x+e)^2+(30*cos(f*x+e
)^3-24*cos(f*x+e)^2-42*cos(f*x+e)+36)*cot(f*x+e)*csc(f*x+e)^2+24*2^(1/2)*(
cos(f*x+e)+1)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*
x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+
e)))-6*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)*ln((-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.23

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \left[-\frac{3\sqrt{2}\sqrt{-a}(\cos(fx+e)-1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3a\cos(fx+e)^2-2a\cos(fx+e)+a}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\text{si}}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
[-1/24*(3*sqrt(2)*sqrt(-a)*(cos(f*x + e) - 1)*log(-(2*sqrt(2)*sqrt(-a)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f
*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*s
in(f*x + e) + 12*sqrt(-a)*(cos(f*x + e) - 1)*log(-(8*a*cos(f*x + e)^3 + 4*
(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x
+ e) - 4*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/12*(3*sqrt
(2)*sqrt(a)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 12*sqrt(a)
*(cos(f*x + e) - 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*si
n(f*x + e) + 2*(11*cos(f*x + e)^2 - 9*cos(f*x + e))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+\sqrt{a\sec(e+fx)+a}}\,dx}{c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - 2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x) + sqrt(a*sec(e + f*x) + a)), x)/c**2
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^2}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3-\sec(fx+e)^2-\sec(fx+e)+1}dx\right)}{a\,c^2}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 - sec(e + f*x)**2 - s
ec(e + f*x) + 1),x))/(a*c**2)
```

3.71 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c^3f} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}c^3f} + \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4ac^3f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^2c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{5a^3c^3f}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/c^3/f-1/8*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(1/2)/
c^3/f+7/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c^3/f-1/2*cot(f*x+e)^3*(a+a*
sec(f*x+e))^(3/2)/a^2/c^3/f+1/5*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/a^3/c^
3/f
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},1-\sec(e-\right.\right.}{5c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

output
```
-1/5*((Hypergeometric2F1[-5/2, 1, -3/2, (1 - Sec[e + f*x])/2] - 2*Hypergeo
metric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]])*Tan[e + f*x])/(c^3*f*(-1 + Sec
[e + f*x])^3*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4392, 3042, 4375, 382, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{5/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$\downarrow$ 4375

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3c^3f}$$

$\downarrow$ 382

$$\frac{2\left(\frac{1}{10}\int -\frac{5a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}\right)}{a^3c^3f}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^3c^3f}$$

$\downarrow$ 445

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{6}\int \frac{3a\cot^2(e+fx)(\sec(e+fx)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}\right.\right.}{a^3c^3f}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\int \frac{\cot^2(e+fx)(\sec(e+fx)a}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}\right.\right.}{a^3c^3f}$$

$\downarrow$ 445

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\,\mathrm{se}}\right.\right.\right.}{a^3c^3f}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec\right.\right.\right.}{a^3c^3f}$$

$\downarrow$ 397

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec\right.\right.\right.}{a^3c^3f}$$

$\downarrow$ 216

$$\frac{2\left(\frac{1}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}-\frac{1}{2}a\left(\frac{1}{2}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec\right.\right.\right.}{a^3c^3f}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^3),x]
```

output
```
(2*((Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/10 - (a*((Cot[e + f*x]^3*(
a + a*Sec[e + f*x])^(3/2))/2 - (a*(-1/2*(a*((-8*ArcTan[(Sqrt[a]*Tan[e + f*
x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2]*Sqrt[a]))) + (7*Cot[e + f*x]*Sqr
t[a + a*Sec[e + f*x]])/2))/2))/2))/(a^3*c^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(167) = 334$.

Time = 0.97 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.84

method	result
default	$-\frac{\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\left(883\cos(fx+e)^5+1557\cos(fx+e)^4-982\cos(fx+e)^3-1978\cos(fx+e)^2+483\cos(fx+e)+805\right)\sqrt{-\frac{2\,c}{\cos}}\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/480/c^3/f/a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(2^(1/2)*(883*cos(f
*x+e)^5+1557*cos(f*x+e)^4-982*cos(f*x+e)^3-1978*cos(f*x+e)^2+483*cos(f*x+e
)+805)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*cot(f*x+e)*csc(f*x+e)^4+(590*cos(f*x+e)^5-260*cos(f*x+e)^4-1920*cos(f
*x+e)^3+1420*cos(f*x+e)^2+1010*cos(f*x+e)-840)*cot(f*x+e)*csc(f*x+e)^4+480
*2^(1/2)*(cos(f*x+e)+1)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/
2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e
)^2-1)^(1/2))+60*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)*ln((-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.10

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
[-1/80*(5*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2*s
qrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*c
os(f*x + e) + 1))*sin(f*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*
sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x
+ e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(49*cos(f*x + e)^3 - 80*cos
(f*x + e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((
a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)),
1/40*(5*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e)))*sin(f*x + e) + 40*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(a)*arct
an(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(49*cos(f
*x + e)^3 - 80*cos(f*x + e)^2 + 35*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*
f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)-3\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+3\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(1/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - 3*sqrt(a*sec(e + f
*x) + a)*sec(e + f*x)**2 + 3*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(
a*sec(e + f*x) + a)), x)/c**3
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \int -\frac{1}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
-integrate(1/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^3}\,dx$$
$$=-\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4-2\sec(fx+e)^3+2\sec(fx+e)-1}dx\right)}{a\,c^3}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x)
```

output
```
( - sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4 - 2*sec(e + f*x)**
3 + 2*sec(e + f*x) - 1),x))/(a*c**3)
```

3.72 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 208

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{12\sqrt{2}c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{14c^4\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{8c^4\tan^3(e+fx)}{3f(a+a\sec(e+fx))^{3/2}} - \frac{2ac^4\tan^5(e+fx)}{f(a+a\sec(e+fx))^{5/2}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output
```
2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+12*2^(1/
2)*c^4*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/
2)/f-14*c^4*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+8/3*c^4*tan(f*x+e)^3/f/(
a+a*sec(f*x+e))^(3/2)-2*a*c^4*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/2)/(2+tan
(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.94

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^4\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\left(-22+20\cos(e+fx)-26\cos(2(e+f\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(c^4*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-22 + 20*Cos[e + f*x] - 26*Cos[2*(
e + f*x)] + 28*Cos[3*(e + f*x)] + 6*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*(Cos[(
e + f*x)/2] + Cos[(3*(e + f*x))/2])^2*Sqrt[-1 + Sec[e + f*x]] + 36*Sqrt[2]
*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*(Cos[(e + f*x)/2] + Cos[(3*(e + f
*x))/2])^2*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x]^2)/(12*a*f*Sqrt[a*(1 + Se
c[e + f*x])])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 444, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{11/2}}dx$$

$\Big\downarrow$ 4375

$$-\frac{2a^3c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\Big\downarrow$ 372

$$-\frac{2a^3c^4\left(\frac{\int \frac{2\tan^4(e+fx)\left(\frac{4a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 27

$$-\frac{2a^3c^4\left(\frac{\int \frac{\tan^4(e+fx)\left(\frac{4a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 444

$$-\frac{2a^3c^4\left(\frac{-\frac{\int \frac{3a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+8\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a}{a\,\mathrm{s}}\right.}\right.}{f}$$

$\Big\downarrow$ 27

$$-\frac{2a^3c^4\left(\frac{-\frac{\int \frac{\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+8\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(e+fx)}{a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a}{a\,s\cdots}\right.}\cdots\right.}{f}$$

$$\Big\downarrow \; 444$$

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{\int \frac{a\left(\frac{13a\tan^2(e+fx)}{\sec(e+fx)a+a}+14\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(\cdots}{a^2(a\sec(e+fx)+a)^{5\cdots}}\cdots\right.}{f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{\int \frac{\frac{13a\tan^2(e+fx)}{\sec(e+fx)a+a}+14}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\frac{\tan^5(\cdots}{a^2(a\sec(e+fx)+a)^{5\cdots}}\cdots\right.}{f}$$

$$\Big\downarrow \; 397$$

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+12\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{a^2}+\cdots\right.}{f}$$

$$\Big\downarrow \; 216$$

$$-\frac{2a^3c^4\left(\frac{-\frac{-\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a}}{a^2}-\frac{4\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}+\frac{\tan^5(e}{a^2(a\sec(e+fx)+a)^{5/}}\right.}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*a^3*c^4*(Tan[e + f*x]^5/(a^2*(a + a*Sec[e + f*x])^(5/2)*(2 + (a*Tan[e
+ f*x]^2)/(a + a*Sec[e + f*x]))) + ((-4*Tan[e + f*x]^3)/(3*a*(a + a*Sec[e
+ f*x])^(3/2)) - (-((-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*
x]]]/Sqrt[a]) - (6*Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[e + f*x]])])/Sqrt[a])/a) - (7*Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f
*x]]))/a)/a^2))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(185) = 370$.

Time = 2.66 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.25

method	result
parts	$\frac{c^4 \sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(fx+\right.}{4f\,a^2}$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^4/f/a^2*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(4*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot
(f*x+e)+csc(f*x+e)))+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc
(f*x+e))-5*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
+1/12*c^4/f/a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*cos(f*x+e)+1)*((-
38*cos(f*x+e)^2-24*cos(f*x+e)+8)*tan(f*x+e)+(33*cos(f*x+e)^2+66*cos(f*x+e)
+33)*2^(1/2)*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)
)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-c^4/f/a^2*(a*(1+sec(f*x+e)))^(1/2)*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*(-cot(f*x+e)+csc(f*x+e))+ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+
e)+csc(f*x+e)))+3/2*c^4/f/a^2*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f
*x+e))+3*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))-4
*c^4/f*(-1/4*(1-cos(f*x+e))^3*csc(f*x+e)^3-7/4*ln((-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+9
/4*csc(f*x+e)-9/4*cot(f*x+e))/a^2*(a*(1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.05

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(18*sqrt(2)*(a*c^4*cos(f*x + e)^3 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*co
s(f*x + e))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x +
 e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 3*(c^4*cos(f*x + e)^3 +
2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2
+ 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 2*(28*c^4*cos(f*x + e)^2
+ 15*c^4*cos(f*x + e) - c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f
*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f*x +
e)), -2/3*(3*(c^4*cos(f*x + e)^3 + 2*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e)
)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqr
t(a)*sin(f*x + e))) + (28*c^4*cos(f*x + e)^2 + 15*c^4*cos(f*x + e) - c^4)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + 18*sqrt(2)*(a*c^4*c
os(f*x + e)^3 + 2*a*c^4*cos(f*x + e)^2 + a*c^4*cos(f*x + e))*arctan(sqrt(2
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e)))/sqrt(a))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2*f*cos(f
*x + e))]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx = c^4\left(\int\left(-\frac{4\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx + \int\frac{6\sec^2(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{4\sec^3(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx + \int\frac{\sec^4(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a*sqrt(a*sec
(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-
4*sec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(
e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a*sqrt(a*sec(e + f*x) + a)*
sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(c\sec(fx+e)-c)^4}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^4/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{a}\,c^4\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)^2+2\sec(fx+e)+1}dx-4\left(\int \frac{\sqrt{\sec(f}}{\sec(fx+}\right.\right.}{}$$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*c**4*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e + f*x)**2
+ 2*sec(e + f*x) + 1),x) - 4*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3)/
(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) + 6*int((sqrt(sec(e + f*x) + 1)*
sec(e + f*x)**2)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) - 4*int((sqrt(s
ec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)))
/a**2
```

3.73 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} + \frac{2\sqrt{2}c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{4c^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{2c^3\tan^3(e+fx)}{f(a+a\sec(e+fx))^{3/2}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output

```
2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f+2*2^(1/2
)*c^3*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2
)/f-4*c^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)+2*c^3*tan(f*x+e)^3/f/(a+a*
sec(f*x+e))^(3/2)/(2+tan(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.75

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^3\left(-3+\arctan\left(\sqrt{-1+\sec(e+fx)}\right)\cot^2\left(\frac{1}{2}(e+fx)\right)\sqrt{-1+\sec(e+}\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*c^3*(-3 + ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cot[(e + f*x)/2]^2*Sqrt[-1 +
Sec[e + f*x]] + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cot[(e + f
*x)/2]^2*Sqrt[-1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[(e + f*x)/2])/(a*f*Sq
rt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^3}{(a\sec(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 4392$$

$$-a^3c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^{9/2}}dx$$

$$\Big\downarrow \; 3042$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$\downarrow$ 4375

$$\frac{2a^2c^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\downarrow$ 372

$$\frac{2a^2c^3\left(\frac{\int \frac{2\tan^2(e+fx)\left(\frac{2a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 27

$$\frac{2a^2c^3\left(\frac{\int \frac{\tan^2(e+fx)\left(\frac{2a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 444

$$\frac{2a^2c^3\left(\frac{-\frac{\int \frac{a\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+4\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\downarrow$ 27

$$\frac{2a^2c^3\left(\frac{-\frac{\int \frac{\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+4}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \; 397$$

$$\frac{2a^2c^3\left(\frac{-\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{2a^2c^3\left(\frac{-\frac{-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a}-\frac{2\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{a^2}+\frac{\tan^3(e+fx)}{a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*c^3*((-((-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/
Sqrt[a]) - (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]])])/Sqrt[a])/a) - (2*Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f*x]]))/
a^2 + Tan[e + f*x]^3/(a^2*(a + a*Sec[e + f*x])^(3/2)*(2 + (a*Tan[e + f*x]^
2)/(a + a*Sec[e + f*x])))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 3.00 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

method	result
default	$\frac{c^3\sqrt{-a(-1-\sec(fx+e))}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}+2(1-\cos(fx+e))^3\csc(fx+e)^3+2\ln\left(\sqrt{\right.}$ $\frac{}{f\,a^2}$
parts	$\frac{c^3\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(fx+\right.}$ $\frac{}{4f\,a^2}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c^3/f/a^2*(-a*(-1-sec(f*x+e)))^(1/2)*(2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)+2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*ln((-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)+4*cot(f*x+e)-4*csc(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.14

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{2}\left(ac^3\cos(fx+e)^2+2\,ac^3\cos(fx+e)+ac^3\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+\cdots}{\cos(fx\cdots}}\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

$$-\frac{2\left(\left(c^3\cos(fx+e)^2+2\,c^3\cos(fx+e)+c^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)+\left(3\,c^3\cos(fx+e)+c\cdots\right.\right.}{a^2f\cos(fx+e)^2+2\,a^2f\,\mathrm{c}\cdots}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*sqrt(-1/a)
*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*
x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) - (c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3
)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e
) + 1)) - 2*(3*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f),
-2*((c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (3
*c^3*cos(f*x + e) + c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
 e) + sqrt(2)*(a*c^3*cos(f*x + e)^2 + 2*a*c^3*cos(f*x + e) + a*c^3)*arctan
(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin
(f*x + e)))/sqrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)
]
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{3/2}}\,dx = -c^3 \left(\int \frac{3\sec(e + fx)}{a\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a\sqrt{a\sec(e + fx) + a}}\,dx + \int \left(-\frac{3\sec^2(e + fx)}{a\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a\sqrt{a\sec(e + fx) + a}} \right) dx + \int \frac{\sec^3(e + fx)}{a\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a\sqrt{a\sec(e + fx) + a}}\,dx + \int \left(-\frac{1}{a\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a\sqrt{a\sec(e + fx) + a}} \right) dx \right)$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a*sqrt(a*se
c(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(
sec(e + f*x)**3/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e
+ f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{3/2}}\,dx = \int -\frac{(c\sec(fx + e) - c)^3}{(a\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\,c^3\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx - \left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right) + 3\left(\int \frac{\sqrt{\mathrm{se}}}{\mathrm{sec}(}\right.\right.}{a^2}$$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*c**3*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1),x) - int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x)**2
+ 2*sec(e + f*x) + 1),x) + 3*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/
(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) - 3*int((sqrt(sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)))/a**2
```

3.74 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{\sqrt{2}c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-2^(1/2)*
c^2*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/
f-2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c^2\left(2\sqrt{1-\sec(e+fx)}-2\text{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)(1+\sec(e+fx))+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)(}{f\sqrt{1-\sec(e+fx)}(a(1+\sec(e+fx)))^{3/2}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c^2*(2*Sqrt[1 - Sec[e + f*x]] - 2*ArcTanh[Sqrt[1 - Sec[e + f*x]]]*(1 +
Sec[e + f*x]) + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]]*(1 + Sec[e
 + f*x]))*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(
3/2)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 303, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \text{4375}$$

$$-\frac{2ac^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{372}$$

$$-\frac{2ac^2\left(\frac{\int \frac{2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2ac^2\left(\frac{\int \frac{1}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \; 303$$

$$-\frac{2ac^2\left(\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$$\Big\downarrow \; 216$$

$$-\frac{2ac^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{a^2}+\frac{\tan(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*a*c^2*((-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt
[a]) + ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(
Sqrt[2]*Sqrt[a]))/a^2 + Tan[e + f*x]/(a^2*Sqrt[a + a*Sec[e + f*x]]*(2 + (a
*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 2.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.43

method	result
default	$\frac{c^2\sqrt{-a(-1-\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(-\cot(fx+e)+\csc(f}{f\,a^2}$
parts	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(-\cot(fx+e)+\csc(fx+}{4f\,a^2}$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c^2/f/a^2*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
(2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x
+e)+csc(f*x+e)))+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x
+e))-ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(102) = 204.

Time = 0.30 (sec) , antiderivative size = 542, normalized size of antiderivative = 4.55

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\sqrt{2}\big(ac^2\cos(fx+e)^2+2\,c\cdots}{\cdots}\right.$$

$$-\frac{2\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+2\left(c^2\cos(fx+e)^2+2\,c^2\cos(fx+e)+c^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cdots}}{\cdots}\right.\cdots}{a^2f\cos(fx+e)^2+2\,a^2f\cos(fx\cdots}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - sqrt(2)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*sqrt(
-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*co
s(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x
+ e)^2 + 2*cos(f*x + e) + 1)) + 2*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e)
 + c^2)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f
*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*
c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 2*
(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2
)*(a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*c^2)*arctan(sqrt(2)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/
sqrt(a))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = c^2\left(\int\left(-\frac{2\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx + \int\frac{\sec^2(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx\right)$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) +
a*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**2/(a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(
a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c\sec(fx+e)-c)^2}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2+2\sec(fx+e)+1}dx-2\left(\int \frac{\sqrt{\sec(f}}{\sec(fx+}\right.\right.}{a^2}$$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*c**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**2
+ 2*sec(e + f*x) + 1),x) - 2*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(se
c(e + f*x)**2 + 2*sec(e + f*x) + 1),x)))/a**2
```

3.75 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{3c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{3/2}f} - \frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-3/2*c*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/
f-c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\left(4c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)(1+\sec(e+fx)) - 3\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec}}{\sqrt{2}}\right.\right.}{2f(a(1+\sec(e+fx)))^{3/2}\sqrt{}}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
((4*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]]*(1 + Sec[e + f*x]) -
 3*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(1
+ Sec[e + f*x]) - 2*c*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(2*f*(a*(1 +
 Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4392, 3042, 4375, 373, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4392}$$

$$-ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4375}$$

$$\frac{2c\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{ 373}$$

$$\frac{2c\left(-\frac{\int \frac{1-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 397

$$\frac{2c\left(-\frac{2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-3\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

$\Big\downarrow$ 216

$$\frac{2c\left(-\frac{\frac{3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*c*(-1/2*((-2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/S
qrt[a] + (3*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[2]*Sqrt[a]))/a - Tan[e + f*x]/(2*a*Sqrt[a + a*Sec[e + f*x]]*(2
+ (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))))/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

method	result
default	$\frac{c\sqrt{-a(-1-\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(f}{2f\,a^2}$
parts	$\frac{c\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(fx+e}{4f\,a^2}$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*c/f/a^2*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot
(f*x+e)+csc(f*x+e)))+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc
(f*x+e))-3*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(96) = 192.

Time = 0.20 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.47

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3\sqrt{2}\left(c\cos(fx+e)^2+2\,c\,c\right.}{}\right.$$

$$-\frac{2\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-3\sqrt{2}\left(c\cos(fx+e)^2+2\,c\cos(fx+e)+c\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{2\left(a^2f\cos(fx+e)^2+2\,a^2\right.}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) + 3*sqrt(2)*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*log(-(2*
sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) + 4*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(-a)*
log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a
^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/2*(2*c*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*sqrt(2)*(c*cos(
f*x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 4*(c*cos(f*
x + e)^2 + 2*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2
+ 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = -c\left(\int \frac{\sec(e+fx)}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\,dx + \int\left(-\frac{1}{a\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a\sqrt{a\sec(e+fx)+a}}\right)dx\right)$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
-c*(Integral(sec(e + f*x)/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sqr
t(a*sec(e + f*x) + a)), x) + Integral(-1/(a*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x) + a*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\int -\frac{c\sec\left(fx+e\right)-c}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{c-\frac{c}{\cos(e+f\,x)}}{{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\,dx = \frac{\sqrt{a}\,c\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx - \left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)\right)}{a^2}$$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*c*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 + 2*sec(e + f*x) +
 1),x) - int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*se
c(e + f*x) + 1),x)))/a**2
```

3.76 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 182

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}cf} - \frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{3/2}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^2cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{2a^2cf\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c/f-7/8*arctan
(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/c/
f+1/4*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c/f+1/2*cot(f*x+e)*(a+a*sec(f*
x+e))^(1/2)/a^2/c/f/(2+tan(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx =$$
$$-\frac{\left(-2+7\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))-8\,\text{Hypergeometric2F1}\left(\cdot\right.\right.}{4cf(-1+\sec(e+fx))(a(1+\sec(e+fx)))^{3/2}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

output
```
-1/4*((-2 + 7*Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]*(1 + S
ec[e + f*x]) - 8*Hypergeometric2F1[-1/2, 1, 1/2, 1 - Sec[e + f*x]]*(1 + Se
c[e + f*x]))*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x])*(a*(1 + Sec[e + f*x]))
^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 374, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{4392}$$
$$-\frac{\int \frac{\cot^2(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{1}{\cot\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2cf}$$

$$\Big\downarrow \text{374}$$

$$\frac{2\left(\frac{\int \frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^2cf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{4}\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^2cf}$$

$$\Big\downarrow \text{445}$$

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+9\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{\cot(e+fx)\sqrt{\ldots}}{4\left(\frac{a\tan^2\ldots}{a\sec(e\ldots}\right.}\right.}{a^2cf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int \frac{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+9}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{\cot(e+fx)\sqrt{\ldots}}{4\left(\frac{a\tan\ldots}{a\sec(\ldots}\right.}\right.}{a^2cf}$$

$$\Big\downarrow \text{397}$$

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(8\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-7\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\mathrm{ta}\ldots}{\sqrt{\sec\ldots}}\right.\right.\right.\right.}{a^2cf}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{8\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)\right)+\frac{\cot(e+fx)\cdots}{4\left(\frac{a\tan\cdots}{a\sec(\cdots}\right.}\cdots}{a^2cf}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])),x]
```

output
```
(2*((-1/2*(a*((-8*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])
/Sqrt[a] + (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*
x]])])/(Sqrt[2]*Sqrt[a]))) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2)/4
+ (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*(2 + (a*Tan[e + f*x]^2)/(a +
a*Sec[e + f*x])))))/(a^2*c*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(157) = 314$.

Time = 4.05 (sec) , antiderivative size = 702, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	702

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*2^(1/2)*(-1/4/f/(cos(1/2*f*x+1/2*e)+1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)
*cos(1/2*f*x+1/2*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*
e)+1)^2)^(1/2)/a*(3*cos(1/2*f*x+1/2*e)*arctan((2*sin(1/2*f*x+1/2*e)-1)/(co
s(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^
(1/2))+(2*cos(1/2*f*x+1/2*e)^3+2*cos(1/2*f*x+1/2*e)^2+2*cos(1/2*f*x+1/2*e)
+2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*sec(1/2*f*
x+1/2*e)*csc(1/2*f*x+1/2*e)+3*cos(1/2*f*x+1/2*e)*arctan((2*sin(1/2*f*x+1/2
*e)+1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2
*e)+1)^2)^(1/2)))-1/f*(4*2^(1/2)*arctan(2^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1
)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*(-csc(1/2*f*x+1/2*e)+cot(1/2*f*x+1/2*e))
)*sin(1/2*f*x+1/2*e)-2*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^
2)^(1/2)*cos(1/2*f*x+1/2*e)+arctan((2*sin(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1
/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*sin(
1/2*f*x+1/2*e)+arctan((2*sin(1/2*f*x+1/2*e)+1)/(cos(1/2*f*x+1/2*e)+1)/((2*
cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*sin(1/2*f*x+1/2*e
)-2*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))/(cos(1/2*
f*x+1/2*e)+1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/((
2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/a*cot(1/2*f*x+1/
2*e))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \left[-\frac{7\sqrt{2}\sqrt{-a}(\cos(fx+e)+1)\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)}{\cos(fx+e)}}}{}\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[-1/16*(7*sqrt(2)*sqrt(-a)*(cos(f*x + e) + 1)*log(-(2*sqrt(2)*sqrt(-a)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f
*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*s
in(f*x + e) + 8*sqrt(-a)*(cos(f*x + e) + 1)*log(-(8*a*cos(f*x + e)^3 + 4*(
2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
 e) - 4*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/8*(7*sqrt(2)*s
qrt(a)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 8*sqrt(a)*(cos(
f*x + e) + 1)*arctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x
+ e) + 2*(3*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{a\sqrt{a\sec{(e+fx)}+a}\sec^2{(e+fx)}-a\sqrt{a\sec{(e+fx)}+a}}\,dx}{c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e +
 f*x) + a)), x)/c
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \int -\frac{1}{(a\sec{(fx+e)}+a)^{\frac{3}{2}}(c\sec{(fx+e)}-c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+\sec(fx+e)^2-\sec(fx+e)-1}dx\right)}{a^2c}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + sec(e + f*x)**2
- sec(e + f*x) - 1),x))/(a**2*c)
```

3.77 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 219

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx &= \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^2f}\\ &- \frac{9\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{3/2}c^2f} + \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{8a^2c^2f}\\ &+ \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{12a^3c^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{2a^3c^2f\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}\end{aligned}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^2/f-9/16*arc
tan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2)
/c^2/f+7/8*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c^2/f+1/12*cot(f*x+e)^3*(
a+a*sec(f*x+e))^(3/2)/a^3/c^2/f-1/2*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3/2)/a^
3/c^2/f/(2+tan(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\left(-6+9\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\sec(e+\right.\right.}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

output
```
((-6 + 9*Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e
 + f*x]) - 8*Hypergeometric2F1[-3/2, 1, -1/2, 1 - Sec[e + f*x]]*(1 + Sec[e
 + f*x]))*Tan[e + f*x])/(12*c^2*f*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*
x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {3042, 4392, 3042, 4375, 374, 25, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \cot^4(e+fx)\sqrt{\sec(e+fx)a+a}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\cot\left(e+fx+\frac{\pi}{2}\right)^4}dx}{a^2c^2}$$

$$\Big\downarrow \; 4375$$

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3c^2f}$$

$$\Big\downarrow \; 374$$

$$-\frac{2\left(\frac{\int -\frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^3c^2f}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\left(\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)}{a^3c^2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^3c^2f}$$

$$\Big\downarrow \; 445$$

$$-\frac{2\left(\frac{1}{4}\left(\frac{1}{6}\int -\frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(7-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3\ldots}\right.\right.}{a^3c^2f}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\int\frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(7-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{1}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}\right.\right.}{a^3c^2f}$$

$\downarrow$ 445

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int\frac{a\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+23\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\frac{1}{6}\right.\right.}{a^3c^2f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int\frac{\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+23}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\frac{1}{6}\right.\right.}{a^3c^2f}$$

$\downarrow$ 397

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(16\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-9\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}\right.\right.\right.\right.}{a^3c^2f}$$

$\downarrow$ 216

$$-\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{9\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{16\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)\right)\right)-\right.}{a^3c^2f}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^2),x]
```

output
```
(-2*((-1/6*(Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2)) - (a*(-1/2*(a*((-16
*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + (9*Arc
Tan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*S
qrt[a]))) + (7*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/2)/4 + (Cot[e +
f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(4*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e
 + f*x])))))/(a^3*c^2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.66

method	result
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left(\left(-96\cos(fx+e)^2-192\cos(fx+e)-96\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)\sqrt{-}\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/96/c^2/f/a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)^2*((-96*cos(f*x+e)^
2-192*cos(f*x+e)-96)*2^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(
f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))*(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)+(-54*cos(f*x+e)^2-108*cos(f*x+e)-54)*ln((-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)+2^(1/2)*(79*cos(f*x+e)^4+232*cos(f*x+e)^3+90*cos(f*x+e)^2-168*cos(
f*x+e)-105)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*cot(f*x+e)*csc(f*x+e)^2+(34*cos(f*x+e)^4+66*cos(f*x+e)^3-150*cos
(f*x+e)^2-34*cos(f*x+e)+84)*cot(f*x+e)*csc(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.56

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
[-1/96*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*co
s(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)
)*sin(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3
 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin
(f*x + e) - 4*(31*cos(f*x + e)^3 - 2*cos(f*x + e)^2 - 21*cos(f*x + e))*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2
*f)*sin(f*x + e)), 1/48*(27*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(sq
rt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*
x + e)))*sin(f*x + e) + 48*(cos(f*x + e)^2 - 1)*sqrt(a)*arctan(2*sqrt(a)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(
f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(31*cos(f*x + e)^3 - 2*
cos(f*x + e)^2 - 21*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))
/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)-a\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)-a\sqrt{a\,\mathrm{s}}}}{c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 - a*sqrt(a*sec(e +
f*x) + a)*sec(e + f*x)**2 - a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a*sq
rt(a*sec(e + f*x) + a)), x)/c**2
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^2}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4-2\sec(fx+e)^2+1}dx\right)}{a^2c^2}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 +
 1),x))/(a**2*c**2)
```

3.78 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 254

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}c^3f} - \frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{3/2}c^3f} + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^2c^3f} - \frac{5\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{24a^3c^3f} - \frac{3\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{20a^4c^3f} + \frac{\cot^5(e+fx)(a+a\sec(e+fx))^{5/2}}{2a^4c^3f\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/c^3/f-11/32*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(3/2
)/c^3/f+21/16*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^2/c^3/f-5/24*cot(f*x+e)^
3*(a+a*sec(f*x+e))^(3/2)/a^3/c^3/f-3/20*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2
)/a^4/c^3/f+1/2*cot(f*x+e)^5*(a+a*sec(f*x+e))^(5/2)/a^4/c^3/f/(2+tan(f*x+e
)^2/(1+sec(f*x+e)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.40

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\left(-10+11\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))-8\,\text{Hypergeometric2F1}\right.}{20c^3f(-1+\sec(e+fx))^3(a(1+\sec(e+fx)}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

output
```
-1/20*((-10 + 11*Hypergeometric2F1[-5/2, 1, -3/2, (1 - Sec[e + f*x])/2]*(1
 + Sec[e + f*x]) - 8*Hypergeometric2F1[-5/2, 1, -3/2, 1 - Sec[e + f*x]]*(1
 + Sec[e + f*x]))*Tan[e + f*x])/(c^3*f*(-1 + Sec[e + f*x])^3*(a*(1 + Sec[e
 + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {3042, 4392, 3042, 4375, 374, 25, 27, 445, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4392}$$

$$-\frac{\int \cot^6(e+fx)(\sec(e+fx)a+a)^{3/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\cot\left(e+fx+\frac{\pi}{2}\right)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{4375}$$

$$\frac{2\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^4c^3f}$$

$$\Big\downarrow \text{374}$$

$$\frac{2\left(\frac{\int -\frac{a\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)}{a^4c^3f}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)}{a^4c^3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int \frac{\cot^6(e+fx)(\sec(e+fx)a+a)^3\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a^4c^3f}$$

$$\Big\downarrow \text{445}$$

$$\frac{2\left(\frac{1}{4}\left(\frac{1}{10}\int -\frac{5a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{3}{10}\cot^5(e+fx)(a\sec(e+fx)+a)\right.\right.}{a^4c^3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{3}{10}\cot^5(e+fx)(a\sec(e+fx)+a)^{5/2}}{a^4c^3f}$$

$\big\downarrow$ 445

$$\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{6}\int \frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+21\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)}}\right.\right.\right.\right.}{a^4c^3f}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+21\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)}}\right.\right.\right.\right.}{a^4c^3f}$$

$\big\downarrow$ 445

$$\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int \frac{a\left(\frac{21a\tan}{\sec(e+}\right.}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+\right.}\right.\right.\right.\right.}{a^4}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int \frac{\frac{21a\tan}{\sec(e+}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+\right.}\right.\right.\right.\right.}{a^4}$$

$\big\downarrow$ 397

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(32\int \frac{1}{\frac{a\tan^2(e+f}{\sec(e+fx)a-}}\right.\right.\right.\right.\right.$$

$\big\downarrow$ 216

$$2\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{5}{6}\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{11\arctan\left(\frac{\sqrt{}}{\sqrt{2}\sqrt{}}\right.}{\sqrt{2}}\right.\right.\right.\right.\right.$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^3),x]
```

output
```
(2*(((-3*Cot[e + f*x]^5*(a + a*Sec[e + f*x])^(5/2))/10 - (a*((5*Cot[e + f*
x]^3*(a + a*Sec[e + f*x])^(3/2))/6 - (a*(-1/2*(a*((-32*ArcTan[(Sqrt[a]*Tan
[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + (11*ArcTan[(Sqrt[a]*Tan[e
+ f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))) + (21*Cot
[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/2))/2))/2)/4 + (Cot[e + f*x]^5*(a + a*
Sec[e + f*x])^(5/2))/(4*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))))/(
a^4*c^3*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 1.35 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\sqrt{a(1+\sec(fx+e))}\left(\left(7680\cos(fx+e)^2+15360\cos(fx+e)+7680\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-}}\right.\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/7680/c^3/f/a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)^2*((7680*cos(f*x
+e)^2+15360*cos(f*x+e)+7680)*2^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e
))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)+(2640*cos(f*x+e)^2+5280*cos(f*x+e)+2640)*ln((-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)+2^(1/2)*(9289*cos(f*x+e)^6+30040*cos(f*x+e)^5+6125*cos
(f*x+e)^4-41200*cos(f*x+e)^3-20085*cos(f*x+e)^2+17304*cos(f*x+e)+10815)*(-
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cot(
f*x+e)*csc(f*x+e)^4+(4210*cos(f*x+e)^6+9630*cos(f*x+e)^5-26980*cos(f*x+e)^
4-8860*cos(f*x+e)^3+29010*cos(f*x+e)^2+3070*cos(f*x+e)-10080)*cot(f*x+e)*c
sc(f*x+e)^4)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.81

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
[-1/960*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*
sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(co
s(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 480*(cos(f*x + e)^3 - c
os(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2
*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
e) - 4*(449*cos(f*x + e)^4 - 351*cos(f*x + e)^3 - 365*cos(f*x + e)^2 + 315
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^2*c^3*f*cos(f*
x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*
sin(f*x + e)), 1/480*(165*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f
*x + e) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 480*(cos(f*x + e)^3
- cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2
+ a*cos(f*x + e) - a))*sin(f*x + e) + 2*(449*cos(f*x + e)^4 - 351*cos(f*x
+ e)^3 - 365*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2
*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{1}{a\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a\sqrt{a\sec(e+fx)+a}\sec^3(e+fx)+2a\sqrt{a\sec(e+fx)+a}\sec(e+fx)-a\sqrt{a\sec(e+fx)+a}}\,dx}{c^3}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**3,x)
```

output
```
-Integral(1/(a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x)**3 + 2*a*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) -
 a*sqrt(a*sec(e + f*x) + a)), x)/c**3
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \int -\frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(c\sec(fx+e)-c)^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
-integrate(1/((a*sec(f*x + e) + a)^(3/2)*(c*sec(f*x + e) - c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^3}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^5-\sec(fx+e)^4-2\sec(fx+e)^3+2\sec(fx+e)^2+\sec(fx+e)-1}dx\right)}{a^2c^3}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x)
```

output
```
( - sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**5 - sec(e + f*x)**4
- 2*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1),x))/(a**2*c**3
)
```

3.79 $\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 265

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx &= \frac{2c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} \\ &- \frac{23\sqrt{2}c^5\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} + \frac{21c^5\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}} \\ &- \frac{19c^5\tan^3(e+fx)}{6af(a+a\sec(e+fx))^{3/2}} + \frac{ac^5\tan^7(e+fx)}{f(a+a\sec(e+fx))^{7/2}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)^2} \\ &+ \frac{3c^5\tan^5(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}\end{aligned}$$

output

```
2*c^5*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23*2^(1/
2)*c^5*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(5/
2)/f+21*c^5*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-19/6*c^5*tan(f*x+e)^3/
a/f/(a+a*sec(f*x+e))^(3/2)+a*c^5*tan(f*x+e)^7/f/(a+a*sec(f*x+e))^(7/2)/(2+
tan(f*x+e)^2/(1+sec(f*x+e)))^2+3/2*c^5*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(5/
2)/(2+tan(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [A] (verified)

Time = 8.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.68

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^5\cot\left(\frac{1}{2}(e+fx)\right)\left((81-30\cos(e+fx)+52\cos(2(e+fx))-66\cos(3(e\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(c^5*Cot[(e + f*x)/2]*((81 - 30*Cos[e + f*x] + 52*Cos[2*(e + f*x)] - 66*Co
s[3*(e + f*x)] - 37*Cos[4*(e + f*x)])*Sec[(e + f*x)/2]^4 + 96*ArcTan[Sqrt[
-1 + Sec[e + f*x]]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f*x]] - 1104*Sqrt[2]*
ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Cos[e + f*x]^2*Sqrt[-1 + Sec[e + f
*x]])*Sec[e + f*x]^2)/(48*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 440, 25, 27, 444, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^5}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \ 4392$$

$$-a^5c^5\int \frac{\tan^{10}(e+fx)}{(\sec(e+fx)a+a)^{15/2}}dx$$

$$\Big\downarrow \ 3042$$

$$-a^5c^5\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^{10}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{15/2}}dx$$

$\big\downarrow$ 4375

$$\frac{2a^3c^5\int \frac{\tan^{10}(e+fx)}{(\sec(e+fx)a+a)^5\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\big\downarrow$ 372

$$\frac{2a^3c^5\left(\frac{\int \frac{2\tan^6(e+fx)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan^7(e+fx)}{2a^2(a\sec(e+fx)+a)^{7/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\big\downarrow$ 27

$$\frac{2a^3c^5\left(\frac{\int \frac{\tan^6(e+fx)\left(\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^7(e+fx)}{2a^2(a\sec(e+fx)+a)^{7/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\big\downarrow$ 440

$$\frac{2a^3c^5\left(\frac{\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int -\frac{a\tan^4(e+fx)\left(\frac{19a\tan^2(e+fx)}{\sec(e+fx)a+a}+15\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}}{2a^2}+\frac{\cdots}{2a^2(a\sec\cdots}\cdots\right)}{f}$$

$\big\downarrow$ 25

$$\frac{2a^3c^5\left(\frac{\frac{\int \frac{a\tan^4(e+fx)\left(\frac{19a\tan^2(e+fx)}{\sec(e+fx)a+a}+15\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2(a\sec(\cdots}\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2a^3c^5\left(\frac{\frac{\int \frac{\tan^4(e+fx)\left(\frac{19a\tan^2(e+fx)}{\sec(e+fx)a+a}+15\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2(a\sec(\cdots}\right.}{f}$$

$$\Big\downarrow \; 444$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{\int \frac{6a\tan^2(e+fx)\left(\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+19\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{3a^2}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+\cdots}{a\sec(e+f\cdots}\right.}}{2a^2}\cdots\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\int \frac{\tan^2(e+fx)\left(\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+19\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(e+fx)}{2a(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e\cdots}{a\sec(e+\cdots}\right.}}{2a^2}\cdots\right.}{f\cdots}$$

$$\Big\downarrow \; 444$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\left(-\frac{\int \frac{2a\left(\frac{22a\tan^2(e+fx)}{\sec(e+fx)a+a}+21\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(\cdots}{2a(a\sec(e+fx)+a)^{5/2}\cdots}}{2a^2}\cdots\right.}{f\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\left(-\frac{2\int \frac{\frac{22a\tan^2(e+fx)}{\sec(e+fx)a+a}+21}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5\cdots}{2a(a\sec(e+fx)+a)^{5/\cdots}}}{2a^2}\cdots\right.}{f\cdots}$$

$$\Big\downarrow \; 397$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\left(-\frac{2\left(23\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}}{2a^2}\right.}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{2a^3c^5\left(\frac{\frac{-\frac{2\left(-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{23\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}\right)}{a}-\frac{21\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}\right)}{a}-\frac{19\tan^3(e+fx)}{3a(a\sec(e+fx)+a)^{3/2}}}{2a}+\frac{3\tan^5(e}{2a(a\sec(e+fx)+a)^{5/2}}}{2a^2}\right.}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^5/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*a^3*c^5*(Tan[e + f*x]^7/(2*a^2*(a + a*Sec[e + f*x])^(7/2)*(2 + (a*Tan[e
 + f*x]^2)/(a + a*Sec[e + f*x]))^2) + ((3*Tan[e + f*x]^5)/(2*a*(a + a*Sec[
e + f*x])^(5/2)*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))) + ((-19*Tan
[e + f*x]^3)/(3*a*(a + a*Sec[e + f*x])^(3/2)) - (2*((-2*(ArcTan[(Sqrt[a]*T
an[e + f*x])/Sqrt[a + a*Sec[e + f*x]]]/Sqrt[a] - (23*ArcTan[(Sqrt[a]*Tan[e
 + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a])))/a - (21*
Tan[e + f*x])/(a*Sqrt[a + a*Sec[e + f*x]])))/a)/(2*a))/(2*a^2)))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(238) = 476$.

Time = 3.25 (sec) , antiderivative size = 1033, normalized size of antiderivative = 3.90

method	result	size
parts	Expression too large to display	1033

input
```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*c^5/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(-2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+
32*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f
*x+e)+csc(f*x+e)))+13*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e))-43*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)
))-5/32*c^5/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*
x+e)+1)*cot(f*x+e)-3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc
(f*x+e))+3*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
+5/16*c^5/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*(-4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*x
+e)+1)*cot(f*x+e)-5*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(
f*x+e))+5*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))-
5/16*c^5/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+1
1*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+19*ln((-2*
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))+5*c^5/f*(-1/16*(1
-cos(f*x+e))^5*csc(f*x+e)^5-17/32*(1-cos(f*x+e))^3*csc(f*x+e)^3-75/32*ln((
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+83/32*csc(f*x+e)-83/32*cot(f*x+e))/a^3*(a*(1+sec(...
```

Fricas [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(69*sqrt(2)*(a*c^5*cos(f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*
cos(f*x + e)^2 + a*c^5*cos(f*x + e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(
f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) -
6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + 3*c^5*cos(f*x + e)^2 + c^5*
cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/
(cos(f*x + e) + 1)) + 4*(37*c^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 2
0*c^5*cos(f*x + e) - c^5)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e
)^2 + a^3*f*cos(f*x + e)), -1/3*(6*(c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e
)^3 + 3*c^5*cos(f*x + e)^2 + c^5*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(37*c
^5*cos(f*x + e)^3 + 70*c^5*cos(f*x + e)^2 + 20*c^5*cos(f*x + e) - c^5)*sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 69*sqrt(2)*(a*c^5*cos(
f*x + e)^4 + 3*a*c^5*cos(f*x + e)^3 + 3*a*c^5*cos(f*x + e)^2 + a*c^5*cos(f
*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^5}{(a + a\sec(e + fx))^{5/2}}\,dx =$$

$$-c^5\left(\int \frac{5\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)}\cdots}\right.$$

$$+\int\left(-\frac{10\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)}+\cdots}\right.$$

$$+\int \frac{10\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)+a}}\,d\cdots$$

$$+\int\left(-\frac{5\sec^4(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)}+\cdots}\right.$$

$$+\int \frac{\sec^5(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)+a}}\,d\cdots$$

$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx) + 2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx) + a^2\sqrt{a\sec(e+fx)}+\cdots}\right.$$

input
```
integrate((c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
-c**5*(Integral(5*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(-10*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a
)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sq
rt(a*sec(e + f*x) + a)), x) + Integral(10*sec(e + f*x)**3/(a**2*sqrt(a*sec
(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-5*sec(e + f*x)**4/(a*
*2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) +
 a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f
*x)**5/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(
e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral
(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^5}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^5}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^5}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^5}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^5/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\,c^5\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx - \left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(}\right.\right.}{}$$

input

```
int((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^(5/2),x)
```

output

```
(sqrt(a)*c**5*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x
)**2 + 3*sec(e + f*x) + 1),x) - int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**
5)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + 5*int((
sqrt(sec(e + f*x) + 1)*sec(e + f*x)**4)/(sec(e + f*x)**3 + 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) + 1),x) - 10*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)*
*3)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + 10*int
((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**3 + 3*sec(e + f*x
)**2 + 3*sec(e + f*x) + 1),x) - 5*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)
)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.80 $\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 235

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} + \frac{7c^4\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}} - \frac{c^4\tan^5(e+fx)}{f(a+a\sec(e+fx))^{5/2}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)^2} - \frac{c^4\tan^3(e+fx)}{2af(a+a\sec(e+fx))^{3/2}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output
```
2*c^4*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/2*c^4
*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(
5/2)/f+7/2*c^4*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)-c^4*tan(f*x+e)^5/f/
(a+a*sec(f*x+e))^(5/2)/(2+tan(f*x+e)^2/(1+sec(f*x+e)))^2-1/2*c^4*tan(f*x+e
)^3/a/f/(a+a*sec(f*x+e))^(3/2)/(2+tan(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.70

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c^4\cot\left(\frac{1}{2}(e+fx)\right)\left((-4+19\cos(e+fx)-12\cos(2(e+fx))-3\cos(3(e-\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(c^4*Cot[(e + f*x)/2]*((-4 + 19*Cos[e + f*x] - 12*Cos[2*(e + f*x)] - 3*Cos
[3*(e + f*x)])*Sec[(e + f*x)/2]^4 + 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Cos
[e + f*x]*Sqrt[-1 + Sec[e + f*x]] - 88*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*
x]]/Sqrt[2]]*Cos[e + f*x]*Sqrt[-1 + Sec[e + f*x]])*Sec[e + f*x])/(16*a^2*f
*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {3042, 4392, 3042, 4375, 372, 27, 440, 25, 27, 444, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^4}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^4c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^8}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{13/2}}dx$$

$\Big\downarrow$ 4375

$$-\frac{2a^2c^4\int \frac{\tan^8(e+fx)}{(\sec(e+fx)a+a)^4\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\Big\downarrow$ 372

$$-\frac{2a^2c^4\left(\frac{\int \frac{2\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan^5(e+fx)}{2a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\Big\downarrow$ 27

$$-\frac{2a^2c^4\left(\frac{\int \frac{\tan^4(e+fx)\left(\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}+5\right)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^5(e+fx)}{2a^2(a\sec(e+fx)+a)^{5/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\Big\downarrow$ 440

$$-\frac{2a^2c^4\left(\frac{\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int -\frac{a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}}{2a^2}+\frac{\ldots}{2a^2(a\,\mathrm{se}\ldots}\right)}{f}$$

$\Big\downarrow$ 25

$$-\frac{2a^2c^4\left(\frac{\frac{\int \frac{a\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2(a\sec\cdots}\right.}{f}$$

$\downarrow$ 27

$$-\frac{2a^2c^4\left(\frac{\frac{\int \frac{\tan^2(e+fx)\left(\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2(a\sec\cdots}\right.}{f}$$

$\downarrow$ 444

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{\int \frac{2a\left(\frac{9a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^2}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\cdots}{2a^2\cdots}\right.}{f}$$

$\downarrow$ 27

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{2\int \frac{\frac{9a\tan^2(e+fx)}{\sec(e+fx)a+a}+7}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\right.}{f}$$

$$\Big\downarrow \text{397}$$

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{2\left(11\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-2\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}}\right.}{2a^2}}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2a^2c^4\left(\frac{\frac{-\frac{2\left(\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}-\frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}\right)}{a}-\frac{7\tan(e+fx)}{a\sqrt{a\sec(e+fx)+a}}}{2a}+\frac{\tan^3(e+fx)}{2a(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\right.}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^4/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*a^2*c^4*(Tan[e + f*x]^5/(2*a^2*(a + a*Sec[e + f*x])^(5/2)*(2 + (a*Tan[
e + f*x]^2)/(a + a*Sec[e + f*x]))^2) + (((-2*((2*ArcTan[(Sqrt[a]*Tan[e + f
*x])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] - (11*ArcTan[(Sqrt[a]*Tan[e + f*x]
)/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a])))/a - (7*Tan[e +
f*x])/(a*Sqrt[a + a*Sec[e + f*x]]))/(2*a) + Tan[e + f*x]^3/(2*a*(a + a*Sec
[e + f*x])^(3/2)*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/(2*a^2)))
/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(210) = 420$.

Time = 2.40 (sec) , antiderivative size = 851, normalized size of antiderivative = 3.62

method	result	size
parts	Expression too large to display	851

input

```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/32*c^4/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(-2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+
32*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f
*x+e)+csc(f*x+e)))+13*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e))-43*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)
))+c^4/f*(-1/16*(1-cos(f*x+e))^5*csc(f*x+e)^5-17/32*(1-cos(f*x+e))^3*csc(f
*x+e)^3-75/32*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e
))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+83/32*csc(f*x+e)-83/32*cot(f*x+e))
/a^3*(a*(1+sec(f*x+e)))^(1/2)-1/8*c^4/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*(4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*cot(f*x+e)-3*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+3*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)-cot(f*x+e)+csc(f*x+e)))+3/16*c^4/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*cot(f*x+e)-5*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+5*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-cot(f*x+e)+csc(f*x+e)))-1/8*c^4/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*(2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos
(f*x+e))^3*csc(f*x+e)^3+11*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+
e)+csc(f*x+e))+19*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+cs...
```

Fricas [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.79

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/4*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f
*x + e) + c^4)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*
x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 4*(c^4*cos(f*x + e)^3
 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(-a)*log((2*a*cos(
f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*c^4*cos(f
*x + e)^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3
*f*cos(f*x + e) + a^3*f), 1/2*(11*sqrt(2)*(c^4*cos(f*x + e)^3 + 3*c^4*cos(
f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 4*(c^4*
cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + 3*c^4*cos(f*x + e) + c^4)*sqrt(a)*
arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e))) + 2*(3*c^4*cos(f*x + e)^2 + 9*c^4*cos(f*x + e) + 2*c^4)*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^
3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = c^4\left(\int\left(-\frac{4\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\,\mathrm{se}}\right.\right.$$

$$+\int\frac{6\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,d$$

$$+\int\left(-\frac{4\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+}}\right.$$

$$+\int\frac{\sec^4(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,d$$

$$+\int\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,d$$

input
```
integrate((c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
c**4*(Integral(-4*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(6*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*
sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt
(a*sec(e + f*x) + a)), x) + Integral(-4*sec(e + f*x)**3/(a**2*sqrt(a*sec(e
 + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x
) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**4/(a**2*sq
rt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*s
ec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a
*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e
 + f*x) + a**2*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^4}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^4/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c - c\sec(e + fx))^4}{(a + a\sec(e + fx))^{5/2}}\,dx = \frac{\sqrt{a}\,c^4\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx + \int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(f$$

input
```
int((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*c**4*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x
)**2 + 3*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**
4)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) - 4*int((
sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x)**3 + 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) + 1),x) + 6*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**
2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) - 4*int((
sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2
+ 3*sec(e + f*x) + 1),x)))/a**3
```

3.81 $\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{7c^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{5/2}f} + \frac{c^3\tan^3(e+fx)}{af(a+a\sec(e+fx))^{3/2}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)^2} - \frac{c^3\tan(e+fx)}{2a^2f\sqrt{a+a\sec(e+fx)}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output
```
2*c^3*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-7/4*c^3*
arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(5
/2)/f+c^3*tan(f*x+e)^3/a/f/(a+a*sec(f*x+e))^(3/2)/(2+tan(f*x+e)^2/(1+sec(f
*x+e)))^2-1/2*c^3*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(2+tan(f*x+e)^2/
(1+sec(f*x+e)))
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{5/2}}\, dx =$$

$$-\frac{c^3 \cot\left(\frac{1}{2}(e + fx)\right)\left((-5 + 8\cos(e + fx) - 3\cos(2(e + fx)))\sec^4\left(\frac{1}{2}(e + fx)\right) - 32\arctan\left(\sqrt{-1 + \sec(\right.\right.}{16a^2 f\sqrt{a(1 + \sec(e + f}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/16*(c^3*Cot[(e + f*x)/2]*((-5 + 8*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Se
c[(e + f*x)/2]^4 - 32*ArcTan[Sqrt[-1 + Sec[e + f*x]]]*Sqrt[-1 + Sec[e + f*
x]] + 28*Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[-1 + Sec[e +
 f*x]]))/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4392, 3042, 4375, 372, 27, 440, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sec(e + fx))^3}{(a\sec(e + fx) + a)^{5/2}}\, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(c - c\csc\left(e + fx + \frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e + fx + \frac{\pi}{2}\right) + a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4392$$

$$-a^3c^3\int \frac{\tan^6(e + fx)}{(\sec(e + fx)a + a)^{11/2}}dx$$

$$\Big\downarrow \; 3042$$

$$-a^3c^3\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^6}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{11/2}}dx$$

$\Big\downarrow$ 4375

$$\frac{2ac^3\int \frac{\tan^6(e+fx)}{(\sec(e+fx)a+a)^3\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\Big\downarrow$ 372

$$\frac{2ac^3\left(\frac{\int \frac{2\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\Big\downarrow$ 27

$$\frac{2ac^3\left(\frac{\int \frac{\tan^2(e+fx)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+3\right)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\Big\downarrow$ 440

$$\frac{2ac^3\left(\frac{-\frac{\int \frac{a\left(1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\Big\downarrow$ 27

$$\frac{2ac^3\left(\frac{-\frac{\int \frac{1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\downarrow$ 397

$$\frac{2ac^3\left(\frac{-\frac{4\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-7\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\downarrow$ 216

$$\frac{2ac^3\left(\frac{-\frac{\frac{7\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{4\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}}{2a}-\frac{\tan(e+fx)}{2a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan^3(e+fx)}{2a^2(a\sec(e+fx)+a)^{3/2}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*a*c^3*(Tan[e + f*x]^3/(2*a^2*(a + a*Sec[e + f*x])^(3/2)*(2 + (a*Tan[e +
 f*x]^2)/(a + a*Sec[e + f*x]))^2) + (-1/2*((-4*ArcTan[(Sqrt[a]*Tan[e + f*x
])/Sqrt[a + a*Sec[e + f*x]]])/Sqrt[a] + (7*ArcTan[(Sqrt[a]*Tan[e + f*x])/(
Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))/a - Tan[e + f*x]/(2
*a*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x])))
)/(2*a^2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 2.68 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05

method	result
default	$-\frac{c^3\sqrt{-a(-1-\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(1-\cos(fx+e))^3\csc(fx+e)^3-\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(-\cot(fx+e)+\mathrm{cs}\right.}{4fa^3}$
parts	$\frac{c^3\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(1-\cos(fx+e))^3\csc(fx+e)^3+32\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(f}{\sqrt{(1-\cos(fx+e))^2\csc(fx}}\right.\right.}{32fa^3}$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*c^3/f/a^3*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*(2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3
-(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-4*2^(1/2)*a
rctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*
x+e)))+7*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.16

$$\int \frac{(c - c\sec(e + fx))^3}{(a + a\sec(e + fx))^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/8*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*
x + e) + c^3)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x
 + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^3
+ 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f
*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(3*c^3*cos(f*
x + e)^2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f
*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x
+ e) + a^3*f), 1/4*(7*sqrt(2)*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 +
 3*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 8*(c^3*cos(f*x + e)
^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) -
2*(3*c^3*cos(f*x + e)^2 - c^3*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3
*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx =$$
$$-c^3\left(\int \frac{3\sec(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)-}\right.$$
$$+\int\left(-\frac{3\sec^2(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+}\right.$$
$$+\int\frac{\sec^3(e+fx)}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+a}}\,a$$
$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+2a^2\sqrt{a\sec(e+fx)+a}\sec(e+fx)+a^2\sqrt{a\sec(e+fx)+}\right.$$

input
```
integrate((c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
-c**3*(Integral(3*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(-3*sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)
*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqr
t(a*sec(e + f*x) + a)), x) + Integral(sec(e + f*x)**3/(a**2*sqrt(a*sec(e +
 f*x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
+ a**2*sqrt(a*sec(e + f*x) + a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*
x) + a)*sec(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a
**2*sqrt(a*sec(e + f*x) + a)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\,c^3\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(}\right.\right.}{}$$

input
```
int((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x)
```

output

```
(sqrt(a)*c**3*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x
)**2 + 3*sec(e + f*x) + 1),x) - int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**
3)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + 3*int((
sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**3 + 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) + 1),x) - 3*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/
(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.82 $\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 203

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{11c^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{4\sqrt{2}a^{5/2}f} - \frac{c^2\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)^2} - \frac{3c^2\tan(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output

```
2*c^2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-11/8*c^2
*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(
5/2)/f-c^2*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(2+tan(f*x+e)^2/(1+sec(
f*x+e)))^2-3/4*c^2*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(2+tan(f*x+e)^2
/(1+sec(f*x+e)))
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.71

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{c^2\left(22\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)-8\text{arctanh}\left(\sqrt{1-\sec(e+fx)}\right)(1+\sec(}{4f\sqrt{1-\sec(e+fx)}(a(1+\sec(e+fx)))^5}$$

input
```
Integrate[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/4*(c^2*(22*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]]*Cos[(e + f*x
)/2]^4*Sec[e + f*x]^2 - 8*ArcTanh[Sqrt[1 - Sec[e + f*x]]]*(1 + Sec[e + f*x
])^2 + Sqrt[1 - Sec[e + f*x]]*(7 + 3*Sec[e + f*x]))*Tan[e + f*x])/(f*Sqrt[
1 - Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4392, 3042, 4375, 372, 27, 402, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^2}{(a\sec(e+fx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$a^2c^2\int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2 c^2 \int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^4}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{9/2}}dx$$

$\Big\downarrow$ 4375

$$-\frac{2c^2 \int \frac{\tan^4(e+fx)}{(\sec(e+fx)a+a)^2\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$\Big\downarrow$ 372

$$-\frac{2c^2\left(\frac{\int \frac{2\left(1-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\Big\downarrow$ 27

$$-\frac{2c^2\left(\frac{\int \frac{1-\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{f}$$

$\Big\downarrow$ 402

$$-\frac{2c^2\left(\frac{\frac{\int \frac{a\left(5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e}{a\sec(e+}\right.}\right.}{f}$$

$\Big\downarrow$ 27

$$-\frac{2c^2\left(\frac{\frac{1}{4}\int \frac{5-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(\ldots}\right.\ldots}\right.}{f}$$

$\downarrow$ 397

$$-\frac{2c^2\left(\frac{\frac{1}{4}\left(8\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-11\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\ldots\right.}{f}$$

$\downarrow$ 216

$$-\frac{2c^2\left(\frac{\frac{1}{4}\left(\frac{11\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{8\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)+\frac{3\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{2a^2}+\frac{\tan(e+fx)}{2a^2\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2\ldots}{a\sec(\ldots}\right.\ldots}\right.}{f}$$

input
```
Int[(c - c*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*c^2*(Tan[e + f*x]/(2*a^2*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]
^2)/(a + a*Sec[e + f*x]))^2) + (((-8*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
+ a*Sec[e + f*x]]])/Sqrt[a] + (11*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))/4 + (3*Tan[e + f*x])/(4*Sqrt
[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/(2*a^
2)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x]])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.09

method	result
default	$\frac{c^2\sqrt{-a(-1-\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-\frac{4\sqrt{2}(\cos(fx+e)-1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cot(fx+e)}{\cos(fx+e)+1}+8\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(f}{\sqrt{(1-\cos(fx+e))^2\csc(fx}}\right.\right.}{8fa^3}$
parts	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(1-\cos(fx+e))^3\csc(fx+e)^3+32\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(f}{\sqrt{(1-\cos(fx+e))^2\csc(fx}}\right.\right.}{32fa^3}$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*c^2/f/a^3*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*(-4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*x
+e)+1)*cot(f*x+e)+8*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*(-cot(f*x+e)+csc(f*x+e))-11*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f
*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.18

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/16*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(
f*x + e) + c^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f
*x + e) + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 16*(c^2*cos(f*x + e)
^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(7*c^2*cos
(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos
(f*x + e) + a^3*f), 1/8*(11*sqrt(2)*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x +
e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 16*(c^2*cos(f
*x + e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arcta
n(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e))) - 2*(7*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e))*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
 e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^2}{(a + a\sec(e + fx))^{5/2}} \, dx = c^2 \left(\int \left(-\frac{2\sec(e + fx)}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\,\mathrm{se}} \right. \right.$$
$$+ \int \frac{\sec^2(e + fx)}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) + a}}\,d$$
$$+ \int \frac{1}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) + a}}\,d$$

input
```
integrate((c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
c**2*(Integral(-2*sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)
**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f
*x) + a)), x) + Integral(sec(e + f*x)**2/(a**2*sqrt(a*sec(e + f*x) + a)*se
c(e + f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a
*sec(e + f*x) + a)), x) + Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e
+ f*x)**2 + 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec
(e + f*x) + a)), x))
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(c\sec\left(fx+e\right)-c)^2}{(a\sec\left(fx+e\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sec(f*x + e) - c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(f}\right.}{a^3}$$

input
```
int((c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*c**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x
)**2 + 3*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**
2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) - 2*int((
sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2
+ 3*sec(e + f*x) + 1),x)))/a**3
```

3.83 $\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f} - \frac{23c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{8\sqrt{2}a^{5/2}f} - \frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} - \frac{7c\tan(e+fx)}{8af(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-23/16*c*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(5/2
)/f-1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-7/8*c*tan(f*x+e)/a/f/(a+a*se
c(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(-64c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{c}}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)+46\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\cos^4}{8f(a(1+\sec(e+fx)))^{5/2}\sqrt{c-c}}$$

input
```
Integrate[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/8*((-64*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]/Sqrt[c]]*Cos[(e + f*x)
/2]^4*Sec[e + f*x]^2 + 46*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c*Sec[e + f*x]]
/(Sqrt[2]*Sqrt[c])]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^2 + c*(11 + 7*Sec[e +
f*x])*Sqrt[c - c*Sec[e + f*x]])*Tan[e + f*x])/(f*(a*(1 + Sec[e + f*x]))^(5
/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 4392, 3042, 4375, 373, 402, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-c\sec(e+fx)}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4392}$$

$$-ac\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)^{7/2}}dx$$

$$\Big\downarrow \ 3042$$

$$-ac\int \frac{\cot\left(e+fx+\frac{\pi}{2}\right)^2}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{7/2}}dx$$

$$\Big\downarrow \ 4375$$

$$\frac{2c\int \frac{\tan^2(e+fx)}{(\sec(e+fx)a+a)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{af}$$

$$\Big\downarrow \ 373$$

$$\frac{2c\left(-\frac{\int \frac{1-\frac{3a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{af}$$

$$\Big\downarrow \ 402$$

$$\frac{2c\left(-\frac{\frac{\int \frac{a\left(9-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+\cdots}{a\sec(e+fx)\cdots}\cdots\right.}\cdots\right.}{af}$$

$$\Big\downarrow \ 27$$

$$\frac{2c\left(-\frac{\frac{1}{4}\int \frac{9-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(\cdots}{a\sec(e+\cdots}\cdots\right.}\cdots\right.}{af}$$

$$\Big\downarrow \ 397$$

$$2c\left(-\frac{\frac{1}{4}\left(16\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-23\int \frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\right.$$
$$\overline{\qquad\qquad af\qquad\qquad}$$

$$\downarrow \text{216}$$

$$2c\left(-\frac{\frac{1}{4}\left(\frac{23\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{16\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)+\frac{7\tan(e+fx)}{4\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}}{4a}-\frac{\tan(e+fx)}{4a\sqrt{a\sec(e+fx)+a}\left(\frac{a\tan^2(\ldots}{a\sec(e+\ldots}\right.}\right.$$
$$\overline{\qquad\qquad af\qquad\qquad}$$

input
```
Int[(c - c*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*c*(-1/4*Tan[e + f*x]/(a*Sqrt[a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2
)/(a + a*Sec[e + f*x]))^2) - (((-16*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
 a*Sec[e + f*x]]])/Sqrt[a] + (23*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))/4 + (7*Tan[e + f*x])/(4*Sqrt[
a + a*Sec[e + f*x]]*(2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/(4*a))
)/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 1.85 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.49

method	result
default	$\frac{c\sqrt{-a(-1-\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-\frac{4\sqrt{2}\,(\cos(fx+e)-1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cot(fx+e)}{\cos(fx+e)+1}+16\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(f}{\sqrt{(1-\cos(fx+e))^2\csc(fx}}\right.\right.}{16f\,a^3}$
parts	$\frac{c\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(1-\cos(fx+e))^3\csc(fx+e)^3+32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx}{\sqrt{(1-\cos(fx+e))^2\csc(fx+}}\right.\right.}{32f\,a^3}$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*c/f/a^3*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(-4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*x+
e)+1)*cot(f*x+e)+16*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+7*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*(-cot(f*x+e)+csc(f*x+e))-23*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f
*x+e)+csc(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(123) = 246$.

Time = 0.39 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.09

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/32*(23*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x +
e) + c)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e)
+ a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 32*(c*cos(f*x + e)^3 + 3*c*c
os(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2
*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 4*(11*c*cos(f*x + e)^2 + 7*c
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*
f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f),
 1/16*(23*sqrt(2)*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e
) + c)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(sqrt(a)*sin(f*x + e))) - 32*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)
^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(11*c*cos(f*x + e)^2 + 7*
c*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3
*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
]
```

Sympy [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx =$$

$$-c\left(\int \frac{\sec(e + fx)}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) +}\right.$$

$$+\int\left(-\frac{1}{a^2\sqrt{a\sec(e + fx) + a}\sec^2(e + fx) + 2a^2\sqrt{a\sec(e + fx) + a}\sec(e + fx) + a^2\sqrt{a\sec(e + fx) +}}\right.$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
-c*(Integral(sec(e + f*x)/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 +
 2*a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) +
 a)), x) + Integral(-1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**2 + 2*
a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x) + a**2*sqrt(a*sec(e + f*x) + a)
), x))
```

Maxima [F]

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\, dx = \int -\frac{c\sec(fx + e) - c}{(a\sec(fx + e) + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-integrate((c*sec(f*x + e) - c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c - c\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\, dx = \int \frac{c - \frac{c}{\cos(e+f\,x)}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\, dx$$

input
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{c-c\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\,c\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx - \left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(f}\right.\right.}{a^3}$$

input
```
int((c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*c*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x)**
2 + 3*sec(e + f*x) + 1),x) - int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.84 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 238

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}cf} - \frac{71\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{32\sqrt{2}a^{5/2}cf} - \frac{7\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{32a^3cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{4a^3cf\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)^2} + \frac{13\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{16a^3cf\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c/f-71/64*arct
an(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(5/2)/
c/f-7/32*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c/f+1/4*cot(f*x+e)*(a+a*sec
(f*x+e))^(1/2)/a^3/c/f/(2+tan(f*x+e)^2/(1+sec(f*x+e)))^2+13/16*cot(f*x+e)*
(a+a*sec(f*x+e))^(1/2)/a^3/c/f/(2+tan(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx =$$
$$-\frac{\left(71\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^2-2\big(17+13\sec(e+fx)+32}{32cf(-1+\sec(e+fx))(a(1+\sec(}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

output
```
-1/32*((71*Hypergeometric2F1[-1/2, 1, 1/2, (1 - Sec[e + f*x])/2]*(1 + Sec[
e + f*x])^2 - 2*(17 + 13*Sec[e + f*x] + 32*Hypergeometric2F1[-1/2, 1, 1/2,
 1 - Sec[e + f*x]]*(1 + Sec[e + f*x])^2))*Tan[e + f*x])/(c*f*(-1 + Sec[e +
 f*x])*(a*(1 + Sec[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4392, 3042, 4375, 374, 27, 441, 25, 27, 445, 25, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4392}$$

$$-\frac{\int \frac{\cot^2(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx}{ac}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{1}{\cot\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{ac}$$

$$\Big\downarrow \; 4375$$

$$\frac{2\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^3cf}$$

$$\Big\downarrow \; 374$$

$$\frac{2\left(\frac{\int \frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(3-\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{8a}+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^3cf}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{8}\int \frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(3-\frac{5a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^3cf}$$

$$\Big\downarrow \; 441$$

$$\frac{2\left(\frac{1}{8}\left(\frac{\int -\frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{39a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{13\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}\right.}\right.}{a^3cf}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{8}\left(\frac{13\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{39a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)+\frac{\cot(e+fx)\sqrt{a\sec(e+}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+\right.}\right.}{a^3cf}$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{1}{8}\left(\frac{13\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int\frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(\frac{39a\tan^2(e+fx)}{\sec(e+fx)a+a}+7\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\frac{\cot(e+fx)\sqrt{}}{8\left(\frac{a\tan^2(}{a\sec(e+}\right.}\right.$$
$$\overline{\qquad a^3cf \qquad}$$

$\downarrow$ 445

$$2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{2}\int-\frac{a\left(57-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13\cot}{4}\right.\right.$$
$$\overline{\qquad a^3cf \qquad}$$

$\downarrow$ 25

$$2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}\int\frac{a\left(57-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13\cot}{4}\right.\right.$$
$$\overline{\qquad a^3cf \qquad}$$

$\downarrow$ 27

$$2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\int\frac{57-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13\cot}{}\right.\right.$$
$$\overline{\qquad a^3cf \qquad}$$

$\downarrow$ 397

$$2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(64\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-71\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)-\frac{7}{2}\cot(e+fx\right.\right.\right.$$
$$\overline{\qquad a^3cf \qquad}$$

$\downarrow$ 216

$$2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{71\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{64\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right)-\frac{7}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}\right)+\frac{13}{}\right.\right.$$
$$\overline{\qquad a^3cf \qquad}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])),x]
```

output
```
(2*((Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(8*(2 + (a*Tan[e + f*x]^2)/(a
+ a*Sec[e + f*x]))^2) + ((-1/2*(a*((-64*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt
[a + a*Sec[e + f*x]]])/Sqrt[a] + (71*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2
]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))) - (7*Cot[e + f*x]*Sqrt[a
 + a*Sec[e + f*x]])/2)/4 + (13*Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(4*(
2 + (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/8))/(a^3*c*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(209) = 418$.

Time = 4.76 (sec) , antiderivative size = 737, normalized size of antiderivative = 3.10

method	result	size
default	Expression too large to display	737

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/8/c*2^(1/2)*(-1/16/f/(cos(1/2*f*x+1/2*e)+1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1
)*cos(1/2*f*x+1/2*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2
*e)+1)^2)^(1/2)/a^2*(15*cos(1/2*f*x+1/2*e)*arctan((2*sin(1/2*f*x+1/2*e)-1)
/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)
^2)^(1/2))+(-6*cos(1/2*f*x+1/2*e)^5-6*cos(1/2*f*x+1/2*e)^4+26*cos(1/2*f*x+
1/2*e)^3+26*cos(1/2*f*x+1/2*e)^2-4*cos(1/2*f*x+1/2*e)-4)*((2*cos(1/2*f*x+1
/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*
x+1/2*e)+15*cos(1/2*f*x+1/2*e)*arctan((2*sin(1/2*f*x+1/2*e)+1)/(cos(1/2*f*
x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)))-
1/2/f/(cos(1/2*f*x+1/2*e)+1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2
*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)/a
^2*(7*cos(1/2*f*x+1/2*e)*arctan((2*sin(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*
e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))+7*cos(1
/2*f*x+1/2*e)*arctan((2*sin(1/2*f*x+1/2*e)+1)/(cos(1/2*f*x+1/2*e)+1)/((2*c
os(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))+16*2^(1/2)*cos(1/2
*f*x+1/2*e)*arctan(2^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)
+1)^2)^(1/2)*(-csc(1/2*f*x+1/2*e)+cot(1/2*f*x+1/2*e)))+(-6*cos(1/2*f*x+1/2
*e)^3-6*cos(1/2*f*x+1/2*e)^2+2*cos(1/2*f*x+1/2*e)+2)*((2*cos(1/2*f*x+1/2*e
)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*
e)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.55

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[-1/128*(71*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-a)*log(-(2
*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(cos(f*x + e)^2 + 2
*cos(f*x + e) + 1))*sin(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2*cos(f*x + e)^2 - cos(f*x + e))*
sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 7*a*cos(f*
x + e) + a)/(cos(f*x + e) + 1))*sin(f*x + e) - 4*(27*cos(f*x + e)^3 + 12*c
os(f*x + e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/(
(a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)),
 1/64*(71*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x
 + e)))*sin(f*x + e) + 64*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(a)*ar
ctan(2*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*
x + e)/(2*a*cos(f*x + e)^2 + a*cos(f*x + e) - a))*sin(f*x + e) + 2*(27*cos
(f*x + e)^3 + 12*cos(f*x + e)^2 - 7*cos(f*x + e))*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c
*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{1}{a^2\sqrt{a\sec{(e+fx)}+a}\sec^3{(e+fx)}+a^2\sqrt{a\sec{(e+fx)}+a}\sec^2{(e+fx)}-a^2\sqrt{a\sec{(e+fx)}+a}\sec{(e+fx)}-a^2\sqrt{a\sec{(e+fx)}+a}}\,dx}{c}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e)),x)
```

output
```
-Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**3 + a**2*sqrt(a*s
ec(e + f*x) + a)*sec(e + f*x)**2 - a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f
*x) - a**2*sqrt(a*sec(e + f*x) + a)), x)/c
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \int -\frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4+2\sec(fx+e)^3-2\sec(fx+e)-1}dx\right)}{a^3c}$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x)
```

output
```
( - sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4 + 2*sec(e + f*x)**
3 - 2*sec(e + f*x) - 1),x))/(a**3*c)
```

3.85 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 277

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}c^2f} - \frac{107\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{64\sqrt{2}a^{5/2}c^2f} + \frac{21\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{64a^3c^2f} + \frac{43\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{96a^4c^2f} - \frac{\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{4a^4c^2f\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)^2} - \frac{15\cot^3(e+fx)(a+a\sec(e+fx))^{3/2}}{16a^4c^2f\left(2+\frac{\tan^2(e+fx)}{1+\sec(e+fx)}\right)}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/c^2/f-107/128*
arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^(5
/2)/c^2/f+21/64*cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a^3/c^2/f+43/96*cot(f*x+
e)^3*(a+a*sec(f*x+e))^(3/2)/a^4/c^2/f-1/4*cot(f*x+e)^3*(a+a*sec(f*x+e))^(3
/2)/a^4/c^2/f/(2+tan(f*x+e)^2/(1+sec(f*x+e)))^2-15/16*cot(f*x+e)^3*(a+a*se
c(f*x+e))^(3/2)/a^4/c^2/f/(2+tan(f*x+e)^2/(1+sec(f*x+e)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.40

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\cot^3(e+fx)\left(107\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1-\right.\right.}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

output
```
(Cot[e + f*x]^3*(107*Hypergeometric2F1[-3/2, 1, -1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^2 - 2*(57 + 45*Sec[e + f*x] + 32*Hypergeometric2F1[-3
/2, 1, -1/2, 1 - Sec[e + f*x]]*(1 + Sec[e + f*x])^2)))/(96*a^2*c^2*f*Sqrt[
a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {3042, 4392, 3042, 4375, 374, 27, 441, 25, 27, 445, 27, 445, 27, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4392}$$

$$\frac{\int \frac{\cot^4(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\cot\left(e+fx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a^2c^2}$$

$\Big\downarrow$ 4375

$$-\frac{2\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^3}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{a^4c^2f}$$

$\Big\downarrow$ 374

$$-\frac{2\left(\frac{\int \frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(1-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{8a}+\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^4c^2f}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{1}{8}\int \frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(1-\frac{7a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)^2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)+\frac{\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{8\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)^2}\right)}{a^4c^2f}$$

$\Big\downarrow$ 441

$$-\frac{2\left(\frac{1}{8}\left(\frac{\int -\frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{75a\tan^2(e+fx)}{\sec(e+fx)a+a}+43\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}+\frac{15\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}\right)+\frac{\cot^3(e+fx)}{8\left(\frac{a\,\text{ta}}{a\,\text{sec}}\right.}\right.}{a^4c^2f}$$

$\Big\downarrow$ 25

$$-\frac{2\left(\frac{1}{8}\left(\frac{15\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{\int \frac{a\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{75a\tan^2(e+fx)}{\sec(e+fx)a+a}+43\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{4a}\right)+\frac{\cot^3(e+fx)(}{8\left(\frac{a\tan^3}{a\sec(}\right.}\right.}{a^4c^2f}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{1}{8}\left(\frac{15\cot^3(e+fx)(a\sec(e+fx)+a)^{3/2}}{4\left(\frac{a\tan^2(e+fx)}{a\sec(e+fx)+a}+2\right)}-\frac{1}{4}\int\frac{\cot^4(e+fx)(\sec(e+fx)a+a)^2\left(\frac{75a\tan^2(e+fx)}{\sec(e+fx)a+a}+43\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)+\cot^3\cdots}{a^4c^2f}$$

$\downarrow$ 445

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{6}\int-\frac{3a\cot^2(e+fx)(\sec(e+fx)a+a)\left(21-\frac{43a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{43}{6}\cot^3(e+fx)(a\sec(e+fx)\cdots}{a^4c^2f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\int\frac{\cot^2(e+fx)(\sec(e+fx)a+a)\left(21-\frac{43a\tan^2(e+fx)}{\sec(e+fx)a+a}\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-\frac{43}{6}\cot^3(e+fx)(a\sec(e+fx)\cdots}{a^4c^2f}$$

$\downarrow$ 445

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}\int\frac{a\left(\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+149\right)}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)\cdots}{a^4c^2f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\int\frac{\frac{21a\tan^2(e+fx)}{\sec(e+fx)a+a}+149}{\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1\right)\left(\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+2\right)}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)\right)\cdots}{a^4c^2f}$$

$\downarrow$ 397

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(128\int\frac{1}{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)-107\int\frac{\cdots}{\frac{a\tan^2\cdots}{\sec(e+\cdots}}\cdots}{\cdots}$$

$\downarrow$ 216

$$-\frac{2\left(\frac{1}{8}\left(\frac{1}{4}\left(-\frac{1}{2}a\left(\frac{21}{2}\cot(e+fx)\sqrt{a\sec(e+fx)+a}-\frac{1}{2}a\left(\frac{107\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}}-\frac{128\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}}\right.\right.\right.\right.\right.}{a^4c}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^2),x]
```

output
```
(-2*((Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(8*(2 + (a*Tan[e + f*x]^2
)/(a + a*Sec[e + f*x]))^2) + (((-43*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3
/2))/6 - (a*(-1/2*(a*((-128*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e
 + f*x]]])/Sqrt[a] + (107*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]))) + (21*Cot[e + f*x]*Sqrt[a + a*Sec[e
 + f*x]])/2))/2)/4 + (15*Cot[e + f*x]^3*(a + a*Sec[e + f*x])^(3/2))/(4*(2
+ (a*Tan[e + f*x]^2)/(a + a*Sec[e + f*x]))))/8))/(a^4*c^2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

Maple [A] (warning: unable to verify)

Time = 1.31 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.53

method	result
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left(\left(-1536\cos(fx+e)^3-4608\cos(fx+e)^2-4608\cos(fx+e)-1536\right)\sqrt{2}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{\ }}{\sqrt{\cot(fx+e)^2-}}\right.\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/1536/c^2/f/a^3*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)/(cos(f*x+e)^2+2*c
os(f*x+e)+1)*((-1536*cos(f*x+e)^3-4608*cos(f*x+e)^2-4608*cos(f*x+e)-1536)*
2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-c
sc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))+(-
1284*cos(f*x+e)^3-3852*cos(f*x+e)^2-3852*cos(f*x+e)-1284)*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+cs
c(f*x+e))+2^(1/2)*(485*cos(f*x+e)^5+3037*cos(f*x+e)^4+3542*cos(f*x+e)^3-85
8*cos(f*x+e)^2-3003*cos(f*x+e)-1155)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)*csc(f*x+e)^2+(-670*cos(f*x+e
)^5+1256*cos(f*x+e)^4+396*cos(f*x+e)^3-1376*cos(f*x+e)^2-110*cos(f*x+e)+50
4)*cot(f*x+e)*csc(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.55

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
[-1/768*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*
sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)/(co
s(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 384*(cos(f*x + e)^3 + c
os(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a)*log(-(8*a*cos(f*x + e)^3 + 4*(2
*cos(f*x + e)^2 - cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sin(f*x + e) - 7*a*cos(f*x + e) + a)/(cos(f*x + e) + 1))*sin(f*x +
e) - 4*(205*cos(f*x + e)^4 + 71*cos(f*x + e)^3 - 149*cos(f*x + e)^2 - 63*c
os(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x
+ e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*si
n(f*x + e)), 1/384*(321*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x
 + e) - 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 384*(cos(f*x + e)^3 +
cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(a)*arctan(2*sqrt(a)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(2*a*cos(f*x + e)^2 +
a*cos(f*x + e) - a))*sin(f*x + e) + 2*(205*cos(f*x + e)^4 + 71*cos(f*x + e
)^3 - 149*cos(f*x + e)^2 - 63*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2
*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{1}{a^2\sqrt{a\sec(e+fx)+a}\sec^4(e+fx)-2a^2\sqrt{a\sec(e+fx)+a}\sec^2(e+fx)+a^2}}{c^2}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**2,x)
```

output
```
Integral(1/(a**2*sqrt(a*sec(e + f*x) + a)*sec(e + f*x)**4 - 2*a**2*sqrt(a*
sec(e + f*x) + a)*sec(e + f*x)**2 + a**2*sqrt(a*sec(e + f*x) + a)), x)/c**
2
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(c\sec(fx+e)-c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(5/2)*(c*sec(f*x + e) - c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^2}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^5+\sec(fx+e)^4-2\sec(fx+e)^3-2\sec(fx+e)^2+\sec(fx+e}}{a^3c^2}$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2
*sec(e + f*x)**3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1),x))/(a**3*c**2)
```

3.86 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 185

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\frac{ac^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{ac^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}-\frac{ac(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
a*c^4*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-a*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-1/2
*a*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-1/3*a*c*
(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.45

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\frac{ac^4(6\log(\cos(e+fx))+18\sec(e+fx)-9\sec^2(e+fx)+2\sec^3(e+fx))\tan(e+fx)}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(a*c^4*(6*Log[Cos[e + f*x]] + 18*Sec[e + f*x] - 9*Sec[e + f*x]^2 + 2*Sec[e
 + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
+ f*x]])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 4394, 3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4394}$$

$$c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{5/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4394

$$c\left(c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4394

$$c\left(c\left(c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-}{2f\sqrt{a\sec(e}}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$c\left(c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan}{}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 4393

$$c\left(c\left(\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(}{2f\sqrt{a\sec(e+fx)}}\right.$$
$$\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 25

$$c\left(c\left(-\frac{ac^2\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$c\left(c\left(-\frac{ac^2\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3956

$$c\left(c\left(\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/3*(a*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f
*x]]) + c*(-1/2*(a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[a +
a*Sec[e + f*x]]) + c*((a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a
*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*T
an[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.83

method	result
risch	$\frac{c^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(-18ie^{i(fx+e)}-3\,e^{6i(fx+e)}fx-3i\ln\left(e^{2i(fx+e)}+1\right)-6\,e^{6i(fx+e)}e-9\,e^{4i(fx+e)}fx-9ie^{4i(fx\cdots\right.}{\cdots}$
default	$-\frac{\left(\left(24\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6-36\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+18\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-48\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6+72\cos\left(\cdots\right.\right.}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*c^3*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(c*(exp(I*(f*x
+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(-18*I*exp(I*(f*x+e))-3*exp(6*I*(f*x
+e))*f*x-3*I*ln(exp(2*I*(f*x+e))+1)-6*exp(6*I*(f*x+e))*e-9*exp(4*I*(f*x+e)
)*f*x-9*I*exp(4*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)-18*exp(4*I*(f*x+e))*e-9*
exp(2*I*(f*x+e))*f*x+18*I*exp(2*I*(f*x+e))+18*I*exp(4*I*(f*x+e))-18*I*exp(
5*I*(f*x+e))-3*I*exp(6*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)-9*I*exp(2*I*(f*x+
e))*ln(exp(2*I*(f*x+e))+1)-44*I*exp(3*I*(f*x+e))-18*exp(2*I*(f*x+e))*e-3*f
*x-6*e)/(exp(I*(f*x+e))+1)/(exp(I*(f*x+e))-1)/(exp(2*I*(f*x+e))+1)^2/f
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.55

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx = \left[-\frac{\left(11\,c^3\cos\left(fx+e\right)^2-7\,c^3\cos\left(fx+e\right)+2\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)-}{6}\right.$$

$$-\frac{\left(11\,c^3\cos\left(fx+e\right)^2-7\,c^3\cos\left(fx+e\right)+2\,c^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)+6\left(c^3\cos\left(fx+\right.\right.}{6\left(f\cos\left(fx+e\right)^3+f\cos\left(f\right.\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
[-1/6*((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
+ e) - 3*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a*c)*log(1/2*(a*c
*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x
 + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2), -1/6*
((11*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) + 2*c^3)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) +
 6*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(a*c)*arctan((cos(f*x + e
)^3 + cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a*c)*sin(f*x +
e))))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(165) = 330$.

Time = 0.25 (sec) , antiderivative size = 1289, normalized size of antiderivative = 6.97

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*c^3*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*c^3*cos(4*f*x + 4*
e)^2 + 27*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*c^3*sin(6*f*x + 6
*e)^2 + 27*(f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*c^3*sin(2*f*x +
 2*e)^2 + 18*(f*x + e)*c^3*cos(2*f*x + 2*e) + 3*(f*x + e)*c^3 + 18*c^3*sin
(2*f*x + 2*e) - 3*(c^3*cos(6*f*x + 6*e)^2 + 9*c^3*cos(4*f*x + 4*e)^2 + 9*c
^3*cos(2*f*x + 2*e)^2 + c^3*sin(6*f*x + 6*e)^2 + 9*c^3*sin(4*f*x + 4*e)^2
+ 18*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*c^3*sin(2*f*x + 2*e)^2 + 6*
c^3*cos(2*f*x + 2*e) + c^3 + 2*(3*c^3*cos(4*f*x + 4*e) + 3*c^3*cos(2*f*x +
 2*e) + c^3)*cos(6*f*x + 6*e) + 6*(3*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*f*x
 + 4*e) + 6*(c^3*sin(4*f*x + 4*e) + c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e)
)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*c^3*cos
(4*f*x + 4*e) + 3*(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 + 3*c^3*s
in(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x +
e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3)*cos(4*f*x + 4*e) + 18*(c^3*sin(6*
f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 44*(c^3*sin(6*f*x + 6*e) + 3*c^
3*sin(4*f*x + 4*e) + 3*c^3*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 18*(c^3*sin(6*f*x + 6*e) + 3*c^3*sin(4*f*x + 4*e
) + 3*c^3*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 18*((f*x + e)*c^3*sin(4*f*x + 4*e) + (f*x + e)*c^3*sin(2*f*x +...
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(7/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx&=\sqrt{c}\,\sqrt{a}\,c^3\left(-\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)\right.\\&+3\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)\\&-3\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)\\&\left.+\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}dx\right)\end{aligned}$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(7/2),x)
```

output
```
sqrt(c)*sqrt(a)*c**3*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
 1)*sec(e + f*x)**3,x) + 3*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**2,x) - 3*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*
x) + 1)*sec(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
+ 1),x))
```

3.87 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 139

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \frac{ac^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{ac^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{ac(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

output
```
a*c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-a*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-1/2
*a*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{ac^3(-2\log(\cos(e+fx))-4\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*c^3*(-2*Log[Cos[e + f*x]] - 4*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
 + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4394}$$

$$c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4394}$$

$$c\left(c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4393}$$

$$c\left(\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$c\left(-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\left(-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3956}$$

$$c\left(\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f
*x]]) + c*((a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x]
)/(f*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{c^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(i e^{4i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+e^{4i(fx+e)}fx+2i e^{2i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+2\,e^{4i(fx+e)}e+\right.}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\left(e^{2i(fx+e)}\right.}$
default	$\frac{\left(\left(8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+2\right)\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\right.\right.}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-c^2*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(c*(exp(I*(f*x+e)
)-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(I*ln(exp(2*I*(f*x+e))+1)*exp(4*I*(f*x+
e))+exp(4*I*(f*x+e))*f*x+2*I*ln(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))+2*exp
(4*I*(f*x+e))*e+2*exp(2*I*(f*x+e))*f*x+4*I*exp(3*I*(f*x+e))+I*ln(exp(2*I*(
f*x+e))+1)-2*I*exp(2*I*(f*x+e))+4*I*exp(I*(f*x+e))+4*exp(2*I*(f*x+e))*e+f*
x+2*e)/(exp(I*(f*x+e))+1)/(exp(I*(f*x+e))-1)/(exp(2*I*(f*x+e))+1)/f
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.14

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \left[-\frac{\left(3\,c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)-\left(c^2\cos\left(fx+e\right)^2+c^2\right.}{2\left(f\cos\left(fx+\right.\right.}\right.$$

$$-\frac{\left(3\,c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)+2\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\sqrt{}}{2\left(f\cos\left(fx+e\right)^2+f\cos\left(fx+e\right)\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
[-1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) - (c^2*cos(f*x + e)^2
 + c^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e
)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/
(f*cos(f*x + e)^2 + f*cos(f*x + e)), -1/2*((3*c^2*cos(f*x + e) - c^2)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)
)*sin(f*x + e) + 2*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(a*c)*arcta
n((cos(f*x + e)^3 + cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a
*c)*sin(f*x + e))))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(125) = 250$.

Time = 0.20 (sec) , antiderivative size = 710, normalized size of antiderivative = 5.11

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*cos(2*f*x + 2*e)^2 +
(f*x + e)*c^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^2*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*c^2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + 2*c^2*sin(2*f*x + 2*e) -
(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(4*f*x + 4*e)^
2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4
*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x +
 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*c^
2*cos(2*f*x + 2*e) + (f*x + e)*c^2 + c^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e
) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f
*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*
x + e)*c^2*sin(2*f*x + 2*e) - c^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) - 4*(
c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*
x + 2*e) + c^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt
(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*
e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(
2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\sqrt{c}\,\sqrt{a}\,c^2\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx-2\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)+\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*c**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)
*sec(e + f*x)**2,x) - 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1),
x))
```

3.88 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
a*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-a*c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{ac^2(\log(\cos(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a*c^2*(Log[Cos[e + f*x]] + Sec[e + f*x])*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec
[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4394}$$

$$c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ac^2\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a
 + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(85) = 170$.

Time = 3.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.57

method	result
default	$\frac{\left(\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-2\right)\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right.}{f}$
risch	$\frac{c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}-\frac{2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}-\frac{2ic\sqrt{}}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((4*cos(1/2*f*x+1/2*e)^2-2)*ln(2/(cos(1/2*f*x+1/2*e)+1))+(-2*cos(1/2*f
*x+1/2*e)^2+1)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+
1/2*e)+1))+(-2*cos(1/2*f*x+1/2*e)^2+1)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f
*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))-2*sin(1/2*f*x+1/2*e)^2)*(a/(2*cos(1/2*f
*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*
sin(1/2*f*x+1/2*e)^2)^(1/2)*c*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(85) = 170$.

Time = 0.15 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.88

$$\int\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\left[-\frac{2\,c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)-\sqrt{-ac}(c\cos\left(fx+e\right)+c)\log\left(\frac{ac\cos(fx+e)^{4}-}{}\right.}{2\left(f\cos\left(fx+e\right)+f\right)}\right.$$

$$-\frac{c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(fx+e\right)+\sqrt{ac}(c\cos\left(fx+e\right)+c)\arctan\left(\frac{\left(\cos(fx+e)^{3}+\cos(fx+e)\right)\sqrt{ac}\sqrt{\frac{a\cos}{c}}}{\left(ac\cos(fx+e)^{2}-ac\right)}\right.}{f\cos\left(fx+e\right)+f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[-1/2*(2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e))*sin(f*x + e) - sqrt(-a*c)*(c*cos(f*x + e) + c)*log(1/2*(a
*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), -(c*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x +
e) + sqrt(a*c)*(c*cos(f*x + e) + c)*arctan((cos(f*x + e)^3 + cos(f*x + e))
*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e))))/(f*cos(f*x +
e) + f)]
```

Sympy [F]

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}\,dx = \int \sqrt{a\left(\sec\left(e + fx\right) + 1\right)}\left(-c\left(\sec\left(e + fx\right) - 1\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(85) = 170$.

Time = 0.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.61

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}\,dx =$$

$$-\frac{\left((fx + e)c\cos\left(2\,fx + 2\,e\right)^2 + (fx + e)c\sin\left(2\,fx + 2\,e\right)^2 + 2\,(fx + e)c\cos\left(2\,fx + 2\,e\right) + 2\,c\cos\left(\frac{1}{2}\,\mathrm{arcta}\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c*cos(2*f*x + 2*e)^2 + (f*x + e)*c*sin(2*f*x + 2*e)^2 + 2*(f*x
 + e)*c*cos(2*f*x + 2*e) + 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*c - (c*cos(2*f*x + 2*e)^2 + c*sin(2
*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1) - 2*(c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(e + fx)}(c - c\sec(e + fx))^{3/2}\,dx = \int \sqrt{a + \frac{a}{\cos(e + f\,x)}}\,\left(c - \frac{c}{\cos(e + f\,x)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)+\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*c*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)
*sec(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1),x)
)
```

3.89 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 48

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \frac{ac\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \frac{c\log(\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(c*Log[Cos[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[
c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4393}$$

$$\frac{ac\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{ac\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ac\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{ac\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.81

method	result
default	$-\frac{\left(\ln(-\cot(fx+e)+\csc(fx+e)-1)+\ln(-\cot(fx+e)+\csc(fx+e)+1)-\ln\left(\frac{2}{\cos(fx+e)+1}\right)\right)\sqrt{-c(-1+\sec(fx+e))}\,\sqrt{a(1+\sec(fx+e}}{f}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)x}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)} - \frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f} - \frac{i\sqrt{\frac{a(e}{e}}}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(ln(-cot(f*x+e)+csc(f*x+e)-1)+ln(-cot(f*x+e)+csc(f*x+e)+1)-ln(2/(cos(
f*x+e)+1)))*(-c*(-1+sec(f*x+e)))^(1/2)*(a*(1+sec(f*x+e)))^(1/2)*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(44) = 88.

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 4.44

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\left[\frac{\sqrt{-ac}\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e)+ac}{2\cos(fx+e)^2}\right)}{2f},\right.$$
$$\left.-\frac{\sqrt{ac}\arctan\left(\frac{\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{\left(ac\cos(fx+e)^2-ac\right)\sin(fx+e)}\right)}{f}\right]$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x +
e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)/f, -sqrt(a*c)*arc
tan((cos(f*x + e)^3 + cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 -
 a*c)*sin(f*x + e)))/f]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{(fx+e-\arctan(\sin(2\,fx+2\,e),\cos(2\,fx+2\,e)+1))\sqrt{a}\sqrt{c}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sqrt(a)*sqrt(
c)/f
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\sec(e + fx)}\sqrt{c - c\sec(e + fx)}\,dx$$
$$= \int \sqrt{a + \frac{a}{\cos(e + f\,x)}}\,\sqrt{c - \frac{c}{\cos(e + f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a + a\sec(e + fx)}\sqrt{c - c\sec(e + fx)}\,dx$$
$$= \sqrt{c}\,\sqrt{a}\left(\int \sqrt{\sec(fx + e) + 1}\,\sqrt{-\sec(fx + e) + 1}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1),x)
```

3.90 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 51

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a*ln(1-cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{(\log(\cos(e+fx))+\log(1-\sec(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
((Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4399}$$

$$\frac{ac\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(47) = 94$.

Time = 3.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.37

method	result
default	$-\frac{2\left(\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}-\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)\ln(e^{i(fx+e}}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(ln(2/(cos(1/2*f*x+1/2*e)+1))-ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e
)))*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1
/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*tan(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x +
e) - c), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}} - \frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
(2*sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1))/sqrt(c) - sqrt(-a)*log(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c))/f
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\frac{\sqrt{-ac}a\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c|a|}-\frac{\sqrt{-ac}a\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c|a|}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
-(sqrt(-a*c)*a*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c*abs(a)) - sqrt(-a*c)*
a*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c*abs(a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)-1}dx\right)}{c}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x) - 1),x))/c
```

3.91 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-a*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a*ln(1-cos(f
*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{1}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + (-1 + Sec[e + f*x])^(-1))*
Tan[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4395}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4399}$$

$$\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2))
) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\left(8\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-8\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-\sec\left(\frac{f}{2}\right.\right.}{4f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c}$
risch	$-\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+e^{2i(fx+e)}fx-4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2\,e^{2i(fx+e)}e-2\,e^{i(fx+e)}fx-2ie^{i(fx+\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*co
s(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(8*ln(2/(cos(1/2*f*x+1
/2*e)+1))*tan(1/2*f*x+1/2*e)-8*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*t
an(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e)-sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)
)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x +
 e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(88) = 176.

Time = 0.19 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left((fx+e)\cos\left(2\,fx+2\,e\right)^2+4\,(fx+e)\cos\left(fx+e\right)^2+(fx+e)\sin\left(2\,fx+2\,e\right)^2+4\,(fx+e)\sin\left(fx+\right.\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*si
n(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(2*cos(f*x + e)
 - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x + e)^2 - sin(2*f*x
 + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e)^2 + 4*cos(f*x
 + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 2*(f*x + e)*
cos(f*x + e) + e + sin(f*x + e))*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f*x +
e) - 2*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e + 2*
sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(f*x + e
)^2 + c^2*sin(2*f*x + 2*e)^2 - 4*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2
*sin(f*x + e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(f*x + e) - c^2)*
cos(2*f*x + 2*e))*f)
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-ac}a\log\left(2\left|a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right|\right)}{c^2|a|} - \frac{2\sqrt{2}\sqrt{-ac}a\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^2|a|} - \frac{\sqrt{2}\left(2\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)\sqrt{-ac}a+\sqrt{-a}\right.}{ac^2|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right.}{4f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
-1/4*sqrt(2)*(2*sqrt(2)*sqrt(-a*c)*a*log(2*abs(a*tan(1/2*f*x + 1/2*e)^2))/
(c^2*abs(a)) - 2*sqrt(2)*sqrt(-a*c)*a*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 -
a))/(c^2*abs(a)) - sqrt(2)*(2*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a
+ sqrt(-a*c)*a^2)/(a*c^2*abs(a)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^2-2\sec(fx+e)+1}dx\right)}{c^2}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x)**2 - 2*sec(e + f*x) + 1),x))/c**2
```

3.92 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 142

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-1/2*a*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)-a*tan(f*
x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a*ln(1-cos(f*x+e))*
tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\left(-2\log(\cos(e+fx))-2\log(1-\sec(e+fx))+\frac{3-2\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*(-2*Log[Cos[e + f*x]] - 2*Log[1 - Sec[e + f*x]] + (3 - 2*Sec[e + f
*x])/(-1 + Sec[e + f*x])^2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])
]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 4395

$$\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 4399

$$\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 16

$$\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/
2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\left(-128\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+128\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+29\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{64f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(-2i\ln\left(e^{i(fx+e)}-1\right)-e^{4i(fx+e)}fx+4ie^{i(fx+e)}-2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx-12ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+8\,e\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/f*(-128*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)
^4+128*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^4+29*cos(1/2*f*x+1/
2*e)^4+6*cos(1/2*f*x+1/2*e)^2-27)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*
x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/
2)/c^2*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^3
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x
+ e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(128) = 256$.

Time = 0.28 (sec) , antiderivative size = 1173, normalized size of antiderivative = 8.26

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c - c\sec(e + fx))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
 + e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f
*x + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2
*e)^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x + 2*(2*(4*cos(3*f*x + 3*e) - 6*c
os(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^
2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 16*cos(
3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 36*cos(2*f*x +
 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) +
 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x +
 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 16*sin(3*f*x + 3*e)^2 - 36*sin(
2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) - 16*sin(f*x + e)^2 + 8*
cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*(f*x - 4*(f*
x + e)*cos(3*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f
*x + e) + e + 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*co
s(4*f*x + 4*e) - 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) - 4*(f*x + e)*cos(f
*x + e) + e)*cos(3*f*x + 3*e) + 12*(f*x - 4*(f*x + e)*cos(f*x + e) + e)*co
s(2*f*x + 2*e) - 8*(f*x + e)*cos(f*x + e) - 2*(4*(f*x + e)*sin(3*f*x + 3*e
) - 6*(f*x + e)*sin(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x
+ 3*e) - 3*cos(2*f*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) - 4*(12*(f*
x + e)*sin(2*f*x + 2*e) - 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e...
```

Giac [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{8\sqrt{2}\sqrt{-ac}a\log\left(2\left|a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right|\right)}{c^3|a|}-\frac{8\sqrt{2}\sqrt{-ac}a\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^3|a|}-\frac{\sqrt{2}\left(12\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-ac}a+18\right.}{a^2c^3|a|\tan}\right.}{16f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-1/16*sqrt(2)*(8*sqrt(2)*sqrt(-a*c)*a*log(2*abs(a*tan(1/2*f*x + 1/2*e)^2))
/(c^3*abs(a)) - 8*sqrt(2)*sqrt(-a*c)*a*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 -
 a))/(c^3*abs(a)) - sqrt(2)*(12*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c
)*a + 18*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^2 + 7*sqrt(-a*c)*a^3)
/(a^2*c^3*abs(a)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)}{c^3}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x))/c**3
```

3.93 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = &-\frac{a\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\ &-\frac{a\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\ &-\frac{a\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\ &+\frac{a\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
-1/3*a*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2)-1/2*a*ta
n(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)-a*tan(f*x+e)/c^
2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a*ln(1-cos(f*x+e))*tan(f
*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a\left(-6\log(\cos(e+fx))-6\log(1-\sec(e+fx))+\frac{-11+15\sec(e+fx)-6\sec^2(e+fx)}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{6c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/6*(a*(-6*Log[Cos[e + f*x]] - 6*Log[1 - Sec[e + f*x]] + (-11 + 15*Sec[e
+ f*x] - 6*Sec[e + f*x]^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4395, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4395}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\downarrow$ 4395

$$\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\downarrow$ 4395

$$\frac{\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\downarrow$ 4399

$$\frac{\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow 16$$

$$\frac{\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/3*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/
2)) + (-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*S
ec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c)/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

method	result
default	$\frac{\left(-768\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^6+768\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^6+239\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6-141\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{384f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,c^3}$
risch	$\frac{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(9\,e^{5i(fx+e)}-27\,e^{4i(fx+e}\right.}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-\right.}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/f*(-768*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^6+768*ln(csc
(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^6+239*cos(1/2*f*x+1
/2*e)^6-141*cos(1/2*f*x+1/2*e)^4-291*cos(1/2*f*x+1/2*e)^2+209)*(a/(2*cos(1
/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^3*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^
5
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x +
 e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) +
 c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2444 vs. $2(168) = 336$.

Time = 1.59 (sec) , antiderivative size = 2444, normalized size of antiderivative = 13.00

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*cos(6*f*x + 6*e)^2 + 108*(f*x + e)*cos(5*f*x + 5*e)^2 +
675*(f*x + e)*cos(4*f*x + 4*e)^2 + 1200*(f*x + e)*cos(3*f*x + 3*e)^2 + 675
*(f*x + e)*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*cos(f*x + e)^2 + 3*(f*x + e)
*sin(6*f*x + 6*e)^2 + 108*(f*x + e)*sin(5*f*x + 5*e)^2 + 675*(f*x + e)*sin
(4*f*x + 4*e)^2 + 1200*(f*x + e)*sin(3*f*x + 3*e)^2 + 675*(f*x + e)*sin(2*
f*x + 2*e)^2 + 108*(f*x + e)*sin(f*x + e)^2 + 3*f*x + 6*(2*(6*cos(5*f*x +
5*e) - 15*cos(4*f*x + 4*e) + 20*cos(3*f*x + 3*e) - 15*cos(2*f*x + 2*e) + 6
*cos(f*x + e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 + 12*(15*cos(4*f*
x + 4*e) - 20*cos(3*f*x + 3*e) + 15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)
*cos(5*f*x + 5*e) - 36*cos(5*f*x + 5*e)^2 + 30*(20*cos(3*f*x + 3*e) - 15*c
os(2*f*x + 2*e) + 6*cos(f*x + e) - 1)*cos(4*f*x + 4*e) - 225*cos(4*f*x + 4
*e)^2 + 40*(15*cos(2*f*x + 2*e) - 6*cos(f*x + e) + 1)*cos(3*f*x + 3*e) - 4
00*cos(3*f*x + 3*e)^2 + 30*(6*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - 225*cos
(2*f*x + 2*e)^2 - 36*cos(f*x + e)^2 + 2*(6*sin(5*f*x + 5*e) - 15*sin(4*f*x
 + 4*e) + 20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x + e))*sin(
6*f*x + 6*e) - sin(6*f*x + 6*e)^2 + 12*(15*sin(4*f*x + 4*e) - 20*sin(3*f*x
 + 3*e) + 15*sin(2*f*x + 2*e) - 6*sin(f*x + e))*sin(5*f*x + 5*e) - 36*sin(
5*f*x + 5*e)^2 + 30*(20*sin(3*f*x + 3*e) - 15*sin(2*f*x + 2*e) + 6*sin(f*x
 + e))*sin(4*f*x + 4*e) - 225*sin(4*f*x + 4*e)^2 + 120*(5*sin(2*f*x + 2*e)
 - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 400*sin(3*f*x + 3*e)^2 - 225*sin(...
```

Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\sqrt{2}\left(\frac{24\sqrt{2}\sqrt{-ac}a\log\left(2\left|a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right|\right)}{c^4|a|} - \frac{24\sqrt{2}\sqrt{-ac}a\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^4|a|} - \frac{\sqrt{2}\left(44\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^3\sqrt{-ac}a+\right.}{}\right.}{48f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
-1/48*sqrt(2)*(24*sqrt(2)*sqrt(-a*c)*a*log(2*abs(a*tan(1/2*f*x + 1/2*e)^2)
)/(c^4*abs(a)) - 24*sqrt(2)*sqrt(-a*c)*a*log(abs(-a*tan(1/2*f*x + 1/2*e)^2
 - a))/(c^4*abs(a)) - sqrt(2)*(44*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a
*c)*a + 111*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^2 + 96*(a*tan(1/
2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^3 + 28*sqrt(-a*c)*a^4)/(a^3*c^4*abs(a)*
tan(1/2*f*x + 1/2*e)^6))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c - c/cos(e + f*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^4-4\sec(fx+e)^3+6\sec(fx+e)^2-4\sec(fx+e)+1}dx\right)}{c^4}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) +
1),x))/c**4
```

3.94 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 190

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{a^2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{a^2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
a^2*c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)-a^2*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
-1/2*a^2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/3*
a^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a^2c^3(2+6\log(\cos(e+fx))+6\sec(e+fx)+3\sec^2(e+fx)-2\sec^3(e+fx))\tan}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^2*c^3*(2 + 6*Log[Cos[e + f*x]] + 6*Sec[e + f*x] + 3*Sec[e + f*x]^2 - 2*
Sec[e + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*S
ec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 4397, 3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4397$$

$$a\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{5/2}dx+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 3042$$

$$a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4394}$$

$$a\left(c\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{ac\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}\right)+\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4394}$$

$$a\left(c\left(c\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-}{2f\sqrt{a\sec(e}}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(c\left(c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan}{}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4393}$$

$$a\left(c\left(\frac{ac^2\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec(}{2f\sqrt{a\sec(e+fx}}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{25}$$

$$a\left(c\left(-\frac{ac^2\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec}{2f\sqrt{a\sec(e+f}}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$a\left(c\left(-\frac{ac^2\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec}{2f\sqrt{a\sec(e+f}}\right.$$
$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 3956

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}+$$
$$a\left(c\left(\frac{ac^2\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}\right)-\frac{ac\tan(e+fx)(c-c\sec}{2f\sqrt{a\sec(e+f}}\right.$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]
]) + a*(-1/2*(a*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[a + a*S
ec[e + f*x]]) + c*((a*c^2*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[c - c*Sec[e + f*x]]*Tan[
e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

rule 4397
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a   Int[Sqrt[a +
b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.78

method	result
risch	$-\frac{a\,c^{2}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(6i\mathrm{e}^{i(fx+e)}+3\,\mathrm{e}^{6i(fx+e)}fx+3i\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)+6\,\mathrm{e}^{6i(fx+e)}e+9\,\mathrm{e}^{4i(fx+e)}fx+9i\mathrm{e}^{4i(f\cdots\right.}{\cdots}$
default	$-\frac{\left(\left(24\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-36\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+18\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(24\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-36\cos\left(\cdots\right.\right.}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*c^2*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(c*(exp(I*(
f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(6*I*exp(I*(f*x+e))+3*exp(6*I*(f*
x+e))*f*x+3*I*ln(exp(2*I*(f*x+e))+1)+6*exp(6*I*(f*x+e))*e+9*exp(4*I*(f*x+e
))*f*x+9*I*exp(4*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)+18*exp(4*I*(f*x+e))*e+9
*exp(2*I*(f*x+e))*f*x+6*I*exp(2*I*(f*x+e))+6*I*exp(4*I*(f*x+e))+6*I*exp(5*
I*(f*x+e))+3*I*exp(6*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)+9*I*exp(2*I*(f*x+e)
)*ln(exp(2*I*(f*x+e))+1)+4*I*exp(3*I*(f*x+e))+18*exp(2*I*(f*x+e))*e+3*f*x+
6*e)/(exp(I*(f*x+e))+1)/(exp(2*I*(f*x+e))+1)^2/(exp(I*(f*x+e))-1)/f
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.52

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \left[-\frac{\left(7\,ac^2\cos\left(fx+e\right)^2+ac^2\cos\left(fx+e\right)-2\,ac^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin\left(\right.}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
[-1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f
*x + e) - 3*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a*c)*log(1
/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*
sin(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)
, -1/6*((7*a*c^2*cos(f*x + e)^2 + a*c^2*cos(f*x + e) - 2*a*c^2)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(
f*x + e) + 6*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(a*c)*arcta
n((cos(f*x + e)^3 + cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a
*c)*sin(f*x + e))))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. $2(170) = 340$.

Time = 0.25 (sec) , antiderivative size = 1356, normalized size of antiderivative = 7.14

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a*c^2*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a*c^2*cos(4*f*x
+ 4*e)^2 + 27*(f*x + e)*a*c^2*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a*c^2*sin(6
*f*x + 6*e)^2 + 27*(f*x + e)*a*c^2*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a*c^2
*sin(2*f*x + 2*e)^2 + 18*(f*x + e)*a*c^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a*
c^2 - 6*a*c^2*sin(2*f*x + 2*e) - 3*(a*c^2*cos(6*f*x + 6*e)^2 + 9*a*c^2*cos
(4*f*x + 4*e)^2 + 9*a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(6*f*x + 6*e)^2 +
9*a*c^2*sin(4*f*x + 4*e)^2 + 18*a*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
9*a*c^2*sin(2*f*x + 2*e)^2 + 6*a*c^2*cos(2*f*x + 2*e) + a*c^2 + 2*(3*a*c^2
*cos(4*f*x + 4*e) + 3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(6*f*x + 6*e) + 6
*(3*a*c^2*cos(2*f*x + 2*e) + a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(4*f*x
+ 4*e) + a*c^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*a*c^2*cos(4*f*x + 4*e) + 3*(f*x
+ e)*a*c^2*cos(2*f*x + 2*e) + (f*x + e)*a*c^2 - a*c^2*sin(4*f*x + 4*e) - a
*c^2*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a*c^2*cos(2*f*x
+ 2*e) + (f*x + e)*a*c^2)*cos(4*f*x + 4*e) + 6*(a*c^2*sin(6*f*x + 6*e) + 3
*a*c^2*sin(4*f*x + 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4
*f*x + 4*e) + 3*a*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 6*(a*c^2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e)
+ 3*a*c^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x...
```

Giac [F(-2)]

Exception generated.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{3/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{5/2}\,dx = \sqrt{c}\,\sqrt{a}\,a\,c^2\left(\left(\int \sqrt{\sec(fx + e) + 1}\,\sqrt{-\sec(fx + e) + 1}\,\sec(fx+e)^3\,dx - \left(\int \sqrt{\mathrm{sec}\right.\right.\right.$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*c**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**3,x) - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2,x) - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1),
x))
```

3.95 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^2c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^2c^2\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^2*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+1/2*a^2*c^2*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^2c^2(2\log(\cos(e+fx))+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^2*c^2*(2*Log[Cos[e + f*x]] + Sec[e + f*x]^2)*Tan[e + f*x])/(2*f*Sqrt[a*
(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4393, 25, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4393}$$

$$-\frac{a^2c^2\tan(e+fx)\int -\tan^3(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2c^2\tan(e+fx)\int \tan^3(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c^2\tan(e+fx)\int \tan(e+fx)^3dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{a^2c^2\tan(e+fx)\left(\frac{\tan^2(e+fx)}{2f}-\int \tan(e+fx)dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c^2\tan(e+fx)\left(\frac{\tan^2(e+fx)}{2f}-\int\tan(e+fx)dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^2c^2\tan(e+fx)\left(\frac{\tan^2(e+fx)}{2f}+\frac{\log(\cos(e+fx))}{f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^2*c^2*Tan[e + f*x]*(Log[Cos[e + f*x]]/f + Tan[e + f*x]^2/(2*f)))/(Sqrt[
a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.31

method	result
risch	$-\frac{ac\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(ie^{4i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+e^{4i(fx+e)}fx+2ie^{2i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+2\,e^{4i(fx+e)}e+\right.}{\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$
default	$\frac{\left(\left(8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+2\right)\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\right.\right.}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}\right.}{}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-a*c*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(c*(exp(I*(f*x+e)
)-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(I*ln(exp(2*I*(f*x+e))+1)*exp(4*I*(f*x+
e))+exp(4*I*(f*x+e))*f*x+2*I*ln(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))+2*exp
(4*I*(f*x+e))*e+2*exp(2*I*(f*x+e))*f*x+2*I*exp(2*I*(f*x+e))+I*ln(exp(2*I*(
f*x+e))+1)+4*exp(2*I*(f*x+e))*e+f*x+2*e)/(exp(I*(f*x+e))+1)/(exp(2*I*(f*x+
e))+1)/(exp(I*(f*x+e))-1)/f
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.47

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \left[\frac{\sqrt{-ac}ac\cos\left(fx+e\right)\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(f}{\cos(}}}{2\cos(fx+e)^2}\right.}{2\,f\cos\left(fx+e\right)}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[1/2*(sqrt(-a*c)*a*c*cos(f*x + e)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x +
 e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2)
 - a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)), -1/2*(2*sqrt(a*c)*a*c*arctan(
(cos(f*x + e)^3 + cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a*c
)*sin(f*x + e)))*cos(f*x + e) + a*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(93) = 186.

Time = 0.19 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.63

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{3/2}\,dx =$$
$$-\frac{((fx + e)ac\cos(4\,fx + 4\,e)^2 + 4\,(fx + e)ac\cos(2\,fx + 2\,e)^2 + (fx + e)ac\sin(4\,fx + 4\,e)^2 + 4\,(fx + e)}{}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a*c*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*cos(2*f*x + 2*e)^2 +
(f*x + e)*a*c*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a*c*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*a*c*cos(2*f*x + 2*e) + (f*x + e)*a*c - 2*a*c*sin(2*f*x + 2*e) -
(a*c*cos(4*f*x + 4*e)^2 + 4*a*c*cos(2*f*x + 2*e)^2 + a*c*sin(4*f*x + 4*e)^
2 + 4*a*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a*c*sin(2*f*x + 2*e)^2 + 4
*a*c*cos(2*f*x + 2*e) + a*c + 2*(2*a*c*cos(2*f*x + 2*e) + a*c)*cos(4*f*x +
 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a*
c*cos(2*f*x + 2*e) + (f*x + e)*a*c - a*c*sin(2*f*x + 2*e))*cos(4*f*x + 4*e
) + 2*(2*(f*x + e)*a*c*sin(2*f*x + 2*e) + a*c*cos(2*f*x + 2*e))*sin(4*f*x
+ 4*e))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + co
s(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*
f)
```

Giac [F(-2)]

Exception generated.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^{3/2}\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{3/2}\left(c - \frac{c}{\cos(e + f\,x)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,ac\left(-\left(\int\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)+\int\sqrt{\text{s}}$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*c*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1),x))
```

3.96 $\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{a^2c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{ac\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

output
```
a^2*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-a*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=-\frac{ac\sqrt{a(1+\sec(e+fx))}(-\log(\cos(e+fx))+\sec(e+fx))\tan(e+fx)}{f(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((a*c*Sqrt[a*(1 + Sec[e + f*x])]*(-Log[Cos[e + f*x]] + Sec[e + f*x])*Tan[
e + f*x])/(f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4394}$$

$$a\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{a^2c\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2c\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2c\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(85) = 170$.

Time = 3.01 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.61

method	result
default	$\frac{\sqrt{2}\left(\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-2\right)\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\cdots\right.\right.}{2f\cdots}$
risch	$\frac{a\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,x}{\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)}-\frac{2a\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,(fx+e)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)f}+\frac{2ia\cdots}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)/f*((4*cos(1/2*f*x+1/2*e)^2-2)*ln(2/(cos(1/2*f*x+1/2*e)+1))+(-2
*cos(1/2*f*x+1/2*e)^2+1)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(co
s(1/2*f*x+1/2*e)+1))+(-2*cos(1/2*f*x+1/2*e)^2+1)*ln(-2*(cos(1/2*f*x+1/2*e)
+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+2*sin(1/2*f*x+1/2*e)^2)*(a/(2
*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*(-2*c/(2*cos(1/2*f*x+
1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1
/2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(85) = 170$.

Time = 0.15 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.87

$$\int (a + a\sec(e + fx))^{3/2}\sqrt{c - c\sec(e + fx)}\,dx = \left[\frac{2\,a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e) + \sqrt{-ac}(a\cos(fx+e) + a}{2\,(f\,co} \right.$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*(2*a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e))*sin(f*x + e) + sqrt(-a*c)*(a*cos(f*x + e) + a)*log(1/2*(a*
c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e) + f), (a*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e)
 - sqrt(a*c)*(a*cos(f*x + e) + a)*arctan((cos(f*x + e)^3 + cos(f*x + e))*s
qrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e))))/(f*cos(f*x + e)
 + f)]
```

Sympy [F]

$$\int (a + a\sec(e + fx))^{3/2}\sqrt{c - c\sec(e + fx)}\,dx = \int (a(\sec(e + fx) + 1))^{\frac{3}{2}}\sqrt{-c(\sec(e + fx) - 1)}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(85) = 170$.

Time = 0.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.61

$$\int (a + a\sec(e + fx))^{3/2} \sqrt{c - c\sec(e + fx)}\, dx =$$
$$-\frac{((fx + e)a\cos(2fx + 2e)^2 + (fx + e)a\sin(2fx + 2e)^2 + 2(fx + e)a\cos(2fx + 2e) - 2a\cos\left(\frac{1}{2}\operatorname{arct}\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a*cos(2*f*x + 2*e)^2 + (f*x + e)*a*sin(2*f*x + 2*e)^2 + 2*(f*x
 + e)*a*cos(2*f*x + 2*e) - 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e)))*sin(2*f*x + 2*e) + (f*x + e)*a - (a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1) + 2*(a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a + a\sec(e + fx))^{3/2} \sqrt{c - c\sec(e + fx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\sqrt{c}\,\sqrt{a}\,a\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx+\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*se
c(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1),x))
```

3.97 $\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 104

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a(\log(\cos(e+fx))+2\log(1-\sec(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e\right.}{f\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a*(Log[Cos[e + f*x]] + 2*Log[1 - Sec[e + f*x]])*Sqrt[a*(1 + Sec[e + f*x])
]*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a \sec(e+fx)+a)^{3/2}}{\sqrt{c-c \sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a \csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{c-c \csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac \tan(e+fx) \int \frac{a \cos(e+fx)(\sec(e+fx)+1)}{c(1-\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2 \tan(e+fx) \int \frac{\cos(e+fx)(\sec(e+fx)+1)}{1-\sec(e+fx)}d\sec(e+fx)}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{a^2 \tan(e+fx) \int \left(\cos(e+fx)-\frac{2}{\sec(e+fx)-1}\right)d\sec(e+fx)}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 \tan(e+fx)(\log(\sec(e+fx))-2\log(1-\sec(e+fx)))}{f\sqrt{a \sec(e+fx)+a}\sqrt{c-c \sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((a^2*(-2*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]])*Tan[e + f*x])/(f*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(96) = 192$.

Time = 2.86 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\left(2\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-4\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\right.\right.}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right.\right.}{f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}}$
risch	$\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{4ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(f\right.}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(2*ln(2/(cos(1/2*f*x+1/2*e)+1))+ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x
+1/2*e))/(cos(1/2*f*x+1/2*e)+1))-4*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e
))+ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1)))*
(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a/(-c/(2*cos(1/2
*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*tan(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x
+ e) - c), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/sqrt(-c*(sec(e + f*x) - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx =$$
$$-\frac{((fx+e)a + a\arctan(\sin(2fx+2e), \cos(2fx+2e)+1) - 4a\arctan(\sin(fx+e), \cos(fx+e)-1}{\sqrt{c}f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a + a*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*a*ar
ctan2(sin(f*x + e), cos(f*x + e) - 1))*sqrt(a)/(sqrt(c)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)-1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)-1}dx\right)}{c}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x))/(sec(e + f*x) - 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt(
 - sec(e + f*x) + 1))/(sec(e + f*x) - 1),x)))/c
```

3.98 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*a^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a^2*ln(1
-cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a^2\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{2}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^2*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + 2/(-1 + Sec[e + f*x]))*T
an[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4396, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4396}$$

$$\frac{a\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{a^2\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{a^2\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/
2)) + (a^2*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.64

method	result
default	$\frac{\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\, a\left(4\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\sec\left(\frac{fx}{2}\cdots\right.\right.}{2f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\, c}$
risch	$-\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+e^{2i(fx+e)}fx-4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2e^{2i(fx+e)}e-2e^{i(fx+e)}fx-4ie^{i(fx\cdots\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\, f}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a/(-c/(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(4*ln(csc(1/2*f*x+1/2
*e)-cot(1/2*f*x+1/2*e))*tan(1/2*f*x+1/2*e)-4*ln(2/(cos(1/2*f*x+1/2*e)+1))*
tan(1/2*f*x+1/2*e)+cot(1/2*f*x+1/2*e)+sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e
))
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x
 + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{2\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{-a}a(\cos(fx+e)+1)^2}{c^{\frac{3}{2}}\sin(fx+e)^2}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
(2*sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(3/2) - sqrt(-a)*a*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + sqrt(-a)*a*(cos(f*x +
 e) + 1)^2/(c^(3/2)*sin(f*x + e)^2))/f
```

Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\frac{\sqrt{-ac}a^2\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^2|a|} - \frac{\sqrt{-ac}a^2\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^2|a|} - \frac{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)\sqrt{-ac}a}{c^2|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
-(sqrt(-a*c)*a^2*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^2*abs(a)) - sqrt(-a
*c)*a^2*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^2*abs(a)) - (a*tan(1/2*
f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a/(c^2*abs(a)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e}}{\sec(fx+e)^2-2\sec(fx+e)+}\right.}{c^2}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x) + int((sqrt(sec(e
+ f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x)**2 - 2*sec(e + f*x) +
 1),x)))/c**2
```

3.99 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-a^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)-a^2*tan(f*
x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a^2*ln(1-cos(f*x+e)
)*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^2\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))+\frac{-2+\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^2*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + (-2 + Sec[e + f*x])/(-1
+ Sec[e + f*x])^2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
- c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4396, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4396}$$

$$\frac{a\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 4395

$$\frac{a\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{a\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 4399

$$\frac{a\left(\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$\Big\downarrow$ 16

$$\frac{a\left(\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c}-\frac{a^2\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-((a^2*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
))) + (a*(-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.28

method	result
default	$\frac{\left(64\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-64\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-21\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+19\right)}{32f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}}$
risch	$-\frac{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(2i\ln\left(e^{i(fx+e)}-1\right)+e^{4i(fx+e)}fx-8ie^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2e^{4i(fx+e)}e-4e^{3i(fx+e)}fx-6ie^{i(fx+e)}-8e\right.}{}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/f*(64*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^4-
64*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^4-21*cos(1/2*f*x+1/2*e)
^4-6*cos(1/2*f*x+1/2*e)^2+19)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/
2*e)^2)^(1/2)*a/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
/c^2*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^3
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*
x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(-c(\sec(e+fx)-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(-c*(sec(e + f*x) - 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(134) = 268$.

Time = 0.32 (sec) , antiderivative size = 1786, normalized size of antiderivative = 12.23

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 16*
(f*x + e)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f
*x + e)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x +
e)*a*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*a*sin(2*f*x + 2*e)^2 + 16*(f*x + e)
*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*a
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*a*c
os(2*f*x + 2*e) + (f*x + e)*a - 2*(a*cos(4*f*x + 4*e)^2 + 36*a*cos(2*f*x +
 2*e)^2 + 16*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16
*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a*sin(4*f*x +
4*e)^2 + 12*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*a*sin(2*f*x + 2*e)^2
+ 16*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*a*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*a*cos(2*f*x + 2*e
) + a)*cos(4*f*x + 4*e) + 12*a*cos(2*f*x + 2*e) - 8*(a*cos(4*f*x + 4*e) +
6*a*cos(2*f*x + 2*e) - 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a*cos
(4*f*x + 4*e) + 6*a*cos(2*f*x + 2*e) + a)*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - 4*a*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(a*sin(4*f*x + 4*e) + 6*a*sin(2*f*x +
 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*arctan...
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{4\sqrt{-ac}a^2\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^3|a|}-\frac{4\sqrt{-ac}a^2\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^3|a|}-\frac{6\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-ac}a^2+8\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}{a^2c^3|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}{4f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-1/4*(4*sqrt(-a*c)*a^2*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^3*abs(a)) - 4
*sqrt(-a*c)*a^2*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^3*abs(a)) - (6*
(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^2 + 8*(a*tan(1/2*f*x + 1/2*e
)^2 - a)*sqrt(-a*c)*a^3 + 3*sqrt(-a*c)*a^4)/(a^2*c^3*abs(a)*tan(1/2*f*x +
1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^{3}-3\sec(fx+e)^{2}+3\sec(fx+e)-1}dx+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^{3}-3\sec(fx+e)^{2}+3\sec(fx+e)-1}dx\right)}{c^{3}}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x))/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1
),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x)))/c**3
```

3.100 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 196

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{2a^2\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^2\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^2\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2/3*a^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2)-1/2*a^
2*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)-a^2*tan(f*x
+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a^2*ln(1-cos(f*x+e
))*tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^2\left(-6\log(\cos(e+fx))-6\log(1-\sec(e+fx))+\frac{-13+15\sec(e+fx)-6\sec^2(e+fx)}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{6c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/6*(a^2*(-6*Log[Cos[e + f*x]] - 6*Log[1 - Sec[e + f*x]] + (-13 + 15*Sec[
e + f*x] - 6*Sec[e + f*x]^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*S
qrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4396, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4396}$$

$$\frac{a\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \frac{\sqrt{\csc(e+fx+\frac{\pi}{2})a+a}}{(c-c\csc(e+fx+\frac{\pi}{2}))^{5/2}}dx}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\big\downarrow$ 4395

$$\frac{a\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\big\downarrow$ 3042

$$\frac{a\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\big\downarrow$ 4395

$$\frac{a\left(\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\big\downarrow$ 3042

$$\frac{a\left(\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\big\downarrow$ 4399

$$\frac{a\left(\frac{\frac{a\tan(e+fx)\int\frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow 16$$

$$\frac{a\left(\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c}-\frac{2a^2\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(-2*a^2*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(
7/2)) + (a*(-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
 + f*x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*S
ec[e + f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.02

method	result
default	$\frac{\left(48 \ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right) \sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-48 \ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right) \sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+19 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-9 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-24 \mathrm{co}}{24 f \sqrt{-\frac{c \sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2 \cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{3}}$
risch	$\frac{a \sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right) x}{c^{3}\left(\mathrm{e}^{i(fx+e)}+1\right) \sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}}-\frac{2 a \sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)(fx+e)}{c^{3}\left(\mathrm{e}^{i(fx+e)}+1\right) \sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\, f}+\frac{2 i a \sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(12\,\mathrm{e}^{5i(fx+e)}-33\,\mathrm{e}^{4i(}\right.}{3 c^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx\right.}}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f*(48*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^6-
48*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^6+19*cos(1/2*f*x+1/2*e)
^6-9*cos(1/2*f*x+1/2*e)^4-24*cos(1/2*f*x+1/2*e)^2+16)*(a/(2*cos(1/2*f*x+1/
2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin
(1/2*f*x+1/2*e)^2)^(1/2)/c^3*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^5
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x
 + e)^4 - 4*c^4*sec(f*x + e)^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e)
 + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3480 vs. $2(176) = 352$.

Time = 1.60 (sec) , antiderivative size = 3480, normalized size of antiderivative = 17.76

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c - c\sec(e + fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a*cos(4*f*x + 4*e)^
2 + 675*(f*x + e)*a*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a*cos(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a*sin(6*f*x + 6*e)^2 +
675*(f*x + e)*a*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a*sin(2*f*x + 2*e)^2 +
108*(f*x + e)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1
200*(f*x + e)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1
08*(f*x + e)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90
*(f*x + e)*a*cos(2*f*x + 2*e) + 3*(f*x + e)*a - 6*(a*cos(6*f*x + 6*e)^2 +
225*a*cos(4*f*x + 4*e)^2 + 225*a*cos(2*f*x + 2*e)^2 + 36*a*cos(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 36*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + a*sin(6*f*x + 6*e)^2 + 225*a*sin(4*f*x + 4*e)^2 + 450*a
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*a*sin(2*f*x + 2*e)^2 + 36*a*sin(5
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 2*(15*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2
*e) + a)*cos(6*f*x + 6*e) + 30*(15*a*cos(2*f*x + 2*e) + a)*cos(4*f*x + 4*e
) + 30*a*cos(2*f*x + 2*e) - 12*(a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4...
```

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.02

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx =$$

$$-\frac{\frac{24\sqrt{-ac}a^2\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^4|a|}-\frac{24\sqrt{-ac}a^2\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^4|a|}-\frac{44\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^3\sqrt{-ac}a^2+108\left(a\tan\left(\frac{1}{2}fx\right.\right.}{a^3c}}{24f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
-1/24*(24*sqrt(-a*c)*a^2*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^4*abs(a)) -
 24*sqrt(-a*c)*a^2*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^4*abs(a)) -
(44*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*c)*a^2 + 108*(a*tan(1/2*f*x +
 1/2*e)^2 - a)^2*sqrt(-a*c)*a^3 + 93*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-
a*c)*a^4 + 27*sqrt(-a*c)*a^5)/(a^3*c^4*abs(a)*tan(1/2*f*x + 1/2*e)^6))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c - c/cos(e + f*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^4-4\sec(fx+e)^3+6\sec(fx+e)^2-4\sec(fx+e)+1}dx+\int \frac{\sqrt{\sec \cdots}}{\sec(fx+e)^4-4\sec \cdots}\cdots\right.}{c^4}$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*
sec(e + f*x))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4
*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
 1))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e +
f*x) + 1),x)))/c**4
```

3.101 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 153

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a^3c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a^3c^3\tan^3(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^3c^3\tan^5(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^3*c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+1/2*a^3*c^3*tan(f*x+e)^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)-1/4*a^3*c^3*tan(f*x+e)^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.50

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{a^3c^3(-4\log(\cos(e+fx))-4\sec^2(e+fx)+\sec^4(e+fx))\tan(e+fx)}{4f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-1/4*(a^3*c^3*(-4*Log[Cos[e + f*x]] - 4*Sec[e + f*x]^2 + Sec[e + f*x]^4)*T
an[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.56, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4393, 25, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4393}$$

$$\frac{a^3c^3\tan(e+fx)\int -\tan^5(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3c^3\tan(e+fx)\int \tan^5(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^3c^3\tan(e+fx)\int\tan(e+fx)^5dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3954$$

$$-\frac{a^3c^3\tan(e+fx)\left(\frac{\tan^4(e+fx)}{4f}-\int\tan^3(e+fx)dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^3c^3\tan(e+fx)\left(\frac{\tan^4(e+fx)}{4f}-\int\tan(e+fx)^3dx\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3954$$

$$-\frac{a^3c^3\tan(e+fx)\left(\int\tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{a^3c^3\tan(e+fx)\left(\int\tan(e+fx)dx+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3956$$

$$-\frac{a^3c^3\tan(e+fx)\left(\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f}-\frac{\log(\cos(e+fx))}{f}\right)}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
-((a^3*c^3*Tan[e + f*x]*(-(Log[Cos[e + f*x]]/f) - Tan[e + f*x]^2/(2*f) + T
an[e + f*x]^4/(4*f)))/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 29.53 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.38

method	result
risch	$-\frac{a^2 c^2 \sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\left(i\ln\left(e^{2i(fx+e)}+1\right)e^{8i(fx+e)}+e^{8i(fx+e)}fx+4ie^{6i(fx+e)}+2\,e^{8i(fx+e)}e+4\,e^{6i(fx+e)}fx\cdots\right.}{\cdots}$
default	$-\frac{\left(\left(16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^8-32\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^6+24\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(16\cos\left(\frac{fx}{2}\cdots\right.\right.}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2*c^2*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(c*(exp(I*(f*
x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(I*ln(exp(2*I*(f*x+e))+1)*exp(8*I*(
f*x+e))+exp(8*I*(f*x+e))*f*x+4*I*exp(6*I*(f*x+e))+2*exp(8*I*(f*x+e))*e+4*e
xp(6*I*(f*x+e))*f*x+I*ln(exp(2*I*(f*x+e))+1)+8*exp(6*I*(f*x+e))*e+6*exp(4*
I*(f*x+e))*f*x+6*I*ln(exp(2*I*(f*x+e))+1)*exp(4*I*(f*x+e))+12*exp(4*I*(f*x
+e))*e+4*exp(2*I*(f*x+e))*f*x+4*I*exp(2*I*(f*x+e))+4*I*exp(4*I*(f*x+e))+4*
I*ln(exp(2*I*(f*x+e))+1)*exp(6*I*(f*x+e))+4*I*ln(exp(2*I*(f*x+e))+1)*exp(2
*I*(f*x+e))+8*exp(2*I*(f*x+e))*e+f*x+2*e)/(exp(I*(f*x+e))+1)/(exp(2*I*(f*x
+e))+1)^3/(exp(I*(f*x+e))-1)/f
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.72

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \left[\frac{2\sqrt{-ac}a^2c^2\cos\left(fx+e\right)^3\log\left(\frac{ac\cos(fx+e)^4-\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{}}{2\cos(fx+e)^2}\right.}{4}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
[1/4*(2*sqrt(-a*c)*a^2*c^2*cos(f*x + e)^3*log(1/2*(a*c*cos(f*x + e)^4 - (c
os(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x
 + e)^2) - (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^3), -1/4*(4*sqrt(a*c)*a^2*c^2*arctan((cos(f*x + e)^3 + cos(f*x +
 e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)))*cos(f*x +
e)^3 + (3*a^2*c^2*cos(f*x + e)^2 - a^2*c^2)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1619 vs. 2(137) = 274.

Time = 0.30 (sec) , antiderivative size = 1619, normalized size of antiderivative = 10.58

$$\int (a + a\sec(e + fx))^{5/2}(c - c\sec(e + fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a^2*c^2*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2*c^2*cos(6*f*x +
6*e)^2 + 36*(f*x + e)*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*a^2*c^2*co
s(2*f*x + 2*e)^2 + (f*x + e)*a^2*c^2*sin(8*f*x + 8*e)^2 + 16*(f*x + e)*a^2
*c^2*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*a^2*c^2*sin(4*f*x + 4*e)^2 + 16*(f*
x + e)*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) +
 (f*x + e)*a^2*c^2 - 4*a^2*c^2*sin(2*f*x + 2*e) - (a^2*c^2*cos(8*f*x + 8*e
)^2 + 16*a^2*c^2*cos(6*f*x + 6*e)^2 + 36*a^2*c^2*cos(4*f*x + 4*e)^2 + 16*a
^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(8*f*x + 8*e)^2 + 16*a^2*c^2*sin(6*
f*x + 6*e)^2 + 36*a^2*c^2*sin(4*f*x + 4*e)^2 + 48*a^2*c^2*sin(4*f*x + 4*e)
*sin(2*f*x + 2*e) + 16*a^2*c^2*sin(2*f*x + 2*e)^2 + 8*a^2*c^2*cos(2*f*x +
2*e) + a^2*c^2 + 2*(4*a^2*c^2*cos(6*f*x + 6*e) + 6*a^2*c^2*cos(4*f*x + 4*e
) + 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(8*f*x + 8*e) + 8*(6*a^2*c^2*
cos(4*f*x + 4*e) + 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(6*f*x + 6*e)
+ 12*(4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*cos(4*f*x + 4*e) + 4*(2*a^2*c^
2*sin(6*f*x + 6*e) + 3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*x + 2*
e))*sin(8*f*x + 8*e) + 16*(3*a^2*c^2*sin(4*f*x + 4*e) + 2*a^2*c^2*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
) + 2*(4*(f*x + e)*a^2*c^2*cos(6*f*x + 6*e) + 6*(f*x + e)*a^2*c^2*cos(4*f*
x + 4*e) + 4*(f*x + e)*a^2*c^2*cos(2*f*x + 2*e) + (f*x + e)*a^2*c^2 - 2*a^
2*c^2*sin(6*f*x + 6*e) - 2*a^2*c^2*sin(4*f*x + 4*e) - 2*a^2*c^2*sin(2*f...
```

Giac [F(-2)]

Exception generated.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^2c^2\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^4\,dx-2\left(\int \sqrt{s}\right.\right.$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*c**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**4,x) - 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*
x) + 1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*
x) + 1),x))
```

3.102 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 190

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^3c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}+\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

output
```
a^3*c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)-a^2*c^2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
-1/2*a*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+1/3*
c^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^3c^2(2+6\log(\cos(e+fx))-6\sec(e+fx)+3\sec^2(e+fx)+2\sec^3(e+fx))\tan}{6f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^3*c^2*(2 + 6*Log[Cos[e + f*x]] - 6*Sec[e + f*x] + 3*Sec[e + f*x]^2 + 2*
Sec[e + f*x]^3)*Tan[e + f*x])/(6*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*S
ec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 4397, 3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4397}$$

$$c\int (\sec(e+fx)a+a)^{5/2}\sqrt{c-c\sec(e+fx)}dx+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$c\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\downarrow \text{4394}$$

$$c\left(a\int (\sec(e+fx)a+a)^{3/2}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}\right)+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\downarrow \text{3042}$$

$$c\left(a\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}\right)+\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\downarrow \text{4394}$$

$$c\left(a\left(a\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\,s}{2f\sqrt{c-c\,s}}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\downarrow \text{3042}$$

$$c\left(a\left(a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan}{}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\downarrow \text{4393}$$

$$c\left(a\left(\frac{a^2c\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+}{2f\sqrt{c-c\sec(e-}}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$$\downarrow \text{25}$$

$$c\left(a\left(-\frac{a^2c\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e}{2f\sqrt{c-c\sec(e}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 3042

$$c\left(a\left(-\frac{a^2c\tan(e+fx)\int\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e}{2f\sqrt{c-c\sec(e}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

$\downarrow$ 3956

$$c\left(a\left(\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e}{2f\sqrt{c-c\sec(e}\right.$$
$$\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]
]) + c*(-1/2*(a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*S
ec[e + f*x]]) + a*((a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[
e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

rule 4397
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a   Int[Sqrt[a +
b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.78

method	result
risch	$-\frac{a^{2} c \sqrt{\frac{a \left(e^{i (f x+e)}+1\right)^{2}}{e^{2 i (f x+e)}+1}}\, \sqrt{\frac{c \left(e^{i (f x+e)}-1\right)^{2}}{e^{2 i (f x+e)}+1}}\, \left(-6 i e^{i (f x+e)}+3\, e^{6 i (f x+e)} f x+3 i \ln\left(e^{2 i (f x+e)}+1\right)+6\, e^{6 i (f x+e)} e+9\, e^{4 i (f x+e)} f x+9 i e^{4 i (\cdots}\right.}{\cdots}$
default	$-\frac{\left(\left(24 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}-36 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+18 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-3\right) \ln\left(-\frac{2 \left(\cos\left(\frac{f x}{2}+\frac{e}{2}\right)-\sin\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{f x}{2}+\frac{e}{2}\right)+1}\right)+\left(-48 \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}+72 \cos\left(\cdots\right.\right.}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*c*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(c*(exp(I*(
f*x+e))-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(-6*I*exp(I*(f*x+e))+3*exp(6*I*(f
*x+e))*f*x+3*I*ln(exp(2*I*(f*x+e))+1)+6*exp(6*I*(f*x+e))*e+9*exp(4*I*(f*x+
e))*f*x+9*I*exp(4*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)+18*exp(4*I*(f*x+e))*e+
9*exp(2*I*(f*x+e))*f*x+6*I*exp(2*I*(f*x+e))+6*I*exp(4*I*(f*x+e))-6*I*exp(5
*I*(f*x+e))+3*I*exp(6*I*(f*x+e))*ln(exp(2*I*(f*x+e))+1)+9*I*exp(2*I*(f*x+e
))*ln(exp(2*I*(f*x+e))+1)-4*I*exp(3*I*(f*x+e))+18*exp(2*I*(f*x+e))*e+3*f*x
+6*e)/(exp(I*(f*x+e))+1)/(exp(2*I*(f*x+e))+1)^2/(exp(I*(f*x+e))-1)/f
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.52

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \left[\frac{\left(a^2c\cos(fx+e)^2-5\,a^2c\cos(fx+e)-2\,a^2c\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(f\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) + 3*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a*c)*log(1/
2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*s
in(f*x + e) + a*c)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2),
 1/6*((a^2*c*cos(f*x + e)^2 - 5*a^2*c*cos(f*x + e) - 2*a^2*c)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*
x + e) - 6*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(a*c)*arctan(
(cos(f*x + e)^3 + cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a*c
)*sin(f*x + e))))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1356 vs. 2(170) = 340.

Time = 0.25 (sec) , antiderivative size = 1356, normalized size of antiderivative = 7.14

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a^2*c*cos(6*f*x + 6*e)^2 + 27*(f*x + e)*a^2*c*cos(4*f*x
+ 4*e)^2 + 27*(f*x + e)*a^2*c*cos(2*f*x + 2*e)^2 + 3*(f*x + e)*a^2*c*sin(6
*f*x + 6*e)^2 + 27*(f*x + e)*a^2*c*sin(4*f*x + 4*e)^2 + 27*(f*x + e)*a^2*c
*sin(2*f*x + 2*e)^2 + 18*(f*x + e)*a^2*c*cos(2*f*x + 2*e) + 3*(f*x + e)*a^
2*c - 6*a^2*c*sin(2*f*x + 2*e) - 3*(a^2*c*cos(6*f*x + 6*e)^2 + 9*a^2*c*cos
(4*f*x + 4*e)^2 + 9*a^2*c*cos(2*f*x + 2*e)^2 + a^2*c*sin(6*f*x + 6*e)^2 +
9*a^2*c*sin(4*f*x + 4*e)^2 + 18*a^2*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
9*a^2*c*sin(2*f*x + 2*e)^2 + 6*a^2*c*cos(2*f*x + 2*e) + a^2*c + 2*(3*a^2*c
*cos(4*f*x + 4*e) + 3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(6*f*x + 6*e) + 6
*(3*a^2*c*cos(2*f*x + 2*e) + a^2*c)*cos(4*f*x + 4*e) + 6*(a^2*c*sin(4*f*x
+ 4*e) + a^2*c*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1) + 6*(3*(f*x + e)*a^2*c*cos(4*f*x + 4*e) + 3*(f*x
+ e)*a^2*c*cos(2*f*x + 2*e) + (f*x + e)*a^2*c - a^2*c*sin(4*f*x + 4*e) - a
^2*c*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 18*(3*(f*x + e)*a^2*c*cos(2*f*x
+ 2*e) + (f*x + e)*a^2*c)*cos(4*f*x + 4*e) - 6*(a^2*c*sin(6*f*x + 6*e) + 3
*a^2*c*sin(4*f*x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4
*f*x + 4*e) + 3*a^2*c*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 6*(a^2*c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e)
+ 3*a^2*c*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x...
```

Giac [F(-2)]

Exception generated.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^2c\left(-\left(\int\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)-\left(\int\right.\right.$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*c*( - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3,x) - int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**2,x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1),x))
```

3.103 $\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 139

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{a^3c\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{a^2c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{ac(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

output
```
a^3*c*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-a^2*c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/2
*a*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=-\frac{a^3c(-2\log(\cos(e+fx))+4\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-1/2*(a^3*c*(-2*Log[Cos[e + f*x]] + 4*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
 + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4394, 3042, 4394, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4394}$$

$$a\int (\sec(e+fx)a+a)^{3/2}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4394}$$

$$a\left(a\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4393}$$

$$a\left(\frac{a^2c\tan(e+fx)\int -\tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$a\left(-\frac{a^2c\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(-\frac{a^2c\tan(e+fx)\int \tan(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$a\left(\frac{a^2c\tan(e+fx)\log(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{ac\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}\right)-\frac{ac\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-1/2*(a*c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f
*x]]) + a*((a^2*c*Log[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*
x]]*Sqrt[c - c*Sec[e + f*x]]) - (a*c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x]
)/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4394
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[c   Int[Sqrt[a
 + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[n, 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{a^{2} \sqrt{\frac{a \left({\mathrm e}^{i \left(f x +e \right)}+1\right)^{2}}{{\mathrm e}^{2 i \left(f x +e \right)}+1}}\, \sqrt{\frac{c \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{2}}{{\mathrm e}^{2 i \left(f x +e \right)}+1}}\, \left(i {\mathrm e}^{4 i \left(f x +e \right)} \ln \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)+{\mathrm e}^{4 i \left(f x +e \right)} f x +2 i {\mathrm e}^{2 i \left(f x +e \right)} \ln \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)+2 \,{\mathrm e}^{4 i \left(f x +e \right)} e+\right.}{\left({\mathrm e}^{i \left(f x +e \right)}+1\right) \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right) \left({\mathrm e}^{i \left(f x +e \right)}\right.}$
default	$\frac{\sqrt{2}\, \left(\left(8 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}-8 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+2\right) \ln \left(\frac{2}{\cos \left(\frac{f x}{2}+\frac{e}{2}\right)+1}\right)+\left(-4 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+4 \cos \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right) \ln \left(-\frac{2 \left(\cos \left(\frac{f x}{2}+\frac{e}{2}\right)+\mathrm{si}\right.}{\cos \left(\frac{f x}{2}+\frac{e}{2}\right.}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2*(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(c*(exp(I*(f*x+e)
)-1)^2/(exp(2*I*(f*x+e))+1))^(1/2)*(I*ln(exp(2*I*(f*x+e))+1)*exp(4*I*(f*x+
e))+exp(4*I*(f*x+e))*f*x+2*I*ln(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))+2*exp
(4*I*(f*x+e))*e+2*exp(2*I*(f*x+e))*f*x-2*I*exp(2*I*(f*x+e))-4*I*exp(I*(f*x
+e))-4*I*exp(3*I*(f*x+e))+I*ln(exp(2*I*(f*x+e))+1)+4*exp(2*I*(f*x+e))*e+f*
x+2*e)/(exp(I*(f*x+e))+1)/(exp(2*I*(f*x+e))+1)/(exp(I*(f*x+e))-1)/f
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.11

$$\int (a + a\sec(e + fx))^{5/2}\sqrt{c - c\sec(e + fx)}\,dx = \left[\frac{(5\,a^2\cos(fx+e) + a^2)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sin(fx+e) + (a^2 c}{}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + (a^2*cos(f*x + e)^2
+ a^2*cos(f*x + e))*sqrt(-a*c)*log(1/2*(a*c*cos(f*x + e)^4 - (cos(f*x + e)
^3 + cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a*c)/cos(f*x + e)^2))/(
f*cos(f*x + e)^2 + f*cos(f*x + e)), 1/2*((5*a^2*cos(f*x + e) + a^2)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*
sin(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt(a*c)*arctan(
(cos(f*x + e)^3 + cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a*c*cos(f*x + e)^2 - a*c
)*sin(f*x + e))))/(f*cos(f*x + e)^2 + f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}\sqrt{c - c\sec(e + fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(125) = 250$.

Time = 0.21 (sec) , antiderivative size = 710, normalized size of antiderivative = 5.11

$$\int (a + a\sec(e + fx))^{5/2}\sqrt{c - c\sec(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 +
(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + 2*a^2*sin(2*f*x + 2*e) -
(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(4*f*x + 4*e)^
2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4
*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x +
 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(2*(f*x + e)*a^
2*cos(2*f*x + 2*e) + (f*x + e)*a^2 + a^2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e
) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f
*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*(f*
x + e)*a^2*sin(2*f*x + 2*e) - a^2*cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + 4*(
a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*
x + 2*e) + a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt
(a)*sqrt(c)/((2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*
e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(
2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a + a\sec(e + fx))^{5/2}\sqrt{c - c\sec(e + fx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=\sqrt{c}\,\sqrt{a}\,a^2\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2\,dx+2\left(\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)+\int \sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,dx\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)
*sec(e + f*x)**2,x) + 2*int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1),
x))
```

3.104 $\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 152

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
a^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+4*a^3*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)+a^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\frac{a^3(\log(\cos(e+fx))+4\log(1-\sec(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
(a^3*(Log[Cos[e + f*x]] + 4*Log[1 - Sec[e + f*x]] + Sec[e + f*x])*Tan[e +
f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2}{c(1-\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2}{1-\sec(e+fx)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{a^3\tan(e+fx)\int\left(\cos(e+fx)-\frac{4}{\sec(e+fx)-1}-1\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\tan(e+fx)(-\sec(e+fx)-4\log(1-\sec(e+fx))+\log(\sec(e+fx)))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/Sqrt[c - c*Sec[e + f*x]],x]
```

output
```
-((a^3*(-4*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]] - Sec[e + f*x])*Tan[e
 + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(140) = 280$.

Time = 2.99 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.89

method	result
default	$-\frac{\left(\left(6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2\right)\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\cdots\right.\right.}{f\left(2\cos\left(\frac{fx}{2}\cdots\right.\right.}$
risch	$-\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(-8i\ln\left(e^{i(fx+e)}-1\right)-2ie^{i(fx+e)}+e^{3i(fx+e)}fx+8i\ln\left(e^{i(fx+e)}-1\right)e^{3i(fx+e)}+3i\ln\left(e^{2i(fx+e)}+1\right)+8ie^{i(fx\cdots}\right.}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*((6*cos(1/2*f*x+1/2*e)^2-3)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2
*e))/(cos(1/2*f*x+1/2*e)+1))+(4*cos(1/2*f*x+1/2*e)^2-2)*ln(2/(cos(1/2*f*x+
1/2*e)+1))+(-16*cos(1/2*f*x+1/2*e)^2+8)*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+
1/2*e))+(6*cos(1/2*f*x+1/2*e)^2-3)*ln(2*(-cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1
/2*e))/(cos(1/2*f*x+1/2*e)+1))-2*sin(1/2*f*x+1/2*e)^2)*(a/(2*cos(1/2*f*x+1
/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a^2/(2*cos(1/2*f*x+1/2*e)^2-1)/(-c/
(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*tan(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
 e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{\sqrt{c - c\sec(e + fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{\sqrt{c - c\sec(e + fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{\sqrt{c - c\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)-1}dx\right)-2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{}}{\mathrm{se}}\right.\right.}{c}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**2)/(sec(e + f*x) - 1),x) - 2*int((sqrt(sec(e + f*x) +
1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) - 1),x) - int((sq
rt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x) - 1),x)))/c
```

3.105 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{4a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(\cos(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-4*a^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a^3*ln(c
os(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a^3\left(\log(\cos(e+fx))+\frac{4}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(a^3*(Log[Cos[e + f*x]] + 4/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(c*f*Sqrt[a
*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4398, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx}{c^2}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c^2}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{a^3\tan(e+fx)\int -\tan(e+fx)dx}{c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^3\tan(e+fx)\int \tan(e+fx)dx}{c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^3\tan(e+fx)\int\tan(e+fx)dx}{c\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^3\tan(e+fx)\log(\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(3/2),x]
```

output
```
(-4*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/
2)) + (a^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(88) = 176$.

Time = 3.00 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.33

method	result
default	$\frac{\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,a^2\left(\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{\cos\left(\cdots\right.}\right.\right.}{f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,c}$
risch	$-\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(ie^{2i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+e^{2i(fx+e)}fx-2ie^{i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+2e^{2i(fx+e)}e-2e^{i(fx+e)}fx-8ie^{i(\cdots}\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a^2/(-c/(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c*(ln(-2*(cos(1/2*f*x+1
/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*tan(1/2*f*x+1/2*e)-2*ln(
2/(cos(1/2*f*x+1/2*e)+1))*tan(1/2*f*x+1/2*e)+ln(-2*(cos(1/2*f*x+1/2*e)-sin
(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*tan(1/2*f*x+1/2*e)+cot(1/2*f*x+1/
2*e)+sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.

Time = 0.15 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.75

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{3/2}}\,dx = \left[\frac{(a^2c\cos(fx + e) - a^2c)\sqrt{-\frac{a}{c}}\log\left(\frac{a\cos(fx+e)^4 - \left(\cos(fx+e)^3 + \cos(fx+e)\right)\sqrt{-\frac{a}{c}}\sqrt{\frac{a}{}}}{2\cos(fx+e}\right.}{} \right.$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[1/2*((a^2*c*cos(f*x + e) - a^2*c)*sqrt(-a/c)*log(1/2*(a*cos(f*x + e)^4 -
(cos(f*x + e)^3 + cos(f*x + e))*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + a)/cos(f*x
 + e)^2)*sin(f*x + e) + 4*(a^2*cos(f*x + e)^2 + a^2*cos(f*x + e))*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(
(c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), -((a^2*c*cos(f*x + e) - a^2*c)
*sqrt(a/c)*arctan((cos(f*x + e)^3 + cos(f*x + e))*sqrt(a/c)*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a*cos(
f*x + e)^2 - a)*sin(f*x + e)))*sin(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a^2*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2-2\sec(fx+e)+1}dx+2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(}}{\sec(fx+e)^2-2\,\mathrm{s}}\right.\right.}{c^2}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x) + 2*int((sqr
t(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)*
*2 - 2*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e +
f*x) + 1))/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1),x)))/c**2
```

3.106 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{2a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*a^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)+a^3*ln(1
-cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2
)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^3\left(\log(\cos(e+fx))+\log(1-\sec(e+fx))-\frac{2}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(a^3*(Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] - 2/(-1 + Sec[e + f*x])^2)
*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4398, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c^2}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{a^3\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{a^3\tan(e+fx)\log(1-\cos(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/
2)) + (a^3*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(92) = 184$.

Time = 3.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.89

method	result
default	$-\frac{\left(32\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-32\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+13\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-11}{16f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{2}}$
risch	$-\frac{a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(2i\ln\left(e^{i(fx+e)}-1\right)+e^{4i(fx+e)}fx-8ie^{i(fx+e)}+2\,e^{4i(fx+e)}e-4\,e^{3i(fx+e)}fx+2ie^{4i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-8}{}$

input

```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/16/f*(32*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^4-32*ln(csc(1/
2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^4+13*cos(1/2*f*x+1/2*e
)^4+6*cos(1/2*f*x+1/2*e)^2-11)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1
/2*e)^2)^(1/2)*a^2/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1
/2)/c^2*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^3
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input

```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output

```
integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
 e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e
)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\frac{\frac{4\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}}-\frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{5}{2}}}-\frac{\left(\sqrt{-a}a^2\sqrt{c}-\frac{2\sqrt{-a}a^2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right.}{c^3\sin(fx+e}}{2f}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/2*(4*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) - 2*sqrt(
-a)*a^2*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) - (sqrt(-a)*a
^2*sqrt(c) - 2*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(
cos(f*x + e) + 1)^4/(c^3*sin(f*x + e)^4))/f
```

Giac [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.70

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=$$

$$-\frac{\frac{2\sqrt{-ac}a^3\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^3|a|}-\frac{2\sqrt{-ac}a^3\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^3|a|}-\frac{3\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^2\sqrt{-ac}a^3+4\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}{a^2c^3|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}{2f}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-1/2*(2*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^3*abs(a)) - 2
*sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^3*abs(a)) - (3*
(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^3 + 4*(a*tan(1/2*f*x + 1/2*e
)^2 - a)*sqrt(-a*c)*a^4 + 2*sqrt(-a*c)*a^5)/(a^2*c^3*abs(a)*tan(1/2*f*x +
1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3-3\sec(fx+e)^2+3\sec(fx+e)-1}dx\right)-2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{}}{\sec(fx+e)^3-3\,\mathrm{se}}\right.\right.}{c^3}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**2)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*
x) - 1),x) - 2*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e
 + f*x))/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x) - i
nt((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x)**3 - 3
*sec(e + f*x)**2 + 3*sec(e + f*x) - 1),x)))/c**3
```

3.107 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 148

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{4a^3\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}} - \frac{a^3\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-4/3*a^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2)-a^3*ta
n(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a^3*ln(1-cos(
f*x+e))*tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^3\left(-3\log(\cos(e+fx))-3\log(1-\sec(e+fx))+\frac{-4-3(-1+\sec(e+fx))^2}{(-1+\sec(e+fx))^3}\right)\tan(e+fx)}{3c^3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
-1/3*(a^3*(-3*Log[Cos[e + f*x]] - 3*Log[1 - Sec[e + f*x]] + (-4 - 3*(-1 +
Sec[e + f*x])^2)/(-1 + Sec[e + f*x])^3)*Tan[e + f*x])/(c^3*f*Sqrt[a*(1 + S
ec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4398, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \; 4398$$

$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 4395

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 4399

$$\frac{a^2\left(\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

$\Big\downarrow$ 16

$$\frac{a^2\left(\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(7/2),x]
```

output
```
(-4*a^3*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(
7/2)) + (a^2*(-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
+ f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*S
ec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])))/c^2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

method	result
default	$\frac{\left(-192\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+192\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+89\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-27\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-}{96f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c^{3}}$
risch	$\frac{a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^{3}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}}-\frac{2a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^{3}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(15\,e^{5i(fx+e)}-36\right.}{3c^{3}\left(e^{i(fx+e)}+1\right)\left(e^{i(\right.}}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/f*(-192*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^6+192*ln(csc(
1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^6+89*cos(1/2*f*x+1/2
*e)^6-27*cos(1/2*f*x+1/2*e)^4-117*cos(1/2*f*x+1/2*e)^2+71)*(a/(2*cos(1/2*f
*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a^2/(-c/(2*cos(1/2*f*x+1/2*e)^2
-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^3*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^
5
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^4*sec(f*x + e)^4 - 4*c^4*sec(f*x + e)
^3 + 6*c^4*sec(f*x + e)^2 - 4*c^4*sec(f*x + e) + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3738 vs. $2(134) = 268$.

Time = 1.68 (sec) , antiderivative size = 3738, normalized size of antiderivative = 25.26

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
-1/3*(3*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 675*(f*x + e)*a^2*cos(4*f*x + 4
*e)^2 + 675*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*cos(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3*(f*x + e)*a^2*sin(6*f
*x + 6*e)^2 + 675*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 675*(f*x + e)*a^2*sin
(2*f*x + 2*e)^2 + 108*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 1200*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 108*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 90*(f*x + e)*a^2*cos(2*f*x + 2*e) + 3*(f*x + e)*a^2
- 72*a^2*sin(2*f*x + 2*e) - 6*(a^2*cos(6*f*x + 6*e)^2 + 225*a^2*cos(4*f*x
+ 4*e)^2 + 225*a^2*cos(2*f*x + 2*e)^2 + 36*a^2*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 36*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + a^2*sin(6*f*x + 6*e)^2 + 225*a^2*sin(4*f*x + 4*e)^2 + 450*a^2*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*a^2*sin(2*f*x + 2*e)^2 + 36*a^2*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*a^2*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*a^2*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*a^2*cos(2*f*x + 2*e) + a^2 + 2*(15*a
^2*cos(4*f*x + 4*e) + 15*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) +...
```

Giac [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.35

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\frac{6\sqrt{-ac}a^3\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^4|a|} - \frac{6\sqrt{-ac}a^3\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^4|a|} - \frac{11\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^3\sqrt{-ac}a^3+27\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{a^3c^4|a|}}{6f}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
-1/6*(6*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^4*abs(a)) - 6
*sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^4*abs(a)) - (11
*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*c)*a^3 + 27*(a*tan(1/2*f*x + 1/2
*e)^2 - a)^2*sqrt(-a*c)*a^4 + 24*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)
*a^5 + 7*sqrt(-a*c)*a^6)/(a^3*c^4*abs(a)*tan(1/2*f*x + 1/2*e)^6))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{7/2}}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{7/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^4-4\sec(fx+e)^3+6\sec(fx+e)^2-4\sec(fx+e)+1}dx + 2\left(\int \frac{\sqrt{\sec(}}{\sec(fx+e)^4}\right.\right.}{}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)*
*2 - 4*sec(e + f*x) + 1),x) + 2*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e
+ f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
 f*x)**2 - 4*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - se
c(e + f*x) + 1))/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2
- 4*sec(e + f*x) + 1),x)))/c**4
```

3.108 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 194

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}} - \frac{a^3\tan(e+fx)}{2c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{a^3\tan(e+fx)}{c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-a^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(9/2)-1/2*a^3*ta
n(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)-a^3*tan(f*x+e
)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a^3*ln(1-cos(f*x+e))
*tan(f*x+e)/c^4/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.65 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.55

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3\left(-2\log(\cos(e+fx))-2\log(1-\sec(e+fx))+\frac{2+(-1+\sec(e+fx))^2-2(-1+\sec(e+fx))^3}{(-1+\sec(e+fx))^4}\right)\tan(e+fx)}{2c^4f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

output
```
-1/2*(a^3*(-2*Log[Cos[e + f*x]] - 2*Log[1 - Sec[e + f*x]] + (2 + (-1 + Sec
[e + f*x])^2 - 2*(-1 + Sec[e + f*x])^3)/(-1 + Sec[e + f*x])^4)*Tan[e + f*x
])/(c^4*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4398, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2 \int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{c^2} - \frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 4395

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2} - \frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2} - \frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 4395

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c} - \frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2} - \frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c} - \frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c} - \frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2} - \frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 4399

$$\frac{a^2\left(\frac{\frac{a\tan(e+fx)\int\frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

$$\Big\downarrow 16$$

$$\frac{a^2\left(\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}\right)}{c^2}-\frac{a^3\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(9/2),x]
```

output
```
-((a^3*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(9/2
))) + (a^2*(-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e
 + f*x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*S
ec[e + f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a
+ a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c))/c^2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\left(4096\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-4096\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+2109\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}-2292\cos\left(\frac{f}{2}\right.\right.}{2048f\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-}}}$
risch	$\frac{a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)x}{c^{4}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}}-\frac{2a^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)(fx+e)}{c^{4}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(6\,e^{7i(fx+e)}-23\,e\right.}{c^{4}\left(e^{i(f\right.}}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2048/f*(4096*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^8-4096*ln(
csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^8+2109*cos(1/2*f
*x+1/2*e)^8-2292*cos(1/2*f*x+1/2*e)^6-2450*cos(1/2*f*x+1/2*e)^4+4364*cos(1
/2*f*x+1/2*e)^2-1603)*(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^
(1/2)*a^2/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^4*s
ec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^7
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="fric
as")
```

output
```
integral(-(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
 e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^5*sec(f*x + e)^5 - 5*c^5*sec(f*x + e
)^4 + 10*c^5*sec(f*x + e)^3 - 10*c^5*sec(f*x + e)^2 + 5*c^5*sec(f*x + e) -
 c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6134 vs. $2(176) = 352$.

Time = 11.09 (sec) , antiderivative size = 6134, normalized size of antiderivative = 31.62

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*a^2*cos(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*cos(6*f*x + 6*e)^2
+ 4900*(f*x + e)*a^2*cos(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*cos(2*f*x + 2*
e)^2 + 64*(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 3136*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 3136*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 64*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + (f*x + e)*a^2*sin(8*f*x + 8*e)^2 + 784*(f*x + e)*a^2*sin(6*f*x
+ 6*e)^2 + 4900*(f*x + e)*a^2*sin(4*f*x + 4*e)^2 + 784*(f*x + e)*a^2*sin(2
*f*x + 2*e)^2 + 64*(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 3136*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + 3136*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 64*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 56*(f*x + e)*a^2*cos(2*f*x + 2*e) + (f*x + e)*a^2 - 46
*a^2*sin(2*f*x + 2*e) - 2*(a^2*cos(8*f*x + 8*e)^2 + 784*a^2*cos(6*f*x + 6*
e)^2 + 4900*a^2*cos(4*f*x + 4*e)^2 + 784*a^2*cos(2*f*x + 2*e)^2 + 64*a^2*c
os(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*a^2*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*a^2*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(8*f*x + 8*e)^2 + 784*a^2*sin(6*f*x +
 6*e)^2 + 4900*a^2*sin(4*f*x + 4*e)^2 + 3920*a^2*sin(4*f*x + 4*e)*sin(2...
```

Giac [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{\frac{48\sqrt{-ac}a^3\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^5|a|}-\frac{48\sqrt{-ac}a^3\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^5|a|}-\frac{100\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^4\sqrt{-ac}a^3+352\left(a\tan\left(\frac{1}{2}f\right.\right.}{}}{48f}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac
")
```

output
```
-1/48*(48*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^5*abs(a)) -
 48*sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^5*abs(a)) -
(100*(a*tan(1/2*f*x + 1/2*e)^2 - a)^4*sqrt(-a*c)*a^3 + 352*(a*tan(1/2*f*x
+ 1/2*e)^2 - a)^3*sqrt(-a*c)*a^4 + 480*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sq
rt(-a*c)*a^5 + 292*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^6 + 67*sqrt
(-a*c)*a^7)/(a^4*c^5*abs(a)*tan(1/2*f*x + 1/2*e)^8))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c - c\sec(e + fx))^{9/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^5-5\sec(fx+e)^4+10\sec(fx+e)^3-10\sec(fx+e)^2+5\sec(fx+e)-1}dx\right)\right.}{}$$

input

```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**2)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f
*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1),x) - 2*int((sqrt(sec(e +
 f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**5 - 5*se
c(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) -
 1),x) - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f
*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*s
ec(e + f*x) - 1),x)))/c**5
```

3.109 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx =& -\frac{4a^3\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{11/2}}\\&-\frac{a^3\tan(e+fx)}{3c^2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\\&-\frac{a^3\tan(e+fx)}{2c^3f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\\&-\frac{a^3\tan(e+fx)}{c^4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\\&+\frac{a^3\log(1-\cos(e+fx))\tan(e+fx)}{c^5f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
-4/5*a^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(11/2)-1/3*a
^3*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2)-1/2*a^3*
tan(f*x+e)/c^3/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2)-a^3*tan(f*x
+e)/c^4/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2)+a^3*ln(1-cos(f*x+e
))*tan(f*x+e)/c^5/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.49

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx =$$
$$-\frac{a^3\left(-30\log(\cos(e+fx))-30\log(1-\sec(e+fx))+\frac{-24-10(-1+\sec(e+fx))^2+15(-1+\sec(e+fx))^3-30(-1+\sec(e+fx}{(-1+\sec(e+fx))^5}\right.}{30c^5f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

output
```
-1/30*(a^3*(-30*Log[Cos[e + f*x]] - 30*Log[1 - Sec[e + f*x]] + (-24 - 10*(
-1 + Sec[e + f*x])^2 + 15*(-1 + Sec[e + f*x])^3 - 30*(-1 + Sec[e + f*x])^4
)/(-1 + Sec[e + f*x])^5)*Tan[e + f*x])/(c^5*f*Sqrt[a*(1 + Sec[e + f*x])]*S
qrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {3042, 4398, 3042, 4395, 3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c-c\sec(e+fx))^{11/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{11/2}}dx$$
$$\Big\downarrow \text{ 4398}$$
$$\frac{a^2\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{7/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{5/2}}dx}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{(c-c\sec(e+fx))^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right)}{c^2}-\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \text{4395}$$

$$a^2\left(\frac{\frac{\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c-c\sec(e+fx)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}\right.$$

$$\overline{\qquad c^2 \qquad}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \ 3042$$

$$a^2\left(\frac{\frac{\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx)}\right.$$

$$\overline{\qquad c^2 \qquad}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \ 4399$$

$$a^2\left(\frac{\frac{\frac{a\tan(e+fx)\int \frac{1}{c\cos(e+fx)-c}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e-}{3f\sqrt{a\sec(e+fx)+a}(}\right.$$

$$\overline{\qquad c^2 \qquad}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

$$\Big\downarrow \ 16$$

$$a^2\left(\frac{\frac{\frac{a\tan(e+fx)\log(1-\cos(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}}{c}-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}}{c}-\frac{a\tan(e+fx)}{3f\sqrt{a\sec(e+fx)+a}(c-c\,\mathrm{se}}\right.$$

$$\overline{\qquad c^2 \qquad}$$

$$\frac{4a^3\tan(e+fx)}{5f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{11/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c - c*Sec[e + f*x])^(11/2),x]
```

output
```
(-4*a^3*Tan[e + f*x])/(5*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(
11/2)) + (a^2*(-1/3*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Se
c[e + f*x])^(7/2)) + (-1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c
 - c*Sec[e + f*x])^(5/2)) + (-((a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]
]*(c - c*Sec[e + f*x])^(3/2))) + (a*Log[1 - Cos[e + f*x]]*Tan[e + f*x])/(c
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))/c)/c))/c^2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\left(61440\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-61440\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}-34489\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+6492\cdots\right.}{30720f\sqrt{\cdots}}$
risch	$\frac{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)x}{c^5\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)(fx+e)}{c^5\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(105\,e^{9i(fx+e)}-5\cdots\right.}{\cdots}$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/30720/f*(61440*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^10-61440
*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^10-34489*cos
(1/2*f*x+1/2*e)^10+64925*cos(1/2*f*x+1/2*e)^8+12230*cos(1/2*f*x+1/2*e)^6-1
13350*cos(1/2*f*x+1/2*e)^4+95075*cos(1/2*f*x+1/2*e)^2-25159)*(a/(2*cos(1/2
*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*a^2/(-c/(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/c^5*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e
)^9
```

Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{(-c\sec(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="fri
cas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^6*sec(f*x + e)^6 - 6*c^6*sec(f*x + e)
^5 + 15*c^6*sec(f*x + e)^4 - 20*c^6*sec(f*x + e)^3 + 15*c^6*sec(f*x + e)^2
 - 6*c^6*sec(f*x + e) + c^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9150 vs. $2(218) = 436$.

Time = 66.61 (sec) , antiderivative size = 9150, normalized size of antiderivative = 37.50

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="max
ima")
```

output
```
-1/15*(15*(f*x + e)*a^2*cos(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*cos(8*f
*x + 8*e)^2 + 661500*(f*x + e)*a^2*cos(6*f*x + 6*e)^2 + 661500*(f*x + e)*a
^2*cos(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*cos(2*f*x + 2*e)^2 + 1500*(f*x
 + e)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*
(f*x + e)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952
560*(f*x + e)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 216000*(f*x + e)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 1500*(f*x + e)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 15*(f*x + e)*a^2*sin(10*f*x + 10*e)^2 + 30375*(f*x + e)*a^2*sin(8*f
*x + 8*e)^2 + 661500*(f*x + e)*a^2*sin(6*f*x + 6*e)^2 + 661500*(f*x + e)*a
^2*sin(4*f*x + 4*e)^2 + 30375*(f*x + e)*a^2*sin(2*f*x + 2*e)^2 + 1500*(f*x
 + e)*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 216000*
(f*x + e)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 952
560*(f*x + e)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 216000*(f*x + e)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 1500*(f*x + e)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 1350*(f*x + e)*a^2*cos(2*f*x + 2*e) + 15*(f*x + e)*a^2 - 1110*a^2*s
in(2*f*x + 2*e) - 30*(a^2*cos(10*f*x + 10*e)^2 + 2025*a^2*cos(8*f*x + 8*e)
^2 + 44100*a^2*cos(6*f*x + 6*e)^2 + 44100*a^2*cos(4*f*x + 4*e)^2 + 2025*a^
2*cos(2*f*x + 2*e)^2 + 100*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*...
```

Giac [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.07

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{\frac{120\sqrt{-ac}a^3\log\left(|a|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c^6|a|} - \frac{120\sqrt{-ac}a^3\log\left(\left|-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right|\right)}{c^6|a|} - \frac{274\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)^5\sqrt{-ac}a^3+1250\left(a\tan\left(\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="gia
c")
```

output
```
-1/120*(120*sqrt(-a*c)*a^3*log(abs(a)*tan(1/2*f*x + 1/2*e)^2)/(c^6*abs(a))
 - 120*sqrt(-a*c)*a^3*log(abs(-a*tan(1/2*f*x + 1/2*e)^2 - a))/(c^6*abs(a))
 - (274*(a*tan(1/2*f*x + 1/2*e)^2 - a)^5*sqrt(-a*c)*a^3 + 1250*(a*tan(1/2*
f*x + 1/2*e)^2 - a)^4*sqrt(-a*c)*a^4 + 2320*(a*tan(1/2*f*x + 1/2*e)^2 - a)
^3*sqrt(-a*c)*a^5 + 2165*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*sqrt(-a*c)*a^6 +
 1015*(a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*c)*a^7 + 191*sqrt(-a*c)*a^8)/
(a^5*c^6*abs(a)*tan(1/2*f*x + 1/2*e)^10))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c - c/cos(e + f*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^6-6\sec(fx+e)^5+15\sec(fx+e)^4-20\sec(fx+e)^3+15\sec(fx+e)^2-6\sec(fa}\right.}{}$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e + f*x)
**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),x) + 2
*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(
e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3
+ 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1
)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec
(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) +
1),x)))/c**6
```

3.110 $\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 204

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{8c^4\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{4c^4\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{c^4\sec^2(e+fx)\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c^4*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+8*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)-4*c^4*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+1/2*c^4*sec(f*x+e)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.41

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^4(2(\log(\cos(e+fx))+8\log(1+\sec(e+fx)))-8\sec(e+fx)+\sec^2(e+$$
$$2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^4*(2*(Log[Cos[e + f*x]] + 8*Log[1 + Sec[e + f*x]]) - 8*Sec[e + f*x] + S
ec[e + f*x]^2)*Tan[e + f*x])/(2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Se
c[e + f*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^3\cos(e+fx)(1-\sec(e+fx))^3}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))^3}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 93$$

$$-\frac{c^4\tan(e+fx)\int\left(\cos(e+fx)-\sec(e+fx)-\frac{8}{\sec(e+fx)+1}+4\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 2009$$

$$-\frac{c^4\tan(e+fx)\left(-\frac{1}{2}\sec^2(e+fx)+4\sec(e+fx)+\log(\sec(e+fx))-8\log(\sec(e+fx)+1)\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(7/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c^4*(Log[Sec[e + f*x]] - 8*Log[1 + Sec[e + f*x]] + 4*Sec[e + f*x] - Sec
[e + f*x]^2/2)*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e
+ f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(186) = 372$.

Time = 2.97 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.05

method	result
default	$\frac{\left(\left(28\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-28\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+7\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(28\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-28\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2+7\right)\ln\left(-\frac{2(\cdots}{\cdots}\right.}{\cdots}$
risch	$\frac{c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)}-\frac{2c^3\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}+\frac{2ic^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(4e^{2i(fx+e)}-e^{i(fx\cdots}\right.}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\cdots\right.}$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((28*cos(1/2*f*x+1/2*e)^4-28*cos(1/2*f*x+1/2*e)^2+7)*ln(-2*(cos(1/2*f*
x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(28*cos(1/2*f*x+1/2*e
)^4-28*cos(1/2*f*x+1/2*e)^2+7)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e
))/(cos(1/2*f*x+1/2*e)+1))+(-64*cos(1/2*f*x+1/2*e)^4+64*cos(1/2*f*x+1/2*e)
^2-16)*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)+(-64*cos(1/2*f*x+1/2*e
)^4+64*cos(1/2*f*x+1/2*e)^2-16)*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-
1)+(8*cos(1/2*f*x+1/2*e)^4-8*cos(1/2*f*x+1/2*e)^2+2)*ln(2/(cos(1/2*f*x+1/2
*e)+1))+(14*cos(1/2*f*x+1/2*e)^2-8)*sin(1/2*f*x+1/2*e)^2)*(-c/(2*cos(1/2*f
*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^3/(a/(2*cos(1/2*f*x+1/2*e)^2-
1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(4*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e
)^2+1)*cot(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{7}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e)
- c^3)*sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^3\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)+1}dx\right)+3\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{}}{\mathrm{se}}\right.\right.}{}$$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**3*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3)/(sec(e + f*x) + 1),x) + 3*int((sqrt(sec(e + f*x) +
1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x) - 3*in
t((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e +
 f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(se
c(e + f*x) + 1),x)))/a
```

3.111 $\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c^3\sec(e+fx)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c^3*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+4*c^3*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)-c^3*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.46

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{c^3(\log(\cos(e+fx))+4\log(1+\sec(e+fx))-\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^3*(Log[Cos[e + f*x]] + 4*Log[1 + Sec[e + f*x]] - Sec[e + f*x])*Tan[e +
f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^2\cos(e+fx)(1-\sec(e+fx))^2}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^3\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))^2}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 93$$

$$-\frac{c^3\tan(e+fx)\int\left(\cos(e+fx)-\frac{4}{\sec(e+fx)+1}+1\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 2009$$

$$-\frac{c^3\tan(e+fx)(\sec(e+fx)+\log(\sec(e+fx))-4\log(\sec(e+fx)+1))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(5/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c^3*(Log[Sec[e + f*x]] - 4*Log[1 + Sec[e + f*x]] + Sec[e + f*x])*Tan[e
+ f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(139) = 278.

Time = 2.96 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.17

method	result
default	$\frac{\left(\left(6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3\right)\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-2\right)\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+8\right)}{}$
risch	$-\frac{c^{2}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\left(8ie^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)-2ie^{i(fx+e)}+e^{3i(fx+e)}fx+8ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)-3i\ln\left(e^{2i(fx+e)}+1\right)+8\right.}{}$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((6*cos(1/2*f*x+1/2*e)^2-3)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*
e))/(cos(1/2*f*x+1/2*e)+1))+(4*cos(1/2*f*x+1/2*e)^2-2)*ln(2/(cos(1/2*f*x+1
/2*e)+1))+(-16*cos(1/2*f*x+1/2*e)^2+8)*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+
1/2*e)-1)+(-16*cos(1/2*f*x+1/2*e)^2+8)*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+
1/2*e)+1)+(6*cos(1/2*f*x+1/2*e)^2-3)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x
+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+2*sin(1/2*f*x+1/2*e)^2)*(-c/(2*cos(1/2*f*
x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^2/(2*cos(1/2*f*x+1/2*e)^2-1)/(
a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)*cot(1/2*f*x+1/2*e
)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(-c*sec(f*x +
 e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\Big(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}dx - 2\Big(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(}{\sec(fx+e}}{a}$$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x) - 2*int((sqrt(sec(e + f*x) + 1)*
sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x) + int((sqrt(
sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x) + 1),x)))/a
```

3.112 $\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 102

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^2\log(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c^2*ln(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*
x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c^2(\log(\cos(e+fx))+2\log(1+\sec(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c^2*(Log[Cos[e + f*x]] + 2*Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a
*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c\cos(e+fx)(1-\sec(e+fx))}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^2\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{c^2\tan(e+fx)\int \left(\cos(e+fx)-\frac{2}{\sec(e+fx)+1}\right)d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\tan(e+fx)(\log(\sec(e+fx))-2\log(\sec(e+fx)+1))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(3/2)/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c^2*(Log[Sec[e + f*x]] - 2*Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt
[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(94) = 188$.

Time = 2.93 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.17

method	result
default	$\frac{\left(\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\right.\right.}{f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}}$
risch	$\frac{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)}-\frac{2c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}-\frac{4ic\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\ln(e^{i(fx}}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))
-4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)+2*ln(2/(cos(1/2*f*x+1/2*e)
+1))-4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)+ln(-2*(cos(1/2*f*x+1/2
*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1)))*(-c/(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*
f*x+1/2*e)^2)^(1/2)*cot(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral((-c*sec(f*x + e) + c)^(3/2)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c(\sec{(e+fx)}-1))^{\frac{3}{2}}}{\sqrt{a\left(\sec{(e+fx)}+1\right)}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{((fx+e)c+c\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-4\,c\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)}{\sqrt{a}f}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c + c*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*c*ar
ctan2(sin(f*x + e), cos(f*x + e) + 1))*sqrt(c)/(sqrt(a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(}}{\sec(fx+e)+1}\right.}{a}$$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x))/(sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt(
 - sec(e + f*x) + 1))/(sec(e + f*x) + 1),x)))/a
```

3.113 $\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
c*ln(1+cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{c(\log(\cos(e+fx))+\log(1+\sec(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c*(Log[Cos[e + f*x]] + Log[1 + Sec[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a*(1
+ Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4399}$$

$$\frac{ac\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[c - c*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{-c(-1+\sec(fx+e))}\,\sqrt{a(1+\sec(fx+e))}\,\ln\left(\frac{2}{\cos(fx+e)+1}\right)\cot(fx+e)}{fa}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)} - \frac{2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f} - \frac{2i\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\ln\left(e^{i(fx+e)}\right.}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/a*(-c*(-1+sec(f*x+e)))^(1/2)*(a*(1+sec(f*x+e)))^(1/2)*ln(2/(cos(f*x+e)
+1))*cot(f*x+e)
```

Fricas [F]

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\sqrt{-c\sec\left(fx + e\right) + c}}{\sqrt{a\sec\left(fx + e\right) + a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\sqrt{-c\left(\sec\left(e + fx\right) - 1\right)}}{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}{\sqrt{-a}f}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\sqrt{c - \frac{c}{\cos(e+f\,x)}}}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{\sqrt{a + a\sec(e + fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x) + 1),x))/a
```

3.114 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\log(\sin(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(sin(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{(\log(\cos(e+fx))+\log(\tan(e+fx)))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
((Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[
e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4393, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4393}$$

$$-\frac{\tan(e+fx)\int -\cot(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \cot(e+fx)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx}{\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan(e+fx)\log(-\sin(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
(Log[-Sin[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*S
ec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(42) = 84$.

Time = 2.82 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.33

method	result
default	$-\frac{\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\left(2\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{i\left(e^{i(fx+e)}-\right.}{\sqrt{\frac{a\left(e^{i(fx+e)}+\right.}{e^{2i(fx+e)}+}}}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/f*sin(1/2*f*x+1/2*e)*(2*ln(2/(cos(1/2*f*x+1/2*e)+1))-ln(-cot(1/2*f*x+1/
2*e)+csc(1/2*f*x+1/2*e)+1)-ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))-ln(-c
ot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1))*cos(1/2*f*x+1/2*e)/(2*cos(1/2*f*x
+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c
/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.91

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(256\cos(fx+e)^5-512\cos(fx+e)^3+175\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}-\left(256\,ac\cos(fx+e)^4-5\right.}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right.}{2\,acf}\right.$$
$$\left.-\frac{\sqrt{ac}\arctan\left(\frac{\left(16\cos(fx+e)^3-7\cos(fx+e)\right)\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{\left(16\,ac\cos(fx+e)^2-25\,ac\right)\sin(fx+e)}\right)}{acf}\right]$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/2*sqrt(-a*c)*log(-8*((256*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*co
s(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x +
 e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f
), -sqrt(a*c)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/
((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{fx+e-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-(f*x + e - arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt
(c)*f)
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\frac{\sqrt{-ac}\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{a|c|} - \frac{2\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+c\right|\right)}{a|c|}}{2\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
1/2*(sqrt(-a*c)*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a*abs(c)) - 2*sqrt(-a*
c)*log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(a*abs(c)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^2-1}dx\right)}{ac}$$

input

```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output

```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x)**2 - 1),x))/(a*c)
```

3.115 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{2cf(1-\cos(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{3\log(1-\cos(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1+\cos(e+fx))\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*tan(f*x+e)/c/f/(1-cos(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)+3/4*ln(1-cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+1/4*ln(1+cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(
c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\left(4\log(\cos(e+fx))+3\log(1-\sec(e+fx))+\log(1+\right.}{4cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
((4*Log[Cos[e + f*x]] + 3*Log[1 - Sec[e + f*x]] + Log[1 + Sec[e + f*x]] +
2/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(4*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqr
t[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{ac^2(1-\sec(e+fx))^2(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^2(\sec(e+fx)+1)}d\sec(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\tan(e+fx)\int\left(\cos(e+fx)-\frac{3}{4(\sec(e+fx)-1)}-\frac{1}{4(\sec(e+fx)+1)}+\frac{1}{2(\sec(e+fx)-1)^2}\right)d\sec(e+fx)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{1}{2(1-\sec(e+fx))}-\frac{3}{4}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{1}{4}\log(\sec(e+fx)+1)\right)}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
-((((-3*Log[1 - Sec[e + f*x]])/4 + Log[Sec[e + f*x]] - Log[1 + Sec[e + f*x
]]/4 + 1/(2*(1 - Sec[e + f*x])))*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.48

method	result
default	$\frac{\left(-16\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+12\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c\sqrt{}}$
risch	$\frac{3ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-3i\ln\left(e^{i(fx+e)}-1\right)-2\,e^{3i(fx+e)}fx+ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+2ie^{i(fx+e)}-4\,e^{3i(fx+e)}e+2\,e^{2i(fx+e)}}{2}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/8/f*(-16*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^2+12*ln(csc(1/2
*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2+4*ln(-cot(1/2*f*x+1/2
*e)+csc(1/2*f*x+1/2*e)-1)*sin(1/2*f*x+1/2*e)^2+4*ln(-cot(1/2*f*x+1/2*e)+cs
c(1/2*f*x+1/2*e)+1)*sin(1/2*f*x+1/2*e)^2+cos(1/2*f*x+1/2*e)^2+1)/(2*cos(1/
2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2
)/c/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*cot(1/2*f*x
+1/2*e)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^2*sec(f*x
 + e)^3 - a*c^2*sec(f*x + e)^2 - a*c^2*sec(f*x + e) + a*c^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(148) = 296$.

Time = 0.21 (sec) , antiderivative size = 818, normalized size of antiderivative = 4.87

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="ma
xima")
```

output
```
-1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x
+ e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - (cos
(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 1) - 3*(cos(2*f*x + 2*e)^2 - 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 - 4*sin(2*f*x + 2*e)*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(4*f*x +
4*(f*x + e)*cos(2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(
2*f*x + 2*e) - 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2
*e)/((c*cos(2*f*x + 2*e)^2 + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(...
```

Giac [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\frac{3\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}|c|}-\frac{4\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}|c|}-\frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}c|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{4f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gi
ac")
```

output
```
-1/4*(3*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*abs(c)) - 4*log(abs
(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*abs(c)) - (3*c*tan(1/2*f*x + 1
/2*e)^2 - c)/(sqrt(-a*c)*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^{3}-\sec(fx+e)^{2}-\sec(fx+e)+1}dx\right)}{a\,c^{2}}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x)**3 - sec(e + f*x)**2 - sec(e + f*x) + 1),x))/(a*c**2)
```

3.116 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 274

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\frac{\log(\cos(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{7\log(1-\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(1+\sec(e+fx))\tan(e+fx)}{8c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4c^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)}{4c^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+7/8*ln(1-sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+1/8*ln(1+sec(f*x+e))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)
/(c-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))^2/(a+a*sec(f*x
+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/c^2/f/(1-sec(f*x+e))/(a+a
*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(8\log(\cos(e+fx))+7\log(1-\sec(e+fx))+\log(1+\right.}{8c^2f\sqrt{a(1+\sec(e+}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
((8*Log[Cos[e + f*x]] + 7*Log[1 - Sec[e + f*x]] + Log[1 + Sec[e + f*x]] -
2/(-1 + Sec[e + f*x])^2 + 6/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(8*c^2*f*Sq
rt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{ac^3(1-\sec(e+fx))^3(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^3(\sec(e+fx)+1)}d\sec(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\tan(e+fx)\int\left(\cos(e+fx)-\frac{7}{8(\sec(e+fx)-1)}-\frac{1}{8(\sec(e+fx)+1)}+\frac{3}{4(\sec(e+fx)-1)^2}-\frac{1}{2(\sec(e+fx)-1)^3}\right)d\sec(e+fx)}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{3}{4(1-\sec(e+fx))}+\frac{1}{4(1-\sec(e+fx))^2}-\frac{7}{8}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{1}{8}\log(\sec(e+fx)+\right.}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
-((((-7*Log[1 - Sec[e + f*x]])/8 + Log[Sec[e + f*x]] - Log[1 + Sec[e + f*x
]]/8 + 1/(4*(1 - Sec[e + f*x])^2) + 3/(4*(1 - Sec[e + f*x])))*Tan[e + f*x]
)/(c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\left(-224\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-32\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-32\ln\left(-\cot\left(\frac{fx}{2}+\right.\right.}{128f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{c^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{c^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)f}+\frac{}{2c^{2}\sqrt{\frac{a\left(e^{i(\right.}}{e^{2i}}}}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/128/f*(-224*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e
)^4-32*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*sin(1/2*f*x+1/2*e)^4-3
2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*sin(1/2*f*x+1/2*e)^4+256*ln
(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^4+37*cos(1/2*f*x+1/2*e)^4+6*
cos(1/2*f*x+1/2*e)^2-35)/(2*cos(1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*
e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2
*f*x+1/2*e)^2)^(1/2)/c^2*cot(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)^2
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a*c^3*sec(f*
x + e)^4 - 2*a*c^3*sec(f*x + e)^3 + 2*a*c^3*sec(f*x + e) - a*c^3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{a(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(5/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(242) = 484$.

Time $= 0.31$ (sec) , antiderivative size $= 2206$, normalized size of antiderivative $= 8.05$

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="ma
xima")
```

output
```
-1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 +
64*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(
f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x +
 e)*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - (2*(6*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^
2 - 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) - 4*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 -
8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 3
6*sin(2*f*x + 2*e)^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) - 4*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 - 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), ...
```

Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{14\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}c|c|}-\frac{16\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}c|c|}-\frac{21\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+34\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+14c^2}{\sqrt{-ac}c^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}{16f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gi
ac")
```

output
```
-1/16*(14*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*c*abs(c)) - 16*lo
g(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*c*abs(c)) - (21*(c*tan(1/
2*f*x + 1/2*e)^2 - c)^2 + 34*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 14*c^2)/(s
qrt(-a*c)*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\,\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^4-2\sec(fx+e)^3+2\sec(fx+e)-1}dx\right)}{a\,c^3}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x) - 1),x))/(a*c**3)
```

3.117 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 215

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &\quad - \frac{4c^4\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &\quad + \frac{c^4\sec(e+fx)\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\ &\quad - \frac{8c^4\tan(e+fx)}{af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
c^4*ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)-4*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*se
c(f*x+e))^(1/2)+c^4*sec(f*x+e)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)-8*c^4*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/
2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.45

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-c^3\log(\cos(e+fx))+4c^3\log(1+\sec(e+fx))-c^3\sec(e+fx)+\frac{8c^3}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*(-(c^3*Log[Cos[e + f*x]]) + 4*c^3*Log[1 + Sec[e + f*x]] - c^3*Sec[e +
 f*x] + (8*c^3)/(1 + Sec[e + f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e +
 f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^3\cos(e+fx)(1-\sec(e+fx))^3}{a^2(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\tan(e+fx)\int\frac{\cos(e+fx)(1-\sec(e+fx))^3}{(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{c^4\tan(e+fx)\int\left(\cos(e+fx)+\frac{4}{\sec(e+fx)+1}-\frac{8}{(\sec(e+fx)+1)^2}-1\right)d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4\tan(e+fx)\left(-\sec(e+fx)+\frac{8}{\sec(e+fx)+1}+\log(\sec(e+fx))+4\log(\sec(e+fx)+1)\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c^4*(Log[Sec[e + f*x]] + 4*Log[1 + Sec[e + f*x]] - Sec[e + f*x] + 8/(1
+ Sec[e + f*x]))*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Se
c[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.85

method	result
default	$\frac{\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\,c^3\left(\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-8\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(-10\cos\left(\frac{fx}{2}+\frac{e}{2}\right.\right.\right.}{}$
risch	$-\frac{c^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(2e+fx+4\,e^{2i(fx+e)}e+18ie^{i(fx+e)}+5i\ln\left(e^{2i(fx+e)}+1\right)-16i\ln\left(e^{i(fx+e)}+1\right)e^{3i(fx+e)}+5ie^{4i(fx+e)}\ln\left(e^{2i(\right.\right.}{}$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^3/(2*cos(
1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1
/2)/a*(ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*(16*cos(1/2*f*x+1/2*e)
^2*cot(1/2*f*x+1/2*e)-8*cot(1/2*f*x+1/2*e))+(-10*cos(1/2*f*x+1/2*e)^2+5)*l
n(2*(-cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1
/2*f*x+1/2*e)+(-10*cos(1/2*f*x+1/2*e)^2+5)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1
/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)+(4*cos(1/2*f*x+1
/2*e)^2-2)*ln(2/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)+ln(-cot(1/2*f*x
+1/2*e)+csc(1/2*f*x+1/2*e)-1)*(16*cos(1/2*f*x+1/2*e)^2*cot(1/2*f*x+1/2*e)-
8*cot(1/2*f*x+1/2*e))+(-10*cos(1/2*f*x+1/2*e)^2+4)*tan(1/2*f*x+1/2*e))
```

Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{(-c\sec(fx + e) + c)^{\frac{7}{2}}}{(a\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e)
- c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e
)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2393 vs. $2(199) = 398$.

Time = 0.35 (sec) , antiderivative size = 2393, normalized size of antiderivative = 11.13

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c^3*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*cos(2*f*x + 2*e)^2 +
(f*x + e)*c^3*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*c^3*sin(2*f*x + 2*e)^2 + 4*
(f*x + e)*c^3*cos(2*f*x + 2*e) + (f*x + e)*c^3 - 4*c^3*sin(2*f*x + 2*e) +
4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*
cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3
- 5*c^3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 4*((f*x + e)*c^3 - 5*c^3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 - 5*(c^3*cos(4*f*x + 4*e)^2 + 4*c^3*cos(2*f*x + 2*e)^2 + c^3*sin(4*f*x
 + 4*e)^2 + 4*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^3*sin(2*f*x + 2*
e)^2 + 4*c^3*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x + 2*e) + c^3)*cos
(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 8*(c^3*co
s(4*f*x + 4*e)^2 + 4*c^3*cos(2*f*x + 2*e)^2 + 4*c^3*cos(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + c^3*sin(4*f*x + 4*e)^2 + 4*c^3*sin(4*f*x + 4*e)*si
n(2*f*x + 2*e) + 4*c^3*sin(2*f*x + 2*e)^2 + 4*c^3*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 4*c^3*cos(2*f*x + 2*e) + c^3 + 2*(2*c^3*cos(2*f*x...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c^3\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)+3\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{}}{\sec(fx+e}\right.\right.}{}$$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**3*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) + 3*int((
sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e +
 f*x)**2 + 2*sec(e + f*x) + 1),x) - 3*int((sqrt(sec(e + f*x) + 1)*sqrt( -
sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)
+ int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x)**2
+ 2*sec(e + f*x) + 1),x)))/a**2
```

3.118 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{4c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-4*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(c
os(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^3(-4+\log(\cos(e+fx))+\log(\cos(e+fx))\sec(e+fx))\tan(e+fx)}{f(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(c^3*(-4 + Log[Cos[e + f*x]] + Log[Cos[e + f*x]]*Sec[e + f*x])*Tan[e + f*x
])/(f*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4398, 3042, 4393, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4398}$$

$$\frac{c^2\int \sqrt{\sec(e+fx)a+a}\sqrt{c-c\sec(e+fx)}dx}{a^2}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4393}$$

$$\frac{c^3\tan(e+fx)\int -\tan(e+fx)dx}{a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c^3\tan(e+fx)\int \tan(e+fx)dx}{a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^3\tan(e+fx)\int\tan(e+fx)dx}{a\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx))}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{4c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-4*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x
]]) + (c^3*Log[Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(88) = 176$.

Time = 3.01 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\, c^2\left(\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-2}{f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}}\, a}$
risch	$-\frac{c^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(ie^{2i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+e^{2i(fx+e)}fx+2ie^{i(fx+e)}\ln\left(e^{2i(fx+e)}+1\right)+2\,e^{2i(fx+e)}e+2\,e^{i(fx+e)}fx+8ie^{i(}\right.}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^2/(a/(2*
cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a*(ln(-2*(cos(1/2*f*x+
1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)+ln(-
2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*
f*x+1/2*e)-2*ln(2/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)+2*tan(1/2*f*x
+1/2*e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(88) = 176.

Time = 0.15 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.84

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,c^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\left(ac^2\cos(fx\right.}{2\,(}\right.$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[-1/2*(4*c^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (a*c^2*cos(f*x + e)^2 + 2*a
*c^2*cos(f*x + e) + a*c^2)*sqrt(-c/a)*log(1/2*(c*cos(f*x + e)^4 - (cos(f*x
 + e)^3 + cos(f*x + e))*sqrt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sin(f*x + e) + c)/cos(f*x + e)^2)
)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -(2*c^2*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) + (a*c^2*cos(f*x + e)^2 + 2*a*c^2*cos(f*x + e) + a*
c^2)*sqrt(c/a)*arctan((cos(f*x + e)^3 + cos(f*x + e))*sqrt(c/a)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c*
cos(f*x + e)^2 - c)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*
x + e) + a^2*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{3/2}} \, dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}} \, dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2+2\sec(fx+e)+1}dx - 2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(}{\sec(fx+e)^2+2\,\mathrm{s}}\right.\right.}{a^2}$$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) - 2*int((sqr
t(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)*
*2 + 2*sec(e + f*x) + 1),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e +
f*x) + 1))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)))/a**2
```

3.119 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{2c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-2*c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1
+cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-c\log(\cos(e+fx)) - c\log(1+\sec(e+fx)) + \frac{2c}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*(-(c*Log[Cos[e + f*x]]) - c*Log[1 + Sec[e + f*x]] + (2*c)/(1 + Sec[e
+ f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4396, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4396}$$

$$\frac{c\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{c^2\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{c^2\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x
]]) + (c^2*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.85

method	result
default	$\frac{\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,c\left(2\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-2\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-2\ln\left(-\cot\left(\frac{fx}{2}+\cdots\right.\right.\right.}{f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a}$
risch	$-\frac{c\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+e^{2i(fx+e)}fx+4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+2e^{2i(fx+e)}e+2e^{i(fx+e)}fx+4ie^{i(fx\cdots\right.}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c/(a/(2*cos
(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a*(2*ln(2/(cos(1/2*f*x+1/
2*e)+1))*cot(1/2*f*x+1/2*e)-2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)
*cot(1/2*f*x+1/2*e)-2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*cot(1/2
*f*x+1/2*e)-tan(1/2*f*x+1/2*e))
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^2*sec(f*x
 + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}a}-\frac{c^{\frac{3}{2}}\sin(fx+e)^2}{\sqrt{-a}a(\cos(fx+e)+1)^2}}{f}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
(c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a) - c^(3/
2)*sin(f*x + e)^2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^{2}+2\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec}}{\sec(fx+e)^{2}+2\sec(fx}\right.}{a^{2}}$$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x) + int((sqrt(sec
(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1),x)))/a**2
```

3.120 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f
*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{1}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + (1 + Sec[e + f*x])^(-1)
)*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{c\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]])
) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(86) = 172$.

Time = 3.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.96

method	result
default	$\frac{\sqrt{2}\,\sqrt{-\frac{2c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,\left(4\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)-4\ln\left(-\cot\left(\frac{f}{2}\right.\right.\right.}{4f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a}$
risch	$-\frac{\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(2ie^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+e^{2i(fx+e)}fx+4ie^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+2\,e^{2i(fx+e)}e+2\,e^{i(fx+e)}fx+2ie^{i(fx+\right.}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/f*(-2*c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a*(4*ln(2/(cos(
1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)-4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x
+1/2*e)+1)*cot(1/2*f*x+1/2*e)-4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-
1)*cot(1/2*f*x+1/2*e)-tan(1/2*f*x+1/2*e))
```

Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x +
 e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\sqrt{-c\,(\sec\left(e + fx\right) - 1)}}{\left(a\left(\sec\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(86) = 172.

Time = 0.20 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{3/2}}\,dx =$$
$$-\frac{\left((fx + e)\cos\left(2\,fx + 2\,e\right)^2 + 4\,(fx + e)\cos\left(fx + e\right)^2 + (fx + e)\sin\left(2\,fx + 2\,e\right)^2 + 4\,(fx + e)\sin\left(fx + \right.\right.}{}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(2*f*x + 2*e)^2 + 4*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*si
n(2*f*x + 2*e)^2 + 4*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(2*cos(f*x + e)
 + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x + e)^2 + sin(2*f*x
 + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x + e)^2 + 4*cos(f*x
 + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 2*(f*x + e)*
cos(f*x + e) + e - sin(f*x + e))*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f*x +
e) + 2*(2*(f*x + e)*sin(f*x + e) + cos(f*x + e))*sin(2*f*x + 2*e) + e - 2*
sin(f*x + e))*sqrt(a)*sqrt(c)/((a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(f*x + e
)^2 + a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2
*sin(f*x + e)^2 + 4*a^2*cos(f*x + e) + a^2 + 2*(2*a^2*cos(f*x + e) + a^2)*
cos(2*f*x + 2*e))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x)**2 + 2*sec(e + f*x) + 1),x))/a**2
```

3.121 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 215

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\log(\cos(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(1-\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{3\log(1+\sec(e+fx))\tan(e+fx)}{4af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2
)+1/4*ln(1-sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+
e))^(1/2)+3/4*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*
sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)
/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c\left(-\frac{\log(\cos(e+fx))}{c}-\frac{\log(1-\sec(e+fx))}{4c}-\frac{3\log(1+\sec(e+fx))}{4c}+\frac{1}{2c(1+\sec(e+fx))}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-((c*(-(Log[Cos[e + f*x]]/c) - Log[1 - Sec[e + f*x]]/(4*c) - (3*Log[1 + Se
c[e + f*x]])/(4*c) + 1/(2*c*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a
*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.43, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^2c(1-\sec(e+fx))(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\tan(e+fx)\int \left(\cos(e+fx)-\frac{1}{4(\sec(e+fx)-1)}-\frac{3}{4(\sec(e+fx)+1)}-\frac{1}{2(\sec(e+fx)+1)^2}\right)d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{1}{2(\sec(e+fx)+1)}-\frac{1}{4}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{3}{4}\log(\sec(e+fx)+1)\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-(((-1/4*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]] - (3*Log[1 + Sec[e + f*
x]])/4 + 1/(2*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.15

method	result
default	$\frac{\left(6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+6\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+c\right.\right.}{4f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\sqrt{-\frac{c}{2c}}}$
risch	$-\frac{-i\ln\left(\mathrm{e}^{i(fx+e)}-1\right)-2i\mathrm{e}^{i(fx+e)}+2\,\mathrm{e}^{3i(fx+e)}fx+3i\ln\left(\mathrm{e}^{i(fx+e)}+1\right)\mathrm{e}^{3i(fx+e)}+i\mathrm{e}^{2i(fx+e)}\ln\left(\mathrm{e}^{i(fx+e)}-1\right)+4\,\mathrm{e}^{3i(fx+e)}e+2\,\mathrm{e}^{2i(fx-}}{}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/4/f*(6*cos(1/2*f*x+1/2*e)^2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)
-8*cos(1/2*f*x+1/2*e)^2*ln(2/(cos(1/2*f*x+1/2*e)+1))+6*cos(1/2*f*x+1/2*e)^
2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)+2*cos(1/2*f*x+1/2*e)^2*ln(c
sc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))+sin(1/2*f*x+1/2*e)^2)/(2*cos(1/2*f*x
+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c
/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a*tan(1/2*f*x+1/2*
e)
```

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c*sec(f*
x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e) - a^2*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(193) = 386$.

Time = 0.20 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.80

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/2*(2*(f*x + e)*cos(2*f*x + 2*e)^2 + 8*(f*x + e)*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(f*x + e)*sin(2*f*x + 2*e)^2 + 8*(f*x
+ e)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*f*x - 3*(c
os(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 1) - (cos(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e) + 1)*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 1) + 4*(f*x + e)*cos(2*f*x + 2*e) + 2*(4*f*x +
4*(f*x + e)*cos(2*f*x + 2*e) + 4*e + sin(2*f*x + 2*e))*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(4*(f*x + e)*sin(2*f*x + 2*e) - cos(
2*f*x + 2*e) - 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2
*e)/((a*cos(2*f*x + 2*e)^2 + 4*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(...
```

Giac [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.46

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}a|c|} - \frac{\sqrt{-ac}\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^2|c|} + \frac{4\sqrt{-ac}\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{a^2|c|}}{4f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a*abs(c)) - sqrt(-a*c)*lo
g(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(a^2*abs(c)) + 4*sqrt(-a*c)*log(abs(c*tan
(1/2*f*x + 1/2*e)^2 + c))/(a^2*abs(c)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^3+\sec(fx+e)^2-\sec(fx+e)-1}dx\right)}{a^2c}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) - 1),x))/(a**2*c)
```

3.122 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 101

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\cot(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(\sin(e+fx))\tan(e+fx)}{acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*cot(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(sin(
f*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\cot(e+fx)+2(\log(\cos(e+fx))+\log(\tan(e+fx)))}{2acf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+f}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
(Cot[e + f*x] + 2*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Tan[e + f*x])/(2
*a*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4393, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4393}$$

$$\frac{\tan(e+fx)\int -\cot^3(e+fx)dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\int \cot^3(e+fx)dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)^3dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}-\int -\cot(e+fx)dx\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\tan(e+fx)\left(\int \cot(e+fx)dx+\frac{\cot^2(e+fx)}{2f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\left(\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\cot^2(e+fx)}{2f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}-\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan(e+fx)\left(\frac{\cot^2(e+fx)}{2f}+\frac{\log(-\sin(e+fx))}{f}\right)}{ac\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
((Cot[e + f*x]^2/(2*f) + Log[-Sin[e + f*x]]/f)*Tan[e + f*x])/(a*c*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.40

method	result
risch	$-\frac{ie^{4i(fx+e)}\ln\left(e^{2i(fx+e)}-1\right)+e^{4i(fx+e)}fx+2\,e^{4i(fx+e)}e-2ie^{2i(fx+e)}\ln\left(e^{2i(fx+e)}-1\right)-2\,e^{2i(fx+e)}fx-4\,e^{2i(fx+e)}e-2ie^{2i(fx+e)}}{ac\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}$
default	$\frac{\left(16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-32\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+16\cos}{16f(}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-(I*exp(4*I*(f*x+e))*ln(exp(2*I*(f*x+e))-1)+exp(4*I*(f*x+e))*f*x+2*exp(4*I
*(f*x+e))*e-2*I*exp(2*I*(f*x+e))*ln(exp(2*I*(f*x+e))-1)-2*exp(2*I*(f*x+e))
*f*x-4*exp(2*I*(f*x+e))*e-2*I*exp(2*I*(f*x+e))+I*ln(exp(2*I*(f*x+e))-1)+f*
x+2*e)/a/c/(exp(I*(f*x+e))+1)/(a*(exp(I*(f*x+e))+1)^2/(exp(2*I*(f*x+e))+1)
)^(1/2)/(exp(2*I*(f*x+e))+1)/(exp(I*(f*x+e))-1)/(c*(exp(I*(f*x+e))-1)^2/(e
xp(2*I*(f*x+e))+1))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(91) = 182$.

Time = 0.31 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.87

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{9\sqrt{-ac}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{8\left(\left(256\,\cos(fx+e)^5\right.\right.}{}\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fr
icas")
```

output
```
[-1/18*(9*sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-8*((256*cos(f*x + e)^5 - 51
2*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) - (256*a*c*cos(f*x +
 e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x + e))/((cos(f*x + e)^2 -
 1)*sin(f*x + e)))*sin(f*x + e) + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), -1/18*(18*sqrt
(a*c)*(cos(f*x + e)^2 - 1)*arctan((16*cos(f*x + e)^3 - 7*cos(f*x + e))*sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(f*x + e)))*sin(f*x + e)
 + (16*cos(f*x + e)^3 - 25*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2
 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1))**(3/2)),
 x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(91) = 182$.

Time = 0.21 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.81

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left((fx+e)\cos(4fx+4e)^2+4(fx+e)\cos(2fx+2e)^2+(fx+e)\sin(4fx+4e)^2+4(fx+e)\sin(2\ldots\right.}{}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="ma
xima")
```

output
```
-((f*x + e)*cos(4*f*x + 4*e)^2 + 4*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e
)*sin(4*f*x + 4*e)^2 + 4*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(2*cos(2*
f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(2*f*x + 2*e)
^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) - 4*sin(2*f*
x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) - 1) + 2*(f*x - 2*(f*x + e)*cos(2*f*x + 2*e) + e + sin(2*f*x + 2*e))
*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) - 2*(2*(f*x + e)*sin(2*f*
x + 2*e) + cos(2*f*x + 2*e))*sin(4*f*x + 4*e) + e + 2*sin(2*f*x + 2*e))*sq
rt(a)*sqrt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f*x + 2*e)^2
+ a^2*c^2*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*a^2*c^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2 - 2
*(2*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

Giac [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\frac{4\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a|c|}-\frac{8\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}a|c|}+\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}ac|c|}-\frac{4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{\sqrt{-ac}ac|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{8f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gi
ac")
```

output
```
-1/8*(4*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*abs(c)) - 8*log(a
bs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*a*abs(c)) + (c*tan(1/2*f*x +
 1/2*e)^2 - c)/(sqrt(-a*c)*a*c*abs(c)) - (4*c*tan(1/2*f*x + 1/2*e)^2 - c)/
(sqrt(-a*c)*a*c*abs(c)*tan(1/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^4-2\sec(fx+e)^2+1}dx\right)}{a^2c^2}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1),x))/(a**2*c**2)
```

3.123 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 347

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{11\log(1-\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1+\sec(e+fx))\tan(e+fx)}{16ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1-\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2ac^2f(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8ac^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)+11/16*ln(1-sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)+5/16*ln(1+sec(f*x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+
e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^2/f/(1-sec(f*x+e))^2/(
a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a/c^2/f/(1-sec
(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a/c^
2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(16\log(\cos(e+fx))+11\log(1-\sec(e+fx))+5\log\right.}{16ac^2f\sqrt{a(}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
((16*Log[Cos[e + f*x]] + 11*Log[1 - Sec[e + f*x]] + 5*Log[1 + Sec[e + f*x]
] - 2/(-1 + Sec[e + f*x])^2 + 8/(-1 + Sec[e + f*x]) - 2/(1 + Sec[e + f*x])
)*Tan[e + f*x])/(16*a*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4400$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^2c^3(1-\sec(e+fx))^3(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^3(\sec(e+fx)+1)^2}d\sec(e+fx)}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$\big\downarrow$ 99

$$-\frac{\tan(e+fx)\int \left(\cos(e+fx)-\frac{11}{16(\sec(e+fx)-1)}-\frac{5}{16(\sec(e+fx)+1)}+\frac{1}{2(\sec(e+fx)-1)^2}-\frac{1}{8(\sec(e+fx)+1)^2}-\frac{1}{4(\sec(e+fx)-}\right.}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$\big\downarrow$ 2009

$$-\frac{\tan(e+fx)\left(\frac{1}{2(1-\sec(e+fx))}+\frac{1}{8(\sec(e+fx)+1)}+\frac{1}{8(1-\sec(e+fx))^2}-\frac{11}{16}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{5}{16}\right.}{ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
-((((-11*Log[1 - Sec[e + f*x]])/16 + Log[Sec[e + f*x]] - (5*Log[1 + Sec[e
+ f*x]])/16 + 1/(8*(1 - Sec[e + f*x])^2) + 1/(2*(1 - Sec[e + f*x])) + 1/(8
*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\left(512\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-160\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-3\cdots\right.}{\cdots}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a\,c^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a\,c^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}+\frac{\cdots}{4a\,c^{2}\left(e^{i\cdots}\right.}$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/256/f*(512*cos(1/2*f*x+1/2*e)^2*sin(1/2*f*x+1/2*e)^4*ln(2/(cos(1/2*f*x+
1/2*e)+1))-160*cos(1/2*f*x+1/2*e)^2*sin(1/2*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1
/2*e)+csc(1/2*f*x+1/2*e)+1)-352*cos(1/2*f*x+1/2*e)^2*sin(1/2*f*x+1/2*e)^4*
ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))-160*cos(1/2*f*x+1/2*e)^2*sin(1/2
*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)+61*cos(1/2*f*x+
1/2*e)^6-42*cos(1/2*f*x+1/2*e)^4+5*cos(1/2*f*x+1/2*e)^2-16)/(2*cos(1/2*f*x
+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/(-c
/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/a/c^2*sec(1/2*f*x+
1/2*e)*csc(1/2*f*x+1/2*e)^3
```

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*c^3*sec(
f*x + e)^5 - a^2*c^3*sec(f*x + e)^4 - 2*a^2*c^3*sec(f*x + e)^3 + 2*a^2*c^3
*sec(f*x + e)^2 + a^2*c^3*sec(f*x + e) - a^2*c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4272 vs. $2(309) = 618$.

Time = 1.66 (sec) , antiderivative size = 4272, normalized size of antiderivative = 12.31

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="ma
xima")
```

output
```
-1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*
(f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x
 + 4*e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*f*x + 5*(2*(cos(4*f*x + 4*e) + cos(2*f
*x + 2*e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e)
 - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 + 4*(cos(
6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 4*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 8*(cos(6*f*x
 + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 2*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 4*(cos(6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f*...
```

Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{22\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}ac|c|}-\frac{32\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\right|\right)}{\sqrt{-ac}ac|c|}+\frac{2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\sqrt{-ac}ac^2|c|}-\frac{33\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+56\left(c\tan\left(\frac{1}{2}fx\right.\right.}{\sqrt{-ac}ac^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}}{32f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gi
ac")
```

output
```
-1/32*(22*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a*c*abs(c)) - 32*
log(abs(c*tan(1/2*f*x + 1/2*e)^2 + c))/(sqrt(-a*c)*a*c*abs(c)) + 2*(c*tan(
1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a*c^2*abs(c)) - (33*(c*tan(1/2*f*x + 1
/2*e)^2 - c)^2 + 56*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 24*c^2)/(sqrt(-a*c)
*a*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^5-\sec(fx+e)^4-2\sec(fx+e)^3+2\sec(fx+e)^2+\sec(fx+e)-1}dx\right)}{a^2c^3}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x)**
2 + sec(e + f*x) - 1),x))/(a**2*c**3)
```

3.124 $\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 220

$$\begin{aligned}\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx &= \frac{c^4\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &+ \frac{2c^4\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &- \frac{4c^4\tan(e+fx)}{a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &+ \frac{4c^4\tan(e+fx)}{a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

output
```
c^4*ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)+2*c^4*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-
c*sec(f*x+e))^(1/2)-4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e
))^(1/2)/(c-c*sec(f*x+e))^(1/2)+4*c^4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a
*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.39

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^4\left(-\log(\cos(e+fx))-2\log(1+\sec(e+fx))-\frac{4\sec(e+fx)}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^4*(-Log[Cos[e + f*x]] - 2*Log[1 + Sec[e + f*x]] - (4*Sec[e + f*x])/(1
 + Sec[e + f*x])^2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{c^3\cos(e+fx)(1-\sec(e+fx))^3}{a^3(\sec(e+fx)+1)^3}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c^4\tan(e+fx)\int \frac{\cos(e+fx)(1-\sec(e+fx))^3}{(\sec(e+fx)+1)^3}d\sec(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{c^4\tan(e+fx)\int \left(\cos(e+fx)-\frac{2}{\sec(e+fx)+1}+\frac{4}{(\sec(e+fx)+1)^2}-\frac{8}{(\sec(e+fx)+1)^3}\right)d\sec(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{c^4\tan(e+fx)\left(-\frac{4}{\sec(e+fx)+1}+\frac{4}{(\sec(e+fx)+1)^2}+\log(\sec(e+fx))-2\log(\sec(e+fx)+1)\right)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(7/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^4*(Log[Sec[e + f*x]] - 2*Log[1 + Sec[e + f*x]] + 4/(1 + Sec[e + f*x])
^2 - 4/(1 + Sec[e + f*x]))*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 3.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.25

method	result
default	$\frac{\sqrt{-\frac{c\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\, c^{3}\left(\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(-\frac{2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+2\ln\right.}{}$
risch	$-\frac{c^{3}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\left(-4i\ln\left(e^{2i(fx+e)}+1\right)e^{3i(fx+e)}+8ie^{i(fx+e)}+e^{4i(fx+e)}fx+4ie^{4i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)-i\ln\left(e^{2i(fx+e)}+1\right)\right.}{}$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*c^3/(a/(2*c
os(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a^2*(ln(-2*(cos(1/2*f*x
+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)+ln(
-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))*cot(1/2
*f*x+1/2*e)+2*ln(2/(cos(1/2*f*x+1/2*e)+1))*cot(1/2*f*x+1/2*e)-4*ln(-cot(1/
2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*cot(1/2*f*x+1/2*e)-4*ln(-cot(1/2*f*x+1/
2*e)+csc(1/2*f*x+1/2*e)+1)*cot(1/2*f*x+1/2*e)+tan(1/2*f*x+1/2*e)^3)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{7}{2}}}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(-(c^3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e)
- c^3)*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e
)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.42

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2\sqrt{-ac}\,a^2|c|+2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\sqrt{-ac}\,a^2c|c|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3\right.}{a^5f}$$

input
```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
-((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*abs(c) + 2*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c*abs(c))*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e))/(a^5*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(7/2)/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c - c\sec(e + fx))^{7/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^3\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right) + 3\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{}}{\sec(fx+e)^3+3\,\mathrm{se}}\right.\right.}{}$$

input
```
int((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**3*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x)
 + 1)*sec(e + f*x)**3)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*
x) + 1),x) + 3*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e
 + f*x)**2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)
- 3*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e + f*x))/(s
ec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + int((sqrt(se
c(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x)**3 + 3*sec(e + f*
x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.125 $\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2c^3\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^3\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
-2*c^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+c^3*ln(1
+cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2
)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{2}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^3*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + 2/(1 + Sec[e + f*x])^
2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]
]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4398, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4398}$$

$$\frac{c^2\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a^2}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c^2\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a^2}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4399}$$

$$\frac{c^3\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{c^3\tan(e+fx)\log(\cos(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^3\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(5/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*c^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x
]]) + (c^3*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4398
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(5/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-8*a^3*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a^2/c^2   Int[Sqr
t[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)
]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(90) = 180$.

Time = 3.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.31

method	result
default	$-\frac{\left(4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-4\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right.}{2f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}}$
risch	$-\frac{c^{2}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\left(8ie^{i(fx+e)}+e^{4i(fx+e)}fx+8i\ln\left(e^{i(fx+e)}+1\right)e^{3i(fx+e)}+2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx+8ie^{i(fx+e)}\ln\left(e^{i(fx+e)}\right.\right.}{}$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*cos(1/2*f*x+1/2*e)^
4+4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*cos(1/2*f*x+1/2*e)^4-4*ln
(2/(cos(1/2*f*x+1/2*e)+1))*cos(1/2*f*x+1/2*e)^4-3*cos(1/2*f*x+1/2*e)^4+4*c
os(1/2*f*x+1/2*e)^2-1)*c^2*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*
e)^2)^(1/2)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a^2*
sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c\sec\left(fx+e\right)+c)^{\frac{5}{2}}}{(a\sec\left(fx+e\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral((c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2)*sqrt(a*sec(f*x +
e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)
^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \frac{\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{-a}a^2} + \frac{\frac{2\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{a^3}}{2f}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/2*(2*c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(sqrt(-a)*a^2)
 + (2*sqrt(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - sqrt(-a)*c^(5
/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c - c\sec(e + fx))^{5/2}}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(5/2)/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx-2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(\cdots}{\sec(fx+e)^3+3\sec(fx+\cdots}\right.\right.}{a^3}$$

input
```
int((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x)**2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1),x) - 2*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1)*sec(e +
f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x) + int(
(sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x)**3 + 3*se
c(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.126 $\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\tan(e+fx)}{f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c^2\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-c^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c^2*tan(f*
x+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c^2*ln(1+cos(f*x+e)
)*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^2\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{2+\sec(e+fx)}{(1+\sec(e+fx))^2}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^2*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + (2 + Sec[e + f*x])/(1
 + Sec[e + f*x])^2)*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4396, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \ 4396$$

$$\frac{c\int \frac{\sqrt{c-c\sec(e+fx)}}{(\sec(e+fx)a+a)^{3/2}}dx}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{c\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4395}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4399}$$

$$\frac{c\left(\frac{c\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{c\left(\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}\right)}{a}-\frac{c^2\tan(e+fx)}{f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]
])) + (c*(-((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[
e + f*x]])) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4396
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-4*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a/c   Int[Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d +
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.56

method	result
default	$\frac{\left(8\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-8\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-8\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}{4f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}}\,a^{2}}$
risch	$-\frac{c\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,(6ie^{i(fx+e)}+e^{4i(fx+e)}fx+8ie^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx+8i\ln\left(e^{i(fx+e)}+1\right)e^{3i(f}}{}$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(8*ln(2/(cos(1/2*f*x+1/2*e)+1))*cos(1/2*f*x+1/2*e)^4-8*ln(-cot(1/2*f
*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*cos(1/2*f*x+1/2*e)^4-8*ln(-cot(1/2*f*x+1/2
*e)+csc(1/2*f*x+1/2*e)-1)*cos(1/2*f*x+1/2*e)^4+5*cos(1/2*f*x+1/2*e)^4-6*co
s(1/2*f*x+1/2*e)^2+1)*c*(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^
2)^(1/2)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a^2*sec
(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)/(a^3*sec(f*x
 + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c(\sec{(e+fx)}-1))^{\frac{3}{2}}}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**(3/2)/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(132) = 264$.

Time = 0.34 (sec) , antiderivative size = 1786, normalized size of antiderivative = 12.40

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*c*cos(4*f*x + 4*e)^2 + 36*(f*x + e)*c*cos(2*f*x + 2*e)^2 + 16*
(f*x + e)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f
*x + e)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + (f*x +
e)*c*sin(4*f*x + 4*e)^2 + 36*(f*x + e)*c*sin(2*f*x + 2*e)^2 + 16*(f*x + e)
*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*(f*x + e)*c
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 12*(f*x + e)*c*c
os(2*f*x + 2*e) + (f*x + e)*c - 2*(c*cos(4*f*x + 4*e)^2 + 36*c*cos(2*f*x +
 2*e)^2 + 16*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16
*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(4*f*x +
4*e)^2 + 12*c*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36*c*sin(2*f*x + 2*e)^2
+ 16*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 16*c*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*(6*c*cos(2*f*x + 2*e
) + c)*cos(4*f*x + 4*e) + 12*c*cos(2*f*x + 2*e) + 8*(c*cos(4*f*x + 4*e) +
6*c*cos(2*f*x + 2*e) + 4*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + c)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*cos
(4*f*x + 4*e) + 6*c*cos(2*f*x + 2*e) + c)*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x + 2*e) + 4*c*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 8*(c*sin(4*f*x + 4*e) + 6*c*sin(2*f*x +
 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c)*arctan...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)+\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-}}{\sec(fx+e)^3+3\sec(fx+e}\right.}{a^3}$$

input
```
int((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) +
1)*sec(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1
),x) + int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(sec(e + f*x
)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)))/a**3
```

3.127 $\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 140

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{c\log(1+\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
-1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)-c*tan(f*
x+e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)+c*ln(1+cos(f*x+e))*
tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\left(-\log(\cos(e+fx))-\log(1+\sec(e+fx))+\frac{1}{2(1+\sec(e+fx))^2}+\frac{1}{1+\sec(e+fx)}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

input
```
Integrate[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((c*(-Log[Cos[e + f*x]] - Log[1 + Sec[e + f*x]] + 1/(2*(1 + Sec[e + f*x])
^2) + (1 + Sec[e + f*x])^(-1))*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*
x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4395, 3042, 4395, 3042, 4399, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4395}$$

$$\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{(\sec(e+fx)a+a)^{3/2}}dx}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4395}$$

$$\frac{\frac{\int \frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{\sec(e+fx)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{4399}$$

$$\frac{\frac{c\tan(e+fx)\int \frac{1}{\cos(e+fx)a+a}d\cos(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{c\tan(e+fx)\log(\cos(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)}{f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}}{a}-\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[Sqrt[c - c*Sec[e + f*x]]/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/2*(c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x
]]) + (-((c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e +
 f*x]])) + (c*Log[1 + Cos[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]]))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4395
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(n_), x_Symbol] :> Simp[-2*a*Cot[e + f*x]*((c + d*Csc[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[1/c   Int[Sqrt[a + b*C
sc[e + f*x]]*(c + d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, 
f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)]
```

rule 4399
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[(-a)*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[
e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(b + a*x)^(m - 1/2)*((d + 
c*x)^(n - 1/2)/x^(m + n)), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] && 
EqQ[m + n, 0]
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.61

method	result
default	$-\frac{\sqrt{2}\left(16\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+16\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}-16\ln\left(\frac{}{\cos(}\right.\right.}{16f\sqrt{\frac{a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}}}$
risch	$-\frac{\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(4ie^{i(fx+e)}+e^{4i(fx+e)}fx+8i\ln\left(e^{i(fx+e)}+1\right)e^{3i(fx+e)}+2\,e^{4i(fx+e)}e+4\,e^{3i(fx+e)}fx+12ie^{2i(fx+e)}\ln\left(e^{i(fx+\right.\right.}{}$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*2^(1/2)/f*(16*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)+1)*cos(1/2*f
*x+1/2*e)^4+16*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*cos(1/2*f*x+1/
2*e)^4-16*ln(2/(cos(1/2*f*x+1/2*e)+1))*cos(1/2*f*x+1/2*e)^4-7*cos(1/2*f*x+
1/2*e)^4+8*cos(1/2*f*x+1/2*e)^2-1)*(-2*c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/
2*f*x+1/2*e)^2)^(1/2)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^
(1/2)/a^2*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{-c\sec\left(fx+e\right)+c}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x +
 e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(-c*(sec(e + f*x) - 1))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(126) = 252$.

Time = 0.28 (sec) , antiderivative size = 1165, normalized size of antiderivative = 8.32

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
-((f*x + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(3*f*x + 3*e)^2 + 36*(f*x
 + e)*cos(2*f*x + 2*e)^2 + 16*(f*x + e)*cos(f*x + e)^2 + (f*x + e)*sin(4*f
*x + 4*e)^2 + 16*(f*x + e)*sin(3*f*x + 3*e)^2 + 36*(f*x + e)*sin(2*f*x + 2
*e)^2 + 16*(f*x + e)*sin(f*x + e)^2 + f*x - 2*(2*(4*cos(3*f*x + 3*e) + 6*c
os(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^
2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(
3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x +
 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) +
 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x +
 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e) + 16*sin(3*f*x + 3*e)^2 + 36*sin(
2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e) + 16*sin(f*x + e)^2 + 8*
cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*(f*x + 4*(f*
x + e)*cos(3*f*x + 3*e) + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f
*x + e) + e - 2*sin(3*f*x + 3*e) - 3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*co
s(4*f*x + 4*e) + 8*(f*x + 6*(f*x + e)*cos(2*f*x + 2*e) + 4*(f*x + e)*cos(f
*x + e) + e)*cos(3*f*x + 3*e) + 12*(f*x + 4*(f*x + e)*cos(f*x + e) + e)*co
s(2*f*x + 2*e) + 8*(f*x + e)*cos(f*x + e) + 2*(4*(f*x + e)*sin(3*f*x + 3*e
) + 6*(f*x + e)*sin(2*f*x + 2*e) + 4*(f*x + e)*sin(f*x + e) + 2*cos(3*f*x
+ 3*e) + 3*cos(2*f*x + 2*e) + 2*cos(f*x + e))*sin(4*f*x + 4*e) + 4*(12*(f*
x + e)*sin(2*f*x + 2*e) + 8*(f*x + e)*sin(f*x + e) - 1)*sin(3*f*x + 3*e...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{\sqrt{c - \frac{c}{\cos(e+f\,x)}}}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c - c/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c - c\sec(e + fx)}}{(a + a\sec(e + fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)}{a^3}$$

input
```
int((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x))/a**3
```

3.128 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 270

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\log(1-\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{7\log(1+\sec(e+fx))\tan(e+fx)}{8a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\tan(e+fx)}{4a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
ln(cos(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)+1/8*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(
f*x+e))^(1/2)+7/8*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)
/(c-c*sec(f*x+e))^(1/2)-1/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x
+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-3/4*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a
*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\left(8\log(\cos(e+fx))+\log(1-\sec(e+fx))+7\log(1+\right.}{8a^2f\sqrt{a(1+\sec(e+f}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
((8*Log[Cos[e + f*x]] + Log[1 - Sec[e + f*x]] + 7*Log[1 + Sec[e + f*x]] -
2/(1 + Sec[e + f*x])^2 - 6/(1 + Sec[e + f*x]))*Tan[e + f*x])/(8*a^2*f*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\sqrt{c-c\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^3c(1-\sec(e+fx))(\sec(e+fx)+1)^3}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))(\sec(e+fx)+1)^3}d\sec(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{93}$$

$$-\frac{\tan(e+fx)\int\left(\cos(e+fx)-\frac{1}{8(\sec(e+fx)-1)}-\frac{7}{8(\sec(e+fx)+1)}-\frac{3}{4(\sec(e+fx)+1)^2}-\frac{1}{2(\sec(e+fx)+1)^3}\right)d\sec(e+fx)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{3}{4(\sec(e+fx)+1)}+\frac{1}{4(\sec(e+fx)+1)^2}-\frac{1}{8}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{7}{8}\log(\sec(e+fx)+\right.}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

output
```
-(((-1/8*Log[1 - Sec[e + f*x]] + Log[Sec[e + f*x]] - (7*Log[1 + Sec[e + f*
x]])/8 + 1/(4*(1 + Sec[e + f*x])^2) + 3/(4*(1 + Sec[e + f*x])))*Tan[e + f*
x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.02

method	result
default	$\frac{\left(28\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-32\ln\left(\frac{2}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-c\right.}{16f\left(2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\sqrt{-\frac{c}{2c}}}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{}{2a^2\left(e^{i(fx+}\right.}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/16/f*(28*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)*cos(1/2*f*x+1/2*e)
^4-32*ln(2/(cos(1/2*f*x+1/2*e)+1))*cos(1/2*f*x+1/2*e)^4+4*cos(1/2*f*x+1/2*
e)^4*ln(csc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))+28*ln(-cot(1/2*f*x+1/2*e)+c
sc(1/2*f*x+1/2*e)+1)*cos(1/2*f*x+1/2*e)^4-9*cos(1/2*f*x+1/2*e)^4+10*cos(1/
2*f*x+1/2*e)^2-1)/(2*cos(1/2*f*x+1/2*e)^2-1)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1
)*sin(1/2*f*x+1/2*e)^2)^(1/2)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/
2*e)^2)^(1/2)/a^2*tan(1/2*f*x+1/2*e)*sec(1/2*f*x+1/2*e)^2
```

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c*sec(f*
x + e)^4 + 2*a^3*c*sec(f*x + e)^3 - 2*a^3*c*sec(f*x + e) - a^3*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x) - 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2206 vs. $2(242) = 484$.

Time = 0.32 (sec) , antiderivative size = 2206, normalized size of antiderivative = 8.17

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/4*(4*(f*x + e)*cos(4*f*x + 4*e)^2 + 144*(f*x + e)*cos(2*f*x + 2*e)^2 +
64*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(
f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*(f*x +
 e)*sin(4*f*x + 4*e)^2 + 144*(f*x + e)*sin(2*f*x + 2*e)^2 + 64*(f*x + e)*s
in(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*(f*x + e)*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*f*x - 7*(2*(6*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e
)^2 + 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 4*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ 8*(cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 16*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + sin(4*f*x + 4*e)^2 + 12*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 36*sin(2*f*x + 2*e)^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e) + 4*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 16*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 8*(sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 16*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 12*cos(2*f*x + 2*e) + 1)*arctan2(sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e)...
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2\sqrt{-ac}a^2c|c|-6\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}a^2c^2|c|}{16\,a^5c^5f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-1/16*((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c*abs(c) - 6*(c*tan
(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c^2*abs(c))/(a^5*c^5*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^4+2\sec(fx+e)^3-2\sec(fx+e)-1}dx\right)}{a^3c}$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x) - 1),x))/(a**3*c)
```

3.129 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\log(\cos(e+fx))\tan(e+fx)}{a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{5\log(1-\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{11\log(1+\sec(e+fx))\tan(e+fx)}{16a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8a^2cf(1-\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{8a^2cf(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{2a^2cf(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output

```
ln(cos(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^
(1/2)+5/16*ln(1-sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c
*sec(f*x+e))^(1/2)+11/16*ln(1+sec(f*x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+
e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1-sec(f*x+e))/(a+
a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/8*tan(f*x+e)/a^2/c/f/(1+sec(f
*x+e))^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*tan(f*x+e)/a^2/
c/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\left(16\log(\cos(e+fx))+5\log(1-\sec(e+fx))+11\log\right.}{16a^2cf\sqrt{a(1}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
((16*Log[Cos[e + f*x]] + 5*Log[1 - Sec[e + f*x]] + 11*Log[1 + Sec[e + f*x]
] + 2/(-1 + Sec[e + f*x]) - 2/(1 + Sec[e + f*x])^2 - 8/(1 + Sec[e + f*x]))
*Tan[e + f*x])/(16*a^2*c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f
*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4400$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)}{a^3c^2(1-\sec(e+fx))^2(\sec(e+fx)+1)^3}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(1-\sec(e+fx))^2(\sec(e+fx)+1)^3}d\sec(e+fx)}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$\Big\downarrow$ 99

$$-\frac{\tan(e+fx)\int \left(\cos(e+fx)-\frac{5}{16(\sec(e+fx)-1)}-\frac{11}{16(\sec(e+fx)+1)}+\frac{1}{8(\sec(e+fx)-1)^2}-\frac{1}{2(\sec(e+fx)+1)^2}-\frac{1}{4(\sec(e+fx)+\right.}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$\Big\downarrow$ 2009

$$-\frac{\tan(e+fx)\left(\frac{1}{8(1-\sec(e+fx))}+\frac{1}{2(\sec(e+fx)+1)}+\frac{1}{8(\sec(e+fx)+1)^2}-\frac{5}{16}\log(1-\sec(e+fx))+\log(\sec(e+fx))-\frac{1}{1}\right.}{a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

output
```
-((((-5*Log[1 - Sec[e + f*x]])/16 + Log[Sec[e + f*x]] - (11*Log[1 + Sec[e
+ f*x]])/16 + 1/(8*(1 - Sec[e + f*x])) + 1/(8*(1 + Sec[e + f*x])^2) + 1/(2
*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.97

method	result
default	$\frac{\left(44\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+20\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right.\right.\right.}{}$
risch	$\frac{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)x}{a^{2}c\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}}-\frac{2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)(fx+e)}{a^{2}c\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}-\frac{}{4a^{2}c\left(e^{i(\right.}}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/32/f*(44*cos(1/2*f*x+1/2*e)^4*sin(1/2*f*x+1/2*e)^2*ln(-cot(1/2*f*x+1/2*e
)+csc(1/2*f*x+1/2*e)+1)+20*cos(1/2*f*x+1/2*e)^4*sin(1/2*f*x+1/2*e)^2*ln(cs
c(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))-64*cos(1/2*f*x+1/2*e)^4*sin(1/2*f*x+1
/2*e)^2*ln(2/(cos(1/2*f*x+1/2*e)+1))+44*cos(1/2*f*x+1/2*e)^4*sin(1/2*f*x+1
/2*e)^2*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2*e)-1)+12*cos(1/2*f*x+1/2*e)
^6-22*cos(1/2*f*x+1/2*e)^4+13*cos(1/2*f*x+1/2*e)^2-1)/(2*cos(1/2*f*x+1/2*e
)^2-1)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/(a/(2*co
s(1/2*f*x+1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/a^2/c*sec(1/2*f*x+1/2*e)
^3*csc(1/2*f*x+1/2*e)
```

Fricas [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*c^2*sec(f
*x + e)^5 + a^3*c^2*sec(f*x + e)^4 - 2*a^3*c^2*sec(f*x + e)^3 - 2*a^3*c^2*
sec(f*x + e)^2 + a^3*c^2*sec(f*x + e) + a^3*c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4272 vs. $2(309) = 618$.

Time = 1.69 (sec) , antiderivative size = 4272, normalized size of antiderivative = 12.38

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="ma
xima")
```

output
```
-1/8*(8*(f*x + e)*cos(6*f*x + 6*e)^2 + 8*(f*x + e)*cos(4*f*x + 4*e)^2 + 8*
(f*x + e)*cos(2*f*x + 2*e)^2 + 32*(f*x + e)*cos(5/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 8*(f*x + e)*sin(6*f*x + 6*e)^2 + 8*(f*x + e)*sin(4*f*x
 + 4*e)^2 + 8*(f*x + e)*sin(2*f*x + 2*e)^2 + 32*(f*x + e)*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 128*(f*x + e)*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 32*(f*x + e)*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*f*x + 11*(2*(cos(4*f*x + 4*e) + cos(2*
f*x + 2*e) - 1)*cos(6*f*x + 6*e) - cos(6*f*x + 6*e)^2 - 2*(cos(2*f*x + 2*e
) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - cos(2*f*x + 2*e)^2 - 4*(cos
(6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) - 4*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*(cos(6*f*
x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 2*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 1)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 16*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
- 4*(cos(6*f*x + 6*e) - cos(4*f*x + 4*e) - cos(2*f*x + 2*e) + 1)*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*cos(1/2*arctan2(sin(2*f...
```

Giac [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{10\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{\sqrt{-ac}a^2|c|} - \frac{32\,\log\left(\left|-c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right|\right)}{\sqrt{-ac}a^2|c|} - \frac{2\left(5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)}{\sqrt{-ac}a^2c|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2} + \frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2\sqrt{-ac}a^2c^2|c|-}{a^5}}{32\,f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="gi
ac")
```

output
```
-1/32*(10*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*abs(c)) - 32*
log(abs(-c*tan(1/2*f*x + 1/2*e)^2 - c))/(sqrt(-a*c)*a^2*abs(c)) - 2*(5*c*t
an(1/2*f*x + 1/2*e)^2 - c)/(sqrt(-a*c)*a^2*c*abs(c)*tan(1/2*f*x + 1/2*e)^2
) + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c^2*abs(c) - 8*(c*tan
(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*a^2*c^3*abs(c))/(a^5*c^7))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^5+\sec(fx+e)^4-2\sec(fx+e)^3-2\sec(fx+e)^2+\mathrm{se}}\right.}{a^3c^2}$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))/(s
ec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**3 - 2*sec(e + f*x)**2 +
 sec(e + f*x) + 1),x))/(a**3*c**2)
```

3.130 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx=\frac{\cot(e+fx)}{2a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\cot^3(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\log(\sin(e+fx))\tan(e+fx)}{a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

output
```
1/2*cot(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4
*cot(f*x+e)^3/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+ln(s
in(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{2\cot(e+fx)-\cot^3(e+fx)+4(\log(\cos(e+fx))+}{4a^2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c}}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
(2*Cot[e + f*x] - Cot[e + f*x]^3 + 4*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]
])*Tan[e + f*x])/(4*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
+ f*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 4393, 25, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4393}$$

$$-\frac{\tan(e+fx)\int -\cot^5(e+fx)dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(e+fx)\int \cot^5(e+fx)dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(e+fx)\int -\tan\left(e+fx+\frac{\pi}{2}\right)^5dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5dx}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\int -\cot^3(e+fx)dx\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(\int \cot^3(e+fx)dx+\frac{\cot^4(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tan(e+fx)\left(\int -\tan\left(e+fx+\frac{\pi}{2}\right)^3dx+\frac{\cot^4(e+fx)}{4f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3dx\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$-\frac{\tan(e+fx)\left(\int -\cot(e+fx)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(-\int \cot(e+fx)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\tan(e+fx)\left(-\int -\tan\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(e+fx)\left(\int \tan\left(\frac{1}{2}(2e+\pi)+fx\right)dx+\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\tan(e+fx)\left(\frac{\cot^4(e+fx)}{4f}-\frac{\cot^2(e+fx)}{2f}-\frac{\log(-\sin(e+fx))}{f}\right)}{a^2c^2\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

output
```
-(((-1/2*Cot[e + f*x]^2/f + Cot[e + f*x]^4/(4*f) - Log[-Sin[e + f*x]]/f)*T
an[e + f*x])/(a^2*c^2*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4393
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(m_), x_Symbol] :> Simp[((-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[
a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[Cot[e + f*x]^(2*m), x]
, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IntegerQ[m + 1/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(135) = 270$.

Time = 3.00 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.32

method	result
default	$-\frac{\left(-512\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4 \ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^4-512\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4 \ln\left(-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.\right.}{}$
risch	$-\frac{-4i e^{2i(fx+e)}\ln\left(e^{2i(fx+e)}-1\right)+e^{8i(fx+e)}fx+2\,e^{8i(fx+e)}e-4i e^{6i(fx+e)}-4\,e^{6i(fx+e)}fx-8\,e^{6i(fx+e)}e+i\ln\left(e^{2i(fx+e)}-1\right)+6\,e^{4i(}}{a^2c^2\left(e^{i(fx+e)}+1\right)^3}$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/512/f*(-512*cos(1/2*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1/2*e)+csc(1/2*f*x+1/2
*e)-1)*sin(1/2*f*x+1/2*e)^4-512*cos(1/2*f*x+1/2*e)^4*ln(-cot(1/2*f*x+1/2*e
)+csc(1/2*f*x+1/2*e)+1)*sin(1/2*f*x+1/2*e)^4-512*cos(1/2*f*x+1/2*e)^4*ln(c
sc(1/2*f*x+1/2*e)-cot(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^4+1024*cos(1/2*f*
x+1/2*e)^4*ln(2/(cos(1/2*f*x+1/2*e)+1))*sin(1/2*f*x+1/2*e)^4+157*cos(1/2*f
*x+1/2*e)^8-314*cos(1/2*f*x+1/2*e)^6+285*cos(1/2*f*x+1/2*e)^4-128*cos(1/2*
f*x+1/2*e)^2+8)/(2*cos(1/2*f*x+1/2*e)^2-1)/(a/(2*cos(1/2*f*x+1/2*e)^2-1)*c
os(1/2*f*x+1/2*e)^2)^(1/2)/(-c/(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*
e)^2)^(1/2)/a^2/c^2*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)^3
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.74

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fr
icas")
```

output
```
[-1/324*(162*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-8*((2
56*cos(f*x + e)^5 - 512*cos(f*x + e)^3 + 175*cos(f*x + e))*sqrt(-a*c)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)
) - (256*a*c*cos(f*x + e)^4 - 512*a*c*cos(f*x + e)^2 + 337*a*c)*sin(f*x +
e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) + (832*cos(f*x + e)^
5 - 1988*cos(f*x + e)^3 + 1075*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x +
e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e)), -1/324*(324*
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(a*c)*arctan((16*cos(f*x + e)^
3 - 7*cos(f*x + e))*sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) - c)/cos(f*x + e))/((16*a*c*cos(f*x + e)^2 - 25*a*c)*sin(
f*x + e)))*sin(f*x + e) + (832*cos(f*x + e)^5 - 1988*cos(f*x + e)^3 + 1075
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e
)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(135) = 270$.

Time $= 0.31$ (sec) , antiderivative size $= 1386$, normalized size of antiderivative $= 9.18$

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="ma
xima")
```

output
```
-((f*x + e)*cos(8*f*x + 8*e)^2 + 16*(f*x + e)*cos(6*f*x + 6*e)^2 + 36*(f*x
 + e)*cos(4*f*x + 4*e)^2 + 16*(f*x + e)*cos(2*f*x + 2*e)^2 + (f*x + e)*sin
(8*f*x + 8*e)^2 + 16*(f*x + e)*sin(6*f*x + 6*e)^2 + 36*(f*x + e)*sin(4*f*x
 + 4*e)^2 + 16*(f*x + e)*sin(2*f*x + 2*e)^2 + f*x + (2*(4*cos(6*f*x + 6*e)
 - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f
*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e
) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e)
- 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x +
8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 1
6*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) - 1) + 2*(f*x - 4*(f*x + e)*cos(6*f*x + 6*e)
+ 6*(f*x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e + 2*sin(
6*f*x + 6*e) - 2*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) -
 8*(f*x + 6*(f*x + e)*cos(4*f*x + 4*e) - 4*(f*x + e)*cos(2*f*x + 2*e) + e
+ sin(4*f*x + 4*e))*cos(6*f*x + 6*e) + 4*(3*f*x - 12*(f*x + e)*cos(2*f*x +
 2*e) + 3*e + 2*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 8*(f*x + e)*cos(2*f*x
 + 2*e) - 4*(2*(f*x + e)*sin(6*f*x + 6*e) - 3*(f*x + e)*sin(4*f*x + 4*e) +
 2*(f*x + e)*sin(2*f*x + 2*e) + cos(6*f*x + 6*e) - cos(4*f*x + 4*e) + c...
```

Giac [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{32\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{\sqrt{-ac}a^2c|c|}-\frac{64\log\left(\left|-c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right|\right)}{\sqrt{-ac}a^2c|c|}-\frac{48\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+84\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+37c^2}{\sqrt{-ac}a^2c^3|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}+\frac{(c\tan}{}}{64f}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="gi
ac")
```

output
```
-1/64*(32*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/(sqrt(-a*c)*a^2*c*abs(c)) - 6
4*log(abs(-c*tan(1/2*f*x + 1/2*e)^2 - c))/(sqrt(-a*c)*a^2*c*abs(c)) - (48*
(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 84*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 3
7*c^2)/(sqrt(-a*c)*a^2*c^3*abs(c)*tan(1/2*f*x + 1/2*e)^4) + ((c*tan(1/2*f*
x + 1/2*e)^2 - c)^2*sqrt(-a*c)*a^2*c^3*abs(c) - 10*(c*tan(1/2*f*x + 1/2*e)
^2 - c)*sqrt(-a*c)*a^2*c^4*abs(c))/(a^5*c^9))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sqrt{-\sec(fx+e)+1}}{\sec(fx+e)^6-3\sec(fx+e)^4+3\sec(fx+e)^2-1}dx\right)}{a^3c^3}$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(sec(e + f*x) + 1)*sqrt( - sec(e + f*x) + 1))
/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x)**2 - 1),x))/(a**3*c
**3)
```

3.131 $\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2}+n,\frac{1}{2}-m,1,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2^(1/2+m)*AppellF1(1/2+n,1,1/2-m,3/2+n,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(c
-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)
```

Mathematica [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4400, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 4400}$$

$$-\frac{c\tan(e+fx)\int \cos(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{\sec(e+fx)+1}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 153}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^n\operatorname{AppellF1}\left(n+\frac{1}{2},\frac{1}{2}-m,1,n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(1 + Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2 + n, 1/2 - m, 1, 3/2 + n, (1 - Sec[e + f*x])/2,
1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1
 + Sec[e + f*x]])
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (1+\sec(fx+e))^m (c-c\sec(fx+e))^n \, dx$$

input
```
int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$= \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$= \int (-c(\sec(e+fx)-1))^n\,(\sec(e+fx)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**n*(sec(e + f*x) + 1)**m, x)
```

Maxima [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$= \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Giac [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (-c\sec(fx+e)+c)^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n*(sec(f*x + e) + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int \left(\frac{1}{\cos(e+fx)}+1\right)^m\left(c-\frac{c}{\cos(e+fx)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (1+\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = \int (\sec(fx+e)+1)^m(-\sec(fx+e)c+c)^n\,dx$$

input
```
int((1+sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int((sec(e + f*x) + 1)**m*( - sec(e + f*x)*c + c)**n,x)
```

3.132 $\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 109

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$= \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2}-n,1,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+}{f(1+2m)}$$

output
```
2^(1/2+n)*c*AppellF1(1/2+m,1,1/2-n,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+
e)/f/(1+2*m)
```

Mathematica [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$= \int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int\cos(e+fx)(\sec(e+fx)a+a)^{m-\frac{1}{2}}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}(\sec(e+}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2}-n,1,n\right.}{f(2m+1)}$$

input
```
Int[(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[1/2 + m, 1/2 - n, 1, 3/2 + m, (1 + Sec[e + f*x])/2
, 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c
 - c*Sec[e + f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a + a\sec(fx + e))^m (c - c\sec(fx + e))^n \, dx$$

input
```
int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a(\sec(e+fx)+1))^m\,(-c(\sec(e+fx)-1))^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n, x)
```

Maxima [F]

$$\int (a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a + a\sec(e + fx))^m (c - c\sec(e + fx))^n \, dx = \int (a\sec(fx + e) + a)^m (-c\sec(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^m (c - c\sec(e + fx))^n \, dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^m \left(c - \frac{c}{\cos(e + f\,x)}\right)^n \, dx$$

input
```
int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (a + a\sec(e + fx))^m (c - c\sec(e + fx))^n \, dx = \int (\sec(fx + e)\,a + a)^m \,(-\sec(fx + e)\,c + c)^n \, dx$$

input
```
int((a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

output
```
int((sec(e + f*x)*a + a)**m*( - sec(e + f*x)*c + c)**n,x)
```

3.133 $\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{7}{2},\frac{1}{2}-n,1,\frac{9}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+$$
$$7f$$

output
```
1/7*2^(1/2+n)*c*AppellF1(7/2,1,1/2-n,9/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+
e)/f
```

Mathematica [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx = \int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n, x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int\cos(e+fx)(\sec(e+fx)a+a)^{5/2}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}(\sec(e+$$
$$f\sqrt{a\sec(e+fx)+a}$$

$$\Big\downarrow \text{153}$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^3(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(\frac{7}{2},\frac{1}{2}-n,1,\frac{9}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+\right.}{7f}$$

input
```
Int[(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[7/2, 1/2 - n, 1, 9/2, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^3*(c - c*Sec
[e + f*x])^(-1 + n)*Tan[e + f*x])/(7*f)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !G
tQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a + a\sec(fx + e))^3 (c - c\sec(fx + e))^n dx$$

input
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a\sec(e + fx))^3 (c - c\sec(e + fx))^n \, dx = \int (a\sec(fx + e) + a)^3 (-c\sec(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int (a + a\sec(e + fx))^3 (c - c\sec(e + fx))^n \, dx = a^3 \left(\int 3(-c\sec(e + fx) + c)^n \sec(e + fx) \, dx + \int 3(-c\sec(e + fx) + c)^n \sec^2(e + fx) \, dx + \int (-c\sec(e + fx) + c)^n \sec^3(e + fx) \, dx + \int (-c\sec(e + fx) + c)^n \, dx \right)$$

input
```
integrate((a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**n,x)
```

output
```
a**3*(Integral(3*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral(3*(-
c*sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c
)**n*sec(e + f*x)**3, x) + Integral((-c*sec(e + f*x) + c)**n, x))
```

Maxima [F]

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^n\, dx$$
$$= \int (a\sec(fx + e) + a)^3(-c\sec(fx + e) + c)^n\, dx$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^n\, dx$$
$$= \int (a\sec(fx + e) + a)^3(-c\sec(fx + e) + c)^n\, dx$$

input
```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^3(c - c\sec(e + fx))^n\, dx$$
$$= \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^3 \left(c - \frac{c}{\cos(e + f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^3(c-c\sec(e+fx))^n\,dx$$
$$= a^3\left(\int (-\sec(fx+e)\,c+c)^n\,dx + \int (-\sec(fx+e)\,c+c)^n \sec(fx+e)^3\,dx + 3\left(\int (-\sec(fx+e)\,c+c)^n \sec(fx+e)^2\,dx\right) + 3\left(\int (-\sec(fx+e)\,c+c)^n \sec(fx+e)\,dx\right)\right)$$

input

```
int((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^n,x)
```

output

```
a**3*(int(( - sec(e + f*x)*c + c)**n,x) + int(( - sec(e + f*x)*c + c)**n*s
ec(e + f*x)**3,x) + 3*int(( - sec(e + f*x)*c + c)**n*sec(e + f*x)**2,x) +
3*int(( - sec(e + f*x)*c + c)**n*sec(e + f*x),x))
```

3.134 $\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$$
$$= \frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{5}{2},\frac{1}{2}-n,1,\frac{7}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+}{5f}$$

output
```
1/5*2^(1/2+n)*c*AppellF1(5/2,1,1/2-n,7/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+
e)/f
```

Mathematica [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$$
$$= \int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n, x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 4400}$$

$$-\frac{ac\tan(e+fx)\int\cos(e+fx)(\sec(e+fx)a+a)^{3/2}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}(\sec(e+}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 153}$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)^2(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(\frac{5}{2},\frac{1}{2}-n,1,\frac{7}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+\right.}{5f}$$

input
```
Int[(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[5/2, 1/2 - n, 1, 7/2, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^2*(c - c*Sec
[e + f*x])^(-1 + n)*Tan[e + f*x])/(5*f)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !G
tQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a + a\sec(fx + e))^2 (c - c\sec(fx + e))^n \, dx$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(-c*sec(f*x + e)
+ c)^n, x)
```

Sympy [F]

$$\int (a+a\sec(e+fx))^2(c-c\sec(e+fx))^n\,dx = a^2\left(\int 2(-c\sec(e+fx)+c)^n\sec(e+fx)\,dx + \int (-c\sec(e+fx)+c)^n\sec^2(e+fx)\,dx + \int (-c\sec(e+fx)+c)^n\,dx\right)$$

input
```
integrate((a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**n,x)
```

output
```
a**2*(Integral(2*(-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*
sec(e + f*x) + c)**n*sec(e + f*x)**2, x) + Integral((-c*sec(e + f*x) + c)*
*n, x))
```

Maxima [F]

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^n\, dx = \int (a\sec(fx + e) + a)^2 (-c\sec(fx + e) + c)^n\, dx$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^n\, dx = \int (a\sec(fx + e) + a)^2 (-c\sec(fx + e) + c)^n\, dx$$

input
```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^n\, dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^2 \left(c - \frac{c}{\cos(e + f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (a + a\sec(e + fx))^2 (c - c\sec(e + fx))^n \, dx$$
$$= a^2 \left(\int (-\sec(fx + e)\, c + c)^n \, dx + \int (-\sec(fx + e)\, c + c)^n \sec(fx + e)^2 \, dx + 2 \left(\int (-\sec(fx + e)\, c + c)^n \sec(fx + e) \, dx \right) \right)$$

input
```
int((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^n,x)
```

output
```
a**2*(int(( - sec(e + f*x)*c + c)**n,x) + int(( - sec(e + f*x)*c + c)**n*s
ec(e + f*x)**2,x) + 2*int(( - sec(e + f*x)*c + c)**n*sec(e + f*x),x))
```

3.135 $\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$
$$=\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{2}-n,1,\frac{5}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+}{3f}$$

output
```
1/3*2^(1/2+n)*c*AppellF1(3/2,1,1/2-n,5/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*
(1-sec(f*x+e))^(1/2-n)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)
/f
```

Mathematica [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx=\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```

output
```
Integrate[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n, x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \cos(e+fx)\sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}\sqrt{\sec(e}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(a\sec(e+fx)+a)(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2}-n,1,\frac{5}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+}{3f}$$

input
```
Int[(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + n)*c*AppellF1[3/2, 1/2 - n, 1, 5/2, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])*(c - c*Sec[e
 + f*x])^(-1 + n)*Tan[e + f*x])/(3*f)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a + a\sec(fx + e))(c - c\sec(fx + e))^n \, dx$$

input
```
int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n \, dx = \int (a\sec(fx + e) + a)(-c\sec(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n \, dx = a\left(\int (-c\sec(e + fx) + c)^n \sec(e + fx) \, dx + \int (-c\sec(e + fx) + c)^n \, dx\right)$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))**n,x)
```

output
```
a*(Integral((-c*sec(e + f*x) + c)**n*sec(e + f*x), x) + Integral((-c*sec(e
 + f*x) + c)**n, x))
```

Maxima [F]

$$\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n \, dx = \int (a\sec(fx + e) + a)(-c\sec(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))(c-c\sec(e+fx))^n\,dx$$
$$=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (a + a\sec(e + fx))(c - c\sec(e + fx))^n dx$$
$$= a\left(\int (-\sec (fx + e) c + c)^n dx + \int (-\sec (fx + e) c + c)^n \sec (fx + e) dx\right)$$

input
```
int((a+a*sec(f*x+e))*(c-c*sec(f*x+e))^n,x)
```

output
```
a*(int(( - sec(e + f*x)*c + c)**n,x) + int(( - sec(e + f*x)*c + c)**n*sec(
e + f*x),x))
```

3.136 $\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = -\frac{2^{\frac{1}{2}+n}c\,\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2}-n,1,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{-1+n}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

output
```
-2^(1/2+n)*c*AppellF1(-1/2,1,1/2-n,1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1
-sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Mathematica [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

output
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \frac{\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{3/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}\operatorname{AppellF1}\left(-\frac{1}{2},\frac{1}{2}-n,1,\frac{1}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-c}{f(a\sec(e+fx)+a)}$$

input
```
Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

output
```
-((2^(1/2 + n)*c*AppellF1[-1/2, 1/2 - n, 1, 1/2, (1 + Sec[e + f*x])/2, 1 +
 Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 + n)*
Tan[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{(c - c\sec(fx + e))^n}{a + a\sec(fx + e)}dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx=\int \frac{(-c\sec\left(fx+e\right)+c)^n}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{(-c\sec\left(e+fx\right)+c)^n}{\sec\left(e+fx\right)+1}\,dx}{a}$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx=\int \frac{(-c\sec\left(fx+e\right)+c)^n}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0]%%%} / %%%{2,[0,0,1]%%%} Error: Bad Argument Val
ue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{(-\sec(fx+e)c+c)^n}{\sec(fx+e)+1}dx}{a}$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

output
```
int(( - sec(e + f*x)*c + c)**n/(sec(e + f*x) + 1),x)/a
```

3.137 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left(-\frac{3}{2},\frac{1}{2}-n,1,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\,\mathrm{se}}{3f(a+a\sec(e+fx))^2}$$

output
```
-1/3*2^(1/2+n)*c*AppellF1(-3/2,1,1/2-n,-1/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e
))*(1-sec(f*x+e))^(1/2-n)*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(a+a*sec(f*
x+e))^2
```

Mathematica [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4400, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{154}$$

$$-\frac{ac2^{n-\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(c-c\sec(e+fx))^{n-1}\int \frac{\cos(e+fx)\left(\frac{1}{2}-\frac{1}{2}\sec(e+fx)\right)^{n-\frac{1}{2}}}{(\sec(e+fx)a+a)^{5/2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2}-n,1,-\frac{1}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)(c-}{3f(a\sec(e+fx)+a)^2}$$

input
```
Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(2^(1/2 + n)*c*AppellF1[-3/2, 1/2 - n, 1, -1/2, (1 + Sec[e + f*x])/2,
 1 + Sec[e + f*x]]*(1 - Sec[e + f*x])^(1/2 - n)*(c - c*Sec[e + f*x])^(-1 +
 n)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^2)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{(c - c\sec(fx + e))^n}{(a + a\sec(fx + e))^2}dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^2}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e)
+ a^2), x)
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\,dx = \frac{\int \frac{(-c\sec(e+fx)+c)^n}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral((-c*sec(e + f*x) + c)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
x)/a**2
```

Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^2}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{(a\sec(fx + e) + a)^2}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%}+%%%{-3,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%}
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{(-\sec(fx+e)c+c)^n}{\sec(fx+e)^2+2\sec(fx+e)+1}dx}{a^2}$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int(( - sec(e + f*x)*c + c)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)/a
**2
```

3.138 $\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\frac{6a^3(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2a^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{2a^3(c-c\sec(e+fx))^{1+n}\tan(e+fx)}{cf(3+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
6*a^3*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^3
*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/
f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)-2*a^3*(c-c*sec(f*x+e))^(1+n)*tan(f*x+e)/c
/f/(3+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.59

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\frac{2a^3(c-c\sec(e+fx))^n\left(4(2+n)+(3+2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1\right.\right.}{f(1+2n)(3+2n)\sqrt{a(1+\sec(e+}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*a^3*(c - c*Sec[e + f*x])^n*(4*(2 + n) + (3 + 2*n)*Hypergeometric2F1[1,
1/2 + n, 3/2 + n, 1 - Sec[e + f*x]] + (1 + 2*n)*Sec[e + f*x])*Tan[e + f*x]
)/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3042, 4400, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4400$$

$$-\frac{ac\tan(e+fx)\int a^2\cos(e+fx)(\sec(e+fx)+1)^2(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^3c\tan(e+fx)\int \cos(e+fx)(\sec(e+fx)+1)^2(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 99$$

$$-\frac{a^3c\tan(e+fx)\int\left(\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}+3(c-c\sec(e+fx))^{n-\frac{1}{2}}-\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}}{c}\right)d\sec(e}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow 2009$$

$$-\frac{a^3c\tan(e+fx)\left(\frac{2(c-c\sec(e+fx))^{n+\frac{3}{2}}}{c^2(2n+3)}-\frac{2(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{c(2n+1)}-\frac{6(c-c\sec(e+fx}{c(2n+1)}\right.}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
-((a^3*c*((-6*(c - c*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)) - (2*Hypergeom
etric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1/2
+ n))/(c*(1 + 2*n)) + (2*(c - c*Sec[e + f*x])^(3/2 + n))/(c^2*(3 + 2*n)))*
Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a+a\sec(fx+e))^{\frac{5}{2}}(c-c\sec(fx+e))^n\,dx$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^n\,dx=\sqrt{a}\,a^2\left(\int \sqrt{\sec(fx+e)+1}\,(-\sec(fx+e)\,c+c)^n\sec(fx+e)^2\,dx+2\left(\int \sqrt{\sec(fx+e)+1}\,(-\sec(fx+e)\,c+c)^n\sec(fx+e)\,dx\right)+\int \sqrt{\sec(fx+e)+1}\,(-\sec(fx+e)\,c+c)^n\,dx\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n*sec(e
+ f*x)**2,x) + 2*int(sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n*sec
(e + f*x),x) + int(sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n,x))
```

3.139 $\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=\frac{2a^2(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2
*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/
f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx=$$
$$-\frac{2a^2\left(-6-4n+(1+2n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+n,\frac{5}{2}+n,1-\sec(e+fx)\right)(-1+\sec(e+fx))\right)(c-}{f(1+2n)(3+2n)\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
(-2*a^2*(-6 - 4*n + (1 + 2*n)*Hypergeometric2F1[1, 3/2 + n, 5/2 + n, 1 - S
ec[e + f*x]]*(-1 + Sec[e + f*x]))*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*
(1 + 2*n)*(3 + 2*n)*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4397, 3042, 4400, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4397}$$

$$a\int \sqrt{\sec(e+fx)a+a}(c-c\sec(e+fx))^n dx+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4400}$$

$$\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^2c\tan(e+fx)\int \cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(c-c\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*a^2*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e
+ f*x]]) + (2*a^2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]
*(c - c*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]
])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4397
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[-2*a^2*Cot[e + f*x]*((c + d*Csc[e + f
*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[a   Int[Sqrt[a +
b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LeQ[n, -2^(-1)]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a+a\sec(fx+e))^{\frac{3}{2}} (c-c\sec(fx+e))^n \, dx$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n \, dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^n \, dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n \, dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^n\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (a + a\sec(e + fx))^{3/2}(c - c\sec(e + fx))^n \, dx = \sqrt{a}\, a\left(\int \sqrt{\sec(fx + e) + 1}\,(-\sec(fx + e)\, c + c)^n \sec(fx + e)\, dx + \int \sqrt{\sec(fx + e) + 1}\,(-\sec(fx + e)\, c + c)^n \, dx\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n*sec(e + f
*x),x) + int(sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n,x))
```

3.140 $\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1, 1/2+n],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+
e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))^n\tan\left(\frac{1}{2}\right.}{f+2fn}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Se
c[e + f*x])]*(c - c*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f + 2*f*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 4400, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 4400}$$

$$-\frac{ac\tan(e+fx)\int\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{2a\tan(e+fx)(c-c\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^n,x]
```

output
```
(2*a*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e
 + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \sqrt{a+a\sec\left(fx+e\right)}\left(c-c\sec\left(fx+e\right)\right)^{n}dx$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx = \int \sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^{n}dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c(\sec\left(e+fx\right)-1)\right)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**n,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**n, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a\sec\left(fx+e\right)+a}\left(-c\sec\left(fx+e\right)+c\right)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c-\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^n\,dx$$
$$=\sqrt{a}\left(\int \sqrt{\sec(fx+e)+1}\,(-\sec(fx+e)\,c+c)^n\,dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^n,x)
```

output
```
sqrt(a)*int(sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n,x)
```

3.141 $\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
-hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f
*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n],[3/2+n],1-se
c(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\left(\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)-2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1\right.\right.}{(f+2fn)\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-(((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2] - 2*Hyper
geometric2F1[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]])*(c - c*Sec[e + f*x])^
n*Tan[e + f*x])/((f + 2*f*n)*Sqrt[a*(1 + Sec[e + f*x])]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4400, 27, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{a(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{97}$$

$$-\frac{c\tan(e+fx)\left(\int \cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)-\int \frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$-\frac{c\tan(e+fx)\left(-\int \frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)-\frac{2(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},1-\sec(e+fx)\right)}{c(2n+1)}\right)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{78}$$

$$-\frac{c\tan(e+fx)\left(\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}\left(1,n+\frac{1}{2},n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{c(2n+1)}-\frac{2(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}}{c(2n+1)}\right.}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((c*((Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c
*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)) - (2*Hypergeometric2F1[1, 1/2 + n,
 3/2 + n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)))
*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{(c - c\sec(fx + e))^n}{\sqrt{a + a\sec(fx + e)}} dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}} dx = \int \frac{(-c\sec(fx + e) + c)^n}{\sqrt{a\sec(fx + e) + a}} dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}} dx = \int \frac{(-c(\sec(e + fx) - 1))^n}{\sqrt{a(\sec(e + fx) + 1)}} dx$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{\sqrt{a\sec(fx + e) + a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{(-c\sec(fx + e) + c)^n}{\sqrt{a\sec(fx + e) + a}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^n}{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c - c\sec(e + fx))^n}{\sqrt{a + a\sec(e + fx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,(-\sec(fx+e)c+c)^n}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n)/(sec(e +
f*x) + 1),x))/a
```

3.142 $\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 205

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{(5-2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(c-c\sec(e+fx))^n\tan(e+fx)}{4af(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,1-\sec(e+fx)\right)(c-c\sec(e+fx))^n\tan(e+fx)}{af(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{(c-c\sec(e+fx))^n\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$

output
```
-1/4*(5-2*n)*hypergeom([1, 1/2+n],[3/2+n],1/2-1/2*sec(f*x+e))*(c-c*sec(f*x
+e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*hypergeom([1, 1/2+n
],[3/2+n],1-sec(f*x+e))*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+2*n)/(a+a*sec
(f*x+e))^(1/2)-1/2*(c-c*sec(f*x+e))^n*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*s
ec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{(c - c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{(c-c\sec(e+fx))^n\left(-2-4n+(-5+2n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\right.\right.}{}$$

input
```
Integrate[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
((c - c*Sec[e + f*x])^n*(-2 - 4*n + (-5 + 2*n)*Hypergeometric2F1[1, 1/2 +
n, 3/2 + n, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x]) + 8*Hypergeometric2F1
[1, 1/2 + n, 3/2 + n, 1 - Sec[e + f*x]]*(1 + Sec[e + f*x]))*Tan[e + f*x])/
(4*(f + 2*f*n)*(a*(1 + Sec[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4400, 27, 114, 27, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sec(e+fx))^n}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c-c\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4400}$$

$$-\frac{ac\tan(e+fx)\int \frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{a^2(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{c\tan(e+fx)\int\frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}}{(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{114}$$

$$-\frac{c\tan(e+fx)\left(\frac{\int\frac{c\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}(4-(1-2n)\sec(e+fx))}{2(\sec(e+fx)+1)}d\sec(e+fx)}{2c}+\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}}{2c(\sec(e+fx)+1)}\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\int\frac{\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}(4-(1-2n)\sec(e+fx))}{\sec(e+fx)+1}d\sec(e+fx)+\frac{(c-c\sec(e+fx))^{n+\frac{1}{2}}}{2c(\sec(e+fx)+1)}\right)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{174}$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\left(4\int\cos(e+fx)(c-c\sec(e+fx))^{n-\frac{1}{2}}d\sec(e+fx)-(5-2n)\int\frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+\right.\right.}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\left(-(5-2n)\int\frac{(c-c\sec(e+fx))^{n-\frac{1}{2}}}{\sec(e+fx)+1}d\sec(e+fx)-\frac{8(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}(1,n+\frac{1}{2},n+\frac{3}{2},1}{c(2n+1)}\right.\right.}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

$$\Big\downarrow \text{78}$$

$$-\frac{c\tan(e+fx)\left(\frac{1}{4}\left(\frac{(5-2n)(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hypergeometric2F1}(1,n+\frac{1}{2},n+\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)))}{c(2n+1)}-\frac{8(c-c\sec(e+fx))^{n+\frac{1}{2}}\text{Hyperge}}{}\right.\right.}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

input
```
Int[(c - c*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((c*((c - c*Sec[e + f*x])^(1/2 + n)/(2*c*(1 + Sec[e + f*x])) + (((5 - 2*n
)*Hypergeometric2F1[1, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x])/2]*(c - c*Sec[
e + f*x])^(1/2 + n))/(c*(1 + 2*n)) - (8*Hypergeometric2F1[1, 1/2 + n, 3/2
+ n, 1 - Sec[e + f*x]]*(c - c*Sec[e + f*x])^(1/2 + n))/(c*(1 + 2*n)))/4)*T
an[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4400
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a*c*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^(n - 1/2)/x), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{(c-c\sec(fx+e))^n}{(a+a\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^n/(a^2*sec(f*x + e
)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c(\sec(e+fx)-1))^n}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-c*(sec(e + f*x) - 1))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c-c\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*sec(f*x + e) + c)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{\left(c - \frac{c}{\cos(e+f\,x)}\right)^n}{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c - c/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c - c\sec(e + fx))^n}{(a + a\sec(e + fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}\,(-\sec(fx+e)c+c)^n}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int((c-c*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x) + 1)*( - sec(e + f*x)*c + c)**n)/(sec(e +
f*x)**2 + 2*sec(e + f*x) + 1),x))/a**2
```

3.143 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 91

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{cf}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f-2^(1/2)*a^
(1/2)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = -\frac{i\sqrt{1+e^{2i(e+fx)}}\left(\operatorname{arcsinh}\left(e^{i(e+fx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)-\operatorname{arctanh}\left(\sqrt{1+e^{2i(e+fx)}}\right)\right)\sqrt{a(1+}}{c\left(1+e^{i(e+fx)}\right)f}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

output
```
((-I)*Sqrt[1 + E^((2*I)*(e + f*x))]*(ArcSinh[E^(I*(e + f*x))] - Sqrt[2]*Ar
cTanh[(-1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] - Ar
cTanh[Sqrt[1 + E^((2*I)*(e + f*x))]])*Sqrt[a*(1 + Sec[e + f*x])])/(c*(1 +
E^(I*(e + f*x)))*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2011, 3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{c\sec(e+fx)+c}dx$$

$$\Big\downarrow \text{2011}$$

$$\frac{a\int \frac{1}{\sqrt{\sec(e+fx)a+a}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{c}$$

$$\Big\downarrow \text{4263}$$

$$\frac{a\left(\frac{\int \sqrt{\sec(e+fx)a+a}dx}{a}-\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)}{c}$$

$$\Big\downarrow \text{4261}$$

$$\frac{a\left(-\frac{2\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)}{c}$$

$$\Big\downarrow \; 216$$

$$\frac{a\left(\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)}{c}$$

$$\Big\downarrow \; 4282$$

$$\frac{a\left(\frac{2\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}\right)}{c}$$

$$\Big\downarrow \; 216$$

$$\frac{a\left(\frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}\right)}{c}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + c*Sec[e + f*x]),x]
```

output
```
(a*((2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f
) - (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x
]])])/(Sqrt[a]*f)))/c
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

method	result
default	$-\frac{\sqrt{-a\left(-1-\sec\left(fx+e\right)\right)}\,\sqrt{-\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\left(\ln\left(\sqrt{-\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}-\cot\left(fx+e\right)+\csc\left(fx+e\right)\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\left(-\cot\left(fx+e\right)+\csc(fx}{\sqrt{\left(1-\cos\left(fx+e\right)\right)^{2}\csc(fx+}}\right.\right.}{cf}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/c/f*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(ln
((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))-2^(1/2)*arcta
nh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)
)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \left[\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+3\,a\cos(fx+e)^2+2\,a\cos(fx+e)-a}{\cos(fx+e)^2+2\,\cos(fx+e)+1}\right)+2\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)}{\cdots}\right.}{2\,cf}\right.$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x
+ e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a)*log((2*a*cos
(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), (sqrt(2
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)/(sqrt(a)*sin(f*x + e))) - 2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{\int \frac{\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)+1}\,dx}{c}$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) + 1), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{c\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(c*sec(f*x + e) + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(76) = 152.

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{c|a|}+\frac{\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}\right.}\right.\right.}{c}\right.}{2f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
- sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(
sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4
*sqrt(2)*abs(a) - 6*a))/(c*abs(a)) + sqrt(-a)*log((sqrt(-a)*tan(1/2*f*x +
1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2)/c)*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + c\sec(e + fx)}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{c + \frac{c}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + c/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + c\sec(e + fx)}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)}{c}$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x))/c
```

3.144 $\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 231

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = -\frac{2c\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\sec(e+fx))}{a\sqrt{c+d}f} - \frac{(c-d)E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

output
```
-2*c*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-
d)/(c+d))^(1/2))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)*(d*(1+sec(f*x+
e))/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/a/(c+d)^(1/2)/f-(c-d)*Ellipti
cE(tan(f*x+e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)
*(c+d*sec(f*x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 810 vs. $2(231) = 462$.

Time = 32.53 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.51

$$\int \frac{(c + d\sec(e + fx))^{3/2}}{a + a\sec(e + fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

output
```
(Cos[e/2 + (f*x)/2]^2*(c + d*Sec[e + f*x])^(3/2)*(2*Sec[(e + f*x)/2]*(-(c*
Sin[(e + f*x)/2]) + d*Sin[(e + f*x)/2]) - 2*(-c + d)*Sin[e + f*x]))/(f*(d
+ c*Cos[e + f*x])*(a + a*Sec[e + f*x])) + (2*Cos[e/2 + (f*x)/2]^2*(c + d*S
ec[e + f*x])^(3/2)*(c^2*Tan[(e + f*x)/2] - d^2*Tan[(e + f*x)/2] - 2*c^2*Ta
n[(e + f*x)/2]^3 + 2*c*d*Tan[(e + f*x)/2]^3 + c^2*Tan[(e + f*x)/2]^5 - 2*c
*d*Tan[(e + f*x)/2]^5 + d^2*Tan[(e + f*x)/2]^5 - 4*c^2*EllipticPi[-1, ArcS
in[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(
c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] - 4*c^2*Elli
pticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Tan[(e + f*x)/2]^2*S
qrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e
+ f*x)/2]^2)/(c + d)] + (c^2 - d^2)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c
 - d)/(c + d)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[
(c + d - c*Tan[(e + f*x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(c + d)] + 2*c*(c -
d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)]*Sqrt[1 - Tan[(e +
f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(c + d - c*Tan[(e + f*x)/2]^2 + d
*Tan[(e + f*x)/2]^2)/(c + d)]))/(f*(d + c*Cos[e + f*x])^(3/2)*Sqrt[Sec[e +
 f*x]]*(a + a*Sec[e + f*x])*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[
(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(c + d - c*Tan[(e + f*
x)/2]^2 + d*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4415, 3042, 4267, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a\sec(e+fx)+a}dx$$

$\downarrow$ 3042

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$\downarrow$ 4415

$$\frac{c\int \sqrt{c+d\sec(e+fx)}dx}{a}-(c-d)\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx$$

$\downarrow$ 3042

$$\frac{c\int \sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a}-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$\downarrow$ 4267

$$-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right.}{af\sqrt{c+d}}$$

$\downarrow$ 4456

$$-\frac{2c\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-}{c+}\right.}{af\sqrt{c+d}}$$
$$\frac{(c-d)\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

input
```
Int[(c + d*Sec[e + f*x])^(3/2)/(a + a*Sec[e + f*x]),x]
```

output
```
(-2*c*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec
[e + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e +
 f*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e +
f*x]))/(a*Sqrt[c + d]*f) - ((c - d)*EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec
[e + f*x])], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec
[e + f*x]])/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

rule 4415
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Simp[a/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[(b*c - a*d)/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Cs
c[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.11

method	result
default	$\frac{\left(2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c^2-2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)cd+\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e)\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/a/f*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c^2-2*Ellipt
icF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c*d+EllipticE(cot(f*x+e)-cs
c(f*x+e),((c-d)/(c+d))^(1/2))*c^2-EllipticE(cot(f*x+e)-csc(f*x+e),((c-d)/(
c+d))^(1/2))*d^2-4*c^2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+d))^(
1/2)))*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(
f*x+e))/(cos(f*x+e)+1))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(d+c*cos(f*x+e))
```

Fricas [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)+1}\,dx + \int \frac{d\sqrt{c+d\sec(e+fx)}\sec(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate((c+d*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

output
```
(Integral(c*sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x) + Integral(d*s
qrt(c + d*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)),x)
```

output
```
int((c + d/cos(e + f*x))^(3/2)/(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{\left(\int \frac{\sqrt{\sec(fx+e)d+c}}{\sec(fx+e)+1}dx\right)c + \left(\int \frac{\sqrt{\sec(fx+e)d+c}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)d}{a}$$

input

```
int((c+d*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x)
```

output

```
(int(sqrt(sec(e + f*x)*d + c)/(sec(e + f*x) + 1),x)*c + int((sqrt(sec(e +
f*x)*d + c)*sec(e + f*x))/(sec(e + f*x) + 1),x)*d)/a
```

3.145 $\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(1+\sec(e+fx))}{c+d\sec(e+fx)}}(c+d\sec(}{a\sqrt{c+d}f} - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

output
```
-2*cot(f*x+e)*EllipticPi((c+d)^(1/2)/(c+d*sec(f*x+e))^(1/2),c/(c+d),((c-d)
/(c+d))^(1/2))*(-d*(1-sec(f*x+e))/(c+d*sec(f*x+e)))^(1/2)*(d*(1+sec(f*x+e)
)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/a/(c+d)^(1/2)/f-EllipticE(tan(f
*x+e)/(1+sec(f*x+e)),((c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*se
c(f*x+e))^(1/2)/a/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Mathematica [A] (verified)

Time = 6.70 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\left((c+d)E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+\right.\right.\right.\right.}{a(c+d)f(1+\cos(e+fx}$$

input
```
Integrate[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

output
```
(-4*Cos[(e + f*x)/2]^4*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d
)/(c + d)] + 2*(c - d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)
] - 4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sqrt[(1
 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(c + d)*f*(1 + Cos[e +
 f*x])^2*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4413, 3042, 4267, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4413}$$

$$\frac{\int \sqrt{c+d\sec(e+fx)}dx}{a}-\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx$$

$\downarrow$ 3042

$$\frac{\int \sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{a}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$\downarrow$ 4267

$$-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}$$

$\downarrow$ 4456

$$-\frac{2\cot(e+fx)\sqrt{-\frac{d(1-\sec(e+fx))}{c+d\sec(e+fx)}}\sqrt{\frac{d(\sec(e+fx)+1)}{c+d\sec(e+fx)}}(c+d\sec(e+fx))\,\text{EllipticPi}\left(\frac{c}{c+d},\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{c-d}{c+d}\right)}{af\sqrt{c+d}}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

input
```
Int[Sqrt[c + d*Sec[e + f*x]]/(a + a*Sec[e + f*x]),x]
```

output
```
(-2*Cot[e + f*x]*EllipticPi[c/(c + d), ArcSin[Sqrt[c + d]/Sqrt[c + d*Sec[e
 + f*x]]], (c - d)/(c + d)]*Sqrt[-((d*(1 - Sec[e + f*x]))/(c + d*Sec[e + f
*x]))]*Sqrt[(d*(1 + Sec[e + f*x]))/(c + d*Sec[e + f*x])]*(c + d*Sec[e + f*
x]))/(a*Sqrt[c + d]*f) - (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])
], (c - d)/(c + d)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]]
)/(a*f*Sqrt[(c + d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

rule 4413
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 -
 b^2, 0] || EqQ[c^2 - d^2, 0])
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.10

method	result
default	$\frac{\left(2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)c-2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{c-d}{c+d}}\right)d+c\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/a/f*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c-2*Elliptic
F(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE(cot(f*x+e)-csc(
f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d)
)^(1/2))-4*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+d))^(1/2)))*(co
s(f*x+e)+1)*(c+d*sec(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(c
+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\frac{\int \frac{\sqrt{c+d\sec\left(e+fx\right)}}{\sec\left(e+fx\right)+1}\,dx}{a}$$

input
```
integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

output
```
Integral(sqrt(c + d*sec(e + f*x))/(sec(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}}{a\sec\left(fx+e\right)+a}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)),x)
```

output
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{\sqrt{\sec(fx+e)d+c}}{\sec(fx+e)+1}dx}{a}$$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x)
```

output
```
int(sqrt(sec(e + f*x)*d + c)/(sec(e + f*x) + 1),x)/a
```

3.146 $\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 319

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{a(c-d)f} - \frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{c+d}{c},\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sec(e+fx))}{c-d}}}{acf} - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{c-d}{c+d}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{c+d\sec(e+fx)}}{a(c-d)f\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(1+\sec(e+fx))}}}$$

output
```
2*(c+d)^(1/2)*cot(f*x+e)*EllipticF((c+d*sec(f*x+e))^(1/2)/(c+d)^(1/2),((c+
d)/(c-d))^(1/2))*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(-d*(1+sec(f*x+e))/(c-d))^
(1/2)/a/(c-d)/f-2*(c+d)^(1/2)*cot(f*x+e)*EllipticPi((c+d*sec(f*x+e))^(1/2)
/(c+d)^(1/2),(c+d)/c,((c+d)/(c-d))^(1/2))*(d*(1-sec(f*x+e))/(c+d))^(1/2)*(
-d*(1+sec(f*x+e))/(c-d))^(1/2)/a/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),(
(c-d)/(c+d))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/(c-d
)/f/((c+d*sec(f*x+e))/(c+d)/(1+sec(f*x+e)))^(1/2)
```

Mathematica [A] (verified)

Time = 7.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx =$$

$$-\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left((c+d)E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{c-d}{c+d}\right)+2(c-2d)\,\text{EllipticF}\,(\right.}{a(c-}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(-4*Cos[(e + f*x)/2]^4*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x
]))]*((c + d)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 2*(c
- 2*d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)] + 4*(-c + d)*E
llipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (c - d)/(c + d)])*Sec[e + f*x]^2*(
(1 + Sec[e + f*x])^(-1))^(3/2))/(a*(c - d)*f*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4417, 25, 3042, 4409, 3042, 4271, 4319, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4417}$$

$$-\frac{\int -\frac{a(c-d)+ad\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}dx}{a^2(c-d)}-\frac{\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx}{c-d}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{a(c-d)+ad\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}dx}{a^2(c-d)}-\frac{\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sec(e+fx)a+a}dx}{c-d}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{a(c-d)+ad\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a^2(c-d)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \ 4409$$

$$\frac{a(c-d)\int \frac{1}{\sqrt{c+d\sec(e+fx)}}dx+ad\int \frac{\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}dx}{a^2(c-d)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \ 3042$$

$$\frac{a(c-d)\int \frac{1}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+ad\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a^2(c-d)}-\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \ 4271$$

$$\frac{ad\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2a(c-d)\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\text{EllipticPi}\left(\frac{c+d}{c},\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right.}{cf}}{a^2(c-d)}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \ 4319$$

$$\frac{\frac{2a\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{f}-\frac{2a(c-d)\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx}{c+d}}}{}}{a^2(c-d)}$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c-d}$$

$$\Big\downarrow \ 4456$$

$$\frac{\frac{2a\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx))}{c+d}}\sqrt{-\frac{d(\sec(e+fx)+1)}{c-d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}}\right),\frac{c+d}{c-d}\right)}{f}-\frac{2a(c-d)\sqrt{c+d}\cot(e+fx)\sqrt{\frac{d(1-\sec(e+fx}{c+d}}}{}}{a^2(c-d)}$$

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{c+d\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{c-d}{c+d}\right)}{af(c-d)\sqrt{\frac{c+d\sec(e+fx)}{(c+d)(\sec(e+fx)+1)}}}$$

input
```
Int[1/((a + a*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
-((EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (c - d)/(c + d)]*Sqr
t[(1 + Sec[e + f*x])^(-1)]*Sqrt[c + d*Sec[e + f*x]])/(a*(c - d)*f*Sqrt[(c
+ d*Sec[e + f*x])/((c + d)*(1 + Sec[e + f*x]))])) + ((2*a*Sqrt[c + d]*Cot[
e + f*x]*EllipticF[ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(
c - d)]*Sqrt[(d*(1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))
/(c - d))])/f - (2*a*(c - d)*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(c + d)/c
, ArcSin[Sqrt[c + d*Sec[e + f*x]]/Sqrt[c + d]], (c + d)/(c - d)]*Sqrt[(d*(
1 - Sec[e + f*x]))/(c + d)]*Sqrt[-((d*(1 + Sec[e + f*x]))/(c - d))])/(c*f)
)/(a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4417
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Simp[1/(c*(b*c - a*d))   Int[(b*c - a*d - b*d*
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[d^2/(c*(b*c - a*d))
 Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] |
| EqQ[c^2 - d^2, 0])
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))])))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.90

method	result
default	$\frac{\left(2 \operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e), \sqrt{\frac{c-d}{c+d}}\right) c-4 \operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e), \sqrt{\frac{c-d}{c+d}}\right) d+c \operatorname{EllipticE}\left(\cot(fx+e)-\csc(fx+e), \right.\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/f/(c-d)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*c-4*El
lipticF(cot(f*x+e)-csc(f*x+e),((c-d)/(c+d))^(1/2))*d+c*EllipticE(cot(f*x+e
)-csc(f*x+e),((c-d)/(c+d))^(1/2))+d*EllipticE(cot(f*x+e)-csc(f*x+e),((c-d)
/(c+d))^(1/2))-4*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+d))^(1/2)
)+4*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((c-d)/(c+d))^(1/2))*d)*(cos(f*x+e
)+1)*(c+d*sec(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+
c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{1}{\sqrt{c+d\sec(e+fx)}\sec(e+fx)+\sqrt{c+d\sec(e+fx)}}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(c + d*sec(e + f*x))*sec(e + f*x) + sqrt(c + d*sec(e + f*x
))), x)/a
```

Maxima [F]

$$\int \frac{1}{(a + a\sec(e + fx))\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{1}{(a\sec(fx + e) + a)\sqrt{d\sec(fx + e) + c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{1}{(a + a\sec(e + fx))\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{1}{(a\sec(fx + e) + a)\sqrt{d\sec(fx + e) + c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\sec(e + fx))\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{1}{\left(a + \frac{a}{\cos(e+f\,x)}\right)\,\sqrt{c + \frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sqrt{\sec(fx+e)d+c}}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx}{a}$$

input
```
int(1/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)*d + c)/(sec(e + f*x)**2*d + sec(e + f*x)*c + sec(e +
 f*x)*d + c),x)/a
```

3.147 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 27, antiderivative size = 271

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx &= \frac{2ad(2c+d)\left(2c^2+2cd+d^2\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} \\ &\quad+\frac{2a^{3/2}c^4\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &\quad-\frac{2d^2\left(6c^2+8cd+3d^2\right)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} \\ &\quad+\frac{2d^3(4c+3d)(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}} \\ &\quad-\frac{2d^4(a-a\sec(e+fx))^3\tan(e+fx)}{7a^2f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
2*a*d*(2*c+d)*(2*c^2+2*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3
/2)*c^4*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+
e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(6*c^2+8*c*d+3*d^2)*(a-a*sec(f*x+
e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/5*d^3*(4*c+3*d)*(a-a*sec(f*x+e))
^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2/7*d^4*(a-a*sec(f*x+e))^3*tan(f*
x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 8.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.85

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$= \frac{2\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^4\left(105c^4\arctan\left(\frac{\tan(\frac{1}{2}(e+fx))}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\sqrt{\frac{\sec(e+fx)}{(1+\sec(e+fx))^2}}\sqrt{1+\sec(e+f\cdots}\right.}{}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

output
```
(2*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^4*(105*c^4*ArcTan[Tan[(
e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]*Sqrt[Sec[e + f*x]/(1 +
Sec[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + d*Sqrt[Sec[e + f*x]]*(420*c^3 +
420*c^2*d + 224*c*d^2 + 48*d^3 + 2*d*(105*c^2 + 56*c*d + 12*d^2)*Sec[e + f
*x] + 6*d^2*(14*c + 3*d)*Sec[e + f*x]^2 + 15*d^3*Sec[e + f*x]^3)*Tan[(e +
f*x)/2]))/(105*f*(d + c*Cos[e + f*x])^4*Sec[e + f*x]^(9/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 4428, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int\frac{\cos(e+fx)(c+d\sec(e+fx))^4}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\big\downarrow$ 99

$$-\frac{a^2\tan(e+fx)\int\left(\frac{\cos(e+fx)c^4}{\sqrt{a-a\sec(e+fx)}}-\frac{d^4(a-a\sec(e+fx))^{5/2}}{a^3}+\frac{d^3(4c+3d)(a-a\sec(e+fx))^{3/2}}{a^2}-\frac{d^2(6c^2+8dc+3d^2)\sqrt{a-a\sec(e+f}}{a}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\big\downarrow$ 2009

$$-\frac{a^2\tan(e+fx)\left(\frac{2d^4(a-a\sec(e+fx))^{7/2}}{7a^4}-\frac{2d^3(4c+3d)(a-a\sec(e+fx))^{5/2}}{5a^3}+\frac{2d^2(6c^2+8cd+3d^2)(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^4\text{arctan}}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^4,x]
```

output
```
-((a^2*((-2*c^4*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*d*
(2*c + d)*(2*c^2 + 2*c*d + d^2)*Sqrt[a - a*Sec[e + f*x]])/a + (2*d^2*(6*c^
2 + 8*c*d + 3*d^2)*(a - a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d^3*(4*c + 3*d
)*(a - a*Sec[e + f*x])^(5/2))/(5*a^3) + (2*d^4*(a - a*Sec[e + f*x])^(7/2))
/(7*a^4))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]))
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 2.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.87

method	result
default	$\frac{2\sqrt{a(1+\sec(fx+e))}\left((105\cos(fx+e)+105)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,c^4\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)+420c\right.}{}$
parts	$\frac{c^4\sqrt{2}\,\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}+\frac{2d^4\left(16\cos(fx+e)^3+8\cos(fx+e)^2+6\cos\right.}{35}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
2/105/f*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*((105*cos(f*x+e)+105)*(-co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^4*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x
+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+420*c^3*d*s
in(f*x+e)+c^2*d^2*(420*sin(f*x+e)+210*tan(f*x+e))+c*d^3*(224*sin(f*x+e)+11
2*tan(f*x+e)+84*sec(f*x+e)*tan(f*x+e))+(48*cos(f*x+e)^3+24*cos(f*x+e)^2+18
*cos(f*x+e)+15)*d^4*tan(f*x+e)*sec(f*x+e)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.74

$$\int \sqrt{a + a\sec(e + fx)}(c + d\sec(e + fx))^4 \, dx$$
$$= \left[\frac{105 \left(c^4 \cos\left(fx + e\right)^4 + c^4 \cos\left(fx + e\right)^3\right) \sqrt{-a} \log\left(\frac{2\,a\cos(fx+e)^2 - 2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) + a\cos(f}{\cos(fx+e)+1}\right.}{} \right.$$
$$- \frac{2\left(105 \left(c^4 \cos\left(fx + e\right)^4 + c^4 \cos\left(fx + e\right)^3\right) \sqrt{a} \arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) - \left(15\,d^4 + 4\,(105\,c^3 d -\right.}{}$$

input

```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

output

```
[1/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*sqrt(-a)*log((2*a*co
s(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*d^4 +
4*(105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^3 + 2*(105*c^
2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
^4 + f*cos(f*x + e)^3), -2/105*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)
^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - (15*d^4 + 4*(105*c^3*d + 105*c^2*d^2 + 56*c*d^3 +
12*d^4)*cos(f*x + e)^3 + 2*(105*c^2*d^2 + 56*c*d^3 + 12*d^4)*cos(f*x + e)^
2 + 6*(14*c*d^3 + 3*d^4)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sin(f*x + e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^4\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**4,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**4, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$=\int \sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)^4\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
-1/210*(16*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(7*(15*c^3*d*sin(6*f*x + 6*e) + 5*(9*c^3*d + 3*c^2*d^2 + 4*c*d^3
)*sin(4*f*x + 4*e) + (45*c^3*d + 30*c^2*d^2 + 28*c*d^3 + 6*d^4)*sin(2*f*x
+ 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (105*c^
3*d*cos(6*f*x + 6*e) + 105*c^3*d + 105*c^2*d^2 + 56*c*d^3 + 12*d^4 + 35*(9
*c^3*d + 3*c^2*d^2 + 4*c*d^3)*cos(4*f*x + 4*e) + 7*(45*c^3*d + 30*c^2*d^2
+ 28*c*d^3 + 6*d^4)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)))*sqrt(a) + 105*((c^4*cos(2*f*x + 2*e)^4 + c^4*sin(2*f
*x + 2*e)^4 + 4*c^4*cos(2*f*x + 2*e)^3 + 6*c^4*cos(2*f*x + 2*e)^2 + 4*c^4*
cos(2*f*x + 2*e) + c^4 + 2*(c^4*cos(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e
) + c^4)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) + 1) - (c^4*cos(2*f*x + 2*e)^4 + c^4*sin(2*f*x + 2*e)^4 + 4*c^4*cos(2*f*
x + 2*e)^3 + 6*c^4*cos(2*f*x + 2*e)^2 + 4*c^4*cos(2*f*x + 2*e) + c^4 + 2*(
c^4*cos(2*f*x + 2*e)^2 + 2*c^4*cos(2*f*x + 2*e) + c^4)*sin(2*f*x + 2*e)^2)
*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(250) = 500$.

Time = 0.77 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.97

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
-1/105*(105*sqrt(-a)*a*c^4*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(420*sqrt(2)*a^4*c^3*d*sgn(
cos(f*x + e)) + 630*sqrt(2)*a^4*c^2*d^2*sgn(cos(f*x + e)) + 420*sqrt(2)*a^
4*c*d^3*sgn(cos(f*x + e)) + 105*sqrt(2)*a^4*d^4*sgn(cos(f*x + e)) - (1260*
sqrt(2)*a^4*c^3*d*sgn(cos(f*x + e)) + 1470*sqrt(2)*a^4*c^2*d^2*sgn(cos(f*x
 + e)) + 700*sqrt(2)*a^4*c*d^3*sgn(cos(f*x + e)) + 105*sqrt(2)*a^4*d^4*sgn
(cos(f*x + e)) - (1260*sqrt(2)*a^4*c^3*d*sgn(cos(f*x + e)) + 1050*sqrt(2)*
a^4*c^2*d^2*sgn(cos(f*x + e)) + 476*sqrt(2)*a^4*c*d^3*sgn(cos(f*x + e)) +
147*sqrt(2)*a^4*d^4*sgn(cos(f*x + e)) - (420*sqrt(2)*a^4*c^3*d*sgn(cos(f*x
 + e)) + 210*sqrt(2)*a^4*c^2*d^2*sgn(cos(f*x + e)) + 196*sqrt(2)*a^4*c*d^3
*sgn(cos(f*x + e)) + 27*sqrt(2)*a^4*d^4*sgn(cos(f*x + e)))*tan(1/2*f*x + 1
/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2
*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))
)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)^4\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^4, x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^4\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\sec(fx+e)+1}dx\right)c^4+\left(\int\sqrt{\sec(fx+e)+1}\sec(fx+e)^4\,dx\right)d^4 + 4\left(\int\sqrt{\sec(fx+e)+1}\sec(fx+e)^3\,dx\right)c\,d^3 + 6\left(\int\sqrt{\sec(fx+e)+1}\sec(fx+e)^2\,dx\right)c^2d^2 + 4\left(\int\sqrt{\sec(fx+e)+1}\sec(fx+e)\,dx\right)c^3d\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^4,x)
```

output
```
sqrt(a)*(int(sqrt(sec(e + f*x) + 1),x)*c**4 + int(sqrt(sec(e + f*x) + 1)*s
ec(e + f*x)**4,x)*d**4 + 4*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x)*c
*d**3 + 6*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*c**2*d**2 + 4*int(
sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c**3*d)
```

3.148 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 205

$$\begin{aligned}\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx &= \frac{2ad(3c^2+3cd+d^2)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{3/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\&\quad-\frac{2d^2(3c+2d)(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}\\&\quad+\frac{2d^3(a-a\sec(e+fx))^2\tan(e+fx)}{5af\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
2*a*d*(3*c^2+3*c*d+d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^3*
arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)-2/3*d^2*(3*c+2*d)*(a-a*sec(f*x+e))*tan(f*x+e)/f/(
a+a*sec(f*x+e))^(1/2)+2/5*d^3*(a-a*sec(f*x+e))^2*tan(f*x+e)/a/f/(a+a*sec(f
*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 8.64 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.94

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$
$$= \frac{2\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^3\left(15c^3\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\sqrt{\frac{\sec(e+fx)}{(1+\sec(e+fx))^2}}\sqrt{1+\sec(e+fx}\right.}{15f(d+c\cos(e+}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

output
```
(2*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x])^3*(15*c^3*ArcTan[Tan[(e
 + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]]*Sqrt[Sec[e + f*x]/(1 + S
ec[e + f*x])^2]*Sqrt[1 + Sec[e + f*x]] + d*Sqrt[Sec[e + f*x]]*(45*c^2 + 30
*c*d + 8*d^2 + d*(15*c + 4*d)*Sec[e + f*x] + 3*d^2*Sec[e + f*x]^2)*Tan[(e
+ f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 4428, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int\frac{\cos(e+fx)(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 99

$$-\frac{a^2\tan(e+fx)\int\left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3(a-a\sec(e+fx))^{3/2}}{a^2}-\frac{d^2(3c+2d)\sqrt{a-a\sec(e+fx)}}{a}+\frac{d(3c^2+3dc+d^2)}{\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 2009

$$-\frac{a^2\tan(e+fx)\left(-\frac{2d^3(a-a\sec(e+fx))^{5/2}}{5a^3}+\frac{2d^2(3c+2d)(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d(3c^2+3cd+d^2\cdots}{\cdots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^2*((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*d*
(3*c^2 + 3*c*d + d^2)*Sqrt[a - a*Sec[e + f*x]])/a + (2*d^2*(3*c + 2*d)*(a
- a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d^3*(a - a*Sec[e + f*x])^(5/2))/(5*a
^3))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.90

method	result
default	$\frac{2\sqrt{a(1+\sec(fx+e))}\left(d^3(3\sec(fx+e)\tan(fx+e)+8\sin(fx+e)+4\tan(fx+e))+15(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{}{\sqrt{\cot(}}\right.\right.}{15f(\cos(fx+e)+1)}$
parts	$\frac{c^3\sqrt{2}\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}+\frac{d^3(16\sin(fx+e)+8\tan(fx+e)+6\sec(fx}{f(15+15\cos(}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/15/f*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(d^3*(3*sec(f*x+e)*tan(f*x+
e)+8*sin(f*x+e)+4*tan(f*x+e))+15*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^
2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*c^3+45*c^2*d*sin(f*x+e)+c*d^2*(30*sin
(f*x+e)+15*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.91

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$

$$= \left[\frac{15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx}{\cos(fx+e)+1}\right.}{15\left(f\cos(fx\right.}\right.$$

$$-\frac{2\left(15\left(c^3\cos(fx+e)^3+c^3\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3\,d^3+\left(45\,c^2d+30\,c\right.\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*sqrt(-a)*log((2*a*cos(
f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*d^3 + (45
*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2 + (15*c*d^2 + 4*d^3)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3
 + f*cos(f*x + e)^2), -2/15*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*
sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(
a)*sin(f*x + e))) - (3*d^3 + (45*c^2*d + 30*c*d^2 + 8*d^3)*cos(f*x + e)^2
+ (15*c*d^2 + 4*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$
$$=\int \sqrt{a\left(\sec(e+fx)+1\right)}\left(c+d\sec(e+fx)\right)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$
$$=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/30*(15*((c^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*
f*x + 2*e) + c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c
^3*cos(2*f*x + 2*e)^2 + c^3*sin(2*f*x + 2*e)^2 + 2*c^3*cos(2*f*x + 2*e) +
c^3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*(c^3*f*cos(2*
f*x + 2*e)^2 + c^3*f*sin(2*f*x + 2*e)^2 + 2*c^3*f*cos(2*f*x + 2*e) + c^3*f
)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(
2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e)
+ 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e
) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)...
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.78

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx =$$

$$-\frac{\frac{15\sqrt{-a}ac^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(\left(\sqrt{2}(45a^3c^2d\mathrm{sgn}(\cos(fx+e))+15\right.\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(-a)*a*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-
a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*((sqrt(2)*(45*a^3*c^2*d*sgn(c
os(f*x + e)) + 15*a^3*c*d^2*sgn(cos(f*x + e)) + 7*a^3*d^3*sgn(cos(f*x + e)
))*tan(1/2*f*x + 1/2*e)^2 - 10*sqrt(2)*(9*a^3*c^2*d*sgn(cos(f*x + e)) + 6*
a^3*c*d^2*sgn(cos(f*x + e)) + a^3*d^3*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/
2*e)^2 + 15*sqrt(2)*(3*a^3*c^2*d*sgn(cos(f*x + e)) + 3*a^3*c*d^2*sgn(cos(f
*x + e)) + a^3*d^3*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*
x + 1/2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$
$$=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3\,dx$$
$$= \sqrt{a}\left(\left(\left(\int \sqrt{\sec(fx+e)+1}dx\right)c^3+\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)d^3 + 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c\,d^2 + 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c^2d\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*(int(sqrt(sec(e + f*x) + 1),x)*c**3 + int(sqrt(sec(e + f*x) + 1)*s
ec(e + f*x)**3,x)*d**3 + 3*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*c
*d**2 + 3*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c**2*d)
```

3.149 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx = \frac{2ad(2c+d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{3/2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^2(a-a\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*d*(2*c+d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(3/2)*c^2*arctanh((a
-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2)-2/3*d^2*(a-a*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.23 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.08

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$
$$=\frac{\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}(c+d\sec(e+fx))^2\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(e+fx)\right)}}\sqrt{1-2\sin^2}}{}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

output
```
(Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec
[e + f*x])^2*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x
)/2]^2]*(256*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(e + f*x)/2]
^2]*Sin[(e + f*x)/2]^6*(c + d - 2*c*Sin[(e + f*x)/2]^2)^2 + 1024*Hypergeom
etric2F1[3/2, 7/2, 9/2, 2*Sin[(e + f*x)/2]^2]*Sin[(e + f*x)/2]^6*(d^2 + c*
d*(2 - 3*Sin[(e + f*x)/2]^2) + c^2*(1 - 3*Sin[(e + f*x)/2]^2 + 2*Sin[(e +
f*x)/2]^4)) - (7*Sqrt[2]*(-3*ArcSin[Sqrt[2]*Sqrt[Sin[(e + f*x)/2]^2]] + Sq
rt[2]*Sqrt[Sin[(e + f*x)/2]^2]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*(3 + 4*Sin[(
e + f*x)/2]^2))*(15*d^2 + 10*c*d*(3 - 2*Sin[(e + f*x)/2]^2) + c^2*(15 - 20
*Sin[(e + f*x)/2]^2 + 12*Sin[(e + f*x)/2]^4)))/Sqrt[Sin[(e + f*x)/2]^2]))/
(672*f*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 4428, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int\frac{\cos(e+fx)(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{99}$$

$$-\frac{a^2\tan(e+fx)\int\left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}-\frac{d^2\sqrt{a-a\sec(e+fx)}}{a}+\frac{d(2c+d)}{\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\tan(e+fx)\left(\frac{2d^2(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d(2c+d)\sqrt{a-a\sec(e+fx)}}{a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^2*((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*d*
(2*c + d)*Sqrt[a - a*Sec[e + f*x]])/a + (2*d^2*(a - a*Sec[e + f*x])^(3/2))
/(3*a^2))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x
]]))
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
default	$-\frac{2\sqrt{a(1+\sec(fx+e))}\left(3(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)c^2+d^2(-2$ ... $}{3f(\cos(fx+e)+1)}$
parts	$\frac{c^2\sqrt{2}\,\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}+\frac{d^2(4\sin(fx+e)+2\tan(fx+e))\sqrt{a(1+\sec}$ ... $}{f(3\cos(fx+e)+3)}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(3*(cos(f*x+e)+1)*(-cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)-csc(f*x+e))/(cot(f*x
+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))*c^2+d^2*(-2*sin(f*x+e
)-tan(f*x+e))-6*c*d*sin(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.22

$$\int \sqrt{a + a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$
$$= \left[\frac{3\left(c^2\cos\left(fx+e\right)^2 + c^2\cos\left(fx+e\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) + a\cos(fx+e}{\cos(fx+e)+1}\right.}{3\left(f\cos\left(fx+e\right)^2 + f\cos\left(fx+e\right)\right.}\right.$$
$$- \frac{2\left(3\left(c^2\cos\left(fx+e\right)^2 + c^2\cos\left(fx+e\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right) - \left(d^2 + 2\left(3\,cd + d^2\right)\cos\left(\right.\right.\right.}{3\left(f\cos\left(fx+e\right)^2 + f\cos\left(fx+e\right)\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/3*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(d^2 + 2*(3*c*d
+ d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(f*cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(c^2*cos(f*x + e)^2 + c^2*co
s(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e))) - (d^2 + 2*(3*c*d + d^2)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos
(f*x + e))]
```

Sympy [F]

$$\int \sqrt{a + a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$
$$= \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int \sqrt{a + a\sec(e + fx)}(c + d\sec(e + fx))^2 \, dx$$
$$= \int \sqrt{a\sec(fx + e) + a}(d\sec(fx + e) + c)^2 \, dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/6*(8*(3*c*d*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*si
n(2*f*x + 2*e) - (3*c*d*cos(2*f*x + 2*e) + 3*c*d + d^2)*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sqrt(a) + 3*((c^2*cos(2*f*x + 2*e)^
2 + c^2*sin(2*f*x + 2*e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1)) + 1) - (c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2
*f*x + 2*e)^2 + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2((cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)) - 1) - 2*(c^2*f*cos(2*f*x + 2*e)^2 + c^2*f*sin(2*f*x +
2*e)^2 + 2*c^2*f*cos(2*f*x + 2*e) + c^2*f)*integrate((((cos(6*f*x + 6*e)*c
os(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2
 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*
e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(129) = 258$.

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.86

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx =$$

$$-\frac{\frac{3\sqrt{-a}ac^2\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\operatorname{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(6\sqrt{2}a^2cd\operatorname{sgn}(\cos(fx+e))+3\sqrt{2}a^2c\cdots\right.}{}}{3f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(-a)*a*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(6*sqrt(2)*a^2*c*d*sgn(cos(f*x
+ e)) + 3*sqrt(2)*a^2*d^2*sgn(cos(f*x + e)) - (6*sqrt(2)*a^2*c*d*sgn(cos(f
*x + e)) + sqrt(2)*a^2*d^2*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(
1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$
$$= \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\sec(fx+e)+1}dx\right)c^2+\left(\int\sqrt{\sec(fx+e)+1}\sec(fx+e)^2\,dx\right)d^2+2\left(\int\sqrt{\sec(fx+e)+1}\sec(fx+e)\,dx\right)cd\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*(int(sqrt(sec(e + f*x) + 1),x)*c**2 + int(sqrt(sec(e + f*x) + 1)*s
ec(e + f*x)**2,x)*d**2 + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c*d)
```

3.150 $\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx = \frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^(1/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2*a*d*tan(
f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx = \frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\left(\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\cos(e+fx)}+2d\sin\left(\frac{1}{2}(e+f\right.\right.}{f}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin
[(e + f*x)/2]]*Sqrt[Cos[e + f*x]] + 2*d*Sin[(e + f*x)/2]))/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4403}$$

$$c\int \sqrt{\sec(e+fx)a+a}dx+d\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+d\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4261}$$

$$d\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{2ac\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$d\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2ad\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(2*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f +
(2*a*d*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

method	result
default	$\frac{\sqrt{-a(-1-\sec(fx+e))}\left((2\csc(fx+e)-2\cot(fx+e))d+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{2}\,c\right)}{f}$
parts	$\frac{c\sqrt{2}\,\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)}{f}-\frac{2d\sqrt{a(1+\sec(fx+e))}\,(\cot(fx+e)-\csc(fx\cdot}{f}$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-1-sec(f*x+e)))^(1/2)*((2*csc(f*x+e)-2*cot(f*x+e))*d+(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*2^(1/2)*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.56

$$\int\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$=\left[\frac{(c\cos(fx+e)+c)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+a\cos(fx+e)-a}{\cos(fx+e)+1}\right)+2\,d\sqrt{\frac{a\cos(}{\cos}}}{f\cos(fx+e)+f}\right.$$
$$\left.-\frac{2\left((c\cos(fx+e)+c)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)\right)}{f\cos(fx+e)+f}\right]$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[((c*cos(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x +
 e) - a)/(cos(f*x + e) + 1)) + 2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e) + f), -2*((c*cos(f*x + e) + c)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) - d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e) + f)]
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$=\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}(c+d\sec\left(e+fx\right))\;dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(58) = 116$.

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$=\frac{\sqrt{a}c\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^2+\sin\left(2\,fx+2\,e\right)^2+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
sqrt(a)*c*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
+ sin(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
cos(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(58) = 116.

Time = 0.42 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.92

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx =$$

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\,ad\,\mathrm{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a}+\frac{\sqrt{-a}ac\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a\right.}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a\right.}\right.}{|a|}}{f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
-(2*sqrt(2)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)*a*d*sgn(cos(f*x + e))*tan(
1/2*f*x + 1/2*e)/(a*tan(1/2*f*x + 1/2*e)^2 - a) + sqrt(-a)*a*c*log(abs(2*(
sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4
*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(
1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/ab
s(a))/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$=\sqrt{a}\left(\left(\left(\int \sqrt{\sec(fx+e)+1}dx\right)c+\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)d\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x)
```

output
```
sqrt(a)*(int(sqrt(sec(e + f*x) + 1),x)*c + int(sqrt(sec(e + f*x) + 1)*sec(
e + f*x),x)*d)
```

3.151 $\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} - \frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{c+d}f}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f-2*a^(1/2)*
d^(1/2)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/
2))/c/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\left(\sqrt{c+d}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}\arctan\left(\frac{\sqrt{d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{a(1+\sec(e+fx))}}{c\sqrt{c+d}f}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output
```
(2*(Sqrt[c + d]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x
])]] - Sqrt[d]*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e +
 f*x]/(1 + Cos[e + f*x])]])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[a*
(1 + Sec[e + f*x])])/(c*Sqrt[c + d]*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4413, 3042, 4261, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4413}$$

$$\frac{\int \sqrt{\sec(e+fx)a+a}dx}{c}-\frac{d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2a\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{d\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}$$

$$\Big\downarrow \text{4455}$$

$$\frac{2ad\int\frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2\sqrt{a}\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{cf\sqrt{c+d}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f)
- (2*Sqrt[a]*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])])/(c*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4413
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 -
 b^2, 0] || EqQ[c^2 - d^2, 0])
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(85) = 170$.

Time = 5.72 (sec) , antiderivative size = 499, normalized size of antiderivative = 4.75

method	result
default	$\frac{\sqrt{2}\left(2\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{\frac{d}{c-d}}-d\ln\left(-\frac{2\left(\sqrt{2}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\sqrt{\frac{d}{c-d}}\,c-\sqrt{2}\sqrt{\right.}{-c(-\cot(fx+e)+\csc}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)/c*(2*((c+d)*(c-d))^(1/2)
*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(
f*x+e)))*(d/(c-d))^(1/2)-d*ln(-2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-
cot(f*x+e)+csc(f*x+e))+d*(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))+d*
ln(2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c-2^
(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-
d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-d*(-co
t(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2))))*((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.37

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + d\sec(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[(sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e
)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
+ e) + d)) + sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(
cos(f*x + e) + 1)))/(c*f), -(2*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(-a*d/(c + d))*log(
(2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*c
os(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*f), (2*sqr
t(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))) + sqrt(-a)*log((2*a*cos(f*x
 + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*
sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*(sqrt(a)
*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(
f*x + e))) - sqrt(a*d/(c + d))*arctan((c + d)*sqrt(a*d/(c + d))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*d*sin(f*x + e))))/(c*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{c + d\sec(e + fx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}{c + d\sec\left(e + fx\right)}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(85) = 170.

Time = 0.68 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.64

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{\sqrt{2}\left(\frac{2\sqrt{2}\sqrt{-a}d\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac+3\,ac}{4\sqrt{-cd-d^2}a}\right)}{\sqrt{-cd-d^2}c}\right.}{2f}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(2*sqrt(2)*sqrt(-a)*d*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2
*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(s
qrt(-c*d - d^2)*a))/(sqrt(-c*d - d^2)*c) + sqrt(2)*sqrt(-a)*a*log(abs(2*(s
qrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*
sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1
/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(c*abs(a)))*sgn(cos(f
*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)\,d+c}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)*d + c),x)
```

3.152 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}(3c+2d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
2*a^(3/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(3/2)*d^(1/2)*(3*c+2*d)*arctanh(d
^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c+d)^(3
/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a*d*tan(f*x+e)/c/(c+d)
/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 4.58 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{(d+c\cos(e+fx))^2\sec^{\frac{3}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(\frac{2\left(2(c+d)^{3/2}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{d}(3c+2d)\arctan\left(\frac{\cdots}{\sqrt{\cdots}}\right)\cdots}{(c+d)^{3/2}}\cdots\right)}{2c^2f(c+d\sec(e+fx))^2}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

output
```
((d + c*Cos[e + f*x])^2*Sec[e + f*x]^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*((2*
(2*(c + d)^(3/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*
x])]] - Sqrt[d]*(3*c + 2*d)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]
*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f
*x])^2]*Sqrt[1 + Sec[e + f*x]])/(c + d)^(3/2) - (2*c*d*Tan[(e + f*x)/2])/(
(c + d)*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]])))/(2*c^2*f*(c + d*Sec[e +
 f*x])^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4428, 114, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c+d\sec(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4428$$

$$-\frac{a^2 \tan(e+fx) \int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 114$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\int \frac{a\cos(e+fx)(2(c+d)-d\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\int \frac{\cos(e+fx)(2(c+d)-d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\frac{2(c+d)\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}-\frac{d(3c+2d)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\frac{2d(3c+2d)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\int \frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^2 \tan(e+fx) \left(\frac{\frac{2d(3c+2d)\int \frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{2\sqrt{d}(3c+2d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{c+d}}-\frac{4(c+d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^2*(((-4*(c + d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c
) + (2*Sqrt[d]*(3*c + 2*d)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqr
t[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[c + d]))/(2*c*(c + d)) + (d*Sqrt[a - a
*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[
a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 9497 vs. $2(189) = 378$.

Time = 6.08 (sec) , antiderivative size = 9498, normalized size of antiderivative = 43.37

method	result	size
default	Expression too large to display	9498

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 1413, normalized size of antiderivative = 6.45

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 + (3*c^2 + 5*c*d +
2*d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sqrt(-a*d/(c + d))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c +
 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
 + (c + d)*cos(f*x + e) + d)) - 2*((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2
+ (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f*x + e)^2
 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -1/2*(
2*c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
4*((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x +
e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - ((3*c^2 + 2*c*d)*cos(f*x + e)^2 + 3*c*d + 2*d^2 +
(3*c^2 + 5*c*d + 2*d^2)*cos(f*x + e))*sqrt(-a*d/(c + d))*log((2*(c + d)*sq
rt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/
(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4 + c^3*d)*f*cos(f*x +
 e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f), -
(c*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) ...
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(189) = 378$.

Time = 0.77 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*(3*sqrt(-a)*a*c*d + 2*sqrt(-a)*a*d^2)*arctan(1/4*sqr
t(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)
)^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 +
a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((c^3 + c^2*d)*sqrt(-c*d - d^
2)*a) + sqrt(2)*sqrt(-a)*a*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/(c^2*abs(a)) - 4*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(
-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a*c*d + 3*(sqrt(-a)*tan(1/2*f*x
 + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a*d^2 + sqrt(-
a)*a^2*c*d - sqrt(-a)*a^2*d^2)/(((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^4*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-
a*tan(1/2*f*x + 1/2*e)^2 + a))^4*d + 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*c + 6*(sqrt(-a)*tan(1/2*f*x + 1/2*e
) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*d + a^2*c - a^2*d)*(c^3 - c*d
^2)))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^2}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2d^2+2\sec(fx+e)cd+c^2}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2*d**2 + 2*sec(e + f*x)*
c*d + c**2),x)
```

3.153 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 287

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{3/2}\sqrt{d}(15c^2+20cd+8d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{ad\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{ad(7c+4d)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
2*a^(3/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*a^(3/2)*d^(1/2)*(15*c^2+20*c*d+
8*d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x
+e)/c^3/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a*
d*tan(f*x+e)/c/(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2-1/4*a*d*(
7*c+4*d)*tan(f*x+e)/c^2/(c+d)^2/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 6.76 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(d+c\cos(e+fx))^3\sec\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{5}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(\frac{\left(8(c+d)^{5/2}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)-\sqrt{}\right.}{}\right.}{8c}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]*Sec[e + f*x]^(5/2)*Sqrt[a*(1 + Se
c[e + f*x])]*(((8*(c + d)^(5/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/
(1 + Cos[e + f*x])]] - Sqrt[d]*(15*c^2 + 20*c*d + 8*d^2)*ArcTan[(Sqrt[d]*T
an[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])]])*Sqr
t[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sec[(e + f*x)/2]*Sqrt[1 + Sec[e + f*x]]
)/((c + d)^(5/2)*Sqrt[(1 + Cos[e + f*x])^(-1)]) - (2*c*d*(d*(7*c + 4*d) +
3*c*(3*c + 2*d)*Cos[e + f*x])*Sec[e + f*x]^(3/2)*Sin[(e + f*x)/2])/((c + d
)^2*(c + d*Sec[e + f*x])^2)))/(8*c^3*f*(c + d*Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 4428, 114, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{(c+d\sec(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4428$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 114$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{a\cos(e+fx)(4(c+d)-3d\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2ac(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\int \frac{\cos(e+fx)(4(c+d)-3d\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{4c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 168$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{a\cos(e+fx)\left(8(c+d)^2-d(7c+4d)\sec(e+fx)\right)}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\int \frac{\cos(e+fx)\left(8(c+d)^2-d(7c+4d)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}+\frac{d\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 174$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{8(c+d)^2\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}-\frac{d\left(15c^2+20cd+8d^2\right)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec}}{ac(c+d)(c+d\sec(e}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{2d\left(15c^2+20cd+8d^2\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\int\frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{\cdots}}{ac(c+d)(c\cdots}}{4c(c+d)}\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 219$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{2d\left(15c^2+20cd+8d^2\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec\cdots}}{ac(c+d)(c+d\sec(e\cdots}}{4c(c+d)}\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(e+fx)\left(\frac{\frac{\frac{2\sqrt{d}\left(15c^2+20cd+8d^2\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{c+d}}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}+\frac{d(7c+4d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}\cdots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^2*((d*Sqrt[a - a*Sec[e + f*x]])/(2*a*c*(c + d)*(c + d*Sec[e + f*x])^2
) + (((-16*(c + d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c
) + (2*Sqrt[d]*(15*c^2 + 20*c*d + 8*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e
 + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[c + d]))/(2*c*(c + d)) +
 (d*(7*c + 4*d)*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e + f*x]
)))/(4*c*(c + d)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Se
c[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 21395 vs. $2(249) = 498$.

Time = 8.49 (sec) , antiderivative size = 21396, normalized size of antiderivative = 74.55

method	result	size
default	Expression too large to display	21396

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(249) = 498$.

Time = 4.00 (sec) , antiderivative size = 2368, normalized size of antiderivative = 8.25

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/8*((15*c^2*d^2 + 20*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos
(f*x + e)^3 + (15*c^4 + 50*c^3*d + 48*c^2*d^2 + 16*c*d^3)*cos(f*x + e)^2 +
 (30*c^3*d + 55*c^2*d^2 + 36*c*d^3 + 8*d^4)*cos(f*x + e))*sqrt(-a*d/(c + d
))*log((2*(c + d)*sqrt(-a*d/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c +
 a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 8*(c^
2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)^3 + (c^4 +
4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c^2*d^2 + 4*c
*d^3 + d^4)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*
x + e) - a)/(cos(f*x + e) + 1)) - 2*(3*(3*c^3*d + 2*c^2*d^2)*cos(f*x + e)^
2 + (7*c^2*d^2 + 4*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4
*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 +
4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f),
-1/8*(16*(c^2*d^2 + 2*c*d^3 + d^4 + (c^4 + 2*c^3*d + c^2*d^2)*cos(f*x + e)
^3 + (c^4 + 4*c^3*d + 5*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^2 + (2*c^3*d + 5*c
^2*d^2 + 4*c*d^3 + d^4)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*c^2*d^2 + 20
*c*d^3 + 8*d^4 + (15*c^4 + 20*c^3*d + 8*c^2*d^2)*cos(f*x + e)^3 + (15*c...
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^3} dx = \int \frac{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}{\left(c + d\sec\left(e + fx\right)\right)^3} dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^3} dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1381 vs. $2(249) = 498$.

Time = 1.12 (sec) , antiderivative size = 1381, normalized size of antiderivative = 4.81

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{(c + d\sec(e + fx))^3} dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(sqrt(2)*(15*sqrt(-a)*a*c^2*d + 20*sqrt(-a)*a*c*d^2 + 8*sqrt(
-a)*a*d^3)*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((c^
5 + 2*c^4*d + c^3*d^2)*sqrt(-c*d - d^2)*a) + 4*sqrt(2)*sqrt(-a)*a*log(abs(
2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
- 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(c^3*abs(a)) - 4*
(9*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6
*sqrt(-a)*a*c^4*d + 25*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*
x + 1/2*e)^2 + a))^6*sqrt(-a)*a*c^3*d^2 - 33*(sqrt(-a)*tan(1/2*f*x + 1/2*e
) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a*c^2*d^3 - 13*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*
a*c*d^4 + 12*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)
^2 + a))^6*sqrt(-a)*a*d^5 + 27*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*c^4*d + 87*(sqrt(-a)*tan(1/2*f*x
 + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*c^3*d^2 +
149*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^
4*sqrt(-a)*a^2*c^2*d^3 - 59*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1
/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^2*c*d^4 - 76*(sqrt(-a)*tan(1/2*f*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^3}\,dx$$
$$=\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3*d**3 + 3*sec(e + f*x)*
*2*c*d**2 + 3*sec(e + f*x)*c**2*d + c**3),x)
```

3.154 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 267

$$\begin{aligned}\int (a+a\sec(e+fx))^{3/2}(c &+ d\sec(e+fx))^3\,dx = \frac{4a^2(36c^3+243c^2d+189cd^2+52d^3)\tan(e+fx)}{105f\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2a^2d(24c^2+111cd+52d^2)\sec(e+fx)\tan(e+fx)}{105f\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2a^{5/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2a^2(6c+13d)(c+d\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2a^2(c+d\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
4/105*a^2*(36*c^3+243*c^2*d+189*c*d^2+52*d^3)*tan(f*x+e)/f/(a+a*sec(f*x+e)
)^(1/2)+2/105*a^2*d*(24*c^2+111*c*d+52*d^2)*sec(f*x+e)*tan(f*x+e)/f/(a+a*s
ec(f*x+e))^(1/2)+2*a^(5/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan
(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2/35*a^2*(6*c+13*d
)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/7*a^2*(c+d*sec(
f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.82

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}c^3\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[2]
*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 2*(210*c^2*d +
378*c*d^2 + 164*d^3 + 9*(35*c^3 + 175*c^2*d + 154*c*d^2 + 52*d^3)*Cos[e +
f*x] + 2*d*(105*c^2 + 189*c*d + 52*d^2)*Cos[2*(e + f*x)] + 105*c^3*Cos[3*(
e + f*x)] + 525*c^2*d*Cos[3*(e + f*x)] + 378*c*d^2*Cos[3*(e + f*x)] + 104*
d^3*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 4428, 27, 170, 27, 170, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^3\tan(e+fx)\int\frac{\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 170$$

$$-\frac{a^3\tan(e+fx)\left(-\frac{2\int-\frac{a\cos(e+fx)(c+d\sec(e+fx))^2(7c+(6c+13d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{7a}-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{7a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\int\frac{\cos(e+fx)(c+d\sec(e+fx))^2(7c+(6c+13d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}{7a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 170$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\left(-\frac{2\int-\frac{a\cos(e+fx)(c+d\sec(e+fx))\left(35c^2+\left(24c^2+111dc+52d^2\right)\sec(e+fx)\right)}{2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a}-\frac{2(6c+13d)\sqrt{a-a\sec(e+fx)}(}{5a}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\left(\frac{1}{5}\int\frac{\cos(e+fx)(c+d\sec(e+fx))(35c^2+(24c^2+111dc+52d^2)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2(6c+13d)\sqrt{a-a\sec(e}}{5}\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 164$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\left(\frac{1}{5}\left(35c^3\int\frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(d(24c^2+111cd+52d^2)\sec(e+fx)+2(36c^3+2}{3a}\right.\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec}}$$

$$\Big\downarrow 73$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\left(\frac{1}{5}\left(-\frac{70c^3\int\frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{a}-\frac{2\sqrt{a-a\sec(e+fx)}(d(24c^2+111cd+52d^2)\sec(e+fx)+2(36c^3}{3a}\right.\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,se}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{7}\left(\frac{1}{5}\left(-\frac{70c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a-a\sec(e+fx)}\left(d\left(24c^2+111cd+52d^2\right)\sec(e+fx)+2\left(36c^3+243c^2d\right.\right.}{3a}\right.\right.\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^3*((-2*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3)/(7*a) + ((-2*
(6*c + 13*d)*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(5*a) + ((-7
0*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[a - a*S
ec[e + f*x]]*(2*(36*c^3 + 243*c^2*d + 189*c*d^2 + 52*d^3) + d*(24*c^2 + 11
1*c*d + 52*d^2)*Sec[e + f*x]))/(3*a))/5)/7)*Tan[e + f*x])/(f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 2.36 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.19

method	result
parts	$\frac{c^3 a\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)+2\sin(f}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/f*a*(a*(1+sec(f*x+e)))^(1/2)*(2^(1/2)*(cos(f*x+e)+1)*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+
csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))+2*sin(f*x+e))/(cos(f*x+e)+
1)+2/105*d^3/f*a*(104*cos(f*x+e)^3+52*cos(f*x+e)^2+39*cos(f*x+e)+15)*(a*(1
+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*tan(f*x+e)*sec(f*x+e)^2+3*c*d^2/f*(12*s
in(f*x+e)+6*tan(f*x+e)+2*sec(f*x+e)*tan(f*x+e))/(5+5*cos(f*x+e))*a*(a*(1+s
ec(f*x+e)))^(1/2)+3*c^2*d/f*(10*sin(f*x+e)+2*tan(f*x+e))/(3*cos(f*x+e)+3)*
a*(a*(1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.81

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \left[\frac{105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cdots}{\cos(fx+\cdots}\right.\cdots}{\cdots}\right.$$

$$-\frac{2\left(105\left(ac^3\cos(fx+e)^4+ac^3\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15\,ad^3+\left(105\,ac^3+\cdots\right.\right.\right.}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/105*(105*(a*c^3*cos(f*x + e)^4 + a*c^3*cos(f*x + e)^3)*sqrt(-a)*log((2*
a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(15*a*
d^3 + (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 +
 (105*a*c^2*d + 189*a*c*d^2 + 52*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 1
3*a*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e
))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a*c^3*cos(f*x + e)^
4 + a*c^3*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a*d^3 + (105*a*c^3 + 525
*a*c^2*d + 378*a*c*d^2 + 104*a*d^3)*cos(f*x + e)^3 + (105*a*c^2*d + 189*a*
c*d^2 + 52*a*d^3)*cos(f*x + e)^2 + 3*(21*a*c*d^2 + 13*a*d^3)*cos(f*x + e))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4 +
 f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx=\int\left(a(\sec(e+fx)+1)\right)^{\frac{3}{2}}(c+d\sec(e+fx))^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx=\int(a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/210*(4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(1/4)*(7*(15*(a*c^3 + 3*a*c^2*d)*sin(6*f*x + 6*e) + 5*(9*a*c^3 + 33*a*c
^2*d + 18*a*c*d^2 + 4*a*d^3)*sin(4*f*x + 4*e) + (45*a*c^3 + 195*a*c^2*d +
144*a*c*d^2 + 52*a*d^3)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) - (105*a*c^3 + 525*a*c^2*d + 378*a*c*d^2 + 104*a*
d^3 + 105*(a*c^3 + 3*a*c^2*d)*cos(6*f*x + 6*e) + 35*(9*a*c^3 + 33*a*c^2*d
+ 18*a*c*d^2 + 4*a*d^3)*cos(4*f*x + 4*e) + 7*(45*a*c^3 + 195*a*c^2*d + 144
*a*c*d^2 + 52*a*d^3)*cos(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)))*sqrt(a) + 105*((a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*si
n(2*f*x + 2*e)^4 + 4*a*c^3*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2
 + 4*a*c^3*cos(2*f*x + 2*e) + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^
3*cos(2*f*x + 2*e) + a*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)) + 1) - (a*c^3*cos(2*f*x + 2*e)^4 + a*c^3*sin(2*f*x +
2*e)^4 + 4*a*c^3*cos(2*f*x + 2*e)^3 + 6*a*c^3*cos(2*f*x + 2*e)^2 + 4*a*c^3
*cos(2*f*x + 2*e) + a*c^3 + 2*(a*c^3*cos(2*f*x + 2*e)^2 + 2*a*c^3*cos(2*f*
x + 2*e) + a*c^3)*sin(2*f*x + 2*e)^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(241) = 482$.

Time = 0.96 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.97

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/105*(105*sqrt(-a)*a^2*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqr
t(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(105*sqrt(2)*a^5*c^3*sgn(
cos(f*x + e)) + 630*sqrt(2)*a^5*c^2*d*sgn(cos(f*x + e)) + 630*sqrt(2)*a^5*
c*d^2*sgn(cos(f*x + e)) + 210*sqrt(2)*a^5*d^3*sgn(cos(f*x + e)) - (315*sqr
t(2)*a^5*c^3*sgn(cos(f*x + e)) + 1680*sqrt(2)*a^5*c^2*d*sgn(cos(f*x + e))
+ 1260*sqrt(2)*a^5*c*d^2*sgn(cos(f*x + e)) + 280*sqrt(2)*a^5*d^3*sgn(cos(f
*x + e)) - (315*sqrt(2)*a^5*c^3*sgn(cos(f*x + e)) + 1470*sqrt(2)*a^5*c^2*d
*sgn(cos(f*x + e)) + 882*sqrt(2)*a^5*c*d^2*sgn(cos(f*x + e)) + 266*sqrt(2)
*a^5*d^3*sgn(cos(f*x + e)) - (105*sqrt(2)*a^5*c^3*sgn(cos(f*x + e)) + 420*
sqrt(2)*a^5*c^2*d*sgn(cos(f*x + e)) + 252*sqrt(2)*a^5*c*d^2*sgn(cos(f*x +
e)) + 76*sqrt(2)*a^5*d^3*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/
2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2
*f*x + 1/2*e)^2 - a)^3*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3\,dx = \sqrt{a}\,a\Bigg(\left(\int\sqrt{\sec(fx+e)+1}dx\right)c^3 + \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right)d^3 + 3\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)c\,d^2 + \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)d^3 + 3\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c^2d + 3\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c\,d^2 + \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c^3 + 3\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c^2d\Bigg)$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1),x)*c**3 + int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x)**4,x)*d**3 + 3*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x)
*c*d**2 + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x)*d**3 + 3*int(sqrt(
sec(e + f*x) + 1)*sec(e + f*x)**2,x)*c**2*d + 3*int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x)**2,x)*c*d**2 + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c*
*3 + 3*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c**2*d)
```

3.155 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 202

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \frac{4a^2(6c^2+25cd+9d^2)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2d(4c+9d)\sec(e+fx)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}$$

output

```
4/15*a^2*(6*c^2+25*c*d+9*d^2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2
*d*(4*c+9*d)*sec(f*x+e)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(5/2)*c^2*
arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)+2/5*a^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(
f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.72

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\left(30\sqrt{2}c^2\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2]*
c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(15*c^2 + 50*c
*d + 24*d^2 + 2*d*(10*c + 9*d)*Cos[e + f*x] + (15*c^2 + 50*c*d + 18*d^2)*C
os[2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4428, 27, 170, 27, 164, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 170

$$-\frac{a^3\tan(e+fx)\left(-\frac{2\int -\frac{a\cos(e+fx)(c+d\sec(e+fx))(5c+(4c+9d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{5a}-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{5a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 27

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\int \frac{\cos(e+fx)(c+d\sec(e+fx))(5c+(4c+9d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}{5a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 164

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\left(5c^2\int \frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)-\frac{2\sqrt{a-a\sec(e+fx)}(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))}{3a}\right)-2\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 73

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\left(-\frac{10c^2\int \frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{a}-\frac{2\sqrt{a-a\sec(e+fx)}(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))}{3a}\right)-\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 219

$$-\frac{a^3\tan(e+fx)\left(\frac{1}{5}\left(-\frac{10c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a-a\sec(e+fx)}(2(6c^2+25cd+9d^2)+d(4c+9d)\sec(e+fx))}{3a}\right)-2\sqrt{a-}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^3*((-2*Sqrt[a - a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2)/(5*a) + ((-10
*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[a - a*Se
c[e + f*x]]*(2*(6*c^2 + 25*c*d + 9*d^2) + d*(4*c + 9*d)*Sec[e + f*x]))/(3*
a))/5)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

method	result
default	$\frac{2a\sqrt{a(1+\sec(fx+e))}\left(15(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)c^2+d^2(18\,\mathrm{s}}{15f(\cos(fx+e)+1)}$
parts	$\frac{c^2a\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)+2\sin(f}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15/f*a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(15*(cos(f*x+e)+1)*(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*co
t(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*c^2+d^2*(18*sin(f
*x+e)+9*tan(f*x+e)+3*sec(f*x+e)*tan(f*x+e))+15*c^2*sin(f*x+e)+c*d*(50*sin(
f*x+e)+10*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.97

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \left[\frac{15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\cdots}{\cos(fx+e\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(15\left(ac^2\cos(fx+e)^3+ac^2\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3\,ad^2+\left(15\,ac^2+50\cdots\right.\right.}{15\left(f\cos(fx+e)^3+f\cos\cdots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*cos(f*x + e)^2)*sqrt(-a)*log((2*a*
cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a*d^2
 + (15*a*c^2 + 50*a*c*d + 18*a*d^2)*cos(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos
(f*x + e)^3 + f*cos(f*x + e)^2), -2/15*(15*(a*c^2*cos(f*x + e)^3 + a*c^2*c
os(f*x + e)^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(sqrt(a)*sin(f*x + e))) - (3*a*d^2 + (15*a*c^2 + 50*a*c*d + 18*a*
d^2)*cos(f*x + e)^2 + (10*a*c*d + 9*a*d^2)*cos(f*x + e))*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)
]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \int (a(\sec(e+fx)+1))^{\frac{3}{2}}(c+d\sec(e+fx))^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))^2\,dx = \int (a\sec(fx + e) + a)^{\frac{3}{2}}(d\sec(fx + e) + c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/30*(15*((a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*
cos(2*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
 1) - (a*c^2*cos(2*f*x + 2*e)^2 + a*c^2*sin(2*f*x + 2*e)^2 + 2*a*c^2*cos(2
*f*x + 2*e) + a*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) -
 2*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*co
s(2*f*x + 2*e) + a*c^2*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*...
```

Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.78

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx =$$

$$-\frac{\frac{15\sqrt{-a}a^2c^2\log\left(\left|\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a}\right|\right)\operatorname{sgn}(\cos(fx+e))}{|a|}-\frac{2\left(\left(\sqrt{2}(15a^4c^2\operatorname{sgn}(\cos(fx+e))+40a^4c\right.\right.}{}}{}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(-a)*a^2*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*((sqrt(2)*(15*a^4*c^2*sgn(c
os(f*x + e)) + 40*a^4*c*d*sgn(cos(f*x + e)) + 12*a^4*d^2*sgn(cos(f*x + e))
)*tan(1/2*f*x + 1/2*e)^2 - 10*sqrt(2)*(3*a^4*c^2*sgn(cos(f*x + e)) + 10*a^
4*c*d*sgn(cos(f*x + e)) + 3*a^4*d^2*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*
e)^2 + 15*sqrt(2)*(a^4*c^2*sgn(cos(f*x + e)) + 4*a^4*c*d*sgn(cos(f*x + e))
 + 2*a^4*d^2*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/
2*e)^2 - a)^2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (a + a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sec(fx+e)+1}dx\right)c^2 + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)d^2 + 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)cd + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)d^2 + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c^2 + 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)cd\right)$$

input

```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x)
```

output

```
sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1),x)*c**2 + int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x)**3,x)*d**2 + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)
*c*d + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*d**2 + int(sqrt(sec(e
 + f*x) + 1)*sec(e + f*x),x)*c**2 + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f
*x),x)*c*d)
```

3.156 $\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\frac{2a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}+\frac{2ad\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{3f}$$

output
```
2*a^(3/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/3*a^2*(3
*c+4*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/3*a*d*(a+a*sec(f*x+e))^(1/2)
*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\frac{a\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\left(3\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right.}{3f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(a*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*(3*Sqrt[2]*c*A
rcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(3/2) + 2*(d + (3*c + 5*d)*Co
s[e + f*x])*Sin[(e + f*x)/2]))/(3*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4405, 27, 3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4405}$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{\sec(e+fx)a+a}(3ac+a(3c+4d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{\sec(e+fx)a+a}(3ac+a(3c+4d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(3ac+a(3c+4d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{4403}$$

$$\frac{1}{3}\left(a(3c+4d)\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}dx+3ac\int \sqrt{\sec(e+fx)a+a}dx\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a(3c+4d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+3ac\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{4261}$$

$$\frac{1}{3}\left(a(3c+4d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{6a^2c\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(a(3c+4d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{6a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{4279}$$

$$\frac{1}{3}\left(\frac{6a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^2(3c+4d)\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}}\right)+\frac{2ad\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(2*a*d*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f) + ((6*a^(3/2)*c*ArcTan
[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (2*a^2*(3*c + 4*d)*
Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4405
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
 (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2
*m]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.34

method	result
default	$\frac{2a\sqrt{a(1+\sec(fx+e))}\left(3c\sin(fx+e)+d(5\sin(fx+e)+\tan(fx+e))+3(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\mathrm{c}}{\sqrt{\cot(fx+e)^{2}-2\csc}}\right.\right.}{3f(\cos(fx+e)+1)}$
parts	$\frac{ca\sqrt{a(1+\sec(fx+e))}\left(\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^{2}-1}}\right)+2\sin(f\right.}{f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/3/f*a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(3*c*sin(f*x+e)+d*(5*sin(f
*x+e)+tan(f*x+e))+3*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arct
anh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-
cot(f*x+e)+csc(f*x+e)))*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.01

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \left[\frac{3\left(ac\cos(fx+e)^{2}+ac\cos(fx+e)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+}{\cos(fx+e)+1}\right.}{3\left(f\cos(fx+e\right.}\right.$$

$$-\frac{2\left(3\left(ac\cos(fx+e)^{2}+ac\cos(fx+e)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-(ad+(3\,ac+5\,ad)\cos(f\right.}{3\left(f\cos(fx+e)^{2}+f\cos(fx+e)\right)}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/3*(3*(a*c*cos(f*x + e)^2 + a*c*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x
+ e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(a*d + (3*a*c +
5*a*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))
/(f*cos(f*x + e)^2 + f*cos(f*x + e)), -2/3*(3*(a*c*cos(f*x + e)^2 + a*c*co
s(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e))) - (a*d + (3*a*c + 5*a*d)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2 + f*cos
(f*x + e))]
```

Sympy [F]

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))\,dx = \int \left(a(\sec(e + fx) + 1)\right)^{\frac{3}{2}}(c + d\sec(e + fx))\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(91) = 182$.

Time = 0.20 (sec) , antiderivative size = 998, normalized size of antiderivative = 9.50

$$\int (a + a\sec(e + fx))^{3/2}(c + d\sec(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))) + 1) - a*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))) - 1) - a*arctan2((cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e),...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(91) = 182$.

Time = 0.51 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.50

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = -\frac{\frac{3\sqrt{-a}a^2c\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|}+\frac{2\left(3\sqrt{2}a^3c\,\mathrm{sgn}(\cos(fx+e))+6\sqrt{2}a^3d\,\mathrm{sgn}(\cdots\right.}{\cdots}}{3f}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(-a)*a^2*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*a
bs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(3*sqrt(2)*a^3*c*sgn(cos(f*x +
e)) + 6*sqrt(2)*a^3*d*sgn(cos(f*x + e)) - (3*sqrt(2)*a^3*c*sgn(cos(f*x + e
)) + 4*sqrt(2)*a^3*d*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*
x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)*sqrt(-a*tan(1/2*f*x + 1/2*e)^2
+ a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\sqrt{a}\,a\left(\left(\int\sqrt{\sec(fx+e)+1}dx\right)c+\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)d+\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c+\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)d\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1),x)*c + int(sqrt(sec(e + f*x) + 1)*se
c(e + f*x)**2,x)*d + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c + int(sq
rt(sec(e + f*x) + 1)*sec(e + f*x),x)*d)
```

3.157 $\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2a^{3/2}(c-d)\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{c\sqrt{d}\sqrt{c+d}f}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f+2*a^(3/2)*
(c-d)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2)
)/c/d^(1/2)/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{\sqrt{2}a\left(\sqrt{d}\sqrt{c+d}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(c-d)\arctan\left(\frac{\sqrt{2}\sqrt{d}\sin(\frac{1}{2}(e+}{\sqrt{c+d}\sqrt{\cos(e+}}\right.\right.}{c\sqrt{d}\sqrt{c+d}f}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

output
```
(Sqrt[2]*a*(Sqrt[d]*Sqrt[c + d]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (c - d)
*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]]
)])*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(c*Sqr
t[d]*Sqrt[c + d]*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4415, 3042, 4261, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4415}$$

$$\frac{a(c-d)\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{c}+\frac{a\int \sqrt{\sec(e+fx)a+a}dx}{c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{a\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{c}$$

$$\Big\downarrow \text{ 4261}$$

$$\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}-\frac{2a^2\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{a(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

$$\Big\downarrow \textcolor{blue}{4455}$$

$$\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}-\frac{2a^2(c-d)\int\frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf}$$

$$\Big\downarrow \textcolor{blue}{218}$$

$$\frac{2a^{3/2}(c-d)\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{c\sqrt{d}f\sqrt{c+d}}+\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{cf}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

output
```
(2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(c*f)
+ (2*a^(3/2)*(c - d)*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])])/(c*Sqrt[d]*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4415
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Simp[a/c   Int[Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[(b*c - a*d)/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Cs
c[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& (EqQ[a^2 - b^2, 0] || EqQ[c^2 - d^2, 0])
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(90) = 180$.

Time = 6.01 (sec) , antiderivative size = 840, normalized size of antiderivative = 7.64

method	result
default	$\frac{\sqrt{2}\, a \left(2 \sqrt{(c+d)(c-d)}\, \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^{2} \csc(fx+e)^{2}-1}}\right) \sqrt{\frac{d}{c-d}}+\ln\left(-\frac{2\left(\sqrt{2}\, \sqrt{(1-\cos(fx+e))^{2} \csc(fx+e)^{2}-1}\, \sqrt{\frac{d}{c-d}}\, c-\sqrt{2}\, \sqrt{\cdots}\right.}{-c(-\cot(fx+e)+\csc \cdots}\right.\right.}{}$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*2^(1/2)*a/((c+d)*(c-d))^(1/2)/c/(d/(c-d))^(1/2)*(2*((c+d)*(c-d))^(1/
2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+cs
c(f*x+e)))*(d/(c-d))^(1/2)+ln(-2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-
cot(f*x+e)+csc(f*x+e))+d*(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))*c-
d*ln(-2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c
-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*
(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+d*
(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))-ln(2*(2^(1/2)*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(
f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-d*(-cot(f*x+e)+csc(f*x+e))+((c+d)
*(c-d))^(1/2)))*c+d*ln(2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1
)^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e
)+csc(f*x+e))-d*(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2))))*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 731, normalized size of antiderivative = 6.65

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[-((a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2)
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c
 + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)
^2 + (c + d)*cos(f*x + e) + d)) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*s
qrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -(2*a^(3/2)*arctan(sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) +
(a*c - a*d)*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c +
2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2
+ (c + d)*cos(f*x + e) + d)))/(c*f), -(2*(a*c - a*d)*sqrt(a/(c*d + d^2))*a
rctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)/(a*sin(f*x + e))) - sqrt(-a)*a*log((2*a*cos(f*x + e)^2 - 2*sq
rt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) +
 a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*f), -2*((a*c - a*d)*sqrt(a/(c
*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))) + a^(3/2)*arctan(sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))))/(c*f)]
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\int \frac{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)\,d+c}dx + \int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)\,d+c}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)*d + c),x) + int((sqrt(
sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)*d + c),x))
```

3.158 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^{5/2}(c^2-3cd-2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2\sqrt{d}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^2(c-d)\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
2*a^(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(5/2)*(c^2-3*c*d-2*d^2)*arctanh(d
^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/d^(1/2)/
(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^2*(c-d)*tan(
f*x+e)/c/(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 3.79 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.28

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{(d+c\cos(e+fx))^2\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}(a(1+\sec(e+fx)))^{3/2}}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
((d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[
e + f*x]))^(3/2)*(((2*Sqrt[d]*(c + d)^(3/2)*ArcTan[Tan[(e + f*x)/2]/Sqrt[C
os[e + f*x]/(1 + Cos[e + f*x])]] + (c^2 - 3*c*d - 2*d^2)*ArcTan[(Sqrt[d]*T
an[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Sqr
t[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sec[(e + f*x)/2]*Sqrt[1 + Sec[e + f*x]]
)/(Sqrt[d]*(c + d)^(3/2)*Sqrt[(1 + Cos[e + f*x])^(-1)]) + (2*c*(c - d)*Sqr
t[Sec[e + f*x]]*Sin[(e + f*x)/2])/((c + d)*(c + d*Sec[e + f*x]))))/(4*c^2*
f*(c + d*Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4428, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int\frac{a\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^3\tan(e+fx)\int\frac{\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 168$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int\frac{a\cos(e+fx)(2(c+d)+(c-d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int\frac{\cos(e+fx)(2(c+d)+(c-d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 174$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\left(c^2-3cd-2d^2\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}+\frac{2(c+d)\int\frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 73$$

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2\left(c^2-3cd-2d^2\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\int\frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 219$$

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2\left(c^2-3cd-2d^2\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{4(c+d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 221$$

$$-\frac{a^3\tan(e+fx)\left(\frac{-\frac{2(c^2-3cd-2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{d}\sqrt{c+d}}-\frac{4(c+d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^3*(((-4*(c + d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c
) - (2*(c^2 - 3*c*d - 2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(S
qrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[d]*Sqrt[c + d]))/(2*c*(c + d)) - ((c
 - d)*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e + f*x])))*Tan[e
+ f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8517 vs. $2(199) = 398$.

Time = 6.45 (sec) , antiderivative size = 8518, normalized size of antiderivative = 37.20

method	result	size
default	Expression too large to display	8518

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 1640, normalized size of antiderivative = 7.16

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^2} dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
e)*sin(f*x + e) - (a*c^2*d - 3*a*c*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*
a*c*d^2)*cos(f*x + e)^2 + (a*c^3 - 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*
x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*
d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c
 + d)*cos(f*x + e) + d)) + 2*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)
^2 + (a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-a)*log((2*a*cos(f*x + e
)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/((c^4 + c^3*d)*f*cos(f
*x + e)^2 + (c^4 + 2*c^3*d + c^2*d^2)*f*cos(f*x + e) + (c^3*d + c^2*d^2)*f
), 1/2*(2*(a*c^2 - a*c*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) - 4*(a*c*d + a*d^2 + (a*c^2 + a*c*d)*cos(f*x + e)^2 + (a
*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (a*c^2*d - 3*a*c
*d^2 - 2*a*d^3 + (a*c^3 - 3*a*c^2*d - 2*a*c*d^2)*cos(f*x + e)^2 + (a*c^3 -
 2*a*c^2*d - 5*a*c*d^2 - 2*a*d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((
2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a
*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/((c^4...
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}}{(c+d\sec(e+fx))^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(199) = 398$.

Time = 0.95 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.00

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-(sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*
f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) -
6*a))*sgn(cos(f*x + e))/(c^2*abs(a)) - sqrt(2)*(sqrt(-a)*a^2*c^2*sgn(cos(f
*x + e)) - 3*sqrt(-a)*a^2*c*d*sgn(cos(f*x + e)) - 2*sqrt(-a)*a^2*d^2*sgn(c
os(f*x + e)))*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a
*tan(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(
-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/(
(sqrt(2)*c^3 + sqrt(2)*c^2*d)*sqrt(-c*d - d^2)*a) + 4*((sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(-a)*a^2*c*sgn(co
s(f*x + e)) + 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^2*sqrt(-a)*a^2*d*sgn(cos(f*x + e)) + sqrt(-a)*a^3*c*sgn(cos(f*
x + e)) - sqrt(-a)*a^3*d*sgn(cos(f*x + e)))/(((sqrt(-a)*tan(1/2*f*x + 1/2*
e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*c - (sqrt(-a)*tan(1/2*f*x + 1/
2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*d + 2*(sqrt(-a)*tan(1/2*f*x
+ 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*c + 6*(sqrt(-a)*tan(1/
2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a*d + a^2*c - a^2*
d)*(sqrt(2)*c^2 + sqrt(2)*c*d)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^2}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2 d^2+2\sec(fx+e)\,cd+c^2}dx + \int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2 d^2+2\sec(fx+e)\,cd+c^2}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2*d**2 + 2*sec(e + f*
x)*c*d + c**2),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x
)**2*d**2 + 2*sec(e + f*x)*c*d + c**2),x))
```

3.159 $\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 310

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx &= \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &+ \frac{a^{5/2}(3c^3-15c^2d-20cd^2-8d^3)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c^3\sqrt{d}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &+ \frac{a^2(c-d)\tan(e+fx)}{2c(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\\ &+ \frac{a^2(3c^2-7cd-4d^2)\tan(e+fx)}{4c^2(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\end{aligned}$$

output
```
2*a^(5/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+1/4*a^(5/2)*(3*c^3-15*c^2*d-20*c*d^
2-8*d^3)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f
*x+e)/c^3/d^(1/2)/(c+d)^(5/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1
/2)+1/2*a^2*(c-d)*tan(f*x+e)/c/(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x
+e))^2+1/4*a^2*(3*c^2-7*c*d-4*d^2)*tan(f*x+e)/c^2/(c+d)^2/f/(a+a*sec(f*x+e
))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 4.77 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{(d+c\cos(e+fx))^3\sec^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(a(1+\sec(e+fx)))^{3/2}}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^3*Sec[e + f*x]^(3/2)*(a*(1 + Sec[
e + f*x]))^(3/2)*((Sqrt[2]*(8*Sqrt[d]*(c + d)^(5/2)*ArcTan[Tan[(e + f*x)/2
]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] + (3*c^3 - 15*c^2*d - 20*c*d^2 -
8*d^3)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1
 + Cos[e + f*x])])])*Sec[(e + f*x)/2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2
]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])/(Sqrt[d]*(c + d)^(5/2)*Sqrt[Sec[(
e + f*x)/2]^2]) + (2*c*Sqrt[Sec[e + f*x]]*(c*(5*c^2 - 7*c*d - 6*d^2) + d*(
3*c^2 - 7*c*d - 4*d^2)*Sec[e + f*x])*Sin[(e + f*x)/2])/((c + d)^2*(c + d*S
ec[e + f*x])^2)))/(16*c^3*f*(c + d*Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {3042, 4428, 27, 168, 27, 168, 27, 174, 73, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{3/2}}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 4428$$

$$-\frac{a^2\tan(e+fx)\int \frac{a\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^3\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 168$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int \frac{a\cos(e+fx)(4(c+d)+3(c-d)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{2ac(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\int \frac{\cos(e+fx)(4(c+d)+3(c-d)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{4c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 168$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\int \frac{a\cos(e+fx)\left(8(c+d)^2+\left(3c^2-7dc-4d^2\right)\sec(e+fx)\right)}{2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{ac(c+d)}-\frac{\left(3c^2-7cd-4d^2\right)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\int \frac{\cos(e+fx)\left(8(c+d)^2+\left(3c^2-7dc-4d^2\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{2c(c+d)}-\frac{\left(3c^2-7cd-4d^2\right)\sqrt{a-a\sec(e+fx)}}{ac(c+d)(c+d\sec(e+fx))}}{4c(c+d)}-\frac{(c-d)\sqrt{a-a\sec(e+fx)}}{2ac(c+d)(c+d\sec(e+fx))^2}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 174$$

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{\frac{\left(3c^3-15c^2d-20cd^2-8d^3\right)\int\frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{c}+\frac{8(c+d)^2\int\frac{\cos(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{c}}{2c(c+d)}-\frac{\left(3c^2-7cd-4\right.}{ac(c+d)}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 73

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{-\frac{2\left(3c^3-15c^2d-20cd^2-8d^3\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\int\frac{1}{1-\frac{a-a\sec(e+fx)}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}}{2c(c+d)}-\frac{\left(3c\right.}{}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 219

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{-\frac{2\left(3c^3-15c^2d-20cd^2-8d^3\right)\int\frac{1}{c+d-\frac{d(a-a\sec(e+fx))}{a}}d\sqrt{a-a\sec(e+fx)}}{ac}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{\left(3c^2-7cd-\right.}{ac(c+}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 221

$$-\frac{a^3\tan(e+fx)\left(\frac{\frac{-\frac{2\left(3c^3-15c^2d-20cd^2-8d^3\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c\sqrt{d}\sqrt{c+d}}-\frac{16(c+d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}}{2c(c+d)}-\frac{\left(3c^2-7cd-4d^2\right)\sqrt{a-}}{ac(c+d)(c+d\sec}}{4c(c+d)}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^3*(-1/2*((c - d)*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c + d)*(c + d*Sec[e
+ f*x])^2) + (((-16*(c + d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(
Sqrt[a]*c) - (2*(3*c^3 - 15*c^2*d - 20*c*d^2 - 8*d^3)*ArcTanh[(Sqrt[d]*Sqr
t[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*Sqrt[d]*Sqrt[c +
 d]))/(2*c*(c + d)) - ((3*c^2 - 7*c*d - 4*d^2)*Sqrt[a - a*Sec[e + f*x]])/(
a*c*(c + d)*(c + d*Sec[e + f*x])))/(4*c*(c + d)))*Tan[e + f*x])/(f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 18831 vs. $2(272) = 544$.

Time = 8.28 (sec) , antiderivative size = 18832, normalized size of antiderivative = 60.75

method	result	size
default	Expression too large to display	18832

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(272) = 544$.

Time = 10.98 (sec) , antiderivative size = 2729, normalized size of antiderivative = 8.80

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[-1/8*((3*a*c^3*d^2 - 15*a*c^2*d^3 - 20*a*c*d^4 - 8*a*d^5 + (3*a*c^5 - 15*
a*c^4*d - 20*a*c^3*d^2 - 8*a*c^2*d^3)*cos(f*x + e)^3 + (3*a*c^5 - 9*a*c^4*
d - 50*a*c^3*d^2 - 48*a*c^2*d^3 - 16*a*c*d^4)*cos(f*x + e)^2 + (6*a*c^4*d
- 27*a*c^3*d^2 - 55*a*c^2*d^3 - 36*a*c*d^4 - 8*a*d^5)*cos(f*x + e))*sqrt(-
a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e
)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x
+ e) + d)) - 8*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2
*d^2)*cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f
*x + e)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sq
rt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) +
1)) - 2*((5*a*c^4 - 7*a*c^3*d - 6*a*c^2*d^2)*cos(f*x + e)^2 + (3*a*c^3*d -
 7*a*c^2*d^2 - 4*a*c*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sin(f*x + e))/((c^7 + 2*c^6*d + c^5*d^2)*f*cos(f*x + e)^3 + (c^7 + 4
*c^6*d + 5*c^5*d^2 + 2*c^4*d^3)*f*cos(f*x + e)^2 + (2*c^6*d + 5*c^5*d^2 +
4*c^4*d^3 + c^3*d^4)*f*cos(f*x + e) + (c^5*d^2 + 2*c^4*d^3 + c^3*d^4)*f),
-1/8*(16*(a*c^2*d^2 + 2*a*c*d^3 + a*d^4 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*
cos(f*x + e)^3 + (a*c^4 + 4*a*c^3*d + 5*a*c^2*d^2 + 2*a*c*d^3)*cos(f*x + e
)^2 + (2*a*c^3*d + 5*a*c^2*d^2 + 4*a*c*d^3 + a*d^4)*cos(f*x + e))*sqrt(...
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx=\int \frac{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}{(c+d\sec{(e+fx)})^3}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1592 vs. $2(272) = 544$.

Time = 1.53 (sec) , antiderivative size = 1592, normalized size of antiderivative = 5.14

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/4*(4*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs
(a) - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) - sqrt(2)*(3*sqrt(-a)*a^2*c^3*s
gn(cos(f*x + e)) - 15*sqrt(-a)*a^2*c^2*d*sgn(cos(f*x + e)) - 20*sqrt(-a)*a
^2*c*d^2*sgn(cos(f*x + e)) - 8*sqrt(-a)*a^2*d^3*sgn(cos(f*x + e)))*arctan(
1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)
^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*
e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((sqrt(2)*c^5 + 2*sqrt
(2)*c^4*d + sqrt(2)*c^3*d^2)*sqrt(-c*d - d^2)*a) + 4*(5*(sqrt(-a)*tan(1/2*
f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2*c^4*sgn
(cos(f*x + e)) + 13*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x +
 1/2*e)^2 + a))^6*sqrt(-a)*a^2*c^3*d*sgn(cos(f*x + e)) - 29*(sqrt(-a)*tan(
1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2*c^2
*d^2*sgn(cos(f*x + e)) - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*
f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^2*c*d^3*sgn(cos(f*x + e)) + 12*(sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^
2*d^4*sgn(cos(f*x + e)) + 15*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(
1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-a)*a^3*c^4*sgn(cos(f*x + e)) + 35*(sqrt(-
a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(-...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{3/2}}{(c + d\sec(e + fx))^3}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\left(c + \frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^3}\,dx=\sqrt{a}\,a\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx + \int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*a*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3*d**3 + 3*sec(e + f*
x)**2*c*d**2 + 3*sec(e + f*x)*c**2*d + c**3),x) + int((sqrt(sec(e + f*x) +
 1)*sec(e + f*x))/(sec(e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec
(e + f*x)*c**2*d + c**3),x))
```

3.160 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 336

$$\begin{aligned}\int (a+a\sec(e+fx))^{5/2}(c&+d\sec(e+fx))^3\,dx=\frac{2a^3(3c^3+12c^2d+12cd^2+4d^3)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\\&+\frac{2a^{7/2}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\&+\frac{2ad(3c^2+15cd+13d^2)\,(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}\\&-\frac{6d^2(c+2d)(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}}\\&+\frac{2d^3(a-a\sec(e+fx))^4\tan(e+fx)}{9af\sqrt{a+a\sec(e+fx)}}\\&-\frac{2(c^3+12c^2d+24cd^2+12d^3)\,(a^3-a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
2*a^3*(3*c^3+12*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+
2*a^(7/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2/5*a*d*(3*c^2+15*c*d+13*d^2)*(a-a*
sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-6/7*d^2*(c+2*d)*(a-a*sec
(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/9*d^3*(a-a*sec(f*x+e))^4*
tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-2/3*(c^3+12*c^2*d+24*c*d^2+12*d^3)*(
a^3-a^3*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.48 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.85

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\left(2520\sqrt{2}c^3\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

output
```
(a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*(2520*Sqrt
[2]*c^3*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(9/2) + 2*(2520*c^3
+ 8883*c^2*d + 8370*c*d^2 + 2908*d^3 + (630*c^3 + 5292*c^2*d + 7290*c*d^2
+ 2792*d^3)*Cos[e + f*x] + 4*(840*c^3 + 2898*c^2*d + 2610*c*d^2 + 803*d^3)
*Cos[2*(e + f*x)] + 210*c^3*Cos[3*(e + f*x)] + 1764*c^2*d*Cos[3*(e + f*x)]
 + 2070*c*d^2*Cos[3*(e + f*x)] + 584*d^3*Cos[3*(e + f*x)] + 840*c^3*Cos[4*
(e + f*x)] + 2709*c^2*d*Cos[4*(e + f*x)] + 2070*c*d^2*Cos[4*(e + f*x)] + 5
84*d^3*Cos[4*(e + f*x)])*Sin[(e + f*x)/2]))/(2520*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^3\,dx$$

$\Big\downarrow$ 3042

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$\Big\downarrow$ 4428

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 27

$$-\frac{a^4\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^3}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 198

$$-\frac{a^4\tan(e+fx)\int \left(\frac{d^3(a-a\sec(e+fx))^{7/2}}{a^4}-\frac{3d^2(c+2d)(a-a\sec(e+fx))^{5/2}}{a^3}+\frac{d(3c^2+15dc+13d^2)(a-a\sec(e+fx))^{3/2}}{a^2}+\frac{(-c^3-12dc\ldots}{\ldots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+f\ldots}}$$

$\Big\downarrow$ 2009

$$-\frac{a^4\tan(e+fx)\left(-\frac{2d^3(a-a\sec(e+fx))^{9/2}}{9a^5}+\frac{6d^2(c+2d)(a-a\sec(e+fx))^{7/2}}{7a^4}-\frac{2d(3c^2+15cd+13d^2)(a-a\sec(e+fx))^{5/2}}{5a^3}+\frac{2(c^3+12\ldots}{\ldots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\,s\ldots}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^4*((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*(3
*c^3 + 12*c^2*d + 12*c*d^2 + 4*d^3)*Sqrt[a - a*Sec[e + f*x]])/a + (2*(c^3
+ 12*c^2*d + 24*c*d^2 + 12*d^3)*(a - a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d
*(3*c^2 + 15*c*d + 13*d^2)*(a - a*Sec[e + f*x])^(5/2))/(5*a^3) + (6*d^2*(c
 + 2*d)*(a - a*Sec[e + f*x])^(7/2))/(7*a^4) - (2*d^3*(a - a*Sec[e + f*x])^
(9/2))/(9*a^5))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
 + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 72.96 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.10

method	result
parts	$\frac{c^3a^2\sqrt{a(1+\sec(fx+e))}\left(16\sin(fx+e)+2\tan(fx+e)+3\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^3/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(16*sin(f*x+e)+2*tan
(f*x+e)+3*2^(1/2)*(cos(f*x+e)+1)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arct
anh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2)*(-
cot(f*x+e)+csc(f*x+e))))+2/315*d^3/f*a^2*(584*cos(f*x+e)^4+292*cos(f*x+e)^
3+219*cos(f*x+e)^2+130*cos(f*x+e)+35)*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)
+1)*tan(f*x+e)*sec(f*x+e)^3+2/7*c*d^2/f*a^2*(46*cos(f*x+e)^3+23*cos(f*x+e)
^2+12*cos(f*x+e)+3)*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*tan(f*x+e)*sec
(f*x+e)^2+3*c^2*d/f*(86*sin(f*x+e)+28*tan(f*x+e)+6*sec(f*x+e)*tan(f*x+e))/
(15+15*cos(f*x+e))*a^2*(a*(1+sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.85

$$\int (a + a\sec(e + fx))^{5/2}(c + d\sec(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3
5*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*a^2*d^3)*
cos(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 + 292*a^2*d
^3)*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3)*cos(f*x
 + e)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x + e)^4), -
2/315*(315*(a^2*c^3*cos(f*x + e)^5 + a^2*c^3*cos(f*x + e)^4)*sqrt(a)*arcta
n(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
e))) - (35*a^2*d^3 + (840*a^2*c^3 + 2709*a^2*c^2*d + 2070*a^2*c*d^2 + 584*
a^2*d^3)*cos(f*x + e)^4 + (105*a^2*c^3 + 882*a^2*c^2*d + 1035*a^2*c*d^2 +
292*a^2*d^3)*cos(f*x + e)^3 + 3*(63*a^2*c^2*d + 180*a^2*c*d^2 + 73*a^2*d^3
)*cos(f*x + e)^2 + 5*(27*a^2*c*d^2 + 26*a^2*d^3)*cos(f*x + e))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5 + f*cos(f*x +
 e)^4)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \int (a(\sec{(e+fx)}+1))^{\frac{5}{2}}\,(c+d\sec{(e+fx)})^3\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.82

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/315*(315*sqrt(-a)*a^3*c^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqr
t(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(945*sqrt(2)*a^7*c^3*sgn(
cos(f*x + e)) + 3780*sqrt(2)*a^7*c^2*d*sgn(cos(f*x + e)) + 3780*sqrt(2)*a^
7*c*d^2*sgn(cos(f*x + e)) + 1260*sqrt(2)*a^7*d^3*sgn(cos(f*x + e)) - (3570
*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)) + 12600*sqrt(2)*a^7*c^2*d*sgn(cos(f*x +
 e)) + 10080*sqrt(2)*a^7*c*d^2*sgn(cos(f*x + e)) + 2520*sqrt(2)*a^7*d^3*sg
n(cos(f*x + e)) - (5040*sqrt(2)*a^7*c^3*sgn(cos(f*x + e)) + 15876*sqrt(2)*
a^7*c^2*d*sgn(cos(f*x + e)) + 11340*sqrt(2)*a^7*c*d^2*sgn(cos(f*x + e)) +
3276*sqrt(2)*a^7*d^3*sgn(cos(f*x + e)) - (3150*sqrt(2)*a^7*c^3*sgn(cos(f*x
 + e)) + 9072*sqrt(2)*a^7*c^2*d*sgn(cos(f*x + e)) + 6480*sqrt(2)*a^7*c*d^2
*sgn(cos(f*x + e)) + 1872*sqrt(2)*a^7*d^3*sgn(cos(f*x + e)) - (735*sqrt(2)
*a^7*c^3*sgn(cos(f*x + e)) + 2016*sqrt(2)*a^7*c^2*d*sgn(cos(f*x + e)) + 14
40*sqrt(2)*a^7*c*d^2*sgn(cos(f*x + e)) + 416*sqrt(2)*a^7*d^3*sgn(cos(f*x +
 e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)
^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^
2 - a)^4*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(e + fx))^{5/2}(c + d\sec(e + fx))^3\,dx = \int \left(a + \frac{a}{\cos(e + f\,x)}\right)^{5/2}\left(c + \frac{d}{\cos(e + f\,x)}\right)^3 dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3\,dx = \sqrt{a}\,a^2\Bigg(\left(\int \sqrt{\sec(fx+e)+1}dx\right)c^3 + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^5\,dx\right)d^3 + 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right)c\,d^2 + 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right)d^3 + 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)c^2d + 6\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)c\,d^2 + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)d^3 + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c^3 + 6\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c^2d + 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c\,d^2 + 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c^3 + 3\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c^2d\Bigg)$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1),x)*c**3 + int(sqrt(sec(e + f*x) +
 1)*sec(e + f*x)**5,x)*d**3 + 3*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**4
,x)*c*d**2 + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**4,x)*d**3 + 3*int(
sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x)*c**2*d + 6*int(sqrt(sec(e + f*x)
 + 1)*sec(e + f*x)**3,x)*c*d**2 + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)*
*3,x)*d**3 + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*c**3 + 6*int(sq
rt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*c**2*d + 3*int(sqrt(sec(e + f*x) +
 1)*sec(e + f*x)**2,x)*c*d**2 + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),
x)*c**3 + 3*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c**2*d)
```

3.161 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 258

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\frac{2a^3(c+2d)(3c+2d)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2a^{7/2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}+\frac{2ad(2c+5d)(a-a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{a+a\sec(e+fx)}}-\frac{2d^2(a-a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{a+a\sec(e+fx)}}-\frac{2(c^2+8cd+8d^2)\left(a^3-a^3\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^3*(c+2*d)*(3*c+2*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*c^2*
arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2)+2/5*a*d*(2*c+5*d)*(a-a*sec(f*x+e))^2*tan(f*x+e)/f
/(a+a*sec(f*x+e))^(1/2)-2/7*d^2*(a-a*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f
*x+e))^(1/2)-2/3*(c^2+8*c*d+8*d^2)*(a^3-a^3*sec(f*x+e))*tan(f*x+e)/f/(a+a*
sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.74

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\left(420\sqrt{2}c^2\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
(a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*(420*Sqrt[
2]*c^2*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(7/2) + 4*(35*c^2 + 1
96*c*d + 145*d^2 + (420*c^2 + 987*c*d + 465*d^2)*Cos[e + f*x] + (35*c^2 +
196*c*d + 115*d^2)*Cos[2*(e + f*x)] + 140*c^2*Cos[3*(e + f*x)] + 301*c*d*C
os[3*(e + f*x)] + 115*d^2*Cos[3*(e + f*x)])*Sin[(e + f*x)/2]))/(420*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a^4\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2(c+d\sec(e+fx))^2}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 198$$

$$-\frac{a^4\tan(e+fx)\int \left(-\frac{d^2(a-a\sec(e+fx))^{5/2}}{a^3}+\frac{d(2c+5d)(a-a\sec(e+fx))^{3/2}}{a^2}+\frac{(-c^2-8dc-8d^2)\sqrt{a-a\sec(e+fx)}}{a}+\frac{3c^2+8dc+4d^2}{\sqrt{a-a\sec(e+f}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^4\tan(e+fx)\left(\frac{2d^2(a-a\sec(e+fx))^{7/2}}{7a^4}-\frac{2d(2c+5d)(a-a\sec(e+fx))^{5/2}}{5a^3}+\frac{2(c^2+8cd+8d^2)(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^2\text{arctanh}\big(}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^4*((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] - (2*(c
 + 2*d)*(3*c + 2*d)*Sqrt[a - a*Sec[e + f*x]])/a + (2*(c^2 + 8*c*d + 8*d^2)
*(a - a*Sec[e + f*x])^(3/2))/(3*a^2) - (2*d*(2*c + 5*d)*(a - a*Sec[e + f*x
])^(5/2))/(5*a^3) + (2*d^2*(a - a*Sec[e + f*x])^(7/2))/(7*a^4))*Tan[e + f*
x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (verified)

Time = 18.18 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.86

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(fx+e))}\left(\left(230\cos(fx+e)^3+115\cos(fx+e)^2+60\cos(fx+e)+15\right)d^2\tan(fx+e)\sec(fx+e)^2+c^2(280\sin(fx+e)+35\ldots}{\ldots}$
parts	$\frac{c^2a^2\sqrt{a(1+\sec(fx+e))}\left(16\sin(fx+e)+2\tan(fx+e)+3\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\ldots}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot\ldots}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
2/105/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*((230*cos(f*x+e)^3+115
*cos(f*x+e)^2+60*cos(f*x+e)+15)*d^2*tan(f*x+e)*sec(f*x+e)^2+c^2*(280*sin(f
*x+e)+35*tan(f*x+e))+105*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*arctanh(2^(1/2)/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/
2)*(-cot(f*x+e)+csc(f*x+e)))*c^2+c*d*(602*sin(f*x+e)+196*tan(f*x+e)+42*sec
(f*x+e)*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.94

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \left[\frac{105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{-a}\log\left(\frac{2a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+\cdots}{\cos(fx+e)}}\cdots}{\cos(f\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(105\left(a^2c^2\cos(fx+e)^4+a^2c^2\cos(fx+e)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(15a^2d^2+2(140a\cdots\right.\right.}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/105*(105*(a^2*c^2*cos(f*x + e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(-a)*log
((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(1
5*a^2*d^2 + 2*(140*a^2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (
35*a^2*c^2 + 196*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10
*a^2*d^2)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
e))/(f*cos(f*x + e)^4 + f*cos(f*x + e)^3), -2/105*(105*(a^2*c^2*cos(f*x +
e)^4 + a^2*c^2*cos(f*x + e)^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - (15*a^2*d^2 + 2*(140*a^
2*c^2 + 301*a^2*c*d + 115*a^2*d^2)*cos(f*x + e)^3 + (35*a^2*c^2 + 196*a^2*
c*d + 115*a^2*d^2)*cos(f*x + e)^2 + 6*(7*a^2*c*d + 10*a^2*d^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
 + f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int \left(a(\sec(e+fx)+1)\right)^{\frac{5}{2}}\left(c+d\sec(e+fx)\right)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)^2\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/210*(105*((a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 + 2*
a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 1) - (a^2*c^2*cos(2*f*x + 2*e)^2 + a^2*c^2*sin(2*f*x + 2*e)^2 +
 2*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)) - 1) - 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + a^2*c^2*f*sin(2*f*x +
 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*integrate((((cos(6*f*x
 + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin
(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*...
```

Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.71

$$\int (a + a\sec(e + fx))^{5/2}(c + d\sec(e + fx))^2 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/105*(105*sqrt(-a)*a^3*c^2*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq
rt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqr
t(2)*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) + 2*(315*sqrt(2)*a^6*c^2*sgn(
cos(f*x + e)) + 840*sqrt(2)*a^6*c*d*sgn(cos(f*x + e)) + 420*sqrt(2)*a^6*d^
2*sgn(cos(f*x + e)) - (875*sqrt(2)*a^6*c^2*sgn(cos(f*x + e)) + 1960*sqrt(2
)*a^6*c*d*sgn(cos(f*x + e)) + 700*sqrt(2)*a^6*d^2*sgn(cos(f*x + e)) - (805
*sqrt(2)*a^6*c^2*sgn(cos(f*x + e)) + 1568*sqrt(2)*a^6*c*d*sgn(cos(f*x + e)
) + 560*sqrt(2)*a^6*d^2*sgn(cos(f*x + e)) - (245*sqrt(2)*a^6*c^2*sgn(cos(f
*x + e)) + 448*sqrt(2)*a^6*c*d*sgn(cos(f*x + e)) + 160*sqrt(2)*a^6*d^2*sgn
(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)^2)*tan(1/2*f*
x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^3*sqrt(
-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2 dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2\,dx = \sqrt{a}\,a^2\left(\left(\int\sqrt{\sec(fx+e)+1}dx\right)c^2 + \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^4\,dx\right)d^2 + 2\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)cd + 2\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)d^2 + \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c^2 + 4\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)cd + \left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)d^2 + 2\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c^2 + 2\left(\int\sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)cd\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1),x)*c**2 + int(sqrt(sec(e + f*x) +
 1)*sec(e + f*x)**4,x)*d**2 + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3
,x)*c*d + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3,x)*d**2 + int(sqrt(
sec(e + f*x) + 1)*sec(e + f*x)**2,x)*c**2 + 4*int(sqrt(sec(e + f*x) + 1)*s
ec(e + f*x)**2,x)*c*d + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*d**2
 + 2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*c**2 + 2*int(sqrt(sec(e +
f*x) + 1)*sec(e + f*x),x)*c*d)
```

3.162 $\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{2a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f} + \frac{2a^3(35c+32d)\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(5c+8d)\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{15f} + \frac{2ad(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

output
```
2*a^(5/2)*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/f+2/15*a^3*(
35*c+32*d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2*(5*c+8*d)*(a+a*sec
(f*x+e))^(1/2)*tan(f*x+e)/f+2/5*a*d*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \frac{a^2\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\Big(30\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(a^2*Sec[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*(30*Sqrt[2
]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Cos[e + f*x]^(5/2) + 2*(40*c + 49*d +
 2*(5*c + 14*d)*Cos[e + f*x] + (40*c + 43*d)*Cos[2*(e + f*x)])*Sin[(e + f*
x)/2]))/(30*f)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4405, 27, 3042, 4405, 27, 3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 4405}$$

$$\frac{2}{5}\int \frac{1}{2}(\sec(e+fx)a+a)^{3/2}(5ac+a(5c+8d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int (\sec(e+fx)a+a)^{3/2}(5ac+a(5c+8d)\sec(e+fx))dx+\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(5ac+a(5c+8d)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx+\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{ 4405}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{1}{2}\sqrt{\sec(e+fx)a+a}\left(15ca^2+(35c+32d)\sec(e+fx)a^2\right)dx+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)}}{3f}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{3}\int \sqrt{\sec(e+fx)a+a}\left(15ca^2+(35c+32d)\sec(e+fx)a^2\right)dx+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)}}{3f}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(15ca^2+(35c+32d)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx+\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a}}{3f}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \; 4403$$

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \sec(e+fx)\sqrt{\sec(e+fx)a+a}dx+15a^2c\int \sqrt{\sec(e+fx)a+a}dx\right)+\frac{2a^2(5c+8d)\tan}{}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+15a^2c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx\right)\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \; 4261$$

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-\frac{30a^3c\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a-}}\right.}{f}\right.\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(a^2(35c+32d)\int \csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx+\frac{30a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}\right)+\frac{2}{}\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{4279}$$

$$\frac{1}{5}\left(\frac{2a^2(5c+8d)\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{3f}+\frac{1}{3}\left(\frac{30a^{5/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}+\frac{2a^3(35c+32d)\tan(e}{f\sqrt{a\sec(e+fx)}}\right.\right.$$
$$\frac{2ad\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{5f}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(2*a*d*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f) + ((2*a^2*(5*c + 8*d
)*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(3*f) + ((30*a^(5/2)*c*ArcTan[(Sq
rt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f + (2*a^3*(35*c + 32*d)*Ta
n[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4405
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
 (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2
*m]
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.19

method	result
default	$-\frac{2a^{2}\sqrt{a(1+\sec(fx+e))}\left(c(-40\sin(fx+e)-5\tan(fx+e))+d(-43\sin(fx+e)-14\tan(fx+e)-3\sec(fx+e)\tan(fx+e))+15(\cos(f\right.}{15f(\cos(fx+e)+1)}$
parts	$\frac{c\,a^{2}\sqrt{a(1+\sec(fx+e))}\left(16\sin(fx+e)+2\tan(fx+e)+3\sqrt{2}\,(\cos(fx+e)+1)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+c}{\sqrt{\cot(fx+e)^{2}-2\csc(fx+e)\cot(}}\right.\right.}{3f(\cos(fx+e)+1)}$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/15/f*a^2*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(c*(-40*sin(f*x+e)-5*t
an(f*x+e))+d*(-43*sin(f*x+e)-14*tan(f*x+e)-3*sec(f*x+e)*tan(f*x+e))+15*(co
s(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)*(cot(f*x+e)
-csc(f*x+e))/(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x+e)^2-1)^(1/2))*
c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.75

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \left[\frac{15\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(fx+e)^2-2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\cdots}{\cos(fx+e)\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{2\left(15\left(a^2c\cos(fx+e)^3+a^2c\cos(fx+e)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\left(3\,a^2d+\left(40\,a^2c+43\cdots\right.\right.\right.}{15\left(f\cos(fx+e)^3+f\cos(fx\cdots\right.}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)^2)*sqrt(-a)*log((2*a*
cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) + 2*(3*a^2*d
 + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 + (5*a^2*c + 14*a^2*d)*cos(f*x + e
))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3
 + f*cos(f*x + e)^2), -2/15*(15*(a^2*c*cos(f*x + e)^3 + a^2*c*cos(f*x + e)
^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e))) - (3*a^2*d + (40*a^2*c + 43*a^2*d)*cos(f*x + e)^2 +
(5*a^2*c + 14*a^2*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sin(f*x + e))/(f*cos(f*x + e)^3 + f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int \left(a(\sec(e+fx)+1)\right)^{\frac{5}{2}}(c+d\sec(e+fx))\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(124) = 248$.

Time = 0.23 (sec) , antiderivative size = 1396, normalized size of antiderivative = 9.83

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x +
 2*e) - 3*a^2*sin(2*f*x + 2*e) - 4*(3*a^2*cos(2*f*x + 2*e) + 4*a^2)*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)) + (12*a^2*sin(2*f*x + 2*e)*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2*cos(2*f*x + 2*e) - a^2 + 4*(3
*a^2*cos(2*f*x + 2*e) + 4*a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(
a) + 3*((a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*x + 2*e)^2 + 2*a^2*cos(2*f*x
 + 2*e) + a^2)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))) + 1) - (a^2*cos(2*f*x + 2*e)^2 + a^2*sin(2*f*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(124) = 248$.

Time = 0.69 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.18

$$\int (a + a\sec(e+fx))^{5/2}(c + d\sec(e+fx))\,dx = -\frac{\frac{15\sqrt{-a}a^3c\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(fx+e))}{|a|} - \frac{2\left(45\sqrt{2}a^5c\mathrm{sgn}(\cos(fx+e))+60\sqrt{2}a^5ds\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(-a)*a^3*c*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-
a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)
*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))*sgn(cos(f*x + e))/abs(a) - 2*(45*sqrt(2)*a^5*c*sgn(cos(f*x
 + e)) + 60*sqrt(2)*a^5*d*sgn(cos(f*x + e)) - (80*sqrt(2)*a^5*c*sgn(cos(f*
x + e)) + 80*sqrt(2)*a^5*d*sgn(cos(f*x + e)) - (35*sqrt(2)*a^5*c*sgn(cos(f
*x + e)) + 32*sqrt(2)*a^5*d*sgn(cos(f*x + e)))*tan(1/2*f*x + 1/2*e)^2)*tan
(1/2*f*x + 1/2*e)^2)*tan(1/2*f*x + 1/2*e)/((a*tan(1/2*f*x + 1/2*e)^2 - a)^
2*sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)))/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx=\int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int (a + a\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \sqrt{a}\,a^2\left(\left(\left(\int \sqrt{\sec(fx+e)+1}dx\right)c + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3\,dx\right)d + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)c + 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2\,dx\right)d + 2\left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)c + \left(\int \sqrt{\sec(fx+e)+1}\,\sec(fx+e)\,dx\right)d\right)\right.$$

input
```
int((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1),x)*c + int(sqrt(sec(e + f*x) + 1)
*sec(e + f*x)**3,x)*d + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*c +
2*int(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2,x)*d + 2*int(sqrt(sec(e + f*x
) + 1)*sec(e + f*x),x)*c + int(sqrt(sec(e + f*x) + 1)*sec(e + f*x),x)*d)
```

3.163 $\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \frac{2a^3\tan(e+fx)}{df\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^{7/2}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^3*tan(f*x+e)/d/f/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*arctanh((a-a*sec(f*x
+e))^(1/2)/a^(1/2))*tan(f*x+e)/c/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))
^(1/2)-2*a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c
+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.80 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.69

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \frac{\cos^{\frac{3}{2}}(e+fx)(d+c\cos(e+fx))\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}\left(\right.}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

output
```
(Cos[e + f*x]^(3/2)*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e
+ f*x]))^(5/2)*((10*(c - d)^2*(c + 3*d + 2*c*Cos[e + f*x])*Csc[(e + f*x)/2
]*(-ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]] + Sqrt[-((d*(-1 + Se
c[e + f*x]))/(c + d))]))/(d*(c + d)*Sqrt[Cos[e + f*x]]*Sqrt[-((d*(-1 + Sec
[e + f*x]))/(c + d))]) + (20*(3*c - d)*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]
] - (16*(c - d)^2*d*(d + c*Cos[e + f*x])*Hypergeometric2F1[2, 5/2, 7/2, (-
2*d*Sec[e + f*x]*Sin[(e + f*x)/2]^2)/(c + d)]*Sin[(e + f*x)/2]^3)/((c + d)
^3*Cos[e + f*x]^(5/2)) + 10*c*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] -
(2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]])))/(40*c^2*f*(c + d*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a \csc \left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{c+d \csc \left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2 \tan(e+fx) \int \frac{a^2 \cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a^4 \tan(e+fx) \int \frac{\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{a^4 \tan(e+fx) \int \left(-\frac{(c-d)^2}{cd\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{\cos(e+fx)}{c\sqrt{a-a\sec(e+fx)}}+\frac{1}{d\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4 \tan(e+fx)\left(\frac{2(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}cd^{3/2}\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c}-\frac{2\sqrt{a-a\sec(e+fx)}}{ad}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x]),x]
```

output
```
-((a^4*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c) + (2*(c
 - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])
/(Sqrt[a]*c*d^(3/2)*Sqrt[c + d]) - (2*Sqrt[a - a*Sec[e + f*x]])/(a*d))*Tan
[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1405 vs. $2(173) = 346$.

Time = 13.38 (sec) , antiderivative size = 1406, normalized size of antiderivative = 6.93

method	result	size
default	Expression too large to display	1406

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
1/2/f*a^2/c/((c+d)*(c-d))^(1/2)/d/(d/(c-d))^(1/2)*(-2^(1/2)*((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-c
ot(f*x+e)+csc(f*x+e))+d*(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))*c^2
-2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln(-2*(2^(1/2)*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((
1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+c
sc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+d*(-cot(f*x+e)+csc(f*x+e))+((
c+d)*(c-d))^(1/2)))*d^2+2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*ln
(2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1/2)*c-2^(1
/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((c+d)*(c-d)
)^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e))-d*(-cot(
f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))*c^2+2^(1/2)*((1-cos(f*x+e))^2*csc
(f*x+e)^2-1)^(1/2)*ln(2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(
d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)
+csc(f*x+e))-d*(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))*d^2+2*2^(1/2
)*((c+d)*(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*arctanh(2^(1
/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(...
```

Fricas [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 1140, normalized size of antiderivative = 5.62

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input

```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[(2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (a^2*c^2
- 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt
(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x +
 e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*
x + e) + d)) + (a^2*d*cos(f*x + e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)
^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(c*d*f*cos(f*x + e) + c
*d*f), (2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 2*(
a^2*d*cos(f*x + e) + a^2*d)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + (a^2*c^2 - 2*a^2*c*d + a^2
*d^2 + (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*
log((2*(c*d + d^2)*sqrt(-a/(c*d + d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*cos(f*x + e)*sin(f*x + e) + (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*
c + a*d)*cos(f*x + e))/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)))/(c*
d*f*cos(f*x + e) + c*d*f), (2*a^2*c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sin(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2 + (a^2*c^2 - 2*a^2*c*d +
 a^2*d^2)*cos(f*x + e))*sqrt(a/(c*d + d^2))*arctan((c + d)*sqrt(a/(c*d + d
^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e))
) + (a^2*d*cos(f*x + e) + a^2*d)*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*s...
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx=\int \frac{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{c+d\sec(e+fx)}\,dx = \sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)\,d+c}dx + \int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)\,d+c}dx + 2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)\,d+c}dx\right)\right)$$

input

```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x)
```

output

```
sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)*d + c),x) + int((sq
rt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)*d + c),x) + 2*int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)*d + c),x))
```

3.164 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 329

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{2a^{7/2}\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^{7/2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{cd^{3/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^{7/2}(c-d)\sqrt{c+d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^3(c-d)^2\tan(e+fx)}{cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a^(7/2)*(c-d)^2*arctanh(d^(1/2)*(a-
a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d)^(3/2)/
f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*a^(7/2)*(c-d)*(c+d)^(1/2
)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c
^2/d^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-a^3*(c-d)^2*tan
(f*x+e)/c/d/(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 2.90 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \frac{\sqrt{\cos(e+fx)}(d+c\cos(e+fx))^2\sec^5\left(\frac{1}{2}(e+fx)\right)(a(1+\sec(e+fx)))^{5/2}}{}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
(Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x])^2*Sec[(e + f*x)/2]^5*(a*(1 + Sec[
e + f*x]))^(5/2)*(2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (4*Sqrt[2]*
(c - d)*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e
+ f*x]])])/(Sqrt[d]*Sqrt[c + d]) - ((c - d)^2*(2*c*Cos[e + f*x] - (2*(c +
2*d)*ArcTanh[Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]]*(d + c*Cos[e + f*x]
))/((c + d)*Sqrt[-((d*(-1 + Sec[e + f*x]))/(c + d))]))*Sin[(e + f*x)/2])/(
d*(c + d)*Sqrt[Cos[e + f*x]]*(d + c*Cos[e + f*x]))))/(8*c^2*f*(c + d*Sec[e
 + f*x])^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 4428$$

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{a^4\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 198$$

$$-\frac{a^4\tan(e+fx)\int \left(-\frac{(c-d)^2}{cd\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}+\frac{c^2-d^2}{c^2d\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}\right)d\sec(e}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^4\tan(e+fx)\left(-\frac{2\sqrt{c+d}(c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2d^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{(c-d)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e}{\sqrt{a}\sqrt{c+d}}\right.}{\sqrt{a}cd^{3/2}(c+d)^{3/2}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^2,x]
```

output
```
-((a^4*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + ((c
 - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])
/(Sqrt[a]*c*d^(3/2)*(c + d)^(3/2)) - (2*(c - d)*Sqrt[c + d]*ArcTanh[(Sqrt[
d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*d^(3/2))
 + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(a*c*d*(c + d)*(c + d*Sec[e + f*x]
)))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 6166 vs. $2(281) = 562$.

Time = 39.98 (sec) , antiderivative size = 6167, normalized size of antiderivative = 18.74

method	result	size
default	Expression too large to display	6167

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 12.52 (sec) , antiderivative size = 2031, normalized size of antiderivative = 6.17

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^2*c^3 - 2*a^2*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*
c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*
cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^
2*d^4)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*log((2*(c*d + d^2)*sqrt(-a/(c*d
+ d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ (a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))/(c*cos(f*
x + e)^2 + (c + d)*cos(f*x + e) + d)) - 2*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*
d + a^2*c*d^2)*cos(f*x + e)^2 + (a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*
x + e))*sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(-a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f
*x + e) + 1)))/((c^4*d + c^3*d^2)*f*cos(f*x + e)^2 + (c^4*d + 2*c^3*d^2 +
c^2*d^3)*f*cos(f*x + e) + (c^3*d^2 + c^2*d^3)*f), -1/2*(2*(a^2*c^3 - 2*a^2
*c^2*d + a^2*c*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) + 4*(a^2*c*d^2 + a^2*d^3 + (a^2*c^2*d + a^2*c*d^2)*cos(f*x + e
)^2 + (a^2*c^2*d + 2*a^2*c*d^2 + a^2*d^3)*cos(f*x + e))*sqrt(a)*arctan(sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
+ (a^2*c^3*d + 4*a^2*c^2*d^2 - 3*a^2*c*d^3 - 2*a^2*d^4 + (a^2*c^4 + 4*a^2*
c^3*d - 3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*x + e)^2 + (a^2*c^4 + 5*a^2*c^3
*d + a^2*c^2*d^2 - 5*a^2*c*d^3 - 2*a^2*d^4)*cos(f*x + e))*sqrt(-a/(c*d ...
```

Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}{(c+d\sec{(e+fx)})^2}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(5/2)/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(281) = 562$.

Time = 1.19 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.01

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*sqrt(-a)*a^5*(sqrt(2)*(c^3 + 4*c^2*d - 3*c*d^2 - 2*d^3)*arctan
(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e
)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))/((a^2*c^3*d + a^2*c^2
*d^2)*sqrt(-c*d - d^2)*a) + 4*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*c^2 + 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt
(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*c*d - 3*(sqrt(-a)*tan(1/2*f*x + 1/2*e)
- sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d^2 + a*c^2 - 2*a*c*d + a*d^2)/((
(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*c
- (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*
d + 2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a)
)^2*a*c + 6*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^
2 + a))^2*a*d + a^2*c - a^2*d)*(a^2*c^2*d + a^2*c*d^2)) - sqrt(2)*log(abs(
2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2
- 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*t
an(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(a^2*c^2*abs(a)))
*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^2}\,dx = \sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2 d^2+2\sec(fx+e)\,cd+c^2}dx + \int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2 d^2+2\sec(fx+e)\,cd+c^2}dx + 2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2 d^2+2\sec(fx+e)\,cd+c^2}dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2*d**2 + 2*sec(e +
 f*x)*c*d + c**2),x) + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e
 + f*x)**2*d**2 + 2*sec(e + f*x)*c*d + c**2),x) + 2*int((sqrt(sec(e + f*x)
 + 1)*sec(e + f*x))/(sec(e + f*x)**2*d**2 + 2*sec(e + f*x)*c*d + c**2),x))
```

3.165 $\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 536

$$\begin{aligned}\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx &= \frac{2a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{3a^{7/2}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4cd^{3/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{a^{7/2}(c-d)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2d^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{2a^{7/2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{a^3(c-d)^2\tan(e+fx)}{2cd(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} \\ &+ \frac{a^3(c-d)\tan(e+fx)}{c^2df\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} \\ &- \frac{3a^3(c-d)^2\tan(e+fx)}{4cd(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\end{aligned}$$

output
```
2*a^(7/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/4*a^(7/2)*(c-d)^2*arctanh(d^(1/2)
*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/d^(3/2)/(c+d)^(5
/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(7/2)*(c-d)*arctanh(
d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/d^(3/2)
/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*a^(7/2)*d^(
1/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e
)/c^3/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/2*a^3*
(c-d)^2*tan(f*x+e)/c/d/(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2+a
^3*(c-d)*tan(f*x+e)/c^2/d/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))-3/4*a^
3*(c-d)^2*tan(f*x+e)/c/d/(c+d)^2/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 9.61 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.03

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx = \frac{\left(8d^{3/2}(c+d)^2\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right) - \frac{(c^4+10c^3d-15c^2d^2-20cd^3-8d^4)\operatorname{arctanh}\left(\cdots\right.}{\sqrt{-c-d}}\right.}{\cdots} + \frac{(d+c\cos(e+fx))^3\sec^5\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(a(1+\sec(e+fx)))^{5/2}\left(-\frac{(c^3-12c^2d+5cd^2+6d^3)\sin\left(\frac{1}{2}(e+fx)\right)}{16c^3d(c+d)^2}\cdots\right.}{f(c+d\cdots}$$

input
```
Integrate[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
((8*d^(3/2)*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e
 + f*x])]] - ((c^4 + 10*c^3*d - 15*c^2*d^2 - 20*c*d^3 - 8*d^4)*ArcTanh[(Sq
rt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])
])])/Sqrt[-c - d])*(d + c*Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sqrt[Cos[e +
f*x]*Sec[(e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e
+ f*x]]*(a*(1 + Sec[e + f*x]))^(5/2))/(16*Sqrt[2]*c^3*d^(3/2)*(c + d)^2*f*
Sqrt[Sec[(e + f*x)/2]^2]*(c + d*Sec[e + f*x])^3) + ((d + c*Cos[e + f*x])^3
*Sec[(e + f*x)/2]^5*Sec[e + f*x]*(a*(1 + Sec[e + f*x]))^(5/2)*(-1/16*((c^3
 - 12*c^2*d + 5*c*d^2 + 6*d^3)*Sin[(e + f*x)/2])/(c^3*d*(c + d)^2) + (-(c^
2*d*Sin[(e + f*x)/2]) + 2*c*d^2*Sin[(e + f*x)/2] - d^3*Sin[(e + f*x)/2])/(
8*c^3*(c + d)*(d + c*Cos[e + f*x])^2) + (3*c^3*Sin[(e + f*x)/2] - 14*c^2*d
*Sin[(e + f*x)/2] + 3*c*d^2*Sin[(e + f*x)/2] + 8*d^3*Sin[(e + f*x)/2])/(16
*c^3*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*(c + d*Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^{5/2}}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{a^2\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^4\tan(e+fx)\int \frac{\cos(e+fx)(\sec(e+fx)+1)^2}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 198

$$-\frac{a^4\tan(e+fx)\int\left(-\frac{(c-d)^2}{cd\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}+\frac{\cos(e+fx)}{c^3\sqrt{a-a\sec(e+fx)}}-\frac{d}{c^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{}{c^2d\sqrt{a-}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 2009

$$-\frac{a^4\tan(e+fx)\left(\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^3\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}-\frac{(c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2d^{3/2}\sqrt{c+d}}+\frac{3}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

input
```
Int[(a + a*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^3,x]
```

output
```
-((a^4*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + (3*
(c - d)^2*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])
])/(4*Sqrt[a]*c*d^(3/2)*(c + d)^(5/2)) - ((c - d)*ArcTanh[(Sqrt[d]*Sqrt[a
- a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*d^(3/2)*Sqrt[c + d
]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c
 + d])])/(Sqrt[a]*c^3*Sqrt[c + d]) + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/
(2*a*c*d*(c + d)*(c + d*Sec[e + f*x])^2) - ((c - d)*Sqrt[a - a*Sec[e + f*x
]])/(a*c^2*d*(c + d*Sec[e + f*x])) + (3*(c - d)^2*Sqrt[a - a*Sec[e + f*x]]
)/(4*a*c*d*(c + d)^2*(c + d*Sec[e + f*x]))))*Tan[e + f*x])/(f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 17960 vs. $2(460) = 920$.

Time = 2.14 (sec) , antiderivative size = 17961, normalized size of antiderivative = 33.51

output too large to display

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 21.66 (sec) , antiderivative size = 3351, normalized size of antiderivative = 6.25

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1606 vs. $2(460) = 920$.

Time $= 1.71$ (sec) , antiderivative size $= 1606$, normalized size of antiderivative $= 3.00$

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/4*(4*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 4*sqrt(2)*abs
(a) - 6*a))*sgn(cos(f*x + e))/(c^3*abs(a)) - sqrt(2)*(sqrt(-a)*a^3*c^4*sgn
(cos(f*x + e)) + 10*sqrt(-a)*a^3*c^3*d*sgn(cos(f*x + e)) - 15*sqrt(-a)*a^3
*c^2*d^2*sgn(cos(f*x + e)) - 20*sqrt(-a)*a^3*c*d^3*sgn(cos(f*x + e)) - 8*s
qrt(-a)*a^3*d^4*sgn(cos(f*x + e)))*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f
*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2
*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(s
qrt(-c*d - d^2)*a))/((sqrt(2)*c^5*d + 2*sqrt(2)*c^4*d^2 + sqrt(2)*c^3*d^3)
*sqrt(-c*d - d^2)*a) - 4*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2
*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^3*c^4*sgn(cos(f*x + e)) + (sqrt(-a)*tan
(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^3*c^
3*d*sgn(cos(f*x + e)) - 25*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/
2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^3*c^2*d^2*sgn(cos(f*x + e)) + 11*(sqrt
(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a
)*a^3*c*d^3*sgn(cos(f*x + e)) + 12*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-
a*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(-a)*a^3*d^4*sgn(cos(f*x + e)) + 3*(s
qrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt
(-a)*a^4*c^4*sgn(cos(f*x + e)) - 17*(sqrt(-a)*tan(1/2*f*x + 1/2*e) - sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^3}\,dx = \int \frac{\left(a + \frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\left(c + \frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^3}\,dx=\sqrt{a}\,a^2\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx\right.$$
$$+\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx$$
$$\left.+2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx\right)\right)$$

input
```
int((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3*d**3 + 3*sec(e +
 f*x)**2*c*d**2 + 3*sec(e + f*x)*c**2*d + c**3),x) + int((sqrt(sec(e + f*x
) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 +
 3*sec(e + f*x)*c**2*d + c**3),x) + 2*int((sqrt(sec(e + f*x) + 1)*sec(e +
f*x))/(sec(e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec(e + f*x)*c*
*2*d + c**3),x))
```

3.166 $\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 258

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2(3c-d)d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+\frac{2d^3\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{2d^3(1-\sec(e+fx))\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
2*(3*c-d)*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*d^3*tan(f*x+e)/f/(a+a*
sec(f*x+e))^(1/2)-2/3*d^3*(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/
2)+2*a^(1/2)*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a
*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*a^(1/2)*(c-d)^3*arctanh(
1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.34 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.05

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Cos[(e + f*x)/2]*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^2)
^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((2*c*(c^2 + 3*d^2)*Sin[(e + f*x)/2]
)/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) - (4*c^2*(c + 3*d)*Sin[(e + f*x)/2]
^3)/(3*(1 - 2*Sin[(e + f*x)/2]^2)^(3/2)) + (4*c*(c^2 + 3*d^2)*Sin[(e + f*x
)/2])/(3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) + (c^3*Csc[(e + f*x)/2]*Sqrt[1 -
2*Sin[(e + f*x)/2]^2]*((4*Sin[(e + f*x)/2]^4)/(1 - 2*Sin[(e + f*x)/2]^2)^2
 - (6*Sin[(e + f*x)/2]^2)/(1 - 2*Sin[(e + f*x)/2]^2) + (3*Sqrt[2]*ArcSin[S
qrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2])
)/3 - ((c - d)^3*Csc[(e + f*x)/2]^5*(-12*Cos[(e + f*x)/2]^4*Hypergeometric
PFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2)
)]*Sin[(e + f*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin[(e + f*x)/
2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]^8*(4 - 7*Sin[(e + f*x)/
2]^2 + 3*Sin[(e + f*x)/2]^4) + 7*Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e +
 f*x)/2]^2))]*(1 - 2*Sin[(e + f*x)/2]^2)^3*(15 - 20*Sin[(e + f*x)/2]^2 + 8
*Sin[(e + f*x)/2]^4)*((3 - 7*Sin[(e + f*x)/2]^2)*Sqrt[-(Sin[(e + f*x)/2]^2
/(1 - 2*Sin[(e + f*x)/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2
*Sin[(e + f*x)/2]^2))]]*(1 - 2*Sin[(e + f*x)/2]^2))))/(63*(1 - 2*Sin[(e +
f*x)/2]^2)^(7/2))))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^(5/2)*Sqrt[a*(1
 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a\sec(e+fx)+a}}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$\Big\downarrow$ 4428

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 27

$$-\frac{a\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 198

$$-\frac{a\tan(e+fx)\int \left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{(3c-d)d^2}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3\sec(e+fx)}{\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^3}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\Big\downarrow$ 2009

$$-\frac{a\tan(e+fx)\left(\frac{2d^3(a-a\sec(e+fx))^{3/2}}{3a^2}-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2(3c-d)\sqrt{}}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(c + d*Sec[e + f*x])^3/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((a*((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2
]*(c - d)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] -
 (2*(3*c - d)*d^2*Sqrt[a - a*Sec[e + f*x]])/a - (2*d^3*Sqrt[a - a*Sec[e +
f*x]])/a + (2*d^3*(a - a*Sec[e + f*x])^(3/2))/(3*a^2))*Tan[e + f*x])/(f*Sq
rt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 2.42 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.67

method	result
parts	$\frac{c^3\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+\cdots}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e\cdots}}\cdots\right.\right.}{fa}$
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left(-6(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}}\right)c^3-3\sqrt{2}\,(\cos\cdots\right.}{\cdots}$

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c^3/f/a*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-ln
((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))+2^(1/2)*arcta
nh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)
)))+1/3*d^3/f/a*(a*(1+sec(f*x+e)))^(1/2)/(cos(f*x+e)+1)*(-2*sin(f*x+e)+2*t
an(f*x+e)+3*2^(1/2)*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))+3*c*d^2/f*(-ln
((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)-2*cot(f*x+e)+2*csc(f*x+e))/a*(a*(1+sec(f*x+e)))^(1
/2)+3*c^2*d/f/a*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a*(1+sec(f*x+e)))^(1
/2)*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.40

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 +
 (a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*sqrt(-1/a)*log(-(2*
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*si
n(f*x + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)) + 6*(c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*sqrt(-a)*log(
(2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*c
os(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(d^
3 + (9*c*d^2 - d^3)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e)), -1/3*(6*(c^3*cos(f*
x + e)^2 + c^3*cos(f*x + e))*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(d^3 + (9*c*d^2 - d^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - 3*sqr
t(2)*((a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e)^2 + (a*c^3 - 3*
a*c^2*d + 3*a*c*d^2 - a*d^3)*cos(f*x + e))*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/(a*f
*cos(f*x + e)^2 + a*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c+d\sec(e+fx))^3}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**3/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e)+c)^3}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^3/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^3}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)c^3+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)+1}dx\right)d^3+3\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}dx\right)c\,d^2+3\left(\int \right.\right.}{a}$$

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x)*c**3 + int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x) + 1),x)*d**3 + 3*int((s
qrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x)*c*d**2 + 3*in
t((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)*c**2*d))/a
```

3.167 $\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2d^2\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + \frac{2\sqrt{a}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}\sqrt{a}(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
2*d^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2*a^(1/2)*c^2*arctanh((a-a*sec(f
*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))
^(1/2)-2^(1/2)*a^(1/2)*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*tan(f*x+e)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.47 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.61

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos^{\frac{3}{2}}(e+fx)(c+d\sec(e+fx))^2\left(-\frac{(c-d)^2\sqrt{-1+\cos(e+fx)}(2+\cos(e+fx))\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-2\text{arcta}\right.}{2\sqrt{2}}\right.}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Cos[(e + f*x)/2]*Cos[e + f*x]^(3/2)*(c + d*Sec[e + f*x])^2*(-1/2*((c -
d)^2*Sqrt[-1 + Cos[e + f*x]]*(2 + Cos[e + f*x])*Csc[(e + f*x)/2]^3*(-2*Arc
Tanh[Sqrt[-(Sec[e + f*x]*Sin[(e + f*x)/2]^2)]] + Sqrt[2 - 2*Sec[e + f*x]])
)/Sqrt[2] + (4*c*d*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]] + c^2*(Sqrt[2]*Arc
Sin[Sqrt[2]*Sin[(e + f*x)/2]] - (2*Sin[(e + f*x)/2])/Sqrt[Cos[e + f*x]]) -
 ((c - d)^2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[e + f*x]*Sin[(e + f*x)/2]
^2)]*Sin[(e + f*x)/2]*Sin[e + f*x]^2)/(10*Cos[e + f*x]^(5/2))))/(f*(d + c*
Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a\sec(e+fx)+a}}\,dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{a\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{a\tan(e+fx)\int \left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}+\frac{d^2}{\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\tan(e+fx)\left(-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}(c-d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\sqrt{a-a\sec(e+fx)}}{a}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(c + d*Sec[e + f*x])^2/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((a*((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2
]*(c - d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] -
 (2*d^2*Sqrt[a - a*Sec[e + f*x]])/a)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 1.96 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.63

method	result
default	$\frac{\left(2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)cd+(2\csc(fx+e)-2\cot(fx+e))d^2+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{arctanh}\left(\right.\right.}{}$
parts	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e}}\right.\right.}{fa}$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)-cot(f*x+e)+csc(f*x+e))*c*d+(2*csc(f*x+e)-2*cot(f*x+e))*d^2+(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*2^(1/2)*c^2-(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+
e))*c^2-(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*d^2)/a*(-a*(-1-sec(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.63

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[\frac{4\,d^2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)+\sqrt{2}(ac^2-2\,acd+ad^2+(ac^2-2\,acd+ad^2)\cos(fx+e))\sqrt{-\frac{1}{a}}\log\Big(\right.$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(4*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + sqrt(2)
*(a*c^2 - 2*a*c*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*sqrt(-
1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos
(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x +
 e)^2 + 2*cos(f*x + e) + 1)) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*
a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)))/(a*f*cos(
f*x + e) + a*f), (2*d^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
e) - 2*(c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + sqrt(2)*(a*c^2 - 2*a*c
*d + a*d^2 + (a*c^2 - 2*a*c*d + a*d^2)*cos(f*x + e))*arctan(sqrt(2)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqr
t(a))/(a*f*cos(f*x + e) + a*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(c+d\sec{(e+fx)})^2}{\sqrt{a\,(\sec{(e+fx)}+1)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**2/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec{(fx+e)}+c)^2}{\sqrt{a\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^2/sqrt(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^2}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)c^2+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)+1}dx\right)d^2+2\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)cd\right)}{a}$$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x)*c**2 + int((sqr
t(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x) + 1),x)*d**2 + 2*int((s
qrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)*c*d))/a
```

3.168 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/f-2^(1/2)*(c
-d)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/
f
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{2\left(\sqrt{2}c\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+(-c+d)\arctan\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*(Sqrt[2]*c*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]] + (-c + d)*ArcTan[Sin[(e +
f*x)/2]/Sqrt[Cos[e + f*x]]])*Cos[(e + f*x)/2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[
a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 4408}$$

$$\frac{c\int \sqrt{\sec(e+fx)a+a}dx}{a}-(c-d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{a}-(c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{ 4261}$$

$$-\frac{2c\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-\left((c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-(c-d)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2(c-d)\int\frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f}$$

input
```
Int[(c + d*Sec[e + f*x])/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*f)
- (Sqrt[2]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e
 + f*x]])])/(Sqrt[a]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(76) = 152$.

Time = 1.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\sqrt{-a(-1-\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(c\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)-d\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\right.\right.}{fa}$
parts	$\frac{c\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-\ln\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cot(fx+e)+\csc(fx+e)\right)+\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2}}\right.\right.}{fa}$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(c*
ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))-d*ln((-2*co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))-c*2^(1/2)*arctanh(2^
(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))))
```

Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.45

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{2}(ac-ad)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)-3\cos(fx+e)^2-2\cos(fx+e)+1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)+2\sqrt{-a}c\log}{2af}\right.$$

$$\left.-\frac{2\sqrt{a}c\arctan\left(\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)-\frac{\sqrt{2}(ac-ad)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)}{\sqrt{a}}}{af}\right]$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(2)*(a*c - a*d)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^
2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-a
)*c*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))
)/(a*f), -(2*sqrt(a)*c*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(
f*x + e)/(sqrt(a)*sin(f*x + e))) - sqrt(2)*(a*c - a*d)*arctan(sqrt(2)*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/s
qrt(a))/(a*f)]
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{c+d\sec{(e+fx)}}{\sqrt{a(\sec{(e+fx)}+1)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 699, normalized size of antiderivative = 7.68

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*f*x + I*e) + 2)^4 + 16*cos(f*x + e)
^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^2 - 2*cos(f*x +
e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 + 32*(cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)^2 - 64*cos(f*x
+ e) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(f*x + e)/abs(2*e
^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*cos(f*x + e)^2 -
4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*e) + 2)^2)) + 2*
sin(f*x + e))/abs(2*e^(I*f*x + I*e) + 2), ((abs(2*e^(I*f*x + I*e) + 2)^4 +
 16*cos(f*x + e)^4 + 16*sin(f*x + e)^4 + 8*(cos(f*x + e)^2 - sin(f*x + e)^
2 - 2*cos(f*x + e) + 1)*abs(2*e^(I*f*x + I*e) + 2)^2 - 64*cos(f*x + e)^3 +
 32*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)^2 + 96*cos(f*x + e)
^2 - 64*cos(f*x + e) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(f*x + e) - 1)*sin(
f*x + e)/abs(2*e^(I*f*x + I*e) + 2)^2, (abs(2*e^(I*f*x + I*e) + 2)^2 + 4*c
os(f*x + e)^2 - 4*sin(f*x + e)^2 - 8*cos(f*x + e) + 4)/abs(2*e^(I*f*x + I*
e) + 2)^2)) + 2*cos(f*x + e) - 2)/abs(2*e^(I*f*x + I*e) + 2)) - sqrt(a)*ar
ctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x +
e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*x + e)
))*c/(a*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c + d\sec(e + fx)}{\sqrt{a + a\sec(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)c+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)+1}dx\right)d\right)}{a}$$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x) + 1),x)*c + int((sqrt(s
ec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x) + 1),x)*d))/a
```

3.169 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}c(c-d)\sqrt{c+d}f}$$

output
```
2*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/c/f-2^(1/2)*ar
ctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/a^(1/2)/(c-d)/
f+2*d^(3/2)*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))
^(1/2))/a^(1/2)/c/(c-d)/(c+d)^(1/2)/f
```

Mathematica [A] (warning: unable to verify)

Time = 2.97 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\left(-c\sqrt{c+d}\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{2}\left((c-d)\sqrt{c+d}\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)+d^{3/2}\arctan\left(\frac{\sqrt{d}\tan\ldots}{\sqrt{c+d}\ldots}\right.\right.\right.}{c(c-d)\sqrt{c+d}f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a(1+\sec(e\ldots}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(2*(-(c*Sqrt[c + d]*ArcSin[Tan[(e + f*x)/2]]) + Sqrt[2]*((c - d)*Sqrt[c +
d]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] + d^(3/2
)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + Co
s[e + f*x])])]))*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*(d + c*Cos[e + f*x]
)*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*x]])/(c*(c - d)*Sqrt[c + d]*f*Sqrt
[Sec[(e + f*x)/2]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c + d*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {3042, 4417, 3042, 4408, 3042, 4261, 216, 4282, 216, 4455, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4417}$$

$$\frac{d^2\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{c+d\sec(e+fx)}dx}{ac(c-d)}+\frac{\int \frac{a(c-d)-ad\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{\int \frac{a(c-d)-ad\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{4408}$$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{(c-d)\int \sqrt{\sec(e+fx)a+a}dx-ac\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{(c-d)\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}$$

$$\Big\downarrow \text{4261}$$

$$\frac{-\frac{2a(c-d)\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}+\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-ac\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{ac(c-d)}+\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{2ac\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}+\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{d^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{ac(c-d)}+\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}$$

$$\Big\downarrow \textbf{4455}$$

$$\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}-\frac{2d^2\int \frac{1}{\frac{a^2d\tan^2(e+fx)}{\sec(e+fx)a+a}+a(c+d)}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{cf(c-d)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{2d^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf(c-d)\sqrt{c+d}}+\frac{\frac{2\sqrt{a}(c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{ac(c-d)}$$

input

```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output

```
((2*Sqrt[a]*(c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]
])/f - (Sqrt[2]*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sec[e + f*x]])])/f)/(a*c*(c - d)) + (2*d^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]*T
an[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*c*(c - d)*S
qrt[c + d]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4417
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Simp[1/(c*(b*c - a*d))   Int[(b*c - a*d - b*d*
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[d^2/(c*(b*c - a*d))
 Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] |
| EqQ[c^2 - d^2, 0])
```

rule 4455
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[-2*(b/f)   Subst[In
t[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(137) = 274$.

Time = 5.94 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.97

method	result
default	$\frac{\left(2\sqrt{2}\,\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)\sqrt{\frac{d}{c-d}}\,c-2\sqrt{2}\,\sqrt{(c+d)(c-d)}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e)}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2}}\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(d/(c-d))^(1/2)/(c-d)/c/((c+d)*(c-d))^(1/2)/a*(2*2^(1/2)*((c+d)*(c-d
))^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x
+e)+csc(f*x+e)))*(d/(c-d))^(1/2)*c-2*2^(1/2)*((c+d)*(c-d))^(1/2)*arctanh(2
^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)))*(
d/(c-d))^(1/2)*d+2^(1/2)*ln(-2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^
(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-co
t(f*x+e)+csc(f*x+e))+d*(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))*d^2-
2^(1/2)*ln(2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(d/(c-d))^(1
/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d-((
c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc(f*x+e)
)-d*(-cot(f*x+e)+csc(f*x+e))+((c+d)*(c-d))^(1/2)))*d^2-2*((c+d)*(c-d))^(1/
2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(d/(c
-d))^(1/2)*c)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((1-cos(f*x+e)
)^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 1050, normalized size of antiderivative = 6.33

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

input

```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output

```
[-1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e)^2 - 2*co
s(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*a*d*sqrt(-d/(a*
c + a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)
*cos(f*x + e) - d)/(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) + 2*sqrt
(-a)*(c - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a
)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x +
 e) + 1)))/((a*c^2 - a*c*d)*f), -1/2*(sqrt(2)*a*c*sqrt(-1/a)*log(-(2*sqrt(
2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x
 + e) - 3*cos(f*x + e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x
 + e) + 1)) + 4*a*d*sqrt(d/(a*c + a*d))*arctan((c + d)*sqrt(d/(a*c + a*d))
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(d*sin(f*x + e))) +
2*sqrt(-a)*(c - d)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos
(f*x + e) + 1)))/((a*c^2 - a*c*d)*f), -(a*d*sqrt(-d/(a*c + a*d))*log((2*(c
 + d)*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e) + (c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/
(c*cos(f*x + e)^2 + (c + d)*cos(f*x + e) + d)) - sqrt(2)*sqrt(a)*c*arctan(
sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*s...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$=\int \frac{1}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)}{a}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2*d + sec(e + f*x)*c + 
sec(e + f*x)*d + c),x))/a
```

3.170 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 416

$$\begin{aligned}
\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx
&= \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad - \frac{\sqrt{2}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\sqrt{a}d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c(c-d)(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{2\sqrt{a}(2c-d)d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{d^2\tan(e+fx)}{c\,(c^2-d^2)\,f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}
\end{aligned}$$

output
```
2*a^(1/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^2/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*a^(1/2)*arctanh(1/2*(a-a*se
c(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/(c-d)^2/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)+a^(1/2)*d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))
^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)/(c+d)^(3/2)/f/(a-a*sec(f*x+
e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*a^(1/2)*(2*c-d)*d^(3/2)*arctanh(d^(1/2)
*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c-d)^2/(c+d)^
(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+d^2*tan(f*x+e)/c/(c^
2-d^2)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 9.91 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{(d+c\cos(e+fx))^2\sec^{\frac{5}{2}}(e+fx)\left(\frac{c(c-d)d^2\sin(e+fx)}{(c+d)(d+c\cos(e+fx))\sqrt{\sec(e+fx)}}+\frac{\left(\sqrt{2}\left(2(c-d)^2(c+d)^{3/2}\operatorname{arctanh}\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{-\frac{\cos(e+fx)}{1+\cos(e+fx}}}\right.\right.\right.}{}\right.}{c^2(c-d)^2f\sqrt{a(}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

output
```
((d + c*Cos[e + f*x])^2*Sec[e + f*x]^(5/2)*((c*(c - d)*d^2*Sin[e + f*x])/(
(c + d)*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + ((Sqrt[2]*(2*(c - d)^2*
(c + d)^(3/2)*ArcTanh[Tan[(e + f*x)/2]/Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*
x]))]] + d^(3/2)*(5*c^2 + c*d - 2*d^2)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/
(Sqrt[c + d]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))])]) - 2*c^2*(c + d)^(
3/2)*ArcTanh[Tan[(e + f*x)/2]/Sqrt[-1 + Tan[(e + f*x)/2]^2]])*Sqrt[Cos[(e
+ f*x)/2]^2*Sec[e + f*x]]*Sqrt[-1 + Tan[(e + f*x)/2]^2])/((c + d)^(3/2)*Sq
rt[Sec[(e + f*x)/2]^2])))/(c^2*(c - d)^2*f*Sqrt[a*(1 + Sec[e + f*x])]*(c +
 d*Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(e+fx)+a}(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{a\tan(e+fx)\int \left(\frac{(2c-d)d^2}{c^2(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{d^2}{c(c-d)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}-\ldots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\tan(e+fx)\left(-\frac{2d^{3/2}(2c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^2\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2}-\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)(c+d)^{3/2}}\ldots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((a*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + (Sqrt
[2]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^
2) - (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c +
 d])])/(Sqrt[a]*c*(c - d)*(c + d)^(3/2)) - (2*(2*c - d)*d^(3/2)*ArcTanh[(S
qrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c -
 d)^2*Sqrt[c + d]) - (d^2*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c^2 - d^2)*(c +
d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 11906 vs. $2(355) = 710$.

Time = 6.56 (sec) , antiderivative size = 11907, normalized size of antiderivative = 28.62

method	result	size
default	Expression too large to display	11907

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 103.67 (sec) , antiderivative size = 2508, normalized size of antiderivative = 6.03

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
[1/2*(2*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
+ e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a*c^4 + a*c^3*d)*cos(f
*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x + e))*sqrt(-1/a)*log((
2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*
sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)^2 + 2*
cos(f*x + e) + 1)) - (5*a*c^2*d^2 + a*c*d^3 - 2*a*d^4 + (5*a*c^3*d + a*c^2
*d^2 - 2*a*c*d^3)*cos(f*x + e)^2 + (5*a*c^3*d + 6*a*c^2*d^2 - a*c*d^3 - 2*
a*d^4)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*log((2*(c + d)*sqrt(-d/(a*c + a*
d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (c
 + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)/(c*cos(f*x + e)^2 + (c
+ d)*cos(f*x + e) + d)) - 2*(c^3*d - c^2*d^2 - c*d^3 + d^4 + (c^4 - c^3*d
- c^2*d^2 + c*d^3)*cos(f*x + e)^2 + (c^4 - 2*c^2*d^2 + d^4)*cos(f*x + e))*
sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
+ 1)))/((a*c^6 - a*c^5*d - a*c^4*d^2 + a*c^3*d^3)*f*cos(f*x + e)^2 + (a*c^
6 - 2*a*c^4*d^2 + a*c^2*d^4)*f*cos(f*x + e) + (a*c^5*d - a*c^4*d^2 - a*c^3
*d^3 + a*c^2*d^4)*f), 1/2*(2*(c^2*d^2 - c*d^3)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) + sqrt(2)*(a*c^3*d + a*c^2*d^2 + (a
*c^4 + a*c^3*d)*cos(f*x + e)^2 + (a*c^4 + 2*a*c^3*d + a*c^2*d^2)*cos(f*x +
 e))*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a + a\sec(e + fx)}(c + d\sec(e + fx))^2}\,dx$$
$$= \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3 d^2+2\sec(fx+e)^2 cd+\sec(fx+e)^2 d^2+\sec(fx+e)c^2+2\sec(fx+e)cd+c^2}dx\right)}{a}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3*d**2 + 2*sec(e + f*x)
**2*c*d + sec(e + f*x)**2*d**2 + sec(e + f*x)*c**2 + 2*sec(e + f*x)*c*d +
c**2),x))/a
```

3.171 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 653

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{3\sqrt{a}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4c(c-d)(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{\sqrt{a}(2c-d)d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^2(c-d)^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{2\sqrt{a}d^{3/2}(3c^2-3cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{c^3(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{d^2\tan(e+fx)}{2c(c^2-d^2)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2}$$
$$+ \frac{3d^2\tan(e+fx)}{4c(c-d)(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$
$$+ \frac{(2c-d)d^2\tan(e+fx)}{c^2(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
2*a^(1/2)*arctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/c^3/f/(a-a*se
c(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*a^(1/2)*arctanh(1/2*(a-a*se
c(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)+3/4*a^(1/2)*d^(3/2)*arctanh(d^(1/2)*(a-a*sec(f*x
+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c/(c-d)/(c+d)^(5/2)/f/(a-a*sec(
f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+a^(1/2)*(2*c-d)*d^(3/2)*arctanh(d^(1/
2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/c^2/(c-d)^2/(c+d
)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*a^(1/2)*d^(3/2)*
(3*c^2-3*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/
2))*tan(f*x+e)/c^3/(c-d)^3/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)+1/2*d^2*tan(f*x+e)/c/(c^2-d^2)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*
sec(f*x+e))^2+3/4*d^2*tan(f*x+e)/c/(c-d)/(c+d)^2/f/(a+a*sec(f*x+e))^(1/2)/
(c+d*sec(f*x+e))+(2*c-d)*d^2*tan(f*x+e)/c^2/(c-d)^2/(c+d)/f/(a+a*sec(f*x+e
))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2940 vs. $2(653) = 1306$.

Time = 21.83 (sec) , antiderivative size = 2940, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

output
```
(Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^4*(-1/2*(d^2*(-13*c^
2 - c*d + 6*d^2)*Sin[(e + f*x)/2])/(c^3*(-c + d)^2*(c + d)^2) - (d^4*Sin[(
e + f*x)/2])/(c^3*(-c + d)*(c + d)*(d + c*Cos[e + f*x])^2) + (-15*c^2*d^3*
Sin[(e + f*x)/2] - c*d^4*Sin[(e + f*x)/2] + 8*d^5*Sin[(e + f*x)/2])/(2*c^3
*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x]))))/(f*Sqrt[a*(1 + Sec[e + f*x])
]*(c + d*Sec[e + f*x])^3) - (Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])^3*((Sqr
t[2]*d^(3/2)*(35*c^4 + 14*c^3*d - 21*c^2*d^2 - 4*c*d^3 + 8*d^4)*ArcTan[(Sq
rt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x
]))])])/(Sqrt[-c - d]*(c - d)) - 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[(e + f*x)
/2]^2*(-1 + 2*Cos[e + f*x] - 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Si
n[e + f*x])] + 2*Sqrt[2]*(c^2 - d^2)^2*Log[Sec[(e + f*x)/2]^2*(-1 + 2*Cos[
e + f*x] + 2*Sqrt[-(Cos[e + f*x]/(1 + Cos[e + f*x]))]*Sin[e + f*x])] + (8*
c^3*(c + d)^2*Log[Tan[(e + f*x)/2] + Sqrt[-1 + Tan[(e + f*x)/2]^2]])/(c -
d))*((-2*c*d*Sec[(e + f*x)/2])/((-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])*
Sqrt[Sec[e + f*x]]) - (13*d^2*Sec[(e + f*x)/2])/(8*(-c + d)^2*(c + d)^2*(d
 + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + (d^3*Sec[(e + f*x)/2])/(8*c*(-c +
 d)^2*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) + (d^4*Sec[(e + f
*x)/2])/(2*c^2*(-c + d)^2*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]
]) + (c^2*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(2*(-c + d)^2*(c + d)^2*(d
+ c*Cos[e + f*x])) + (3*d^2*Sec[(e + f*x)/2]*Sqrt[Sec[e + f*x]])/(8*(-c...
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sec(e+fx)+a}(c+d \sec(e+fx))^3} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a \csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(c+d \csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 27$$

$$-\frac{a\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow 198$$

$$-\frac{a\tan(e+fx)\int \left(\frac{(3c^2-3dc+d^2)d^2}{c^3(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{(2c-d)d^2}{c^2(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{d^2}{c(c-d)\sqrt{a-a\sec(e+f}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

$$\Big\downarrow 2009$$

$$-\frac{a\tan(e+fx)\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}-\frac{d^{3/2}(2c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^2(c+d)^{3/2}}-\frac{2d^{3/2}(3c^2-3cd+d^2)\text{arctanh}\left(\frac{\sqrt{d}}{}\right.}{\sqrt{a}c^3(c-d)^3\sqrt{c+d}}\right.}{}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((a*((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + (Sqrt
[2]*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^
3) - (3*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c
 + d])])/(4*Sqrt[a]*c*(c - d)*(c + d)^(5/2)) - ((2*c - d)*d^(3/2)*ArcTanh[
(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c
 - d)^2*(c + d)^(3/2)) - (2*d^(3/2)*(3*c^2 - 3*c*d + d^2)*ArcTanh[(Sqrt[d]
*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^3*(c - d)^3*
Sqrt[c + d]) - (d^2*Sqrt[a - a*Sec[e + f*x]])/(2*a*c*(c^2 - d^2)*(c + d*Se
c[e + f*x])^2) - (3*d^2*Sqrt[a - a*Sec[e + f*x]])/(4*a*c*(c - d)*(c + d)^2
*(c + d*Sec[e + f*x])) - ((2*c - d)*d^2*Sqrt[a - a*Sec[e + f*x]])/(a*c^2*(
c - d)^2*(c + d)*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e
+ f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 24368 vs. 2(564) = 1128.

Time = 9.67 (sec) , antiderivative size = 24369, normalized size of antiderivative = 37.32

method	result	size
default	Expression too large to display	24369

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^3}\,dx$$
$$=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4 d^3+3\sec(fx+e)^3 c\,d^2+\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c^2 d+3\sec(fx+e)^2 c\,d^2+\sec(fx+e)c^3+3\sec(fx+e)c^2 d+c^3}dx\right)}{a}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4*d**3 + 3*sec(e + f*x)
**3*c*d**2 + sec(e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c**2*d + 3*sec(e + f
*x)**2*c*d**2 + sec(e + f*x)*c**3 + 3*sec(e + f*x)*c**2*d + c**3),x))/a
```

3.172 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 324

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{2d^3\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c-d)^3\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\sqrt{2}(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output

```
2*d^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^3*tan(f*x+e)/a/f/(1+
sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arctanh((a-a*sec(f*x+e))^(1/2)/a^
(1/2))*tan(f*x+e)/a^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-
1/4*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)
*2^(1/2)/a^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*(
c-d)^2*(c+2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x
+e)/a^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.22 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.64

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output

```
(2*Cos[(e + f*x)/2]^3*(c + d*Sec[e + f*x])^3*Sqrt[(1 - 2*Sin[(e + f*x)/2]^
2)^(-1)]*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]*((-3*(c - d)^3*ArcTan[(1 - 2*Sin[(
e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 + (3*(c - d)^3*ArcTan[(1 +
 2*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2]])/2 - (4*c^2*(c - 3*d)
*Sin[(e + f*x)/2])/Sqrt[1 - 2*Sin[(e + f*x)/2]^2] + ((c - d)^3*(1 - 2*Sin[
(e + f*x)/2]))/(4*(1 + Sin[(e + f*x)/2])*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]) -
 ((c - d)^3*(1 + 2*Sin[(e + f*x)/2]))/(4*(1 - Sin[(e + f*x)/2])*Sqrt[1 - 2
*Sin[(e + f*x)/2]^2]) - ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 - Si
n[(e + f*x)/2]) + ((c - d)^3*Sqrt[1 - 2*Sin[(e + f*x)/2]^2])/(1 + Sin[(e +
 f*x)/2]) - (2*c^3*(-(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]) + 2*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + 2*Sin[(e + f*x)/2]
*Sqrt[1 - 2*Sin[(e + f*x)/2]^2]))/(1 - 2*Sin[(e + f*x)/2]^2) - ((c - d)^2*
(11*c + d)*Sin[(e + f*x)/2]*((2*Cos[(e + f*x)/2]^2*Hypergeometric2F1[2, 5/
2, 7/2, -(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*Sin[(e + f*x)/2]
^2)/(1 - 2*Sin[(e + f*x)/2]^2) + 5*Csc[(e + f*x)/2]^4*Sqrt[-(Sin[(e + f*x)
/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))]*(1 - 2*Sin[(e + f*x)/2]^2)^2*(3 - 2*Sin
[(e + f*x)/2]^2)*(-ArcTanh[Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/
2]^2))]] + Sqrt[-(Sin[(e + f*x)/2]^2/(1 - 2*Sin[(e + f*x)/2]^2))])))/(10*(
1 - 2*Sin[(e + f*x)/2]^2)^(3/2))))/(f*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^
(3/2)*(a*(1 + Sec[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^3}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3}{\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2(c+2d)}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^3}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}\right)d\text{s}$$

$$f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}}+\frac{\sqrt{2}(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+f}}{\sqrt{2}\sqrt{a}}\right.}{\sqrt{a}}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input

```
Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + ((c - d)^
3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a])
 + (Sqrt[2]*(c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*
Sqrt[a])])/Sqrt[a] - (2*d^3*Sqrt[a - a*Sec[e + f*x]])/a + ((c - d)^3*Sqrt[
a - a*Sec[e + f*x]])/(2*a*(1 + Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a
*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 2.30 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.63

method	result
parts	$\frac{c^3\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(fx+}{4f\,a^2}$

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^3/f/a^2*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(4*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot
(f*x+e)+csc(f*x+e)))+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc
(f*x+e))-5*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
+d^3/f*(-1/4*(1-cos(f*x+e))^3*csc(f*x+e)^3-7/4*ln((-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+9
/4*csc(f*x+e)-9/4*cot(f*x+e))/a^2*(a*(1+sec(f*x+e)))^(1/2)+3/4*c*d^2/f/a^2
*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+3*ln((-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))-3/4*c^2*d/f/a^2*(a*(1+sec(f*x+
e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.16

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(2)*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c
*d^2 + 7*d^3)*cos(f*x + e)^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f
*x + e))*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*cos(f*x + e)*sin(f*x + e) - 3*a*cos(f*x + e)^2 - 2*a*cos(f*x + e)
 + a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c^3*cos(f*x + e)^2 + 2*c
^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x +
 e) - a)/(cos(f*x + e) + 1)) - 4*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3
)*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^2
*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), 1/4*(sqrt(2)*(5*c^3 - 3
*c^2*d - 9*c*d^2 + 7*d^3 + (5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e
)^2 + 2*(5*c^3 - 3*c^2*d - 9*c*d^2 + 7*d^3)*cos(f*x + e))*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f
*x + e))) - 8*(c^3*cos(f*x + e)^2 + 2*c^3*cos(f*x + e) + c^3)*sqrt(a)*arct
an(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x +
 e))) + 2*(4*d^3 - (c^3 - 3*c^2*d + 3*c*d^2 - 5*d^3)*cos(f*x + e))*sqrt((a
*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a
^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c+d\sec{(e+fx)})^3}{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e)+c)^3}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^3/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)c^3+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^3}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)d^3+3\left(\right.\right.}{a^2}$$

input

```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(3/2),x)
```

output

```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1
),x)*c**3 + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**3)/(sec(e + f*x)**2
+ 2*sec(e + f*x) + 1),x)*d**3 + 3*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)
**2)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)*c*d**2 + 3*int((sqrt(sec(e
+ f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)*c**2*d
))/a**2
```

3.173 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 290

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx &= -\frac{(c-d)^2\tan(e+fx)}{2af(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\sqrt{2}(c^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
-1/2*(c-d)^2*tan(f*x+e)/a/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*ar
ctanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/f/(a-a*sec(f*x+e)
)^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)-2^(1/2)*(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x
+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.61

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{-\sqrt{2}(5c^2-2cd-3d^2)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec(e+f\ldots}{af(\ldots}$$

input
```
Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(-(Sqrt[2]*(5*c^2 - 2*c*d - 3*d^2)*ArcSin[Tan[(e + f*x)/2]]*Cos[(e + f*x)/
2]^4*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + 8*c^2*ArcTan[Tan[(e + f
*x)/2]/Sqrt[(1 + Sec[e + f*x])^(-1)]]*Cos[(e + f*x)/2]^4*Sec[e + f*x]*Sqrt
[(1 + Sec[e + f*x])^(-1)] - ((c - d)^2*Sin[e + f*x])/2)/(a*f*(1 + Cos[e +
f*x])*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^2}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}+\frac{d^2-c^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}\right)d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{\sqrt{2}(c^2-d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(c-d)^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}}+\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + ((c - d)^
2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a])
 + (Sqrt[2]*(c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])
])/Sqrt[a] + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(2*a*(1 + Sec[e + f*x]))
)*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 2.06 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.16

method	result
default	$\frac{\sqrt{-a(-1-\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4c^2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,c^2(-\cot(fx+e)+\csc}{}$
parts	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(fx+}{4f\,a^2}$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/a^2*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
(4*c^2*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-c
ot(f*x+e)+csc(f*x+e)))+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c^2*(-cot(f*x+
e)+csc(f*x+e))-2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*d*(-cot(f*x+e)+csc
(f*x+e))+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d^2*(-cot(f*x+e)+csc(f*x+e))
-5*c^2*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))+2*c*
d*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))+3*d^2*ln(
(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.14

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(c^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5
*c^2 - 2*c*d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(-a)*lo
g((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2
 + 2*cos(f*x + e) + 1)) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2
)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e
) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f), -1/4*(2*(c
^2 - 2*c*d + d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e) - sqrt(2)*((5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e)^2 + 5*c^2 - 2*c*
d - 3*d^2 + 2*(5*c^2 - 2*c*d - 3*d^2)*cos(f*x + e))*sqrt(a)*arctan(sqrt(2)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e
))) + 8*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqr
t((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))
/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(c+d\sec{(e+fx)})^2}{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e)+c)^2}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\,\Big(\Big(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\Big)\,c^2+\Big(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)^2}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\Big)\,d^2+2\Big(}{a^2}$$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1
),x)*c**2 + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**2
+ 2*sec(e + f*x) + 1),x)*d**2 + 2*int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)
)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),x)*c*d))/a**2
```

3.174 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{3/2}f} - \frac{(5c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(c-d)\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(3/2)/f-1/4*(5*c-d
)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/a^
(3/2)/f-1/2*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.81

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(-\left((5c-d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

input
```
Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(Cos[(e + f*x)/2]*Sec[e + f*x]*(-((5*c - d)*ArcSin[Tan[(e + f*x)/2]]*Cos[(
e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[1 + Sec[
e + f*x]]) + Sqrt[2]*(4*c*ArcTan[Tan[(e + f*x)/2]/Sqrt[(1 + Sec[e + f*x])^
(-1)]]*Cos[(e + f*x)/2]*Sqrt[Sec[e + f*x]]*Sqrt[(1 + Sec[e + f*x])^(-1)]*S
qrt[1 + Sec[e + f*x]] - (c - d)*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sin[(e + f*x
)/2])))/(f*Sqrt[Sec[(e + f*x)/2]^2]*(a*(1 + Sec[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4410, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4410}$$

$$-\frac{\int -\frac{4ac-a(c-d)\sec(e+fx)}{2\sqrt{\sec(e+fx)a+a}}dx}{2a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4ac-a(c-d)\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4ac-a(c-d)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{4408}$$

$$\frac{4c\int \sqrt{\sec(e+fx)a+a}dx-a(5c-d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4c\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-a(5c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{-\frac{8ac\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}-a(5c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{8\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-a(5c-d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{2a(5c-d)\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{8\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{8\sqrt{a}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}\sqrt{a}(5c-d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{(c-d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}$$

input
```
Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
((8*Sqrt[a]*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f -
 (Sqrt[2]*Sqrt[a]*(5*c - d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[e + f*x]])])/f)/(4*a^2) - ((c - d)*Tan[e + f*x])/(2*f*(a + a*Sec[e
 + f*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(106) = 212$.

Time = 1.74 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.95

method	result
default	$\frac{\sqrt{-a(-1-\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4c\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+c\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(\right.}{}$
parts	$\frac{c\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(fx+e))}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(-\cot(fx+e)+\csc(fx+e\right.}{4f\,a^2}$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/a^2*(-a*(-1-sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
(4*c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot
(f*x+e)+csc(f*x+e)))+c*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+c
sc(f*x+e))-d*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))
-5*c*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e))+d*ln((
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(106) = 212$.

Time = 1.59 (sec) , antiderivative size = 548, normalized size of antiderivative = 4.31

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \left[-\frac{4\,(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\big((5\,c-d)\cos\left(fx\right.}{}\right.$$

$$-\frac{2\,(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}\big((5\,c-d)\cos\left(fx+e\right)^2+2\,(5\,c-d)\cos\left(fx+e\right)-}{4\left(a^2f\cos\left(fx\right.\right.}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e) +
5*c - d)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e)
- a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(c*cos(f*x + e)^2 + 2*c*co
s(f*x + e) + c)*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a
)/(cos(f*x + e) + 1)))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*
f), -1/4*(2*(c - d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e) - sqrt(2)*((5*c - d)*cos(f*x + e)^2 + 2*(5*c - d)*cos(f*x + e)
 + 5*c - d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) + 8*(c*cos(f*x + e)^2 + 2*c*cos(f*x
+ e) + c)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x +
 e)/(sqrt(a)*sin(f*x + e))))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e)
+ a^2*f)]
```

Sympy [F]

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{c + d\sec(e + fx)}{(a(\sec(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\,dx = \int \frac{d\sec(fx + e) + c}{(a\sec(fx + e) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)c+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx+e)}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)d\right)}{a^2}$$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1
),x)*c + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**2 + 2*se
c(e + f*x) + 1),x)*d))/a**2
```

3.175 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 394

$$\begin{aligned}
\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx &= -\frac{\tan(e+fx)}{2a(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\
&+ \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{\sqrt{2}(c-2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&- \frac{2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

output
```
-1/2*tan(f*x+e)/a/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh(
(a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/c/f/(a-a*sec(f*x+e))^(1
/2)/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/(c-d)^2/f/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*tan(f*x+e)*2^(1/2)/a^(1/2)/(c-d)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x
+e))^(1/2)-2*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^
(1/2))*tan(f*x+e)/a^(1/2)/c/(c-d)^2/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.50 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^{\frac{5}{2}}(e+fx)\left(\frac{\left(-c\right.}{}\right.}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

output
```
(Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e + f*x]^(5/2)*(((-(c*(5*c -
9*d)*Sqrt[c + d]*ArcSin[Tan[(e + f*x)/2]]) + 4*Sqrt[2]*((c - d)^2*Sqrt[c +
 d]*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - d^(5/
2)*ArcTan[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f*x]/(1 + C
os[e + f*x])])]))*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[1 + Sec[e + f
*x]])/(c*Sqrt[c + d]*Sqrt[Sec[(e + f*x)/2]^2]) + (c - d)*Sqrt[Sec[e + f*x]
]*(-Sin[e + f*x] + Tan[(e + f*x)/2])))/((c - d)^2*f*(a*(1 + Sec[e + f*x]))
^(3/2)*(c + d*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{\tan(e+fx)\int \left(-\frac{d^3}{c(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{\cos(e+fx)}{c\sqrt{a-a\sec(e+fx)}}+\frac{2d-c}{(c-d)^2(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{}{(c-d}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\tan(e+fx)\left(\frac{2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)^2\sqrt{c+d}}+\frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}(c-d)}+\frac{\sqrt{2}(c-2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}(c-d)^2}-\frac{2a}{}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c) + (Sqrt[2]*
(c - 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c
 - d)^2) + ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*
Sqrt[a]*(c - d)) + (2*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(
Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*(c - d)^2*Sqrt[c + d]) + Sqrt[a - a*Sec[
e + f*x]]/(2*a*(c - d)*(1 + Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2504 vs. $2(334) = 668$.

Time = 6.74 (sec) , antiderivative size = 2505, normalized size of antiderivative = 6.36

method	result	size
default	Expression too large to display	2505

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*(-1/2/f*(c+d)/(2^(1/2)*(c*(c+d))^(1/2)-c-d)^2/(2^(1/2)*(c*(c+d
))^(1/2)+c+d)^2/(-d/(c+d))^(1/2)/(cos(1/2*f*x+1/2*e)+1)/(a/(2*cos(1/2*f*x+
1/2*e)^2-1)*cos(1/2*f*x+1/2*e)^2)^(1/2)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1
/2*f*x+1/2*e)+1)^2)^(1/2)/a*(4*2^(1/2)*cos(1/2*f*x+1/2*e)*ln(-(2^(1/2)*(-d
/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*
c*cos(1/2*f*x+1/2*e)+2^(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/
(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)*d+2^(1/2)*c*(-d/(c+d))^
(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)+2^(1/2)*
(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/
2)*d+2^(1/2)*(c*(c+d))^(1/2)-2*c*sin(1/2*f*x+1/2*e)-2*sin(1/2*f*x+1/2*e)*d
)/(2^(1/2)*(c*(c+d))^(1/2)*sin(1/2*f*x+1/2*e)-c-d))*d^2+4*2^(1/2)*cos(1/2*
f*x+1/2*e)*ln((2^(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1
/2*f*x+1/2*e)+1)^2)^(1/2)*c*cos(1/2*f*x+1/2*e)+2^(1/2)*(-d/(c+d))^(1/2)*((
2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*
e)*d+2^(1/2)*c*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1
/2*e)+1)^2)^(1/2)+2^(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(co
s(1/2*f*x+1/2*e)+1)^2)^(1/2)*d-2^(1/2)*(c*(c+d))^(1/2)-2*c*sin(1/2*f*x+1/2
*e)-2*sin(1/2*f*x+1/2*e)*d)/(2^(1/2)*(c*(c+d))^(1/2)*sin(1/2*f*x+1/2*e)+c+
d))*d^2+cos(1/2*f*x+1/2*e)*arctan((2*sin(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/
2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2))*(-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}\,(c+d\sec(e+fx))}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3d+\sec(fx+e)^2c+2\sec(fx+e)^2d+2\sec(fx+e)c+\sec(fx}\right.}{a^2}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3*d + sec(e + f*x)**2*c
 + 2*sec(e + f*x)**2*d + 2*sec(e + f*x)*c + sec(e + f*x)*d + c),x))/a**2
```

3.176 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 560

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx &= -\frac{\tan(e+fx)}{2a(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}\\
&+\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{\sqrt{2}(c-3d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c(c-d)^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{2(3c-d)d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{d^3\tan(e+fx)}{ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\end{aligned}$$

output
```
-1/2*tan(f*x+e)/a/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctan
h((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/c^2/f/(a-a*sec(f*x+e)
)^(1/2)/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*(c-3*d)*arctanh(1/2*(a-a*sec(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*tan(f*x+e)*2^(1/2)/a^(1/2)/(c-d)^2/f/(a-a*sec(f*x+e))^(1/2)/(a+a*s
ec(f*x+e))^(1/2)-d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c
+d)^(1/2))*tan(f*x+e)/a^(1/2)/c/(c-d)^2/(c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)-2*(3*c-d)*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e
))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(1/2)/c^2/(c-d)^3/(c+d)^(1/2)/f
/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-d^3*tan(f*x+e)/a/c/(c-d)^2/
(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 11.41 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \frac{\left((-c-d)^{3/2}\left(-c^2(5c-13d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{} + \frac{\cos^3\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))^2\sec^4(e+fx)\left(-\frac{2(c^3+c^2d+2d^3)\sin\left(\frac{1}{2}(e+fx)\right)}{c^2(-c+d)^2(c+d)}+\frac{4d^4\sin\left(\frac{1}{2}(e+fx)\right)}{c^2(-c+d)^2(c+d)(d+c\cos(e+fx))}\right.}{f(a(1+\sec(e+fx)))^{3/2}(c+d\sec(e+fx))^2}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
(((-c - d)^(3/2)*(-(c^2*(5*c - 13*d)*ArcSin[Tan[(e + f*x)/2]]) + 4*Sqrt[2]
*(c - d)^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])
 + 2*Sqrt[2]*d^(5/2)*(-7*c^2 - 3*c*d + 2*d^2)*ArcTanh[(Sqrt[d]*Tan[(e + f*
x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])*(d + c*Cos[e
 + f*x])^2*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(7/2)*Sqrt[C
os[(e + f*x)/2]^2*Sec[e + f*x]])/(c^2*(-c - d)^(3/2)*(c - d)^3*f*(Sec[(e +
 f*x)/2]^2)^(3/2)*(a*(1 + Sec[e + f*x]))^(3/2)*(c + d*Sec[e + f*x])^2) + (
Cos[(e + f*x)/2]^3*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^4*((-2*(c^3 + c^2*d
 + 2*d^3)*Sin[(e + f*x)/2])/(c^2*(-c + d)^2*(c + d)) + (4*d^4*Sin[(e + f*x
)/2])/(c^2*(-c + d)^2*(c + d)*(d + c*Cos[e + f*x])) + (Sec[(e + f*x)/2]*Ta
n[(e + f*x)/2])/(-c + d)^2))/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(c + d*Sec[e
+ f*x])^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{\tan(e+fx)\int \left(-\frac{(3c-d)d^3}{c^2(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}-\frac{d^3}{c(c-d)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}+\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+f}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\tan(e+fx)\left(\frac{2d^{5/2}(3c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^3\sqrt{c+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)^2(c+d)^{3/2}}-\right.}{f\sqrt{a-a\sec(e+f}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + (Sqrt[2
]*(c - 3*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*
(c - d)^3) + ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2
]*Sqrt[a]*(c - d)^2) + (d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])
/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*(c - d)^2*(c + d)^(3/2)) + (2*(3*c - d
)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])
])/(Sqrt[a]*c^2*(c - d)^3*Sqrt[c + d]) + Sqrt[a - a*Sec[e + f*x]]/(2*a*(c
- d)^2*(1 + Sec[e + f*x])) + (d^3*Sqrt[a - a*Sec[e + f*x]])/(a*c*(c - d)^2
*(c + d)*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 14150 vs. 2(480) = 960.

Time = 7.34 (sec) , antiderivative size = 14151, normalized size of antiderivative = 25.27

method	result	size
default	Expression too large to display	14151

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx=\int \frac{1}{(a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^2}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(}}{\sec(fx+e)^4d^2+2\sec(fx+e)^3cd+2\sec(fx+e)^3d^2+\sec(fx+e)^2c^2+}\right.}{a^2}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4*d**2 + 2*sec(e + f*x)
**3*c*d + 2*sec(e + f*x)**3*d**2 + sec(e + f*x)**2*c**2 + 4*sec(e + f*x)**
2*c*d + sec(e + f*x)**2*d**2 + 2*sec(e + f*x)*c**2 + 2*sec(e + f*x)*c*d +
c**2),x))/a**2
```

3.177 $\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 802

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = -\frac{\tan(e+fx)}{2a(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{\sqrt{a}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}\sqrt{a}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4\sqrt{a}c(c-d)^2(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(3c-d)d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2d^{5/2}(6c^2-4cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d^3\tan(e+fx)}{2ac(c-d)^2(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} - \frac{(3c-d)d^3\tan(e+fx)}{ac^2(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} - \frac{3d^3\tan(e+fx)}{4ac\left(c^2-d^2\right)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-1/2*tan(f*x+e)/a/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctan
h((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/c^3/f/(a-a*sec(f*x+e)
)^(1/2)/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*(c-4*d)*arctanh(1/2*(a-a*sec(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(1/2)/(c-d)^4/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*tan(f*x+e)*2^(1/2)/a^(1/2)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*s
ec(f*x+e))^(1/2)-3/4*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2
)/(c+d)^(1/2))*tan(f*x+e)/a^(1/2)/c/(c-d)^2/(c+d)^(5/2)/f/(a-a*sec(f*x+e))
^(1/2)/(a+a*sec(f*x+e))^(1/2)-(3*c-d)*d^(5/2)*arctanh(d^(1/2)*(a-a*sec(f*x
+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(1/2)/c^2/(c-d)^3/(c+d)^(3/2)
/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-2*d^(5/2)*(6*c^2-4*c*d+d^
2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/
a^(1/2)/c^3/(c-d)^4/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^
(1/2)-1/2*d^3*tan(f*x+e)/a/c/(c-d)^2/(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*s
ec(f*x+e))^2-(3*c-d)*d^3*tan(f*x+e)/a/c^2/(c-d)^3/(c+d)/f/(a+a*sec(f*x+e))
^(1/2)/(c+d*sec(f*x+e))-3/4*d^3*tan(f*x+e)/a/c/(c^2-d^2)^2/f/(a+a*sec(f*x+
e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2632 vs. $2(802) = 1604$.

Time = 18.15 (sec) , antiderivative size = 2632, normalized size of antiderivative = 3.28

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
(Cos[(e + f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^5*(-(((-2*c^5 - 4*
c^4*d - 2*c^3*d^2 - 17*c^2*d^3 - 5*c*d^4 + 6*d^5)*Sin[(e + f*x)/2])/(c^3*(
-c + d)^3*(c + d)^2)) - (2*d^5*Sin[(e + f*x)/2])/(c^3*(-c + d)^2*(c + d)*(
d + c*Cos[e + f*x])^2) + (-19*c^2*d^4*Sin[(e + f*x)/2] - 5*c*d^5*Sin[(e +
f*x)/2] + 8*d^6*Sin[(e + f*x)/2])/(c^3*(-c + d)^3*(c + d)^2*(d + c*Cos[e +
 f*x])) - (Sec[(e + f*x)/2]*Tan[(e + f*x)/2])/(-c + d)^3))/(f*(a*(1 + Sec[
e + f*x]))^(3/2)*(c + d*Sec[e + f*x])^3) - ((2*c^3*(5*c - 17*d)*(c + d)^2*
ArcSin[Tan[(e + f*x)/2]] - 8*Sqrt[2]*(c - d)^4*(c + d)^2*ArcTan[Tan[(e + f
*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]] - (Sqrt[2]*d^(5/2)*(63*c^4 +
 54*c^3*d - 17*c^2*d^2 - 12*c*d^3 + 8*d^4)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/
2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])/Sqrt[-c - d])*C
os[(e + f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2
]^2]*((c^3*Sec[(e + f*x)/2])/(2*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])*
Sqrt[Sec[e + f*x]]) - (c^2*d*Sec[(e + f*x)/2])/((-c + d)^3*(c + d)^2*(d +
c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (19*c*d^2*Sec[(e + f*x)/2])/(2*(-c +
 d)^3*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (33*d^3*Sec[(e
+ f*x)/2])/(4*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]
) - (3*d^4*Sec[(e + f*x)/2])/(4*c*(-c + d)^3*(c + d)^2*(d + c*Cos[e + f*x]
)*Sqrt[Sec[e + f*x]]) + (d^5*Sec[(e + f*x)/2])/(c^2*(-c + d)^3*(c + d)^2*(
d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (c^3*Sec[(e + f*x)/2]*Sqrt[Se...
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{3/2}(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^2(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \textbf{198}$$

$$-\frac{\tan(e+fx)\int \left(-\frac{(6c^2-4dc+d^2)d^3}{c^3(c-d)^4\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}-\frac{(3c-d)d^3}{c^2(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}-\frac{d}{c(c-d)^2\sqrt{a-a\sec(e+}\right.}{f\sqrt{a-a\sec(e}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\tan(e+fx)\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}+\frac{d^{5/2}(3c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^3(c+d)^{3/2}}+\frac{2d^{5/2}(6c^2-4cd+d^2)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{}}{}\right.}{\sqrt{a}c^3(c-d)^4\sqrt{c+d}}\right.}{}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + (Sqrt[2
]*(c - 4*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*
(c - d)^4) + ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2
]*Sqrt[a]*(c - d)^3) + (3*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]
])/(Sqrt[a]*Sqrt[c + d])])/(4*Sqrt[a]*c*(c - d)^2*(c + d)^(5/2)) + ((3*c -
 d)*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d
])])/(Sqrt[a]*c^2*(c - d)^3*(c + d)^(3/2)) + (2*d^(5/2)*(6*c^2 - 4*c*d + d
^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sq
rt[a]*c^3*(c - d)^4*Sqrt[c + d]) + Sqrt[a - a*Sec[e + f*x]]/(2*a*(c - d)^3
*(1 + Sec[e + f*x])) + (d^3*Sqrt[a - a*Sec[e + f*x]])/(2*a*c*(c - d)^2*(c
+ d)*(c + d*Sec[e + f*x])^2) + ((3*c - d)*d^3*Sqrt[a - a*Sec[e + f*x]])/(a
*c^2*(c - d)^3*(c + d)*(c + d*Sec[e + f*x])) + (3*d^3*Sqrt[a - a*Sec[e + f
*x]])/(4*a*c*(c^2 - d^2)^2*(c + d*Sec[e + f*x])))*Tan[e + f*x])/(f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 27298 vs. 2(694) = 1388.

Time = 11.32 (sec) , antiderivative size = 27299, normalized size of antiderivative = 34.04

method	result	size
default	Expression too large to display	27299

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{3}{2}}(c+d\sec(e+fx))^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(3/2)*(c + d*sec(e + f*x))**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \text{Hanged}$$

input
```
int(1/((a + a/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x))^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{3/2}(c+d\sec(e+fx))^3}\,dx = \frac{\sqrt{a}\left(\int \frac{}{\sec(fx+e)^5 d^3+3\sec(fx+e)^4 c\,d^2+2\sec(fx+e)^4 d^3+3\sec(fx+e)^3 c}\right.}{}$$

input
```
int(1/(a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**5*d**3 + 3*sec(e + f*x)
**4*c*d**2 + 2*sec(e + f*x)**4*d**3 + 3*sec(e + f*x)**3*c**2*d + 6*sec(e +
 f*x)**3*c*d**2 + sec(e + f*x)**3*d**3 + sec(e + f*x)**2*c**3 + 6*sec(e +
f*x)**2*c**2*d + 3*sec(e + f*x)**2*c*d**2 + 2*sec(e + f*x)*c**3 + 3*sec(e
+ f*x)*c**2*d + c**3),x))/a**2
```

3.178 $\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 480

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = &-\frac{(c-d)^3\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}\\
&-\frac{3(c-d)^3\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}\\
&-\frac{(c-d)^2(c+2d)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}\\
&+\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{3(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{(c-d)^2(c+2d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&-\frac{\sqrt{2}(c^3-d^3)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
-1/4*(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16
*(c-d)^3*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c-d)^
2*(c+2*d)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^3*arc
tanh((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))
^(1/2)/(a+a*sec(f*x+e))^(1/2)-3/32*(c-d)^3*arctanh(1/2*(a-a*sec(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(
a+a*sec(f*x+e))^(1/2)-1/4*(c-d)^2*(c+2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a
+a*sec(f*x+e))^(1/2)-2^(1/2)*(c^3-d^3)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+
e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.91

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\left(\left(-43c^3+9c^2d+15cd^2+19d^3\right)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+32\sqrt{2}c^3\,\text{arctan}\cdots\right.}{4f(d+c\cos(e+fx))^3\sqrt{\sec^2\left(\frac{1}{2}(e\cdots\right.}}$$
$$+\frac{\cos^5\left(\frac{1}{2}(e+fx)\right)(c+d\sec(e+fx))^3\left(-\frac{3}{2}(-c+d)^2(5c+3d)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{1}{2}\sec^4\left(\frac{1}{2}(e+fx)\right)\left(-c^3\,\text{s}\cdots\right.\right.}{\cdots}$$

input
```
Integrate[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(((-43*c^3 + 9*c^2*d + 15*c*d^2 + 19*d^3)*ArcSin[Tan[(e + f*x)/2]] + 32*Sq
rt[2]*c^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*
Cos[(e + f*x)/2]^4*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[1 + Sec[e +
f*x]]*(c + d*Sec[e + f*x])^3)/(4*f*(d + c*Cos[e + f*x])^3*Sqrt[Sec[(e + f*
x)/2]^2]*Sqrt[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)) + (Cos[(e + f*x)
/2]^5*(c + d*Sec[e + f*x])^3*((-3*(-c + d)^2*(5*c + 3*d)*Sin[(e + f*x)/2])
/2 + (Sec[(e + f*x)/2]^4*(-(c^3*Sin[(e + f*x)/2]) + 3*c^2*d*Sin[(e + f*x)/
2] - 3*c*d^2*Sin[(e + f*x)/2] + d^3*Sin[(e + f*x)/2]))/2 + (Sec[(e + f*x)/
2]^2*(19*c^3*Sin[(e + f*x)/2] - 33*c^2*d*Sin[(e + f*x)/2] + 9*c*d^2*Sin[(e
 + f*x)/2] + 5*d^3*Sin[(e + f*x)/2]))/4))/(f*(d + c*Cos[e + f*x])^3*(a*(1
+ Sec[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^3}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^3}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{\cos(e+fx)c^3}{\sqrt{a-a\sec(e+fx)}}+\frac{d^3-c^3}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2(c+2d)}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^3}{(\sec(e+fx)+1)^3\sqrt{a-a}}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\tan(e+fx)\left(\frac{\sqrt{2}(c^3-d^3)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2c^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{3(c-d)^3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\sqrt{a}}+\right.}{af\sqrt{a-a\sec(e+f}}$$

input
```
Int[(c + d*Sec[e + f*x])^3/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((((-2*c^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (3*(c - d
)^3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*Sqrt[
a]) + ((c - d)^2*(c + 2*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[
a])])/(2*Sqrt[2]*Sqrt[a]) + (Sqrt[2]*(c^3 - d^3)*ArcTanh[Sqrt[a - a*Sec[e
+ f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] + ((c - d)^3*Sqrt[a - a*Sec[e + f*x]])
/(4*a*(1 + Sec[e + f*x])^2) + (3*(c - d)^3*Sqrt[a - a*Sec[e + f*x]])/(16*a
*(1 + Sec[e + f*x])) + ((c - d)^2*(c + 2*d)*Sqrt[a - a*Sec[e + f*x]])/(2*a
*(1 + Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 2.48 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.48

method	result
parts	$\frac{c^3\sqrt{a(1+\sec(fx+e))}\,\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(1-\cos(fx+e))^3\csc(fx+e)^3+32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(f}{\sqrt{(1-\cos(fx+e))^2\csc(fx}}\right.\right.}{32 f\, a^3}$

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*c^3/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(-2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+
32*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f
*x+e)+csc(f*x+e)))+13*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e))-43*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)
))+1/32*d^3/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^
3+11*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+19*ln((
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))-3/32*c*d^2/f/a
^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(4*2^(1/2
)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*cot(f*x
+e)+5*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-5*ln((
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))+3/32*c^2*d/f/a
^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(4*2^(1/2
)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)*cot(f*x
+e)-3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+3*ln((
-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 33.05 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.83

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(sqrt(2)*((43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43
*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d
^3)*cos(f*x + e)^2 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)
)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(c
os(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(
f*x + e)^2 + 3*c^3*cos(f*x + e) + c^3)*sqrt(-a)*log((2*a*cos(f*x + e)^2 +
2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*(3*(5*c^3 - 7*c^2*d - c*d
^2 + 3*d^3)*cos(f*x + e)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f
*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(
sqrt(2)*((43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e)^3 + 43*c^3 -
9*c^2*d - 15*c*d^2 - 19*d^3 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos
(f*x + e)^2 + 3*(43*c^3 - 9*c^2*d - 15*c*d^2 - 19*d^3)*cos(f*x + e))*sqrt(
a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sq
rt(a)*sin(f*x + e))) - 64*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c
^3*cos(f*x + e) + c^3)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*(3*(5*c^3 - 7*c^2*d - c*d^2 +
 3*d^3)*cos(f*x + e)^2 + (11*c^3 - 9*c^2*d - 15*c*d^2 + 13*d^3)*cos(f*x...
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(c+d\sec{(e+fx)})^3}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**3/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^3}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))^3/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)c^3+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(f}{\sec(fx+e)^3+3\sec(fx+e)^2+3}\right.\right.}{}$$

input
```
int((c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2
+ 3*sec(e + f*x) + 1),x)*c**3 + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**
3)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)*d**3 + 3*
int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**2)/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1),x)*c*d**2 + 3*int((sqrt(sec(e + f*x) + 1)*se
c(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)*
c**2*d))/a**3
```

3.179 $\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 468

$$\begin{aligned}\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx &= -\frac{(c-d)^2\tan(e+fx)}{4a^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\ &- \frac{3(c-d)^2\tan(e+fx)}{16a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c^2-d^2)\tan(e+fx)}{2a^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\sqrt{2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{3(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c^2-d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
-1/4*(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-3/16
*(c-d)^2*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-1/2*(c^2-d
^2)*tan(f*x+e)/a^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*c^2*arctanh((
a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)
/(a+a*sec(f*x+e))^(1/2)-2^(1/2)*c^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))
^(1/2)-3/32*(c-d)^2*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*ta
n(f*x+e)*2^(1/2)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)-1
/4*(c^2-d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e
)*2^(1/2)/a^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 5.43 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.56

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}(c+d\sec(e+fx))^2\left(\frac{\left(\left(-43c^2+6cd+5d^2\right)\arcsin(\mathrm{t}\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(Cos[(e + f*x)/2]^4*Sqrt[Sec[e + f*x]]*(c + d*Sec[e + f*x])^2*((((-43*c^2
+ 6*c*d + 5*d^2)*ArcSin[Tan[(e + f*x)/2]] + 32*Sqrt[2]*c^2*ArcTan[Tan[(e +
 f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Sqrt[Cos[e + f*x]/(1 + Co
s[e + f*x])]*Sqrt[1 + Sec[e + f*x]])/Sqrt[Sec[(e + f*x)/2]^2] + ((c - d)*(
11*c + 5*d + (15*c + d)*Cos[e + f*x])*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]
]*(Sin[(e + f*x)/2] - Sin[(3*(e + f*x))/2]))/4))/(4*f*(d + c*Cos[e + f*x])
^2*(a*(1 + Sec[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^2}{(a\sec(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)(c+d\sec(e+fx))^2}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int \left(\frac{\cos(e+fx)c^2}{\sqrt{a-a\sec(e+fx)}}-\frac{c^2}{(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}+\frac{d^2-c^2}{(\sec(e+fx)+1)^2\sqrt{a-a\sec(e+fx)}}-\frac{(c-d)^2}{(\sec(e+fx)+1)^3\sqrt{a-a}}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\tan(e+fx)\left(\frac{(c^2-d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\sqrt{a}}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}c^2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}+\frac{3(c-}{}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\,\text{s}}}$$

input
```
Int[(c + d*Sec[e + f*x])^2/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-((((-2*c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[2]*
c^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] + (3*(c -
 d)^2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*Sqr
t[a]) + ((c^2 - d^2)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/
(2*Sqrt[2]*Sqrt[a]) + ((c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(4*a*(1 + Sec[e
 + f*x])^2) + (3*(c - d)^2*Sqrt[a - a*Sec[e + f*x]])/(16*a*(1 + Sec[e + f*
x])) + ((c^2 - d^2)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(1 + Sec[e + f*x])))*Ta
n[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 1.49 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.17

method	result
parts	$\frac{c^2\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(1-\cos(fx+e))^3\csc(fx+e)^3+32\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,(-\cot(fx+e)+\csc(f}{\sqrt{(1-\cos(fx+e))^2\csc(fx}}\right.\right.}{32f\,a^3}$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*c^2/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(-2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+
32*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f
*x+e)+csc(f*x+e)))+13*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e))-43*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)
))-1/32*d^2/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*
x+e)+1)*cot(f*x+e)+5*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc
(f*x+e))-5*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
+1/16*c*d/f/a^3*(a*(1+sec(f*x+e)))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*(4*2^(1/2)*(cos(f*x+e)-1)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(cos(f*x+
e)+1)*cot(f*x+e)-3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cot(f*x+e)+csc(f
*x+e))+3*ln((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cot(f*x+e)+csc(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.67

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(sqrt(2)*((43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*
d - 5*d^2)*cos(f*x + e)^2 + 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5
*d^2)*cos(f*x + e))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3*a*cos(f*x + e)^2 + 2*a*co
s(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 64*(c^2*cos(f*x +
 e)^3 + 3*c^2*cos(f*x + e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(-a)*log((2*a
*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1)) - 4*((15*c^
2 - 14*c*d - d^2)*cos(f*x + e)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^
3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*
((43*c^2 - 6*c*d - 5*d^2)*cos(f*x + e)^3 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(
f*x + e)^2 + 43*c^2 - 6*c*d - 5*d^2 + 3*(43*c^2 - 6*c*d - 5*d^2)*cos(f*x +
 e))*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*
x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c^2*cos(f*x + e)^3 + 3*c^2*cos(f*x +
e)^2 + 3*c^2*cos(f*x + e) + c^2)*sqrt(a)*arctan(sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c^2 - 14*c*d -
 d^2)*cos(f*x + e)^2 + (11*c^2 - 6*c*d - 5*d^2)*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*
cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(c+d\sec{(e+fx)})^2}{(a(\sec{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**2/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e)+c)^2}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)^2/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^2}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))^2/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)c^2+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(f}{\sec(fx+e)^3+3\sec(fx+e)^2+3}\right.\right.}{a^3}$$

input
```
int((c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2
+ 3*sec(e + f*x) + 1),x)*c**2 + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x)**
2)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)*d**2 + 2*
int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(sec(e + f*x)**3 + 3*sec(e + f*x
)**2 + 3*sec(e + f*x) + 1),x)*c*d))/a**3
```

3.180 $\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{2c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{a^{5/2}f}-\frac{(43c-3d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(c-d)\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}-\frac{(11c-3d)\tan(e+fx)}{16af(a+a\sec(e+fx))^{3/2}}$$

output
```
2*c*arctan(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/a^(5/2)/f-1/32*(43*c
-3*d)*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2
)/a^(5/2)/f-1/4*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-1/16*(11*c-3*d)*
tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(164) = 328$.

Time = 7.27 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.09

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\left((-43c+3d)\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+32\sqrt{2}c\arctan\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)\text{co}}{4f(d+c\cos(e+fx))\sqrt{\sec^2\left(\frac{1}{2}\right.}}$$
$$+\frac{\cos^5\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)(c+d\sec(e+fx))\left(\frac{1}{2}(-15c+7d)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{1}{4}\sec^2\left(\frac{1}{2}(e+fx)\right)(19\right.}{f(d+c\cos(e+fx))(}$$

input
```
Integrate[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
(((-43*c + 3*d)*ArcSin[Tan[(e + f*x)/2]] + 32*Sqrt[2]*c*ArcTan[Tan[(e + f*
x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])*Cos[(e + f*x)/2]^4*Sqrt[Cos[
e + f*x]/(1 + Cos[e + f*x])]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e + f*x]]*(c
+ d*Sec[e + f*x]))/(4*f*(d + c*Cos[e + f*x])*Sqrt[Sec[(e + f*x)/2]^2]*(a*(
1 + Sec[e + f*x]))^(5/2)) + (Cos[(e + f*x)/2]^5*Sec[e + f*x]^2*(c + d*Sec[
e + f*x])*(((-15*c + 7*d)*Sin[(e + f*x)/2])/2 + (Sec[(e + f*x)/2]^2*(19*c*
Sin[(e + f*x)/2] - 11*d*Sin[(e + f*x)/2]))/4 + (Sec[(e + f*x)/2]^4*(-(c*Si
n[(e + f*x)/2]) + d*Sin[(e + f*x)/2]))/2))/(f*(d + c*Cos[e + f*x])*(a*(1 +
 Sec[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4410, 27, 3042, 4410, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a\sec(e+fx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4410}$$

$$-\frac{\int -\frac{8ac-3a(c-d)\sec(e+fx)}{2(\sec(e+fx)a+a)^{3/2}}dx}{4a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{8ac-3a(c-d)\sec(e+fx)}{(\sec(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{8ac-3a(c-d)\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{4410}$$

$$\frac{-\frac{\int -\frac{32a^2c-a^2(11c-3d)\sec(e+fx)}{2\sqrt{\sec(e+fx)a+a}}dx}{2a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{32a^2c-a^2(11c-3d)\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{32a^2c-a^2(11c-3d)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{4408}$$

$$\frac{\frac{32ac\int \sqrt{\sec(e+fx)a+a}dx-a^2(43c-3d)\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{32ac\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx-a^2(43c-3d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{\frac{-\left(a^2(43c-3d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx\right)-\frac{64a^2c\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{64a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-a^2(43c-3d)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{\frac{2a^2(43c-3d)\int \frac{1}{\frac{a^2\tan^2(e+fx)}{\sec(e+fx)a+a}+2a}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}+\frac{64a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{64a^{3/2}c\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}-\frac{\sqrt{2}a^{3/2}(43c-3d)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{f}}{4a^2}-\frac{a(11c-3d)\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}}}{8a^2}-\frac{(c-d)\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}}$$

input
```
Int[(c + d*Sec[e + f*x])/(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
-1/4*((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])^(5/2)) + (((64*a^(3/2)
*c*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f - (Sqrt[2]*a
^(3/2)*(43*c - 3*d)*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]])])/f)/(4*a^2) - (a*(11*c - 3*d)*Tan[e + f*x])/(2*f*(a + a*Sec[e
+ f*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(139) = 278$.

Time = 1.85 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.29

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2c\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(-\cot(fx+e)+\csc(\right.}{}$
parts	$\frac{c\sqrt{a(1+\sec(fx+e))}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(-2\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}(1-\cos(fx+e))^3\csc(fx+e)^3+32\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(fx+e)+\csc(fx}{\sqrt{(1-\cos(fx+e))^2\csc(fx+}}\right.\right.}{32fa^3}$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/f/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)^(1/2)*(2*c*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-co
t(f*x+e)+csc(f*x+e))-2*d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(-cot(f*x
+e)+csc(f*x+e))-11*c*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+
csc(f*x+e))+3*d*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f
*x+e))-32*c*2^(1/2)*arctanh(2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2
)*(-cot(f*x+e)+csc(f*x+e)))+43*c*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2))-3*d*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(139) = 278$.

Time = 3.74 (sec) , antiderivative size = 670, normalized size of antiderivative = 4.09

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(sqrt(2)*((43*c - 3*d)*cos(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^
2 + 3*(43*c - 3*d)*cos(f*x + e) + 43*c - 3*d)*sqrt(-a)*log((2*sqrt(2)*sqrt
(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 3
*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/(cos(f*x + e)^2 + 2*cos(f*x + e)
 + 1)) - 64*(c*cos(f*x + e)^3 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)
*sqrt(-a)*log((2*a*cos(f*x + e)^2 + 2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e)
 + 1)) - 4*((15*c - 7*d)*cos(f*x + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3
*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f), 1/32*(sqrt(2)*((43*
c - 3*d)*cos(f*x + e)^3 + 3*(43*c - 3*d)*cos(f*x + e)^2 + 3*(43*c - 3*d)*c
os(f*x + e) + 43*c - 3*d)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 64*(c*cos(f*x + e)^3
 + 3*c*cos(f*x + e)^2 + 3*c*cos(f*x + e) + c)*sqrt(a)*arctan(sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e))) - 2*((15*c
 - 7*d)*cos(f*x + e)^2 + (11*c - 3*d)*cos(f*x + e))*sqrt((a*cos(f*x + e) +
 a)/cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)]
```

Sympy [F]

$$\int \frac{c + d\sec(e + fx)}{(a + a\sec(e + fx))^{5/2}}\,dx = \int \frac{c + d\sec(e + fx)}{(a(\sec(e + fx) + 1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a*(sec(e + f*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{d\sec\left(fx+e\right)+c}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(a*sec(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{c+\frac{d}{\cos(e+f\,x)}}{{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + a/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{c+d\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^3+3\sec(fx+e)^2+3\sec(fx+e)+1}dx\right)c+\left(\int \frac{\sqrt{\sec(fx+e)+1}\,\sec(fx}{\sec(fx+e)^3+3\sec(fx+e)^2+3\,\mathrm{s}}\right.\right.\right.}{a^3}$$

input
```
int((c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2
+ 3*sec(e + f*x) + 1),x)*c + int((sqrt(sec(e + f*x) + 1)*sec(e + f*x))/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1),x)*d))/a**3
```

3.181 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 592

$$\begin{aligned}\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx &= -\frac{\tan(e+fx)}{4a^2(c-d)f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c-2d)\tan(e+fx)}{2a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &- \frac{3\tan(e+fx)}{16a^2(c-d)f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}cf\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{(c-2d)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &- \frac{\sqrt{2}(c^2-3cd+3d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\ &+ \frac{2d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

output
```
-1/4*tan(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*(c
-2*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/16*
tan(f*x+e)/a^2/(c-d)/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh((a-
a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c/f/(a-a*sec(f*x+e))^(1/2)
/(a+a*sec(f*x+e))^(1/2)-1/4*(c-2*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/(c-d)^2/f/(a-a*sec(f*x+e))^(1/2)/
(a+a*sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/(c-d)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f
*x+e))^(1/2)-2^(1/2)*(c^2-3*c*d+3*d^2)*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a+a*
sec(f*x+e))^(1/2)+2*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)
/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.23 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^{\frac{7}{2}}(e+fx)\left(-\frac{4(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

output
```
(Cos[(e + f*x)/2]^4*(d + c*Cos[e + f*x])*Sec[e + f*x]^(7/2)*((-4*(Sqrt[-c
- d]*(c*(43*c^2 - 126*c*d + 115*d^2)*ArcSin[Tan[(e + f*x)/2]] - 32*Sqrt[2]
*(c - d)^3*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])
 + 32*Sqrt[2]*d^(7/2)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqr
t[Cos[e + f*x]/(1 + Cos[e + f*x])])])*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])
]*Sqrt[1 + Sec[e + f*x]])/(c*Sqrt[-c - d]*Sqrt[Sec[(e + f*x)/2]^2]) + (c -
 d)*(11*c - 19*d + (15*c - 23*d)*Cos[e + f*x])*Sec[(e + f*x)/2]^3*Sqrt[Sec
[e + f*x]]*(Sin[(e + f*x)/2] - Sin[(3*(e + f*x))/2])))/(16*(c - d)^3*f*(a*
(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 4428$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \ 198$$

$$-\frac{\tan(e+fx)\int \left(\frac{d^4}{c(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{\cos(e+fx)}{c\sqrt{a-a\sec(e+fx)}}+\frac{-c^2+3dc-3d^2}{(c-d)^3(\sec(e+fx)+1)\sqrt{a-a\sec(e+fx)}}+\frac{}{(c-d)^2}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\tan(e+fx)\left(\frac{\sqrt{2}(c^2-3cd+3d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}(c-d)^3}-\frac{2d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c(c-d)^3\sqrt{c+d}}+\frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\sqrt{a}(c-d)}\right.}{af\sqrt{a-a}}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c) + ((c - 2*d
)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a]*
(c - d)^2) + (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16*S
qrt[2]*Sqrt[a]*(c - d)) + (Sqrt[2]*(c^2 - 3*c*d + 3*d^2)*ArcTanh[Sqrt[a -
a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^3) - (2*d^(7/2)*ArcTa
nh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c*(
c - d)^3*Sqrt[c + d]) + Sqrt[a - a*Sec[e + f*x]]/(4*a*(c - d)*(1 + Sec[e +
 f*x])^2) + ((c - 2*d)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(c - d)^2*(1 + Sec[e
 + f*x])) + (3*Sqrt[a - a*Sec[e + f*x]])/(16*a*(c - d)*(1 + Sec[e + f*x]))
)*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3509 vs. $2(509) = 1018$.

Time = 8.82 (sec) , antiderivative size = 3510, normalized size of antiderivative = 5.93

method	result	size
default	Expression too large to display	3510

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*(1/8/f*(c+d)^2/(2^(1/2)*(c*(c+d))^(1/2)-c-d)^3/(2^(1/2)*(c*(c+
d))^(1/2)+c+d)^3/(-d/(c+d))^(1/2)*(32*2^(1/2)*cos(1/2*f*x+1/2*e)^4*ln(-(2^
(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^
2)^(1/2)*c*cos(1/2*f*x+1/2*e)+2^(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2
*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(1/2*f*x+1/2*e)*d+2^(1/2)*c*(-
d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)
+2^(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+
1)^2)^(1/2)*d+2^(1/2)*(c*(c+d))^(1/2)-2*c*sin(1/2*f*x+1/2*e)-2*sin(1/2*f*x
+1/2*e)*d)/(2^(1/2)*(c*(c+d))^(1/2)*sin(1/2*f*x+1/2*e)-c-d))*d^3+32*2^(1/2
)*cos(1/2*f*x+1/2*e)^4*ln((2^(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)
^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*c*cos(1/2*f*x+1/2*e)+2^(1/2)*(-d/(c+
d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*cos(
1/2*f*x+1/2*e)*d+2^(1/2)*c*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2*e)^2-1)/(c
os(1/2*f*x+1/2*e)+1)^2)^(1/2)+2^(1/2)*(-d/(c+d))^(1/2)*((2*cos(1/2*f*x+1/2
*e)^2-1)/(cos(1/2*f*x+1/2*e)+1)^2)^(1/2)*d-2^(1/2)*(c*(c+d))^(1/2)-2*c*sin
(1/2*f*x+1/2*e)-2*sin(1/2*f*x+1/2*e)*d)/(2^(1/2)*(c*(c+d))^(1/2)*sin(1/2*f
*x+1/2*e)+c+d))*d^3-3*cos(1/2*f*x+1/2*e)^4*(-d/(c+d))^(1/2)*arctan((2*sin(
1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1/2*e)^2-1)/(cos(
1/2*f*x+1/2*e)+1)^2)^(1/2))*c^3+11*cos(1/2*f*x+1/2*e)^4*(-d/(c+d))^(1/2)*a
rctan((2*sin(1/2*f*x+1/2*e)-1)/(cos(1/2*f*x+1/2*e)+1)/((2*cos(1/2*f*x+1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}\,(c+d\sec(e+fx))}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{(a\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^4 d+\sec(fx+e)^3 c+3\sec(fx+e)^3 d+3\sec(fx+e)^2 c+3\sec(}\right.}{a^3}$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**4*d + sec(e + f*x)**3*c
 + 3*sec(e + f*x)**3*d + 3*sec(e + f*x)**2*c + 3*sec(e + f*x)**2*d + 3*sec
(e + f*x)*c + sec(e + f*x)*d + c),x))/a**3
```

3.182 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 756

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx =$$
$$-\frac{\tan(e+fx)}{4a^2(c-d)^2f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(c-3d)\tan(e+fx)}{2a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{3\tan(e+fx)}{16a^2(c-d)^2f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{(c-3d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$-\frac{\sqrt{2}(c^2-4cd+6d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c(c-d)^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{2(4c-d)d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+\frac{d^4\tan(e+fx)}{a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-1/4*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*
(c-3*d)*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/1
6*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh
((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c^2/f/(a-a*sec(f*x+e))
^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-3*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/(c-d)^3/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/(c-d)^2/f/(a-a*sec(f*x+e))^(1/2)/(a
+a*sec(f*x+e))^(1/2)-2^(1/2)*(c^2-4*c*d+6*d^2)*arctanh(1/2*(a-a*sec(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^4/f/(a-a*sec(f*x+e))^(1/
2)/(a+a*sec(f*x+e))^(1/2)+d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a
^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/(c+d)^(3/2)/f/(a-a*sec(f*
x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*(4*c-d)*d^(7/2)*arctanh(d^(1/2)*(a-a*
sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^2/(c-d)^4/(c+d
)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+d^4*tan(f*x+e)/a^2
/c/(c-d)^3/(c+d)/f/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 12.27 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))^2\sec^{\frac{9}{2}}(e+fx)\left(-\right.}{}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
(Cos[(e + f*x)/2]^4*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^(9/2)*(-(((c^2*(43
*c^3 - 123*c^2*d + 53*c*d^2 + 219*d^3)*ArcSin[Tan[(e + f*x)/2]] - 32*Sqrt[
2]*(c - d)^4*(c + d)*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f*x]/(1 + Cos[e
+ f*x])]] + (16*Sqrt[2]*d^(7/2)*(9*c^2 + 5*c*d - 2*d^2)*ArcTanh[(Sqrt[d]*T
an[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]])]/Sq
rt[-c - d])*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*
Sec[e + f*x]])/(c^2*(c - d)^4*(c + d)*Sqrt[Sec[(e + f*x)/2]^2])) + (Cos[(e
 + f*x)/2]*Sqrt[Sec[e + f*x]]*((2*(15*c^2 - 16*c*d - 31*d^2 - (16*d^4*Cos[
e + f*x])/(c*(d + c*Cos[e + f*x])))*Sin[(e + f*x)/2])/(c + d) + 32*(c - d)
*Csc[e + f*x]^4*Sin[(e + f*x)/2]^5 + (-19*c + 35*d)*Sec[(e + f*x)/2]*Tan[(
e + f*x)/2]))/(-c + d)^3))/(4*f*(a*(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e
+ f*x])^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{198}$$

$$-\frac{\tan(e+fx)\int\left(\frac{(4c-d)d^4}{c^2(c-d)^4\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{d^4}{c(c-d)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{\cos(e+fx)}{c^2\sqrt{a-a\sec(e+fx)}}+\frac{}{(}\right.}{af\sqrt{a-a\sec(e+}}$$

$\downarrow$ 2009

$$-\frac{\tan(e+fx)\left(-\frac{2d^{7/2}(4c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^4\sqrt{c+d}}+\frac{\sqrt{2}(c^2-4cd+6d^2)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}(c-d)^4}-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a}}{}\right.}{\sqrt{a}c^2}\right.}{}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^2),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^2) + ((c - 3
*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a
]*(c - d)^3) + (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16
*Sqrt[2]*Sqrt[a]*(c - d)^2) + (Sqrt[2]*(c^2 - 4*c*d + 6*d^2)*ArcTanh[Sqrt[
a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^4) - (d^(7/2)*Arc
Tanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c
*(c - d)^3*(c + d)^(3/2)) - (2*(4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[a -
 a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c - d)^4*Sqrt[c +
d]) + Sqrt[a - a*Sec[e + f*x]]/(4*a*(c - d)^2*(1 + Sec[e + f*x])^2) + ((c
- 3*d)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(c - d)^3*(1 + Sec[e + f*x])) + (3*S
qrt[a - a*Sec[e + f*x]])/(16*a*(c - d)^2*(1 + Sec[e + f*x])) - (d^4*Sqrt[a
 - a*Sec[e + f*x]])/(a*c*(c - d)^3*(c + d)*(c + d*Sec[e + f*x])))*Tan[e +
f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 16552 vs. $2(653) = 1306$.

Time = 7.82 (sec) , antiderivative size = 16553, normalized size of antiderivative = 21.90

method	result	size
default	Expression too large to display	16553

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))^2}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="maxima
")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^2}\,dx = \frac{\sqrt{a}\left(\int \frac{}{\sec(fx+e)^5d^2+2\sec(fx+e)^4cd+3\sec(fx+e)^4d^2+\sec(fx+e)^3c^2+$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**5*d**2 + 2*sec(e + f*x)
**4*c*d + 3*sec(e + f*x)**4*d**2 + sec(e + f*x)**3*c**2 + 6*sec(e + f*x)**
3*c*d + 3*sec(e + f*x)**3*d**2 + 3*sec(e + f*x)**2*c**2 + 6*sec(e + f*x)**
2*c*d + sec(e + f*x)**2*d**2 + 3*sec(e + f*x)*c**2 + 2*sec(e + f*x)*c*d +
c**2),x))/a**3
```

3.183 $\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 999

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = -\frac{\tan(e+fx)}{4a^2(c-d)^3f(1+\sec(e+fx))^2\sqrt{a+a\sec(e+fx)}} - \frac{(c-4d)\tan(e+fx)}{2a^2(c-d)^4f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} - \frac{3\tan(e+fx)}{16a^2(c-d)^3f(1+\sec(e+fx))\sqrt{a+a\sec(e+fx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}c^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-4d)\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{2\sqrt{2}a^{3/2}(c-d)^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{3\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{16\sqrt{2}a^{3/2}(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\sqrt{2}(c^2-5cd+10d^2)\,\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\tan(e+fx)}{a^{3/2}(c-d)^5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{3d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{4a^{3/2}c(c-d)^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(4c-d)d^{7/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^2(c-d)^4(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2d^{7/2}(10c^2-5cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)\tan(e+fx)}{a^{3/2}c^3(c-d)^5\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d^4\tan(e+fx)}{2a^2c(c-d)^3(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))^2} + \frac{3d^4\tan(e+fx)}{4a^2c(c-d)^3(c+d)^2f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))} + \frac{(4c-d)d^4\tan(e+fx)}{a^2c^2(c-d)^4(c+d)f\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}$$

output
```
-1/4*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))^2/(a+a*sec(f*x+e))^(1/2)-1/2*
(c-4*d)*tan(f*x+e)/a^2/(c-d)^4/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)-3/1
6*tan(f*x+e)/a^2/(c-d)^3/f/(1+sec(f*x+e))/(a+a*sec(f*x+e))^(1/2)+2*arctanh
((a-a*sec(f*x+e))^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/c^3/f/(a-a*sec(f*x+e))
^(1/2)/(a+a*sec(f*x+e))^(1/2)-1/4*(c-4*d)*arctanh(1/2*(a-a*sec(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/(c-d)^4/f/(a-a*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2)-3/32*arctanh(1/2*(a-a*sec(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*tan(f*x+e)*2^(1/2)/a^(3/2)/(c-d)^3/f/(a-a*sec(f*x+e))^(1/2)/(a
+a*sec(f*x+e))^(1/2)-2^(1/2)*(c^2-5*c*d+10*d^2)*arctanh(1/2*(a-a*sec(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*tan(f*x+e)/a^(3/2)/(c-d)^5/f/(a-a*sec(f*x+e))^(1
/2)/(a+a*sec(f*x+e))^(1/2)+3/4*d^(7/2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1
/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c/(c-d)^3/(c+d)^(5/2)/f/(a-a*s
ec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+(4*c-d)*d^(7/2)*arctanh(d^(1/2)*(a
-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*tan(f*x+e)/a^(3/2)/c^2/(c-d)^4/(
c+d)^(3/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)+2*d^(7/2)*(10*c
^2-5*c*d+d^2)*arctanh(d^(1/2)*(a-a*sec(f*x+e))^(1/2)/a^(1/2)/(c+d)^(1/2))*
tan(f*x+e)/a^(3/2)/c^3/(c-d)^5/(c+d)^(1/2)/f/(a-a*sec(f*x+e))^(1/2)/(a+a*s
ec(f*x+e))^(1/2)+1/2*d^4*tan(f*x+e)/a^2/c/(c-d)^3/(c+d)/f/(a+a*sec(f*x+e))
^(1/2)/(c+d*sec(f*x+e))^2+3/4*d^4*tan(f*x+e)/a^2/c/(c-d)^3/(c+d)^2/f/(a+a*
sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))+(4*c-d)*d^4*tan(f*x+e)/a^2/c^2/(c-d)...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2904 vs. $2(999) = 1998$.

Time = 18.98 (sec) , antiderivative size = 2904, normalized size of antiderivative = 2.91

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
(Cos[(e + f*x)/2]^5*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*((-3*(5*c^6 - 3*
c^5*d - 21*c^4*d^2 - 13*c^3*d^3 - 28*c^2*d^4 - 12*c*d^5 + 8*d^6)*Sin[(e +
f*x)/2])/(2*c^3*(-c + d)^4*(c + d)^2) - (4*d^6*Sin[(e + f*x)/2])/(c^3*(-c
+ d)^3*(c + d)*(d + c*Cos[e + f*x])^2) + (Sec[(e + f*x)/2]^2*(19*c*Sin[(e
+ f*x)/2] - 43*d*Sin[(e + f*x)/2]))/(4*(-c + d)^4) + (2*(-23*c^2*d^5*Sin[(
e + f*x)/2] - 9*c*d^6*Sin[(e + f*x)/2] + 8*d^7*Sin[(e + f*x)/2]))/(c^3*(-c
 + d)^4*(c + d)^2*(d + c*Cos[e + f*x])) + (Sec[(e + f*x)/2]^3*Tan[(e + f*x
)/2])/(2*(-c + d)^3)))/(f*(a*(1 + Sec[e + f*x]))^(5/2)*(c + d*Sec[e + f*x]
)^3) - ((c^3*(c + d)^2*(43*c^2 - 206*c*d + 355*d^2)*ArcSin[Tan[(e + f*x)/2
]] - 32*Sqrt[2]*(c - d)^5*(c + d)^2*ArcTan[Tan[(e + f*x)/2]/Sqrt[Cos[e + f
*x]/(1 + Cos[e + f*x])]] + (4*Sqrt[2]*d^(7/2)*(99*c^4 + 110*c^3*d - 5*c^2*
d^2 - 20*c*d^3 + 8*d^4)*ArcTanh[(Sqrt[d]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*S
qrt[Cos[e + f*x]/(1 + Cos[e + f*x])])])/Sqrt[-c - d])*Cos[(e + f*x)/2]^5*(
d + c*Cos[e + f*x])^3*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*((-11*c^4*Sec[
(e + f*x)/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*
x]]) + (45*c^3*d*Sec[(e + f*x)/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e +
f*x])*Sqrt[Sec[e + f*x]]) - (5*c^2*d^2*Sec[(e + f*x)/2])/(8*(-c + d)^4*(c
+ d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (317*c*d^3*Sec[(e + f*x)
/2])/(8*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sqrt[Sec[e + f*x]]) - (6
9*d^4*Sec[(e + f*x)/2])/(2*(-c + d)^4*(c + d)^2*(d + c*Cos[e + f*x])*Sq...
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 729, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4428, 27, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(e+fx)+a)^{5/2}(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4428}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\cos(e+fx)}{a^3(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 27

$$-\frac{\tan(e+fx)\int \frac{\cos(e+fx)}{(\sec(e+fx)+1)^3\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^3}d\sec(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$\downarrow$ 198

$$-\frac{\tan(e+fx)\int \left(\frac{(10c^2-5dc+d^2)d^4}{c^3(c-d)^5\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))}+\frac{(4c-d)d^4}{c^2(c-d)^4\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^2}+\frac{d^4}{c(c-d)^3\sqrt{a-a\sec(e+fx}}\right.}{}$$

$\downarrow$ 2009

$$-\frac{\tan(e+fx)\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}c^3}-\frac{d^{7/2}(4c-d)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(e+fx)}}{\sqrt{a}\sqrt{c+d}}\right)}{\sqrt{a}c^2(c-d)^4(c+d)^{3/2}}+\frac{\sqrt{2}(c^2-5cd+10d^2)\text{arctanh}\left(\frac{\sqrt{a-a}}{\sqrt{}}\right.}{\sqrt{a}(c-d)^5}\right.}{}$$

input
```
Int[1/((a + a*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x])^3),x]
```

output
```
-((((-2*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a]])/(Sqrt[a]*c^3) + ((c - 4
*d)*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*Sqrt[a
]*(c - d)^4) + (3*ArcTanh[Sqrt[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(16
*Sqrt[2]*Sqrt[a]*(c - d)^3) + (Sqrt[2]*(c^2 - 5*c*d + 10*d^2)*ArcTanh[Sqrt
[a - a*Sec[e + f*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[a]*(c - d)^5) - (3*d^(7/2)*
ArcTanh[(Sqrt[d]*Sqrt[a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(4*Sqrt
[a]*c*(c - d)^3*(c + d)^(5/2)) - ((4*c - d)*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[
a - a*Sec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^2*(c - d)^4*(c + d
)^(3/2)) - (2*d^(7/2)*(10*c^2 - 5*c*d + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a - a*S
ec[e + f*x]])/(Sqrt[a]*Sqrt[c + d])])/(Sqrt[a]*c^3*(c - d)^5*Sqrt[c + d])
+ Sqrt[a - a*Sec[e + f*x]]/(4*a*(c - d)^3*(1 + Sec[e + f*x])^2) + ((c - 4*
d)*Sqrt[a - a*Sec[e + f*x]])/(2*a*(c - d)^4*(1 + Sec[e + f*x])) + (3*Sqrt[
a - a*Sec[e + f*x]])/(16*a*(c - d)^3*(1 + Sec[e + f*x])) - (d^4*Sqrt[a - a
*Sec[e + f*x]])/(2*a*c*(c - d)^3*(c + d)*(c + d*Sec[e + f*x])^2) - (3*d^4*
Sqrt[a - a*Sec[e + f*x]])/(4*a*c*(c - d)^3*(c + d)^2*(c + d*Sec[e + f*x]))
 - ((4*c - d)*d^4*Sqrt[a - a*Sec[e + f*x]])/(a*c^2*(c - d)^4*(c + d)*(c +
d*Sec[e + f*x])))*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4428
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[a^2*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e
 + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d
*x)^n/(x*Sqrt[a - b*x])), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && IntegerQ[m - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 30638 vs. 2(868) = 1736.

Time = 12.63 (sec) , antiderivative size = 30639, normalized size of antiderivative = 30.67

method	result	size
default	Expression too large to display	30639

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \int \frac{1}{(a(\sec(e+fx)+1))^{\frac{5}{2}}(c+d\sec(e+fx))^3}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral(1/((a*(sec(e + f*x) + 1))**(5/2)*(c + d*sec(e + f*x))**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="maxima
")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \text{Hanged}$$

input
```
int(1/((a + a/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x))^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(a+a\sec(e+fx))^{5/2}(c+d\sec(e+fx))^3}\,dx = \frac{\sqrt{a}\left(\int \frac{}{\sec(fx+e)^6 d^3+3\sec(fx+e)^5 c\,d^2+3\sec(fx+e)^5 d^3+3\sec(fx+e)^4 c}\right.}{}$$

input
```
int(1/(a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x)
```

output
```
(sqrt(a)*int(sqrt(sec(e + f*x) + 1)/(sec(e + f*x)**6*d**3 + 3*sec(e + f*x)
**5*c*d**2 + 3*sec(e + f*x)**5*d**3 + 3*sec(e + f*x)**4*c**2*d + 9*sec(e +
 f*x)**4*c*d**2 + 3*sec(e + f*x)**4*d**3 + sec(e + f*x)**3*c**3 + 9*sec(e
+ f*x)**3*c**2*d + 9*sec(e + f*x)**3*c*d**2 + sec(e + f*x)**3*d**3 + 3*sec
(e + f*x)**2*c**3 + 9*sec(e + f*x)**2*c**2*d + 3*sec(e + f*x)**2*c*d**2 +
3*sec(e + f*x)*c**3 + 3*sec(e + f*x)*c**2*d + c**3),x))/a**3
```

3.184 $\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 123

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\sqrt{a}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

output
```
2*a^(1/2)*c^(1/2)*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)
/(c+d*sec(f*x+e))^(1/2))/f+2*a^(1/2)*d^(1/2)*arctanh(a^(1/2)*d^(1/2)*tan(f
*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/f
```

Mathematica [A] (verified)

Time = 13.69 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.95

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}\left(-2\sqrt{c}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{d}\sqrt{c-c\cos(e+fx)}}\right)\sqrt{c(1+c\right.}{f\sqrt{c(1+\cos(e+fx))}\sqrt{c-}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(-2*Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]]*(-2*S
qrt[c]*Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c
- c*Cos[e + f*x]])]*Sqrt[c*(1 + Cos[e + f*x])]*Sin[(e + f*x)/2]^2 + ArcTan
[(Sqrt[c*(1 + Cos[e + f*x])]*Sqrt[d + c*Cos[e + f*x]])/Sqrt[c^2*Sin[e + f*
x]^2]]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[c^2*Sin[e + f*x]^2]))/(f*Sqrt[c*(1 +
Cos[e + f*x])]*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4420, 3042, 4422, 216, 4468, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sec(e+fx)+a}\sqrt{c+d \sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c+d \csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4420}$$

$$c\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d \sec(e+fx)}}dx+d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d \sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d \csc\left(e+fx+\frac{\pi}{2}\right)}}dx+d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d \csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4422}$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2ac\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{4468}$$

$$\frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}-\frac{2ad\int \frac{1}{1-\frac{ad\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{a}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/f + (2*Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[
a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]
])])/f
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4420
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
 d*Csc[e + f*x]], x], x] + Simp[d   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*
x]]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4468
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*(b/f)   Subs
t[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
 + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(99) = 198$.

Time = 1.14 (sec) , antiderivative size = 1680, normalized size of antiderivative = 13.66

method	result	size
default	Expression too large to display	1680

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)/(c^2-2*c*d+d^2)*(a*(1+sec(f*x+e)))^(1
/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(c+d*sec(f*x+e))^(1/2)*(ln(1/(c-d)^(
1/2)*((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)-c*cot(f*x+e)+
d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d)^(1/2)*c^3-3*ln(1/(c-
d)^(1/2)*((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)-c*cot(f*x
+e)+d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d)^(1/2)*c^2*d+3*ln
(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)-c*c
ot(f*x+e)+d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d)^(1/2)*c*d^
2-ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)
-c*cot(f*x+e)+d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d)^(1/2)*
d^3+ln(-2*((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(-d)^(1/2)*2^(1/2)*c
os(f*x+e)+2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)-c*
cos(f*x+e)+c*sin(f*x+e)-cos(f*x+e)*d-d*sin(f*x+e)-c-d)/(cos(f*x+e)-sin(f*x
+e)+1))*(c-d)^(1/2)*c^2*d-2*ln(-2*((-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1
/2)*(-d)^(1/2)*2^(1/2)*cos(f*x+e)+2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/
(1+cos(f*x+e)))^(1/2)-c*cos(f*x+e)+c*sin(f*x+e)-cos(f*x+e)*d-d*sin(f*x+e)-
c-d)/(cos(f*x+e)-sin(f*x+e)+1))*(c-d)^(1/2)*c*d^2+ln(-2*((-2*(d+c*cos(f*x+
e))/(1+cos(f*x+e)))^(1/2)*(-d)^(1/2)*2^(1/2)*cos(f*x+e)+2^(1/2)*(-d)^(1/2)
*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)-c*cos(f*x+e)+c*sin(f*x+e)-cos(
f*x+e)*d-d*sin(f*x+e)-c-d)/(cos(f*x+e)-sin(f*x+e)+1))*(c-d)^(1/2)*d^3-l...
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 806, normalized size of antiderivative = 6.55

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[(sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a*c - a*d)
*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)^2 + cos(
f*x + e))) + sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c)*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos
(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x +
e) + 1)))/f, -(2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(
f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*
x + e))) - sqrt(a*d)*log((2*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (a
*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e)
^2 + cos(f*x + e))))/f, -(2*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e
)/(a*d*sin(f*x + e))) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 - 2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*
x + e))*cos(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/
(cos(f*x + e) + 1)))/f, -2*(sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/
(a*c*sin(f*x + e))) + sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(...
```

Sympy [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$= \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$=\int \sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$=\int \sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \sqrt{a}\left(\int \sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)+1}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1),x)
```

3.185 $\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*se
c(f*x+e))^(1/2))/c^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx =\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{d+c\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{c}f\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(
Sqrt[c]*f*Sqrt[c + d*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3042, 4422, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(e+fx)+a}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4422}$$

$$-\frac{2a\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{c}f}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[c]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(49) = 98$.

Time = 0.50 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.70

method	result	size
default	$\frac{2\sqrt{2}\,\sqrt{-(c-d)^4 c}\,\sqrt{a(1+\sec(fx+e))}\,\sqrt{c+d\sec(fx+e)}\,\arctan\left(\frac{(c-d)^2 c\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{-(c-d)^4 c}\,\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}}\right)\cos(fx+e)}{f(c^2-2cd+d^2)c(1+\cos(fx+e))\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}}$	165

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*2^(1/2)*(-(c-d)^4*c)^(1/2)/(c^2-2*c*d+d^2)/c*(a*(1+sec(f*x+e)))^(1/2)*
(c+d*sec(f*x+e))^(1/2)*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)/(-2*(d+
c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(cot(f*x+e)-csc(f*x+e)))*cos(f*x+e)/(1
+cos(f*x+e))/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \left[\frac{\sqrt{-\frac{a}{c}}\log\left(-\frac{2\,c\sqrt{-\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-2\,ac\cos(fx+e)^2+ac-ad-(ac+ad)\cos(fx+e)}{\cos(fx+e)+1}\right)}{f}, -\frac{2\sqrt{\frac{a}{c}}\arctan\left(\frac{\sqrt{\frac{a}{c}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{a\sin(fx+e)}\right)}{f}\right]$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[sqrt(-a/c)*log(-(2*c*sqrt(-a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*c
os(f*x + e)^2 + a*c - a*d - (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1))/
f, -2*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*sin(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\,(\sec\,(e+fx)+1)}}{\sqrt{c+d\sec\,(e+fx)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/sqrt(c + d*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{\sqrt{c + d\sec(e + fx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{\sqrt{a\sec(fx + e) + a}}{\sqrt{d\sec(fx + e) + c}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\sec(e + fx)}}{\sqrt{c + d\sec(e + fx)}}\,dx = \int \frac{\sqrt{a + \frac{a}{\cos(e+f\,x)}}}{\sqrt{c + \frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)+1}}{\sec(fx+e)\,d+c}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1))/(sec(e + f*x
)*d + c),x)
```

3.186 $\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2ad\tan(e+fx)}{c(c+d)f\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

output
```
2*a^(1/2)*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*se
c(f*x+e))^(1/2))/c^(3/2)/f-2*a*d*tan(f*x+e)/c/(c+d)/f/(a+a*sec(f*x+e))^(1/
2)/(c+d*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\left(-\sqrt{2}(c+d)^{3/2}\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}+2\sqrt{c}d\sin}{c^{3/2}(c+d)f\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
-((Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*(-(Sqrt[2]*(c + d)^(3/2)*Ar
cSin[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f
*x])/(c + d)]) + 2*Sqrt[c]*d*Sin[(e + f*x)/2]))/(c^(3/2)*(c + d)*f*Sqrt[c
+ d*Sec[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4427, 3042, 4422, 216, 4475, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a \sec(e+fx)+a}}{(c+d \sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a \csc\left(e+fx+\frac{\pi}{2}\right)+a}}{\left(c+d \csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4427}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d \sec(e+fx)}}dx}{c}-\frac{d\int \frac{\sec(e+fx)\sqrt{\sec(e+fx)a+a}}{(c+d \sec(e+fx))^{3/2}}dx}{c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d \csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c+d \csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}$$

$$\Big\downarrow \text{ 4422}$$

$$-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{2a\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{cf}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}$$

$\Big\downarrow$ 4475

$$\frac{a^2d\tan(e+fx)\int \frac{1}{\sqrt{a-a\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}d\sec(e+fx)}{cf\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}$$

$\Big\downarrow$ 48

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2ad\tan(e+fx)}{cf(c+d)\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}$$

input
```
Int[Sqrt[a + a*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(c^(3/2)*f) - (2*a*d*Tan[e + f*x])/(c*(c + d)
*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4427
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt
[c + d*Csc[e + f*x]], x], x] - Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[
e + f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

rule 4475
```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))
 Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x
], x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || In
tegerQ[m - 1/2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(95) = 190$.

Time = 0.38 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.10

method	result
default	$-\frac{\left(2c^3d\sin(fx+e)-4c^2d^2\sin(fx+e)+2c\,d^3\sin(fx+e)+(1+\cos(fx+e))\arctan\left(\frac{(c-d)^2c\sqrt{2}\,(\cot(fx+e)-\csc(fx+e))}{\sqrt{-(c-d)^4c}\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}}\right)\sqrt{-\frac{2(d+c\cos}{1+\cos(}}\right.}{f(c-}$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(c+d)/c^2/(c^2-2*c*d+d^2)*(2*c^3*d*sin(f*x+e)-4*c^2*d^2*sin(f*x+e)+2*
c*d^3*sin(f*x+e)+(1+cos(f*x+e))*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2
)/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(cot(f*x+e)-csc(f*x+e)))*(-2*
(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(-(c-d)^4*c)^(1/2)*2^(1/2)*c+(1+cos
(f*x+e))*arctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)/(-2*(d+c*cos(f*x+e))/
(1+cos(f*x+e)))^(1/2)*(cot(f*x+e)-csc(f*x+e)))*(-2*(d+c*cos(f*x+e))/(1+cos
(f*x+e)))^(1/2)*(-(c-d)^4*c)^(1/2)*2^(1/2)*d)*(c+d*sec(f*x+e))^(1/2)*(a*(1
+sec(f*x+e)))^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/(d+c*cos(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(95) = 190$.

Time = 0.20 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.66

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \left[-\frac{2\,d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-\left((c^2+cd)\cos\right.}{}\right.$$

$$-\frac{2\left(d\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+\left((c^2+cd)\cos(fx+e)^2+cd+d^2+(c^2+2\right.\right.}{(c^3+c^2d)f\cos(fx+e)^2+(c^3+2\,c^2d+cd^2)f\cos(}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
[-(2*d*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/c
os(f*x + e))*cos(f*x + e)*sin(f*x + e) - ((c^2 + c*d)*cos(f*x + e)^2 + c*d
 + d^2 + (c^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(-a/c)*log(-(2*c*sqrt(-a/c)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) - 2*a*c*cos(f*x + e)^2 + a*c - a*d - (a*c
 + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/((c^3 + c^2*d)*f*cos(f*x + e)^2
 + (c^3 + 2*c^2*d + c*d^2)*f*cos(f*x + e) + (c^2*d + c*d^2)*f), -2*(d*sqrt
((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e)
)*cos(f*x + e)*sin(f*x + e) + ((c^2 + c*d)*cos(f*x + e)^2 + c*d + d^2 + (c
^2 + 2*c*d + d^2)*cos(f*x + e))*sqrt(a/c)*arctan(sqrt(a/c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x +
 e)/(a*sin(f*x + e))))/((c^3 + c^2*d)*f*cos(f*x + e)^2 + (c^3 + 2*c^2*d +
c*d^2)*f*cos(f*x + e) + (c^2*d + c*d^2)*f)]
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))/(c + d*sec(e + f*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\sec(fx+e)+a}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2\,d^2+2\sec(fx+e)\,cd+c^2}dx\right)$$

input
```
int((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1))/(sec(e + f*x
)**2*d**2 + 2*sec(e + f*x)*c*d + c**2),x)
```

3.187 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} - \frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
2*c^(1/2)*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*se
c(f*x+e))^(1/2))/a^(1/2)/f-2^(1/2)*(c-d)^(1/2)*arctan(1/2*a^(1/2)*(c-d)^(1
/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/a^(1
/2)/f
```

Mathematica [A] (verified)

Time = 12.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$$
$$=\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{-c+d}\,\text{arctanh}\left(\frac{\sqrt{-c+d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)+\frac{\sqrt{2}\sqrt{c}\sqrt{c+d}\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}}{\sqrt{d+c\cos(e+fx)}}\right)}{f\sqrt{d+c\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

input
```
Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Cos[(e + f*x)/2]*(Sqrt[-c + d]*ArcTanh[(Sqrt[-c + d]*Sin[(e + f*x)/2])/
Sqrt[d + c*Cos[e + f*x]]] + (Sqrt[2]*Sqrt[c]*Sqrt[c + d]*ArcSin[(Sqrt[2]*S
qrt[c]*Sin[(e + f*x)/2])/Sqrt[c + d]]*Sqrt[(d + c*Cos[e + f*x])/(c + d)])/
Sqrt[d + c*Cos[e + f*x]])*Sqrt[c + d*Sec[e + f*x]])/(f*Sqrt[d + c*Cos[e +
f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4423, 3042, 4422, 216, 4471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a\sec(e+fx)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \text{4423}$$

$$\frac{c\int\frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx}{a}-(c-d)\int\frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}dx$$

$\Big\downarrow$ 3042

$$\frac{c\int\frac{\sqrt{\csc(e+fx+\frac{\pi}{2})a+a}}{\sqrt{c+d\csc(e+fx+\frac{\pi}{2})}}dx}{a}-(c-d)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 4422

$$-\left((c-d)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2c\int\frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}-(c-d)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 4471

$$\frac{2(c-d)\int\frac{1}{\frac{a(c-d)\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}-\frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

input
```
Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) - (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sq
rt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c +
 d*Sec[e + f*x]])])/(Sqrt[a]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4423
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[a/c   Int[Sqrt[c + d*Csc[e + f*x]]/Sqrt[a
 + b*Csc[e + f*x]], x], x] + Simp[(b*c - a*d)/c   Int[Csc[e + f*x]/(Sqrt[a
+ b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

rule 4471
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*(a/(b*f))
Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e +
f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(114) = 228$.

Time = 0.36 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.87

method	result
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\sqrt{c+d\sec(fx+e)}\left(\sqrt{2}\sqrt{-(c-d)^4c}\arctan\left(\frac{(c-d)^2c\sqrt{2}(\csc(fx+e)-\cot(fx+e)}{\sqrt{-(c-d)^4c}\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}}\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)*(a*(1+sec(f*x+e)))^(1/2)*((1-cos(f*x+e))
^2*csc(f*x+e)^2-1)*(c+d*sec(f*x+e))^(1/2)*(2^(1/2)*(-(c-d)^4*c)^(1/2)*arct
an((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)
))^(1/2)*(csc(f*x+e)-cot(f*x+e)))*(c-d)^(1/2)-ln((c-d)^(1/2)*(csc(f*x+e)-c
ot(f*x+e))+(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2))*c^3+3*ln((c-d)^(1/2
)*(csc(f*x+e)-cot(f*x+e))+(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2))*c^2*
d-3*ln((c-d)^(1/2)*(csc(f*x+e)-cot(f*x+e))+(-2*(d+c*cos(f*x+e))/(1+cos(f*x
+e)))^(1/2))*c*d^2+ln((c-d)^(1/2)*(csc(f*x+e)-cot(f*x+e))+(-2*(d+c*cos(f*x
+e))/(1+cos(f*x+e)))^(1/2))*d^3)/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 883, normalized size of antiderivative = 6.26

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
[1/2*(sqrt(2)*sqrt(-(c - d)/a)*log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) -
 c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(-c/a)*log(-(2*sq
rt(-c/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)
/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*co
s(f*x + e) + c - d)/(cos(f*x + e) + 1)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*
log((2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + (3*c - d
)*cos(f*x + e)^2 + 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*c
os(f*x + e) + 1)) - 4*sqrt(c/a)*arctan(sqrt(c/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(c*sin
(f*x + e))))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
+ e))*cos(f*x + e)/((c - d)*sin(f*x + e))) - sqrt(-c/a)*log(-(2*sqrt(-c/a)
*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e) - 2*c*cos(f*x + e)^2 - (c + d)*cos(f*x +
e) + c - d)/(cos(f*x + e) + 1)))/f, -(sqrt(2)*sqrt((c - d)/a)*arctan(-sqrt
(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*
x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + 2*sqrt...
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+d\sec{(e+fx)}}}{\sqrt{a\,(\sec{(e+fx)}+1)}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(c + d*sec(e + f*x))/sqrt(a*(sec(e + f*x) + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-c>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\sec(fx+e)+c}}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^(1/2)/(a + a/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)d+c}\,\sqrt{\sec(fx+e)+1}}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1))/(sec(e + f*
x) + 1),x))/a
```

3.188 $\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

output
```
2*arctan(a^(1/2)*c^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))/a^(1/2)/c^(1/2)/f-2^(1/2)*arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+
e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/a^(1/2)/(c-d)^(1
/2)/f
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\left(\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)-\sqrt{c}\arctan\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+}{\sqrt{c}\sqrt{c-d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(2*(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sin[(e + f*x)/2])/Sqrt[d +
 c*Cos[e + f*x]]] - Sqrt[c]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d +
 c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])
/(Sqrt[c]*Sqrt[c - d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]
])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3042, 4426, 3042, 4422, 216, 4471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a \sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a \csc\left(e+fx+\frac{\pi}{2}\right)+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4426}$$

$$\frac{\int \frac{\sqrt{\sec(e+fx)a+a}}{\sqrt{c+d\sec(e+fx)}}dx}{a}-\int \frac{\sec(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4422}$$

$$-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2\int \frac{1}{\frac{ac\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+1}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4471}$$

$$\frac{2\int \frac{1}{\frac{a(c-d)\tan^2(e+fx)}{(\sec(e+fx)a+a)(c+d\sec(e+fx))}+2}d\left(-\frac{\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}\sqrt{c+d\sec(e+fx)}}\right)}{f}+\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{c}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c}f}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

input
```
Int[1/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[c]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
+ d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c]*f) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c
 - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f
*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4422
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[-2*(a/f)   Subst[Int[1/(1 + a*c*x^2), x],
 x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0]
```

rule 4426
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sq
rt[c + d*Csc[e + f*x]], x], x] - Simp[b/a   Int[Csc[e + f*x]/(Sqrt[a + b*Cs
c[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0]
```

rule 4471
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*(a/(b*f))
Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e +
f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(114) = 228$.

Time = 0.36 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.51

method	result
default	$\frac{\sqrt{a(1+\sec(fx+e))}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\sqrt{c+d\sec(fx+e)}\left(\sqrt{2}\sqrt{-(c-d)^4c}\arctan\left(\frac{(c-d)^2c\sqrt{2}(\csc(fx+e)-\cot(fx+e)}{\sqrt{-(c-d)^4c}\sqrt{-\frac{2(d+c\cos(fx+e))}{1+\cos(fx+e)}}}\right.\right.}{}$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/f/a/(c-d)^(1/2)/(c^2-2*c*d+d^2)/c*(a*(1+sec(f*x+e)))^(1/2)*((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)*(c+d*sec(f*x+e))^(1/2)*(2^(1/2)*(-(c-d)^4*c)^(1/2)*ar
ctan((c-d)^2*c*2^(1/2)/(-(c-d)^4*c)^(1/2)/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+
e)))^(1/2)*(csc(f*x+e)-cot(f*x+e)))*(c-d)^(1/2)-ln((c-d)^(1/2)*(csc(f*x+e)
-cot(f*x+e))+(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2))*c^3+2*ln((c-d)^(1
/2)*(csc(f*x+e)-cot(f*x+e))+(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2))*c^
2*d-ln((c-d)^(1/2)*(csc(f*x+e)-cot(f*x+e))+(-2*(d+c*cos(f*x+e))/(1+cos(f*x
+e)))^(1/2))*c*d^2)/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 913, normalized size of antiderivative = 6.48

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
[1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-
1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c
 + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*s
qrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(-a*c)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x + e))/(cos(f*x + e) + 1)))/(a*c
*f), 1/2*(sqrt(2)*a*c*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*
cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sq
rt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 +
2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) -
 4*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(a*c*sin(f*x + e))))/(a*c
*f), (sqrt(2)*a*c*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f
*x + e)))/sqrt(a*c - a*d) - sqrt(-a*c)*log((2*a*c*cos(f*x + e)^2 + 2*sqrt(
-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/co
s(f*x + e))*cos(f*x + e)*sin(f*x + e) - a*c + a*d + (a*c + a*d)*cos(f*x +
e))/(cos(f*x + e) + 1)))/(a*c*f), (sqrt(2)*a*c*arctan(sqrt(2)*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\int \frac{1}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\frac{\sqrt{a}\left(\int \frac{\sqrt{\sec(fx+e)d+c}\,\sqrt{\sec(fx+e)+1}}{\sec(fx+e)^2d+\sec(fx+e)c+\sec(fx+e)d+c}dx\right)}{a}$$

input
```
int(1/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x) + 1))/(sec(e + f*
x)**2*d + sec(e + f*x)*c + sec(e + f*x)*d + c),x))/a
```

3.189 $\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \frac{ax}{c}+\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}f}$$

output
```
a*x/c+2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c/(
c-d)^(1/2)/(c+d)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \frac{a(e+fx)+\frac{2(-bc+ad)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}}{cf}$$

input
```
Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

output
```
(a*(e + f*x) + (2*(-(b*c) + a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2])/(c*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4407}$$

$$\frac{(bc-ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{4318}$$

$$\frac{(bc-ad)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{cd}+\frac{ax}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{cd}+\frac{ax}{c}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(bc-ad)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{cdf}+\frac{ax}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{ax}{c}$$

input
```
Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x]),x]
```

output
```
(a*x)/c + (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d
]])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{2(ad-bc)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}}{f}$
default	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c}-\frac{2(ad-bc)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c\sqrt{(c+d)(c-d)}}}{f}$
risch	$\frac{ax}{c}+\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)ad}{\sqrt{c^2-d^2}\,fc}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)b}{\sqrt{c^2-d^2}\,f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-}}{\sqrt{c^2-d^2}\,c}\right.}{\sqrt{c^2-d^2}\,fc}$

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a/c*arctan(tan(1/2*f*x+1/2*e))-2*(a*d-b*c)/c/((c+d)*(c-d))^(1/2)*ar
ctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.73

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx$$
$$=\left[\frac{2\,(ac^2-ad^2)fx-(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right.}{2\,(c^3-cd^2)f}\right.$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(2*(a*c^2 - a*d^2)*f*x - (b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f
*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e)
 + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e)
 + d^2)))/((c^3 - c*d^2)*f), ((a*c^2 - a*d^2)*f*x + (b*c - a*d)*sqrt(-c^2
+ d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x
+ e))))/((c^3 - c*d^2)*f)]
```

Sympy [F]

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx=\int \frac{a+b\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(58) = 116$.

Time = 0.16 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.09

$$\int \frac{a + b\sec(e + fx)}{c + d\sec(e + fx)}\,dx$$
$$= \frac{\frac{\left(\sqrt{-c^2+d^2}a(c-2d)|-c+d|+\sqrt{-c^2+d^2}bc|-c+d|-\sqrt{-c^2+d^2}a|c||-c+d|+\sqrt{-c^2+d^2}b|c||-c+d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-\frac{d+\sqrt{(c+d)(c-d)+d}}{c-d}}}\right.\right.}{(c^2-2cd+d^2)c^2+(c^2d-2cd^2+d^3)|c|}}{f}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
((sqrt(-c^2 + d^2)*a*(c - 2*d)*abs(-c + d) + sqrt(-c^2 + d^2)*b*c*abs(-c +
 d) - sqrt(-c^2 + d^2)*a*abs(c)*abs(-c + d) + sqrt(-c^2 + d^2)*b*abs(c)*ab
s(-c + d))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)
/sqrt(-(d + sqrt((c + d)*(c - d) + d^2))/(c - d))))/((c^2 - 2*c*d + d^2)*c
^2 + (c^2*d - 2*c*d^2 + d^3)*abs(c)) + (a*c + b*c - 2*a*d + a*abs(c) - b*a
bs(c))*(pi*floor(1/2*(f*x + e)/pi + 1/2) + arctan(tan(1/2*f*x + 1/2*e)/sqr
t(-(d - sqrt((c + d)*(c - d) + d^2))/(c - d))))/(c^2 - d*abs(c)))/f
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 573, normalized size of antiderivative = 8.55

$$\int \frac{a + b\sec(e + fx)}{c + d\sec(e + fx)}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))/(c + d/cos(e + f*x)),x)
```

output
```
(b*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/
2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (b*d^2*
log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2
 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*a*c*atan(si
n(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (b*log((c*cos(e/2
+ (f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/
cos(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) - (a*c*d*log(
(c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*a*d^2*atan(sin(
e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)) + (a*d^3*log((c*sin(
e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(c*f*(c^2 - d^2)^(3/2)) + (a*d*log((c*cos(e/2 + (
f*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos
(e/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(c*f*(c^2 - d^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.06

$$\int \frac{a+b\sec(e+fx)}{c+d\sec(e+fx)}\,dx = \frac{-2\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)ad+2\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)bc+a\,c^2f\cdots}{cf\,(c^2-d^2)}$$

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

output
```
( - 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*a*d + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*
c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*b*c + a*c**2*f*x - a*d**2*f*
x)/(c*f*(c**2 - d**2))
```

3.190 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{ax}{c^2}+\frac{2(bc^3-2ac^2d+ad^3)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} - \frac{d(bc-ad)\tan(e+fx)}{c\,(c^2-d^2)\,f(c+d\sec(e+fx))}$$

output
```
a*x/c^2+2*(-2*a*c^2*d+a*d^3+b*c^3)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/
(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f-d*(-a*d+b*c)*tan(f*x+e)/c/(c^2-
d^2)/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{-\frac{2(bc^3+ad(-2c^2+d^2))\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{ad(c^2-d^2)(e+fx)+ac(c^2-d^2)(e+fx)\cos(e+fx)-cd(bc-ad)\sin(e+fx)}{d+c\cos(e+fx)}}{c^2(c-d)(c+d)f}$$

input
```
Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

output
```
((-2*(b*c^3 + a*d*(-2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt
[c^2 - d^2]])/Sqrt[c^2 - d^2] + (a*d*(c^2 - d^2)*(e + f*x) + a*c*(c^2 - d^
2)*(e + f*x)*Cos[e + f*x] - c*d*(b*c - a*d)*Sin[e + f*x])/(d + c*Cos[e + f
*x]))/(c^2*(c - d)*(c + d)*f)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4411, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4411}$$

$$-\frac{\int -\frac{a(c^2-d^2)+c(bc-ad)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c\,(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{cf\,(c^2-d^2)\,(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(c^2-d^2)+c(bc-ad)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c\,(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{cf\,(c^2-d^2)\,(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(c^2-d^2)+c(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c\,(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{cf\,(c^2-d^2)\,(c+d\sec(e+fx))}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{\left(bc^3-ad(2c^2-d^2)\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\left(bc^3-ad(2c^2-d^2)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\left(bc^3-ad(2c^2-d^2)\right)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{cd}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\left(bc^3-ad(2c^2-d^2)\right)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{cd}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{2\left(bc^3-ad(2c^2-d^2)\right)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{cdf}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2\left(bc^3-ad(2c^2-d^2)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{ax(c^2-d^2)}{c}}{c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{cf\left(c^2-d^2\right)(c+d\sec(e+fx))}$$

input

```
Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^2,x]
```

output

```
((a*(c^2 - d^2)*x)/c + (2*(b*c^3 - a*d*(2*c^2 - d^2))*ArcTanh[(Sqrt[c - d]
*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d]*f))/(c*(c^2 -
d^2)) - (d*(b*c - a*d)*Tan[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x])
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2d(ad-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\right.}{\sqrt{(c+d)(c-d)}}\right.}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
default	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2d(ad-bc)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a\,c^2d-a\,d^3-b\,c^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\right.}{\sqrt{(c+d)(c-d)}}\right.}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
risch	$\frac{ax}{c^2}-\frac{2id(-ad+bc)\left(d\,e^{i(fx+e)}+c\right)}{c^2(c^2-d^2)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{2\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)ad}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}-\frac{\ln\left(e^{i(fx+e)}+\frac{-ic^2+}{}\right.}{\sqrt{c^2-d^2}\,(c+}$

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a/c^2*arctan(tan(1/2*f*x+1/2*e))+2/c^2*(-d*(a*d-b*c)*c/(c^2-d^2)*ta
n(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-(2*a*
c^2*d-a*d^3-b*c^3)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f
*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(114) = 228.

Time = 0.12 (sec) , antiderivative size = 561, normalized size of antiderivative = 4.56

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$$
$$=\left[\frac{2\left(ac^5-2\,ac^3d^2+acd^4\right)fx\cos\left(fx+e\right)+2\left(ac^4d-2\,ac^2d^3+ad^5\right)fx-\left(bc^3d-2\,ac^2d^2+ad^4+\left(bc^4-\right.\right.}{2\left(\left(c\right.\right.}\right.$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a*c^5 - 2*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + 2*(a*c^4*d - 2*
a*c^2*d^3 + a*d^5)*f*x - (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3
*d + a*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2
 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
+ e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(
b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin(f*x + e))/((c^7 - 2*c^5*d^2
 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a*c^5 - 2
*a*c^3*d^2 + a*c*d^4)*f*x*cos(f*x + e) + (a*c^4*d - 2*a*c^2*d^3 + a*d^5)*f
*x + (b*c^3*d - 2*a*c^2*d^2 + a*d^4 + (b*c^4 - 2*a*c^3*d + a*c*d^3)*cos(f*
x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c
^2 - d^2)*sin(f*x + e))) - (b*c^4*d - a*c^3*d^2 - b*c^2*d^3 + a*c*d^4)*sin
(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^
3 + c^2*d^5)*f)]
```

Sympy [F]

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \int \frac{a+b\sec{(e+fx)}}{(c+d\sec{(e+fx)})^2}\,dx$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.63

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \frac{\frac{2\,(bc^3-2\,ac^2d+ad^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-c^2d^2)\sqrt{-c^2+d^2}}+\frac{(fx+e)a}{c^2}+\frac{2\,\left(bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{(c^3-cd^2)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}}{f}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
(2*(b*c^3 - 2*a*c^2*d + a*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c
+ 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c
^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(-c^2 + d^2)) + (f*x + e)*a/c^2 + 2*(b*c*
d*tan(1/2*f*x + 1/2*e) - a*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan
(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 3763, normalized size of antiderivative = 30.59

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^2,x)
```

output
```
(2*a*atan(((a*((a*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^
2 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 -
 c^4*d^2) - (a*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6
*d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2))
)*1i)/c^2 + (32*tan(e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*
c*d^5 - 2*a^2*c^5*d - 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*
b*c^5*d + 2*a*b*c^3*d^3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2 - (a*((a
*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2 - b*c^6*d^3 + b
*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 - c^4*d^2) + (a*ta
n(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4 - 4*c^7*d^3
- 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i)/c^2 - (32*t
an(e/2 + (f*x)/2)*(a^2*c^6 + 2*a^2*d^6 + b^2*c^6 - 2*a^2*c*d^5 - 2*a^2*c^5
*d - 5*a^2*c^2*d^4 + 4*a^2*c^3*d^3 + 3*a^2*c^4*d^2 - 4*a*b*c^5*d + 2*a*b*c
^3*d^3))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))/c^2)/((64*(a^3*d^5 + a*b^2*c^
5 - a^2*b*c^5 - a^3*c*d^4 + 2*a^3*c^4*d - 3*a^3*c^2*d^3 + 2*a^3*c^3*d^2 +
a^2*b*c^2*d^3 + a^2*b*c^3*d^2 - 3*a^2*b*c^4*d))/(c^5*d + c^6 - c^3*d^3 - c
^4*d^2) + (a*((a*((32*(a*c^4*d^5 - b*c^9 - a*c^9 - 3*a*c^6*d^3 + a*c^7*d^2
 - b*c^6*d^3 + b*c^7*d^2 + 2*a*c^8*d + b*c^8*d))/(c^5*d + c^6 - c^3*d^3 -
c^4*d^2) - (a*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*
d^4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.34

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{-4\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d}{\sqrt{-c^2+d^2}}\right)\cos(fx+e)\,a\,c^3d+2\sqrt{-c^2+d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{\sqrt{-c^2+d^2}}\right.}{}$$

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

output

```
( - 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*cos(e + f*x)*a*c**3*d + 2*sqrt( - c**2 + d**2)*atan(
(tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*
x)*a*c*d**3 + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f
*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*b*c**4 - 4*sqrt( - c**2 + d**
2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*a*
c**2*d**2 + 2*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x
)/2)*d)/sqrt( - c**2 + d**2))*a*d**4 + 2*sqrt( - c**2 + d**2)*atan((tan((e
 + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*b*c**3*d + cos(e
+ f*x)*a*c**5*f*x - 2*cos(e + f*x)*a*c**3*d**2*f*x + cos(e + f*x)*a*c*d**4
*f*x + sin(e + f*x)*a*c**3*d**2 - sin(e + f*x)*a*c*d**4 - sin(e + f*x)*b*c
**4*d + sin(e + f*x)*b*c**2*d**3 + a*c**4*d*f*x - 2*a*c**2*d**3*f*x + a*d*
*5*f*x)/(c**2*f*(cos(e + f*x)*c**5 - 2*cos(e + f*x)*c**3*d**2 + cos(e + f*
x)*c*d**4 + c**4*d - 2*c**2*d**3 + d**5))
```

3.191 $\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \frac{ax}{c^3} + \frac{(bc^3(2c^2+d^2)-ad(6c^4-5c^2d^2+2d^4))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} - \frac{d(bc-ad)\tan(e+fx)}{2c\,(c^2-d^2)\,f(c+d\sec(e+fx))^2} - \frac{d(3bc^3-5ac^2d+2ad^3)\tan(e+fx)}{2c^2\,(c^2-d^2)^2\,f(c+d\sec(e+fx))}$$

output

```
a*x/c^3+(b*c^3*(2*c^2+d^2)-a*d*(6*c^4-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2
)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d)^(5/2)/f-1/2*d*(-a*
d+b*c)*tan(f*x+e)/c/(c^2-d^2)/f/(c+d*sec(f*x+e))^2-1/2*d*(-5*a*c^2*d+2*a*d
^3+3*b*c^3)*tan(f*x+e)/c^2/(c^2-d^2)^2/f/(c+d*sec(f*x+e))
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.31

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))\left(2a(e+fx)(d+c\cos(e+fx))^2-\frac{2(bc^3(2c^2+d^2)+ad(-}{}\right.}{2c^3f(b+a\cos}$$

input
```
Integrate[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])*(2*a*(e + f*x)*(
d + c*Cos[e + f*x])^2 - (2*(b*c^3*(2*c^2 + d^2) + a*d*(-6*c^4 + 5*c^2*d^2
- 2*d^4))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[
e + f*x])^2)/(c^2 - d^2)^(5/2) + (c*d^2*(b*c - a*d)*Sin[e + f*x])/((c - d)
*(c + d)) - (c*d*(4*b*c^3 - 6*a*c^2*d - b*c*d^2 + 3*a*d^3)*(d + c*Cos[e +
f*x])*Sin[e + f*x])/((c - d)^2*(c + d)^2)))/(2*c^3*f*(b + a*Cos[e + f*x])*
(c + d*Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4411, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4411}$$

$$-\frac{\int -\frac{-d(bc-ad)\sec^2(e+fx)+2c(bc-ad)\sec(e+fx)+2a(c^2-d^2)}{(c+d\sec(e+fx))^2}dx}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-d(bc-ad)\sec^2(e+fx)+2c(bc-ad)\sec(e+fx)+2a(c^2-d^2)}{(c+d\sec(e+fx))^2}dx}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-d(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2+2c(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)+2a(c^2-d^2)}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{4548}$$

$$\frac{-\frac{\int -\frac{2a\left(c^2-d^2\right)^2-c\left(ad\left(4c^2-d^2\right)-bc\left(2c^2+d^2\right)\right)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2a\left(c^2-d^2\right)^2-c\left(ad\left(4c^2-d^2\right)-bc\left(2c^2+d^2\right)\right)\sec(e+fx)}{c+d\sec(e+fx)}dx}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2a\left(c^2-d^2\right)^2-c\left(ad\left(4c^2-d^2\right)-bc\left(2c^2+d^2\right)\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{\frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{c}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c(c^2-d^2)}-\frac{d(bc-ad)\tan(e+fx)}{2cf(c^2-d^2)(c+d\sec(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{1}{\frac{c\cos(e+fx)}{d}+1}dx}{cd}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{1}{\frac{c\sin\left(e+fx+\frac{\pi}{2}\right)}{d}+1}dx}{cd}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{2\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\int \frac{1}{\left(1-\frac{c}{d}\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+\frac{c+d}{d}}d\tan\left(\frac{1}{2}(e+fx)\right)}{cdf}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{2\left(bc^3\left(2c^2+d^2\right)-ad\left(6c^4-5c^2d^2+2d^4\right)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}+\frac{2ax\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}-\frac{d(-5ac^2d+2ad^3+3bc^3)\tan(e+fx)}{cf(c^2-d^2)(c+d\sec(e+fx))}}{2c\left(c^2-d^2\right)}-\frac{d(bc-ad)\tan(e+fx)}{2cf\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

input

```
Int[(a + b*Sec[e + f*x])/(c + d*Sec[e + f*x])^3,x]
```

output
```
-1/2*(d*(b*c - a*d)*Tan[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^2)
 + (((2*a*(c^2 - d^2)^2*x)/c + (2*(b*c^3*(2*c^2 + d^2) - a*d*(6*c^4 - 5*c^
2*d^2 + 2*d^4))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sq
rt[c - d]*Sqrt[c + d]*f))/(c*(c^2 - d^2)) - (d*(3*b*c^3 - 5*a*c^2*d + 2*a*
d^3)*Tan[e + f*x])/(c*(c^2 - d^2)*f*(c + d*Sec[e + f*x])))/(2*c*(c^2 - d^2
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^{3}}+\frac{2\left(-\frac{\left(6a\,c^{2}d+ac\,d^{2}-2a\,d^{3}-4b\,c^{3}-b\,c^{2}d\right)cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2(c-d)\left(c^{2}+2cd+d^{2}\right)}+\frac{dc\left(6a\,c^{2}d-ac\,d^{2}-2a\,d^{3}-4b\,c^{3}+b\,c^{2}d\right)\tan}{2(c+d)(c-d)^{2}}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}\ c^{3}}}{f}$
default	$\frac{\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^{3}}+\frac{2\left(-\frac{\left(6a\,c^{2}d+ac\,d^{2}-2a\,d^{3}-4b\,c^{3}-b\,c^{2}d\right)cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2(c-d)\left(c^{2}+2cd+d^{2}\right)}+\frac{dc\left(6a\,c^{2}d-ac\,d^{2}-2a\,d^{3}-4b\,c^{3}+b\,c^{2}d\right)\tan}{2(c+d)(c-d)^{2}}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}\ c^{3}}}{f}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a*c^2*d+a*c*d^2-2*
a*d^3-4*b*c^3-b*c^2*d)*c*d/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*
d*c*(6*a*c^2*d-a*c*d^2-2*a*d^3-4*b*c^3+b*c^2*d)/(c+d)/(c-d)^2*tan(1/2*f*x+
1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/2*(6*a*c^4
*d-5*a*c^2*d^3+2*a*d^5-2*b*c^5-b*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d)
)^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(191) = 382$.

Time = 0.16 (sec) , antiderivative size = 1152, normalized size of antiderivative = 5.65

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x + e)^2
 + 8*(a*c^7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e) + 4*
(a*c^6*d^2 - 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x - (2*b*c^5*d^2 - 6*a*c
^4*d^3 + b*c^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*c^5*
d^2 + 5*a*c^4*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c^5*d
^2 + b*c^4*d^3 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(c^2 - d^2)*lo
g((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(
d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*
d*cos(f*x + e) + d^2)) - 2*(3*b*c^6*d^2 - 5*a*c^5*d^3 - 3*b*c^4*d^4 + 7*a*
c^3*d^5 - 2*a*c*d^7 + (4*b*c^7*d - 6*a*c^6*d^2 - 5*b*c^5*d^3 + 9*a*c^4*d^4
 + b*c^3*d^5 - 3*a*c^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 - 3*c^9*d^2
 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10*d - 3*c^8*d^3 + 3*c^6*d
^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^4 + 3*c^5*d^6 - c^3*d^8)
*f), 1/2*(2*(a*c^8 - 3*a*c^6*d^2 + 3*a*c^4*d^4 - a*c^2*d^6)*f*x*cos(f*x +
e)^2 + 4*(a*c^7*d - 3*a*c^5*d^3 + 3*a*c^3*d^5 - a*c*d^7)*f*x*cos(f*x + e)
+ 2*(a*c^6*d^2 - 3*a*c^4*d^4 + 3*a*c^2*d^6 - a*d^8)*f*x + (2*b*c^5*d^2 - 6
*a*c^4*d^3 + b*c^3*d^4 + 5*a*c^2*d^5 - 2*a*d^7 + (2*b*c^7 - 6*a*c^6*d + b*
c^5*d^2 + 5*a*c^4*d^3 - 2*a*c^2*d^5)*cos(f*x + e)^2 + 2*(2*b*c^6*d - 6*a*c
^5*d^2 + b*c^4*d^3 + 5*a*c^3*d^4 - 2*a*c*d^6)*cos(f*x + e))*sqrt(-c^2 + d^
2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +...
```

Sympy [F]

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx=\int \frac{a+b\sec{(e+fx)}}{(c+d\sec{(e+fx)})^3}\,dx$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(191) = 382$.

Time = 0.21 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.24

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \frac{\frac{(2\,bc^5-6\,ac^4d+bc^3d^2+5\,ac^2d^3-2\,ad^5)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^7-2\,c^5d^2+c^3d^4)\sqrt{-c^2+d^2}}+\frac{(fx+e)a}{c^3}+\frac{4\,bc^4d\,\mathrm{t}}{}}{}$$

input
```
integrate((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
((2*b*c^5 - 6*a*c^4*d + b*c^3*d^2 + 5*a*c^2*d^3 - 2*a*d^5)*(pi*floor(1/2*(
f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*t
an(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*sqrt(
-c^2 + d^2)) + (f*x + e)*a/c^3 + (4*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 6*a*c
^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a*c
^2*d^3*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 3*a*c*d
^4*tan(1/2*f*x + 1/2*e)^3 - 2*a*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b*c^4*d*tan
(1/2*f*x + 1/2*e) + 6*a*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*b*c^3*d^2*tan(1/2
*f*x + 1/2*e) + 5*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + b*c^2*d^3*tan(1/2*f*x +
 1/2*e) - 3*a*c*d^4*tan(1/2*f*x + 1/2*e) - 2*a*d^5*tan(1/2*f*x + 1/2*e))/(
(c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/
2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.70 (sec) , antiderivative size = 6909, normalized size of antiderivative = 33.87

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))/(c + d/cos(e + f*x))^3,x)
```

output
```
(2*a*atan(((a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d^10 + 4*b^2*c^10
 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3*d^7 + 57*a^2*c^
4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 + 24*a^2*c^8*d^2
+ b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d^7 + 2*a*b*c^5*d
^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^
7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) + (a*((8*(4*a*c^15 + 4*b*c^15 - 4*a*c^6*d^9
 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10*d^5 + 6*a*c^11*d^4
 + 34*a*c^12*d^3 - 8*a*c^13*d^2 - 2*b*c^8*d^7 + 2*b*c^9*d^6 + 6*b*c^12*d^3
 - 6*b*c^13*d^2 - 12*a*c^14*d - 4*b*c^14*d))/(c^12*d + c^13 - c^6*d^7 - c^
7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) - (a*tan(e/2 + (f
*x)/2)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c
^10*d^6 + 48*c^11*d^5 + 32*c^12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(
c^10*d + c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*
c^9*d^2)))*1i)/c^3))/c^3 + (a*((8*tan(e/2 + (f*x)/2)*(4*a^2*c^10 + 8*a^2*d
^10 + 4*b^2*c^10 - 8*a^2*c*d^9 - 8*a^2*c^9*d - 32*a^2*c^2*d^8 + 32*a^2*c^3
*d^7 + 57*a^2*c^4*d^6 - 48*a^2*c^5*d^5 - 52*a^2*c^6*d^4 + 32*a^2*c^7*d^3 +
 24*a^2*c^8*d^2 + b^2*c^6*d^4 + 4*b^2*c^8*d^2 - 24*a*b*c^9*d - 4*a*b*c^3*d
^7 + 2*a*b*c^5*d^5 + 8*a*b*c^7*d^3))/(c^10*d + c^11 - c^4*d^7 - c^5*d^6 +
3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2) - (a*((8*(4*a*c^15 + 4*b*c^
15 - 4*a*c^6*d^9 + 2*a*c^7*d^8 + 18*a*c^8*d^7 - 4*a*c^9*d^6 - 36*a*c^10...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1542, normalized size of antiderivative = 7.56

$$\int \frac{a+b\sec(e+fx)}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

output

```
( - 24*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*cos(e + f*x)*a*c**5*d**2 + 20*sqrt( - c**2 + d**2)*
atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e
 + f*x)*a*c**3*d**4 - 8*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - ta
n((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a*c*d**6 + 8*sqrt( -
c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 +
 d**2))*cos(e + f*x)*b*c**6*d + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)
/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*b*c**4*d**3
 + 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*sin(e + f*x)**2*a*c**6*d - 10*sqrt( - c**2 + d**2)*a
tan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e
+ f*x)**2*a*c**4*d**3 + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*a*c**2*d**5 - 4*
sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
- c**2 + d**2))*sin(e + f*x)**2*b*c**7 - 2*sqrt( - c**2 + d**2)*atan((tan(
(e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2
*b*c**5*d**2 - 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e +
 f*x)/2)*d)/sqrt( - c**2 + d**2))*a*c**6*d - 2*sqrt( - c**2 + d**2)*atan((
tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*a*c**4*d**3
 + 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d...
```

3.192 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{a^2x}{c^2} + \frac{2(bc-ad)\left(2ac^2-bcd-ad^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^2(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)^2\sin(e+fx)}{c\left(c^2-d^2\right)f(d+c\cos(e+fx))}$$

output
```
a^2*x/c^2+2*(-a*d+b*c)*(2*a*c^2-a*d^2-b*c*d)*arctanh((c-d)^(1/2)*tan(1/2*f
*x+1/2*e)/(c+d)^(1/2))/c^2/(c-d)^(3/2)/(c+d)^(3/2)/f+(-a*d+b*c)^2*sin(f*x+
e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{a^2(e+fx)+\frac{2(-2abc^3+b^2c^2d+a^2(2c^2d-d^3))\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}}+\frac{c(bc-ad)^2\sin(e+fx)}{(c-d)(c+d)(d+c\cos(e+fx))}}{c^2f}$$

input
```
Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

output
```
(a^2*(e + f*x) + (2*(-2*a*b*c^3 + b^2*c^2*d + a^2*(2*c^2*d - d^3))*ArcTanh
[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + (c*(b*c
 - a*d)^2*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e + f*x])))/(c^2*f)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4429, 3042, 3269, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{(a\cos(e+fx)+b)^2}{(c\cos(e+fx)+d)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^2}dx$$

$\Big\downarrow$ 3269

$$\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)}-\frac{\int -\frac{\left(c^2-d^2\right)\cos(e+fx)a^2+c\left(2abc-\left(a^2+b^2\right)d\right)}{d+c\cos(e+fx)}dx}{c\left(c^2-d^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\left(c^2-d^2\right)\cos(e+fx)a^2+c\left(2abc-\left(a^2+b^2\right)d\right)}{d+c\cos(e+fx)}dx}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)a^2+c\left(2abc-\left(a^2+b^2\right)d\right)}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)}$$

$\Big\downarrow$ 3214

$$\frac{\frac{(bc-ad)\left(2ac^2-ad^2-bcd\right)\int \frac{1}{d+c\cos(e+fx)}dx}{c}+\frac{a^2x\left(c^2-d^2\right)}{c}}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(bc-ad)\left(2ac^2-ad^2-bcd\right)\int \frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c}+\frac{a^2x\left(c^2-d^2\right)}{c}}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)}$$

$\Big\downarrow$ 3138

$$\frac{\frac{2(bc-ad)\left(2ac^2-ad^2-bcd\right)\int \frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan\left(\frac{1}{2}(e+fx)\right)}{cf}+\frac{a^2x\left(c^2-d^2\right)}{c}}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)}$$

$\Big\downarrow$ 221

$$\frac{\frac{a^2x\left(c^2-d^2\right)}{c}+\frac{2(bc-ad)\left(2ac^2-ad^2-bcd\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}}{c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)}{cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)}$$

input
```
Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^2,x]
```

output
```
((a^2*(c^2 - d^2)*x)/c + (2*(b*c - a*d)*(2*a*c^2 - b*c*d - a*d^2)*ArcTanh[
(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sqrt[c - d]*Sqrt[c + d]*f)
)/(c*(c^2 - d^2)) + ((b*c - a*d)^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*C
os[e + f*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3269
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Simp[
1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)
*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1
) + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2\left(a^2d^2-2abdc+b^2c^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a^2c^2d-a^2d^3-2ab\,c^3+b^2c^2d\right)\operatorname{arctanh}\left(\frac{(c-}{}\right.}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
default	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^2}+\frac{-\frac{2\left(a^2d^2-2abdc+b^2c^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{2\left(2a^2c^2d-a^2d^3-2ab\,c^3+b^2c^2d\right)\operatorname{arctanh}\left(\frac{(c-}{}\right.}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{c^2}}{f}$
risch	$\frac{a^2x}{c^2}+\frac{2i\left(a^2d^2-2abdc+b^2c^2\right)\left(d\,e^{i(fx+e)}+c\right)}{c^2\left(c^2-d^2\right)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{2\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)a^2d}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}-\frac{\ln\left(e^{i(fx+e)}+\frac{-ic}{}\right.}{\sqrt{c^2-d^2}\,(}$

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^2/c^2*arctan(tan(1/2*f*x+1/2*e))+2/c^2*(-(a^2*d^2-2*a*b*c*d+b^2*c
^2)*c/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2
*e)^2*d-c-d)-(2*a^2*c^2*d-a^2*d^3-2*a*b*c^3+b^2*c^2*d)/(c+d)/(c-d)/((c+d)*
(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(124) = 248.

Time = 0.12 (sec) , antiderivative size = 672, normalized size of antiderivative = 5.05

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{2\,(a^2c^5-2\,a^2c^3d^2+a^2cd^4)fx\cos\left(fx+e\right)+2\,(a^2c^4d-2\,a^2c^2d^3+a^2d^5)fx-(2\,abc^3d+a^2d^4-(2\,a^2+\right.$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2*c^5 - 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + 2*(a^2*c^
4*d - 2*a^2*c^2*d^3 + a^2*d^5)*f*x - (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2
)*c^2*d^2 + (2*a*b*c^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sq
rt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*s
qrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f
*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^5 - 2*a*b*c^4*d + 2*a*b*
c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin(f*x + e))/((c^7 - 2*c^5*d^2
 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^3 + c^2*d^5)*f), ((a^2*c^5 -
 2*a^2*c^3*d^2 + a^2*c*d^4)*f*x*cos(f*x + e) + (a^2*c^4*d - 2*a^2*c^2*d^3
+ a^2*d^5)*f*x + (2*a*b*c^3*d + a^2*d^4 - (2*a^2 + b^2)*c^2*d^2 + (2*a*b*c
^4 + a^2*c*d^3 - (2*a^2 + b^2)*c^3*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arcta
n(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b^
2*c^5 - 2*a*b*c^4*d + 2*a*b*c^2*d^3 - a^2*c*d^4 + (a^2 - b^2)*c^3*d^2)*sin
(f*x + e))/((c^7 - 2*c^5*d^2 + c^3*d^4)*f*cos(f*x + e) + (c^6*d - 2*c^4*d^
3 + c^2*d^5)*f)]
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a+b\sec\left(e+fx\right))^2}{(c+d\sec\left(e+fx\right))^2}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

output
```
Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{\frac{(fx+e)a^2}{c^2} + \frac{2\,(2\,abc^3-2\,a^2c^2d-b^2c^2d+a^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-c^2d^2)\sqrt{-c^2+d^2}} - \frac{2\,(b^2c^2\tan(}{(c^3-}}{f}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
((f*x + e)*a^2/c^2 + 2*(2*a*b*c^3 - 2*a^2*c^2*d - b^2*c^2*d + a^2*d^3)*(pi
*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x +
1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^4 - c^2*d^2)*sqrt(
-c^2 + d^2)) - 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1
/2*e) + a^2*d^2*tan(1/2*f*x + 1/2*e))/((c^3 - c*d^2)*(c*tan(1/2*f*x + 1/2*
e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)))/f
```

Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 4934, normalized size of antiderivative = 37.10

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^2,x)
```

output
```
(2*a^2*atan(((a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*d
^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*c
^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^4
*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2) +
 (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d^
3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2*
a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*d
^2) - (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^4
 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1i
)/c^2))/c^2 + (a^2*((32*tan(e/2 + (f*x)/2)*(a^4*c^6 + 2*a^4*d^6 - 2*a^4*c*
d^5 - 2*a^4*c^5*d + 4*a^2*b^2*c^6 - 5*a^4*c^2*d^4 + 4*a^4*c^3*d^3 + 3*a^4*
c^4*d^2 + b^4*c^4*d^2 + 4*a^3*b*c^3*d^3 - 2*a^2*b^2*c^2*d^4 + 4*a^2*b^2*c^
4*d^2 - 4*a*b^3*c^5*d - 8*a^3*b*c^5*d))/(c^4*d + c^5 - c^2*d^3 - c^3*d^2)
- (a^2*((32*(2*a^2*c^8*d - a^2*c^9 + b^2*c^8*d + a^2*c^4*d^5 - 3*a^2*c^6*d
^3 + a^2*c^7*d^2 + b^2*c^5*d^4 - b^2*c^6*d^3 - b^2*c^7*d^2 - 2*a*b*c^9 + 2
*a*b*c^8*d - 2*a*b*c^6*d^3 + 2*a*b*c^7*d^2))/(c^5*d + c^6 - c^3*d^3 - c^4*
d^2) + (a^2*tan(e/2 + (f*x)/2)*(2*c^9*d - 2*c^4*d^6 + 2*c^5*d^5 + 4*c^6*d^
4 - 4*c^7*d^3 - 2*c^8*d^2)*32i)/(c^2*(c^4*d + c^5 - c^2*d^3 - c^3*d^2)))*1
i)/c^2))/c^2)/((64*(a^6*d^5 - 2*a^5*b*c^5 - a^6*c*d^4 + 2*a^6*c^4*d + 4*a^
4*b^2*c^5 - 3*a^6*c^2*d^3 + 2*a^6*c^3*d^2 - 4*a^3*b^3*c^4*d - a^4*b^2*c...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 715, normalized size of antiderivative = 5.38

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

output
```
( - 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*cos(e + f*x)*a**2*c**3*d + 2*sqrt( - c**2 + d**2)*at
an((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e +
 f*x)*a**2*c*d**3 + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan(
(e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a*b*c**4 - 2*sqrt( - c*
*2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d
**2))*cos(e + f*x)*b**2*c**3*d - 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x
)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*a**2*c**2*d**2 + 2*sqrt
( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c*
*2 + d**2))*a**2*d**4 + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*a*b*c**3*d - 2*sqrt( - c**2 + d*
*2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*b
**2*c**2*d**2 + cos(e + f*x)*a**2*c**5*f*x - 2*cos(e + f*x)*a**2*c**3*d**2
*f*x + cos(e + f*x)*a**2*c*d**4*f*x + sin(e + f*x)*a**2*c**3*d**2 - sin(e
+ f*x)*a**2*c*d**4 - 2*sin(e + f*x)*a*b*c**4*d + 2*sin(e + f*x)*a*b*c**2*d
**3 + sin(e + f*x)*b**2*c**5 - sin(e + f*x)*b**2*c**3*d**2 + a**2*c**4*d*f
*x - 2*a**2*c**2*d**3*f*x + a**2*d**5*f*x)/(c**2*f*(cos(e + f*x)*c**5 - 2*
cos(e + f*x)*c**3*d**2 + cos(e + f*x)*c*d**4 + c**4*d - 2*c**2*d**3 + d**5
))
```

3.193 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx &= \frac{a^2x}{c^3} \\ &\quad - \frac{(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5))\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} \\ &\quad - \frac{d(bc-ad)^2\sin(e+fx)}{2c^2\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} \\ &\quad - \frac{(bc-ad)\left(3ad(2c^2-d^2)-bc(2c^2+d^2)\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}\end{aligned}$$

output
```
a^2*x/c^3-(3*b^2*c^4*d-2*a*b*c^3*(2*c^2+d^2)+a^2*(6*c^4*d-5*c^2*d^3+2*d^5)
)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3/(c-d)^(5/2)/(c+d
)^(5/2)/f-1/2*d*(-a*d+b*c)^2*sin(f*x+e)/c^2/(c^2-d^2)/f/(d+c*cos(f*x+e))^2
-1/2*(-a*d+b*c)*(3*a*d*(2*c^2-d^2)-b*c*(2*c^2+d^2))*sin(f*x+e)/c^2/(c^2-d^
2)^2/f/(d+c*cos(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 493 vs. $2(237) = 474$.

Time = 2.46 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^2\left(\frac{4(3b^2c^4d-2abc^3(2c^2+d^2)+a^2(6c^4d-5c^2d^3+2d^5))\operatorname{arctanh}\left(\frac{(-c+}{}\right.}{(c^2-d^2)^{5/2}}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^2*((4*(3*b^2*c^4*d
 - 2*a*b*c^3*(2*c^2 + d^2) + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[((
-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2)/(c^2 -
d^2)^(5/2) + (2*a^2*c^6*e - 6*a^2*c^2*d^4*e + 4*a^2*d^6*e + 2*a^2*c^6*f*x
- 6*a^2*c^2*d^4*f*x + 4*a^2*d^6*f*x + 8*a^2*c*d*(c^2 - d^2)^2*(e + f*x)*Co
s[e + f*x] + 2*a^2*c^2*(c^2 - d^2)^2*(e + f*x)*Cos[2*(e + f*x)] + 2*b^2*c^
5*d*Sin[e + f*x] - 12*a*b*c^4*d^2*Sin[e + f*x] + 10*a^2*c^3*d^3*Sin[e + f*
x] + 4*b^2*c^3*d^3*Sin[e + f*x] - 4*a^2*c*d^5*Sin[e + f*x] + 2*b^2*c^6*Sin
[2*(e + f*x)] - 8*a*b*c^5*d*Sin[2*(e + f*x)] + 6*a^2*c^4*d^2*Sin[2*(e + f*
x)] + b^2*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^
2*c^2*d^4*Sin[2*(e + f*x)])/(c^2 - d^2)^2))/(4*c^3*f*(b + a*Cos[e + f*x])^
2*(c + d*Sec[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4429, 3042, 3467, 25, 3042, 3500, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4429}$$

$$\int \frac{\cos(e+fx)(a\cos(e+fx)+b)^2}{(c\cos(e+fx)+d)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^3}dx$$

$$\Big\downarrow \text{ 3467}$$

$$-\frac{\int -\frac{2c(bc-ad)^2+2a^2c\left(c^2-d^2\right)\cos^2(e+fx)-\left(\left(2c^2d-d^3\right)a^2-2bc\left(2c^2-d^2\right)a+b^2c^2d\right)\cos(e+fx)}{(d+c\cos(e+fx))^2}dx}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2c(bc-ad)^2+2a^2c\left(c^2-d^2\right)\cos^2(e+fx)-\left(\left(2c^2d-d^3\right)a^2-2bc\left(2c^2-d^2\right)a+b^2c^2d\right)\cos(e+fx)}{(d+c\cos(e+fx))^2}dx}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2c(bc-ad)^2+2a^2c\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-\left(\left(2c^2d-d^3\right)a^2\right)+2bc\left(2c^2-d^2\right)a-b^2c^2d\right)\sin\left(e+fx+\frac{\pi}{2}\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{\frac{\int \frac{(bc-ad)\left(4ac^2-3bdc-ad^2\right)c^2+2a^2\left(c^2-d^2\right)^2\cos(e+fx)c}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{(bc-ad)\left(4ac^2-3bdc-ad^2\right)c^2+2a^2\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)c}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \; 3214$$

$$\frac{\frac{2a^2x(c^2-d^2)^2-\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\int\frac{1}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2a^2x(c^2-d^2)^2-\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\int\frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e+fx)}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}-$$

$$\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{2a^2x(c^2-d^2)^2-\frac{2\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\int\frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc\right.}{f(c^2-d^2)(c\cos(e-}}{2c^2\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2a^2x(c^2-d^2)^2-\frac{2\left(a^2\left(6c^4d-5c^2d^3+2d^5\right)-2abc^3\left(2c^2+d^2\right)+3b^2c^4d\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}}}{c(c^2-d^2)}+\frac{(bc-ad)\left(-6ac^2d+3ad^3+2bc^3+bcd^2\right)\sin(e}{f(c^2-d^2)(c\cos(e+fx)+d)}}{2c^2\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)^2\sin(e+fx)}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

input
```
Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^3,x]
```

output
```
-1/2*(d*(b*c - a*d)^2*Sin[e + f*x])/(c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x]
)^2) + ((2*a^2*(c^2 - d^2)^2*x - (2*(3*b^2*c^4*d - 2*a*b*c^3*(2*c^2 + d^2)
 + a^2*(6*c^4*d - 5*c^2*d^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2
])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*f))/(c*(c^2 - d^2)) + ((b*c - a*
d)*(2*b*c^3 - 6*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sin[e + f*x])/((c^2 - d^2)*f*
(d + c*Cos[e + f*x])))/(2*c^2*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{\frac{2a^2 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2d^3c-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c\right.}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{}$
default	$\frac{\frac{2a^2 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^2c^2d^2+a^2d^3c-2a^2d^4-8ab\,c^3d-2ab\,c^2d^2+2b^2c^4+b^2c^3d+2b^2c^2d^2\right)c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{c\left(6a^2c\right.}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^2/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a^2*c^2*d^2+a^2*
c*d^3-2*a^2*d^4-8*a*b*c^3*d-2*a*b*c^2*d^2+2*b^2*c^4+b^2*c^3*d+2*b^2*c^2*d^
2)*c/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*c*(6*a^2*c^2*d^2-a^2*c
*d^3-2*a^2*d^4-8*a*b*c^3*d+2*a*b*c^2*d^2+2*b^2*c^4-b^2*c^3*d+2*b^2*c^2*d^2
)/(c+d)/(c-d)^2*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/
2*e)^2*d-c-d)^2-1/2*(6*a^2*c^4*d-5*a^2*c^2*d^3+2*a^2*d^5-4*a*b*c^5-2*a*b*c
^3*d^2+3*b^2*c^4*d)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*
tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(223) = 446$.

Time = 0.17 (sec) , antiderivative size = 1409, normalized size of antiderivative = 5.95

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*d^4 - a^2*c^2*d^6)*f*x*cos(f*
x + e)^2 + 8*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a^2*c^3*d^5 - a^2*c*d^7)*f*x*c
os(f*x + e) + 4*(a^2*c^6*d^2 - 3*a^2*c^4*d^4 + 3*a^2*c^2*d^6 - a^2*d^8)*f*
x - (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^2*d^5 - 2*a^2*d^7 - 3*(2*a^2
+ b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^2 + 5*a^2*c^4*d^3 - 2*a^2*c^2*d^
5 - 3*(2*a^2 + b^2)*c^6*d)*cos(f*x + e)^2 + 2*(4*a*b*c^6*d + 2*a*b*c^4*d^3
 + 5*a^2*c^3*d^4 - 2*a^2*c*d^6 - 3*(2*a^2 + b^2)*c^5*d^2)*cos(f*x + e))*sq
rt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*s
qrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f
*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^2*c^7*d - 6*a*b*c^6*d^2 + 6*
a*b*c^4*d^4 + 2*a^2*c*d^7 + (5*a^2 + b^2)*c^5*d^3 - (7*a^2 + 2*b^2)*c^3*d^
5 + (2*b^2*c^8 - 8*a*b*c^7*d + 10*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + 3*a^2*c^2*
d^6 + (6*a^2 - b^2)*c^6*d^2 - (9*a^2 + b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x
 + e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^1
0*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d
^4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(a^2*c^8 - 3*a^2*c^6*d^2 + 3*a^2*c^4*
d^4 - a^2*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a^2*c^7*d - 3*a^2*c^5*d^3 + 3*a
^2*c^3*d^5 - a^2*c*d^7)*f*x*cos(f*x + e) + 2*(a^2*c^6*d^2 - 3*a^2*c^4*d^4
+ 3*a^2*c^2*d^6 - a^2*d^8)*f*x + (4*a*b*c^5*d^2 + 2*a*b*c^3*d^4 + 5*a^2*c^
2*d^5 - 2*a^2*d^7 - 3*(2*a^2 + b^2)*c^4*d^3 + (4*a*b*c^7 + 2*a*b*c^5*d^...
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx=\int \frac{(a+b\sec{(e+fx)})^2}{(c+d\sec{(e+fx)})^3}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(223) = 446$.

Time = 0.23 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
((4*a*b*c^5 - 6*a^2*c^4*d - 3*b^2*c^4*d + 2*a*b*c^3*d^2 + 5*a^2*c^2*d^3 -
2*a^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*
tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 -
2*c^5*d^2 + c^3*d^4)*sqrt(-c^2 + d^2)) + (f*x + e)*a^2/c^3 - (2*b^2*c^5*ta
n(1/2*f*x + 1/2*e)^3 - 8*a*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - b^2*c^4*d*tan(
1/2*f*x + 1/2*e)^3 + 6*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*b*c^3*d^2*
tan(1/2*f*x + 1/2*e)^3 + b^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^2*c^2*d^
3*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b^2*c^
2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*a^2*
d^5*tan(1/2*f*x + 1/2*e)^3 - 2*b^2*c^5*tan(1/2*f*x + 1/2*e) + 8*a*b*c^4*d*
tan(1/2*f*x + 1/2*e) - b^2*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^2*c^3*d^2*tan(
1/2*f*x + 1/2*e) + 6*a*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - b^2*c^3*d^2*tan(1/
2*f*x + 1/2*e) - 5*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*a*b*c^2*d^3*tan(1/
2*f*x + 1/2*e) - 2*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 3*a^2*c*d^4*tan(1/2*
f*x + 1/2*e) + 2*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4
)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 8682, normalized size of antiderivative = 36.63

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^3,x)
```

output
```
((tan(e/2 + (f*x)/2)^3*(2*b^2*c^4 - 2*a^2*d^4 + a^2*c*d^3 + b^2*c^3*d + 6*
a^2*c^2*d^2 + 2*b^2*c^2*d^2 - 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c^2*d - c^3)
*(c + d)^2) - (tan(e/2 + (f*x)/2)*(2*a^2*d^4 - 2*b^2*c^4 + a^2*c*d^3 + b^2
*c^3*d - 6*a^2*c^2*d^2 - 2*b^2*c^2*d^2 + 8*a*b*c^3*d - 2*a*b*c^2*d^2))/((c
 + d)*(c^4 - 2*c^3*d + c^2*d^2)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2
- 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) - (2*a^2
*atan(((a^2*((a^2*((8*(4*a^2*c^15 - 12*a^2*c^14*d - 6*b^2*c^14*d - 4*a^2*c
^6*d^9 + 2*a^2*c^7*d^8 + 18*a^2*c^8*d^7 - 4*a^2*c^9*d^6 - 36*a^2*c^10*d^5
+ 6*a^2*c^11*d^4 + 34*a^2*c^12*d^3 - 8*a^2*c^13*d^2 + 6*b^2*c^9*d^6 - 6*b^
2*c^10*d^5 - 12*b^2*c^11*d^4 + 12*b^2*c^12*d^3 + 6*b^2*c^13*d^2 + 8*a*b*c^
15 - 8*a*b*c^14*d - 4*a*b*c^8*d^7 + 4*a*b*c^9*d^6 + 12*a*b*c^12*d^3 - 12*a
*b*c^13*d^2))/(c^12*d + c^13 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 -
 3*c^10*d^3 - 3*c^11*d^2) - (a^2*tan(e/2 + (f*x)/2)*(8*c^15*d - 8*c^6*d^10
 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 - 48*c^10*d^6 + 48*c^11*d^5 + 32*c^
12*d^4 - 32*c^13*d^3 - 8*c^14*d^2)*8i)/(c^3*(c^10*d + c^11 - c^4*d^7 - c^5
*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)))*1i)/c^3 + (8*tan(e
/2 + (f*x)/2)*(4*a^4*c^10 + 8*a^4*d^10 - 8*a^4*c*d^9 - 8*a^4*c^9*d + 16*a^
2*b^2*c^10 - 32*a^4*c^2*d^8 + 32*a^4*c^3*d^7 + 57*a^4*c^4*d^6 - 48*a^4*c^5
*d^5 - 52*a^4*c^6*d^4 + 32*a^4*c^7*d^3 + 24*a^4*c^8*d^2 + 9*b^4*c^8*d^2 -
12*a*b^3*c^7*d^3 - 8*a^3*b*c^3*d^7 + 4*a^3*b*c^5*d^5 + 16*a^3*b*c^7*d^3...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1969, normalized size of antiderivative = 8.31

$$\int \frac{(a + b \sec(e + fx))^2}{(c + d \sec(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

output

```
( - 24*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*cos(e + f*x)*a**2*c**5*d**2 + 20*sqrt( - c**2 + d**
2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*co
s(e + f*x)*a**2*c**3*d**4 - 8*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*
c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a**2*c*d**6 + 1
6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt
( - c**2 + d**2))*cos(e + f*x)*a*b*c**6*d + 8*sqrt( - c**2 + d**2)*atan((t
an((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)
*a*b*c**4*d**3 - 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e
 + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*b**2*c**5*d**2 + 12*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*sin(e + f*x)**2*a**2*c**6*d - 10*sqrt( - c**2 + d**2)*atan((tan
((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**
2*a**2*c**4*d**3 + 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((
e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*a**2*c**2*d**5 - 8*sq
rt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( -
c**2 + d**2))*sin(e + f*x)**2*a*b*c**7 - 4*sqrt( - c**2 + d**2)*atan((tan(
(e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2
*a*b*c**5*d**2 + 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e
+ f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*b**2*c**6*d - 12*sqr...
```

3.194 $\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 377

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \frac{a^2x}{c^4} - \frac{(b^2c^4d(4c^2+d^2)-ab(4c^7+6c^5d^2)+a^2(8c^6d-8c^4d^3+7c^2d^5-2d^7))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^4(c-d)^{7/2}(c+d)^{7/2}f} + \frac{d^2(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3} - \frac{d(bc-ad)\,(6bc^3-8ac^2d-bcd^2+3ad^3)\sin(e+fx)}{6c^3\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^2} - \frac{(2abcd(18c^4-5c^2d^2+2d^4)-a^2d^2(34c^4-28c^2d^2+9d^4)-b^2(6c^6+10c^4d^2-c^2d^4))\sin(e+fx)}{6c^3\,(c^2-d^2)^3\,f(d+c\cos(e+fx))}$$

output
```
a^2*x/c^4-(b^2*c^4*d*(4*c^2+d^2)-a*b*(4*c^7+6*c^5*d^2)+a^2*(8*c^6*d-8*c^4*
d^3+7*c^2*d^5-2*d^7))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/
c^4/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*d^2*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2
-d^2)/f/(d+c*cos(f*x+e))^3-1/6*d*(-a*d+b*c)*(-8*a*c^2*d+3*a*d^3+6*b*c^3-b*
c*d^2)*sin(f*x+e)/c^3/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(2*a*b*c*d*(18*
c^4-5*c^2*d^2+2*d^4)-a^2*d^2*(34*c^4-28*c^2*d^2+9*d^4)-b^2*(6*c^6+10*c^4*d
^2-c^2*d^4))*sin(f*x+e)/c^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))
```

Mathematica [A] (verified)

Time = 4.02 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))^2\left(6a^2(e+fx)(d+c\cos(e+fx))^3+\frac{6(b^2c^4d(4c^2+d^2)-\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^2*(6*a^2*(e + f*
x)*(d + c*Cos[e + f*x])^3 + (6*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2
 + 3*d^2) + a^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[((-c +
d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3)/(c^2 - d^2)^
(7/2) + (2*c*d^2*(b*c - a*d)^2*Sin[e + f*x])/(c^2 - d^2) - (c*d*(a^2*d^2*(
12*c^2 - 7*d^2) + b^2*(6*c^4 - c^2*d^2) + a*b*(-18*c^3*d + 8*c*d^3))*(d +
c*Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-2*a*b*c*d*(18*c^4 - 5*c
^2*d^2 + 2*d^4) + a^2*d^2*(36*c^4 - 32*c^2*d^2 + 11*d^4) + b^2*(6*c^6 + 10
*c^4*d^2 - c^2*d^4))*(d + c*Cos[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3))/
(6*c^4*f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^4)
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4429, 3042, 3527, 25, 3042, 3510, 3042, 3500, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{\cos^2(e+fx)(a\cos(e+fx)+b)^2}{(c\cos(e+fx)+d)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^2}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^4}dx$$

$$\Big\downarrow \text{3527}$$

$$\frac{\int -\frac{(b+a\cos(e+fx))\left(-3a\left(c^2-d^2\right)\cos^2(e+fx)-\left(3bc^2-3adc-bd^2\right)\cos(e+fx)+d(3bc-2ad)\right)}{(d+c\cos(e+fx))^3}dx}{3c\left(c^2-d^2\right)}+\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\int \frac{(b+a\cos(e+fx))\left(-3a\left(c^2-d^2\right)\cos^2(e+fx)+\left(3acd-b\left(3c^2-d^2\right)\right)\cos(e+fx)+d(3bc-2ad)\right)}{(d+c\cos(e+fx))^3}dx}{3c\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(-3a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(3acd-b\left(3c^2-d^2\right)\right)\sin\left(e+fx+\frac{\pi}{2}\right)+d(3bc-2ad)\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{3c\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{3510}$$

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-$$

$$\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{6a^2c\left(c^2-d^2\right)^2\cos^2(e+fx)-\big(\big(\left(3d^5-10c^2d^3+12c^4d\right)a^2-2bc\left(6c^4-3d^2c^2+2d^4\right)a+b^2c^2d\big(6c^2-d^2\big)}{(d+c\cos(e+fx))^2}}{2c^2(c^2-d^2)}}{3c\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2 \sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{6a^2c\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-\left(\left(3d^5-10c^2d^3+12c^4d\right)a^2\right)+2bc\left(6c^4-3d^2c^2+2d^4\right)a-b^2c^2d\left(\cdots\right.\right.}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\cdots\right.\right.}}{2c^2(c^2-d^2)\cdots}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3500

$$\frac{d^2 \sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{\int -\frac{3\left(c^2\left(-2ab\left(2c^2+3d^2\right)c^3+b^2d\left(4c^2+d^2\right)c^2+a^2\left(d^5-2c^2d^3+6c^4d\right)\right)-2a^2c\left(c^2-d^2\right)^3\cos(e+fx)\right)\cdots}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}\cdots}{2c^2(c\cdots}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 27

$$\frac{d^2 \sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\int \frac{c^2\left(-2ab\left(2c^2+3d^2\right)c^3+b^2d\left(4c^2+d^2\right)c^2+a^2d\left(6c^4-2d^2c^2+d^4\right)\right)-2a^2c\left(c^2-d^2\right)^3\cos(e+fx)}{d+c\cos(e+fx)}dx\cdots}{c\left(c^2-d^2\right)}\cdots}{2c^2(c^2\cdots}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3042

$$\frac{d^2 \sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\int \frac{c^2\left(-2ab\left(2c^2+3d^2\right)c^3+b^2d\left(4c^2+d^2\right)c^2+a^2d\left(6c^4-2d^2c^2+d^4\right)\right)-2a^2c\left(c^2-d^2\right)^3\sin\left(e+fx+\frac{\pi}{2}\cdots\right.}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}\cdots}{c\left(c^2-d^2\right)}\cdots}{2c^2(\cdots}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3214

$$\frac{d^2 \sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\int \frac{1}{d+c\cos(e+fx)}dx-2a^2\cdots\right.}{c\left(c^2-d^2\right)}\cdots}{2\cdots}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\int\frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx-\cdots\right.}{c\left(c^2-d^2\right)}}{\cdots}}{3c\left(c^2-d^2\right)}$$

$\downarrow$ 3138

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\frac{2\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\int\frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+f\cdots\right.\right.}}{f}\right.}{c\left(c^2-d^2\right)}}{\cdots}}{3c\left(c^2-d^2\cdots\right.}$$

$\downarrow$ 221

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\frac{d(bc-ad)\left(-8ac^2d+3ad^3+6bc^3-bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{-\frac{3\left(\frac{2\left(a^2\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-2abc^5\left(2c^2+3d^2\right)+b^2c^4d\left(4c^2+d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\cdots\right.}{\sqrt{c\cdots}}\right.}{f\sqrt{c-d}\sqrt{c+d}}\right.}{c\left(c^2-d^2\right)}}{\cdots}}{3c\left(c^2-d^2\right)}$$

input
```
Int[(a + b*Sec[e + f*x])^2/(c + d*Sec[e + f*x])^4,x]
```

output
```
(d^2*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e
+ f*x])^3) - ((d*(b*c - a*d)*(6*b*c^3 - 8*a*c^2*d - b*c*d^2 + 3*a*d^3)*Sin
[e + f*x])/(2*c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) - ((-3*(-2*a^2*(c^
2 - d^2)^3*x + (2*(b^2*c^4*d*(4*c^2 + d^2) - 2*a*b*c^5*(2*c^2 + 3*d^2) + a
^2*(8*c^6*d - 8*c^4*d^3 + 7*c^2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e
+ f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*f)))/(c*(c^2 - d^2)) - (
(2*a*b*c*d*(18*c^4 - 5*c^2*d^2 + 2*d^4) - a^2*d^2*(34*c^4 - 28*c^2*d^2 + 9
*d^4) - b^2*(6*c^6 + 10*c^4*d^2 - c^2*d^4))*Sin[e + f*x])/((c^2 - d^2)*f*(
d + c*Cos[e + f*x])))/(2*c^2*(c^2 - d^2)))/(3*c*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12abd\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+\right.}{2(c-d)\left(c^3+3c^2d+3d^2c+d^3\right)}}{}}{}$
default	$\frac{\frac{2a^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^2c^4d^2+4a^2c^3d^3-6a^2d^4c^2-a^2c\,d^5+2a^2d^6-12abd\,c^5-6ab\,c^4d^2-4ab\,d^3c^3+2b^2c^6+2b^2c^5d+\right.}{2(c-d)\left(c^3+3c^2d+3d^2c+d^3\right)}}{}}{}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^2/c^4*arctan(tan(1/2*f*x+1/2*e))+2/c^4*((-1/2*(12*a^2*c^4*d^2+4*a
^2*c^3*d^3-6*a^2*c^2*d^4-a^2*c*d^5+2*a^2*d^6-12*a*b*c^5*d-6*a*b*c^4*d^2-4*
a*b*c^3*d^3+2*b^2*c^6+2*b^2*c^5*d+6*b^2*c^4*d^2+b^2*c^3*d^3)*c/(c-d)/(c^3+
3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(18*a^2*c^4*d^2-11*a^2*c^2*d
^4+3*a^2*d^6-18*a*b*c^5*d-2*a*b*c^3*d^3+3*b^2*c^6+7*b^2*c^4*d^2)*c/(c^2-2*
c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/2*(12*a^2*c^4*d^2-4*a^2*c^
3*d^3-6*a^2*c^2*d^4+a^2*c*d^5+2*a^2*d^6-12*a*b*c^5*d+6*a*b*c^4*d^2-4*a*b*c
^3*d^3+2*b^2*c^6-2*b^2*c^5*d+6*b^2*c^4*d^2-b^2*c^3*d^3)*c/(c+d)/(c^3-3*c^2
*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/
2*e)^2*d-c-d)^3-1/2*(8*a^2*c^6*d-8*a^2*c^4*d^3+7*a^2*c^2*d^5-2*a^2*d^7-4*a
*b*c^7-6*a*b*c^5*d^2+4*b^2*c^6*d+b^2*c^4*d^3)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6
)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(362) = 724$.

Time = 0.25 (sec) , antiderivative size = 2362, normalized size of antiderivative = 6.27

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^2*c^11 - 4*a^2*c^9*d^2 + 6*a^2*c^7*d^4 - 4*a^2*c^5*d^6 + a^2*
c^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^2*c^10*d - 4*a^2*c^8*d^3 + 6*a^2*c^6*d
^5 - 4*a^2*c^4*d^7 + a^2*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^2*c^9*d^2 - 4
*a^2*c^7*d^4 + 6*a^2*c^5*d^6 - 4*a^2*c^3*d^8 + a^2*c*d^10)*f*x*cos(f*x + e
) + 12*(a^2*c^8*d^3 - 4*a^2*c^6*d^5 + 6*a^2*c^4*d^7 - 4*a^2*c^2*d^9 + a^2*
d^11)*f*x - 3*(4*a*b*c^7*d^3 + 6*a*b*c^5*d^5 - 7*a^2*c^2*d^8 + 2*a^2*d^10
- 4*(2*a^2 + b^2)*c^6*d^4 + (8*a^2 - b^2)*c^4*d^6 + (4*a*b*c^10 + 6*a*b*c^
8*d^2 - 7*a^2*c^5*d^5 + 2*a^2*c^3*d^7 - 4*(2*a^2 + b^2)*c^9*d + (8*a^2 - b
^2)*c^7*d^3)*cos(f*x + e)^3 + 3*(4*a*b*c^9*d + 6*a*b*c^7*d^3 - 7*a^2*c^4*d
^6 + 2*a^2*c^2*d^8 - 4*(2*a^2 + b^2)*c^8*d^2 + (8*a^2 - b^2)*c^6*d^4)*cos(
f*x + e)^2 + 3*(4*a*b*c^8*d^2 + 6*a*b*c^6*d^4 - 7*a^2*c^3*d^7 + 2*a^2*c*d^
9 - 4*(2*a^2 + b^2)*c^7*d^3 + (8*a^2 - b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^
2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*b^2*c^9*d^2 - 22*a*b*c^8*d^3 + 14
*a*b*c^6*d^5 + 8*a*b*c^4*d^7 + 23*a^2*c^3*d^8 - 6*a^2*c*d^10 + (26*a^2 + 1
1*b^2)*c^7*d^4 - (43*a^2 + 13*b^2)*c^5*d^6 + (6*b^2*c^11 - 36*a*b*c^10*d +
 46*a*b*c^8*d^3 - 14*a*b*c^6*d^5 + 4*a*b*c^4*d^7 - 11*a^2*c^3*d^8 + 4*(9*a
^2 + b^2)*c^9*d^2 - (68*a^2 + 11*b^2)*c^7*d^4 + (43*a^2 + b^2)*c^5*d^6)*co
s(f*x + e)^2 + 3*(2*b^2*c^10*d - 18*a*b*c^9*d^2 + 16*a*b*c^7*d^4 + 2*a*...
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx=\int \frac{(a+b\sec{(e+fx)})^2}{(c+d\sec{(e+fx)})^4}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

output
```
Integral((a + b*sec(e + f*x))**2/(c + d*sec(e + f*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(362) = 724$.

Time = 0.24 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.19

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
1/3*(3*(4*a*b*c^7 - 8*a^2*c^6*d - 4*b^2*c^6*d + 6*a*b*c^5*d^2 + 8*a^2*c^4*
d^3 - b^2*c^4*d^3 - 7*a^2*c^2*d^5 + 2*a^2*d^7)*(pi*floor(1/2*(f*x + e)/pi
+ 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
 1/2*e))/sqrt(-c^2 + d^2)))/((c^10 - 3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt
(-c^2 + d^2)) + 3*(f*x + e)*a^2/c^4 - (6*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 -
36*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 6*b^2*c^7*d*tan(1/2*f*x + 1/2*e)^5 +
 36*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 54*a*b*c^6*d^2*tan(1/2*f*x + 1/2*
e)^5 + 12*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^2*c^5*d^3*tan(1/2*f*x
+ 1/2*e)^5 - 12*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 27*b^2*c^5*d^3*tan(1/
2*f*x + 1/2*e)^5 - 6*a^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*c^4*d^4*ta
n(1/2*f*x + 1/2*e)^5 + 12*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 45*a^2*c^3*
d^5*tan(1/2*f*x + 1/2*e)^5 - 12*a*b*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b^2
*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 1
5*a^2*c*d^7*tan(1/2*f*x + 1/2*e)^5 + 6*a^2*d^8*tan(1/2*f*x + 1/2*e)^5 - 12
*b^2*c^8*tan(1/2*f*x + 1/2*e)^3 + 72*a*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72
*a^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^3 - 16*b^2*c^6*d^2*tan(1/2*f*x + 1/2*e)^
3 - 64*a*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 + 116*a^2*c^4*d^4*tan(1/2*f*x +
1/2*e)^3 + 28*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*a*b*c^3*d^5*tan(1/2*f
*x + 1/2*e)^3 - 56*a^2*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 12*a^2*d^8*tan(1/2
*f*x + 1/2*e)^3 + 6*b^2*c^8*tan(1/2*f*x + 1/2*e) - 36*a*b*c^7*d*tan(1/2...
```

Mupad [B] (verification not implemented)

Time = 23.43 (sec) , antiderivative size = 12818, normalized size of antiderivative = 34.00

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^2/(c + d/cos(e + f*x))^4,x)
```

output
```
(2*a^2*atan(((a^2*((8*tan(e/2 + (f*x)/2)*(4*a^4*c^14 + 8*a^4*d^14 - 8*a^4*
c*d^13 - 8*a^4*c^13*d + 16*a^2*b^2*c^14 - 48*a^4*c^2*d^12 + 48*a^4*c^3*d^1
1 + 117*a^4*c^4*d^10 - 120*a^4*c^5*d^9 - 164*a^4*c^6*d^8 + 160*a^4*c^7*d^7
 + 156*a^4*c^8*d^6 - 120*a^4*c^9*d^5 - 92*a^4*c^10*d^4 + 48*a^4*c^11*d^3 +
 44*a^4*c^12*d^2 + b^4*c^8*d^6 + 8*b^4*c^10*d^4 + 16*b^4*c^12*d^2 - 12*a*b
^3*c^9*d^5 - 56*a*b^3*c^11*d^3 + 24*a^3*b*c^5*d^9 - 68*a^3*b*c^7*d^7 + 40*
a^3*b*c^9*d^5 - 32*a^3*b*c^11*d^3 - 4*a^2*b^2*c^4*d^10 - 2*a^2*b^2*c^6*d^8
 + 40*a^2*b^2*c^8*d^6 - 12*a^2*b^2*c^10*d^4 + 112*a^2*b^2*c^12*d^2 - 32*a*
b^3*c^13*d - 64*a^3*b*c^13*d))/(c^16*d + c^17 - c^6*d^11 - c^7*d^10 + 5*c^
8*d^9 + 5*c^9*d^8 - 10*c^10*d^7 - 10*c^11*d^6 + 10*c^12*d^5 + 10*c^13*d^4
- 5*c^14*d^3 - 5*c^15*d^2) + (a^2*((8*(4*a^2*c^21 - 16*a^2*c^20*d - 8*b^2*
c^20*d - 4*a^2*c^8*d^13 + 2*a^2*c^9*d^12 + 26*a^2*c^10*d^11 - 14*a^2*c^11*
d^10 - 70*a^2*c^12*d^9 + 30*a^2*c^13*d^8 + 110*a^2*c^14*d^7 - 30*a^2*c^15*
d^6 - 110*a^2*c^16*d^5 + 20*a^2*c^17*d^4 + 64*a^2*c^18*d^3 - 12*a^2*c^19*d
^2 - 2*b^2*c^11*d^10 + 2*b^2*c^12*d^9 - 2*b^2*c^13*d^8 + 2*b^2*c^14*d^7 +
18*b^2*c^15*d^6 - 18*b^2*c^16*d^5 - 22*b^2*c^17*d^4 + 22*b^2*c^18*d^3 + 8*
b^2*c^19*d^2 + 8*a*b*c^21 - 8*a*b*c^20*d + 12*a*b*c^12*d^9 - 12*a*b*c^13*d
^8 - 28*a*b*c^14*d^7 + 28*a*b*c^15*d^6 + 12*a*b*c^16*d^5 - 12*a*b*c^17*d^4
 + 12*a*b*c^18*d^3 - 12*a*b*c^19*d^2))/(c^19*d + c^20 - c^9*d^11 - c^10*d^
10 + 5*c^11*d^9 + 5*c^12*d^8 - 10*c^13*d^7 - 10*c^14*d^6 + 10*c^15*d^5 ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3809, normalized size of antiderivative = 10.10

$$\int \frac{(a+b\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x)
```

output

```
( - 48*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a**2*c**9*d + 48*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a**2*c**7*d**3 - 42*sqrt( - c**2 +
 d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2)
)*cos(e + f*x)*sin(e + f*x)**2*a**2*c**5*d**5 + 12*sqrt( - c**2 + d**2)*at
an((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e +
 f*x)*sin(e + f*x)**2*a**2*c**3*d**7 + 24*sqrt( - c**2 + d**2)*atan((tan((
e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin
(e + f*x)**2*a*b*c**10 + 36*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c
- tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a
*b*c**8*d**2 - 24*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e +
 f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*b**2*c**9*d
 - 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/s
qrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*b**2*c**7*d**3 + 48*sqrt
( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c*
*2 + d**2))*cos(e + f*x)*a**2*c**9*d + 96*sqrt( - c**2 + d**2)*atan((tan((
e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a**
2*c**7*d**3 - 102*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e +
 f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a**2*c**5*d**5 + 114*sqr...
```

3.195 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \frac{a^3x}{c^3} - \frac{(bc-ad)\left(2abcd(4c^2-d^2)-b^2c^2(c^2+2d^2)-a^2(6c^4-5c^2d^2+2d^4)\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{c^3(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)^2(b+a\cos(e+fx))\sin(e+fx)}{2c\left(c^2-d^2\right)f(d+c\cos(e+fx))^2} + \frac{(bc-ad)^2\left(5ac^2-3bcd-2ad^2\right)\sin(e+fx)}{2c^2\left(c^2-d^2\right)^2f(d+c\cos(e+fx))}$$

output
```
a^3*x/c^3-(-a*d+b*c)*(2*a*b*c*d*(4*c^2-d^2)-b^2*c^2*(c^2+2*d^2)-a^2*(6*c^4
-5*c^2*d^2+2*d^4))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c^3
/(c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)^2*(b+a*cos(f*x+e))*sin(f*x+e)/c/
(c^2-d^2)/f/(d+c*cos(f*x+e))^2+1/2*(-a*d+b*c)^2*(5*a*c^2-2*a*d^2-3*b*c*d)*
sin(f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 517 vs. $2(254) = 508$.

Time = 2.10 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{-\frac{4(-9ab^2c^4d+3a^2bc^3(2c^2+d^2)+b^3c^3(c^2+2d^2)+a^3(-6c^4d+5c^2d^3-2d^5))\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}}+\frac{2a^3c^6e-6a^3c^2d^4e+4a^3d^6e}{}}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

output
```
((-4*(-9*a*b^2*c^4*d + 3*a^2*b*c^3*(2*c^2 + d^2) + b^3*c^3*(c^2 + 2*d^2) +
 a^3*(-6*c^4*d + 5*c^2*d^3 - 2*d^5))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(c^2 - d^2)^(5/2) + (2*a^3*c^6*e - 6*a^3*c^2*d^4*e + 4*a^
3*d^6*e + 2*a^3*c^6*f*x - 6*a^3*c^2*d^4*f*x + 4*a^3*d^6*f*x + 8*a^3*c*d*(c
^2 - d^2)^2*(e + f*x)*Cos[e + f*x] + 2*a^3*(c^3 - c*d^2)^2*(e + f*x)*Cos[2
*(e + f*x)] + 2*b^3*c^6*Sin[e + f*x] + 6*a*b^2*c^5*d*Sin[e + f*x] - 18*a^2
*b*c^4*d^2*Sin[e + f*x] - 8*b^3*c^4*d^2*Sin[e + f*x] + 10*a^3*c^3*d^3*Sin[
e + f*x] + 12*a*b^2*c^3*d^3*Sin[e + f*x] - 4*a^3*c*d^5*Sin[e + f*x] + 6*a*
b^2*c^6*Sin[2*(e + f*x)] - 12*a^2*b*c^5*d*Sin[2*(e + f*x)] - 3*b^3*c^5*d*S
in[2*(e + f*x)] + 6*a^3*c^4*d^2*Sin[2*(e + f*x)] + 3*a*b^2*c^4*d^2*Sin[2*(
e + f*x)] + 3*a^2*b*c^3*d^3*Sin[2*(e + f*x)] - 3*a^3*c^2*d^4*Sin[2*(e + f*
x)])/((c^2 - d^2)^2*(d + c*Cos[e + f*x])^2))/(4*c^3*f)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4429, 3042, 3271, 3042, 3500, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{(a\cos(e+fx)+b)^3}{(c\cos(e+fx)+d)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^3}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^3}dx$$

$$\Big\downarrow \text{3271}$$

$$\frac{\int \frac{d^2a^3+2\left(c^2-d^2\right)\cos^2(e+fx)a^3-4bcda^2+5b^2c^2a-2b^3cd+\left(-2cda^3+b\left(6c^2-d^2\right)a^2-4b^2cda+b^3c^2\right)\cos(e+fx)}{(d+c\cos(e+fx))^2}dx}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{d^2a^3+2\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2a^3-4bcda^2+5b^2c^2a-2b^3cd+\left(-2cda^3+b\left(6c^2-d^2\right)a^2-4b^2cda+b^3c^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\frac{\int -\frac{c\left(\left(4c^2d-d^3\right)a^3-3bc\left(2c^2+d^2\right)a^2+9b^2c^2da-b^3c\left(c^2+2d^2\right)\right)-2a^3\left(c^2-d^2\right)^2\cos(e+fx)}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}+\frac{(5ac^2-2ad^2-3bcd)(bc-ad)^2\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\int \frac{c\left(\left(4c^2d-d^3\right)a^3-3bc\left(2c^2+d^2\right)a^2+9b^2c^2da-b^3c\left(c^2+2d^2\right)\right)-2a^3\left(c^2-d^2\right)^2\cos(e+fx)}{d+c\cos(e+fx)}dx}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\int \frac{c\left(\left(4c^2d-d^3\right)a^3-3bc\left(2c^2+d^2\right)a^2+9b^2c^2da-b^3c\left(c^2+2d^2\right)\right)-2a^3\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^2}$$

$\Big\downarrow$ 3214

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\int\frac{1}{d+c\cos(e+fx)}dx}{c}-\frac{2a^3x\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\int\frac{1}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{c}-\frac{2a^3x\left(c^2-d^2\right)^2}{c}}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{(bc-ad)^2(5ac^2-2ad^2-3bcd)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{2\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\int\frac{1}{-\left((c-d)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+c+d}d\tan}{cf}}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}+\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{(bc-ad)^2\left(5ac^2-2ad^2-3bcd\right)\sin(e+fx)}{cf(c^2-d^2)(c\cos(e+fx)+d)}-\frac{\frac{2\left(a^3\left(6c^4d-5c^2d^3+2d^5\right)-3a^2bc^3\left(2c^2+d^2\right)+9ab^2c^4d-b^3c^3\left(c^2+2d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{cf\sqrt{c-d}\sqrt{c+d}}-}{c(c^2-d^2)}}{2c\left(c^2-d^2\right)}$$
$$\frac{(bc-ad)^2\sin(e+fx)(a\cos(e+fx)+b)}{2cf\left(c^2-d^2\right)\left(c\cos(e+fx)+d\right)^2}$$

input
```
Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^3,x]
```

output
```
((b*c - a*d)^2*(b + a*Cos[e + f*x])*Sin[e + f*x])/(2*c*(c^2 - d^2)*f*(d +
c*Cos[e + f*x])^2) + (-(((-2*a^3*(c^2 - d^2)^2*x)/c + (2*(9*a*b^2*c^4*d -
3*a^2*b*c^3*(2*c^2 + d^2) - b^3*c^3*(c^2 + 2*d^2) + a^3*(6*c^4*d - 5*c^2*d
^3 + 2*d^5))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(c*Sqrt[
c - d]*Sqrt[c + d]*f))/(c*(c^2 - d^2))) + ((b*c - a*d)^2*(5*a*c^2 - 3*b*c*
d - 2*a*d^2)*Sin[e + f*x])/(c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])))/(2*c*(c
^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{\frac{2a^3 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^3c^2d^2+a^3d^3c-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)}{2(c-d)\left(c^2+2cd+d^2\right)}\right.}{\left(\tan\left(\right.\right.}}{}$
default	$\frac{\frac{2a^3 \arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^3}+\frac{2\left(-\frac{\left(6a^3c^2d^2+a^3d^3c-2a^3d^4-12a^2b\,c^3d-3a^2b\,c^2d^2+6a\,b^2c^4+3a\,b^2c^3d+6a\,b^2c^2d^2-b^3c^4-4b^3c^3d\right)}{2(c-d)\left(c^2+2cd+d^2\right)}\right.}{\left(\tan\left(\right.\right.}}{}$
risch	Expression too large to display

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^3/c^3*arctan(tan(1/2*f*x+1/2*e))+2/c^3*((-1/2*(6*a^3*c^2*d^2+a^3*
c*d^3-2*a^3*d^4-12*a^2*b*c^3*d-3*a^2*b*c^2*d^2+6*a*b^2*c^4+3*a*b^2*c^3*d+6
*a*b^2*c^2*d^2-b^3*c^4-4*b^3*c^3*d)*c/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/
2*e)^3+1/2*c*(6*a^3*c^2*d^2-a^3*c*d^3-2*a^3*d^4-12*a^2*b*c^3*d+3*a^2*b*c^2
*d^2+6*a*b^2*c^4-3*a*b^2*c^3*d+6*a*b^2*c^2*d^2+b^3*c^4-4*b^3*c^3*d)/(c+d)/
(c-d)^2*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d
-c-d)^2-1/2*(6*a^3*c^4*d-5*a^3*c^2*d^3+2*a^3*d^5-6*a^2*b*c^5-3*a^2*b*c^3*d
^2+9*a*b^2*c^4*d-b^3*c^5-2*b^3*c^3*d^2)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^
(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(238) = 476.

Time = 0.21 (sec) , antiderivative size = 1629, normalized size of antiderivative = 6.41

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d^4 - a^3*c^2*d^6)*f*x*cos(f*
x + e)^2 + 8*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^3*c^3*d^5 - a^3*c*d^7)*f*x*c
os(f*x + e) + 4*(a^3*c^6*d^2 - 3*a^3*c^4*d^4 + 3*a^3*c^2*d^6 - a^3*d^8)*f*
x - (5*a^3*c^2*d^5 - 2*a^3*d^7 + (6*a^2*b + b^3)*c^5*d^2 - 3*(2*a^3 + 3*a*
b^2)*c^4*d^3 + (3*a^2*b + 2*b^3)*c^3*d^4 + (5*a^3*c^4*d^3 - 2*a^3*c^2*d^5
+ (6*a^2*b + b^3)*c^7 - 3*(2*a^3 + 3*a*b^2)*c^6*d + (3*a^2*b + 2*b^3)*c^5*
d^2)*cos(f*x + e)^2 + 2*(5*a^3*c^3*d^4 - 2*a^3*c*d^6 + (6*a^2*b + b^3)*c^6
*d - 3*(2*a^3 + 3*a*b^2)*c^5*d^2 + (3*a^2*b + 2*b^3)*c^4*d^3)*cos(f*x + e)
)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 -
 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*c
os(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b^3*c^8 + 3*a*b^2*c^7*d +
2*a^3*c*d^7 - (9*a^2*b + 5*b^3)*c^6*d^2 + (5*a^3 + 3*a*b^2)*c^5*d^3 + (9*a
^2*b + 4*b^3)*c^4*d^4 - (7*a^3 + 6*a*b^2)*c^3*d^5 + 3*(2*a*b^2*c^8 - a^2*b
*c^3*d^5 + a^3*c^2*d^6 - (4*a^2*b + b^3)*c^7*d + (2*a^3 - a*b^2)*c^6*d^2 +
 (5*a^2*b + b^3)*c^5*d^3 - (3*a^3 + a*b^2)*c^4*d^4)*cos(f*x + e))*sin(f*x
+ e))/((c^11 - 3*c^9*d^2 + 3*c^7*d^4 - c^5*d^6)*f*cos(f*x + e)^2 + 2*(c^10
*d - 3*c^8*d^3 + 3*c^6*d^5 - c^4*d^7)*f*cos(f*x + e) + (c^9*d^2 - 3*c^7*d^
4 + 3*c^5*d^6 - c^3*d^8)*f), 1/2*(2*(a^3*c^8 - 3*a^3*c^6*d^2 + 3*a^3*c^4*d
^4 - a^3*c^2*d^6)*f*x*cos(f*x + e)^2 + 4*(a^3*c^7*d - 3*a^3*c^5*d^3 + 3*a^
3*c^3*d^5 - a^3*c*d^7)*f*x*cos(f*x + e) + 2*(a^3*c^6*d^2 - 3*a^3*c^4*d^...
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx=\int \frac{(a+b\sec{(e+fx)})^3}{(c+d\sec{(e+fx)})^3}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

output
```
Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(238) = 476$.

Time = 0.25 (sec) , antiderivative size = 818, normalized size of antiderivative = 3.22

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
((f*x + e)*a^3/c^3 + (6*a^2*b*c^5 + b^3*c^5 - 6*a^3*c^4*d - 9*a*b^2*c^4*d
+ 3*a^2*b*c^3*d^2 + 2*b^3*c^3*d^2 + 5*a^3*c^2*d^3 - 2*a^3*d^5)*(pi*floor(1
/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) -
 d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^7 - 2*c^5*d^2 + c^3*d^4)*s
qrt(-c^2 + d^2)) - (6*a*b^2*c^5*tan(1/2*f*x + 1/2*e)^3 - b^3*c^5*tan(1/2*f
*x + 1/2*e)^3 - 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^4*d*tan(
1/2*f*x + 1/2*e)^3 - 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^3*c^3*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*a*b^2*c^
3*d^2*tan(1/2*f*x + 1/2*e)^3 + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*a^
3*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^3
- 6*a*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e
)^3 + 2*a^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*a*b^2*c^5*tan(1/2*f*x + 1/2*e)
- b^3*c^5*tan(1/2*f*x + 1/2*e) + 12*a^2*b*c^4*d*tan(1/2*f*x + 1/2*e) - 3*a
*b^2*c^4*d*tan(1/2*f*x + 1/2*e) + 3*b^3*c^4*d*tan(1/2*f*x + 1/2*e) - 6*a^3
*c^3*d^2*tan(1/2*f*x + 1/2*e) + 9*a^2*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 3*a
*b^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 4*b^3*c^3*d^2*tan(1/2*f*x + 1/2*e) - 5
*a^3*c^2*d^3*tan(1/2*f*x + 1/2*e) - 3*a^2*b*c^2*d^3*tan(1/2*f*x + 1/2*e) -
 6*a*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 3*a^3*c*d^4*tan(1/2*f*x + 1/2*e) +
 2*a^3*d^5*tan(1/2*f*x + 1/2*e))/((c^6 - 2*c^4*d^2 + c^2*d^4)*(c*tan(1/2*f
*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 10759, normalized size of antiderivative = 42.36

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^3,x)
```

output
```
(atan(((((8*tan(e/2 + (f*x)/2)*(4*a^6*c^10 + 8*a^6*d^10 + b^6*c^10 - 8*a^6
*c*d^9 - 8*a^6*c^9*d + 12*a^2*b^4*c^10 + 36*a^4*b^2*c^10 - 32*a^6*c^2*d^8
+ 32*a^6*c^3*d^7 + 57*a^6*c^4*d^6 - 48*a^6*c^5*d^5 - 52*a^6*c^6*d^4 + 32*a
^6*c^7*d^3 + 24*a^6*c^8*d^2 + 4*b^6*c^6*d^4 + 4*b^6*c^8*d^2 - 36*a*b^5*c^7
*d^3 - 120*a^3*b^3*c^9*d - 12*a^5*b*c^3*d^7 + 6*a^5*b*c^5*d^5 + 24*a^5*b*c
^7*d^3 + 12*a^2*b^4*c^6*d^4 + 111*a^2*b^4*c^8*d^2 - 8*a^3*b^3*c^3*d^7 + 16
*a^3*b^3*c^5*d^5 - 68*a^3*b^3*c^7*d^3 + 36*a^4*b^2*c^4*d^6 - 81*a^4*b^2*c^
6*d^4 + 144*a^4*b^2*c^8*d^2 - 18*a*b^5*c^9*d - 72*a^5*b*c^9*d))/(c^10*d +
c^11 - c^4*d^7 - c^5*d^6 + 3*c^6*d^5 + 3*c^7*d^4 - 3*c^8*d^3 - 3*c^9*d^2)
+ ((a*d - b*c)*((8*(4*a^3*c^15 + 2*b^3*c^15 + 12*a^2*b*c^15 - 12*a^3*c^14*
d - 2*b^3*c^14*d - 4*a^3*c^6*d^9 + 2*a^3*c^7*d^8 + 18*a^3*c^8*d^7 - 4*a^3*
c^9*d^6 - 36*a^3*c^10*d^5 + 6*a^3*c^11*d^4 + 34*a^3*c^12*d^3 - 8*a^3*c^13*
d^2 - 4*b^3*c^8*d^7 + 4*b^3*c^9*d^6 + 6*b^3*c^10*d^5 - 6*b^3*c^11*d^4 + 18
*a*b^2*c^9*d^6 - 18*a*b^2*c^10*d^5 - 36*a*b^2*c^11*d^4 + 36*a*b^2*c^12*d^3
 + 18*a*b^2*c^13*d^2 - 6*a^2*b*c^8*d^7 + 6*a^2*b*c^9*d^6 + 18*a^2*b*c^12*d
^3 - 18*a^2*b*c^13*d^2 - 18*a*b^2*c^14*d - 12*a^2*b*c^14*d))/(c^12*d + c^1
3 - c^6*d^7 - c^7*d^6 + 3*c^8*d^5 + 3*c^9*d^4 - 3*c^10*d^3 - 3*c^11*d^2) -
 (4*tan(e/2 + (f*x)/2)*(a*d - b*c)*((c + d)^5*(c - d)^5)^(1/2)*(6*a^2*c^4
+ 2*a^2*d^4 + b^2*c^4 - 5*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + 2*a*b*c*d^3 - 8*a*
b*c^3*d)*(8*c^15*d - 8*c^6*d^10 + 8*c^7*d^9 + 32*c^8*d^8 - 32*c^9*d^7 -...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2529, normalized size of antiderivative = 9.96

$$\int \frac{(a + b\sec(e + fx))^3}{(c + d\sec(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

output

```
( - 24*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*cos(e + f*x)*a**3*c**5*d**2 + 20*sqrt( - c**2 + d**
2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*co
s(e + f*x)*a**3*c**3*d**4 - 8*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*
c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a**3*c*d**6 + 2
4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt
( - c**2 + d**2))*cos(e + f*x)*a**2*b*c**6*d + 12*sqrt( - c**2 + d**2)*ata
n((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e +
f*x)*a**2*b*c**4*d**3 - 36*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
 tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*a*b**2*c**5*d**2 +
 4*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqr
t( - c**2 + d**2))*cos(e + f*x)*b**3*c**6*d + 8*sqrt( - c**2 + d**2)*atan(
(tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*
x)*b**3*c**4*d**3 + 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan
((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*a**3*c**6*d - 10*sq
rt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( -
c**2 + d**2))*sin(e + f*x)**2*a**3*c**4*d**3 + 4*sqrt( - c**2 + d**2)*atan
((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f
*x)**2*a**3*c**2*d**5 - 12*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c -
 tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*sin(e + f*x)**2*a**2*b*c**7 ...
```

3.196 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 412

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \frac{a^3x}{c^4}$$
$$-\frac{(3ab^2c^4d(4c^2+d^2)-b^3c^5(c^2+4d^2)-a^2b(6c^7+9c^5d^2)+a^3(8c^6d-8c^4d^3+7c^2d^5-2d^7))\operatorname{arctanh}\left(\sqrt{\ldots}\right.}{c^4\sqrt{c-d}\sqrt{c+d}\,(c^2-d^2)^3 f}$$
$$-\frac{d(bc-ad)(b+a\cos(e+fx))^2\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))^3}$$
$$+\frac{(bc-ad)^2\,(3bc^3-8ac^2d+2bcd^2+3ad^3)\sin(e+fx)}{6c^3\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^2}$$
$$-\frac{(bc-ad)\,(b^2c^2d(13c^2+2d^2)-abc(18c^4+17c^2d^2-5d^4)+a^2(34c^4d-28c^2d^3+9d^5))\sin(e+fx)}{6c^3\,(c^2-d^2)^3\,f(d+c\cos(e+fx))}$$

output
```
a^3*x/c^4-(3*a*b^2*c^4*d*(4*c^2+d^2)-b^3*c^5*(c^2+4*d^2)-a^2*b*(6*c^7+9*c^
5*d^2)+a^3*(8*c^6*d-8*c^4*d^3+7*c^2*d^5-2*d^7))*arctanh((c-d)^(1/2)*tan(1/
2*f*x+1/2*e)/(c+d)^(1/2))/c^4/(c-d)^(1/2)/(c+d)^(1/2)/(c^2-d^2)^3/f-1/3*d*
(-a*d+b*c)*(b+a*cos(f*x+e))^2*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))^3+
1/6*(-a*d+b*c)^2*(-8*a*c^2*d+3*a*d^3+3*b*c^3+2*b*c*d^2)*sin(f*x+e)/c^3/(c^
2-d^2)^2/f/(d+c*cos(f*x+e))^2-1/6*(-a*d+b*c)*(b^2*c^2*d*(13*c^2+2*d^2)-a*b
*c*(18*c^4+17*c^2*d^2-5*d^4)+a^2*(34*c^4*d-28*c^2*d^3+9*d^5))*sin(f*x+e)/c
^3/(c^2-d^2)^3/f/(d+c*cos(f*x+e))
```

Mathematica [A] (verified)

Time = 4.38 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec(e+fx)(a+b\sec(e+fx))^3\left(6a^3(e+fx)(d+c\cos(e+fx))^3 - \frac{6\left(-3ab^2c^4d(4c^2+d^2\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]*(a + b*Sec[e + f*x])^3*(6*a^3*(e + f*x)
*(d + c*Cos[e + f*x])^3 - (6*(-3*a*b^2*c^4*d*(4*c^2 + d^2) + b^3*c^5*(c^2
+ 4*d^2) + a^2*b*(6*c^7 + 9*c^5*d^2) + a^3*(-8*c^6*d + 8*c^4*d^3 - 7*c^2*d
^5 + 2*d^7))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*C
os[e + f*x])^3)/(c^2 - d^2)^(7/2) - (2*c*d*(b*c - a*d)^3*Sin[e + f*x])/(c^
2 - d^2) + (c*(b*c - a*d)^2*(3*b*c^3 - 12*a*c^2*d + 2*b*c*d^2 + 7*a*d^3)*(
d + c*Cos[e + f*x])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(-(b^3*c^3*d*(13*c^2
+ 2*d^2)) + 3*a*b^2*c^2*(6*c^4 + 10*c^2*d^2 - d^4) - 3*a^2*b*c*d*(18*c^4 -
 5*c^2*d^2 + 2*d^4) + a^3*(36*c^4*d^2 - 32*c^2*d^4 + 11*d^6))*(d + c*Cos[e
 + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3))/(6*c^4*f*(b + a*Cos[e + f*x])^3*(
c + d*Sec[e + f*x])^4)
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4429, 3042, 3468, 3042, 3510, 3042, 3500, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{\cos(e+fx)(a\cos(e+fx)+b)^3}{(c\cos(e+fx)+d)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^3}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^4}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{\int \frac{(b+a\cos(e+fx))\left(3a^2\left(c^2-d^2\right)\cos^2(e+fx)-\left(3cda^2-b\left(6c^2-d^2\right)a+2b^2cd\right)\cos(e+fx)+(3bc-2ad)(bc-ad)\right)}{(d+c\cos(e+fx))^3}dx}{3c\left(c^2-d^2\right)}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(3a^2\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-3cda^2+b\left(6c^2-d^2\right)a-2b^2cd\right)\sin\left(e+fx+\frac{\pi}{2}\right)+(3bc-2ad)(bc-ad)\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^3}dx}{3c\left(c^2-d^2\right)}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{3510}$$

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{-6c\left(c^2-d^2\right)^2\cos^2(e+fx)a^3+2c(bc-ad)\left(\left(8c^2d-3d^3\right)a^2-bc\left(9c^2+d^2\right)a+5b^2c^2d\right)+\left(\left(3d^5-10c^2\right.\right.}{(d+c\cos(e+}}{2c^2(c^2-}}{3c\left(c^2-d^2\right)}-\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\int \frac{-6c\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)^2a^3+2c(bc-ad)\left(\left(8c^2d-3d^3\right)a^2-bc\left(9c^2+d^2\right)a+5b^2c^2d\right)+\left(\left(3d^5-\right.\right.}{\left(d+c\sin\left(e\right.\right.}}{2c^2(}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3500

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{\int \frac{3\left(c^2(bc-ad)\left(-\left(\left(6c^4-2d^2c^2+d^4\right)a^2\right)+bcd\left(11c^2-d^2\right)a-b^2c^2\left(c^2+4d^2\right)\right)-2a^3c\left(c^2-d^2\right)^3\cos(e\right.}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{2}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 27

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{3\int \frac{c^2(bc-ad)\left(-\left(\left(6c^4-2d^2c^2+d^4\right)a^2\right)+bcd\left(11c^2-d^2\right)a-b^2c^2\left(c^2+4d^2\right)\right)-2a^3c\left(c^2-d^2\right)^3\cos(e+}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{2c}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{\frac{3\int \frac{c^2(bc-ad)\left(-\left(\left(6c^4-2d^2c^2+d^4\right)a^2\right)+bcd\left(11c^2-d^2\right)a-b^2c^2\left(c^2+4d^2\right)\right)-2a^3c\left(c^2-d^2\right)^3\sin\left(e+\right.}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{c\left(c^2-d^2\right)}}{}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3214

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\int \frac{}{d+c}\right.}{c\left(c^2-d^2\right)}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3042

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\int\frac{}{d+c}\right.}{c\left(c^2-d^2\right)}}{3c\left(c^2-d^2\right)}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 3138

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\frac{2\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\int -}{f}\right.}{c\left(c^2-d^2\right)}}{3c(}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

$\downarrow$ 221

$$\frac{\frac{(bc-ad)^2\left(-8ac^2d+3ad^3+3bc^3+2bcd^2\right)\sin(e+fx)}{2c^2f(c^2-d^2)(c\cos(e+fx)+d)^2}-\frac{3\left(\frac{2\left(a^3\left(8c^6d-8c^4d^3+7c^2d^5-2d^7\right)-3a^2bc^5\left(2c^2+3d^2\right)+3ab^2c^4d\left(4c^2+d^2\right)-b^3c^5\left(c^2+4d^2\right)\right)\text{arc}}{f\sqrt{c-d}\sqrt{c+d}}\right.}{c\left(c^2-d^2\right)}}{3c\left(c^2-\right.}$$

$$\frac{d(bc-ad)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}$$

input
```
Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^4,x]
```

output
```
-1/3*(d*(b*c - a*d)*(b + a*Cos[e + f*x])^2*Sin[e + f*x])/(c*(c^2 - d^2)*f*
(d + c*Cos[e + f*x])^3) + (((b*c - a*d)^2*(3*b*c^3 - 8*a*c^2*d + 2*b*c*d^2
 + 3*a*d^3)*Sin[e + f*x])/(2*c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) - (
(3*(-2*a^3*(c^2 - d^2)^3*x + (2*(3*a*b^2*c^4*d*(4*c^2 + d^2) - 3*a^2*b*c^5
*(2*c^2 + 3*d^2) - b^3*c^5*(c^2 + 4*d^2) + a^3*(8*c^6*d - 8*c^4*d^3 + 7*c^
2*d^5 - 2*d^7))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt
[c - d]*Sqrt[c + d]*f)))/(c*(c^2 - d^2)) - ((b*c - a*d)*(18*a*b*c^5 - 34*a
^2*c^4*d - 13*b^2*c^4*d + 17*a*b*c^3*d^2 + 28*a^2*c^2*d^3 - 2*b^2*c^2*d^3
- 5*a*b*c*d^4 - 9*a^2*d^5)*Sin[e + f*x])/((c^2 - d^2)*f*(d + c*Cos[e + f*x
])))/(2*c^2*(c^2 - d^2)))/(3*c*(c^2 - d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\cdots\right.}{2(c-d)\left(c^3+3c^2d+3d\cdots\right.}\cdots\right.}{\cdots}$
default	$\frac{2a^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{c^4}+\frac{2\left(-\frac{\left(12a^3c^4d^2+4a^3c^3d^3-6a^3c^2d^4-a^3c\,d^5+2a^3d^6-18a^2b\,c^5d-9a^2b\,c^4d^2-6a^2b\,c^3d^3+6a\,b^2c^6+6a\cdots\right.}{2(c-d)\left(c^3+3c^2d+3d\cdots\right.}\cdots\right.}{\cdots}$
risch	Expression too large to display

input

```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

output

```
1/f*(2*a^3/c^4*arctan(tan(1/2*f*x+1/2*e))+2/c^4*((-1/2*(12*a^3*c^4*d^2+4*a
^3*c^3*d^3-6*a^3*c^2*d^4-a^3*c*d^5+2*a^3*d^6-18*a^2*b*c^5*d-9*a^2*b*c^4*d^
2-6*a^2*b*c^3*d^3+6*a*b^2*c^6+6*a*b^2*c^5*d+18*a*b^2*c^4*d^2+3*a*b^2*c^3*d
^3-b^3*c^6-6*b^3*c^5*d-2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c/(c-d)/(c^3+3*c^2*d+3
*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(18*a^3*c^4*d^2-11*a^3*c^2*d^4+3*a^3*
d^6-27*a^2*b*c^5*d-3*a^2*b*c^3*d^3+9*a*b^2*c^6+21*a*b^2*c^4*d^2-7*b^3*c^5*
d-3*b^3*c^3*d^3)*c/(c^2-2*c*d+d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/
2*(12*a^3*c^4*d^2-4*a^3*c^3*d^3-6*a^3*c^2*d^4+a^3*c*d^5+2*a^3*d^6-18*a^2*b
*c^5*d+9*a^2*b*c^4*d^2-6*a^2*b*c^3*d^3+6*a*b^2*c^6-6*a*b^2*c^5*d+18*a*b^2*
c^4*d^2-3*a*b^2*c^3*d^3+b^3*c^6-6*b^3*c^5*d+2*b^3*c^4*d^2-2*b^3*c^3*d^3)*c
/(c+d)/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2
*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3-1/2*(8*a^3*c^6*d-8*a^3*c^4*d^3+7*a^3*c^2*
d^5-2*a^3*d^7-6*a^2*b*c^7-9*a^2*b*c^5*d^2+12*a*b^2*c^6*d+3*a*b^2*c^4*d^3-b
^3*c^7-4*b^3*c^5*d^2)/(c^6-3*c^4*d^2+3*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*ar
ctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(396) = 792$.

Time = 0.31 (sec) , antiderivative size = 2776, normalized size of antiderivative = 6.74

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^3*c^11 - 4*a^3*c^9*d^2 + 6*a^3*c^7*d^4 - 4*a^3*c^5*d^6 + a^3*
c^3*d^8)*f*x*cos(f*x + e)^3 + 36*(a^3*c^10*d - 4*a^3*c^8*d^3 + 6*a^3*c^6*d
^5 - 4*a^3*c^4*d^7 + a^3*c^2*d^9)*f*x*cos(f*x + e)^2 + 36*(a^3*c^9*d^2 - 4
*a^3*c^7*d^4 + 6*a^3*c^5*d^6 - 4*a^3*c^3*d^8 + a^3*c*d^10)*f*x*cos(f*x + e
) + 12*(a^3*c^8*d^3 - 4*a^3*c^6*d^5 + 6*a^3*c^4*d^7 - 4*a^3*c^2*d^9 + a^3*
d^11)*f*x + 3*(7*a^3*c^2*d^8 - 2*a^3*d^10 - (6*a^2*b + b^3)*c^7*d^3 + 4*(2
*a^3 + 3*a*b^2)*c^6*d^4 - (9*a^2*b + 4*b^3)*c^5*d^5 - (8*a^3 - 3*a*b^2)*c^
4*d^6 + (7*a^3*c^5*d^5 - 2*a^3*c^3*d^7 - (6*a^2*b + b^3)*c^10 + 4*(2*a^3 +
 3*a*b^2)*c^9*d - (9*a^2*b + 4*b^3)*c^8*d^2 - (8*a^3 - 3*a*b^2)*c^7*d^3)*c
os(f*x + e)^3 + 3*(7*a^3*c^4*d^6 - 2*a^3*c^2*d^8 - (6*a^2*b + b^3)*c^9*d +
 4*(2*a^3 + 3*a*b^2)*c^8*d^2 - (9*a^2*b + 4*b^3)*c^7*d^3 - (8*a^3 - 3*a*b^
2)*c^6*d^4)*cos(f*x + e)^2 + 3*(7*a^3*c^3*d^7 - 2*a^3*c*d^9 - (6*a^2*b + b
^3)*c^8*d^2 + 4*(2*a^3 + 3*a*b^2)*c^7*d^3 - (9*a^2*b + 4*b^3)*c^6*d^4 - (8
*a^3 - 3*a*b^2)*c^5*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x
+ e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) +
c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) +
d^2)) + 2*(b^3*c^10*d + 6*a*b^2*c^9*d^2 + 23*a^3*c^3*d^8 - 6*a^3*c*d^10 -
11*(3*a^2*b + b^3)*c^8*d^3 + (26*a^3 + 33*a*b^2)*c^7*d^4 + (21*a^2*b + 4*b
^3)*c^6*d^5 - (43*a^3 + 39*a*b^2)*c^5*d^6 + 6*(2*a^2*b + b^3)*c^4*d^7 + (1
8*a*b^2*c^11 + 6*a^2*b*c^4*d^7 - 11*a^3*c^3*d^8 - (54*a^2*b + 13*b^3)*c...
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \int \frac{(a+b\sec{(e+fx)})^3}{(c+d\sec{(e+fx)})^4}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

output
```
Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(396) = 792$.

Time = 0.28 (sec) , antiderivative size = 1572, normalized size of antiderivative = 3.82

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

output

```
1/3*(3*(6*a^2*b*c^7 + b^3*c^7 - 8*a^3*c^6*d - 12*a*b^2*c^6*d + 9*a^2*b*c^5
*d^2 + 4*b^3*c^5*d^2 + 8*a^3*c^4*d^3 - 3*a*b^2*c^4*d^3 - 7*a^3*c^2*d^5 + 2
*a^3*d^7)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*t
an(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^10 -
3*c^8*d^2 + 3*c^6*d^4 - c^4*d^6)*sqrt(-c^2 + d^2)) + 3*(f*x + e)*a^3/c^4 -
 (18*a*b^2*c^8*tan(1/2*f*x + 1/2*e)^5 - 3*b^3*c^8*tan(1/2*f*x + 1/2*e)^5 -
 54*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^5 - 18*a*b^2*c^7*d*tan(1/2*f*x + 1/2*
e)^5 - 12*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 36*a^3*c^6*d^2*tan(1/2*f*x +
1/2*e)^5 + 81*a^2*b*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 + 36*a*b^2*c^6*d^2*tan(
1/2*f*x + 1/2*e)^5 + 27*b^3*c^6*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^3*c^5*d^
3*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 81*a*
b^2*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 - 12*b^3*c^5*d^3*tan(1/2*f*x + 1/2*e)^5
 - 6*a^3*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 9*a^2*b*c^4*d^4*tan(1/2*f*x + 1/
2*e)^5 + 36*a*b^2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 + 6*b^3*c^4*d^4*tan(1/2*f
*x + 1/2*e)^5 + 45*a^3*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 18*a^2*b*c^3*d^5*t
an(1/2*f*x + 1/2*e)^5 + 9*a*b^2*c^3*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*b^3*c^3
*d^5*tan(1/2*f*x + 1/2*e)^5 - 6*a^3*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 15*a^
3*c*d^7*tan(1/2*f*x + 1/2*e)^5 + 6*a^3*d^8*tan(1/2*f*x + 1/2*e)^5 - 36*a*b
^2*c^8*tan(1/2*f*x + 1/2*e)^3 + 108*a^2*b*c^7*d*tan(1/2*f*x + 1/2*e)^3 + 2
8*b^3*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 72*a^3*c^6*d^2*tan(1/2*f*x + 1/2*e...
```

Mupad [B] (verification not implemented)

Time = 24.25 (sec) , antiderivative size = 15647, normalized size of antiderivative = 37.98

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

input

```
int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^4,x)
```

output
```
((tan(e/2 + (f*x)/2)^5*(b^3*c^6 - 2*a^3*d^6 - 6*a*b^2*c^6 + a^3*c*d^5 + 6*
b^3*c^5*d + 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 - 12*a^3*c^4*d^2 + 2*b^3*c^3*d^3
 + 2*b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 - 18*a*b^2*c^4*d^2 + 6*a^2*b*c^3*d^3 +
9*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d + 18*a^2*b*c^5*d))/((c^3*d - c^4)*(c + d)^
3) + (4*tan(e/2 + (f*x)/2)^3*(7*b^3*c^5*d - 9*a*b^2*c^6 - 3*a^3*d^6 + 11*a
^3*c^2*d^4 - 18*a^3*c^4*d^2 + 3*b^3*c^3*d^3 - 21*a*b^2*c^4*d^2 + 3*a^2*b*c
^3*d^3 + 27*a^2*b*c^5*d))/(3*(c + d)^2*(c^5 - 2*c^4*d + c^3*d^2)) - (tan(e
/2 + (f*x)/2)*(2*a^3*d^6 + b^3*c^6 + 6*a*b^2*c^6 + a^3*c*d^5 - 6*b^3*c^5*d
 - 6*a^3*c^2*d^4 - 4*a^3*c^3*d^3 + 12*a^3*c^4*d^2 - 2*b^3*c^3*d^3 + 2*b^3*
c^4*d^2 - 3*a*b^2*c^3*d^3 + 18*a*b^2*c^4*d^2 - 6*a^2*b*c^3*d^3 + 9*a^2*b*c
^4*d^2 - 6*a*b^2*c^5*d - 18*a^2*b*c^5*d))/((c + d)*(3*c^5*d - c^6 + c^3*d^
3 - 3*c^4*d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d
^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 +
 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3
))) - (2*a^3*atan(((a^3*((a^3*((8*(4*a^3*c^21 + 2*b^3*c^21 + 12*a^2*b*c^21
 - 16*a^3*c^20*d - 2*b^3*c^20*d - 4*a^3*c^8*d^13 + 2*a^3*c^9*d^12 + 26*a^3
*c^10*d^11 - 14*a^3*c^11*d^10 - 70*a^3*c^12*d^9 + 30*a^3*c^13*d^8 + 110*a^
3*c^14*d^7 - 30*a^3*c^15*d^6 - 110*a^3*c^16*d^5 + 20*a^3*c^17*d^4 + 64*a^3
*c^18*d^3 - 12*a^3*c^19*d^2 + 8*b^3*c^12*d^9 - 8*b^3*c^13*d^8 - 22*b^3*c^1
4*d^7 + 22*b^3*c^15*d^6 + 18*b^3*c^16*d^5 - 18*b^3*c^17*d^4 - 2*b^3*c^1...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4706, normalized size of antiderivative = 11.42

$$\int \frac{(a + b\sec(e + fx))^3}{(c + d\sec(e + fx))^4} dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x)
```

output
```
( - 48*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)
/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a**3*c**9*d + 48*sqrt(
 - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**
2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a**3*c**7*d**3 - 42*sqrt( - c**2 +
 d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2)
)*cos(e + f*x)*sin(e + f*x)**2*a**3*c**5*d**5 + 12*sqrt( - c**2 + d**2)*at
an((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e +
 f*x)*sin(e + f*x)**2*a**3*c**3*d**7 + 36*sqrt( - c**2 + d**2)*atan((tan((
e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin
(e + f*x)**2*a**2*b*c**10 + 54*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)
*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**
2*a**2*b*c**8*d**2 - 72*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - ta
n((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a*b**
2*c**9*d - 18*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x
)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a*b**2*c**7*d**
3 + 6*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/
sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*b**3*c**10 + 24*sqrt( -
 c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2
+ d**2))*cos(e + f*x)*sin(e + f*x)**2*b**3*c**8*d**2 + 48*sqrt( - c**2 + d
**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2...
```

3.197 $\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 622

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \frac{a^3x}{c^5}$$
$$-\frac{(15ab^2c^6d(4c^2+3d^2)-3a^2bc^5(8c^4+24c^2d^2+3d^4)-b^3c^5(4c^4+27c^2d^2+4d^4)+a^3(40c^8d-40c^6d^3+}{4c^5\sqrt{c-d}\sqrt{c+d}\,(c^2-d^2)^4 f}$$
$$+\frac{d^2(b+a\cos(e+fx))^3\sin(e+fx)}{4c\,(c^2-d^2)\,f(d+c\cos(e+fx))^4}$$
$$-\frac{d(8bc^3-11ac^2d-bcd^2+4ad^3)\,(b+a\cos(e+fx))^2\sin(e+fx)}{12c^2\,(c^2-d^2)^2\,f(d+c\cos(e+fx))^3}$$
$$-\frac{(bc-ad)\,(2abcd(32c^4+c^2d^2+2d^4)-a^2d^2(58c^4-35c^2d^2+12d^4)-b^2(12c^6+25c^4d^2-2c^2d^4))\sin(e}{24c^4\,(c^2-d^2)^3\,f(d+c\cos(e+fx))^2}$$
$$-\frac{(b^3c^3d(68c^4+39c^2d^2-2d^4)+a^2bcd(272c^6+10c^4d^2+49c^2d^4-16d^6)-3ab^2c^2(24c^6+84c^4d^2-5c^2d}{24c^4\,(c^2-d^2)^4\,f(d+c\cos(e+fx))}$$

output
```
a^3*x/c^5-1/4*(15*a*b^2*c^6*d*(4*c^2+3*d^2)-3*a^2*b*c^5*(8*c^4+24*c^2*d^2+
3*d^4)-b^3*c^5*(4*c^4+27*c^2*d^2+4*d^4)+a^3*(40*c^8*d-40*c^6*d^3+63*c^4*d^
5-36*c^2*d^7+8*d^9))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/c
^5/(c-d)^(1/2)/(c+d)^(1/2)/(c^2-d^2)^4/f+1/4*d^2*(b+a*cos(f*x+e))^3*sin(f*
x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))^4-1/12*d*(-11*a*c^2*d+4*a*d^3+8*b*c^3-
b*c*d^2)*(b+a*cos(f*x+e))^2*sin(f*x+e)/c^2/(c^2-d^2)^2/f/(d+c*cos(f*x+e))^
3-1/24*(-a*d+b*c)*(2*a*b*c*d*(32*c^4+c^2*d^2+2*d^4)-a^2*d^2*(58*c^4-35*c^2
*d^2+12*d^4)-b^2*(12*c^6+25*c^4*d^2-2*c^2*d^4))*sin(f*x+e)/c^4/(c^2-d^2)^3
/f/(d+c*cos(f*x+e))^2-1/24*(b^3*c^3*d*(68*c^4+39*c^2*d^2-2*d^4)+a^2*b*c*d*
(272*c^6+10*c^4*d^2+49*c^2*d^4-16*d^6)-3*a*b^2*c^2*(24*c^6+84*c^4*d^2-5*c^
2*d^4+2*d^6)-a^3*(212*c^6*d^2-210*c^4*d^4+139*c^2*d^6-36*d^8))*sin(f*x+e)/
c^4/(c^2-d^2)^4/f/(d+c*cos(f*x+e))
```

Mathematica [A] (verified)

Time = 8.20 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^2(e+fx)(a+b\sec(e+fx))^3\left(24a^3(e+fx)(d+c\cos(e+fx))^4-\frac{6(-15ab^2c^6d(4c^2}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

output
```
((d + c*Cos[e + f*x])*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^3*(24*a^3*(e + f
*x)*(d + c*Cos[e + f*x])^4 - (6*(-15*a*b^2*c^6*d*(4*c^2 + 3*d^2) + 3*a^2*b
*c^5*(8*c^4 + 24*c^2*d^2 + 3*d^4) + b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) +
 a^3*(-40*c^8*d + 40*c^6*d^3 - 63*c^4*d^5 + 36*c^2*d^7 - 8*d^9))*ArcTanh[(
(-c + d)*Tan[(e + f*x)/2]]/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^4)/(c^2 -
 d^2)^(9/2) + (6*c*d^2*(b*c - a*d)^3*Sin[e + f*x])/(c^2 - d^2) - (2*c*d*(b
*c - a*d)^2*(8*b*c^3 - 20*a*c^2*d - b*c*d^2 + 13*a*d^3)*(d + c*Cos[e + f*x
])*Sin[e + f*x])/(c^2 - d^2)^2 + (c*(a^3*d^3*(-120*c^4 + 131*c^2*d^2 - 46*
d^4) - 3*a*b^2*c^2*d*(36*c^4 - 3*c^2*d^2 + 2*d^4) + 3*a^2*b*c*d^2*(72*c^4
- 55*c^2*d^2 + 18*d^4) + b^3*(12*c^7 + 25*c^5*d^2 - 2*c^3*d^4))*(d + c*Cos
[e + f*x])^2*Sin[e + f*x])/(c^2 - d^2)^3 + (c*(b^3*c^3*d*(-68*c^4 - 39*c^2
*d^2 + 2*d^4) - 3*a^2*b*c*d*(96*c^6 - 8*c^4*d^2 + 23*c^2*d^4 - 6*d^6) + 3*
a*b^2*c^2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) + 5*a^3*(48*c^6*d^2 -
56*c^4*d^4 + 39*c^2*d^6 - 10*d^8))*(d + c*Cos[e + f*x])^3*Sin[e + f*x])/(c
^2 - d^2)^4))/(24*c^5*f*(b + a*Cos[e + f*x])^3*(c + d*Sec[e + f*x])^5)
```

Rubi [A] (verified)

Time = 3.12 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.10, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4429, 3042, 3527, 25, 3042, 3526, 25, 3042, 3510, 3042, 3500, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{4429}$$

$$\int \frac{\cos^2(e+fx)(a\cos(e+fx)+b)^3}{(c\cos(e+fx)+d)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2 \left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)^3}{\left(c\sin\left(e+fx+\frac{\pi}{2}\right)+d\right)^5}dx$$

$\big\downarrow$ 3527

$$\frac{\int -\frac{(b+a\cos(e+fx))^2\left(-4a\left(c^2-d^2\right)\cos^2(e+fx)-\left(4bc^2-4adc-bd^2\right)\cos(e+fx)+d(4bc-3ad)\right)}{(d+c\cos(e+fx))^4}dx}{4c\left(c^2-d^2\right)}+\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}$$

$\big\downarrow$ 25

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\int \frac{(b+a\cos(e+fx))^2\left(-4a\left(c^2-d^2\right)\cos^2(e+fx)+\left(4acd-b\left(4c^2-d^2\right)\right)\cos(e+fx)+d(4bc-3ad)\right)}{(d+c\cos(e+fx))^4}dx}{4c\left(c^2-d^2\right)}$$

$\big\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(-4a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(4acd-b\left(4c^2-d^2\right)\right)\sin\left(e+fx+\frac{\pi}{2}\right)+d(4bc-3ad)\right)}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^4}dx}{4c\left(c^2-d^2\right)}$$

$\big\downarrow$ 3526

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\int -\frac{(b+a\cos(e+fx))\left(12b^2c^4-40abdc^3+22a^2d^2c^2+9b^2d^2c^2+5abd^3c-8a^2d^4+12a^2\left(c^2-d^2\right)^2\cos^2(e+fx)-\left(3\left(8c^3d-cd^3\right)a^2-b\left(24c^4+7d^2c^2+4d^4\right)a+2b^2cd\left(8c \cdots\right.\right.\right.}{(d+c\cos(e+fx))^3}}{3c\left(c^2-d^2\right)} \cdots \Big/ 4c\left(c^2-d^2\right)$$

$\big\downarrow$ 25

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-\frac{\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^3}-\frac{\int \frac{(b+a\cos(e+fx))\left(3\left(4c^4+3d^2c^2\right)b^2-5acd\left(8c^2-d^2\right)b+12a^2\left(c^2-d^2\right)^2\cos^2(e+fx)+2a \cdots\right.}{(d+c\,c \cdots}}{3c(\cdots}}{4c\left(c^2-d^2\right)}$$

$\big\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{\int\frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)\left(3\left(4c^4+3d^2c^2\right)b^2-5acd\left(8c^2-d^2\right)b+12a^2\left(c^2-d^2\right)^2\sin\left(e+fx+\cdots\right.\right.}{(\cdots}}{\cdots}}{4c\left(c^2-d^2\right)}$$

$\downarrow$ 3510

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{\int\frac{-24a^3c\cos^2(e+fx)\left(c^2-d^2\right)^3+2c\left(-\left(\left(12d^6-35c^2d^4+58c^4d^2\right)a^3\right)+bcd\left(100c^4-3d^2\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{\int\frac{-24a^3c\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(c^2-d^2\right)^3+2c\left(-\left(\left(12d^6-35c^2d^4+58c^4d^2\right)a^3\right)+bcd\left(100c^4\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3500

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{\frac{\int\frac{3\left(c\left(15ab^2d\left(4c^2+3d^2\right)c^5-3a^2b\left(8c^4+24d^2c^2+3d^4\right)c^4-b^3\left(4c^4+27d^2c^2+4d^4\right)c^4+c\cdots\right.\right.}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{\cdots}}{\cdots}$$

$\downarrow$ 27

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{-\frac{\frac{3\int\frac{c\left(15ab^2d\left(4c^2+3d^2\right)c^5-3a^2b\left(8c^4+24d^2c^2+3d^4\right)c^4-b^3\left(4c^4+27d^2c^2+4d^4\right)c^4+a^3\cdots\right.}{d+c\cos(e+fx)}}{c\left(c^2-d^2\right)}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{3\int\frac{c\left(15ab^2d\left(4c^2+3d^2\right)c^5-3a^2b\left(8c^4+24d^2c^2+3d^4\right)c^4-b^3\left(4c^4+27d^2c^2+4d^4\right)c^4+a^3\cdots}{d+c\sin\left(e+fx+\frac{\pi}{2}\right)\cdots}}{c\left(c^2-d^2\right)\cdots}}{\cdots}$$

$\downarrow$ 3214

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{\frac{3\left(\left(a^3\left(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9\right)-3a^2bc^5\left(8c^4+24c^2d^2+3d^4\right)+15ab^2c^6\cdots\right.\right.}{c\left(c^2-d^2\cdots\right.}}{\cdots}$$

$\downarrow$ 3042

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{\frac{3\left(\left(a^3\left(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9\right)-3a^2bc^5\left(8c^4+24c^2d^2+3d^4\right)+15ab^2c^6\cdots\right.\right.}{c\left(c^2-\cdots\right.}}{\cdots}$$

$\downarrow$ 3138

$$\frac{d^2\sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4}-$$

$$\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3}-\frac{3\left(\frac{2\left(a^3\left(40c^8d-40c^6d^3+63c^4d^5-36c^2d^7+8d^9\right)-3a^2bc^5\left(8c^4+24c^2d^2+3d^4\right)+15ab^2c\cdots\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 221

$$\frac{d^2 \sin(e+fx)(a\cos(e+fx)+b)^3}{4cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^4} -$$

$$\frac{\frac{d\left(-11ac^2d+4ad^3+8bc^3-bcd^2\right)\sin(e+fx)(a\cos(e+fx)+b)^2}{3cf(c^2-d^2)(c\cos(e+fx)+d)^3} - \frac{-\frac{(bc-ad)\left(-a^2d^2\left(58c^4-35c^2d^2+12d^4\right)+2abcd\left(32c^4+c^2d^2+2d^4\right)-\left(b^2\left(12c^6+25c^4d^2-\right.\right.\right.}{2c^2f\left(c^2-d^2\right)(c\cos(e+fx)+d)^2}}{}}{}$$

input
```
Int[(a + b*Sec[e + f*x])^3/(c + d*Sec[e + f*x])^5,x]
```

output
```
(d^2*(b + a*Cos[e + f*x])^3*Sin[e + f*x])/(4*c*(c^2 - d^2)*f*(d + c*Cos[e
+ f*x])^4) - ((d*(8*b*c^3 - 11*a*c^2*d - b*c*d^2 + 4*a*d^3)*(b + a*Cos[e +
 f*x])^2*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^3) - (-1/2*
((b*c - a*d)*(2*a*b*c*d*(32*c^4 + c^2*d^2 + 2*d^4) - a^2*d^2*(58*c^4 - 35*
c^2*d^2 + 12*d^4) - b^2*(12*c^6 + 25*c^4*d^2 - 2*c^2*d^4))*Sin[e + f*x])/(
c^2*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^2) - ((3*(-8*a^3*(c^2 - d^2)^4*x +
(2*(15*a*b^2*c^6*d*(4*c^2 + 3*d^2) - 3*a^2*b*c^5*(8*c^4 + 24*c^2*d^2 + 3*d
^4) - b^3*c^5*(4*c^4 + 27*c^2*d^2 + 4*d^4) + a^3*(40*c^8*d - 40*c^6*d^3 +
63*c^4*d^5 - 36*c^2*d^7 + 8*d^9))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/S
qrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*f)))/(c*(c^2 - d^2)) + ((b^3*c^3*d*(
68*c^4 + 39*c^2*d^2 - 2*d^4) + a^2*b*c*d*(272*c^6 + 10*c^4*d^2 + 49*c^2*d^
4 - 16*d^6) - 3*a*b^2*c^2*(24*c^6 + 84*c^4*d^2 - 5*c^2*d^4 + 2*d^6) - a^3*
(212*c^6*d^2 - 210*c^4*d^4 + 139*c^2*d^6 - 36*d^8))*Sin[e + f*x])/((c^2 -
d^2)*f*(d + c*Cos[e + f*x])))/(2*c^2*(c^2 - d^2)))/(3*c*(c^2 - d^2)))/(4*c
*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4429
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f
*x])^n/Sin[e + f*x]^(m + n)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IntegerQ[n] && LeQ[-2, m + n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(603) = 1206$.

Time = 1.34 (sec) , antiderivative size = 1345, normalized size of antiderivative = 2.16

method	result	size
derivativedivides	Expression too large to display	1345
default	Expression too large to display	1345
risch	Expression too large to display	5103

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2*a^3/c^5*arctan(tan(1/2*f*x+1/2*e))+2/c^5*((-1/8*(80*a^3*c^6*d^2+40*
a^3*c^5*d^3-40*a^3*c^4*d^4-15*a^3*c^3*d^5+32*a^3*c^2*d^6+4*a^3*c*d^7-8*a^3
*d^8-96*a^2*b*c^7*d-72*a^2*b*c^6*d^2-96*a^2*b*c^5*d^3-15*a^2*b*c^4*d^4+24*
a*b^2*c^8+36*a*b^2*c^7*d+144*a*b^2*c^6*d^2+51*a*b^2*c^5*d^3+24*a*b^2*c^4*d
^4-4*b^3*c^8-32*b^3*c^7*d-21*b^3*c^6*d^2-32*b^3*c^5*d^3-4*b^3*c^4*d^4)*c/(
c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d^4)*tan(1/2*f*x+1/2*e)^7+1/24*c*(720*
a^3*c^6*d^2+120*a^3*c^5*d^3-520*a^3*c^4*d^4-69*a^3*c^3*d^5+320*a^3*c^2*d^6
+12*a^3*c*d^7-72*a^3*d^8-864*a^2*b*c^7*d-216*a^2*b*c^6*d^2-480*a^2*b*c^5*d
^3+27*a^2*b*c^4*d^4+216*a*b^2*c^8+108*a*b^2*c^7*d+1008*a*b^2*c^6*d^2+81*a*
b^2*c^5*d^3+120*a*b^2*c^4*d^4-12*b^3*c^8-224*b^3*c^7*d-39*b^3*c^6*d^2-224*
b^3*c^5*d^3-12*b^3*c^4*d^4)/(c^3+3*c^2*d+3*c*d^2+d^3)/(c-d)^2*tan(1/2*f*x+
1/2*e)^5-1/24*c*(720*a^3*c^6*d^2-120*a^3*c^5*d^3-520*a^3*c^4*d^4+69*a^3*c^
3*d^5+320*a^3*c^2*d^6-12*a^3*c*d^7-72*a^3*d^8-864*a^2*b*c^7*d+216*a^2*b*c^
6*d^2-480*a^2*b*c^5*d^3-27*a^2*b*c^4*d^4+216*a*b^2*c^8-108*a*b^2*c^7*d+100
8*a*b^2*c^6*d^2-81*a*b^2*c^5*d^3+120*a*b^2*c^4*d^4+12*b^3*c^8-224*b^3*c^7*
d+39*b^3*c^6*d^2-224*b^3*c^5*d^3+12*b^3*c^4*d^4)/(c-d)^3/(c^2+2*c*d+d^2)*t
an(1/2*f*x+1/2*e)^3+1/8*(80*a^3*c^6*d^2-40*a^3*c^5*d^3-40*a^3*c^4*d^4+15*a
^3*c^3*d^5+32*a^3*c^2*d^6-4*a^3*c*d^7-8*a^3*d^8-96*a^2*b*c^7*d+72*a^2*b*c^
6*d^2-96*a^2*b*c^5*d^3+15*a^2*b*c^4*d^4+24*a*b^2*c^8-36*a*b^2*c^7*d+144*a*
b^2*c^6*d^2-51*a*b^2*c^5*d^3+24*a*b^2*c^4*d^4+4*b^3*c^8-32*b^3*c^7*d+21...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(603) = 1206$.

Time = 0.49 (sec) , antiderivative size = 4346, normalized size of antiderivative = 6.99

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx=\int \frac{(a+b\sec{(e+fx)})^3}{(c+d\sec{(e+fx)})^5}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

output
```
Integral((a + b*sec(e + f*x))**3/(c + d*sec(e + f*x))**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3173 vs. $2(603) = 1206$.

Time = 0.38 (sec) , antiderivative size = 3173, normalized size of antiderivative = 5.10

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="giac")
```

output
```
1/12*(3*(24*a^2*b*c^9 + 4*b^3*c^9 - 40*a^3*c^8*d - 60*a*b^2*c^8*d + 72*a^2
*b*c^7*d^2 + 27*b^3*c^7*d^2 + 40*a^3*c^6*d^3 - 45*a*b^2*c^6*d^3 + 9*a^2*b*
c^5*d^4 + 4*b^3*c^5*d^4 - 63*a^3*c^4*d^5 + 36*a^3*c^2*d^7 - 8*a^3*d^9)*(pi
*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x +
1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^13 - 4*c^11*d^2 +
6*c^9*d^4 - 4*c^7*d^6 + c^5*d^8)*sqrt(-c^2 + d^2)) + 12*(f*x + e)*a^3/c^5
- (72*a*b^2*c^11*tan(1/2*f*x + 1/2*e)^7 - 12*b^3*c^11*tan(1/2*f*x + 1/2*e)
^7 - 288*a^2*b*c^10*d*tan(1/2*f*x + 1/2*e)^7 - 108*a*b^2*c^10*d*tan(1/2*f*
x + 1/2*e)^7 - 60*b^3*c^10*d*tan(1/2*f*x + 1/2*e)^7 + 240*a^3*c^9*d^2*tan(
1/2*f*x + 1/2*e)^7 + 648*a^2*b*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 + 324*a*b^2*
c^9*d^2*tan(1/2*f*x + 1/2*e)^7 + 189*b^3*c^9*d^2*tan(1/2*f*x + 1/2*e)^7 -
600*a^3*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 504*a^2*b*c^8*d^3*tan(1/2*f*x + 1
/2*e)^7 - 891*a*b^2*c^8*d^3*tan(1/2*f*x + 1/2*e)^7 - 183*b^3*c^8*d^3*tan(1
/2*f*x + 1/2*e)^7 + 240*a^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 459*a^2*b*c^7
*d^4*tan(1/2*f*x + 1/2*e)^7 + 801*a*b^2*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 1
83*b^3*c^7*d^4*tan(1/2*f*x + 1/2*e)^7 + 435*a^3*c^6*d^5*tan(1/2*f*x + 1/2*
e)^7 - 513*a^2*b*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 189*a*b^2*c^6*d^5*tan(1/
2*f*x + 1/2*e)^7 - 189*b^3*c^6*d^5*tan(1/2*f*x + 1/2*e)^7 - 249*a^3*c^5*d^
6*tan(1/2*f*x + 1/2*e)^7 + 153*a^2*b*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 63*a
*b^2*c^5*d^6*tan(1/2*f*x + 1/2*e)^7 + 60*b^3*c^5*d^6*tan(1/2*f*x + 1/2*...
```

Mupad [B] (verification not implemented)

Time = 24.79 (sec) , antiderivative size = 21021, normalized size of antiderivative = 33.80

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(e + f*x))^3/(c + d/cos(e + f*x))^5,x)
```

output
```
(atan(((((c + d)^9*(c - d)^9)^(1/2)*((tan(e/2 + (f*x)/2)*(64*a^6*c^18 + 12
8*a^6*d^18 + 16*b^6*c^18 - 128*a^6*c*d^17 - 128*a^6*c^17*d + 192*a^2*b^4*c
^18 + 576*a^4*b^2*c^18 - 1024*a^6*c^2*d^16 + 1024*a^6*c^3*d^15 + 3584*a^6*
c^4*d^14 - 3584*a^6*c^5*d^13 - 6968*a^6*c^6*d^12 + 7168*a^6*c^7*d^11 + 838
5*a^6*c^8*d^10 - 8960*a^6*c^9*d^9 - 7024*a^6*c^10*d^8 + 7168*a^6*c^11*d^7
+ 4848*a^6*c^12*d^6 - 3584*a^6*c^13*d^5 - 1920*a^6*c^14*d^4 + 1024*a^6*c^1
5*d^3 + 1152*a^6*c^16*d^2 + 16*b^6*c^10*d^8 + 216*b^6*c^12*d^6 + 761*b^6*c
^14*d^4 + 216*b^6*c^16*d^2 - 360*a*b^5*c^11*d^7 - 2910*a*b^5*c^13*d^5 - 36
00*a*b^5*c^15*d^3 - 3200*a^3*b^3*c^17*d - 144*a^5*b*c^5*d^13 - 504*a^5*b*c
^7*d^11 + 3666*a^5*b*c^9*d^9 - 6624*a^5*b*c^11*d^7 + 2016*a^5*b*c^13*d^5 -
 3840*a^5*b*c^15*d^3 + 72*a^2*b^4*c^10*d^8 + 3087*a^2*b^4*c^12*d^6 + 9552*
a^2*b^4*c^14*d^4 + 5472*a^2*b^4*c^16*d^2 - 64*a^3*b^3*c^5*d^13 - 144*a^3*b
^3*c^7*d^11 + 1376*a^3*b^3*c^9*d^9 - 3604*a^3*b^3*c^11*d^7 - 6224*a^3*b^3*
c^13*d^5 - 12640*a^3*b^3*c^15*d^3 + 720*a^4*b^2*c^6*d^12 - 2280*a^4*b^2*c^
8*d^10 + 1431*a^4*b^2*c^10*d^8 + 5256*a^4*b^2*c^12*d^6 + 4416*a^4*b^2*c^14
*d^4 + 8256*a^4*b^2*c^16*d^2 - 480*a*b^5*c^17*d - 1920*a^5*b*c^17*d))/(2*(
c^22*d + c^23 - c^8*d^15 - c^9*d^14 + 7*c^10*d^13 + 7*c^11*d^12 - 21*c^12*
d^11 - 21*c^13*d^10 + 35*c^14*d^9 + 35*c^15*d^8 - 35*c^16*d^7 - 35*c^17*d^
6 + 21*c^18*d^5 + 21*c^19*d^4 - 7*c^20*d^3 - 7*c^21*d^2)) + (((32*a^3*c^27
 + 16*b^3*c^27 + 96*a^2*b*c^27 - 160*a^3*c^26*d - 16*b^3*c^26*d - 32*a^...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7989, normalized size of antiderivative = 12.84

$$\int \frac{(a+b\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
int((a+b*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x)
```

output

```
( - 960*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d
)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a**3*c**11*d**2 + 960
*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt(
 - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a**3*c**9*d**4 - 1512*sqrt(
- c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2
 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a**3*c**7*d**6 + 864*sqrt( - c**2 +
 d**2)*atan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2)
)*cos(e + f*x)*sin(e + f*x)**2*a**3*c**5*d**8 - 192*sqrt( - c**2 + d**2)*a
tan((tan((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e
+ f*x)*sin(e + f*x)**2*a**3*c**3*d**10 + 576*sqrt( - c**2 + d**2)*atan((ta
n((e + f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*
sin(e + f*x)**2*a**2*b*c**12*d + 1728*sqrt( - c**2 + d**2)*atan((tan((e +
f*x)/2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e +
 f*x)**2*a**2*b*c**10*d**3 + 216*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/
2)*c - tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)
**2*a**2*b*c**8*d**5 - 1440*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c
- tan((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a
*b**2*c**11*d**2 - 1080*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - ta
n((e + f*x)/2)*d)/sqrt( - c**2 + d**2))*cos(e + f*x)*sin(e + f*x)**2*a*b**
2*c**9*d**4 + 96*sqrt( - c**2 + d**2)*atan((tan((e + f*x)/2)*c - tan((e...
```

3.198 $\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 320

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = -\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf} + \frac{2\sqrt{a+b}(b(c-d)+ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{bf} - \frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

output
```
-2*(a-b)*(a+b)^(1/2)*d*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e)
)/(a-b))^(1/2)/b/f+2*(a+b)^(1/2)*(b*(c-d)+a*d)*cot(f*x+e)*EllipticF((a+b*s
ec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))
^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*(a+b)^(1/2)*c*cot(f*x+e)*Elli
pticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*
(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Mathematica [A] (verified)

Time = 8.79 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.18

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$= \frac{\cos(e+fx)\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\left(2d\sin(e+fx)-\frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\left(4(a+b)d\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b}{(a+}}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])*(2*d*Sin[e + f
*x] - (Cos[(e + f*x)/2]^2*(4*(a + b)*d*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x]
)]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticE[ArcSi
n[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 4*(a*(c - d) - b*(c + d))*Sqrt[Cos
[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[
e + f*x]))]*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 8*a*c*S
qrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1
 + Cos[e + f*x]))]*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b
)] + d*(b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^3*(-Sin[(e + f*x)/2] + Sin[(3
*(e + f*x))/2])))/(b + a*Cos[e + f*x])))/(f*(d + c*Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4404, 3042, 4271, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4404}$$

$$\int \frac{\sec(e+fx)(bc+ad+bd\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}dx+ac\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(bc+ad+bd\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+ac\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4271}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(bc+ad+bd\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$\Big\downarrow \text{4493}$$

$$(ad+b(c-d))\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+bd\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$\Big\downarrow \text{3042}$$

$$(ad+b(c-d))\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+bd\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$\Big\downarrow \text{4319}$$

$$bd\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}$$

$$\Big\downarrow \text{4492}$$

$$\frac{2\sqrt{a+b}(ad+b(c-d))\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{2d(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bf}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x]),x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*d*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
 f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a + b]*(b*(c - d
) + a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b
]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[
(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4404
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[a*c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Int[Csc[e + f*x]*((b*c + a*d + b*d*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]
), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(293) = 586$.

Time = 18.06 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.02

method	result
parts	$\frac{2c\left(\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a-\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)b-2a\operatorname{EllipticPi}\left(\cot(fx+e)-\csc(fx+e\right.\right.}{f(b+a\cos(fx+e))}$
default	$\frac{(1+\cos(fx+e))^2\left(\left(\left((1-\cos(fx+e))^3\csc(fx+e)^3-\csc(fx+e)+\cot(fx+e)\right)ad+\left(-(1-\cos(fx+e))^3\csc(fx+e)^3-\csc(fx+e)+\cot(f\right.\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2*c/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a-EllipticF(co
t(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(cot(f*x+e)-csc(f
*x+e),-1,((a-b)/(a+b))^(1/2)))*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/
2)/(b+a*cos(f*x+e))-1/2*d/f/(b+a*cos(f*x+e))*(1+cos(f*x+e))^2*((-(1-cos(f*
x+e))^3*csc(f*x+e)^3+csc(f*x+e)-cot(f*x+e))*a+((1-cos(f*x+e))^3*csc(f*x+e)
^3+csc(f*x+e)-cot(f*x+e))*b-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(
b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-
b)/(a+b))^(1/2))*a-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f
*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*b+2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1
+cos(f*x+e)))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a
+2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+
e)))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b)*((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)*(a+b*sec(f*x+e))^(1/2)*sec(f*x+e)
```

Fricas [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{a+b\sec\left(e+fx\right)}(c+d\sec\left(e+fx\right))\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{b\sec\left(fx+e\right)+a}(d\sec\left(fx+e\right)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \int \sqrt{a+\frac{b}{\cos(e+f\,x)}}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))\,dx = \left(\int \sqrt{\sec(fx+e)\,b+a}dx\right)c+\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)\,dx\right)d$$

input
```
int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e)),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a),x)*c + int(sqrt(sec(e + f*x)*b + a)*sec(e + f
*x),x)*d
```

3.199 $\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 220

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{cf} + \frac{2(bc-ad)\,\mathrm{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
-2*(a+b)^(1/2)*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a
+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e
))/(a-b))^(1/2)/c/f+2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/
2),2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f
*x+e)/c/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 7.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(-((a-b)c(c+d)\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output
```
(4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(-((a - b)*c*(c + d)*EllipticF[A
rcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]) + 2*a*(c^2 - d^2)*EllipticPi[-1
, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*d*(-(b*c) + a*d)*Elliptic
Pi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b
*Sec[e + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4414, 3042, 4271, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4414}$$

$$\frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{c}+\frac{a\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{c}+\frac{a\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}$$

$$\Big\downarrow \text{4271}$$

$$\frac{(bc-ad)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{c}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf}$$

$$\Big\downarrow \text{4461}$$

$$\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf}$$

input

```
Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x]),x]
```

output

```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
 + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(c*f) + (2*(b*c - a*d)*Ellipti
cPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*
Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*S
ec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4414
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[a/c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[(b*c - a*d)/c   Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Cs
c[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.82

method	result
default	$\frac{2\left(\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)a\,c^2+\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)acd-\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/f/c/(c-d)/(c+d)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*
c^2+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*c*d-EllipticF(c
ot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c^2-EllipticF(cot(f*x+e)-csc(f
*x+e),((a-b)/(a+b))^(1/2))*b*c*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a
-b)/(a+b))^(1/2))*a*c^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b)
)^(1/2))*a*d^2-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b)
)^(1/2))*a*d^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b)
)^(1/2))*b*c*d)*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*
(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x
+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{d\sec\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)\,d+c}dx$$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a)/(sec(e + f*x)*d + c),x)
```

3.200 $\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 380

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = -\frac{2(a-b)\sqrt{a+b}(3bc+4ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{3bf} + \frac{2\sqrt{a+b}(ab(6c-4d)-b^2(3c-d)+3a^2d)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\cdots}{3bf} - \frac{2a\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f} + \frac{2bd\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{3f}$$

output
```
-2/3*(a-b)*(a+b)^(1/2)*(4*a*d+3*b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))
^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/3*(a+b)^(1/2)*(a*b*(6*c-4*d)-b^2*(3*c-d
)+3*a^2*d)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/
2)/b/f-2*a*(a+b)^(1/2)*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b
)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1
+sec(f*x+e))/(a-b))^(1/2)/f+2/3*b*d*(a+b*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6063 vs. 2(380) = 760.

Time = 22.14 (sec) , antiderivative size = 6063, normalized size of antiderivative = 15.96

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4406, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4406}$$

$$\frac{2}{3}\int \frac{3ca^2+b(3bc+4ad)\sec^2(e+fx)+\left(3da^2+6bca+b^2d\right)\sec(e+fx)}{2\sqrt{a+b\sec(e+fx)}}dx+\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3ca^2+b(3bc+4ad)\sec^2(e+fx)+\left(3da^2+6bca+b^2d\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\int \frac{3ca^2+b(3bc+4ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(3da^2+6bca+b^2d\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \textbf{4546}$$

$$\frac{1}{3}\left(\int \frac{3ca^2+\left(3da^2+6bca+b^2d-b(3bc+4ad)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+b(4ad+3bc)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\left(\int \frac{3ca^2+\left(3da^2+6bca+b^2d-b(3bc+4ad)\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b(4ad+3bc)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e\right.\right.}{\sqrt{a+b\csc\left(e+\right.}}\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \textbf{4409}$$

$$\frac{1}{3}\left(\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+3a^2c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx+b(4ad+3bc)\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{3}\left(\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+3a^2c\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b(\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{3}\left(\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b(4ad+3bc)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e\right.\right.}{\sqrt{a+b\csc\left(e+\right.}}\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{3}\left(b(4ad+3bc)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\cot}{}\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+b}\left(3a^2d+ab(6c-4d)-b^2(3c-d)\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\surd}{}\right.\right.}{bf}\right.$$
$$\frac{2bd\tan(e+fx)\sqrt{a+b\sec(e+fx)}}{3f}$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)*(c + d*Sec[e + f*x]),x]
```

output
```
((-2*(a - b)*Sqrt[a + b]*(3*b*c + 4*a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e +
f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a
 + b]*(a*b*(6*c - 4*d) - b^2*(3*c - d) + 3*a^2*d)*Cot[e + f*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) -
(6*a*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f)/3 + (2*b*d*Sqrt[a + b*Se
c[e + f*x]]*Tan[e + f*x])/(3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4406
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
 + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
 - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*
c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(345) = 690$.

Time = 24.60 (sec) , antiderivative size = 1239, normalized size of antiderivative = 3.26

method	result	size
parts	Expression too large to display	1239
default	Expression too large to display	1240

input
```
int((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*c/f/(b+a*cos(f*x+e))*(1+cos(f*x+e))^2*(((1-cos(f*x+e))^3*csc(f*x+e)^3-
csc(f*x+e)+cot(f*x+e))*a*b+(-(1-cos(f*x+e))^3*csc(f*x+e)^3-csc(f*x+e)+cot(
f*x+e))*b^2-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/
(1+cos(f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))
*a^2+4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(
f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b+2*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e))
)^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2-2*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b-2*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*Ellipt
icE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2+4*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticPi(co
t(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^2)*((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)*(a+b*sec(f*x+e))^(1/2)*sec(f*x+e)-2/3*d/f*(a+b*sec(f*x+e))^(1/2)/
(cos(f*x+e)^2*a+a*cos(f*x+e)+b*cos(f*x+e)+b)*((-4*cos(f*x+e)^2-8*cos(f*x+e
)-4)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*
x+e)))^(1/2)*a^2*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+(-4*
cos(f*x+e)^2-8*cos(f*x+e)-4)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b
+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b*EllipticE(cot(f*x+e)-csc(f*x+e...
```

Fricas [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e))*sqrt(b*sec(
f*x + e) + a), x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (a+b\sec{(e+fx)})^{\frac{3}{2}}\,(c+d\sec{(e+fx)})\,\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)*(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec{(fx+e)}+a)^{\frac{3}{2}}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx = \int (b\sec{(fx+e)}+a)^{\frac{3}{2}}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)*(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int (a+b\sec(e+fx))^{3/2}(c+d\sec(e+fx))\,dx=\left(\int \sqrt{\sec(fx+e)\,b+a}dx\right)ac+\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)^2\,dx\right)bd+\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)\,dx\right)ad+\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)\,dx\right)bc$$

input
```
int((a+b*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a),x)*a*c + int(sqrt(sec(e + f*x)*b + a)*sec(e +
 f*x)**2,x)*b*d + int(sqrt(sec(e + f*x)*b + a)*sec(e + f*x),x)*a*d + int(s
qrt(sec(e + f*x)*b + a)*sec(e + f*x),x)*b*c
```

3.201 $\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 326

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx}{a+b}}}{df}$$
$$-\frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{cf}$$
$$-\frac{2(bc-ad)^2\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{cd(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
2*b*(a+b)^(1/2)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b)
)^(1/2)/d/f-2*a*(a+b)^(1/2)*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(
a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/c/f-2*(-a*d+b*c)^2*EllipticPi(1/2*(1-sec(f*x+
e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+
b))^(1/2)*tan(f*x+e)/c/d/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 8.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx =$$
$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)^2c(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

output
```
(-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*C
os[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)^2*c*(c + d)*EllipticF[
ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*(a^2*(c^2 - d^2)*EllipticPi
[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + (b*c - a*d)^2*EllipticPi
[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))*Sqrt[a + b*
Sec[e + f*x]])/(c*(c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4416, 3042, 4409, 3042, 4271, 4319, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4416}$$

$$\frac{\int \frac{da^2+b^2c\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{cd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{da^2+b^2c\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{ 4409}$$

$$\frac{a^2d\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx+b^2c\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2d\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b^2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{ 4271}$$

$$\frac{b^2c\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2ad\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{ 4319}$$

$$\frac{\frac{2bc\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{2ad\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{}}{}}{cd}-\frac{(bc-ad)^2\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{cd}$$

$$\Big\downarrow \text{ 4461}$$

$$\frac{\frac{2bc\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{2ad\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{}}{}}{cd}$$

$$\frac{2(bc-ad)^2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cdf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x]),x]
```

output
```
((2*b*Sqrt[a + b]*c*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[e + f*x]))/(a - b))])/f - (2*a*Sqrt[a + b]*d*Cot[e + f*x]*El
lipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x])
)/(a - b))])/f)/(c*d) - (2*(b*c - a*d)^2*EllipticPi[(2*d)/(c + d), ArcSin[
Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])/
(a + b)]*Tan[e + f*x])/(c*d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e
 + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4416
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_)), x_Symbol] :> Simp[1/(c*d)   Int[(a^2*d + b^2*c*Csc[e + f*x])/
Sqrt[a + b*Csc[e + f*x]], x], x] - Simp[(b*c - a*d)^2/(c*d)   Int[Csc[e + f
*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.64

method	result
default	$-\frac{2\left(2\,\mathrm{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)a^2c^2-2\,\mathrm{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)a^2d^2+2\,\mathrm{EllipticPi}\left(\cot\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f/c/(c+d)/(c-d)*(2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1
/2))*a^2*c^2-2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^
2*d^2+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*
a^2*d^2-4*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2)
)*a*b*c*d+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/
2))*b^2*c^2-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^2*c^2-E
llipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a^2*c*d+2*EllipticF(co
t(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c^2+2*EllipticF(cot(f*x+e)-cs
c(f*x+e),((a-b)/(a+b))^(1/2))*a*b*c*d-EllipticF(cot(f*x+e)-csc(f*x+e),((a-
b)/(a+b))^(1/2))*b^2*c^2-EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/
2))*b^2*c*d)*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+
a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \int \frac{(a+b\sec(e+fx))^{\frac{3}{2}}}{c+d\sec(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{d\sec(fx+e)+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{c+d\sec(e+fx)}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)\,d+c}dx\right)a + \left(\int \frac{\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)\,d+c}dx\right)b$$

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a)/(sec(e + f*x)*d + c),x)*a + int((sqrt(sec(e +
 f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)*d + c),x)*b
```

3.202 $\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 442

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(35abc+23a^2d+9b^2d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(}{}}}{15bf}$$

$$+\frac{2\sqrt{a+b}(a^2b(45c-23d)-ab^2(35c-17d)+b^3(5c-9d)+15a^3d)\cot(e+fx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec}}{\sqrt{a-}}\right.\right.}{15bf}$$

$$-\frac{2a^2\sqrt{a+b}c\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$+\frac{2b(5bc+8ad)\sqrt{a+b\sec(e+fx)}\tan(e+fx)}{15f}+\frac{2bd(a+b\sec(e+fx))^{3/2}\tan(e+fx)}{5f}$$

output
```
-2/15*(a-b)*(a+b)^(1/2)*(23*a^2*d+35*a*b*c+9*b^2*d)*cot(f*x+e)*EllipticE((
a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(
a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f+2/15*(a+b)^(1/2)*(a^2*b*(4
5*c-23*d)-a*b^2*(35*c-17*d)+b^3*(5*c-9*d)+15*a^3*d)*cot(f*x+e)*EllipticF((
a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(
a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b/f-2*a^2*(a+b)^(1/2)*c*cot(f*
x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(
1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+2/1
5*b*(8*a*d+5*b*c)*(a+b*sec(f*x+e))^(1/2)*tan(f*x+e)/f+2/5*b*d*(a+b*sec(f*x
+e))^(3/2)*tan(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7138 vs. $2(442) = 884$.

Time = 24.68 (sec) , antiderivative size = 7138, normalized size of antiderivative = 16.15

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4406, 27, 3042, 4544, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4406}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sec(e+fx)}\left(5ca^2+b(5bc+8ad)\sec^2(e+fx)+\left(5da^2+10bca+3b^2d\right)\sec(e+fx)\right)dx+\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\sec(e+fx)}\left(5ca^2+b(5bc+8ad)\sec^2(e+fx)+\left(5da^2+10bca+3b^2d\right)\sec(e+fx)\right)dx+\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(5ca^2+b(5bc+8ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(5da^2+10bca+3b^2d\right)\csc\left(e+fx+\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{4544}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15ca^3+b\left(23da^2+35bca+9b^2d\right)\sec^2(e+fx)+\left(15da^3+45bca^2+17b^2da+5b^3c\right)\sec(e+fx)}{2\sqrt{a+b\sec(e+fx)}}dx+\frac{2b}{\ }\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15ca^3+b\left(23da^2+35bca+9b^2d\right)\sec^2(e+fx)+\left(15da^3+45bca^2+17b^2da+5b^3c\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx+\frac{2b}{\ }\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15ca^3+b\left(23da^2+35bca+9b^2d\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2+\left(15da^3+45bca^2+17b^2da+5b^3c\right)\csc\left(e+fx+\right.}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 4546

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int\frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx+\int\frac{15ca^3+\left(15da^3+45bca^2+17b^2da+5b\right.}{\sqrt{a+b}}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\int\frac{15ca^3+\left(15da^3+45bca^2+\right.}{}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 4409

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^3c\int\frac{1}{\sqrt{a+b\sec(e+fx)}}dx+b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^3c\int\frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+\right.}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 4271

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+\left(15a^3d+a^2b(45c-23d)-ab^2\right.\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\Big\downarrow$ 4319

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2d+35abc+9b^2d\right)\int\frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{30a^2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-}{}}}{}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

$\Big\downarrow$ 4492

$$\frac{1}{5}\left(\frac{1}{3}\left(-\frac{2(a-b)\sqrt{a+b}\left(23a^2d+35abc+9b^2d\right)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec}}{\sqrt{a}}\right.\right.}{bf}\right.\right.$$
$$\frac{2bd\tan(e+fx)(a+b\sec(e+fx))^{3/2}}{5f}$$

input
```
Int[(a + b*Sec[e + f*x])^(5/2)*(c + d*Sec[e + f*x]),x]
```

output
```
(2*b*d*(a + b*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f) + (((-2*(a - b)*Sqrt
[a + b]*(35*a*b*c + 23*a^2*d + 9*b^2*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) + (2*Sqrt[a
+ b]*(a^2*b*(45*c - 23*d) - a*b^2*(35*c - 17*d) + b^3*(5*c - 9*d) + 15*a^3
*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e
+ f*x]))/(a - b))])/(b*f) - (30*a^2*Sqrt[a + b]*c*Cot[e + f*x]*EllipticPi[
(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f)/3 + (2*b*(5*b*c + 8*a*d)*Sqrt[a + b*Sec[e + f*x]]*Tan[e + f*x])/(3*
f))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4406
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
 + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
 - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*
c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1738 vs. $2(403) = 806$.

Time = 36.16 (sec) , antiderivative size = 1739, normalized size of antiderivative = 3.93

method	result	size
default	Expression too large to display	1739
parts	Expression too large to display	1765

input
```
int((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/15/f*(a+b*sec(f*x+e))^(1/2)/(cos(f*x+e)^2*a+a*cos(f*x+e)+b*cos(f*x+e)+b)
*(30*(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^3*c*EllipticPi(cot(f*x+e)-cs
c(f*x+e),-1,((a-b)/(a+b))^(1/2))+23*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*
a^3*d*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+35*(cos(f*x+e)^
2+2*cos(f*x+e)+1)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+
e))/(1+cos(f*x+e)))^(1/2)*a^2*b*c*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))+23*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*b*d*EllipticE(co
t(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+35*(cos(f*x+e)^2+2*cos(f*x+e)+1)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e))
)^(1/2)*a*b^2*c*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+9*(co
s(f*x+e)^2+2*cos(f*x+e)+1)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a
*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b^2*d*EllipticE(cot(f*x+e)-csc(f*x+e)
,((a-b)/(a+b))^(1/2))+9*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*b^3*d*Ellipt
icE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+15*(cos(f*x+e)^2+2*cos(f*x+
e)+1)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f
*x+e)))^(1/2)*a^3*c*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2)...
```

Fricas [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx=\int (b\sec(fx+e)+a)^{\frac{5}{2}}(d\sec(fx+e)+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b^2*d*sec(f*x + e)^3 + a^2*c + (b^2*c + 2*a*b*d)*sec(f*x + e)^2
+ (2*a*b*c + a^2*d)*sec(f*x + e))*sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (a+b\sec{(e+fx)})^{\frac{5}{2}}\,(c+d\sec{(e+fx)})\,\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)*(c+d*sec(f*x+e)),x)
```

output
```
Integral((a + b*sec(e + f*x))**(5/2)*(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (b\sec{(fx+e)}+a)^{\frac{5}{2}}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx = \int (b\sec{(fx+e)}+a)^{\frac{5}{2}}(d\sec{(fx+e)}+c)\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)*(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx=\int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{5/2}\left(c+\frac{d}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)),x)
```

output
```
int((a + b/cos(e + f*x))^(5/2)*(c + d/cos(e + f*x)), x)
```

Reduce [F]

$$\int (a+b\sec(e+fx))^{5/2}(c+d\sec(e+fx))\,dx=\left(\int \sqrt{\sec(fx+e)\,b+a}dx\right)a^2c+\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)^3\,dx\right)b^2d+2\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)^2\,dx\right)abd+\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)^2\,dx\right)b^2c+\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)\,dx\right)a^2d+2\left(\int \sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)\,dx\right)abc$$

input
```
int((a+b*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a),x)*a**2*c + int(sqrt(sec(e + f*x)*b + a)*sec(
e + f*x)**3,x)*b**2*d + 2*int(sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2,x)*
a*b*d + int(sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2,x)*b**2*c + int(sqrt(
sec(e + f*x)*b + a)*sec(e + f*x),x)*a**2*d + 2*int(sqrt(sec(e + f*x)*b + a
)*sec(e + f*x),x)*a*b*c
```

3.203 $\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 208

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}d\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bf} - \frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

output
```
2*(a+b)^(1/2)*d*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b)
)^(1/2)/b/f-2*(a+b)^(1/2)*c*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(
a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/a/f
```

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.70

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((-c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+\right.}{f\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-c + d)*EllipticF[ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)] + 2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2
]], (a - b)/(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4409, 3042, 4271, 4319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4409}$$
$$c\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx+d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$c\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4271$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}$$

$$\Big\downarrow \; 4319$$

$$\frac{2d\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bf}-\frac{2c\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}$$

input
```
Int[(c + d*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a + b]*d*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*c*Cot[e + f*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))
/(a - b))])/(a*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 12.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.88

method	result
default	$\frac{2\left(\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)c-\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)d-2\operatorname{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-\right.\right.}{f(b+a\cos(fx+e))}$
parts	$\frac{2c\left(\operatorname{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)-2\operatorname{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)\right)(1+\cos(fx+e))\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{}}{f(b+a\cos(fx+e))}$

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c-EllipticF(cot(
f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e
),-1,((a-b)/(a+b))^(1/2))*c)*(1+cos(f*x+e))*(cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)
/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c + d\sec(e + fx)}{\sqrt{a + b\sec(e + fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{c + d\sec(e + fx)}{\sqrt{a + b\sec(e + fx)}}\,dx = \int \frac{c + d\sec{(e + fx)}}{\sqrt{a + b\sec{(e + fx)}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{c+d\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)\,b+a}dx\right)c + \left(\int \frac{\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)\,b+a}dx\right)d$$

input

```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

output

```
int(sqrt(sec(e + f*x)*b + a)/(sec(e + f*x)*b + a),x)*c + int((sqrt(sec(e +
 f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)*b + a),x)*d
```

3.204 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 216

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{acf} - \frac{2d\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{c(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

output
```
-2*(a+b)^(1/2)*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a
+b)/a,((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e
))/(a-b))^(1/2)/a/c/f-2*d*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/
(c+d),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/c
/(c+d)/f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 20.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx =$$
$$-\frac{2\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}(d+c\cos(e+fx))\left(c(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-2((c^2-\right.}{c(c-}$$

input
```
Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(-2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(d + c*Cos[e +
 f*x])*(c*(c + d)*EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2
*((c^2 - d^2)*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] +
d^2*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]
))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[1 + Sec[e
 + f*x]])/(c*(c - d)*(c + d)*f*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e +
 f*x]]*(c + d*Sec[e + f*x]))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4418, 3042, 4271, 4461}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \ 4418$$

$$\frac{\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx}{c}-\frac{d\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{c}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{c}$$

$\Big\downarrow$ 4271

$$-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)}dx}{c}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{acf}$$

$\Big\downarrow$ 4461

$$-\frac{2d\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{cf(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{acf}$$

input
```
Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

output
```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
 + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*c*f) - (2*d*EllipticPi[(2*d
)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a
+ b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/(c*(c + d)*f*Sqrt[a + b*Sec[e + f
*x]]*Sqrt[-Tan[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4418
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))), x_Symbol] :> Simp[1/c   Int[1/Sqrt[a + b*Csc[e + f*x]], x],
 x] - Simp[d/c   Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0]
```

rule 4461
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\left(2c^2\,\mathrm{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)-2\,\mathrm{EllipticPi}\left(\cot(fx+e)-\csc(fx+e),-1,\sqrt{\frac{a-b}{a+b}}\right)d^2+2\,\mathrm{EllipticPi}\left(\cot(fx+\right.\right.}{}$

input
```
int(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f/c/(c+d)/(c-d)*(2*c^2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b)
)^(1/2))-2*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b))^(1/2))*d^2+2*
EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d^2-Elli
pticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*c^2-EllipticF(cot(f*x+e)-
csc(f*x+e),((a-b)/(a+b))^(1/2))*c*d)*(1+cos(f*x+e))*(a+b*sec(f*x+e))^(1/2)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)
))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

input
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

output
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$= \int \frac{\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^2\,bd+\sec(fx+e)\,ad+\sec(fx+e)\,bc+ac}dx$$

input
```
int(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a)/(sec(e + f*x)**2*b*d + sec(e + f*x)*a*d + sec
(e + f*x)*b*c + a*c),x)
```

3.205 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 376

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(}{}}}{ab\sqrt{a+b}f}$$
$$-\frac{2(bc-ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{ab\sqrt{a+b}f}$$
$$-\frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^2f}$$
$$+\frac{2b(bc-ad)\tan(e+fx)}{a(a^2-b^2)f\sqrt{a+b\sec(e+fx)}}$$

output
```
2*(-a*d+b*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(
1/2)/a/b/(a+b)^(1/2)/f-2*(-a*d+b*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*
(1+sec(f*x+e))/(a-b))^(1/2)/a/b/(a+b)^(1/2)/f-2*(a+b)^(1/2)*c*cot(f*x+e)*E
llipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*
(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b*(
-a*d+b*c)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1138 vs. $2(376) = 752$.

Time = 12.81 (sec) , antiderivative size = 1138, normalized size of antiderivative = 3.03

$$\int \frac{c + d\sec(e + fx)}{(a + b\sec(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
((b + a*Cos[e + f*x])^2*Sec[e + f*x]*(c + d*Sec[e + f*x])*((2*(-(b*c) + a*
d)*Sin[e + f*x])/(a*(a^2 - b^2)) - (2*(-(b^2*c*Sin[e + f*x]) + a*b*d*Sin[e
 + f*x]))/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(d + c*Cos[e + f*x])*(
a + b*Sec[e + f*x])^(3/2)) - (2*(b + a*Cos[e + f*x])^(3/2)*Sqrt[Sec[e + f*
x]]*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f
*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(-(a*b*c*Tan[(e + f*x)/2]) - b^2*c*Tan
[(e + f*x)/2] + a^2*d*Tan[(e + f*x)/2] + a*b*d*Tan[(e + f*x)/2] + 2*a*b*c*
Tan[(e + f*x)/2]^3 - 2*a^2*d*Tan[(e + f*x)/2]^3 - a*b*c*Tan[(e + f*x)/2]^5
 + b^2*c*Tan[(e + f*x)/2]^5 + a^2*d*Tan[(e + f*x)/2]^5 - a*b*d*Tan[(e + f*
x)/2]^5 - 2*a^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)
]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[
(e + f*x)/2]^2)/(a + b)] + 2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f
*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - 2*a^2*c*EllipticPi[-1, ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f
*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a +
b)] + 2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Ta
n[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x
)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (a + b)*(-(b*c) + a*d)*EllipticE
[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2...
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4411, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4411}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2\int -\frac{-b(bc-ad)\sec^2(e+fx)-a(bc-ad)\sec(e+fx)+\left(a^2-b^2\right)c}{2\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-b(bc-ad)\sec^2(e+fx)-a(bc-ad)\sec(e+fx)+\left(a^2-b^2\right)c}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2-a(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)+\left(a^2-b^2\right)c}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{4546}$$

$$\frac{\int \frac{\left(a^2-b^2\right)c+(b(bc-ad)-a(bc-ad))\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx-b(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a^2-b^2)c+(b(bc-ad)-a(bc-ad))\csc(e+fx+\frac{\pi}{2})}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx-b(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx}{a(a^2-b^2)}+\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 4409

$$\frac{c(a^2-b^2)\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx-b(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx-(a-b)(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a^2-b^2)}+\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{c(a^2-b^2)\int \frac{1}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx-(a-b)(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx-b(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2}}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}}{a(a^2-b^2)}+\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 4271

$$\frac{-(a-b)(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx-b(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx-\frac{2c\sqrt{a+b}(a^2-b^2)\cot(e+fx)\sqrt{}}{}}{a(a^2-b^2)}+\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 4319

$$\frac{-b(bc-ad)\int \frac{\csc(e+fx+\frac{\pi}{2})(\csc(e+fx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(e+fx+\frac{\pi}{2})}}dx-\frac{2c\sqrt{a+b}(a^2-b^2)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\text{arc}\right.}{af}}{a(a^2}+\frac{2b(bc-ad)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}$$

$\downarrow$ 4492

$$-\frac{2c\sqrt{a+b}(a^2-b^2)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}-\frac{2(a-b)\sqrt{a+b}(bc-ad)\cot}{}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

input
```
Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
((2*(a - b)*Sqrt[a + b]*(b*c - a*d)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*(a - b)*Sqr
t[a + b]*(b*c - a*d)*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b*f) - (2*Sqrt[a + b]*(a^2 - b^2)*c*
Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[e + f*x]))/(a - b))])/(a*f))/(a*(a^2 - b^2)) + (2*b*(b*c - a*d)*Tan[
e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. 2(347) = 694.

Time = 11.86 (sec) , antiderivative size = 1070, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	1070
parts	Expression too large to display	1134

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/a/(a+b)/(a-b)*((-(1-cos(f*x+e))^3*csc(f*x+e)^3+csc(f*x+e)-cot(f*x+e))*
a*b*c+((1-cos(f*x+e))^3*csc(f*x+e)^3-csc(f*x+e)+cot(f*x+e))*d*a^2+((1-cos(
f*x+e))^3*csc(f*x+e)^3-csc(f*x+e)+cot(f*x+e))*c*b^2+(-(1-cos(f*x+e))^3*csc
(f*x+e)^3+csc(f*x+e)-cot(f*x+e))*a*b*d-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(
f*x+e),((a-b)/(a+b))^(1/2))*a^2*c+2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e
),((a-b)/(a+b))^(1/2))*a^2*d-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*
(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a
-b)/(a+b))^(1/2))*a*b*c+2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*
cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))*a*b*d-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f
*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))
^(1/2))*a^2*d+2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e)
)/(1+cos(f*x+e)))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2
))*a*b*c-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+
cos(f*x+e)))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*
b*d+2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f
*x+e)))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b^2*c+4
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x...
```

Fricas [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^2*sec(f*x + e)^2
 + 2*a*b*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{c+d\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx=\left(\int \frac{\sqrt{\sec\left(fx+e\right)b+a}}{\sec\left(fx+e\right)^{2}b^{2}+2\sec\left(fx+e\right)ab+a^{2}}dx\right)c+\left(\int \frac{\sqrt{\sec\left(fx+e\right)b+a}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)^{2}b^{2}+2\sec\left(fx+e\right)ab+a^{2}}dx\right)d$$

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(3/2),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a)/(sec(e + f*x)**2*b**2 + 2*sec(e + f*x)*a*b +
a**2),x)*c + int((sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)**2*
b**2 + 2*sec(e + f*x)*a*b + a**2),x)*d
```

3.206 $\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 495

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \frac{2(7a^2bc-3b^3c-4a^3d)\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec\ldots}{a\ldots}}}{3a^2(a-b)b(a+b)^{3/2}f}$$
$$-\frac{2(6a^2bc-ab^2c-3b^3c-3a^3d+a^2bd)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx)\ldots}{a+b}}}{3a^2(a-b)b(a+b)^{3/2}f}$$
$$-\frac{2\sqrt{a+b}c\cot(e+fx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a^3f}$$
$$+\frac{2b(bc-ad)\tan(e+fx)}{3a(a^2-b^2)f(a+b\sec(e+fx))^{3/2}}+\frac{2b(7a^2bc-3b^3c-4a^3d)\tan(e+fx)}{3a^2(a^2-b^2)^2f\sqrt{a+b\sec(e+fx)}}$$

output

```
2/3*(-4*a^3*d+7*a^2*b*c-3*b^3*c)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2/3*(-3*a^3*d+6*a^2*b*c
+a^2*b*d-a*b^2*c-3*b^3*c)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x
+e))/(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/f-2*(a+b)^(1/2)*c*cot(f*x+e)*Ell
ipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(b
*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a^3/f+2/3*b*(
-a*d+b*c)*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+2/3*b*(-4*a^3*d+
7*a^2*b*c-3*b^3*c)*tan(f*x+e)/a^2/(a^2-b^2)^2/f/(a+b*sec(f*x+e))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1589 vs. $2(495) = 990$.

Time = 14.04 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.21

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

output

```
((b + a*Cos[e + f*x])^3*Sec[e + f*x]^2*(c + d*Sec[e + f*x])*((2*(-7*a^2*b*
c + 3*b^3*c + 4*a^3*d)*Sin[e + f*x])/(3*a^2*(a^2 - b^2)^2) - (2*(b^3*c*Sin
[e + f*x] - a*b^2*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[e + f*x])
^2) - (2*(-8*a^2*b^2*c*Sin[e + f*x] + 4*b^4*c*Sin[e + f*x] + 5*a^3*b*d*Sin
[e + f*x] - a*b^3*d*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[e + f*x
]))))/(f*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(5/2)) - (2*(b + a*Cos[
e + f*x])^(5/2)*Sec[e + f*x]^(3/2)*(c + d*Sec[e + f*x])*Sqrt[(a + b - a*Ta
n[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)]*(-7*a^3
*b*c*Tan[(e + f*x)/2] - 7*a^2*b^2*c*Tan[(e + f*x)/2] + 3*a*b^3*c*Tan[(e +
f*x)/2] + 3*b^4*c*Tan[(e + f*x)/2] + 4*a^4*d*Tan[(e + f*x)/2] + 4*a^3*b*d*
Tan[(e + f*x)/2] + 14*a^3*b*c*Tan[(e + f*x)/2]^3 - 6*a*b^3*c*Tan[(e + f*x)
/2]^3 - 8*a^4*d*Tan[(e + f*x)/2]^3 - 7*a^3*b*c*Tan[(e + f*x)/2]^5 + 7*a^2*
b^2*c*Tan[(e + f*x)/2]^5 + 3*a*b^3*c*Tan[(e + f*x)/2]^5 - 3*b^4*c*Tan[(e +
 f*x)/2]^5 + 4*a^4*d*Tan[(e + f*x)/2]^5 - 4*a^3*b*d*Tan[(e + f*x)/2]^5 - 6
*a^4*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2
]^2)/(a + b)] + 12*a^2*b^2*c*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a -
 b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]
^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - 6*b^4*c*EllipticPi[-1, ArcSin[Tan[(e
 + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b ...
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4411, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4411}$$

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}-\frac{2\int -\frac{b(bc-ad)\sec^2(e+fx)-3a(bc-ad)\sec(e+fx)+3\left(a^2-b^2\right)c}{2(a+b\sec(e+fx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(bc-ad)\sec^2(e+fx)-3a(bc-ad)\sec(e+fx)+3\left(a^2-b^2\right)c}{(a+b\sec(e+fx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)^2-3a(bc-ad)\csc\left(e+fx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)c}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\frac{2b\left(-4a^3d+7a^2bc-3b^3c\right)\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2\int -\frac{3c\left(a^2-b^2\right)^2-b\left(-4da^3+7bca^2-3b^3c\right)\sec^2(e+fx)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\sec(e+fx)}{2\sqrt{a+b\sec(e+fx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2-b\left(-4da^3+7bca^2-3b^3c\right)\sec^2(e+fx)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a^2-b^2)}+\frac{2b(-4a^3d+7a^2bc-3b^3c)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2-b\left(-4da^3+7bca^2-3b^3c\right)\csc\left(e+fx+\frac{\pi}{2}\right)^2-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b(-4a^3d+7a^2bc-3b^3c)\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}}{3a\left(a^2-b^2\right)}+\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4546}$$

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2+\left(b\left(-4da^3+7bca^2-3b^3c\right)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}}dx}{a(a^2-b^2)}+\frac{2b(-4}{aj}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{3c\left(a^2-b^2\right)^2+\left(b\left(-4da^3+7bca^2-3b^3c\right)-a\left(-3da^3+6bca^2-b^2da-2b^3c\right)\right)\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4409}$$

$$\frac{\frac{3c(a^2-b^2)^2\int \frac{1}{\sqrt{a+b\sec(e+fx)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+(a-b)(3a^3d-a^2b(6c+d)+ab^2c+3b^3c)\int \frac{\sec(e}{\sqrt{a+b\,se}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$
$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3c(a^2-b^2)^2\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+(a-b)(3a^3d-a^2b(6c+d)+ab^2c+3b^3c)\int \frac{\ldots}{\sqrt{a\ldots}}\ldots}{a(a^2-b^2)}\ldots}{3a\left(a^2-b^2\right)}\ldots$$

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4271

$$\frac{\frac{-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+(a-b)(3a^3d-a^2b(6c+d)+ab^2c+3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{6c\sqrt{a+b}\left(a^2-b^2\right)\ldots}{\ldots}}{a(a^2-b^2)}\ldots}{3a\left(a^2-b^2\right)\ldots}\ldots$$

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4319

$$\frac{\frac{-b(-4a^3d+7a^2bc-3b^3c)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{6c\sqrt{a+b}\left(a^2-b^2\right)^2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\text{arcs}\ldots\right.}{af}\ldots}{\ldots}}{\ldots}\ldots$$

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4492

$$\frac{2b(bc-ad)\tan(e+fx)}{3af\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}+$$

$$\frac{\frac{-\frac{6c\sqrt{a+b}\left(a^2-b^2\right)^2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af}+\frac{2(a-b)\sqrt{a+b}\left(-4a^3d+7a^2bc-3b^3c\right)\ldots}{\ldots}}{\ldots}}{\ldots}\ldots$$

input
```
Int[(c + d*Sec[e + f*x])/(a + b*Sec[e + f*x])^(5/2),x]
```

output
```
(2*b*(b*c - a*d)*Tan[e + f*x])/(3*a*(a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/
2)) + (((2*(a - b)*Sqrt[a + b]*(7*a^2*b*c - 3*b^3*c - 4*a^3*d)*Cot[e + f*x
]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b
))])/(b*f) + (2*(a - b)*Sqrt[a + b]*(a*b^2*c + 3*b^3*c + 3*a^3*d - a^2*b*(
6*c + d))*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[e + f*x]))/(a - b))])/(b*f) - (6*Sqrt[a + b]*(a^2 - b^2)^2*c*Cot[e + f
*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
 f*x]))/(a - b))])/(a*f))/(a*(a^2 - b^2)) + (2*b*(7*a^2*b*c - 3*b^3*c - 4*
a^3*d)*Tan[e + f*x])/(a*(a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]]))/(3*a*(a^2
 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2818 vs. $2(456) = 912$.

Time = 18.13 (sec) , antiderivative size = 2819, normalized size of antiderivative = 5.69

method	result	size
default	Expression too large to display	2819
parts	Expression too large to display	2862

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f/(a-b)^2/(a+b)^2/a^2*((cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a
*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^5*c*EllipticF(cot(f*x+e)-csc(f*x+e),(
(a-b)/(a+b))^(1/2))*(3*cos(f*x+e)^3+6*cos(f*x+e)^2+3*cos(f*x+e))-7*a^4*b*c
*cos(f*x+e)^2*sin(f*x+e)-4*a^3*b^2*cos(f*x+e)*d*sin(f*x+e)+4*a^5*cos(f*x+e
)^2*d*sin(f*x+e)-3*b^5*c*cos(f*x+e)*sin(f*x+e)+(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^5*d*EllipticF(cot
(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(-3*cos(f*x+e)^3-6*cos(f*x+e)^2-3*
cos(f*x+e))+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1
+cos(f*x+e)))^(1/2)*a^5*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-1,((a-b)/(a+b)
)^(1/2))*(-6*cos(f*x+e)^3-12*cos(f*x+e)^2-6*cos(f*x+e))+(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^5*d*Elli
pticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*(4*cos(f*x+e)^3+8*cos(f*x
+e)^2+4*cos(f*x+e))+sin(f*x+e)*cos(f*x+e)*(-5*cos(f*x+e)+3)*a^4*b*d+sin(f*
x+e)*cos(f*x+e)*(1+cos(f*x+e))*a^2*b^3*d+(3*cos(f*x+e)^2+6*cos(f*x+e)+3)*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))
^(1/2)*b^5*c*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+(-6*cos(
f*x+e)^2-12*cos(f*x+e)-6)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*
cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*b^5*c*EllipticPi(cot(f*x+e)-csc(f*x+e),-
1,((a-b)/(a+b))^(1/2))+sin(f*x+e)*cos(f*x+e)*(-4*cos(f*x+e)+2)*a*b^4*c+sin
(f*x+e)*cos(f*x+e)*(8*cos(f*x+e)-6)*a^3*b^2*c+sin(f*x+e)*cos(f*x+e)*(3*...
```

Fricas [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)/(b^3*sec(f*x + e)^3
 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e) + a^3), x)
```

Sympy [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \int \frac{c+d\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*sec(e + f*x))/(a + b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx = \int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{d\sec\left(fx+e\right)+c}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e) + c)/(b*sec(f*x + e) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\int \frac{c+\frac{d}{\cos(e+f\,x)}}{{\left(a+\frac{b}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

input
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2),x)
```

output
```
int((c + d/cos(e + f*x))/(a + b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{c+d\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx=\left(\int \frac{\sqrt{\sec\left(fx+e\right)b+a}}{\sec\left(fx+e\right)^{3}b^{3}+3\sec\left(fx+e\right)^{2}a\,b^{2}+3\sec\left(fx+e\right)a^{2}b+a^{3}}dx\right)c+\left(\int \frac{\sqrt{\sec\left(fx+e\right)b+a}\,\sec\left(fx+e\right)}{\sec\left(fx+e\right)^{3}b^{3}+3\sec\left(fx+e\right)^{2}a\,b^{2}+3\sec\left(fx+e\right)a^{2}b+a^{3}}dx\right)d$$

input
```
int((c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(5/2),x)
```

output
```
int(sqrt(sec(e + f*x)*b + a)/(sec(e + f*x)**3*b**3 + 3*sec(e + f*x)**2*a*b
**2 + 3*sec(e + f*x)*a**2*b + a**3),x)*c + int((sqrt(sec(e + f*x)*b + a)*s
ec(e + f*x))/(sec(e + f*x)**3*b**3 + 3*sec(e + f*x)**2*a*b**2 + 3*sec(e +
f*x)*a**2*b + a**3),x)*d
```

3.207 $\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 389

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\operatorname{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{\sqrt{a+b}f} + \frac{2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-}{(c-d}}}{\sqrt{\frac{a+b}{c+d}}f}$$

output
```
-2*(c+d)^(1/2)*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c
+d)^(1/2)/(a+b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))
^(1/2))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/(a+b)^(1
/2)/f+2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(
a+b*sec(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(-(
-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(
f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/((a+b)/(c+d))^(1/2)
/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.78 (sec) , antiderivative size = 26982, normalized size of antiderivative = 69.36

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 4420, 3042, 4424, 4470}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4420}$$

$$c\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx+d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx+d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4424}$$

$$d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx - \frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{f\sqrt{a+b}}$$

$$\Big\downarrow \text{4470}$$

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c\ldots}}{\sqrt{a+b\ldots}}\right.\right.}{f\sqrt{\frac{a+b}{c+d}}} - \frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{f\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*f)
+ (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)
/(c + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c
 + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)
*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a
 + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[(a + b)/(c + d)]*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4420
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[c +
 d*Csc[e + f*x]], x], x] + Simp[d   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*
x]]/Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4470
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 9.06 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+e)-\cot(fx+e)),\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)ac-\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(El
lipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+
d))^(1/2))*a*c-EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b
)*(c-d)/(a-b)/(c+d))^(1/2))*a*d-EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-
cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*c+EllipticF(((a-b)/(a+b))^(
1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b*d-2*Ellipt
icPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(a+b)/(a-b),((c-d)/(c+d))
^(1/2)/((a-b)/(a+b))^(1/2))*b*d-2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+
e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c)*
(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/
(1+cos(f*x+e)))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(
f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$=\int \sqrt{a+b\sec{(e+fx)}}\sqrt{c+d\sec{(e+fx)}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$=\int \sqrt{b\sec{(fx+e)}+a}\sqrt{d\sec{(fx+e)}+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$=\int \sqrt{b\sec{(fx+e)}+a}\sqrt{d\sec{(fx+e)}+c}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$= \int \sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}\,dx$$
$$= \int \sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}dx$$

input
```
int((a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2),x)
```

output
```
int(sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a),x)
```

3.208 $\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\,\sqrt{}}{\sqrt{a+b}cf}$$

output
```
-2*(c+d)^(1/2)*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c
+d)^(1/2)/(a+b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))
^(1/2))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/(a+b)^(1
/2)/c/f
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{4\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\csc(e+fx)\left((a+b)c\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(a+b)(d+c\ldots}{\ldots}}\right.\right.\right.}{(a+\ldots}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*((a + b)*c*Ellipti
cF[ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) +
 a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))] - a*(c + d)*EllipticPi
[(b*c - a*d)/(a*c + b*c), ArcSin[Sqrt[((a + b)*(d + c*Cos[e + f*x])*Csc[(e
 + f*x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]
)*Sqrt[a + b*Sec[e + f*x]]*Sin[(e + f*x)/2]^2)/((a + b)*c*f*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[c + d*Sec[e + f
*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {3042, 4424}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4424}$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\right.\right.}{cf\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c*f
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.63

method	result
default	$\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\left(2\,\text{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+e)-\cot(fx+e)),-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)a-\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+\right.\right.}{f\sqrt{}}$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(2*E
llipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/
(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x
+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a+EllipticF(((a-b)/(a+b))
^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b)*(1/(c+d
)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(
f*x+e)))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{b\sec(fx+e)+a}}{\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+b\sec{(e+fx)}}}{\sqrt{c+d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/sqrt(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}}{\sqrt{d\sec{(fx+e)}+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}}{\sqrt{d\sec{(fx+e)}+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)d+c}\,\sqrt{\sec\left(fx+e\right)b+a}}{\sec\left(fx+e\right)d+c}dx$$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)*d +
c),x)
```

3.209 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 598

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(b}{(}}}{\sqrt{a+b}c^2f}$$

$$-\frac{2\sqrt{a+b}d\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)(1+\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}}{c(c-d)\sqrt{c+d}f\sqrt{-\frac{(bc-ad)(1+\sec(e+fx))}{(a-b)(c+d\sec(e+fx))}}}$$

$$-\frac{2(a-b)\sqrt{a+b}d\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-}}{c(c-d)\sqrt{c+d}(bc-ad)f}$$

output
```
-2*(c+d)^(1/2)*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c
+d)^(1/2)/(a+b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))
^(1/2))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/(a+b)^(1
/2)/c^2/f-2*(a+b)^(1/2)*d*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e)
)^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)*(1+sec(f*x+e))*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)/
c/(c-d)/(c+d)^(1/2)/f/(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^
(1/2)-2*(a-b)*(a+b)^(1/2)*d*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+
e))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/
2))*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/c/(c-d)/(c+d
)^(1/2)/(-a*d+b*c)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1708 vs. $2(598) = 1196$.

Time = 13.44 (sec) , antiderivative size = 1708, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
((d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]]*((4*b*c*
(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b +
a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos
[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[
Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[
2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c +
d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(a*c
 + b*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*
Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e
 + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sq
rt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]
], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)
*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sqrt[((c + d)*Cot[(
e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2
]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/
(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)*c), ArcSin[Sqrt[
((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]],
(2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*c*Sqrt[b +
 a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) + 2*a*d*((Sqrt[(-a + b)/(a + b
)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*EllipticE[ArcSin[(...
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4427, 3042, 4424, 4474, 3042, 4472, 4482}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4427}$$

$$\frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx}{c}-\frac{d\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}dx}{c}$$

$\downarrow$ 3042

$$\frac{\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c}-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}$$

$\downarrow$ 4424

$$-\frac{d\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{c^2f\sqrt{a+b}}$$

$\downarrow$ 4474

$$-\frac{d\left(\frac{(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx}{c-d}+\frac{(a-b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx}{c-d}\right)}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{c^2f\sqrt{a+b}}$$

$\downarrow$ 3042

$$-\frac{d\left(\frac{(a-b)\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{c-d}+\frac{(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx}{c-d}\right)}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{c^2f\sqrt{a+b}}$$

$\downarrow$ 4472

$$-\frac{d\left(\frac{(bc-ad)\int \frac{\sec(e+fx)(\sec(e+fx)+1)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx}{c-d}+\frac{2(a-b)\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)}{(a-b)(c+d\sec(e+f}}}{f(c-d)\sqrt{c+d}(bc-ad)}\right.}{c}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{c^2f\sqrt{a+b}}$$

$$\Big\downarrow \text{ 4482}$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\cdots\right.\right.}{c^2f\sqrt{a+b}}$$

$$\frac{d\left(\frac{2(a-b)\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c\cdots}{(a-b)(c\cdots}\right.}{f(c-d)\sqrt{c+d}(bc-ad)}\cdots\right.}{c}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(Sqrt[a + b]*c^2
*f) - (d*((2*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a
 + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*(1 + Sec[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sec[e +
f*x]))/((a + b)*(c + d*Sec[e + f*x]))])/((c - d)*Sqrt[c + d]*f*Sqrt[-(((b*
c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]) + (2*(a - b
)*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
+ f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a -
b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f
*x])))]*(c + d*Sec[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)))/c
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4427
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_))^(3/2), x_Symbol] :> Simp[1/c   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt
[c + d*Csc[e + f*x]], x], x] - Simp[d/c   Int[Csc[e + f*x]*(Sqrt[a + b*Csc[
e + f*x]]/(c + d*Csc[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[c^2 - d^2, 0]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4474
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(3/2), x_Symbol] :> Simp[(a - b)/(c -
d)   Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
x], x] + Simp[(b*c - a*d)/(c - d)   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/(S
qrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])^(3/2))), x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^
2, 0]
```

rule 4482
```
Int[(sec[(e_.) + (f_.)*(x_)]*((A_) + (B_.)*sec[(e_.) + (f_.)*(x_)]))/(Sqrt[
(a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sec[(e_.) + (f_.)*(x_)]
)^(3/2)), x_Symbol] :> Simp[2*A*(1 + Sec[e + f*x])*(Sqrt[(b*c - a*d)*((1 -
Sec[e + f*x])/((a + b)*(c + d*Sec[e + f*x])))]/(f*(b*c - a*d)*Rt[(c + d)/(a
 + b), 2]*Tan[e + f*x]*Sqrt[(-(b*c - a*d))*((1 + Sec[e + f*x])/((a - b)*(c
+ d*Sec[e + f*x])))]))*EllipticE[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*
Sec[e + f*x]]/Sqrt[c + d*Sec[e + f*x]])], (a + b)*((c - d)/((a - b)*(c + d)
))], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1599 vs. $2(553) = 1106$.

Time = 6.61 (sec) , antiderivative size = 1600, normalized size of antiderivative = 2.68

method	result	size
default	Expression too large to display	1600

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/c/(c-d)/((a-b)/(a+b))^(1/2)/(c+d)*((-2*cos(f*x+e)^2-4*cos(f*x+e)-2)*(
1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(
1+cos(f*x+e)))^(1/2)*a*c^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(
f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))+(2*cos(f*x+e
)^2+4*cos(f*x+e)+2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+
d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*d^2*EllipticPi(((a-b)/(a+b))^(
1/2)*(csc(f*x+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b)
)^(1/2))+(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f
*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*c*d*Ellipt
icE(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^
(1/2))+(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x
+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*d^2*Elliptic
E(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1
/2))+(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)
))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*b*c*d*EllipticE((
(a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)+(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^
(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*b*d^2*EllipticE(((a-
b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))+(
cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a+b\sec{(e+fx)}}}{(c+d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{b\sec{(fx+e)}+a}}{(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{b\sec{(fx+e)}+a}}{(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^2\,d^2+2\sec(fx+e)\,cd+c^2}dx$$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)**2*d
**2 + 2*sec(e + f*x)*c*d + c**2),x)
```

3.210 $\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 899

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx = \frac{2(a-b)\sqrt{a+b}d(6bc^3-7ac^2d-2bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(b}{(a}}}{3c^2(c-d)^2(c+d)}$$

$$+\frac{2\sqrt{a+b}(bc^2(3c^2+3cd-2d^2)-ad(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx}{(a-b)(d+c\cos(e+fx}}}{3c^3(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b}}$$

$$-\frac{2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)}\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+}}$$

$$+\frac{2d^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
2/3*(a-b)*(a+b)^(1/2)*d*(-7*a*c^2*d+3*a*d^3+6*b*c^3-2*b*c*d^2)*(-(-a*d+b*c
)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))
/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE
((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^2/(c-d)^2/(c+d)^(3/
2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/3*(a+b)^
(1/2)*(b*c^2*(3*c^2+3*c*d-2*d^2)-a*d*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(-(-a*
d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*
x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*Elli
pticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2
),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^3/(c-d)^2/(c+d
)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*(a+b)
^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b
*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*cs
c(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*co
s(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(
f*x+e))^(1/2)/c^3/(c+d)^(1/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1
/2)+2/3*d^2*(a+b*sec(f*x+e))^(1/2)*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e
))/(c+d*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1990 vs. $2(899) = 1798$.

Time = 7.13 (sec) , antiderivative size = 1990, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a + b\sec(e + fx)}}{(c + d\sec(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

output

```
((d + c*Cos[e + f*x])^3*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*Si
n[e + f*x])/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*x])^2) - (2*(6*b*c^3*d*Sin[e
 + f*x] - 7*a*c^2*d^2*Sin[e + f*x] - 2*b*c*d^3*Sin[e + f*x] + 3*a*d^4*Sin[
e + f*x]))/(3*c*(b*c - a*d)*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(c +
d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*Sqrt[a
 + b*Sec[e + f*x]]*((4*(b*c - a*d)*(3*b^2*c^4 - 3*a*b*c^3*d - a^2*c^2*d^2
+ b^2*c^2*d^2 - a*b*c*d^3 + a^2*d^4)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c
- d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*
Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e
 + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)
/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*
x]]) + 4*(b*c - a*d)*(3*a*b*c^4 - 3*a^2*c^3*d + 6*b^2*c^3*d - 7*a*b*c^2*d^
2 - a^2*c*d^3 - 2*b^2*c*d^3 + 4*a*b*d^4)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^
2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e
+ f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin
[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[
e + f*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*...
```

Rubi [A] (verified)

Time = 2.92 (sec) , antiderivative size = 826, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 4430, 3042, 3527, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\cos^2(e+fx)\sqrt{b+a\cos(e+fx)}}{(d+c\cos(e+fx))^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3527}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int-\frac{-3a\left(c^2-d^2\right)\cos^2(e+fx)-\left(3bc^2-3adc-2bd^2\right)\cos(e+fx)+d(3bc-ad)}{2\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{3c(c^2-d^2)}+\frac{2d^2\sin(e+fx)\sqrt{}}{3cf(c^2-d^2)(c\,c}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-3a\left(c^2-d^2\right)\cos^2(e+fx)-\left(3bc^2-3adc-2bd^2\right)\cos(e+fx)-}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-3a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-3bc^2+3adc+2bd^2\right)\sin\left(e\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right.}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3532}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\int\frac{d\left(3bc^3+2adc^2-3ad^3\right)-c(bc-3ad)\left(3c^2-2d^2\right)\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}d}{c^2}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\int\frac{d\left(3bc^3+2adc^2-3ad^3\right)-c(bc-3ad)\left(3c^2-2d^2\right)\sin\left(e+fx+\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}}{c^2}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3290

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{\int\frac{d\left(3bc^3+2adc^2-3ad^3\right)-c(bc-3ad)\left(3c^2-2d^2\right)\sin\left(e+fx+\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}}{c^2}}{}\right.}{\sqrt{a\cos(e+f}}$$

$\downarrow$ 3477

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{cd\left(-7ac^2d+3ad^3+6bc^3-2bcd^2\right)\int\frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos}}{c-d}}{}\right.}{}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{cd\left(-7ac^2d+3ad^3+6bc^3-2bcd^2\right)\int\frac{\sin\left(e+fx+\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+\right.}}{c-d}}{}\right.}{}$$

$\downarrow$ 3297

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf(c^2-d^2)(c\cos(e+fx)+d)^{3/2}}-\frac{\frac{cd\left(-7ac^2d+3ad^3+6bc^3-2bcd^2\right)\int\frac{\sin\left(e+fx+\cdots\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+\cdots\right.}}{c-d}\cdots}{\cdots}\right.$$

$\downarrow$ 3475

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))^{3/2}}-\frac{-\frac{2(a-b)\sqrt{a+b}cd\left(6bc^3-7adc^2-2bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1\cdots}{(a+b)(d+\cdots}}}{\cdots}}{\cdots}\right.$$

input

```
Int[Sqrt[a + b*Sec[e + f*x]]/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*(-1/3*(((-2*(a - b)*Sqr
t[a + b]*c*d*(6*b*c^3 - 7*a*c^2*d - 2*b*c*d^2 + 3*a*d^3)*Sqrt[-(((b*c - a*
d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d
)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x]
)*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sq
rt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
)/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) - (2*Sqrt[a + b]*(c*(b*c - 3*a*d)*
(3*c^2 - 2*d^2) + d*(3*b*c^3 + 2*a*c^2*d - 3*a*d^3))*Sqrt[-(((b*c - a*d)*(
1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1
 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Cs
c[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a
 + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((
c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 + (6*Sqrt[a + b]*(c^2 - d^2)*Sqrt[-
(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(
((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*C
os[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sqrt
[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])],
 ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c + d]*f))/(c*(c^2 - d^2)
) + (2*d^2*Sqrt[b + a*Cos[e + f*x]]*Sin[e + f*x])/(3*c*(c^2 - d^2)*f*(d +
c*Cos[e + f*x])^(3/2))))/(Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7331 vs. $2(820) = 1640$.

Time = 8.31 (sec) , antiderivative size = 7332, normalized size of antiderivative = 8.16

method	result	size
default	Expression too large to display	7332

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(d^3*sec(f*x +
e)^3 + 3*c*d^2*sec(f*x + e)^2 + 3*c^2*d*sec(f*x + e) + c^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{a+b\sec\left(e+fx\right)}}{\left(c+d\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{b\sec\left(fx+e\right)+a}}{\left(d\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(1/2)/(c + d/cos(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)d+c}\,\sqrt{\sec\left(fx+e\right)b+a}}{\sec\left(fx+e\right)^{3}d^{3}+3\sec\left(fx+e\right)^{2}c\,d^{2}+3\sec\left(fx+e\right)c^{2}d+c^{3}}dx$$

input
```
int((a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)**3*d
**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec(e + f*x)*c**2*d + c**3),x)
```

3.211 $\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 593

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx =$$

$$-\frac{2\sqrt{c+d}(-bc+ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(-bc+ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\ \cdots}{\sqrt{a+b}cdf}$$

$$-\frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{-\frac{(-bc+ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\ \cdots}{c\sqrt{c+d}f}$$

$$+\frac{2b\cot(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\frac{\sqrt{\frac{c+d}{a+b}}\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{-\frac{(-bc+ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{\frac{(-bc+\cdots}{(a-b\cdots}}}{d\sqrt{\frac{c+d}{a+b}}f}$$

output
```
-2*(c+d)^(1/2)*(a*d-b*c)*cot(f*x+e)*EllipticF((a+b)^(1/2)*(c+d*sec(f*x+e))
^(1/2)/(c+d)^(1/2)/(a+b*sec(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))
*((a*d-b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*(-(a*d-b*c)*(1+se
c(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/(a+b)^(1/2)/c/d/f
-2*a*(a+b)^(1/2)*cot(f*x+e)*EllipticPi((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/
(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d
))^(1/2))*(-(a*d-b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*((a*d-b
*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/c/(c+d)^
(1/2)/f+2*b*cot(f*x+e)*EllipticPi(((c+d)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/
2)/(c+d*sec(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*(-(a*d-b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*((a*d-b*c)*(1+se
c(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/d/((c+d)/(a+b))^(
1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 37.30 (sec) , antiderivative size = 50041, normalized size of antiderivative = 84.39

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{ 4433}$$
$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/Sqrt[c + d*Sec[e + f*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 12.07 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\left(2\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+e)-\cot(fx+e)),-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)a^2+2\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(\right.\right.}{}$

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(2*E
llipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/
(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a^2+2*EllipticPi(((a-b)/(a+b))^(1/2)*(cs
c(f*x+e)-cot(f*x+e)),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*
b^2-EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-
b)/(c+d))^(1/2))*a^2+2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e
)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a*b-EllipticF(((a-b)/(a+b))^(1/2)*(csc
(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b^2)*(1/(c+d)*(d+c*co
s(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^
(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{c+d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)/sqrt(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/sqrt(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{\sqrt{d\sec(fx+e)+c}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/sqrt(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)\,d+c}dx\right)b + \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)\,d+c}dx\right)a$$

input

```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output

```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(
e + f*x)*d + c),x)*b + int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b +
 a))/(sec(e + f*x)*d + c),x)*a
```

3.212 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 744

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = -\frac{2(a-b)\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)E\Big(\arcsin\Big(}{c(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+fx)}}$$

$$-\frac{2\sqrt{a+b}(bc-a(2c-d))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{E}}{c^2(c-d)\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\,\mathrm{s}}}$$

$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\Big(\frac{(a+b)}{a(c+d}}{c^2\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+}}$$

output
```
-2*(a-b)*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(
1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*
x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^
(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x
+e))^(1/2)/c/(c-d)/(c+d)^(1/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(
1/2)-2*(a+b)^(1/2)*(b*c-a*(2*c-d))*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*
cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/
2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e)
)^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)*(a+b*sec(f*x+e))^(1/2)/c^2/(c-d)/(c+d)^(1/2)/f/(b+a*cos(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2)-2*a*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+
c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(
1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x
+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)
/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^2/(c+d)^(1/2)/f/(b+a*cos(f*x
+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1750 vs. $2(744) = 1488$.

Time = 18.78 (sec) , antiderivative size = 1750, normalized size of antiderivative = 2.35

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*(d + c*Cos[e + f*x])*(a + b*Sec[e + f*x])^(3/2)*(-(b*c*Sin[e + f*x]) +
a*d*Sin[e + f*x]))/((-c^2 + d^2)*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x
])^(3/2)) + ((d + c*Cos[e + f*x])^(3/2)*(a + b*Sec[e + f*x])^(3/2)*((4*(b*
c - a*d)*(a*b*c - b^2*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[(
(c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a -
b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*Elli
pticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
- a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/(
(a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(a^
2*c - b^2*c)*(b*c - a*d)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt
[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a
- b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*El
lipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)
/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (Sq
rt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x]
)*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a + b)
*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c -
 a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)...
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 691, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4430, 3042, 3277, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3277}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{a^2\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}+\frac{(bc-ad)\int\frac{bc+2a\cos(e+fx)c+ad}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{a^2\int\frac{\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}+\frac{(bc-ad)\int\frac{bc+2a\sin\left(e+fx+\frac{\pi}{2}\right)c+ad}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3290}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\int\frac{bc+2a\sin\left(e+fx+\frac{\pi}{2}\right)c+ad}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-\frac{2a\sqrt{a+b}\csc(e+fx)(c\cos(e+fx}{\cdots}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\text{s}\cdots}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(\frac{c(bc-ad)\int\frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c-d}-\frac{(bc-a(2c-d))\int\frac{1}{\sqrt{b+a\cos(e+fx)}\sqrt{\cdots}}}{c-d}\right.}{c^2}\right.}{\sqrt{a\cos\cdots}}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(\frac{c(bc-ad)\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{(bc-a(2c-d))\int\frac{}{\sqrt{b+a\sin}}}{}\right.}{c^2}\right.$$

$\downarrow$ 3297

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(\frac{c(bc-ad)\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{2\sqrt{a+b}(bc-a(2c-d))\csc(e}{}\right.}{}\right.$$

$\downarrow$ 3475

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{(bc-ad)\left(-\frac{2\sqrt{a+b}(bc-a(2c-d))\csc(e+fx)(c\cos(e+fx)+d)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)}{(a-b)(c}}}{f(c-d)\sqrt{c+d}(bc-}\right.}{}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(3/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*(((b*c - a*d)*((-2*(a - b)*Sqrt[a + b]*c*Sqrt[-(
((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e +
f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)
*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) - (2*Sqrt[a + b]*(b*c - a*
(2*c - d))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e +
 f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*S
qrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f)))/c^2 -
(2*a*Sqrt[a + b]*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a
*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt
[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c
+ d]*f))*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Se
c[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3277
```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[d^2/b^2   Int[Sqrt[a + b*Sin[e + f*x
]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[(b*c - a*d)/b^2   Int[Simp[b*c +
 a*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2390 vs. $2(675) = 1350$.

Time = 10.08 (sec) , antiderivative size = 2391, normalized size of antiderivative = 3.21

method	result	size
default	Expression too large to display	2391

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/(c+d)/((a-b)/(a+b))^(1/2)/(c-d)/c*((2*cos(f*x+e)^2+4*cos(f*x+e)+2)*(1/
(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+
cos(f*x+e)))^(1/2)*a^2*c^2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(
f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))+(-2*cos(f*x+
e)^2-4*cos(f*x+e)-2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a
+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*d^2*EllipticPi(((a-b)/(a+b)
)^(1/2)*(csc(f*x+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a
+b))^(1/2))+(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos
(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*c*d*El
lipticE(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+
d))^(1/2))+(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a^2*d^2*Ell
ipticE(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d
))^(1/2))+(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b*c^2*Ell
ipticE(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d
))^(1/2))+(-2*cos(f*x+e)^2-4*cos(f*x+e)-2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+co
s(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*a*b*c*d*E
llipticE(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c
+d))^(1/2))+(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(1/(c+d)*(d+c*cos(f*x+e))/(1...
```

Fricas [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="fric
as")
```

output
```
integral((b*sec(f*x + e) + a)^(3/2)*sqrt(d*sec(f*x + e) + c)/(d^2*sec(f*x
+ e)^2 + 2*c*d*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{(a+b\sec(e+fx))^{\frac{3}{2}}}{(c+d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{3/2}}\,dx=\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)^{2}d^{2}+2\sec(fx+e)\,cd+c^{2}}dx\right)b+\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^{2}d^{2}+2\sec(fx+e)\,cd+c^{2}}dx\right)a$$

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(
e + f*x)**2*d**2 + 2*sec(e + f*x)*c*d + c**2),x)*b + int((sqrt(sec(e + f*x
)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)**2*d**2 + 2*sec(e + f*x)*
c*d + c**2),x)*a
```

3.213 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 919

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(3bc^3-7ac^2d+bcd^2+3ad^3)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+}{3c^2(c-d)^2(c+d)^{3/2}(bc-ad)f\sqrt{b+a\cos(e+}$$

$$-\frac{2\sqrt{a+b}(b^2c^3(3c+d)-2abc^2(3c^2+2cd-d^2)+a^2d(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{}}{3c^3(c-d)^2(c+d)^{3/2}(bc-}$$

$$-\frac{2a\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)}{a(c+d}\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+}$$

$$-\frac{2d(bc-ad)\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c\,(c^2-d^2)\,f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
-2/3*(a-b)*(a+b)^(1/2)*(-7*a*c^2*d+3*a*d^3+3*b*c^3+b*c*d^2)*(-(-a*d+b*c)*(
1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a
-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c
+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)
*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^2/(c-d)^2/(c+d)^(3/2)/
(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2/3*(a+b)^(1/2)
*(b^2*c^3*(3*c+d)-2*a*b*c^2*(3*c^2+2*c*d-d^2)+a^2*d*(9*c^3-2*c^2*d-6*c*d^2
+3*d^3))*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d
+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*
csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*c
os(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c
^3/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2)-2*a*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e))
)^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos
(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a
+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^3/(c+d)^(1/2)/f/(b+a*cos(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2)-2/3*d*(-a*d+b*c)*(a+b*sec(f*x+e))^(1/2)*sin(f*x+e)/c/
(c^2-d^2)/f/(d+c*cos(f*x+e))/(c+d*sec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1960 vs. 2(919) = 1838.

Time = 6.99 (sec) , antiderivative size = 1960, normalized size of antiderivative = 2.13

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
((d + c*Cos[e + f*x])^3*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((2*(-(b*c
*d*Sin[e + f*x]) + a*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d + c*Cos[e + f*
x])^2) + (2*(3*b*c^3*Sin[e + f*x] - 7*a*c^2*d*Sin[e + f*x] + b*c*d^2*Sin[e
 + f*x] + 3*a*d^3*Sin[e + f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x])))
)/(f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*
x])^(5/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((4*(b*c - a*d)*(3*a*b*c
^3 + a^2*c^2*d - 4*b^2*c^2*d + a*b*c*d^2 - a^2*d^3)*Sqrt[((c + d)*Cot[(e +
 f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2
)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x]
)*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c -
 d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d
 + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(3*a^2*c^3 - 3*b^2*c^3 + 4*a*b*c^2*d +
 a^2*c*d^2 - b^2*c*d^2 - 4*a*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c
 - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]
*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[
e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x
)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e +
 f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f
*x]]) - (Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + ...
```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 850, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 4430, 3042, 3468, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\cos(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3468

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int\frac{3a^2\left(c^2-d^2\right)\cos^2(e+fx)-\left(3cda^2-2b\left(3c^2-d^2\right)a+b^2cd\right)\cos(e+fx)+(bc-ad)(3bc-ad)}{2\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{3c(c^2-d^2)}-\frac{2d}{}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{3a^2\left(c^2-d^2\right)\cos^2(e+fx)-\left(3cda^2-2b\left(3c^2-d^2\right)a+b^2cd\right)\cos(e+fx)+(bc-ad)(3bc-ad)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{3c(c^2-d^2)}-\frac{2d(b}{3}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{3a^2\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-3cda^2+2b\left(3c^2-d^2\right)a-b^2cd\right)\sin\left(e+fx+\frac{\pi}{2}\right)+(bc-ad)(3bc-ad)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}d}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3532

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\int\frac{3b^2c^4-4abdc^3-\left(\left(9c^2d-6d^3\right)a^2-b\left(6c^3-2cd^2\right)a+b^2c^2d\right)\cos(e+fx)c-a^2d^2\left(2c^2-3d^2\right)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}+\frac{3a^2(}{}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\int \frac{3b^2c^4-4abdc^3-\left(\left(9c^2d-6d^3\right)a^2-b\left(6c^3-2cd^2\right)a+b^2c^2d\right)\sin\left(e+fx+\frac{\pi}{2}\right)c-a^2d^2\left(2c^2-3d^2\right)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}+}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3290

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\int \frac{3b^2c^4-4abdc^3-\left(\left(9c^2d-6d^3\right)a^2-b\left(6c^3-2cd^2\right)a+b^2c^2d\right)\sin\left(e+fx+\frac{\pi}{2}\right)c-a^2d^2\left(2c^2-3d^2\right)}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-}{}\right.}{}$$

$\downarrow$ 3477

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{c(bc-ad)\left(-7ac^2d+3ad^3+3bc^3+bcd^2\right)\int\frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c-d}-\frac{\left(a^2d\left(9c^3-2c^2d-\right.\right.}{}}{c^2}\right.}{}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{c(bc-ad)\left(-7ac^2d+3ad^3+3bc^3+bcd^2\right)\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{\left(a^2d\left(9c\right.\right.}{}}{}\right.}{}$$

$\downarrow$ 3297

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{c(bc-ad)\left(-7ac^2d+3ad^3+3bc^3+bcd^2\right)\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}-\frac{2\sqrt{a+b}(\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 3475

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{-\frac{2(a-b)\sqrt{a+b}c\left(3bc^3-7adc^2+bd^2c+3ad^3\right)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(d+c\cos(e+fx))}}(d\ldots}{(c-d)\sqrt{c+d}(bc-ad)f}}{\ldots}\right.$$

input

```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output

```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((((-2*(a - b)*Sqrt[a +
 b]*c*(3*b*c^3 - 7*a*c^2*d + b*c*d^2 + 3*a*d^3)*Sqrt[-(((b*c - a*d)*(1 - C
os[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Co
s[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e +
 f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]
*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d
)*Sqrt[c + d]*(b*c - a*d)*f) - (2*Sqrt[a + b]*(b^2*c^3*(3*c + d) - 2*a*b*c
^2*(3*c^2 + 2*c*d - d^2) + a^2*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sqrt
[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[
-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c
*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e
 + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a -
 b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 - (6*a*Sqrt[a + b]
*(c^2 - d^2)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
 + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e
+ f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c
+ d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d +
 c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c + d]
*f))/(3*c*(c^2 - d^2)) - (2*d*(b*c - a*d)*Sqrt[b + a*Cos[e + f*x]]*Sin[e +
 f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(3/2))))/(Sqrt[b + a*Cos...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 6041 vs. $2(840) = 1680$.

Time = 12.58 (sec) , antiderivative size = 6042, normalized size of antiderivative = 6.57

method	result	size
default	Expression too large to display	6042

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\sec(e+fx))^{\frac{3}{2}}}{(c+d\sec(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x))**(3/2)/(c + d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)^3\,d^3+3\sec(fx+e)^2\,c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx\right)b + \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^3\,d^3+3\sec(fx+e)^2\,c\,d^2+3\sec(fx+e)\,c^2d+c^3}dx\right)a$$

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(
e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec(e + f*x)*c**2*d + c**3
),x)*b + int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e +
f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec(e + f*x)*c**2*d + c**3),x)
*a
```

3.214 $\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1122

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\text{Too large to display}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(2*a*b*c*d*(35*c^4-8*c^2*d^2+5*d^4)-a^2*d^2*(58*c^4
-41*c^2*d^2+15*d^4)-b^2*(15*c^6+19*c^4*d^2-2*c^2*d^4))*(-(-a*d+b*c)*(1-cos
(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(
d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(
1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d
)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^3/(c-d)^3/(c+d)^(5/2)/(-a*d
+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2/15*(a+b)^(1/2)*(
b^2*c^3*(15*c^3+10*c^2*d+9*c*d^2-2*d^3)-2*a*b*c^2*(15*c^4+20*c^3*d-4*c^2*d
^2-4*c*d^3+5*d^4)+a^2*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d^3+30*c*d^4-15*
d^5))*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*
c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc
(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(
f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^4/
(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(
1/2)-2*a*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(
1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*
x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)
^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1
/2))*(a+b*sec(f*x+e))^(1/2)/c^4/(c+d)^(1/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*
sec(f*x+e))^(1/2)+2/5*d^2*(b+a*cos(f*x+e))*(a+b*sec(f*x+e))^(1/2)*sin(f...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2385 vs. $2(1122) = 2244$.

Time = 7.75 (sec) , antiderivative size = 2385, normalized size of antiderivative = 2.13

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

output
```
((d + c*Cos[e + f*x])^4*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2)*((-2*(-(
b*c*d^2*Sin[e + f*x]) + a*d^3*Sin[e + f*x]))/(5*c^2*(c^2 - d^2)*(d + c*Cos
[e + f*x])^3) - (4*(5*b*c^3*d*Sin[e + f*x] - 8*a*c^2*d^2*Sin[e + f*x] - b*
c*d^3*Sin[e + f*x] + 4*a*d^4*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^2*(d + c*C
os[e + f*x])^2) + (2*(15*b^2*c^6*Sin[e + f*x] - 70*a*b*c^5*d*Sin[e + f*x]
+ 58*a^2*c^4*d^2*Sin[e + f*x] + 19*b^2*c^4*d^2*Sin[e + f*x] + 16*a*b*c^3*d
^3*Sin[e + f*x] - 41*a^2*c^2*d^4*Sin[e + f*x] - 2*b^2*c^2*d^4*Sin[e + f*x]
 - 10*a*b*c*d^5*Sin[e + f*x] + 15*a^2*d^6*Sin[e + f*x]))/(15*c^2*(b*c - a*
d)*(c^2 - d^2)^3*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])*(c + d*Se
c[e + f*x])^(7/2)) + ((d + c*Cos[e + f*x])^(7/2)*Sec[e + f*x]^2*(a + b*Sec
[e + f*x])^(3/2)*((4*(b*c - a*d)*(-15*a*b^2*c^6 + 5*a^2*b*c^5*d + 25*b^3*c
^5*d + 13*a^3*c^4*d^2 - 38*a*b^2*c^4*d^2 + 25*a^2*b*c^3*d^3 + 7*b^3*c^3*d^
3 - 18*a^3*c^2*d^4 - 11*a*b^2*c^2*d^4 + 2*a^2*b*c*d^5 + 5*a^3*d^6)*Sqrt[((
c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
 f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d +
 c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d)
)/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e
 + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)*(-15*a^2*b*c^6 + 15*b^3
*c^6 + 15*a^3*c^5*d - 55*a*b^2*c^5*d + 33*a^2*b*c^4*d^2 + 19*b^3*c^4*d^...
```

Rubi [A] (verified)

Time = 4.36 (sec) , antiderivative size = 1053, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {3042, 4430, 3042, 3527, 27, 3042, 3526, 27, 3042, 3532, 25, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\cos^2(e+fx)(b+a\cos(e+fx))^{3/2}}{(d+c\cos(e+fx))^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3527$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int-\frac{\sqrt{b+a\cos(e+fx)}\left(-5a\left(c^2-d^2\right)\cos^2(e+fx)-\left(5bc^2-5adc-2bd^2\right)\cos(e+fx)+d(5bc-3ad)\right)}{2(d+c\cos(e+fx))^{5/2}}dx}{5c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\int\frac{\sqrt{b+a\cos(e+fx)}\left(-5a\left(c^2-d^2\right)\cos^2(e+fx)-\left(5bc^2-5ac\right.\right.}{(d+c\cos(e+fx))^{5/2}}}{5c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\int\frac{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(-5a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(\right.\right.}{\left(d+c\sin\left(e+fx\right.\right.}}{5c(c^2-}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \; 3526$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{2\int-\frac{3\left(5c^4+3d^2c^2\right)b^2-8acd\left(5c^2-d^2\right)b+15a^2\left(c^2-d^2\right)^2}{}}{}\right.}{\sqrt{a\cos(e+fx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.}{\sqrt{a\cos(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.}{\sqrt{a\cos(e+}}$$

$\downarrow$ 3532

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.}{\sqrt{}}$$

$\downarrow$ 25

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.}{\sqrt{a}}$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3290

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3477

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{3/2}}{5cf(c^2-d^2)(c\cos(e+fx)+d)^{5/2}}-\frac{\frac{2d\left(-13ac^2d+5ad^3+10bc^3-2bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+f}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}}{}\right.$$

$\downarrow$ 3297

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{3/2}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^{5/2}}-\frac{\frac{2d\left(10bc^3-13adc^2-2bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}}{}\right.$$

$\downarrow$ 3475

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{3/2}\sin(e+fx)}{5c(c^2-d^2)f(d+c\cos(e+fx))^{5/2}}-\frac{\frac{2d\left(10bc^3-13adc^2-2bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}}{}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(3/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*(b + a*Cos[e +
f*x])^(3/2)*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(5/2)) -
 (-1/3*(-(((-2*(a - b)*Sqrt[a + b]*c*(2*a*b*c*d*(35*c^4 - 8*c^2*d^2 + 5*d^
4) - a^2*d^2*(58*c^4 - 41*c^2*d^2 + 15*d^4) - b^2*(15*c^6 + 19*c^4*d^2 - 2
*c^2*d^4))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e +
 f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e +
f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*S
qrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) + (2*S
qrt[a + b]*(b^2*c^3*(15*c^3 + 10*c^2*d + 9*c*d^2 - 2*d^3) - 2*a*b*c^2*(15*
c^4 + 20*c^3*d - 4*c^2*d^2 - 4*c*d^3 + 5*d^4) + a^2*d*(60*c^5 - 2*c^4*d -
66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos
[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[
e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f
*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*S
qrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*
Sqrt[c + d]*(b*c - a*d)*f))/c^2) - (30*a*Sqrt[a + b]*(c^2 - d^2)^2*Sqrt[-(
((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-((
(b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Co
s[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(Sq...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 14938 vs. 2(1037) = 2074.

Time = 15.39 (sec) , antiderivative size = 14939, normalized size of antiderivative = 13.31

method	result	size
default	Expression too large to display	14939

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{3}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(3/2)/(c + d/cos(e + f*x))^(7/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{3/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)^4 d^4+4\sec(fx+e)^3 c\,d^3+6\sec(fx+e)^2 c^2d^2+4\sec(fx+e)\,c^3d+c^4}dx\right)b + \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^4 d^4+4\sec(fx+e)^3 c\,d^3+6\sec(fx+e)^2 c^2d^2+4\sec(fx+e)\,c^3d+c^4}dx\right)a$$

input
```
int((a+b*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^(7/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(
e + f*x)**4*d**4 + 4*sec(e + f*x)**3*c*d**3 + 6*sec(e + f*x)**2*c**2*d**2
+ 4*sec(e + f*x)*c**3*d + c**4),x)*b + int((sqrt(sec(e + f*x)*d + c)*sqrt(
sec(e + f*x)*b + a))/(sec(e + f*x)**4*d**4 + 4*sec(e + f*x)**3*c*d**3 + 6*
sec(e + f*x)**2*c**2*d**2 + 4*sec(e + f*x)*c**3*d + c**4),x)*a
```

3.215 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 891

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}(7ac^2-4bcd-3ad^2)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\,\mathrm{c}}{3c^2(c-d)^2(c+d)^{3/2}f\sqrt{b+a\cos(e+fx)}\sqrt{c-}}$$

$$+\frac{2\sqrt{a+b}(b^2c^2(c+3d)-abc(7c^2+4cd-3d^2)+a^2(9c^3-2c^2d-6cd^2+3d^3))\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-}}{3c^3(c-d)^2(c+d)^{3/2}f}$$

$$-\frac{2a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(d+c\cos(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\cos(e+fx))}{(a-b)(d+c\cos(e+fx))}}(d+c\cos(e+fx))^{3/2}\csc(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+}{a(c+}\right.}{c^3\sqrt{c+d}f\sqrt{b+a\cos(e+fx)}\sqrt{c+d\sec(e+}}$$

$$+\frac{2(bc-ad)^2\sqrt{a+b\sec(e+fx)}\sin(e+fx)}{3c(c^2-d^2)f(d+c\cos(e+fx))\sqrt{c+d\sec(e+fx)}}$$

output
```
-2/3*(a-b)*(a+b)^(1/2)*(7*a*c^2-3*a*d^2-4*b*c*d)*(-(-a*d+b*c)*(1-cos(f*x+e
))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*co
s(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(
b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b
)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^2/(c-d)^2/(c+d)^(3/2)/f/(b+a*cos(
f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/3*(a+b)^(1/2)*(b^2*c^2*(c+3*d)-a*b*
c*(7*c^2+4*c*d-3*d^2)+a^2*(9*c^3-2*c^2*d-6*c*d^2+3*d^3))*(-(-a*d+b*c)*(1-c
os(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)
/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)
^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c
-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^3/(c-d)^2/(c+d)^(3/2)/f/(
b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2*a^2*(a+b)^(1/2)*(-(-a*d+b*c
)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))
/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticP
i((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a
+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^3/
(c+d)^(1/2)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/3*(-a*d+b*c)
^2*(a+b*sec(f*x+e))^(1/2)*sin(f*x+e)/c/(c^2-d^2)/f/(d+c*cos(f*x+e))/(c+d*s
ec(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2026 vs. $2(891) = 1782$.

Time = 6.89 (sec) , antiderivative size = 2026, normalized size of antiderivative = 2.27

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output
```
((d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2*c^2*Sin[e + f*
x] - 2*a*b*c*d*Sin[e + f*x] + a^2*d^2*Sin[e + f*x]))/(3*c*(c^2 - d^2)*(d +
 c*Cos[e + f*x])^2) + (2*(7*a*b*c^3*Sin[e + f*x] - 7*a^2*c^2*d*Sin[e + f*x
] - 4*b^2*c^2*d*Sin[e + f*x] + a*b*c*d^2*Sin[e + f*x] + 3*a^2*d^3*Sin[e +
f*x]))/(3*c*(c^2 - d^2)^2*(d + c*Cos[e + f*x]))))/(f*(b + a*Cos[e + f*x])^
2*(c + d*Sec[e + f*x])^(5/2)) + ((d + c*Cos[e + f*x])^(5/2)*(a + b*Sec[e +
 f*x])^(5/2)*((4*(b*c - a*d)*(2*a^2*b*c^3 + b^3*c^3 + a^3*c^2*d - 8*a*b^2*
c^2*d + 2*a^2*b*c*d^2 + 3*b^3*c*d^2 - a^3*d^3)*Sqrt[((c + d)*Cot[(e + f*x)
/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*
c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a
*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc
[(e + f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]
*Sin[(e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*
Cos[e + f*x]]) + 4*(b*c - a*d)*(3*a^3*c^3 - 7*a*b^2*c^3 + 4*b^3*c^2*d + a^
3*c*d^2 + 3*a*b^2*c*d^2 - 4*a^2*b*d^3)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)
/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*
d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*C
sc[e + f*x]*EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e +
f*x)/2]^2)/(b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(
e + f*x)/2]^4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos...
```

Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 853, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {3042, 4430, 3042, 3271, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3271

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int\frac{d^2a^3+3\left(c^2-d^2\right)\cos^2(e+fx)a^3-5bcda^2+7b^2c^2a-3b^3cd+\left(-3cda^3+b\left(9c^2-2d^2\right)a^2-5b^2cda+b^3c\right.}{2\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{d^2a^3+3\left(c^2-d^2\right)\cos^2(e+fx)a^3-5bcda^2+7b^2c^2a-3b^3cd+\left(-3cda^3+b\left(9c^2-2d^2\right)a^2-5b^2cda+b^3c^2\right.}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{d^2a^3+3\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2a^3-5bcda^2+7b^2c^2a-3b^3cd+\left(-3cda^3+b\left(9c^2-2d^2\right)a^2-5b^2cda+\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3532

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{3a^3\left(c^2-d^2\right)\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}+\frac{\int\frac{7ab^2c^4-3b^3dc^3-5a^2bdc^3+(bc-ad)\left(9a^2c^2+b^2c^2-4abdc-6a\right.}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx)}}{c^2}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{3a^3\left(c^2-d^2\right)\int\frac{\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}+\frac{\int\frac{7ab^2c^4-3b^3dc^3-5a^2bdc^3+(bc-ad)\left(9a^2c^2+b^2c^2-4abdc\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\right.}}{c^2}}{3c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\Big\downarrow$ 3290

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{7ab^2c^4-3b^3dc^3-5a^2bdc^3+(bc-ad)\left(9a^2c^2+b^2c^2-4abdc-6a^2d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)c-a^3\left(2c^2d^2-3d^4\right.}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}}{c^2}\right.}{}$$

$\Big\downarrow$ 3477

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\left(a^3(-d)\left(9c^3-2c^2d-6cd^2+3d^3\right)+a^2bc^2\left(9c^2+5cd-2d^2\right)-ab^2c^3(7c+5d)+b^3c^3(c+3d)\right)\int\frac{}{\sqrt{b+a\cos(\right.}}}{c-d}}{c}\right.}{}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\left(a^3(-d)\left(9c^3-2c^2d-6cd^2+3d^3\right)+a^2bc^2\left(9c^2+5cd-2d^2\right)-ab^2c^3(7c+5d)+b^3c^3(c+3d)\right)\int\frac{}{\sqrt{b+a\sin\left(\right.}}}{c-d}\right.}{}$$

$\Big\downarrow$ 3297

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{\frac{c\left(7ac^2-3ad^2-4bcd\right)(bc-ad)^2\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c-d}+\frac{2\sqrt{a+b}\left(a^3(-d)\right.\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{ 3475}$$

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2\sqrt{b+a\cos(e+fx)}\sin(e+fx)(bc-ad)^2}{3c(c^2-d^2)f(d+c\cos(e+fx))^{3/2}}+\frac{\frac{2\sqrt{a+b}\left(-d\left(9c^3-2dc^2-6d^2c+3d^3\right)a^3+bc^2\left(9c^2+5dc\right.\right.\cdots}{\cdots}}{\cdots}\right.$$

input

```
Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(5/2),x]
```

output

```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((((-2*(a - b)*Sqrt[a +
 b]*c*(7*a*c^2 - 4*b*c*d - 3*a*d^2)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x])
)/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))
/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*Ellipt
icE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*
Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d
]*f) + (2*Sqrt[a + b]*(b^3*c^3*(c + 3*d) - a*b^2*c^3*(7*c + 5*d) + a^2*b*c
^2*(9*c^2 + 5*c*d - 2*d^2) - a^3*d*(9*c^3 - 2*c^2*d - 6*c*d^2 + 3*d^3))*Sq
rt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqr
t[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d +
 c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos
[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
 - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 - (6*a^2*Sqrt[a
+ b]*(c^2 - d^2)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a
*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt
[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(c^2*Sqrt[c
+ d]*f))/(3*c*(c^2 - d^2)) + (2*(b*c - a*d)^2*Sqrt[b + a*Cos[e + f*x]]*Sin
[e + f*x])/(3*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(3/2))))/(Sqrt[b + a...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3271
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin
[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^
2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2
+ b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
 d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] ||
IntegersQ[2*m, 2*n])
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7309 vs. $2(812) = 1624$.

Time = 12.46 (sec) , antiderivative size = 7310, normalized size of antiderivative = 8.20

method	result	size
default	Expression too large to display	7310

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{5/2}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)^2}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2 d+c^3}dx\right)b^2 + 2\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2 d+c^3}dx\right)ab + \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^3 d^3+3\sec(fx+e)^2 c\,d^2+3\sec(fx+e)\,c^2 d+c^3}dx\right)a^2$$

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(5/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)/(s
ec(e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec(e + f*x)*c**2*d + c
**3),x)*b**2 + 2*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*se
c(e + f*x))/(sec(e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec(e + f
*x)*c**2*d + c**3),x)*a*b + int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x
)*b + a))/(sec(e + f*x)**3*d**3 + 3*sec(e + f*x)**2*c*d**2 + 3*sec(e + f*x
)*c**2*d + c**3),x)*a**2
```

3.216 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1150

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

output
```
2/15*(a-b)*(a+b)^(1/2)*(b^2*c^2*d*(29*c^2+3*d^2)-a*b*c*(35*c^4+34*c^2*d^2-
5*d^4)+a^2*(58*c^4*d-41*c^2*d^3+15*d^5))*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)
/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)
))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticE((c+d)^(1/2)*(b+a*cos(
f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^3/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*c
os(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/15*(a+b)^(1/2)*(b^3*c^4*(5*c^2+2
4*c*d+3*d^2)-a*b^2*c^3*(35*c^3+42*c^2*d+21*c*d^2-2*d^3)+a^2*b*c^2*(45*c^4+
48*c^3*d+c^2*d^2-8*c*d^3+10*d^4)-a^3*d*(60*c^5-2*c^4*d-66*c^3*d^2+25*c^2*d
^3+30*c*d^4-15*d^5))*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(
1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*
x+e))^(3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^
(1/2)/(d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x
+e))^(1/2)/c^4/(c-d)^3/(c+d)^(5/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+
d*sec(f*x+e))^(1/2)-2*a^2*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d
+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^
(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*
x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d
)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^4/(c+d)^(1/2)/f/(b+a*cos(f*
x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)-2/5*d*(-a*d+b*c)*(b+a*cos(f*x+e))*(a...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2344 vs. 2(1150) = 2300.

Time = 7.40 (sec) , antiderivative size = 2344, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

output
```
((d + c*Cos[e + f*x])^4*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((-2*(b^2*
c^2*d*Sin[e + f*x] - 2*a*b*c*d^2*Sin[e + f*x] + a^2*d^3*Sin[e + f*x]))/(5*
c^2*(c^2 - d^2)*(d + c*Cos[e + f*x])^3) + (2*(5*b^2*c^4*Sin[e + f*x] - 21*
a*b*c^3*d*Sin[e + f*x] + 16*a^2*c^2*d^2*Sin[e + f*x] + 3*b^2*c^2*d^2*Sin[e
 + f*x] + 5*a*b*c*d^3*Sin[e + f*x] - 8*a^2*d^4*Sin[e + f*x]))/(15*c^2*(c^2
 - d^2)^2*(d + c*Cos[e + f*x])^2) + (2*(35*a*b*c^5*Sin[e + f*x] - 58*a^2*c
^4*d*Sin[e + f*x] - 29*b^2*c^4*d*Sin[e + f*x] + 34*a*b*c^3*d^2*Sin[e + f*x
] + 41*a^2*c^2*d^3*Sin[e + f*x] - 3*b^2*c^2*d^3*Sin[e + f*x] - 5*a*b*c*d^4
*Sin[e + f*x] - 15*a^2*d^5*Sin[e + f*x]))/(15*c^2*(c^2 - d^2)^3*(d + c*Cos
[e + f*x]))))/(f*(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(7/2)) + ((d
+ c*Cos[e + f*x])^(7/2)*Sec[e + f*x]*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c -
 a*d)*(10*a^2*b*c^5 + 5*b^3*c^5 + 13*a^3*c^4*d - 48*a*b^2*c^4*d + 15*a^2*b
*c^3*d^2 + 27*b^3*c^3*d^2 - 18*a^3*c^2*d^3 - 16*a*b^2*c^2*d^3 + 7*a^2*b*c*
d^4 + 5*a^3*d^5)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[Ar
cSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)
*(15*a^3*c^5 - 35*a*b^2*c^5 + 23*a^2*b*c^4*d + 29*b^3*c^4*d + 13*a^3*c^...
```

Rubi [A] (verified)

Time = 4.28 (sec) , antiderivative size = 1086, normalized size of antiderivative = 0.94, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {3042, 4430, 3042, 3468, 27, 3042, 3526, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\cos(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\big\downarrow$ 3468

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int \frac{\sqrt{b+a\cos(e+fx)}\left(5a^2\left(c^2-d^2\right)\cos^2(e+fx)-\left(5cda^2-2b\left(5c^2-d^2\right)a+3b^2cd\right)\cos(e+fx)+(5bc-3a}{2(d+c\cos(e+fx))^{5/2}}}{5c(c^2-d^2)}\right.$$
$$\overline{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\big\downarrow$ 27

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int \frac{\sqrt{b+a\cos(e+fx)}\left(5a^2\left(c^2-d^2\right)\cos^2(e+fx)-\left(5cda^2-2b\left(5c^2-d^2\right)a+3b^2cd\right)\cos(e+fx)+(5bc-3ad)}{(d+c\cos(e+fx))^{5/2}}}{5c(c^2-d^2)}\right.$$
$$\overline{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\big\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int \frac{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(5a^2\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)^2+\left(-5cda^2+2b\left(5c^2-d^2\right)a-3b^2cd\right)\sin\left(e+fx+\right.\right.}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}}{5c(c^2-d^2)}\right.$$
$$\overline{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$\big\downarrow$ 3526

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int -\frac{-15\left(c^2-d^2\right)^2\cos^2(e+fx)a^3+(bc-ad)\left(\left(13c^2d-5d^3\right)a^2-b\left(35c^3-3cd^2\right)a+24b^2c^2d\right)+\left(6cd\left(5c^2\right.\right.}{2\sqrt{b+a\cos(e+fx)}(d+c\cos}}{3c\left(c^2-d^2\right)}\right.$$

$\big\downarrow$ 27

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-15\left(c^2-d^2\right)^2\cos^2(e+fx)a^{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\sqrt{(\cdots}$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{-15\left(c^2-d^2\right)^2\sin\left(e+fx+\frac{\pi}{2}\right)\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3532

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{15\left(c^2d-d^3\right)^2a^3+c^2(bc-ad\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{15\left(c^2d-d^3\right)^2a^3+c^2(bc-ad\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 3290

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{\int\frac{15\left(c^2d-d^3\right)^2a^3+c^2(bc-ad}{}}{}\right.$$

$\downarrow$ 3477

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2(bc-ad)\left(-13ac^2d+5ad^3+5bc^3+3bcd^2\right)\sin(e+fx)\sqrt{a\cos(e+fx)+b}}{3cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{3/2}}-\frac{-\frac{c(bc-ad)\left(-58a^2c^4d+41a^2c\right.}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2(bc-ad)\left(5bc^3-13adc^2+3bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}-\frac{30a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e}{(a+b)(d+c\cos(e}}}{}\right.$$

$\downarrow$ 3297

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2(bc-ad)\left(5bc^3-13adc^2+3bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}-\frac{30a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e}{(a+b)(d+c\cos(e}}}{}\right.$$

$\downarrow$ 3475

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{\frac{2(bc-ad)\left(5bc^3-13adc^2+3bd^2c+5ad^3\right)\sqrt{b+a\cos(e+fx)}\sin(e+fx)}{3c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{3/2}}-\frac{30a^2\sqrt{a+b}\sqrt{-\frac{(bc-ad)(1-\cos(e}{(a+b)(d+c\cos(e}}}{}}{}\right.$$

input

```
Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(7/2),x]
```

output

```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((-2*d*(b*c - a*d)*(b +
 a*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x
])^(5/2)) + (-1/3*(((2*(a - b)*Sqrt[a + b]*c*(35*a*b*c^5 - 58*a^2*c^4*d -
29*b^2*c^4*d + 34*a*b*c^3*d^2 + 41*a^2*c^2*d^3 - 3*b^2*c^2*d^3 - 5*a*b*c*d
^4 - 15*a^2*d^5)*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*C
os[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Co
s[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c
+ d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((
a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f) -
(2*Sqrt[a + b]*(b^3*c^4*(5*c^2 + 24*c*d + 3*d^2) - a*b^2*c^3*(35*c^3 + 42*
c^2*d + 21*c*d^2 - 2*d^3) + a^2*b*c^2*(45*c^4 + 48*c^3*d + c^2*d^2 - 8*c*d
^3 + 10*d^4) - a^3*d*(60*c^5 - 2*c^4*d - 66*c^3*d^2 + 25*c^2*d^3 + 30*c*d^
4 - 15*d^5))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e
 + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e
+ f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]
*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2 +
 (30*a^2*Sqrt[a + b]*(c^2 - d^2)^2*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))
/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/
((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*Elli...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 12380 vs. 2(1065) = 2130.

Time = 15.43 (sec) , antiderivative size = 12381, normalized size of antiderivative = 10.77

method	result	size
default	Expression too large to display	12381

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx=\text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(7/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{7/2}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)^{2}}{\sec(fx+e)^{4}d^{4}+4\sec(fx+e)^{3}c\,d^{3}+6\sec(fx+e)^{2}c^{2}d^{2}+4\sec(fx$$

$$+2\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)^{4}d^{4}+4\sec(fx+e)^{3}c\,d^{3}+6\sec(fx+e)^{2}c^{2}d^{2}+4\sec(fx+e)\,c^{3}d+c^{4}}dx\right)ab$$

$$+\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^{4}d^{4}+4\sec(fx+e)^{3}c\,d^{3}+6\sec(fx+e)^{2}c^{2}d^{2}+4\sec(fx+e)\,c^{3}d+c^{4}}dx\right)a^{2}$$

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(7/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)/(s
ec(e + f*x)**4*d**4 + 4*sec(e + f*x)**3*c*d**3 + 6*sec(e + f*x)**2*c**2*d*
*2 + 4*sec(e + f*x)*c**3*d + c**4),x)*b**2 + 2*int((sqrt(sec(e + f*x)*d +
c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(e + f*x)**4*d**4 + 4*sec(e
+ f*x)**3*c*d**3 + 6*sec(e + f*x)**2*c**2*d**2 + 4*sec(e + f*x)*c**3*d + c
**4),x)*a*b + int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec
(e + f*x)**4*d**4 + 4*sec(e + f*x)**3*c*d**3 + 6*sec(e + f*x)**2*c**2*d**2
 + 4*sec(e + f*x)*c**3*d + c**4),x)*a**2
```

3.217 $\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1428

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

output
```
2/105*(a-b)*(a+b)^(1/2)*(2*b^3*c^3*d*(133*c^4+62*c^2*d^2-3*d^4)+2*a^2*b*c*
d*(406*c^6+73*c^4*d^2+132*c^2*d^4-35*d^6)-a*b^2*c^2*(245*c^6+852*c^4*d^2+4
1*c^2*d^4+14*d^6)-a^3*(582*c^6*d^2-485*c^4*d^4+392*c^2*d^6-105*d^8))*(-(-a
*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f
*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*Ell
ipticE((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(d+c*cos(f*x+e))^(1/
2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1/2)/c^4/(c-d)^4/(c+
d)^(7/2)/(-a*d+b*c)^2/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2)+2/10
5*(a+b)^(1/2)*(b^3*c^4*(35*c^4+231*c^3*d+67*c^2*d^2+57*c*d^3-6*d^4)-a*b^2*
c^3*(245*c^5+413*c^4*d+439*c^3*d^2+53*c^2*d^3-12*c*d^4+14*d^5)+a^2*b*c^2*(
315*c^6+497*c^5*d+219*c^4*d^2-73*c^3*d^3+208*c^2*d^4+56*c*d^5-70*d^6)-a^3*
d*(525*c^7+57*c^6*d-699*c^5*d^2+214*c^4*d^3+672*c^3*d^4-280*c^2*d^5-210*c*
d^6+105*d^7))*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(f*x+e)))^(1/2)*(-
(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(d+c*cos(f*x+e))^(
3/2)*csc(f*x+e)*EllipticF((c+d)^(1/2)*(b+a*cos(f*x+e))^(1/2)/(a+b)^(1/2)/(
d+c*cos(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(a+b*sec(f*x+e))^(1
/2)/c^5/(c-d)^4/(c+d)^(7/2)/(-a*d+b*c)/f/(b+a*cos(f*x+e))^(1/2)/(c+d*sec(f
*x+e))^(1/2)-2*a^2*(a+b)^(1/2)*(-(-a*d+b*c)*(1-cos(f*x+e))/(a+b)/(d+c*cos(
f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+cos(f*x+e))/(a-b)/(d+c*cos(f*x+e)))^(1/2)*(
d+c*cos(f*x+e))^(3/2)*csc(f*x+e)*EllipticPi((c+d)^(1/2)*(b+a*cos(f*x+e)...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2979 vs. $2(1428) = 2856$.

Time = 8.55 (sec) , antiderivative size = 2979, normalized size of antiderivative = 2.09

$$\int \frac{(a + b\sec(e + fx))^{5/2}}{(c + d\sec(e + fx))^{9/2}}\, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

output
```
((d + c*Cos[e + f*x])^5*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((2*(b^2
*c^2*d^2*Sin[e + f*x] - 2*a*b*c*d^3*Sin[e + f*x] + a^2*d^4*Sin[e + f*x]))/
(7*c^3*(c^2 - d^2)*(d + c*Cos[e + f*x])^4) + (2*(-14*b^2*c^4*d*Sin[e + f*x
] + 43*a*b*c^3*d^2*Sin[e + f*x] - 29*a^2*c^2*d^3*Sin[e + f*x] + 2*b^2*c^2*
d^3*Sin[e + f*x] - 19*a*b*c*d^4*Sin[e + f*x] + 17*a^2*d^5*Sin[e + f*x]))/(
35*c^3*(c^2 - d^2)^2*(d + c*Cos[e + f*x])^3) + (2*(35*b^2*c^6*Sin[e + f*x]
 - 224*a*b*c^5*d*Sin[e + f*x] + 234*a^2*c^4*d^2*Sin[e + f*x] + 67*b^2*c^4*
d^2*Sin[e + f*x] + 52*a*b*c^3*d^3*Sin[e + f*x] - 209*a^2*c^2*d^4*Sin[e + f
*x] - 6*b^2*c^2*d^4*Sin[e + f*x] - 20*a*b*c*d^5*Sin[e + f*x] + 71*a^2*d^6*
Sin[e + f*x]))/(105*c^3*(c^2 - d^2)^3*(d + c*Cos[e + f*x])^2) + (2*(245*a*
b^2*c^8*Sin[e + f*x] - 812*a^2*b*c^7*d*Sin[e + f*x] - 266*b^3*c^7*d*Sin[e
+ f*x] + 582*a^3*c^6*d^2*Sin[e + f*x] + 852*a*b^2*c^6*d^2*Sin[e + f*x] - 1
46*a^2*b*c^5*d^3*Sin[e + f*x] - 124*b^3*c^5*d^3*Sin[e + f*x] - 485*a^3*c^4
*d^4*Sin[e + f*x] + 41*a*b^2*c^4*d^4*Sin[e + f*x] - 264*a^2*b*c^3*d^5*Sin[
e + f*x] + 6*b^3*c^3*d^5*Sin[e + f*x] + 392*a^3*c^2*d^6*Sin[e + f*x] + 14*
a*b^2*c^2*d^6*Sin[e + f*x] + 70*a^2*b*c*d^7*Sin[e + f*x] - 105*a^3*d^8*Sin
[e + f*x]))/(105*c^3*(b*c - a*d)*(c^2 - d^2)^4*(d + c*Cos[e + f*x]))))/(f*
(b + a*Cos[e + f*x])^2*(c + d*Sec[e + f*x])^(9/2)) + ((d + c*Cos[e + f*x])
^(9/2)*Sec[e + f*x]^2*(a + b*Sec[e + f*x])^(5/2)*((4*(b*c - a*d)*(-70*a^2*
b^2*c^8 - 35*b^4*c^8 - 77*a^3*b*c^7*d + 427*a*b^3*c^7*d + 162*a^4*c^6*d...
```

Rubi [A] (verified)

Time = 6.34 (sec) , antiderivative size = 1353, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {3042, 4430, 3042, 3527, 27, 3042, 3526, 27, 3042, 3526, 27, 3042, 3532, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\cos^2(e+fx)(b+a\cos(e+fx))^{5/2}}{(d+c\cos(e+fx))^{9/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3527}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2\int-\frac{(b+a\cos(e+fx))^{3/2}\left(-7a\left(c^2-d^2\right)\cos^2(e+fx)-\left(7bc^2-7adc-2bd^2\right)\cos(e+fx)+d(7bc-5ad)\right)}{2(d+c\cos(e+fx))^{7/2}}d}{7c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\int\frac{(b+a\cos(e+fx))^{3/2}\left(-7a\left(c^2-d^2\right)\cos^2(e+fx)-\left(7bc^2-\right.\right.}{(d+c\cos(e+fx))^{7/}}}{7c(c^2-d^2)}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\int\frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(-7a\left(c^2-d^2\right)\sin\left(e+fx+\frac{\pi}{2}\right)\right.}{\left(d+c\sin\left(e+\right.\right.}}{7c(c^2}\right.}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3526}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{2\int-\frac{\sqrt{b+a\cos(e+fx)}\left(35b^2c^4-112abdc^3+57a^2d^2c^2+25b\right.}{}}{}\right.}{\sqrt{a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.}{\sqrt{a\,c}}$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3526

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 27

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3532

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{2d^2\sin(e+fx)(a\cos(e+fx)+b)^{5/2}}{7cf(c^2-d^2)(c\cos(e+fx)+d)^{7/2}}-\frac{\frac{2d\left(-19ac^2d+7ad^3+14bc^3-2bcd^2\right)\sin(e+fx)(a\cos(e+fx)}{5cf\left(c^2-d^2\right)(c\cos(e+fx)+d)^{5/2}}}{}\right.$$

$\downarrow$ 3290

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

$\downarrow$ 3477

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{}\right.$$

$\downarrow$ 3297

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(\ldots}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{\ldots}\right.$$

$\downarrow$ 3475

$$\sqrt{d+c\cos(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2d^2(b+a\cos(e+fx))^{5/2}\sin(e+fx)}{7c(c^2-d^2)f(d+c\cos(e+fx))^{7/2}}-\frac{\frac{2d\left(14bc^3-19adc^2-2bd^2c+7ad^3\right)(b+a\cos(e+fx))^{3/2}\sin(\ldots}{5c\left(c^2-d^2\right)f(d+c\cos(e+fx))^{5/2}}}{\ldots}\right.$$

input
```
Int[(a + b*Sec[e + f*x])^(5/2)/(c + d*Sec[e + f*x])^(9/2),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*d^2*(b + a*Cos[e +
f*x])^(5/2)*Sin[e + f*x])/(7*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(7/2)) -
 ((2*d*(14*b*c^3 - 19*a*c^2*d - 2*b*c*d^2 + 7*a*d^3)*(b + a*Cos[e + f*x])^
(3/2)*Sin[e + f*x])/(5*c*(c^2 - d^2)*f*(d + c*Cos[e + f*x])^(5/2)) - (-1/3
*(((-2*(a - b)*Sqrt[a + b]*c*(2*b^3*c^3*d*(133*c^4 + 62*c^2*d^2 - 3*d^4) +
 2*a^2*b*c*d*(406*c^6 + 73*c^4*d^2 + 132*c^2*d^4 - 35*d^6) - a*b^2*c^2*(24
5*c^6 + 852*c^4*d^2 + 41*c^2*d^4 + 14*d^6) - a^3*(582*c^6*d^2 - 485*c^4*d^
4 + 392*c^2*d^6 - 105*d^8))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a +
b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b
)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticE[ArcS
in[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e +
f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c -
 a*d)^2*f) - (2*Sqrt[a + b]*(b^3*c^4*(35*c^4 + 231*c^3*d + 67*c^2*d^2 + 57
*c*d^3 - 6*d^4) - a*b^2*c^3*(245*c^5 + 413*c^4*d + 439*c^3*d^2 + 53*c^2*d^
3 - 12*c*d^4 + 14*d^5) + a^2*b*c^2*(315*c^6 + 497*c^5*d + 219*c^4*d^2 - 73
*c^3*d^3 + 208*c^2*d^4 + 56*c*d^5 - 70*d^6) - a^3*d*(525*c^7 + 57*c^6*d -
699*c^5*d^2 + 214*c^4*d^3 + 672*c^3*d^4 - 280*c^2*d^5 - 210*c*d^6 + 105*d^
7))*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x]))
)]*Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))
]*(d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3526
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m -
 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*
d)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x
] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 24929 vs. 2(1337) = 2674.

Time = 18.56 (sec) , antiderivative size = 24930, normalized size of antiderivative = 17.46

method	result	size
default	Expression too large to display	24930

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{5}{2}}}{(d\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(5/2)/(d*sec(f*x + e) + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(5/2)/(c + d/cos(e + f*x))^(9/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+b\sec(e+fx))^{5/2}}{(c+d\sec(e+fx))^{9/2}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\mathrm{sec}}{\sec(fx+e)^5 d^5 + 5\sec(fx+e)^4 c\,d^4 + 10\sec(fx+e)^3 c^2d^3 + 10\sec(}\right.$$
$$+2\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)^5 d^5 + 5\sec(fx+e)^4 c\,d^4 + 10\sec(fx+e)^3 c^2d^3 + 10\sec(fx+e)^2 c^3d^2 + 5\sec(fx+e}\right.$$
$$+\left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)^5 d^5 + 5\sec(fx+e)^4 c\,d^4 + 10\sec(fx+e)^3 c^2d^3 + 10\sec(fx+e)^2 c^3d^2 + 5\sec(fx+e)}\right.$$

input
```
int((a+b*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^(9/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x)**2)/(s
ec(e + f*x)**5*d**5 + 5*sec(e + f*x)**4*c*d**4 + 10*sec(e + f*x)**3*c**2*d
**3 + 10*sec(e + f*x)**2*c**3*d**2 + 5*sec(e + f*x)*c**4*d + c**5),x)*b**2
 + 2*int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/
(sec(e + f*x)**5*d**5 + 5*sec(e + f*x)**4*c*d**4 + 10*sec(e + f*x)**3*c**2
*d**3 + 10*sec(e + f*x)**2*c**3*d**2 + 5*sec(e + f*x)*c**4*d + c**5),x)*a*
b + int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)*
*5*d**5 + 5*sec(e + f*x)**4*c*d**4 + 10*sec(e + f*x)**3*c**2*d**3 + 10*sec
(e + f*x)**2*c**3*d**2 + 5*sec(e + f*x)*c**4*d + c**5),x)*a**2
```

3.218 $\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 652

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2c(c+d)\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(1+\sec(e+fx))}{(c-d)(a+b\sec(e+fx))}}(a}{a(a+b)f\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2d(c+d)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(1+\sec(e+fx))}{(c-d)(a+b\sec(e+fx))}}(a}{b(a+b)f\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2(bc-ad)\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{\frac{(bc-ad)(-1+\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad}{(c-d)(}}}{abf\sqrt{\frac{(a+b)(c+d\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}$$

output
```
-2*c*(c+d)*cot(f*x+e)*EllipticPi(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*
x+e)))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*((-a*d+b*c)*
(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(3/2)*((a+b)
*(-a*d+b*c)*(-1+sec(f*x+e))*(c+d*sec(f*x+e))/(c+d)^2/(a+b*sec(f*x+e))^2)^(
1/2)/a/(a+b)/f/(c+d*sec(f*x+e))^(1/2)+2*d*(c+d)*cot(f*x+e)*EllipticPi(((a+
b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(
c+d)/(a+b)/(c-d))^(1/2))*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e))
)^(1/2)*(a+b*sec(f*x+e))^(3/2)*(-(a+b)*(a*d-b*c)*(-1+sec(f*x+e))*(c+d*sec(
f*x+e))/(c+d)^2/(a+b*sec(f*x+e))^2)^(1/2)/b/(a+b)/f/(c+d*sec(f*x+e))^(1/2)
+2*(-a*d+b*c)*cot(f*x+e)*EllipticF(((a+b)*(c+d*sec(f*x+e))/(c+d)/(a+b*sec(
f*x+e)))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*((-a*d+b*c)*(-1+sec(f*x+e)
)/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(
f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/a/b/f/((a+b)*
(c+d*sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.85 (sec) , antiderivative size = 50041, normalized size of antiderivative = 76.75

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}dx$$

input
```
Int[(c + d*Sec[e + f*x])^(3/2)/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [A] (verified)

Time = 12.06 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+e)-\cot(fx+e)),\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)c^2-2\,\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx\right.\right.}{}$

input
```
int((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(El
lipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+
d))^(1/2))*c^2-2*EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*c*d+EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e
)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*d^2-2*EllipticPi(((a-b)/(a+
b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/
(a+b))^(1/2))*c^2-2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))
,(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*d^2)*(1/(c+d)*(d+c*c
os(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))
^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(c+d\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*sec(e + f*x))**(3/2)/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{(d\sec(fx+e)+c)^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e) + c)^(3/2)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^(3/2)/(a + b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+d\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx = \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}\,\sec(fx+e)}{\sec(fx+e)\,b+a}dx\right)d + \left(\int \frac{\sqrt{\sec(fx+e)\,d+c}\,\sqrt{\sec(fx+e)\,b+a}}{\sec(fx+e)\,b+a}dx\right)c$$

input
```
int((c+d*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a)*sec(e + f*x))/(sec(
e + f*x)*b + a),x)*d + int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b +
 a))/(sec(e + f*x)*b + a),x)*c
```

3.219 $\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{}}{a\sqrt{c+d}f}$$

output
```
-2*(a+b)^(1/2)*cot(f*x+e)*EllipticPi((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a
+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),(a+b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b
*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/a/(c+d)^
(1/2)/f
```

Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$= \frac{4\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(e+fx)\right)}{a-b}}\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\csc(e+fx)\left(a(c+d)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{(c+d)(b+a}{}}\right.\right.\right.}{a(c}$$

input
```
Integrate[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(4*Sqrt[((a + b)*Cot[(e + f*x)/2]^2)/(a - b)]*Sqrt[((c + d)*(b + a*Cos[e +
 f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*(a*(c + d)*EllipticF[
ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(2*b*c - 2*a
*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))] - (a + b)*c*EllipticPi[(-(b*c
) + a*d)/(a*(c + d)), ArcSin[Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f
*x)/2]^2)/(2*b*c - 2*a*d)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))])*Sqrt[c
 + d*Sec[e + f*x]]*Sin[(e + f*x)/2]^2)/(a*(c + d)*f*Sqrt[((a + b)*(d + c*C
os[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3042, 4424}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4424}$$

$$-\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\right.\right.}{af\sqrt{c+d}}$$

input
```
Int[Sqrt[c + d*Sec[e + f*x]]/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
(-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSin[(S
qrt[c + d]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]]
)], ((a + b)*(c - d))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*
x]))/((a + b)*(c + d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]
))/((a - b)*(c + d*Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*f
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 7.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.63

method	result
default	$\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\left(2\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+e)-\cot(fx+e)),-\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)c-\mathrm{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+\right.\right.}{f\sqrt{}}$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(2*E
llipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/
(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*c-EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x
+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*c+EllipticF(((a-b)/(a+b))
^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*d)*(1/(c+d
)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(
f*x+e)))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+d\sec{(e+fx)}}}{\sqrt{a+b\sec{(e+fx)}}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(c + d*sec(e + f*x))/sqrt(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec{(fx+e)}+c}}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{d\sec{(fx+e)}+c}}{\sqrt{b\sec{(fx+e)}+a}}\,dx$$

input
```
integrate((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(d*sec(f*x + e) + c)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((c + d/cos(e + f*x))^(1/2)/(a + b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)d+c}\,\sqrt{\sec\left(fx+e\right)b+a}}{\sec\left(fx+e\right)b+a}dx$$

input
```
int((c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)*b +
a),x)
```

3.220 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 398

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx =$$

$$-\frac{2\sqrt{c+d}\cot(e+fx)\,\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\cdots}}{a\sqrt{a+b}cf}$$

$$-\frac{2b\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\cdots}}{a\sqrt{c+d}(bc-ad)f}$$

output

```
-2*(c+d)^(1/2)*cot(f*x+e)*EllipticPi((a+b)^(1/2)*(c+d*sec(f*x+e))^(1/2)/(c
+d)^(1/2)/(a+b*sec(f*x+e))^(1/2),a*(c+d)/(a+b)/c,((a-b)*(c+d)/(a+b)/(c-d))
^(1/2))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b
*c)*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))/a/(a+b)^
(1/2)/c/f-2*b*(a+b)^(1/2)*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e)
)^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2)
)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1
+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/a/(c+d)^(1/2)/
(-a*d+b*c)/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{4i\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\sqrt{\frac{d+c\cos(e+fx)}{(c+d)(1+\cos(e+fx))}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{-a+b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{\sqrt{\frac{-a+b}{a+b}}f\sqrt{a+b\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
((4*I)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e +
f*x]))]*Sqrt[(d + c*Cos[e + f*x])/((c + d)*(1 + Cos[e + f*x]))]*(EllipticF
[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a
 - b)*(c + d))] - 2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(e + f*x)/2]], ((a + b)*(c - d))/((a - b)*(c + d))])*Sec[e +
 f*x])/(Sqrt[(-a + b)/(a + b)]*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e
 + f*x]])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 4426, 3042, 4424, 4472}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4426$$

$$\frac{\int \frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}dx}{a}-\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \ 4424$$

$$-\frac{b\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{acf\sqrt{a+b}}$$

$$\Big\downarrow \ 4472$$

$$-\frac{2b\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{}}{\sqrt{a+b}\sqrt{}}\right.\right.}{af\sqrt{c+d}(bc-ad)}$$
$$\frac{2\sqrt{c+d}\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\text{EllipticPi}\left(\frac{a(c+d)}{(a+b)c},\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{acf\sqrt{a+b}}$$

input
```
Int[1/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

output
```
(-2*Sqrt[c + d]*Cot[e + f*x]*EllipticPi[(a*(c + d))/((a + b)*c), ArcSin[(S
qrt[a + b]*Sqrt[c + d*Sec[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*x]]
)], ((a - b)*(c + d))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e +
f*x]))/((c + d)*(a + b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x
]))/((c - d)*(a + b*Sec[e + f*x]))]*(a + b*Sec[e + f*x]))/(a*Sqrt[a + b]*c
*f) - (2*b*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a +
 b*Sec[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d
))/((a - b)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c +
d*Sec[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*
Sec[e + f*x])))]*(c + d*Sec[e + f*x]))/(a*Sqrt[c + d]*(b*c - a*d)*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4424
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.) + (c_)], x_Symbol] :> Simp[2*((a + b*Csc[e + f*x])/(c*f*Rt[(a + b)/(
c + d), 2]*Cot[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Csc[e + f*x])/((c - d)*(a
+ b*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Csc[e + f*x])/((c + d)*(a +
b*Csc[e + f*x])))]*EllipticPi[a*((c + d)/(c*(a + b))), ArcSin[Rt[(a + b)/(c
 + d), 2]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]])], (a - b)*((c
 + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4426
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x
_)]*(d_.) + (c_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sq
rt[c + d*Csc[e + f*x]], x], x] - Simp[b/a   Int[Csc[e + f*x]/(Sqrt[a + b*Cs
c[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0]
```

rule 4472
```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 9.47 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(1+\cos(fx+e))}}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(1+\cos(fx+e))}}\,\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(\csc(fx+e)-\cot(fx+e)),\sqrt{\right.\right.}{f\sqrt{\frac{a-b}{a+b}}\,(d+c\cos(fx+e))(b+a\,c}$

input
```
int(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/f/((a-b)/(a+b))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/
(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+
cos(f*x+e)))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),
((a+b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x
+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))/(d+
c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\sqrt{c+d\sec{(e+fx)}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}+a}\sqrt{d\sec{(fx+e)}+c}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}+a}\sqrt{d\sec{(fx+e)}+c}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\int \frac{\sqrt{\sec (fx+e)\,d+c}\,\sqrt{\sec (fx+e)\,b+a}}{\sec (fx+e)^2\,bd+\sec (fx+e)\,ad+\sec (fx+e)\,bc+ac}dx$$

input
```
int(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)**2*b
*d + sec(e + f*x)*a*d + sec(e + f*x)*b*c + a*c),x)
```

3.221 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 622

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2(a-b)\sqrt{a+b}d^2\cot(e+fx)E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(1\cdots}{(a-b)(c+\cdots}}}{c(c-d)\sqrt{c+d}(bc-ad)^2f}$$

$$-\frac{2\sqrt{a+b}(2c-d)d\cot(e+fx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\cdots}}{c^2(c-d)\sqrt{c+d}(bc-ad)f}$$

$$-\frac{2\sqrt{a+b}\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{(a+b)c}{a(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(\cdots}{(\cdots}}}{ac^2\sqrt{c+d}f}$$

output
```
-2*(a-b)*(a+b)^(1/2)*d^2*cot(f*x+e)*EllipticE((c+d)^(1/2)*(a+b*sec(f*x+e))
^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))
*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+
sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+e))/c/(c-d)/(c+d)^(
1/2)/(-a*d+b*c)^2/f-2*(a+b)^(1/2)*(2*c-d)*d*cot(f*x+e)*EllipticF((c+d)^(1/
2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),((a+b)*(c-d)/
(a-b)/(c+d))^(1/2))*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/
2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)*(c+d*sec(f*x+
e))/c^2/(c-d)/(c+d)^(1/2)/(-a*d+b*c)/f-2*(a+b)^(1/2)*cot(f*x+e)*EllipticPi
((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sec(f*x+e))^(1/2),(a+
b)*c/a/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*((-a*d+b*c)*(1-sec(f*x+e))/(
a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sec(f*x+e))/(a-b)/(c+d*sec(f*
x+e)))^(1/2)*(c+d*sec(f*x+e))/a/c^2/(c+d)^(1/2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1761 vs. $2(622) = 1244$.

Time = 19.12 (sec) , antiderivative size = 1761, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

output
```
(Sqrt[b + a*Cos[e + f*x]]*(d + c*Cos[e + f*x])^(3/2)*Sec[e + f*x]^2*((-4*b
*c*d*(b*c - a*d)*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*
(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d +
c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticF[Ar
cSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]/
Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/((a + b)*
(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) + 4*(b*c - a*d)
*(b*c^2 - a*c*d - 2*b*d^2)*((Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sq
rt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-
a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*
EllipticF[ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(
b*c - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^
4)/((a + b)*(c + d)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]]) - (
Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((c + d)*(b + a*Cos[e + f*
x])*Csc[(e + f*x)/2]^2)/(b*c - a*d)]*Sqrt[((-a - b)*(d + c*Cos[e + f*x])*C
sc[(e + f*x)/2]^2)/(b*c - a*d)]*Csc[e + f*x]*EllipticPi[(b*c - a*d)/((a +
b)*c), ArcSin[Sqrt[((-a - b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(b*c
 - a*d)]/Sqrt[2]], (2*(b*c - a*d))/((a + b)*(c - d))]*Sin[(e + f*x)/2]^4)/
((a + b)*c*Sqrt[b + a*Cos[e + f*x]]*Sqrt[d + c*Cos[e + f*x]])) - 2*a*d^2*(
(Sqrt[(-a + b)/(a + b)]*(a + b)*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x...
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 4430, 3042, 3533, 25, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4430}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\cos^2(e+fx)}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^2}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3533}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int-\frac{d^2+2c\cos(e+fx)d}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}+\frac{\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{\sqrt{d+c\cos(e+fx)}}{\sqrt{b+a\cos(e+fx)}}dx}{c^2}-\frac{\int\frac{d^2+2c\cos(e+fx)d}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(\frac{\int\frac{\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c^2}-\frac{\int\frac{d^2+2c\sin\left(e+fx+\frac{\pi}{2}\right)d}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(e+fx)}}$$

$$\Big\downarrow \text{3290}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{\int\frac{d^2+2c\sin\left(e+fx+\frac{\pi}{2}\right)d}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{c^2}-\frac{2\sqrt{a+b}\csc(e+fx)(c\cos(e+fx)+d)\sqrt{\cdots}}{\cdots}\cdots\right)}{\sqrt{a\cos(e+fx)+b}\sqrt{c+d\sec(\cdots}}$$

$$\Big\downarrow \text{3477}$$

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{\frac{d(2c-d)\int\frac{1}{\sqrt{b+a\cos(e+fx)}\sqrt{d+c\cos(e+fx)}}dx}{c-d}-\frac{cd^2\int\frac{\cos(e+fx)+1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))^{3/2}}dx}{c-d}}{c^2}-\cdots\right)}{\sqrt{a\cos(e+fx)\cdots}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{\frac{d(2c-d)\int \frac{1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\sqrt{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{c-d}-\frac{cd^2\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)+1}{\sqrt{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}\left(d+c\sin\left(e+\cdots\right.\right.}}{c-d}}{c^2}\right.}{\sqrt{a\cos(e\cdots}}$$

$\downarrow$ 3297

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{2d\sqrt{a+b}(2c-d)\csc(e+fx)(c\cos(e+fx)+d)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\,\mathrm{E}\cdots}{f(c-d)\sqrt{c+d}(bc-ad)}\right.}{\cdots}$$

$\downarrow$ 3475

$$\frac{\sqrt{a+b\sec(e+fx)}\sqrt{c\cos(e+fx)+d}\left(-\frac{2cd^2(a-b)\sqrt{a+b}\csc(e+fx)(c\cos(e+fx)+d)\sqrt{-\frac{(bc-ad)(1-\cos(e+fx))}{(a+b)(c\cos(e+fx)+d)}}\sqrt{-\frac{(bc-ad)(\cos(e+fx)+1)}{(a-b)(c\cos(e+fx)+d)}}\cdots}{f(c-d)\sqrt{c+d}(bc-ad)^2}\right.}{\cdots}$$

input
```
Int[1/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])^(3/2)),x]
```

output
```
(Sqrt[d + c*Cos[e + f*x]]*(-(((2*(a - b)*Sqrt[a + b]*c*d^2*Sqrt[-(((b*c -
a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt[-(((b*c - a
*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d + c*Cos[e + f*
x])*Csc[e + f*x]*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(
Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d)
)])/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) + (2*Sqrt[a + b]*(2*c - d)*d*Sqr
t[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*Sqrt
[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(d +
c*Cos[e + f*x])*Csc[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[b + a*Cos[
e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f*x]])], ((a + b)*(c - d))/((a
- b)*(c + d))])/((c - d)*Sqrt[c + d]*(b*c - a*d)*f))/c^2) - (2*Sqrt[a + b]
*Sqrt[-(((b*c - a*d)*(1 - Cos[e + f*x]))/((a + b)*(d + c*Cos[e + f*x])))]*
Sqrt[-(((b*c - a*d)*(1 + Cos[e + f*x]))/((a - b)*(d + c*Cos[e + f*x])))]*(
d + c*Cos[e + f*x])*Csc[e + f*x]*EllipticPi[((a + b)*c)/(a*(c + d)), ArcSi
n[(Sqrt[c + d]*Sqrt[b + a*Cos[e + f*x]])/(Sqrt[a + b]*Sqrt[d + c*Cos[e + f
*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(a*c^2*Sqrt[c + d]*f))*Sqrt[
a + b*Sec[e + f*x]])/(Sqrt[b + a*Cos[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3533
```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[1/b^2   Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e +
 f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4430
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sqrt[d + c*Sin[e + f*x]]*(Sqrt[a + b*Cs
c[e + f*x]]/(Sqrt[b + a*Sin[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]))   Int[(b +
 a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^(m + n)), x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && IntegerQ[m + 1/
2] && IntegerQ[n + 1/2] && LeQ[-2, m + n, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1870 vs. $2(577) = 1154$.

Time = 12.34 (sec) , antiderivative size = 1871, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	1871

input
```
int(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/f/(a*d-b*c)/(c+d)/((a-b)/(a+b))^(1/2)/(c-d)/c*(a+b*sec(f*x+e))^(1/2)*((
1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(c+d*sec(f*x+e))^(1/2)*((-(1-cos(f*x+e))^3
*csc(f*x+e)^3+csc(f*x+e)-cot(f*x+e))*d^3*a*((a-b)/(a+b))^(1/2)+((1-cos(f*x
+e))^3*csc(f*x+e)^3+csc(f*x+e)-cot(f*x+e))*d^3*b*((a-b)/(a+b))^(1/2)+((1-c
os(f*x+e))^3*csc(f*x+e)^3-csc(f*x+e)+cot(f*x+e))*d^2*c*a*((a-b)/(a+b))^(1/
2)+(-(1-cos(f*x+e))^3*csc(f*x+e)^3-csc(f*x+e)+cot(f*x+e))*d^2*c*b*((a-b)/(
a+b))^(1/2)+4*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+
c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*
x+e)-cot(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*c
^2*d-4*(1/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f
*x+e))/(1+cos(f*x+e)))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-co
t(f*x+e)),-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*a*d^3-4*(1
/(a+b)*(b+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1
+cos(f*x+e)))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))
,-(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c^3+4*(1/(a+b)*(b
+a*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+
e)))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),-(a+b)/(
a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b*c*d^2-2*(1/(a+b)*(b+a*cos(
f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1
/2)*EllipticF(((a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),((a+b)*(c-d)/...
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)/(b*d^2*sec(f*x
+ e)^3 + a*c^2 + (2*b*c*d + a*d^2)*sec(f*x + e)^2 + (b*c^2 + 2*a*c*d)*sec(
f*x + e)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a+b\sec{(e+fx)}}\,(c+d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(3/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,{\left(c+\frac{d}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{\sec\left(fx+e\right)d+c}}{\sec\left(fx+e\right)^{3}b\,d^{2}+\sec\left(fx+e\right)^{2}a\,d^{2}+2\sec\left(fx+e\right.}$$

input
```
int(1/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(3/2),x)
```

output
```
int((sqrt(sec(e + f*x)*d + c)*sqrt(sec(e + f*x)*b + a))/(sec(e + f*x)**3*b
*d**2 + sec(e + f*x)**2*a*d**2 + 2*sec(e + f*x)**2*b*c*d + 2*sec(e + f*x)*
a*c*d + sec(e + f*x)*b*c**2 + a*c**2),x)
```

3.222 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \frac{\sqrt[3]{d+c\cos(e+fx)}\sqrt[3]{a+b\sec(e+fx)}\text{Int}\left(\frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}},x\right)}{\sqrt[3]{b+a\cos(e+fx)}\sqrt[3]{c+d\sec(e+fx)}}$$

output
```
(d+c*cos(f*x+e))^(1/3)*(a+b*sec(f*x+e))^(1/3)*Defer(Int)((b+a*cos(f*x+e))^
(1/3)/(d+c*cos(f*x+e))^(1/3),x)/(b+a*cos(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1
/3)
```

Mathematica [N/A]

Not integrable

Time = 14.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4431, 3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt[3]{c+d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4431}$$

$$\frac{\sqrt[3]{a+b\sec(e+fx)}\sqrt[3]{c\cos(e+fx)+d}\int \frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}}dx}{\sqrt[3]{a\cos(e+fx)+b}\sqrt[3]{c+d\sec(e+fx)}}$$

$$\downarrow \text{3042}$$

$$\frac{\sqrt[3]{a+b\sec(e+fx)}\sqrt[3]{c\cos(e+fx)+d}\int \frac{\sqrt[3]{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}}{\sqrt[3]{d+c\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{\sqrt[3]{a\cos(e+fx)+b}\sqrt[3]{c+d\sec(e+fx)}}$$

$$\downarrow \text{3304}$$

$$\frac{\sqrt[3]{a+b\sec(e+fx)}\sqrt[3]{c\cos(e+fx)+d}\int \frac{\sqrt[3]{b+a\cos(e+fx)}}{\sqrt[3]{d+c\cos(e+fx)}}dx}{\sqrt[3]{a\cos(e+fx)+b}\sqrt[3]{c+d\sec(e+fx)}}$$

input
```
Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(1/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4431
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sin[e + f*x]^(m + n)*(a + b*Csc[e + f*x
])^m*((c + d*Csc[e + f*x])^n/((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n
))   Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^Simpli
fy[m + n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d,
0] && EqQ[m + n, 0] &&  !IntegerQ[2*m]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a + b\sec(fx + e))^{\frac{1}{3}}}{(c + d\sec(fx + e))^{\frac{1}{3}}} dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a + b\sec(e + fx)}}{\sqrt[3]{c + d\sec(e + fx)}} dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a + b\sec(e + fx)}}{\sqrt[3]{c + d\sec(e + fx)}} dx = \int \frac{\sqrt[3]{a + b\sec(e + fx)}}{\sqrt[3]{c + d\sec(e + fx)}} dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(1/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 90.97 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{\sqrt[3]{c+d\sec(e+fx)}}\,dx=\int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(1/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(1/3)/(sec(e + f*x)*d + c)**(1/3),x)
```

3.223 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}},x\right)$$

output
```
Defer(Int)((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Mathematica [N/A]

Not integrable

Time = 83.87 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt[3]{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$
$$\Big\downarrow \text{4433}$$
$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

output
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 11.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(4/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(1/3)/(c + d*sec(e + f*x))**(4/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(b\sec\left(fx+e\right)+a)^{\frac{1}{3}}}{(d\sec\left(fx+e\right)+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(b\sec\left(fx+e\right)+a)^{\frac{1}{3}}}{(d\sec\left(fx+e\right)+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 94.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{1/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3),x)
```

output
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{4/3}}\,dx = \int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)\,d+(\sec(fx+e)\,d+c)^{\frac{1}{3}}\,c}dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(4/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(1/3)/((sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*
d + (sec(e + f*x)*d + c)**(1/3)*c),x)
```

3.224 $\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

output
```
Defer(Int)((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Mathematica [N/A]

Not integrable

Time = 80.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt[3]{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/3}}dx$$

$$\Big\downarrow \ 4433$$

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(1/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{1}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

output
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(1/3)/(c+d*sec(f*x+e))**(7/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{1}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(1/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(1/3)/(c + d/cos(e + f*x))^(7/3),x)
```

output
```
\text{Hanged}
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.76

$$\int \frac{\sqrt[3]{a+b\sec(e+fx)}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)^2\,d^2+2\,(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+$$

input
```
int((a+b*sec(f*x+e))^(1/3)/(c+d*sec(f*x+e))^(7/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(1/3)/((sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*
*2*d**2 + 2*(sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*c*d + (sec(e + f*x)*d
 + c)**(1/3)*c**2),x)
```

3.225 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\frac{(d+c\cos(e+fx))^{2/3}(a+b\sec(e+fx))^{2/3}\text{Int}\left(\frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}},x\right)}{(b+a\cos(e+fx))^{2/3}(c+d\sec(e+fx))^{2/3}}$$

output
```
(d+c*cos(f*x+e))^(2/3)*(a+b*sec(f*x+e))^(2/3)*Defer(Int)((b+a*cos(f*x+e))^
(2/3)/(d+c*cos(f*x+e))^(2/3),x)/(b+a*cos(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2
/3)
```

Mathematica [N/A]

Not integrable

Time = 14.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4431, 3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 4431}$$

$$\frac{(a+b\sec(e+fx))^{2/3}(c\cos(e+fx)+d)^{2/3}\int \frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}}dx}{(a\cos(e+fx)+b)^{2/3}(c+d\sec(e+fx))^{2/3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\sec(e+fx))^{2/3}(c\cos(e+fx)+d)^{2/3}\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{(a\cos(e+fx)+b)^{2/3}(c+d\sec(e+fx))^{2/3}}$$

$$\Big\downarrow \text{ 3304}$$

$$\frac{(a+b\sec(e+fx))^{2/3}(c\cos(e+fx)+d)^{2/3}\int \frac{(b+a\cos(e+fx))^{2/3}}{(d+c\cos(e+fx))^{2/3}}dx}{(a\cos(e+fx)+b)^{2/3}(c+d\sec(e+fx))^{2/3}}$$

input
```
Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(2/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4431
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sin[e + f*x]^(m + n)*(a + b*Csc[e + f*x
])^m*((c + d*Csc[e + f*x])^n/((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n
))   Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^Simpli
fy[m + n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d,
0] && EqQ[m + n, 0] &&  !IntegerQ[2*m]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

output
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{(a+b\sec{(e+fx)})^{\frac{2}{3}}}{(c+d\sec{(e+fx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(2/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(2/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{2}{3}}}{(d\sec(fx+e)+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(2/3), x)
```

Mupad [N/A]

Not integrable

Time = 97.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3),x)
```

output
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(2/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{2/3}}\,dx = \int \frac{(\sec(fx+e)\,b+a)^{\frac{2}{3}}}{(\sec(fx+e)\,d+c)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(2/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(2/3)/(sec(e + f*x)*d + c)**(2/3),x)
```

3.226 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\text{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}},x\right)$$

output
```
Defer(Int)((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

Mathematica [N/A]

Not integrable

Time = 84.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$
$$\Big\downarrow \text{4433}$$
$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(5/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [N/A]

Not integrable

Time = 63.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{\frac{2}{3}}}{(c+d\sec(e+fx))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(5/3),x)
```

output
```
Integral((a + b*sec(e + f*x))**(2/3)/(c + d*sec(e + f*x))**(5/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(5/3), x)
```

Mupad [N/A]

Not integrable

Time = 95.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{2/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(5/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{5/3}}\,dx=\int \frac{(\sec(fx+e)\,b+a)^{\frac{2}{3}}}{(\sec(fx+e)\,d+c)^{\frac{2}{3}}\sec(fx+e)\,d+(\sec(fx+e)\,d+c)^{\frac{2}{3}}\,c}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(5/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(2/3)/((sec(e + f*x)*d + c)**(2/3)*sec(e + f*x)*
d + (sec(e + f*x)*d + c)**(2/3)*c),x)
```

3.227 $\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\text{Int}\left(\frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}},x\right)$$

output
```
Defer(Int)((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

Mathematica [N/A]

Not integrable

Time = 81.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{2/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{8/3}}dx$$
$$\Big\downarrow \text{ 4433}$$
$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(2/3)/(c + d*Sec[e + f*x])^(8/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{2}{3}}}{(c+d\sec(fx+e))^{\frac{8}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

output
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(2/3)/(c+d*sec(f*x+e))**(8/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{8}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

Giac [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\int \frac{(b\sec{(fx+e)}+a)^{\frac{2}{3}}}{(d\sec{(fx+e)}+c)^{\frac{8}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(2/3)/(d*sec(f*x + e) + c)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(2/3)/(c + d/cos(e + f*x))^(8/3),x)
```

output
```
\text{Hanged}
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.76

$$\int \frac{(a+b\sec(e+fx))^{2/3}}{(c+d\sec(e+fx))^{8/3}}\,dx=\int \frac{(\sec(fx+e)\,b+a)^{\frac{2}{3}}}{(\sec(fx+e)\,d+c)^{\frac{2}{3}}\sec(fx+e)^{2}\,d^{2}+2(\sec(fx+e)\,d+c)^{\frac{2}{3}}\sec(fx+$$

input
```
int((a+b*sec(f*x+e))^(2/3)/(c+d*sec(f*x+e))^(8/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(2/3)/((sec(e + f*x)*d + c)**(2/3)*sec(e + f*x)*
*2*d**2 + 2*(sec(e + f*x)*d + c)**(2/3)*sec(e + f*x)*c*d + (sec(e + f*x)*d
 + c)**(2/3)*c**2),x)
```

3.228 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\frac{(d+c\cos(e+fx))^{4/3}(a+b\sec(e+fx))^{4/3}\text{Int}\left(\frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}},x\right)}{(b+a\cos(e+fx))^{4/3}(c+d\sec(e+fx))^{4/3}}$$

output
```
(d+c*cos(f*x+e))^(4/3)*(a+b*sec(f*x+e))^(4/3)*Defer(Int)((b+a*cos(f*x+e))^
(4/3)/(d+c*cos(f*x+e))^(4/3),x)/(b+a*cos(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4
/3)
```

Mathematica [N/A]

Not integrable

Time = 71.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4431, 3042, 3304}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{4431}$$

$$\frac{(a+b\sec(e+fx))^{4/3}(c\cos(e+fx)+d)^{4/3}\int \frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}}dx}{(a\cos(e+fx)+b)^{4/3}(c+d\sec(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+b\sec(e+fx))^{4/3}(c\cos(e+fx)+d)^{4/3}\int \frac{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(d+c\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{(a\cos(e+fx)+b)^{4/3}(c+d\sec(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3304}$$

$$\frac{(a+b\sec(e+fx))^{4/3}(c\cos(e+fx)+d)^{4/3}\int \frac{(b+a\cos(e+fx))^{4/3}}{(d+c\cos(e+fx))^{4/3}}dx}{(a\cos(e+fx)+b)^{4/3}(c+d\sec(e+fx))^{4/3}}$$

input
```
Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3304
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4431
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_), x_Symbol] :> Simp[Sin[e + f*x]^(m + n)*(a + b*Csc[e + f*x
])^m*((c + d*Csc[e + f*x])^n/((b + a*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n
))   Int[(b + a*Sin[e + f*x])^m*((d + c*Sin[e + f*x])^n/Sin[e + f*x]^Simpli
fy[m + n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d,
0] && EqQ[m + n, 0] &&  !IntegerQ[2*m]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

output
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(4/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Giac [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(4/3), x)
```

Mupad [N/A]

Not integrable

Time = 100.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{4/3}}{\left(c+\frac{d}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3),x)
```

output
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(4/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.97

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{4/3}}\,dx = \left(\int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)\,d+(\sec(fx+e)\,d+c)^{\frac{1}{3}}\,c}dx\right)a + \left(\int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}\sec(fx+e)}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)\,d+(\sec(fx+e)\,d+c)^{\frac{1}{3}}\,c}dx\right)b$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(4/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(1/3)/((sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*
d + (sec(e + f*x)*d + c)**(1/3)*c),x)*a + int(((sec(e + f*x)*b + a)**(1/3)
*sec(e + f*x))/((sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*d + (sec(e + f*x)
*d + c)**(1/3)*c),x)*b
```

3.229 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}},x\right)$$

output
```
Defer(Int)((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Mathematica [N/A]

Not integrable

Time = 92.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/3}}dx$$
$$\Big\downarrow \text{4433}$$
$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(7/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

output
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(7/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="maxi
ma")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Giac [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x, algorithm="giac
")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(7/3),x)
```

output
```
\text{Hanged}
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 5.90

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{7/3}}\,dx = \left(\int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)^{2}d^{2}+2(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+\right.$$
$$+\left(\int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}\sec(fx+e)}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)^{2}d^{2}+2(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)\,cd+(\sec(fx+e)\,d+c)\right.$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(7/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(1/3)/((sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*
*2*d**2 + 2*(sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*c*d + (sec(e + f*x)*d
 + c)**(1/3)*c**2),x)*a + int(((sec(e + f*x)*b + a)**(1/3)*sec(e + f*x))/(
(sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)**2*d**2 + 2*(sec(e + f*x)*d + c)*
*(1/3)*sec(e + f*x)*c*d + (sec(e + f*x)*d + c)**(1/3)*c**2),x)*b
```

3.230 $\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\text{Int}\left(\frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}},x\right)$$

output
```
Defer(Int)((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

Mathematica [N/A]

Not integrable

Time = 121.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4433}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{4/3}}{\left(c+d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{4433}$$

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}dx$$

input
```
Int[(a + b*Sec[e + f*x])^(4/3)/(c + d*Sec[e + f*x])^(10/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4433
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Unintegrable[(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sec(fx+e))^{\frac{4}{3}}}{(c+d\sec(fx+e))^{\frac{10}{3}}}dx$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

output
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e))**(4/3)/(c+d*sec(f*x+e))**(10/3),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{10}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="max
ima")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

Giac [N/A]

Not integrable

Time = 3.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx=\int \frac{(b\sec(fx+e)+a)^{\frac{4}{3}}}{(d\sec(fx+e)+c)^{\frac{10}{3}}}\,dx$$

input
```
integrate((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x, algorithm="gia
c")
```

output
```
integrate((b*sec(f*x + e) + a)^(4/3)/(d*sec(f*x + e) + c)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \text{Hanged}$$

input
```
int((a + b/cos(e + f*x))^(4/3)/(c + d/cos(e + f*x))^(10/3),x)
```

output
```
\text{Hanged}
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 7.83

$$\int \frac{(a+b\sec(e+fx))^{4/3}}{(c+d\sec(e+fx))^{10/3}}\,dx = \left(\int \frac{(\sec(f}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)^{3}\,d^{3}+3\,(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx}\right.$$
$$+\left(\int \frac{(\sec(fx+e)\,b+a)^{\frac{1}{3}}\sec(fx+e)}{(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)^{3}\,d^{3}+3\,(\sec(fx+e)\,d+c)^{\frac{1}{3}}\sec(fx+e)^{2}\,c\,d^{2}+3\,(\sec(fx+e)\,d-}\right.$$

input
```
int((a+b*sec(f*x+e))^(4/3)/(c+d*sec(f*x+e))^(10/3),x)
```

output
```
int((sec(e + f*x)*b + a)**(1/3)/((sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*
*3*d**3 + 3*(sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)**2*c*d**2 + 3*(sec(e
+ f*x)*d + c)**(1/3)*sec(e + f*x)*c**2*d + (sec(e + f*x)*d + c)**(1/3)*c**
3),x)*a + int(((sec(e + f*x)*b + a)**(1/3)*sec(e + f*x))/((sec(e + f*x)*d
+ c)**(1/3)*sec(e + f*x)**3*d**3 + 3*(sec(e + f*x)*d + c)**(1/3)*sec(e + f
*x)**2*c*d**2 + 3*(sec(e + f*x)*d + c)**(1/3)*sec(e + f*x)*c**2*d + (sec(e
 + f*x)*d + c)**(1/3)*c**3),x)*b
```

3.231 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx$

Optimal result

Integrand size = 27, antiderivative size = 116

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},1-np,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\sec^{1-np}(e+fx)\,(c(d\sec(e+f}{f}$$

output
```
2^(1/2+m)*AppellF1(1/2,-n*p+1,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*s
ec(f*x+e)^(-n*p+1)*(c*(d*sec(f*x+e))^p)^n*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec
(f*x+e))^m*sin(f*x+e)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2425 vs. 2(116) = 232.

Time = 13.79 (sec) , antiderivative size = 2425, normalized size of antiderivative = 20.91

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p)*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^(m + n*p)*(c*(d*Sec[e + f*x])^p)^n*(a*(1 + Sec[e + f*x]))^m*Tan[(
e + f*x)/2])/(f*(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m
 + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(n*p)*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^(m + n*p))/(3*AppellF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1
 + m + n*p, 1 - n*p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n*p)*AppellF1[1/2, m + n*p, 1 - n*p, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n*p
)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n*p)*Tan[(e + f*x)/2]^2)/(3*Appel
lF1[1/2, m + n*p, 1 - n*p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((-1 + n*p)*AppellF1[3/2, m + n*p, 2 - n*p, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (m + n*p)*AppellF1[3/2, 1 + m + n*p, 1 - n*p, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 +...
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4436, 3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m \left(c(d\sec(e+fx))^p\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(e+fx)+a)^m \left(c(d\sec(e+fx))^p\right)^n dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)^m dx$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx$$

$\downarrow$ 4315

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{np}(\sec(e+fx)+1)^m dx$$

$\downarrow$ 3042

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$\downarrow$ 4314

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \frac{(d\sec(e+fx))}{\sqrt{}}}{f\sqrt{1-\sec(e+fx)}}$$

$\downarrow$ 150

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \operatorname{AppellF1}\left(np,\frac{1}{2},\frac{1}{2}-m,np+1,\sec(e+fx),-\sec(e-\right.}{fnp\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n*p, 1/2, 1/2 - m, 1 + n*p, Sec[e + f*x], -Sec[e + f*x]]*(c*(d
*Sec[e + f*x])^p)^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*T
an[e + f*x])/(f*n*p*Sqrt[1 - Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))^m\,dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \int (a(\sec(e+fx)+1))^m (c(d\sec(e+fx))^p)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(c*(d*sec(e + f*x))**p)**n, x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \int ((d\sec(fx+e))^p c)^n (a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(a*sec(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^m\,dx$$
$$= d^{np}c^n\left(\int \sec(fx+e)^{np}(\sec(fx+e)a+a)^m\,dx\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^m,x)
```

output
```
d**(n*p)*c**n*int(sec(e + f*x)**(n*p)*(sec(e + f*x)*a + a)**m,x)
```

3.232 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 275

$$\begin{aligned}
&\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx \\
&= \frac{a^3(7+4np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp(2+np)\sqrt{\sin^2(e+fx)}} \\
&\quad -\frac{a^3(1+4np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx)}{f\left(1-n^2p^2\right)\sqrt{\sin^2(e+fx)}} \\
&\quad +\frac{a^3(5+2np)\,(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)} \\
&\quad +\frac{(c(d\sec(e+fx))^p)^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2+np)}
\end{aligned}$$

output
```
a^3*(4*n*p+7)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*s
ec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1/2)-a^3*(4*n*p+1
)*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c
*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^3*(2
*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)/(n*p+2)+(c*(d*sec(f*x+
e))^p)^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(n*p+2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx$$
$$= \frac{a^3\cot(e+fx)\,(c(d\sec(e+fx))^p)^n \left((2+9np+4n^2p^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(a^3*Cot[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*((2 + 9*n*p + 4*n^2*p^2)*Hyperg
eometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^
2] + n*p*((6 + 3*n*p + (1 + n*p)*Sec[e + f*x])*Tan[e + f*x]^2 + (7 + 4*n*p
)*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]*Sec[e +
 f*x]*Sqrt[-Tan[e + f*x]^2])))/(f*n*p*(1 + n*p)*(2 + n*p))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4436, 3042, 4301, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3 (c(d\sec(e+fx))^p)^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sec(e+fx)+a)^3 (c(d\sec(e+fx))^p)^n\,dx$$
$$\Big\downarrow \text{4436}$$
$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)^3dx$$
$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3dx$$

$\downarrow$ 4301

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\int(d\sec(e+fx))^{np}(\sec(e+fx)a+a)(2a(np+1)+a(2np+5)\sec(e+fx))dx}{np+2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(2a(np+1)+a(2np+5)\csc\right.}{np+2}\right.$$

$\downarrow$ 4485

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{\int(d\sec(e+fx))^{np}\left((np+2)(4np+1)a^2+(np+1)(4np+7)\sec(e+fx)a^2\right)dx}{np+1}+\frac{a^2(2np+5)\tan(e+fx)(}{f(np+1)}\right.}{np+2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left((np+2)(4np+1)a^2+(np+1)(4np+7)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{np+1}+\frac{a^2(2np+5)\tan(}{}\right.}{np+2}\right.$$

$\downarrow$ 4274

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\int(d\sec(e+fx))^{np}dx+\frac{a^2(np+1)(4np+7)\int(d\sec(e+fx))^{np+1}dx}{d}}{np+1}+\frac{a^2(2np+5)\tan(}{f}\right.}{np+2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{a^2(np+1)(4np+7)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}}{np+1}+\frac{a^2(2np}{}\right.}{np+2}\right.$$

$\downarrow$ 4259

$$(d\sec(e+ f x))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{a^2(np+1)(4np+7)\left(\frac{\cos(e+fx)}{d}\right.}{}}{np+1}\right.}{np+2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+ f x))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a\left(\frac{a^2(np+2)(4np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{a^2(np+1)(4np+7)\left(\cos\right.}{}}{np+1}\right.}{np+}\right.$$

$\downarrow$ 3122

$$(d\sec(e+ f x))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)(d\sec(e+fx))^{np}}{f(np+2)}+\frac{a\left(\frac{a^2(np+1)(4np+7)\sin(e+fx)(d\,\mathrm{s}}{}\right.}{}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*(((d*Sec[e + f*x])^(n*p)*(a^3 + a^3*Sec[e + f*x]
)*Tan[e + f*x])/(f*(2 + n*p)) + (a*(((a^2*(1 + n*p)*(7 + 4*n*p)*Hypergeome
tric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*
p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a^2*d*(2 + n*p)*(1 + 4*n*
p)*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec
[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2]))/(1
 + n*p) + (a^2*(5 + 2*n*p)*(d*Sec[e + f*x])^(n*p)*Tan[e + f*x])/(f*(1 + n*
p))))/(2 + n*p)))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))^3 \, dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = \int (a\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = a^3 \left(\int (c(d\sec(e+fx))^p)^n \, dx + \int 3(c(d\sec(e+fx))^p)^n \sec(e+fx) \, dx + \int 3(c(d\sec(e+fx))^p)^n \sec^2(e+fx) \, dx + \int (c(d\sec(e+fx))^p)^n \sec^3(e+fx) \, dx \right)$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(3*(c*(d*sec(e + f
*x))**p)**n*sec(e + f*x), x) + Integral(3*(c*(d*sec(e + f*x))**p)**n*sec(e
 + f*x)**2, x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3 \, dx = \int (a\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx = \int (a\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^3 dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^3\,dx = d^{np}c^n a^3\left(\int \sec(fx+e)^{np}\,dx + \int \sec(fx+e)^{np}\sec(fx+e)^3\,dx + 3\left(\int \sec(fx+e)^{np}\sec(fx+e)^2\,dx\right) + 3\left(\int \sec(fx+e)^{np}\sec(fx+e)\,dx\right)\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^3,x)
```

output

```
d**(n*p)*c**n*a**3*(int(sec(e + f*x)**(n*p),x) + int(sec(e + f*x)**(n*p)*s
ec(e + f*x)**3,x) + 3*int(sec(e + f*x)**(n*p)*sec(e + f*x)**2,x) + 3*int(s
ec(e + f*x)**(n*p)*sec(e + f*x),x))
```

3.233 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 205

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx$$
$$= \frac{2a^2 \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{np}{2}, \frac{1}{2}(2-np), \cos^2(e+fx)\right) (c(d\sec(e+fx))^p)^n \sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$
$$- \frac{a^2(1+2np)\cos(e+fx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1-np), \frac{1}{2}(3-np), \cos^2(e+fx)\right) (c(d\sec(e+fx)}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+ \frac{a^2(c(d\sec(e+fx))^p)^n \tan(e+fx)}{f(1+np)}$$

output
```
2*a^2*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e
))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a^2*(2*n*p+1)*cos(f*x+e)*hyp
ergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^
p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+a^2*(c*(d*sec(f*x+e))^
p)^n*tan(f*x+e)/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$$
$$= \frac{a^2\cot(e+fx)\,(c(d\sec(e+fx))^p)^n\left((1+2np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\sqrt{-\tan}\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*Cot[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*((1 + 2*n*p)*Hypergeometric2F1[
1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + n*p*(Ta
n[e + f*x]^2 + 2*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e +
f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2])))/(f*n*p*(1 + n*p))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4436, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2 (c(d\sec(e+fx))^p)^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sec(e+fx)+a)^2 (c(d\sec(e+fx))^p)^n\,dx$$

$$\Big\downarrow \text{ 4436}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$\downarrow$ 4275

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\int(d\sec(e+fx))^{np}\left(\sec^2(e+fx)a^2+a^2\right)dx+\frac{2a^2\int(d\sec(e+fx))^{np+1}dx}{d}\right)$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{2a^2\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}+\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\right.\right.\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{np}}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{np}}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\sin(e+fx)(d}{}\right.$$

$\downarrow$ 4534

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a^2(2np+1)\int(d\sec(e+fx))^{np}dx}{np+1}+\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{np}\text{Hypergeom}}{fnp\sqrt{\sin^2}}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a^2(2np+1)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx}{np+1}+\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{np}\text{Hype}}{fnp}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a^2(2np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{np+1}+\frac{2a^2\sin(e+f$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a^2(2np+1)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin(e+fx+\frac{\pi}{2})}{d}\right)^{-np}dx}{np+1}+\frac{2a^2\sin(e+$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{np}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+\right.}{fnp\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((2*a^2*Hypergeometric2F1[1/2, -1/2*(n*p), (2 -
n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[S
in[e + f*x]^2]) - (a^2*d*(1 + 2*n*p)*Hypergeometric2F1[1/2, (1 - n*p)/2, (
3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(
1 - n*p)*(1 + n*p)*Sqrt[Sin[e + f*x]^2]) + (a^2*(d*Sec[e + f*x])^(n*p)*Tan
[e + f*x])/(f*(1 + n*p))))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))^2\,dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx = \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*((d*sec(f*x + e))
^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 \, dx = a^2 \left(\int (c(d\sec(e+fx))^p)^n \, dx + \int 2(c(d\sec(e+fx))^p)^n \sec(e+fx) \, dx + \int (c(d\sec(e+fx))^p)^n \sec^2(e+fx) \, dx \right)$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral(2*(c*(d*sec(e + f
*x))**p)**n*sec(e + f*x), x) + Integral((c*(d*sec(e + f*x))**p)**n*sec(e +
 f*x)**2, x))
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 dx$$
$$= \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 dx$$
$$= \int (a\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2 dx$$
$$= \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)^2 dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))^2\,dx$$
$$= d^{np}c^n a^2 \left(\int \sec(fx+e)^{np}\,dx + \int \sec(fx+e)^{np}\sec(fx+e)^2\,dx + 2\left(\int \sec(fx+e)^{np}\sec(fx+e)\,dx\right)\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e))^2,x)
```

output
```
d**(n*p)*c**n*a**2*(int(sec(e + f*x)**(n*p),x) + int(sec(e + f*x)**(n*p)*s
ec(e + f*x)**2,x) + 2*int(sec(e + f*x)**(n*p)*sec(e + f*x),x))
```

3.234 $\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

output
```
a*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p
)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/
2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/
f/(-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx$$
$$= \frac{a\csc(e+fx)\left((1+np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+np\,\text{Hypergeome}\right.}{fnp(1+np)}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

output
```
(a*Csc[e + f*x]*((1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1
+ (n*p)/2, Sec[e + f*x]^2] + n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 +
n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(
f*n*p*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4436, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)\,(c(d\sec(e+fx))^p)^n\ dx$$

$$\Big\downarrow \ 3042$$

$$\int (a\sec(e+fx)+a)\,(c(d\sec(e+fx))^p)^n\,dx$$

$$\Big\downarrow \ 4436$$

$$(d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\int (d\sec(e+fx))^{np}(\sec(e+fx)a+a)dx$$

$$\Big\downarrow \ 3042$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)dx$$

$\downarrow$ 4274

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(a\int(d\sec(e+fx))^{np}dx+\frac{a\int(d\sec(e+fx))^{np+1}dx}{d}\right)$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}\right)$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{a\left(\frac{\cos(e+fx)}{d}\right)^{np}(\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{a\left(\frac{\cos(e+fx)}{d}\right.}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{a\sin(e+fx)(d\sec(e+fx))^{np}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+\right.}{fnp\sqrt{\sin^2(e+fx)}}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + a*Sec[e + f*x]),x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((a*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)
/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e
 + f*x]^2]) - (a*d*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e
+ f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[
e + f*x]^2])))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+a\sec(fx+e))\,dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx)) \, dx = \int (a \sec(fx+e)+a)((d \sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx)) \, dx = a\left(\int (c(d\sec(e+fx))^p)^n \, dx + \int (c(d\sec(e+fx))^p)^n \sec(e+fx) \, dx\right)$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+a*sec(f*x+e)),x)
```

output
```
a*(Integral((c*(d*sec(e + f*x))**p)**n, x) + Integral((c*(d*sec(e + f*x))*
*p)**n*sec(e + f*x), x))
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx)) \, dx = \int (a \sec(fx+e)+a)((d \sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+a\sec(e+fx))\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{a}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$
\begin{aligned}
&\int (c(d\sec(e+fx))^p)^n\,(a+a\sec(e+fx))\,dx \\
&= d^{np}c^n a\left(\int \sec(fx+e)^{np}\,dx+\int \sec(fx+e)^{np}\sec(fx+e)\,dx\right)
\end{aligned}
$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+a*sec(f*x+e)),x)
```

output
```
d**(n*p)*c**n*a*(int(sec(e + f*x)**(n*p),x) + int(sec(e + f*x)**(n*p)*sec(
e + f*x),x))
```

3.235 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 208

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{(c(d\sec(e+fx))^p)^n\sin(e+fx)}{f(a+a\sec(e+fx))}$$
$$-\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+}{af\sqrt{\sin^2(e+fx)}}$$
$$+\frac{(1-np)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2-np),\frac{1}{2}(4-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)}{af(2-np)\sqrt{\sin^2(e+fx)}}$$

output
```
(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*hypergeom(
[1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*si
n(f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)+(-n*p+1)*cos(f*x+e)^2*hypergeom([1/2, -1
/2*n*p+1],[-1/2*n*p+2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/a/f
/(-n*p+2)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx$$
$$= \frac{\cot\left(\frac{1}{2}(e+fx)\right)(c(d\sec(e+fx))^p)^n\left(-\left((-1+np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)\sqrt{}\right.\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Cot[(e + f*x)/2]*(c*(d*Sec[e + f*x])^p)^n*(-((-1 + n*p)*Hypergeometric2F1
[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]) + n*p*(
1 - Cos[e + f*x] + Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + n*p)/2, (1 +
n*p)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2])))/(a*f*n*p*(1 + Sec[e + f*x
]))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 4436, 3042, 4307, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a\sec(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c(d\sec(e+fx))^p)^n}{a\sec(e+fx)+a}dx$$
$$\Big\downarrow \text{4436}$$
$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\int \frac{(d\sec(e+fx))^{np}}{\sec(e+fx)a+a}dx$$
$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$\downarrow$ 4307

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\int(d\sec(e+fx))^{np-1}(a-a\sec(e+\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np-1}(a-a\right.$$

$\downarrow$ 4274

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\int(d\sec(e+fx))^{np-1}dx-\frac{a\int(d\right.}{a^2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np-1}dx-\right.}{a^2}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\right.}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\right.}{}\right.$$

$\big\downarrow$ 3122

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{np-1}}{f(a\sec(e+fx)+a)}-\frac{d(1-np)\left(\frac{a\sin(e+fx)(d\sec(e+fx))^{np-1}\text{Hypergeom}}{f(1-np)}\right.}{}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x]),x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*(-((d*(1 - n*p)*(-((a*d*Hypergeometric2F1[1/2, (
2 - n*p)/2, (4 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-2 + n*p)*Sin[e
 + f*x])/(f*(2 - n*p)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeometric2F1[1/2, (
1 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e
 + f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2])))/a^2) + (d*(d*Sec[e + f*x])^(
-1 + n*p)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{a+a\sec(fx+e)}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{(c(d\sec(e+fx))^p)^n}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0]%%%} / %%%{2,[0,0,1]%%%} Error: Bad Argument Val
ue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+a\sec(e+fx)}\,dx = \frac{d^{np}c^n\left(\int \frac{\sec(fx+e)^{np}}{\sec(fx+e)+1}dx\right)}{a}$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e)),x)
```

output
```
(d**(n*p)*c**n*int(sec(e + f*x)**(n*p)/(sec(e + f*x) + 1),x))/a
```

3.236 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 248

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \frac{2(2-np)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{(3-2np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{2(2-np)(c(d\sec(e+fx))^p)^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(c(d\sec(e+fx))^p)^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

output
```
2/3*(-n*p+2)*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*se
c(f*x+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(-2*n*p+3)*cos(f*
x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(
f*x+e))^p)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-2/3*(-n*p+2)*(c*(d*sec(
f*x+e))^p)^n*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*(c*(d*sec(f*x+e))^p)^n*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.87

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{(c(d\sec(e+fx))^p)^n \left(-np(1+np)\tan(e+fx)+(1+\sec(e+fx))\left(2np(-2+np)(1+np)\tan(e+fx\right.\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*(-(n*p*(1 + n*p)*Tan[e + f*x]) + (1 + Sec[e + f*
x])*(2*n*p*(-2 + n*p)*(1 + n*p)*Tan[e + f*x] + ((-1 + n*p)*(1 + n*p)*(-3 +
 2*n*p)*Cot[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e +
f*x]^2] - 2*n^2*p^2*(-2 + n*p)*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 + n*
p)/2, (3 + n*p)/2, Sec[e + f*x]^2])*(1 + Sec[e + f*x])*Sqrt[-Tan[e + f*x]^
2])))/(3*a^2*f*n*p*(1 + n*p)*(1 + Sec[e + f*x])^2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4436, 3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a\sec(e+fx)+a)^2}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a\sec(e+fx)+a)^2}dx$$
$$\Big\downarrow \ 4436$$
$$(d\sec(e+fx))^{-np}\,(c(d\sec(e+fx))^p)^n\int \frac{(d\sec(e+fx))^{np}}{(\sec(e+fx)a+a)^2}dx$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$\Big\downarrow$ 4304

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(-\frac{\int-\frac{(d\sec(e+fx))^{np}(a(3-np)-a(1-np)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{np}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 25

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\frac{(d\sec(e+fx))^{np}(a(3-np)-a(1-np)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{np}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a(3-np)-a(1-np)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{n}}{3f(a\sec(e+fx)+a)^2}\right.$$

$\Big\downarrow$ 4508

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{\int(d\sec(e+fx))^{np}\left((3-2np)(1-np)a^2+2np(2-np)\sec(e+fx)a^2\right)dx}{a^2}-\frac{2(2-np)\tan(e+fx)(d\sec(e+fx}{f(\sec(e+fx)+1)}}{3a^2}\right.$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left((3-2np)(1-np)a^2+2np(2-np)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{a^2}-\frac{2(2-np)\tan(e+fx)(d\,\mathrm{s}}{f(\sec(e+fx)-}}{3a^2}\right.$$

$\Big\downarrow$ 4274

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\int(d\sec(e+fx))^{np}dx+\frac{2a^2np(2-np)\int(d\sec(e+fx))^{np+1}dx}{d}}{a^2}-\frac{2(2-np)\tan(e+fx)(d\,\mathrm{se}}{f(\sec(e+fx)+}}{3a^2}\right.$$

$\Big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{2a^2np(2-np)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}}{a^2}-\frac{2(2-np)\tan(e}{f(\sec}}{3a^2}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{2a^2np(2-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\,s}{}}{a^2}}{3a^2}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{a^2(3-2np)(1-np)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{2a^2np(2-np)\left(\frac{\cos(e+fx)}{d}\right)}{}}{a^2}}{3a^2}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{2a^2(2-np)\sin(e+fx)(d\sec(e+fx))^{np}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{a^2d(3-2np)\sin(e+fx)}{}}{a^2}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*(-1/3*((d*Sec[e + f*x])^(n*p)*Tan[e + f*x])/(f*(
a + a*Sec[e + f*x])^2) + (((2*a^2*(2 - n*p)*Hypergeometric2F1[1/2, -1/2*(n
*p), (2 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*
Sqrt[Sin[e + f*x]^2]) - (a^2*d*(3 - 2*n*p)*Hypergeometric2F1[1/2, (1 - n*p
)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x]
)/(f*Sqrt[Sin[e + f*x]^2]))/a^2 - (2*(2 - n*p)*(d*Sec[e + f*x])^(n*p)*Tan[
e + f*x])/(f*(1 + Sec[e + f*x])))/(3*a^2)))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{(a+a\sec(fx+e))^2}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left((d\sec\left(fx+e\right))^p c\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e)
 + a^2), x)
```

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{(c(d\sec(e+fx))^p)^n}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1)
, x)/a**2
```

Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left((d\sec\left(fx+e\right))^p c\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(a*sec(f*x + e) + a)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0]%%%}+%%%{-3,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%}
 Error: B
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + a/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+a\sec(e+fx))^2}\,dx = \frac{d^{np}c^n\left(\int \frac{\sec(fx+e)^{np}}{\sec(fx+e)^2+2\sec(fx+e)+1}dx\right)}{a^2}$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+a*sec(f*x+e))^2,x)
```

output

```
(d**(n*p)*c**n*int(sec(e + f*x)**(n*p)/(sec(e + f*x)**2 + 2*sec(e + f*x) +
 1),x))/a**2
```

3.237 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = (d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \operatorname{Int}((d\sec(e+fx))^{np}(a+b\sec(e+fx))^m, x)$$

output
```
(c*(d*sec(f*x+e))^p)^n*Defer(Int)((d*sec(f*x+e))^(n*p)*(a+b*sec(f*x+e))^m,
x)/((d*sec(f*x+e))^(n*p))
```

Mathematica [N/A]

Not integrable

Time = 5.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4436, 3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^m\left(c(d\sec(e+fx))^p\right)^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+b\sec(e+fx))^m\left(c(d\sec(e+fx))^p\right)^n dx$$

$$\Big\downarrow \ 4436$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^m dx$$

$$\Big\downarrow \ 3042$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \ 4357$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^m dx$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(c (d \sec \left(f x +e \right))^{p}\right)^{n} \left(a +b \sec \left(f x +e \right)\right)^{m}d x$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d \sec(e+fx))^p)^n (a+b\sec(e+fx))^m \, dx = \int \left(\left(d \sec \left(f x + e\right)\right)^{p} c\right)^{n} \left(b \sec \left(f x + e\right) + a\right)^{m} \,d x$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Sympy [N/A]

Not integrable

Time = 6.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**m, x)
```

Maxima [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx = \int ((d\sec(fx+e))^p c)^n (b\sec(fx+e)+a)^m dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m\,dx = \int ((d\sec(fx+e))^p c)^n (b\sec(fx+e)+a)^m\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n*(b*sec(f*x + e) + a)^m, x)
```

Mupad [N/A]

Not integrable

Time = 12.80 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m\,dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+f\,x)}\right)^m dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^m dx$$
$$= d^{np}c^n\left(\int \sec(fx+e)^{np}(\sec(fx+e)b+a)^m dx\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^m,x)
```

output
```
d**(n*p)*c**n*int(sec(e + f*x)**(n*p)*(sec(e + f*x)*b + a)**m,x)
```

3.238 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 296

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx$$
$$= \frac{b(b^2(1+np)+3a^2(2+np)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n}{fnp(2+np)\sqrt{\sin^2(e+fx)}}$$
$$- \frac{a(3b^2np+a^2(1+np))\cos(e+fx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\,\mathrm{se}}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+ \frac{ab^2(5+2np)(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)(2+np)}$$
$$+ \frac{b^2(c(d\sec(e+fx))^p)^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+np)}$$

output
```
b*(b^2*(n*p+1)+3*a^2*(n*p+2))*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f
*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/n/p/(n*p+2)/(sin(f*x+e)^2)^(1
/2)-a*(3*b^2*n*p+a^2*(n*p+1))*cos(f*x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1
/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)
/(sin(f*x+e)^2)^(1/2)+a*b^2*(2*n*p+5)*(c*(d*sec(f*x+e))^p)^n*tan(f*x+e)/f/
(n*p+1)/(n*p+2)+b^2*(c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))*tan(f*x+e)/f/(
n*p+2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.94

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 \, dx =$$
$$-\frac{\csc^3(e+fx)\left(a^3(6+11np+6n^2p^2+n^3p^3)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+\right.\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
-((Csc[e + f*x]^3*(a^3*(6 + 11*n*p + 6*n^2*p^2 + n^3*p^3)*Cos[e + f*x]^3*H
ypergeometric2F1[1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(3*a*b
*(3 + 4*n*p + n^2*p^2)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1 + (n*p)/2, 2
+ (n*p)/2, Sec[e + f*x]^2] + (2 + n*p)*(3*a^2*(3 + n*p)*Cos[e + f*x]^2*Hyp
ergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2] + b^2*(1 + n
*p)*Hypergeometric2F1[1/2, (3 + n*p)/2, (5 + n*p)/2, Sec[e + f*x]^2])))*(c
*(d*Sec[e + f*x])^p)^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*p*(1 + n*p)*(2 + n*p)
*(3 + n*p)))
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4436, 3042, 4329, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^3 \left(c(d\sec(e+fx))^p\right)^n \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\sec(e+fx))^3 \left(c(d\sec(e+fx))^p\right)^n dx$$

$$\Big\downarrow \text{ 4436}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int(d\sec(e+fx))^{np}(a+b\sec(e+fx))^3dx$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$\downarrow$ 4329

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int(d\sec(e+fx))^{np}\left(ab^2d(2np+5)\sec^2(e+fx)+bd\left(3(np+2)a^2+b^2(np+1)\right)\mathrm{s}\right.}{d(np+2)}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+bd\left(3(np+2)a^2+b\right.\right.}{d(np+2)}\right.$$

$\downarrow$ 4535

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int(d\sec(e+fx))^{np}\left(ab^2d(2np+5)\sec^2(e+fx)+ad\left((np+2)a^2+b^2np\right)\right)dx+b}{d(np+2)}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{b\left(3a^2(np+2)+b^2(np+1)\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx+\int\left(d\csc\left(e+fx+\right.\right.}{d(np+2)}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left((np+2)a^2+b^2\right.\right.}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left((np+2)a^2+b^2\right.\right.}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(ab^2d(2np+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left((np+2)a^2+b^2\right.\right.}{}\right.$$

$\downarrow$ 4534

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\int(d\sec(e+fx))^{np}dx}{np+1}+\frac{bd\left(3a^2(np+2)+b^2(np+1)\right)\sin(e+fx)(d\sec(e+f}{fnp}}{d(np}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx}{np+1}+\frac{bd\left(3a^2(np+2)+b^2(np+1)\right)\sin(e+fx)(d\sec(}{}}{d(n}\right.$$

$\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx}{np+1}+\frac{bd\left(3a^2(np+2)+b\right.}{}}{}\right.$$

$\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{\frac{ad(np+2)\left(a^2(np+1)+3b^2np\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx}{np+1}+\frac{bd\left(3a^2(np+\right.}{}}{}\right.$$

$\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{-\frac{ad^2(np+2)\left(a^2(np+1)+3b^2np\right)\sin(e+fx)(d\sec(e+fx))^{np-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-n\right.}{f(1-np)(np+1)\sqrt{\sin^2(e+fx)}}}{}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((b^2*(d*Sec[e + f*x])^(n*p)*(a + b*Sec[e + f*x]
)*Tan[e + f*x])/(f*(2 + n*p)) + ((b*d*(b^2*(1 + n*p) + 3*a^2*(2 + n*p))*Hy
pergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f
*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e + f*x]^2]) - (a*d^2*(2 + n*p)*(
3*b^2*n*p + a^2*(1 + n*p))*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2
, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*(
1 + n*p)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*d*(5 + 2*n*p)*(d*Sec[e + f*x])^(n*
p)*Tan[e + f*x])/(f*(1 + n*p)))/(d*(2 + n*p))))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+b\sec(fx+e))^3\,dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x +
e) + a^3)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx
= \int (c(d\sec{(e+fx)})^p)^n (a+b\sec{(e+fx)})^3\;dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**3,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**3, x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3\,dx
= \int (b\sec{(fx+e)}+a)^3((d\sec{(fx+e)})^p\,c)^n\;dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 dx$$
$$= \int (b\sec(fx+e)+a)^3((d\sec(fx+e))^p c)^n dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^3*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 dx$$
$$= \int \left(c\left(\frac{d}{\cos(e+fx)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+fx)}\right)^3 dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^3, x)
```

Reduce [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^3 dx$$
$$= d^{np}c^n\left(\left(\int \sec(fx+e)^{np} dx\right)a^3 + \left(\int \sec(fx+e)^{np}\sec(fx+e)^3 dx\right)b^3 + 3\left(\int \sec(fx+e)^{np}\sec(fx+e)^2 dx\right)a b^2 + 3\left(\int \sec(fx+e)^{np}\sec(fx+e) dx\right)a^2 b\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^3,x)
```

output
```
d**(n*p)*c**n*(int(sec(e + f*x)**(n*p),x)*a**3 + int(sec(e + f*x)**(n*p)*s
ec(e + f*x)**3,x)*b**3 + 3*int(sec(e + f*x)**(n*p)*sec(e + f*x)**2,x)*a*b*
*2 + 3*int(sec(e + f*x)**(n*p)*sec(e + f*x),x)*a**2*b)
```

3.239 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 211

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx$$
$$= \frac{2ab\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}}$$
$$-\frac{(b^2np+a^2(1+np))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(}{f(1-n^2p^2)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{b^2(c(d\sec(e+fx))^p)^n\tan(e+fx)}{f(1+np)}$$

output
```
2*a*b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e
))^p)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-(b^2*n*p+a^2*(n*p+1))*cos(f*
x+e)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(
f*x+e))^p)^n*sin(f*x+e)/f/(-n^2*p^2+1)/(sin(f*x+e)^2)^(1/2)+b^2*(c*(d*sec(
f*x+e))^p)^n*tan(f*x+e)/f/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx$$
$$= \frac{\csc(e+fx)\left(a^2(2+3np+n^2p^2)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+bnp(b\right.}{}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
(Csc[e + f*x]*(a^2*(2 + 3*n*p + n^2*p^2)*Cos[e + f*x]^2*Hypergeometric2F1[
1/2, (n*p)/2, 1 + (n*p)/2, Sec[e + f*x]^2] + b*n*p*(b*(1 + n*p)*Hypergeome
tric2F1[1/2, 1 + (n*p)/2, 2 + (n*p)/2, Sec[e + f*x]^2] + 2*a*(2 + n*p)*Cos
[e + f*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sec[e + f*x]^2]
))*Sec[e + f*x]*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])/(f*n*p*(1
+ n*p)*(2 + n*p))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4436, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^2 (c(d\sec(e+fx))^p)^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sec(e+fx))^2 (c(d\sec(e+fx))^p)^n dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(a+b\sec(e+fx))^2 dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2dx$$

$\big\downarrow$ 4275

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int(d\sec(e+fx))^{np}\left(a^2+b^2\sec^2(e+fx)\right)dx+\frac{2ab\int(d\sec(e+fx))^{np+1}dx}{d}\right)$$

$\big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\int\left(d\csc\left(e+f\right.\right.}{d}\right.$$

$\big\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{np}}{}\right.$$

$\big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{np}}{}\right.$$

$\big\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\sin(e+fx)(d}{}\right.$$

$\big\downarrow$ 4534

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\int(d\sec(e+fx))^{np}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^{np}\,\text{Hyperg}}{fnp\sqrt{}}\right.$$

$\big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^{np}}{}\right.$$

$$\Big\downarrow \text{4259}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{2}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(\left(a^2+\frac{b^2np}{np+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx\right.$$

$$\Big\downarrow \text{3122}$$

$$(d\sec(e+fx))^{-np}(c(d\sec(e+fx))^p)^n\left(-\frac{d\left(a^2+\frac{b^2np}{np+1}\right)\sin(e+fx)(d\sec(e+fx))^{np-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np\right.}{f(1-np)\sqrt{\sin^2(e+fx)}}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((2*a*b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 -
n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[S
in[e + f*x]^2]) - (d*(a^2 + (b^2*n*p)/(1 + n*p))*Hypergeometric2F1[1/2, (1
 - n*p)/2, (3 - n*p)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e
+ f*x])/(f*(1 - n*p)*Sqrt[Sin[e + f*x]^2]) + (b^2*(d*Sec[e + f*x])^(n*p)*T
an[e + f*x])/(f*(1 + n*p))))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+b\sec(fx+e))^2 \, dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = \int (b\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*((d*sec(f*x + e))
^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x))**2, x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2\,dx = \int (b\sec(fx+e)+a)^2((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int (b\sec(fx+e)+a)^2 ((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^2*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = \int \left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+f\,x)}\right)^2 dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))^2 \, dx = d^{np}c^n\left(\left(\int \sec(fx+e)^{np}\, dx\right)a^2+\left(\int \sec(fx+e)^{np}\sec(fx+e)^2\, dx\right)b^2 + 2\left(\int \sec(fx+e)^{np}\sec(fx+e)\, dx\right)ab\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e))^2,x)
```

output
```
d**(n*p)*c**n*(int(sec(e + f*x)**(n*p),x)*a**2 + int(sec(e + f*x)**(n*p)*s
ec(e + f*x)**2,x)*b**2 + 2*int(sec(e + f*x)**(n*p)*sec(e + f*x),x)*a*b)
```

3.240 $\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 156

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+fx)}{fnp\sqrt{\sin^2(e+fx)}} - \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1-np),\frac{1}{2}(3-np),\cos^2(e+fx)\right)(c(d\sec(e+fx))^p)^n\sin(e+
}{f(1-np)\sqrt{\sin^2(e+fx)}}$$

output
```
b*hypergeom([1/2, -1/2*n*p],[-1/2*n*p+1],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p
)^n*sin(f*x+e)/f/n/p/(sin(f*x+e)^2)^(1/2)-a*cos(f*x+e)*hypergeom([1/2, -1/
2*n*p+1/2],[-1/2*n*p+3/2],cos(f*x+e)^2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/
f/(-n*p+1)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$
$$= \frac{\csc(e+fx)\left(a(1+np)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{np}{2},1+\frac{np}{2},\sec^2(e+fx)\right)+bnp\,\text{Hypergeome}\right.}{fnp(1+np)}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

output
```
(Csc[e + f*x]*(a*(1 + n*p)*Cos[e + f*x]*Hypergeometric2F1[1/2, (n*p)/2, 1
+ (n*p)/2, Sec[e + f*x]^2] + b*n*p*Hypergeometric2F1[1/2, (1 + n*p)/2, (3
+ n*p)/2, Sec[e + f*x]^2])*(c*(d*Sec[e + f*x])^p)^n*Sqrt[-Tan[e + f*x]^2])
/(f*n*p*(1 + n*p))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4436, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx)) (c(d\sec(e+fx))^p)^n \, dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\sec(e+fx)) (c(d\sec(e+fx))^p)^n dx$$

$$\Big\downarrow \; 4436$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int (d\sec(e+fx))^{np}(a+b\sec(e+fx))dx$$

$$\Big\downarrow \; 3042$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx$$

$\big\downarrow$ 4274

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\int(d\sec(e+fx))^{np}dx+\frac{b\int(d\sec(e+fx))^{np+1}dx}{d}\right)$$

$\big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}dx+\frac{b\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np+1}dx}{d}\right)$$

$\big\downarrow$ 4259

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\cos(e+fx)}{d}\right)^{-np}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right)^{np}(\right.$$

$\big\downarrow$ 3042

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(a\left(\frac{\cos(e+fx)}{d}\right)^{np}(d\sec(e+fx))^{np}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-np}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right.}{}\right.$$

$\big\downarrow$ 3122

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\left(\frac{b\sin(e+fx)(d\sec(e+fx))^{np}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{np}{2},\frac{1}{2}(2-np),\cos^2(e+f\right.}{fnp\sqrt{\sin^2(e+fx)}}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n*(a + b*Sec[e + f*x]),x]
```

output
```
((c*(d*Sec[e + f*x])^p)^n*((b*Hypergeometric2F1[1/2, -1/2*(n*p), (2 - n*p)
/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(n*p)*Sin[e + f*x])/(f*n*p*Sqrt[Sin[e
 + f*x]^2]) - (a*d*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[e
+ f*x]^2]*(d*Sec[e + f*x])^(-1 + n*p)*Sin[e + f*x])/(f*(1 - n*p)*Sqrt[Sin[
e + f*x]^2])))/(d*Sec[e + f*x])^(n*p)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (c(d\sec(fx+e))^p)^n (a+b\sec(fx+e))\,dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Sympy [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n*(a+b*sec(f*x+e)),x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n*(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Giac [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$
$$= \int (b\sec(fx+e)+a)((d\sec(fx+e))^p c)^n \, dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)*((d*sec(f*x + e))^p*c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$
$$= \int \left(c\left(\frac{d}{\cos(e+fx)}\right)^p\right)^n \left(a+\frac{b}{\cos(e+fx)}\right) dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n*(a + b/cos(e + f*x)), x)
```

Reduce [F]

$$\int (c(d\sec(e+fx))^p)^n (a+b\sec(e+fx))\,dx$$
$$= d^{np}c^n\left(\left(\int \sec(fx+e)^{np}\,dx\right)a+\left(\int \sec(fx+e)^{np}\sec(fx+e)\,dx\right)b\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n*(a+b*sec(f*x+e)),x)
```

output
```
d**(n*p)*c**n*(int(sec(e + f*x)**(n*p),x)*a + int(sec(e + f*x)**(n*p)*sec(
e + f*x),x)*b)
```

3.241 $\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{np}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}\left(c(d\sec(e+fx))^p\right)^n\sin(e+fx)}{(a^2-b^2)\,f} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}\left(c(d\sec(e\right.}{(a^2-b^2)\,f}$$

output
```
-b*AppellF1(1/2,1/2*n*p,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(co
s(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)/f+a*Appe
llF1(1/2,1/2*n*p-1/2,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*cos(f*
x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b
^2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5411 vs. $2(206) = 412$.

Time = 31.13 (sec) , antiderivative size = 5411, normalized size of antiderivative = 26.27

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {3042, 4436, 3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \frac{(d\sec(e+fx))^{np}}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np}\left(c(d\sec(e+fx))^p\right)^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\frac{\cos^{1-np}(e+fx)}{b+a\cos(e+fx)}dx$$

$\Big\downarrow$ 3042

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-np}}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$\Big\downarrow$ 3302

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(b\int\frac{\cos^{1-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx-a\int\frac{\cos^{2-np}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx\right)$$

$\Big\downarrow$ 3042

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(b\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-np}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx-a\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-np}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx\right)$$

$\Big\downarrow$ 3668

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(\frac{b\cos^{-np}(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\int-\frac{\left(1-\sin^2(e+fx)\right)^{-\frac{np}{2}}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{a\cos^{1-np}(e+\cdots}{\cdots}\right.$$

$\Big\downarrow$ 25

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a\cos^{1-np}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\int\frac{\left(1-\sin^2(e+fx)\right)^{\frac{1}{2}(1-np)}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{b\cos^{-np}\cdots}{\cdots}\right.$$

$\Big\downarrow$ 333

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(\frac{a\sin(e+fx)\cos^{1-np}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(np-1)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-1),1,\frac{3}{2},\sin^2(e\cdots\right.}{f\left(a^2-b^2\right)}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x]),x]
```

output
```
Cos[e + f*x]^(n*p)*(c*(d*Sec[e + f*x])^p)^n*(-((b*AppellF1[1/2, (n*p)/2, 1
, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^
((n*p)/2)*Sin[e + f*x])/((a^2 - b^2)*f*Cos[e + f*x]^(n*p))) + (a*AppellF1[
1/2, (-1 + n*p)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2
)]*Cos[e + f*x]^(1 - n*p)*(Cos[e + f*x]^2)^((-1 + n*p)/2)*Sin[e + f*x])/((
a^2 - b^2)*f))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] && !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{a+b\sec(fx+e)}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{(c(d\sec{(e+fx)})^p)^n}{a+b\sec{(e+fx)}}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e)),x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{((d\sec{(fx+e)})^p\,c)^n}{b\sec{(fx+e)}+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{((d\sec{(fx+e)})^p\,c)^n}{b\sec{(fx+e)}+a}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+f\,x)}\right)^p\right)^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)),x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{a+b\sec(e+fx)}\,dx = d^{np}c^n\left(\int \frac{\sec(fx+e)^{np}}{\sec(fx+e)\,b+a}dx\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e)),x)
```

output
```
d**(n*p)*c**n*int(sec(e + f*x)**(n*p)/(sec(e + f*x)*b + a),x)
```

3.242 $\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 322

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = -\frac{2ab\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{\frac{np}{2}}(c(d\sec(e+fx))^p)^n\sin(\ldots}{(a^2-b^2)^2 f} + \frac{a^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(d\sec(\ldots}{(a^2-b^2)^2 f} + \frac{b^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+np),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+np)}(c(d\sec(\ldots}{(a^2-b^2)^2 f}$$

output
```
-2*a*b*AppellF1(1/2,1/2*n*p-1,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2
))*(cos(f*x+e)^2)^(1/2*n*p)*(c*(d*sec(f*x+e))^p)^n*sin(f*x+e)/(a^2-b^2)^2/
f+a^2*AppellF1(1/2,1/2*n*p-3/2,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^
2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+e))^p)^n*sin(f*x
+e)/(a^2-b^2)^2/f+b^2*AppellF1(1/2,1/2*n*p-1/2,2,3/2,sin(f*x+e)^2,a^2*sin(
f*x+e)^2/(a^2-b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*n*p-1/2)*(c*(d*sec(f*x+
e))^p)^n*sin(f*x+e)/(a^2-b^2)^2/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 14108 vs. $2(322) = 644$.

Time = 45.64 (sec) , antiderivative size = 14108, normalized size of antiderivative = 43.81

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2} dx = \text{Result too large to show}$$

input
```
Integrate[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4436, 3042, 4356, 3042, 3303, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2} dx$$

$$\Big\downarrow \text{4436}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int \frac{(d\sec(e+fx))^{np}}{(a+b\sec(e+fx))^2} dx$$

$$\Big\downarrow \text{3042}$$

$$(d\sec(e+fx))^{-np} (c(d\sec(e+fx))^p)^n \int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{np}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2} dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\frac{\cos^{2-np}(e+fx)}{(b+a\cos(e+fx))^2}dx$$

$\Big\downarrow$ 3042

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-np}}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$\Big\downarrow$ 3303

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\int\left(\frac{b^2\cos^{2-np}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}-\frac{2ab\cos^{3-np}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}+\frac{a^2\cos^{4-np}(e+fx)}{(a^2\cos^2(e+fx)-b^2)^2}\right)dx$$

$\Big\downarrow$ 2009

$$\cos^{np}(e+fx)\left(c(d\sec(e+fx))^p\right)^n\left(-\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{np}{2}}\cos^{-np}(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}(np-2),2,\frac{3}{2},\sin^2(e\right.}{f\left(a^2-b^2\right)^2}\right.$$

input
```
Int[(c*(d*Sec[e + f*x])^p)^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Cos[e + f*x]^(n*p)*(c*(d*Sec[e + f*x])^p)^n*((-2*a*b*AppellF1[1/2, (-2 + n
*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e +
f*x]^2)^((n*p)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f*Cos[e + f*x]^(n*p)) + (a^
2*AppellF1[1/2, (-3 + n*p)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]^(1 - n*p)*(Cos[e + f*x]^2)^((-1 + n*p)/2)*Sin[e
 + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 + n*p)/2, 2, 3/2, Sin[
e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 - n*p)*(Cos[
e + f*x]^2)^((-1 + n*p)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3303
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4436
```
Int[((c_.)*((d_.)*sec[(e_.) + (f_.)*(x_)])^(p_))^(n_)*((a_.) + (b_.)*sec[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Simp[c^IntPart[n]*((c*(d*Sec[e + f*x
])^p)^FracPart[n]/(d*Sec[e + f*x])^(p*FracPart[n]))   Int[(a + b*Sec[e + f*
x])^m*(d*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(c(d\sec(fx+e))^p)^n}{(a+b\sec(fx+e))^2}dx$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

output
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec(fx+e))^p\,c)^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(((d*sec(f*x + e))^p*c)^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
 + a^2), x)
```

Sympy [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c(d\sec{(e+fx)})^p)^n}{(a+b\sec{(e+fx)})^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))**p)**n/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((c*(d*sec(e + f*x))**p)**n/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{((d\sec{(fx+e)})^p\,c)^n}{(b\sec{(fx+e)}+a)^2}\,dx$$

input
```
integrate((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(((d*sec(f*x + e))^p*c)^n/(b*sec(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{\left(c\left(\frac{d}{\cos(e+fx)}\right)^p\right)^n}{\left(a+\frac{b}{\cos(e+fx)}\right)^2}\,dx$$

input
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2,x)
```

output
```
int((c*(d/cos(e + f*x))^p)^n/(a + b/cos(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c(d\sec(e+fx))^p)^n}{(a+b\sec(e+fx))^2}\,dx = d^{np}c^n\left(\int \frac{\sec(fx+e)^{np}}{\sec(fx+e)^2 b^2+2\sec(fx+e)\,ab+a^2}dx\right)$$

input
```
int((c*(d*sec(f*x+e))^p)^n/(a+b*sec(f*x+e))^2,x)
```

output
```
d**(n*p)*c**n*int(sec(e + f*x)**(n*p)/(sec(e + f*x)**2*b**2 + 2*sec(e + f*
x)*a*b + a**2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file